Weighted Multi-resource Minority Games

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Abstract Game theory and its application in multi-agent systems continues to attract a considerable number of scientists and researchers around the globe. Moreover, the need for distributed resource allocation is increasing at a high pace and multi-agent systems are known to be suitable to deal with these problems. In this chapter, we investigate the presence of multiple resources in minority games where each resource can be given a weight (importance). In this context, we investigate different settings of the parameters and how they change the results of the game. In spite of some previous works on multi-resource minority games, we explain why they should be referred as *multi-option* games. Through exploring various scenarios of multi-resource situations, we take into account two important issues: *(i)* degree of freedom to choose strategy, and *(ii)* the effect of resource capacity on the different evaluation criteria. Besides, we introduce a new criterion named *resource usage* to understand the behavior of the system and the performance of agents in utilizing each resource. We find that although using a single strategy may involve less computation, using different strategies is more effective when employing multiple resources simultaneously. In addition, we investigate the system behavior as the importance of resources are different; we find that by adjusting the weight of resources, it is possible to attract agents towards a particular resource.

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1 Introduction

With the expansion of computer systems, the need for distributed resources seems to be a vital issue. Therefore, we need an effective resource allocation system that preferably meets two important requirements: simplicity and distribution. From a different point of view, we have the concept of intelligent agents. If a system employs several agents to reach its goal, it is called a *Multi-agent System* (MAS) [21]. Given that agents in a multi-agent systems can be designed to coordinate or even compete with each other, they are suitable for use in distributed scenarios. Therefore, a resource allocation problem could involve intelligent agents with bounded rationality competing for limited resources [15]. These scenarios exist across countless disciplines. For example, sellers and buyers can be considered as agents who complete on price of items on a market. Moreover, animals can be observed as agents that are competing on their territories. In fact, these are some typical example of complex adaptive systems with self-organizing agents whose behaviors dynamically adapt over time [18]. It is arduous to model numerous different scenarios of resource allocation problems, and this makes it challenging for determining the best approach to solve them.

Latterly, the minority game has been adopted as an effective model for resource allocation using inductive reasoning [15]. We believe the literature has been incorrectly proposing multi-option games as multi-resources. In Section 2 we clarify this issue in more details.

1.1 The Minority Game

Back 1994, Arthur [1] introduced a resource allocation model, the *El Farol Bar problem*, that paved the way for the minority game. In the El Farol bar problem, there are a number of people who want to enjoy their evening in a bar. The bar has a limited capacity, so if people go to an overcrowded bar, they cannot enjoy their evening, and the ones who stay at home will have a better evening. Two years after the introduction of El Farol Bar problem, challet and zhang [6] mathematically modeled El Farol bar problem. They assumed that a population of N agents want to use a resource and the capacity of the resource is half of the population. The agents have two options: to go or not to go. If agents end up to an option chosen by less than half of the population, they they are winners. Agents should try to be in the winner group using the previous outcome of the game and their best strategy for each round of the game.

Agents pick their favorite choice with respect to their best strategy. An strategy is a mapping table from the last m outcome of the game to a decision in a round of the game. The last m outcome is also called the history of the game which is global in a traditional classic minority game. There are 2^m possible inputs for a strategy and 2^{2^m} possible strategies. Agents reward their strategy if it leads them in the minority group of the previous round of the game, otherwise, the strategy loses points. Before choosing the best strategy in each round, agents sort their strategies based on the scores they could received.

At the end of each round, the outcome of the game is determined based on the minority rule. Consequently, winning agents are given points and the losers are taken points according to a prize-to-fine ratio. The individual strategies of the agents are reinforced in this manner as well. This payoff process is the essence of learning in the minority game, whereas agents receive feedback from environment and adjust their strategies' scores in order to detect the best strategy. Finally, the outcome of the game constructs the global memory. The game is normally repeated for a certain number of iterations. The system containing this game is preferred to have the agents' attendance close to the capacity of the resource and agents have won almost 50% of the rounds [12].

1.2 Evaluation Criteria

In order to understand the behavior of a minority game we can make use of several evaluation criteria. The conventional criteria used in the literature include attendance, variance, and winning rate. Attendance $A(t)$ is a measure of how many agents chose to attend a resource in t^{th} iteration [15]. Usually, attendance rates are measured over many iterations. Since there is a reinforcement process for agents, the simulation should end to a state with low attendance fluctuation around the resource capacity which is considered as the threshold for the minority rule.

The variance of attendance rates are used to measure the stability of resource attendance [15]. Low variance is indicative of a stable attendance rate, while high variance is a sign of volatility. The trends in variance for complete simulations are often plotted against different memory sizes. Many works use the value called α (a function of memory size, calculated as $\alpha = 2^m/N$) to express the dynamics of the attendance rate variance (as shown in Figure 1). The figure demonstrates the existence of memory sizes optimal for minimizing the variance for different agent population sizes. The figure also shows that after a certain value of α, increasing the memory size does not decrease the variance.

Another important criterion in minority games is the winning rate defined as a ratio between the number of times the agent was in the minority for a resource (a win) and the total number of game iterations. When resource allocation is efficient and resource capacity is 50% of the population size, winning rates among the agents would be almost 50% of the population.

The above metrics are some of the most important criteria for the minority game according to the standard literature [7, 13], but how can one claim that a resource has been utilized effectively in a game or during some iterations? In our study, we want to take into account the general concept of *resource usage* [8, 9] to analyze system behavior. If the number of participants in a resource is less than the resource capacity, the resource is being used effectively. Consequently, the agents that use the under-crowded resource are the winner of game. For more illustrations, we ex-

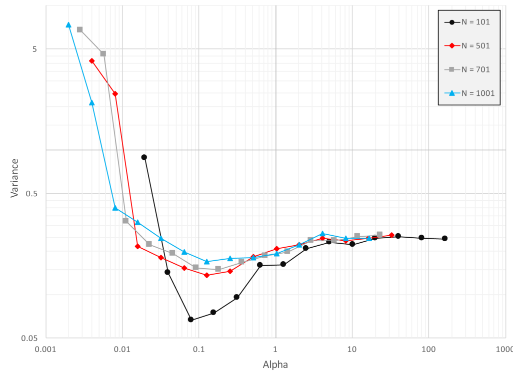

Fig. 1: Association of σ^2 and $alpha$ for the classic minority game, $N = 101, 501, 701, 1001$, $s = 6$, $t = 1000$, resource capacity $= 500$.

plain an example of a computer system in which some processes need to use shared resources. We would prefer the processes to hold all of the resources that they need, but the operating system may put them in a sleep mode or waiting list if they attempt to use an overcrowded resource [20, 22]. Therefor, if a resource is overcrowded most of the times, it is not used properly.

2 Multi-option Versus Multi-resource

We begin explaining the difference between multi-resource and multi-option approach with a real-world example. We consider a situation in which “Diego" wants to go to a movie theater. We assume that all movies in the theater start at the same time so he cannot watch all of them at the same night, thus, he must choose one of the movies. However, regardless of his choice, he will be using the capacity of just one single resource. If he chooses a movie playing in a movie theater that is overcrowded, he will not get in and hence not be able to watch any movie. As a result, *Diego* will not enjoy the night and will "fine" the strategy that led him to choose that movie. This situation is an example of a multi-option game in which multiple options are applied to a single resource, in this case the movie theater.

Now imagine the situation in which *Diego* wants to watch a specific movie that is being played in more than one movie theater. If one theater is overcrowded, other theaters could still be available. This situation may be considered as a multi-resource minority game. The theaters are only equivalent when *Diego* and his friends choose not to split and necessarily go all together to the same theater. In this example, if we consider *Diego* (or either of his friends) individually as an agent, we are facing a multi-option game. On the contrary, if we look at the entire group as an agent, we have a multi-resource game. In fact, the multi-resource game is a more general case of a multi-option case where agents can use more than one resource/option at each round of the game.

It should be noted that in resource allocation game studies, we can consider one single resource with two options as two resource with one option per each. If we generalize it, a game with one resource and k options for that resource is equivalent to a game with k mutual exclusive resources, i.e. agents cannot contribute to more than one resource at each round of the game. Fig. 2 demonstrates the difference between multi-resource and multi-option minority games. Unlike a multi-option (MO) model in which agents make decision for one resource, in multi-resource (MR) model, each agent can decide over different resources and pick more than one of them. In other words, an agent chooses one of the k resources available in the MR model.

For more illustrations, We assume that $agent_i$ wants to pick one resource in an MR model. If the strategies of the agent suggest action a_i where $a_i = (0,1,1,0,1,\ldots,0)$, we can say this agent decided to use the 2^{nd}, 3^{rd} and 5^{th} resources (If we count the options/decisions from zero, then $a_{i,1} = 1$). Nonetheless, $agent_i$ may decide to choose the 2^{nd} option for resource R in an MO model. In this case, the suggested action is $a_i = (0,1,0,0,0,\ldots,0)$ or $a_i = 1$. As it can be seen, MO and MR models may be mistakingly assumed to be equal, but this is true if and only if agents are allowed to choose one resource for each round of the game. Moreover, the multi-resource model is a more generalized version of the multi-option one. Indeed, MO games can be classified as a sub-category of the MR games.

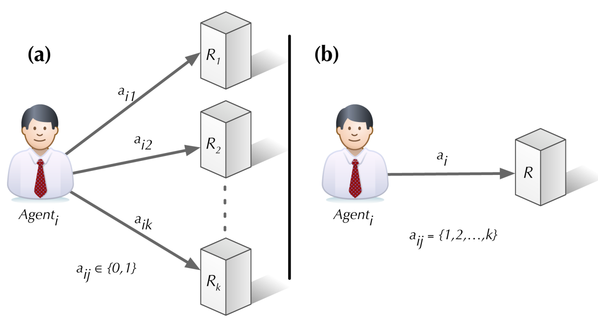

Fig. 2: Multi-option vs multi-resource: (a) Agent i should decide to choose more than one resource in the multi-resource model, (b) There are k options corresponding to the resource R and agent i should choose one of them. The action a_i can be in 1 dimension -a single value- where $a_i \in \{1,2,3,\cdots,k\}$.

A summary of our discussion over MO and Mr models is shown in Fig. 3. As it is shown, the classic minority game - the game with one single resource - is a specific form of the multi-option (MO) model, because the agents can use one resource with two options. Furthermore, multi-option games are a specific form of multi-resource

(MR) model, because the agents can choose exactly one resource/option at each iteration.

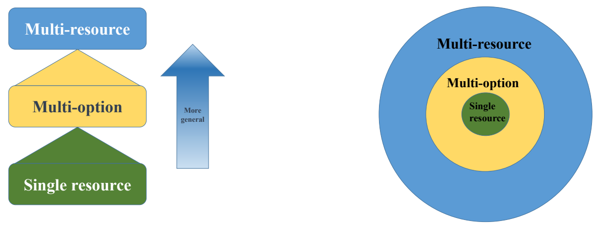

Fig. 3: From left to right: The hierarchical and Venn diagram of the relation among single resource, multi-option and multi resource model.

3 Related Works

One of the earliest variations introduced for minority game is an evolutionary model in which agents' strategies are targeted to be evolved during the game in order to utilize the resources in a more efficient way [11]. In this model, instead of using binary or bipolar representation for agents' strategies, a probability p is assigned to each agent as an strategy. Agents can either refer to local information with probability p or act alone with probability of $p-1$ [10]. The system results in self-organizing agents who choose strategies in groups. The significant part of the work of Huant et al. [10] is that they actually tried to investigate a *multi-option* model; in their model, each agent is allowed to choose only one resource at a time, which is a representation of the multi-option model (as depicted in Figure 2(b)). The other part of their work focuses on the observed grouping phenomenon among the agents on a lattice. They detect this phenomenon on a stock market dataset and analyze it by mean field theory.

The structure underlying the agents' relation to each other is an interesting topic, because networks allow researchers to represent the relations between agents. In real resource allocation problems, agents may often communicate over a social network. The study of structures of social networks has received much recently in the so-called field of Network Science [3]. By using different network structures in agent communication in conjunction with the minority game, agents might influence each other in groups. Remondino and Cappellini [16] connected agents using a random network. The simulation provides the agents two modes of decision-making on a resource: synchronous and asynchronous. With asynchronous decision-making, each agent in sequence makes a decision on a resource. While in the synchronous decision-making, each agent broadcasts its proposed decision to their neighbors.

Subsequently, all agents consider their neighbors' proposed decision and choose what decision to make simultaneously.

Shang and Wang [18] probed the evolutionary minority game on four different network topologies connecting agents to the neighbors: a star network, a regular network (lattice), a random network, and a scale-free network [2]. Similar to the work in [11], they assign a probability p as strategy to each agent. Then, they investigate the distribution of probability p among agents with respect to the different values of the mutation variable L and the variation of the prize-to-fine ratio ρ. With $\rho = 1$, most networks showed the agents segregating to polar probability values, while the star network showed agents with a probability close to 0.5. With $\rho = 0.9$, all network topologies showed agent probabilities clustered around 0.5, suggesting that regular, random, and scale-free networks are sensitive to the value of ρ, while the star network is sensitive to the value of L.

Caridi and Ceva [4] looked at underlying networks in the minority game by connecting agents who have similar strategies together. They found that there are different phases with different networks (e.g. different link definition). Furthermore, in the phase where the system performs like in a game of random decisions, the underlying network behaves as a random one with the same network characteristics.

Applications of the minority game consist allocating energy to rooms in smart buildings with varying energy needs from multiple energy sources [23]. Additionally, computers could use the minority game in allocating re-configurable multi-core processors to maximize efficiency [17]; the results demonstrate that the minority-game policy achieves on average 2-times higher application performance and a 5-times improved efficiency of resource utilization compared to state-of-the-art. Furthermore, the minority game may also be used in cognitive radios and other wireless networks to facilitate cooperation between devices without as much energy or communication overhead[14, 9].

Catteeuw and Manderick [5] conducted a research in which the reinforcement process of Q-learning is used in the minority game. In their version of minority game, agents decide to attend (or use) *only one* of the multiple resources, what is called here "multi-option". Agents will be punished or rewarded based on the resource that they chose to attend. The reinforcement process allows agents to balance between exploration and exploration of new strategies and their best known strategy. In fact, in their research, agents are allowed to choose just one of the resources in each round of the game. Genuinely, their approach may be considered as a minority game in which there is just one resource and more than one option to choose over that resource.

Recall that in the classic minority game, one considers the game for one resource and two mutual exclusive options. However, in distributed systems, a process may need to obtain several different resources at once (such as CPU, RAM, and bus channels) before executing [19]. Although few studies have considered multi-option instead of multiple resources in their research, none have done an in-depth exploration of the variation of the minority game parameters with multiple resources in which an agent is able to apply for more than one resource at each round.

4 The Multi-resource Minority Game Model

In our model of multi-resource minority game, we have more than two resources and two options for each one. The resources have their specific capacity and memory. The minority threshold is the capacity of each resource. Using this model, we explore scenarios that can be taken into account only in a multi-resource model. First, the way that the agents may use their strategies could be different. Here, agents can use different strategies for different resources while in another scenario they can act like simple agents in a classic single resource minority game and use the same strategy for all the resources. We use the aforementioned criteria to analyze the system behavior.

There exists many different possible scenarios one can now consider in this variation. In the following, we express a number of scenarios that deal with real world applications. A real-world example is provided with each scenario. The basis of the examples is a computer system in which a number of processes need to hold some resources to finish their task.

1. Agents need to win all the resources in order to win the game in each round. For example, a process simultaneously needs CPU, RAM, and access to the network.
2. Agents need to win at least one resource in order to win the game in each round. For example, in a multi-processor system, a process needs to win at least one of the processors.
3. Winners are the agents who win the most valuable resources. For example, holding more CPU may be better than having access to different network channels for a particular process.

There are two major variations that can be applied on the aforementioned scenarios. On the one hand, one can now *vary the memory size* independently for each resource. On the other hand, *variations on the capacity* which means we have games where the total capacity of all resources equals to N (number of agents), or are less than N, or more than N.

4.1 Weighted resource model

After we explain the concept of multi-resource, one may ask: what if the resources are valued different by the agents? We examine the effect of two different kinds of resource-weighting methods, linear and exponential. In fact, weights could be used to prioritize the acquisition of one resource over another. In these models, each resource has a weight that is applied to strategy scores during the payoff phase of the game. In the linear method, in every iteration, the weight of every resource is either added to the old score when the agent is the winner, or it is subtracted from the old score when the agent is not in the minority for the resource. This is accomplished with:

$$S(t+1) = S(t) + w \times \beta, \quad (1)$$

where $S(t)$ is the score of the agent at iteration t, w is the resource weight, and β is 1 when the agent is in the minority, and it is -1 when the agent is not in the minority. Increasingly larger values for resource weights alter agent and strategy scores at a rate linearly proportional to the value of the changing weight itself.

With the exponential method, updating the score is a multi-step process. First, for every resource, the old agent and strategy scores are multiplied by the resource weight to the power of β (equation 2). Second, the value obtained by this equation for each resource is averaged to compute the new score. In other words, the new agent and strategy score is calculated by equation 3.

$$S(t+1) = S(t) \times w^{\beta}, \quad (2)$$

where $S_(t)$ is the score of the agent at iteration t, w is the resource weight, and β determines positive or negative payoff.

$$S_k(t+1) = \frac{\sum_{j=1}^{R} S_j(t) \times w_j^{\beta}}{K}, \quad (3)$$

where K is the number of resources and w_j is the base/weight of the r^{th} resource, where w must be greater than 1. More details on resource weighting are found in Section 5.

5 Experimental Results

In this section, we try to compare the situation in which agents are allowed to use one strategy for all resources with the situation where agents can use different strategies for resources. Further, the different weighting methods will be compared with each other. In the same strategy simulation, agents use their best strategy to make decision over all the resources in the system. On the other hand, in the different-strategies simulation, each agent refers to its best strategies with respect to the resource on which it makes decision. As it is mentioned before, we can have different minority rules, here we consider an agent a winner if it is in the minority for all resources. If we use other types of the minority rule we will mention them. In all simulations, $N = 501$ is the population size, $S = 3$ is the number of strategy, $m = 4$ is the memory size and $T = 1000$ is the maximum iteration of the game. We consider $k = 3$ resources for our simulations because having 3 resources is enough to satisfy our assumption about the multi-resource model.

5.1 Multi-resource Simulation

The first experiment is about the attendance of the agents which use the same strategy, i.e. $A(t)$. In this experiment, the capacity distribution is $c = (\frac{N}{4}, \frac{N}{2}, \frac{3N}{4})$, i.e. capacity of resource 1 is 25% of the population (125), resource 2 is 50% (250), and resource 3 is 75% (375). Figure 4(left) shows the impact of the resource capacity on the attendance in that resource. The obvious trends are steady over-attendance for resource 1 and steady under-attendance for resource 3. This is because the capacity is the lowest one in resource 1 and agents have a difficult time winning by attending in it. Meanwhile, it is easy to win by attending resource 3 because its capacity is the highest (375). This phenomenon causes the history to consistently suggest agents avoid attending resource 1, and always attend resource 3. In this simulation, the strategies which have zero in their first element and 1 in the last elements can lead their corresponding agents to win the rounds of the game.[1]

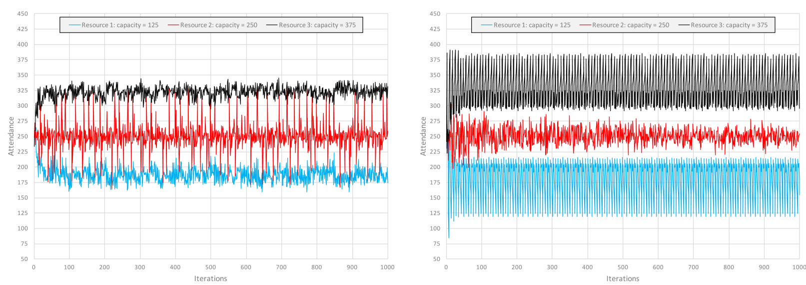

Fig. 4: Attendance vs. iterations for $N = 501$, $T = 1000$, $S = 3$, and $m = 4$. Left: Agents are allowed to use just one strategy for all resources (SS approach). Right: Agents are allowed to use different strategies for different resources (DS).

In order to start the analysis, we look at $A(t)$ when agents are allowed to use different strategies in Fig. 4(right). The attendance rate is still heavily under the influence of the resource's capacity; but by allowing agents to change their strategy we can observe variation in the outcomes. The dashed lines in Fig. 5 imply that losing agents tend to change their strategies in order to avoid attending resource 1. That is to say, the number of agents using resource 1 is decreased and that allows those agents to win. Therefor, a considerable number of agents reward the strategy that helps them avoid this resource. Similarly, we can observe a repetitive pattern with resource 3 where agents that have not attempt for resource 3 try to switch their strategy to a new strategy in order to be in the resource 3. As a result, a periodic over-

[1] The first element of the strategy taken into account corresponds to the situation where all previous outcomes of the game (or the recent ones in memory) should be zero (i.e. no agent chooses to use or go for the resource). Similarly, the last element represents the situation where all of the previous outcomes of the game is one (i.e. all agents choose to use the resource).

attendance happens for resource 3. Figure 5 also allows us to observe the differences between the use of different strategies (DS) (dashed lines) and the same strategy (SS) (solid lines). One main difference between the DS and the SS approach is that if agents use SS, then resource 1 is mostly over-utilized while resource 3 is mostly under-utilized. Another key fact to remember is that the variance in the utilization of resource 2 in DS approach is lower than the SS approach.

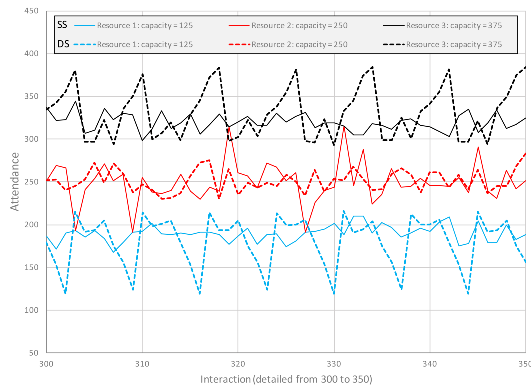

Fig. 5: A zoomed in view of what is happening from iteration 300 to iteration 350. Dashed lines and solid lines show the attendance for different strategy and same strategy approaches receptively.

Furthermore, the impact of memory size in attendance for the SS and DS approaches is investigated in Fig. 6. As it is shown in Fig. 6, using different strategies let the agents have consistent variances over different memory size. Additionally, it is observed that in the same strategy approach, there are more fluctuations on the resource with capacity of 50% of the population. The reason behind this phenomenon is that agents are using just one strategy, and they try to use the strategy that gives them the most for all three resources.

In Figure 7(left), we explore the resource usage for the same strategy simulation. As a result, the resource with lowest capacity has been never successfully used during the game. That happens because it is mostly overcrowded and difficult to win; so, agents decide to not attend to this resource. However, the resource with highest capacity has been 100% successfully utilized.

The story is different when agents are allowed to use different strategies for different resources. Evidently, it is more possible for agents to obtain all resources successfully. Since the different strategies used for resources are independent, agents have the opportunity to use their best strategy for individual resources. Therefor, resource 1 can be periodically under-capacity, which was never the case when using the same strategy. As shown in Table 1 the number of iterations in which agents successfully utilize resource 1 increases to about 14%. Figure 7(right) develops the

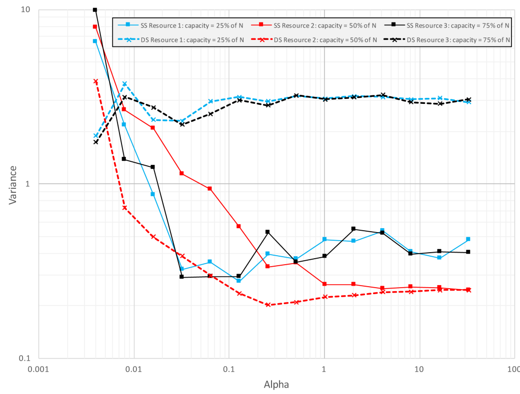

Fig. 6: σ^2 vs. α for $N = 501$, $T = 1000$, and $S = 3$ for the same-strategy simulation (SS) and different-strategies simulation (DS). The variance is quite high for 25% of the capacity (125) and 75% of the capacity (375) when using different strategies.

claim that if agents need all resources in order to accomplish their tasks, then using different strategies is beneficial for them.

Table 1: Percentage of iterations with under-capacity attendance for $N = 501$, $T = 1000$, $S = 3$, and $m = 4$.

Same Strategy			Different Strategies		
Resource #			Resource #		
1	2	3	1	2	3
0.000	0.520	1.000	0.140	0.500	0.870

Another mentioned criterion is the average resource usage that is stated in Table 2. The results from SS analysis shows that if you want your agents to use the resources, and the resources are the same, it is better to use the same strategy. However, the results suggest to consider different strategies in the case that you need to utilize different resources. The average resource usage (ARU) is calculated based on Equation 4.

$$ARU_i = \frac{\sum_{t=1}^{T} A_i(t).\delta_i(t)}{T.C_i}, \tag{4}$$

where i and t represents the resource index and iteration number, $A_i(t)$ is the number of agents using resource i at iteration t (aka attendance rate), C_i is the capacity of resource i, T is the total number of iterations, $\delta_i(t) = 0$ if resource i is overcrowded at iteration t, and $\delta_i(t) = 1$ if resource i is not overcrowded at iteration t.

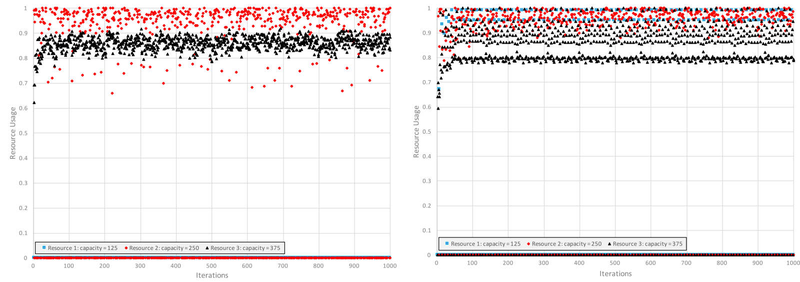

Fig. 7: Resource usage vs. iterations for $N = 501$, $T = 1000$, $S = 3$, and $m = 4$. Left: Same strategy Method. Right: Different strategy method. unlike in the same-strategy simulation, resource 1 is successfully utilized.

Table 2: Average resource usage for $N = 501$, $T = 1000$, $S = 3$, and $m = 4$. Resource 1 is never successfully used in the same-strategy simulation. Note that here we do care about the number of agents (attendance rate) in a resource that is not overcrowded.

Same Strategy			Different Strategies		
Resource #			Resource #		
1	2	3	1	2	3
0.000	0.494	0.858	0.137	0.479	0.748

When agents use the same strategy for each resource, they have a higher average winning rate, as seen in Table 3, The pitfall is the fact that some agents cannot win even one single round of the game. In order to understand this phenomenon, we should take a closer look into the strategies for each resource separately. Since most of the times th resource number 1 in over crowded, the one which decide not to use it are the winners. As a result, most of the times the memory is as $m = (0, 0, \dots, 0)$ for resource 1. So agents need to have the value 0 in the element number 0 in their strategy to be able to win the next round with a high probability.

What's more, we consider the resource 3 (the resource with capacity of 75% of the population). In this case, it is more likely that the agents that use this resource are the winners. So the outcome will be 1 and it should happens frequently. Consequently, the agent's memory will be $m = (1, 1, \dots, 1)$ for the third resource. Then, the winners will be the agents which have 1 in the last element of their strategy. All facts considered, the agents that have a strategy $s = (0, \dots, 1)$ are guarantied to make the correct decision for two resources. They just need to adjust their strategy for the resource 2 (with capacity of 50% of the population). In other words, if agents have strategies that meet these requirements (i.e. having 0 and 1 in their first and last element of strategy respectively), the deciding factor is their attendance in resource

2. The advantage of using the same strategy for each resource happens when agents have strategies meeting this criterion given that they only need to make a decision for resource 2 in order to win. The disadvantage of using the same strategy is the considerable number of agents which never win a round of game (almost one-third of the population based on Table 3). Agents tend to have lower average winning rates when they can use different strategies for each resource. Although it is more difficult for agents to become winner, we found that every agent is able to win at least one round of the game.

Table 3: Average winning rate of agents and the average percentage of agents with no wins in all simulation iterations.

	Avg. Winning Rate	Agents w/o wins
Same Strategy	19.8	29.1
Different Strategies	15.8	0.0

The resource usage percentage is shown in Table 4. We considered two other types of capacity distribution over the resources. In one of them, all the resources have different capacity but all at the level that is less than the half of the population. In this situation, agents are expected to have more chance to win all resources. In the other distribution, resources have the same capacity (33%) at a level less than half of the population size. In a general case, the capacity of resource c_i can be considered as $c_i = \frac{N}{K}$, where N is the population size and K is the number of resources. We also cover a scenario with a different winning rule in which agents have only to win two resources out of three available ones. In this scenario, we consider the same distributions we use in the last simulation.

Table 4: Resource usage for different capacity distribution. Terms "DS" and "SS" are refer to different strategy and same strategy respectively.

Need to win 3 out of 3 resources									
	20	**30**	**40**	**25**	**50**	**75**	**33**	**33**	**33**
SS	0	19.9	19.9	0	49.4	85.8	14.3	14.3	14.29
DS	12.5	14.2	35.5	13.7	47.9	74.8	14.3	14.3	14.3

Need to win 2 out of 3 resources									
	20	**30**	**40**	**25**	**50**	**75**	**33**	**33**	**33**
SS	5.8	19.9	37.6	11.79	53.0	88.3	14.3	14.3	14.3
DS	12.5	14.2	35.5	14.2	50.2	87.5	12.5	14.2	35.5

First we discuss other types of the distribution in the scenario of “win 3 out of 3” situation. As it can be seen in Table 4, using different strategies (DS) can improve the resource usage significantly where resources have different capacity distributions and the capacities are less than half of the population. However, when the capacity is uniformly distributed, there is no significant difference between SS and DS. Since the same strategy approach requires less computational power, it is better to follow it. This situation can be modeled as assigning similar resources to the processes in a computer system.

In the “winning two out of three resources" scenario, we expect that agents will be biased to the resources with higher capacity. The results in Table 4 show that using SS works better in this scenario. Nonetheless, using the DS approach helps the system to have a better load balance over the resources. However, we will prefer to use the SS approach in the systems that the capacity distribution is $c = (\frac{N}{4}, \frac{N}{2}, \frac{3N}{4})$, because it is as effective as the DS approach.

The lower part of Table 4 refers to the scenario in which agents should win two out of three resources. Since there is a higher chance for them to win or utilize resources with higher capacity, we anticipate that the agents tend to participate in those resources. Furthermore, we expect that simulations with DS approach may show much better results, because agents have more degree of freedom to choose a suitable strategy. Surprisingly, the results show that in this experiment, using the same strategy (SS) performs better than DS approach. Particularly, using SS seems to be more efficient where the capacities are less, equal and more than half of the population size. Notwithstanding, it is better to use different strategy (DS) when a balanced load on the resources is more important than the computational complexity of DS approach. Similar to the previous scenario, there is no significant difference between SS and DS when the capacity is uniformly distributed.

5.2 Multiple Weighted Resources

In this section we investigate the linear and exponential methods of weighting resources. In the weighted model, either the high capacity resource has a greater weight or the low capacity resource has the greater weight. We refer to the former as heavier high capacity resource and the latter as heavier low capacity. Note that resource weighting simulations were initially performed with three resources, two resources with a capacity of 250 and one resource with a capacity of 166 $1/2$ and $1/3$ of the agent population of 501, respectively). Weights were chosen to sum to 1, with the two identical resources having identical weight values. For clarity of the description, only two resources are presented in the figures.

Figure 8 shows a situation in which the higher capacity resource is weighted higher (.4) than the lower capacity resource (.2). As expected, the attendance rates for both resources stay close to the capacity of the more heavily weighted higher capacity resource 1. This causes significant and consistent over-attendance for resource 2. In contrast, Fig 9 shows a situation in which the lower capacity resource is weighted higher (.8) than the higher capacity resource (.1). The attendance rate of resource 2 quickly drops to its capacity of 166 over the course of the simulation,

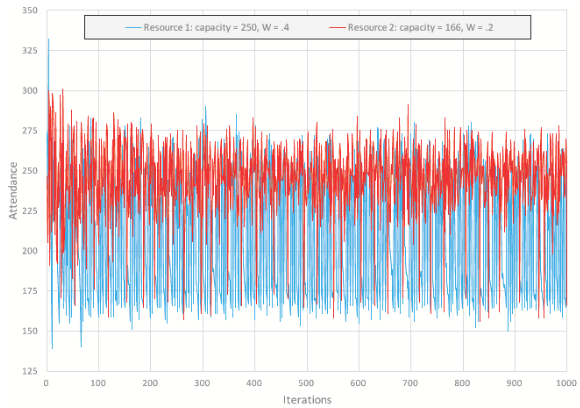

Fig. 8: Attendance rate for resources with different capacities with heavier high capacity resource using a linear resource weighting method, N = 501, T = 1000, S = 6, and m = 3.

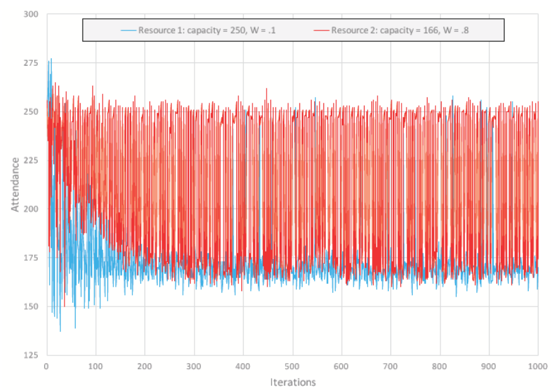

Fig. 9: Attendance rate for differing capacity resources with heavier low capacity resource using a linear resource weighting method, N = 501, T = 1000, S = 6, and m = 3. Note that attendances are influenced by the heavily weighted low capacity resource.

while attendance for Resource 1, with a capacity of 250, swings back and forth from around 250 to around 166. The attendance rate is more predictable for both resources and more importantly, both resources enjoy efficient utilization, as attendance more frequently is at or below capacity.

Figure 10 shows a situation in which the low capacity resource is weighted at 2 and the higher capacity resource is weighted at 1.5. The plot looks remarkably similar to the baseline plot, however the point at which the attendance spikes meet rise from approximately 220 to 230. This is unexpected, as higher weights for the

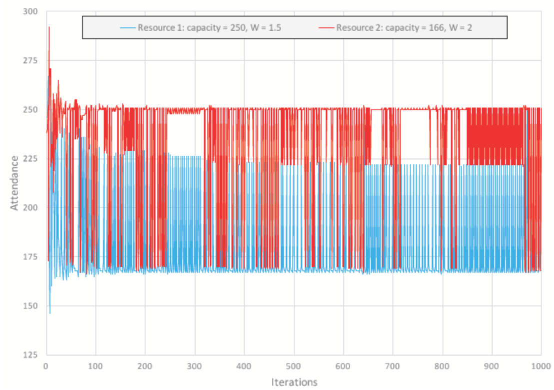

Fig. 10: Attendance rate for differing capacity resource resources with heavier low capacity resource using resource weighting method 2, N = 501, T = 1000, S = 6, and m = 3.

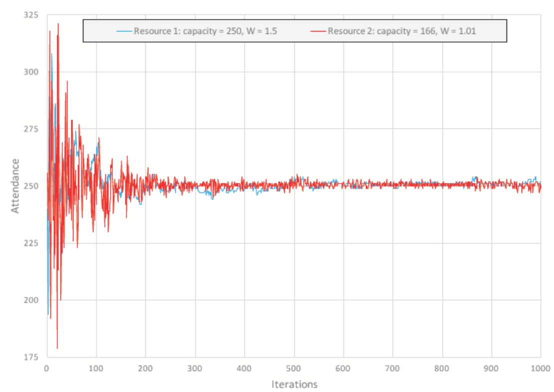

Fig. 11: Attendance rate for differing capacity resource resources with heavier high capacity resource using resource weighting method 2, N = 501, T = 1000, S = 6, and m = 3. Shows attendance of low capacity resource pulled to capacity of resource with heavier weight.

low capacity resource should have resulted in a stronger pull towards the low capacity. It is unclear why this occurred. However, when the low capacity resource is weighted at 1.01 and the higher capacity resource is weighted at 1.5, there is a dramatic change in plot appearance, as seen in Figure 11. The attendance of both plots approach equilibrium about highly weighted resource 1's capacity of 250. Variance around the attendance mean minimizes very rapidly. This suggests that the effect of a lower capacity resource on the attendance rate of a higher capacity resource can be minimized if the weight of the low capacity resource is reduced.

Based on these experiments, the weighting methods appear to have different strengths. The linear weighting method can increase resource utilization for all resources in the system by weighting low capacity resources more heavily. The exponential weighting method can dramatically decrease the attendance variance of a high capacity resource by weighting low capacity resource less heavily.

Another interesting result for the weighted multi-resource minority game simulation has been observed when we try two other weight distributions. In this simulation, we investigate a different game with a uniform capacity distribution. In one system, the capacity of each resource is $\frac{1}{3}$ of the population, i.e. $c_i = \frac{N}{3}$; in the other one, the capacity is half of the population, i.e. $c_i = \frac{N}{2}$. Surprisingly, the resource usage is the same for SS and DS approaches in the case that resource capacity is $\frac{N}{2}$, and neither using different weights for resources nor different weighting methods can make difference. Apparently, it is more important for agents to be a winner than taking the risk to win a resource with higher weight.

In the simulation with the capacity $c_i = \frac{N}{3}$, the resource usage is almost the same for the resources with different weights in each simulation. Table 5 shows the results for this experiment. In the simulation with 3 strategies per agent and memory size of 4, the resource usage is 0.14 for all conditions. While changing the number of strategy per agent does not make any significant difference, increasing the memory size causes dropping down in resource usage except the situation in which we use weighting method 2 and different strategy approach. One reason behind this phenomenon is the fact that agents avoid utilizing resources because they can easily be overcrowded. However, increasing the number of strategies per agent can slightly improve the resource usage.

Table 5: Resource usage for weighting methods 1 and 2 with SS and DS approaches. Each number in the table represents the resource usage for all three resources, because they have the same resource usage. Capacity of each resource is $\frac{N}{3}$, $N = 501$, $T = 1000$, and $k = 3$; the weight distribution is $w = (0.4, 0.35, 0.25)$ for weighting method 1, and $w = (2, 1.5, 1.01)$ for weighting method 2.

Strategy per agent	3	5	7	3	3
Memory size	4	4	4	6	8
Weighting method 1 + SS	0.14	0.27	0.28	0.11	0.08
Weighting method 1 + DS	0.14	0.26	0.29	0.11	0.09
Weighting method 2 + SS	0.14	0.26	0.28	0.11	0.09
Weighting method 2 + DS	0.14	0.26	0.28	0.16	0.11

6 Conclusion and Future Work

This study proposed an extension of the minority game in which agents need to compete for more than two resources and are able of utilizing multiple resources at the same time. We introduced the idea of a multi-resource minority game and investigated the behavior of agents and systems for several cases using 3 resources. We clarified that multi-resource as we describe is different than what we call multi-option (but other authors insist in calling multi-resource). We have made the case for the naming as we use in this text.

Additionally, we investigate the weighted resources approach and we observed that a linear method is potentially able to improve resource utilization by adjusting the weights of the resources. Further, the impact of exponential weighting seems to be crucial to the attendance variance of high capacity resources. However, when the resource distribution is uniform, different weighting methods cannot make significant changes in resource usage. More research with these weighting methods has the potential to unveil many more practical uses of resource weighting with multiple resources. For instance, having different method of rewarding in a single close system can change agent behavior in favor of a particular resource.

A further potential work can focus on the mathematical analysis of the evolutionary multi-resource minority game. In this case, instead of using a traditional binary/bipolar representation of strategies, agents can use a probability to chose the resource. Additionally, analyzing the significance of different memory size with respect to the umber of strategies needs more explorations. Besides, analyzing the influence of connected agents over different networks may be useful for the situation in which the agents share information.

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