# Assessing Implicit Large Eddy Simulation for Two-Dimensional Flow

Submitted by

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I certify that all material in this thesis which is not my own work has been identified and that no material is included for which a degree has previously been conferred upon me.

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James Kent

**Abstract**

Implicit large eddy simulation (ILES) has been shown, in the literature, to have some success for three-dimensional flow (e.g. see [Grinstein, F. F., Margolin, L. G. and Rider, W. Implicit Large Eddy Simulation. *Cambridge*. 2007]), but it has not previously been examined for two-dimensional flow. This thesis investigates whether ILES can be applied successfully to two-dimensional flow. Modified equation analysis is used to demonstrate the similarities between the truncation errors of certain numerical schemes and the subgrid terms of the barotropic vorticity equation (BVE). This presents a theoretical motivation for the numerical testing.

Burgers equation is first used as a model problem to develop the ideas and methodology. Numerical schemes that are known to model Burgers equation well (shock capturing schemes) are shown to be implicitly capturing the subgrid terms of the one-dimensional inviscid Burgers equation through their truncation errors.

Numerical tests are performed on three equation sets (BVE, Euler equations and the quasi-geostrophic potential vorticity equation) to assess the application of ILES to two-dimensional flow. The results for each of these equation sets show that the schemes considered for ILES are able to capture some of the subgrid terms through their truncation errors. In terms of accuracy, the ILES schemes are comparable (or outperform) schemes with simple explicit subgrid models when comparing vorticity solutions with a high resolution reference vorticity solution. The results suggest that conservation of vorticity is important to the successful application of ILES to two-dimensional flow, whereas conservation of momentum is not. The schemes considered for ILES are able to successfully model the downscale enstrophy transfer, but none of the schemes considered for ILES (or the schemes with simple subgrid models) can model the correct upscale energy transfer from the subgrid to the resolved scales.

Energy backscatter models are considered and are used with the ILES schemes. It is shown that it is possible to create an energy conserving and enstrophy dissipating scheme, composed of an ILES scheme and a backscatter model, that improves the accuracy of the vorticity solution (when compared with the corresponding ILES scheme without backscatter).

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# Acronyms

| | |
|---|---|
| APVM | Anticipated Potential Vorticity Method |
| BVE | Barotropic Vorticity Equation |
| CR | Coarse Resolution |
| DNS | Direct Numerical Simulation |
| EFS | Equations of Finite Scale |
| ENO | Essentially Non-Oscillatory |
| GCM | General Circulation Model |
| HR | High Resolution |
| ILES | Implicit Large Eddy Simulation |
| LES | Large Eddy Simulation |
| MEA | Modified Equation Analysis |
| MILES | Monotonically Integrated Large Eddy Simulation |
| PDE | Partial Differential Equation |
| PV | Potential Vorticity |
| QGPV | Quasi-Geostrophic Potential Vorticity |
| QGSWE | Quasi-Geostrophic Shallow Water Equations |
| SG | Subgrid |
| SGM | Subgrid Model |
| SL | Semi-Lagrangian |
| TE | Truncation Error |
| TVD | Total Variation Diminishing |
| UNO | Uniformly high order Non-Oscillatory |