

Giant planet formation and migration

Numerical simulations with SPH

Benjamin Anthony Ayliffe

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Abstract

This thesis describes efforts to improve the realism of numerical models of giant planet formation and migration in an attempt to better understand these processes. A new approach has been taken to the modelling of accretion, designed to mimic reality by allowing gas to accumulate upon a protoplanetary surface. Implementing this treatment in three-dimensional self-gravity radiation hydrodynamics calculations provides an excellent model for planet growth, allowing an exploration of the factors that affect accretion. Moreover, these calculations have also been extended to investigate the migration of protoplanets through their parent discs as they grow.

When focusing on the growth of non-migrating protoplanets, the models are performed using small sections of disc, enabling excellent resolution right down to the core; gas structures and flow can be resolved on scales from $\sim 10^4$ to 10^{11} metres. Using radiative transfer, these models reveal the importance of opacity in determining the accretion rates. For the low mass protoplanets, equivalent in mass to a giant planet core ($\sim 10 M_{\oplus}$), the accretion rates were found to increase by up to an order of magnitude for a factor of 100 reduction in the grain opacity of the parent circumstellar disc. However, even these low opacities lead to growth rates that are an order of magnitude slower than those obtained in locally-isothermal conditions. For high mass protoplanets ($\gtrsim 100M_{\oplus}$), the accretion rates show very little dependence upon opacity. Nevertheless, the rates obtained using radiative transfer are still lower than those obtained in locally-isothermal models by a factor of ~ 2 , due to the release of accretion energy as heat.

Only high mass protoplanets are found to be capable of developing circumplanetary discs, and this ability is dependent upon the opacity, as are the scaleheights of such discs. However, their radial extents were found to be independent of the opacity and the protoplanet mass, all reaching $\approx R_H/3$, inline with analytic predictions.

Migration is investigated using global models, ensuring a self-consistently evolved disc. Using locally-isothermal calculations, it was found that the capture radius of an accreting sink particle, used to model a protoplanet without a surface, must be small ($\ll R_H$) to yield migration timescales consistent with linear theory of Type I migration. In the low mass regime of Type I migration, accreting sinks with such small radii yield timescales consistent with those models in which a protoplanetary surface is used. However, for high mass protoplanets, undergoing Type II migration, the surface treatment leads to faster rates of migration, indicating the importance of a realistic accretion model. Using radiative transfer, with high opacities, leads to a factor of ~ 3 increase in the migration timescale of the lowest mass protoplanets, improving their chances of survival. As suitable gas giant progenitors, their survival is key to understanding the growth of giant planets. An unexpected result of the radiative transfer was a reduction in the migration timescale of high mass planets. This appears to be a result of the less thoroughly evacuated gaps created by planets in non-locally-isothermal discs, which affects the corotation torque.

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*“Imagination will often carry us to worlds that never were.
But without it we go nowhere.”*

– Carl Sagan (1934 - 1996)

Declaration

Much of the work described in this thesis has been published in scientific journals in the form of papers, written cooperatively by myself and my supervisor, Matthew Bate. The results of growth models discussed in chapter 3 were published in the Monthly Notices of the Royal Astronomical Society (MNRAS), 2009, volume 393, pages 49 to 64. Also published were the circumplanetary disc properties presented in chapter 4, which can be found in MNRAS, 2009, volume 397, pages 657-665.

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