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Effective Hamiltonians for Discrete Time Crystals

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Abstract. We analyze the effective Hamiltonian for the $2T$ discrete time crystal ($2T$-DTC or DTC). This effective Hamiltonian is given by spin 1/2 many-body Hamiltonian which includes all-to-all coupling terms, thus being of infinite range. We describe the possible structure of the Hamiltonian, including many-body localized version which prevents thermalization. Finally, we show how the DTC melts when symmetry breaking terms are added.

INTRODUCTION

The domain of quantum time crystals can be divided into two realms: 1) original quantum time crystal, breaking a continuous time translation symmetry in closed systems, and 2) discrete time crystals, breaking a discrete time translation symmetry in driven system. The original idea of a quantum time crystal was proposed by Frank Wilczek [1]. Later, a number of studies have shown that the proposed breaking a time translation symmetry in equilibrium is ruled out by the no-go theorem [2]. The way to bypass this no-go theorem was proposed by the authors of the current paper in Ref. [3]. In the present work we focus on $2T$ discrete time crystals ($2T$-DTC or DTC). The idea of $2T$-DTC relies on the application of driver and interaction unitary operators in turns [4]. For the chain of spin 1/2 qubits, the DTC-behavior is expressed in subharmonic oscillation of magnetization which is stable to perturbations of the drive [4, 5]. In this work we consider the $2T$-DTC and derive the effective Hamiltonian which repeats dynamics of the system in stroboscopic time. The effective Hamiltonian may contain interactions that were initially absent in the system thus making it useful in the context of quantum simulations [6]. More details on the effective Hamiltonian that we consider in this work and its relation to original quantum time crystals can be found in [3]. We also note, that one could consider generalized spin models that are studied in high-energy physics, see for example [7].

EFFECTIVE HAMILTONIAN FOR THE DISCRETE TIME CRYSTAL

Let us consider the example of a discrete time crystal. In the discrete or Floquet case, the time translation invariance is broken for the stroboscopic time, and the DTC behavior is generated by the set of gates: 1) starting with all-down or all-up spin string, the spins are flipped by the unitary operator $\hat{U}_{\rm X} = \exp(-i\pi/2\sum_j \hat{X}_j) = \prod_j(-i)\hat{X}$; 2) after, the states evolves during the time τ with the Ising Hamiltonian $\hat{U}_{\rm Ising} = \exp(-i\tau\sum_{j=1}^{N} J_{j,j+1}\hat{Z}_j\hat{Z}_{j+1})$, completing the single step, given by the unitary $\hat{\mathscr{U}}_{\rm step} = \hat{U}_{\rm Ising}\hat{U}_{\rm X}$; 3) application of $\hat{\mathscr{U}}_{\rm step}$ continues for many stroboscopic periods $\hat{\mathscr{U}}_{\rm DTC} = \hat{\mathscr{U}}_{\rm step}\cdot\hat{\mathscr{U}}_{\rm step}\cdot\ldots$ The resulting stroboscopic dynamics then shows persistent oscillations of the total Z magnetization $M_z = \sum_{j=1}^{N}\hat{Z}_j/N$ with the period $2T$, where T is the period of the dive.

Now we consider the effective Hamiltonian, that reproduces the dynamics of the system at $t = 0, T, 2T, 3T\ldots$, namely $\hat{\mathscr{U}}_{\rm step} = \exp(-iT\hat{\mathscr{H}}_{\rm eff})$ where we defined the static Hamiltonian $\hat{\mathscr{H}}_{\rm eff}$ which we try to find. Formally, this can be done by taking the logarithm of the single step unitary as (here we assume that $T = 1$)

$$\hat{\mathscr{H}}_{\rm eff} = i\log(\hat{U}_{\rm Ising}\hat{U}_{\rm X}) = i\log(\hat{\mathscr{U}}_{\rm step}), \tag{1}$$

where $\hat{\mathscr{U}}_{\rm step}$ is a matrix which can in principle be obtained from the Baker-Campbell-Hausdorff relations. The latter however is cumbersome and cannot be resummed up to an infinite order, thus precluding an analytic answer.

What we can do instead is consider the fixed size system with Ising interaction fixed $\phi_{j,j+1} = J_{j,j+1}\tau = \phi$ to some value, and we can rescale it to unity. Then, the matrix exponentials and logarithm can be calculated numerically, thus providing the structure of the effective time crystal Hamiltonian. This corresponds to elements only occupying diagonal and antidiagonal for $\hat{\mathscr{H}}_{\rm eff}$, irrespective of the size of the system and boundary conditions.

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Given the freedom of choosing propagation time and interaction strength, we can choose units as $J\tau = 2\pi$. The unitary for a single DTC step then reads

$$\hat{\mathscr{U}}_{\text{step}} = \exp(-i2\pi \sum_{j=1}^{N} \hat{Z}_j \hat{Z}_{j+1}) \exp(-i\pi/2 \sum_{j=1}^{N} \hat{X}_j). \tag{2}$$

The Hamiltonian in this case reduces to the simple form, for $N = 4l$ ($l = 1, 2..$) it reads

$$\hat{\mathscr{H}}_{\text{eff},(N=4l)} = \begin{bmatrix} -\pi/2 & 0 & \dots & 0 & \pi/2 \\ 0 & -\pi/2 & \dots & \pi/2 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \pi/2 & \dots & -\pi/2 & 0 \\ \pi/2 & 0 & \dots & 0 & -\pi/2 \end{bmatrix}, \tag{3}$$

and for $N = 4l + 1$ ($l = 1, 2..$) it reads

$$\hat{\mathscr{H}}_{\text{eff},(N=4l+1)} = \begin{bmatrix} 0 & 0 & \dots & 0 & \pi/2 \\ 0 & 0 & \dots & \pi/2 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \pi/2 & \dots & 0 & 0 \\ \pi/2 & 0 & \dots & 0 & 0 \end{bmatrix}, \tag{4}$$

and similar simple X-shaped forms $\hat{\mathscr{H}}_{\text{eff},(N=4l+2)}$ and $\hat{\mathscr{H}}_{\text{eff},(N=4l+3)}$ with periodicity with respect to N. However, we find it to be highly symmetric, and not representative of the DTC phase.

Moreover, the disorder introduced in factors $J_{j,j+1}$ can also be treated numerically and does not change the Hamiltonian qualitatively (see below). At the same time, the deviation from ideal π pulses for $\hat{X}$ rotations generates extra terms, which do not anymore occupy only diagonal or antidiagonal. We considered the many-body localized system where the Ising couplings are drawn from the random distribution. We choose $J_{j,j+1} = [0.5, 1.5]$ and consider 500 configurations. We observe that for the case of a zero magnetic field in the Z-direction ($h^z = 0$), both distributions of the diagonal and antidiagonal elements are symmetric, meaning that $a_{kl} = a_{lk}$ and $d_{kk} = d_{(N-k+1)(N-k+1)}$, where a_{kl} and d_{kl} denote antidiagonal and diagonal matrix elements respectively, N is the size of the matrix. For the $h^z \neq 0$ the symmetry in antidiagonal is removed, and we conclude that it is not essential for DTC observation. On the contrary, symmetry at the diagonal persists. This can be explained if we consider only expansion of $\hat{Z}$ string of even length which originate from Ising interactions.

LEVEL STATISTICS, ROBUSTNESS, AND THERMALIZATION.

The sequence of gates shown discussed above represents an "ideal" setting for the DTC. The corresponding time crystal Hamiltonian $\hat{\mathscr{H}}_{\text{eff}}$ can be diagonalized and its energy spectrum obeys Poissonian statistics, thus identifying its integrable nature [8]. This prevents the system from heating up at late time, and allows for true DTC order.

In more generic case, the Ising unitary can be considered with longitudinal magnetic field,

$$\hat{U}_{\text{Ising}} = \exp(-i\tau \sum_{j=1}^{N} J_{j,j+1} \hat{Z}_j \hat{Z}_{j+1} + h_j^z \hat{Z}_j). \tag{5}$$

For non-zero h^z (we consider homogeneous setting for simplicity) the energy spectrum of Hamiltonian $\hat{\mathscr{H}}_{\text{eff}}$ still obeys Poissonian statistics, while the magnitude alters.

Finally, we consider the deviation for the ideal Hamiltonian of the X-shape and observe how the DTC order melts. This can done by adding the transverse field h^x to the Ising Hamiltonian unitary as $\hat{U}_{\text{Ising}} = \exp(-i\tau \sum_{j=1}^{N} J_{j,j+1} \hat{Z}_j \hat{Z}_{j+1} + h_j^z \hat{Z}_j + h_j^x \hat{X}_j)$. The degeneracies in the energy spectrum of $\hat{\mathscr{H}}_{\text{eff}}$ disappear in this case, and gap distribution becomes of the Wigner-Dyson type, signalling the ergodic nature. At the same time, for small h^x it allows for time crystalline behavior at the pre-thermal time scale, while ultimately melting away into featureless infinite temperature state.

To study the structure of Hamiltonian with deviations from ideal DTC phase let us plot matrix $\hat{\mathscr{H}}_{\text{eff}}$ as a density plot for different transverse fields. The main difference to all previously considered situations is the change of $\hat{\mathscr{H}}_{\text{eff}}$

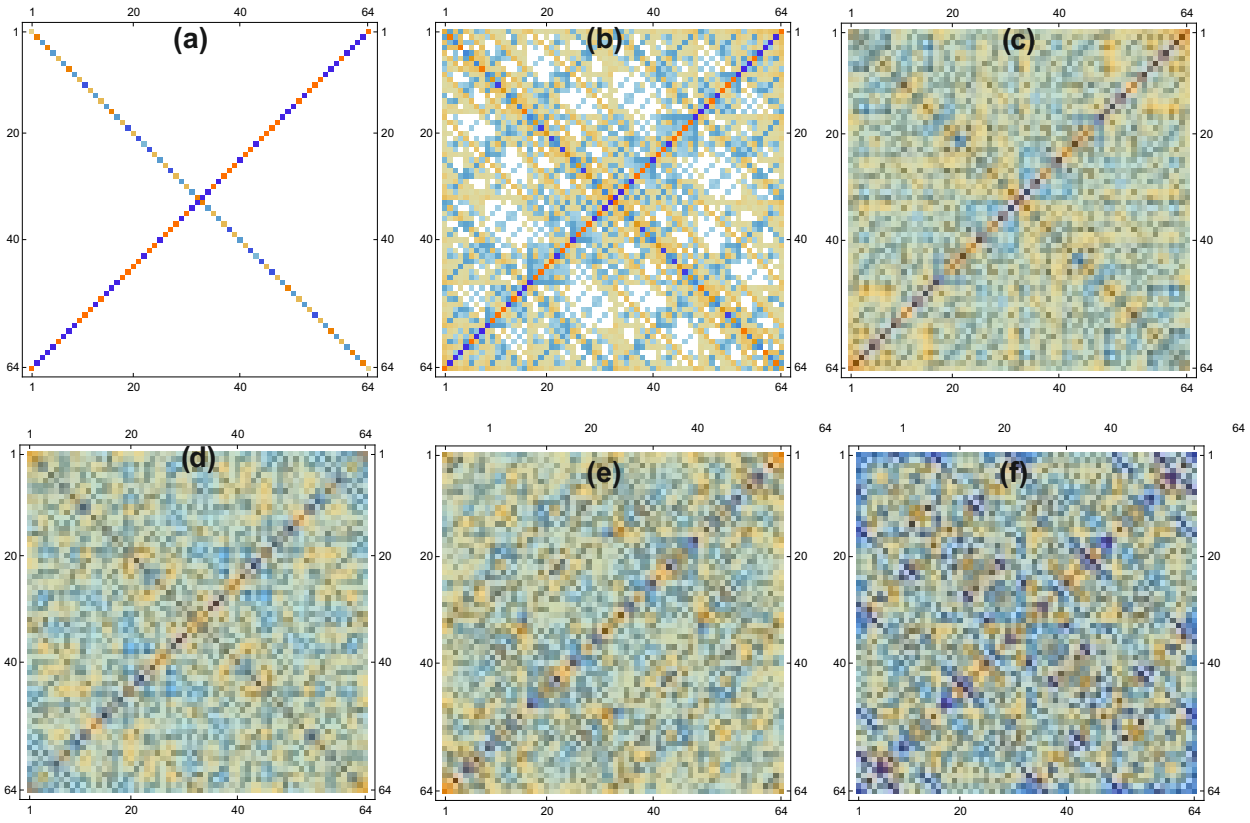

FIGURE 1. Density plots for matrix elements of $\hat{\mathscr{H}}_{\text{eff}}$ for different transverse fields h^x, $J\tau = 1$, $h^z = 0$, $N = 6$. Values of $h^x\tau$ for plots (a)-(e) are: 0, 0.01, 0.1, 0.2, 0.5, 0.8.

from the form the X-shape, and other matrix elements start to appear. The results are shown in Fig. 1. In the ideal case of $h^x = 0$ the pure x-shaped Hamiltonian is observed. Already at small increment of transverse magnetic field (b, $h^x\tau = 0.01$) off-diagonal elements appear, while x-shape remains dominant. The corresponding Fourier spectrum of the response of the magnetization M_z is dominated by single peak [not shown]. Same behavior is noticed for largely increased $h^x\tau$ being 0.1 (c) and 0.2 (d), with dynamics which can be contributed to DTC. For even larger $h^x\tau$ of 0.5 (e) and 0.8 (f) the DTC order is melted, and simultaneously the X-shaped structure disappears.

SUMMARY

We studied the matrix structure of effective Hamiltonian corresponding to the discrete time crystal. We have shown that the DTC phase can be associatied with the X-shape of the effective Hamiltonian and investigated how it changes in the presence of perturbations. It was shown that the many-body localized regime does not break the X-shape of the effective Hamiltonian.

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