Non-dominated Sorting on Performance Indicators for Evolutionary Many-objective Optimization

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Abstract

Much attention has been paid to solving real-world engineering problems with multiple conflicting objectives efficiently using evolutionary approaches. However, in such approaches the loss of selection pressure and the non-uniformity in the distribution of the Pareto optimal solutions can cause issues when there are many objectives. This has been observed in both dominance-based and decomposition-based multi-objective optimizers. We confront this performance is in this work we circumvent by exploiting two quality indicators, and use these in an optimizer's environmental selection via non-dominated sorting. This effectively converts the original many-objective problem into a bi-objective one. Our convergence performance criterion tries to balance the performance of individuals in different parts of the objective space. The angle between solutions on objective space is adopted to measure the diversity of each individual. Using these two measures, solutions can be separated into different layers easily, which is often not possible for the original many-objective optimization representation. The performance of the proposed method is evaluated on the DTLZ benchmark problems with up to 30 objectives, and MaF test suite with 10, 15, 20 and 30 objectives. The experimental results show that our proposed method is competitive compared to six recently proposed algorithms, especially for solving problems with a large number of objectives.

Keywords: Many-objective optimization problems, performance indicator, non-dominated sorting, environmental selection
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1. Introduction

Many real-world applications, such as industrial scheduling [1], controller design [2], and design optimization [3], often have multiple objectives that are in conflict with one another. Without loss of generality, multi-objective opti-

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mization problems (MOPs) can be modeled as follows:

$$\begin{aligned} &\text{Minimize} \quad \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})) \\ &\text{Subject to:} \quad \mathbf{x} \in \mathcal{X} \end{aligned} \tag{1}$$

where $\mathcal{X}$ is the feasible decision (variable) space (here in D dimensions) and $f_i(\mathbf{x}), i = 1, 2, \ldots, M$ are the objectives to be optimized. When the objective functions are in conflict it is not possible to define a single optimal solution, but a set of non-dominated solutions instead, which is known as Pareto optimal set (PS) in the decision space. The image in objective space of the Pareto optimal set under $\mathbf{F}(\cdot)$ is known as the Pareto front (PF). The main goal of multi-objective optimization is to find a set of solutions whose objective vectors form a uniformly distributed non-dominated set as close as possible to the PF.

Evolutionary multi-objective optimization (EMO) [4, 5, 6, 7, 8, 9] has garnered widespread attention because of its capability to find multiple tradeoff solutions simultaneously [10, 4, 6, 11]. However, it is often difficult to obtain good approximations to the Pareto set when problems have four or more objectives. Such problems are called many-objective problems (MaOPs) [12], and the issue with solving them stems from the loss of selection pressure [13] in high-dimensional objective spaces (which causes nearly all solutions to be incomparable with each other under a Pareto dominance comparison) [14].

Generally speaking, evolutionary optimization algorithms for many-objective problems can be divided into dominance based multi-objective evolutionary algorithms (MOEAs) [15, 16], decomposition based MOEAs [17, 18, 19, 20], and performance indicator-based MOEAs [21, 22, 23]. As indicated in [3, 8], dominance based MOEAs lose selection pressure significantly in environmental selection because the number of non-dominated individuals increases dramatically when the dimension of the objective space increases [24, 25, 26]. Therefore, the environmental selection behaves akin to a random selection process. This results in a final population whose members are distributed widely over the objective space but, which objective vectors lying far from the desired PF [27]. The straightforward way for confronting this problem is to modify the Pareto dominance relation. Some interesting attempts include loosening the dominance condition or dominance relation, such as α-dominance [28], and dominance area control [29]. These parameterized dominance relations are able to provide sufficient selection pressure towards the Pareto front. However, a crucial aspect of such methods is determining *a priori* an appropriate value of the parameter which determines the relaxation degree. This has been highlighted as an area needing further research [30]. For decomposition-based MOEAs, [24] shows that their performance strongly depends on the shape of the Pareto front. Hence the choice of their reference vectors is particularly important to achieve a good performance. However, as the dimension of the objective space increases, it is difficult to divide the objective space evenly into sub-objective spaces. Furthermore, it is also difficult to adapt the distribution of search directions when the Pareto front is irregular. In performance indicator approaches, such as the hypervolume (HV) [7] and R2 indicator [31], a fitness value is assigned to each individual based on the indicator before environmental selection. These approaches are popular as because both HV and R2 are able to account for convergence and diversity in parallel. For example, HyPE [21] and SMS-MOEA [7] use the hypervolume to evaluate the convergence and diversity of a solution. Unfortunately however, the computation of the hypervolume indicator can be relatively time-consuming compared to the other operations required during optimization, especially when the number of objectives is large. Recently, Li et al. [32] utilized the stochastic ranking technique to balance the search biases of different indicators. Tian et al. [33] developed an improved inverted generational distance indicator and designed a strategy to adaptively alter the reference vectors according to the indicator contributions of candidate solutions in an external archive. Sun et al. [23] proposed using IGD for environmental selection. Zhou et al. [34] designed a co-guided MaOEA (many-objective evolutionary optimizer) and used an indicator together with reference points to evaluate the convergence and diversity of the solutions. A promising-region based MaOEA with the ratio based indicator was proposed in [35], in which a ratio based indicator with infinite norm was used to identify the promising region and the parallel distance was adopted to select individuals in the promising region to ensure the diversity of the population.

In the evolutionary process, the contributions to convergence and the diversity performances of some solutions may be in conflict with one other, i.e., where many solutions might be located close to ideal (optimal) objective combinations, those solutions with good convergence may collectively have poor diversity. Therefore, in this paper, we regard the convergence performance and diversity performance as two separate objectives. A new population is selected according to non-dominated sorting on these two objectives. Note that although often the case, convergence and diversity are not *required* to be in conflict with each other. If both convergence and diversity are good, the individual will be definitely be in the first front. Relatedly, Li et al. [36] also proposed to convert a many-objective problem into a bi-goal problem encompassing proximity and diversity, called BiGE. In BigE, the summation of all elements of $\mathbf{F}(\mathbf{x})$ is taken as the first objective, and the crowding degree calculated niching is the second objective used for environmental selection. As the first objective compresses together all the values in the original objectives, this can lead to the loss of some important individuals, such as the extreme solutions of a convex Pareto front, or the solutions in the center of a concave Pareto front. Similarly, most existing convergence performance indicators are based only on the distance to the ideal or nadir point, which may also result in the loss of such solutions. Thus, in this paper, we propose a new convergence measure, which tries to balance the convergence performance of the solutions on the boundaries and in the center of the approximated Pareto front. For the diversity performance, we propose to use the angle to measure the degree of crowdedness between individuals, which has been shown more precise than the Euclidean distance in high-dimensional objective space [17]. In contrast, in BiGE [36] the diversity of solution is evaluated by a niching technique, which requires the setting of an additional parameter. The main contributions of this paper can be summarized as follows:

1. A new method to measure of convergence performance is proposed, which makes use of both an ideal point and a nadir point to balance the convergence performance of individuals that are located in different regions of the objective space.
2. The angle between an individual and its closest neighbour, is used as the diversity performance as an objective to be maximized together with the convergence performance for guiding the search.
3. The effectiveness of contributions 1) and 2) are evaluated on a range of well-known test problems, and shown to be competitive with state-of-the-art methods, particularly for many-objective problem instances.

The rest of this paper is organized as follows. Section 2 provides a detailed description of our proposed approach, named non-dominated sorting on performance indicators for evolutionary many-objective optimization (NSPI-EMO). The performance of the experimental results on DTLZ and MaF test problems are presented and analyzed in Section 3. Section 4 summarises the paper and outlines future work leading on from this study.

2. Non-dominated Sorting on Performance Indicators for Evolutionary Many-objective Optimization

2.1. A General Framework

Environmental selection plays an important role in solving many-objective problems. Performance on diversity and convergence are normally integrated into a single indicator for environmental selection. However, as discussed above, these measures are often in conflict with one another. We address this here by casting this two measures as separate objectives in the environmental selection, and use non-dominated sorting to rank individuals on these measures. The pseudocode of the proposed method is given in Algorithm 1.

In Algorithm 1, the convergence and diversity of each individual in the initial population are calculated (line 2)and the non-dominated solutions (according to their original objective values) in the current population P are saved in the archive A (line 3). A mating pool is generated using tournament selection, the details of which are given in Algorithm 2. After this, an offspring population is generated using the canonical simulated binary crossover and polynomial mutation. The archive A is then updated by individuals in the offspring population after the original objectives are evaluated (line 8). To be specific, a union is made of archive A and the offspring population, and is subsequently non-dominated sorted according to the objective values. The resulting non-dominated solutions will replace all solutions in archive A. Next, the convergence and diversity of each individual in the combined population P' is calculated, and a new population is selected according to the environmental selection strategy (see Algorithm 2). This process repeats until the stopping criterion is met. Finally, N reference vectors are generated using the method proposed in MOEA/DD [37], and N solutions will be selected using these as the output. These are determined by the minimum perpendicular distance to the ray defined by each of the reference vectors and the ideal point.

As in NSPI-EMO each solution has two performance values with respect to convergence and diversity, which are treated as objectives to be optimized, standard MOEAs such as NSGA-II can be used for solving the problem. It should be pointed out, however, that solutions selected based on the two metrics are not necessarily non-dominated in the sense of Pareto dominance. Therefore, we use Pareto dominance to preserve the non-dominated solutions on the original objectives in an external archive.

Algorithm 1 Pseudocode of the proposed NSPI-EMO method

Initialize a population P;
Calculate the convergence indicator $Conv(\mathbf{x}_i)$ and diversity indicator $Div(\mathbf{x}_i)$ for each $\mathbf{x}_i$, $\mathbf{x}_i \in P$, respectively;
Save all non-dominated individuals in P in an archive A;
while the stopping criterion is not met **do**
    Select individuals from the parent population P based on tournament selection strategy and save them to a mating pool for mating selection. (**Refer to Algorithm 2**);
    Generate offspring Q;
    Evaluate the objective values for each individual in Q;
    Update the archive A by individuals in Q;
    Combine the parent and offspring populations, denoted as $P' = P \cup Q$;
    Calculate the convergence indicator $Conv(\mathbf{x}_i)$ and diversity indicator $Div(\mathbf{x}_i)$ of each individual i in the combined population P';
    Perform environmental selection to set P. (**Refer to Algorithm 3**);
end while
Generate N reference vectors and select N solutions from archive A as the output;

We now give a detailed description of mating selection and the environmental selection in NSPI-EMO algorithm.

2.2. Mating Selection

Like other state-of-the-art methods, for example, BiGE [36], AR-MOEA [33], and MOEA-CSS [38], in NSPI-EMO the mating pool is also utilized for the offspring generation. As detailed in Algorithm 2, two solutions, $\mathbf{x}_i$ and $\mathbf{x}_j$, will be randomly selected. Then, if $\mathbf{x}_i$ is not worse than $\mathbf{x}_j$ with respect to both convergence $Conv$ and diversity Div, then $\mathbf{x}_i$ will be kept in the mating pool Z, and vice versa. However, when the pair of solutions is mutually non-dominating using these two objectives, then one of the pair will be randomly selected and put into the mating pool Z. The procedure is repeated until the number of the solutions in Z reaches the required population size N.

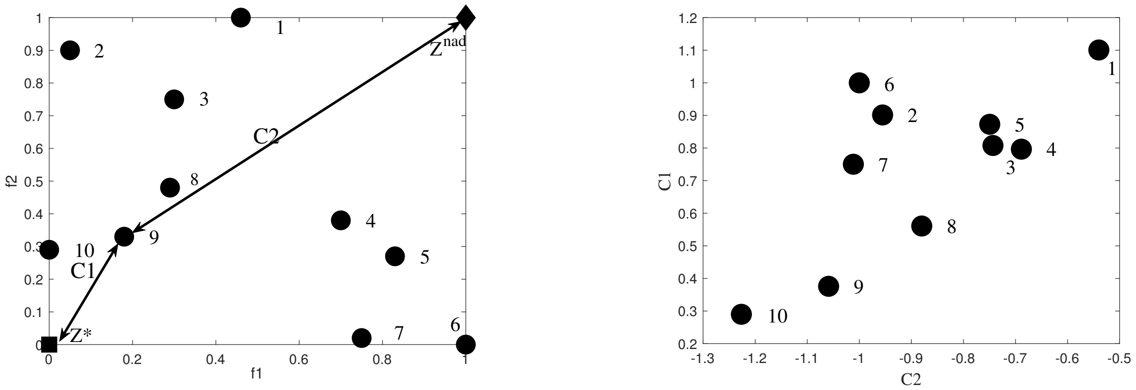
Algorithm 2 Mating Selection

Input: population P, convergence indicator $Conv$, diversity indicator Div;
Output: parent population Z;

Z = empty list to store $|P|$ parent solutions;
$index = 1$;
while $|Z| \le |P|$ **do**
    Randomly select two solutions, $\mathbf{x}_i$ and $\mathbf{x}_j$, from the current population P;
    if $Con(\mathbf{x}_i) \le Con(\mathbf{x}_j)$ and $Div(\mathbf{x}_i) \le Div(\mathbf{x}_j)$ **then**
        $Z_{index} = \mathbf{x}_i$;
    else if $Con(\mathbf{x}_i) \ge Con(\mathbf{x}_j)$ and $Div(\mathbf{x}_i) \ge Div(\mathbf{x}_j)$ **then**
        $Z_{index} = \mathbf{x}_j$;
    else
        Randomly choose one solution, $\mathbf{x}_i$ or $\mathbf{x}_j$, and insert in Z_{index};
    end if
    $index = index + 1$;
end while
output Z;

2.3. Environmental Selection

Environmental selection plays a key role in solving multi-/many-objective problems. It is well-known that dominance-based MOEAs often fail in optimizing MaOPs because of the loss of selection pressure when the dimension of objectives increases. As there are only two objectives for non-dominated sorting in this work, we can prevent the loss



(a) The objective space (minimized) (b) The convergence space (minimized)

Figure 1: An example to show the method to calculate the convergence performance of each solution. A black dot with a number represents an individual in the population. The square and diamond in black are the ideal point Z^* and the nadir point Z^{nad}, respectively.

of selection pressure and it is much easier to separate individuals into different layers, irrespective of whether the number of original objectives is high or not. In the following, we will describe in detail the methods to calculate the convergence and diversity performance, for each individual, and how they are utilized in the environmental selection.

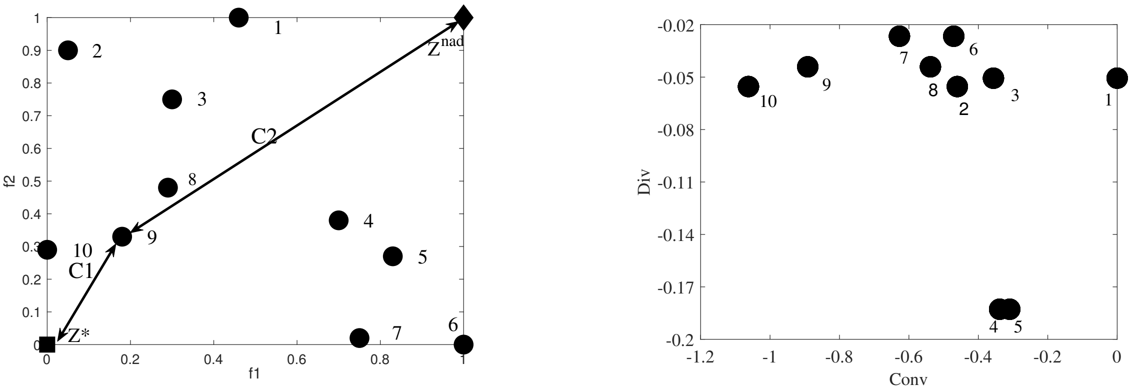
2.3.1. The convergence indicator

The convergence performance measures the quality of an individual in terms of its closeness to the PF. Therefore, the better the convergence performance is, the closer the individual is to the PF. The distance to the ideal point, or from the nadir point, is usually adopted to evaluate the convergence performance of a solution, however, some essential solutions may be lost if only one of these is considered. For example, an individual at the edge of the objective space will not be selected for the convex problem if only the distance to the ideal point is considered. Conversely, an individual at the center of the PF will be discarded if the problem is concave and only the distance from the nadir point is used as the convergence performance. Therefore, in order not to lose solutions that may be important in searching for an accurate approximation to the PF, we first propose two new convergence performance indicators, C_1 and C_2, which are utilized to keep solutions that locate at the middle and the edge parts of the front, respectively, in the objective space for convex problems, or vice versa for concave problems. C_1 and C_2 are then integrated into a unary convergence performance indicator, *Conv*. Fig. 1 provides an illustration of how the value of convergence performance of solutions is calculated. Suppose there are ten solutions in the population, whose objective vectors are given in black solid circles (as shown in Fig. 1(a)). Z^* and Z^{nad} are the ideal and nadir points, the value of each dimension of these points is the minimum and maximum objective value of the population, respectively. The performance indicators C_1 and C_2 of solution i, denoted as C_1^i and $C_2^i, i = 1, 2, \dots, N$, where N is the population size, are calculated as shown in Eq. (2) and Eq. (3).

$$C_1^i = \sqrt{\sum_{m=1}^{M} (f_m(\mathbf{x}_i) - Z_m^*)^2} \tag{2}$$

$$C_2^i = -\sqrt{\sum_{m=1}^{M} (f_m(\mathbf{x}_i) - Z_m^{nad})^2} \tag{3}$$

Fig. 1(b) shows the solutions in Fig. 1(a) in the convergence space which is composed of C_1 and C_2. We can clearly see from Fig. 1(b) that individual 2 will be lost if only C_1 is used to measure the convergence performance of a solution for a convex problem if the population size is five, and individual 8 will be lost if only C_2 is utilized for a concave problem. However, it can be seen from Fig. 1(a) that we are likely to experience better subsequent



(a) The objective space (minimized)

(b) The performance indicator space (minimized)

Figure 2: The convergence and diversity of each solution in the current population. A black dot with a number represents an individual in the population. The square and diamond in black are the Z^* and Z^{nad}, respectively.

exploration if individual 2 is kept. Additionally, if individual 8 is kept, it should be able to assist convergence to the central portion of the PF. That is, both of these individuals are likely to play an important role in the search process, and we would like to keep them in the population. From Fig. 1(b), we find that solutions far away from the nadir point of convergence space are those solutions either close to the ideal point or at the edge position in the objective space. Therefore, we integrate C_1 and C_2 to a unary measure, *Conv*, as the final convergence criterion, which is given in Eq. (4). In Eq. (4), $\mathbf{C}^i = (C_1^i, C_2^i), i = 1, 2, \ldots, N$, $\mathbf{C}^{nad} = (\max\{C_1^i, i = 1, 2, \ldots, N\}), \max\{C_2^i, i = 1, 2, \ldots, N\})$, $\|\cdot\|$ represents the Euclidean distance.

$$Conv^i = \|\mathbf{C}^i - \mathbf{C}^{nad}\| \tag{4}$$

2.3.2. Diversity

The population diversity is important as a common aim is to find a diverse and accurate approximation to the PF. As indicated in [17], the angle is a better measure for assessing the diversity of a population in high-dimensional objective spaces. As such, in this work the angle between the solution and its closest neighbor will be used to measure the crowding degree (and diversity) of a solution. Eq. (5) shows how this is calculated for each solution [39], which is denoted by *Div*. In Eq. (5), A_j^i is the angle between individuals i and j. Different to RVEA [17], in which the angle between an individual and a reference vector was first proposed to be used as a diversity measurement, in our method the angle between two *individuals* is adopted to measure the diversity performance of the population.

$$Div^i = \min_{j \in \{1,2,\ldots,N\}, j \neq i} A_j^i \tag{5}$$

where

$$A_j^i = \arccos \frac{\sum_{m=1}^{M} [(f_m^i - z_m^*)(f_m^j - z_m^*)]}{\sqrt{\sum_{m=1}^{M} (f_m^i - z_m^*)^2} \sqrt{\sum_{m=1}^{M} (f_m^j - z_m^*)^2}} \tag{6}$$

2.3.3. Individual selection

After the evaluation of each solution in terms of convergence and diversity, we select a new population by simultaneously maximizing the convergence (*Conv*) and the diversity (*Div*) based on non-dominated sorting. To ensure consistency with the rest of this work, the maximization of the convergence (*Conv*) and the diversity (*Div*) is converted into minimization of the negative *Conv* and *Div*, i.e., $-Conv$ and $-Div$. Fig. 2(b) shows the positions of each solution given in Fig. 2(a) in the convergence-diversity space, which are non-dominated sorted. Solutions are progressively selected from the first front to the next until the number of individuals is equal to the desired population size. From

Fig. 2(b), we can see that the individuals selected under our environmental selection strategy are the same as we would expect from Fig. 2(a). Note that as individuals 2, 4, 7 and 8 are in the final layer to be considered, and three of them will be randomly selected from it to be placed in the next population. From Fig. 2(a), we can easily understand that individuals 2, 7 and 8 are preferred to be kept because individuals 2 and 7 are at the edge of the objective space and individual 8 falls in the center part of the objective space. While the individual 4 is also possible to be selected because of its good diversity.

The pseudocode of our environmental selection approach is given in Algorithm 3. The convergence performance *Conv* and the diversity performance *Div* are first calculated for each individual in the combined population P'. Then all individuals are non-dominated sorted on these two performance indicators (line 3). The individuals are selected sequentially from the first non-dominated front to the critical front (the k-th layer in Algorithm 3), on which the individual will be selected randomly, until the size of the population reaches N.

Algorithm 3 Environmental selection

Input: the combined population P';
Output: the population P to be passed to the next generation;

```
1: Evaluate the convergence indicator, Conv^i, for each solution in population P';
2: Evaluate the diversity indicator, Div^i, for each solution in population P';
3: Perform non-dominated sorting on population P' according to Conv and Div, suppose there are L non-dominated
   layers after sorting and L_k represents those solutions located in the k-th layer.
4: Set P := ∅, k = 1;
5: while |P| + |L_k| < N do
6:     P := P ∪ L_k;
7:     k := k + 1;
8: end while
9: K := randomly select N − |P| individuals from the k-th layer (L_k);
10: P := P ∪ K
```

3. Experimental Results and Analysis

In order to assess whether the proposed environmental selection strategy is efficient for many-objective optimization, we firstly conducted experiments on the DTLZ1 and DTLZ3 test problems with different numbers of objectives. DTLZ1 and DTLZ3 are known to be hard problems to converge to when the function evaluation budget is limited. We compared versions of the algorithm with three different environmental selection strategies: (i) the convergence only (called convergence-strategy); (ii) diversity only (called diversity-strategy); and, (iii) dominated sorting on the two convergence measures and diversity (called C_1-C_2-Div-strategy) [40].

Following the initial environmental selection study, we conducted several experiments on the unconstrained DTLZ [41] problems with 3, 5, 8, 10, 15, 20, 30 objectives, and the MaF [42] test problems with 10, 15, 20, 30 objectives, to evaluate the performance of our proposed algorithm. Note that the 30-objective variants of DTLZ7, MaF7 and MaF10 are not tested due to the difficulty of sampling their complex Pareto front uniformly — which we need access to in order to calculate the performance indicator. Results are compared with six state-of-the-art algorithms, namely NSGA-III [3], SPEA/R [18], MaOEAIGD [23], NMPSO [43], MOEA/D-DE [44], and BiGE [36]. These algorithms have all been proposed for solving multi/many-objective problems, and cover the different categories of evolutionary many-objective optimization algorithms discussed in Section 1. NSGA-III (nondominated sorting genetic algorithm-III) [3] is an extended version of NSGA-II [4]. It incorporates a number of changes in the selection mechanism compared to NSGA-II (notably decomposition-based niching rather than crowding), and it was adopted in our experiments as a representative dominance based method. SPEA/R [18] and MOEA/D-DE [44] are two decomposition based approaches. In SPEA/R, each solution is assigned a fitness based on the local raw fitness and density for environmental selection. MOEA/D-DE is a modification of MOEA/D [6], in which a differential evolution algorithm is employed in place of a genetic algorithm. MaOEAIGD [23], NMPSO [43] and BiGE [36] are three performance indicator based MOEAs. In MaOEAIGD, the IGD indicator is utilized in each generation to select

Table 1: The characteristics of test problems

Problem	Characteristics
DTLZ1	Linear, Multimodal
DTLZ2	Concave
DTLZ3	Concave, Multimodal
DTLZ4	Concave, Biased
DTLZ5	Concave, Degenerate
DTLZ6	Concave, Degenerate, Biased
DTLZ7	Mixed, Disconnected, Multimodal, Scaled
MaF1	Linear
MaF2	Concave
MaF3	Convex, Multimodal
MaF4	Concave, Multimodal
MaF5	Convex, Biased
MaF6	Concave, Degenerate
MaF7	Mixed, Disconnected, Multimodal
MaF8	Linear, Degenerate
MaF9	Linear, Degenerate
MaF10	Mixed, Biased
MaF11	Convex, Disconnected, Nonseparable
MaF12	Concave, Nonseparable, Biased deceptive
MaF13	Concave, Unimodal, Nonseparable, Degenerate
MaF14	Linear, Partially separable, Large scale
MaF15	Convex, Partially separable, Large scale

solutions with good convergence and diversity. In NMPSO both convergence and diversity are considered together in fitness estimation to address the curse of dimensionality in MaOPs. BiGE [36] also utilizes the NSGA-II framework to select solutions layer by layer. In BiGE, all solutions in the last layer are non-dominated sorted on proximity and diversity and the required number of solutions are selected layer by layer. The main contributions of BiGE is the utilization of two indicators to select individuals in the last layer, therefore. As such it is of particular interest to compare the performance of BiGE to our proposed approach, as there are similar concepts underpinning the method, albeit via different implementation routes.

3.1. Test Problems

The DTLZ benchmark problems [41] are widely used for testing multi- and many-objective optimization algorithms, and therefore are adopted for empirical comparisons in this work. These test suites are composed of optimization problems with linear, concave, multimodal, disconnected, biased, or degenerate Pareto optimal fronts. The characteristics of the DTLZ test problems are summarized in Table 1. The number of decision variables is set to $D = M + L - 1$, as recommended in [45], where M is the number of objectives, $L = 5$ for DTLZ1, $L = 10$ for DTLZ 2-6 and $L = 20$ for DTLZ7.

The MaF problems were proposed in [42] for testing the efficiency of optimization algorithms on many-objective problems. The number of decision variable is set to $D = M + L - 1$ for MaF1-7 and MaF10-12, where $L = 10$ for all these problems except $L = 20$ for MaF7. For other MaF problems, $D = 2$ for MaF8 and MaF9, $D = 5$ for MaF13, and $D = M \times 20$ for MaF14 and MaF15.

3.2. Performance Indicator

The inverted generational distance (IGD), which can offer performance measures of the convergence and diversity simultaneously, is popular in performance assessment of evolutionary algorithms for MaOPs. As such, the IGD is adopted here as the performance indicator to evaluate the ability of each algorithm to solve many-objective problems.

Table 2: Setting of the population size, where p_1 and p_2 are parameters controlling the number of reference points along the boundary and inside of the Pareto optimal front, respectively.

M	p_1	p_2	N
3	16	0	153
5	6	0	210
8	3	2	156
10	3	2	275
15	2	1	135
20	2	1	230
30	1	1	60

IGD is calculated as follows:

$$IGD(F, \mathbf{P}^*) = \frac{1}{|\mathbf{P}^*|} \sum_{\mathbf{v} \in \mathbf{P}^*} dist(\mathbf{v}, \mathbf{p}) \quad (7)$$

where $\mathbf{P}^*$ represents a set of solutions uniformly sampled from the true Pareto optimal front, and $dist(\mathbf{v}, \mathbf{p})$ is the Euclidean distance between the solution $\mathbf{v}$ in $\mathbf{P}^*$ and its nearest point $\mathbf{p}$ from the approximating front F. When the number of solutions in $\mathbf{P}^*$, i.e., $|\mathbf{P}^*|$, is large enough to cover the true Pareto optimal front at a high resoultion, $IGD(F, \mathbf{P}^*)$ measures both the diversity and convergence of final solutions. Note, as IGD requires access to the actual Pareto front for a problem, which is not typically accessible in a real-world optimization, it is not an indicator which can be reasonably be embedded within an optimization algorithm to drive its selection mechanisms.

3.3. Parameter Settings

All algorithms under comparison are run on the PlatEMO 2.0 platform [46]. SBX [47] and polynomial mutation [48] are employed as the genetic operators in NSPI-EMO, and the probabilities of crossover and mutation are set to be 1 and $1/D$, respectively. The distribution parameters of mutation and crossover are set to be 20. In order to make a fair comparison with other algorithms, the population size of all approaches is set to be the same. The population size is set according to parameters p_1 and p_2 for the different number of objectives. These are listed in Table 2 for each problem dimension. The maximum number of function evaluations (the termination condition) is set to $30,000$ for each run. Each algorithm is run independently 20 times on each test problem. The Wilcoxon rank-sum test [39] with Bonferroni correction for a significance level of 0.05 is applied to assess whether the expected performance of a solution obtained by one of the two compared algorithms is significantly different to another [49]. In the tabulated results, the symbols $+$, $\approx$, and $-$ indicate where the compared algorithms are significantly better, equivalent to, or worse than NSPI-EMO, respectively, according to the Wilcoxon rank-sum test on median IGD values [50].

3.4. Experimental Results

We now discuss the results obtained from the different experiments.

3.4.1. Performance comparison on environmental selection

Table 3 presents the median and median absolute deviation (MAD) obtained on DTLZ1 and DTLZ3 using different environmental selection strategies set out at the start of Sec. 3. Recall that NSPI-EMO uses the convergence-diversity-strategy. The best median result is shaded for each problem, along with those which are not statistically different from it. We can see that the proposed convergence-diversity-strategy obtains better or competitive results on both problems across a range of M, which shows that the method to combine two convergence performance into one indicator is effective. From Table 3, we can also see that the proposed convergence-diversity-strategy obtains much better results than the diversity-strategy on both DTLZ1 and DTLZ3 problems. Compared to the convergence-strategy, we can clearly see that our proposed strategy can obtain better performance on DTLZ1. Although the convergence-diversity-strategy failed to obtain better results on the DTLZ3 problem with three objectives, it achieved better results on DTLZ3 with 15, 20 and 30 objectives, and equivalent results for 5, 8 and 10 objectives. The reason might be because the Euclidean distance is utilized to measure the convergence performance in our proposed method. When the dimension of objective is increased, it will gradually become more difficult to select individuals using the convergence strategy

Table 3: Median and MAD of IGD obtained by different environmental selection strategies on DTLZ1 and DTLZ3. The best median result in each row is shown with a gray background, along with any results not significantly different from it.

Problem	M	C1-C2-Div-strategy	Convergence-strategy	Diversity-strategy	Convergence-diversity-strategy
DTLZ1	3	2.4811e-2 (4.01e-2) –	1.6281e-1 (6.19e-2) –	2.4811e-2 (4.01e-2) –	1.9649e-2 (1.45e-3)
	5	7.4814e-2 (3.54e-2) ≈	2.9603e-1 (6.02e-2) –	1.3146e+1 (4.03e+0) –	6.5513e-2 (2.27e-3)
	8	1.7757e-1 (6.58e-2) –	3.6757e-1 (4.32e-2) –	2.3974e+1 (5.98e+0) –	1.2537e-1 (6.51e-3)
	10	2.8177e-1 (7.26e-2) –	3.6177e-1 (3.82e-2) –	2.4000e+1 (9.09e+0) –	1.3210e-1 (6.59e-3)
	15	2.8341e-1 (6.14e-2) –	3.9341e-1 (4.37e-2) –	3.4521e+1 (1.12e+1) –	1.8742e-1 (1.52e-2)
	20	3.5847e-1 (8.21e-2) –	4.2847e-1 (7.69e-2) –	2.9199e+1 (5.78e+0) –	2.5702e-1 (1.32e-2)
	30	3.8341e-1 (6.14e-2) –	4.6527e-1 (8.09e-2) –	3.2591e+1 (6.28e+0) –	2.8437e-1 (4.72e-2)
DTLZ3	3	2.6253e+0 (1.51e-1) –	9.8311e-1 (4.73e-1) +	1.9546e+2 (4.24e+1) –	1.8513e+0 (8.62e-1)
	5	5.4487e+0 (3.53e+0) –	1.5863e+1 (4.58e+1) ≈	2.8532e+2 (4.67e+1) –	1.5196e+0 (1.21e+0)
	8	5.8712e+0 (5.59e+0) –	3.1061e+1 (5.49e+1) ≈	4.1929e+2 (7.56e+1) –	1.2527e+0 (8.34e-1)
	10	6.2475e+0 (6.21e+0) –	3.7920e+1 (7.51e+1) ≈	4.4413e+2 (8.00e+1) –	1.4948e+0 (1.11e+0)
	15	6.8204e+0 (5.04e+0) –	2.1914e+2 (2.03e+2) –	5.0859e+2 (6.27e+1) –	2.2473e+0 (9.27e-1)
	20	7.2012e+0 (3.24e+0) –	1.4644e+2 (1.53e+2) –	4.7804e+2 (7.65e+1) –	3.1255e+0 (1.13e+0)
	30	7.6241e+0 (4.01e+0) –	2.4532e+2 (1.21e+2) –	4.8212e+2 (5.25e+1) –	3.1076e+0 (1.64e+0)
+/ ≈ /−		0/1/13	1/3/10	0/0/14	

only. In the diversity strategy, by contrast, the angle is utilized to measure the crowding degree of each individual, which has been shown to be better than the Euclidean distance for identifying the crowdedness of an individual in a high-dimensional objective space [17]. Therefore, we conclude that the convergence-diversity-strategy proposed in this work is the most suitable for solving many-objective problems of the four variants considered here.

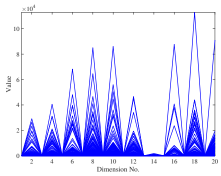
3.4.2. Results on DTLZ test problems

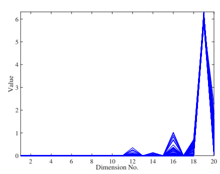
Table 4 presents the median and MAD on 48 DTLZ test problem instances, of NSPI-EMO together with those of the five other MOEAs for MaOPs being compared. The best median result and those results not significantly different to it are shaded for each problem. From Table 4, we can see that NSPI-EMO obtained better median results on 26 problems out of the 48 instances than the other six algorithms, and is statistically equivalent to the best performing on a further five test instances. The proportion of the test instances on which our proposed NSPI-EMO algorithm outperforms NSGA-III, MaOEAIGD, NMPSO, BiGE, SPEA/R and MOEA/D-DE with statistical significance is 38/48, 41/48, 39/48, 38/48, 42/48 and 29/48, respectively. However, we can see from Table 4 that NSPI-EMO is not well suited to solving problems with degenerate Pareto fronts, such as DTLZ5 and DTLZ6. This is expected because the two proposed performance criteria equally contribute and if the Pareto front is degenerate, the environmental selection strategy may fail to effectively guide the search towards the Pareto front. Furthermore, we can see in Table 4 that the NSPI-EMO algorithm is outperformed by other algorithms on most problems with three objectives. This is also not unexpected: in the lower-dimensional objective space the solutions can be separated well for environmental selection by the Pareto dominance based MOEAs, and the space can be uniformly divided in the decomposition based MOEAs.

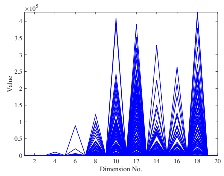
3.4.3. Results on MaF test problems

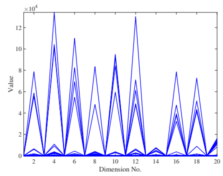
Table 5 gives the statistical results on the MaF problems considered. We can see that NSPI-EMO outperforms, or is statistically comparable to, the other algorithms on 23 out of the 58 test problem instances considered. The proportion that NSPI-EMO performs better than NSGA-III, MaOEAIGD, NMPSO, BiGE, SPEA/R, and MOEA/D-DE are 33/58, 47/58, 35/58, 27/58, 47/58 and 26/58, respectively. Interestingly, we find that NSPI-EMO with two performance indicators is much better than MaOEAIGD and NMPSO (which utilize only one performance indicator in their environmental selection). Furthermore, it can be clearly seen that the win/loss ratio of NSPI-EMO and BiGE, both of which utilize two performance indicators in environmental selection, is 27/19. This win/loss ratio suggests that our proposed algorithm, to some extent, can address the weakness of BiGE and achieve better results. However, the proposed algorithm is not able to achieve better results on MaF8, MaF9 and MaF13 problems whose Pareto fronts are degenerate (as was observed in the Sec. 3.4.2 for degenerate DTLZ problems).

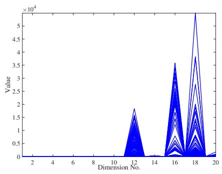
Looking further at Table 5, we find NSPI-EMO obtained better results on MaF14 and MaF15 which have $20 \times M$ decision variables. The decision space for these problems therefore gets very large when the number of objectives increases. In order to understand why the NSPI-EMO algorithm can perform better than the other algorithms on high-dimensional many-objective problems, we graphically plot the parallel coordinates of the final solutions obtained by the algorithms for the 20-objective MaF14 and MaF15 problems in Fig. 3 and Fig. 4, respectively. The horizontal axis represent each objective, and vertical axis the values obtains on each objective in the final returned solutions. We can see clearly that the convergence and diversity of NSPI-EMO is much better than others on these two high-dimensional many-objective problems.

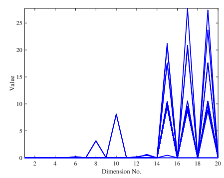

(a) NSGA-III on 20 objectives of MaF14


(b) MaOEAIGD on 20 objectives of MaF14


(c) NMPSO on 20 objectives of MaF14


(d) BiGE on 20 objectives of MaF14


(e) SPEA/R on 20 objectives of MaF14


(f) MOEA/D-DE on 20 objectives of MaF14

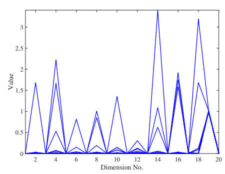
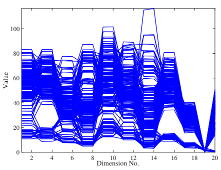
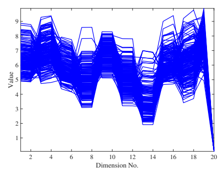
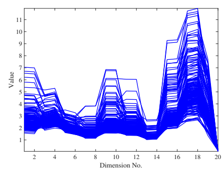
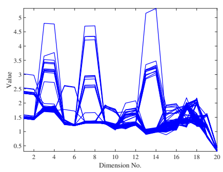

(g) NSPI-EMO on 20 objectives of MaF14

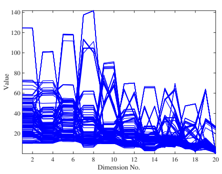
Figure 3: Parallel coordinates of the final solutions obtained by five compared algorithms for the 20-objective MaF14 instance. (a) NSGA-III. (b) MaOEAIGD. (c) NMPSO. (d) BiGE. (e) SPEA/R. (f) MOEA/D-DE. (g) NSPI-EMO.

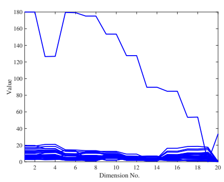

(a) NSGA-III on 20 objectives of MaF15


(b) MaOEAIGD on 20 objectives of MaF15


(c) NMPSO on 20 objectives of MaF15


(d) BiGE on 20 objectives of MaF15


(e) SPEA/R on 20 objectives of MaF15


(f) MOEA/D-DE on 20 objectives of MaF15

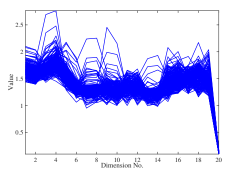

(g) NSPI-EMO on 20 objectives of MaF15

Figure 4: Parallel coordinates of the final solutions obtained by five compared algorithms for the 20-objective MaF15 instance. (a) NSGA-III. (b) MaOEAIGD. (c) NMPSO. (d) BiGE. (e) SPEA/R. (f) MOEA/D-DE. (g) NSPI-EMO

Table 4: Median and MAD of the IGD values obtained by NSGA-III, MaOEAIGD, NMPSO, BiGE, SPEA/R, MOEA/D-DE and NSPI-EMO on DTLZ1-7. The best median result in each row is shown with a gray background, along with any results not significantly different from it.

Problem	M	NSGAIII	MaOEAIGD	NMPSO	BiGE	SPEAR	MOEA/D-DE	NSPI-EMO
DTLZ1	3	2.0851e-2 (1.56e-4) –	3.2258e-1 (2.17e-1) –	2.3735e-2 (9.90e-2) –	3.4119e-2 (1.23e-2) –	2.6143e-2 (1.75e-2) –	2.3103e-2 (1.48e-1) –	1.9649e-2 (1.45e-3)
	5	6.9589e-2 (2.06e-2) –	3.4483e-1 (2.52e-1) –	7.0993e-2 (4.28e-2) –	1.7062e-1 (5.63e-2) –	1.2465e-1 (1.05e-1) –	3.9341e-1 (2.73e-1) –	6.5513e-2 (2.27e-3)
	8	1.5579e-1 (1.06e-1) –	3.2576e-1 (2.31e-1) –	3.0187e-1 (5.38e+0) –	9.4519e-1 (5.42e-1) –	5.0246e-1 (2.27e-1) –	1.3337e+0 (8.57e-1) –	1.2537e-1 (6.51e-3)
	10	2.4752e-1 (1.55e-1) –	4.5146e-1 (1.97e+0) –	3.7615e+0 (1.29e+1) –	1.5717e+0 (8.74e-1) –	5.8784e-1 (5.51e-1) –	4.9824e-1 (5.15e-1) –	1.3210e-1 (6.59e-3)
	15	2.8848e-1 (1.50e-1) –	4.7190e-1 (2.64e-1) –	4.1943e+1 (7.08e+0) –	2.5853e+0 (1.53e+0) –	5.1957e-1 (2.03e-1) –	3.4395e-1 (3.21e-1) –	1.8742e-1 (1.52e-2)
	20	5.5224e-1 (3.52e-1) –	6.0313e-1 (3.81e-1) –	4.1470e+1 (5.32e+0) –	2.0358e+0 (1.87e+0) –	7.9883e-1 (8.76e-1) –	9.9762e-1 (5.23e-1) –	2.5702e-1 (1.32e-2)
	30	5.3089e-1 (3.36e-1) –	1.0363e+0 (5.85e-1) –	4.3627e+1 (7.30e+0) –	2.8440e+0 (1.54e+0) –	1.9315e+0 (1.52e+0) –	4.4108e-1 (5.45e-1) ≈	2.8437e-1 (4.72e-2)
DTLZ2	3	5.4478e-2 (7.56e-6) ≈	1.7123e-1 (5.39e-4) –	7.6773e-2 (2.25e-3) –	7.8719e-2 (3.13e-3) –	5.7141e-2 (8.88e-4) ≈	5.6150e-2 (4.21e-4) ≈	5.4702e-2 (7.37e-3)
	5	2.1234e-1 (4.54e-5) –	2.2252e-1 (7.31e-3) –	2.2912e-1 (1.74e-3) –	2.7230e-1 (9.10e-3) –	2.1755e-1 (1.46e-3) –	3.3294e-1 (1.29e-2) –	1.6804e-1 (3.42e-3)
	8	3.8760e-1 (2.29e-2) –	4.0093e-1 (2.74e-2) –	3.9270e-1 (1.99e-3) –	4.4260e-1 (9.22e-3) –	3.8933e-1 (1.39e-3) –	6.8108e-1 (4.35e-2) –	3.4177e-1 (3.16e-3)
	10	6.1411e-1 (4.65e-2) –	6.4100e-1 (5.82e-2) –	5.8802e-1 (4.83e-2) –	5.4897e-1 (1.12e-2) –	5.3968e-1 (7.54e-3) –	6.4951e-1 (1.53e-2) –	4.1310e-1 (1.26e-2)
	15	7.5802e-1 (1.67e-2) –	7.3074e-1 (1.80e-2) –	8.5004e-1 (6.81e-2) –	6.7260e-1 (7.65e-3) ≈	6.9998e-1 (1.59e-3) –	9.5482e-1 (4.49e-2) –	6.7141e-1 (4.29e-2)
	20	1.0070e+0 (2.54e-2) –	8.5628e-1 (5.78e-2) –	9.5962e-1 (1.28e-1) –	7.4923e-1 (1.10e-2) +	7.6481e-1 (2.04e-3) +	1.0748e+0 (2.38e-2) –	7.8040e-1 (2.32e-2)
	30	8.4814e-1 (1.02e-2) +	9.6376e-1 (6.13e-2) +	2.9764e+0 (3.19e-1) –	9.4810e-1 (1.85e-2) +	8.2339e-1 (8.74e-3) +	1.2108e+0 (1.76e-2) +	1.2592e+0 (3.87e-2)
DTLZ3	3	1.4104e-1 (4.33e-1) +	1.0073e+1 (2.97e+0) –	1.8898e+1 (9.03e+0) –	1.1318e-1 (3.20e-1) +	1.3871e+0 (7.45e-1) ≈	2.4040e+0 (7.38e+0) ≈	1.8513e+0 (8.62e-1)
	5	3.1693e+0 (1.59e+0) –	1.1968e+1 (5.72e+0) –	1.7705e+1 (6.84e+0) –	2.0597e+0 (6.77e-1) ≈	5.8395e+0 (2.44e+0) –	8.9400e+0 (1.66e+1) –	1.5196e+0 (1.21e+0)
	8	8.9607e+0 (4.30e+0) –	9.1686e+0 (3.96e+0) –	1.3248e+1 (1.48e+1) –	2.6664e+1 (1.29e+1) –	2.1994e+1 (6.03e+0) –	3.5113e+1 (2.25e+1) –	1.2527e+0 (8.34e-1)
	10	1.1221e+1 (5.84e+0) –	8.0502e+0 (3.94e+1) –	6.6194e+1 (4.16e+1) –	4.2786e+1 (9.64e+0) –	5.3340e+1 (1.14e+1) –	4.7734e+0 (8.13e+0) –	1.4948e+0 (1.11e+0)
	15	6.6528e+0 (4.19e+0) –	6.4309e+0 (4.07e+0) –	1.3936e+2 (5.41e+1) –	4.5353e+1 (7.76e+0) –	4.1042e+1 (1.93e+1) –	4.5568e+0 (8.52e+0) –	2.2473e+0 (9.27e-1)
	20	2.2102e+1 (1.66e+1) –	6.6413e+0 (4.64e+0) –	2.2952e+2 (3.42e+1) –	4.8012e+1 (1.06e+1) –	6.1387e+1 (2.82e+1) –	7.9118e+0 (1.25e+1) –	3.1255e+0 (1.13e+0)
	30	1.4165e+1 (6.56e+0) –	1.4668e+1 (7.54e+0) –	2.5090e+2 (1.01e+1) –	4.9403e+1 (1.86e+1) –	9.2535e+1 (1.96e+1) –	2.4578e+0 (1.45e+1) ≈	3.1076e+0 (1.64e+0)
DTLZ4	3	5.4496e-2 (2.13e-1) –	5.4143e-1 (2.10e-1) –	7.6781e-2 (8.39e-2) –	7.8686e-2 (4.40e-2) –	5.7824e-2 (1.11e-3) –	7.2483e-2 (5.88e-2) –	4.2294e-2 (2.84e-4)
	5	2.1270e-1 (1.15e-1) –	4.2712e-1 (1.85e-1) –	2.3307e-1 (6.73e-2) –	2.6439e-1 (4.33e-3) –	2.2018e-1 (2.04e-3) –	4.0584e-1 (1.81e-2) –	1.7298e-1 (2.24e-3)
	8	5.2262e-1 (7.50e-2) –	4.8384e-1 (6.11e-2) –	5.1415e-1 (8.16e-2) –	4.3539e-1 (4.81e-3) –	3.8983e-1 (1.65e-3) –	7.1183e-1 (2.43e-2) –	3.5997e-1 (2.04e-2)
	10	5.5929e-1 (5.47e-2) –	7.0896e-1 (6.14e-2) –	5.8650e-1 (1.01e-1) –	5.3645e-1 (5.73e-3) –	5.4447e-1 (3.21e-3) –	7.3222e-1 (1.91e-2) –	4.4626e-1 (1.49e-2)
	15	7.7816e-1 (1.36e-2) –	8.7404e-1 (8.40e-2) –	7.7859e-1 (1.37e-1) –	6.5377e-1 (3.78e-3) ≈	7.4239e-1 (8.12e-3) –	9.3621e-1 (3.57e-2) –	6.5404e-1 (1.22e-2)
	20	9.7627e-1 (3.72e-2) –	8.6667e-1 (6.28e-2) –	1.1139e+0 (1.89e-1) –	6.9110e-1 (6.45e-3) –	7.9713e-1 (1.47e-2) –	1.0455e+0 (1.92e-2) –	6.6484e-1 (1.58e-2)
	30	8.4679e-1 (6.65e-3) +	8.8481e-1 (1.90e-2) +	2.7775e+0 (2.11e-1) –	9.3198e-1 (3.75e-2) +	8.3502e-1 (8.06e-3) +	1.1709e+0 (1.50e-2) –	1.1478e+0 (3.03e-2)
DTLZ5	3	1.1968e-2 (1.44e-3) +	6.7110e-1 (1.63e-1) –	1.4384e-2 (1.39e-3) +	1.4241e-2 (2.74e-3) +	3.0958e-2 (1.80e-3) –	8.7409e-3 (6.80e-5) +	2.0001e-2 (1.29e-3)
	5	1.0008e-1 (2.50e-2) –	6.5747e-1 (1.42e-1) –	4.1836e-2 (4.80e-3) ≈	1.0762e-1 (1.46e-2) –	2.3494e-1 (6.52e-2) –	3.8045e-2 (1.31e-3) +	4.1485e-2 (5.48e-3)
	8	2.3220e-1 (6.17e-2) –	6.8872e-1 (1.24e-1) –	6.2540e-1 (1.65e-1) –	1.9881e-1 (4.98e-2) –	4.0172e-1 (7.63e-2) –	1.6469e-1 (1.33e-2) –	1.5403e-1 (1.79e-2)
	10	2.3825e-1 (5.21e-2) –	6.9919e-1 (1.49e-1) –	7.7251e-1 (7.02e-2) –	2.7919e-1 (5.52e-2) –	6.2606e-1 (1.25e-1) –	1.6448e-1 (4.84e-3) –	1.5264e-1 (1.66e-2)
	15	2.7151e-1 (8.30e-2) ≈	7.1308e-1 (1.27e-1) –	7.5008e-1 (1.56e-2) –	4.3758e-1 (6.45e-2) –	9.5744e-1 (2.57e-1) –	8.4310e-2 (1.01e-2) +	3.0302e-1 (4.52e-2)
	20	1.0400e+0 (5.23e-1) –	9.2065e-2 (1.38e-1) +	7.4209e-1 (5.01e-2) –	4.7555e-1 (3.25e-2) –	1.0486e+0 (2.17e-1) –	1.0261e-1 (1.20e-2) +	3.4203e-1 (3.80e-2)
	30	3.0515e-1 (4.39e-2) –	9.2424e-2 (1.73e-1) +	7.4209e-1 (1.64e-1) –	3.3766e-1 (5.00e-2) –	9.9886e-1 (1.21e-1) –	9.1735e-2 (1.19e-2) +	2.6772e-1 (8.82e-2)
DTLZ6	3	2.0064e-2 (2.25e-3) +	6.7105e-1 (9.38e-2) –	1.3894e-2 (1.68e-3) +	6.9697e-1 (5.43e-2) –	3.5106e-2 (2.83e-3) –	8.8312e-3 (2.83e-5) +	2.5416e-2 (3.99e-3)
	5	2.5571e-1 (1.47e-1) –	6.5765e-1 (2.13e-1) –	4.9194e-2 (4.04e-3) +	7.1623e-1 (5.38e-2) –	6.6312e-1 (3.21e-1) –	3.5596e-2 (2.07e-4) +	5.8807e-2 (9.04e-3)
	8	1.6884e+0 (8.41e-1) –	7.1514e-1 (1.41e-1) –	7.4209e-1 (1.48e-1) –	6.9109e-1 (6.75e-2) –	1.1323e+0 (4.05e-1) –	1.0860e-1 (3.00e-2) +	1.6512e-1 (4.66e-2)
	10	1.7287e+0 (8.67e-1) –	6.9862e-1 (1.09e+0) –	7.4209e-1 (2.22e-16) –	7.3573e-1 (2.50e-2) –	7.6373e+0 (3.84e-1) –	2.9513e-2 (1.96e-3) +	2.1262e-1 (3.90e-2)
	15	2.0576e+0 (5.24e-1) –	7.1294e-1 (3.74e-2) –	7.4209e-1 (2.22e-16) –	7.4209e-1 (8.71e-2) –	5.8250e+0 (1.04e+0) –	7.8845e-2 (1.47e-2) +	3.7468e-1 (6.98e-2)
	20	4.7738e+0 (1.53e+0) –	9.1472e-2 (3.22e-1) +	7.4209e-1 (4.17e-2) –	7.5006e-1 (1.90e-1) –	7.3298e+0 (1.11e+0) –	7.6743e-2 (7.49e-3) +	5.0962e-1 (3.35e-2)
	30	1.8926e+0 (5.51e-1) –	8.8534e-1 (4.47e-1) –	2.1385e+0 (2.16e+0) –	7.7031e-1 (1.44e-1) –	8.4674e+0 (5.99e-1) –	7.2398e-2 (2.86e-3) +	7.4209e-1 (6.72e-3)
DTLZ7	3	7.7938e-2 (2.54e-3) +	6.8294e-1 (1.77e-1) –	6.9102e-2 (3.07e-3) +	8.2868e-2 (2.72e-2) +	9.5148e-2 (2.22e-3) +	1.5661e-1 (2.44e-2) –	1.2764e-1 (1.87e-2)
	5	3.9254e-1 (1.73e-2) +	7.5723e-1 (3.68e-2) –	3.0717e-1 (8.78e-3) +	5.1642e-1 (1.32e-1) ≈	5.0987e-1 (1.15e-2) –	1.2010e+0 (1.80e-1) –	4.6981e-1 (2.02e-2)
	8	9.9476e-1 (7.48e-2) +	1.2247e+0 (5.42e-2) +	9.9108e-1 (1.51e-1) +	2.2005e+0 (3.23e-1) –	2.1235e+0 (5.09e-1) –	1.4118e+0 (2.12e-1) –	1.3292e+0 (4.11e-2)
	10	2.2820e+0 (4.23e-1) –	1.6596e+0 (3.01e+0) ≈	1.1843e+0 (1.20e-1) +	4.2476e+0 (5.77e-1) –	3.3375e+0 (2.47e-2) –	1.6200e+0 (1.04e-1) ≈	1.6839e+0 (8.60e-2)
	15	6.5148e+0 (8.56e-1) –	2.7468e+0 (1.37e-1) –	2.9614e+0 (2.28e+0) –	1.1187e+1 (4.31e-1) –	1.4699e+1 (3.97e+0) –	2.0258e+0 (5.28e-2) +	2.5357e+0 (1.37e-1)
	20	1.5324e+1 (2.49e-1) –	3.5378e+0 (6.32e-1) –	2.8879e+0 (1.31e+0) ≈	1.5646e+1 (1.81e-1) –	1.7139e+1 (3.79e+0) –	2.4473e+0 (6.10e-2) ≈	2.8951e+0 (1.93e-1)
	30	-	-	-	-	-	-	-
+/ ≈ /−		8/2/38	6/1/41	7/2/39	6/4/38	4/2/42	14/5/29	

3.5. Computational Complexity Analysis

The computational complexity of NSPI-EMO in one generation depends mainly on four parts: (i) calculation of the performance indicators; (ii) the formation of the mating pool; (iii) environmental selection; and (iv) the updating of the archive. Suppose the number of objectives is M and the size of the population is N. The computational complexity for calculating the performance indicators consists of the calculation of the diversity and convergence performances, which will cost $O(N)$ and $O(N^2)$, respectively. To form a mating pool, N binary tournament selections are required. Therefore, a complexity of $O(N)$ is required. In the environmental selection, the next parent population is selected based on the non-dominated sorting according to the proposed two indicators. Thus, the time complexity of the environmental selection is $O(N^2)$. The archive is updated based on the non-dominated sorting on the objective values. Therefore, the time complexity will be $O(M \times |A|)$ for each solution, where $|A|$ represents the number of solutions in the archive. Thus, the total computational complexity is $O(M \times N \times |A|)$ for all solutions for updating the archive.

To summarize, as $M << N$ in general, the overall computational complexity of NSPI-EMO for one generation is $O(N \times |A|)$ if $|A| > N$, otherwise it will be $O(N^2)$.

Table 5: Median and MAD of the IGD values obtained by NSGA-III, MaOEAIGD, NMPSO, SPEA/R, BiGE, MOEA/D-DE and NSPI-EMO on MaF1–15. The best median result in each row is shown with a gray background, along with any results not significantly different from it.

Problem	M	NSGAIII	MaOEAIGD	NMPSO	BiGE	SPEAR	MOEA/D-DE	NSPI-EMO
MaF1	10	3.2584e-1 (1.05e-2) ≈	3.6272e-1 (9.57e-3) –	3.3981e-1 (1.28e-2) ≈	2.9254e-1 (9.79e-3) +	4.5218e-1 (5.05e-2) –	2.9786e-1 (2.06e-2) +	3.1874e-1 (2.23e-2)
	15	3.9370e-1 (1.32e-2) +	4.0122e-1 (1.63e-2) +	4.3901e-1 (2.08e-2) ≈	3.5300e-1 (1.23e-2) +	4.9078e-1 (5.40e-2) –	4.1585e-1 (1.61e-2) ≈	4.3489e-1 (1.89e-2)
	20	5.2069e-1 (2.34e-2) ≈	5.2621e-1 (1.63e-2) ≈	5.2888e-1 (2.44e-2) ≈	4.2642e-1 (1.74e-2) +	6.7199e-1 (4.23e-2) –	5.2043e-1 (1.69e-2) ≈	5.2944e-1 (2.36e-2)
	30	4.8439e-1 (9.04e-3) +	5.5654e-1 (2.00e-2) +	6.1403e-1 (3.18e-2) +	4.2825e-1 (4.56e-3) +	6.5135e-1 (4.43e-2) +	5.5737e-1 (2.17e-2) +	8.1923e-1 (7.12e-2)
MaF2	10	2.6361e-1 (3.06e-2) –	4.3413e-1 (5.54e-2) –	2.1146e-1 (9.88e-3) +	2.4662e-1 (1.68e-2) ≈	2.4968e-1 (2.44e-3) ≈	3.2475e-1 (3.09e-2) –	2.4666e-1 (2.19e-2)
	15	3.1211e-1 (3.52e-2) ≈	5.0002e-1 (3.81e-2) –	3.6016e-1 (1.27e-2) –	2.6490e-1 (2.28e-2) +	6.5950e-1 (8.56e-2) –	4.7130e-1 (3.62e-2) –	3.0567e-1 (1.64e-2)
	20	3.0114e-1 (5.66e-2) +	4.8725e-1 (1.02e-1) –	4.5283e-1 (3.43e-2) –	3.2724e-1 (2.34e-2) +	4.7071e-1 (9.07e-2) –	3.6405e-1 (6.58e-2) ≈	3.5465e-1 (1.57e-2)
	30	2.5083e-1 (6.20e-2) +	5.2277e-1 (5.59e-2) +	8.9519e-1 (1.33e-2) +	6.0414e-1 (9.34e-2) +	3.4248e-1 (4.48e-2) +	4.1232e-1 (6.54e-2) +	9.1314e-1 (9.60e-3)
MaF3	10	1.1525e+3 (1.04e+4) –	7.1062e+0 (3.42e+1) ≈	3.0755e+5 (4.88e+7) –	1.1395e+6 (1.77e+6) –	2.1341e+4 (1.64e+7) –	2.1994e+1 (4.08e+2) ≈	5.2862e+0 (1.01e+1)
	15	1.4182e+3 (5.45e+3) –	4.6025e+1 (6.26e+1) –	1.0471e+8 (1.51e+9) –	2.1165e+6 (2.66e+6) –	9.5453e+4 (1.18e+8) –	2.6810e+1 (9.68e+1) ≈	1.1511e+1 (1.60e+1)
	20	7.8321e+3 (1.40e+5) –	1.2688e+1 (5.78e+1) ≈	2.4945e+9 (1.49e+9) –	3.6440e+6 (7.01e+6) –	1.1223e+5 (1.30e+5) –	2.9656e+0 (2.65e+2) ≈	1.8652e+1 (1.27e+1)
	30	1.3246e+3 (6.54e+3) –	2.5442e+2 (3.78e+2) –	1.4166e+9 (1.32e+9) –	5.4872e+6 (3.94e+7) –	9.4663e+4 (7.13e+8) –	1.5711e+1 (2.11e+2) –	9.3180e+0 (7.72e+1)
MaF4	10	1.6596e+2 (3.95e+2) ≈	7.8332e+2 (5.47e+2) –	2.6245e+4 (6.30e+3) –	3.7935e+2 (5.14e+2) ≈	2.3737e+3 (1.41e+3) –	4.5860e+3 (6.15e+3) –	4.3431e+2 (1.43e+2)
	15	6.0097e+3 (8.59e+3) +	7.4076e+4 (7.58e+4) –	1.0466e+6 (4.49e+5) –	9.8981e+3 (1.20e+4) ≈	7.9201e+4 (5.21e+4) –	1.0593e+5 (3.29e+5) ≈	1.4273e+4 (3.04e+3)
	20	2.0551e+5 (1.21e+5) +	2.1107e+6 (1.74e+6) –	4.0867e+7 (1.42e+7) –	3.8382e+5 (5.46e+5) ≈	3.5532e+6 (1.97e+6) –	2.7688e+6 (8.79e+6) –	5.1833e+5 (1.26e+5)
	30	5.0284e+8 (1.16e+9) ≈	6.6256e+9 (2.74e+9) –	2.2784e+10 (1.28e+10) –	9.3785e+8 (6.37e+8) ≈	3.3020e+9 (2.55e+9) –	6.3162e+8 (4.26e+9) –	7.0075e+8 (1.17e+8)
MaF5	10	1.3205e+2 (6.74e+0) +	3.0397e+2 (3.72e+1) –	1.0357e+2 (3.55e+1) +	9.3710e+1 (4.00e+0) +	1.2763e+2 (9.80e+0) +	3.0605e+2 (4.45e-1) –	1.3887e+2 (8.03e+0)
	15	4.6126e+3 (6.92e+2) +	7.3260e+3 (2.90e+2) –	4.2261e+3 (1.47e+3) +	2.3046e+3 (1.87e+2) +	4.9896e+3 (3.16e+2) ≈	7.3260e+3 (9.30e-2) –	5.2216e+3 (3.91e+2)
	20	7.7024e+4 (1.57e+4) +	1.7095e+5 (3.16e-1) –	1.1474e+5 (3.24e+4) ≈	4.0663e+4 (6.51e+3) +	1.1063e+5 (1.67e+4) ≈	1.7095e+5 (4.63e-2) –	1.1795e+5 (7.61e+3)
	30	7.2313e+7 (8.85e+6) +	1.3021e+8 (1.01e+0) –	1.3021e+8 (4.36e+0) –	9.0255e+7 (2.00e+7) +	6.8452e+7 (7.60e+6) +	1.3021e+8 (4.16e-4) –	1.2956e+8 (7.43e+6)
MaF6	10	3.5647e-1 (7.18e-2) –	7.0303e-1 (1.63e-1) –	1.3194e+0 (9.21e-1) –	3.9713e-1 (1.18e-1) –	3.0251e-1 (7.77e-2) –	4.2628e-2 (4.99e-4) –	3.2703e-2 (1.68e-2)
	15	3.4234e-1 (1.66e-1) –	7.4226e-1 (9.23e-3) –	3.7161e+0 (2.06e+0) –	5.7048e-1 (3.11e-1) –	3.0621e-1 (7.05e+0) –	5.3466e-2 (4.95e-3) +	8.3593e-2 (6.44e-4)
	20	3.7756e-1 (5.43e+0) –	9.2334e-2 (1.58e-1) –	3.7002e+0 (7.92e+0) –	6.4031e-1 (1.96e-1) –	3.4251e-1 (2.68e+1) –	8.4767e-2 (1.14e-2) –	8.3338e-2 (5.74e-4)
	30	3.9166e-1 (1.46e-1) –	1.1760e-1 (2.45e-1) ≈	1.1583e+2 (7.43e+1) –	3.4567e-1 (7.47e-2) ≈	3.7990e+1 (2.57e+1) –	7.3318e-2 (8.64e-4) +	3.8775e-1 (2.30e-1)
MaF7	10	2.9200e+0 (7.90e-1) –	1.6682e+0 (1.42e+0) ≈	1.3458e+0 (8.31e-1) +	4.2526e+0 (5.29e-1) –	3.3454e+0 (9.74e-1) –	1.6614e+0 (1.46e-1) ≈	1.6736e+0 (8.40e-2)
	15	7.2229e+0 (1.85e+0) –	2.7790e+0 (4.32e-1) –	2.9324e+0 (1.78e+0) –	1.1257e+1 (2.16e-1) –	1.8246e+1 (7.80e+0) –	2.0332e+0 (4.16e-2) +	2.5449e+0 (1.33e-1)
	20	1.5583e+1 (1.44e+1) –	3.9094e+0 (1.26e+0) –	3.0179e+0 (6.92e-1) ≈	1.5563e+1 (1.30e-1) –	1.9989e+1 (9.36e+0) –	2.4519e+0 (7.02e-2) +	2.9038e+0 (1.39e-1)
	30	-	-	-	-	-	-	-
MaF8	10	4.7141e-1 (4.93e-2) –	1.5250e+0 (2.25e-1) –	2.1054e-1 (1.02e-2) +	3.1988e-1 (3.84e-2) –	1.7960e+3 (1.55e+3) –	1.3631e-1 (1.51e-3) +	2.2823e-1 (2.58e-2)
	15	8.6141e-1 (1.21e-1) –	1.9316e+0 (1.11e+0) –	5.5495e-1 (3.91e-2) –	3.7645e-1 (4.77e-2) +	2.7730e+3 (3.07e+3) –	2.0125e-1 (2.55e-3) +	4.5584e-1 (2.18e-2)
	20	8.1291e-1 (1.51e-1) –	2.2562e+0 (3.03e-1) –	6.8300e-1 (9.49e-2) –	4.5386e-1 (4.59e-2) ≈	3.6886e+3 (3.13e+3) –	1.8501e-1 (2.44e-3) +	4.4945e-1 (2.15e-2)
	30	8.5110e-1 (9.34e-2) +	2.9908e+0 (4.92e-1) –	8.9741e-1 (1.12e-1) +	7.4505e-1 (1.15e-1) +	4.2768e+3 (3.38e+3) –	1.1293e+0 (1.23e-2) +	1.7087e+0 (1.65e-1)
MaF9	10	1.1015e+0 (8.95e-1) –	1.4342e+0 (1.35e+0) –	2.1377e-1 (1.08e-2) +	2.4100e+0 (1.35e-1) –	9.1386e+0 (4.51e+0) –	1.7975e-1 (6.21e-3) +	3.2297e-1 (1.32e-1)
	15	1.3331e+0 (5.33e+0) –	1.2785e+1 (7.57e+0) –	5.7353e-1 (6.22e-1) +	2.3320e+0 (3.19e+0) –	3.2579e+1 (3.54e+1) –	4.6052e-1 (3.03e-2) +	7.6106e-1 (1.04e+0)
	20	1.8508e+1 (6.04e+0) –	1.6941e+1 (6.48e+0) –	1.1807e+0 (3.79e-1) ≈	1.7167e+1 (7.06e+0) –	3.3831e+1 (4.72e+1) –	2.9258e-1 (2.57e-2) +	9.4062e-1 (4.69e+0)
	30	1.7788e+0 (2.99e+0) ≈	3.2367e+1 (1.56e+1) –	4.2489e+0 (2.81e+0) ≈	4.0536e+0 (8.08e+0) ≈	5.4726e+1 (1.85e+1) –	2.7290e+0 (1.57e+1) ≈	1.6924e+1 (1.53e+1)
MaF10	10	2.5184e+0 (2.06e-1) ≈	2.3624e+0 (1.80e-1) +	2.8031e+0 (2.39e-1) ≈	1.5186e+0 (8.14e-2) +	2.9676e+0 (1.42e-1) –	3.2947e+0 (3.10e-2) –	2.6288e+0 (1.62e-1)
	15	2.6435e+0 (1.19e-1) +	3.5292e+0 (2.23e-1) –	3.6920e+0 (1.66e-1) –	2.0147e+0 (8.76e-2) +	3.7105e+0 (1.65e-1) –	3.3957e+0 (1.15e-1) –	3.1979e+0 (1.11e-1)
	20	5.2991e+0 (1.24e-1) –	5.5008e+0 (1.37e-1) –	5.3163e+0 (1.34e-1) –	3.9549e+0 (6.33e-2) +	5.4331e+0 (8.04e-2) –	5.7142e+0 (8.63e-2) –	4.8733e+0 (6.36e-2)
	30	-	-	-	-	-	-	
MaF11	10	1.4683e+0 (7.20e-2) –	2.3923e+0 (9.21e-1) –	1.7704e+0 (2.11e-1) –	1.4956e+0 (5.59e-2) –	1.6500e+0 (8.24e-2) –	1.7923e+0 (5.58e-2) –	1.1992e+0 (7.12e-2)
	15	2.8759e+0 (6.10e-1) –	2.3971e+0 (2.54e+0) –	2.7730e+0 (3.75e-1) –	1.9114e+0 (5.60e-2) –	2.5398e+0 (5.53e-1) –	3.1822e+0 (4.23e-1) –	1.7478e+0 (6.66e-2)
	20	8.5064e+0 (2.10e+0) –	5.0447e+0 (5.89e+0) –	4.7852e+0 (5.36e-1) –	3.7939e+0 (6.93e-2) –	5.0498e+0 (1.42e+0) –	5.1234e+0 (2.14e-1) –	3.4428e+0 (1.39e-1)
	30	5.0771e+0 (3.30e-1) ≈	3.9501e+1 (2.01e+1) –	1.4410e+1 (1.65e+0) –	5.1274e+0 (4.56e-1) –	5.0738e+0 (2.46e-1) ≈	7.7823e+0 (1.06e+0) –	5.2310e+0 (1.78e-1)
MaF12	10	5.7192e+0 (8.97e-2) –	6.8356e+0 (4.67e+0) –	4.8361e+0 (5.85e-2) –	5.6426e+0 (2.93e-1) –	5.7886e+0 (4.35e-2) –	6.4873e+0 (2.86e-1) –	4.3383e+0 (9.17e-2)
	15	1.1372e+1 (2.45e-1) –	1.3635e+1 (8.39e+0) –	9.0472e+0 (4.10e-1) ≈	1.0081e+1 (2.78e-1) –	1.1978e+1 (7.52e-2) –	1.3297e+1 (6.59e-1) –	9.3609e+0 (3.75e-1)
	20	1.8488e+1 (9.12e-1) –	3.7932e+1 (1.07e+1) –	1.6837e+1 (1.79e+0) –	1.4880e+1 (3.40e-1) –	1.7433e+1 (6.98e-2) –	2.3725e+1 (1.23e+0) –	1.3555e+1 (7.88e-1)
	30	2.5372e+1 (4.30e-1) +	2.4277e+1 (1.28e+1) +	5.0963e+1 (1.12e+0) –	3.3679e+1 (2.53e+0) +	2.6124e+1 (9.51e-2) +	4.9752e+1 (2.13e+0) –	4.3287e+1 (3.64e+0)
MaF13	10	4.0709e-1 (9.80e-2) ≈	1.2675e+0 (1.40e-1) –	2.6293e-1 (3.60e-2) +	4.2239e-1 (1.05e-1) ≈	7.5559e-1 (1.46e-1) –	2.7357e-1 (9.98e-3) +	4.2214e-1 (2.80e-2)
	15	7.3486e-1 (1.46e-1) ≈	1.6600e+0 (1.26e-1) –	3.8626e-1 (7.98e-2) +	5.1312e-1 (1.71e-1) +	8.0964e-1 (3.11e-1) ≈	5.9186e-1 (4.12e-2) +	7.9590e-1 (6.68e-2)
	20	7.0308e-1 (1.36e-1) +	1.9522e+0 (2.14e-1) –	5.2701e-1 (7.45e-2) +	6.7750e-1 (3.60e-1) ≈	1.4403e+0 (3.34e-1) –	5.8756e-1 (4.20e-2) +	9.0035e-1 (1.06e-1)
	30	7.0705e-1 (1.97e-1) +	2.3505e+0 (5.10e-1) –	1.1403e+0 (8.73e-1) ≈	1.4751e+0 (6.06e-1) –	1.6239e+0 (4.60e-1) ≈	8.6231e-1 (1.38e-1) +	1.3416e+0 (5.22e-1)
MaF14	10	1.5974e+1 (1.47e+1) –	2.5174e+0 (8.61e-1) –	3.0517e+1 (2.45e+1) –	2.2273e+1 (8.11e+1) –	2.6065e+1 (2.54e+1) –	6.2109e+0 (1.28e+0) –	1.3177e+0 (2.15e-1)
	15	1.4174e+1 (1.07e+1) –	1.7189e+1 (1.50e+1) –	4.2469e+1 (2.38e+2) –	1.7516e+1 (9.63e+0) –	2.8183e+1 (1.16e+1) –	7.6009e+0 (3.73e+0) –	1.5240e+0 (5.22e-1)
	20	6.1421e+0 (1.30e+3) –	2.0554e+0 (1.55e+0) –	5.5266e+1 (1.84e+1) –	3.1027e+1 (1.02e+2) –	6.2389e+1 (7.72e+2) –	1.2425e+0 (1.75e-1) ≈	1.2364e+0 (1.35e-1)
	30	4.5151e+0 (4.05e+1) –	3.1596e+0 (2.79e+0) –	6.5877e+1 (3.10e+4) –	3.6061e+2 (7.84e+2) –	1.9305e+2 (3.42e+2) –	1.4726e+0 (5.28e-1) ≈	1.5568e+0 (5.32e-1)
MaF15	10	1.4045e+1 (3.26e+0) –	1.6060e+0 (2.26e-1) –	1.5157e+0 (2.40e-1) –	5.7483e+0 (1.19e+0) –	2.3887e+1 (3.74e+0) –	7.1632e+0 (6.75e-1) –	1.0561e+0 (4.58e-2)
	15	2.2302e+1 (1.13e+1) –	1.4231e+0 (2.34e+0) ≈	1.7777e+0 (1.79e+0) –	5.4214e+0 (1.63e+0) –	7.1913e+1 (1.03e+1) –	1.2038e+1 (8.92e-1) –	1.4286e+0 (4.34e-2)
	20	4.8716e+1 (4.87e+0) –	5.0182e+0 (1.20e+0) –	4.5258e+0 (4.34e-1) –	2.7834e+0 (3.12e-1) –	9.6359e+1 (6.35e+0) –	1.5339e+1 (8.28e-1) –	1.7247e+0 (4.54e-2)
	30	5.8912e+1 (3.27e+0) –	2.2315e+1 (4.06e+0) –	8.7668e+0 (9.91e-1) –	5.1510e+0 (7.47e-1) +	1.0496e+2 (6.91e+0) –	2.5043e+1 (2.49e+0) –	6.3738e+0 (8.90e-1)
+/ ≈ /−		15/9/34	5/6/47	13/10/35	19/12/27	5/6/47	16/13/29	

4. Conclusion

Two new performance indicators, one focusing on convergence and the other on diversity, have been proposed in this paper, and used in the environmental selection after non-dominated sorting for many-objective optimization. These performance indicators are helpful in maintaining solutions located in both the middle and edge parts in the objective space, thereby effectively enhancing the diversity of the population. On the basis of these performance indicators, a new MOEA for many-objective optimization, termed NSPI-EMO has been developed. The experimental results on DTLZ test functions with 3, 5, 8, 10, 15, 20 and 30 objectives, and on MaF test instances with 10, 15, 20 and 30 objectives, show that the performance of proposed NSPI-EMO algorithm is highly competitive compared to a number of state-of-the-art many-objective optimizers, especially when the number of objectives and/or design space is large.

However, as discussed, the NSPI-EMO algorithm is (relatively) less adept at solving problems when the number of objectives is low. We posit that in the lower-dimensional objective space it may be better to use the distance instead of angle to measure the crowdedness of a solution. We intend to analyze the characteristics of distance-based and angle-based diversities further in order to propose even better strategies to evaluate the performance on diversity for each solution in our future work. Furthermore, the experimental results show that NSPI-EMO is inefficient in solving problems with irregular/degenerate Pareto fronts. Therefore, we look forward to designing new strategies for identifying and dealing with such Pareto front properties.

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