A Hidden Markov Model for Seismocardiography

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Abstract—We propose a hidden Markov model approach for processing seismocardiograms. The seismocardiogram morphology is learned using the expectation-maximization algorithm, and the state of the heart at a given time instant is estimated by the Viterbi algorithm. From the obtained Viterbi sequence, it is then straightforward to estimate instantaneous heart rate, heart rate variability measures, and cardiac time intervals (the latter requiring a small number of manual annotations). As is shown in the conducted experimental study, the presented algorithm outperforms the state-of-the-art in seismocardiogram-based heart rate and heart rate variability estimation. Moreover, the isovolumic contraction time and the left ventricular ejection time are estimated with mean absolute errors of about 5 [ms] and 9 [ms], respectively. The proposed algorithm 1) can be applied to any set of inertial sensors; 2) does not require access to any additional sensor modalities; 3) does not make any assumptions on the seismocardiogram morphology; and 4) explicitly models sensor noise and beat-to-beat variations (both in amplitude and temporal scaling) in the seismocardiogram morphology. As such, it is well-suited for low-cost implementations using off-the-shelf inertial sensors and targeting e.g., at-home medical services.

Index Terms—Seismocardiogram, heart rate variability, cardiac time intervals, hidden Markov model.

I. INTRODUCTION

Cardiovascular disease is the leading global cause of death, accounting for about 17 million deaths per year and with estimated annual costs of $320 billion [1]. The most commonly employed modality for assessing cardiac function is the electrocardiogram (ECG), which is a recording of the time-dependent voltage measured by electrodes placed on the body. In each cardiac cycle, the ECG demonstrates a distinctive waveform whose largest deflections can be found in the QRS complex. Often, the beat-to-beat interval is estimated as the time interval between two successive R peaks, the so-called RR-interval. Later, we will also make use of the term NN-interval, referring to the RR-interval between two successive beats produced by sinus node depolarizations, i.e., excluding intervals affected by ectopic beats [2]. Although the ECG remains the standard diagnostic tool for cardiovascular disease, there is an increasing interest in alternative methods based on e.g., the photoplethysmogram (PPG) [3], the impedance cardiogram (ICG) [4], the phonocardiogram (PCG) [5], or the ballistocardiogram (BCG) [6]. In this study, our primary concern is the seismocardiogram (SCG)\(^1\) [8]. Although SCGs traditionally have been recorded using accelerometers, recent studies have also employed gyroscopes [9], [10]. The vibrations can be studied along the superior-inferior axis (head-to-foot), the sinistro-dexter axis (left-to-right), and the dorsoventral axis (back-to-front). Most of the translational vibrations are concentrated along the dorsoventral axis, and hence, this has been considered the most important axis in accelerometer-based SCGs\(^2\) [11]. SCGs offer several advantages over competing sensor modalities. These include the possibility of utilizing sensors embedded into personal devices such as smartphones (enabling a variety of at-home medical services) [12], the non-invasive nature of the method, and the low development cost of the sensors. The cost of a mass-produced inertial measurement unit (IMU), comprising three accelerometers and three gyroscopes, can be expected to be less than $1 [13]. To conclude, the SCG compares favorably to the ECG in terms of both availability and cost (but typically not in terms of accuracy), and can to some extent also be said to provide a different type of information (mechanical rather than electrical).

Normally, the data recorded from IMUs fixed to the sternum will reflect both respiratory motions and cardiac vibrations.

\(^1\)While some studies use the terms BCG and SCG somewhat interchangeably, the general agreement is that the BCG represents recoil forces of the body to blood ejected by the heart, while the SCG represents local vibrations caused by the heart’s contractions and relaxations [7].

\(^2\)On the other hand, the data collected in this study indicates that the rotational vibrations are primarily concentrated along the superior-inferior and sinistro-dexter axes.
This is exemplified in Fig. 1, which shows accelerometer and gyroscope measurements along the dorsoventral and sinistrodexter axes, respectively. The measurements were recorded over two respiratory cycles. As should be intuitive, the respiratory contribution in Fig. 1 (a) bears close resemblance to a sine wave. Moreover, each heartbeat in Fig. 1 can be seen to generate a characteristic wave pattern. While several attempts have been made to map different parts of this pattern to events in the cardiac cycle, this is still not a fully resolved issue [11], [14]. However, it should be noted that the characterizing features of the most important cardiac events in the SCG waveform (along the dorsoventral axis of the accelerometer measurements) has been widely agreed upon in the literature [15]. The morphology of the SCG signal is affected by several factors, including the sensor placement [11], the heart rate [12], the state of respiration [16], medical conditions of the subject [17], the age of the subject [18], and the occurrence of ectopic events [19]. Generally, SCGs will be more intricate and have more interpersonal variations than ECGs. As a result, heartbeat detectors adapted to ECGs tend to perform poorly when applied to SCGs [20]. Next, we will discuss three potential capabilities of seismocardiograms, namely, heart rate estimation, the assessment of heart rate variability (HRV), and the estimation of cardiac time intervals.

A. Heart Rate Estimation

One method for estimating heart rate from SCGs is to detect each heartbeat individually as a sampling instance around which large signal deflections are concentrated. For this purpose, different combinations of band-pass filters, wavelet transforms, and envelope functions have been used [21], [22]. The shortcoming of these methods is that they detect heartbeats solely based on vibration peaks and their time points, while discarding any additional information provided by the recurrent vibration pattern.

An alternative method is to take sliding windows of the signal and transform them, one by one, to the frequency domain. The average heart rate over each window is then estimated as the frequency that maximizes the power spectral density (PSD) of the corresponding windowed signal. The window should include at least two heartbeats, with previously employed window lengths chosen in the interval (20, 30) s [23], [24]. A similar method was used in [25], which attempted to find local maximums of the autocorrelation function of a windowed BCG. In [25], the window length was chosen as the smallest interval that was expected to fit at least two heartbeats. Unfortunately, these methods are only heuristically motivated, and it is not clear how their performance is affected when the vibrational pattern resulting from each heartbeat is stretched in time (events of this kind has previously been described in [14] and [26]).

B. Assessment of Heart Rate Variability

Variations in heart rate reflect complex interactions between the parasympathetic nervous system, which works to decrease the heart rate, and the sympathetic nervous system, which works to increase the heart rate. Even though the exact nature of these interactions remain the subject of active debate, measures of HRV have found use in a variety of clinical applications. For example, studies of HRV can be used for detecting increased mortality risks following a heart attack [27], for classifying sleep stages [28], and for assessing the risk of developing hypertension [29]. Common measures of HRV include time domain features such as standard deviations of the beat-to-beat interval, and frequency domain features such as ratios of spectral powers (of the beat-to-beat interval) at low and high frequencies [30]. Normally, a healthy heart exhibits a high HRV and utilizes a wide range of heart rates to cope with the stresses of daily life.

C. Time Interval Estimation

It is often of interest to estimate the time interval between two events in the cardiac cycle [31]. Cardiac time intervals have for example been used for the prognosis of patients recovering from myocardial infarction and for the detection of hypovolemia [32]. Many of the considered events can be identified from the SCG. These include the mitral valve closure (MC), the isovolumic moment (IM), the aortic valve opening (AO), and the aortic valve closure (AC)3 [15]. Typically, these events are detected as sampling instances where the accelerometer signal along the dorsoventral axis attains local maximums or minimums. Often, the data is preprocessed using a combination of band-pass filters, wavelet transforms, and envelope functions [15], [20], [33]–[36]. Commonly studied time intervals which can be estimated using only cardiac events detected from the SCG include the isovolumic contraction time (IVCT), defined as the time from the MC to AO, and the left ventricular ejection time (LVET), defined as the time from the AO to AC. Most methods for the identification of cardiac events from an SCG first detect fiducial points from an ECG and then make assumptions on the temporal distance between these fiducial points and specific cardiac events.

D. Contributions

We propose a hidden Markov model (HMM) approach to SCGs. The idea is to divide the cardiac vibration into a set of hidden states, which only are allowed to be traversed in a sequential manner. This enables the estimation of instantaneous heart rate, HRV indices, and cardiac time intervals, using information from the full waveform of cardiac vibrations. Performance evaluations are carried out using data from 67 subjects. The proposed method is shown to outperform the current state-of-the-art among envelope and spectral-based methods for heart rate estimation, while providing highly accurate estimates of standard HRV measures and cardiac time intervals.

Reproducible research: The data used in this paper is available at www.kth.se/profile/jwahlst together with a Matlab implementation of the proposed method.

3The set of cardiac events that can be detected will depend on the considered modality. For example, the ECG cannot be used to detect the AO.
II. MODEL AND ESTIMATION FRAMEWORK

The heartbeat vibrations are described using a HMM. Since the HMM depends on a number of unknown parameters, we learn these parameters using the Baum-Welch algorithm. Following this, the most likely sequence of states is found by the Viterbi algorithm. Last, we estimate the time point of each individual heartbeat. While similar models have been proposed for ECGs, these have typically relied on manual annotations during the training phase [37], [38], and the associated publications have seldom presented any performance characterizations of beat-to-beat interval estimates.

A. State-space Model

Consider a stochastic process \( \{x_k\}_{k=0}^K \). At each sampling instance \( k \), the variable \( x_k \) is in one of the “hidden” states \( \{1, 2, \ldots, N + 1\} \). Here, we make the interpretation

\[
\text{State } 1: \text{ No vibration due to heartbeats.} \\
\text{State } n + 1: \text{ The } n\text{th step in the wave pattern of vibrations due to a heartbeat.}
\]

Formulated in words, the wave pattern seen in the inertial measurements during a heartbeat is divided into \( N \) states. In addition, we use a dummy state to capture noise in between the vibrations caused by two heartbeats. For short, we shall write \( x_k = n \) as \( x_k^{[n]} \) for any \( k \) and \( n \).

The probabilities of different state transitions are given by \( A_{nm} \triangleq p(x_{k+1}^{[m]}|x_k^{[n]}) \), where \( p(\cdot|\cdot) \) denotes a conditional probability mass function (pmf). In every cardiac cycle, the Markov chain is expected to traverse each of the \( N + 1 \) states in a sequential manner. Therefore, it is assumed that the HMM is cyclic, so that \( A_{nm} = 0 \) for all \( n \) and \( m \) except \( n = m, n + 1 = m \), and \( [n \ m] = [N + 1 \ 1] \). By letting \( A_{nm} \) be nonzero for \( n > 1 \), we allow for some variability in the time length of different subpatterns in the SCG waveform.

To separate the cardiac vibrations from the respiratory motion, we first subtract the mean from each accelerimeter and gyroscope signal, and then high-pass filter the signals using a Butterworth filter of order three and with a cutoff frequency of 2 [Hz]. This will not only remove the larger part of the respiratory contribution, but also part of the cardiac signal (some spectral overlap is typically present [39]). The practice of high-pass filtering the raw SCG signal has been well established in the literature [21]–[23]. The high-pass filtered inertial measurements are used as input in the HMM, and are modeled as

\[
y_k = h(x_k^{[n]}) + \epsilon_k.
\]

Here, we could have \( y_k \triangleq [y_k^a \ y_k^g]^\top \), where \( y_k^a \) includes both three-dimensional accelerometer measurements \( y_k^a \) and three-dimensional gyroscope measurements \( y_k^g \). However, the presented framework can easily be modified to only use a subset of these measurements, e.g., only accelerometer measurements along the dorsoventral axis. For notational convenience, we define \( \mu_n \triangleq h(x_k^{[n]}) \).

The measurement noise \( \epsilon_k \) is assumed to be normally distributed and white. If the measurements are taken from a 6-degrees of freedom IMU, the covariance becomes

\[
R \triangleq \begin{bmatrix} \sigma_a^2 I_3 & 0_3 \\ 0_3 & \sigma_o^2 I_3 \end{bmatrix}
\]

where the parameters \( \sigma_a \) and \( \sigma_o \) need to take both sensor noise and variations in the SCG morphology into account. Here, \( 0_\ell \) and \( I_\ell \) denote \( \ell \times \ell \)-dimensional zero and identity matrices, respectively.

B. Baum-Welch Algorithm

The expectation-maximization (EM) algorithm is an iterative method for estimating the parameters of a statistical model that depends on some hidden states. When applied to HMMs, the algorithm is known as the Baum-Welch algorithm [40]. We use the Baum-Welch algorithm to find an approximation to the maximum likelihood (ML) estimate

\[
\hat{\lambda} = \arg \max_{\lambda} p(y_0, \ldots, y_L | \lambda).
\]

Here, \( \lambda \triangleq \{\{\pi_n\}, \{A_{nm}\}, \{\mu_n\}\} \) collects the model parameters, \( \pi_n \triangleq p(x_0^{[n]}) \) is the initial state probability, and \( p(\cdot|\cdot) \) denotes a conditional probability density function (pdf). For computational reasons, the Baum-Welch algorithm is only applied to a subset \( \{y_k\}_{k=0}^L \) of the total set of available measurements \( \{y_k\}_{k=0}^K \). Note that the measurement covariance matrix \( R \) is excluded from \( \lambda \) since both \( \sigma_a \) and \( \sigma_o \) are considered to be known.

Assuming that the sequential parameter estimates are \( \{\hat{\lambda}^{(i)}, \hat{\lambda}^{(2)}, \ldots\} \), the parameter estimates are updated until

\[
\frac{\log p(y_0, \ldots, y_L | \hat{\lambda}^{(i+1)}) - \log p(y_0, \ldots, y_L | \hat{\lambda}^{(i)})}{\log p(y_0, \ldots, y_L | \hat{\lambda}^{(i)})} < \eta
\]

where \( \log(\cdot) \) denotes the natural logarithm. For further details on the Baum-Welch algorithm, refer to [41] and references therein.

C. Initialization of the Baum-Welch Algorithm

As noted in [41], there is usually no obvious way of initializing the Baum-Welch recursions. In this paper, we set
To initialize \( \hat{\mu}_n \), we attempt to find intervals of length \( N \) over which the signal energy is locally maximized. The initialization is then made based on the signal values within these intervals. Hence, assuming that the measurements are taken from a 6-degrees of freedom IMU, we begin by computing the normalized energy function

\[
E_k = \frac{\sum_{\ell_1=k}^{k+N-1} \|y_{\ell_1}^\omega\|^2 + \sum_{\ell_2=k}^{L+N-1} \|y_{\ell_2}^\omega\|^2}{\sum_{\ell_1=0}^{L+N-1} \|y_{\ell_1}^\omega\|^2 + \sum_{\ell_2=0}^{L+N-N-1} \|y_{\ell_2}^\omega\|^2}
\]

where \( \| \cdot \| \) denotes the Euclidean norm. Following this, we identify all sampling instances where \( E_k = \max(E_{k-\delta k}, \ldots, E_{k+\delta k}) \). These sampling instances are denoted \( \{j_1, j_2, \ldots, j_J\} \). Here, \( \delta k \) is some chosen positive integer. Normally, \( \{(j_1, j_1+N-1), (j_2, j_2+N-1), \ldots, (j_J, j_J+N-1)\} \) will be sets of samples over which similar cardiac vibrations have been recorded. This is illustrated in Fig. 2 which shows high-pass filtered accelerometer measurements along the dorsoventral axis, with the aforementioned intervals shaded in grey. The measurements are taken from the same time interval as in Fig. 1. Finally, \( \hat{\mu}_n \) is initialized as \( \hat{\mu}_1^{(0)} = 0_{3,1} \) and

\[
\hat{\mu}_n^{(0)} = \frac{1}{J} \sum_{i=1}^{J} y_{j_i+n-2}
\]

for \( n = 2, \ldots, N+1 \). Here, \( 0_{\ell_1, \ell_2} \) denotes the zero matrix of dimension \( \ell_1 \times \ell_2 \).

Even if the specified HMM would have been the true underlying model of the studied data, the Baum-Welch algorithm would not have been guaranteed to converge to the globally optimal solution, i.e., the maximum likelihood estimate [42]. However, the specified initialization always resulted in parameter estimates that agreed with basic intuition.

### D. Viterbi Algorithm

Now, assume that the Baum-Welch algorithm has converged and that the final estimate is \( \hat{\lambda} \). We then use the Viterbi algorithm [41] to find the maximum a posteriori (MAP) estimate

\[
\{\hat{x}_k\} \equiv \arg \max_{x_0, \ldots, x_K} p(x_0, \ldots, x_K | y_0, \ldots, y_K, \hat{\lambda})
\]

of the complete sequence of states \( \{x_k\}_{k=0}^K \). Note that Viterbi sequence always is valid in the sense that \( p(\hat{x}_0, \ldots, \hat{x}_K | y_0, \ldots, y_K, \hat{\lambda}) > 0 \) (all its state transitions will be permissible). Hence, the sequence \( \{\hat{x}_k\}_{k=0}^K \) is guaranteed to unambiguously indicate when a heartbeat is considered to have been detected. A typical Viterbi sequence over five heartbeats is illustrated in Fig. 3. As indicated in Fig. 3, the number of self-transitions \( \{\hat{x}_k^{(n)}, \hat{x}_k^{(n+1)}\} \) for \( n > 1 \) over a single heartbeat is usually small in comparison to \( N \).

### E. Heart Rate Estimation

When the Viterbi sequence \( \{\hat{x}_k\}_{k=0}^K \) has been identified, we can go on to estimate the beat-to-beat interval. Intuitively, the beat-to-beat interval is equal to the horizontal distance between two adjacent tilted stripes in Fig. 3. Based on this observation, we first compute the average time point \( t_i^{(n)} \) at which \( \{\hat{x}_k\} \) is in state \( n \) during the \( i \)th cardiac cycle (only states \( n > 1 \) are considered). The computations are shown in Algorithm 1, where \( f_s \) denotes the sampling rate. Since the nominal beat-to-beat interval \( t_{i+1}^{(n)} - t_i^{(n)} \) may differ depending on which state \( n \) that is considered, the final estimates of the beat-to-beat intervals are computed by averaging over all states \( n = 2, \ldots, N+1 \). Hence, the estimated beat-to-beat interval and instantaneous heart rate as implied by heartbeats \( i \) and \( i+1 \) are

\[
\delta t_i \equiv t_{i+1} - t_i
\]

and

\[
HR_i \equiv \frac{1}{t_{i+1} - t_i},
\]

respectively, where the estimated time point of heartbeat \( i \) is

\[
t_i \equiv \frac{1}{N} \sum_{n=2}^{N+1} t_i^{(n)}.
\]

The sequences \( \{\delta t_i\} \) and \( \{HR_i\} \) can then be used to obtain estimates of standard HRV indices [2].
Algorithm 2: Summary of data inference.

1. **High-pass filtering**: Subtract the mean from each IMU signal, and then high-pass filter the signals using a Butterworth filter of order 3 and with a cutoff frequency of 2 [Hz]. The resulting signal is \( \{y_k\}_{k=0}^n \).
2. **Baum-Welch algorithm**: Approximate the ML estimate
   \[ \hat{\lambda} = \arg \max _{\lambda} p(y_0, \ldots, y_L | \lambda) \]
   of the model parameters \( \lambda \) by employing the following two steps:
   a) Compute the initial parameter estimate \( \hat{\lambda}(0) \) as described in Section II-C.
   b) Update the parameter estimate using the Baum-Welch algorithm until (5) is satisfied.
3. **Viterbi algorithm**: Find the MAP estimate
   \[ \{\hat{x}_k\} \triangleq \arg \max _{x_0, \ldots, x_K} p(x_0, \ldots, x_K | y_0, \ldots, y_K, \hat{\lambda}) \]
   of the complete sequence of states \( \{x_k\}_{k=0}^K \) by using the Viterbi algorithm.
4. **Heart rate and cardiac time interval estimation**: Estimate the time point \( t_i^{(n)} \) of state \( n \) during heartbeat \( i \) as outlined in Algorithm 1. Finally, (10) and (11) give the estimated beat-to-beat interval and instantaneous heart rate, and (12) gives the estimated cardiac time intervals.

### F. Time Interval Estimation

The estimation of cardiac time intervals can be performed in a manner analogous to that of the heart rate estimation. However, we first need to identify which states in the HMM that may decide that the MC can be expected to occur at state \( i \).

#### 1: Subtract the mean from each IMU signal, and then high-pass filter the signals using a Butterworth filter of order 3 and with a cutoff frequency of 2 [Hz]. The resulting signal is \( \{y_k\}_{k=0}^n \).

#### 2: Approximate the ML estimate
\[ \hat{\lambda} = \arg \max _{\lambda} p(y_0, \ldots, y_L | \lambda) \]

of the model parameters \( \lambda \) by employing the following two steps:

- a) Compute the initial parameter estimate \( \hat{\lambda}(0) \) as described in Section II-C.

- b) Update the parameter estimate using the Baum-Welch algorithm until (5) is satisfied.

#### 3: Find the MAP estimate
\[ \{\hat{x}_k\} \triangleq \arg \max _{x_0, \ldots, x_K} p(x_0, \ldots, x_K | y_0, \ldots, y_K, \hat{\lambda}) \]

of the complete sequence of states \( \{x_k\}_{k=0}^K \) by using the Viterbi algorithm.

#### 4: Estimate the time point \( t_i^{(n)} \) of state \( n \) during heartbeat \( i \) as outlined in Algorithm 1. Finally, (10) and (11) give the estimated beat-to-beat interval and instantaneous heart rate, and (12) gives the estimated cardiac time intervals.

### III. Experimental study

We begin this section by describing the two employed data sets. Then, the design parameters are presented, and the SCG morphology measured by a 6-degrees of freedom IMU is illustrated. Finally, we present the results of the estimation of heart rate, HRV measures, and cardiac time intervals.

#### A. Data Sets

Table I describes the studied data sets. All data was recorded with the subject in supine position. The first data set was recorded from an inertial array with 32 InvenSense MPU-9150 IMUs, placed about five centimeters to the left (as seen from the subject) of the lower sternum midpoint [43]. Details about the sensor array can be found in [44]. By averaging the data over the IMUs, the array was made to function as a single high-sensitive 6-degrees of freedom IMU. ECG data was recorded from a myPatch 3 Holter monitor. The subject was a healthy male, aged 26 years. The second data set consists of accelerometer measurements along the dorsoventral axis from 66 subjects. For further details on this data set, refer to the description of the SFU_GYM study in [15]. The R peaks in the ECG data were detected using the Pan-Tompkins algorithm [45], and the resulting beat-to-beat intervals were used as ground truth in Sections III-C and III-D.

The benefit of the first data set is that it can be used to compute a large number of realizations of standard HRV measures, while complying with the standard convention of computing the measures using NN-intervals obtained from five-minute data segments [2]. In addition, data from a 6-degrees of freedom IMU allows us to study both translational and rotational SCG morphology in all spatial directions. On the other hand, the second data set makes it possible to evaluate the presented algorithm on a diverse set of subjects with differing SCG characteristics. Moreover, it also includes manual annotations of the MC, IM, AO, and AC over all recorded data, which were used as ground truth in the evaluation of the cardiac time interval estimates in Section III-E. Since the manual annotations used in the training phase described in Section II-F were made based on parameter...
estimates $\hat{\mu}_n$ obtained from the first $L/f_s = 20 \,[s]$ of data from each subject, the accuracy of the cardiac time interval estimates was evaluated on the remaining $40 \,[s]$ of data.

B. Design Parameters and the SCG Morphology

The employed design parameters are detailed in Table II. Further, the number of states was chosen as $N = \left\lceil 0.6 f_s / \overline{HR_S} \right\rceil$, where $\lfloor \cdot \rfloor$ denotes the floor function and $\overline{HR_S}$ is the average heart rate as estimated by the spectral-based method presented in Appendix B. This means that the Markov process will be in the first state approximately $40 \,[\%]$ of the time. Since the Viterbi sequence is expected to pass through each of the $N + 1$ states during each individual cardiac cycle, there will always exist an upper bound, inversely proportional to $N$, on the estimated heart rate. However, a too small $N$ will increase the risk of overestimating the heart rate.

Fig. 4 illustrates the SCG morphology in a randomly chosen segment in data set 1. For clarity, the translational vibrations, measured by the accelerometers, and the rotational vibrations, measured by the gyroscopes, are displayed separately. The estimated parameters $\hat{\mu}_{n}^a$ and $\hat{\mu}_{n}^w$ are displayed in thick lines. Here, $\hat{\mu}_n = [ (\hat{\mu}_{n}^a)^\top \ (\hat{\mu}_{n}^w)^\top ]^\top$. Further, Fig. 4 also shows the corresponding measurements from 50 heartbeats in thin lines. In doing this, the state at a given sampling instance was estimated using the Viterbi algorithm. None of the displayed measurements were used in the estimation of $\mu_n$.

C. Heart Rate Estimation

The HMM-based method for heart rate estimation presented in Section II was evaluated on both data sets. To benchmark its performance, we also applied previously proposed envelope and spectral-based methods, described in Appendix A and Appendix B, respectively, to the same data sets. Data from the first data set was processed in five-minute segments. In the second data set, data from each individual subject was processed independently.

Table III displays error measures of the estimated instantaneous heart rate $HR$ and beat-to-beat interval $\delta t$ for both data sets. The errors are quantified in terms of their mean absolute error (MAE) and normalized MAE (nMAE), i.e., the MAE divided by the time-averaged ECG-derived value of the variable. In addition, standard deviations of the absolute errors, with or without normalization, are added to characterize the error distributions. As can be seen from Table III, the HMM yields a substantial performance gain over previously proposed methods. Although the performance seems to be better on the first data set than on the second, it should be noted that this primarily is due to a small number of individuals in the second data set who had irregular SCG waveforms or other anomalies. To exemplify this, we mention that for 41 of the 66 subjects in second data set, the HMM resulted in a MAE of the beat-to-beat interval that was lower than the corresponding MAE obtained on the first data set (4.28 $[ms]$). However, one should also take into account the comparatively low sampling rate of the ECG in the first data set. Since the detected R peaks always are chosen among the existing samples, the timing of an individual detected peak could have an absolute error up to 3.9 $[ms]$ on the first data set (the sampling rate was 128 $[Hz]$), even when assuming that the chosen sample is the one that is closest to the true peak. This uncertainty has most likely increased the apparent MAEs of the HMM on the first data set.

To further characterize the beat-to-beat intervals errors we display Bland-Altman and scatter plots for the HMM applied
Based on previous studies, the HMM also performed well in comparison to PPGs. For example, in [50], the PPG-based estimates of the beat-to-beat interval had a MAE of 6.88 [ms], while another study reported a MAE of 1.40 [ms] reported in [47]. In accordance with [47], Fig. 5 and associated measures did not consider any absolute beat-to-beat interval errors that were larger than 100 [ms] (these were considered to be false positives or missed beats). To illustrate the heavy-tailed nature of the beat-to-beat interval errors we visualize the empirical distribution functions (edfs) of the absolute heart rate errors, collected from both data sets, in Fig. 6.

It is encouraging to compare the accuracy of the HMM with results reported in studies using BCG signals. For example, the beat-to-beat interval has previously been estimated with nMAEs in the intervals (0.47, 1.06) [%] [6] (8 subjects; sampling rate: 200 [Hz]) and (0.49, 3.64) [%] [48] (1 subject; sampling rate: 200 [Hz]), while another study reported a MAE of 14.16 [ms] [49] (17 subjects; sampling rate: 128 [Hz]). Based on previous studies, the HMM also performed well in comparison to PPGs. For example, in [50], the PPG-based estimates of the beat-to-beat interval had a MAE of 6.88 [ms]. The study in [50] excluded false alarms and missed detections, and was conducted on 10 subjects with a sampling rate of 1 [kHz]. None of the studies reported any documented cardiac abnormality among the subjects.

Due to disparities in data quality and the absence of established reporting standards, it is often difficult to compare the accuracies reported in different SCG studies. Many studies will for example exclude estimates with too large errors when computing error measures [21], [47], while others only report errors of time-averaged heart rates [9], [10], [23].

The variables were estimated using five minutes (data set 1) or one minute (data set 2) of data.

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<td>Normalized absolute errors (mean ± standard deviation)</td>
</tr>
<tr>
<td>HR [%]</td>
</tr>
<tr>
<td>δt [%]</td>
</tr>
</tbody>
</table>
studied subjects. The clinical applicability of SCG-based HRV estimates will be highly dependent on the extent to which the reliability of individual estimates can be assessed, a topic which will need to be addressed in future studies.

E. Cardiac Time Intervals

Table V shows the MAEs of the estimated time points for the four cardiac events MC, IM, AO, and AC, and of the estimated time intervals IVCT and LVET. Fig. 8 displays Bland-Altman and scatter plots for the HMM estimates and the ground truth of the LVET. The estimates were obtained by applying the method described in Section II-F to the second data set. As can be seen, the MAE of the time points for the cardiac events is in the order of \(5 \text{[ms]}\). This can be compared to MAEs of the IM, AO, and AC at \(9 \text{[ms]}, 9 \text{[ms]},\) and \(6 \text{[ms]},\) respectively, for a previously presented SCG-based method conducted on 18 subjects with a sampling rate of \(1 \text{[kHz]}\) [52], and MAEs of the MC, AO, and AC at \(12 \text{[ms]}, 17 \text{[ms]},\) and \(9 \text{[ms]},\) respectively, for a PCG-based method conducted on 60 subjects, (51 of which suffered from congestive heart failure) with a sampling rate of \(10 \text{[kHz]}\) [35]. Similarly, the MAE of 8.51 \(\text{[ms]}\) for the LVET estimates compare well to previously published results. Specifically, [53] reported LVET MAEs of 29.9 \(\text{[ms]}, 14.4 \text{[ms]},\) and 11.5 \(\text{[ms]},\) when using the ICG, PCG, and PPG, respectively (17 subjects; PCG sampling rate: 44.1 \(\text{[kHz]}\)). While the PCG-based method presented in [54] resulted in MAEs of 11.39 \(\text{[ms]}\) and 17.51 \(\text{[ms]}\) for populations of 23 healthy subjects and 12 subjects suffering from different cardiovascular diseases, respectively (sampling rate: 44.1 \(\text{[kHz]}\)). However, as a word of caution, we note that there is some uncertainty both in the manual annotation, and in the relation between the cardiac events and the SCG waveform [55].

IV. SUMMARY

We have developed an HMM-based method for processing SCGs. Each state in the HMM represents a stage in the cardiac cycle. After estimating the cardiac state at each sampling instance, the obtained sequence of states can be used to estimate heart rate, HRV measures, and cardiac time intervals. The proposed algorithm was evaluated on data from 67 subjects. For the estimation of heart rate and HRV measures, comparisons were made against previously proposed envelope-based and spectral-based methods. The proposed HMM-based method demonstrated a superior performance in all respects, achieving a MAE of the beat-to-beat interval in the order of \(5 \text{[ms]}\). Estimated cardiac time intervals and time points for cardiac events were benchmarked against manual annotations. The resulting MAEs were about \(5 \text{[ms]}\) for the cardiac events, and \(5 \text{[ms]}\) and \(9 \text{[ms]}\) for the IVCT and LVET intervals, respectively. Continued studies of SCGs were motivated by favorable comparisons with results reported in studies using competing sensor modalities.

To assess the inter-annotation variability of the ground truth, we let an independent expert annotate the same events on data from two subjects. The mean absolute differences of the timings obtained from the two annotators were less than 0.4 \(\text{[ms]}\) for all events, and hence, they were well below the typical estimation error.
and then use machine-learning methods to obtain a general function which takes an estimated waveform as input and outputs the HMM states associated with the considered cardiac events. Moreover, it may also be possible to apply rule-based annotation methods similar to those presented in e.g., [52] and [59], to $\mu_n$. Either way, once the desired HMM states have been found, the time intervals can be estimated as described by (12).

Cardiac arrhythmia is a generic term for disturbances of the heart rhythm. The arrhythmias can in many instances be identified as sequences of heartbeats with unusual timing or ECG morphology. Often, features from the ECG waveform are used to classify each heartbeat into one of several arrhythmia classes [60]. Presumably, similar methods can be used to detect arrhythmias from SCGs [61], [62]. However, if each arrhythmia class gives rise to a unique SCG waveform, it may also be possible to model the occurrence of arrhythmic beats by using a hierarchical HMM (HHMM) [63]. Naturally, the top level would include one state for each arrhythmia class, and an additional resting state [38]. The substates of each state representing an arrhythmia class could then be modeled as in Section II. Similar models have previously been used in e.g., pedestrian activity classification [64].

Amplitudes and peak-to-peak amplitudes in the SCG waveform have been used to estimate cardiac output [12] and assess myocardial contractility [65]. Commonly, the estimation of amplitudes utilizes fiducial points detected from an ECG. However, since each state in the HMM presented in Section II corresponds to a specific stage in the cardiac cycle, it may be possible to estimate these amplitudes from measurements at sampling instances when the Viterbi sequence traverses a given state. For example, if the AO is determined to correspond to state $\hat{n}$, the AO amplitude could be estimated as the average of the measurements (in the considered directions) recorded while $\hat{x}_k = \hat{n}$ in the considered cardiac cycle. This would then eliminate the need to record a concurrent ECG.

**TABLE V**

<table>
<thead>
<tr>
<th>Timing/time interval [ms]</th>
<th>$t^{(E)}$</th>
<th>$\delta t^{(EE')}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC</td>
<td>5.49 ± 14.00</td>
<td>5.50 ± 9.03</td>
</tr>
<tr>
<td>IM</td>
<td>4.95 ± 13.91</td>
<td>5.36 ± 9.53</td>
</tr>
<tr>
<td>AO</td>
<td>5.24 ± 14.03</td>
<td>8.51 ± 15.98</td>
</tr>
<tr>
<td>AC</td>
<td>5.36 ± 9.53</td>
<td>8.51 ± 15.98</td>
</tr>
<tr>
<td>IVCT</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>LVET</td>
<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

**V. POSSIBLE EXTENSIONS**

Most likely, there are several extensions that can be made to the presented estimation framework to increase its accuracy and extend its area of application. Here, we will briefly discuss non-exponential state duration densities, automated cardiac time interval estimation, the detection of cardiac arrhythmia, and beat-to-beat estimation of amplitudes in the SCG waveform. The use of non-exponential state duration densities, i.e., hidden semi-Markov models (HSMMs) [41], can be expected to enable a more realistic model of the time spent in state 1. HSMMs have previously been used for ECG delineation in [37] and [38]. Moreover, the state duration distributions could be adjusted to account for the temporal correlation of the beat-to-beat intervals [56].

The method for estimation of cardiac time intervals outlined in Section II-F suffers from the fact it requires new manual annotations for each considered subject. While this may be acceptable in some applications [57], [58], automated methods are more likely to achieve widespread recognition [15]. One way to resolve this issue while still utilizing the proposed HMM could be to first make manual annotations on SCG waveforms $\mu_n$, obtained from a restricted number of subjects, and then use machine-learning methods to obtain a general function which takes an estimated waveform as input and outputs the HMM states associated with the considered cardiac events. Moreover, it may also be possible to apply rule-based annotation methods similar to those presented in e.g., [52] and [59], to $\mu_n$. Either way, once the desired HMM states have been found, the time intervals can be estimated as described by (12).

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**APPENDIX A**

This appendix describes the envelope-based heart rate estimation algorithm that was employed in Section III. Similar algorithms have previously been presented in [21], [22], and [66].

To begin with, all IMU signals are high-pass filtered using the same method as in step 1 in Algorithm 2. Following this, we compute the envelope of each IMU signal [67]. Assuming that the resulting signals are $\{\hat{y}_k^e\}_{k=0}^K$ and $\{\hat{y}_k^a\}_{k=0}^K$, the normalized energy function is computed as

$$
\hat{E}_k = \frac{\|\hat{y}_k^e\|^2}{\sum_{k=0}^K \|\hat{y}_k^e\|^2} + \frac{\|\hat{y}_k^a\|^2}{\sum_{k=0}^K \|\hat{y}_k^a\|^2}.
$$

(13)
for all \( k \). Now, heartbeats are detected as time instances \( k \) where \( \hat{E}_k = \max(\hat{E}_{k-\delta k}, \ldots, \hat{E}_{k+\delta k}) \). Finally, false alarms are removed by iterating through all heartbeats from start to finish and rejecting any heartbeat occurring less than 0.5 [s] after the previous heartbeat.

**APPENDIX B**

This appendix describes the spectral-based heart rate estimation algorithm that was employed in Section III. Similar algorithms have previously been presented in [9], [10], [23] and [24].

First, the envelope-based method is used to divide the data into segments. A segment is considered to start 0.25 [s] before a heartbeat (as detected by the envelope-based method) and end 0.5 [s] after the subsequent heartbeat. The data from each segment is processed independently. To start with, the accelerometer measurements (normalized to have zero mean) are high-pass filtered using a Butterworth filter of order \( 0 \) and with a cutoff frequency of 0.5 [Hz]. While gyroscopes also could be used, we will follow [23] and only employ accelerometers. Following this, the signals are reduced to scalar form by computing the signal which is the norm of the band-pass filtered accelerometer signals at each sampling instance. This has the advantage of creating a signal which is not dependent on the orientation in which the accelerometer was placed on the body. Last, the discrete-time Fourier transform of the signal is computed, and the heart rate is estimated as the frequency that, among the frequencies giving a local maximum of the PSD, is closest to the heart rate implied by the envelope-based method (i.e., the inverse of the estimated beat-to-beat interval of the two heartbeats).

**REFERENCES**


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