

Three Essays on Incentive Design

Submitted by Jordan James Large, to the University of Exeter as a thesis for
the degree of Doctor of Philosophy in Economics, April 2013.

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ACKNOWLEDGEMENTS

First and foremost I would like to thank my supervisor, Doctor Surajeet Chakravarty. Without his guidance and persistent help this thesis would not have been possible. I would also like to thank both Professor Tigran Melkonyan and Professor Robin Mason for their useful discussions that helped shape the workings of two of the chapters to follow. I am indebted to the ESRC for providing the financial support necessary to pursue this venture. I also offer sincere gratitude to my colleagues and seminar participants at the University of Exeter Business School whose insight and criticism was utterly invaluable. Finally, it gives me great pleasure in acknowledging the help of my loving family and friends. In particular Agathe, whose support and patience gave me the strength when I needed it most.

Three Essays on Incentive Design

Abstract

We present three distinct works on the subject of incentive design. The first focuses on a fundamental aspect of all principal-agent models, the participation constraint. We endogenise the constraint, allowing the agent to influence his outside option, albeit at some detriment to the project he is contracted to work upon. We compare the optimal contract to the literature on the supposed trade-off between risk and incentives. We find support for the Prendergast (2002) observation of a positive relationship between the two variables and offer an explanation through the use of said influence activities. The second contribution introduces another principal-agent framework for models with both adverse selection and moral hazard, with the novel inclusion of limited liability. Described in a target-setting environment, the findings are related to and support the use of tenure contracts in academia. This is justified by the fact that pooling equilibria maximise the value to the principal and fully separating equilibria are implemented with non-monotonic wage structures. Finally, in opposition to conventional literature, those of low type make rent gains over and above their reservation utility, while the high types break even. The final chapter studies organisational design and allocation of control. We offer conditions whereby firms would wish to integrate, or profit-share, with another, given varying degrees of control allocation. We show that integration comes at a lower cost for the decision-making firm when control is contractible as opposed to transferable. Also we show that the level of incompatibility between firms, unrelated to financial gain, can affect the integration decision.

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CHAPTER 1

Introduction and Literature

Most economic problems arise because of the choices facing economic agents. Whether those consumers are deciding which brand of orange juice to buy, how hard to work while sat in the office, or which asset in the stock market to invest in, they all are faced with a decision about how to allocate their particular resources. Consumers may believe they have complete control over the decisions that they make and are not influenced by the actions of others around them, however, in many economic interactions, when another party has a vested interest in the decisions made by the consumer, steps can be taken to manipulate the decision making process and hence the outcome, to the gain of this second party. If we consider most contracting procedures, ensuring that the employed party chooses the preferred choice of action is at the heart of the process, and it is how we cope with this that is broadly referred to in economics as the incentive problem. It must be said at this juncture, that our work here covers several topics, so to introduce and explain in detail all of them would be both excessive and unnecessary. We therefore provide only a brief overview of the literature, drawing attention to several key pieces of work that we deem pertinent.

The analysis of many incentive problems takes place under a principal-agent scenario. That is to say, an agent (employee) is required to perform some task(s) for a

principal (employer) in return for payment. The principal is considered as having the required assets to enter an industry but is normally lacking the correct expertise to facilitate such transactions themselves. This is where the human capital, in the form of an agent, enters the process. In order to fully utilise the skills of an agent, several creative payment schemes have been devised. These include bonuses, piece rates, or even vested shares. Such payments are used to mitigate the problems that arise through how to share both the risk and earnings. It is the construction of such methods that we study here, labelled as incentive design.

One must consider, firstly, why would the principal wish to manipulate an agent? Namely, because, there is some action that is preferable for the principal, that under normal conditions, i.e. without interference, would not be chosen. Furthermore, it can be made in the interest of both parties that this deviation actually occur. Secondly, all pertinent information concerning the agent may not be available to the principal. In which case, knowing how to and to what extent the agent needs to be manipulated may be difficult or in fact impossible to determine.

If for example, the agent were to ordinarily carry out the principal's preferred action choice, then there is no reason for him to intervene, and so there is no incentive problem. Similarly, if there were perfect information, then exactly how to and to what degree the agent needs to be manipulated would be known. In which case, the agent's actions could then be directly controlled through various measures of perhaps quite autocratic incentive and punishment structures, so that he would act in the way the principal prefers, as if of his own accord. Once again, the incentive problem disappears. Therefore, we focus on cases where there are differing objectives between two parties, coupled with information asymmetry. Under these cases, the relevance of incentive provision becomes apparent.

The standard assumption that follows is that each party acts in their own interests only and is extrinsically motivated, i.e. by financial reward¹. Perhaps, in some situations, such as a family business, where concern for others plays some role, this may not be entirely true. We do not consider any altruistic tendencies in our work here, the only possible reason for cooperation on any part from our agents, would be due to

¹For an interesting discussion on intrinsic motivation see Makris (2009).

a long-term relationship, that could be damaged by unnecessary rent-seeking. Before to call for the use of incentives, one must be sure of their effectiveness. The theory itself seems sound, when one is paid according to the output that they contribute to production, then increasing one's production results in a higher pay. If working harder can allow an agent to produce more, then higher incentives should result in more effort and hence more output for the principal. This was tested by Lazear in one of his many significant contributions on worker incentives. The author found significant effects upon productivity when firms moved from hourly wages to piece rates². The minimum level of ability will not change, but more able workers, who are aware of their greater productivity, are attracted by the new payment structure. This results in average output per worker rising. It was estimated that incentive effects were responsible for around half of the 44 per cent productivity increase observed by Lazear in his informative example. Given the importance of private incentives, especially when motivating behaviour, it is imperative that the use of incentives does not encourage the wrong sort of behaviour. See also Asch (1989) and Chevalier and Ellison (1995) for other interesting discussions.

The approach that is now conventionally used when solving such problems was formally set out by Ross (1973), before the more recent improvements of Mirrlees (1975). We follow the idea that principals wish to maximise the value of their projects subject to an agent carrying out the desired action and finding it in his interests to do so. The incentives of an agent are normally governed by two main constraints; Incentive Compatibility (*IC* hereafter) and Individual Rationality (also called Participation Constraint, *PC* hereafter). As we have described earlier, more often than not, the interests of both parties are not aligned. In which case, it is up to the principal to structure the agent's payment so that he is encouraged to select the 'correct' type of effort. The *IC* constraint specifies precisely that. It says that for all types of effort that an agent can exert, the level of effort that the principal desires is at least as valuable to the agent, in terms of his wage, as any other choice he may make. In which case, given that he is a profit maximiser, will choose this particular effort level.

However, this is not the whole story. Agents are not coerced into entering the contract and therefore it must be in their interests to do so. This is the relevance

²When a worker is paid an extra sum for each additional unit of production.

of the *PC* constraint. This says that entering into the contract must be at least as profitable for the agent as not doing so. Therefore its value to him must be greater than or equal to some reservation utility, or outside option. This is the value that he could obtain elsewhere, outside of the contract. This can be thought of as the utility from not working, or even the utility from working elsewhere, that is affected by the number and value of contract offers received. Conventional literature normalises this to zero, exogenously imposing some acceptable level of utility for the agent.

In the first chapter to follow, we allow a richer examination of this constraint, whereby the agent can influence the value of his outside option. The rationale behind such a model is related to the Sociology side of the literature, in particular, Milgrom and Roberts (1988) who analysed agents who may undertake influence activities rather than expend effort on the current project. The two agents can choose to focus on their own current work, or to establish their credentials for the new job or influence activities. In order to reduce the inefficiency arising from such influence activities, Milgrom and Roberts show that the firm or principal should adopt a promotion policy where the criteria should be more on the productive activities or incentives should be direct. Rasul and Sonderegger (2010) have studied the role of the outside option in principal-agent relationships. In an adverse selection framework where the ex ante outside option can be different from the ex post outside option for the agent, Rasul and Sonderegger show that this allows the principal an additional instrument for screening and therefore improves the efficiency of the contract.

When considering how to approach contract design, the type of information problem plays a key role in determining the solution. Private information itself can take two potential forms. Firstly, when the agent can make choices that are unobserved by the principal, known as moral hazard, sometimes referred to as hidden action. This has major implications for contract incentives when insuring an agent. A fully insured agent has no incentive to comply, as he is not risking any wage by his action choice. He is paid the same amount irrespective of the outcome. In which case, balancing the trade-off between insurance and incentives is an important medium against the effects of moral hazard. Arrow quite clearly highlights the incentive problem here in his early work and although an already well-known concept outside of our area, he introduced

the notion of moral hazard to our literature, arguing that it led to a market failure because some insurance markets would not emerge. “Once a machinery for making social choices from individual tastes is established, individuals will find it profitable, from a rational point of view, to misrepresent their tastes by their actions or, more usually, because some other individual will be made so much better off by the first individual’s misrepresentation that he could compensate the first individual in such a way that both are better off than if everyone really acted in direct accordance with his tastes”.

An enormous body of work followed, notably, Holmström (1979), who considered the role of imperfect information subject to moral hazard, deriving a necessary and sufficient condition for imperfect information to improve on contracts based on the payoff alone. This work was built upon by Baker whose study of incentive contracts focused on imperfect measurements of performance. Contracts in which the agent’s payoff is not based upon the principal’s objective will in general not provide first-best incentives. This is the case even when the agent is risk-neutral. The form of the optimal contract therefore depends upon the relationship between the performance measure used and the principal’s objective.

The second type of information problem is known as adverse selection, when an agent has access to some information that the principal does not. This is perhaps most eminently described in George Akerlof’s model of the American second hand car market. He explains how dealers of faulty second hand cars can under some trivial conditions, drive good dealers out of the market. A buyer’s inability to determine the quality of a second hand car, and a seller’s propensity to conceal it, hence the information asymmetry, are central in causing this problem to occur. These two information problems are rarely considered in tandem, as we do in our second chapter.

The method of obtaining solutions and the testing of their validity, was revolutionised by the seminal work of Grossman and Hart (1983). Previously, the principal chose an incentive scheme to maximise expected utility, subject to the agent’s utility being at a stationary point. This satisfied the first-order conditions, but ignored the second order, which meant that generally, such solutions were invalid, unless already at

an optimum³. Therefore, they separated the problem into two elements for the principal; first calculate the minimal cost of eliciting each action choice from the agent, then from this list choose the effort level (and hence the minimum wage cost) that would be preferential for the principal, from a profit maximising perspective. Although perhaps a tedious method, such a process undeniably leads to optimal solutions. Nonetheless, the regularity with which solutions were declared ‘optimal’ previously to this work and the rigour that has since followed, highlights the enormity of such a contribution.

As was alluded to earlier, risk preferences of an agent also play a key role in the solutions to such contracting problems. Typically, we assume that the principal is risk-neutral. This means that he has no preference over the level of risk in the contract, but essentially cares only for the expected value that it brings him. If we think of one principal venturing upon many projects at once, then in theory, he should have no concern of the individual risk they bring, as through diversification, they will cancel each other out. For an agent however, his livelihood will normally depend greatly on his performance in the project that he is employed to undertake. Therefore, we expect his approach to be a little more cautious in nature, and hence he is modelled as having risk-averse preferences. He would prefer to be insured on the project, and if he were to be encouraged to take any risk, would have to be given an extra payment, labelled a risk premium. If he were fully insured this would mean he receives a fixed payment, independent of outcome, and so has no incentive to exert any effort. A large amount of literature therefore, considers the case where agents also have risk-neutral preferences. Problems arise when a project is extremely unsuccessful and the decision of how to reward (or in this case punish), the agent is to be made. Ultimately, agents should not be punished for poor luck when the ‘correct’ effort has been exerted. In this line of thought, Innes (1990), building on the work of Sappington (1983), highlights the importance of limited liability. That is to say, “security-holders are not liable for firm losses over and above their investment”, looking at specifically when the agent’s effort choice is made before the state of nature is realised. Such a constraint therefore, should always be enforced, when adopting an agent with risk-neutral preferences, as we do in our later work. In essence, as long as agents are paid a non-negative wage, then we

³Mirrlees shows this to be the case.

have satisfied limited liability.

The informational asymmetry problems however are not solely confined to those inside the contract, but also to those on the outside. In some cases, when disputes arise over the actual value of the outcome, an individual or body, independent of the contract must be brought in to determine the true state. This can be problematic when we have issues of nonverifiability. The standard form of this occurs when both the principal and the agent know the outcome that has occurred but no third party, and in particular, no law enforcing court can verify it. It may lie in the interests of one or both parties to claim a different outcome, in the hope of achieving some renegotiated, preferential outcome, perhaps even at the expense of the other. This particular inefficiency has come to be known as the hold-up problem⁴. MacLeod and Malcomson (1989) consider the enforceability of employment contracts when performance cannot be verified and so piece rate contracts are not legally enforceable. Their results more closely resemble actual labour contracts than the standard previous principal-agent literature. Other significant and related pieces of work (See Prendergast and Topel (1996), Levin (2003) and MacLeod (2003)) have studied non-verifiability (and subjectivity) of outcomes and the resulting inefficiencies and conditions in which the inefficiencies can be reduced. Levin studies the design of self-enforced relational contracts, showing that optimal contracts can often take a simple form, but self-enforcement restricts promised compensation and affects incentive provision. Baker, Gibbons and Murphy (2002) also emphasise the importance of relational contracts within and between firms. These informal agreements, that rely upon the value of future relationships are shown to be affected by integration decisions, which has implications for joint ventures and networks et cetera.

This brings us onto the extremely vast areas of literature on incomplete contracts and renegotiation. Initially set out by the work of Williamson (1975) and Klein et al (1978), transaction cost economics argues that firms are important when contracts are incomplete. The risk of ex post renegotiation when an unspecified event occurs can lead to underinvestment in transaction-specific capital. Property rights theory, Grossman and Hart (1986) and Hart and Moore (1990) delves a little deeper, arguing that the

⁴Extensions of this idea could be considered, where one party, normally the agent, truly knows the outcome and the other does not precisely. Therefore, the ‘threat’ of renegotiation has to be applied in order to resolve the situation.

owner of an asset possesses residual control rights over that asset. They find optimal allocations of such residual control rights, in which case, not all activities should take place inside a single firm. Chung (1991) helps us to further understand when and how renegotiation can be a useful tool, especially when contracts are incomplete, which is often the case. He shows that when used to facilitate trade between two risk-neutral parties, first-best can be implemented by a simple contract, but when used to share risk, it is not generally possible to implement the first-best. When one party is risk-neutral however, it is possible to implement the first-best by assigning all the ex post decision rights to that party, which gives us an indication of the optimal bargaining procedure within our game. He argues in favour of renegotiation, as new information becomes available, with the possibility of explicitly stating each parties' ex post bargaining positions in the contract, as a contractual term. In this strand of literature, others have also analysed cooperative or selfish investment by the contracting parties, see De Meza and Lockwood (1998) and Che and Hausch (1999). Most papers in this framework have looked at the design of the renegotiation game and the efficiency implication it has on the cooperative or selfish investment made by the contracting parties.

We must at this juncture, mention the paper of Fudenberg and Tirole (1990). Before their work, the common method of dealing with the potential problem of renegotiation, was to assume that parties could commit to a contract under which renegotiation will not occur. However, they find an optimal contract which allows for renegotiation, specifically between the agent choosing his action and the consequences of this choice becoming known. The agent is actually offered a menu of contracts, in the interim renegotiation period, and according to his sunk effort, chooses to accept more or less risk, where it is preferential for a high effort worker to choose a risky one. Although the agent cannot be induced to take a high level of effort with certainty (as the principal would renegotiate to shelter him from risk), this optimal contract may give the agent a positive rent, in contrast to that without renegotiation. A later paper that expands upon this topic is Maskin and Moore (1999), who characterise choice rules that can be implemented, precisely when agents cannot commit not to renegotiate.

The most crucial work when considering such bargaining games however, must be Binmore, Rubinstein and Wolinsky (1986) that considers strategic models of alternating

offers, with differing incentives to reach agreement; time preference and risk of breakdown of negotiation respectively. They find that, when the motivation to reach agreement is made negligible, the unique perfect equilibrium approaches the Nash bargaining solution. This proves useful when determining the outcome of any (and in particular our own to follow later) renegotiation process. We now begin our contribution to the literature with the first of our three chapters.

Influence Activities and the Outside Option

2.1 Introduction

It is a well known result in moral hazard analysis that providing incentives to a risk-averse agent is costly. If it is in the principal's interest that the agent exerts high effort, then the principal has to choose a contract such that the agent will find it in his interest to exert the high effort. The agent will find such a contract acceptable if his expected payoff from entering the contract is higher than his outside option. In this paper we extend the standard moral hazard analysis, where the agent can influence his outside option once he signs the contract. We find that the ability of the agent to influence the outside option at the cost of productive effort for the principal's project exacerbates the agency problem, but it also provides the agent an instrument to insure himself. The latter allows the principal to provide more incentives to the agent compared to the case when the agent cannot influence his outside option. In addition, our analysis here can provide a possible reasoning for the Prendergast (2002) observation that higher bonuses are observed in riskier environments.

In a number of specialist professions there is only a small pool of potential employees. In such professions employees may find it beneficial to have good connections

with others in their profession and increase their likeability outside of the work place and subsequently increase their employability. Employees can spend time at industry workshops or conferences, invest time and resources in skills which are more easily observable to outsiders but may not be as useful to the current task or simply socialising with other potential employers. This may result in these employees being well regarded in the profession, and as a result such well connected employees may have a good outside option or in other words may find alternative employment quite easily. So if an employee performs poorly at his current employment but is well regarded by the competitors and peers out of office, he may use the threat of leaving for alternative employment to renegotiate his wage. This ability to influence the outside option and use it as a threat of alternative employment provides the employee with opportunity to extract additional rent from the employer. This therefore provides incentive to the employee to improve opinions of outsiders, perhaps at some cost to his own current work. We model such an employee who has an incentive to work on his outside option and analyse the implication of this on the nature of the contract signed between the principal and the agent.

The root of our particular problem focuses on an important part of principal-agent analysis that is, the role played by the outside option in contract negotiations. The contract which the principal offers should ensure that the expected payoff is at least as much as the outside option or the next best payoff available from another alternative. The risk-averse agent, in our analysis, has a fixed total amount of effort he can exert. From this fixed amount of effort the agent can choose how much he wants to exert on the project he has been contracted to work on and how much to exert on activities that affect his outside option. We assume that these actions are entirely separate from his normal work and as such are not valued by the current employer. While this opportunity to influence the outside option may exacerbate the incentive problem, there is a secondary effect that the ability to influence the outside option may provide an insurance to the agent from the random project output. We study optimal contracts given this particular problem.

Once the output and the outside options have been realised, the agent can renegotiate his compensation. Here the contract will be renegotiated in cases where there

is conflicting evidence from the output and outside option. While the output provides incomplete information about the effort of agent, the outside option provides information about how good the person is in the job. Note that the latter does not depend on how much the agent produces in the project he is working on. The outside option can however, be driven by the agent's reputation. High levels of each measure, output and outside option, indicate that it is likely the agent is hard working and good. Correspondingly, low levels of both measures indicate likely low effort levels and a poor quality agent. So for the contracting parties the states in which both the project value and the outside option are high or both low, seem congruent and they agree on the outcome, in particular about the payment of the agent. However, when there is a discrepancy in the two 'measures', so under a high project value and a low outside option or vice versa, it becomes more problematic to assess the worthiness of the agent. When either of these states arise, which they do with positive probability, the parties renegotiate the payment.

We show that the agent's ability to influence the outside option will lead to higher wage costs than the standard moral hazard problem. When the outside option influenced by the agent's activities is random, the moral hazard problem increases and the contract is such that the wage payments exceed the outside options. When the outside option is influenced deterministically by influence activities, the optimal contract consists of a bonus for producing the high output which is much higher than the case when the outside option is random. Thus the principal has to pay more to induce high effort. There are two effects here worth noting: one, given the incentive to divert effort to increase the outside option the agent has to be paid more and second, by increasing his outside option the agent insures against the random output and hence his wage. Therefore the contract offered can provide higher incentives and there is less risk sharing between the principal and the agent.

We next discuss the literature, after which we introduce the model and present our findings. We then discuss the implications of the risk and incentives trade-off and we conclude.

2.2 Risk and Incentives Trade-Off in the Literature

In most cases agents are modelled as risk-averse. Therefore, as the environment becomes more and more uncertain, if payments are linked to the outcome, then an agent's compensation too becomes more and more variable. In order to encourage an agent to accept such a risky payment he must therefore be offered what is known as a risk premium, some extra payment in compensation for accepting such a risk. In which case, it is in the interests of the principal to insure the agent, thereby protecting him from any undue risk he may be subject to. This trade-off was accepted as the standard theory within incentive design. Namely, that in areas of high risk, we expect to see low levels of incentives, i.e. high levels of insurance within the contract. The first to put such a theory to the test was Prendergast (2002), who noticed in fact the opposite was being used in practice. Namely, that whenever the environment was considered more risky, agents were highly incentivised and offered payments that were linked very closely to output. A summary of his suggested explanations in a variety of situations follows. The author first attempted to explain this unexpected empirical positive relationship with a theory based around delegation. According to Prendergast in a more uncertain environment, a principal is not sure what actions the agent he employs should take to ensure the best possible outcome. Therefore, he delegates action choices to the agent and monitors his performance by the outcome of the project. Given that he monitors outputs and not inputs, the agent's payment is therefore more closely linked to his output, hence higher incentives.

In many situations, firms have a fairly good idea of what exactly an employee should be doing. Therefore, how should they proceed? By instructing the agent of his duties, then heavily monitoring his behaviour, firms can feel confident in the fact that private and social benefits should be aligned. In which case, as long as his effort is directed as the firms decrees, he cannot be held responsible for any results that do not favour the firm. However, in the likely absence of an effective mechanism for revealing this information, firms must respond by changing the payment structure. This was summed up rather nicely by Prendergast, "the more uncertainty there is in the environment, the more important it is to induce the agent to choose the correct activity rather than assigning him one, which can only be done by basing pay on output."

Introducing the idea of subjective evaluation, it becomes quite obvious that supervisors are able to and often quite willing to distort their reports on workers based on their personal preferences. There is compelling evidence that favouritism, i.e. positive performance appraisals independent of performance, are widespread. Furthermore, that supervisors are unwilling to assign their subordinates low performance ratings, known as "leniency bias". It has also been shown that such distortions are also much more commonplace when pay is tied to performance, i.e. when such evaluations have a real bearing on the compensation levels that agents receive. If we introduce risk into the environment, then the value of the supervisor's report falls, as the information is much more noisy. In which case, we should expect to see pay tied much more closely to output. Conversely, where there is little risk, a truthful report is extremely valuable and so we avoid the use of incentive pay, thereby protecting the sympathetic supervisor. Either way, this logic coincides with a positive relationship between risk and incentives.

Additionally, we must consider the impact of endogenous monitoring and investigations. It had been put forward that costless signals about the effort of the agent were readily available and the principal always monitored the performance of the agent. This however, is not too realistic, monitoring itself is sporadic and normally in response to some complaint made about the performance of the agent and hence endogenous. When the environment is more noisy, the link is even less clear, and so the agent believes he will escape unpunished more often. Furthermore, investigation would then normally only follow if performance had been poor. The agent would then suffer less from a damning investigation, because it is clear that the environment itself is risky. The response to this on the part of the principal should be to place greater incentives when there is greater risk, therefore mitigating such an effect.

Rasul and Sonderegger (2010) have studied the role of the outside option in principal-agent relationships. In an adverse selection framework where the ex ante outside option can be different from the ex post outside option for the agent, Rasul and Sonderegger show that this allows the principal an additional instrument for screening and therefore improves the efficiency of the contract. While we too study the outside option, unlike Rasul and Sonderegger, we focus on moral hazard and a risk-averse agent. This allows us to analyse the impact of the outside option on incentive provision and risk sharing.

Milgrom and Roberts (1988) have analysed a similar source of transaction cost in employment relationships where agents can undertake influence activities rather than expend effort on the current project. The two agents can choose to focus on their own current work, or to establish their credentials for the new job (influence activities). Establishing credentials is purely self-interested behaviour, but is rational up to a point, as it can be used to distinguish between those who are better qualified for the upcoming promotion. Therefore, the first-best involves some level of establishing credentials, however, typically agents will have the incentive to spend too much time on these other credential creating activities. In order to reduce the inefficiency arising from such influence activities, Milgrom and Roberts show that the firm or principal should adopt a promotion policy where the criteria should be more on the productive activities or incentives should be direct.

Finally, our work can be related to the incomplete contract and the non-verifiability of output literature. When the output is realised there are two signals, output and outside option, which are publicly observable. When the two signals indicate opposite views about the agent then the contract gets renegotiated. Some of the papers which have looked into the implications of contract renegotiations are Hart and Moore (1990) and MacLeod and Malcomson (1993). Most papers in this framework have looked at the design of the renegotiation game and the efficiency implication it has on the cooperative or selfish investment made by the contracting parties. Here our focus is not on the design of renegotiation, but on the size of the incentive/bonus provided in the contract. Secondly, the cooperative effort the agent puts in, in our model can be diverted to self serving effort to increase the agent's outside option.

Another class of models (See Baker (1992), Prendergast and Topel (1996), Levin (2003) and MacLeod (2003)) have studied non-verifiability (and subjectivity) of outcomes and the resulting inefficiencies and conditions in which the inefficiencies can be reduced. Here we just assume that conflicting signals result in a bargaining game where the outcome is determined by the outside option. So in our framework the mechanism through which the agent manipulates the outcome to his advantage is quite different.

A further consideration under incentive design, that we discuss briefly in this chapter was introduced by Gibbons and Murphy (1992), where agents have career concerns.

This means that they have concerns about the effect of current performance on future compensation. They break incentives up into two parts, implicit incentives, that are driven by career concerns and explicit incentives from the compensation contract. The optimal compensation contract therefore optimises total incentives. The implication of this finding is that explicit incentives should be strongest for those closest to retirement because career concerns are weakest for these workers. Our work offers a contradictory suggestion but for the same reasons, essentially that incentives to deviate¹ are greatest early on in a career, hence more incentives are needed.

Our contribution to the above work is another plausible explanation for the relationship between risk and incentives. We look at a different framework to study the supposed trade-off when the agent may influence his outside option and this can work as an insurance for the risk he faces within the contract. Furthermore when there is scope for the agent to more dramatically influence his outside option, which we present as a situation in which there is more risk, we observe higher incentives. We therefore conclude that our work is in support of the Prendergast hypotheses explaining the positive relationship between risk and incentives.

2.3 Model

We consider a risk neutral principal and a risk averse agent. The principal employs an agent to carry out a project that has two possible outcomes, good \bar{q} , which he prefers to the other, bad \underline{q} . The probability of a good outcome depends on the level of costly effort, e , exerted by the agent. Unobservable effort can take one of two levels, high $e = \bar{e}$, or low $e = \underline{e}$. Each individual effort level $e \in [0, 1]$. Only exerting high effort incurs a cost $C(\bar{e}) = c$, whereas $C(\underline{e}) = 0$. Given that the principal prefers a high outcome, he would like the agent to exert a high level of effort, as the probability of a good outcome is increasing in effort. More formally, $\Pr(q = \bar{q} \mid e = \bar{e}) = p_1$ and $\Pr(q = \bar{q} \mid e = \underline{e}) = p_0$, with $p_1 > p_0$.

Simultaneous to choosing e , the agent also chooses another action, n , which like Milgrom and Roberts (1988), we call influence activities. Influence activity n affects the

¹We explain what we mean by such a term in the work to follow.

value of the agent's outside option \bar{U} . The chosen value of action $n \in [0, 1]$ can be high \bar{n} , or low \underline{n} . The outside option, \bar{U} , is stochastic and can take two values, $\bar{U}_H > \bar{U}_L$, and is increasing in n .² Formally, $\Pr(\bar{U} = \bar{U}_H \mid n = \bar{n}) = \pi_1$ and $\Pr(\bar{U} = \bar{U}_H \mid n = \underline{n}) = \pi_0$, with $\pi_1 > \pi_0$. The realised outside option is public information. As mentioned earlier in the introduction, the situation we have in mind for our analysis is where the number of potential agents is small. In such a case the potential employers may have the relevant information about the small pool of agents/employees.³

We assume the agent has a binding limit on the amount of time available to him. This implies the agent has to choose between either working hard, \bar{e} , or exerting a high level of influence activities, \bar{n} . All other combinations of e and n are permissible ((\bar{e}, \underline{n}) , (\underline{e}, \bar{n}) and $(\underline{e}, \underline{n})$). Although exerting effort is costly, influence activities are not. Therefore, putting high effort into the principal's project is more costly for the agent than choosing a high level of influence activity. This we believe is reasonable since working on the project causes more disutility to the agent than networking/socialising to increase the agent's employability. As long as this is true our results hold. Therefore, the agent has one of two strategies and focuses either on high effort (and therefore low influence activities), or high influence activities (and therefore low effort)⁴.

Given that the participation constraint must be satisfied ex ante, the lowest possible wage that can be set is equal to the agent's outside option. Therefore, the outside option can be thought of as some lower bound and so the benefit of these influence activities quickly becomes apparent. The influence activities allow the agent to increase the expected lower bound for his wage, adding an insurance effect to his wage. If the agent chooses not to engage in influence activities and focus solely on the project then he exposes himself to the risk of the contract.

The agent is paid a monetary value m by the principal and his utility is, separable

²Note that while the agent can choose to exert effort in the project or on his outside option (influence activities) unlike Holmström and Milgrom (1991), only the former activity increases the benefit of the principal.

³Here we maintain the assumption in a number of papers where contract terms get renegotiated, that the renegotiation occurs under symmetric information (Hart and Moore (1988), Fudenberg and Tirole (1990)).

⁴ $e + n = 1$, with $e, n \in [0, 1]$. Assuming a binding time constraint allows us to keep our analysis simple, by focusing on one choice variable of the agent. This also helps us to keep our analysis distinct from multi-tasking problems as the second effort level does not affect the principal's output. This is reinforced by the fact that higher influence activities are feasible, costless and beneficial.

between money and effort, $U = u(m) - C(e)$, with $u' > 0$ and $u'' < 0$. Let $h(u)$ be the inverse function of u (hence $h' > 0$ and $h'' > 0$). $S(q)$ is the value of the project to the principal. To simplify notation we now write $S(\bar{q}) = \bar{S}$ and $S(\underline{q}) = \underline{S}$. The principal wishes to maximise the difference between his expected revenue and the expected wage of the agent. There are four possible realisations of the states of the world that are $((\bar{q}, \bar{U}_H), (\bar{q}, \bar{U}_L), (\underline{q}, \bar{U}_H)$ and $(\underline{q}, \bar{U}_L))$. A wage \bar{m} is paid when (\bar{q}, \bar{U}_H) is observed, $\bar{\bar{m}}$ for (\bar{q}, \bar{U}_L) , $\underline{\underline{m}}$ for $(\underline{q}, \bar{U}_H)$ and \underline{m} is paid when $(\underline{q}, \bar{U}_L)$ is observed. For the agent, entering into a contract with the principal must be preferred to the expected outside option at the time of signing the contract, which in this case is $\pi_1 \bar{U}_H + (1 - \pi_1) \bar{U}_L$ ⁵. The real innovation comes from endogenising this latter constraint, permitting the agent to influence his outside option, an otherwise constant variable.

We present the time line of actions taken and events:

1. At date 1, the principal and the agent write a contract.
2. After the contract is signed, the agent chooses effort, e , and influence activity, n .
3. At date 2, the output, q , and the agent's outside option, \bar{U} , are realised.
4. At date 3, the agent delivers the project to the principal and gets paid.

The principal's value functions, for high and low effort choices by the agent respectively, are

$$V_1 = p_1 \pi_0 (\bar{S} - \bar{m}) + p_1 (1 - \pi_0) (\bar{S} - \bar{\bar{m}}) + (1 - p_1) \pi_0 (\underline{S} - \underline{\underline{m}}) + (1 - p_1) (1 - \pi_0) (\underline{S} - \underline{m})$$

and

$$V_0 = p_0 \pi_1 (\bar{S} - \bar{m}) + p_0 (1 - \pi_1) (\bar{S} - \bar{\bar{m}}) + (1 - p_0) \pi_1 (\underline{S} - \underline{\underline{m}}) + (1 - p_0) (1 - \pi_1) (\underline{S} - \underline{m})$$

respectively.

Effort and influence activities result in two signals for the principal and the agent at date 2. One signal which provides information about the output and productive effort

⁵This is because influence activities are costless, feasible and beneficial, $\pi_1 > \pi_0$. Therefore the agent's best outside option occurs when employing high levels of influence activities.

and the second tells the principal the agent's market value. If either state $(\underline{q}, \overline{U}_L)$ or $(\overline{q}, \overline{U}_H)$ occurs then the initial contract is used and the agent delivers the good to the principal and gets paid the amount he has been promised. This is when both the signals are aligned and the principal and agent agree on the outcome. When the output and market value are not similarly aligned the contract gets renegotiated. The principal and the agent will not agree upon the payment due and the agent holds back delivery of the good.⁶ In other words, when discrepancies arise between the two measures, and so when either of the states $(\underline{q}, \overline{U}_H)$ or $(\overline{q}, \overline{U}_L)$ occur, then agent's true worth is not clear and therefore the contract has to be renegotiated. The bargaining outcome during renegotiation is driven by the agent's outside option. The principal makes a take it or leave it offer to the agent, and the agent will accept as long as it is greater than or equal to the outside option. So the renegotiation outcome is in fact the payment equivalent of the respective outside options. The result of the renegotiation game under the outcome $(\overline{q}, \overline{U}_L)$ is the payment equivalent, namely $\underline{\underline{U}}$. Conversely, when the outcome $(\underline{q}, \overline{U}_H)$ arises, the result is the payment equivalent, $\overline{\overline{U}}$. Given the outcome of the contract renegotiation, we can therefore rewrite the principal's value functions and the agent's wage as:

$$V_1 = p_1\pi_0(\overline{S} - \overline{m}) + p_1(1 - \pi_0)(\overline{S} - \underline{\underline{U}}) + (1 - p_1)\pi_0(\underline{S} - \overline{\overline{U}}) + (1 - p_1)(1 - \pi_0)(\underline{S} - \underline{m})$$

if the agent puts in high effort, and the expected payoff for the principal if the agent puts in low effort is

$$V_0 = p_0\pi_1(\overline{S} - \overline{m}) + p_0(1 - \pi_1)(\overline{S} - \underline{\underline{U}}) + (1 - p_0)\pi_1(\underline{S} - \overline{\overline{U}}) + (1 - p_0)(1 - \pi_1)(\underline{S} - \underline{m}),$$

and the corresponding expected payoff for the agent for high and low effort, respectively, is

$$p_1\pi_0u(\overline{m}) + p_1(1 - \pi_0)u(\underline{\underline{U}}) + (1 - p_1)\pi_0u(\overline{\overline{U}}) + (1 - p_1)(1 - \pi_0)u(\underline{m}) - c$$

and

$$p_0\pi_1u(\overline{m}) + p_0(1 - \pi_1)u(\underline{\underline{U}}) + (1 - p_0)\pi_1u(\overline{\overline{U}}) + (1 - p_0)(1 - \pi_1)u(\underline{m}).$$

⁶We have in mind a sales contract. So ownership of the good changes from the agent to the principal only after delivery of the good to the principal and payment to the agent.

Define $\bar{u} = u(\bar{m})$, $\bar{U}_H = u(\bar{\bar{U}})$, $\underline{u} = u(\underline{m})$, and $\bar{U}_L = u(\underline{U})$, and so equivalently, $h(\bar{u}) = \bar{m}$, $h(\bar{U}_H) = \bar{\bar{U}}$, $h(\underline{u}) = \underline{m}$, and $h(\bar{U}_L) = \underline{U}$. The new variables are the levels of ex post utility obtained by the agent in each outcome. The principal would like to encourage the agent to exert high effort and therefore make his contractual payments, (the contract offered is renegotiation-proof) dependent also upon his effort level. In which case, we have; $\bar{u}_1 = u(\bar{m}(\bar{e}))$, $\bar{u}_0 = u(\bar{m}(\underline{e}))$, $\underline{u}_1 = u(\underline{m}(\bar{e}))$ and $\underline{u}_0 = u(\underline{m}(\underline{e}))$ ⁷. Now that the wage in each state of the world is determined, we can solve the problem and describe the optimal contract offered to the agent⁸.

We first describe the first-best contract, where the effort is observable. This result also relates our framework to the standard moral hazard problem where the agent undertakes no influence activity to improve his outside option.

Proposition 1 *The first-best contract offers full insurance. But the total surplus under it is strictly worse than the standard case, when the agent does not undertake influence activities, first-best setting by an amount exactly equal to the agent's expected outside option.*

All proofs are provided in Appendix A.

The proof here is trivial. The agent is offered a flat wage under first-best. Therefore, no matter the outcome, he is guaranteed a particular wage level, full insurance. This is because, in the first-best, we can ensure that the agent complies, and so exerts high effort. It therefore follows, due to his risk aversion, to insure him completely and offer him a wage independent of the outcome, given his compliance. It is also straightforward to show that this wage exceeds that in the standard first-best setting. Moreover, the wage that is paid to the agent is larger by an amount exactly equal to the expected outside option of the agent. The intuition for this is simple. Under the standard model, the agent's outside option is normalised to zero. That is, his wage must only be non-zero to satisfy his participation constraint. Here, we allow for the agent to influence his outside option, so that it can take some non-zero value, in expectation $\pi_1 \bar{U}_H + (1 - \pi_1) \bar{U}_L$. Therefore, his wage in this setting must be at least this value⁹.

⁷Note that the subscript on u relates to the level of effort exerted by the agent.

⁸Fudenberg and Tirole (1990).

⁹In the standard problem, the result is that the first-best wage is equal to the cost of exerting high effort plus his outside option (which is zero, therefore irrelevant), so the agent himself extracts no rent.

This first-best wage is strictly greater than in the standard setting. This highlights rather simply the implications of introducing influence activities in the more standard moral hazard problem.

Proposition 2 *The second-best insures the agent's choice of effort within the contract, although he is still subject to risk outside of the contract.*

When the effort of the agent is not observable, the wage offered has to ensure that the agent is given the incentive to put in high effort if that is desirable for the principal. The cost of this will be the wage level that would have to be offered to the agent in order that the agent exert high effort. The agent is given insurance of his effort level. The wage level offered to the agent is such that he will receive a higher bonus for the higher outcome in case he expends high effort. So as expected the degree of incentives offered is much more for inducing high effort.

The second aspect of the contract is, given that the contract may be renegotiated, a higher outside option will result in a relatively higher pay out for the agent. This provides the agent an added incentive and consequently, an added rationale to exert low effort. This is due to the fact that agents may look to insure themselves more and hence for those who prefer 'safer' contracts. Agents may therefore insure oneself and focus upon a higher outside option, given that these cases of renegotiation may occur, and if they do one would be less protected had they worked hard on the project.

Proposition 3 (i) *The payment under low effort is a convex combination of the high and low outside options. (ii) The payment under high effort is increasing in the cost borne by the agent for exerting high effort and for large enough c , we have the ranking $u_1 > \bar{U}_H > u_0 > \bar{U}_L$.*

Corollary 4 *The principal will offer the bonus contract u_1 and u_0 rather than a flat wage, if:*

$$\begin{aligned}
x \cdot (p_1 - p_0) \cdot (\bar{S} - \underline{S}) &\geq \\
&x \cdot u_1 \cdot (p_1 \cdot \pi_0 + (1 - p_1) \cdot (1 - \pi_0)) + x \cdot (p_1 \cdot (1 - \pi_0) \cdot \bar{U}_L + (1 - p_1) \cdot \pi_0 \cdot \bar{U}_H) + \\
&(1 - x) \cdot u_0 \cdot (p_0 \cdot \pi_1 + (1 - p_0) \cdot (1 - \pi_1)) + (1 - x) \cdot (p_0 \cdot (1 - \pi_1) \cdot \bar{U}_L + (1 - p_0) \cdot \pi_1 \cdot \bar{U}_H) \\
&- (\pi_1 \cdot \bar{U}_H + (1 - \pi_1) \cdot \bar{U}_L)
\end{aligned}$$

i.e. the expected gain exceeds the cost.

The ranking above provides us with some important information about the magnitude of the wage in the low effort case. It lies strictly between the high and low outside options, which again is useful for effort incentives, that we deal with later. We show this by determining that it is in fact a convex combination of the two outside options in Appendix A. One implication of this is that, if the outside option were a fixed value, so if $\bar{U}_H = \bar{U}_L$, then the wage under low effort would be equal to the outside option. In the next part of the proposition we then show that the wage under high effort would then strictly exceed this value.

The second part of the proposition relates to the incentives to encourage high effort. For the agent to exert high effort he must incur the cost of effort, c . If this cost were small, then it would not be such a burden to take and perhaps the agent would more readily accept it. Therefore, the agent would need less encouragement to comply and so the incentives on high effort could be relatively small. Conversely, if this cost were very large, then the agent may need more convincing that it would be worth his while to exert high effort, in which case the incentives may have to be very large. Therefore, we would expect his wage under high effort to be increasing in the cost that he has to bear in order to exert high effort. We can in fact express his wage under high effort as a function of the respective probabilities of states, his high and low outside option, and his cost of effort. Furthermore, we can indeed show that, for all else equal, an increase in this cost, increases the level of wage required under high effort. If this cost is large enough, then we can show that the following ranking occurs; $u_1 > \bar{U}_H > u_0 > \bar{U}_L$. What is the significance of this particular ranking? Immediately, we can see that $u_1 > u_0$, therefore we have monotonic wages. The agent would have a higher wage in high effort outcomes in comparison to low effort outcomes. Also, the wage under a high (low) outcome is preferential to the high (low) outside option. This shows that an agent would strictly prefer to remain within the firm and get his contractual wage than to take up his outside option. These rent gains are uncommon in the literature and a little unexpected. We explain the reason behind this finding in the section to follow, using a risk and incentives argument. The principal can set the wages under high and low effort, such that a hard working agent can exert high effort and find this

preferential to his high outside option, whereas a lazy agent can exert low effort and find this preferential to his low outside option.

2.4 Risk And Incentives

We have a model in which an agent can affect their outside option, but by doing so, restricts his available time for work upon a project that he is contracted to complete. This results in a contract in which the agent is paid more than his high outside option for high effort, that is, he spends the majority of his time focusing on the work he is paid to do. The effort the agent chooses to exert to influence his outside option may provide him with insurance from the contractual outcome. Given this insurance device, the principal can provide the agent with greater incentives.

Standard theory suggests that one should expect to see a trade-off between the risk and incentives (Grossman and Hart (1983)). The risk averse agent would prefer his payment to be tied less to the noisy outcome and so incentives are reduced. In our framework we have a situation where the agency problem worsens due to the opportunity to influence the outside option. The agent in order to increase the ex post bargaining outcome may work less on the principal's project¹⁰. However, this also provides the agent with an opportunity to insure against a random payment. We study the impact on incentive provision if the agent could provide himself with such insurance. Note that risk faced by the agent regarding the outside option is outside the contract. It is this feature which can be used by the agent to insure the risk in the contract.

The outside option is the lower bound for an agent's wage, and so by increasing its value, the agent will increase his minimum wage, albeit perhaps sacrificing potentially higher maximum wages. An agent who chooses to work hard on his project is therefore also making a choice to exert low amounts of influence activities. Therefore, he would expect to have a lower outside option and so be less insured. Given that he is now in a more risky environment, but carrying out the principal's desired action choices, the

¹⁰Here in our analysis the effort for the principal's project and the influence activity are substitutes. The other case would be if these two actions were complements. While we do not solve the case when the two are complements, our conjecture is that the agency problem may not worsen as much as our case.

firm will allow him to take a larger proportion of the earnings and so give him larger incentives. In which case, if our results are to fit with those of Prendergast, we expect a lower outside option to coincide with higher incentives. To show this we make a small alteration to our model, solve again and then compare results to those of our earlier proposition.

In our work above, the outside option of the agent is modelled in a similar way as the project, however, now the relationship between the influence activity and the outside option is set to be a deterministic relationship. In particular, if the level of influence activity is high, then the outside option is high ($\pi_1 = 1$), and vice versa ($\pi_0 = 0$). Remember that we have a time constraint, and so this implies that some states of the world are now not possible. For example, with low effort and high influence activity, we cannot have a low outside option and therefore under this action choice only $(\bar{q}, \bar{U}_H, \underline{e})$ and $(\underline{q}, \bar{U}_H, \underline{e})$ are possible. This simplifies the analysis, and allows us to say a bit more about the risk and incentives trade-off.

Proposition 5 *When the outside option is deterministic, the optimal wage is such that the payment under high effort is greater than the payment under low effort, and the latter should be equivalent to the high outside option.*

Corollary 6 *When the agent faces a deterministic outside option then, the principal will prefer a bonus contract over a flat wage contract if $(p_1 - p_0) \cdot (\bar{S} - \underline{S}) > c$.*

In other words $u_1 > \bar{U}_H = u_0 > \bar{U}_L$. So once again, the payment under high effort exceeds the value of the high outside option. What is interesting to us however is the relationship between the two contractual payments under low and high effort between the two differing models for an outside option. Trivially, it is clear that the payment under low effort is higher in the deterministic case. Low effort, due to the time constraint, implies a high level of influence activity, which therefore means that the agent has a high outside option. It therefore follows that the agent expects nothing less than his lower bound, now his high outside option. Under the stochastic case, low effort implies that a high outside option occurs with a high but less than certain probability. In which case, the agent can be rewarded with less than his high outside option.

After a little algebra we can show that a sufficient but not necessary condition for the payment under high effort to be larger for the deterministic case is $p_1 \geq \frac{1}{2}$. Given that the agent needs to be provided with incentives $u_1 > u_0$. To reason our result, we compare the two outside options. In the deterministic case, high effort implies a low level of influence activity and hence a low outside option. For the stochastic case, there is a high chance of a low outside option, but still some chance of a high outside option. In which case, the latter has a larger outside option and this in the stochastic case may lead to a higher wage. If an agent puts in more effort on the contracted project and consequently less on influence activity they are subjected to higher risk or lesser insurance. In order to compensate the agent for the higher risk level he has to be paid more. So due to the agent's ability to influence the outside option he earns a higher rent than the standard moral hazard case. In the deterministic case, the outside option is more severely weakened when high effort is chosen and we observe an even higher wage payment. Therefore, in cases where the outside option is more strongly weakened, this relationship should be more pronounced. Thus we provide an alternative reasoning to Prendergast to explain higher pay for more risky environments. We believe that this may provide us a testable hypothesis that higher incentives are found within professions where agents have the opportunity to influence the outside options.

Next we analyse the intensity of incentives. For this we look at the difference between wages under high and low effort or the bonus which needs to be paid to provide incentives. While not surprisingly and as discussed above the bonus is positive but differs in value when the outside option changes from a stochastic to a deterministic relationship.

Proposition 7 *The bonus paid when the outside option is deterministic is higher than when the outside option is random if $p_1 \geq \frac{1}{2}$ and $\frac{(1-\pi_1)}{(1-\pi_0)} \geq p_1$.*

One must remember that it is not only the cost of exerting high effort that is important, but also the incremental cost of inducing an increase in effort level compared to the incremental gain from moving to a higher effort level for the principal. This determines whether the principal would find it preferential to elicit a high or low effort choice from the agent. This gives rise to certain peculiarities. The cost of inducing high effort is larger under the deterministic outside option than under the stochastic.

It may hold however, that the principal prefers high effort under the deterministic case and low effort under the stochastic case. This is due to the wage gap being larger in the stochastic case than the expected gain for the principal, which highlights the importance of the wage under low effort. The larger the wage gap therefore, the larger the incentive problem and the more likely the principal is to prefer a choice of low over high effort.

Therefore, with a deterministic outside option if high effort is induced by the contract then the agent will have a low outside option, and is subject to the risk of the random output from his high effort. Whereas, under a stochastic outside option, although there may only be a small probability of a high outside option, this probability is strictly positive, in which case, his outside option is strictly greater. So if the principal were to induce high effort from the agent, the agent would be subject to a higher degree of risk in the case of deterministic outside option. Since we see that with deterministic outside options incentives are stronger or bonuses are higher we observe higher incentives coincide with higher risk.

The condition $(1 - \pi_1)/(1 - \pi_0) \geq p_1$ states that the marginal effect of influence activities on the probability of increasing one's outside option be not too large. This adds more weight to our argument. Not only in cases where a choice of high effort imposes more relative risk on an agent, but also in those where switching to a strategy of high influence activities does not have too large a gain, should we expect to see higher incentives. When the marginal gain from these influence activities is smaller, once again, it is clear that the agent is in a more risky environment. Hence, this supports further the logic of higher risk, higher incentives.

To summarise this section, we find that agency problems get worse with influence activities in comparison to the moral hazard problem without influence activities. When the outside option which the agent can influence is stochastic then the principal can provide incentives to the agent but the cost of compensation goes up in comparison to the case when the agent cannot influence his outside option. If, instead, the outside option is deterministic then the principal will provide even higher incentives to the agent, given that the cost of effort is less than the marginal gain for the principal¹¹.

¹¹ $(p_1 - p_0) \cdot (\bar{S} - \underline{S}) > c.$

2.5 Conclusion

We propose a model whereby the agent can influence his outside option and this results in a wage strictly greater than this value. Furthermore, the weaker one's outside option and the greater the consequences of not improving said outside option, the larger incentives one should receive. Our analysis presented here can provide an alternative theory to Prendergast (2002) for the supposed peculiarity found in the data set that we often observe higher incentives with higher risk. According to our results if agents can influence their outside options then this can work as an insurance for the risk the agent faces. Given the insurance outside of the contract, the principal can provide higher incentives to the agent within the contract. Therefore we believe the observation made by Prendergast (2002), that we do not observe a risk and incentives trade-off can be explained by our reasoning.

Another related concept was explained by Gibbons and Murphy (1992), where agents have career concerns. This means that they have concerns about the effect of current performance on future compensation. They break incentives up into two parts, implicit incentives, that are driven by career concerns and explicit incentives from the compensation contract. The optimal compensation contract therefore optimises total incentives. The implication of this finding is that explicit incentives should be strongest for those closest to retirement because career concerns are weakest for these workers. We could explain agents exerting high levels of influence activities, not only to maximise current wage, but also because of the effect this has on future wage, because of career concerns. This therefore, has the converse logic, the potential rewards from these influence activities are greater earlier in the career, leading perhaps to the opposite conclusion about explicit incentives, that has already been put forward by Milgrom and Roberts.

We believe that our results are empirically testable, as we can construct a hypothesis that employment where there is a greater opportunity to participate in influence activities will result in higher incentive contracts under greater risk. There are further opportunities to expand our theoretical model such as the outside option of the agent may be private information and the influence activities as suggested by Milgrom and Roberts may add to firm value. These are left for future work.

Target Setting with Moral Hazard and Adverse Selection

3.1 Introduction

In our work to date, we have discussed the importance of the PC , not only for the agent, but also how it can dramatically affect the principal. Endogenising the constraint and allowing for moral hazard can lead to more expensive contracts, with less risk sharing between the principal and agent. What we consider now is a new angle on the problems associated with adverse selection, namely when agents are of differing types and the measures that principals will go to in order to encourage them to truthfully reveal their private information.

In most professions, ability matters. Imagine a University department is looking to expand and take on several young, new, bright researchers. The ability of the individual in question is clearly highly important. Although it can be seen if the applicant has some history with publications, and where his doctoral research was completed is public knowledge, the actual talent is private information. One does not know if this person was spoon-fed their entire PhD, or if the glowing reference they produce is in fact because they have spent the last months of their course supplying the head of the department with expensive bottles of his favourite wine. So when two

identical candidates appear, determining who is of ‘better type’ is of utmost importance to the department. We aim to model academia by introducing ability into moral hazard through adverse selection. We use a specific contract structure, that introduces target setting to counter the problems associated with different types of agents.

One could also think of this model in the field of public work contracts. Take for example the highly publicised and controversial bidding process for rights of construction for Wembley stadium. Firms submitted bids according to forecasted costs of production and much debate was made over the final choice of Multiplex. Initially, Bovis Lend Lease and Multiplex were working on a joint venture and considered as the preferred bidders. The deal however broke down after a row between the firms over the cost of construction and then only days later Multiplex made a bid on their own, that undercut the previous value. The claim of target heavily influenced the decision of who won the contract, however of course, the claim itself could not be met. The project was delayed by several years and ran massively over budget, with Multiplex making significant losses. The process also led to several legal disputes over who was at fault for the problem with an out of court settlement resulting in Multiplex reclaiming a portion of the losses they had made. One must therefore seriously consider the implications of such claims in relation to information about one’s type. Other examples include:

When a rail franchise is due for renewal in the UK, the Department for Transport (DfT) invites bidders to tender. The bids from all competitors includes such information as the requirements of public funding, projected levels of services and rail ticket prices. The DfT then judges each bid on the above criteria and awards the franchise thusly. Both the claims made by the potential franchisee and their ability to deliver them are of extreme importance in such a case.

With the recent trend of emphasis on sustainability, firms’ (or even country’s) carbon footprints or energy usage are under significant scrutiny. There is a large amount of pressure to reduce both these levels and if not, severe financial sanctions are intended to be put into place. Take for example the legally binding emission-reduction targets for the UK in the Climate Change Act of 2008. They aim to reduce greenhouse gas emissions by 34% by 2020 and by 80% by 2050. Firstly, one must consider how achievable these targets in fact are and secondly how declaring them upfront has affected the

potential future sanctions faced.

Reverting back to the original idea, adverse selection problems are pertinent when we consider what one must achieve for tenure and how readily it is offered by departments. Academic tenure is the prospect of securing the job of a researcher, to the extent that removing him from the post then becomes very difficult. In the USA, a researcher normally has a limited period of time to prove his worth and establish his track record so that he may gain such a status. This process is primarily intended to give the candidate academic freedom and economic security in the future, thereby removing any external pressures from him.

Furthermore it can help to attract those of high ability to the profession and ensure retention of their services over a prolonged period of time. If this is the case he may be able to pursue sensitive lines of research or those that he deems of interest. With more freedom to research, his ideas should become more original and his reports more honest, as he has no (potential politically biased) benefactor to answer to. Quite obviously, departments wish to offer such positions only to those who truly merit it and therefore the adverse selection issues come to the fore.

It is a well-known fact how difficult the tenure application process can be. The publication demands and time constraints make it an incredibly taxing process for all candidates involved. If we take note of recent trends, there has been a much slower growth in tenure track positions than those of non-tenure¹. Those who oppose the current system argue that by granting tenure, all incentives are removed on the part of the researcher. Given how difficult it becomes to dismiss the newly promoted professor he will then neglect all teaching and research obligations he would otherwise have had to complete.

We focus not on the merits of the tenure system², but the demands that it should place on the researcher. We suppose firstly that a department has hired two new recruits, one of which is highly able the other less so. However, this information is private to the individuals and also valuable to the department; a researcher who is more able is more likely to produce a higher quality output. In this case that is equivalent to

¹See list of figures for relevant data.

²Opinion is divided on this issue, see Holley (1977) and Carmichael (1988).

being published in a higher ranked journal. To try and counter this problem we allow for a public signal to be sent by the researcher, in the form of a target declaration. This signal has no direct cost to the researcher, but may limit his potential output, depending on his choice. He may choose to declare a high or low target, akin to announcing that he will attempt to publish either in *American Economic Review*, or in *Economic Letters*. Researchers can choose to work hard or not, however this too is private to the individual and clearly subject to moral hazard³.

Our approach is therefore to determine what set of requirements should be placed on a researcher, given that he may vary in type, so as to maximise the value of research output in a department. Considering various combinations of targets for different types, we attempt to comment upon the features of optimal contracts given a particular target choice. A simple comparison across cases will therefore offer some straightforward analysis of what is most desirable in the eyes of the principal. We find, rather interestingly, that to ensure fully separating equilibria, one must offer non-monotonic wages. We suggest that this is so in order to guarantee non-mimicking of type⁴. Precisely, we reward those of low type for unproductive behaviour so that we may heavily incentivise those who are more able. However, it appears that pooling equilibria offer larger returns to the principal, implemented with an incentive scheme that places strong demands upon all agents. This we say is in support of the current tenure system for academia, vis-à-vis the tough requirements that candidates face. Finally, in opposition to the classical solution in adverse selection models, our high type agents are indifferent between the contract and their outside option, it is the low type agents that make rent gains. We next discuss the literature, introduce the model, then present our results. Finally, we conclude.

³The problem therefore that we are attempting to answer here deals with both moral hazard and adverse selection, highly uncommon in the literature.

⁴That it is not in the interests of either type, high or low, to conceal their true ability, and act as if of another type.

3.2 Discussion and Relation to Literature

We must first discuss two types of equilibria that exist under the paradigm of adverse selection; pooling and separating. For a formal discussion on both these topics see Spence (1973) and Cho and Kreps (1987). Ordinarily, we consider a game whereby one player has some private information about their type, which would be of some value to the other player. The player with the hidden information then chooses whether or not to send a costly signal to the other player indicating his type. If all players find it optimal to send the signal then we cannot distinguish between types and so we call this a pooling equilibrium. If, on the other hand, only some players find it in their interests to suffer the cost, typically just those of higher type, then we can determine something about the type of the agent we are dealing with, known as a separating equilibrium.

We however, consider the problem from a slightly different perspective, that will be discussed in more detail later, and allow the player without the private information to set out some targets that he require from the other player in order to elicit some information, a form of screening. Therefore, in some cases, we may choose not to distinguish at all between different types. We may find it preferential to offer all employees the exact same contract, i.e. all researchers are required to publish four papers in three years, that are done so within journals that are at least three star. This would normally occur because the cost of encouraging the agent to reveal his type is too expensive for the principal and so he just treats all the same. This however, has two unsavoury consequences. Those of good type are now discouraged to try too hard. They are being treated in exactly the same way as those who are of lesser ability, and so feel undervalued by the contract they have signed. In which case, as a principal, our output may fall, as good agents are merely satisficing, rather than realising their potential. Also, related to the above, so that we may deal with some good types, we have had to over pay the bad types, leading to efficiency losses. In no way have we solved the problem associated with adverse selection, we have chosen only to ignore it. This we label as a pooling equilibrium, as all types of agent are pooled together and treated as one.

Ideally, we would like to separate types of agents out and reward them differently. This is potentially possible by offering a menu (literally a choice of options) of contracts

to agents. We hope to design these in such a way that one such item on the menu is preferred by one type and another preferred by another type. That way, when an agent makes his choice what item he would like on the menu, we should be able to fully determine his type. If we can design such a menu, then have what is known as a separating equilibrium.

Modelling ability is not an innovation, there exists a large amount of literature with both moral hazard and ability (see Prendergast and Topel (1996), Prendergast (1999)), however such papers model the latter as a latent variable in the model. They do not use contract design to resolve the unobserved variable of ability, as we do. We choose to include the adverse selection problem in our model according to ability.

What moves our problem away from the main body of literature is the inclusion of hidden action in addition to hidden information. Not only are the agents of different types (abilities), but they may also choose to work hard or not. This costly effort level is private to the agent and affects the probability of a good outcome occurring. We model how to circumvent the transaction costs associated with each type of information problem. In order to combat the problems with adverse selection we choose to set targets for the agents, whereas bonuses for particular outcomes are used to deal with the moral hazard present.

There does however exist a small strain of literature that covers the topic of both moral hazard and adverse selection. The most significant contribution is Guesnerie et al (1988) (see also Picard (1987) and Melumad and Reichelstein(1989)) who present a model between two risk-neutral agents. Their main message is that, in comparison to the case of pure adverse selection, the addition of moral hazard does not create welfare losses. They describe a problem when the agent's action choice is observable and show that the same mechanisms are implementable when actions are not perfectly observed. There are several departures from our work. The authors model the outcome as an additively noisy measure on the actions that the agent chooses to take, that also does not depend on his type. Furthermore, several technical differences such as the assumption of multiple levels of differentiability of utility functions and 'wages'. They also place restrictions on payment schedules, such as either a linear or a quadratic form, whereas we look only for points in a space. Our results also include pooling

equilibria as a possible solution while they search only for separating cases. What perhaps drives their result however is the assumption that agents can be punished. Since the publication of this paper, Innes (1990) showed for moral hazard to have any effect, one needs either risk-aversion, or risk-neutrality with limited liability. As Guesnerie et al require large punishments for low action choices by agents, they are therefore not protected by limited liability. We however, include an assumption of limited liability.

One other point that we should mention is a quick comment on monotonicity of wages. This rather simply means that my wage as an agent should be increasing in my output. If not, it leads to some rather perverse incentives on the part of the agent. For example, I may wish to destroy some of my output, known as burning money, so that I end up at an outcome in which I will receive a higher wage. Furthermore, I may also have no incentive to work hard, as if I did, I may achieve the higher outcome, that rewards me less so. This is clearly not desirable for the principal, as the whole reasoning behind incentive design is to align the goals of the two parties. In this contribution, we find that in order to achieve separating equilibria, we must have some wage structures that are non-monotonic. The justification is that we reward unproductiveness of those who are less able in order to encourage those of high ability to be more productive.

3.3 Model

We consider a standard principal-agent problem, where an agent is employed to work on a project for the principal. Both parties are risk-neutral and the agent is covered by limited liability. The agent can be of two types high ability $\bar{\theta}$ (H hereafter) or low $\underline{\theta}$ (L hereafter), that are private, and is able to perform two private actions; costly (binary) effort that directly increases the chance of success of the project and target declaration, whereby he notifies the principal of his goal for the project. The cost of effort is increasing ($c(e_H, \theta) > c(e_L, \theta)$ for all θ), but decreasing in the type of the agent ($c(e_H, \underline{\theta}) > c(e_H, \bar{\theta})$ for e_H and e_L). The target declaration can be to either declare a high target of output or a low target of output. We assume that if an agent declares that he is targeting the low output, then he has no chance of realising a high level of

output, for all effort levels and types. In which case the probability of a high outcome after a low target $e_H.H(\theta, l) = e_L.H(\theta, l) = 0$ for all θ ⁵. This emphasises the importance of the target declaration, as the consequences of it are clearly verifiable. Given a high target and effort levels, the probability of a high output is increasing in the type of the agent ($e_H.H(\bar{\theta}, h) > e_H.H(\underline{\theta}, h)$ and $e_L.H(\bar{\theta}, h) > e_L.H(\underline{\theta}, h)$). Similarly, given a low target and effort levels, the probability of a low output is increasing in the type of the agent ($e_H.L(\bar{\theta}, l) > e_H.L(\underline{\theta}, l)$ and $e_L.L(\bar{\theta}, l) > e_L.L(\underline{\theta}, l)$). Furthermore, the marginal probability of effort on a high output for a high target is larger for type H . Finally, the marginal probability of effort on a low output for a low target is larger for type H . The project itself can take one of 3 values, high output (H) that is preferred by the principal to low output (L), that is itself preferred to zero output (0). The value that each outcome brings to the principal is therefore as follows; $V(H) > V(L) > V(0) = 0$. The principal offers wages that are conditional both upon the output and targets that the agents publicly declared themselves. The contract therefore consists of five wages, that are; $w(H, h), w(L, h), w(0, h), w(L, l)$ and $w(0, l)$. She seeks to maximise her expected profit, i.e. the expected difference between value and wages. Meanwhile agents maximise their expected payoff, given that they know their type and effort choice.

In order to better understand our framework we offer a timeline below:

| Agent Realises Type | Contract Signed | Target Declared | Effort Completed | Outcomes Reported | Payoffs Given |
|---------------------|-----------------|-----------------|------------------|-------------------|---------------|
| <i>Date 0</i> | <i>Date 1</i> | <i>Date 2</i> | <i>Date 3</i> | <i>Date 4</i> | <i>Date 5</i> |

Note that we introduce both moral hazard, in the choice of effort, and adverse selection, from the ability of the agent. The output is fully verifiable and the target is publicly declared, therefore in this contribution we see no reason to introduce renegotiation.

For us it is clear that H should work hard and target the high outcome. Therefore, in this problem, one must give consideration as to the desired action choices of L . Should he be required to work hard like H ? Should he be aiming for the perhaps unattainable high levels of success? We therefore consider four cases and look to determine the solution in each. These cases drive our IC constraints and the workings of our problem.

⁵In the example of research, if one were to send a paper to a mid-range journal, we do not expect to suddenly be published in a 4 star publication. This could be relaxed to the case whereby a low target declaration may result in only an epsilon chance of high output. We leave this to further work.

The low type therefore has four possible combinations of action choices available to them ranging from, entirely mimic H (high effort choice and high target) to behave oppositely (low effort choice with low target), and everything in between. Due to the complexity of the problem we now have more than the standard two incentive constraints in each case of our problem. There are however two constraints that are common in all of our cases that govern the behaviour of H . Firstly, H should choose their effort level such that they maximise their expected wage, given they are targeting a high output. As we are considering a binary choice of effort, this results in H exerting high effort. In addition, H should always prefer to target the high output given that he exerts high effort and is of type H . Therefore in all cases, there should be no incentive for a unilateral deviation in either effort or target for H . Now the IC constraints for L depend on which case we are in. Suppose we are in the case where L is required to mimic the behaviour of H , then the IC constraints should be constructed so that he is encouraged to do just that. Namely, given he is targeting high output, he should prefer to exert high effort and given that he is exerting high effort he should prefer to target a high output, in both cases assuming he is of low type. If the case we are considering requires some other behaviour, then this should be captured by the IC constraints. One should also ask whether it is in the interests of an agent to renege on both action choices simultaneously. We show in Appendix B that for all relevant cases, if both are independently satisfied at the solution, then this guarantees that no agent will wish to do so. In which case we can say that, at the solution, no type of agent will prefer to portray themselves as the other type. In addition, the wage of each type of agent should satisfy their respective participation constraints. So, in particular, the optimal wage level for each type has to be at least as high as their respective outside options, with the reservation utility of H larger than that of L ⁶.

The use of the target declaration in this model is quite evident, it is a mechanism to allow the principal to gather more information about the type of agent he is contracting with. Of course, the agent may lie, however, wages are to be constructed in such a way so that it will not be in his interests to do so. Conversely, it can also be used as a guide for the principal's expectations of the agent. Suppose that the agent in this case is a

⁶ $\bar{U} > \underline{U}$.

rail network franchisee, then the target might be an upper limit on the number of late arrivals in their franchise. In which case, the principal may only consider the bids for the franchise that include a target of this many or fewer late trains. It must also be made clear that there is no direct conflict between the two actions here, a high choice in one does not preclude a high choice in the other - there is no need for a time constraint.

Let us first offer a general form to the algebra to make it clear the problem we are solving⁷. We consider only a bilateral choice of effort, therefore either high or low. The utility of the agent is a function of his effort, target, type and wages in each state. If he chooses a larger effort level it costs him more personally, but he is now more likely to achieve a larger output. Similarly, being of a higher type makes it more possible for high outcomes to occur. In each of the three states he is paid a wage that depends on the outcome but also according to the target claim that he made:

$$U_A(e, t, \theta, w(., t)) = p(H|e, t, \theta).w(H, t) + p(L|e, t, \theta).w(L, t) + p(0|e, t, \theta).w(0, t) - c(e, \theta)$$

So, onto the contract design, firstly, the *IC* constraints for effort. Each agent chooses their own levels of effort to maximise their own utility. Given that they behave in this way, their contracts should be designed so that the agent is compensated in such a way that he should prefer to choose the level of effort that we want him too. Note too that we should account for particular choices of target too in our constraints. Therefore, suppose that we would like high effort from the able agents, but low effort from the lower quality agents, then given target requirements, the payoffs should have the following structure:

$$U_A(\bar{e}, t, \bar{\theta}, w(., t)) \geq U_A(\underline{e}, t, \bar{\theta}, w(., t)) \quad (IC1)$$

$$U_A(\underline{e}, t, \underline{\theta}, w(., t)) \geq U_A(\bar{e}, t, \underline{\theta}, w(., t)) \quad (IC2)$$

If the payoffs are set in such a way, then we should see those of high type exerting high effort and those of low type exerting low effort, in order to maximise their own utility.

Now to elaborate on the target setting constraints we implement. Not only do we

⁷The algebra listed below for each specific case can be found in Appendix B.

require certain types to choose particular levels of effort, but we also aim to encourage certain types to focus their efforts on particular targets. These seems reasonable as in some examples there may be some leeway as to what sort of performance is acceptable. Perhaps in the academic case listed above, it may be acceptable for some publications to be in lower rank journals and this would not harm the reputation of an institution. In this case, the focus might be on the volume of output and although some journals would clearly be preferred to others, some lower valued publications might be considered acceptable. However if we are thinking of a case where doctors are targeting the number of their patients who survive, then the individual level of performance becomes more important. Rather than accepting a ‘mediocre’ success rate that contributes to total value, all employed doctors should be aiming to save every life they are assigned to. Therefore, not only should we design contracts so that particular effort choices are important, but also that their effort is focused on the ‘correct’ areas. Suppose we are using the latter example, where every employee should be targeting the high outcome, then our target setting constraints should be designed so that:

$$U_A(e, h, \bar{\theta}, w(., t)) \geq U_A(e, l, \bar{\theta}, w(., t)) \quad (IC3)$$

$$U_A(e, h, \underline{\theta}, w(., t)) \geq U_A(e, l, \underline{\theta}, w(., t)) \quad (IC4)$$

Note once again, that these constraints must take into account the other variable, in this case, effort. Therefore, the two sets of incentive constraints, when used in conjunction, should govern the behaviour of both types of agents in the way that the principal deems preferable.

In addition to the *IC* constraints given above, each player themselves must find it in their individual interests to sign the contract and so we introduce two participation constraints. We assume that those of higher type have a larger reservation utility and so, given their respective choices of target and effort the following conditions must be satisfied:

$$U_A(e, t, \bar{\theta}, w(., t)) \geq \bar{U} \quad (PC1)$$

$$U_A(e, t, \underline{\theta}, w(., t)) \geq \underline{U} \quad (PC2)$$

We must also mention that each agent is protected by limited liability - no agent shall lose more than his investment from the contract. In previous literature such as Guesnerie et al (1988), agents can be severely punished in some cases and incentivised heavily in others. We do not allow such contracts. This simply reduces to ensuring that no wage is negative and as such agents are not punished for bad outcomes, only perhaps not rewarded for them.

The principal therefore designs such a contract to elicit the ‘right’ sort of behaviour from his employees, but what is it that he cares about? This is where we introduce the objective function, the value of the contract to the principal. The principal pays a wage in each state designed to incentivise the agent, described above. However, as is the case with all economic agents, she seeks to maximise her own welfare and therefore targets as high as possible a level of profit. This is the difference between the expected value of each state, minus the expected wage in each state.

$$U_P(e, t, \theta, w(\cdot, t)) =$$

$$p(H|e, t, \theta).(V(H) - w(H, t)) + p(L|e, t, \theta).(V(L) - w(L, t)) + p(0|e, t, \theta).(V(0) - w(0, t))$$

The problem for the principal now is obvious, the remaining surplus left for him is clearly decreasing in the wages that he pays, however, wages are needed to provide incentives. All such problems can therefore be thought of as a cost minimisation procedure⁸.

Our proposal towards a solution, follows a very specific format of contract design, using target declaration. Not only do we wish to determine how agents should be incentivised to act in a particular way, but also how placing different demands upon them affects the solution. As was made reference to above, the particular choices of target will vary from situation to situation. In some cases we may accept lower targets as, if successful, they add to the total value of the project. These lower targets may also be a more realistic aim of those agents of lower ability and as we care of total value, we ensure that their claims are more attainable. In addition, such claims can also be used to separate between types and help ease the adverse selection problem in

⁸As in Grossman and Hart (1983).

our model. That way, we should hopefully feel more confident that by rewarding for certain outcomes we can go a long way towards rewarding certain types.

In others cases however, setting a low target may not be feasible. That may be equivalent to accepting a larger loss of life in hospitals, a lowering of health and safety standards on a train franchise, or delaying the completion date of an Olympic venue until after the event is due to take place. In these cases, all types should be required to submit a high target and although a low outcome adds to the ‘value’, it is expected not to be rewarded in the same manner. Furthermore, in order to separate out types and reward them accordingly may prove too expensive. Therefore, given our objective of maximising the value of the contract for the principal, it may be in our interests to operate a pooling equilibrium.

Thinking along these lines we separate the problem into four smaller problems. In each problem we outline the set of targets and effort levels we require of each agent. Solving each case individually we can say something about the degree of incentives one should offer. Then on a comparative level, we can discuss why the solution has changed when we change the demands placed upon the agent and finally how the value to the principal changes across cases. This methodology allows us to separate the effects of changing effort and target requirements individually and try to be a little more specific in our analysis.

3.4 Solution

So, we begin by constructing the four cases’ constraints and attempt to determine which Lagrange Multipliers are binding and which are slack. We first show that at least one of the participation constraints are binding in any of the cases. If both are binding, then the principal is able to fully extract all rent. We aim to concentrate on the various inefficiencies, therefore focus on all potential subcases where exactly one participation constraint is binding, to decide if any solutions exist.

We present a table of the four menus considered, that lists the demands placed upon both types of agents, that we will refer to by number throughout:

The desired behaviour of H remains constant, we only vary our demands of L . So

Table 3.1: Summary of Cases

| | High Type Demands | | Low Type Demands | |
|--------|-------------------|-------------|------------------|-------------|
| Case 1 | High Effort | High Target | High Effort | Low Target |
| Case 2 | High Effort | High Target | Low Effort | Low Target |
| Case 3 | High Effort | High Target | High Effort | High Target |
| Case 4 | High Effort | High Target | Low Effort | High Target |

for example under Case 1 we require both high effort and a high target declaration from H , but from L we ask for high effort and a low target declaration. Moving to Case 4 we would require the same behaviour from H , but now ask L to choose a low level of effort and a high target. Some restrictions need to be placed otherwise we end up with sixteen potential combinations, some of which seem quite irrelevant. For example, it seems sensible to require monotonicity of types, but how this is characterised is also up for debate. Does this refer solely to the effort level, to the claimed target, or to both? For example, if we require L to target the high outcome with high effort, then H should not be permitted to target the low outcome, but what effort levels should be permitted? Or perhaps we require high effort but not a high target. Therefore, how we enforce ‘monotonicity’ is important. We therefore restrict the actions of H and study only, perhaps more importantly, the various permutations for L .

The four cases that we study impact directly upon the IC constraints that we utilise and therefore should lead to some cases being more or less ‘expensive’ to enforce. A comparison across all four could then provide some discussion as to which is the most favourable to implement, i.e. what sort of targets should be set for employees. It should be made clear, that the target declaration is made publicly and is therefore observed by all.

Case 3 can be thought of as a complete pooling equilibrium, as we require L to completely mimic H . Case 2 is that of a complete separating equilibrium, as neither action choice is required to be the same. At this juncture it is worth reiterating the ‘cost’ of each action choice. Naturally, choosing a high level of effort is costly for either agent, although not to the same degree, but there is also a cost borne out of the target choice. Choosing a low target completely denies the agent any chance of a high outcome, however, a choice of high target builds expectation about the type.

One could ask at this point why we use indirect mechanisms as opposed to using the

Revelation Principle to characterise type-dependent incentive schemes, the approach that seems much more prevalent in the literature. We justify this by the fact that we find our process a little more interesting, essentially because it allows us to study non-incentive compatible schemes. We therefore offer a much broader solution to the problem.

3.4.1 What Constraints Should be Binding?

We now discuss some of the general findings to our solutions and the forms that they may take. We describe potential solutions and use simulations to guide our final results. As in all the cases to follow it is relatively straightforward to show that at least one of the participation constraints must bind. This is of course obvious as both types of agents would be making rent gains if not and their wages could be reduced and they would still be willing to sign the contract. If both were binding then all rent would be extracted and we are in the first-best case. However, typically we see just one of these constraints binding. In most previous literature, what we expect is that the participation constraint for L would bind and H 's to be slack. This is because of the hidden information about the agent's type. High types must be given enough to want to sign the contract but the cost of adverse selection, not knowing who is of what type, is that they make rent gains. However, we observe in our findings that in our separating equilibria, it is in fact those of low type who make rent gains and the participation constraint for H is binding. We now present the findings over which constraints may bind in the form of several lemmas below. The exact algebra of the constraints for all cases can be found in Appendix B.

Binding Constraints - Case 1

Let us first look at case 1, where we require high effort from both types, but encourage L to target the low outcome. This seems the most plausible case to enact given its structure when the target is less important for the principal, perhaps in cases where such a claim is not a life-or-death matter. Both types of agents are working hard, so should contribute to a large output and those of lower ability are able to focus their efforts on a more attainable outcome. Proofs of the following lemmas can be found in

Appendix B.

Lemma 8 *In case 1, at least one participation constraint must bind. Furthermore, if the target setting constraint for L binds then either; the target setting constraint for H binds, the participation constraint for H binds, or both do so.*

The first part of the lemma is of course quite obvious, as explained above, the second part seems to point towards a potential solution. One would expect a participation constraint to bind, but it seems that H 's does so partially because the target setting constraint for the low type does. This constraint wants us to force L to abandon the high target, meaning that his wages under the low target must be set high enough. It is therefore this additional cost of separation that forces a non-binding participation constraint for L and therefore H 's must bind. One would also expect an IC constraint for effort to bind, perhaps that of L , this we explain later on.

Binding Constraints - Case 2

Moving onto case 2, we now fully separate the agents and so in addition to the target requirements above, want L to only exert low effort. We explain the rationale for this case in that it is not worthwhile encouraging L to try hard. His productive capacity is perhaps quite limited and rather than letting him waste effort, for which he will have to be compensated, we let him slack off, easing the moral hazard problem.

Lemma 9 *In case 2, at least one participation constraint must bind. Neither the IC constraint for effort for L nor the target setting constraint for H bind. Finally, if the target setting constraint for L binds then so does the participation constraint for H .*

Once again the initial statement is obvious, however the second part leads to some interesting conclusions. Given that we have reversed the effort requirements for L , it would appear we have eased the moral hazard problem to some extent. In the case above, it is entirely possible that L could be indifferent between high and low effort, now, the IC constraint for L cannot bind, in which case he must now strictly prefer to be lazy. In conjunction, H must now strictly prefer the high target. What from this can we predict about the contract? With low effort from L we would have a lower expected

return for the principal, in which case she may reduce some of the wages designed for that particular agent. If also H strictly prefers a high target then perhaps the total level of incentives under a low target has been reduced. However L now strictly prefers low effort, therefore given a low target we expect the incentives offered to reflect this. The last statement is a stronger case of the above. So it is certainly the target setting constraint for L , through the logic described above, that forces a binding participation constraint for H .

Binding Constraints - Case 3

This we describe as the fully pooling equilibrium, as both agents are required to behave in exactly the same manner: high effort and high target declaration. Why might one expect such a setting? This we relate to the examples whereby the principal has more to lose from a low target and so expects each type of agent to at least attempt to achieve the highest possible outcome, even if it is at the very limit of L 's potential.

Lemma 10 *In case 3, at least one of the participation constraints must bind and neither of the target setting constraints bind.*

Once again, an obvious finding with regards to the participation constraints, however it is perhaps quite interesting that neither target setting constraint binds. Both are required to aim for the high target and once again, due to adverse selection it seems to make sense that H has a non-binding target constraint, but the fact that L 's does not bind either is puzzling. We suggest that, given the target requirements, agents are not rewarded at all for claims of a low target⁹. In which case, as they receive some payment for claiming a high target, both must therefore strictly prefer to do so.

Binding Constraints - Case 4

We now pool agents according to their target claims, but allow L to exert low levels of effort. We use the same rationale as in case 2 - that it may be too expensive to reward high effort for L , but here we require a high target as above. This is perhaps the most perverse of all systems that we study, but interesting nonetheless.

⁹Such wages still satisfy limited liability however.

Lemma 11 *In case 4, at least one of the participation constraints must bind and neither of the target setting constraints bind. Furthermore, IC constraints for effort and participation constraints bind in pairs of different types.*

We follow the above intuition for neither of the target setting constraints binding, and of course for the participation constraints too. We now clarify the last statement. The *IC* constraint for effort for *L* binds if and only if the participation constraint for *H* does. Conversely, the *IC* constraint for effort for *H* binds if and only if the participation constraint for *L* does. What does this mean? If *L* is indifferent between his choice of effort, then *H* is indifferent between signing the contract and not at all and vice versa. Therefore the level of incentives that are offered to each agent, when it comes to effort, affects the willingness to enter into the contract of the other agent. Or from another perspective, incentivising one agent more heavily to exert the required degree of effort comes at the cost of over-rewarding the other to sign the contract. We also go on to show that if only one pair were to bind then it would be the former in the analysis above¹⁰, again an interesting finding in opposition to the main body of literature.

Given what we have looked at, we now want to simulate possible outcomes. In order to do this we must make some assumptions on the functional form of several parameters in our model, however these are chosen to fit in with the assumptions we have introduced earlier. After running the simulations we then discuss our findings and use these to form algebraic expressions for our solutions.

3.4.2 Simulation and Findings

We will now present the results of the simulations, that we use as a template for our general form solutions. Firstly, we include a list of all parameters used in our simulation followed by an explanation and functional forms.

Table 3.2: List of Parameter Values

| Effort Level | Type | Target | Value | Outside Option |
|---------------------|----------------------------|---------------|--------------|-----------------------|
| $e_H = 0.7$ | $\bar{\theta} = 0.7$ | $h = 0.7$ | $V(H) = 10$ | $\bar{U} = 2$ |
| $e_L = 0.5$ | $\underline{\theta} = 0.5$ | $l = 0.5$ | $V(L) = 5$ | $\underline{U} = 0$ |

¹⁰Namely *IC2* and *PC1*.

Our cost of effort function should be increasing in the level of effort, but decreasing in the type, therefore we use the form $c(e, \theta) = \frac{e^2}{\theta}$. So for example, the cost to an agent of high type of exerting high effort is $c(e_H, \bar{\theta}) = \frac{0.7^2}{0.7} = 0.7$. Our probabilities of success should be increasing in type and effort, therefore we take the forms $e.H(\theta, t) = e \cdot \frac{\theta^t}{2}$ and $e.L(\theta, t) = e \cdot \frac{\theta^t}{4}$. One can see that $H(\bar{\theta}, h) = \frac{0.7^{0.7}}{2} > \frac{0.5^{0.7}}{2} = H(\underline{\theta}, h)$ and so on. In addition, we impose our constraint that whenever a low target is declared, a high output is impossible. Therefore, $H(\theta, l) = 0$, for both types of agent. Finally, the value to the principal is increasing in the outcome and the outside option of the agent is increasing in his type. Throughout we assume that the probability of a high type is $\rho = 0.5$.

We now present a table below summarising our results, followed by a discussion of each individual case. The discussion will take a comparative form as we believe in that manner one can more clearly understand what it is that drives this particular solution.

Table 3.3: Simulation Results

| | $w(H, h)$ | $w(L, h)$ | $w(0, h)$ | $w(L, l)$ | $w(0, l)$ | Expected Value |
|---------------|-----------|-----------|-----------|-----------|-----------|----------------|
| Case 1 | 9.9021 | 0 | 0 | 14.0299 | 0.453409 | -0.403161 |
| Case 2 | 9.9021 | 0 | 0 | 0 | 1.67161 | -0.186776 |
| Case 3 | 9.9021 | 0 | 0 | 0 | 0 | 0.634044 |
| Case 4 | 8.65836 | 0 | 0.573825 | 0 | 0 | 0.429104 |

Firstly, let us make a comparison between cases 1 and 2. The difference between these two is that we only ask for low effort from agent L , but in both cases allow him to set a low target. It should be made clear too that case 2 is a fully separating case where we require entirely different behaviour from both types of agent. Obviously, in both cases we use the same sets of parameters and our findings are quite interesting.

What can we say of note? Firstly that under case 1 we have monotonic wages in output, if we separate the wages into two, i.e. those under the same target claim. Precisely, $w(H, h) \succeq w(L, h) \succeq w(0, h)$ and $w(L, l) \succeq w(0, l)$. However, $w(L, h) \geq w(L, l)$ and $w(0, h) \succeq w(0, l)$ do not necessarily hold. In fact in our simulation the opposite is true in both cases. The fact that $w(L, l)$ is larger than $w(H, h)$ is driven by the form we have placed on our probabilities in the simulations. Secondly, the wages under a high target do not differ among cases. In all, a wage is paid only in the ‘best’ outcome and in all else under a high target the wage is 0. It therefore appears that

there are no effects on any of the constraints for the high type by moving from case 1 to Case 2. His incentives to conceal his type however may be another matter. This is exemplified by the fact that we have non-monotonic wages in output, in the form described above¹¹. We believe this is driven by the incentives of the high type and is to prevent him from mimicking the low type agent and acting lazily and fixing a low target. As was noted earlier on, a wage is paid under a high target only when the high outcome is achieved. This may be difficult for agent L and therefore a payment scheme in this manner should deter him from attempting to copy the behaviour of H . Furthermore, we require in Case 2 both low effort and a low target from agent L . If the able agent was to behave as such he would perhaps be more likely to achieve a low but non-zero outcome. In which case, to prevent the able agent from being lazy, we reward unproductive behaviour from L , so that we may separate between the two.

One must also mention the total cost of each scheme. Clearly, case 1 should be much more expensive to run than case 2, the wages under a low target and low outcome are significantly higher in the former and wages under the ‘worst’ possible outcome are only slightly more expensive in the latter. This can be backed up by the values to the objective function. Although both types can be separated in these cases, the former is subject to a greater degree of moral hazard. In the latter, we allow low type agents to work less hard and so they need not be compensated so highly. Although this reduces the contribution made by low types this may be outweighed by the gain from saving on the cost of wages. In which case, we may be able to suggest that allowing the less able to slack off and further distinguish themselves from those of higher ability actually helps the principal to incentivise the latter and reduces the efficiency loss from moral hazard¹². To summarise, in case 2 we are able to reduce wages under a low claim, as this should coincide with low effort, however, must now offer non-monotonic wages to incentivise such behaviour.

Let us now mention the last two cases. We introduce case 3, which is a pooling equilibrium in the truest sense, where we require identical behaviour from both types. Additionally, case 4, where we wish for the low type agent to target a high outcome

¹¹ $w(0, l) > w(L, l)$.

¹²Even though this results in the perverse outcome of non-monotonic wages.

but allow him to exert a low level of effort. Initially, we must take note that the wage under the ‘best’ outcome in case 3 is the same as in the two previous cases and that under case 4 is slightly lower. This is perhaps due to the fact that we can satisfy a low type agent’s *PC* through the use of a different wage in case 4. This alternative wage we use again gives rise to non-monotonicity of wages, which we suggest is once again designed to separate the types. Recall that we require a high target but low effort from the low types, in which case he should be rewarded with the correct incentives. A low, but positive wage is therefore offered when no output has been produced under a high target. This has been designed to satisfy the low type’s *PC* but not the high type’s, in which case we expect no mimicking on the part of the more able. Furthermore, in case 3, all other wages are set to zero, and so the agent receives a wage if and only if the best outcome occurs. This reflects the pooling nature of this particular case as we choose not to distinguish in any way between types.

One final point is to compare the values of objectives functions, not only in these last two cases, but across all. For this particular set of parameters we find that case 3 then 4 are the largest of all. The latter is lower as although we reward the agents less in the ‘best’ outcome, we offer a small payment in another¹³ and encourage low type agents to exert less effort, therefore contributing to a lower expected output. Rather interestingly though, both of these values are higher than those in the previous two cases. Although this holds true for this set of parameters, some comparative statics would be interesting to see if this is in fact a general rule. That the pooling equilibrium gives rise to the highest level of profit for the principal is also quite interesting. This could be support for the rather high requirements set upon researchers seeking tenure in the American academic system¹⁴.

The simulations above were highly informative, however we would like to extend these to a more general rule. We take the constraints of the simulation and offer some propositions that hold true under the above structure. It is straightforward to determine which *IC* and *PC* constraints are slack in the results and so can provide some interesting conditions.

¹³ $w(0, h) > 0$.

¹⁴Either our agents are paid well and in only the best outcome (tenure), or they receive nothing (contracts end and they seek work elsewhere).

3.4.3 Results

We now use the results of our simulations to aid us in characterising general expressions of solutions. We list our key results below and in the following pages offer some explanation as to what is behind such a solution. From the simulations carried out, if the restrictions above were in place, then the results below would follow:

Proposition 12 *Wages are non-monotonic when encouraging low effort from the low type agent.*

We must first consider why we would ever want an agent to exert low effort. Low effort reduces the probability of success, but high effort is more costly for the agent. Therefore we have to balance the additional compensation required for high effort versus the additional benefit to the principal. We consider only agent L exerting low effort therefore it may be the case that he is so ‘unable’ that it would only be a waste of resources to incentivise him to try hard. In other words the marginal benefit may be exceeded by the marginal cost.

Yet, another possible reasoning exists. We have at heart an adverse selection problem into which we have introduced moral hazard. We can use the moral hazard to our advantage in order to solve the adverse selection problem. It is well-known in the literature, that to solve moral hazard one should offer bonuses. If these bonuses are too large then one may prefer to allow the agents to shirk. Therefore, one can offer perverse incentives, encouraging some agents to shirk, removing the problem of moral hazard, thereby reducing the entire wage bill. We allow only those of low type, and already less valuable to the principal, to shirk. If the reduction of the wage bill is greater than the loss of value then these perverse incentives may actually be preferential. The proposition above states that whenever we decide to allow the low types to shirk, the principal should offer, at least partially, non-monotonic wages. Therefore he should take advantage of the opportunity to reduce the wage bill by offering some smaller but perverse incentive schemes. Take for example the workings of a kitchen. We have both talented, such as Guy Savoy, and less talented ‘chefs’, like myself. As hard as I try, the chance of preparing an artichoke soup with truffles to the level that he sets is almost zero. Our solution states that I should be rewarded for understanding the ‘impossibility’ of such

a claim and therefore paid whenever complete failure has occurred. This however, does encourage me to ‘burn’ output in order to receive higher wages - clearly not a desirable solution.

Proposition 13 *The principal’s payoff is larger under the pooling equilibria of targets, namely cases 3 and 4. Furthermore, the preferred setting for the principal is the most ‘demanding’ for the agent¹⁵.*

How can we interpret this? One must remember that the case we are in is determined by the principal. She decides what requirements to place on the agents she may employ. Therefore, given that she cares only of the value to her of employing said agents, she should choose the payment scheme that offers the largest return. Under the set of parameters used in our simulation this turned out to be case 3 and we present a general case below for when this holds. Essentially whenever an agent of low type adds more value by targeting the high outcome we should enforce case 3. Recall that when targeting the low outcome that is the only possible contribution one can make to total output. However when targeting the high outcome, even if not successful in that regard one may still end up with a low output. Combined with the fact that there is no cost to the agent for claiming a high target then the support for aiming high begins to build. However, as we showed above, our solution for case 3 rewarded agents only in the high outcome and their wage was zero otherwise. We therefore refer to this case as the most demanding for an agent. Given that the most valuable scheme for a principal coincides with the most demanding for an agent, we relate this to the current academic tenure system. Agents compete for the very large bonuses (tenure), knowing that if they do not succeed then in most cases they will not be able to continue working at that institution (zero wage).

Proposition 14 *In all cases, an increase in the probability of a high output for a good agent under a high target decreases the wage $w(H, h)$ and increases the principal’s payoff.*

¹⁵By this we mean that the agent is paid a wage only when the best outcome occurs, otherwise his wage is zero.

This may seem quite obvious but has much wider implications. First and foremost, the wage under the high outcome can be shown to decrease in the probability of the high type achieving that outcome. This seems quite logical, the more likely an event is to occur, the less an agent should be rewarded for it. However, in the latter two cases, we also encourage L to target the high outcome and so he may be ‘punished’ for the ability of the high type agents. In addition to this, the value to the principal is increasing in this probability. Therefore she benefits twice; firstly from a reduced wage bill, then also from a higher return.

Proposition 15 *The participation constraint for the high type agent always binds.*

This is in direct opposition to most of the literature. Typically in adverse selection problems, one would see the PC of the low agent binding and the high type would be making rent gains on his wage. Furthermore, if there were several types then the rent gains would be increasing in type. We have the opposite case. When L is requested to target the low outcome, although he incurs no direct cost, he is giving up any chance of achieving the highest possible outcome. Therefore he is also giving up any possibility of being paid the wage $w(H, h)$ and receiving the potentially high bonus on offer. We therefore have to offer compensation for forgoing such a possibility and so make sure that acting according to type dominates targeting the high bonus.

To explain the propositions above we now proceed on a case-by-case analysis. All working of the following algebra can be found in Appendix B.

Results - Case 1

From our simulations we can see that in case 1, $IC2$, $IC4$ and $PC1$ bind. Furthermore, that wages $w(H, h)$, $w(L, l)$ and $w(0, l)$ are non-zero. If we substitute the above into our binding constraints we can after a little algebra reduce to:

$$\begin{aligned}
 w(H, h) &= \frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H \cdot H(\bar{\theta}, h)} \\
 w(L, l) &= (c(e_H, \bar{\theta}) + \bar{U}) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)} + \frac{(1 - e_H \cdot L(\underline{\theta}, l)) \cdot \Delta c}{L(\underline{\theta}, l) \cdot (e_H - e_L)} \\
 w(0, l) &= (c(e_H, \bar{\theta}) + \bar{U}) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)} - \frac{e_H \cdot \Delta c}{(e_H - e_L)}
 \end{aligned}$$

What can we say of note about our solution? Firstly the wage for the able agent under a high claim is decreasing in the probability of the high outcome. This seems quite obvious. The more likely an event is to occur, the less reward the agent should receive when it does. Some interesting points however can be made about the wages under a low claim. It is decreasing in the probability of a high outcome when of high ability and increasing in the probability of a high outcome when of low ability. If it was extremely likely that one would achieve the high outcome when of low ability then in this case one must be compensated to a larger degree. This is because we wish to force L to target the low claim and so he must forgo the potential higher output in order to separate himself from H . By telling the truth and owning up, he must be rewarded. However, this is offset by the ‘discounting’ of the high type’s probability, which should mean that it is in the interests for one to act according to type.

The wage gap for the low target¹⁶ is decreasing in the effort gap, but increasing in the difference in cost of effort. As we take a binary choice of effort, the difference in magnitude between the two, affects significantly the probability of each respective outcome occurring. If high effort were only a little larger than low, then the increased probability of the larger outcome is small. In which case, agents need to be incentivised an enormous amount in order to choose to work hard. Conversely, should there be a large difference between the two, then the agent would already benefit greatly from exerting high effort, therefore does not need to be convinced financially. In addition, the more disutility from exerting higher effort, the more one should be compensated.

The wages under a low target are also increasing in the outside option of the high type. This highlights the fact that L makes rent gains, i.e. earns more than his outside option and his payoff is in some way tied to the outside option of H .

What about the value of this to the principal? After some work it can be reduced to:

$$\rho \cdot e_H \cdot H(\bar{\theta}, h) \cdot V(H) + (\rho \cdot e_H \cdot L(\bar{\theta}, h) + (1 - \rho) \cdot e_H \cdot L(\underline{\theta}, l)) \cdot V(L) - (c(e_H, \bar{\theta}) + \bar{U}) \cdot (\rho + (1 - \rho) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)})$$

¹⁶By this we mean $w(L, l) - w(0, l)$.

This is increasing in the probability of a good type whenever:

$$H(\bar{\theta}, h).V(H) + (L(\bar{\theta}, h) - L(\underline{\theta}, l)).V(L) > w(H, h).(H(\bar{\theta}, h) - H(\underline{\theta}, h))$$

We interpret this as follows. Where the extra value one brings to the firm, by an increase in type¹⁷, is greater than the additional cost incurred by the firm by employing a better type¹⁸.

The value to the principal is also increasing in $H(\bar{\theta}, h)$ and decreasing in $H(\underline{\theta}, h)$. So the principal would earn more from this scheme the more likely H is to achieve the best possible outcome, and the less successful L is. This clearly fits with the objectives we set out in this particular case. Finally and rather obviously, it is decreasing in both the cost of high effort for the high type and his outside option. Trivially - if either of these are larger he needs to be compensated more in order to sign the contract.

Results - Case 2

In case 2 we can see that constraints $IC4$ and $PC1$ bind. Therefore, in moving from case 1 to case 2 we have slackened $IC2$, the IC constraint for effort for the low type. This is obvious, since the difference between the two cases is that in the latter, we require only low effort from the low type. We also know that wages $w(H, h)$ and $w(0, l)$ are non-zero, so substituting into our binding constraints we can show that:

$$\begin{aligned} w(H, h) &= \frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H.H(\bar{\theta}, h)} \\ w(0, l) &= \frac{e_L.H(\underline{\theta}, h).(c(e_H, \bar{\theta}) + \bar{U})}{e_H.H(\bar{\theta}, h).(1 - e_L.L(\underline{\theta}, l))} \end{aligned}$$

As above, the wage for the best possible outcome under a high claim is decreasing in the probability of the high type achieving this outcome. The same can also be said for the now non-monotonically structured low target wage, the logic is as previously. This latter wage now also decreases in the probability of that particular outcome. This is consistent with the objectives placed upon the agent. We wish to encourage him to

¹⁷Given the agents stick to their target requirements.

¹⁸As in increase in type results in a larger probability of success and therefore a larger expected payoff.

exert low effort and the probability of some non-zero outcome is increasing in effort. Therefore, he must be compensated once again for complying with our demands and forgoing a potentially larger contribution to total output. His compensation therefore is larger, the less likely this outcome is to occur. Finally, as above, the wage in both states, in particular that under the zero outcome with a low target, is increasing in the outside option of the high type.

What about the value of this to the principal? After some work it can be reduced to:

$$\rho \cdot e_H \cdot H(\bar{\theta}, h) \cdot V(H) + (\rho \cdot e_H \cdot L(\bar{\theta}, h) + (1 - \rho) \cdot e_L \cdot L(\underline{\theta}, l)) \cdot V(L) - (c(e_H, \bar{\theta}) + \bar{U}) \cdot (\rho + (1 - \rho) \cdot \frac{e_L \cdot H(\underline{\theta}, h)}{e_H \cdot H(\bar{\theta}, h)})$$

If we compare this to the case above, case 1 is preferred whenever:

$$(e_H - e_L) \cdot L(\underline{\theta}, l) \cdot V(L) > (e_H - e_L) \cdot H(\underline{\theta}, h) \cdot w(H, h)$$

This means that the marginal gain of agent L exerting more effort¹⁹ should increase the total value of a low output more than his wage under a high target²⁰. The opposite condition therefore would mean that the principal prefers to use case 2 and therefore the saving in costs is larger than the gain in output. In other words, whether or not it is too expensive to encourage L to work hard.

In addition, it is increasing in the probability of a high type whenever:

$$e_H \cdot H(\bar{\theta}, h) \cdot V(H) + (e_H \cdot L(\bar{\theta}, h) - e_L \cdot L(\underline{\theta}, l)) \cdot V(L) > w(H, h) \cdot (e_H \cdot H(\bar{\theta}, h) - e_L \cdot H(\underline{\theta}, h))$$

We offer the same interpretation as previously. Where the extra value one brings to the firm, by an increase in type, is greater than the additional cost incurred by the firm by employing a better type. The only difference here is that the effort requirements for L are weaker and this is reflected in the above condition.

The value to the principal is also increasing in $H(\bar{\theta}, h)$ and decreasing in $H(\underline{\theta}, h)$. Finally and rather obviously, it is decreasing in both the cost of high effort for the high

¹⁹Note that in both cases 1 and 2, we ask for L to target the low outcome, but in the former we ask for high effort.

²⁰Under a high target a wage is paid only in the case of the ‘best’ outcome.

type and his outside option.

Results - Case 3

In case 3 we can see that, rather oddly, just *PC1* binds. Does this actually make any sense? Surely the principal can tighten one of the *IC* constraints and leave the other *PC* constraint slack. Should we not expect both a *PC* and *IC* to bind together? Nonetheless, knowing that solely $w(H, h)$ is non-zero, we can substitute this into our binding constraint to determine:

$$w(H, h) = \frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H \cdot H(\bar{\theta}, h)}$$

This does not differ from either of the previous cases, so we have nothing to add here. What about the value of this to the principal? After some work it can be reduced to:

$$\begin{aligned} & (\rho \cdot e_H \cdot H(\bar{\theta}, h) + (1 - \rho) \cdot e_H \cdot H(\underline{\theta}, h)) \cdot V(H) + (\rho \cdot e_H \cdot L(\bar{\theta}, h) + (1 - \rho) \cdot e_H \cdot L(\underline{\theta}, h)) \cdot V(L) \\ & - (c(e_H, \bar{\theta}) + \bar{U}) \cdot (\rho + (1 - \rho) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}) \end{aligned}$$

Furthermore, this case is preferred to case 1 whenever:

$$e_H \cdot H(\underline{\theta}, h) \cdot V(H) + e_H \cdot L(\underline{\theta}, h) \cdot V(L) > e_H \cdot L(\underline{\theta}, l) \cdot V(L)$$

This has quite a straightforward meaning. Whenever an *L* adds more value by targeting the high outcome than the low outcome, then the principal prefers case 3 to case 1. This could be the case when the difference between a high and low value for the principal is quite large, say the value of a human life in the example of health care above.

Once again, this value is also increasing in the probability of a high type whenever:

$$(H(\bar{\theta}, h) - H(\underline{\theta}, h)) \cdot V(H) + (L(\bar{\theta}, h) - L(\underline{\theta}, h)) \cdot V(L) > w(H, h) \cdot (H(\bar{\theta}, h) - H(\underline{\theta}, h))$$

Quite simply, when the additional value added by an increase in type, outweighs

the additional cost.

The value to the principal is certainly increasing in both $H(\bar{\theta}, h)$ and $H(\underline{\theta}, h)$ when:

$$\rho.H(\bar{\theta}, h).V(H) + (1 - \rho).H(\underline{\theta}, h).w(H, h) > 0 \quad \text{AND} \quad V(H) > w(H, h)$$

Therefore, it is increasing in the probability of H achieving a high outcome whenever the expected return of H exceeds the expected cost of L . Furthermore, it is increasing in the probability of L achieving a high outcome whenever that outcome returns more value than wage costs for the principal. Finally, it is decreasing in both the cost of high effort for the high type and his outside option.

We can make one final comment on the solution here. We find conditions for which this equilibrium offers the largest payoff for the principal, but we must note the wage structure. The agent is paid only in the high outcome. We consider this the most stringent of all solutions, but it coincides with the highest value. Therefore we suggest that this is in support of the academic tenure system, that demands be extremely tough on all, in order to produce the most valuable amount of research.

Results - Case 4

In case 4 we can see that $IC2$ and $PC1$ bind. Also that wages $w(H, h)$ and $w(0, h)$ are non-zero, therefore after some manipulation we can derive wages:

$$\begin{aligned} w(H, h) &= \frac{\Delta c}{(e_H - e_L)} \cdot \frac{(1 - e_H.(L(\bar{\theta}, h) + H(\bar{\theta}, h)))}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)} \\ &\quad + \left(\frac{(H(\underline{\theta}, h) + L(\underline{\theta}, h)).(c(e_H, \bar{\theta}) + \bar{U})}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)} \right) \\ w(0, h) &= \frac{(e_H - e_L).H(\underline{\theta}, h).(c(e_H, \bar{\theta}) + \bar{U}) - e_H.H(\bar{\theta}, h).\Delta c}{(e_H - e_L).(H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h))} \end{aligned}$$

Unfortunately, in this case, the solution does not simplify a great deal and so interpreting the algebra is a little tricky. We can at least say that both of the above wage payments are increasing in the cost of high effort and outside option of H , as per usual. However, there is a difference when we look at the marginal cost of effort for the low type. The wage under the high outcome is increasing in this difference and the wage under a zero outcome is decreasing in this difference.

Perhaps of more interest would be to determine the ‘wage gap’, then we can comment upon what affects the difference between these two wages:

$$w(H, h) - w(0, h) = \frac{\Delta c}{(e_H - e_L)} \cdot \frac{(1 - e_H \cdot L(\bar{\theta}, h))}{H(\underline{\theta}, h) \cdot (1 - e_H \cdot L(\bar{\theta}, h)) + e_H \cdot H(\bar{\theta}, h) \cdot L(\underline{\theta}, h)} + \left(\frac{L(\underline{\theta}, h) \cdot (c(e_H, \bar{\theta}) + \bar{U})}{H(\underline{\theta}, h) \cdot (1 - e_H \cdot L(\bar{\theta}, h)) + e_H \cdot H(\bar{\theta}, h) \cdot L(\underline{\theta}, h)} \right)$$

As mentioned above, the marginal cost of effort for L increases the level of incentives. The cost of high effort and outside option of H clearly affects the wage in the high outcome to a larger degree, as we can see that the level of incentives grows with both of these values. The level of incentives also decreases with the effort gap. This is similar to an explanation used previously, where the difference between the two effort levels is already very large, then the effect on outcome is significant. In which case, the agent does not need to be motivated financially to the same degree. Beyond the above, the algebra becomes quite cumbersome, and so, we refer the reader to Appendix B, in particular the condition for case 3 being preferred to case 4.

3.5 Discussion

Recent trends have seen a decline in the proportion of tenure track positions offered in the American academic sector. This is perhaps a response to general criticism of the system. Some argue that by offering tenure one removes all incentives to work hard after it has been granted. Others suggest that the requirements are much too stringent and demand standards that are too high and therefore not realistically achievable. Furthermore, perfectly good researchers, who miss out on the post then move on to a different institution meaning that they are ‘lost’ by the department who wished to employ them in the first place.

For each of these claims, there is however a reasonable response. Although perhaps difficult, it is certainly not impossible to remove an individual who has been granted tenure. In addition, several studies show that output actually increases once tenure has been given. The demands may be high, but in a competitive environment, the search for those who are most able goes on, and unfortunately, some ‘collateral damage’ is deemed acceptable. What’s more, the process itself of pushing employees to the limit

means that although some do not find themselves with permanent employment at the end, in the interim period they have maximised their research output and therefore that of the department. Surely this has to be one of the predominant concerns of a research orientated institution?

This contribution is in support of the tenure track system. We have shown conditions whereby pooling equilibria are preferred and therefore tough requirements should be placed on all, if the target is to maximise research output. Limited previous research has introduced the topics of moral hazard and adverse selection, but focuses solely on the aforementioned pooling equilibria and without limited liability. We allow also for the potential to separate out those of differing abilities and this may apply to other target-setting environments. However, we have shown that these are dominated from an output perspective by the current status quo. The suggestion is therefore a return to the widespread use of tenure-track as a means to boost research output and quality.

Furthermore, we show that should one wish to fully separate among types, then it would come at the cost of non-monotonic wage structures. These have severe negative consequences because they encourage the burning of money, literally destroying value, in order to increase one's wages. An example of this would be the case where one is required to publish in a well respected journal such as American Economic Review. If the paper is only accepted into Economic Letters, then due to the perverse wage structure, it is actually in one's interests to not publish the paper at all than to publish in this 'lesser' journal. Obviously allowing the paper to be published would increase the research output of the institution and add value to its standing. However the logic here is that it would actually tarnish said reputation, by diluting the high quality output of the department.

In addition to this, we also end up with the undesirable consequence that those of lesser ability will benefit more from this setting. If one had to choose, it would clearly be preferable to reward those who are more able above and beyond what they perhaps are worth, as opposed to those weaker candidates. This is the standard result of an adverse selection problem. However, we find that it is the type L who make rent gains and therefore benefit more from this wage structure. This appears to be driven by our target setting constraints, that incur no direct cost but can preclude a high outcome if

low targets are set. In which case, weak researchers are being retained and remunerated too highly, at the expense of those more able.

3.6 Conclusion

We have provided the framework and initial solutions towards a model that features both moral hazard and adverse selection with the inclusion of limited liability. The concept was conceived in the field of academia, in particular the demands for tenure placed upon researchers, but lends itself to almost any ‘target setting’ problem. The first point of interest that we notice is the fact that our solution is contrary to the majority of literature on adverse selection. Typically, the high type is paid more than his reservation utility and it is the low type who breaks even. In our work, the opposite holds true, with the low type making rent gains - perhaps due to the presence of moral hazard. We provide conditions whereby pooling equilibria are the preferred case for a profit-maximising principal and relate the solution to tenure contracts. We suggest the solution supports the current demanding structure of the tenure system for academia. By making more intensive demands on all candidates, an institution shall produce a better output of research. Finally, in order to separate types, in some cases we must offer wages with a non-monotonic structure. This seems perverse, but it must be used in order to encourage low levels of effort from weaker quality agents. This helps us to separate out types and actually eases both the moral hazard and adverse selection issues.

4.1 Introduction

We have looked at endogenising *PC*'s, and what should be the make up of our *IC* constraints, now we turn our attention to collaborations. More specifically we look at various forms of contractibility when two companies have a working relationship and what should affect their decision or not to integrate. Integration we think of as a profit-sharing mechanism. Firms adopt a working relationship that has many positive effects and the resulting profit that is made is shared in some way between the two.

The motives behind two firms merging are both obvious and numerous. Not only from the perspective of the firms as a whole, but also the individuals involved, seek to benefit from the decision. They should lead to various types of economies of scale, reductions in cost, technology spillovers, spreading of risk, barriers to entry and market power, but also to increased sales and share value, that are perhaps linked to the payment structure of management. The potential benefits are there for all to see, but it is clearly the latter, the search for personal gain that drives such investment decisions. This is even more evident when it is estimated¹ that between 66% and 75% of mergers

¹https://docs.google.com/viewer?a=v&q=cache:swlO62iGpAIJ:www.mckinsey.com/client_service/

fail.

Why do some mergers fail?

Several reasons have been proposed, the most prevalent when firms cite cultural differences. Firms have their own way of functioning and sometimes a more relaxed, entrepreneurial atmosphere such as Google has, may not fit with the more corporate, traditional structure found under a company such as IBM. This clash of cultures, so to speak, may lead to firms becoming aggrieved with the newfound working conditions they have to face and in response may drag their heels on joint ventures, or perhaps even choose not to share valuable working practices, that would benefit their new partners. This is perhaps most evident in the Daimler Benz/Chrysler merger of the late 1990's. The firms originated from different countries, with different languages and styles and it became impossible for any synergies to exist. Chrysler had intended to use Daimler parts and architecture to cut costs of production of future vehicles. However, Daimler's luxury division who had been earmarked to share components did not take too kindly to the idea. A few token packages were handed over with the expectation that Chrysler would leap up the standings in the North American car market. However, due to tough competition from Asian producers, little progress was made. Although the merger had been broadcast as that of equals, Daimler quickly took control and panicked, trying to resurrect falling sales. Eventually, after nearly 9 years, Chrysler was sold to a Capital Management firm who specialised in restructuring troubled companies at a loss of around \$30 billion.

Similarly, when two thriving firms in similar markets choose to merge, the lack of a clear plan for the future can also cause problems. Initially, there may have been a clear perspective on the direction that the firms individually wished to take, but when joining as one, resolving such important decisions may prove problematic. The issue therefore of how control is allocated must be determined, so that the focus on the long term goal is not lost. Take for example the merger between Quaker and Snapple. Quaker Oats, an

organization/latest_thinking/~media/mckinsey/dotcom/client_service/organization/pdfs/775084%2520merger%2520management%2520article%2520compendium.ashx+mckinsey+study+merger+failure&hl=en&gl=uk&pid=bl&srcid=ADGEESi7-XdT8oh7tG-ByVnruaJRYa3xirgOU_hliYsetoWNFMk0CoY9vdnMmpemDpoh8BJHl887F_2parbYAlVQc3Xp4RDVzSJK2VZAI-3R1ETwYZKOBlpqXUfNVWty1GY60_oRJO5w&sig=AHIEtbQkS99synL5JXdWbg0zJuqEQrJIDA

American food conglomerate had just recently been involved in a successful acquisition of Gatorade, a sports drink, and was looking to diversify within the drinks market. It decided to purchase an up-and-coming, highly successful juice drink named Snapple for \$1.7 billion in 1994, that it had intended to popularise to the same degree. A heavy outlay was made on advertising as Quaker Oats intended to stock Snapple in every grocery shop and restaurant chain that they could. However, what had made Snapple successful had been lost in the merger process and the new direction they took was disastrous. Snapple initially succeeded because they marketed to small, independent shops, the brand however could not compete in the wider drinks market in large retailers nationally. Competitors such as Pepsi and Coca-Cola themselves released similar style drinks and the general public moved away from their love of Snapple until eventually Quaker Oats cut their losses. In 1997 they decided to sell the business to Triarc for \$300 million, quite a significant loss.

Mergers may also fail because of a lack of disclosure or misrepresentation of both relevant and important information. BMW purchased Rover in 1994 for £800 million and although it cannot be regarded as a total failure, some aspects of the deal should be more closely scrutinised. At one point, a factory in Birmingham was losing the equivalent of £2 million a day and so was sold to Phoenix Consortium² for the mere sum of £10. There are several suggested reasons why the acquisition was not entirely successful. As above, a difference in cultures meant that Rover were sceptical to BMW's proposed new methods and a strong trade union was unwilling to make job cuts on the grounds of efficiency. Additionally, there were disagreements about the buyout itself of Rover. In the year after the acquisition, more directors resigned at BMW than in the previous forty. However, perhaps the most important is the hurried process with which the deal was completed. It took only 10 days for the negotiation to end and on reflection, BMW clearly had not done enough due diligence. If they had done so, they would have noticed the inaccurate sales data³ and painted a much clearer picture of the problems at Rover. They certainly would not have paid as high a price had they

²A company founded by four British businessmen who personally went on to make a significant profit from the deal.

³<http://www.tutor2u.net/blog/index.php/business-studies/comments/buss4-mergers-acquiitions-one-that-went-wrong.-bmw-and-rover>

taken longer over the decision⁴. This example of a firm presenting information that is not wholly accurate can be used for support of ‘bad’ types in our game below.

For these reasons, it is obvious, that in the early stages of a new working relationship, the actions that the party in control takes must be carefully chosen. Upsetting your new colleagues, belittling their methods, or even choosing to ignore their advice on direction of the firm can cause frictions in the partnership. Once such damage has been made it can become difficult to undo and can lead to retaliatory effects as mentioned above. Holding back important information, or being secretive over new technology can affect the profitability of the firm. The loss created that is in response to perhaps ignorant or self-interested behaviour is known as shading. The Daimler/Chrysler merger above is a quite clear example of these actions in practice.

In a recent interview with CNN, Rajan Tata⁵, the former chief of Tata Group a multinational conglomerate, made some interesting comments about his decision-making process behind a merger. Before choosing whether or not to buy a company, alot of time is spent studying the compatibility of the working ethics of the potential new acquisition with their own. “If there’s a chemistry problem, or a business policy deviation from us, we have walked away from companies which would be good business fits for us, but its method of operation would be too alien, so we have stayed away from it”. It can be seen clearly, that it is more than just financial gain that plays a role in the merger decision of some companies. There exists some sort of private benefit of like-mindedness that firms must take into account. We suggest that this private benefit actually plays an important role and may determine ultimately which party is in control of the relationship.

Who has control?

Before we can talk about giving away control we must first define the term. Control itself can take two forms. After buying, the parent company will usually introduce its own staff over time and gradually ‘fade-out’ the existing and underperforming employees.

⁴It was however not a complete disaster. Land Rover was sold to Ford for £1800 million and the Mini brand continues to be a success under BMW’s ownership.

⁵<http://www.dnaindia.com/money/1821853/report-tata-group-would-never-favour-a-hostile-takeover-ratan-tata>

In the interim period one must decide who makes the decisions. By acquiring the company, Quaker Oats gained decision-making rights, known as formal authority. They have the final say over investment strategy and all other choices that may affect the firm's performance. In the example above, had Quaker Oats let Snapple continue to run the firm as they had done, then the result may have been different. Delegating the decision-making process to Snapple directors may have meant that they kept the old but successful strategy of targeting small, independent stores that had fared so well. This is known as granting real authority. It is normally used to test the intentions of the employees, as if their jobs are at stake they should strive to work hard and for the benefit of their new employers.

When one large firm buys out another smaller company, with it, they tend to acquire decision-making rights too. However, when the firms are of similar size and instead they choose to merge, then the actual process of determining who has control is not as straightforward as one might think. It may be determined by some rule, or may actually be used as a way of encouraging one party to act in the 'greater good'. For example, two car companies may merge to form one super company, but who makes the decisions, is up for debate. Typically one may assume that those with the largest stake⁶ should have control rights and so the decision is made according to some 'measure' then contractually enforced. For example, who produces the most cars in a calendar month determines managerial decisions. This way there can be no debate over who deserves such a right and no party then has any frustration over being railroaded out of the decision-making process. This is contractible control. However, a new stream of thought and one not without merit, focuses upon the potential inability not to renege on some agreements. The suggestion is that sometimes there are no such contractible measures one can use, but still giving control away may actually lead to some sort of benefit. If we were to allow our new partners to make some or all of the decisions in a bid to prove themselves to us, it should in theory, force them to behave in a manner that should benefit the entire firm as a whole. If not, we have now learnt of their untrustworthiness and so are able to entirely remove their rights to make decisions. This is known as transferability of control.

⁶See literature below for comments on this topic.

This problem can be thought of not only as a merger between two firms, but also between divisions within a firm. Intra firm mergers are common in the restructuring process of companies, especially in the case of government reform. Take for example the UK Border Agency that was formed as a merger of the Border and Immigration Agency (BIA), UKvisas and the Detection functions of HM Revenue and Customs, all branches of the British Government. A Cabinet Office report suggested that a single border control organisation would better serve its purpose. However, the home secretary Theresa May has since declared that this led to a “secretive culture”, that “struggles with the volume of its casework”. The decision was therefore taken to split up the agency into two separate parts that focus on the visa system and on immigration law enforcement. Unsuccessful mergers therefore need not be between firms but also within.

Our study of mergers is not designed to explain the failures of the past, but how to prevent them in the future. We look at how authority should be allocated, given that these mergers will occur. We have incorporated the concept of a clash of cultures into our work with the inclusion of shading. One has prior beliefs about how a firm should be run and perhaps is used to a certain modus operandi, that when challenged displeases that party. For example, Google is renowned for its innovative culture where ‘creativity rooms’, including football tables, beanbags and lava lamps try to foster an environment for ideas. Furthermore, employee opinion is highly valued, to the extent that regular surveys about the performance of their managers are made which may result in coaching and support for those deemed the worst. Trying to integrate such a firm with one with more ‘traditional’ values and approach to business would prove difficult. Not only would Google employees be reluctant to give up some of these perks, it may not be possible to adapt the working practices of the new firm. Such tensions then give rise to disruptive or retaliatory behaviour aimed at the other party. As in the Daimler Benz/Chrysler merger above one firm may choose not to share information or technology that would benefit the firm as a whole. The acquisition of a potentially ‘bad’ firm can also be related to the BMW/Rover deal where not all information regarding the firm’s finances is entirely accurate. Had the sales data been a truer reflection of reality then the deal would have either not gone ahead, or under less costly terms. We next discuss the literature, introduce the model, then present our results. Finally, we

conclude.

4.2 Literature

When studying how to allocate control, one must first look to the work of Aghion and Tirole (1997) who noted the marked difference between real and formal authority. They defined formal authority as the right to decide, whereas real authority is the effective control over decisions. They show, under a principal-agent scenario that increasing an agent's real authority promotes initiative but results in a loss of control for the principal and additionally that the amount of communication in a firm depends on the allocation of formal authority. This work was expanded upon by Aghion et al (2004) who introduced to the literature the initial concept of transferable control. The authors defined this as a situation where one party can transfer control to another party but cannot commit herself to do so. They study the extent to which control transfers may allow an agent to reveal information regarding his ability or willingness to cooperate with the principal in the future. Under the normal process of contractible control, revelation mechanisms can be used to acquire information. Whereas when this is not possible but control is still transferable, it can be optimal to unconditionally give control to the other party and learn instead from the way that party exercises control.

Grossman and Hart (1986) show that distributing ownership rights to one party typically increases the incentives to invest of that party whilst it reduces the investment incentives of the other party. Aghion et al (2002) show that it can enhance both incentives to invest. They introduce an idea of partial contracting, as formal contracts can only influence the underlying game between the two parties. Furthermore, some actions are non-verifiable ex post and therefore not contractible, so those actions cannot be delegated by a social planner, and yet control over such actions may be (partly) contractible. Introducing ex post non-verifiability of actions reduces the power of message games and thus restores a role for ownership and control allocations. As above, they distinguish between two types of control; contractible control where one cannot contract upon an action, but on who will control the action, and transferable control where one cannot contract upon control, but can contract upon how control can be contracted.

In the latter, one party optimally decides to give some control at the start to the other in order to learn her trustworthiness and future willingness to cooperate, so that they may develop a reputation. The transferable outcome is less efficient for the party with control than the contractible case, therefore only when control is non-contractible but transferable should they choose this optimally. The main innovation is that transferable control allows control allocation to act as a reputation-building device.

Hart and Holmström (2010) develop a model based on shading in which the use of authority has a central role. There is a great deal of literature that argues that the boundaries of firms and the allocation of asset ownership can be understood in terms of incomplete contracts and property rights. Parties write contracts that are ex ante incomplete but can be completed ex post. The ability to exercise residual control rights improves the ex post bargaining position of an asset owner and thereby increases his or her incentive to make relationship-specific investments. As a consequence, it is optimal to assign asset ownership to those who have the most important relationship-specific investments. One of the limitations focused on in the paper is the assumption that ex post conflicts are resolved through bargaining with side payments. It is rare for example, for a firm to go to a competitor with the intention of extracting side payments for avoiding aggressive moves. They use authority rather than bargaining power and follow the "contracts as reference points" approach. Parties do not feel entitled to outcomes outside the contract, but may have different views of what they are entitled to within the contract. Not getting one's most favourable outcome within the contract leads them to feel aggrieved and so they shade, creating deadweight losses. Within their framework, two units have a lateral relationship and each unit is operated by a manager whose decision of action affects the other unit too. For example, the units may be deciding whether to adopt a common standard or platform for their technology or product. They consider only two aggregate outcomes, "coordination" or "noncoordination", from a binary decision of "yes" or "no" from each manager. The decision is ex ante non-contractible, but ex post contractible. The boss of each unit has the right to make the decision in that unit ex post. Under the first organisational form, non-integration, the units are separate firms and the unit managers are the bosses. Under integration the units are part of a single firm and an outside manager is the boss. They

assume that the monetary profit generated is transferable with ownership but private benefits are nontransferable. Initially they rule out profit sharing as a way to influence incentives, although it would alleviate but not eliminate the effects. Integration results in less weight being placed on private benefits than under non-integration, this is offset by the fact that, under integration, total profits, rather than individual unit profits, are maximised. They also make a distinction between "non-integration without cooperation" and "non-integration with cooperation", where shading can only occur in the latter. Conversely, shading can always occur under integration. Non-integration leads to too little coordination when the benefits from coordination are unevenly divided across the units, i.e. it may be vetoed even though it is collectively beneficial. In addition, under the assumption that coordination causes a fall in private benefits, integration leads to too much coordination. For an interesting study on limited power at firm headquarters see Rajan et al.

Hart and Holmström raise some questions about ex post non-contractibility. If a decision is ex post non-contractible, how does a boss get it carried out except by doing it herself? Second, even if decisions are ex post non-contractible, as long as decision rights can be traded ex post, it is unclear why ex ante organisational form matters (in the absence of non-contractible investments). The parties could just rely on ex post bargaining of decision rights to achieve an optimum. Finally, the "ex post non-contractibility" approach by itself does not yield an analysis of delegation.

We attempt to mix two of the papers mentioned above, that is to introduce organisational design into the Aghion et al (2002) framework. We allow for shading and private benefits à la Hart and Holmström and consider a decision to integrate on the part of the two parties (A and B) ex ante. Integration is considered solely as profit sharing, and may be used to alter the incentives of players. If the parties do not integrate, then we follow the "non-integration with cooperation" model, so that shading may still occur. As in Aghion et al we consider two types of verifiability problems, and therefore account for both methods of control (contractible and transferable). We follow their solutions when determining how to allocate control. Our focus is on when to integrate and the implications therein. Our initial idea is to use integration when a player cannot be trusted in order to alter his incentives.

4.3 Model

We consider two firms or organisations (henceforth A and B) that have a lateral relationship and are deciding whether or not to integrate. We model integration as a profit sharing mechanism, the degree to which to be determined ex ante at the contracting stage. A can take only one type, but firm B can be considered either good or bad (with probability μ), which is known privately to him, the implications of which we will explain later. A has formal authority, but may choose to give B real authority in order to learn some information about his type. They play a two stage game, where, in the first period one of the two players makes a decision about the outcome that occurs, which will determine the payoff to both firms.

Whoever has control, which is determined by A , can decide either to choose the selfish action, thereby receiving a higher payoff at the expense of the other player, or to compromise, guaranteeing both good firms some non-zero return. A bad B derives no benefit from the compromise action in the first stage. Specifically, if A were in control, then the selfish action choice would be a and if B were in control, then his selfish action choice would be b . There are no thoughts here of altruistic behaviour and therefore it is never in the interests of one player to choose the other player's selfish action choice for them. The compromise decision, action c , is equivalent to a high outcome in the project for both good players. Therefore A receives H_A , a good B receives H_B and a bad B receives 0. If the player chooses the selfish outcome then they 'steal' all profit plus some private benefit. Therefore A would receive $H_A + H_B + P_A$ and B would receive $H_A + H_B + P_B$, with the level of private benefits depending on his type. We then assume that good players will suffer, perhaps through some sort of guilt associated with punishing the other player, while bad players enjoy the fact that they have stolen from the other. Therefore private benefits for A and for good B are negative ($P_A, P_B < 0$), while private benefit for bad B is positive ($\overline{P_B} > 0$). We do however limit the punishment that good players put on themselves to less than the amount they have 'stolen', in which case, they still have an incentive to act selfishly should the game last only one period. This means that $H_A + H_B + P_A > H_A$ and $H_A + H_B + \underline{P_B} > H_B$. The incentives not to compromise are therefore greater for a bad type B than for a good type.

If the player with control in the first period has chosen to act selfishly, then the other player feels aggrieved by this decision. Therefore when we come to the second period, the aggrieved player chooses to shade, directly reducing the selfish player's payoff by that amount. For example, if B acts selfishly in period one then A shades in period two, reducing B 's payoff by an amount S_A . Similarly if A acts selfishly in the first period then his payoff in period two will be reduced by B shading by an amount S_B . Shading reduces only the private benefit, not the public benefit, so under integration, shading by one player harms only the other. This of course can clearly be extended to the public benefit being destroyed by shading. However, we considered shading as making the lives difficult of one's new co-workers, excluding them from social situations and generally creating an unpleasant environment. We therefore suggest there is no necessary impact on firm performance.

Furthermore, in the second period both players have a role in what the outcome will be as they play a variation of a prisoner's dilemma game. Now they can both choose whether or not to compromise. One can think of this as accommodating the working practices of the other and focusing on the entire firm's performance. Both players compromising rewards each player with a high outcome, H_A and H_B respectively. If they do not, then they receive some private benefit that will solely accrue to that particular player. Once again, good players would like to compromise, because they gain from working with a like-minded partner. Bad B 's however do not agree with the new structure that the firm is under, in which case they prefer not to compromise. This is thought of as the cultural differences explained by the quotes of Rajan Tata above. Also, when one player has chosen not to compromise, then the other ends up with a reduced payoff, some low outcome L_A and L_B respectively.

If both players choose not to compromise, public payoffs are reduced from the compromise state, but preferable to being 'cheated' by the other. They therefore each receive some mid-range payoff, M_A and M_B respectively. The public values from the relationship can therefore be ranked $H_i > M_i > L_i$ for $i \in A, B$. In the non-integration response, all players receive their own public benefit from the company that cannot be taken from them in period two. We use the Hart and Holmström concept of "non-integration with cooperation" where firms can still have a working relationship without

sharing profits. When they do integrate, A receives a fraction α ($0 < \alpha < 1$) of the total public benefit, and the remaining fraction of $1 - \alpha$ goes to B . One can therefore adjust the incentives of a bad B by offering him a large enough share of the total public benefits. We decide to take α as given in our model⁷. This parameter is essentially the contract between the two parties as it determines who receives what share of the profits. One could think of it's value being bargained over before the contract is signed, although we do not model this particular stage. It would therefore depend on the relative bargaining powers of the two parties. We can however make some restrictions on it's value. In the work to follow we derive several conditions, that place both upper and lower limits on the level of α . The level of profit sharing must therefore be chosen somewhere within this range.

In our setting control can either be contractible or transferable⁸ and player A responds to this state of the world with a decision to integrate with B or not. We choose to separate our problem into four different 'games' and compare across all in order to determine how agents should respond given the state of the world. The four settings are therefore all potential combinations of the type of control and the integration decision. For example, control may be transferable and A may have chosen to integrate with B . A comparison across cases will therefore allow us to determine when A would prefer to integrate given the type of control measures available. Furthermore, when he may wish to alter the incentives of a bad B too, by granting him a larger share of the profits $1 - \alpha$.

As in Aghion et al, the problem is of hidden information over the type of player B that A is contracting with. We therefore begin with A in control and place the integration decision in his hands. Our timeline is similar to their's, with an additional organisational design stage:

- Nature determines whether control is transferable or contractible
- Player A then decides whether or not to integrate at a given α
- A message game is played⁹:

⁷We leave it to further work to incorporate α as an endogenous variable into the model.

⁸Which is known by both players.

⁹As in Aghion et al.

- A (who has control) asks B his type
 - B reports (truthfully)¹⁰
- If control is contractible then it is allocated according to the game above
 - If control is transferable then the decision to transfer control is made
 - The period one game is played
 - Finally, the period two game is played

The payoffs in each period are now shown below. In Aghion et al, in the first period, all players would prefer to choose the selfish action. We follow a similar payoff structure in period one to their work.

Table 4.1: Period One Payoffs

| Action | A 's Payoff | Good B 's Payoff | Bad B 's Payoff |
|--------|-------------------|-------------------------------|------------------------------|
| c | H_A | H_B | 0 |
| a | $H_A + H_B + P_A$ | 0 | 0 |
| b | 0 | $H_A + H_B + \underline{P_B}$ | $H_A + H_B + \overline{P_B}$ |

In the second period the payoffs of the game take one of two types, either we have or have not integrated. The values below do not include the levels of shading that are a response to the action choice in period one. Furthermore, B 's private benefit depends on their type as mentioned above.

Table 4.2: Period Two Payoffs

| | Integration | B | |
|-----|------------------------|--|--|
| | | Compromise | Non-Compromise |
| A | Compromise | $\alpha.(H_A + H_B), (1 - \alpha).(H_A + H_B)$ | $\alpha.(L_A + H_B), (1 - \alpha).(L_A + H_B) + P_B$ |
| | Non-Compromise | $\alpha.(H_A + L_B) + P_A, (1 - \alpha).(H_A + L_B)$ | $\alpha.(M_A + M_B) + P_A, (1 - \alpha).(M_A + M_B) + P_B$ |
| | Non-Integration | B | |
| | | Compromise | Non-Compromise |
| A | Compromise | H_A, H_B | $L_A, H_B + P_B$ |
| | Non-Compromise | $H_A + P_A, L_B$ | $M_A + P_A, M_B + P_B$ |

¹⁰As in Aghion et al, we focus only on the equilibria in which B has the incentive to truthfully report his type. We include in the Appendix the conditions that must be satisfied for the results to hold true, both when control is contractible and transferable.

As is quite clear from the above tables, following the setup of Aghion et al, all good players have an incentive to compromise in the second period. However, a bad B will definitely prefer to not compromise in the case of non-integration. What we can show is that under integration, we can change the incentives of a bad B by offering him a large enough share of the profits. What however is missing from the table above is the inclusion of shading¹¹. If for example A had control in period 1 and chose the selfish action a , then every payoff of A 's in period 2 would be reduced by the value of B 's shading S_B ¹². For each of the treatments (contractible and transferable control) we can determine which organisational design is preferred and hence how to respond to either case.

4.4 Solution and Discussion

We follow the form of solution in Aghion et al and determine conditions whereby player A would prefer integration over non-integration. We summarise with the following propositions, the proofs of which can be found in Appendix C.

Proposition 16 *Integration increases the incentives for a bad B to truthfully signal his type, independent of the level of profit sharing. The same can only be said of a good B if his share of the profits is sufficiently large.*

Proposition 17 *Under contractibility player A would prefer to integrate with player B whenever*

$$\frac{\alpha}{1 - \alpha} > \frac{\frac{(H_A + H_B + P_A - S_B) \cdot \mu \cdot L_A}{H_A + H_B + P_B} + \mu \cdot M_A + (1 - \mu) \cdot H_A}{\frac{(H_A + H_B + P_A - S_B) \cdot \mu \cdot H_B}{H_A + H_B + P_B} + \mu \cdot M_B + (1 - \mu) \cdot H_B}$$

Proposition 18 *Under transferability player A would prefer to integrate with player B whenever*

$$\frac{\alpha}{1 - \alpha} > \frac{\mu \cdot M_A + (1 - \mu) \cdot H_A}{\mu \cdot M_B + (1 - \mu) \cdot H_B}$$

In both cases we can quite clearly show that the payoff under contractibility is preferred to transferability as in Aghion et al. One must also take note of some results of the above propositions.

¹¹The exact payoffs in all possible states of the world can be found in Appendix C.

¹²Similarly, A shades B 's payoff by the value S_A whenever B had control and choose the selfish action.

Corollary 19 *In both cases, whenever B is certainly of good type, then for A to strictly prefer integration, he must take more than his contribution to total profits under the outcome of compromise.*

What does this mean? A knows that B is of good type, in which case we can be certain that the outcome will be to compromise for both players. Therefore, if the firms do not integrate, they each get the high outcome in the second period, H_A and H_B respectively. If they integrate then this is shared between them according to the level of α . For A to prefer to integrate then he must claim more than his contribution to public benefits. If for example, $H_A = H_B$, then α must be more than half. Once the active firm A has investigated in great detail the behaviour of their new partner and are assured that he is a good fit for them, then the only reason for them to integrate would be if they took a share of B 's public benefit. This is justification for an acquisition. One good company may buy another and therefore claim all rights to the profits for themselves. Clearly, in reality, discovering the true identity of B is time-consuming and costly. Note the example of Rover and BMW. BMW had a prior opinion over the type of partner they were dealing with and so decided not to invest in verifying their belief. Had they done so, they may have chosen to motivate Rover differently.

Corollary 20 *Under transferability, whenever B is certainly of bad type, then for A to strictly prefer integration, he must take more than his contribution to total profits under the non-compromise outcome. Furthermore, A would be willing to accept a smaller fraction of the profits under contractibility (in comparison to transferability) and still prefer integration.*

The important part of this proposition is the second half. This tells us that whenever control rights are contractible, A would prefer to integrate for a smaller fraction of the profits than if control was only transferable. This clearly makes a lot of sense. Transferability involves a game of trust where B is encouraged to prove himself. A has no real knowledge of B 's type and therefore in order for him to wish to share profits, A must be compensated with a larger proportion himself. This is premium for taking a chance on B .

Now let us look at the individual incentives of a bad type B . Under non-integration, he will always prefer to not compromise and given the structure, there is no way that

we can incentivise him to change his mind. However, under the case of integration, his incentives can be altered just by being given a larger share of the profits. This however, is only possible if the value of private benefit a bad B receives is not too large. Specifically, if we set $1 - \alpha = \frac{\overline{P}_B}{H_A - L_A}$ then a bad B will be indifferent between compromising and not. It must be noted however, this is only possible if $\overline{P}_B < H_A - L_A$. We know also that if B were to choose to compromise rather than not compromise, then A 's payoff would rise. Therefore, if, for this particular level of profit sharing then the above condition is satisfied, then not only would A prefer to integrate, but also a bad B would compromise. This gives rise to the following proposition:

Proposition 21 *If B 's fraction of the profits is larger than his contribution to total public profits (including a non-compromising second period) then for A to prefer to integrate and B to certainly compromise, irrespective of B 's type, under transferability, then the probability of a bad B must exceed that of a good B .*

We must make clear that this is a necessary, but not sufficient condition. Let us now explain this proposition, and then we can begin to interpret its meaning. A good B does not need convincing to alter his choice of actions, but a bad B does. Player A however does not know the type of player he is dealing with. Therefore, to guarantee that all B 's compromise, he should promise a certain fraction of the profits, given above¹³. This is only possible if a bad B 's private benefits are not too large. If this can be done however, B is paid this amount and A may then choose to integrate. If the resulting share B ends up with is more than his contribution to public benefits, then A will only integrate, with all B 's compromising, if B is more likely to be bad than good. A is then left with a fraction less than what he contributed to total profits under a non-compromise second period outcome. Although now, the level of total profits has increased, which may offset this. In plain terms, a smaller fraction of a larger number may exceed a larger fraction of a smaller number.

It is a large private benefit that forces the fraction B must receive to compromise to increase. Whenever A chooses to integrate, he also chooses to heavily incentivise B . In a perfect world he would only wish to do this whenever B is bad, but given that this

¹³If he does so, a bad B will be indifferent between compromise and not, so we assume that he will compromise.

is private he must do so all of the time. He only makes potential gains when B is bad, therefore he weighs up the level of incentives he has to offer against the chance that B is bad. It is only worth his while to offer B a larger fraction than his contribution if B is more likely to be bad than good. Player B then may benefit from attempting to portray himself as a bad type. That way, if A falls for his ruse, he may integrate with B and offer large incentives to alter his action choice. A good B will therefore end up benefitting significantly from such behaviour. This perhaps explains the reasoning behind the Tata Group's reluctance to merge with some firms even if from a business sense there could be some profitability. There was a clash of culture to such a degree that working together would be so uncomfortable for them, that one party would prefer not to compromise on any working practices. The only way that their methods could be changed would be through significant financial rewards, compensating them for their 'private benefits', that would prove too expensive for Tata. They therefore chose to abandon such potential mergers.

4.5 Conclusion

We offer a first step towards a model that mixes two important papers on organisational design. We introduce profit sharing (through integration), shading and private benefits into a model that studies two different types of control allocation. Firstly, we show that integration can help to separate types. More specifically, it increases the incentives of a bad partner to truthfully signal his type, independent of the share of profits he receives. We find that transferable control would result in less profit sharing for the acquired firm in order to compensate one for the trust it has placed in the other. Incorporating several features of merger behaviour into a prisoner's dilemma style game, we offer an explanation as to why mergers fail and why some may not even go ahead, even if profitability may not be a concern. The final proposition is then related to the comments of Rajan Tata who suggested that many other variables, such as human chemistry, must be considered before a merger of two firms may go ahead.

CHAPTER 5

Concluding Remarks

We have presented three chapters that, themselves are unrelated, but can all be placed within the wider topic of incentive design. A summary of their results follows. Our initial contribution aims to target and augment something fundamental within contract theory - the participation constraint. The participation constraint is designed to ensure that all agents who sign the contract do so because it is in their interests to. Specifically, so that they receive more than some pre-determined acceptable value, known as a reservation utility or outside option. This can take the form of alternative contract offers, or the utility one receives from doing something else outside of the contract signed. This value is exogenously imposed on the agent and we believe is a major shortcoming of incentive design. Our innovation is to suggest that the agent themselves can affect the contract offers that they receive and therefore their outside option through the use of influence activities. This however comes at the cost of spending less time on the work that one is contracted to do. Allowing the agent to behave in such a way therefore worsens the agency problem, but also provides an instrument to insure himself. The resulting contract design is then related to the literature on risk and incentives trade-off.

Typically, agents are assumed not to like much variability in their wage payments

and so are modelled as risk-averse. Furthermore, it is argued that increasing the risk in the contractual environment should result in less incentives. Suppose we fully incentivise the agent and say that he is paid a fraction of the outcome of the project he is working on. The more risky the project, the more variability in its value and hence the more variability in the wage payments to the agent. Given that he is risk-averse he suffers from such a method of remuneration and therefore must be offered a larger payment, known as a risk premium. In which case, the principal should shelter him from some of the risk and offer some fixed payment, with a smaller fraction of the wage depending on the outcome. Therefore, more risk, less incentives. However, Prendergast (2002) questioned this result and observed that in practice the opposite relationship most often held. Whenever the environment was more risky, agents were in fact offered larger incentives. Many explanations were offered, but perhaps the most resonant was that in more risky settings, the principal themselves had even less idea about what sort of behaviour was required from the agent. Therefore, she delegated action choices to the agent and judged their success on the only variable possible - the outcome. Our contribution is in support of Prendergast's, however we offer an alternative explanation.

We consider the importance of our outside option as an indicator of the risk within the contract. Given that it is the minimum one would be willing to accept from the contract, it can be thought of as a lower bound for the wage of the agent. Increasing its value, increases the lower bound of the agent's wage and therefore provides him with more insurance. In our model, an environment with more insurance is one with less risk. We therefore compare cases where an agent is able to influence his outside option to varying degrees and find that the weaker one's outside option and the greater the consequences of not improving said outside option, the larger incentives one should receive. The basic premise is that if one forgoes the insurance that they could provide themselves outside of the contract through the outside option, then they should take a larger proportion of the earnings. Therefore offering an alternative to the Prendergast observation of more risk more incentives.

There are several areas of further work that one could embark upon. We have modelled the productive effort and influence activities as substitutes, however one could think of them as (at least partial) complements. We have not solved such a case,

however our conjecture is that the agency problem may not worsen as severely as in our case. The influence activities are also costless to the agent and with the use of our time constraint perhaps plays a significant role in the solution we find. Should we make choosing to exert any sort of action costly¹ and only free time as costless this may change our results. This change would also need to be introduced with another enrichment of the model, allowing a continuous level of effort. Finally, we have a relatively stringent bargaining game, that is tied precisely to the outside option of the agent. Although we feel it has been justified, we leave improvements in this area to further work.

In our next contribution we introduced a model that included both moral hazard and adverse selection. Not only is incentive design complicated by the fact that agents are able to conceal their action choice, but also information about themselves. The latter plays an important role in their value to the company they work for. Agents of different types suffer different costs of effort, probabilities of success or willingness to deviate. We consider variation in type through the ability of the agent. An agent that is more able not only has a lower disutility of effort but is also more likely to achieve a higher outcome. Therefore, a principal suffers from the fact that she is unaware whether the agent she employs is in fact able or not. Various methods of signalling and screening have been proposed to solve this problem individually, but little work considers the two problems² together.

The work however that does exist is lacking at least one fundamental refinement that we include. Guesnerie et al (1988) suggested that the addition of moral hazard to adverse selection does not create welfare losses between a risk-neutral agent and principal. However in order to guarantee such a result impose very large punishments on misbehaving agents. Since the publication of their work, Innes (1990) showed that for moral hazard to take any effect one must have either a risk-averse agent or risk-neutrality **and** limited liability. Limited liability is a form of protection to the agent so that he is not liable for anything over and above his level of investment into the project. This therefore means that punishments may not be too severe and ultimately that any wage he receives is non-negative. We impose limited liability in our framework.

¹So both effort and influence activities are equally costly to the agent.

²Moral hazard and adverse selection.

Our results are applied to the tenure track system of academia. Typically in America, researchers compete for tenure, a ‘job-for-life’, and are required to meet several tough criteria in order to guarantee such a position. Should they achieve this, they have academic and economic freedom to pursue any line of research they deem of interest. This should lead to an increase in the quality and output of research but has been criticised by some for removing all incentives on the part of the researcher. He becomes more difficult to remove from his position therefore is more able to neglect all obligations that he may have to the department. The merits of tenure contracts are heavily debated, however our work supports their use.

Our contribution introduces a target-setting problem, that lends itself well to the environment described above. The principal employs an agent that can conceal both his actions and type. In order to deal with the adverse selection problem we encourage the agent to publicly and costlessly declare a target of output, designed to reveal his type. To counter the moral hazard problem, as is normal in the literature we offer large enough bonuses. The principal cares only for the value of output produced by the agent and therefore should design a contract in order to maximise this. However, we solve the problem by first asking what behaviour the principal should wish to encourage from agents of either type. We therefore separate the problem into four cases through a more detailed set of incentive compatibility constraints and reach several interesting conclusions.

Firstly, for given conditions, the pooling equilibria offer the largest returns to the principal and in the strictest possible way. This we conclude is in support of the academic tenure structure. All agents, irrespective of type are expected to target high outcomes and are rewarded only in the case of complete success. Therefore, agents are either highly successful and receive tenure, or are not, so are paid nothing and are expected to look for employment elsewhere. Also, our high type agents only ever break even and it is those of low type who make rent gains, contrary to the literature. It is the target setting constraints in our model that drive such a result. To ensure that low type agents act according to type, they have to be heavily incentivised and therefore earn more than they ‘should’ in expectation. Finally, in order to fully separate among types, low type agents must be offered non-monotonic wage structures. This means that in

some cases agents have an incentive to destroy some value for the principal so as to guarantee themselves a higher wage. How do we explain this? To encourage less able agents to help separate themselves and admit their type, they should be rewarded, and by doing this, we can actually reduce our wage bill and make it even more attractive for high types to act appropriately.

There are however some restrictions in our findings that with further work we hope to address. By declaring a low target, we completely rule out the possibility of a high outcome. We hope to extend this such that there will then be an epsilon chance of a high outcome, not an impossibility. Also, in order to fully characterise a solution, we needed to run a simulation, that guided our form of solution. We hope to find a much more general result without the need for simulations, fully characterising all potential results. Similarly, it would be interesting to see if we do find similar results to the previous literature in terms of welfare losses. We wish to isolate the effects of adverse selection and moral hazard then see if the situation is actually worsened or improved by the introduction of the latter.

The last chapter we have presented, studies the allocation of control in conjunction with integration decisions made by firms. The motivation for two firms merging is obvious. They can benefit in several ways such as economies of scale, spreading of risk and market power. However, why is it that we see a significant proportion, some studies suggest over two thirds, of mergers failing? One suggestion is that cultural differences in working practices mean that some firms are severely incompatible even if they seem profitable on paper. Forcing a previously successful firm to adopt a new procedure when it comes to running their business can upset the new partners and result in the breakdown of any sort of working relationship. They may then respond by purposely becoming difficult to work with and concealing any useful information that may help their new partners. We can think of there existing some measure of how compatible two firms are when it comes to a working relationship, suggesting that it is more than profitability that should determine such integration decisions. From this stream of thought we include a private benefit that captures the like-mindedness of firms and may go some way to deciding whether or not to integrate.

This model is complicated by the variation we put on how control can be allocated

between the two parties. If there exists some measure of how significant the contribution of each party is to the venture, then it may be straightforward to determine who should have the decision rights. Conventionally, that party with the largest investment should be granted such a privilege. However, in some cases, such a measure may not exist, but there may still be a gain from giving control to the other party and allowing them to prove themselves in the relationship. The party who has initial decision rights has formal authority, but by giving up this privilege it gives real authority to the other. This is referred to as transferability of control and is less favourable than the formerly described situation - contractibility of control. We blend together two important works in this subject, Aghion et al (2002) and Hart and Holmström (2010), to introduce organisational design from the latter into the former.

Our contribution indicates when one party would like to integrate, i.e. share profits, with another, who may be either 'good' or 'bad'. Firstly, it can help to identify the type of partner we are dealing with, by increasing a bad partner's incentives to truthfully reveal his type. We can show that the decision to integrate is more likely under a contractible measure of control. By this we mean that a larger share of the profits is required by the decision-making firm when control is only transferable. This is in agreement with the previous literature as transferability requires placing some trust in the other party to prove themselves as a good partner. Furthermore, we show that the incentives of a bad player can be altered only after integration and it may be the case that paying this extra expense in the form of a higher share of the profit is in the interests of the decision-making firm. However, doing so, will only occur whenever the 'bad' partner is more likely than the 'good'.

This chapter is a first step towards a deeper understanding of the failure of mergers. We wish to carry out more analysis, specifically around the impact of shading. In our work, shading is a response to forcing a particular type of behaviour upon the partner early on in the relationship and affects only the other player. We wish to extend this to affecting the total value of the project. The result of this is that integration may actually deter shading, even if one player feels aggrieved by the decisions of the other. The player with decision rights may therefore be able to impose some structure upon the other without any consequences.

Appendix A

Proof of Proposition 1. This is an extremely straightforward result, following the work of Fudenberg and Tirole (1990) and is entirely expected. We omit any formal proof due to its triviality. ■

Proof of Proposition 2. Once again, a straightforward result of the first-order conditions. As the participation constraint is binding, all renegotiated outcomes are tied to the agent's outside option. ■

Proof of Proposition 3(i). It is simple to show that both constraints bind. From the incentive compatibility constraint we can derive a value for the agent's utility under high effort:

$$u_1 = \frac{c + (p_0\pi_1 + (1-p_0)(1-\pi_1)) \cdot u_0 + ((1-p_0)\pi_1 - (1-p_1)\pi_0) \cdot \bar{U}_H + (p_0(1-\pi_1) - p_1(1-\pi_0)) \cdot \bar{U}_L}{(p_1\pi_0 + (1-p_1)(1-\pi_0))}$$

Similarly, from the participation constraint we can derive a similar value:

$$u_1 = \frac{c + (\pi_1 - (1-p_1)\pi_0) \bar{U}_H + ((1-\pi_1) - p_1(1-\pi_0)) \bar{U}_L}{(p_1\pi_0 + (1-p_1)(1-\pi_0))}$$

Equating the two expressions we can rearrange and find an expression for the agent's

utility under low effort:

$$u_0 = \frac{p_0\pi_1\bar{U}_H + (1-p_0)(1-\pi_1)\bar{U}_L}{p_0\pi_1 + (1-p_0)(1-\pi_1)}$$

Clearly, this is a convex combination of the high and low outside option. ■

Proof of Proposition 3(ii). Given that by assumption a high outside option is strictly larger than a low outside option, ($\bar{U}_H > \bar{U}_L$), it follows that the agent's utility under low effort must lie strictly inbetween the two outside options ($\bar{U}_H > u_0 > \bar{U}_L$). Now let us recall one of our expressions for u_1 :

$$u_1 = \frac{c + (\pi_1 - (1-p_1)\pi_0)\bar{U}_H + ((1-\pi_1) - p_1(1-\pi_0))\bar{U}_L}{(p_1\pi_0 + (1-p_1)(1-\pi_0))}$$

After some algebra we can show that $u_1 > \bar{U}_H$, whenever:

$$c > (\bar{U}_H - \bar{U}_L)((1-\pi_1) - p_1(1-\pi_0))$$

In which case, as long as c is at least some fraction of the difference between a high and low outside option, then $u_1 > \bar{U}_H > u_0 > \bar{U}_L$. The coefficient of interest on $\bar{U}_H - \bar{U}_L$ actually is significant later on, and our condition (5.3) to follow, states that this coefficient must be non-negative. It is clearly less than 1, and so is bounded between 0 and 1³. So it is clear that the payment under high effort is increasing in the cost borne by the agent when choosing to exert high effort. ■

Proof of Corollary 4. We must now decide if the principal would actually like to implement either of the contracts described above, i.e. if they are preferred from the point of view of the principal to just offering the agent a flat wage. If the outside option is stochastic and the values to the principal are \bar{S} and \underline{S} and high effort is chosen with probability x , then the expected surplus is:

$$x \cdot (p_1 \cdot \pi_0 \cdot (\bar{S} - u_1) + p_1 \cdot (1 - \pi_0) \cdot (\bar{S} - \bar{U}_L) + (1 - p_1) \cdot \pi_0 \cdot (\underline{S} - \bar{U}_H) + (1 - p_1) \cdot (1 - \pi_0) \cdot (\underline{S} - u_1)) + (1 - x) \cdot (p_0 \cdot \pi_1 \cdot (\bar{S} - u_0) + p_0 \cdot (1 - \pi_1) \cdot (\bar{S} - \bar{U}_L) + (1 - p_0) \cdot \pi_1 \cdot (\underline{S} - \bar{U}_H) + (1 - p_0) \cdot (1 - \pi_1) \cdot (\underline{S} - u_0))$$

In case of a flat wage offered to the agent (equal to his expected outside option),

³For very small values of c , so for example $c = 0$, then it may be true that $u_1 < \bar{U}_H$. Furthermore, if the converse of our condition ** were to be true, then the ranking given in our proposition would hold for sure.

the principal's surplus would be:

$$(p_0 \cdot \pi_1 + p_0 \cdot (1 - \pi_1)) \cdot (\bar{S} - \pi_1 \cdot \bar{U}_H - (1 - \pi_1) \cdot \bar{U}_L) + \\ ((1 - p_0) \cdot \pi_1 + (1 - p_0) \cdot (1 - \pi_1)) \cdot (\underline{S} - \pi_1 \cdot \bar{U}_H - (1 - \pi_1) \cdot \bar{U}_L)$$

So the principal will offer the bonus contract as long as the expected surplus from the bonus contract is greater than the expected surplus from the flat wage. Comparing the two surpluses, gives us the following condition that the principal will offer the bonus contract if:

$$x \cdot (p_1 - p_0) \cdot (\bar{S} - \underline{S}) \geq \\ x \cdot u_1 \cdot (p_1 \cdot \pi_0 + (1 - p_1) \cdot (1 - \pi_0)) + x \cdot (p_1 \cdot (1 - \pi_0) \cdot \bar{U}_L + (1 - p_1) \cdot \pi_0 \cdot \bar{U}_H) + \\ (1 - x) \cdot u_0 \cdot (p_0 \cdot \pi_1 + (1 - p_0) \cdot (1 - \pi_1)) + (1 - x) \cdot (p_0 \cdot (1 - \pi_1) \cdot \bar{U}_L + (1 - p_0) \cdot \pi_1 \cdot \bar{U}_H) \\ - (\pi_1 \cdot \bar{U}_H + (1 - \pi_1) \cdot \bar{U}_L)$$

The expected gain for the principal of encouraging the agent to exert high effort should exceed the expected cost of doing so, namely the difference between the contract and his outside option. See Laffont and Martimort (2002). ■

Proof of Proposition 5. Some states of the world are now no longer viable, which simplifies the possibilities enormously. A high (low) outside option is only possible when high (low) levels of influence activities have been undertaken. It is furthermore trivial to show that the now simplified incentive compatibility and participation constraints are again binding. We proceed as in proposition 3 by deriving and then equating two differing values for utility under high effort from each of the constraints. We thus end up with a value for utility under low effort of:

$$u_0 = \left(\frac{1 - \pi_1}{p_0 \cdot \pi_1} + 1 \right) \cdot \bar{U}_H$$

However, a deterministic outside option means that under high levels of influence activities, a high outside option is certain, in which case, $\pi_1 = 1$, and conversely $\pi_0 = 0$. Therefore $\implies u_0 = \bar{U}_H$

We had previously derived a value for u_1 , which we can now simplify to:

$$u_1 = \frac{c + \bar{U}_H - p_1 \cdot \bar{U}_L}{(1 - p_1)}$$

This is larger than \bar{U}_H whenever:

$$c + (\bar{U}_H - \bar{U}_L) \cdot p_1 > 0$$

Which is clearly true and therefore we have derived the relationship claimed above.

■

Proof of Corollary 6. The expected surplus for the principal if a bonus contract is used:

$$\begin{aligned} & x \cdot (p_1 \cdot (1 - \pi_0) \cdot (\bar{S} - \bar{U}_L) + (1 - p_1) \cdot (1 - \pi_0) \cdot (\underline{S} - u_1)) \\ & + (1 - x) \cdot (p_0 \cdot \pi_1 \cdot (\bar{S} - u_0) + (1 - p_0) \cdot \pi_1 \cdot (\underline{S} - \bar{U}_H)) \end{aligned}$$

Under the flat wage, the agent would receive his outside option, so because it is deterministic, \bar{U}_H . Therefore, once again, as he always exerts low effort, the surplus for the principal of this flat contract is:

$$p_0 \cdot (\bar{S} - \bar{U}_H) + (1 - p_0) \cdot (\underline{S} - \bar{U}_H)$$

Therefore, the bonus contract would be offered whenever the expected surplus for the principal from using a bonus contract is greater than surplus for the principal from using a flat contract. Given $u_1 = \frac{c + \bar{U}_H - p_1 \cdot (1 - \pi_0) \bar{U}_L}{(1 - p_1) \cdot (1 - \pi_0)}$, $u_0 = \bar{U}_H$ and $\pi_0 = 0$, this means that:

$$\begin{aligned} & x \cdot (p_1 \cdot (1 - \pi_0) \cdot (\bar{S} - \bar{U}_L) + (1 - p_1) \cdot (1 - \pi_0) \cdot (\underline{S} - u_1)) \\ & + (1 - x) \cdot (p_0 \cdot \pi_1 \cdot (\bar{S} - u_0) + (1 - p_0) \cdot \pi_1 \cdot (\underline{S} - \bar{U}_H)) \geq \\ & p_0 \cdot (\bar{S} - \bar{U}_H) + (1 - p_0) \cdot (\underline{S} - \bar{U}_H) \end{aligned}$$

Which can be reduced to $(p_1 - p_0) \cdot (\bar{S} - \underline{S}) > c$. So the marginal gain for the principal is larger than that for the agent, which is just the cost he has to be compensated with.

■

Proof of Proposition 7. We wish to show the conditions when the bonus is larger for the deterministic than the stochastic outside option. Under the deterministic outside option, the bonus, or wage gap is:

$$u_1 - u_0 = \frac{c + \bar{U}_H - p_1 \cdot \bar{U}_L}{(1 - p_1)} - \bar{U}_H = \frac{c + (\bar{U}_H - \bar{U}_L) \cdot p_1}{1 - p_1}$$

Similarly, for a stochastic outside option:

$$u_1 - u_0 = \frac{c}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} + \left(\frac{(\pi_1 - \pi_0)}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} + \frac{p_1 \pi_0}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} - \frac{p_0 \pi_1}{p_0 \pi_1 + (1 - p_0)(1 - \pi_1)} \right) \cdot (\bar{U}_H - \bar{U}_L)$$

In which case, the former exceeds the latter whenever:

$$\frac{c + (\bar{U}_H - \bar{U}_L) \cdot p_1}{1 - p_1} > \frac{c}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} + \left(\frac{(\pi_1 - \pi_0)}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} + \frac{p_1 \pi_0}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} - \frac{p_0 \pi_1}{p_0 \pi_1 + (1 - p_0)(1 - \pi_1)} \right) \cdot (\bar{U}_H - \bar{U}_L)$$

We can break this up into two conditions that if they both hold, then this is true:

$$\frac{c}{1 - p_1} > \frac{c}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} \quad (5.1)$$

$$\frac{p_1}{1 - p_1} \cdot (\bar{U}_H - \bar{U}_L) > \left(\frac{(\pi_1 - \pi_0)}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} + \frac{p_1 \pi_0}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} - \frac{p_0 \pi_1}{p_0 \pi_1 + (1 - p_0)(1 - \pi_1)} \right) \cdot (\bar{U}_H - \bar{U}_L) \quad (5.2)$$

After some straightforward algebra, we can show that (5.1) holds $\iff p_1 > \frac{1}{2}$ and (5.2) holds:

$$\iff \frac{p_1}{1 - p_1} > \left(\frac{(\pi_1 - \pi_0)}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} + \frac{p_1 \pi_0}{p_1 \pi_0 + (1 - p_1)(1 - \pi_0)} - \frac{p_0 \pi_1}{p_0 \pi_1 + (1 - p_0)(1 - \pi_1)} \right)$$

Assuming (5.1) holds, then $p_1 > \frac{1}{2} \implies \frac{p_1}{1 - p_1} > 1$.

A sufficient but not necessary condition is therefore to show that the right hand side of (5.2) ≤ 1 . This becomes a little complicated and therefore, we have two further

conditions that if satisfied ensure that the right hand side of (5.2) ≤ 1 . The first is:

$$(1 - p_1)(1 - \pi_0) \cdot (1 - p_0)(1 - \pi_1) \geq (\pi_1 - \pi_0)(1 - p_0)(1 - \pi_1)$$

This can be reduced to the following condition, that we made reference to in an earlier proof:

$$(1 - \pi_1) \geq p_1(1 - \pi_0) \tag{5.3}$$

The second is:

$$2p_0\pi_1 \cdot (1 - p_1)(1 - \pi_0) \geq p_0\pi_1 \cdot (\pi_1 - \pi_0 - p_1\pi_0)$$

This condition can be reduced to:

$$(1 - \pi_1) \geq p_1(1 - \pi_0) - p_1\pi_0$$

Therefore if (5.3) holds, then so does the above relationship.

So if $p_1 > \frac{1}{2}$ and (5.3) holds, then we have that higher incentives are present under the deterministic outside option. The combination of these two conditions together is analogous to the difference between π_1 and π_0 being not too high. Grouped into one, we can place both an upper and lower bound on the potential values of p_1 :

$$\frac{(1 - \pi_1)}{(1 - \pi_0)} \geq p_1 > \frac{1}{2}$$

■

Appendix B

To make our algebra clear in chapter two, under case 1, our constraints are as follows:

IC constraints for effort for both types:

$$e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta})$$

$$\geq e_L.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_L, \bar{\theta})$$

and

$$e_H.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_H, \underline{\theta})$$

$$\geq e_L.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_L, \underline{\theta})$$

IC constraints for target setting for both types:

$$e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta})$$

$$\geq e_H.(L(\bar{\theta}, l).w(L, l) + (1 - L(\bar{\theta}, l)).w(0, l)) - c(e_H, \bar{\theta})$$

and

$$e_H.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_H, \underline{\theta})$$

$$\geq e_H.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_H, \underline{\theta})$$

Participation constraints for both types:

$$\bar{U} \leq e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta})$$

and

$$\underline{U} \leq e_H.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_H, \underline{\theta})$$

Which constrain the maximisation of the principal's problem:

$$\rho.(H(\bar{\theta}, h).e_H.(V(H) - w(H, h)) + L(\bar{\theta}, h).e_H.(V(L) - w(L, h))$$

$$+ (1 - e_H.(H(\bar{\theta}, h) + L(\bar{\theta}, h)).(-w(0, h))))$$

$$+ (1 - \rho).(L(\underline{\theta}, l).e_H.(V(L) - w(L, l)) + (1 - e_H.L(\underline{\theta}, l)).(-w(0, l)))$$

For case 2:

IC constraints for effort for both types:

$$\begin{aligned} & e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta}) \\ \geq & e_L.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_L, \bar{\theta}) \end{aligned}$$

and

$$\begin{aligned} & e_L.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_L, \underline{\theta}) \\ \geq & e_H.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_H, \underline{\theta}) \end{aligned}$$

IC constraints for target setting for both types:

$$\begin{aligned} & e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta}) \\ \geq & e_H.(L(\bar{\theta}, l).w(L, l) + (1 - L(\bar{\theta}, l)).w(0, l)) - c(e_H, \bar{\theta}) \end{aligned}$$

and

$$\begin{aligned} & e_L.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_L, \underline{\theta}) \\ \geq & e_L.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_L, \underline{\theta}) \end{aligned}$$

Participation constraints for both types:

$$\bar{U} \leq e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta})$$

and

$$\underline{U} \leq e_L.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_L, \underline{\theta})$$

Which constrain the maximisation of the principal's problem:

$$\begin{aligned} & \rho.(H(\bar{\theta}, h).e_H.(V(H) - w(H, h)) + L(\bar{\theta}, h).e_H.(V(L) - w(L, h))) \\ & + (1 - e_H).(H(\bar{\theta}, h) + L(\bar{\theta}, h)).(-w(0, h)) \\ & + (1 - \rho).(L(\underline{\theta}, l).e_L.(V(L) - w(L, l)) + (1 - e_L.L(\underline{\theta}, l)).(-w(0, l))) \end{aligned}$$

For case 3:

IC constraints for effort for both types:

$$e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta})$$

$$\geq e_L.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_L, \bar{\theta})$$

and

$$e_H.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_H, \underline{\theta})$$

$$\geq e_L.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_L, \underline{\theta})$$

IC constraints for target setting for both types:

$$e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta})$$

$$\geq e_H.(L(\bar{\theta}, l).w(L, l) + (1 - L(\bar{\theta}, l)).w(0, l)) - c(e_H, \bar{\theta})$$

and

$$e_H.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_H, \underline{\theta})$$

$$\geq e_H.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_H, \underline{\theta})$$

Participation constraints for both types:

$$\bar{U} \leq e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta})$$

and

$$\underline{U} \leq e_H.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_H, \underline{\theta})$$

Which constrain the maximisation of the principal's problem:

$$\rho.(H(\bar{\theta}, h).e_H.(V(H) - w(H, h)) + L(\bar{\theta}, h).e_H.(V(L) - w(L, h)))$$

$$+ (1 - e_H.(H(\bar{\theta}, h) + L(\bar{\theta}, h))).(-w(0, h))$$

$$+ (1 - \rho).(H(\underline{\theta}, h).e_H.(V(H) - w(H, h)) + L(\underline{\theta}, h).e_H.(V(L) - w(L, h)))$$

$$(1 - e_H.(H(\underline{\theta}, h) + L(\underline{\theta}, h))).(-w(0, h))$$

For case 4:

IC constraints for effort for both types:

$$\begin{aligned} & e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta}) \\ \geq & e_L.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_L, \bar{\theta}) \end{aligned}$$

and

$$\begin{aligned} & e_L.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_L, \underline{\theta}) \\ \geq & e_H.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_H, \underline{\theta}) \end{aligned}$$

IC constraints for target setting for both types:

$$\begin{aligned} & e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta}) \\ \geq & e_H.(L(\bar{\theta}, l).w(L, l) + (1 - L(\bar{\theta}, l)).w(0, l)) - c(e_H, \bar{\theta}) \end{aligned}$$

and

$$\begin{aligned} & e_L.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_L, \underline{\theta}) \\ \geq & e_L.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_L, \underline{\theta}) \end{aligned}$$

Participation constraints for both types:

$$\bar{U} \leq e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta})$$

and

$$\underline{U} \leq e_L.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_L, \underline{\theta})$$

Which constrain the maximisation of the principal's problem:

$$\begin{aligned} & \rho.(H(\bar{\theta}, h).e_H.(V(H) - w(H, h)) + L(\bar{\theta}, h).e_H.(V(L) - w(L, h))) \\ & + (1 - e_H).(H(\bar{\theta}, h) + L(\bar{\theta}, h)).(-w(0, h)) \\ & + (1 - \rho).(H(\underline{\theta}, h).e_L.(V(H) - w(H, h)) + L(\underline{\theta}, h).e_L.(V(L) - w(L, h))) \\ & (1 - e_L).(H(\underline{\theta}, h) + L(\underline{\theta}, h)).(-w(0, h)) \end{aligned}$$

We now present the mathematics behind our lemmas, characterising which constraints we expect to bind.

Proof of Lemma 8. Under case 1 simplifying our first-order conditions we end up with:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(H, h)} &= H(\bar{\theta}, h)((\lambda_3 + \lambda_5 - \rho) \cdot e_H + \lambda_1 \cdot (e_H - e_L)) - \lambda_4 \cdot e_H \cdot H(\underline{\theta}, h) \\
\frac{\partial \mathcal{L}}{\partial w(L, h)} &= L(\bar{\theta}, h)((\lambda_3 + \lambda_5 - \rho) \cdot e_H + \lambda_1 \cdot (e_H - e_L)) - \lambda_4 \cdot e_H \cdot L(\underline{\theta}, h) \\
\frac{\partial \mathcal{L}}{\partial w(L, l)} &= L(\underline{\theta}, l)((\lambda_4 + \lambda_6 - (1 - \rho)) \cdot e_H + \lambda_2 \cdot (e_H - e_L)) - \lambda_3 \cdot e_H \cdot L(\bar{\theta}, l) \\
\frac{\partial \mathcal{L}}{\partial w(0, l)} &= L(\underline{\theta}, l)((-\lambda_4 - \lambda_6 + (1 - \rho)) \cdot e_H - \lambda_2 \cdot (e_H - e_L)) + \lambda_3 \cdot e_H \cdot L(\bar{\theta}, l) \\
&\quad + \lambda_4 + \lambda_6 - (1 - \rho) - \lambda_3 \\
\frac{\partial \mathcal{L}}{\partial w(0, h)} &= (H(\bar{\theta}, h) + L(\bar{\theta}, h))((\rho - \lambda_3 - \lambda_5) \cdot e_H - \lambda_1 \cdot (e_H - e_L)) \\
&\quad + \lambda_4 \cdot e_H \cdot (H(\underline{\theta}, h) + L(\underline{\theta}, h)) - \rho + \lambda_3 - \lambda_4 + \lambda_5
\end{aligned}$$

Using the fact that all the above are equal to 0:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(L, l)} + \frac{\partial \mathcal{L}}{\partial w(0, l)} &= \lambda_4 + \lambda_6 - (1 - \rho) - \lambda_3 = 0 \\
&\implies \rho = 1 - \lambda_4 - \lambda_6 + \lambda_3 \\
\frac{\partial \mathcal{L}}{\partial w(H, h)} + \frac{\partial \mathcal{L}}{\partial w(L, h)} + \frac{\partial \mathcal{L}}{\partial w(0, h)} &= -\rho + \lambda_3 - \lambda_4 + \lambda_5 = 0 \\
&\implies \rho = \lambda_3 - \lambda_4 + \lambda_5
\end{aligned}$$

Together we know that:

$$\begin{aligned}
&\implies \lambda_3 - \lambda_4 + \lambda_5 = 1 + \lambda_3 - \lambda_4 - \lambda_6 \\
&\implies \lambda_5 = 1 - \lambda_6 \\
&\implies \lambda_5 + \lambda_6 = 1
\end{aligned}$$

Therefore at least one of λ_5 and λ_6 , the two participation constraints, must bind. Furthermore, as $0 \leq \rho \leq 1$:

$$\begin{aligned}
&\implies 0 \leq \lambda_3 - \lambda_4 + \lambda_5 \leq 1 \\
&\implies \lambda_4 \leq \lambda_3 + \lambda_5
\end{aligned}$$

In which case, if λ_4 binds then at least one of λ_3 and λ_5 bind too. ■

Proof of Lemma 9. Now, for case 2, our first-order conditions reduce to:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(H, h)} &= H(\bar{\theta}, h) \cdot ((\lambda_3 + \lambda_5 - \rho)e_H + \lambda_1 \cdot (e_H - e_L)) - \lambda_4 \cdot e_L \cdot H(\underline{\theta}, h) \\
\frac{\partial \mathcal{L}}{\partial w(L, h)} &= L(\bar{\theta}, h) \cdot ((\lambda_3 + \lambda_5 - \rho)e_H + \lambda_1 \cdot (e_H - e_L)) - \lambda_4 \cdot e_L \cdot L(\underline{\theta}, h) \\
\frac{\partial \mathcal{L}}{\partial w(L, l)} &= L(\underline{\theta}, l) \cdot ((\lambda_4 + \lambda_6 - (1 - \rho))e_L - \lambda_2 \cdot (e_H - e_L)) - \lambda_3 \cdot e_H \cdot L(\bar{\theta}, l) \\
\frac{\partial \mathcal{L}}{\partial w(0, l)} &= L(\underline{\theta}, l) \cdot ((-\lambda_4 - \lambda_6 + (1 - \rho))e_L + \lambda_2 \cdot (e_H - e_L)) + \lambda_3 \cdot e_H \cdot L(\bar{\theta}, l) \\
&\quad - (1 - \rho) - \lambda_3 + \lambda_4 + \lambda_6 \\
\frac{\partial \mathcal{L}}{\partial w(0, h)} &= (H(\bar{\theta}, h) + L(\bar{\theta}, h)) \cdot ((\rho - \lambda_3 - \lambda_5)e_H - \lambda_1 \cdot (e_H - e_L)) \\
&\quad + \lambda_4 \cdot e_L \cdot (H(\underline{\theta}, h) + L(\bar{\theta}, h)) - \rho + \lambda_3 + \lambda_5 - \lambda_4
\end{aligned}$$

Using the fact that all the above expressions are equal to 0:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(L, l)} + \frac{\partial \mathcal{L}}{\partial w(0, l)} &= -(1 - \rho) - \lambda_3 + \lambda_4 + \lambda_6 = 0 \\
\implies (1 - \rho) &= -\lambda_3 + \lambda_4 + \lambda_6
\end{aligned}$$

Substitute the expression for $(1 - \rho)$ into $\frac{\partial \mathcal{L}}{\partial w(L, l)}$:

$$\begin{aligned}
\implies L(\underline{\theta}, l) \cdot ((\lambda_4 + \lambda_6 - (-\lambda_3 + \lambda_4 + \lambda_6))e_L - \lambda_2 \cdot (e_H - e_L)) - \lambda_3 \cdot e_H \cdot L(\bar{\theta}, l) &= 0 \\
\implies L(\underline{\theta}, l) \cdot ((\lambda_3 \cdot e_L - \lambda_2 \cdot (e_H - e_L)) - \lambda_3 \cdot e_H \cdot L(\bar{\theta}, l)) &= 0
\end{aligned}$$

We now have four possible cases; both λ_2 and λ_3 , one of the above, or neither of these constraints bind. Let us first suppose that just λ_3 binds, and therefore that λ_2 does not, therefore, $\lambda_3 > \lambda_2 = 0$. The above equation can now be reduced to:

$$\begin{aligned}
\implies L(\underline{\theta}, l) \cdot \lambda_3 \cdot e_L &= \lambda_3 \cdot e_H \cdot L(\bar{\theta}, l) \\
\implies L(\underline{\theta}, l) \cdot e_L &= L(\bar{\theta}, l) \cdot e_H
\end{aligned}$$

Given that $e_H > e_L$

$$\implies L(\underline{\theta}, l) > L(\bar{\theta}, l)$$

However, this is a direct violation of one of our initial assumptions and therefore cannot be true. Suppose now that just λ_2 binds and not λ_3 , therefore, $\lambda_2 > \lambda_3 = 0$:

$$\implies -L(\underline{\theta}, l) \cdot \lambda_2 \cdot (e_H - e_L) = 0$$

We assumed $L(\underline{\theta}, l) > 0$ and given that $e_H > e_L$, therefore this too cannot be true. Suppose now that both constraints bind:

$$\implies L(\underline{\theta}, l) \cdot \lambda_3 \cdot e_L - \lambda_3 \cdot e_H \cdot L(\bar{\theta}, l) - L(\underline{\theta}, l) \cdot \lambda_2 \cdot (e_H - e_L) = 0$$

$$\implies L(\underline{\theta}, l) \cdot \lambda_3 \cdot e_H - L(\underline{\theta}, l) \cdot \lambda_3 \cdot (e_H - e_L) - \lambda_3 \cdot e_H \cdot L(\bar{\theta}, l) - L(\underline{\theta}, l) \cdot \lambda_2 \cdot (e_H - e_L) = 0$$

$$\implies (L(\underline{\theta}, l) - L(\bar{\theta}, l)) \cdot \lambda_3 \cdot e_H = L(\underline{\theta}, l) \cdot (\lambda_2 + \lambda_3) \cdot (e_H - e_L)$$

From our assumptions, the left hand side is strictly negative, therefore, for this to be true, then $e_H < e_L$, therefore false. In which case we are left with the only possible case, that neither of the constraints are binding. Furthermore:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial w(H, h)} + \frac{\partial \mathcal{L}}{\partial w(L, h)} + \frac{\partial \mathcal{L}}{\partial w(0, h)} &= -\rho + \lambda_3 + \lambda_5 - \lambda_4 = 0 \\ \implies \rho &= \lambda_3 + \lambda_5 - \lambda_4 \end{aligned}$$

Given that $0 \leq \rho \leq 1$:

$$\implies 0 \leq \lambda_3 + \lambda_5 - \lambda_4 \leq 1$$

$$\implies \lambda_4 \leq \lambda_3 + \lambda_5$$

Therefore, if λ_4 binds, then λ_5 must bind, as we have shown that $\lambda_3 = 0$. Equating the two expressions for ρ :

$$\implies 1 + \lambda_3 - \lambda_4 - \lambda_6 = \lambda_3 + \lambda_5 - \lambda_4$$

$$\implies 1 - \lambda_6 = \lambda_5$$

$$\implies 1 = \lambda_5 + \lambda_6$$

In which case, at least one of λ_5 and λ_6 must bind. ■

Proof of Lemma 10. Under case 3 our first-order conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(H, h)} &= H(\bar{\theta}, h) \cdot ((\lambda_3 + \lambda_5 - \rho) \cdot e_H + \lambda_1 \cdot (e_H - e_L)) + H(\underline{\theta}, h) \cdot ((\lambda_4 + \lambda_6 - (1 - \rho)) \cdot e_H \\
&\quad + \lambda_2 \cdot (e_H - e_L)) \\
\frac{\partial \mathcal{L}}{\partial w(L, h)} &= L(\bar{\theta}, h) \cdot ((\lambda_3 + \lambda_5 - \rho) \cdot e_H + \lambda_1 \cdot (e_H - e_L)) + L(\underline{\theta}, h) \cdot ((\lambda_4 + \lambda_6 - (1 - \rho)) \cdot e_H \\
&\quad + \lambda_2 \cdot (e_H - e_L)) \\
\frac{\partial \mathcal{L}}{\partial w(L, l)} &= -\lambda_3 \cdot e_H \cdot L(\bar{\theta}, l) - \lambda_4 \cdot e_H \cdot L(\underline{\theta}, l) \\
\frac{\partial \mathcal{L}}{\partial w(0, l)} &= \lambda_3 \cdot (e_H \cdot L(\bar{\theta}, l) - 1) + \lambda_4 \cdot (e_H \cdot L(\underline{\theta}, l) - 1) \\
\frac{\partial \mathcal{L}}{\partial w(0, h)} &= (H(\bar{\theta}, h) + L(\bar{\theta}, h)) \cdot ((\rho - \lambda_5) \cdot e_H - \lambda_1 \cdot (e_H - e_L)) \\
&\quad + (H(\underline{\theta}, h) + L(\underline{\theta}, h)) \cdot (((1 - \rho) - \lambda_6) \cdot e_H - \lambda_2 \cdot (e_H - e_L)) - \rho - (1 - \rho) + \lambda_5 + \lambda_6
\end{aligned}$$

Using the fact that all of the above are equal to 0, directly, from $\frac{\partial \mathcal{L}}{\partial w(L, l)}$, we can see that the only way for this to hold would be if $\lambda_3 = \lambda_4 = 0$. In which case, neither are binding, which simplifies the first two constraints:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(H, h)} &= H(\bar{\theta}, h) \cdot ((\lambda_5 - \rho) \cdot e_H + \lambda_1 \cdot (e_H - e_L)) + H(\underline{\theta}, h) \cdot ((\lambda_6 - (1 - \rho)) \cdot e_H + \lambda_2 \cdot (e_H - e_L)) \\
\frac{\partial \mathcal{L}}{\partial w(L, h)} &= L(\bar{\theta}, h) \cdot ((\lambda_5 - \rho) \cdot e_H + \lambda_1 \cdot (e_H - e_L)) + L(\underline{\theta}, h) \cdot ((\lambda_6 - (1 - \rho)) \cdot e_H + \lambda_2 \cdot (e_H - e_L))
\end{aligned}$$

Therefore:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(H, h)} + \frac{\partial \mathcal{L}}{\partial w(L, h)} + \frac{\partial \mathcal{L}}{\partial w(0, h)} &= -\rho - (1 - \rho) + \lambda_5 + \lambda_6 = 0 \\
\implies \lambda_5 + \lambda_6 &= 1
\end{aligned}$$

And so, at least one of λ_5 and λ_6 binds. ■

Proof of Lemma 11. For case 4 our first-order conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(H, h)} &= H(\bar{\theta}, h) \cdot ((\lambda_3 + \lambda_5 - \rho) \cdot e_H + \lambda_1 \cdot (e_H - e_L)) + H(\underline{\theta}, h) \cdot ((\lambda_4 + \lambda_6 - (1 - \rho)) \cdot e_L \\
&\quad - \lambda_2 \cdot (e_H - e_L)) \\
\frac{\partial \mathcal{L}}{\partial w(L, h)} &= L(\bar{\theta}, h) \cdot ((\lambda_3 + \lambda_5 - \rho) \cdot e_H + \lambda_1 \cdot (e_H - e_L)) + L(\underline{\theta}, h) \cdot ((\lambda_4 + \lambda_6 - (1 - \rho)) \cdot e_L \\
&\quad - \lambda_2 \cdot (e_H - e_L)) \\
\frac{\partial \mathcal{L}}{\partial w(L, l)} &= -\lambda_3 \cdot e_H \cdot L(\bar{\theta}, l) - \lambda_4 \cdot e_L \cdot L(\underline{\theta}, h) \\
\frac{\partial \mathcal{L}}{\partial w(0, l)} &= \lambda_3 \cdot (e_H \cdot L(\bar{\theta}, l) - 1) + \lambda_4 \cdot (e_L \cdot L(\underline{\theta}, l) - 1) \\
\frac{\partial \mathcal{L}}{\partial w(0, h)} &= (H(\bar{\theta}, h) + L(\bar{\theta}, h)) \cdot ((\rho - \lambda_5) \cdot e_H - \lambda_1 \cdot (e_H - e_L)) \\
&\quad + (H(\underline{\theta}, h) + L(\underline{\theta}, h)) \cdot (((1 - \rho) - \lambda_6) \cdot e_L + \lambda_2 \cdot (e_H - e_L)) - \rho - (1 - \rho) + \lambda_5 + \lambda_6
\end{aligned}$$

Using the fact that all of the above are equal to 0, directly, from $\frac{\partial \mathcal{L}}{\partial w(L, l)}$, we can see that the only way for this to hold would be if $\lambda_3 = \lambda_4 = 0$. In which case, neither are binding, which simplifies the first two constraints:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(H, h)} &= H(\bar{\theta}, h) \cdot ((\lambda_5 - \rho) \cdot e_H + \lambda_1 \cdot (e_H - e_L)) + H(\underline{\theta}, h) \cdot ((\lambda_6 - (1 - \rho)) \cdot e_L - \lambda_2 \cdot (e_H - e_L)) \\
\frac{\partial \mathcal{L}}{\partial w(L, h)} &= L(\bar{\theta}, h) \cdot ((\lambda_5 - \rho) \cdot e_H + \lambda_1 \cdot (e_H - e_L)) + L(\underline{\theta}, h) \cdot ((\lambda_6 - (1 - \rho)) \cdot e_L - \lambda_2 \cdot (e_H - e_L))
\end{aligned}$$

Therefore:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(H, h)} + \frac{\partial \mathcal{L}}{\partial w(L, h)} + \frac{\partial \mathcal{L}}{\partial w(0, h)} &= -\rho - (1 - \rho) + \lambda_5 + \lambda_6 = 0 \\
\implies -1 + \lambda_5 + \lambda_6 &= 0 \\
\implies \lambda_5 + \lambda_6 &= 1
\end{aligned}$$

Therefore, at least one of λ_5 and λ_6 bind. Furthermore, both $\lambda_5 + \lambda_6$ and $\rho + (1 - \rho)$

form convex combinations. Let us rewrite $\frac{\partial \mathcal{L}}{\partial w(H,h)}$:

$$\begin{aligned}
&\implies H(\bar{\theta}, h).((\lambda_5 - \rho).e_H + \lambda_1.(e_H - e_L)) + H(\underline{\theta}, h).((\lambda_6 - (1 - \rho)).e_L - \lambda_2.(e_H - e_L)) \\
&\implies H(\bar{\theta}, h).(\lambda_5 - \rho).e_H + H(\bar{\theta}, h).\lambda_1.(e_H - e_L) + H(\underline{\theta}, h).(\lambda_6 - (1 - \rho)).e_L \\
&\quad - H(\underline{\theta}, h).\lambda_2.(e_H - e_L) \\
&\implies H(\bar{\theta}, h).\lambda_5.e_H + H(\underline{\theta}, h).\lambda_6.e_L - H(\bar{\theta}, h).\rho.e_H - H(\underline{\theta}, h).(1 - \rho).e_L \\
&\quad + (H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2).(e_H - e_L) \\
&\implies (H(\bar{\theta}, h).\lambda_5.e_H + H(\underline{\theta}, h).\lambda_6.e_L) - (H(\bar{\theta}, h).\rho.e_H + H(\underline{\theta}, h).(1 - \rho).e_L) \\
&\quad + (e_H - e_L).(H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2) \\
&\implies (H(\bar{\theta}, h).\lambda_5.e_L + H(\underline{\theta}, h).\lambda_6.e_L) + H(\bar{\theta}, h).\lambda_5.(e_H - e_L) - (H(\bar{\theta}, h).\rho.e_L \\
&\quad + H(\underline{\theta}, h).(1 - \rho).e_L) - H(\bar{\theta}, h).\rho.(e_H - e_L) + (e_H - e_L).(H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2) \\
&\implies (H(\bar{\theta}, h).\lambda_5.e_L + H(\underline{\theta}, h).\lambda_6.e_L) - (H(\bar{\theta}, h).\rho.e_L + H(\underline{\theta}, h).(1 - \rho).e_L) \\
&\quad + (e_H - e_L).H(\bar{\theta}, h).(\lambda_5 - \rho) + (e_H - e_L).(H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2)
\end{aligned}$$

Suppose now that the last two terms in the above equation are of the same sign, therefore either (once again, we assume throughout that $e_H > e_L$):

$$\begin{aligned}
H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2 &> 0 \quad \text{AND} \quad \lambda_5 - \rho > 0 \\
&\text{OR} \\
H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2 &< 0 \quad \text{AND} \quad \lambda_5 - \rho < 0
\end{aligned}$$

Given that the whole equation sums to zero, this must mean that the sum of the first two terms must then take the opposite sign of each case. So, if $\lambda_5 > \rho$, then the sum of the first two terms must be negative. As we showed previously, they are both convex combinations and as the latter must be larger, then the fraction placed on the larger term must itself be larger. We assumed that $H(\bar{\theta}, h) > H(\underline{\theta}, h) \implies H(\bar{\theta}, h).e_L > H(\underline{\theta}, h).e_L \implies \lambda_5 < \rho$, which violates the case we are in, therefore is not possible. Similarly, if we are in the case where $\lambda_5 < \rho$, then the sum of the first two terms must be positive, which by the same logic leads us to require $\lambda_5 > \rho$, again a violation. Therefore, it must be the case that the last two terms in the equation above are of

opposite signs:

$$\begin{aligned}
H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2 &> 0 \quad \text{AND} \quad \lambda_5 - \rho < 0 \\
&\text{OR} \\
H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2 &< 0 \quad \text{AND} \quad \lambda_5 - \rho > 0
\end{aligned}$$

However, recall that both $\lambda_5 + \lambda_6$ and $\rho + (1 - \rho)$ form convex combinations, which means that:

$$\begin{aligned}
H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2 > 0 &\iff \lambda_5 < \rho \quad \text{AND} \quad \lambda_6 > 1 - \rho > 0 \\
H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2 < 0 &\iff \lambda_5 > \rho > 0 \quad \text{AND} \quad \lambda_6 < 1 - \rho
\end{aligned}$$

λ_1 and λ_2 refer to effort *IC* constraints, whereas λ_5 and λ_6 refer to participation constraints. If therefore we were to suppose that one and only one of each type of constraint were to bind at a time, then it is straightforward to determine which constraints bind in pairs:

$$\lambda_1 \text{ binding} \iff \lambda_6 \text{ binding} \quad \text{AND} \quad \lambda_2 \text{ binding} \iff \lambda_5 \text{ binding}$$

From above:

$$\begin{aligned}
0 &= (H(\bar{\theta}, h).\lambda_5.e_L + H(\underline{\theta}, h).\lambda_6.e_L) - (H(\bar{\theta}, h).\rho.e_L + H(\underline{\theta}, h).(1 - \rho).e_L) \\
&\quad + (e_H - e_L).H(\bar{\theta}, h).(\lambda_5 - \rho) + (e_H - e_L).(H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2) \\
\implies 0 &= e_H.H(\bar{\theta}, h).(\lambda_5 - \rho) + e_L.H(\underline{\theta}, h).(\lambda_6 - (1 - \rho)) + (e_H - e_L).(H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2)
\end{aligned}$$

Suppose the last term is positive, then:

$$\begin{aligned}
&\implies e_H.H(\bar{\theta}, h).(\lambda_5 - \rho) + e_L.H(\underline{\theta}, h).(\lambda_6 - (1 - \rho)) < 0 \\
&\implies e_H.H(\bar{\theta}, h).(\lambda_5 - \rho) < e_L.H(\underline{\theta}, h).((1 - \rho) - \lambda_6) \\
&\implies \frac{e_H.H(\bar{\theta}, h)}{e_L.H(\underline{\theta}, h)}.(\lambda_5 - \rho) < (1 - \rho) - \lambda_6 \\
&\implies \lambda_5 - \rho < \frac{e_H.H(\bar{\theta}, h)}{e_L.H(\underline{\theta}, h)}.(\lambda_5 - \rho) < (1 - \rho) - \lambda_6 \\
&\implies \lambda_5 + \lambda_6 < 1 \quad \text{Contradiction}
\end{aligned}$$

Therefore we can conclude that:

$$\begin{aligned}
(e_H - e_L).(H(\bar{\theta}, h).\lambda_1 - H(\underline{\theta}, h).\lambda_2) &\leq 0 \\
&\implies \lambda_1 \leq \lambda_2
\end{aligned}$$

In which case we can say that if only one ‘pair’ of the above binds, then it is λ_2 and λ_5 , fitting with the logic we described. ■

Results - Case 1

If we substitute the results of our simulations into our binding constraints we have:

$$\begin{aligned}
(e_H - e_L).L(\underline{\theta}, l).(w(L, l) - w(0, l)) &= \Delta c \\
w(0, l) &= e_H.(H(\underline{\theta}, h).w(H, h) - L(\underline{\theta}, l).(w(L, l) - w(0, l))) \\
e_H.H(\bar{\theta}, h).w(H, h) &= c(e_H, \bar{\theta}) + \bar{U}
\end{aligned}$$

Directly, from the third and first of these respectively, we can find an expression for $w(H, h)$ and the ‘wage gap’ when claiming a low target:

$$\begin{aligned}
w(H, h) &= \frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H.H(\bar{\theta}, h)} \\
w(L, l) - w(0, l) &= \frac{\Delta c}{L(\underline{\theta}, l).(e_H - e_L)}
\end{aligned}$$

After a little straightforward manipulation we can determine exact values for the

above wages of:

$$w(L, l) = (c(e_H, \bar{\theta}) + \bar{U}) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)} + \frac{(1 - e_H \cdot L(\underline{\theta}, l)) \cdot \Delta c}{L(\underline{\theta}, l) \cdot (e_H - e_L)}$$

$$w(0, l) = (c(e_H, \bar{\theta}) + \bar{U}) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)} - \frac{e_H \cdot \Delta c}{(e_H - e_L)}$$

Furthermore, we can derive the condition that:

$$\frac{c(e_H, \bar{\theta}) + \bar{U}}{c(e_H, \underline{\theta}) + \underline{U}} > \frac{H(\bar{\theta}, h)}{H(\underline{\theta}, h)}$$

What about the value of this to the principal? After some work it can be reduced to:

$$\rho \cdot H(\bar{\theta}, h) \cdot e_H \cdot V(H) + (\rho \cdot L(\bar{\theta}, h) \cdot e_H + (1 - \rho) \cdot L(\underline{\theta}, l) \cdot e_H) \cdot V(L) - (c(e_H, \bar{\theta}) + \bar{U}) \cdot (\rho + (1 - \rho)) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}$$

This is increasing in the probability of a good type whenever:

$$0 < H(\bar{\theta}, h) \cdot e_H \cdot V(H) + L(\bar{\theta}, h) \cdot e_H \cdot V(L) - \rho \cdot L(\underline{\theta}, l) \cdot e_H \cdot V(L) - (c(e_H, \bar{\theta}) + \bar{U})$$

$$+ (c(e_H, \bar{\theta}) + \bar{U}) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}$$

$$\implies 0 < H(\bar{\theta}, h) \cdot V(H) + (L(\bar{\theta}, h) - L(\underline{\theta}, l)) \cdot V(L) - \left(\frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H} \right) + \left(\frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H} \right) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}$$

$$\implies 0 < H(\bar{\theta}, h) \cdot V(H) + (L(\bar{\theta}, h) - L(\underline{\theta}, l)) \cdot V(L) + \left(\frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H} \right) \cdot \left(\frac{H(\underline{\theta}, h) - H(\bar{\theta}, h)}{H(\bar{\theta}, h)} \right)$$

$$\implies H(\bar{\theta}, h) \cdot V(H) + (L(\bar{\theta}, h) - L(\underline{\theta}, l)) \cdot V(L) > (c(e_H, \bar{\theta}) + \bar{U}) \cdot \left(\frac{H(\bar{\theta}, h) - H(\underline{\theta}, h)}{e_H \cdot H(\bar{\theta}, h)} \right)$$

$$\implies H(\bar{\theta}, h) \cdot V(H) + (L(\bar{\theta}, h) - L(\underline{\theta}, l)) \cdot V(L) > \left(\frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H \cdot H(\bar{\theta}, h)} \right) \cdot (H(\bar{\theta}, h) - H(\underline{\theta}, h))$$

$$\implies H(\bar{\theta}, h) \cdot V(H) + (L(\bar{\theta}, h) - L(\underline{\theta}, l)) \cdot V(L) > w(H, h) \cdot (H(\bar{\theta}, h) - H(\underline{\theta}, h))$$

The value to the principal is also increasing in $H(\bar{\theta}, h)$ and decreasing in $H(\underline{\theta}, h)$. Finally and rather obviously, it is decreasing in both the cost of high effort for the high type and his outside option.

Results - Case 2

If we substitute the results of our simulations into our binding constraints we have:

$$\begin{aligned} w(0, l) &= e_L \cdot (H(\underline{\theta}, h) \cdot w(H, h) + L(\underline{\theta}, l) \cdot w(0, l)) \\ e_H \cdot H(\bar{\theta}, h) \cdot w(H, h) &= c(e_H, \bar{\theta}) + \bar{U} \end{aligned}$$

Therefore, directly from the latter, and then substituting into the former we can deduce that:

$$\begin{aligned} w(H, h) &= \frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H \cdot H(\bar{\theta}, h)} \\ w(0, l) &= \frac{e_L \cdot H(\underline{\theta}, h) \cdot (c(e_H, \bar{\theta}) + \bar{U})}{e_H \cdot H(\bar{\theta}, h) \cdot (1 - e_L \cdot L(\underline{\theta}, l))} \end{aligned}$$

Similar to the condition derived above, we can also show that:

$$\frac{c(e_H, \bar{\theta}) + \bar{U}}{c(e_L, \underline{\theta}) + \underline{U}} > \frac{e_H \cdot H(\bar{\theta}, h)}{e_L \cdot H(\underline{\theta}, h)}$$

What about the value of this to the principal? After some work it can be reduced to:

$$\rho \cdot H(\bar{\theta}, h) \cdot e_H \cdot V(H) + (\rho \cdot L(\bar{\theta}, h) \cdot e_H + (1 - \rho) \cdot L(\underline{\theta}, l) \cdot e_L) \cdot V(L) - (c(e_H, \bar{\theta}) + \bar{U}) \cdot (\rho + (1 - \rho) \cdot \frac{e_L \cdot H(\underline{\theta}, h)}{e_H \cdot H(\bar{\theta}, h)})$$

If we compare this to the case above, case 1 is preferred whenever:

$$\begin{aligned}
& \rho.H(\bar{\theta}, h).e_H.V(H) + (\rho.L(\bar{\theta}, h).e_H + (1 - \rho).L(\underline{\theta}, l).e_H).V(L) \\
& - (c(e_H, \bar{\theta}) + \bar{U}).(\rho + (1 - \rho).\frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}) \\
> & \rho.H(\bar{\theta}, h).e_H.V(H) + (\rho.L(\bar{\theta}, h).e_H + (1 - \rho).L(\underline{\theta}, l).e_L).V(L) \\
& - (c(e_H, \bar{\theta}) + \bar{U}).(\rho + (1 - \rho).\frac{e_L.H(\underline{\theta}, h)}{e_H.H(\bar{\theta}, h)}) \\
\implies & (1 - \rho).L(\underline{\theta}, l).e_H.V(L) - (c(e_H, \bar{\theta}) + \bar{U}).(1 - \rho).\frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)} > \\
& (1 - \rho).L(\underline{\theta}, l).e_L.V(L) - (c(e_H, \bar{\theta}) + \bar{U}).(1 - \rho).\frac{e_L.H(\underline{\theta}, h)}{e_H.H(\bar{\theta}, h)} \\
\implies & (e_H - e_L).L(\underline{\theta}, l).V(L) > (c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)} - \frac{e_L.H(\underline{\theta}, h)}{e_H.H(\bar{\theta}, h)}\right) \\
\implies & (e_H - e_L).L(\underline{\theta}, l).V(L) > (c(e_H, \bar{\theta}) + \bar{U}).\frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}.\left(\frac{e_H - e_L}{e_H}\right) \\
\implies & (e_H - e_L).L(\underline{\theta}, l).V(L) > (e_H - e_L).H(\underline{\theta}, h).\left(\frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H.H(\bar{\theta}, h)}\right) \\
\implies & (e_H - e_L).L(\underline{\theta}, l).V(L) > (e_H - e_L).H(\underline{\theta}, h).w(H, h)
\end{aligned}$$

In addition, it is increasing in the probability of a high type whenever:

$$\begin{aligned}
&\implies 0 < H(\bar{\theta}, h).e_H.V(H) + L(\bar{\theta}, h).e_H.V(L) - L(\underline{\theta}, l).e_L.V(L) \\
&\quad - (c(e_H, \bar{\theta}) + \bar{U}) + (c(e_H, \bar{\theta}) + \bar{U}).\frac{e_L.H(\underline{\theta}, h)}{e_H.H(\bar{\theta}, h)} \\
&\implies 0 < e_H.H(\bar{\theta}, h).V(H) + (e_H.L(\bar{\theta}, h) - e_L.L(\underline{\theta}, l)).V(L) \\
&\quad - (c(e_H, \bar{\theta}) + \bar{U}) + (c(e_H, \bar{\theta}) + \bar{U}).\frac{e_L.H(\underline{\theta}, h)}{e_H.H(\bar{\theta}, h)} \\
&\implies 0 < e_H.H(\bar{\theta}, h).V(H) + (e_H.L(\bar{\theta}, h) - e_L.L(\underline{\theta}, l)).V(L) \\
&\quad - (c(e_H, \bar{\theta}) + \bar{U}) + (c(e_H, \bar{\theta}) + \bar{U}).\frac{e_L.H(\underline{\theta}, h)}{e_H.H(\bar{\theta}, h)} \\
&\implies e_H.H(\bar{\theta}, h).V(H) + (e_H.L(\bar{\theta}, h) - e_L.L(\underline{\theta}, l)).V(L) > (c(e_H, \bar{\theta}) + \bar{U}).\left(1 - \frac{e_L.H(\underline{\theta}, h)}{e_H.H(\bar{\theta}, h)}\right) \\
&\implies e_H.H(\bar{\theta}, h).V(H) + (e_H.L(\bar{\theta}, h) - e_L.L(\underline{\theta}, l)).V(L) \\
&\quad > (c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{e_H.H(\bar{\theta}, h) - e_L.H(\underline{\theta}, h)}{e_H.H(\bar{\theta}, h)}\right) \\
&\implies e_H.H(\bar{\theta}, h).V(H) + (e_H.L(\bar{\theta}, h) - e_L.L(\underline{\theta}, l)).V(L) \\
&\quad > \left(\frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H.H(\bar{\theta}, h)}\right).(e_H.H(\bar{\theta}, h) - e_L.H(\underline{\theta}, h)) \\
&\implies e_H.H(\bar{\theta}, h).V(H) + (e_H.L(\bar{\theta}, h) - e_L.L(\underline{\theta}, l)).V(L) \\
&\quad > w(H, h).(e_H.H(\bar{\theta}, h) - e_L.H(\underline{\theta}, h))
\end{aligned}$$

The value to the principal is also increasing in $H(\bar{\theta}, h)$ and decreasing in $H(\underline{\theta}, h)$. Finally and rather obviously, it is decreasing in both the cost of high effort for the high type and his outside option.

Results - Case 3

In case 3 we can see that, rather oddly, just $PC1$ binds. Does this actually make any sense? Surely the principal can tighten one of the IC constraints and leave the other PC slack. Should we not expect both a PC and IC to bind together? Nonetheless, knowing that solely $w(H, h)$ is non-zero, we can substitute this into our binding constraint to determine:

$$e_H.H(\bar{\theta}, h).w(H, h) = c(e_H, \bar{\theta}) + \bar{U}$$

And therefore:

$$w(H, h) = \frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H \cdot H(\bar{\theta}, h)}$$

What about the value of this to the principal? After some work it can be reduced to:

$$\begin{aligned} & (\rho \cdot H(\bar{\theta}, h) \cdot e_H + (1 - \rho) \cdot H(\underline{\theta}, h) \cdot e_H) \cdot V(H) + (\rho \cdot L(\bar{\theta}, h) \cdot e_H + (1 - \rho) \cdot L(\underline{\theta}, h) \cdot e_H) \cdot V(L) \\ & - (c(e_H, \bar{\theta}) + \bar{U}) \cdot (\rho + (1 - \rho) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}) \end{aligned}$$

Furthermore, this is preferred to case 1 whenever:

$$\begin{aligned} & (\rho \cdot H(\bar{\theta}, h) \cdot e_H + (1 - \rho) \cdot H(\underline{\theta}, h) \cdot e_H) \cdot V(H) + (\rho \cdot L(\bar{\theta}, h) \cdot e_H + (1 - \rho) \cdot L(\underline{\theta}, h) \cdot e_H) \cdot V(L) \\ & - (c(e_H, \bar{\theta}) + \bar{U}) \cdot (\rho + (1 - \rho) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}) \\ > & \rho \cdot H(\bar{\theta}, h) \cdot e_H \cdot V(H) + (\rho \cdot L(\bar{\theta}, h) \cdot e_H + (1 - \rho) \cdot L(\underline{\theta}, l) \cdot e_H) \cdot V(L) \\ & - (c(e_H, \bar{\theta}) + \bar{U}) \cdot (\rho + (1 - \rho) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}) \\ \implies & (1 - \rho) \cdot H(\underline{\theta}, h) \cdot e_H \cdot V(H) + (1 - \rho) \cdot L(\underline{\theta}, h) \cdot e_H \cdot V(L) > (1 - \rho) \cdot L(\underline{\theta}, l) \cdot e_H \cdot V(L) \\ \implies & e_H \cdot H(\underline{\theta}, h) \cdot V(H) + e_H \cdot L(\underline{\theta}, h) \cdot V(L) > e_H \cdot L(\underline{\theta}, l) \cdot V(L) \end{aligned}$$

Once again, this value is also increasing in the probability of a high type whenever:

$$\begin{aligned} 0 < & H(\bar{\theta}, h) \cdot e_H \cdot V(H) - H(\underline{\theta}, h) \cdot e_H \cdot V(H) + L(\bar{\theta}, h) \cdot e_H \cdot V(L) - L(\underline{\theta}, h) \cdot e_H \cdot V(L) \\ & - (c(e_H, \bar{\theta}) + \bar{U}) + (c(e_H, \bar{\theta}) + \bar{U}) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)} \\ \implies & (H(\bar{\theta}, h) \cdot e_H - H(\underline{\theta}, h) \cdot e_H) \cdot V(H) + (L(\bar{\theta}, h) \cdot e_H - L(\underline{\theta}, h) \cdot e_H) \cdot V(L) \\ > & (c(e_H, \bar{\theta}) + \bar{U}) \cdot (1 - \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}) \\ \implies & (H(\bar{\theta}, h) - H(\underline{\theta}, h)) \cdot V(H) + (L(\bar{\theta}, h) - L(\underline{\theta}, h)) \cdot V(L) > w(H, h) \cdot (H(\bar{\theta}, h) - H(\underline{\theta}, h)) \end{aligned}$$

$$\begin{aligned}
&\implies (H(\bar{\theta}, h) - H(\underline{\theta}, h)).V(H) + (L(\bar{\theta}, h) - L(\underline{\theta}, h)).V(L) \\
&> \left(\frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H}\right) \cdot \left(\frac{H(\bar{\theta}, h) - H(\underline{\theta}, h)}{H(\bar{\theta}, h)}\right) \\
&\implies (H(\bar{\theta}, h) - H(\underline{\theta}, h)).V(H) + (L(\bar{\theta}, h) - L(\underline{\theta}, h)).V(L) \\
&> \left(\frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H \cdot H(\bar{\theta}, h)}\right) \cdot (H(\bar{\theta}, h) - H(\underline{\theta}, h))
\end{aligned}$$

The value to the principal is certainly increasing in $H(\bar{\theta}, h)$ when:

$$\begin{aligned}
0 &< \rho \cdot e_H \cdot V(H) + (c(e_H, \bar{\theta}) + \bar{U}) \cdot ((1 - \rho) \cdot \frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)^2}) \\
&\implies \rho \cdot V(H) + (1 - \rho) \cdot \left(\frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H \cdot H(\bar{\theta}, h)}\right) \cdot \left(\frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}\right) > 0 \\
&\implies \rho \cdot H(\bar{\theta}, h) \cdot V(H) + (1 - \rho) \cdot H(\underline{\theta}, h) \cdot w(H, h) > 0
\end{aligned}$$

And decreasing in $H(\underline{\theta}, h)$ only when:

$$\begin{aligned}
0 &> (1 - \rho) \cdot e_H \cdot V(H) - (c(e_H, \bar{\theta}) + \bar{U}) \cdot ((1 - \rho) \cdot \frac{1}{H(\bar{\theta}, h)}) \\
&\implies V(H) < \left(\frac{c(e_H, \bar{\theta}) + \bar{U}}{e_H \cdot H(\bar{\theta}, h)}\right) \\
&\implies V(H) < w(H, h)
\end{aligned}$$

Finally, it is decreasing in both the cost of high effort for the high type and his outside option.

With some manipulation we can also establish the conditions:

$$\begin{aligned}
\frac{c(e_H, \bar{\theta}) + \bar{U}}{c(e_H, \underline{\theta}) + \underline{U}} &> \frac{H(\bar{\theta}, h)}{H(\underline{\theta}, h)} \\
\bar{U} &> \frac{e_L \cdot c(e_H, \bar{\theta}) - e_H \cdot c(e_L, \bar{\theta})}{(e_H - e_L)}
\end{aligned}$$

Results - Case 4

If we substitute the results of our simulations into our binding constraints we have:

$$\begin{aligned}
(e_H - e_L) \cdot (H(\underline{\theta}, h) \cdot (w(H, h) - w(0, h)) - L(\underline{\theta}, h) \cdot w(0, h)) &= \Delta \underline{c} \\
e_H \cdot (H(\bar{\theta}, h) \cdot (w(H, h) - w(0, h)) - L(\bar{\theta}, h) \cdot w(0, h)) + w(0, h) &= c(e_H, \bar{\theta}) + \bar{U}
\end{aligned}$$

After some manipulation we can derive wages:

$$\begin{aligned}
w(H, h) &= \frac{\Delta c}{(e_H - e_L)} \cdot \frac{(1 - e_H \cdot (L(\bar{\theta}, h) + H(\bar{\theta}, h)))}{H(\underline{\theta}, h) \cdot (1 - e_H \cdot L(\bar{\theta}, h)) + e_H \cdot H(\bar{\theta}, h) \cdot L(\underline{\theta}, h)} \\
&\quad + \left(\frac{(H(\underline{\theta}, h) + L(\underline{\theta}, h)) \cdot (c(e_H, \bar{\theta}) + \bar{U})}{H(\underline{\theta}, h) \cdot (1 - e_H \cdot L(\bar{\theta}, h)) + e_H \cdot H(\bar{\theta}, h) \cdot L(\underline{\theta}, h)} \right) \\
w(0, h) &= \frac{(e_H - e_L) \cdot H(\underline{\theta}, h) \cdot (c(e_H, \bar{\theta}) + \bar{U}) - e_H \cdot H(\bar{\theta}, h) \cdot \Delta c}{(e_H - e_L) \cdot (H(\underline{\theta}, h) \cdot (1 - e_H \cdot L(\bar{\theta}, h)) + e_H \cdot H(\bar{\theta}, h) \cdot L(\underline{\theta}, h))}
\end{aligned}$$

Therefore we can determine the ‘wage gap’:

$$\begin{aligned}
w(H, h) - w(0, h) &= \frac{\Delta c}{(e_H - e_L)} \cdot \frac{(1 - e_H \cdot L(\bar{\theta}, h))}{H(\underline{\theta}, h) \cdot (1 - e_H \cdot L(\bar{\theta}, h)) + e_H \cdot H(\bar{\theta}, h) \cdot L(\underline{\theta}, h)} \\
&\quad + \left(\frac{L(\underline{\theta}, h) \cdot (c(e_H, \bar{\theta}) + \bar{U})}{H(\underline{\theta}, h) \cdot (1 - e_H \cdot L(\bar{\theta}, h)) + e_H \cdot H(\bar{\theta}, h) \cdot L(\underline{\theta}, h)} \right)
\end{aligned}$$

What about the value of this to the principal? After some significant work it can be reduced to:

$$\begin{aligned}
&(\rho \cdot H(\bar{\theta}, h) \cdot e_H + (1 - \rho) \cdot H(\underline{\theta}, h) \cdot e_L) \cdot V(H) + (\rho \cdot L(\bar{\theta}, h) \cdot e_H + (1 - \rho) \cdot L(\underline{\theta}, h) \cdot e_L) \cdot V(L) \\
&- (c(e_H, \bar{\theta}) + \bar{U}) \cdot \left(\frac{H(\underline{\theta}, h) + \rho \cdot e_H \cdot (H(\bar{\theta}, h) \cdot L(\underline{\theta}, h) - H(\underline{\theta}, h) \cdot L(\bar{\theta}, h))}{H(\underline{\theta}, h) + e_H \cdot (H(\bar{\theta}, h) \cdot L(\underline{\theta}, h) - H(\underline{\theta}, h) \cdot L(\bar{\theta}, h))} \right) \\
&- \frac{\Delta c \cdot (1 - \rho)}{e_H - e_L} \left(\frac{e_L \cdot (H(\underline{\theta}, h) + e_H \cdot (H(\bar{\theta}, h) \cdot L(\underline{\theta}, h) - H(\underline{\theta}, h) \cdot L(\bar{\theta}, h))) - e_H \cdot H(\bar{\theta}, h)}{H(\underline{\theta}, h) + e_H \cdot (H(\bar{\theta}, h) \cdot L(\underline{\theta}, h) - H(\underline{\theta}, h) \cdot L(\bar{\theta}, h))} \right)
\end{aligned}$$

Case 3 is therefore preferred to case 4 as long as:

$$\begin{aligned}
& (\rho.H(\bar{\theta}, h).e_H + (1 - \rho).H(\underline{\theta}, h).e_H).V(H) + (\rho.L(\bar{\theta}, h).e_H + (1 - \rho).L(\underline{\theta}, h).e_H).V(L) \\
& - (c(e_H, \bar{\theta}) + \bar{U}).(\rho + (1 - \rho)).\frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)} \\
> & (\rho.H(\bar{\theta}, h).e_H + (1 - \rho).H(\underline{\theta}, h).e_L).V(H) + (\rho.L(\bar{\theta}, h).e_H + (1 - \rho).L(\underline{\theta}, h).e_L).V(L) \\
& - (c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{H(\underline{\theta}, h) + \rho.e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))}{H(\underline{\theta}, h) + e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))}\right) \\
& - \frac{\Delta c.(1 - \rho)}{e_H - e_L} \left(\frac{e_L.(H(\underline{\theta}, h) + e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))) - e_H.H(\bar{\theta}, h)}{H(\underline{\theta}, h) + e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))}\right) \\
\Rightarrow & (1 - \rho).H(\underline{\theta}, h).(e_H - e_L).V(H) + (1 - \rho).L(\underline{\theta}, h).(e_H - e_L).V(L) > \\
& (c(e_H, \bar{\theta}) + \bar{U}).(\rho + (1 - \rho)).\frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)} - \frac{H(\underline{\theta}, h) + \rho.e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))}{H(\underline{\theta}, h) + e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))} \\
& - \frac{\Delta c.(1 - \rho)}{e_H - e_L} \left(\frac{e_L.(H(\underline{\theta}, h) + e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))) - e_H.H(\bar{\theta}, h)}{H(\underline{\theta}, h) + e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))}\right) \\
\Rightarrow & (e_H - e_L).((1 - \rho).H(\underline{\theta}, h).V(H) + (1 - \rho).L(\underline{\theta}, h).V(L)) > \\
& (1 - \rho).(c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)} - \frac{H(\underline{\theta}, h)}{H(\underline{\theta}, h) + e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))}\right) \\
& - \frac{\Delta c.(1 - \rho)}{e_H - e_L} \left(\frac{e_L.(H(\underline{\theta}, h) + e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))) - e_H.H(\bar{\theta}, h)}{H(\underline{\theta}, h) + e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))}\right) \\
\Rightarrow & (e_H - e_L).(H(\underline{\theta}, h).V(H) + L(\underline{\theta}, h).V(L)) > \\
& (c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}\right) - \frac{\Delta c}{e_H - e_L} \left(\frac{e_L.(H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h))}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) \\
& - (c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{H(\underline{\theta}, h)}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) \\
& + \frac{\Delta c}{e_H - e_L} \left(\frac{e_H.H(\bar{\theta}, h)}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) \\
\Rightarrow & (e_H - e_L).(H(\underline{\theta}, h).V(H) + L(\underline{\theta}, h).V(L)) > (c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{H(\underline{\theta}, h)}{H(\bar{\theta}, h)}\right) - \frac{e_L.\Delta c}{e_H - e_L} - w(0, h)^4
\end{aligned}$$

The value to the principal is increasing in the probability of a high type whenever:

$$\begin{aligned}
0 &< H(\bar{\theta}, h).e_H.V(H) - H(\underline{\theta}, h).e_L.V(H) + L(\bar{\theta}, h).e_H.V(L) - L(\underline{\theta}, h).e_L.V(L) \\
&- (c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) \\
&+ \frac{\Delta c}{e_H - e_L} \left(\frac{e_L.(H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)) - e_H.H(\bar{\theta}, h)}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) \\
\implies 0 &< (H(\bar{\theta}, h).e_H - H(\underline{\theta}, h).e_L).V(H) + (L(\bar{\theta}, h).e_H - L(\underline{\theta}, h).e_L).V(L) \\
&- (c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) \\
&+ \frac{\Delta c}{e_H - e_L} \left(\frac{e_L.(H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)) - e_H.H(\bar{\theta}, h)}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) \\
\implies &(H(\bar{\theta}, h).e_H - H(\underline{\theta}, h).e_L).V(H) + (L(\bar{\theta}, h).e_H - L(\underline{\theta}, h).e_L).V(L) > \\
&(c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h))}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) \\
&- \frac{\Delta c}{e_H - e_L} \left(\frac{e_L.(H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)) - e_H.H(\bar{\theta}, h)}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) \\
\implies &(H(\bar{\theta}, h).e_H - H(\underline{\theta}, h).e_L).V(H) + (L(\bar{\theta}, h).e_H - L(\underline{\theta}, h).e_L).V(L) > \\
&(c(e_H, \bar{\theta}) + \bar{U}).\left(\frac{e_H.(H(\bar{\theta}, h).L(\underline{\theta}, h) - H(\underline{\theta}, h).L(\bar{\theta}, h)) - (H(\underline{\theta}, h) + L(\underline{\theta}, h))}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) \\
&+ \frac{\Delta c}{e_H - e_L} \cdot \left(\frac{e_H.H(\bar{\theta}, h).(1 - e_L.L(\underline{\theta}, h)) - e_L.H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.(H(\bar{\theta}, h) + L(\bar{\theta}, h)) - 1}{H(\underline{\theta}, h).(1 - e_H.L(\bar{\theta}, h)) + e_H.H(\bar{\theta}, h).L(\underline{\theta}, h)}\right) + w(H, h)
\end{aligned}$$

Will agents lie about their type?

At the beginning of the second chapter we introduced the constraints that we implement in order to govern the behaviour of each type of player. We have two *IC* constraints so that each player should select the effort level that we deem appropriate and also the target that fits his type. We consider the constraints independently, given compliance in the other. Might it be in the interests however of either kind of player to violate both simultaneously? This effectively entails attempting to convince the principal that one is of the other type of agent. We in fact now show that, in the solutions in all of our cases, considering the constraints independently guarantees that no player would like to or be able to violate both simultaneously.

Firstly, let us consider cases 3 and 4. In both of these cases, both types of agent are

required to claim a high target. In order to mimic the other player, one in fact has to behave exactly as they should anyway. Furthermore, if either type were to claim a low target then they would be violating the requirements in their contract and so would not be accepted by the principal. We can therefore rule out any potential problems occurring in either of the last two cases and focus solely on the first two. In case 1, both types of players are required to exert high effort. Therefore in this case, we need to check if either type of player would find it in their interests to claim the target of the other player, then exert a low level of effort. Finally, in case 2, we need to check if either type of player would like to completely mimic the other. For the high type this involves a low target and low effort and for the low type, a high target and a high level of effort.

In case 1, we wish to show that the following two conditions must hold:

$$\begin{aligned} & e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta}) \\ \geq & e_L.(L(\bar{\theta}, l).w(L, l) + (1 - L(\bar{\theta}, l)).w(0, l)) - c(e_L, \bar{\theta}) \end{aligned}$$

and

$$\begin{aligned} & e_H.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_H, \underline{\theta}) \\ \geq & e_L.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_L, \underline{\theta}) \end{aligned}$$

Ignoring the *PC*'s, we know that at the solution *IC2* and *IC4* were binding and all others slack, and furthermore that three of the wages are non-zero. Substituting all of that information into our constraints they reduce to:

$$e_H.H(\bar{\theta}, h).w(H, h) - c(e_H, \bar{\theta}) > e_L.H(\bar{\theta}, h).w(H, h) - c(e_L, \bar{\theta}) \quad (5.4)$$

$$e_H.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_H, \underline{\theta}) = e_L.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_L, \underline{\theta}) \quad (5.5)$$

$$e_H.H(\bar{\theta}, h).w(H, h) - c(e_H, \bar{\theta}) > e_H.(L(\bar{\theta}, l).w(L, l) + (1 - L(\bar{\theta}, l)).w(0, l)) - c(e_H, \bar{\theta}) \quad (5.6)$$

$$e_H.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_H, \underline{\theta}) = e_H.H(\underline{\theta}, h).w(H, h) - c(e_H, \underline{\theta}) \quad (5.7)$$

Our conditions that we need to show are satisfied therefore reduce to:

$$e_H.H(\bar{\theta}, h).w(H, h) - c(e_H, \bar{\theta}) \geq e_L.(L(\bar{\theta}, l).w(L, l) + (1 - L(\bar{\theta}, l)).w(0, l)) - c(e_L, \bar{\theta})$$

$$e_H.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_H, \underline{\theta}) \geq e_L.H(\underline{\theta}, h).w(H, h) - c(e_L, \underline{\theta})$$

From our constraints we can derive that:

$$(5.4) + e_L.(5.6) \implies e_H.H(\bar{\theta}, h).w(H, h) - c(e_H, \bar{\theta}) > e_L.(L(\bar{\theta}, l).w(L, l) + (1 - L(\bar{\theta}, l)).w(0, l)) - c(e_L, \bar{\theta})$$

$$(5.5) + e_L.(5.7) \implies e_H.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_H, \underline{\theta}) = e_L.H(\underline{\theta}, h).w(H, h) - c(e_L, \underline{\theta})$$

Which are exactly the two conditions that we were looking to satisfy. The former is strictly satisfied and the latter holds with equality, in which case we assume that the agent's indifference leads him to choose the principal's desired option.

In case 2, the conditions that we need to hold are:

$$\begin{aligned} & e_H.(H(\bar{\theta}, h).w(H, h) + L(\bar{\theta}, h).w(L, h) + (1 - H(\bar{\theta}, h) - L(\bar{\theta}, h)).w(0, h)) - c(e_H, \bar{\theta}) \\ \geq & e_L.(L(\bar{\theta}, l).w(L, l) + (1 - L(\bar{\theta}, l)).w(0, l)) - c(e_L, \bar{\theta}) \end{aligned}$$

and

$$\begin{aligned} & e_L.(L(\underline{\theta}, l).w(L, l) + (1 - L(\underline{\theta}, l)).w(0, l)) - c(e_L, \underline{\theta}) \\ \geq & e_H.(H(\underline{\theta}, h).w(H, h) + L(\underline{\theta}, h).w(L, h) + (1 - H(\underline{\theta}, h) - L(\underline{\theta}, h)).w(0, h)) - c(e_H, \underline{\theta}) \end{aligned}$$

Once again, from our solution and ignoring the *PC*'s, we know that only *IC4* binds and that only two wages are non-zero. Our constraints at the solution therefore reduce to:

$$e_H.H(\bar{\theta}, h).w(H, h) - c(e_H, \bar{\theta}) > e_L.H(\bar{\theta}, h).w(H, h) - c(e_L, \bar{\theta}) \quad (5.8)$$

$$e_L.(1 - L(\underline{\theta}, l)).w(0, l) - c(e_L, \underline{\theta}) > e_H.(1 - L(\underline{\theta}, l)).w(0, l) - c(e_H, \underline{\theta}) \quad (5.9)$$

$$e_H.H(\bar{\theta}, h).w(H, h) - c(e_H, \bar{\theta}) > e_H.(1 - L(\bar{\theta}, l)).w(0, l) - c(e_H, \bar{\theta}) \quad (5.10)$$

$$e_L.(1 - L(\underline{\theta}, l)).w(0, l) - c(e_L, \underline{\theta}) = e_L.H(\underline{\theta}, h).w(H, h) - c(e_L, \underline{\theta}) \quad (5.11)$$

And our conditions that must be satisfied reduce to:

$$e_H.H(\bar{\theta}, h).w(H, h) - c(e_H, \bar{\theta}) \geq e_L.(1 - L(\bar{\theta}, l)).w(0, l) - c(e_L, \bar{\theta})$$

$$e_L.(1 - L(\underline{\theta}, l)).w(0, l) - c(e_L, \underline{\theta}) \geq e_H.H(\underline{\theta}, h).w(H, h) - c(e_H, \underline{\theta})$$

From our constraints we can derive that:

$$(5.8) + e_L.(5.10) \implies e_H.H(\bar{\theta}, h).w(H, h) - c(e_H, \bar{\theta}) > e_L.(1 - L(\bar{\theta}, l)).w(0, l) - c(e_L, \bar{\theta})$$

$$(5.9) + e_H.(5.11) \implies e_L.(1 - L(\underline{\theta}, l)).w(0, l) - c(e_L, \underline{\theta}) > e_H.H(\underline{\theta}, h).w(H, h) - c(e_H, \underline{\theta})$$

Which again are precisely the conditions that we needed. Furthermore, both hold with strict inequality. We can therefore say that in none of the cases that we consider would an agent be able to mislead the principal or find it in his interests to do so and violate both actions simultaneously.

Appendix C

We now calculate total payoffs in all states of the world for our game in chapter three when the two players have not integrated. The left hand column refers to the action choices made by the players in periods 1 and 2. So for example, $a; (C, N)$ means that A was in control and acted selfishly in period 1. Then in period 2, A chose to compromise while B chose not to compromise.

Table 5.1: Total Payoffs in all States

| Non-Integration | | | |
|-----------------|-------------------------------------|---|---|
| Actions | A 's Payoff | Good B 's Payoff | Bad B 's Payoff |
| $a; (C, C)$ | $H_A + H_B + P_A + H_A - S_B$ | H_B | H_B |
| $a; (C, N)$ | $H_A + H_B + P_A + L_A - S_B$ | $H_B + \underline{P_B}$ | $H_B + \overline{P_B}$ |
| $a; (N, C)$ | $H_A + H_B + P_A + H_A + P_A - S_B$ | L_B | L_B |
| $a; (N, N)$ | $H_A + H_B + P_A + M_A + P_A - S_B$ | $M_B + \underline{P_B}$ | $M_B + \overline{P_B}$ |
| $b; (C, C)$ | H_A | $H_A + H_B + \underline{P_B} + H_B - S_A$ | $H_A + H_B + \overline{P_B} + H_B - S_A$ |
| $b; (C, N)$ | L_A | $H_A + H_B + \underline{P_B} + H_B + \underline{P_B} - S_A$ | $H_A + H_B + \overline{P_B} + H_B + \overline{P_B} - S_A$ |
| $b; (N, C)$ | $H_A + P_A$ | $H_A + H_B + \underline{P_B} + L_B - S_A$ | $H_A + H_B + \overline{P_B} + L_B - S_A$ |
| $b; (N, N)$ | $M_A + P_A$ | $H_A + H_B + \underline{P_B} + M_B + \underline{P_B} - S_A$ | $H_A + H_B + \overline{P_B} + M_B + \overline{P_B} - S_A$ |
| $c; (C, C)$ | $H_A + H_A$ | $H_B + H_B$ | H_B |
| $c; (C, N)$ | $H_A + L_A$ | $H_B + H_B + \underline{P_B}$ | $H_B + \overline{P_B}$ |
| $c; (N, C)$ | $H_A + H_A + P_A$ | $H_B + L_B$ | L_B |
| $c; (N, N)$ | $H_A + M_A + P_A$ | $H_B + M_B + \underline{P_B}$ | $M_B + \overline{P_B}$ |

What ensures that our problem remains relevant, is that individually, players' incentives are not the same. More specifically, a good player should look to ensure the compromise outcome, whereas a bad player should be aiming to mislead the other player into believing he is of good type, so that he can make the individually selfish choice of a non-compromising action. However, primarily the distinction between our contribution and that of Aghion et al, is the further complication of an integration decision. We therefore provide the conditions for which our rankings are equivalent to theirs, both under integration and non-integration. For a good B , we need the following period 2 ranking:

$$(C, C) > (N, N) > (N, C) \quad \text{AND} \quad (C, C) > (C, N)$$

Under non-integration, this equates to:

$$H_B > M_B + \underline{P_B} > L_B \quad \text{AND} \quad H_B > H_B + \underline{P_B}$$

Given that $H_B > M_B > L_B$ and $\underline{P}_B < 0$, the only constraint that is important is:

$$\begin{aligned} M_B + \underline{P}_B &> L_B \\ \implies M_B - L_B &> -\underline{P}_B \end{aligned} \tag{5.12}$$

Under integration, the equations become:

$$\begin{aligned} (1 - \alpha).(H_A + H_B) &> (1 - \alpha).(M_A + M_B) + \underline{P}_B > (1 - \alpha).(H_A + L_B) \\ \text{AND} \quad (1 - \alpha).(H_A + H_B) &> (1 - \alpha).(L_A + H_B) + \underline{P}_B \end{aligned}$$

Once again, the only constraint that we must be concerned with is:

$$\begin{aligned} (1 - \alpha).(M_A + M_B) + \underline{P}_B &> (1 - \alpha).(H_A + L_B) \\ \implies (1 - \alpha).(M_A - H_A + M_B - L_B) &> -\underline{P}_B \\ \implies (1 - \alpha).(M_B - L_B + M_A - H_A) &> -\underline{P}_B \end{aligned} \tag{5.13}$$

Therefore, given that $H_A > M_A$ and $0 < \alpha < 1$:

$$\implies M_B - L_B > M_B - L_B + M_A - H_A > (1 - \alpha).(M_B - L_B + M_A - H_A)$$

And so if (5.13) is satisfied then so is (5.12).

For a bad B we need the following rankings:

$$(C, N) > (C, C) > (N, N) > (N, C)$$

Therefore, under non-integration:

$$H_B + \overline{P}_B > H_B > M_B + \overline{P}_B > L_B$$

Given that $H_B > M_B > L_B$ and $\overline{P}_B > 0$, the only constraint that is important is:

$$H_B > M_B + \overline{P}_B$$

$$\implies H_B - M_B > \overline{P_B} \quad (5.14)$$

Under integration the constraints become:

$$(1-\alpha).(L_A+H_B)+\overline{P_B} > (1-\alpha).(H_A+H_B) > (1-\alpha).(M_A+M_B)+\overline{P_B} > (1-\alpha).(H_A+L_B)$$

From the above we can derive three separate conditions:

$$\overline{P_B} > (1-\alpha).(H_A - L_A) \quad (5.15)$$

$$(1-\alpha).(H_A - M_A + H_B - M_B) > \overline{P_B} \quad (5.16)$$

$$\overline{P_B} > (1-\alpha).(H_A - M_A + L_B - M_B) \quad (5.17)$$

Given that $M_A > L_A$ and $M_B > L_B$, then if (5.15) is satisfied then so is (5.17). In which case only the first two need to be satisfied. It is therefore necessary but not sufficient that:

$$H_B - M_B > M_A - L_A$$

If the appropriate conditions above hold, then our rankings are equivalent to Aghion et al and we can conclude that our problem is relevant. We can also derive one more interesting condition from the above relationships. For the rankings to mimic Aghion et al, under integration, we need (5.13), (5.15) and (5.16) to be satisfied. Summing (5.13) and (5.16):

$$\implies (1-\alpha).(M_B - L_B + M_A - H_A + H_A - M_A + H_B - M_B) > \overline{P_B} - \underline{P_B}$$

$$\implies (1-\alpha).(H_B - L_B) > \overline{P_B} - \underline{P_B}$$

$$\implies (1-\alpha) > \frac{\overline{P_B} - \underline{P_B}}{H_B - L_B}$$

From (5.15):

$$\overline{P_B} > (1-\alpha).(H_A - L_A)$$

$$\implies \overline{P_B} - \underline{P_B} > (1-\alpha).(H_A - L_A)$$

$$\implies \frac{\overline{P_B} - \underline{P_B}}{H_A - L_A} > (1 - \alpha)$$

Combining the two:

$$\begin{aligned} \implies \frac{\overline{P_B} - \underline{P_B}}{H_A - L_A} > (1 - \alpha) &> \frac{\overline{P_B} - \underline{P_B}}{H_B - L_B} \\ \implies H_B - L_B > H_A - L_A \end{aligned}$$

We interpret this condition as follows. The consequences of B 's actions are greater than A 's. With this in mind, the signaling game played before control is allocated becomes doubly important. We therefore derive conditions for which each type of player B will find it in their interests to truthfully signal their type.

Proof of Proposition 16. We replicate the equilibria of Aghion et al. Firstly, when control is contractible, if B reports that he is good, then A keeps control. If he reports that he is bad, then he is rewarded with control some of the time. Each player, when granted control, acts selfishly. Therefore, a good B would prefer to truthfully signal his type when the gains from doing so, resulting in the compromise outcome in the second period, outweigh the gains from lying, and being granted control (with some probability) and the opportunity to act selfishly in the first period. Under non-integration this condition is:

$$\begin{aligned} 0 + H_B &\geq (H_A + H_B + \underline{P_B} - S_A) \cdot \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}} \right) + M_B + \underline{P_B} \\ H_B - M_B &\geq (H_A + H_B + \underline{P_B} - S_A) \cdot \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}} \right) + \underline{P_B} \end{aligned} \quad (5.18)$$

Where the probability that a bad claim results in control is $\left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}} \right)$, as in Aghion et al. Under integration, the condition becomes:

$$\begin{aligned} 0 + (1 - \alpha) \cdot (H_A + H_B) &\geq (H_A + H_B + \underline{P_B} - S_A) \cdot \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}} \right) + (1 - \alpha) \cdot (M_A + M_B) + \underline{P_B} \\ (1 - \alpha) \cdot (H_A + H_B - M_A - M_B) &\geq (H_A + H_B + \underline{P_B} - S_A) \cdot \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}} \right) + \underline{P_B} \end{aligned} \quad (5.19)$$

Comparing the two conditions, the right hand side of both is the same. The left hand

sides have the following relationship:

$$H_B - M_B \geq (1 - \alpha).(H_A + H_B - M_A - M_B)$$

$$\alpha.(H_B - M_B) \geq (1 - \alpha).(H_A - M_A)$$

Suppose therefore that B takes a large share of the profit, so that:

$$\alpha.(H_B - M_B) < (1 - \alpha).(H_A - M_A)$$

Then integration strengthens his truth-telling incentives when good. Now, a bad B would have to prefer to admit he is bad, knowing that some of the time he will be rewarded with control, instead of misleading player A by claiming he is good, so that in the second period he can cheat him and result in the (C, N) outcome. Under non-integration this must satisfy:

$$(H_A + H_B + \overline{P}_B - S_A).(\frac{H_B + \overline{P}_B - S_A}{H_A + H_B + \overline{P}_B}) + M_B + \overline{P}_B \geq 0 + H_B + \overline{P}_B$$

$$(H_A + H_B + \overline{P}_B - S_A).(\frac{H_B + \overline{P}_B - S_A}{H_A + H_B + \overline{P}_B}) \geq H_B - M_B \quad (5.20)$$

Under integration, the condition becomes:

$$(H_A + H_B + \overline{P}_B - S_A).(\frac{H_B + \overline{P}_B - S_A}{H_A + H_B + \overline{P}_B}) + (1 - \alpha).(M_A + M_B) + \overline{P}_B \geq 0 + (1 - \alpha).(L_A + H_B) + \overline{P}_B$$

$$(H_A + H_B + \overline{P}_B - S_A).(\frac{H_B + \overline{P}_B - S_A}{H_A + H_B + \overline{P}_B}) \geq (1 - \alpha).(L_A + H_B - M_A - M_B) \quad (5.21)$$

Comparing the two conditions, the left hand side of both are the same. If we compare the two on the right hand side:

$$H_B - M_B \geq (1 - \alpha).(L_A + H_B - M_A - M_B)$$

$$\implies \alpha.(H_B - M_B) > (1 - \alpha).(L_A - M_A)$$

As the right hand side of the above is clearly negative. Therefore, if (5.20) is satisfied,

then so is (5.21). In which case, we can say that integration increases the incentives for truth-telling for a bad B . Note that this condition holds for all levels of α .

We now determine the conditions under which each type of player B would truthfully signal their type, under the setting of transferable control. We therefore focus only on a separating equilibrium. They would truthfully signal their type whenever the gain outweighs the cost. For the good type, he would make gains from choosing the selfish action, however, should he do this, he knows that the second period outcome would be that of non-compromise. Additionally, player A would also shade B 's output. Therefore, the gain from second period compromise should exceed the first period loss from compromise. Under non-integration this can be expressed as:

$$\begin{aligned} H_B - (M_B + \underline{P}_B - S_A) &\geq H_A + H_B + \underline{P}_B - H_B \\ \implies H_B - 2.\underline{P}_B + S_A &\geq H_A + M_B \end{aligned} \quad (5.22)$$

However, when the two parties integrate then the condition needs to satisfy:

$$\begin{aligned} (1 - \alpha).(H_A + H_B) - ((1 - \alpha).(M_A + M_B) + \underline{P}_B - S_A) &\geq H_A + H_B + \underline{P}_B - H_B \\ \implies (H_A + H_B) - \alpha.(H_A + H_B) - (1 - \alpha).(M_A + M_B) - \underline{P}_B + S_A &\geq H_A + \underline{P}_B \\ \implies H_B - 2.\underline{P}_B + S_A &\geq \alpha.(H_A + H_B) + (1 - \alpha).(M_A + M_B) \\ \implies H_B - 2.\underline{P}_B + S_A &\geq H_A + M_B + \alpha.(H_A + H_B) - H_A + (1 - \alpha).(M_A + M_B) - M_B \\ \implies H_B - 2.\underline{P}_B + S_A &\geq H_A + M_B + \alpha.H_B - (1 - \alpha).H_A + (1 - \alpha).M_A - \alpha.M_B \\ \implies H_B - 2.\underline{P}_B + S_A &\geq H_A + M_B + \alpha.(H_B - M_B) - (1 - \alpha).(H_A - M_A) \end{aligned} \quad (5.23)$$

Similarly, for a bad type, the gain from being selfish in the first period should outweigh the potential gain from misleading player A , by compromising in period 1, so that he can cheat him in the second period and result in the (C, N) outcome. Under non-integration, this must satisfy:

$$H_A + H_B + \overline{P}_B \geq H_B + \overline{P}_B - (M_B + \overline{P}_B - S_A)$$

$$H_A + \overline{P}_B \geq S_A - M_B \quad (5.24)$$

Once again, when the two parties integrate, the conditions become:

$$\begin{aligned} H_A + H_B + \overline{P}_B &\geq (1 - \alpha) \cdot (L_A + H_B) + \overline{P}_B - ((1 - \alpha) \cdot (M_A + M_B) + \overline{P}_B - S_A) \\ H_A + H_B + \overline{P}_B &\geq (1 - \alpha) \cdot (L_A + H_B - M_A - M_B) + S_A \\ H_A + \overline{P}_B &\geq S_A + (1 - \alpha) \cdot (L_A - M_A) - (1 - \alpha) \cdot M_B + (1 - \alpha) \cdot H_B - H_B \\ H_A + \overline{P}_B &\geq S_A - M_B + (1 - \alpha) \cdot (L_A - M_A) + \alpha \cdot (M_B - H_B) \\ H_A + \overline{P}_B &\geq S_A - M_B - (\alpha \cdot (H_B - M_B) + (1 - \alpha) \cdot (M_A - L_A)) \end{aligned} \quad (5.25)$$

If the above conditions are satisfied, then each type of B will prefer to truthfully signal their type given transferability, and the ownership rule that follows. Obviously, if (5.24) is satisfied, then so is (5.25). Therefore integration increases the incentives for a bad B to truthfully signal their type. Once again, irrespective of the level of profit sharing α . It is not clear whether this is the case for a good B . If the following is negative, then the same as above can be said for a good B :

$$\alpha \cdot (H_B - M_B) - (1 - \alpha) \cdot (H_A - M_A)$$

As above, this is equivalent to B receiving a large share of the profits $(1 - \alpha)$. ■

Proof of Proposition 17. Following the work of Aghion et al, if any player has control then they act selfishly. This would be guaranteed if the gain from acting selfishly was greater than compromising. Note, that we focus on truthful revelation of type. For these results to hold, the conditions derived in the above proposition need to be satisfied. Under contractibility, we play a message game. Player A asks B his type, if he reports that he is of good type then A keeps control. If he reports bad type, then he gets control with some probability that depends on the payoffs. If we choose non-integration then that probability is $\frac{H_B + \overline{P}_B - S_A}{H_A + H_B + \overline{P}_B}$, under integration it is $\frac{(1 - \alpha) \cdot (L_A + H_B) + \overline{P}_B - S_A}{H_A + H_B + \overline{P}_B}$. If B is of good type then A is selfish in period 1, then we play the compromise outcome with B shading. If B is of bad type, then if A keeps control he is selfish, if B gets control he is selfish, then we play the non-compromise outcome

with B shading whenever A had control in period 1. So A 's expected profit under non-integration is:

$$\begin{aligned}
& (1 - \mu) \cdot (H_A + H_B + P_A + H_A - S_B) + \mu \cdot \left(1 - \frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (H_A + H_B + P_A \\
& + M_A + P_A - S_B) + \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (0 + M_A + P_A) \\
= & (1 - \mu) \cdot (H_A + H_B + P_A + H_A - S_B) + \mu \cdot (H_A + H_B + P_A + M_A + P_A - S_B) \\
& - \mu \cdot \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (H_A + H_B + P_A + M_A + P_A - S_B) \\
& + \mu \cdot \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (M_A + P_A) \\
= & (H_A + H_B + P_A - S_B) + (1 - \mu) \cdot H_A + \mu \cdot (M_A + P_A) \\
& - \mu \cdot \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (H_A + H_B + P_A - S_B) - \mu \cdot \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (M_A + P_A) \\
& + \mu \cdot \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (M_A + P_A) \\
= & (1 - \mu \cdot \left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right)) \cdot (H_A + H_B + P_A - S_B) \\
& + (1 - \mu) \cdot H_A + \mu \cdot (M_A + P_A)
\end{aligned}$$

And A 's expected profit under integration is:

$$\begin{aligned}
& (1 - \mu) \cdot (H_A + H_B + P_A + \alpha \cdot (H_A + H_B) - S_B) \\
& + \mu \cdot \left(1 - \frac{(1 - \alpha) \cdot (L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (H_A + H_B + P_A + \alpha \cdot (M_A + M_B) + P_A - S_B) \\
& + \left(\frac{(1 - \alpha) \cdot (L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (0 + \alpha \cdot (M_A + M_B) + P_A) \\
= & (1 - \mu) \cdot (H_A + H_B + P_A - S_B) + (1 - \mu) \cdot \alpha \cdot (H_A + H_B) + \mu \cdot (H_A + H_B + P_A \\
& + \alpha \cdot (M_A + M_B) + P_A - S_B) - \mu \cdot \left(\frac{(1 - \alpha) \cdot (L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (H_A + H_B + P_A \\
& + \alpha \cdot (M_A + M_B) + P_A - S_B) + \mu \cdot \left(\frac{(1 - \alpha) \cdot (L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) \cdot (\alpha \cdot (M_A + M_B) + P_A)
\end{aligned}$$

$$\begin{aligned}
&= (H_A + H_B + P_A - S_B) + (1 - \mu).\alpha.(H_A + H_B) + \mu.(\alpha.(M_A + M_B) + P_A) \\
&\quad - \mu.\left(\frac{(1 - \alpha).(L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right).(H_A + H_B + P_A - S_B) \\
&\quad - \mu.\left(\frac{(1 - \alpha).(L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right).(\alpha.(M_A + M_B) + P_A) \\
&\quad + \mu.\left(\frac{(1 - \alpha).(L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right).(\alpha.(M_A + M_B) + P_A) \\
&= (1 - \mu.\left(\frac{(1 - \alpha).(L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right)).(H_A + H_B + P_A - S_B) \\
&\quad + (1 - \mu).\alpha.(H_A + H_B) + \mu.(\alpha.(M_A + M_B) + P_A)
\end{aligned}$$

Therefore, integration is preferred to non-integration whenever:

$$\begin{aligned}
&(1 - \mu.\left(\frac{(1 - \alpha).(L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right)).(H_A + H_B + P_A - S_B) \\
&\quad + (1 - \mu).\alpha.(H_A + H_B) + \mu.(\alpha.(M_A + M_B) + P_A) \\
&> (1 - \mu.\left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right)).(H_A + H_B + P_A - S_B) + (1 - \mu).H_A + \mu.(M_A + P_A) \\
\iff &\mu.\left(\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right) - \left(\frac{(1 - \alpha).(L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}\right)).(H_A + H_B + P_A - S_B) \\
&\quad + (1 - \mu).\alpha.(H_A + H_B) + \mu.\alpha.(M_A + M_B) \\
&> (1 - \mu).H_A + \mu.M_A \\
\iff &\mu.\left(\frac{H_B + \overline{P_B} - S_A - (1 - \alpha).(L_A + H_B) - \overline{P_B} + S_A}{H_A + H_B + \overline{P_B}}\right).(H_A + H_B + P_A - S_B) \\
&\quad + (1 - \mu).\alpha.H_A + (1 - \mu).\alpha.H_B + \mu.\alpha.M_A + \mu.\alpha.M_B \\
&> (1 - \mu).H_A + \mu.M_A \\
\iff &\mu.\left(\frac{H_B - (1 - \alpha).H_B - (1 - \alpha).L_A}{H_A + H_B + \overline{P_B}}\right).(H_A + H_B + P_A - S_B) + (1 - \mu).\alpha.H_B + \mu.\alpha.M_B \\
&> (1 - \mu).(1 - \alpha).H_A + \mu.(1 - \alpha).M_A \\
\iff &\mu.\left(\frac{\alpha.H_B - (1 - \alpha).L_A}{H_A + H_B + \overline{P_B}}\right).(H_A + H_B + P_A - S_B) + \alpha.((1 - \mu).H_B + \mu.M_B) \\
&> (1 - \alpha)((1 - \mu).H_A + \mu.M_A)
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow \alpha \cdot (\mu \cdot H_B \cdot (\frac{H_A + H_B + P_A - S_B}{H_A + H_B + \overline{P_B}}) + \mu \cdot M_B + (1 - \mu) \cdot H_B) \\
&> (1 - \alpha) (\mu \cdot L_A \cdot (\frac{H_A + H_B + P_A - S_B}{H_A + H_B + \overline{P_B}}) + \mu \cdot M_A + (1 - \mu) \cdot H_A) \\
&\Leftrightarrow \frac{\alpha}{1 - \alpha} > \frac{(\frac{H_A + H_B + P_A - S_B}{H_A + H_B + \overline{P_B}}) \cdot \mu \cdot L_A + \mu \cdot M_A + (1 - \mu) \cdot H_A}{(\frac{H_A + H_B + P_A - S_B}{H_A + H_B + \overline{P_B}}) \cdot \mu \cdot H_B + \mu \cdot M_B + (1 - \mu) \cdot H_B}
\end{aligned}$$

■

Proof of Proposition 18. In the case of transferability, we require, as in Aghion et al, that B compromises if and only if she is good. If B is of good type then A is selfish in period 1, then we play the compromise outcome with B shading. If B is of bad type, then he acts selfishly, followed by the non-compromise outcome. It should be made clear that we focus on a separating equilibrium here and so the relevant conditions derived in proposition 16 need to be satisfied. Therefore A 's expected profit under non-integration is:

$$\begin{aligned}
&(1 - \mu) \cdot (H_A + H_B + P_A + H_A - S_B) + \mu \cdot (0 + M_A + P_A) \\
&= (1 - \mu) \cdot (H_A + H_B + P_A - S_B) + (1 - \mu) \cdot H_A + \mu \cdot (M_A + P_A)
\end{aligned}$$

On a side note, let us compare this to the case under contractibility:

$$= (1 - \mu \cdot (\frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}})) \cdot (H_A + H_B + P_A - S_B) + (1 - \mu) \cdot H_A + \mu \cdot (M_A + P_A)$$

Therefore contractibility offers a larger payoff if and only if:

$$1 > \frac{H_B + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}$$

Which it clearly does.

Now let us look at A 's expected payoff under integration:

$$\begin{aligned}
&(1 - \mu) \cdot (H_A + H_B + P_A + \alpha \cdot (H_A + H_B) - S_B) + \mu \cdot (0 + \alpha \cdot (M_A + M_B) + P_A) \\
&= (1 - \mu) \cdot (H_A + H_B + P_A - S_B) + (1 - \mu) \cdot \alpha \cdot (H_A + H_B) + \mu \cdot (\alpha \cdot (M_A + M_B) + P_A)
\end{aligned}$$

Once again comparing this to the case under contractibility:

$$\begin{aligned}
&= (1 - \mu \cdot (\frac{(1 - \alpha) \cdot (L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}})) \cdot (H_A + H_B + P_A - S_B) \\
&\quad + (1 - \mu) \cdot \alpha \cdot (H_A + H_B) + \mu \cdot (\alpha \cdot (M_A + M_B) + P_A)
\end{aligned}$$

Therefore contractibility offers a larger payoff if and only if:

$$1 > \frac{(1 - \alpha) \cdot (L_A + H_B) + \overline{P_B} - S_A}{H_A + H_B + \overline{P_B}}$$

Which it clearly does. So we can therefore see that A benefits more from contractibility than transferability. Now let us determine the condition whereby A would prefer to integrate under transferability:

$$\begin{aligned}
&(1 - \mu) \cdot (H_A + H_B + P_A - S_B) + (1 - \mu) \cdot \alpha \cdot (H_A + H_B) + \mu \cdot (\alpha \cdot (M_A + M_B) + P_A) \\
&> (1 - \mu) \cdot (H_A + H_B + P_A - S_B) + (1 - \mu) \cdot H_A + \mu \cdot (M_A + P_A) \\
\iff &(1 - \mu) \cdot \alpha \cdot (H_A + H_B) + \mu \cdot \alpha \cdot (M_A + M_B) > (1 - \mu) \cdot H_A + \mu \cdot M_A \\
\iff &(1 - \mu) \cdot \alpha \cdot H_A + (1 - \mu) \cdot \alpha \cdot H_B + \mu \cdot \alpha \cdot M_A + \mu \cdot \alpha \cdot M_B > (1 - \mu) \cdot H_A + \mu \cdot M_A \\
\iff &(1 - \mu) \cdot \alpha \cdot H_B + \mu \cdot \alpha \cdot M_B > (1 - \mu) \cdot (1 - \alpha) \cdot H_A + \mu \cdot (1 - \alpha) \cdot M_A \\
\iff &\alpha \cdot ((1 - \mu) \cdot H_B + \mu \cdot M_B) > (1 - \alpha) \cdot ((1 - \mu) \cdot H_A + \mu \cdot M_A) \\
\iff &\frac{\alpha}{1 - \alpha} > \frac{\mu \cdot M_A + (1 - \mu) \cdot H_A}{\mu \cdot M_B + (1 - \mu) \cdot H_B}
\end{aligned}$$

■

Proof of Corollaries 19 and 20. Let us now suppose that B is certainly of good type, in which $\mu = 0$. Therefore, both of the conditions derived above reduce to:

$$\frac{\alpha}{1 - \alpha} > \frac{H_A}{H_B}$$

Therefore, the only way that A prefers integration is if his share of the profit is larger than the relationship between H_A and H_B . These are the two contributions to total profit under the outcome where both players choose to compromise. Now let us suppose that B is certainly of bad type, $\mu = 1$. The condition derived above under

transferability reduces to:

$$\frac{\alpha}{1 - \alpha} > \frac{M_A}{M_B}$$

The same intuition as above now holds except for the outcome where both choose not to compromise. The corresponding condition under contractibility reduces to:

$$\frac{\alpha}{1 - \alpha} > \frac{\frac{(H_A + H_B + P_A - S_B) \cdot L_A}{H_A + H_B + \overline{P_B}} + M_A}{\frac{(H_A + H_B + P_A - S_B) \cdot H_B}{H_A + H_B + \overline{P_B}} + M_B}$$

We can see that $\frac{(H_A + H_B + P_A - S_B) \cdot H_B}{H_A + H_B + \overline{P_B}} > \frac{(H_A + H_B + P_A - S_B) \cdot L_A}{H_A + H_B + \overline{P_B}}$ in which case, comparing to the line above, this condition can be satisfied at a lower level of α . Therefore, for given payoffs, player A would accept a lower share of the profits under contractibility and still prefer to integrate. ■

Proof of Proposition 21. Given that A will choose to compromise, when a bad B chooses to compromise in the second period he receives the payoff:

$$(1 - \alpha) \cdot (H_A + H_B)$$

If on the other hand he chooses not to compromise, then he harms the total output, but receives a private benefit that he does not have to share and has the payoff:

$$(1 - \alpha) \cdot (L_A + H_B) + \overline{P_B}$$

Therefore, if given a large enough share of the profits he can be incentivised to compromise so that:

$$(1 - \alpha) \cdot (H_A + H_B) = (1 - \alpha) \cdot (L_A + H_B) + \overline{P_B}$$

$$\implies (1 - \alpha) \cdot (H_A - L_A) = \overline{P_B}$$

$$\implies (1 - \alpha) = \frac{\overline{P_B}}{H_A - L_A}$$

In which case, as long as he receives the above share of total output, then he is happy to compromise. Recalling that the type of B is private information, in order to guarantee

the compromise outcome, in the contract stated at the beginning of the process, A must offer the above the above share to all partners. In the case of transferability, we can rewrite the condition where A prefers to integrate, by offering this share of output, as:

$$\begin{aligned}
\frac{\frac{H_A - L_A - \overline{P_B}}{H_A - L_A}}{\frac{\overline{P_B}}{H_A - L_A}} &> \frac{\mu \cdot M_A + (1 - \mu) \cdot H_A}{\mu \cdot M_B + (1 - \mu) \cdot H_B} \\
\iff (H_A - L_A - \overline{P_B}) \cdot (\mu \cdot M_B + (1 - \mu) \cdot H_B) &> (\mu \cdot M_A + (1 - \mu) \cdot H_A) \cdot \overline{P_B} \\
\iff \mu \cdot M_B \cdot (H_A - L_A - \overline{P_B}) - \mu \cdot M_A \cdot \overline{P_B} &> (1 - \mu) \cdot H_A \cdot \overline{P_B} \\
&\quad - (H_A - L_A - \overline{P_B}) \cdot (1 - \mu) \cdot H_B \\
\iff \mu \cdot (M_B \cdot (H_A - L_A - \overline{P_B}) - M_A \cdot \overline{P_B}) &> (1 - \mu) \cdot (H_A \cdot \overline{P_B} - (H_A - L_A - \overline{P_B}) \cdot H_B) \\
\iff \frac{\mu}{(1 - \mu)} &> \frac{H_A \cdot \overline{P_B} - (H_A - L_A - \overline{P_B}) \cdot H_B}{M_B \cdot (H_A - L_A - \overline{P_B}) - M_A \cdot \overline{P_B}} \\
\iff \frac{\mu}{(1 - \mu)} &> \frac{\overline{P_B} \cdot (H_A + H_B) - H_B \cdot (H_A - L_A)}{M_B \cdot (H_A - L_A) - \overline{P_B} \cdot (M_A + M_B)}
\end{aligned}$$

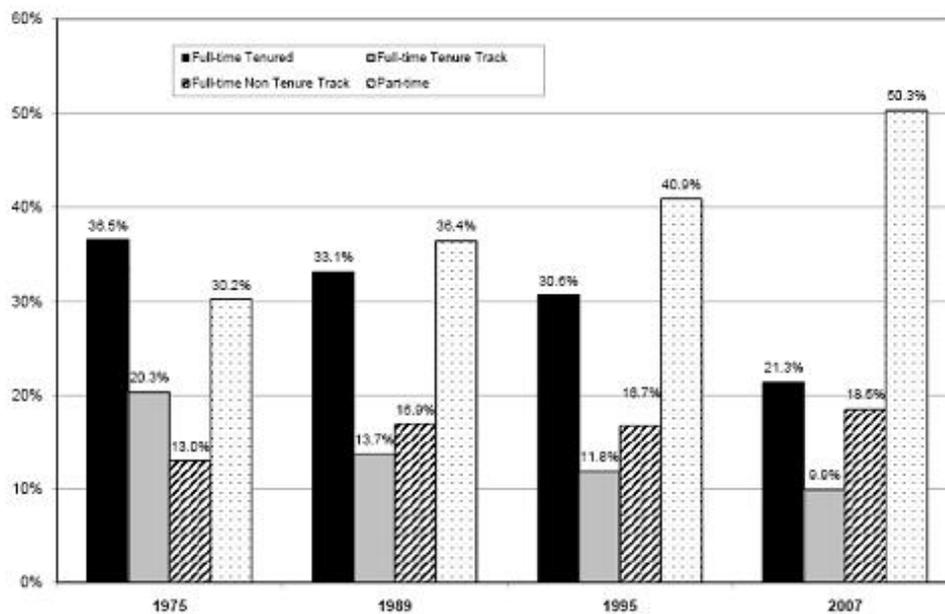
When is the right hand side > 1 ?

$$\begin{aligned}
\iff \overline{P_B} \cdot (H_A + H_B) - H_B \cdot (H_A - L_A) &> M_B \cdot (H_A - L_A) - \overline{P_B} \cdot (M_A + M_B) \\
\iff \overline{P_B} \cdot (H_A + H_B + M_A + M_B) &> (H_B + M_B) \cdot (H_A - L_A) \\
\iff \frac{\overline{P_B}}{H_A - L_A} &> \frac{H_B + M_B}{H_A + H_B + M_A + M_B} \\
\iff 1 - \alpha &> \frac{H_B + M_B}{H_A + H_B + M_A + M_B}
\end{aligned}$$

If this is true, then for A to prefer to integrate we must have that $\frac{\mu}{(1-\mu)} > 1$. ■

List of Figures

Figure 5-1: Trends in Faculty Status, 1975-2007



⁴Source: U.S. Department of Education, IPEDS Fall Staff Survey. Compiled by the American Association of University Professors.

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