Distribution network dynamics with correlated demands

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Abstract
We compare centralised and decentralised distribution network designs for a two-level supply chain. The demand pattern faced by the retailers is modelled as a first order Vector Auto-Regressive process, which is used to represent the progression of and relationship in sets of time series. All participants, i.e. retailers and distribution centres, operate an Order-Up-To policy with the MMSE forecasting. Inventory and capacity costs are considered. The result shows that the demand correlation has a significant impact on decision about consolidation of the distribution system.

Keywords: Distribution network design, Vector Auto-Regressive (VAR), Correlated demand

Introduction

This study is concerned with Distribution Network Design (DND). DND is concerned with determining the number, location and capacity of distribution centres in order to achieve efficient flow of product from suppliers to customers. We develop an analytical model of a two-level supply chain in which the benefit of the consolidation of the distribution network can be evaluated.

The demand pattern faced by the retailer is modelled as a first order Vector Auto-Regressive process, VAR(1). The VAR model is used to represent the progression of and relationships in sets of time series. Thus, the VAR model is used to represent a situation where the demand is correlated over time and between retailers (or with other products). The correlation of demands between retailers complicates distribution network modelling and has been disregarded by most researchers (Chen *et al*., 2002). Since this complexity occurs with real consumer products (Erkip *et al*., 1990) and neglecting it can cause significant deviation from the optimal inventory policy, we are interested in finding an analytical description of the dynamics of this situation.

We are also concerned with the bullwhip effect, (Lee *et al*., 1997). This study captures the relationship between distribution network design, inventory costs and capacity costs in our model. The capacity cost represents the opportunity costs associated with the Bullwhip effect (Disney *et al*., 2006). Using some Control Engineering tools we have derived analytical expressions that describe the dynamics of the net stock levels and order rates over time. These allow us to find the net stock variance and the order variance, which are important inputs for our cost model. From the model we are then able to analyse the costs of network consolidation. In certain circumstances the square root law for inventory is shown to hold, as well as the newly discovered "square root law for bullwhip", Ratanachote and Disney (2008) and Disney *et al.* (2006). In addition, a numerical example is presented in order to evaluate the impact of the demand correlation and consolidation in the distribution network.

Literature review

Although most literature neglects the impact of demand correlation in supply chain models, a number of papers have paid close attention to the matter in a variety of model settings. Erkip *et al.* (1990) develops a depot-warehouse model acting as a centralised distribution system with one supplier, one depot and *n* warehouses. The depot holds no inventory. The warehouses employ base-stock policies. The study shows the impact of demand correlation both across warehouses and in time on the optimal safety stock of a periodic review system. The demand correlation in time is represented by an Auto-Regressive process of the first order, AR(1). This is assisted by a periodic index-variable that represents demand correlation between warehouses.

Güllü (1997) investigates inventory levels and system costs resulting from a proposed forecasting approach. Probabilistic demands models are adopted. The study allows correlation through time and among retailers of both demands and demand forecasts. Chen *et al.* (2002) shows that the inventory position at each location is stationary and uniformly distributed under a lot-size reorder point inventory system in which there is one supplier and multiple retailers. Raghunathan (2003) evaluates the value of and incentives for information sharing in a one-manufacturer and *n*-retailer setting. The retailers' lead-times are set to zero. Demand patterns at each retailer are an AR(1) process. The correlation of demands between retailers is modelled by the correlation coefficient between pairs of retailers. The study shows the magnitude of value of information sharing under different levels of demand correlations and different numbers of retailers.

Our study differs from the previous studies as; (1) we employ the VAR(1) process for the retailer demand. The VAR(1) model uses explicit correlation coefficients in time and between demands of two retailers. (2) We provide a closed form solution of all the order variances and all the net stock variances. (3) We compare two 2-level distribution networks: a decentralised system and a centralised system. The retailer operates Order-Up-To (OUT) inventory replenishment policy with Minimum Mean Squared Error (MMSE) forecasting, as does the distribution centre.

The model

To evaluate the impact of the distribution network design on its dynamic and economic performance, we consider two different distribution networks: a decentralised system and a centralised system. In the lower echelon of each system, there are two retailers operating OUT replenishment policies with MMSE forecasting. For the upper level, there are two distribution centres in the decentralised system. There is one distribution centre in the centralised system. All distribution centres also operate an OUT policy with MMSE forecasting. Unit lead-times are assumed at all locations in both distribution systems. Fig. 1 depicts our model.

The VAR(1) demand process

We assume that the demand at each the retailers follow a first order vector autoregressive (VAR(1)) demand process. Specifically we use the mean centred VAR(1) demand process given by

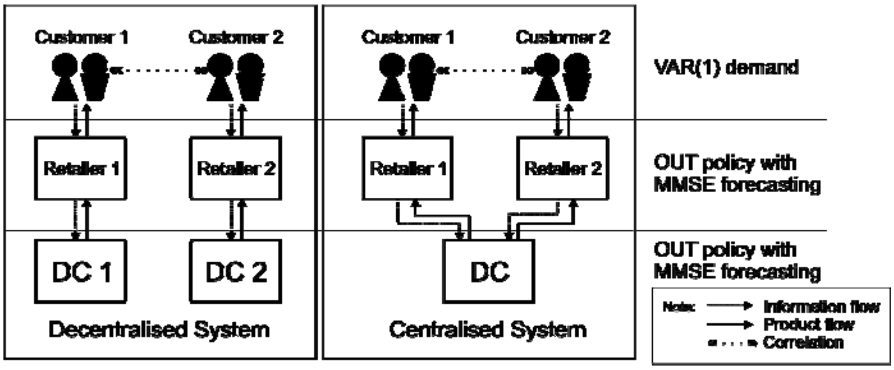

Figure 1 – The decentralised and the centralised distribution networks

$$\begin{aligned} D_{1,t} &= \mu_1 + \phi_{11}(D_{1,t-1} - \mu_1) + \phi_{12}(D_{2,t-1} - \mu_2) + \varepsilon_{1,t} \\ D_{2,t} &= \mu_2 + \phi_{21}(D_{1,t-1} - \mu_1) + \phi_{22}(D_{2,t-1} - \mu_2) + \varepsilon_{2,t} \end{aligned} \quad (1)$$

Here we can see that $D_{1,t}$, the mean centred demand for retailer 1, at time *t*, is given by the sum of four components. The first term is the mean demand level of retailer 1, μ_1. The second term is an autoregressive term of one period with it own mean centred demand, $\phi_{11}(D_{1,t-1} - \mu_1)$. The third term is an autoregressive term with the previous realisation of retailer 2's mean centred demand, $\phi_{12}(D_{2,t-1} - \mu_2)$. The final term is independently and identically distributed (white noise) random process, $\varepsilon_{1,t}$. The demand process at the second retailer, retailer 2, is simply a mirror image of the demand process for retailer 1 with the obvious change in notation, $\{1 \to 2, 2 \to 1\}$. $\varepsilon_{\{1,2\},t}$ can be regarded as the one period forecast error and we assume that it has zero mean and unit variance. We assume from now on that these error terms are uncorrelated as this simplifies the mathematics considerably. In order for the VAR(1) demand process to be stationary the following criteria must be hold

$$\left| \frac{(\phi_{11} + \phi_{22}) \pm \sqrt{(\phi_{11} - \phi_{22})^2 + 4\phi_{12}\phi_{21}}}{2} \right| < 1. \quad (2)$$

We will now simplify the demand model to allow a meaningful exposition of this short conference paper by the following assumptions. (1) The correlation in time at each retailer is the same. That is, $\phi_{11} = \phi_{22} = \phi$. (2) The correlation between the two retailers is the same in both directions. That is $\phi_{12} = \phi_{21} = \theta$.

For stationary processes the variance of the demand at retailer *i* is the given by

$$\frac{\sigma_{i,D}^2}{\sigma_\varepsilon^2} = \frac{1 - \theta^2 - \phi^2}{\theta^4 + \left(\phi^2 - 1\right)^2 - 2\theta^2\left(1 + \phi^2\right)}. \quad (3)$$

The replenishment policies and the forecasting model

Each retailer operates an OUT replenishment policy with MMSE forecasting. The replenishment order at retailer *i* at time *t* is given by

$$OR_{i,t} = F_{i,t} + TNS_i - NS_{i,t} - WIP_{i,t} \quad (4)$$

where ($F_{i,t}$ + TNS_i) is the order-up-to level of retailer i at time t. The first term, $F_{i,t}$, is the forecast of demand. For MMSE forecasting, this forecast is the conditional expectation of demand over the lead-time and review period. For a unit lead-time, the forecast is

$$F_{i,t} = 2\mu_i + (\phi + \phi^2 + \theta^2)(D_{i,t} - \mu_i) + (\theta + 2\phi\theta)(D_{j,t} - \mu_j)\,. \tag{5}$$

The second term in Eq. 4, TNS_i (Target Net Stock at retailer i) can be thought of as a safety stock. The net stock of retailer i at time t is given by

$$NS_{i,t} = NS_{i,t-1} + OR_{i,t-2} - D_{i,t}\,. \tag{6}$$

The work in progress of retailer i at time t is given by

$$WIP_{i,t} = O_{i,t-1}\,. \tag{7}$$

For each of the DCs in the decentralised system, the mathematical expressions of the replenishment system are similar to Eqs (4), (6) and (7) since each DC also employs an OUT policy. A superscripted DC is used to differentiate the DC's variables from the retailer's: $OR_{i,t}^{DC}$, $F_{i,t}^{DC}$, $NS_{i,t}^{DC}$, $WIP_{i,t}^{DC}$ and $D_{i,t}^{DC}$. Note, this is not an exponent. The demand for DC i at time t is actually the retailer i's order that is placed at time t and is passed to DC i. We assume here that orders from the retailer are passed to the DC without delay. We omit the full description of the distributor centres difference equations to save space. It is all rather obvious except for the forecast of demand at DC i which is given by

$$F_{i,t}^{DC} = 2\mu_i + (\phi^3 + \phi^4 + \theta^4 + 3\phi\theta^2 + 6\phi^2\theta^2)(D_{i,t} - \mu_i) + (\theta^3 + 4\phi\theta^3 + 3\phi^2\theta + 4\phi^3\theta)(D_{j,t} - \mu_j) \tag{8}$$

where for retailer *1*, i=1 and j=2; for retailer *2*, i=2 and j=1.

For the DC in the centralised system, the expressions are again similar to Eqs (4), (6) and (7). A superscripted DC is also used here but the subscript i is omitted. The demand for the DC at time t is the sum of orders from the two retailers. The forecast of the DC at time t is given by

$$F_t^{DC} = 2\left(2\mu_1 + 2\mu_2 + ((D_{1,t} - \mu_1) + (D_{2,t} - \mu_2))\begin{pmatrix} \theta^3 + \theta^4 + 3\phi\theta^2 + 4\phi\theta^3 + \\ 3\phi^2\theta + 6\phi^2\theta^2 + \phi^3 + 4\phi^3\theta + \phi^4 \end{pmatrix}\right). \tag{9}$$

The variances of the order rate and the net stock levels

In order to capture the dynamics of our distribution networks to evaluate their economic performance, we will first obtain expressions for the order variances and the net stocks variances using some Control Engineering tools. The block diagram in Fig. 2, Fig. 3 and Fig. 4 represents our replenishment decisions using the discrete time z-transform. We refer interested readers to Nise (1995) for background reading on Control Theory and Hosoda and Disney (2006) for an application to a 3-level supply chain.

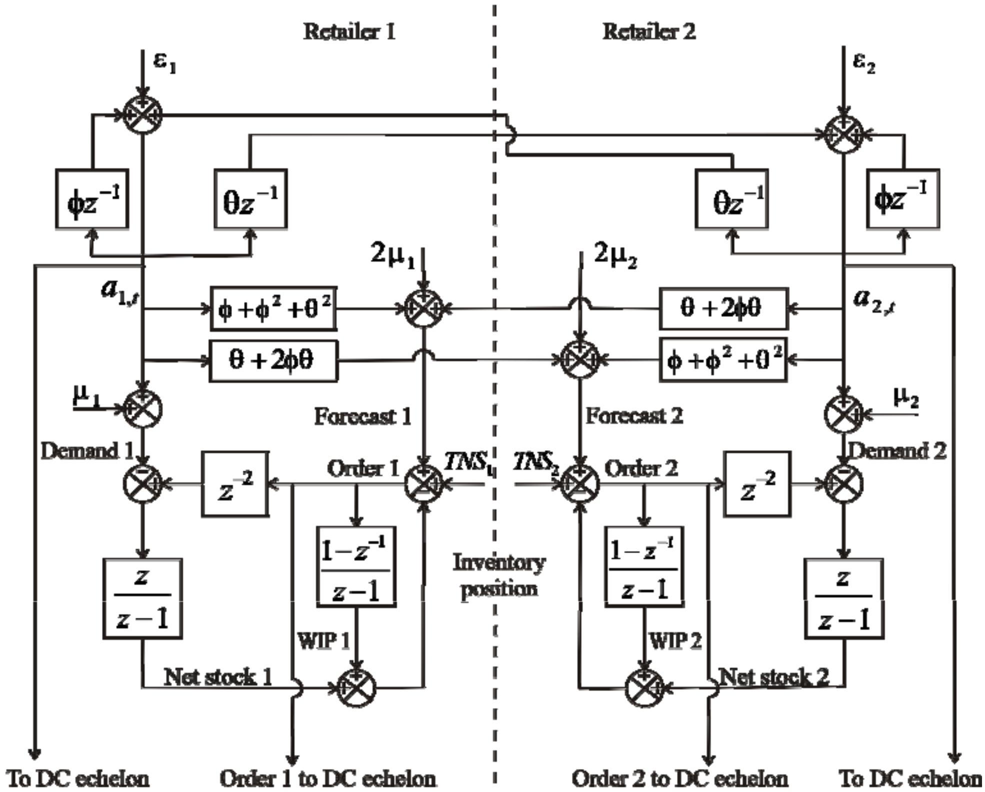

Figure 2 – Block diagram of the retailers' replenishment decision

Retailer Level

We may manipulate the block diagram for the transfer function to find the relationship between the noise (the error term) and the state variables of interest. We then exploit Jury's Inners technique, Disney (2008), to obtain the corresponding variance ratio. This technique utilizes the determinants of certain matrices formed by the coefficients of the transfer function. The variance of the orders for retailer *i*, which is the summation of variances resulted from error *i* and error *j*, is given by

$$\frac{\sigma_{i,OR}^2}{\sigma_\varepsilon^2}=\frac{1}{4}\left(\frac{1}{1+\theta-\phi}+\frac{1}{1-\theta+\phi}+\frac{1}{1-\theta-\phi}+\frac{1}{1+\theta+\phi}+8\theta^2(1+3\phi)+8\phi\left(1+\phi+\phi^2\right)\right). \tag{10}$$

The variance of the net stock for retailer *i* is given by

$$\frac{\sigma_{i,NS}^2}{\sigma_\varepsilon^2}=1+\theta^2+(1+\phi)^2. \tag{11}$$

Distribution Centre Level: Decentralised system

The distribution centre's replenishment decision of the decentralised system is depicted in Fig. 3. The block diagram in Fig. 2 and Fig. 3 are actually connected. The demands fed into the DC are the orders from the retailers.

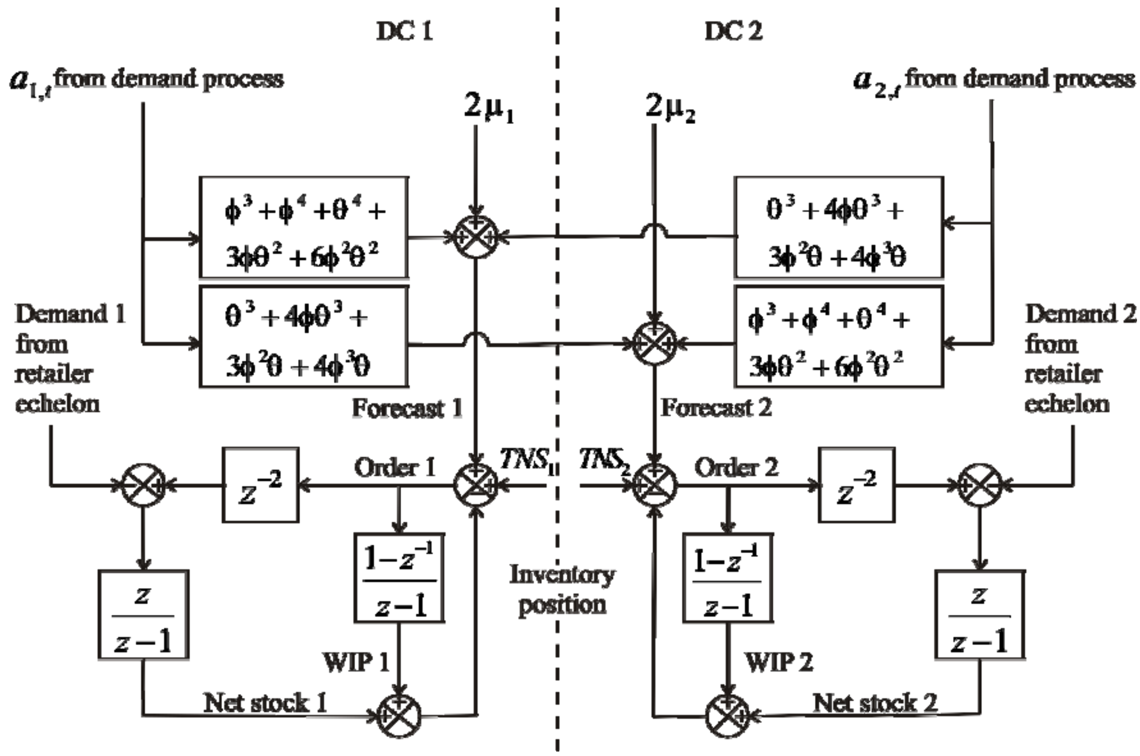

Figure 3 – Block diagram of distribution centres' replenishment decision: decentralised system

The variance of the orders for DC *i* is given by

$$\frac{\sigma^2_{i,DC,OR}}{\sigma^2_\varepsilon}=\left(\begin{array}{l}2\phi-\dfrac{\left(\theta^2+\phi^2-1\right)}{\theta^4+\left(\phi^2-1\right)^2-2\theta^2\left(1+\phi^2\right)}+\\ 2\left(\begin{array}{l}\phi^2\left(1+\phi+\phi^2\right)\left(1+\phi+\phi^3\right)+\theta^4(2+5\phi(2+\phi(3+7\phi)))+\\ \theta^6(1+7\phi)+\theta^2(1+\phi(6+\phi(12+\phi(20+3\phi(5+7\phi)))))\end{array}\right)\end{array}\right). \quad (12)$$

The variance of net stock of DC *i* is given by

$$\frac{\sigma^2_{i,DC,NS}}{\sigma^2_\varepsilon}=\left(\begin{array}{l}\theta^4(4+5\phi(2+3\phi))+\theta^2(6+\phi(18+\phi(24+5\phi(4+3\phi))))+\\ 2+\theta^6+\phi(4+\phi(6+\phi(6+\phi(4+\phi(2+\phi)))))\end{array}\right). \quad (13)$$

Distribution Centre Level: Centralised system

The distribution centre's replenishment decision of the centralised system is depicted in Fig. 4. The block diagrams in Fig. 2 and Fig. 4 are also connected. The demand for this system is the sum of orders of the two retailers. The forecast expression was given by Eq. 9.

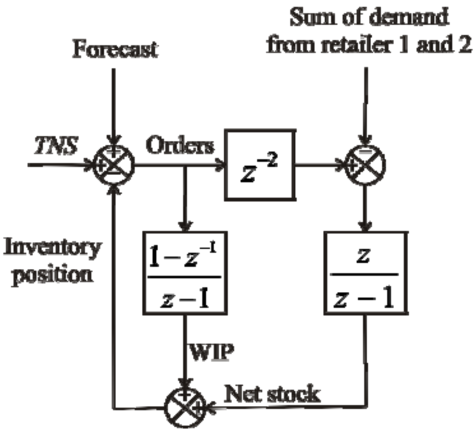

Figure 4 – Block diagram of distribution centres' replenishment decision: centralised system

The variance of the orders for the DC is given by

$$\frac{\sigma^2_{DC,OR}}{\sigma^2_{\varepsilon}} = 4\phi - \frac{2}{(\theta+\phi-1)(1+\theta+\phi)} + 4\left(\begin{array}{l} \theta^7 + \theta^6(1+7\phi) + \phi^2\left(1+\phi+\phi^2\right)\left(1+\phi+\phi^3\right) + \\ \theta^5(2+3\phi(2+7\phi)) + \theta^4(2+5\phi(2+\phi(3+7\phi))) + \\ \theta^3(2+\phi(8+5\phi(4+\phi(4+7\phi)))) + \\ \theta^2(1+\phi(6+\phi(12+\phi(20+3\phi(5+7\phi))))) + \\ \theta(1+\phi(2+\phi(6+\phi(8+\phi(10+\phi(6+7\phi)))))) \end{array}\right). \tag{14}$$

The variance of the net stock for the DC is given by

$$\frac{\sigma^2_{DC,NS}}{\sigma^2_{\varepsilon}} = 2\left(\begin{array}{l} 2+\theta^6+\theta^5(2+6\phi)+\theta^4(4+5\phi(2+3\phi)) + \\ 2\theta^3(3+2\phi(4+5\phi(1+\phi))) + \theta^2(6+\phi(18+\phi(24+5\phi(4+3\phi)))) + \\ \phi(4+\phi(6+\phi(6+\phi(4+\phi(2+\phi))))) + 2\theta(2+\phi(6+\phi(9+\phi(8+\phi(5+3\phi))))) \end{array}\right). \tag{15}$$

The costs in the distribution network

Both a centralised and a decentralised network are considered under two main types of costs; inventory and capacity costs. For inventory costs, we assume that piece-wise linear and convex inventory holding and backlog costs exist,

$$\text{Inventory cost for period } t = \begin{cases} H(NS_t), & \text{when } NS_t \geq 0, \\ B(-NS_t), & \text{when } NS_t < 0. \end{cases} \tag{16}$$

where H and B are the unit costs per period of holding and backlog respectively. The capacity costs have been added to capture the opportunity costs associated with the bullwhip effect. The normal capacity is set to $(\mu_d + S)$ where μ_d is the mean demand and S is spare capacity above (or below) the mean demand. If the order quantity is smaller than the normal capacity, we consider this as a lost capacity situation which has opportunity costs. Then again, if the order is larger than the normal capacity, we will pay a premium: either for overtime capacity or subcontractors. We also assume that piece-wise linear and convex lost capacity and overtime costs exist.

$$\text{Capacity cost for period } t = \begin{cases} N(OR_t - (\mu_d + S)), & \text{when } OR_t \le (\mu_d + S), \\ P((\mu_d + S) - OR_t), & \text{when } OR_t > (\mu_d + S) \end{cases} \quad (17)$$

where N and P are the unit costs of lost capacity and overtime respectively. As the error terms, $\varepsilon_{i,t}$, are assumed to be normally distributed, the inventory levels and the order rates will also be normally distributed. Using this knowledge, Disney *et al.* (2006) derived the optimal target net stock $\left(TNS^*\right) = \sigma_{NS}\Phi^{-1}\left[\frac{B}{B+H}\right]$ and the optimal spare capacity $\left(S^*\right) = \sigma_{OR}\Phi^{-1}\left[\frac{P}{P+N}\right]$that minimises the inventory costs. When the target net stock is set to its optimum, TNS^*, the inventory cost per period is given by

$$I_£ = \sigma_{NS}\left(B+H\right)\varphi\left[\Phi^{-1}\left[\tfrac{B}{B+H}\right]\right] = \sigma_{NS}Y_{NS} \text{ with } Y_{NS} = (B+H)\varphi\left[\Phi^{-1}\left[\tfrac{B}{B+H}\right]\right], \quad (18)$$

where $\varphi\left[\cdot\right]$ is the standard normal density function and $\Phi^{-1}\left[\cdot\right]$ is the inverse of the cumulative probability density function of the normal distribution. Likewise, when the spare capacity is set to S^*, the capacity cost per period is given by

$$C_£ = \sigma_{OR}(N+P)\varphi\left[\Phi^{-1}\left[\tfrac{P}{P+N}\right]\right] = \sigma_{OR}Y_{OR} \text{ with } Y_{OR} = (N+P)\varphi\left[\Phi^{-1}\left[\tfrac{P}{P+N}\right]\right]. \quad (19)$$

The square root law for inventory and bullwhip

In this section, we will show that under certain circumstances the ratio of capacity related costs between the decentralised and the centralised systems equals to the square root of the number of DC's in the decentralised system. Ratanachote & Disney (2008) have shown this quality for the case of AR(1) demands, arbitrary lead-times and n DC's in the decentralised system.

If $\theta = 0$ there is only a correlation of the demands over time. There is no correlation between the two retailer demands. In this particular case, we can see that the following order variances are equivalent,

$$\frac{\sigma^2_{1,DC,OR}}{\sigma^2_\varepsilon} = \frac{\sigma^2_{2,DC,OR}}{\sigma^2_\varepsilon} = \frac{\sigma^2_{DC,OR}}{\sigma^2_\varepsilon} = \frac{1}{4}\left(\frac{(1-\phi)}{(\phi-1)^2} + \frac{1}{1+\phi} + \frac{2}{1-\phi^2} + 8\phi(1-\phi^2)(1+\phi(1+\phi)(1-\phi^2))\right). \quad (20)$$

The ratio we are interested in is given by

$$\text{Cost ratio} = \frac{\text{Capacity cost of the decentralised system}}{\text{Capacity cost of the centralied system}} = \frac{C_{£,1,DC} + C_{£,2,DC}}{C_{£,DC}}$$
$$= \frac{\sqrt{\frac{\sigma^2_{1,DC,OR}}{\sigma^2_\varepsilon}}Y_{OR} + \sqrt{\frac{\sigma^2_{2,DC,OR}}{\sigma^2_\varepsilon}}Y_{OR}}{\sqrt{\frac{\sigma^2_{DC,OR}}{\sigma^2_\varepsilon}}Y_{OR}} = \sqrt{2} \quad . \quad (21)$$

The result shows that under these circumstances the cost ratio equals to the square root of two, the number of distribution centres in the decentralised system. This can be easily extended to n distribution centres and is a "square root law for bullwhip". The inventory cost has the similar character and we can also conclude that the "square root law for inventory", Maister (1976) also holds for the OUT policy with MMSE forecasting.

Numerical example

We now present a numerical example of our system. We assume that the unit costs of H=1, B=9, N=4 and P=6 are present at all locations in our distribution network. We use the ratio of costs between the decentralised and the centralised systems to show the impact of the correlations on the consolidation decision. The capacity and the inventory costs are considered separately and the result for each cost is shown in Fig. 5.

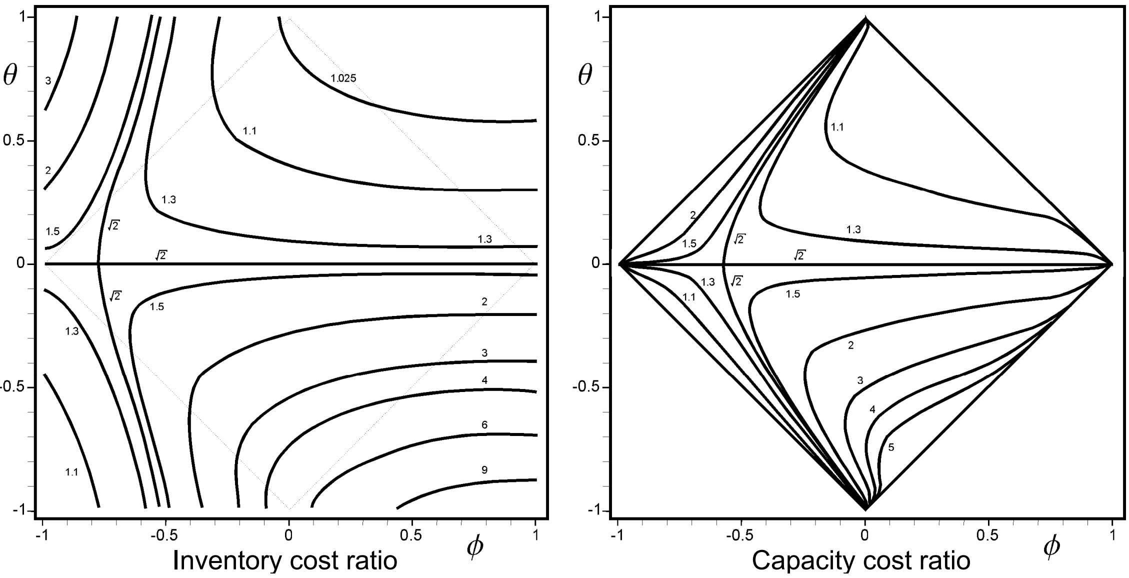

Figure 5 – Ratio of cost between the decentralised and the centralised systems

The result shows that the ratio is greater than unity in all the cases above. That is, the centralised system always has less cost than the decentralised system. When there is no correlation across retailers, $\theta = 0$, the ratio is $\sqrt{2}$ regardless the level of the correlation over time. This means the centralised system provides about 30% less cost than the decentralised system in this particular case. In general, the ratio is smaller when correlation across retailers increases towards +1. This is true for both capacity and inventory costs. However, this relationship is reversed when $\phi \leq\approx -0.5$.

Overall, for negative correlation across retailers, the ratio for both capacity cost and inventory cost increases rapidly when the correlation over time increases. The higher ratio means a greater percentage of cost saving by the network consolidation. In contrast, for positive correlation across retailers, the ratio decreases with correlation over time. We can also see in Fig. 5 that the inventory variance is finite, even for non-stationary demand outside the boundaries give by Eq. (2). However the order variances are infinite for non-stationary demand, hence we can only obtain the cost ratio inside the boundary given by Eq. 2, the diamond shape in Fig. 5.

Implications and Conclusions

We study the impact of demand correlation on a distribution network design. Two-level supply chain models for centralised and decentralised distribution systems have been analysed. The VAR(1) demand is used to represent the progression of and relationship in sets of time series. All participants, i.e. retailers and distribution centres, operate an Order-Up-To policy with the MMSE forecasting. We have derived analytical expressions to describe the dynamics and the variances of the net stock levels and order rates over time. The variance expressions can be embedded into computer programs that practitioners can use to support their decision making.

We use the ratio of costs between the decentralised and the centralised systems to show cost savings resulting from the consolidation of the distribution network. Our results encourage the consolidation of the distribution network as it produces less cost than the decentralised system. The magnitude of the impact of demand correlation depends on the level of correlation. We have also showed that "the square root law for bullwhip" holds in certain circumstances. Again, this law emphasises the benefit of centralised distribution systems. The work can be extended to the case of arbitrary lead-times. Furthermore adding a transportation cost into the objective function may be interesting.

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