Supplementary material

Large-scale Evolutionary Multi-objective Optimization Assisted by Directed Sampling

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A. Experimental results of the methods using different numbers of search directions

The purpose to use two directions in our method is to increase the chance for the search direction vectors to intersect with the Pareto set in the decision space. From Fig. 3(a) in the main body of the paper, we can see if only the direction $\mathbf{v}^{l,1}$ is utilized, no solutions generated on these directions can intersect with the Pareto set. However, if the direction $\mathbf{v}^{u,1}$ is utilized simultaneously, there will be one intersection position between this direction and the Pareto set.

We further conduct experiments on the methods using the direction setting out from $\mathbf{L}$ only or using the direction setting out from $\mathbf{U}$ only. The experimental results of 20 independent running on each problem are given in Table I. In Table I, LMOEA-DS-L and LMOEA-DS-U represent the methods that the search direction is setting out from $\mathbf{L}$ and $\mathbf{U}$, respectively. From Table I, we can see that LMOEA-DS can obtain 34/36 and 36/36 better results, respectively, and no worse results than both of these two methods, which confirms the effectiveness of our method using two search directions.

B. Sensitivity analysis of the upper bound of coefficient s

The values of lower and upper bounds of the coefficient s are set to 0 and $||\mathbf{U}-\mathbf{L}||$, respectively, to ensure that all positions in the upper and lower bounds of the large-scale optimization problem can be sampled. Fig. 1 gives three situations with different upper bounds, i.e., the upper bounds are set to $||\mathbf{U}-\mathbf{L}||$, $\frac{||\mathbf{U}-\mathbf{L}||}{2}$, and a value larger than $||\mathbf{U}-\mathbf{L}||$, respectively. From Fig. 1, we can clearly see that when the upper bound of s is set to no less than $||\mathbf{U}-\mathbf{L}||$, i.e., the cases shown in Fig. 1(a) and 1(c), it is possible to sample any solution in the decision space of a large-scale multi-objective problem. However, from Fig. 1(b), when the upper bound of s is set to $\frac{||\mathbf{U}-\mathbf{L}||}{2}$, we can see that the solutions in two subspaces (two parts shaded in grey in the rectangle) can never be sampled, resulting in worse performance than using $||\mathbf{U}-\mathbf{L}||$. Table II in the following gives the statistical IGD results on LSMOP 1-9 obtained by LSMOEA-DS-half (the upper bound is set to $\frac{||\mathbf{U}-\mathbf{L}||}{2}$) and LSMOEA-DS (the upper bound is set to $||\mathbf{U}-\mathbf{L}||$), respectively. From Table II, we can see that the method using $||\mathbf{U}-\mathbf{L}||$ can obtain better results in 21 test instances than that using $\frac{||\mathbf{U}-\mathbf{L}||}{2}$, and loses only on 4 out of 36 test problems, which is consistent with our previous analysis. Thus, $||\mathbf{U}-\mathbf{L}||$ is adopted as the upper bound of the coefficent s.

C. The effectiveness of double regeneration

Table III gives the statistical IGD values on all nine benchmark problems obtained by LMOEA-DS-one, which utilizes only one reproduction at each generation, and LMOEA-DS. From Table III, we can see that LMOEA-DS is not able to obtain better results on problems LSMOP2 and LSMOP4 with a linear Pareto front. However, the performance of LMOEA-DS is better than LMOEA-DS-one on the majority of the test problems with a convex Pareto front. This can be easily understood because the second reproduction is used to improve the diversity of the population. Therefore, it will be good for those problems with a convex Pareto front to prevent premature convergence. However, for problems with a linear Pareto front, improving the diversity may slow down the convergence to the Pareto front.

D. The effects of the complementary environmental selection

Table IV gives the statistical IGD values on all nine benchmark problems. From Table IV, we can see that LMOEA-DS can obtain 17 and 16 better results out of 36 test problems than LMOEA-DS-decomposition and LMOEA-DS-domination, respectively, while it loses only on 6 out of 36 and 1 out of 36 test problems, respectively.

E. The effectiveness of directed sampling

In order to show the effectiveness of the directed sampling, we conducted experiments on a variant of the proposed method, denoted LMOEA-NoDS, that removes the regeneration of the directed sampling strategy and only keeps the final regeneration. The following table gives the statistical IGD results of LMOEA-DS and LMOEA-NoDS. From the Table,

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TABLE I
THE STATISTICAL RESULTS (MEDIAN AND MEDIAN ABSOLUTE DEVIATION) OBTAINED BY LMOEA-DS-L, LMOEA-DS-U AND LMOEA-DS ON THREE-OBJECTIVE LSMOP TEST PROBLEMS WITH 500, 1000, 2000 AND 5000 DIMENSIONS. THE BEST MEDIAN RESULT IN EACH ROW IS HIGHLIGHTED.

Problem	D	LMOEA-DS-L	LMOEA-DS-U	LMOEA-DS
LSMOP1	500	6.6936e-01(4.51e-02) [+]	8.5995e-01(7.93e-02) [+]	4.5498e-01(2.41e-02)
	1000	8.2344e-01(2.63e-02) [+]	9.8123e-01(1.91e-02) [+]	4.7643e-01(4.11e-02)
	2000	8.4284e-01(1.17e-02) [+]	1.0155e+00(2.36e-02) [+]	4.7684e-01(8.15e-02)
	5000	8.5284e-01(1.21e-02) [+]	1.0321e+00(1.34e-02) [+]	5.5356e-01(8.04e-02)
LSMOP2	500	5.9273e-02(5.77e-04) [+]	5.8848e-02(9.78e-04) [+]	4.4929e-02(8.38e-04)
	1000	4.3478e-02(9.68e-04) [+]	4.3877e-02(6.75e-04) [+]	3.8613e-02(7.89e-04)
	2000	3.6419e-02(7.36e-04) [+]	3.7494e-02(9.54e-04) [+]	3.4450e-02(8.04e-04)
	5000	3.3760e-02(6.79e-04) [≈]	3.5022e-02(7.55e-04) [+]	3.3239e-02(9.80e-04)
LSMOP3	500	8.6064e-01(1.71e-03) [+]	6.0018e+00(7.07e-01) [+]	8.6048e-01(3.62e-03)
	1000	8.6066e-01(1.19e-03) [+]	7.5421e+00(1.20e+00) [+]	8.6049e-01(1.21e-03)
	2000	8.6064e-01(1.68e-03) [+]	7.6414e+00(3.61e+00) [+]	8.6057e-01(3.11e-04)
	5000	8.6069e-01(1.25e-03) [≈]	6.9604e+00(4.86e-01) [+]	8.6064e-01(1.45e-03)
LSMOP4	500	1.3737e-01(4.94e-03) [+]	1.3375e-01(3.92e-03) [+]	1.0818e-01(1.78e-03)
	1000	8.9904e-02(2.03e-03) [+]	8.9887e-02(1.49e-03) [+]	6.8001e-02(8.82e-04)
	2000	6.0074e-02(8.63e-04) [+]	5.9442e-02(8.48e-04) [+]	4.8670e-02(9.49e-04)
	5000	4.1512e-02(8.85e-04) [+]	4.0751e-02(9.19e-04) [+]	3.7368e-02(1.04e-03)
LSMOP5	500	5.4031e-01(2.50e-03) [+]	1.6995e+00(3.05e-01) [+]	5.3450e-01(1.73e-02)
	1000	5.4032e-01(7.83e-04) [+]	1.9226e+00(1.55e-01) [+]	5.3833e-01(1.24e-02)
	2000	5.4034e-01(3.49e-04) [+]	2.1652e+00(3.30e-01) [+]	5.3963e-01(1.66e-03)
	5000	5.4020e-01(5.56e-04) [+]	2.1671e+00(2.75e-01) [+]	5.3883e-01(1.61e-02)
LSMOP6	500	1.2899e+00(1.07e-03) [+]	1.9414e+00(1.65e+01) [+]	7.6982e-01(2.93e-02)
	1000	1.3118e+00(5.06e-04) [+]	1.3756e+02(6.01e+01) [+]	7.6694e-01(2.70e-02)
	2000	1.3215e+00(4.76e-04) [+]	2.6639e+02(1.09e+02) [+]	7.6152e-01(2.36e-02)
	5000	1.3297e+00(4.79e-02) [+]	3.7385e+02(4.55e+02) [+]	7.4373e-01(2.71e-02)
LSMOP7	500	9.1059e-01(3.86e-02) [+]	1.0229e+00(2.62e-02) [+]	8.8589e-01(4.07e-03)
	1000	8.7056e-01(1.75e-03) [+]	1.0107e+00(3.95e-02) [+]	8.5813e-01(1.55e-03)
	2000	8.5191e-01(8.49e-04) [+]	9.8390e-01(2.01e-02) [+]	8.4561e-01(5.58e-04)
	5000	8.4168e-01(5.05e-04) [+]	9.6700e-01(3.20e-03) [+]	8.3922e-01(1.76e-03)
LSMOP8	500	3.1551e-01(1.87e-02) [+]	4.0389e-01(6.84e-02) [+]	2.9319e-01(4.09e-02)
	1000	3.1084e-01(1.46e-02) [+]	5.5438e-01(5.74e-02) [+]	2.1473e-01(2.97e-02)
	2000	3.0263e-01(5.99e-03) [+]	6.1988e-01(7.86e-02) [+]	2.2651e-01(1.29e-02)
	5000	3.0989e-01(1.22e-02) [+]	7.0220e-01(2.29e-01) [+]	2.1994e-01(1.67e-02)
LSMOP9	500	5.8881e-01(1.64e-04) [+]	1.5718e+00(3.63e-01) [+]	5.8575e-01(1.18e-03)
	1000	5.8861e-01(1.87e-04) [+]	7.7674e+00(3.95e+00) [+]	5.8175e-01(1.28e-03)
	2000	5.8822e-01(1.84e-04) [+]	1.5882e+01(4.66e+00) [+]	5.8036e-01(1.57e-03)
	5000	5.8784e-01(1.29e-04) [+]	2.7551e+01(3.08e+00) [+]	5.7991e-01(1.84e-03)
+/≈/-		34/2/0	36/0/0	-

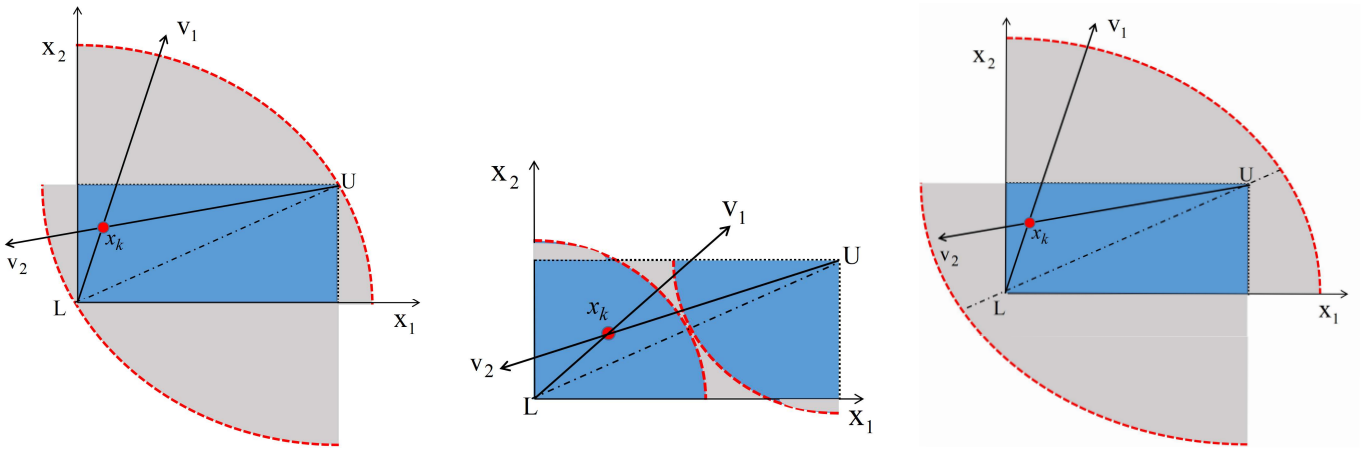

Fig. 1. An example to show the reason why the value of s is selected from the range of $[\mathbf{0}, \|\mathbf{U}-\mathbf{L}\|]$. (a) $\|\mathbf{U}-\mathbf{L}\|$ is utilized as the upper bound of s. (b) $\|\mathbf{U}-\mathbf{L}\|/2$ is utilized as the upper bound of s. (c) the upper bound of s is larger than $\|\mathbf{U}-\mathbf{L}\|$.

we can see that LMOEA-DS can obtain better results on all problems, which shows the importance of the guiding search based on the directed sampling.

Furthermore, we also show the effectiveness of the directed sampling by conducting experiments on three LMOEA-DS variants, in which two of them utilize only one environmental selection, denoted as LMOEA-DS-domination and LMOEA-DS-decomposition, and the other utilizes two reproductions without directed sampling based method but keeps two re-generations, denoted as LMOEA-NoDS-two. The following table gives the statistical results of the IGD values obtained by these three variants. From Table VI, we can see that LMOEA-NoDS-Two obtains 28 out of 36 test instances and on all 36 test instances worse results than LMOEA-DS-domination and LMOEA-DS-decomposition, respectively, indicating that the directed sampling based method plays an important role in the proposed method.

F. Comparison with state-of-the-art large-scale multi-objective algorithms

The statistical results of the HV values obtained by the compared algorithms on bi- and three-objective LSMOP1-9 problems with up to 5000 decision variables are given in Tables VII and VIII, respectively. From Table VII, we can see that LMOEA-DS can obtain 27, 20, 18, 28 and 20 better results among 36 test problems than MOEA/DVA, LSMOF, WOF-NSGA-II, LMOCSO, and OLEA, respectively, but only loses on 8 problems compared to LSMOF and WOF-NSGA-II, and on 4 problems compared to OLEA, respectively. We can also observe from Table VIII that LMOEA-DS can obtain 34/36, 25/36, 27/36, 31/36 and 26/36 better results than the compared algorithms, respectively. Therefore, we can conclude from the HV results that the proposed LMOEA-DS can obtain better results than MOEA/DVA, LSMOF, WOF-NSGA-II, LMOCSO, and OLEA.

TABLE II
THE COMPARED IGD RESULTS OBTAINED BY LMOEA-DS-HALF AND LMOEA-DS ON THREE-OBJECTIVE LSMOP1-9 PROBLEMS WITH 500, 1000, 2000 AND 5000 DECISIONS.

Problem	D	LMOEA-DS-half	LMOEA-DS
LSMOP1	500	6.1319e-01(1.80e-02) [+]	4.5498e-01(2.41e-02)
	1000	6.6016e-01(2.48e-02) [+]	4.7643e-01(4.11e-02)
	2000	6.8117e-01(3.02e-02) [+]	4.7684e-01(8.15e-02)
	5000	7.1367e-01(1.98e-02) [+]	5.5356e-01(8.04e-02)
LSMOP2	500	4.6382e-02(7.89e-04) [+]	4.4929e-02(8.38e-04)
	1000	3.8771e-02(8.95e-04) [≈]	3.8613e-02(7.89e-04)
	2000	3.5421e-02(7.80e-04) [+]	3.4450e-02(8.04e-04)
	5000	3.4055e-02(5.35e-04) [+]	3.3239e-02(9.80e-04)
LSMOP3	500	8.6055e-01(2.43e-03) [≈]	8.6048e-01(3.62e-03)
	1000	8.6059e-01(5.40e-05) [≈]	8.6049e-01(1.21e-03)
	2000	8.6057e-01(1.08e-04) [≈]	8.6057e-01(3.11e-04)
	5000	8.6061e-01(7.71e-05) [≈]	8.6064e-01(1.45e-03)
LSMOP4	500	1.1075e-01(2.11e-03) [+]	1.0818e-01(1.78e-03)
	1000	6.8649e-02(9.75e-04) [+]	6.8001e-02(8.82e-04)
	2000	4.9096e-02(9.26e-04) [≈]	4.8670e-02(9.49e-04)
	5000	3.7894e-02(7.93e-04) [≈]	3.7368e-02(1.04e-03)
LSMOP5	500	5.4030e-01(3.22e-03) [+]	5.3450e-01(1.73e-02)
	1000	5.4047e-01(4.28e-04) [+]	5.3833e-01(1.24e-02)
	2000	5.4054e-01(2.77e-04) [+]	5.3963e-01(1.66e-03)
	5000	5.4057e-01(3.62e-04) [+]	5.3883e-01(1.61e-02)
LSMOP6	500	7.8813e-01(4.60e-02) [≈]	7.6982e-01(2.93e-02)
	1000	7.9176e-01(1.39e-02) [+]	7.6694e-01(2.70e-02)
	2000	7.9352e-01(7.71e-02) [+]	7.6152e-01(2.72e-02)
	5000	7.9510e-01(4.73e-02) [+]	7.4373e-01(2.71e-02)
LSMOP7	500	8.8057e-01(4.51e-03) [-]	8.8589e-01(4.07e-03)
	1000	8.5482e-01(1.04e-02) [-]	8.5813e-01(1.55e-03)
	2000	8.4419e-01(9.16e-03) [-]	8.4561e-01(5.58e-04)
	5000	8.3837e-01(1.77e-02) [-]	8.3922e-01(1.76e-03)
LSMOP8	500	3.1916e-01(4.78e-02) [≈]	2.9319e-01(4.09e-02)
	1000	2.3952e-01(2.73e-02) [+]	2.1473e-01(2.97e-02)
	2000	2.3326e-01(1.82e-02) [≈]	2.2651e-01(1.29e-02)
	5000	2.2263e-01(9.58e-03) [≈]	2.1994e-01(1.67e-02)
LSMOP9	500	5.8875e-01(1.52e-04) [+]	5.8575e-01(1.18e-03)
	1000	5.8841e-01(1.68e-04) [+]	5.8175e-01(1.28e-03)
	2000	5.8815e-01(1.15e-04) [+]	5.8036e-01(1.57e-03)
	5000	5.8782e-01(1.72e-04) [+]	5.7991e-01(1.84e-03)
+/≈/-		21/11/4	-

TABLE III
THE STATISTICAL IGD RESULTS OBTAINED BY LMOEA-DS-ONE AND LMOEA-DS ON THREE-OBJECTIVE LSMOPS WITH 500, 1000, 2000 AND 5000 DECISION VARIABLES. THE BEST ON EACH ROW IS HIGHLIGHTED.

Problem	D	LMOEA-DS-one	LMOEA-DS
LSMOP1	500	6.0554e-01(6.26e-02) [+]	4.5498e-01(2.41e-02)
	1000	6.2847e-01(5.74e-02) [+]	4.7643e-01(4.11e-02)
	2000	6.6456e-01(6.13e-02) [+]	4.7684e-01(8.15e-02)
	5000	6.4850e-01(5.56e-02) [+]	5.5356e-01(8.04e-02)
LSMOP2	500	4.4005e-02(7.87e-04) [-]	4.4929e-02(8.38e-04)
	1000	3.6863e-02(3.58e-04) [-]	3.8613e-02(7.89e-04)
	2000	3.2761e-02(4.08e-04) [-]	3.4450e-02(8.04e-04)
	5000	3.1277e-02(2.33e-04) [-]	3.3239e-02(9.80e-04)
LSMOP3	500	8.6051e-01(2.42e-04) [≈]	8.6048e-01(3.62e-03)
	1000	8.6052e-01(5.97e-04) [≈]	8.6049e-01(1.21e-03)
	2000	8.6054e-01(7.52e-04) [≈]	8.6057e-01(3.11e-04)
	5000	8.6058e-01(2.78e-04) [≈]	8.6064e-01(1.45e-03)
LSMOP4	500	1.0783e-01(1.43e-03) [≈]	1.0818e-01(1.78e-03)
	1000	6.7159e-02(8.59e-04) [≈]	6.8001e-02(8.82e-04)
	2000	4.6646e-02(8.88e-04) [-]	4.8670e-02(9.49e-04)
	5000	3.5242e-02(4.97e-04) [-]	3.7368e-02(1.04e-03)
LSMOP5	500	5.2244e-01(1.28e-02) [-]	5.3450e-01(1.73e-02)
	1000	5.3433e-01(4.03e-03) [-]	5.3833e-01(1.24e-02)
	2000	5.3430e-01(2.33e-03) [-]	5.3963e-01(1.66e-03)
	5000	5.3455e-01(1.56e-03) [-]	5.3883e-01(1.61e-02)
LSMOP6	500	7.8624e-01(2.58e-02) [≈]	7.6982e-01(2.93e-02)
	1000	7.9013e-01(1.27e-02) [+]	7.6694e-01(2.70e-02)
	2000	7.9319e-01(1.34e-02) [+]	7.6152e-01(2.72e-02)
	5000	7.9671e-01(2.43e-03) [+]	7.4373e-01(2.71e-02)
LSMOP7	500	8.8812e-01(4.87e-03) [≈]	8.8589e-01(4.07e-03)
	1000	8.5982e-01(2.42e-03) [+]	8.5813e-01(1.55e-03)
	2000	8.4934e-01(1.49e-03) [+]	8.4561e-01(5.58e-04)
	5000	8.4245e-01(1.29e-03) [+]	8.3922e-01(1.76e-03)
LSMOP8	500	2.8933e-01(4.29e-02) [≈]	2.9319e-01(4.09e-02)
	1000	2.4353e-01(3.84e-02) [+]	2.1473e-01(2.97e-02)
	2000	2.2566e-01(4.32e-02) [≈]	2.2651e-01(1.29e-02)
	5000	2.2261e-01(6.76e-03) [≈]	2.1994e-01(1.67e-02)
LSMOP9	500	5.8534e-01(1.20e-03) [≈]	5.8575e-01(1.18e-03)
	1000	5.8164e-01(1.56e-03) [≈]	5.8175e-01(1.28e-03)
	2000	5.8058e-01(1.25e-03) [≈]	5.8036e-01(1.57e-03)
	5000	5.7944e-01(1.21e-03) [≈]	5.7991e-01(1.84e-03)
+/≈/-		11/15/10	-

TABLE IV
THE STATISTICAL IGD RESULTS OBTAINED BY LMOEA-DS-DECOMPOSITION, LMOEA-DS-DOMINATION AND LMOEA-DS ON LSMOP1-9 PROBLEMS WITH UP TO 5000 DECISION VARIABLES. THE BEST RESULTS ON EACH ROW ARE HIGHLIGHTED.

Problem	D	LMOEA-DS-decomposition	LMOEA-DS-domination	LMOEA-DS
LSMOP1	500	4.4066e-01(2.59e-02) [≈]	5.5297e-01(2.29e-02) [+]	4.5498e-01(2.41e-02)
	1000	4.6985e-01(2.94e-02) [≈]	6.0421e-01(2.30e-02) [+]	4.7643e-01(4.11e-02)
	2000	4.8324e-01(3.62e-02) [≈]	6.5004e-01(2.28e-02) [+]	4.7684e-01(8.15e-02)
	5000	4.9964e-01(4.11e-02) [≈]	6.6624e-01(2.45e-02) [+]	5.5356e-01(8.04e-02)
LSMOP2	500	4.4879e-02(1.39e-03) [≈]	6.5350e-02(1.67e-03) [+]	4.4929e-02(8.38e-04)
	1000	3.8713e-02(1.01e-03) [≈]	5.5886e-02(1.84e-03) [+]	3.8613e-02(7.89e-04)
	2000	3.4702e-02(6.84e-04) [≈]	4.9558e-02(1.85e-03) [+]	3.4450e-02(8.04e-04)
	5000	3.3108e-02(7.12e-04) [≈]	4.6311e-02(1.21e-03) [+]	3.3239e-02(9.80e-04)
LSMOP3	500	8.6056e-01(1.96e+01) [+]	8.6047e-01(9.65e-04) [≈]	8.6048e-01(3.62e-03)
	1000	8.6052e-01(4.45e-05) [≈]	8.6054e-01(2.30e-04) [≈]	8.6049e-01(1.21e-03)
	2000	8.6048e-01(1.65e-04) [-]	8.6055e-01(1.19e-03) [≈]	8.6057e-01(3.11e-04)
	5000	8.6055e-01(4.53e-05) [-]	8.6062e-01(5.02e-04) [≈]	8.6064e-01(1.45e-03)
LSMOP4	500	1.0766e-01(1.37e-03) [≈]	1.3539e-01(2.28e-03) [+]	1.0818e-01(1.78e-03)
	1000	6.8479e-02(8.16e-04) [≈]	9.2675e-02(2.15e-03) [+]	6.8001e-02(8.82e-04)
	2000	4.8911e-02(8.58e-04) [≈]	6.7097e-02(1.45e-03) [+]	4.8670e-02(9.49e-04)
	5000	3.7381e-02(6.88e-04) [≈]	5.2832e-02(1.85e-03) [+]	3.7368e-02(1.04e-03)
LSMOP5	500	4.8822e-01(1.01e-02) [-]	5.3585e-01(9.83e-03) [≈]	5.3450e-01(1.73e-02)
	1000	4.9293e-01(1.64e-02) [-]	5.3912e-01(1.37e-03) [+]	5.3833e-01(1.24e-02)
	2000	4.9402e-01(1.38e-02) [-]	5.3945e-01(2.84e-03) [≈]	5.3963e-01(1.66e-03)
	5000	5.0005e-01(1.17e-02) [-]	5.3965e-01(7.70e-04) [≈]	5.3883e-01(1.61e-02)
LSMOP6	500	9.3287e-01(2.19e-01) [+]	7.5376e-01(3.38e-02) [≈]	7.6982e-01(2.93e-02)
	1000	9.7025e-01(1.71e-01) [+]	7.7854e-01(2.49e-02) [≈]	7.6694e-01(2.70e-02)
	2000	8.9705e-01(1.97e-01) [+]	7.6138e-01(2.73e-02) [≈]	7.6152e-01(2.72e-02)
	5000	8.9529e-01(2.02e-01) [+]	7.7952e-01(2.26e-02) [+]	7.4373e-01(2.71e-02)
LSMOP7	500	1.3250e+00(1.81e-02) [+]	8.8454e-01(4.68e-03) [≈]	8.8589e-01(4.07e-03)
	1000	1.3366e+00(2.68e-02) [+]	8.5835e-01(1.58e-03) [≈]	8.5813e-01(1.55e-03)
	2000	1.3406e+00(3.01e-02) [+]	8.4522e-01(7.08e-04) [≈]	8.4561e-01(5.58e-04)
	5000	1.3441e+00(4.02e-02) [+]	8.3902e-01(4.72e-04) [≈]	8.3922e-01(1.76e-03)
LSMOP8	500	4.4201e-01(5.19e-02) [+]	2.4922e-01(2.84e-02) [≈]	2.9319e-01(4.09e-02)
	1000	4.9016e-01(5.49e-02) [+]	2.4372e-01(2.86e-02) [+]	2.1473e-01(2.97e-02)
	2000	4.7124e-01(6.06e-02) [+]	2.2441e-01(2.04e-02) [≈]	2.2651e-01(1.29e-02)
	5000	4.9348e-01(7.03e-02) [+]	2.1582e-01(4.91e-03) [-]	2.1994e-01(1.67e-02)
LSMOP9	500	6.1799e-01(3.42e-02) [+]	5.8516e-01(2.21e-03) [≈]	5.8575e-01(1.18e-03)
	1000	6.1136e-01(9.87e-02) [+]	5.8284e-01(1.24e-03) [≈]	5.8175e-01(1.28e-03)
	2000	6.2171e-01(2.10e-01) [+]	5.8030e-01(1.99e-03) [≈]	5.8036e-01(1.57e-03)
	5000	6.2362e-01(3.85e-01) [+]	5.8016e-01(1.97e-03) [≈]	5.7991e-01(1.84e-03)
+/≈/-		17/13/6	16/19/1	-

TABLE V
THE COMPARED STATISTICAL IGD RESULTS OBTAINED BY LMOEA-NODS AND LMOEA-DS ON THREE-OBJECTIVE LSMOP TEST PROBLEMS WITH 500, 1000, 2000 AND 5000 DECISION VARIABLES.

Problem	D	LMOEA-NoDS	LMOEA-DS
LSMOP1	500	1.3616e+00(6.89e-01) [+]	4.5498e-01(2.41e-02)
	1000	3.3993e+00(3.74e-01) [+]	4.7643e-01(4.11e-02)
	2000	5.1578e+00(3.78e-01) [+]	4.7684e-01(8.15e-02)
	5000	7.6833e+00(5.30e-01) [+]	5.5356e-01(8.04e-02)
LSMOP2	500	6.0749e-02(1.46e-03) [+]	4.4929e-02(8.38e-04)
	1000	4.6365e-02(1.33e-03) [+]	3.8613e-02(7.89e-04)
	2000	3.9379e-02(1.61e-03) [+]	3.4450e-02(8.04e-04)
	5000	3.6674e-02(1.28e-03) [+]	3.3239e-02(9.80e-04)
LSMOP3	500	1.4893e+01(1.93e+01) [+]	8.6048e-01(3.62e-03)
	1000	2.1153e+01(4.40e+01) [+]	8.6049e-01(1.21e-03)
	2000	2.1300e+01(9.37e+01) [+]	8.6057e-01(3.11e-04)
	5000	3.2848e+01(2.47e+01) [+]	8.6064e-01(1.45e-03)
LSMOP4	500	1.3408e-01(1.80e-03) [+]	1.0818e-01(1.78e-03)
	1000	9.1170e-02(9.76e-04) [+]	6.8001e-02(8.82e-04)
	2000	6.5388e-02(6.66e-04) [+]	4.8670e-02(9.49e-04)
	5000	4.5364e-02(1.19e-03) [+]	3.7368e-02(1.04e-03)
LSMOP5	500	3.7342e+00(1.08e+00) [+]	5.3450e-01(1.73e-02)
	1000	5.6133e+00(1.55e+00) [+]	5.3833e-01(1.24e-02)
	2000	1.5998e+01(4.19e+00) [+]	5.3963e-01(1.66e-03)
	5000	1.9246e+01(2.56e+00) [+]	5.3883e-01(1.61e-02)
LSMOP6	500	3.3064e+01(3.55e+01) [+]	7.6982e-01(2.93e-02)
	1000	8.2804e+02(5.27e+02) [+]	7.6694e-01(2.70e-02)
	2000	8.7671e+03(1.69e+03) [+]	7.6152e-01(2.36e-02)
	5000	1.8635e+04(4.69e+03) [+]	7.4373e-01(2.71e-02)
LSMOP7	500	7.0967e+01(1.36e+01) [+]	8.8589e-01(4.07e-03)
	1000	1.1968e+03(2.97e+02) [+]	8.5813e-01(1.55e-03)
	2000	7.2612e+03(1.20e+03) [+]	8.4561e-01(5.58e-04)
	5000	2.5868e+04(4.66e+03) [+]	8.3922e-01(1.76e-03)
LSMOP8	500	5.7173e-01(5.02e-02) [+]	2.9319e-01(4.09e-02)
	1000	9.1552e-01(3.31e-01) [+]	2.1473e-01(2.97e-02)
	2000	3.4617e+00(1.27e+00) [+]	2.2651e-01(1.29e-02)
	5000	6.8101e+00(1.61e+00) [+]	2.1994e-01(1.67e-02)
LSMOP9	500	2.5194e+00(1.46e-01) [+]	5.8575e-01(1.18e-03)
	1000	5.9856e+00(6.17e-01) [+]	5.8175e-01(1.28e-03)
	2000	1.4229e+01(1.33e+00) [+]	5.8036e-01(1.57e-03)
	5000	2.9423e+01(3.38e+00) [+]	5.7991e-01(1.84e-03)
+/≈/-		36/0/0	

TABLE VI
THE COMPARED STATISTICAL IGD RESULTS OBTAINED BY LMOEA-DS-DOMINATION, LMOEA-DS-DECOMPOSITION AND LMOEA-NODS-TWO ON THREE-OBJECTIVE LSMOP1-7 PROBLEMS WITH UP TO 5000 DECISION VARIABLES.

Problem	D	LMOEA-DS-domination	LMOEA-DS-decomposition	LMOEA-NoDS-Two
LSMOP1	500	5.5297e-01(2.29e-02) [-]	4.4066e-01(2.59e-02) [-]	1.5684e+00(6.45e-01)
	1000	6.0421e-01(2.30e-02) [-]	4.6985e-01(2.94e-02) [-]	3.3994e+00(2.90e-01)
	2000	6.5004e-01(2.28e-02) [-]	4.8324e-01(3.62e-02) [-]	5.1546e+00(4.14e-01)
	5000	6.6624e-01(2.45e-02) [-]	4.9964e-01(4.11e-02) [-]	7.6717e+00(5.38e-01)
LSMOP2	500	6.5350e-02(1.67e-03) [+]	4.4879e-02(1.39e-03) [-]	6.0697e-02(8.28e-04)
	1000	5.5886e-02(1.84e-03) [+]	3.8713e-02(1.01e-03) [-]	4.5734e-02(8.92e-04)
	2000	4.9558e-02(1.85e-03) [+]	3.4702e-02(6.84e-04) [-]	4.0004e-02(1.54e-03)
	5000	4.6311e-02(1.21e-03) [+]	3.3108e-02(7.12e-04) [-]	3.6022e-02(1.46e-03)
LSMOP3	500	8.6047e-01(9.65e-04) [-]	8.6056e-01(1.96e+01) [-]	1.8367e+01(2.52e+01)
	1000	8.6054e-01(2.30e-04) [-]	8.6052e-01(4.45e-05) [-]	1.7736e+01(1.02e+01)
	2000	8.6055e-01(1.19e-03) [-]	8.6048e-01(1.65e-04) [-]	2.4909e+01(4.65e+01)
	5000	8.6062e-01(5.02e-04) [-]	8.6055e-01(4.53e-05) [-]	3.1747e+01(7.84e+01)
LSMOP4	500	1.3539e-01(2.28e-03) [≈]	1.0766e-01(1.37e-03) [-]	1.3445e-01(1.73e-03)
	1000	9.2675e-02(2.15e-03) [+]	6.8479e-02(8.16e-04) [-]	9.1277e-02(1.18e-03)
	2000	6.7097e-02(1.45e-03) [+]	4.8911e-02(8.58e-04) [-]	6.5131e-02(9.61e-04)
	5000	5.2832e-02(1.85e-03) [+]	3.7381e-02(6.88e-04) [-]	4.4326e-02(8.06e-04)
LSMOP5	500	5.3585e-01(9.83e-03) [-]	4.8822e-01(1.01e-02) [-]	3.3670e+00(5.53e-01)
	1000	5.3912e-01(1.37e-03) [-]	4.9293e-01(1.64e-02) [-]	5.3902e+00(1.03e+00)
	2000	5.3945e-01(2.84e-03) [-]	4.9402e-01(1.38e-02) [-]	8.7310e+00(4.17e+00)
	5000	5.3965e-01(7.70e-04) [-]	5.0005e-01(1.17e-02) [-]	1.3600e+01(2.97e+00)
LSMOP6	500	7.5376e-01(3.38e-02) [-]	9.3287e-01(2.19e-01) [-]	2.6849e+01(3.38e+01)
	1000	7.7854e-01(2.49e-02) [-]	9.7025e-01(1.71e-01) [-]	8.9126e+02(6.06e+02)
	2000	7.6138e-01(2.73e-02) [-]	8.9705e-01(1.97e-01) [-]	8.6716e+03(1.58e+03)
	5000	7.7952e-01(2.26e-02) [-]	8.9529e-01(2.02e-01) [-]	2.0181e+04(4.98e+03)
LSMOP7	500	8.8454e-01(4.68e-03) [-]	1.3250e+00(1.81e-02) [-]	6.9801e+01(1.46e+01)
	1000	8.5835e-01(1.58e-03) [-]	1.3366e+00(2.68e-02) [-]	1.0812e+03(2.91e+02)
	2000	8.4522e-01(7.08e-04) [-]	1.3406e+00(3.01e-02) [-]	7.2392e+03(1.36e+03)
	5000	8.3902e-01(4.36e-04) [-]	1.3441e+00(4.02e-02) [-]	2.4384e+04(4.60e+03)
LSMOP8	500	2.4922e-01(2.84e-02) [-]	4.4201e-01(5.19e-02) [-]	5.5328e-01(4.90e-02)
	1000	2.4336e-01(2.86e-02) [-]	4.9016e-01(5.49e-02) [-]	1.3356e+00(3.30e-01)
	2000	2.2441e-01(2.04e-02) [-]	4.7124e-01(6.06e-02) [-]	3.4669e+00(1.13e+00)
	5000	2.1582e-01(4.91e-03) [-]	4.9348e-01(7.03e-02) [-]	6.6648e+00(1.94e+00)
LSMOP9	500	5.8516e-01(2.21e-03) [-]	6.1799e-01(3.42e-02) [-]	2.5447e+00(1.37e-01)
	1000	5.8284e-01(1.24e-03) [-]	6.1136e-01(9.87e-02) [-]	5.6861e+00(8.33e-01)
	2000	5.8030e-01(1.99e-03) [-]	6.2171e-01(2.10e-01) [-]	1.4449e+01(1.12e+00)
	5000	5.8016e-01(1.97e-03) [-]	6.2362e-01(3.85e-01) [-]	2.9761e+01(2.61e+00)
+/≈/-		7/1/28	0/0/36	-

TABLE VII
THE STATISTICAL HV RESULTS (MEDIAN AND MEDIAN ABSOLUTE DEVIATION) OBTAINED BY FIVE COMPARED ALGORITHMS AND LMOEA-DS ON TWO-OBJECTIVE LSMOP1-9 PROBLEMS WITH 500, 1000, 2000 AND 5000 DECISION VARIABLES. THE BEST IS HIGHLIGHTED.

Problem	M	D	MOEA/DVA	LSMOF	WOF-NSGA-II	LMOCSO	OLEA	LMOEA-DS
LSMOP1	2	500	0.0000e+00(0.00e+00) [+]	1.0443e-01(6.29e-03) [+]	9.5602e-02(3.15e-02) [+]	8.4744e-03(8.94e-03) [+]	2.1138e-01(3.79e-02) [≈]	2.3407e-01(8.46e-03)
		1000	0.0000e+00(0.00e+00) [+]	1.0131e-01(4.68e-03) [+]	9.4992e-02(3.10e-02) [+]	0.0000e+00(0.00e+00) [+]	1.6596e-01(4.07e-02) [+]	2.3369e-01(8.85e-03)
		2000	0.0000e+00(0.00e+00) [+]	1.0525e-01(4.57e-03) [+]	9.3829e-02(2.94e-02) [+]	0.0000e+00(0.00e+00) [+]	1.5085e-01(3.90e-02) [+]	2.2711e-01(6.99e-03)
		5000	0.0000e+00(0.00e+00) [+]	1.0262e-01(4.21e-03) [+]	9.5881e-02(3.19e-02) [+]	0.0000e+00(0.00e+00) [+]	1.0567e-01(3.18e-02) [+]	2.2113e-01(5.90e-03)
LSMOP2	2	500	4.9034e-01(4.60e-04) [+]	5.6001e-01(5.09e-04) [+]	5.6519e-01(7.20e-04) [+]	5.2966e-01(8.64e-04) [+]	5.6456e-01(1.36e-03) [+]	5.6739e-01(4.88e-04)
		1000	5.3222e-01(6.24e-04) [+]	5.6706e-01(3.36e-04) [+]	5.6933e-01(5.57e-04) [+]	5.5469e-01(5.28e-04) [+]	5.7453e-01(4.68e-04) [≈]	5.7453e-01(2.84e-04)
		2000	5.5522e-01(5.72e-04) [+]	5.7165e-01(3.34e-04) [+]	5.7068e-01(3.31e-04) [+]	5.6822e-01(2.21e-04) [+]	5.7919e-01(2.33e-04) [-]	5.7859e-01(2.33e-04)
		5000	5.6926e-01(7.86e-04) [+]	5.7545e-01(2.91e-04) [+]	5.7573e-01(4.56e-04) [+]	5.7694e-01(6.84e-05) [+]	5.8177e-01(1.31e-04) [-]	5.8098e-01(1.34e-04)
LSMOP3	2	500	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00)
		1000	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00)
		2000	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00)
		5000	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00)
LSMOP4	2	500	4.2697e-01(7.75e-04) [+]	5.2144e-01(8.53e-04) [+]	5.2647e-01(6.43e-04) [+]	4.7639e-01(9.44e-04) [+]	5.3092e-01(1.73e-03) [+]	5.3210e-01(8.02e-04)
		1000	4.8730e-01(6.39e-04) [+]	5.4745e-01(4.94e-04) [+]	5.5086e-01(7.49e-04) [+]	5.1858e-01(3.99e-04) [+]	5.5397e-01(1.17e-03) [+]	5.5662e-01(4.55e-04)
		2000	5.2733e-01(4.86e-04) [+]	5.6048e-01(5.89e-04) [+]	5.5347e-01(2.37e-03) [+]	5.4674e-01(1.30e-04) [+]	5.6822e-01(5.63e-04) [+]	5.6931e-01(2.24e-04)
		5000	5.5547e-01(4.97e-04) [+]	5.6865e-01(5.39e-04) [+]	5.7005e-01(4.21e-04) [+]	5.6698e-01(6.11e-05) [+]	5.7747e-01(2.19e-04) [-]	5.7726e-01(1.62e-04)
LSMOP5	2	500	0.0000e+00(0.00e+00) [+]	9.0909e-02(4.16e-17) [-]	9.0909e-02(6.08e-07) [-]	0.0000e+00(0.00e+00) [+]	9.0909e-02(8.64e-03) [-]	9.0906e-02(5.67e-06)
		1000	0.0000e+00(0.00e+00) [+]	9.0909e-02(4.16e-17) [-]	9.0909e-02(4.29e-08) [-]	0.0000e+00(0.00e+00) [+]	8.9704e-02(3.89e-02) [≈]	9.0907e-02(4.30e-06)
		2000	0.0000e+00(0.00e+00) [+]	9.0909e-02(4.16e-17) [-]	9.0909e-02(7.63e-03) [≈]	0.0000e+00(0.00e+00) [+]	0.0000e+00(1.30e-02) [+]	9.0903e-02(7.54e-06)
		5000	0.0000e+00(0.00e+00) [+]	9.0909e-02(4.16e-17) [-]	9.0909e-02(3.99e-08) [-]	0.0000e+00(0.00e+00) [+]	3.1756e-02(2.19e-02) [+]	9.0901e-02(8.79e-06)
LSMOP6	2	500	0.0000e+00(0.00e+00) [+]	3.4805e-02(1.74e-03) [+]	6.2341e-02(4.93e-03) [-]	1.2348e-02(6.65e-03) [+]	2.1958e-02(2.44e-03) [+]	3.9994e-02(2.72e-03)
		1000	0.0000e+00(0.00e+00) [+]	6.9687e-02(3.83e-04) [+]	7.7396e-02(7.63e-03) [-]	4.0827e-02(2.57e-03) [+]	5.1543e-02(1.48e-03) [+]	7.1044e-02(1.03e-03)
		2000	0.0000e+00(0.00e+00) [+]	8.9125e-02(4.21e-05) [+]	7.9456e-02(2.17e-03) [+]	6.2376e-02(1.80e-03) [+]	6.9409e-02(6.04e-04) [+]	8.9420e-02(2.62e-04)
		5000	0.0000e+00(0.00e+00) [+]	1.0071e-01(6.51e-05) [+]	8.8339e-02(4.09e-04) [+]	7.8797e-02(2.59e-04) [+]	8.1353e-02(1.82e-04) [+]	1.0079e-01(1.22e-04)
LSMOP7	2	500	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00)
		1000	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00)
		2000	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00)
		5000	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00)
LSMOP8	2	500	0.0000e+00(0.00e+00) [+]	9.0909e-02(4.16e-17) [-]	9.0909e-02(7.14e-03) [-]	0.0000e+00(0.00e+00) [+]	8.0774e-02(1.12e-02) [+]	9.0907e-02(7.05e-06)
		1000	0.0000e+00(0.00e+00) [+]	9.0909e-02(4.16e-17) [-]	9.0909e-02(7.08e-03) [-]	0.0000e+00(0.00e+00) [+]	8.0986e-02(1.77e-02) [+]	9.0906e-02(7.79e-06)
		2000	0.0000e+00(0.00e+00) [+]	9.0909e-02(4.16e-17) [-]	9.0909e-02(8.55e-03) [≈]	0.0000e+00(0.00e+00) [+]	6.5707e-02(2.28e-02) [+]	9.0905e-02(6.28e-06)
		5000	0.0000e+00(0.00e+00) [+]	9.0909e-02(4.16e-17) [-]	9.0909e-02(8.01e-08) [-]	0.0000e+00(0.00e+00) [+]	8.2939e-02(1.34e-02) [+]	9.0905e-02(4.99e-06)
LSMOP9	2	500	0.0000e+00(0.00e+00) [+]	9.0986e-02(4.74e-05) [+]	9.0912e-02(1.51e-05) [+]	3.6470e-02(9.28e-03) [+]	9.7146e-02(2.57e-02) [≈]	9.2464e-02(6.05e-03)
		1000	0.0000e+00(0.00e+00) [+]	9.1210e-02(1.53e-04) [+]	9.1002e-02(7.86e-05) [+]	0.0000e+00(0.00e+00) [+]	7.0850e-02(3.73e-02) [+]	1.1950e-01(6.92e-03)
		2000	0.0000e+00(0.00e+00) [+]	9.1515e-02(1.78e-04) [+]	9.1262e-02(6.57e-04) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(1.69e-02) [+]	1.2642e-01(4.89e-03)
		5000	0.0000e+00(0.00e+00) [+]	9.1824e-02(2.56e-04) [+]	9.1595e-02(2.90e-04) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(9.05e-03) [+]	1.3007e-01(4.72e-03)
+/≈/-			27/9/0	20/8/8	18/10/8	28/8/0	20/12/4	-

TABLE VIII
THE STATISTICAL HV RESULTS (MEDIAN AND MEDIAN ABSOLUTE DEVIATION) OBTAINED BY FIVE COMPARED ALGORITHMS AND LMOEA-DS ON THREE-OBJECTIVE LSMOP1-9 PROBLEMS WITH 500, 1000, 2000 AND 5000 DECISION VARIABLES. THE BEST IS HIGHLIGHTED.

Problem	M	D	MOEA/DVA	LSMOF	WOF-NSGA-II	LMOCSO	OLEA	LMOEA-DS
LSMOP1	3	500	0.0000e+00(0.00e+00) [+]	1.8000e-01(8.69e-03) [+]	2.2205e-01(4.70e-02) [+]	0.0000e+00(0.00e+00) [+]	2.2787e-01(4.34e-02) [+]	2.8659e-01(2.60e-02)
		1000	0.0000e+00(0.00e+00) [+]	1.3316e-01(9.82e-03) [+]	1.5042e-01(2.89e-02) [+]	0.0000e+00(0.00e+00) [+]	1.1341e-01(2.47e-02) [+]	2.7180e-01(4.13e-02)
		2000	0.0000e+00(0.00e+00) [+]	1.0929e-01(5.65e-03) [+]	9.5306e-02(1.11e-02) [+]	0.0000e+00(0.00e+00) [+]	6.9701e-02(2.71e-02) [+]	2.6521e-01(6.61e-02)
		5000	0.0000e+00(0.00e+00) [+]	9.8505e-02(2.90e-04) [+]	1.8441e-02(3.48e-02) [+]	0.0000e+00(0.00e+00) [+]	5.3201e-02(2.20e-02) [+]	2.0580e-01(5.45e-02)
LSMOP2	3	500	7.7882e-01(3.05e-03) [+]	7.8433e-01(1.33e-03) [+]	7.8117e-01(3.22e-03) [+]	8.1829e-01(5.81e-04) [-]	8.2823e-01(4.73e-04) [-]	8.1277e-01(3.00e-03)
		1000	7.9532e-01(4.85e-03) [+]	8.0694e-01(1.80e-03) [+]	8.0771e-01(2.53e-03) [+]	8.3206e-01(3.13e-04) [-]	8.3827e-01(3.36e-04) [-]	8.1891e-01(2.33e-03)
		2000	8.1104e-01(3.92e-03) [+]	8.1891e-01(1.81e-03) [+]	8.1970e-01(2.17e-03) [+]	8.3935e-01(1.75e-04) [-]	8.4299e-01(2.01e-04) [-]	8.2375e-01(2.29e-03)
		5000	8.1494e-01(4.39e-03) [+]	8.2703e-01(2.40e-03) [≈]	8.2490e-01(3.09e-03) [+]	8.4440e-01(6.39e-05) [-]	8.4610e-01(9.44e-05) [-]	8.2838e-01(2.93e-03)
LSMOP3	3	500	0.0000e+00(0.00e+00) [+]	9.0930e-02(2.06e-05) [+]	9.0907e-02(7.06e-04) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(1.64e-02) [+]	9.0968e-02(4.15e-05)
		1000	0.0000e+00(0.00e+00) [+]	9.0929e-02(1.68e-05) [+]	9.0906e-02(2.64e-02) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	9.0978e-02(1.71e-03)
		2000	0.0000e+00(0.00e+00) [+]	9.0914e-02(1.50e-05) [≈]	9.0908e-02(8.60e-03) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	9.0942e-02(5.97e-04)
		5000	0.0000e+00(0.00e+00) [+]	9.0909e-02(1.84e-05) [≈]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	9.0913e-02(1.92e-03)
LSMOP4	3	500	6.1205e-01(6.13e-03) [+]	6.2990e-01(5.36e-03) [+]	6.2891e-01(5.97e-03) [+]	6.9896e-01(9.83e-04) [+]	7.6941e-01(1.78e-03) [-]	7.5101e-01(3.27e-03)
		1000	7.1345e-01(5.01e-03) [+]	7.1940e-01(5.22e-03) [+]	7.1929e-01(3.46e-03) [+]	7.6885e-01(4.79e-04) [+]	8.0971e-01(1.02e-03) [-]	7.9274e-01(2.42e-03)
		2000	7.6919e-01(2.79e-03) [+]	7.7552e-01(3.19e-03) [+]	7.7691e-01(2.84e-03) [+]	8.0859e-01(1.94e-04) [+]	8.2948e-01(3.17e-04) [-]	8.1057e-01(1.96e-03)
		5000	8.0289e-01(4.56e-03) [+]	8.1004e-01(1.94e-03) [+]	8.1152e-01(3.12e-03) [+]	8.3318e-01(1.51e-04) [-]	8.4115e-01(2.40e-04) [-]	8.2311e-01(3.25e-03)
LSMOP5	3	500	0.0000e+00(0.00e+00) [+]	3.3744e-01(7.48e-02) [≈]	3.4702e-01(2.43e-02) [-]	0.0000e+00(0.00e+00) [+]	3.2898e-01(5.50e-03) [≈]	3.4425e-01(7.24e-03)
		1000	0.0000e+00(0.00e+00) [+]	1.7532e-01(6.84e-02) [+]	3.4691e-01(1.71e-04) [-]	0.0000e+00(0.00e+00) [+]	3.2895e-01(2.54e-04) [+]	3.4401e-01(5.12e-03)
		2000	0.0000e+00(0.00e+00) [+]	2.4044e-01(8.90e-02) [+]	3.4685e-01(3.29e-04) [-]	0.0000e+00(0.00e+00) [+]	3.2895e-01(3.19e-04) [+]	3.4408e-01(8.40e-04)
		5000	0.0000e+00(0.00e+00) [+]	3.2141e-01(8.60e-02) [≈]	3.1746e-01(1.05e-01) [+]	0.0000e+00(0.00e+00) [+]	3.2898e-01(4.74e-04) [+]	3.4352e-01(5.46e-03)
LSMOP6	3	500	0.0000e+00(0.00e+00) [+]	1.1459e-02(1.65e-03) [-]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	1.0781e-02(3.14e-03)
		1000	0.0000e+00(0.00e+00) [+]	1.1017e-02(1.45e-03) [≈]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	1.1349e-02(3.01e-03)
		2000	0.0000e+00(0.00e+00) [+]	1.1089e-02(5.53e-04) [≈]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	1.1585e-02(2.31e-03)
		5000	0.0000e+00(0.00e+00) [+]	1.1530e-02(1.08e-03) [≈]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	1.1492e-02(1.81e-03)
LSMOP7	3	500	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(2.01e-04) [≈]	8.2351e-02(3.85e-02) [-]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00)
		1000	0.0000e+00(0.00e+00) [≈]	0.0000e+00(0.00e+00) [≈]	5.4368e-03(5.59e-03) [-]	0.0000e+00(1.60e-02) [≈]	0.0000e+00(0.00e+00) [≈]	0.0000e+00(9.20e-05)
		2000	0.0000e+00(0.00e+00) [+]	3.1905e-02(4.97e-03) [+]	4.4376e-02(2.63e-03) [-]	9.5588e-04(1.16e-02) [+]	3.2541e-02(4.29e-03) [+]	4.0768e-02(2.05e-03)
		5000	0.0000e+00(0.00e+00) [+]	6.7540e-02(1.60e-03) [+]	6.9928e-02(2.07e-03) [+]	5.1032e-02(2.76e-03) [+]	6.5221e-02(1.12e-03) [+]	7.1989e-02(6.16e-04)
LSMOP8	3	500	4.5321e-02(7.75e-03) [+]	2.2173e-01(6.88e-02) [+]	3.8681e-01(4.78e-02) [≈]	7.0451e-02(3.62e-03) [+]	3.7087e-01(7.90e-03) [+]	3.8933e-01(4.61e-03)
		1000	4.9922e-02(1.55e-02) [+]	1.9713e-01(4.81e-02) [+]	3.9692e-01(4.97e-02) [≈]	8.4551e-02(2.32e-03) [+]	3.7798e-01(2.94e-03) [+]	3.9497e-01(5.45e-03)
		2000	6.1790e-02(1.47e-02) [+]	1.8317e-01(3.82e-02) [+]	4.0212e-01(7.16e-02) [≈]	9.0720e-02(3.69e-03) [+]	3.8524e-01(1.93e-03) [+]	3.9620e-01(3.20e-03)
		5000	6.9313e-02(1.49e-02) [+]	2.0798e-01(4.91e-02) [+]	3.7221e-01(7.24e-02) [+]	9.4760e-02(3.18e-03) [+]	3.9102e-01(1.89e-03) [+]	3.9857e-01(3.29e-03)
LSMOP9	3	500	0.0000e+00(0.00e+00) [+]	9.0909e-02(1.02e-02) [+]	1.4764e-01(2.51e-05) [+]	4.9211e-02(3.96e-03) [+]	0.0000e+00(8.29e-04) [+]	1.9069e-01(2.50e-04)
		1000	0.0000e+00(0.00e+00) [+]	9.0909e-02(2.64e-02) [+]	1.4764e-01(1.56e-05) [+]	4.4194e-02(1.18e-02) [+]	0.0000e+00(0.00e+00) [+]	1.9083e-01(2.91e-04)
		2000	0.0000e+00(0.00e+00) [+]	1.4764e-01(2.38e-02) [+]	1.4766e-01(1.03e-02) [+]	5.5531e-04(7.40e-03) [+]	0.0000e+00(0.00e+00) [+]	1.9114e-01(2.52e-04)
		5000	0.0000e+00(0.00e+00) [+]	1.4712e-01(2.42e-02) [+]	1.4699e-01(5.07e-03) [+]	0.0000e+00(0.00e+00) [+]	0.0000e+00(0.00e+00) [+]	1.9127e-01(2.95e-04)
+/≈/-			34/2/0	25/10/1	27/4/5	29/1/6	25/3/8	-