Enhanced Energy Transfer to an Optomechanical Piston from Indistinguishable Photons

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Thought experiments involving gases and pistons, such as Maxwell’s demon and Gibbs’ mixing, are central to our understanding of thermodynamics. Here, we present a quantum thermodynamic thought experiment in which the energy transfer from two photonic gases to a piston membrane grows quadratically with the number of photons for indistinguishable gases, while it grows linearly for distinguishable gases. This signature of bosonic bunching may be observed in optomechanical experiments, highlighting the potential of these systems for the realization of thermodynamic thought experiments in the quantum realm.

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The concept of particle indistinguishability is deeply entwined in the history of both quantum mechanics and thermodynamics. The first remarkable example of the consequences of the difference between distinguishable and indistinguishable particles is found in Gibbs’ thought experiment [1] on the extraction of work from the mixing of gases. Subsequently, the indistinguishability of energy quanta played a central role in the development of quantum mechanics through Planck’s reconciliation of Wien’s law and the Rayleigh-Jeans limit of blackbody radiation. The indistinguishability of elementary particles, fermions and bosons, is now recognised as a fundamental principle, with diverse signatures such as the Pauli blockade [2] or the Hong-Ou-Mandel effect [3], which causes even noninteracting photons to leave beam splitters in pairs, i.e., to bunch.

The role of the statistics of indistinguishable quantum particles in thermodynamics has recently gathered renewed attention. For quantum generalizations of a Szilard engine, the extractable work is independent of whether the working substance is bosonic or fermionic [4], but bosonic bunching can enhance the conversion of information and work [5] and the performance of thermodynamic cycles [6,7].

Although any two fermions or bosons of the same type are intrinsically identical, in practice it is often possible to distinguish such particles via their internal states [8]. In the case of photons, for example, the distinguishability can be carried by a degree of freedom such as polarization that admits coherent superpositions. The distinguishability between two photons—one vertically polarized and the other in the state $|\theta\rangle = \cos \theta |V\rangle + \sin \theta |H\rangle$ with $|V\rangle$ and $|H\rangle$ referring to vertical and horizontal polarizations—can thus be varied continuously, with the photons partially distinguishable for $0 < \theta < \pi/2$.

The possibility of partially distinguishable quantum gases has provided a natural generalization to Gibbs mixing [9,10], with many implications for thermodynamics. For example, it is impossible to perfectly distinguish non-orthogonal quantum states without breaking the second law of thermodynamics [11], the accessible information in Gibbs mixing is limited by the Holevo bound [12], and the extractable work from mixing, defined as the ergotropy [13], can decrease with distinguishability [14].

In this Letter, we present a thought experiment that probes the interplay of distinguishability and particles statistics in quantum thermodynamics. Drawing inspiration from ground-breaking thought experiments involving gases performing work on a membrane attached to a movable piston [1,15,16], we consider the interaction between photon gases and a beam-splitter membrane, which gives the photons access to superpositions of spatial states located on either side of the membrane. We find that the photonic bunching results in striking consequences for how energy is transferred between light and the membrane. Namely, as a result of the Hong-Ou-Mandel effect, the energy transfer grows quadratically with the number of photons for indistinguishable gases, while it grows linearly for distinguishable gases.

The proposed thought experiment may be realized in multimode optomechanical systems in which a microscopic

![Image](image_url)

FIG. 1. Illustration of an optomechanics setup to generalize a variety of classic thermodynamic experiments involving gases and piston membranes. The classical gases are realized by photon gases on either side of a movable beam-splitter membrane.
membrane separates an optical cavity into two parts [17–22], thus highlighting a new avenue for quantum thermodynamic experiments. We argue here that such multimode optomechanical setups go beyond previous proposals in single-mode systems [23–26], by providing a platform both for studying quantum signatures of distinguishability and, more broadly, realizing thermodynamic thought experiments involving the interaction of gases with membranes.

A setup realizing this variety of quantum mechanical generalizations of pistonlike experiments is given by an optomechanical system comprised of a cavity with a membrane that behaves like a beam splitter and that divides the cavity symmetrically into a left and a right part, as sketched in Fig. 1. The photon dynamics resultant from the membrane can be modeled in terms of the Hamiltonian

\[ H_{BS} = \sum_{p=H,V} \frac{\lambda}{2}(R_p^L L_p + L_p^R R_p), \]

where \( \lambda \) is the intercavity coupling strength and the annihilation (creation) operators of both the horizontally, \( R_H^{(1)} \) and \( L_H^{(1)} \), and vertically, \( R_V^{(1)} \) and \( L_V^{(1)} \), polarized photons in the right and left halves of the cavity are explicitly modeled in order to study the effect of distinguishability [27,28]. The membrane has a motional degree of freedom (DOF), like a cantilever, and the interaction between the light field and the motional DOF is given by

\[ H_M = -g(N_L - N_R)X_M, \]

in terms of the total particle number in the left and right parts of the cavity (i.e., \( N_L = L_H^L L_H + L_V^L L_V \), \( N_R = R_H^R R_H + R_V^R R_V \)) and the displacement operator \( X_M \) of the membrane [29]. This optomechanical coupling will allow us to discuss the notion of energy (be it work or heat) transferred to the mechanical DOF in analogy to the extraction of work in the classical setting.

The full system Hamiltonian \( H \) is given by the sum of \( H_{BS} \) and \( H_I \) [defined in Eqs. (1) and (2)] and the non-interacting terms for the four photonic modes \( H_C = \omega(N_L + N_R) \) and single phonon mode \( H_M = \omega_M M^\dagger M \). The eigenfrequencies of both parts of the cavity and of the mechanical DOF are denoted by \( \omega \) and \( \omega_M \), respectively, and the annihilation (creation) operator \( M^{(1)} \) of the mechanical phonons is related to the displacement operator via \( X_M = x_{zp}(M + M^\dagger) \). The prefactor \( x_{zp} \) is the mechanical oscillator’s zero-point uncertainty \( x_{zp} = 1/\sqrt{2m\omega_M} \), with \( m \) the mass of the membrane.

To solve the system dynamics explicitly, despite the high-dimensional Hilbert space, it is helpful to consider the equations of motion for the observables of interest in the Heisenberg picture. The equation of motion for the displacement \( X_M \) of the mechanical DOF,

\[ \frac{d^2X_M}{dt^2} + 2\kappa_M \frac{dX_M}{dt} + \omega_M^2 X_M = \frac{g}{m}(\Delta N_H + \Delta N_V), \]

depends on the photonic mode imbalances \( \Delta N_p = L_H^1 L_p - R_H^1 R_p \) (for \( p = H \) and \( V \)) whose dynamics result from the equations of motion

\[ \frac{dL_p}{dt} = -i(\omega + gX_M - i\kappa)L_p - i\frac{\lambda}{2}R_p, \]

\[ \frac{dR_p}{dt} = -i(\omega - gX_M - i\kappa)R_p - i\frac{\lambda}{2}L_p. \]

In a thought experiment, the damping rates \( \kappa \) and \( \kappa_M \) of the cavity and mechanical modes can be assumed to vanish such that the number of photons in each gas is conserved and the piston membrane is frictionless. However, any experiment would be realized with finite damping.

Solving the coupled differential equations (3)–(5) exactly is prohibitively difficult, but a perturbative solution that describes the dynamics of selected observables for any given initial state can be constructed. Given that the force exerted by a single photon on the membrane is weak, the single photon coupling strength \( g x_{zp} \) is small compared to the intercavity coupling strength \( \lambda \). It is therefore appropriate to solve the dynamics perturbatively in \( g \). With the membrane cooled to cryogenic temperatures (mean phonon occupation number \( \bar{n}_{zh} \) of the order of 10) [30], the higher order contributions are negligible, as discussed in the Supplemental Material [31]. Thus, we focus here on the dynamics to first order; however, similar behavior is observed to higher orders [31].

The initial state of the left and right parts of the cavity and the membrane, \( \rho_L \otimes \rho_R \otimes \sigma_M \), is chosen in analogy to classical thermodynamic thought experiments, and hence the gases are taken to have the same number distribution [and therefore the same average photon number \( \langle N(0) \rangle = \text{tr}_L[N_L(0)\rho_L] = \text{tr}_R[N_R(0)\rho_R] \) and variance \( \delta N(0) \)]. The transition between distinguishable and indistinguishable photon gases can be explored by taking all photons in the left cavity to be in the polarization state \( |V \rangle \) and all photons in the right in \( |\theta \rangle \), where \( \theta \) can be varied continuously between 0 and \( \pi/2 \). For the membrane, an initial state with vanishing displacement, \( \text{tr}_M[X_M(0)\sigma_M] = 0 \), and vanishing momentum is inline with the classical thought experiments; for example, the membrane could be prepared in a thermal state.

The dynamics induced by \( H_{BS} \) entangles the mechanical and optical degrees of freedom, resulting in an energy transfer from the effective energy of the photons, \( (\omega - gX_M)N_L \) and \( (\omega + gX_M)N_R \), to the membrane. To first order in the interaction constant \( g \), the quantum mechanical average of the energy of the membrane is given by
functions (see Fig.4) that depend on the system parameters \( \Delta \) contribution to \( \delta \) number distribution), the initial fluctuations in photon conversely, for high temperature thermal gases (i.e., gases with the difference between these two cases scaling as \( \Delta = \gamma N(0) N(0) \) and \( \cos^2(\theta) \).

For any choice in the number distribution of the photon gases, the energy transfer to the membrane is larger for indistinguishable photons, with the difference between these two cases scaling as \( \Delta = \gamma N(0) N(0) \) and \( \cos^2(\theta) \).

The dependence of the energy transfer \( \Delta H_M(t) \) in Eq. (6) on the distinguishability parameter \( \theta \) is the opposite of Gibbs mixing where work extraction is possible for distinguishable gases but not for indistinguishable gases. The difference in behavior is perhaps unsurprising as the present mechanism does not rely on mixing. What seems striking is the scaling with particle number. Whereas the extractable work in the Gibbs [1], and indeed the Szilard [16] and Maxwell’s demon thought experiments [15], scales linearly with the particle number—i.e., it can be interpreted as “work per particle”—the present situation realizes a quadratic scaling, with a potentially strongly enhanced energy transfer to the membrane.

As we show in the following, this quantum mechanical enhancement of energy transfer, \( \Delta H_M \), between light and the mechanical DOF is a direct consequence of photon bunching as observed in the Hong-Ou-Mandel (HOM) effect [3,32]. To this end, it is instructive to inspect the two-mode second order correlation function [33]

\[
g_{LR}(t) = \frac{\langle N_L(t) N_R(t) \rangle}{\langle N_L(t) \rangle \langle N_R(t) \rangle}.
\]  

(7)

A vanishing value of \( g_{LR} \) indicates that a measurement would find all photons in one cavity, whereas large values of \( g_{LR} \) imply that approximately equal numbers would be found in both halves of the cavity. A small value of \( g_{LR} \) thus indicates bunching, whereas a large value indicates antibunching [31].

The dynamics of \( g_{LR} \) as the light field interacts with the beam-splitter membrane can readily be obtained to first order in \( g \). It is depicted in Fig. 2 in the absence of damping effects \( (\omega = \omega_M = 0) \) for perfectly distinguishable \( (\theta = \pi/2) \), perfectly indistinguishable \( (\theta = 0) \), and partially distinguishable \( (\theta = \pi/4) \) gases. In all three subfigures, corresponding to single photon, coherent, and thermal states of the light field, one can see that, for all times, distinguishable gases result in the largest values of \( g_{LR} \) and indistinguishable gases the smallest. Moreover, the time averaged correlation function [31]

\[
\langle g_{LR}(t) \rangle = \frac{1}{4} (\gamma + 3 - \cos^2(\theta)),
\]

(8)

where \( \gamma = 2 \) for thermal photons, \( \gamma = 1 - (1/n) \) for an \( n \) photon Fock state, and \( \gamma = 1 \) for coherent state photons, has the same \( \cos^2(\theta) \) dependence on distinguishability as the energy transfer to the membrane, Eq. (6). In other words, bunching is most pronounced for indistinguishable gases, as expected.

To understand heuristically how this bunching affects the membrane dynamics, it is instructive to consider the case of single photon gases as sketched in Fig. 3. For both distinguishable and indistinguishable photons, the (quanta) average displacement of the membrane will be zero at all times. However, the fluctuations in the position of the membrane, and therefore the energy of the membrane, will be greater for the case of indistinguishable photons because the probability for the membrane to be displaced to the left or right is double that for distinguishable photons. Moreover, the quadratic scaling of the energy transfer may be explained by the fact that the HOM effect is a pairwise interference effect. Since \( \langle N(0) \rangle \) photons in one gas can interfere with each of the \( \langle N(0) \rangle \) photons in the other gas, the number of pairs of photons that can interfere with one another scales as \( \langle N(0)^2 \rangle \), and this quadratic scaling carries over to the energy transfer.

While considering an initial thermal state for the photon gases realizes a close analogy with classical thermodynamics, including the process of pumping brings the
discussion closer to an experimentally realizable situation. If the cavity is driven on resonance in a pulsed fashion [34–38], with pulses that are shorter than the tunneling time \( \lambda \), the driving processes and tunneling processes occur on different timescales and can be considered independently. Accordingly, driving the left modes of the cavity with a short laser pulse of \( \theta \) polarized photons and the right modes with a short pulse of vertically polarized photons will generate the coherent states \( |\alpha, \theta \rangle \) and \( |\alpha, V \rangle \) in the respective halves of the cavity [39], leading again to an enhanced energy transfer to the membrane for indistinguishable photons as per Eq. (6) with \( \delta N = \langle N \rangle = |\alpha|^2 \).

In the limit in which the cavity damping is much faster than the membrane damping, as is the case in experimental settings such as in Refs. [21,22], the energy of the membrane, as shown in Fig. 4, tends to an approximately constant value on the timescale \( 1/\kappa \ll t \ll 1/\kappa_M \). In this limit, the energy transfer to the membrane after being driven by a single pair of pulses is

\[
\Delta H_{M}^{\alpha \beta / \kappa} = \mu |\alpha|^2 + \eta (|\alpha|^2 + |\alpha|^4 \cos^2(\theta)),
\]

where \( \eta = 1.2 \times 10^{-8} \text{ Hz} \) and \( \mu = 1.3 \times 10^{-18} \text{ Hz} \) for the experimental parameters listed in Fig. 4. For a pulse containing \( 6 \times 10^6 \) indistinguishable photons, the expected energy transfer to the membrane is on the order of 400 kHZ. This effect could be amplified by driving the cavity with a train of laser pulses, increasing the viability of experimentally observing the enhanced energy transfer to the membrane using currently available measurement protocols [30].

It is natural to ask whether this energy transfer to the piston membrane, \( \Delta H_{M} \), should be interpreted as heat or work. While the question of how to define work [40–43] and heat [44,45] in the quantum regime has been discussed extensively, in essence the distinction reduces to the extent to which the energy is “useful” energy as opposed to un-directed fluctuating energy. Since the quantum mechanical average of the mechanical displacement and momentum vanishes at all times, the energy transfer \( \Delta H_{M} \) is entirely given in terms of the fluctuations resulting from the entanglement between light fields and mechanical degrees of freedom [46]. In this vein, one might classify the energy transfer as heat rather than work.

However, the fact that the quantum mechanical average over displacement vanishes can be seen as a direct consequence of the system’s mirror symmetry (i.e., exchange of \( L_p \) and \( R_p \) and simultaneous replacement of \( X_M \) with \( -X_M \)). Given a symmetric initial state, this symmetry is preserved during the dynamics and necessarily needs to be satisfied in the final state. Nonetheless, this symmetry could be broken with a measurement of the photon number in the left or right part of the cavity. As indicated by the correlations depicted in Figs. 2 and 3, a suitable measurement will collapse the symmetric superposition and therefore is likely to find a pronounced imbalance of photons between the left and right corresponding to a substantial instantaneous displacement of the membrane. Indeed, the cross-correlation function

\[
\langle \Delta N(t) X_M(t) \rangle = \nu(t) \delta N + \zeta(t) [\langle N(0) \rangle + \langle N(0) \rangle^2 \cos^2(\theta)]
\]

between the photon number difference \( \Delta N = N_L - N_R \) and the displacement of the membrane, with \( \nu(t) \) and \( \zeta(t) \) oscillatory prefactors depending only the system parameters [31], features the same quadratic enhancement for indistinguishable photons as found for the energy transfer,
Eq. (6). This suggests that a reasonably simple Szilard-type extraction protocol [5], using auxiliary measurements on the light field, would allow one to find a predictable displacement of the membrane that increases with the indistinguishability of the photons in the cavity. The potential energy associated with this displacement is well defined and thus could plausibly be interpreted as a work output.

The bunching enhanced energy transfer to the piston membrane for indistinguishable photons draws a link between iconic thermodynamic experiments conceived by Gibbs, Maxwell, and Szilard, and a paradigmatic example of the impact of indistinguishability in quantum optics, the HOM effect. The optomechanical analysis further gives a flavor of the rich physics that can be explored by explicitly introducing polarization into optomechanical setups while introducing a new platform for quantum thermodynamic experiments. For example, a crucial difference between the present optomechanical setting and classical thermodynamical experiment is the inability of the photons to thermalize via mutual interactions. Interactions with dye molecules, on the other hand, are routinely used to mediate effective interactions between photons resulting in thermalization [47,48]. One may thus envision extensions of the presently discussed setup with thermalization rates as additional parameters, permitting a broad range of future directions. Other open questions include the variation of the initial state, optomechanical coupling regime, and coupling of the photons to the heat bath. Similarly to the present analysis, such settings can be discussed as a thought experiment or even realized in practice with optomechanical systems.

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