‘Voodoo maths’, asymmetric dependency and maths blame: Why collaboration between school science and mathematics teachers is so rare

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Abstract

Mathematical reasoning and tools are intrinsic to science, yet the close and dependent relationship science has to mathematics is not reflected in either school education or science education research. This paper asks what the barriers are to a mutually beneficial relationship between the two disciplines. A two-phase qualitative interview study was used to explore the relationship between school science and mathematics education through the perspectives of science and mathematics education policy-makers and of teachers in departments which are unusual in collaborating. In total there were 36 participants. Interview data were analysed using thematic analysis. Findings show that there is an asymmetry in the dependency between school science and mathematics: science is dependent on mathematics but the reverse is not true. We discuss three consequences of this asymmetric dependency: there is greater benefit for science from any collaboration; ‘maths blame’ can arise from science teacher frustration; and science educators may believe they should have some ownership of the mathematics curriculum. Asymmetry of dependency, and therefore of benefit, will make it very difficult for mathematics and science to work together in a way which is genuinely mutually beneficial.

Key words

Mathematics, curriculum, policy development, secondary/high school

Introduction

It has been claimed that ‘mathematics is the language of science’. It is not an uncontested notion, however, with the biology professor E. O. Wilson writing in the *Wall Street Journal* that ‘many of the most successful scientists in the world today are mathematically no more than semiliterate’ (Wilson, 2013). Even those authors who express contrary opinions to Wilson concede that historically, many scientists, such as Michael Faraday, have known little mathematics (Marcus & Davis, 2013).

In spite of disputes about just how much mathematics one needs in order to progress in a scientific career, there is little disagreement that at least some mathematics is necessary to do science. Two of the eight practices of science identified by Osborne (2011), that now form a core element of the US Framework for K-12 Science Education (NAP, 2012), are mathematical: analysing and interpreting data, and using mathematical tools.

Mathematical reasoning and tools are thus argued to be intrinsic to science. That these two practices are distinct is important: mathematics is not simply a tool for the analysis of data, but a way of thinking and reasoning. In a later paper on the eight practices of science, Osborne further discussed the importance of mathematics for clear communication in science:

> Mathematics and computational thinking are central to science enabling the representation of variables, the symbolic representation of relationships and the prediction of outcomes. As such mathematics supports the description of the material world enabling systematic representation that is the foundation of all scientific modelling and the clear communication of meaning. Thus mathematics serves pragmatic functions as a tool—that is both a communicative function, as one of the languages of science, and a structural function, which allows for logical deduction. Mathematics and numerical representation are the basis of all measurement in science. (Osborne, 2014, p. 187)

One might expect to see this close and dependent relationship reflected in school education and in science education research, but it does not appear to be so. Osborne argues that:

> For too many teachers of science, however, mathematics is not something that is central and core to practice of science. […] But if mathematics is not a core feature of what happens in science classrooms the nature of science will be misrepresented. Avoiding the opportunity to use mathematical forms and representations is a failure to build students [*sic*] competency to make meaning in science. (*ibid*.)

In other words, mathematics is important to science but this importance is not made manifest in school science education.

A number of authors have identified that students have difficulty in using mathematics in science, both at school level (for example, Dodd & Bone, 1995) and at university level (for example, Koenig, 2011). However, aside from identifying that students have difficulties, mathematics in science is largely ignored in education research. In a comprehensive review of mathematics and science education, Orton and Roper concluded that ‘the international science education research journals contain little on the science and mathematics issue’ (2000, p. 143). Similarly, equally little attention is paid to the relationship in the mathematics education literature (*ibid*.).

What evidence therefore justifies the claims for a mutuality between mathematics and science? While there have been calls in the literature for science and mathematics departments in schools to work together more closely (for example, Osborne, 2011), there appears to be limited research about the impacts of closer working. For example:

> There is little research on […] whether more explicit connections or integration across the disciplines significantly improves student learning, retention, achievement, or other valued outcomes. (Honey, Pearson, & Schweingruber, 2014, p. 22)

Osborne also identifies a lack of research and argues that:

> Science and mathematics education exist at a distance from each other – the two communities rarely engage and there is an absence of a literature that explores how they could work symbiotically. (Osborne, 2011, p. 98)

Symbiosis, from the Greek meaning *living together*, is a relationship between two species of organisms and can take a number of forms including mutualism and parasitism. In the discussions that follow we therefore use 'mutually beneficial' in place of Osborne's term 'symbiotic'.

There have been a number of calls in the literature and more widely for school science and mathematics departments to work more closely together. Arguments for such closer alignment are often based on perceived synergies in subject content such that there is, it is argued, substantial overlap between the subjects which consequently makes collaboration useful, not least in saving time (see: Dodd & Bone, 1995; Orton & Roper, 2000; Pang and Good, 2000; Osborne, 2011; Zhang, Orrill, & Campbell, 2015; Boohan, 2016). For instance, Zhang *et al*. argue that '*mathematics and science share a coherent set of values and concepts*' (2015, p. 358) including problem solving and process skills. They suggest that: 'The content of both science and mathematics should encourage teachers to integrate and use new knowledge and skills from across areas of competence' (ibid.). Zhang *et al.* also suggest that it should be relatively easy to find overlap in the content of the two curricula.

The other main arguments for closer working include: shared values and skills (Berlin & White, 1995); a resulting improvement in students' scientific and mathematical understanding (Pang & Good, 2000); an opportunity for teachers to appreciate similarities and differences in the curriculum (Boohan, 2016); that it promotes transfer between the disciplines (Honey, Pearson, & Schweingruber, 2014); and, that it enhances pupil engagement particularly when 'real world' contexts are used (Venville, Wallace, Rennie, & Malone, 2002, Honey *et al*., 2014, Williams *et al.,* 2016,).

Given the lack of research, a study exploring the relationship between school science and mathematics education would appear to be timely. In this study we investigate the relationship between school science and mathematics education through schools engaged in mathematics-science collaboration alongside the production of science and mathematics education policy. We ask: What are the barriers to a mutually beneficial relationship between school science and mathematics?

Context

The study was carried out in England where there is a National Curriculum to the age of 16, with science and mathematics both compulsory subjects. From 16-18 students can choose which subjects to study with none being compulsory. About half the cohort follows the most academic qualification, A-levels, with most studying just three subjects. There are high stakes external examinations at 16 and 18.

Mathematics and science education in the literature

There are some reviews of the literature, including an extensive survey by Honey *et al*. (2014), but on the whole, searching for studies is challenging due to inconsistencies in language. For example, many authors call any attempt at the two disciplines working together 'integration' but a number of authors (for example: Berlin & Lee, 2005, Honey et al., 2014, and Williams *et al*., 2016) problematise the term integration and the lack of an agreed definition. Furthermore, not all the science sources refer to 'mathematics', with some instead using the terms 'numeracy' or 'quantitative skills', the use of neither of which is well-delineated. We discuss a few selected studies and reviews which demonstrate the difficulties that there are in bringing together mathematics and science.

Dodd and Bone (1995) carried out a study with science teachers, mathematics teachers, and pupils aged 11-13 in England. They argue that science teachers consistently overestimate children's ability in mathematics, with pupils themselves often finding the mathematics in science daunting. They suggest that part of the problem children have in using mathematics in science is a lack of understanding or empathy on the part of science teachers, who simply do not understand how challenging it is for many pupils. Pupils were aware that they were required to use mathematics within science which they had not been taught in mathematics and this, unsurprisingly, further reduced their confidence. They found many of the science teachers believed that collaboration between the writers of the science and mathematics curriculum would help to solve the problems, as would collaboration with the mathematics department.

Becker and Park (2011) undertook a meta-analysis of 28 studies into the impact on attainment of integration in STEM subjects (which they define very broadly as teaching/learning between or among any two of the STEM subjects, or any STEM subject with any other subject) by calculating 33 effect sizes. Perhaps their most noteworthy finding is that when science is integrated into technology and engineering the effect sizes were relatively large for all subjects, but when mathematics was integrated the effect sizes were much smaller, particularly for mathematics itself where some of the effect sizes were negative. They call for further research to understand why, of all the STEM subjects, mathematics benefits the least from an integrated approach.

Honey *et al.* (2014) note that there were very few integrated education programmes where the goal of making connections across subjects was stated explicitly, although it was often an implicit aim. One study which is an exception is Frade, Winbourne and Braga (2009) who explored how boundaries between practices can be crossed by students and teachers. They propose that:

> Bernstein can help to explain why it is that a major challenge for teachers and students in schools is to do what looks like transfer or boundary crossing. Boundaries may be socially produced but they are no less real for this in the experience of teachers and students. (2009, p. 17)

Theoretical framework

Boundaries

To look at the relation between categories (here, subjects, A and B), Bernstein uses the idea of classification. To him, classification refers to the relation between categories. He argues that in order for categories (subjects) to be differently specialised there must be space between them:

> A can only be A if it can effectively insulate itself from B. In this sense, there is no A if there is no relationship between A and something else. […] If [the] insulation [between categories] is broken, then a category is in danger of losing its identity. (Bernstein, 2000, p. 6)

In other words, the identity of one subject is reliant on it being different from, separated from, or insulated from another subject.

Bernstein (2000) suggests that when classification (which he uses as a defining characteristic of relations between categories) changes from strong to weak or vice versa, as is the case when departments or subjects collaborate, we should always ask whose interests are served. Considering this question will help to demonstrate how and why it is difficult for mathematics/science collaboration to benefit both departments equally. We will show how using the theory of boundary helps to explain why collaboration between science and mathematics departments is challenging and why, crucially, it is unlikely ever to be genuinely mutually beneficial.

Transfer of learning between mathematics and science

In the educational literature the use or application in one context of knowledge learned in another is known as *transfer.* It is a contested idea with authors expressing a wide range of views as to what it is, whether it exists and if and how it can be promoted by education (Wong, 2018). Transfer is not viewed as straightforward by any of the authors who have seriously investigated it, if it is even considered to take place at all. Even so, some authors and educators still expect transfer of learning to be uncomplicated which, inevitably, leads to a deficit view of students who find using mathematics within science challenging. It can also lead to a deficit view of mathematics teaching, where students' struggles to use mathematics within science is ascribed to inadequate prior teaching and learning in mathematics lessons (*ibid.*).

Redish and Kuo (2015), US university physics lecturers, ask why it is that so many physics undergraduates struggle to use mathematics within physics, even when they may have achieved considerable success in mathematics courses. Like other authors (for example Koenig, 2011) they suggest that sometimes this is because students struggle with basic mathematical concepts. However, they use ideas from linguistics research to argue that the difficulty is more subtle and lack of mathematical understanding is often not the real problem. They argue that although the mathematics looks the same, it is used and interpreted differently and for different purposes. While mathematics is about expressing abstract relationships, in physics, physical knowledge about actual systems is blended into the equations and mathematics, which significantly changes their interpretation.

> The key difference is that loading physical meaning onto symbols does work for physicists and leads to differences in how physicists and mathematicians interpret equations. We not only *use math in doing physics*, we *use physics in doing math*. (Redish and Kuo, 2015, p. 563, italics in original)

As a result, they argue, mathematics in physics has a different semiotics (the way meaning is put into symbols) than mathematics as used by mathematicians. Loading physical meaning onto symbols (for example by giving them units or appreciating the logical limits of what those symbols stand for) allows physicists to use more straightforward mathematics than would be used in the same situation by a mathematician. Physicists also blend physical meaning into the mathematics by 'filtering the equation through the physics' (p. 565).

> The culture of physics expects that each symbol in an equation is to be interpreted in conjunction with its physical meaning. So, part of the acculturation of a physics student is learning to interpret the math physically, not to only focus on mathematical structure and manipulations. (p. 567)

Redish and Kuo argue that this use of mathematics within physics for different purposes to those in pure mathematics amounts to mathematics-in-physics being akin to a different dialect of mathematics. There are likewise differences in how mathematics is used in school science and mathematics itself, for example having units with almost every number and those units conveying physical meaning. These differences could contribute to making the transfer of knowledge between the disciplines less straightforward than teachers sometimes assume.

Beliefs

It is widely recognised that teachers' beliefs play an important role in their pedagogical decision making, both prior to and during a lesson (Wallace, 2014). It has been claimed by many researchers that 'beliefs are the best indicators of the decisions that individuals make throughout their lives' (Pajares, 1992, p. 307). This is in spite of a lack of consensus over the definition of beliefs, described by Pajares as a 'messy construct' (*ibid*.).

Glackin (2016) evaluates teacher responses to a professional development programme and argues that beliefs are important for how teachers respond to suggested changes to their practice. Wallace agrees and contends that teachers may 'review and filter new curriculum innovations for those that resonate with [their] core beliefs' (2014, p. 18). She explains that research has found that when interventions are at odds with beliefs, teachers will either refuse to implement them or do so superficially. This could be expected to be the case with closer collaboration between mathematics and science; as with any other intervention it will be unlikely to happen if teachers do not believe it to be valuable.

In a study of Dutch mathematics and physics teachers,Turşucu et al. (2017) found that science teachers' beliefs about automatic transfer from mathematics to science could lead to routines based on tricks which do not take into account conceptual understanding. They also found that physics teachers were more interested in collaboration than mathematics teachers, believing that 'an ideal collaboration would result in alignment of notations, equations, formulas and algebraic techniques' (p. 595).

Methods

Fensham (2009) argues that science education policy, and the political and cultural context of that policy, is often ignored in science education research. Consequently, the contested nature of science in the curriculum is frequently disregarded as is the interplay between stakeholders in school and beyond who determine both the nature of the science curriculum and the way in which it is enacted. Therefore, a two-phase qualitative approach with two distinct groups was undertaken to try to gain insights into both the policy-making process at a national level and the realities of collaborating across departmental boundaries in school.

In phase one, semi-structured interviews were conducted between in 2013-2014 with 21 long-standing and acknowledged key contributors to the science and mathematics education communities in England, with questions focussed around the development of the original national curriculum for England, the writing of the latest iterations of the national curriculum for mathematics and science (published in 2013 and 2015), and the origins and rise of the STEM agenda in the UK (see Appendix 1 for a sample interview protocol). Interviewees for this study were selected on the basis that they had had some influence on government science or mathematics education policy in the last 30 years. A snowball sampling technique was adopted whereby some initial interviewees were selected and each participant asked for recommendations or introductions to other potential interviewees.

In Phase 2 (2014-2015), a second set of interviews, 15 in total, was conducted in six schools where the science and mathematics departments collaborate to some extent. This approach to working across departments is rare in England and finding such schools was challenging. The aim was to explore the perspectives of the teachers involved in collaborations about the aims of, and benefits and barriers to, such work (see Appendix 2 for a sample interview protocol). The schools were chosen through purposive sampling – examples picked because they possess the particular characteristics being sought. The cases required are highly unusual so there is no pretence that they represent the wider population of schools; the choice is unashamedly selective (Cohen, Manion, & Morrison, 2011).

To enable the interviews to be conversations about a theme of mutual interest (Kvale & Brinkmann, 2009), the interviews were as open-ended as possible to allow 'respondents to demonstrate their unique way of looking at the world – their definition of the situation' (Cohen et al., 2011, p. 205). As such, the sequences of questions and exactly which questions were asked varied between interviews because 'what is a suitable sequence of questions for one respondent might be less suitable for another' (ibid.). When trying to find out what happened and why people acted in the way that they did, Rubin and Rubin (1995) argue that it makes little sense to ask everyone the same questions as the goal is to gain a rich description of people's perspectives in individualistic terms. Leading questions were avoided as far as possible and after each interview the responses were examined to see if the questions were yielding the kind of data hoped for, with the questions being adapted as necessary. A common sequence of questions – about STEM – was included within each policy-maker interview largely unadapted so that there were some questions common across all interviews, allowing norming of the answers to at least some degree as suggested by Rubin and Rubin (1995, p. 84). Some of the points raised by interviewees were incorporated into subsequent interviews in an iterative and self-correcting design (Rubin & Rubin, 1995). There was far less public information about the specific collaborations in schools, or about the teachers themselves, and consequently the interview schedule for schools varied less, but unexpected issues were followed up in the way that they were for the policy makers.

Interviews were recorded following interviewee consent and transcribed intelligent verbatim. The two data sets were analysed separately but both using thematic analysis, seeking to understand a phenomenon as it appears within the dataset collected, as described by Braun and Clarke (2013). The data were coded using a complete coding process in NVIVO. In total, around 100 codes were generated for each data set and a code book written. Each instance of each use of each code was checked against the description from the code book to ensure that it was a genuine fit for the code. In the next phase of the analysis, themes were developed from the codes and coded data by searching for concepts, topics or issues which connected several codes. Further details of the participants and analysis can be found in Wong *et al*. (2016) and Wong (2018).

To try to minimise any ethical dilemmas during the course of the study, BERA's (2011) and King's College London's ethical guidelines were consulted and procedures put in place and followed to ensure a duty of care to all the participants. This research was ethically approved by King's College London.

We have given each of the participants a two or three letter pseudonym to maintain anonymity. For the policy-makers, the first letter represents their discipline (mathematics, science, engineering, civil service), the second a letter to identify them: MA-MH, SA-SK, EA and CSA. For the teacher participants, the first letter represents their school (A-F, all pseudonyms), the second their disciplinary background (mathematics, science, technology), and an L denotes a senior leader. All the senior leaders were also still classroom teachers as is common in England.

Limitations

In interviewing policy makers, the key limitation was that there were fewer mathematics educators who took part due to a lower acceptance rate among mathematicians. Interviewing stopped in part as data saturation was being reached, but also to avoid skewing the data towards science educators. Furthermore, many of the policy makers who declined did so due to a disinterest in the topic. For the mathematics educators this tended to be a dislike of STEM or a lack of interest in the overlap between mathematics and science. Thus the data collected represent mathematicians, in particular, who have more interest in collaborating with science. It is largely fruitless to speculate what might have been said by people who were not interviewed, but nonetheless the policy makers data set is probably skewed towards the views of those with positive conceptions of STEM and mathematics-science collaboration.

Similarly, access to schools was limited to people with whom the lead author had an existing relationship, even if that relationship was very brief in some cases. There may well be schools with interesting collaborations where we unable to gain access. This was not, however, intended to be an in-depth study of every school where collaboration takes place and we ensured that the schools recruited represented a broad geographical range. All provided unique insights into collaboration.

Findings

As there are boundaries around departments in school there are, similarly, boundaries among those seeking to influence policy in England. Collaboration across those boundaries was always described by participants as challenging, and there was even disagreement about where the boundary should be drawn with some policy-maker participants suggesting that science includes mathematics (for instance, 'actually mathematics is part of science,' MD), although the majority of mathematics educators and teachers did not share that view. MA even considered being seen as part of science as dangerous:

> *There's a real danger that maths is seen as a kind of small subset of science; that's how the science people see it always.* [MA]

In spite of disagreement about where the boundaries lie, all participants identified that there are boundaries between subjects and they used the language of divisions, barriers and silos to describe them. Using the lens of boundary we identify three themes: asymmetric benefit; blame and frustration; and service and ownership. Each is discussed in turn.

Asymmetric benefit

Recalling Bernstein's (2000) suggestion that when classification is weakened it is important to ask whose interests are served by the new togetherness, we consider how mathematics and science depend on each other and how this dependency correlates with the levels of perceived benefit gained by each from collaboration. We will show how the dependency is unequal and asymmetric which results in the gains and benefits to the subjects likewise tending to be asymmetric.

A number of teachers noted how important mathematics and mathematical skills were to science, agreeing with Osborne (2011, 2014) who we noted earlier argues that two of the eight practices of science are mathematical in nature. Respondents from both groups suggested two key reasons why mathematics was necessary for science students. Firstly, that mathematics is key for future scientific careers (only an issue for those who choose such careers). For example:

> *Science in schools* [is] *highly dependent on maths. When it came to getting people out into undergraduate degrees and into A-level you actually needed a bit of solid maths there.* [MA]

Secondly, that mathematics is required for all secondary science students to make progress in science, including those only just coping with the demands of the curriculum. FM gave a specific example of how learning aims in a science lesson were thwarted by students struggling with aspects of mathematics:

> *I was talking to a science teacher recently who was doing a lesson on Hooke's Law, which is where you put masses on a spring and then you measure the spring. And she went, 'So my learning intention was for them to learn that the stretch in the spring is proportional to the force that you act on it', but she said, 'right, okay, so the first challenge was none of them could measure the spring properly, because they weren't using the zero on the ruler. Then loads of them had issues drawing the axes on their graph and doing the scale properly, then plotting the graph, then drawing the line of best fit'. So she said 'my whole learning intention just got completely lost in all this numeracy and maths that they were struggling with'.* [FM]

Therefore, in just one lesson, students struggled with measuring using a ruler, choosing a scale for their graph, drawing axes using those scales, plotting the data, and drawing a line of best fit through the points. Lacking these skills, or at least struggling to apply them, was keeping students from the science learning that the teacher hoped for. In other words, the science learning was dependent on students' understanding and use of mathematical skills and thinking.

If the teacher is choosing not to teach those skills in science it shows that they believe that the students should have those skills already; that they should be able to transfer the learning that the teacher assumes they will have from mathematics. Even supposing that the student has covered those skills in mathematics, however, does not necessarily mean that they can use them in science.

While a number of science and mathematics teachers and policy-makers talked about how science was dependent on mathematics, mathematical skills and mathematical reasoning, none of the participants talked about mathematics being dependent on science. Students need to be able to use and apply mathematics in science; they do not need to be able to use and apply science in mathematics. In the 14-16 curriculum in England, for example, there are aspects of mathematics listed for every segment of the science curriculum (Department for Education, 2015), but there is virtually no mention of science in the mathematics curriculum (DfE, 2013). Up to 30 percent of marks in national science assessments for 16 year olds will come from mathematics (Ofqual, 2015), but there is no requirement that there will be science in the mathematics assessments. While some of the content of the mathematics curriculum is used within science, there are large portions of the content of the mathematics curriculum which are not and thus any alignment of teaching strategies is likely to benefit science more broadly and profoundly than it benefits mathematics.

Science was mentioned as a good context in which to do mathematics, and potentially a motivating context, but no one suggested that science was important to the mathematics curriculum or to students' chances of success in mathematics. Even when science was mentioned as an important context for mathematics it tended to be by science teachers who would like to see their mathematics colleagues using scientific contexts to teach aspects of mathematics such as proportional reasoning or rearranging equations, rather than by mathematics teachers themselves.

Some participants suggested that mathematics could be made more accessible by using interesting contexts, and that science could potentially be a source of such contexts. For example:

> *You have the typical statistics in maths questions of 'Ali goes out to a car park and counts how many red, yellow, green and black cars there are'. Where actually there is wonderful data available from* […] *science* [which] *can provide much better examples to carry the maths.* [MF]

SD suggested that resources aimed at giving mathematics a science context often used the science just as a 'story' to wrap around the mathematics:

> *From my perspective, it looked like maths teachers doing* [resources] *with a kind of biology story wrapped around them. They wouldn't help a biology teacher use them and a maths teacher wouldn't teach the context; they wouldn't necessarily be interested in it.* [SD]

In fairness, the resources cited by SD (above) were written by mathematics educators to improve students' understanding of mathematics and not science. The point is that even in these science-context resources mathematics is not dependent on science. Dierdorp, Bakker, van Maanen, and Eijkelhof (2014) similarly suggest that even when mathematics is taught in a context that context is usually subordinated to the mathematics to be learned.

Conversely, MC agreed that while science, and STEM more broadly, provided many good contexts for mathematics, care must be taken not to lose sight of the original mathematics: '[STEM] *really does give that context and excitement* […] *But don't lose the maths'* [MC].

The idea of mathematics getting 'lost', in other words being side-lined or under-emphasised, when it is taught within a STEM context, may help to explain the finding of Becker and Park (2011) that when mathematics is taught in an integrated context with one of the other STEM disciplines, mathematics attainment can be lower than when it is taught separately. Perhaps so much focus can be put on the exciting context that the mathematics, and thus the mathematics learning, gets lost. Furthermore, if learning in science is dependent on mathematics, then a focus on mathematics in science will benefit students' learning in science. If learning in mathematics is not dependent on science then a focus on science in mathematics may not similarly benefit students' learning in mathematics. Collaborating with science is therefore not risk-free for mathematics education and thus the reluctance of many in the mathematics community to have a closer relationship with science is perhaps understandable.

Thus, while for many science educators it is obvious that collaboration with mathematics would potentially be beneficial, mathematics educators do not necessarily have the same view. MG, who had previously worked in science education, explained that *'when you sit in science, you see maths education strongly overlapping with science education'* but that in mathematics you also have important links to other disciplines such as social sciences and vocational mathematics that science educators are, in general, unaware of. MG continued: '*your Venn diagrams look different*', in other words what you see as the area of overlap is just not the same.

For the relationship to be mutually beneficial, mathematics education would need to gain approximately equally from any collaboration. It is worth considering under what circumstances that would be the case. As science includes significant mathematics, it has been suggested (for example by Fairbrother, 2008) that science teachers could use quantitative science to support students' mathematical development, effectively increasing the amount of mathematics teaching that students receive. However, a number of both mathematics teachers and policy-makers in this study suggested that science did not, in general, 'treat its maths well'. For example MC:

> *I found out some horrifying things, actually, in terms of how science treats their mathematical topics* […] *science teachers do not actually use their science* […] *to help kids understand the algebra.* [MC]

Mathematics teachers and educators also pointed to science teachers employing 'voodoo', or 'tricks' when using mathematics within science, meaning that students' mathematical development was not being supported, even though there was mathematics within the science curriculum. For example:

> *The mathematics content that is needed in science [can] be taught what I call 'by voodoo' – so it will be taught by rote rather than from a conceptual understanding because science teachers will want to get that done quickly in order to be able to get onto the science.* [ME]

> *When you do speed, distance, time* […] *and so on and it's always taught as sort of a rule or a trick. Whereas there's a core idea which encompasses all of these about proportionality and scaling.* [MH]

Teaching by voodoo or by rote or by tricks neither teaches students to think mathematically within science as called for by Osborne (2014) nor provides support for the mathematics curriculum. Turşucu et al. (2017) suggest that it is teachers' beliefs about automatic transfer from mathematics to science which could lead to these kinds of trick-based routines which do not take into account conceptual understanding.

Thus there is what we term *asymmetry of dependency*, with science dependent on mathematics but not *vice versa.* This asymmetry of dependency, is rarely, if at all, discussed in the education literature and yet it is critical in understanding the relationship between the disciplines.

The asymmetry of dependency means that when science and mathematics work together more closely – in Bernstein's (2000) terms, the classification weakens – there will tend to be greater benefits for science from such collaboration. This insight comes directly from asking, as Bernstein (2000) suggested, who benefits from the change in classification.

Blame and frustration

The relationship between colleagues across the boundaries of school mathematics and science departments can be strained and characterised by blame and frustration. Mathematics teachers expressed frustration about the way in which school science uses mathematics; specifically, that the way it is used does not reinforce students' overall mathematical development. Science teachers expressed frustration that the mathematics curriculum does not support or underpin the science curriculum.

Mathematics teacher frustration at how school science uses mathematics is exemplified in this description of a science lesson observed by CML. CML could understand the emphasis the teacher placed on practical work and understanding science but, nevertheless, was frustrated at how, in what was to CML clearly a mathematical lesson, mathematical thinking was not promoted:

> *I was observing a* [science] *lesson last term, which was about bouncing. It was about losing energy, I think, and bouncing a ping pong ball and it was coming back up. And rather than talk to the students about what sort of data they were collecting and therefore what would be the best way of them representing that data and getting them to think about it […] they were told instantly […] 'and at the end of this, you are going to draw a graph and it's going to look like this.' And I just thought it was a missed opportunity* […]

> *'Why would you want to do this graph?'* […] *that discussion about why and where that comes from, from the data, those discussions didn't tend to happen. I mean, there's an issue with time and practicals, obviously, but nonetheless, it's a little bit frustrating because you sit there thinking, well, there's some really good maths you could pull out of this.* [CML]

While CML could understand the science teacher's emphasis, there was a lingering frustration that the science teacher was not reinforcing learning that would take place in mathematics. CML suggested this missed opportunity was due to time and pressure to get a practical done, but in actual fact there is no requirement in the science curriculum to explain why a particular graph would be drawn (DfE, 2015).

Frustration was also expressed by science teachers. For example, FS understood that the mathematics curriculum did not include or emphasise mathematics which would be important to science and was consequently frustrated by the content of the mathematics curriculum and the limited support it seemed to offer to the science curriculum. For example:

> *I just feel quite frustrated by the maths [curriculum]; that it doesn't really seem to equip them appropriately. You'll get students who have got their* [grade] *B in maths* [at age 16] *and I teach them* [at 16-18] *and they cannot quickly work out numbers and they'll ask me how to do an average and little things that you're just doing all the time.* [FS]

Frustration about the differences and the lack of support the curricula offer to each other was expressed by teachers from five of the six schools. This frustration can lead to science teachers blaming mathematics teachers, with both the experience and anticipation of being blamed being described by mathematics teachers from three of the schools, exemplified by the following comment from AM, describing a conversation intercepted in the staffroom:

> *A former colleague, science teacher, was sounding off about how awful it is that not everyone in Year 9 [age 13] knows how to solve equations and it transpired that he was teaching some very, very weak pupils who could do some equations, but not the sort that he was wanting. When we had a conversation about how our scheme of work fits together and what sort of things they do at particular times, he realised that actually it was quite reasonable that they couldn't do what he was asking them to do.* [AM]

The teacher challenged by AM was apparently blaming the mathematics department in spite of not knowing about the mathematics curriculum and how it was organised into the scheme of work in that particular school. The science teacher appears to have been expecting too much mathematically of the pupils, both assuming they would have covered aspects of mathematics which actually they had not, and not understanding how difficult students might find solving that type of equation. These two problems (overly high expectations and not understanding how difficult pupils can find mathematics) were previously described by Dodd and Bone (1995). Thus it is apparent that frustration exists on both sides.

Consequently, mathematics teachers both experience and anticipate being blamed when students find it difficult to use mathematics within science, with this blame frequently stemming from the expectation that students will have covered the mathematics that they require for science during mathematics lessons and will be able to use it seamlessly in science; in other words from science teachers' beliefs in transfer. As mathematics education does not require knowledge and skills from science in the way that science needs mathematical skills, there is less expectation that what students have covered in science will support mathematics and consequently less blame.

Service and ownership

While there is broad agreement that science is at least partly dependent on mathematics, it is less clear where such a dependency leaves the relationship between the two disciplines. Several participants with a mathematics background raised the issue of mathematics as a 'service subject.' This idea, which carries within it the idea of being subservient, arises directly from the dependency of science on mathematics. There is a tension within mathematics education between the pressures of being a service subject and the requirements of mathematics as a discipline in its own right (Hoyles, Newman, & Noss, 2001; Smith, 2004). Indeed, the mathematician, astronomer and physicist Gauss (1777-1855) argued that:

> Mathematics is the Queen of the sciences […] She often condescends to offer service to astronomy and other natural sciences, but under all circumstances the first place is her due. (Quoted in Bell, 1951, p. 1).

Bell (1951) titled his book 'Mathematics: Queen and servant of science' in acknowledgement of the dual, and apparently oxymoronic, role mathematics plays in the relationship between the disciplines.

More recently, Hoyles, Newman and Noss identified the:

> tension that is present in mathematics itself, between the utilitarian pressure on mathematics as a service subject for other subjects […] and the requirements of mathematics as a discipline in its own right. (2001, p. 834)

This tension was present for some of the mathematics policy-makers and teachers who, while acknowledging that mathematics was important for science, were also keen to stress it as a separate subject as the following quotes exemplify:

> *I recognise maths as a separate subject, but that some of the things that maths will do are useful things to support science.* [AM]

> *Of course it's fun to do pure maths, I mean that's nice, but actually we wouldn't have five lessons a week […] if it wasn't a big component of other subjects and important for that reason.* [MA]

Unusually among the teachers interviewed, FML, like MB, was not in favour of particularly close relations between mathematics and science departments in school. While this belief was unusual among those interviewed, it should be remembered that the majority of the teachers were closely involved in collaborating and so FML's views may be more representative of many of those who teach mathematics. FML felt that mathematics was applicable more broadly than just to science and that emphasising the link to science risked diminishing those other links:

> *I think I'd really like to see collaboration across the curriculum more than saying maths is maths-and-science.* […] *I think* [maths] *has far more areas it can be in than just maths-and-science and I would actually quite like to break down that image that, oh, it's maths-and-science, maths-and-science faculties.* [FML]

CML, a leader in a school where mathematics and science are in the same faculty, suggested that there were subjects it would be more interesting to be allied to: '*It's a real pity they didn't put maths with something like art* […] *it would have sparked off completely different things'* [CML]. In other words, two out of the six mathematics teachers interviewed did not particularly want or value a closer relationship with science. Given that mathematics is not dependent on science, it is perhaps understandable that mathematics teachers would resist being tied too closely to science, but nevertheless the reluctance of a third of those mathematics teachers interviewed to work more closely with science suggests, at the very least, that it is not a high priority for mathematics departments, although the numbers in this study are small. However, science not being a high priority for mathematics educators and teachers was similarly noted by Orton and Roper (2000).

FML pushed against the notion of mathematics being seen as too closely tied to science, in part not wanting mathematics in science to take priority over mathematics elsewhere: '*I don't want maths just to be seen as the prerogative of maths-and-science'* [FML]. In part, this concern comes from acknowledging that there are other disciplines which are also linked to mathematics, but FML, in using the word 'prerogative', is also expressing disquiet over the increasing closeness of mathematics to science as this would suggest pre-eminence and therefore particular rights or privileges over the mathematics curriculum for science.

Indeed, the dependency of science on mathematics raises the question as to who has, or should have, ownership of the mathematics curriculum. Who should control the content and sequence of the mathematics curriculum: mathematicians or users of mathematics? Bernstein (2000) argues that there is selection in how one discipline 'is to be related to other subjects, and in its sequencing and pacing' (Bernstein, 2000, p. 34). Such selection always involves ideology (*ibid.*); there can, therefore, be differences in ideology and in what people believe to be the ideal sequencing and pacing of the curriculum. The dependency of science on mathematics can lead to science educators believing that they should have some say in the content and sequencing of the mathematics curriculum. For example, SI argued that linking the mathematics and science curricula would bring benefits to science education:

> *I mean, the mathematicians do see themselves as a little bit different, because they always argue that* […] *'We don't just supply maths for scientists and engineers'. Which is fair, but I still think because of the use of the maths, we could benefit more from linkage.* [SI]

SI similarly argued that mathematics A-level should accommodate the needs of the physics A-level (for 16-18s):

> [They should be] *correlated in the sense of using the same symbols and guaranteeing that certain maths will be done by a certain time and vice versa, so that, for example, calculus can be done in year one of maths and the physicists could then use calculus in year two of the A-level.* [SI]

Thus the ideology or belief of some science educators is that the content and sequence of mathematics qualifications (at age 16 and 18) should be arranged to support and benefit science qualifications. These views are similar to those found by Turşucu *et al*. (2017) among Dutch teachers. Indeed, some mathematics educators at least partly agreed. MH felt that when the mathematics curriculum was being reviewed, the government should have included some users of mathematics on the panel, rather than simply mathematicians:

> *I still feel that the government needs to look wider and look at people who may have had more diverse experiences and especially recognise that mathematics is a service subject, alongside being a subject in its own right. I think not bringing on good users of the subject is a weakness.* [MH]

Not all the science educators believed that mathematics should accommodate science. SB suggested that expecting mathematicians to be the 'providers for scientists' was probably not tenable:

> *I think it is difficult for mathematicians to accept enthusiastically the role of being providers for scientists as against people who enjoy and want to convey the beauty and enjoyment of their own subject.* [SB]

It is perhaps understandable if physics and mathematics educators should take a different view of the relationship between the subjects: while 80 percent of physics A-level students take mathematics, only 32 percent of mathematics students take physics, dropping to only 15 percent of female students taking A-level mathematics (Gill, 2012).

The ideology, or even expectation, of ownership of the mathematics curriculum on the part of some science educators is perhaps why MA suggests that being seen as 'a small subset of science' is dangerous. If mathematics is a part of science, especially if only a small part, then the science education community could potentially claim the unwelcome right to exert influence over the mathematics curriculum.

Discussion

Science is dependent on mathematics but the extent of that dependency and exactly which mathematics is depended on varies across the different branches of science. However, the fundamental fact of that dependency is not seriously challenged, either in the literature or by the participants in this study. The converse is not true; mathematics is not dependent on science. There is, thus, what we term 'asymmetric dependency', which does not appear to be discussed in the literature but which has several significant effects on the relationship between mathematics and science. We have identified three effects in the findings: greater benefit for science; maths blame; and curriculum service and ownership.

Transfer and beliefs

Most of the science teachers interviewed who were aware of what students had, and had not, covered in mathematics cited the collaboration as a source of that knowledge. They were, in consequence, actively ensuring any necessary mathematics was covered in science. Osborne suggests that this acceptance of responsibility for students' mathematical knowledge may be unusual among science teachers:

> Many [science teachers], perhaps, operate with the vaccination model of mathematics […] that it is not their responsibility to educate students in the mathematics […] required to understand science. And if students have not been vaccinated, there is little that they, the teacher, can do. (Osborne, 2014, p. 187)

In other words, science teachers expect students to have the mathematical knowledge they need when they arrive in science lessons and do not do much about it if they do not. Although this sounds like rather an unlikely abdication of responsibility, a similar conclusion was reached by the AKSIS project team:

> Many teachers we interviewed recognised the difficulties that pupils had with graphs, but few had made a point of teaching pupils about the construction and use of graphs. (Goldsworthy, Watson, & Wood-Robinson, 1999, p. 2)

Science teachers, it would seem, often expect students to be able to use mathematics within science, recognise that they struggle to do so, and do little about it, although it is widely recognised that students find it hard to use mathematics skills in science. Turşucu *et al*. (2017) found that the majority of physics teachers thought that the problem of students finding it difficult to use mathematics within science should be solved by more intensive practice in the mathematics classroom; in other words they held the 'vaccination' view identified by Osborne (2014).

When students struggle to use mathematics in science smoothly and without help, science teachers may blame their mathematics colleagues for students' deficiencies. This blame can be identified even (and perhaps especially) when students have not yet covered the mathematics that science teachers are expecting. The prevalent idea that students' difficulties in using mathematics in a scientific context can be attributed to their prior, and in some way deficient, mathematical education we articulate as *'maths blame'*. Blame, according to the Oxford English Dictionary, is: an expression of disapprobation; the imputation of demerit on account of a fault or blemish; a charge or accusation; the responsibility for anything wrong. It is also a verb, for example: to lay the blame on; to fix the responsibility upon; to make answerable; to find fault with or censure (OED, n.d.).

Maths blame links directly to the asymmetric dependency between science and mathematics. Science and science education are dependent on mathematics such that difficulties with mathematics can significantly hinder learning in science. The asymmetric dependency is also the reason why the contrasting science blame was not in evidence anywhere in the data. Mathematics teachers were frustrated by some of the ways in which science teachers dealt with mathematics, but their teaching methods were not believed to have a direct impact on whether or not students could access the mathematics curriculum in the way that mathematics teaching was believed to impact science learning.

There is similarly an asymmetric need for knowledge and understanding: science teachers need to know and be able to use mathematics and it is helpful if they understand the order and content of the mathematics curriculum. There is far less need for mathematics teachers to know any science or to understand the order and content of science curriculum. This asymmetric dependency needs to be acknowledged and accounted for in any moves towards STEM education. Furthermore, if there are real difficulties in the relationship across science and mathematics disciplinary practices, it will only be further exacerbated by the additional potential discipline boundaries in STEM education.

Teaching and learning

Another pertinent question is, perhaps, what implications asymmetric dependency has for learning and teaching. Research suggests that students construct their understanding in context and within disciplinary norms. Thus to use knowledge from one discipline within another requires them to reconstruct their learning in the new context (Rebello, et al., 2005). Doing so can prove more challenging for students than many teachers anticipate.

Redish and Kuo (2015) suggest that mathematics is used differently in mathematics itself and in physics – and presumably the other sciences. They argue that this is because physicists use physics in interpreting the mathematics, they do not just use mathematics to interpret physics. Using physical knowledge when applying mathematics to physical systems might involve knowing the limits to the numbers which can be put into an equation. For example, in the ideal gas equation (PV=nRT) none of the values would be negative as pressure, volume, amount of substance, the ideal gas constant and temperature in Kelvin are always positive values. There is no mathematical reason why a negative number could not be introduced into the equation, but there are reasons based on physical knowledge. Redish and Kuo (2015) argue that using this physical knowledge helps to keep the mathematics more straightforward. The idea that mathematics must be learnt, or reconstructed, within scientific disciplines itself is reinforced by the findings of Grove and Pugh (2015) who argue that chemistry undergraduates can be supported to the learn the mathematics they need but, crucially, that this is more likely to be successful if they are taught within the chemistry department by chemistry lecturers. In other words, if the mathematical knowledge is constructed in the context of chemistry.

Tytler, Prain, & Hubber (2013) argue that students need to learn to switch between various ways of representing scientific information, including visual and mathematical modes, because this is critical in how scientific knowledge is built and validated. Using the mathematics physically, to learn to interpret what the results mean and to apply that meaning to a physical (or indeed biological) system, or to another representation, is important in science. It is not a priority in mathematics classrooms.

I have previously demonstrated (Wong, 2017) that graphs are used differently in the two disciplines, meaning there is a need for students to be taught specifically how to construct and use graphs in science, in a scientific way, rather than relying rather hopefully on transfer from mathematics. Leinhardt et al. (1990) suggest that 'real-world' contexts in mathematics do not necessarily support the learning process in mathematics; in science it is very different, or even the reverse, as graphs are an aid to understanding the phenomena being investigated through being representations of observations and aiding the detection of underlying patterns. Science teachers, therefore, need to teach students how to use mathematics in science, and how to switch between mathematical and other representations, and to accept that the responsibility for so doing cannot be outsourced to the mathematics department. Being able to move between representations including, but not limited to, mathematics is a scientific, disciplinary-specific, skill.

Concluding remarks

We have demonstrated the asymmetric dependency which exists between school mathematics and science, leading to asymmetric benefits from collaborating. Asymmetry of dependency, and therefore of benefit, will thus make it very difficult for mathematics and science to work together in a way which is mutually beneficial. Asymmetric dependency will be a useful theoretical tool to allow future researchers to identify benefit in collaboration and when theorising about potential benefits to mathematics and science departments of any joint intervention. It is unlikely to be helpful to recommend that departments work together more closely when the majority of the work is expected to come from the mathematics department and the majority of the benefits accrue to the science department. Such asymmetries are highly unlikely to lead to sustained collaboration. When calling for closer alliances between departments the likely greater benefit for science should be acknowledged and consideration given to ways in which there can also be potential gains for mathematics departments.

Furthermore, we need a research agenda that addresses the interaction between learning in mathematics and in science, and explores how students bring mathematics to bear on a problem in science. Such an agenda may help to lay to rest the notion of simple transfer from mathematics to science, and with it maths blame.

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