Strategic Disaggregation in Matching Markets

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Abstract

Decisions agents make before and after matching can be strategically linked through the match. We demonstrate this linkage in a game where universities either require students to commit to majors before matriculating or allow students to pick majors during their studies. The interaction between "matching forces" (competition for higher quality students) and "principal-agent forces" (moral hazard and adverse selection) leads to two equilibria that mirror the admissions systems in the US and England. With monetary transfers, our model provides insights into athletic scholarships. Payment caps that restrict transfers to potential athletes who decide not to play sports can maximize welfare.

Keywords: Matching with contracts, college admissions, athletic scholarships

JEL Classification Codes: C78, D61, D78, I21, I23

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"You're going to be studying the subject to a very high level for several years so make sure you choose a course you're passionate about and will really enjoy!"

-Cambridge University Undergraduate Study office, England[1]

"You are asked to choose a major by the time you achieve junior status... Even after this point you may change your major if your interests shift."

-Stanford University Office of the Registrar, USA[2]

1 Introduction

High school students applying to university face strikingly different systems depending on where they live. In the United States, universities are what we will call *aggregated*, meaning that they allow incoming students to wait until their second or third year before choosing a major. In England (and many other countries), universities are *disaggregated*, which we use to mean that they force students to commit to a major when applying and have barriers restricting switching to another program after admission. Moreover, English students specialize in their last years of high school, giving them more information on their proposed course of study than American students.[3] How is it that these two arrangements evolved and why do these differences by country continue to exist? What factors lead to shifts in a country from one system to another?

An initial hypothesis is that these two systems – the "English" system of students narrowing their focus in high school before applying to disaggregated universities and the "US" system of students keeping their studies broad in high school before applying to aggregated universities – exist due to different institutions or are outcomes of a coordination game between students and universities. However, these hypotheses do not easily explain why some universities under both systems fail to adhere to the aggregation decisions of the majority of their peer institutions. In the US, certain universities partially disaggregate by disallowing major transfers to business colleges or to certain majors;[4] some engineering, music, and art schools (e.g., Caltech, Juilliard, the Rhode Island School of Design) effectively disaggregate

[1]http://www.study.cam.ac.uk/undergraduate/apply/ 11/17/2014.

[2]http://studentaffairs.stanford.edu/registrar/students/declaring-major 11/17/2014.

[3]We review the evidence to support these claims in Appendix B.1.

[4]E.g., see https://www.bme.jhu.edu/undergraduate/apply/ for a discussion of admissions to Johns Hopkins' biomedical engineering program.

by offering a limited range of programs for study. In some predominantly disaggregated countries, there are recent examples of universities switching to more American-style admissions systems, such as the University of Hong Kong, University of Melbourne,[5] University College London, University of Exeter, University of Birmingham, University of Kent, and King's College London, sometimes explicitly comparing themselves to an "American-style" liberal arts degree. Coordination and institution-based hypotheses do not explain why English universities that have recently switched to aggregation have seen a 25% increase in applications (Guttenplan 2013).[6] A useful model must explain these trends and give predictions on how such deviations change the overall equilibrium.

We propose a *university/major admissions game* played between students and universities and show that the US and English systems are equilibrium outcomes of our model. Before applying, students select whether to pay a cost to learn about their preferences over majors and universities decide whether to aggregate or disaggregate. Our base model simplifies away from many important aspects of university admissions to focus on one potentially important driver of the current systems. We consider many realistic extensions of our model in the appendix.[7] Moreover, our model provides a framework for thinking about this question and layering in additional complexities that we do not consider to understand how various changes in the educational landscape could affect admissions.

Our analysis is built on two important features. First, students and universities sometimes disagree about which major the student should study. For example, a Math Olympiad star may discover a passion for literature to the dismay of her university. That there are differences between a student's *preference* and her *skill*–what a university prefers her to study[8]–appears true by revealed preference as English universities would be better off allowing students to switch majors if this were not the case. The existence of major choice disagreement is also supported by more direct evidence. Wiswall and Zafar (2015) find that

[5]Both the University of Hong Kong and University of Melbourne now allow for more general study followed by more specialized study. For a critical take of the change by the University of Melbourne, see e.g. `https://theconversation.com/undergraduate-education-and-the-melbourne-model-993`.

[6]Such models would also predict no change should a country deregulate its higher education system, whereas ours suggests that deregulation could lead to substantial changes.

[7]The extensions we consider include differences in the overall popularity of universities and a benefit to attending a disaggregated school, which are usually three year programs in England and four years in the US.

[8]While we think of a student's skill major as pertaining to her ability, there are many reasons why universities may want a student to study a particular major. Professors may prefer interacting with more proficient students, the presence of more able students may increase the prestige of a university, universities that rely on alumni donations might want students to pursue more lucrative careers, or universities that rely on governmental funding may feel pressure to demonstrate the impact their students have on society. Our model implies that changes in universities' funding models could have important equilibrium effects through effects on university preferences.

student tastes largely determine major choices above and beyond academic success and future earnings, two measures that are presumably important to universities, especially those that depend on alumni donations. Haggag et al. (2020) find that some American students suffer from attribution bias and are significantly less likely to major in a subject if their first class in that subject is scheduled at an inconvenient time. Our analysis shows that even a small probability of disagreement can have a large impact on market conditions; in two of our results, there is a discontinuity in the probability of disagreement at zero, with any strictly positive probability of disagreement leading to the non-existence of the US system as an equilibrium.

The second important feature is how agents' payoffs depend on their partner(s) and the major studied (the terms of the match). Each student receives a higher payoff from studying her preferred major and each university receives a higher payoff when its students study according to their skills. Agents on both sides of the market also have preferences over partners beyond the terms of the match (e.g., a student prefers one university over another if she can study the same major at each), so there is a trade-off between having a better partner and having matches with better terms.

Students who are unsure of which major they want to study when they apply value the guarantee that they will be able to study their favorite major at aggregated universities, putting disaggregated universities at a competitive disadvantage. Aggregated universities face a moral hazard cost of some enrolled students electing not to study how the university wishes. The US equilibrium exists when the loss in competitiveness outweighs concerns over which majors admitted students study.

In the English equilibrium, a university that deviates suffers from adverse selection without any upside; when students know which major they want to study, providing them with flexibility in their major choice is only useful to students who cannot gain admission under their preferred major. Therefore, universities do not want to deviate. If required to choose a major when applying, students will pay the cost to better learn their preferences in high school to improve their application strategy if that cost is not too high.

To help explain why these two equilibria are prevalent around the world, we give conditions under which they are the only pure strategy equilibria.[9] Generically, the only other pure strategy equilibria that can exist are ones in which students do not acquire information about major preferences before matching and universities make (possibly) asymmetric aggregation decisions. With sufficient competition between students for seats at universities

[9]That the aggregation policies of universities do not shift back and forth from year to year and are largely homogeneous within a country (neither of which we would expect if universities were choosing their aggregation policies according to mixed strategies) justifies restricting our analysis to pure strategy equilibria.

and sufficient competition between universities for top students, such equilibria do not exist.

Our main result concerns which types of markets are conducive to one equilibrium over the other as this helps both to understand current patterns and to project how changes in fundamentals might cause structural shifts. The ability to sort into two equilibria is closely related to competition in the market, which depends on the number of seats, the number of students, and the number of universities. Theorem 1 finds that increasing the number of universities makes the US equilibrium easier to support while increasing the number of seats available (relative to the number of students) makes the English equilibrium easier to support.

As discussed, not all agents necessarily abide by equilibrium strategies. We analyze the robustness of equilibria when students select the "incorrect" decision regarding resolving their preference uncertainty with small probability and when universities make incorrect aggregation decisions with small probability. These deviations can represent, for example, the presence of foreign students and specialized universities offering only one program of study. We find that the US system is robust to small deviations on both sides of the market and is not destablized by the presence of a few specialist schools or students who know their preferences. The English system collapses when the probability of a university deviating is sufficiently large relative to the probability of a student deviating, even when both probabilities are small in absolute terms. This result suggests that markets may move away from the English system over time, which may explain recent trends in predominantly disaggregated countries.[10]

The second half of the paper uses our model to study how to optimally design markets with monetary transfers. This provides a structure to discuss the impacts of limits on tuition, merit-based scholarships, and athletic scholarships on university admissions.[11] Under aggregation, students receive different payments depending on the terms of the contract, creating non-zero costs for "studying" against their skill.

When there are many universities, we show that competition removes the entire surplus from enrolling desirable students compared to the marginal student who is not admitted

[10]It also calls into question a recent policy decision by the British government to limit the number of non-EU students studying at English universities (Fazackerley 2016). Currently, over a quarter of foreign students in the UK come from China (see `http://www.thecompleteuniversityguide.co.uk/international/international-students-the-facts/where-they-come-from/,7/26/2015`). Because of China's general-studies high school education system, students often "have no idea of what their passions are" (Chen 2015). As our Proposition 6 shows, reducing the number of uninformed students without also decreasing the number of aggregated universities in the English system may have the possibly counter-intuitive effect of destabilizing the equilibrium.

[11]For a recent discussion of the history of college athletics in the United States and arguments for why student athletes should be paid competitive wages, please see Sanderson and Siegfried (2015).

anywhere. This price competition eliminates the US equilibrium; a university would prefer to stop competing for the top students, who are no longer preferable to other students due to the expense of retaining them, and deviate to disaggregating, admitting lower quality students and giving these students no scholarships.

In line with these results, many top universities in the United States collude against offering merit based scholarships.[12] Our result shows that a mechanism by which payments to students are constrained may be necessary to maintain the US equilibrium. In US university athletics, the NCAA caps the compensation universities can provide to student athletes at the cost of tuition, room and board, fees, and a small stipend. Increasing the cap but enforcing it as a "sports only" scholarship can lead to a Pareto improvement; the US equilibrium with an appropriately selected transfer cap maximizes aggregate welfare.

Related literature

This paper relates to several strands of the literature. First, our work stems from the centralized matching literature started by Gale and Shapley (1962) and extended to include contracts by Kelso and Crawford (1982), Fleiner (2003), and Hatfield and Milgrom (2005). In this framework, Pakzad-Hurson (2021) finds that a centralized analogue of university aggregation is necessary for the existence of a student efficient matching, which is corroborated in Proposition 3 of the current paper. Yenmez (2018) studies a centralized clearinghouse for college admissions, where contracts can specify majors or monetary transfers. Our paper makes use of both interpretations; student-university matches must specify the intended major of the student and in Section 4, they must also specify monetary transfers.

The second strand of the literature is the recent theoretical work on decentralized matching, which is methodologically most similar to our paper. Lee (2009) and Avery and Levin (2010) study early admissions. Chade et al. (2014) study a setting with costly applications to universities. Che and Koh (2016) discuss admissions strategies of universities when there is aggregate uncertainty regarding the popularity of universities and there is a high but non-prohibitive cost to exceeding capacity. We study similar uncertainty in regards to the popularity of different majors in Appendix C. Our paper is the first, to our knowledge, to study the equilibrium decisions of universities and students when there are multiple majors. We also consider monetary transfers from university to students.

[12]Ivy league universities faced charges from the US Justice Department in 1991 for colluding on financial aid offers (http://www.nytimes.com/1991/05/23/us/ivy-universities-deny-price-fixing-but-agree-to-avoid-it-in-the-future.html, 4/13/15). However, a 1993 ruling by the Third Circuit Court of Appeals allowed the practice of allowing universities to collude on student aid offers for "procompetitive and social welfare justifications" (http://law.justia.com/cases/federal/appellate-courts/F3/5/658/626013/, 4/13/2015).

Third, our paper contributes to the literature on decentralized markets with a "pre-match" phase. Many papers, including Acemoglu (1996), Cole et al. (2001a), Cole et al. (2001b), Peters and Siow (2002), Mailath et al. (2013) and Nöldeke and Samuelson (2015) study strategic investments agents make to improve their quality prior to matching. Rogerson (1992), Bergemann and Välimäki (2003) and Hatfield et al. (2019) study costly information acquisition by agents before arriving to market. Our paper combines the considerations of these papers by allowing heterogeneous agents to either acquire costly information (students can learn their preferences over majors) or improve their own quality (universities can aggregate to become more attractive to students) before matching. Furthermore, we explicitly model what happens after the match is made, which drives pre-match decisions in our setting.

While we believe ours to be the first primarily theoretical paper on the topic of disaggregation in matching markets, there are a number of empirical papers that study major choice in university admissions. Bordon and Fu (2015) estimate a structural model of the welfare impact of moving from a disaggregated system to an aggregated one using Chilean data. Also looking at the Chilean system, Larroucau and Rios (2020) document that 15% of Chilean students reapply even after being given their first choice placement, suggesting that students are initially uncertain about their preferences.[13] Altonji et al. (2012) investigate returns to higher education dependent upon which major a student pursues. Several of our modeling choices are similar to theirs (such as distinguishing between a student's uncertain preferences and her abilities). Other empirical papers focusing on major choice are discussed in Appendix B.1.

The remainder of the paper is organized as follows. In Section 2, we set up the model. Section 3 discusses equilibria without monetary transfers. Section 4 introduces monetary transfers to the model and discusses market interventions. We conclude in Section 5. Proofs are found in Appendix A. The appendix presents stylized facts of the two admissions systems we study (Appendix B.1), contains a brief discussion of the historical development of the two systems suggestive of the plausibility of our proposed explanation (Appendix B.2), and introduces additional results, robustness checks and extensions of our model (Appendix C).

[13] This interpretation of uncertain preferences is based on surveys of students. The instabilities in Chile's centralized application system they document also raise the question of what would happen if applications were decentralized, a question to which our model provides an answer.

2 Model

We introduce a simple game of *university/major admissions*. The base game abstracts from many important features in university admissions to illustrate the strategic implications of aggregation. In Appendix C, we layer in additional complexities of the admissions process and show that the principle forces in our game are present in more complicated and realistic models.

2.1 Setting

There is a unit mass of students S. There is a set of universities $N = \{u_1, \ldots, u_n\}$ with $n \geq 2$, and each $u \in N$ comprises two colleges, M_u and L_u. Each college offers a single major, either M ("Math") or L ("Literature").

Each student $s \in S$ can either study a single major at a single college or not attend university. A contract is a triplet specifying a student, the university she attends, and the major she studies (which defines the college she attends within the university). The set of contracts is $X = S \times N \times \{M, L\}$. For a given contract $x \in X$, x_s is the associated student, x_u is the associated university, and x_m is the associated major. The empty contract $\varnothing$ denotes that a student is unassigned to any college.

Each $s \in S$ has a *type* $(v_s, \theta_s, \rho_s, w_s) \in [0, 1] \times \{M, L\} \times \{M, L\} \times [0, 1]^n$. Student s's *quality* is denoted by $v_s \in [0, 1]$. Each student s's *university preferred major* is denoted by $\theta_s \in \{M, L\}$. We often refer to the university preferred major as the student's *skill type* with the impression that universities often want their students to study what they are good at; however, one could imagine a variety of factors influencing which major a university wants a given student to study and θ_s represents the combination of those factors. Each student s also has a *major preference* $\rho_s \in \{M, L\}$. w_s is an n dimensional vector representing s's cardinal preferences over universities, with $w_s(u) \in [0, 1]$ for each $u \in N$.

The distribution of student types is defined as follows. For any $\tau \in [0, 1]^{n+1}$, let $S(\tau) := \{s | v_s \leq \tau_1, w_s(u_1) \leq \tau_2, ..., w_s(u_n) \leq \tau_{n+1}\}$. Let $S^{i,j} := \{s | \theta_s = i \text{ and } \rho_s = j\}$ for $i \in \{M, L\}$. For any $\tau \in [0, 1]^{n+1}$,

$$|S(\tau) \cap S^{M,M}| = |S(\tau) \cap S^{L,L}| = \frac{\alpha}{2} \prod_{k=1}^{n+1} \tau_k$$

and

$$|S(\tau) \cap S^{M,L}| = |S(\tau) \cap S^{L,M}| = \frac{(1-\alpha)}{2} \prod_{k=1}^{n+1} \tau_k$$

where $\alpha \in (\frac{1}{2}, 1)$ is a given constant.

These marginals represent several features of the distribution of types. First, half of all students have a skill type major of M and the other half of the students have a skill type major of L. With probability $\alpha \in (\frac{1}{2}, 1)$, $\theta_s = \rho_s$; often, but not always, students and universities agree on which major the student should study. We call a student s whose own major preference matches that of universities (i.e. $\rho_s = \theta_s$) *consistent* and a student whose own major preference does not match that of the universities *inconsistent.* Independently of all other components, each student's quality and cardinal value for each university are drawn from $[0, 1]$. Our qualitative results apply to general distributions with full support on [0,1], but to give closed-form conditions, we take each distribution to be uniform.

Payoffs for students, universities, and colleges depend on enrollment and on student types. For a given contract $x \in X$, the utility of student x_s is $U^{x_s}(x) = w_{x_s}(x_u) + b \cdot \mathbb{1}_{\{x_m = \rho_{x_s}\}}$ where $b \leq \frac{1}{1-\alpha}$ is the added benefit a student receives if she studies her preferred major.[14] We normalize the utility a student receives from not attending university to 0. Note that the skill type major θ_s of a student s does not directly enter her objective function. Additionally, each student s pays a cost $c > 0$ if and only if she resolves her uncertainty over her preferred major ρ_s prior to matching, as we discuss further in the following section.

A college's utility is derived from the quality and skill type of each student it enrolls. Each college M_u can enroll students only to study program M. For each contract $x \in X$ with $x_m = M$, college M_{x_u} derives utility $v_{x_s} dx_s$ if $\theta_{x_s} = M$ (i.e. M is the student's skill type) and $\delta(v_{x_s})d_s$ if $\theta_{x_s} = L$, where $0 \leq \delta(v_{x_s}) \leq v_{x_s}$ (with the inequality strict for $v_{x_s} \in (0, 1)$) is an absolutely continuous and strictly increasing discount function and dx_s is an infinitesimal. Analogously, each college L_u can enroll students only to study program L. For each contract $x \in X$ with $x_m = L$, college L_{x_u} derives utility $v_{x_s} d_{x_s}$ if $\theta_{x_s} = L$ and $\delta(v_{x_s})d_{x_s}$ if $\theta_{x_s} = M$. The fact that $\delta(v_{x_s}) < v_{x_s}$ almost everywhere means that colleges (almost always) derive strictly higher utility from a student x_s studying her skill-type major θ_{x_s}.[15]

Each college has a quota of $\frac{q}{2n}$ seats, with $q < 1$. We assume a prohibitively high cost of exceeding the quota. Define $X_{i_u} \subset X$ as a *feasible* set for college i_u for $i \in \{M, L\}$ and $u \in N$ if all contracts $x \in X_{i_u}$ satisfy

[14] The assumption that $b \leq \frac{1}{1-\alpha}$ is not critical. As can be seen in Section 3, if $b > \frac{1}{1-\alpha}$ then several statements would have to be readjusted to avoid claiming events occur with probability greater than 1 or less than 0.

[15] The difference between $\delta(v_{x_s})$ and v_{x_s} can be arbitrarily small, meaning our model encompasses cases in which colleges are nearly indifferent between the skill types. Alternatively, one could model students as having an M specific quality and an L specific quality. θ_s could then be viewed as an indicator for which quality is higher. To align such a model with ours, one would additionally need to assume a specific relationship between a student's M and L qualities, with the student's quality type associated with their skill type distributed as v_s and the relation between their two quality types given by $v_s^{\{M,L\}\setminus\rho_s} = \delta(v_s^{\rho_s})$.

- $x_u = u$,
- $x_m = i$,
- for all $x, y \in X_{i_u}$, $x_s \neq y_s$, and
- the set $X^s_{i_u} \equiv \{s | \exists x_s \in X_{i_u}\}$ is measurable with respect to the Lebesgue measure λ and the measure of $X^s_{i_u}$ is weakly smaller than $\frac{q}{2n}$

The first two conditions require that the relevant college is named in all contracts in x_{i_u}. The third condition requires that no student is assigned more than once to the college. The final condition requires that the college does not exceed its capacity.

For any feasible set of contracts X_{i_u}, college i_u gets total utility

$$\frac{2n}{q} \int_{s \in X^s_{i_u}} \left[\mathbb{1}_{\{i=\theta_s\}} v_s + \mathbb{1}_{\{i \neq \theta_s\}} \delta(v_s) \right] d\lambda$$

where we normalize by the size of the college to facilitate comparisons when changing n.

We define a university's utility as the average of the utilities of its constituent colleges. For any two feasible sets X_{M_u} and X_{L_u} such that for all $x \in X_{M_u}$ and all $y \in X_{L_u}$, $x_s \neq y_s$, the university $u \in U$ receives utility

$$\frac{n}{q} \sum_{i \in \{M,L\}} \int_{s \in X^s_{i_u}} \left[\mathbb{1}_{\{i=\theta_s\}} v_s + \mathbb{1}_{\{i \neq \theta_s\}} \delta(v_s) \right] d\lambda$$

All agents (students, colleges, and universities) maximize expected utility.

2.2 Timing and equilibrium selection

The game has five stages: Nature selects types, universities make aggregation decisions, universities and colleges decide their admissions criteria, students select whether to learn their preferences, and the final matching of students to a college is made.

First, Nature selects all student types according to the aforementioned distribution.

Second, each university u simultaneously selects whether to *aggregate* or *disaggregate* without observing Nature's selections. This decision impacts the next step, admissions. Each university u's first decision is represented by $d_{u_i} \in \{agg, disagg\}$ and the decisions of all universities are given by the vector $d \in \{agg, disagg\}^n$.

The third step is the admissions step. Let $x_i(s, u)$ be the contract with $x_s = s$, $x_u = u$, and $x_m = i$, $i \in \{M, L\}$. If university u is aggregated, u observes the profile of aggregation decisions d and student quality v_s for all s, and can either offer a student s no contract or a contract for both programs but cannot offer the student the contract for just one of the programs. Each university u's admission decision is therefore choosing a function $a^u(d, v_s) \in \{\emptyset, \{x_M(s,u), x_L(s,u)\}\}$, which maps (d, v_s) to either no contract involving s and u or both contracts involving s and u. For each disaggregated university u, for $i \in \{M, L\}$, college i_u observes the d and for each s the student's quality v_s and skill type θ_s. i_u's admission decision is choosing a function $a_i^u(d, v_s, \theta_s) \in \{\emptyset, \{x_i(s,u)\}\}$, which maps (d, v_s, θ_s) to either no contract or the contract involving major i. Throughout, we often refer to "colleges" and "universities" as making admissions decisions rather than "aggregated universities and individual colleges at disaggregated universities" for exposition compactness, as the admitting body is determined entirely by the aggregation decision of the university.

Fourth, each student s, without observing any types (including her own) or the decisions of the universities and colleges, simultaneously selects whether or not to resolve uncertainty about which major is her preferred major (i.e., to learn ρ_s). If she does so, she pays a cost $c > 0$.[16]

Fifth, students enroll in university by selecting at most one contract from the set of all contracts they are offered. The enrollment stage is taken as mechanical and maximizes students' expected utility given knowledge of their skill type and of their preference type if they resolved their uncertainty in the fourth stage. We assume students learn their preferences after enrollment at no cost,[17] so for a student s admitted to contract x where x_u is an aggregated university (and so she is accepted to the contracts for both majors at u), enrolling at u gives the student expected utility $w_s(x_u) + b$ as she can study her preference type major with certainty. For a contract x that s is offered where x_u is disaggregated, a student's expected utility is $w_s(x_u) + \mathbb{1}_{\theta_s = x_m}\alpha b + \mathbb{1}_{\theta_s \neq x_m}(1-\alpha)b$ if she has not learned her preferences and $w_s(x_u) + \mathbb{1}_{\rho_s = x_m} b$ if she has learned her preferences; students who have not resolved their preference uncertainty prior to enrolling know only their skill type major, which is more likely than not to be their preference major, while students who have resolved know both.

We examine pure-strategy Perfect Bayesian equilibria of this game. To deal with pessimistic beliefs on the part of universities discussed below, we make the assumption that the beliefs of universities and colleges over the types of students are the same at all information sets at the admissions stage.

[16] In Appendix C we study an alternative model of preferences over majors in which a student can pay a cost to specialize in one major and realize how much she likes that particular major but learns nothing about the other.

[17] This can be relaxed to students viewing the cost to learn their preferences in the fourth stage as greater than in this stage.

Discussion of modeling choices

We make a few notes on our modeling choices, in particular the stages we choose to make strategic and how we have modelled the admissions stage. These choices are driven by our modeling of students as a continuum.[18] The continuum presents two difficulties: first, sequential equilibrium (which would eliminate equilibria supported by overly pessimistic off-path beliefs) is not defined in this setting and second, measure zero sets of students do not affect a university's or college's utility.

The former issue is a problem as PBE allows for equilibria supported by unrealistic off-path beliefs. For example, various aggregation choices could be supported in equilibrium by each university u believing that all students $s \in S$ have $w_s(u) = 0$ if (and only if) u does not follow the equilibrium prescribed aggregation action. Our restriction that colleges' and universities' aggregation decisions do not affect their beliefs over the distribution of student types achieves the spirit of sequential equilibrium at the admissions stage. Making students' enrollment and major selection choices mechanical serves a similar purpose.[19]

The latter issue of measure zero sets presents two difficulties. First, it leads to a multiplicity of equilibria as we can change the strategies by or concerning any measure zero set of students in one equilibrium to arrive at another. As these "different" equilibria are artifacts of our continuum assumption, we ignore indeterminancies of this sort and view two outcomes of the game as identical if the only difference between which contracts are chosen is over a measure zero set of students. Second, it necessitates our matching procedure wherein aggregated universities and disaggregated colleges can admit any student (i.e., that there is no application stage). In an expanded game in which students select where to apply and universities and colleges admit subsets of applying students, we can construct equilibria wherein students apply nowhere and universities admit no one; because no individual student affects the overall utility of any college or university, the "threat" of admitting no student is credible. Our current game form is similar to this expanded game while restricting attention to equilibria in which students apply to all contracts, so the admissions decision is not restricted by students' application choices.[20] Overall, we believe an interesting takeaway of our

[18]Itself a choice made for analytical tractability in calculating expected utilities.

[19]If students actively chose their contract in the fifth stage of the game, we could support unrealistic resolution decisions in the fourth step by off-path beliefs that any deviating student is inconsistent with probability 1. Alternatively, we could model students as actively choosing their contract in the fifth stage of the game and also add an assumption that students' beliefs about their own types do not vary based on their resolution decision, similar to our assumption on universities' and colleges' beliefs.

[20]If students can apply to at most one college per university, then other than the proliferation of such nuisance equilibria, the analysis of our paper is identical if students learn v immediately before the application stage, as students know on path which colleges/universities will admit them.

paper is that the prematch decisions are of first-order concern in higher-education markets; we show that in addition to the various variants of the decentralized matching framework we describe here that lead to the same equilibrium outcomes, so too does replacing the matching stage with a “well-behaved” centralized matching mechanism (see Online Appendix C.5 for details).

Our decision to model admissions decisions at disaggregated universities as being made by individual colleges instead of by the university itself deserves mention. If a university were able to observe students’ skill type and quality and select contracts with no restriction, it could (possibly) improve its utility by admitting high-quality students only to study their skill type. This would weaken the incentive of students to resolve their preference uncertainty in the English equilibrium that we discuss below, suggesting that this assumption is substantive. However, we feel this assumption is justified for three reasons. First, real-world disaggregated universities typically allow departments to handle their own admissions whereas university-wide admissions offices exist at aggregated universities. Second, a richer model with an explicit application stage and individual students contributing a non-infinitesimal utility to a university would lead to high-quality students applying to only one major per university to avoid such coordination and any threats of rejecting students for applying to the “wrong” major would not be credible.[21] Third, we could obtain the same results with different modeling choices, such as by removing the university as a player and instead framing aggregation as a decision of the individual colleges at a university to commit to admitting the same students.

The observability and timing of information also affects our results. We make the assumption that the pre-match decisions effectively occur simultaneously. We believe this is reasonable as universities do not observe the resolution decisions of individual students and students must decide to learn their preference types by “specializing” their high school studies, which takes place over the course of several years in advance of applications. An excellent example is Victorian College of the Arts (VCA). VCA announced roughly 8 months before the admissions date that it would aggregate, and the VCA administration expressly denied the intent to aggregate one month before the announcement was made.[22] With minor modifications, our analysis is unchanged if universities were to observe student resolution choices

[21] We would also need for students to improve their knowledge of their quality between the preference resolution stage and the application stage, at least following a decision to learn one’s preferences. This dependence of the English equilibrium hints at a certain fragility of the equilibrium, an idea which is expanded on by Proposition 6.

[22] See https://web.archive.org/web/20080720032744/http://www.vcasu.org.au/2008/04/24/arts-college-to-follow-us-model-2/ and https://web.archive.org/web/20080720033033/http://www.vcasu.org.au/2008/04/29/vice-chancellor-lies-about-introduction-of-melbourne-model-at-vca/

prior to making admission decisions. If students observe aggregation decisions prior to making resolution decisions, our results would change as universities would have a "first-mover" advantage in selecting a disaggregation equilibrium, but our analysis is approximately unchanged when there are many universities.[23] If students know their qualities precisely at the beginning of the game, low-quality students would never resolve their uncertainty over majors in any equilibrium, as they are not admitted to any universities, destabilizing the English equilibrium discussed below. However, our results are not knife-edge and hold with modifications to the exact conditions for partially informative signals of quality. This dependence of the English-style equilibrium on students not knowing precisely their quality can be understood as a prediction of how markets might change if students learn their qualities earlier (e.g., as a result of more readily available comparisons from online sources).

We show that in any equilibrium, following any observed aggregation history, all M colleges at disaggregated universities admit the same set of students as one another, all L colleges at disaggregated universities admit the same set of students as one another, and all aggregated universities admit the same set of students as one another. Additionally, we show that the set of students admitted to disaggregated M and L colleges have the same distribution of qualities, i.e. admissions standards are symmetric across colleges at disaggregated universities. This symmetry need not hold following certain histories if we allow aggregated universities to observe student skill types and also allow the admissions functions of aggregated universities to depend on skill type (i.e., if we instead allow universities to pick a mapping $a^u(d, v_s, \theta_s)$). We do not make this assumption in our base model for two reasons. First, as mentioned, admissions at aggregated universities are generally handled by centralized offices which may not have the specialized knowledge needed to discern a student's particular skills. Second, the histories leading to the asymmetric admissions strategies do not occur on path or one step off path in our two focal equilibria. Therefore, the analysis in the more general model is more cumbersome and this assumption does not affect the main intuitions of the paper.[24]

The symmetry in admissions standards also need not hold if we alter the distribution of student types. Our base model assumes that the relative demand for both M and L colleges is

[23]To see this, consider a market with $n = 2$ universities, u_1 and u_2, with only u_1 disaggregating and students observing this prior to making their preference resolution decision. If a student ends up preferring u_1 to u_2 ($w_s(u_1) > w_s(u_2)$), they must decide whether to attend the university they prefer but which restricts their major choice or the university they prefer less but which gives them flexibility, making knowledge of ones preferences beneficial. However, as n increases, a student can achieve expected utility arbitrarily close to $1 + b$ (the maximum possible student utility) by attending their favorite of the $n - 1$ aggregated universities, meaning that the benefit of learning one's preferences is eventually less than any fixed cost c.

[24]Such a change would necessitate an additional condition in the statement of Proposition 4.

equal to the relative capacity of seats. We make this assumption to highlight what we believe are the main strategic features in our environment. Nevertheless, important considerations may be swept under the rug with these assumptions. For example, if the relative popularity of different majors does not match the capacities of different colleges, aggregation may lead to undesirable program unbalance. We study this and other extensions that relax our symmetry assumptions in the online appendix and show that our main intuitions transfer to those settings.

2.3 Preliminaries

In any equilibrium, the set of students with qualities weakly greater than $1-q$ matriculate. Aggregated universities must consider the possibility students will study against their skill types after enrolling. An aggregated university that admits a student who has learned her major preferences and who has been admitted to a college at a disaggregated university must worry that if it "wins" the student in question, the fact that she has chosen the aggregated university implies that she is more likely to study against her skill type. Therefore, the beliefs of universities of the probability that a given student enrolls and which major she will study conditional on enrollment depend on the admissions and aggregation decisions of other universities. However, in all histories such that either no student learns her major preference or all universities make the same aggregation decision, each college sets a threshold and admits all students whose *application quality* exceeds this threshold, where the application quality of student s is defined as v_s for the college of the student's skill type and for an aggregated university and $\delta(v_s)$ for the college not of the student's skill type.[25]

LEMMA **1.**

- *In any equilibrium and following any history, a student s is admitted to one or more universities if and only if $v_s \geq 1-q$.*
- *In any equilibrium and following any history and at any two disaggregated universities u and u', colleges $M_u, L_u, M_{u'}$, and $L_{u'}$ all select the same threshold and admit all students whose application qualities exceed it.*
- *In any equilibrium following any history such that either 1) no student learns her major preferences before matching or 2) all universities make the same aggregation decision, any two aggregated universities u and u' also select the same threshold and admit all students whose application qualities exceed it.*

[25]We discuss in the appendix how colleges use a different type of "threshold" that takes into account adverse selection following other histories of the game.

3 Two equilibria

Throughout this section, we differentiate between an equilibrium of the game and what we call a *university admissions system*. A system is defined as the prescribed actions for the "pre-match" portion of the admissions game: the aggregation decisions of the universities and the major preference uncertainty resolution decisions of the students. We say that a system is supported by an equilibrium if there is an equilibrium with the specified pre-match actions for all players. This distinction is most important in Section 3.4.3, when we examine the resilience of the admissions systems used in the US and England (and elsewhere) to shocks.

3.1 US equilibrium

We define an equilibrium of the university admissions game in which all colleges aggregate and no student resolves her major preferences before matriculating as implementing the *US system*. We call such an equilibrium a *US equilibrium*. In this section, we show that the unique equilibrium outcome implementing the US system involves all universities aggregating and no student learning her major preferences before matriculating at a university, students with $v_s \geq 1-q$ receiving their favorite contract, and all other students going unmatched. We derive conditions under which this outcome is an equilibrium of the university admissions game by specifying actions one-step off equilibrium path; see Lemma 2 in the appendix for the general description of the admissions decisions at all other information sets in equilibrium.

We first calculate the utility of each university on path. As given by Lemma 1, each university admits all students with $v \geq 1-q$ and receives $\frac{1}{n}$ of them. A $1-\alpha$ proportion of enrolling students are inconsistent and elect to study against their skill types. Noting the symmetry in the likelihood of each student having preference type M or L and using the definitions of college and university utility given above, the expected utility of each university is

$$\frac{1}{q}\int_{1-q}^{1} [\alpha v + (1-\alpha)\delta(v)]\, dv \tag{1}$$

Suppose that u_1 disaggregates. u_1 is now less desirable since students who enroll are no longer guaranteed to study their favorite majors. The students that do enroll do so under their skill types because $\alpha > \frac{1}{2}$. Letting $w_s(u_1)$ and $w_s(u_{max})$ be the utilities that student s has for u_1 and for the university other than u_1 at which she has the highest utility, she will most prefer u_1 if and only if $w_s(u_1) > w_s(u_{max}) + (1-\alpha)b$.

From Lemma 2, we know that both colleges at the deviating university set the same admission threshold, $t_{u_1}(n)$. Because u_1 is now the "least popular" university, it must be the case that $t_{u_1}(n)$ is below the threshold used by all other universities, $t_{-u_1}(n)$. Because there are a total of q seats, $t_{u_1}(n) = 1 - q$. Therefore, u_1 will enroll the entire mass of students with quality $v_s \in [t_{u_1}(n), t_{-u_1}(n))$.

Conditional on being admitted to all universities, a student will not enroll in u_1 with probability

$$1 - Pr\left(w_s(u_1) > w_s(u_{max}) + (1-\alpha)b\right) = 1 - \left[\left(1 - (1-\alpha)\,b\right)^n \left(\frac{1}{n}\right)\right]$$

where we use the fact that $w_s(u_1)$ is uniformly distributed to simplify the formula. The admission threshold $t_{-u_1}(n)$ for all other universities must make the university's seat quota, meaning the following equation must hold.

$$(1 - t_{-u_1}) \cdot \frac{1}{n-1}\left(1 - \left[\left(1 - (1-\alpha)\,b\right)^n \left(\frac{1}{n}\right)\right]\right) = \frac{q}{n}$$

Solving,

$$t_{-u_1}(n) = 1 - \frac{q(n-1)}{n\left(1 - \left[\left(1-(1-\alpha)\,b\right)^n \left(\frac{1}{n}\right)\right]\right)} \tag{2}$$

Therefore, u_1's utility is

$$\left(1 - (1-\alpha)\,b\right)^n \left(\frac{1}{q}\right) \int_{t_{-u_1}}^{1} v dv + \frac{n}{q} \int_{1-q}^{t_{-u_1}} v\, dv \tag{3}$$

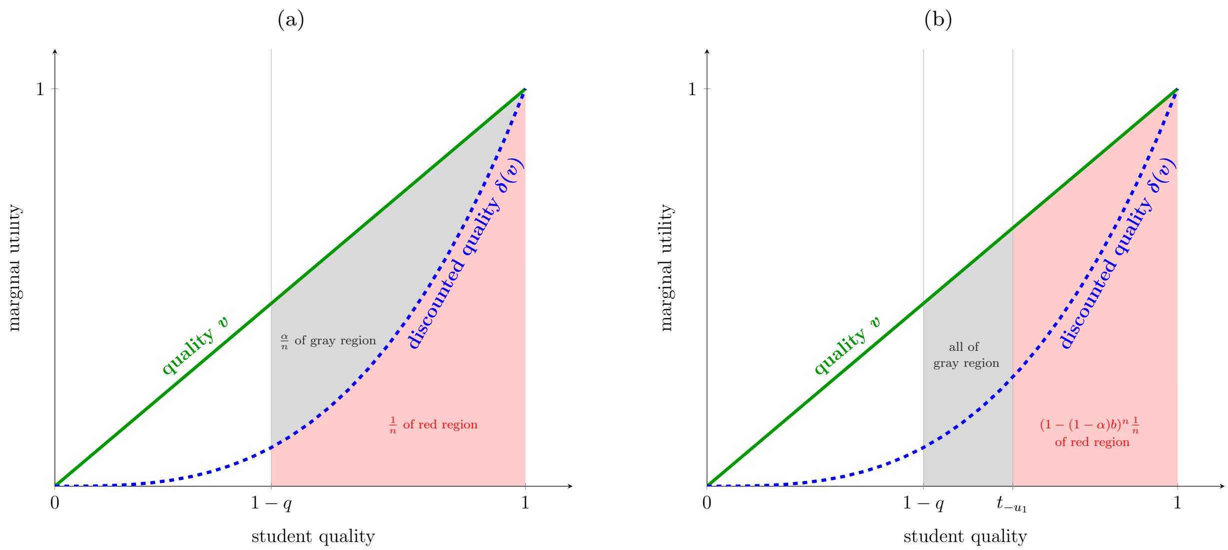
No university wants to deviate from the proposed equilibrium if and only if the (weighted) shaded area in Figure 1 (a) is larger than the (weighted) shaded area in Figure 1 (b); that is, universities do not wish to deviate if competition for better matches outweighs moral hazard concerns. Formally, a necessary condition for the US equilibrium is

$$\int_{1-q}^{1} \left[\alpha v + (1-\alpha)\delta(v)\right] dv \geq \left(1 - (1-\alpha)\,b\right)^n \frac{1 - t_{-u_1}^2}{2} + n\frac{t_{-u_1}^2 - (1-q)^2}{2}. \tag{4}$$

It is very easy to see that no student will deviate to paying cost c to learn her major preferences, as there is no incentive to pay the cost to resolve uncertainty.

Proposition **1.** *There exists an equilibrium of the university admissions game in which*

Figure 1: University utility in US equilibrium vs deviation



Notes: Panel (a) represents university utility in the proposed US equilibrium outcome. Each university admits all students with qualities $\geq 1-q$ and enrolls $\frac{1}{n}$ of them. α proportion of students study their skill type majors. Each university receives utility equal to $\frac{1}{n}$ of the red region and $\frac{\alpha}{n}$ of the grey region. Panel (b) represents the utility that a university receives if it is the lone deviator from the US equilibrium. This deviating disaggregating university admits all students with qualities weakly greater than $1-q$. It enrolls all students with qualities below t_{-u_1} (the grey region) and a $(1-(1-\alpha)b)^n \frac{1}{n}$ fraction of students with qualities above t_{-u_1} (the red region), all to their skill type majors.

all universities aggregate and no student resolves her uncertainty if and only if Inequality 4 holds.

3.2 English equilibrium

We define an equilibrium of the university admissions game in which all colleges disaggregate and all students resolve their major preferences before matriculating as implementing the *English system*. We call such an equilibrium a *English equilibrium*. Define $\delta^{-1}(x) := \sup\{v \in [0,1] | \delta(v) < x\}$ and let $\bar{v} := \delta^{-1}(1-q)$. In the unique equilibrium outcome implementing the English system, each student s such that $v_s \geq \bar{v}$ is admitted to all colleges and so receives her favorite contract (enrolls at her favorite university and studies her preferred major ρ_s). Each student s such that $v_s \in [1-q, \bar{v})$ is matched to her favorite university and studies her skill type major θ_s. All other students are unmatched. This section derives conditions under which this equilibrium exists by specifying actions one-step off equilibrium path; see Lemma 2 in the appendix for the general description of the admissions decisions at all other information sets in equilibrium. We first check that none of the students wish to deviate and second that none of the universities wish to deviate.

Consider two groups of students: those with qualities $v_s \in [1-q, \bar{v})$ and those with qualities $v_s \geq \bar{v}$. Those in the former group are only admitted to the colleges corresponding to their skill types and enroll in their favorite universities. These students gain nothing by realizing their major preferences earlier since they cannot use this information. Students with quality greater than $\bar{v}$ are admitted to all colleges, so knowing their major preferences is useful information in case they are inconsistent. Students have to pay the cost to resolve uncertainty *before* learning their quality. A student has a $1-\bar{v}$ probability of being a top student and a $1-\alpha$ probability of being inconsistent. Therefore, every student resolves her uncertainty if and only if

$$(1-\bar{v})(1-\alpha)b \geq c \tag{5}$$

Consider universities. Suppose that u_1 aggregates. u_1 does not attract any more of the top students in the market (those with quality greater than $\bar{v}$), nor any of the consistent students admitted to other universities. The only students who are more likely to attend u_1 are those with qualities $v_s < \bar{v}$ who are inconsistent, meaning u_1 gets utility $\delta(v_s)$ from such a student. Since $\delta(v_s) < \delta(\bar{v}) = 1-q$, each of these students is less valuable to u_1 than *any* student enrolling in u_1 under the proposed equilibrium. There is a severe adverse selection problem; the only students who are more likely to attend the aggregated university are those who are sure to study against their skill types and are therefore less attractive than students the university could have enrolled.

Because universities never want to aggregate, the necessary condition Equation 5 is also sufficient for the existence of the English equilibrium.

Proposition **2.** *There exists an equilibrium of the university admissions game in which all universities disaggregate and all students resolve their uncertainty if and only if*

$$(1-\bar{v})(1-\alpha)b \geq c.$$

3.3 Ranking of equilibria

How do universities and students rank the different equilibria? Both the US and English equilibria can exist under the same parameters.[26] In the US equilibrium, all students with quality $v_s \geq 1-q$ study their favorite major at their favorite university with certainty. In the counterfactual English market with the same types, qualities, preferences over universities and preferences over majors (which the students have paid cost c to attain), this is not the case. While all students are still admitted to their favorite university, now a $1-\alpha$ proportion of students with $v_s \in [1-q, \bar{v})$ are not able to study their favorite major. Furthermore, all students pay a cost of c, which they did not have to in the US equilibrium.

Consider the same scenario from the point of view of the universities. In both outcomes, universities get the same utility from students with $v_s \geq \bar{v}$. However, the English outcome yields strictly higher utility from inconsistent students with $v_s \in [1-q, \bar{v}]$ to every university as these students are forced to study their skill types. Therefore, every university strictly prefers the English equilibrium outcome to the US equilibrium outcome.[27]

Proposition **3.** *The US equilibrium outcome is Pareto preferred by students to any other equilibrium outcome and every university strictly prefers the English equilibrium outcome to the US equilibrium outcome.*

While the observation that students prefer the American outcome to any other equilibrium outcome is straightforward, the proposition does not state that the English outcome is the most preferred equilibrium outcome for all universities. The following example shows this need not be the case. We provide parameters for which both the English equilibrium and

[26]Parameter c does not appear anywhere in the conditions for the existence of the US equilibrium. Therefore, take any set of parameters for which the US equilibrium holds and then pick c arbitrarily small so that the requirement for the English equilibrium is satisfied. We give an example of parameters that support both equilibria in the online appendix.

[27]If universities have concerns different than what we assume, this point might not hold. See Appendix C for variations on the university utility function.

an equilibrium with asymmetric aggregation decisions exist. We show that the aggregated university receives higher utility in the asymmetric equilibrium than (either) disaggregated university does in the English equilibrium.

Example **1.** : *Let there be 2 universities, $b = 1/(1-\alpha)$, $q = 1/2$, $\alpha = 3/4$, $c = 1/5$, and*

$$\delta(v) = \begin{cases} \epsilon v & v \in [0, .75), \\ \frac{.5-.75\epsilon}{\epsilon}(v - .75) + .75\epsilon & v \in [.75, .75+\epsilon], \\ .5 + \epsilon(v - .75 - \epsilon) & v \in [.75+\epsilon, 1). \end{cases}$$

There exists $E > 0$ such that for all $\epsilon \in (0, E]$, there is an equilibrium in which one university aggregates, one university disaggregates, and no student resolves major uncertainty. In this equilibrium, the aggregated university receives higher utility than a university in the English equilibrium with the same parameters.

The proof of this example is given in the appendix. The aggregated university does better in this mixed aggregation equilibrium than in the English equilibrium due to its competitive advantage over the disaggregated university, letting it admit only high quality students. This improvement in the quality of its enrolled students outweighs the "loss" of letting these students choose their major.

3.4 Which outcome do we expect to see?

Which system do we expect to see in different markets? We answer this in three ways. First, we provide conditions under which the US and English systems are the only systems supported by pure-strategy equilibria. Second, we study the effects of parameter changes on the sustainability of these systems. Third, we analyze whether the two systems are stable in markets that feature small mistakes by participants.

3.4.1 Uniqueness of equilibria

There are three broad classes of equilibria to consider: those in which all agents on both sides of the market play symmetric strategies (such as the US and English equilibria), those in which all agents on only one side of the market play symmetric strategies, and those in which agents on neither side of the market play symmetric strategies.

The US and English equilibrium outcomes are often unique. Generically (that is, for an open and dense subset of the parameter space), the only other equilibrium outcomes that can be supported involve no students learning their major preferences and the universities taking asymmetric actions pre-match. Example 1 is one such equilibria; we give a sufficient condition for the non-existence of this type of equilibrium here.[28] The sufficiency condition focuses on the US outcome (as all other cases can be handled without additional conditions to the existence requirements we derive above) and ensures that disaggregating is not attractive regardless of the aggregation decisions of other universities when students have not paid to resolve their uncertainty over their preferences by ensuring that the cost to disaggregating outweighs the benefit. The "cost" to disaggregating depends on the benefit students receive from attending an aggregated university (which is $(1-\alpha)b$) and the benefit to disaggregating is the guarantee that students study their skill type (and so dependent on δ and α). q plays a part as a university deviating to disaggregation from the US equilibrium fills disproportionately with lower quality students.

Proposition 4. *For any set of parameters, there exists a pure-strategy equilibrium of the university admissions game. Generically, the only pure-strategy equilibria where agents on the same side of the market play asymmetric strategies on path are with no student resolving her uncertainty and universities making different aggregation decisions. If* $\delta(1-q) > 1 - \frac{q}{2(1-\alpha)}\left(1 - \frac{\alpha}{1-((1-\alpha)b)^n+(1-\alpha)bn}\right)$ *and* $(1-\bar{v})(1-\alpha)b > c$*, then the US and English equilibrium outcomes are unique.*

As discussed in greater detail below in the context of Theorem 1, the existence of the US equilibrium outcome is closely related to the extent of competition between universities, which depends on n. As shown in an example in Online Appendix C, the US equilibrium need not be unique when n is small even when it is unique for sufficiently large n, suggesting that market size promotes the uniqueness of our focal equilibrium outcomes. The reason for this is that the competitive loss to disaggregating increases with n because the difference between a student's first and second most preferred universities (i.e., the two universities for which $w_s(u)$ are highest) goes to 0 as n grows. As a result, the probability that a disaggregated college is chosen by a student admitted to all universities goes to 0 exponentially in n.

This exponential loss in competitive edge both complicates the analysis of "small" markets as we must keep track of how a deviating university's popularity with the highest quality students changes with the number of deviators but also suggests that the number of universities need not be overly many for a "large" market approximation.[29] This large

[28]The condition we provide is sufficient but not necessary as it compares the worst case for the in-equilibrium utility to the best case for the deviation utility. A weaker (though still not necessary) condition is given in Corollary 1.

[29]Consider the ratio of the probability that a given aggregated university is chosen by a student with

market approximation allows us to focus on the tradeoffs to disaggregation between ensuring students study a certain major and attracting higher quality students.

As a first example of this improved ability to focus on the main tradeoff, we simplify and weaken the sufficiency condition from Proposition 4. Rather than having to compare the lowest value of δ against the highest possible value of the deviation as we do in Proposition 4, the large market result allows us to compare aggregation to disaggregation on a more equal footing, resulting in a weaker condition on δ.[30] We return to this simplification again later.

Corollary **1.** *Holding the other parameters fixed, there is an n^* such that the US equilibrium outcome is the unique equilibrium outcome with students not resolving their uncertainty over their preferences if $n > n^*$ and $\delta(v) > v - \frac{q}{2(1-\alpha)}$ for all $v \in [1-q, 1]$.*

One application of Proposition 4 is graduate schools, which almost universally admit students to a particular program. To explain this with our framework, we note that students in both US and English systems have either paid a cost in high school to learn their true preferences over majors or learned their preferences over majors after enrolling in an undergraduate university. Therefore, by the time of application to graduate school, all students know their preferences over majors.

Corollary **2.** *Let $c = 0$. For any other parameters, any equilibrium in undominated strategies yields the English equilibrium outcome.*

3.4.2 Comparative statics

We ask what happens to existence of both equilibria when, fixing all other parameters, we make the market more competitive on the university side by increasing the number of universities (n) or on the student side by decreasing the available seats (q).

The necessary and sufficient condition for the English equilibrium does not depend on n, so it is neither easier nor harder to maintain this equilibrium as n increases. However,

unresolved preferences when M universities are disaggregated to the probability the aggregated university is chosen if those M universities did not exist, $\frac{1}{n-M}$. With $M = 0.5n$, $\alpha = 0.75$ and $b = 0.5$ (so that the a priori benefit of attending an aggregated university is 0.125), this ratio is effectively 1 when $n \geq 50$. Another way to see this point is to hold the number of aggregated universities fixed and increase the number of disaggregated universities. As the disaggregated universities are less attractive to students, these additional competitors crowd out other disaggregated universities more than they do aggregated universities. Using the same α and b parameters and fixing the number of aggregated universities at 25, a given student (again admitted to all universities) is only 2% as likely to attend a given disaggregated university as they are to attend a given aggregated university when there are 50 universities in total.

[30] It is important to note that the sufficiency condition of Proposition 4, although not vacuous, is a fairly strict requirement.

increasing n makes it easier to support the US equilibrium, suggesting that the US equilibrium is more likely to be found in larger markets (those with more universities) because a disaggregating university suffers a larger loss in competitiveness for the best students as the guarantee to be able to study one's preferred major at an aggregated university is more likely to sway the student's enrollment selection. As a result, a deviating university admits a stochastically dominated set of applicants for larger n. As $n \to \infty$, the US equilibrium can be sustained if and only if the average utility derived from enrolling students in equilibrium is greater than $1 - q$, the quality of the marginal student not admitted in equilibrium.[31]

Although the existence of the English equilibrium is unaffected by n, it is highly dependent on the mass of total available seats, q. Increased competition on the student side (achieved by decreasing q) makes admission to university more difficult and reduces the chance (ex-ante) that a student can be admitted to study against her skill type at a disaggregated university. Therefore, the benefit of paying the cost c to learn one's major preference increases with q.

Changes in q affect the US equilibrium in a less straightforward manner as q interacts with δ. Increasing q lowers the quality of the marginally admitted students, with whom a deviating university disproportionately fills. However, if the students newly admitted in equilibrium as q increases are of sufficiently low expected value, which happens if $\delta(1-q)$ decreases quickly with q, deviating can become relatively more attractive. We present a condition on δ that ensures this is not the case. In both Theorem 1 and Proposition 5, statements of the form "the set of parameters increases/decreases" are made in the sense of set inclusion.

Theorem **1** (Effect of changes in market size)**.** *As n increases, the set of parameters that sustain the US system in equilibrium also increases. There exists n^* such that for all $n > n^*$ the US system can be sustained in equilibrium if and only if* $\int_{1-q}^{1} \delta(v)dv > q - q^2 \left(\frac{1-\frac{\alpha}{2}}{1-\alpha}\right)$.

As q increases, the set of parameters that sustain the English system in equilibrium also increases. If $\delta(x) > \frac{x(2-\alpha)-1}{1-\alpha}$ for all $x \in [0,1]$, then there exists n^{**} such that if $n > n^{**}$, increasing q increases the set of parameters that sustain the US system in equilibrium.

We make comparative statics statements regarding preference alignment between univer-

[31] This condition is weaker than the condition in Corollary 1, which can be seen in two ways. First, the right-hand side of the condition in Corollary 1 integrates to the right-hand side of the referenced condition in Theorem 1, so any δ which satisfies the condition of Corollary 1 satisfies this condition while there are δ functions which satisfy the condition in Theorem 1 but not the one in Corollary 1. Second, the condition in Corollary 1 is derived by considering incentives to disaggregate when any proportion of the other universities are disaggregated, whereas the condition in Theorem 1 only considers incentives when no other universities are disaggregated.

sities and students and other parameters of our model to illustrate how our model can be used to anticipate the consequences of changes in market fundamentals; these results are immediate from the conditions given in Propositions 1 and 2. We say that $\hat{\delta}(\cdot) \geq \delta(\cdot)$ if and only if $\hat{\delta}(v) \geq \delta(v)$ for all v.

PROPOSITION **5** (Effects of other parameters)**.** *Increasing $\delta(\cdot)$ increases the set of parameters that sustain the US system in equilibrium and the set of parameters that sustain the English system in equilibrium. As α increases, the set of parameters that sustain the English system in equilibrium decreases. There exists n^* such that for $n > n^*$, the set of parameters that sustain the US system in equilibrium increases.*

Increasing b increases the set of parameters that sustain the US system in equilibrium and the set of parameters that sustain the English system in equilibrium. The effect of increasing b on the set of parameters that sustain the US system in equilibrium vanishes as n grows. Decreasing c increases the set of parameters that sustain the English system in equilibrium.

This proposition shows which changes in fundamentals are conducive to both equilibria (though for different reasons, with the US equilibrium usually strengthened on the university side and the English equilibrium strengthened on the student side[32]) and which act in opposite directions. The impact of a change in α is the most subtle. Increasing α weakens the incentive of students to learn their major preferences because their skill type is more likely to be their major preference, harming the English equilibrium. For the US equilibrium, increasing α has two effects; it increases the utility to a university to aggregating by decreasing the likelihood that an enrolled student uses the option to study against her skill type but it also decreases the competitive loss when a university deviates to disaggregation. For sufficiently large n (and for reasons similar to those discussed after Proposition 6), the former effect wins out as n grows.

3.4.3 Robustness to small deviations

We do not always observe all universities and students following the aggregation and information gathering choices of the majority within different countries, motivating a study of which systems are supported by equilibria when there are agents making non-strategic pre-match decisions–the aggregation decision for universities and the preference resolution decision for students–with low but non-zero probability. These non-strategic decisions could be due to mistakes, experimentation, new technologies or feasibility constraints (e.g., new

[32] For example, increasing b increases the competitive loss to deviating to disaggregating in the US equilibrium and increases the value to resolving major preference uncertainty in the English equilibrium.

universities that do not yet have all programs or foreign students who are educated in their home countries) and it is important to understand whether these "dissenters" can shift the equilibrium outcome, particularly as it pertains to the system the equilibrium supports.[33]

We call a system stable if agents do not change their pre-match decisions in the presence of few dissenters. We find that the English system is not stable but that the US system is, which may explain recent aggregation choices by universities in majority-disaggregated countries and foreshadow a move away from disaggregation.

To conduct the robustness analysis, we fix the probability of an initial non-strategic action and then study the existence of equilibria supporting the US and English systems when there are many universities. Universities and students all choose their strategies but with probability $2p$, a given university's aggregation choice is replaced by a non-strategic rule that picks aggregation with probability $\frac{1}{2}$. Therefore, a "mistake" occurs with probability p. Similarly, a mass r of students have their resolution decision replaced by "not resolve" and a mass r have their resolution decision replaced by "resolve." All other parts of the chosen strategy are unaffected.

Let $h_{n,r,p}$ be a vector of parameters (excluding n, r, and p) of the game with n universities and pre-match mistakes with probabilities r and p. A system is *supported* by $h_{n,r,p}$ if there exists an equilibrium of the game with all agents choosing the strategies for aggregation and resolution corresponding to the system of interest, and any deviation in aggregation/resolution leads to a strictly lower (expected) payoff for that agent.

Definition **1.** *A system e is* stochastically stable *if there exists a vector $h_{n,0,0}$ that supports e and for any $h_{n,0,0}$ that supports e, there exists $r^*, p^* > 0$ such that for all $0 < r < r^*$ and $0 < p < p^*$, there is $n(r,p)$ such that for all $n^* > n(r,p)$, e is supported by $h_{n^*,r,p}$.*

We apply this definition of stochastic stability to our two focal systems, the US and English. This definition requires that there be 1) a set of parameters of the game without mistakes that supports the given system and that 2) fixing some sufficiently small likelihood of mistakes, the system continues to be supported *for large enough n*.

If we instead fix n and merely consider the limit as r and p go to zero, any system supported by an equilibrium under $h_{n,0,0}$ (with strict incentives for the chosen pre-match action of each agent), will also be supported by an equilibrium under $h_{n,r,p}$ for r, p sufficiently small; the arguments used in the proof of the following result can be re-purposed to show

[33]Our focus on the stability of systems rather than of equilibria is motivated by the fact that universities' admissions decisions may react to the presence of these "dissenters" even if their aggregation decisions do not. As our paper is motivated by the systems used in the US and England and aims to understand how various changes to the higher education landscape could lead to systemic changes, the focus on system is natural.

that for a fixed n, if p is sufficiently small, then it is as if no universities deviate and that the admission sets of universities are continuous in r around 0. Therefore, the original system can be supported in equilibrium. The dependence on n in our definition leads to more nuanced results.

Proposition **6.** *The US system is stochastically stable; the English system is not.*

In light of Theorem 1 and Proposition 3, it is not surprising that the US system is stochastically stable as increasing n strengthens the incentives of universities to aggregate and students most prefer the US system. What is more subtle is that the English system is not, given the lack of dependence of the existence of the English equilibrium on n. The cause of the instability is students' incentives to resolve their preferences.

The way students' incentives lead to instability underscores the nature of many-to-one matching in our setting. The driver of our result is that students care only about the "best" university they can be matched with whereas universities care about the average quality of the (many) students they enroll, so students care about the *number* of universities aggregating and the universities care about the *proportion* of students resolving. The difference between a student's university-specific utility ($w_s(u)$) for her favorite disaggregated university and for her favorite aggregated university goes in probability to 0 as the number of aggregators and disaggregators grows large. This occurs when n grows large when a fixed proportion of universities aggregate. When the percentage of universities aggregating is large relative to the mass of students not resolving their preference uncertainty (even when both are small in absolute terms), high-quality students can be confident that they will be admitted to all colleges and universities and that at least one of the aggregated universities is almost as good as their favorite disaggregated university, destroying the incentive of students to learn their major preferences.

As emphasized also by Theorem 1 and Proposition 3, this proposition shows that the US system can be thought of as the "competitive" outcome. Theorem 1 shows that increasing competition on the university-side promotes the US system but had nothing to say about the English system. Proposition 6 makes clear that this is not true once mistakes are allowed for, with the English system becoming more fragile as the number of universities grows.

4 Equilibrium with monetary transfers

In this section, we allow agents to exchange money. Payments can be conditioned on which major students ultimately study. By conditioning payments on the major studied, universities can create incentives for students to study their skill-type majors even under aggregation.

Therefore, this section studies a situation in which the cost to switching majors is endogenously determined in the model.[34]

We believe this model is helpful in analyzing athletic scholarships in the United States, in which a student can be thought as being inconsistent if she prefers to focus on academic pursuits instead of athletics.[35] Although there are many factors that affect a student's choice to participate in athletics (for example, successful student athletes may be able to go on to lucrative careers as professional athletes), we believe our simple model sheds light on the equilibrium effects of introducing price competition. Here, we study the case in which transfers are restricted to being weakly positive payments from universities to students, and in the online appendix we consider the unrestricted case.

We allow universities and colleges to make non-negative payments to students. Simultaneously with the decision of which contracts to offer to students, the deciding body (i.e. the university itself if the university has aggregated, or the individual colleges at a disaggregated university) offers a contingent payment to the student for any contract it offers. Payments can depend on the student's quality (v_s), her skill type (θ_s) and the major she studies (x_m) and cannot be reneged upon. We denote payments $p_u(v_s, \theta_s, x_m) \geq 0$.[36] Student s's utility from contract x is given by $w_{x_s}(x_u) + b \cdot \mathbb{1}_{\{x_m = \rho_{x_s}\}} + p_u(v_s, \theta_s, x_m)$. For any feasible set of contracts X_{i_u} that does not violate the capacity constraint, college i_u gets total utility

$$\frac{2n}{q} \int_{s \in X_{i_u}^s} \left[\mathbb{1}_{\{i=\theta_s\}} v_s + \mathbb{1}_{\{i \neq \theta_s\}} \delta(v_s) - p_u(v_s, \theta_s, i) \right] d\lambda$$

We define a university's utility as the average of the utilities of its constituent colleges. For any two feasible sets X_{M_u} and X_{L_u} such that for all $x \in X_{M_u}$ and all $y \in X_{L_u}$, $x_s \neq y_s$, then university $u \in U$ receives utility

$$\frac{n}{q} \sum_{i \in \{M,L\}} \int_{s \in X_{i_u}^s} \left[\mathbb{1}_{\{i=\theta_s\}} v_s + \mathbb{1}_{\{i \neq \theta_s\}} \delta(v_s) - p_u(v_s, \theta_s, i) \right] d\lambda$$

Formally, we modify the game so that in the third step, each aggregated university selects for each $v_s \in [0, 1]$ to either reject student s or admit her with payment $p_u(v_s, \theta_s, M) \geq 0$ and $p_u(v_s, \theta_s, L) \geq 0$, respectively, to both colleges M_u and L_u. Each college i_u at a

[34] Aggregated universities may invest in costly institutions, e.g., mandatory "weed out" courses in each major, designed to incentivize students to study their skill-type majors. However, an aggregated university cannot stop a student from changing majors, differentiating it from a disaggregated university even in the presence of transfers.

[35] Alternatively, a student can be "inconsistent" if she is injured and has to pay cost b to rehabilitate her injury.

[36] We assume that money enters with equal weight in all agents' utility functions. This equal weighting of money is not a substantive assumption with the exception of welfare optimizing calculations.

disaggregated university selects for each $(v_s, \theta_s) \in [0,1] \times \{M, L\}$ whether to admit s with payment $p_u(v_s, \theta_s, i_u) \geq 0$ or to reject s. Note that both aggregated and disaggregated universities can make payments contingent on which major is eventually studied. As before, each student s accepts the contract that maximizes her expected utility, with ties broken arbitrarily. We continue to use "US equilibrium" to refer to equilibria in which all universities aggregate and no student resolves major uncertainty and use "English equilibrium" to refer to equilibria in which all universities disaggregate and all students resolve their major preference uncertainty.

We study the existence and properties of equilibria when n is sufficiently large. Focusing on this case has three advantages. First, it allows for analytic tractability. We show that payments made in equilibrium are capped endogenously when n exceeds a certain threshold, leading to a cleaner analysis. Second, when n exceeds a certain threshold, disparities between the English and US equilibria are heightened, allowing us to explore differences between these systems. Third, as we have argued before, the number of universities in a "large" market need not be prohibitively large; as we are primarily concerned with exploring university admissions within (large) countries, we believe that the case of n "sufficiently large" is the most relevant case. Throughout the analysis and in the appendix, we explicitly solve for the necessary n threshold when appropriate. Elsewhere, we discuss convergences, and in this section all convergences mentioned are with regards to n.

We show that $p_u(v_s, \theta_s, x_m)$ must obey a constant profit condition; all admitted students must be worth the same (in expectation) to the admitting university or college. Otherwise, the university or college could slightly increase its payments for desirable students and fill its seats with these students. Due to capacity constraints, the constant profit condition is relative to the highest quality student not admitted anywhere because any university or college can always enroll this student and pay her nothing.

Let us consider a potential English equilibrium. As before, the admissions threshold at every college is $1-q$. Consider symmetric payments from each college, wherein for any student studying her skill type major the payment offer at university u satisfies

$$\begin{aligned} p_u^*(v_s, \theta_s, x_m = \theta_s) \in \underset{p_u}{\operatorname{argmax}} \quad & (v - p_u) \cdot Pr(p_u + w(u) > p + w(u_j))^{n-1} \\ s.t. \quad & v - p_u \geq 1 - q \end{aligned} \tag{6}$$

where p is the payment offered by other universities. Noting that the random variable $w(u_j) - w(u)$ follows a standard triangular distribution, the above can be solved for the unconstrained maximizer (i.e. ignoring the restriction $v - p_u^*(v_s, \theta_s, x_m = \theta_s) \geq 1 - q$) as

$$v - p_u = \frac{F(p_u - p)}{(n-1)f(p_u - p)}$$

where F and f are the CDF and PDF of the standard triangular distribution, respectively. By imposing $p_u = p$ we arrive at

$$v - p_u = \frac{1}{2(n-1)}$$

For any $v > 1 - q$ there exists n sufficiently large such that the constraint will bind, in which case the equilibrium payment equals $v - (1 - q)$.

The above exercise can be completed for students (with qualities greater than $\bar{v}$) who study against their skill types. The following equation summarizes equilibrium payments for sufficiently large n.

$$p_u^*(v_s, \theta_s, x_m) = \begin{cases} v_s - (1-q) & \text{if } x_m = \theta_s \\ \max\{0, \delta(v_s) - (1-q)\} & \text{if } x_m \neq \theta_s \end{cases} \tag{7}$$

Students with $v_s \in [1 - q, \bar{v})$ remain unable to study against their skill types as they are only admitted to the colleges corresponding to their skill type. However, universities "bribe" inconsistent students with qualities greater than $\bar{v}$ to study their skill types if $v_s - \delta(v_s) \geq b$ (if the gap in payments is greater than the benefit to studying one's preferred major). These bribes make it more difficult to sustain the English system. Even if $v_s - \delta(v_s) < b$, a student's outside option of not resolving (and enrolling at the college of her skill type) has improved, meaning there is less of an incentive for students to resolve their major uncertainty. In the appendix, we complete the existence proof by considering actions following a lone university deviating to aggregation and also show that students do not have an incentive to deviate under the condition given in Proposition 7.

The US system suffers a bigger breakdown. Following a similar argument, equilibrium transfers must make each university indifferent between enrolling students with quality $v_s > 1 - q$ and enrolling students with $v_s = 1 - q$ and paying them nothing. A student of quality $1 - q$ is worth $\alpha(1 - q) + (1 - \alpha)\delta(1 - q)$ in expectation to universities because she may elect to study against her skill type.[37]

[37] This assumes that $\alpha(1 - q) + (1 - \alpha)\delta(1 - q) \geq 1 - q - b$. If not, then the marginal student is worth $1 - q - b$ to an aggregated university as it can pay her b if she studies her skill type and 0 otherwise to bribe her to study her skill type. The logic of the remainder of the argument is largely unchanged in this case and we formally address this case in the appendix.

There is a multiplicity of payment schemes each university can make to students. However, fixing the expected utility a student receives from attending a particular university, the university wants to align the student's incentives to study her skill type with that of the university. When the expected payment a student receives is weakly greater than b, the university can bribe this student to study her skill type regardless of her preferred major by setting $p_s^n(v_s, \theta_s, x_m = \theta_s) - p_s^n(v_s, \theta_s, x_m \neq \theta_s) \geq b$. Note also that all such students must be bribed to study their skill type majors on path in any potential equilibrium; any university that does not do so will be at a competitive disadvantage for these students.

The following payment scheme represents one plan which fulfills these equilibrium requirements for sufficiently large n.

$$p_u^*(v_s, \theta_s, x_m) = \begin{cases} v_s - \alpha(1-q) - (1-\alpha)\delta(1-q) & \text{if } x_m = \theta_s,\ v_s - (\alpha(1-q) + (1-\alpha)\delta(1-q)) \geq b \\ 0 & \text{if } x_m \neq \theta_s,\ v_s - (\alpha(1-q) + (1-\alpha)\delta(1-q)) \geq b \\ v_s - (1-q) & \text{if } x_m = \theta_s,\ v_s - (\alpha(1-q) + (1-\alpha)\delta(1-q)) < b \\ \delta(v_s) - \delta(1-q) & \text{if } x_m \neq \theta_s,\ v_s - (\alpha(1-q) + (1-\alpha)\delta(1-q)) < b \end{cases} \tag{8}$$

In this payment scheme, each university earns $\alpha(1-q)+(1-\alpha)\delta(1-q)$ expected utils from each student it enrolls in the candidate equilibrium. Consider the deviation of a university disaggregating and filling its seats with students of quality arbitrarily close to $1-q$. As these students are all arbitrarily close in quality to the marginally admitted student, almost no payments are required. As a result, the university receives $\approx 1-q > \alpha(1-q)+(1-\alpha)\delta(1-q)$ utils from each student it enrolls and fills its seats. Therefore, the US equilibrium does not exist. Intuitively, the US equilibrium existed without transfers because universities were willing to allow students to study against their skill types so that they could attract higher quality students. With transfers, universities are indifferent (in expectation) between all students they enroll, so the dominant effect is the desire to force students to study their skill types, which is accomplished by disaggregating.[38]

Proposition **7.** *There exists n^* such that for all $n > n^*$, the US system cannot be sustained in equilibrium and the English system can be sustained in equilibrium if and only if*

$$(1-\bar{v})(1-\alpha)\left(b - E\left[\min\left\{b, v_s - \delta(v_s)\right\}|v_s \geq \bar{v}\right]\right) > c$$

Turning to welfare comparisons, universities are better off under the US equilibrium

[38] In Appendix C.11 we show that in a model of buyers and sellers without qualities and capacities, the US equilibrium can be sustained, demonstrating the importance of these features.

without transfers than the English equilibrium with transfers. This follows because the US equilibrium without transfers occurs when the average utility each university receives from students is greater than $1 - q$; in the English equilibrium with transfers, each university receives exactly $1 - q$ from every admitted student.

4.1 Market interventions

One way to retain the US (and English) equilibrium is to place an exogenous cap on transfers.[39] A sufficiently low transfer cap raises the average utility from admitted students above $1 - q$, allowing the US equilibrium to exist. With transfer caps, top students in the English system are less likely to be bribed into studying their skill types so there is more of an incentive to resolve uncertainty over major preferences compared with the unbounded positive transfers case.

We study the properties of two different transfer cap schemes. An *unconditional transfer cap* T_u restricts that for all admitted students $p_u^*(v_s, \theta_s, x_m) \leq T_u$. A *conditional transfer cap* T_c restricts that for all admitted students $p_u^*(v_s, \theta_s, x_m = \theta_s) \leq T_c$ and $p_u^*(v_s, \theta_s, x_m \neq \theta_s) = 0$. In words, conditional transfers allow scholarships up to a value of T_c only if the student studies her skill type and zero otherwise, while an unconditional transfer only requires that no student is paid more than T_u.

Consider transfer caps in the English equilibrium. Proposition 7 characterizes the payments that are made in equilibrium without caps. Similar logic dictates that with an unconditional cap T_u, transfers $p_u^{T_u}(v_s, \theta_s, x_m) \to \min\{T_u,\, p_u^*(v_s, \theta_s, x_m)\}$ where $p_u^*(v_s, \theta_s, x_m)$ is given in Equation 7. Similarly, with a conditional cap T_c, it must hold that $p_u^{T_c}(v_s, \theta_s, x_m = \theta_s) \to \min\{T_c,\, p_u^*(v_s, \theta_s, x_m = \theta_s)\}$.

The US system is more complicated. Uncertain students value transfers for both majors; competition between universities forces transfers to spill over from one major to the other when the cap binds, acting as a form of insurance for students. As a result, students are always compensated such that universities either value them at $\alpha(1-q) + (1-\alpha)\delta(1-q)$ or the cap binds regardless of major chosen. With a conditional cap, this insurance dynamic disappears; the payment for a student studying her skill type satisfies $p_u^{T_c}(v_s, \theta_s, x_m = \theta_s) \to \min\{T_c,\, v_s - \alpha(1-q) + (1-\alpha)\delta(1-q)\}$.

What are the welfare maximizing transfer scheme and system? Transfers from universities to students do not directly affect total welfare, but may bribe high quality, inconsistent students to study their skill types. When student s is bribed to study her skill type, aggregate

[39] This transfer cap could be enforced endogenously in a repeated game by punishing deviators with a "grim trigger" strategy of playing the English equilibrium without a transfer cap.

utility increases by $v_s - \delta(v_s) - b$. Noting that universities will never successfully bribe students to study their skill types when $v_s - \delta(v_s) < b$, the optimal transfer scheme is one that maximizes the set of students universities bribe. This suggests that the US equilibrium should be selected, as some students in the English equilibrium are unable to study against their skill types. Furthermore, the level of the cap selected must be $T \geq b$. Also, since an unconditional cap causes the difference between payments for studying different majors to decrease relative to a conditional cap of the same amount, the latter bribes weakly more students.

This reasoning allows us to consider policy implications of switching from an unconditional to a conditional cap and adjusting the level of the cap. When it is very valuable for students to study their skill types (i.e. b is sufficiently small relative to $v_s - \delta(v_s)$), it is possible to improve the utilities of both the university and student by slightly increasing the transfer limit and switching to a conditional cap. The university is able to pay the student more money when she studies her skill type, which is more valuable to the student than any insurance provided in the unconditional transfer scheme. The university is made better off because it is able to bribe the student to study her skill type at minimal cost.

Theorem **2.**

1. *The US equilibrium with a conditional cap of $T_c \geq b$ yields the welfare-maximizing allocation.*
2. *There exists $b^* > 0$ such that for all $b < b^*$ and for any unconditional transfer cap T_u, there exists a conditional transfer cap $T_c^{'} \geq \min\left\{b,\, T_u + \frac{1-\alpha}{\alpha} \cdot b\right\}$ such that all universities and (ex-ante) all students prefer the conditional cap of $T_c^{'}$ in either equilibrium.*

Because the no transfers case corresponds to an unconditional transfer cap of $T_u = 0$, Theorem 2 shows that for small b, a conditional transfer cap of $T_c = b$ is Pareto improving.

We return to US university athletics. Revenues and, due to the cap on payments students can receive, profits have grown to eight figures and beyond for certain programs.[40] Universities are forbidden to pay market clearing wages for top athletes. To gain or retain a competitive edge against their peers, many universities have begun considering guaranteed scholarships (the spillover dynamic between majors described with unconditional transfers) which pay for the education and stipend of student athletes even if they do not compete on an athletic team. In 2014, the Universities of Maryland, Indiana, and Southern California announced scholarship guarantees for athletes in certain sports, and The Big Ten and

[40] http://www.usatoday.com/sports/college/schools/finances/, 11/7/2015.

Pacific-12 conferences have endorsed the move to guaranteed scholarships.[41] Our analysis indicates that all parties may be better off and more students may participate in athletics if universities were forced to offer conditional scholarships with a slightly higher stipend. While there are other important aspects of student athletics that we have not considered, this model gives caution to the assertion that moving to guaranteed scholarships or removing all caps on scholarships will lead to better outcomes.

Extensions of the model

We have made several simplifying assumptions in our model for clarity, but the qualitative predictions are robust to many complexities that are present in the real world. To demonstrate this, we give parametric restrictions under which US and English equilibria can be maintained under several perturbations of our model in the limiting large market case. We see that for each of these extensions, our original findings of the existence of both equilibria hold. Formal statements of these results and all proofs are relegated to Appendix C.5. We consider the following extensions of our model: heterogeneous intensities of student preferences over the two majors, non-identical university preferences, interdependent university/student preferences, heterogenous major popularity and yield management, arbitrarily many majors, non-homogeneous sizes and popularities of universities, and markets without intrinsic agent qualities.

5 Conclusion

We study the conflict that arises when universities and students–and more broadly, agents on different sides of a market–have different preferences over the terms of matching ex-post. As a result, before matching, universities decide whether to give their students the flexibility to pick their majors after matriculating and students decide whether to pay a cost to learn their preferences over majors. These choices endogenize the value of a match for both parties. Accurately analyzing this market requires a marriage of a matching framework to a principal-agent model.

The US system of full university aggregation and uncertain students is an equilibrium outcome of the university admissions game, with universities discouraged from deviating since they would appear less attractive to students without the option value of studying the

[41] http://www.washingtonpost.com/blogs/terrapins-insider/wp/2014/08/19/maryland-to-guarantee-lifetime-athletic-scholarships/, 11/7/15.

ex-ante less preferred major. The English system of full disaggregation and certain students is also an equilibrium outcome, with universities discouraged from deviating by the concern of adverse selection. Students prefer the US equilibrium as it allows them to study their favorite major without having to pay a cost to discover their preference over majors at the beginning of the game. Universities prefer the English equilibrium as it forces second-tier students to study the major preferred by the university.

These two equilibria are often unique, which sheds light on why we do not observe countries with substantial partial disaggregation. The US equilibrium is easier to maintain with more universities, whereas the English equilibrium becomes harder to maintain when there are more students (equivalently, fewer seats at universities). The US equilibrium is robust to small mistakes and experimentation by agents; the English equilibrium is not. Our framework is useful for understanding the consequences of real-world deviations of universities in their admissions systems, of changes in national educational policies, and of changes in fundamentals such as students' information sets or market size.

One aspect we have not formally studied is which equilibrium is selected when both the US and English systems are viable. A path-dependent explanation seems plausible, with new entrants in the market adopting the admissions policies of their predecessors. Historical reasons for the starting points of these paths may exist, as we discuss in Appendix B.2. In particular, the differences in the social and political climates in the United States and England in the 19^{th} and 20^{th} centuries may have had a large role in selecting each country's equilibrium; varying high school standards and a young education system in the United States may have led to a more broad educational standard, while a desire for strong national efficiency coupled with a relatively centralized secondary education administration could have fostered a more narrowly focused scheme in England. Of course, there may be other historical reasons that contribute to the current university admissions differences between these two countries and a richer consideration of these points may yield new insights.

Our model gives a novel perspective into the debate on whether student athletes should be paid market wages. Universities compete away the surplus from matching with the top athletes. As a result, universities would prefer to disaggregate, avoid competition with other universities and instead force admitted students to participate in athletics. Interestingly, top universities in the United States collude against providing merit-based scholarships and athletic scholarships are capped by the NCAA. As we discuss, this is necessary to maintain the US equilibrium. Granting scholarships to students only if they participate in athletics makes it easier to sustain the US equilibrium and, if the level of the cap is chosen carefully, can lead to a welfare-maximizing outcome. From a policy perspective, Theorem 2 suggests that allowing universities to honor athletic scholarships even if a student does not participate in athletics may not be an optimal way to design the market for athletic scholarships.

Taking a broader view of our model, the aggregation and resolution of major uncertainty decisions give two dimensions along which universities and students can compete, and ultimately, sort into different equilibria. When one of these dimensions is hindered, through students who know their preferences costlessly in graduate admissions or through universities competing away the surplus from desirable students with merit based scholarships, we find that the US equilibrium can no longer be sustained without market interventions. This matches what we observe in the real world.

While the analysis of the paper focuses on university admissions (both with and without money), we believe our model provides a framework for understanding other markets that feature pre- and post-match decisions related to aggregation. Some examples of such markets include:

- **Vendor return policies:** Sellers must decide whether to allow buyers to return items. Buyers can learn about the specifics of an item before buying it, or learn about it through use after buying it. Accepting a return represents a cost for a seller, but is valuable for a buyer who has changed her mind about the purchase or bought the item for a particular task that has since been completed.

- **Worker-to-firm matching:** Different firms and industries offer workers varying levels of self-direction. One example is tenure for academics, which allows a professor to control her own research agenda even when it differs from the preferred research agenda of the university. In spot markets, however, workers are hired for a narrowly defined task. Before matching, workers can learn of their preferences for terms.

- **Refugee settlement:** Are refugees free to relocate after being accepted into a country? In the United States, refugees are allowed to do so, which has led to a large Somali community in Lewiston, Maine, despite initial efforts to settle these refugees in different parts of the country.[42] Discouraging refugees from relocating (as is the case in certain countries, such as Germany) may increase incentives for refugees to gather information about the country to which they seek admission.[43]

[42]http://www.politico.eu/article/migrants-arent-widgets-europe-eu-migrant-refugee-crisis/ 11/17/2014.

[43]Delacrétaz et al. (2016) study centralized matching mechanisms to improve refugee settlement without the notions of relocation within country.

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A Proofs

Proof of Lemma 1

We prove a result more general than presented in the main body. To begin, let r be the proportion of students who have resolved uncertainty, $d_u \in \{agg, disagg\}$ represent whether university u has disaggregated or not, and d represent the vector of such decisions for every university. For each university u, let $(\sigma_u^M(v_s, \theta_s), \sigma_u^L(v_s, \theta_s))$ be a pair of functions mapping quality v_s and skill type θ_s into admission probability to colleges M_u and L_u, respectively. If u is aggregated, it must be the case that the admissions outcomes for $\sigma_u^M(v_s, \cdot)$ and $\sigma_u^L(v_s, \cdot)$ for all v_s are perfectly correlated, so we represent this admission probability function as $\sigma_u(v_s)$.

Define $R_u(v_s, \theta_s, r, d, \sigma_{-u})$ as the expected value of a student of quality v_s and skill type θ_s to university u, defined as the university's expected (infinitesimal) payoff from admitting the student conditional on her enrolling, where σ_{-u} represents the vector of admissions probability functions for all other universities and colleges. For any disaggregated college i_u, $i \in \{M, L\}$, we similarly define $R_{i_u}(v_s, \theta_s, r, d, \sigma_{-i_u})$ as the expected value of a student of quality v_s and skill type θ_s to i_u, defined as the college's expected (infinitesimal) payoff from admitting the student, conditional her enrolling, where σ_{-u} represents the vector of admissions probability functions for all universities and colleges. Note that for any disaggregated college i_u,

$$R_{i_u}(v_s, \theta_s, r, d, \sigma_{-i_u}) = \begin{cases} v & \text{if } i = \theta_s \\ \delta(v_s) & \text{if } i \neq \theta_s \end{cases}$$

because i_u can only admit students to the contract specifying major i_u, that is, disaggregated colleges do not suffer from a winner's curse.

Assuming no capacity constraints are violated, the utility of disaggregated college i_u from admitting a subset of students $S' \subset [0,1] \times \{M, L\}$ according to function σ_u^i given σ_{-i_u} is

$$\frac{2n}{q} \int\limits_{s \in S'} \left[P(\sigma_{-i_u}(v_s, \theta_s)) \cdot R_{i_u}(v_s, \theta_s, r, d, \sigma_{-i_u}) \cdot \sigma_u^i(v_s, \theta_s) \right] d\lambda$$

where $P(\sigma_{-i_u}(v_s, \theta_s))$ is the probability a student of quality v_s and of skill type θ_s enrolls in college i_u conditional on being admitted. Each aggregated university u's utility is given by

$$\frac{n}{q} \int\limits_{s \in S'} \left[P(\sigma_{-u}(v_s, \theta_s)) \cdot R_u(v_s, \theta_s, r, d, \sigma_{-u}) \cdot \sigma_u(v_s) \right] d\lambda$$

where $P(\sigma_{-u}(v_s, \theta_s))$ is the probability a student of quality v_s and of skill type θ_s enrolls in university u conditional on being admitted.

Due to capacity constraints, each university's (college's) utility is concave in its own admissions probability function σ_u (σ_u^i), so no university (college) can benefit by mixing between admission probability functions.

We show that each aggregated university picks a threshold and admits all students with expected value exceeding the threshold. That is, let $S_u^R := \{s|\sigma_u(v_s) > 0 \text{ and } P(\sigma_{-u}(v_s, \theta_s)) > 0\}$. Let τ_u be the infimum expected value conferred to u by students $s \in S_u^R$. Then in equilibrium,

$$\sigma_u(v_s) = \begin{cases} 1 & \text{if } R_u(v_s, \theta_s, r, d, \sigma_{-u}) \geq \tau_u \\ 0 & \text{otherwise} \end{cases}$$

Similarly, we show that each disaggregated college picks a threshold and admits all students with expected value exceeding the threshold. That is, let $S_{i_u}^R := \{s|\sigma_u^i(v_s, \theta_s) > 0 \text{ and } P(\sigma_{-i_u}(v_s, \theta_s)) > 0\}$. Let τ_{i_u} be the infimum expected value conferred to i_u by students $s \in S_{i_u}^R$. Then in equilibrium,

$$\sigma_u^i(v_s, \theta_s) = \begin{cases} 1 & \text{if } R_{i_u}(v_s, \theta_s, r, d, \sigma_{-i_u}) \geq \tau_{i_u} \\ 0 & \text{otherwise} \end{cases}$$

We first consider disaggregated colleges. Note that each disaggregated college i_u must fill all of its seats under σ_u^i as due to the continuum of students, there is no aggregate uncertainty, so if a college is oversubscribed it will merely admit fewer students and if it is under subscribed it will admit more students. Suppose that σ_u^i is not a threshold policy for some disaggregated i_u. Then for some ν there is a positive measure set $S^1 \subset [0,1] \times \{M, L\}$ with expected values $R_{i_u} \geq \nu$ for all $s^1 \in S^1$ who are admitted with probability less than 1. Furthermore, there is some positive measure set $S^2 \subset [0,1] \times \{M, L\}$ with expected values $R_{i_u} < \nu$ for all $s^2 \in S^2$ who are admitted with non-zero probability. Consider the following "tatonnement" process: for some small $\delta_1 > 0$, reject the lowest δ_1 value students in S^2. Meanwhile, admit the top δ_2 students in S^1, where δ_2 is defined to maintain the capacity constraint with equality. Clearly, such δ_1 and δ_2 exist and the utility of each is increased by the adjustment. Contradiction.

Moreover, any two disaggregated M colleges must have the same threshold and any two disaggregated L colleges must have the same threshold, that is, for any disaggregated u, u', $\tau_{i_u} = \tau_{i_{u'}}$ for $i \in \{M, L\}$. This is because for any such colleges, the functions

$R_{i_u}(v_s, \theta_s, r, d, \sigma_{-i_u}) = R_{i_{u'}}(v_s, \theta_s, r, d, \sigma_{-i_{u'}})$ so that if the thresholds are different either one college is not filling all of its seats or one college is overfilled.

Finally, we show that $\tau_{M_u} = \tau_{L_u}$ for any disaggregated university u, which completes the claims regarding these universities. Recall that at any disaggregated universities u, u', $\tau_{i_u} = \tau_{i_{u'}}$ and that at any aggregated university u'' $\sigma_{u''}(v_s)$ does not depend on θ_s. Therefore, because of the symmetric distribution of student types with respect to M and L if WLOG $\tau_{M_u} > \tau_{L_u}$ either all disaggregated colleges M_u will not fill all of its seats, or all disaggregated colleges L_u will not fill all of its seats overfilled.

We now consider aggregated universities. Because all disaggregated colleges pick the same threshold, each college at aggregated universities u also meets its capacity constraint with equality under this admission threshold policy σ_u^i. This is because by the symmetry of the distribution of student types and the continuum of students, there is no aggregate uncertainty, so if a university is oversubscribed it will merely admit fewer students and if it is under subscribed it will admit more students. The remainder of the argument that shows that each aggregated university sets the same admissions threshold is nearly identical to the argument for colleges at disaggregated universities.

It is also easy to see that a student is admitted to university if and only if her quality weakly exceeds $1 - q$. That is, suppose there is some positive measure set of students S^3 with qualities above $1 - q$ who are not admitted to any university. Note that because these students have not been admitted elsewhere, there is no winner's curse associated with admitting them. In other words $R_u(v_s, \theta_s, r, d, \sigma_{-u}) = \alpha v_s + (1 - \alpha)\delta(v_s)$ at any aggregated university for each $s \in S^3$. Therefore, these students have higher expected values than some admitted students, and so universities would rather drop others and admit students from S^3. The fact that aggregate capacity is $1 - q$ and no seats are left vacant in any equilibrium, by our earlier argument, shows that all students with quality weakly greater than $1 - q$ are admitted.

Similarly, the cases in which either no student has resolved major uncertainty, or all universities are aggregated means that $R_u(v_s, \theta_s, r, d, \sigma_{-u}) = \alpha v_s + (1-\alpha)\delta(v_s)$ for all (v_s, θ_s). Disaggregated colleges do not suffer from a winner's curse, as discussed earlier. These special cases, combined with the general proof presented here, imply the statement in the main body.

■

LEMMA **2.** *In equilibrium (both on and off path) disaggregated colleges use thresholds in application quality and the admissions sets of aggregated universities are identical.*

First, colleges at disaggregated universities know exactly the contribution a given admitted student will make conditional on enrolling as there is no room for adverse selection.

That is, for colleges at a disaggregated university, the expected value of a student is equal to her application quality. Thus, colleges at disaggregated universities use application quality thresholds in equilibrium strategies by Lemma 1. Note that this result implies that if a student is admitted to only one college at a disaggregated university, it will be to the college corresponding to her skill type.

For aggregated universities, the admissions decision is less straightforward, as students who have resolved their preference uncertainty and are inconsistent as well as students who have not resolved their preference uncertainty perceive a benefit to an aggregated university and so are more likely to enroll conditional on admission than students who have resolved their preference uncertainty and are consistent. Thus, the expected value to an aggregated university of admitting a student of quality v_s must take into account the information contained in the student's enrollment decision.

The probability that a given student who has not resolved her preference uncertainty chooses an aggregated university u_{m+1} when she is admitted by n' universities, of which m are disaggregated is

$$\omega^a_{n',m} \equiv \text{Prob}(w_s(u_{m+1})+b > \max\{w_s(u_1)+\alpha b, \ldots, w_s(u_m)+\alpha b, w_s(u_{m+2})+b, \ldots, w_s(u_{n'})+b\})$$

Note that $\omega^a_{n',m}$ is increasing in m (holding n' fixed) as increasing m lowers the arguments of the max function, increasing the probability. One can calculate this probability given our assumption of the uniform distribution of the $w_s(u)$ terms. After some simplification, this becomes

$$\omega^a_{n',m} = \frac{1-(1-(1-\alpha)b)^{n'-m}}{n'-m} + \sum_{k=0}^{m}\binom{m}{k}\frac{((1-\alpha)b)^k(1-(1-\alpha)b)^{n'-k}}{n'-k}$$

For a student who has resolved her preference uncertainty, consider first the case of a consistent student. She has no reason to prefer an aggregated university over a disaggregated one as if she is admitted to at least one college at a disaggregated university, it will be to her skill type. So, the probability that a given university receives the student when n' universities admit her is

$$\frac{1}{n'}$$

regardless of the mix of aggregation in the admitting universities.

For an inconsistent student, the calculation for the probability of enrollment is much the same as for an unresolved student but with the added benefit to attending a university that is either aggregated or whose colleges have both admitted her equal to b rather than $(1-\alpha)b$.[44] Thus, the probability that such a student enrolls in either of those two university types when n' universities admit her but m of them admit her only to the college corresponding to her skill type is

$$\omega^c_{n',m} = \frac{1-(1-b)^{n'-m}}{n'-m} + \sum_{k=0}^{m} \binom{m}{k} \frac{((1-b)^k(1-(1-b)^{n'-k}}{n'-k}$$

Using Bayes' formula, the expected value to an aggregated university of admitting a student of quality v_s conditional on the student enrolling given n' total universities admit her, of which m are disaggregated and of which $m' \leq m$ admit the student only to the college of her skill type and when a portion r of students have resolved their preference uncertainty,[45] is

$$\begin{aligned} EV[n', m, m', v_s] = & \frac{\omega^a_{n',m}(1-r)}{\omega^a_{n',m}(1-r) + \frac{1}{n'}r\alpha + \omega^c_{n',m'}r(1-\alpha)}(\alpha v_s + (1-\alpha)\delta(v_s)) + \\ & \frac{\frac{1}{n'}r\alpha}{\omega^a_{n',m}(1-r) + \frac{1}{n'}r\alpha + \omega^c_{n',m'}r(1-\alpha)} v_s + \\ & \frac{\omega^c_{n',m'}r(1-\alpha)}{\omega^a_{n',m}(1-r) + \frac{1}{n'}r\alpha + \omega^c_{n',m'}r(1-\alpha)}\delta(v_s) \end{aligned} \tag{9}$$

where the three fractions are Prob(student is not resolved|student enrolls), Prob(student is resolved and consistent|student enrolls), and Prob(student is resolved and inconsistent|student enrolls), respectively.[46]

Note that given the symmetry in the disaggregated colleges' admissions decisions discussed above, a student is either admitted to only the college of her skill type or admitted to both colleges at all disaggregated universities. Thus, $m' = 0$ or $m' = m$. In the former case, we have $EV[n', m, 0, v_s] = \alpha v_s + (1-\alpha)\delta(v_s)$ (as resolved, inconsistent students are no more likely to choose an aggregated university over a disaggregated university).

[44] The other difference between unresolved students and resolved, inconsistent students is that resolved, inconsistent students view aggregated universities as equal to disaggregated universities to which they've been admitted to both colleges in terms of the ability to study their preference type major. Unresolved students do not know which college corresponds to their preference type, so view all disaggregated universities equally. Given that all disaggregated universities admit a student of a given skill type to the same number of colleges, this difference does not matter for the calculations below.

[45] Which, given the timing of the game, cannot depend on the student's quality.

[46] We note here that $EV[n', m, m', v_s]$ is a similar term to $R_u(v_s, \theta_s, r, d, \sigma_{-i_u})$. We use the new notation here to highlight that given Lemma 1, we know that in equilibrium the EV term does not depend on θ_s, nor are university's admissions strategies random.

Given the equations for the ω terms above, it can be shown that $EV[n'+1, m, m', v_s] > EV[n', m, m', v_s]$ by showing that the last term, Prob(student is resolved and inconsistent|student enrolls), decreases faster than the other two terms (and that the expected value is never greater than $\alpha v_s + (1-\alpha)\delta(v_s)$, so that decreasing this term improves the expected value regardless of the relative changing of the other two terms).

Intuitively, this result can be understood as follows. As mentioned, $\omega^c_{n',m}$ can be thought of as the extreme case of $\omega^a_{n',m}$ with $\alpha = 0$; additionally, the probability that a student enrolls conditional on being resolved and consistent can be thought of as the extreme case of $\omega^a_{n',m}$ with $\alpha = 1$. As α increases, an aggregated universities' "effective" competitors increases. For example, with $b = 1$, an inconsistent, resolved student will never pick a disaggregated university at which they've been admitted to only one college if possible, whereas a consistent, resolved student views all universities equally. As the reduction in the probability that a student enrolls at a given university caused by an additional university admitting that student depends on the number of universities already admitting that student,[47] the reduction for inconsistent, resolved students will be the biggest.

There are two important implications of this monotonicity in $EV[n', m, v_s]$. The first is that we can specify admissions functions in the US and English equilibria at all information sets (i.e., for all disaggregation profiles d) by assuming that all aggregated universities admit the same set of students and maximizing the value of this set given the expected value equation above and subject to seat capacities. The monotonicity ensures that we can accomplish this maximization by using the following algorithm:

1. Assume that all disaggregated colleges admit all students with application quality greater than or equal to $1-q$.

2. Based on the enrollment probabilities, determine the quality cutoff v_1 such that all aggregated universities meet their seat quota if they admit all students with actual qualities $v_s \geq v_1$. As all students are weakly more likely to attend an aggregated university than a disaggregated university and some students are strictly so, $v_1 > 1-q$.

3. Based on the expected values given these admission decisions, determine the quality v_2 such that $EV[n, m, m, v_2] = \alpha(1-q) + (1-\alpha)\delta(1-q)$. This is the student for which the aggregated universities are indifferent between admitting (while competing with the disaggregated colleges) and deviating to admitting the marginal student not admitted anywhere (for which there is no adverse selection, so whose expected value is the RHS).

[47] That is, it acts like $\frac{1}{n}$, and $\frac{1}{n} - \frac{1}{n-1}$ is decreasing in n

4. If $v_2 < v_1$, the admissions decisions for the aggregated universities is to admit all students with quality greater than v_1 and the admissions decisions for the disaggregated colleges is to admit all students with application quality greater than or equal to $1 - q$.

5. If $v_2 > v_1$, as the aggregated universities will not admit any students with quality below v_2 which the disaggregated colleges also admit, the admissions decisions for the aggregated universities is to admit students with quality above v_2 and to find a $v_3 \in (1 - q, v_2)$ such that only aggregated universities admit students with quality in $[1 - q, v_3]$ and aggregated universities meet their seat quotas. Disaggregated colleges admit all students with application quality greater than or equal to v_3. Note that this means that only students with actual quality greater than or equal to $\delta^{-1}(v_3)$ are admitted to both colleges at a disaggregated university.

The second important implication of this monotonicity is that this is not the only possible equilibrium admission function. That is, it can be the case that there is an admission function that is a part of an equilibrium strategy that involves aggregated universities leaving a gap in the quality interval in their admissions functions even though all would prefer to rearrange their admissions to include the students in the quality gap; however, no one aggregated university will change its admission function as it will suffer heavily from adverse selection in the gap.[48] We ignore such possibilities as, given the nature of our results and that the ability of such punishment off-path is to enforce disaggregation, such constructions are not of use to us.

This concept of a "coordination" outcome at the admissions stage (wherein aggregated universities lessen the impact of adverse selection by admitting the same set of students but that set need not be the highest quality students) is also what causes issues if aggregated universities know students' skill types at the time of admission. Following as in the proof of Lemma 1, it can be shown in this case that all disaggregated colleges of the same major type still use the same application quality threshold; however, the threshold used by L colleges can differ from that used by M colleges if more high-quality $\theta_s = L$ students are admitted by aggregated universities. There is a possibility that there is an equilibrium with aggregated universities coordinating on which students to admit by their skill type, leading to just such an imbalance. For this construction to work, there must be students who have resolved their major preference uncertainty, so the main area of our paper where such a concern could lead to complications is in Proposition 4, concerning the uniqueness of the English outcome in the presence of resolved students.

[48]This is similar to the discussion in the proof of Proposition 2 below, but with the important distinction that this gap might not be the utility maximizing outcome for the aggregated universities.

■

Proof of Proposition 1:

The proof is as given in the main text.

Proof of Proposition 2:

There are two cases to consider, following the proof of Lemma 1. First, the expected value threshold corresponds to a quality threshold for the aggregated university, so that it admits all students with qualities greater than some cutoff $t_{u_1}(n)$. Second, the expected value threshold does not correspond to a quality threshold for the aggregated university. The aggregated university chooses not to compete with the disaggregated universities for students below some quality, due to a winners' curse, and instead enrolls students not admitted elsewhere.[49]

In the first case, u_1's aggregation makes no difference to consistent students and all students with qualities above $\bar{v}$ since these students are always able to study their favorite majors. We therefore pay special attention to inconsistent students with $v_s < \bar{v}$ who plan to study against their skill types if they enroll in u_1. Let $w_s(u_1)$ and $w_s(u_{max})$ be the cardinal utilities that student s has for u_1 and the university other than u_1 at which she has the the highest utility. She prefers u_1 if and only if $w_s(u_1) > w_s(u_{max}) - b$. Thus, the probability that an inconsistent student admitted to all universities enrolls in u_1 is

$$Pr\left(w_s(u_1) > w_s(u_{max}) - b\right) = \\ Pr\left(w_s(u_{max}) < b\right) + \int_b^1 Pr\left(w_s(u_1) > y - b\right) \cdot f(y)dy = b + \frac{1}{n}\left(1 - b^n\right) \tag{10}$$

Because the aggregated university is more popular, $t_{-u_1}(n) < t_{u_1}(n)$ for all n. Since a total mass of q seats exists in this market, $t_{-u_1}(n) = 1 - q$ as otherwise at least one university does not enroll exactly a $\frac{q}{n}$ mass of students.

Now we calculate the threshold $t_{u_1}(n)$. Since students with $v_s \geq \bar{v}$ are able to study their favorite majors at any university, $t_{u_1}(n) < \bar{v}$. For our purposes, there are three types of

[49] That is, there will be some set of low quality students that only the deviating university admits, followed by a set of higher quality students which only non-deviating universities admit, then the set of highest quality students which all universities admit.

students to consider:

1. Students with $v_s \geq \bar{v}$. u_1 will get $\frac{1}{n}(1-\bar{v})$ students from this region, as they are no more attracted to u_1 than they were before.

2. Students with $\bar{v} > v_s \geq t_{u_1}(n)$ who are consistent. u_1 will get $\frac{1}{n}\alpha(\bar{v} - t_{u_1}(n))$ students from this region, as they do not find u_1 any more attractive since they do not wish to study against their skill types.

3. Students with $\bar{v} > v_s \geq t_{u_1}(n)$ who are inconsistent. As calculated above, u_1 will get $\left[b + \frac{1}{n}\left(1 - b^n\right)\right](1-\alpha)(\bar{v} - t_{u_1}(n))$ students from this region.

Summing over the mass of students u_1 gets, we can solve for $t_{u_1}(n)$ to match the quota. The result is that

$$t_{u_1}(n) = \bar{v} - \frac{q + \bar{v} - 1}{\alpha + (1-\alpha)\left[bn + 1 - b^n\right]} \tag{11}$$

Since u_1 gets the same mass of (and same utility from) students with qualities $v_s \geq \bar{v}$, for the purposes of comparing this deviation to the proposed equilibrium, we need only consider those students with qualities strictly less than $\bar{v}$. Focusing on those students and eliminating the normalizing constants in the utility functions, we derive the necessary condition for the English equilibrium as

$$\frac{1}{n}\int_{1-q}^{\bar{v}} v dv = \frac{\bar{v}^2 - (1-q)^2}{2n} > \int_{t_{u_1}}^{\bar{v}} \left[\alpha\frac{1}{n}v + (1-\alpha)\left[b + \frac{1}{n}\left(1 - b^n\right)\right]\delta(v)\right] dv \tag{12}$$

where the left-hand side comes from the utility in equilibrium and the right-hand side comes from the utility to deviating to aggregating. This condition is always satisfied, as explained in the main text.

It remains to show that universities do not wish to deviate in the second case in which the winner's curse causes a deviating university to enroll some students not admitted to the disaggregated univerisites. We begin by calculating the beliefs the deviating university has for students who cannot be admitted to a disaggregated university under both majors. From Equation (10) we know that the probability an inconsistent student most prefers the aggregated university is $b + \frac{1}{n}\left(1 - b^n\right)$. A consistent student most prefers the aggregated university with probability $\frac{1}{n}$. Noting that α proportion of students are consistent, applying

Bayes' rule, the belief that an applying student is consistent is

$$\frac{1}{1+(nb+(1-b^n))\,\frac{1-\alpha}{\alpha}} \tag{13}$$

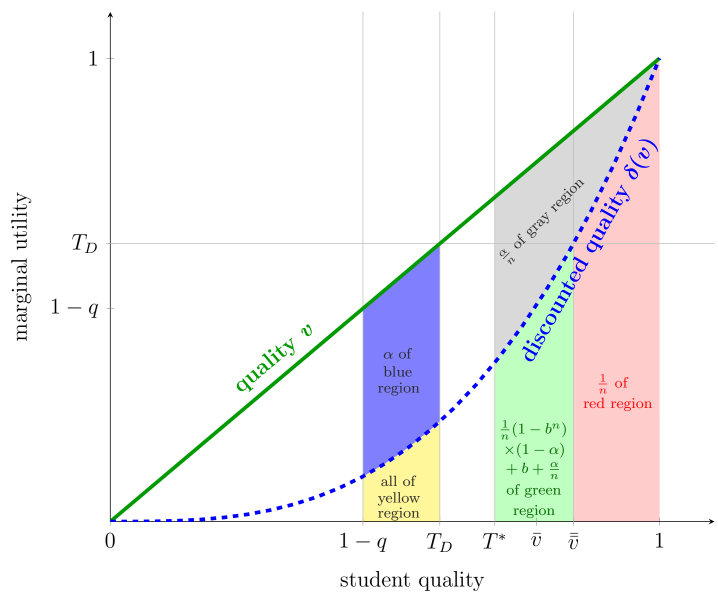
Therefore, the aggregating university would prefer to admit a marginal student not admitted anywhere than incur the winner's curse if

$$\frac{1}{1+(nb+(1-b^n))\,\frac{1-\alpha}{\alpha}}v_s+\left(1-\frac{1}{1+(nb+(1-b^n))\,\frac{1-\alpha}{\alpha}}\right)\delta(v_s)<\alpha(1-q)+(1-\alpha)\delta(1-q) \tag{14}$$

holds for some admitted student with quality v_s. Note that the right hand side arises due to the fact that the aggregated university does not suffer from the winner's curse from students who are not admitted to disaggregated universities. Because no student with quality $v_s < 1-q$ is admitted in equilibrium it must be the case that when the aggregated university credibly wishes to withhold offers of admission to some students and instead admit marginal students, the disaggregated universities raise their own thresholds (for otherwise they would overfill), allowing the aggregated university to enroll students with qualities above $1-q$ without competition and suffering from the winner's curse. Therefore, we construct the following off-path actions: Define T^* such that the student with quality T^* satisfies Equation (14) with equality (this corresponds to v_2 from Lemma 2). The aggregated university competes only for students with qualities weakly greater than T^*. The disaggregated universities adjust for this lack of competition by increasing their cutoffs from $1-q$ to T_D where the latter cutoff is set to match their capacities (T_D corresponds to v_3 from Lemma 2). The aggregated university then admits all students with qualities between $1-q$ and T_D without competition. Note that $\bar{\bar{v}} \equiv \delta^{-1}(T_D) > \delta^{-1}(1-q) = \bar{v}$, meaning that fewer students are able to study either major at disaggregated universities. It is easy to see that $1-q < T_D < T^* < \bar{v} < \bar{\bar{v}}$. The following figure demonstrates the utility of the aggregated universities.

We again see that consistent students with qualities weakly greater than T^* are no more likely to attend the aggregated university, nor are inconsistent students with qualities weakly greater than $\bar{\bar{v}}$. The aggregated university admits more inconsistent students with qualities $v_s \in [T^*, \bar{\bar{v}})$ and all students with qualities $v_s \in [1-q, T_D)$, and admits fewer (0) students with qualities $v_s \in [T_D, T^*)$. Since $T^* < \bar{v}$, all previously admitted students with qualities $v_s \in [T_D, T^*)$ were admitted under their skill types, so they are each more valuable than any additional students admitted with quality $v_s \in [1-q, T_D)$. Furthermore, the additional inconsistent students with qualities $v_s \in [T^*, \bar{\bar{v}})$ are each worth less the to aggregated univer-

Figure A.1: Utility of deviating English university with non-connected admit region



Notes: This figure represents the utility that a university receives if it is the lone deviator from the English equilibrium with a non-connected admit region. This university admits all students with qualities between $1-q$ and T_D and above T^*. All students with qualities between $1-q$ and T_D enroll in the deviating university and α proportion of them study their skill type majors. Consistent students with qualities weakly above T^* and all students with qualities weakly above $\bar{\bar{v}}$ are no more or less likely to attend the deviating university than any other. Inconsistent students with qualities between T^* and $\bar{\bar{v}}$ are more likely to attend the deviating university and study against their skill type majors. As a result, the deviating university enrolls $\frac{1}{n}(1-b^n)(1-\alpha)+b+\frac{\alpha}{n}$ students who study against their skill types in the green region (the area bounded to the left by T^*, to the right by $\bar{\bar{v}}$ and above by $\delta(v)$.

sity then the previously admitted students with qualities $v_s \in [T_D, T^*)$ since $\delta(\bar{\bar{v}}) = T_D$. This means that the newly admitted students are each worth less to the aggregated university than the students given up. In other words, deviation leads to a stochastically dominated set of students in terms of expected value, leaving the deviating university worse off.

$\blacksquare$

Proof of Proposition 3:

Proof given in main text.

Proof of Example 1

As $\epsilon \to 0$, $\delta(\cdot)$ converges pointwise to a step function. This functional form is made purely for computational simplicity.[50]

To show that the asymmetric outcome is supported in equilibrium, we need to check that the disaggregated university does not want to deviate to aggregation, which would result in the US equilibrium, that the aggregated university does not want to disaggregate, and that students do not want to pay to learn their preference types.

We first investigate university deviations. WLOG assume that u_1 disaggregates in the proposed equilibrium, and u_2 aggregates. $b = 1/(1-\alpha)$ implies that no student admitted to u_2 would rather attend u_1. Thus, it must be that u_1 enrolls all students with qualities $v_s \in [1-q, 1-q/2] = [0.5, 0.75)$ while u_2 enrolls all students students with qualities $v_s \in [1-q/2, 1] = [0.75, 1]$. All students who attend u_1 study their skill-type major, since $\alpha > 1/2$ and students have not learned their major preferences.

Under the proposed equilibrium, u_1 receives utility

$$\frac{2}{0.5}\int_{0.5}^{0.75} v\,dv = \frac{5}{8}$$

If u_1 deviated to aggregation, it would lead to the US outcome. Each university would

[50] Of primary importance for the purpose of this example is to ensure that the disaggregated university does not want to aggregate by making the gains from competing with the (currently) aggregated university for the top students less than the benefit of forcing students with quality in the interval 0.5 to 0.75 to study their skill type; this δ makes these students particularly unattractive if they study against their skill type.

then receive

$$\frac{2}{0.5}\frac{1}{2}\int_{0.5}^{1}\alpha v+(1-\alpha)\delta(v)\,dv=\frac{5}{8}-\frac{1}{8}\epsilon+\epsilon\left(\frac{3}{32}+\frac{1}{16}\epsilon+\frac{1}{4}\epsilon^2\right) \tag{15}$$

$$\overset{\epsilon\to 0}{\to}\frac{2}{0.5}\left(\frac{1}{2}\int_{0.5}^{1}\frac{3}{4}v\,dv+\frac{1}{2}\int_{0.75}^{1}\frac{1}{8}\,dv\right)=\frac{5}{8}$$

The utility of each aggregated university in the US outcome approaches the limit of $\frac{5}{8}$ from below as δ approaches 0.5 from below at $v=0.75$ and ϵ goes to zero sufficiently quickly to ensure that the contributions to the aggregated utility from the $[0.5,0.75]$ and $[0.75+\epsilon,1]$ regions in excess of the approximation by the step function are smaller than the loss in the $[0,75,0.75+\epsilon]$ region. Therefore, u_1 does indeed prefer disaggregation for any $\epsilon>0$.[51]

We now check that u_2 does not want to disaggregate. In the proposed equilibrium, it receives utility:

$$\frac{2}{0.5}\int_{0.75}^{1}\alpha v+(1-\alpha)\delta(v)\,dv\overset{\epsilon\to 0}{\to}\frac{2}{0.5}\left(\int_{0.75}^{1}\frac{3}{4}v\,dv+\int_{0.75}^{1}\frac{1}{8}\,dv\right)=0.78125 \tag{16}$$

If it were to disaggregate, it would receive utility

$$\frac{2}{0.5}\frac{1}{2}\int_{0.5}^{1}v\,dv=0.75 \tag{17}$$

where all students enroll in their skill-type major since they have not learned their major preferences. Thus, u_2 will follow the proposed equilibrium strategy.

We now investigate student deviations. As we have argued, in the proposed equilibrium, u_1 sets an admissions threshold of 0.5. This implies that, as $\epsilon\to 0$, $\bar{v}\to 0.75$. Any student s who does not resolve her uncertainty over majors will attend u_2 in the proposed equilibrium if $v_s\geq 0.75$ (i.e. she is accepted to u_2) and will attend u_1 if $v_s\in[0.5,0.75)$. In the latter case, she studies her skill-type major. Therefore, her expected utility from not resolving her uncertainty over majors is

$$(\mathbb{E}(w(u_2))+b)(1-0.75)+(\mathbb{E}(w(u_1))+\alpha b)(0.75-0.5)=0.25\cdot(1+(1+\alpha)b) \tag{18}$$

Any student s who resolves her uncertainty over majors will attend her favorite university and study her preferred major if $v_s\geq\bar{v}=0.75+\epsilon$ or if $v_s\geq 0.75$ and she is consistent, will

[51] The equality of the u_1's utility with the universities' limiting utility under the US equilibrium is not necessary, it simply provides an example with convenient parameters.

attend u_2 if $v_s \in [0.75, \bar{v})$ and she is inconsistent (because $b > 1$, such a student will prefer to study her favorite major than attend her favorite university), and attends u_1 and studies her skill type if $v_s \in [0.5, 0.75)$. Therefore, her expected utility from resolving her uncertainty over majors is

$$\begin{aligned}(\mathbb{E}(\max\{w(u_1), w(u_2)\}) + b)(0.25 + \epsilon(\alpha - 1)) + \\ \mathbb{E}(w(u_2) + b)(1 - \alpha)\epsilon + \mathbb{E}(w(u_1) + \alpha b)0.25 - c \overset{\epsilon \to 0}{\to} \\ 0.25 \cdot \mathbb{E}(\max\{w(u_1), w(u_2)\}) + b) + 0.25 \cdot \mathbb{E}(w(u_1) + \alpha b) - c = \\ 0.25 \cdot (7/6 + (1 + \alpha)b) - c\end{aligned} \tag{19}$$

Therefore, comparing Equations 18 and 19, student will deviate to resolving her uncertainty (for sufficiently small ϵ) if $c < 1/24$. As we have assumed $c = 1/5$, we have completed showing the existence of the asymmetric equilibrium.

To show that the English equilibrium is not the Pareto dominant equilibrium for universities, first note that the English equilibrium exists as

$$(1 - \bar{v})(1 - \alpha)b \overset{\epsilon \to 0}{\to} (1 - 0.75) \cdot 1 = 1/4 > c = 1/5$$

Then, note that the aggregated university in the constructed asymmetric equilibrium achieves a higher utility than it would in either the US equilibrium ($0.78125 > 5/8$, see Equations 16 and 15) or in the English equilibrium ($0.78125 > 0.75$, see Equations 16 and 17, where 0.75 is a weak upper bound on universities' payoff in the English equilibrium as 0.75 is the payoff to each university when both disaggregate and all students study their skill types).

■

Proof of Proposition 4:

First we prove existence of equilibrium. Consider any set of parameters. If $(1-\bar{v})(1-\alpha)b \geq c$ then the English equilibrium exists, as we have shown. If $(1 - \bar{v})(1 - \alpha)b < c$, then it is easy to see that no student will pay cost c to learn her preferences over majors, regardless of the aggregation decisions of universities. Disaggregation is strategically complementary for universities. This is because aggregated universities are more popular than disaggregated universities for uncertain students, since they allow the student to always study her favorite major.

Consider n universities, m of which are disaggregated. A student will select a disaggregated university, WLOG u_m, if and only if

$$w_s^{u_m} + \alpha b > \max\{w_s^{u_1} + \alpha b, ..., w_s^{u_{m-1}} + \alpha b, w_s^{u_{m+1}} + b, ...w_s^{u_n} + b\} \tag{20}$$

Since the w terms are distributed independently of the aggregation decision, a disaggregated university is less likely to be chosen by a student as m increases. This means that the aggregated universities set a higher threshold the higher N is, resulting in higher utility for a disaggregated university as N increases.

So consider the following algorithm to show existence of equilibrium.

Step 1. Suppose all other universities aggregate. Does an arbitrary university wish to disaggregate? If no, terminate the algorithm. If yes, let it disaggregate, and go to Step 2.

⋮

Step t. Suppose exactly $t-1$ universities disaggregate. Does an arbitrary aggregator wish to disaggregate? If no, terminate the algorithm. If yes, let it disaggregate and go to Step t+1.

At any step, a disaggregating university will not want to "go back" to aggregating because its utility is increasing in the number of other universities that disaggregate. So, eventually this process stops at an equilibrium, either when all universities are disaggregated, or with partial disaggregation.

■

We prove the conditional uniqueness results by considering 5 exhaustive cases.

1. **Generically, there are no pure-strategy equilibria in which agents on both sides of the market make asymmetric pre-match actions:** Suppose for contradiction that there is an equilibrium in pure strategies in which some agents on both sides of the market differ in their pre-match actions. Then it must be the case that students are indifferent between resolving uncertainty and not resolving uncertainty, i.e. $(1-\bar{v})B(n^{'}) = c$ where $B : \{1, 2, ..., n\} \to \mathbb{R}_+$ is a decreasing function mapping the number of aggregated universities into the expected benefit that a student with quality greater than $\bar{v}$ receives from paying the cost. Importantly, $B(\cdot)$ can only take on n different values, meaning that generically, it is not the case that $(1-\bar{v})B(n^{'}) = c$ for any $n^{'} \in \{1, 2, ..., n\}$. Contradiction.

2. **Any equilibrium in which no student resolves her uncertainty over major preferences yields the US system if** $\delta(1-q) > 1 - \frac{q}{2(1-\alpha)}\left(1 - \frac{\alpha}{1-((1-\alpha)b)^n+(1-\alpha)bn}\right)$

   To arrive at this condition, we compare the utility of a disaggregated university when m of n universities are disaggregated to the utility of an aggregated university when $m-1$ of n universities are disaggregated (for $m > 1$ and conditional on no student resolving her uncertainty). So long as the latter is always strictly greater than the former, the conclusion follows.

   To express the utility of a disaggregated university, it is useful to consider the aggregated university analogue to Equation 20. Consider an aggregated university i when m of n universities are disaggregated. Then (letting the first m universities be the disaggregated universities and $i = m+1$ WLOG),

$$\omega^a_{n,m} \equiv \text{Prob}(w_s^{u_i} + b > \max\{w_s^{u_1} + \alpha b, \ldots, w_s^{u_m} + \alpha b, w_s^{u_{m+2}} + b, \ldots, w_s^{u_n} + b\}) \quad (21)$$

   Note that $\omega^a_{n,m}$ is increasing in m as increasing m lowers the arguments of the max function, increasing the probability. One can calculate this probability given our assumption of the uniform distribution of the w_s^u terms. After some simplification, this becomes

$$\omega^a_{n,m} = \frac{1-(1-(1-\alpha)b)^{n-m}}{n-m} + \sum_{k=0}^{m} \binom{m}{k} \frac{((1-\alpha)b)^k(1-(1-\alpha)b)^{n-k}}{n-k} \quad (22)$$

   The utility of a university is the average value of its enrolled students. For a disaggregated university, this average can be broken down into the weighted average of the average value of students who are admitted only to disaggregated universities and the average value of students who are admitted to all universities. To compute this, we first need to determine the threshold used by aggregated universities; denote this threshold $t_{m,n}$ (so students with $v \geq t_{n,m}$ are admitted to all universities and students with $v \in [1-q, t_{n,m})$ are admitted only to disaggregated universities). Using the condition that supply equals demand for an aggregated university, we have

$$(1-t_{n,m})\omega^a_{n,m} = \frac{q}{n}$$

   Rearranging, this is

$$t_{n,m} = 1 - \frac{q}{n\omega^a_{n,m}}$$

Of the students with $v \in [1-q, t_{n,m})$, each disaggregated university gets $\frac{1}{m}$ of them as no disaggregated university is more popular than another. Thus, the portion of a disaggregated university's seats taken by these "low" quality students is

$$\frac{\frac{1}{m}(t_{n,m} - (1-q))}{\frac{q}{n}}$$

Rearranging, this is

$$\frac{n}{m}(1 - \frac{1}{n\omega^a_{n,m}})$$

Consequently, the portion of a disaggregated university's seats that are taken by "high" quality students (i.e. students with $v \in [t_{n,m}, 1]$) is 1 minus this value. As students will study their skill type at the disaggregated university (as $\alpha > \frac{1}{2}$, so unresolved students find it more likely that their preference type is the same as their skill type), the average value a disaggregated university gets from a given (quality) interval of students is just the midpoint of that interval. Putting this all together, the utility of a given disaggregated university is

$$U^d_{n,m} = \frac{n}{m}(1 - \frac{1}{n\omega^a_{n,m}})\left(\frac{1-q+t_{n,m}}{2}\right) + (1 - \frac{n}{m}(1 - \frac{1}{n\omega^a_{n,m}}))\left(\frac{t_{n,m}+1}{2}\right)$$

After some rearranging, this simplifies to

$$U^d_{n,m} = \frac{t_{n,m}+1}{2} - \frac{qn}{2m}(1 - \frac{1}{n\omega^a_{n,m}}) \tag{23}$$

For aggregated universities, we do not need to weight two different intervals, but we do need to account for whether a student is consistent (and to take the average value of δ over $[t_{n,m}, 1]$). This means that the utility of a given aggregated university is

$$U^a_{n,m} = \alpha\frac{1+t_{m,n}}{2} + (1-\alpha)Avg[\delta, t_{n,m}, 1] \tag{24}$$

where

$$Avg[\delta, t_{n,m}, 1] \equiv \frac{1}{1-t_{n,m}}\int_{t_{n,m}}^{1} \delta(v)\, dv$$

For the US equilibrium to be the unique equilbrium in which students do not resolve their uncertainty, we need

$$U^d_{n,m} < U^a_{n,m-1} \tag{25}$$

Substituting in the given expressions and rearranging, this becomes

$$2(1-\alpha)Avg[\delta, t_{n,m-1}, 1] > 1-\alpha+t_{n,m}-\alpha t_{n,m-1}-\frac{qn}{m}(1-\frac{1}{n\omega^a_{n,m}})$$

Substituting in the expressions for $t_{n,m}$ and rearranging, this inequality becomes

$$Avg[\delta, t_{n,m-1}, 1] > 1+\frac{q}{2(1-\alpha)}\left[\frac{\alpha}{n\omega^a_{n,m-1}}-\frac{1}{n\omega^a_{n,m}}+\frac{1}{m\omega^a_{n,m}}-\frac{n}{m}\right] \tag{26}$$

The LHS is bounded from below by $\delta(1-q)$. To find a sufficient condition for inequality 25 to hold, we need an upper bound on the RHS. To do this, note that it suffices to maximize the two differences in the brackets. For both differences, it suffices to look at $m=n$ given the formula for $\omega^a_{n,m}$; in this case, we use the convention that $\omega^a_{n,n}=1$, which both is the natural continuation of Equation 22 and ensures the correct calculation of Equation 23. Then, we can bound the RHS by substituting in the formula for $\omega^a_{n,n-1}$ in the first term in the brackets and simplify to have the following sufficient condition for Inequality 26 to hold.

$$\delta(1-q) > 1-\frac{q}{2(1-\alpha)}\left(1-\frac{\alpha}{1-((1-\alpha)b)^n+(1-\alpha)bn}\right) \tag{27}$$

3. **Any equilibrium in which all universities aggregate yields the US system:** This is easy to see, as there is no benefit to a student learning her preferences over majors ex-ante, but there is a cost to doing so.

4. **Any equilibrium in which every student resolves her uncertainty yields the English system:** Suppose that every student resolves her preference uncertainty. Lemma 2 provides the expected value conditional on enrollment of a student of quality v_s for a university that deviates from disaggregation to aggregation, and the algorithm there provides the admissions decisions for both aggregated universities and disaggregated colleges.

   Given a set number of aggregating and disaggregating universities, consider a given university's utility to aggregating vs disaggregating. The exact same argument as presented in the proof of Proposition 2 holds here.[52] Namely, comparing consistent

[52] Given at least one other college is aggregating, the argument from Proposition 2 is in fact stronger here as the v_2 from Lemma 2 is decreasing in the number of aggregating universities as the expected value conditional on enrollment is increasing in the proportion of aggregating universities. Thus, by switching from aggregation to disaggregation, *more* high quality students are admitted to both colleges at disaggregated

students, an aggregated university holds no greater appeal than a disaggregated university, so again a university gains no more of these students by aggregating than by disaggregating, The only difference between the enrollees in an aggregated university and the disaggregated university is that the disaggregated university enrolls a greater portion of inconsistent students who apply knowing that they will definitely study the major that is not their skill type.

5. **Any equilibrium in which all universities disaggregate yields the English system:** As before, the required condition to sustain the English equilibrium from the student's side is $(1-\bar{v})(1-\alpha)b > c$. Now suppose that some proportion of students p does not resolve uncertainty. Then these students apply under their type to every university. Note, however, that this does not change the equilibrium threshold for the universities, and therefore, does not change the condition for students to maintain equilibrium. Since every student faces the same cost and expected benefit to learning her preferences, the equilibrium in which all students learn their preferences ex-ante is unique conditional on all universities disaggregating and $(1-\bar{v})(1-\alpha)b > c$.

■

Proof of Corollary 1:

We begin with a short lemma.

LEMMA **3.** *In any equilibrium with no student resolving her preferences and a fraction* $\frac{m}{n} \equiv \mu$ *universities choosing to disaggregate,* $\lim_{n\to\infty} t_{n,m} = 1-(1-\mu)q$.

From the previous proof, we have

$$t_{n,m} = 1 - \frac{q}{n\omega^a_{n,m}}$$

where

$$\omega^a_{n,m} = \frac{1-(1-(1-\alpha)b)^{n-m}}{n-m} + \sum_{k=0}^{m} \binom{m}{k} \frac{((1-\alpha)b)^k(1-(1-\alpha)b)^{n-k}}{n-k}$$

universities, the consistent students among which will now find aggregated universities no more appealing than disaggregated universities and reducing the only potential upside to aggregating.

To prove the lemma, it suffices to prove that

$$\lim_{n\to\infty} n\omega^a_{n,m} = \frac{1}{1-\mu}$$

This follows from the definition of $\omega^a_{n,m}$ and the fact that $(1-\alpha)b \in (0,1)$, so after distributing n, the first term of $n\omega^a_{n,m}$ goes to the stated limit and the second term goes to zero (as can be seen by bounding the summation by the greatest term in the sum and showing that that goes to zero faster than $\frac{1}{n}$).

■

Returning to the main proof, to see the condition for the large market, we can use the lemma to write the limit of Equation 23 as $n \to \infty$ while holding μ constant as

$$U^d_\mu = \frac{t_\mu + 1 - q}{2}$$

and the limit of Equation 24 as

$$U^a_\mu = \alpha\frac{1+t_\mu}{2} + (1-\alpha)Avg[\delta, t_\mu, 1]$$

where $t_\mu = 1-(1-\mu)q$, the threshold used by aggregated universities, which is derived from equating the supply of seats provided by aggregated universities with the demand from students with quality above t_μ.

Repeating the steps as in Proposition 4, we get the condition

$$Avg[\delta, t_\mu, 1] > \frac{1}{2}(1+t_\mu) - \frac{q}{2(1-\alpha)}$$

We can then rewrite this as

$$\int_{t_\mu}^1 \delta(v)\,dv > \frac{1}{2}(1-t_\mu^2) - \frac{q(1-t_\mu)}{2(1-\alpha)}$$

Because t_μ varies from $1-q$ to 1 as μ varies from 0 to 1, we can think of this condition in terms of t_μ rather than in μ. A sufficient condition for this inequality to hold for all $t_\mu \in [1-q, 1]$ is found by solving for the function for which this condition just holds and then noting that so long as $\delta(\cdot)$ is always above this function, the condition will always hold. Doing this results in

$$\delta(v) > v - \frac{q}{2(1-\alpha)} \text{ for } v \in [1-q, 1]$$

■

Proof of Corollary 2:

As given by Proposition 4, the only equilibria involve students making the same resolution decisions. Given $c = 0$, it is a dominated strategy for students to not resolve their preference uncertainty (as it leads to a worse payoff in the case of universities disaggregating). A second application of Proposition 4 then provides the result.

■

A.1 Proof of Theorem 1:

As we normalize universities' and colleges' utilities for the number of universities n, the average utility of each university in equilibrium is the same regardless of n, so our strategy is to show that the utility to deviation is declining in n by showing that a deviating university receives a decreasing share of the higher quality students for which it is competing with other universities.

Consider university u_1 deviating. As shown in Section 3.1, the deviating university gets $\frac{1}{n}(1-(1-\alpha)b)^n$ of these students. As a portion of u_1's enrolled students, this goes to 0 monotonically as n increases.

Thus, as n increases, there is a set of (high) quality students, of which a deviating university university gains a smaller share. There is a second set of (middle) quality students which, since the threshold of the non-deviating universities is decreasing in n, a deviating university previously enrolled with probability 1 but is now competing for as n increases, and there is the set of (low) quality students which a deviating university continues to enroll with probability 1. Since the deviating university loses the potion of its utility coming from high quality students as n increases, the US equilibrium becomes easier to support.

We now find the necessary and sufficient condition for existence of the US equilibrium as $n \to \infty$. We know that a disaggregating university admits all students above quality $1-q$ and $t_n \to 1-q$ as $n \to \infty$. An important piece of the puzzle is the limiting utility of the disaggregating university. WLOG assume that u_1 disaggregates. To find this, we consider the proportion of students enrolling in u_1 that are not admitted to any other university. This quantity is:

$$\lim_{n\to\infty} \frac{t_n - (1-q)}{\frac{q}{n}} = \lim_{n\to\infty} n - \frac{n-1}{1-[1-(1-\alpha)b]^n \frac{1}{n}} = \lim_{n\to\infty} n - n + 1 = 1$$

The second equality comes from substituting in the value for t_n in Equation 4 of Section 3.1. This means that in the limit, the disaggregated university fills none of its seats with students who are admitted to the other universities. Since $t_n \to 1-q$ as $n \to \infty$, we see that the average utility u_1 receives from each student it enrolls approaches $1-q$ (from above). Therefore, plugging this average utility into the right hand side of the derivation from Proposition 1, the necessary condition is:

$$\frac{1}{q}\int_{1-q}^{1} [\alpha v + (1-\alpha)\delta(v)]\, dv > 1-q \tag{28}$$

Rearranging yields the desired result.

To derive the condition on δ that ensures that increasing q improves utility in equilibrium more than in deviation, take the derivative of both sides of Equation 4 in the main text. Rearranging, this becomes:

$$\delta(x) \geq \frac{x((2-\alpha-(1-(1-\alpha)b)^n)n - 1 + \alpha(1-(1-\alpha)b)^n) - (n-1)(1-(1-(1-\alpha)b)^n)}{(1-\alpha)(n-(1-(1-\alpha)b)^n)}$$

and taking the limit as $n \to \infty$ gives the desired results. Because university utility in equilibrium and in the potential deviation both change continuously in parameters α and b, if the condition holds and q is increased, then there is some ϵ such that if the US equilibrium exists for $\{\alpha, b\}$ then it exists for $\{\alpha-\epsilon, b+\epsilon\}$.

■

Proof of Proposition 5:

For the effect of increasing $\delta(\cdot)$, note that $\bar{v}$ is (weakly) decreasing in $\delta(\cdot)$, increasing the left-hand side of the necessary and sufficient inequality for the existence of the English equilibrium. For the US equilibrium, note that increasing $\delta(\cdot)$ has no impact on the value to deviating to disaggregating as it does not impact the threshold used by aggregating universities nor does it enter the utility of the deviating university. Increasing $\delta(\cdot)$ does increase the equilibrium university utility, establishing the claim.

Increasing α decreases the left-hand side of the necessary and sufficient condition for the English equilibrium. The result for the US equilibrium comes from Theorem 1; the right-hand side of the necessary and sufficient condition for the US equilibrium for sufficiently large n is decreasing in α. In slightly more detail, consider the condition for the US equilibrium

(Inequality 4). On the left-hand side, the marginal effect of α is constant at $\int_{1-q}^{1} v - \delta(v)\, dv$. On the right-hand side, α enters directly and in t_{-u_1}. In both cases, the marginal impact of increasing α goes to 0 (exponentially quickly). Thus, there exists an n large enough that the derivative of the left-hand side w.r.t. α is larger than that of the right-hand side.

Increasing b increases the left-hand side of the necessary and sufficient inequality for the existence of the English equilibrium. For the US equilibrium, increasing b increases the threshold used by aggregating universities in the presence of a deviator and decreases the probability that a student admitted to all universities will choose the disaggregating university, both of which decrease the utility to disaggregating; increasing b has no effect on the utility of universities in the US equilibrium. Moreover, note that b enters as $(1-(1-\alpha)b)^n$ in both the threshold and probability mentioned, so the effect of increasing b goes to 0 exponentially in n.

Decreasing c decreases the right-hand side of the necessary and sufficient inequality for the existence of the English equilibrium.

■

Proof of Proposition 6:

US equilibrium

As in the definition of stochastic stability, assume that the US system is supported by an equilibrium when the parameters are $h_{n,0,0}$. By Theorem 1, the US system is supported by an equilibrium for all parameters $h_{n',0,0}$ for $n' \geq n$ and the utility of aggregating can be compared to the utility of disaggregating given all other universities are aggregating and no students resolve their preferences as in Inequality 28, with the inequality holding strictly by the assumption of the definition of stochastic stability. Let Δ be the difference between the left and right hand sides of Inequality 28.

Our method is to show that for sufficiently large n, the corresponding comparison for the market with small mistakes can be made arbitrarily close to the inequality for the market with no mistakes by taking the mistakes sufficiently small. That is, we show that the limiting value as $n \to \infty$ of aggregating when all other universities intend to aggregate and the value to disaggregating when all other universities intend to aggregate can be made arbitrarily close to the no mistakes case by choosing r and p sufficiently small. As we know the inequality with no mistakes holds, so will the inequality for the market with mistakes.

As discussed in Lemma 2, all disaggregated universities make symmetric admissions decisions and all aggregated universities make symmetric admissions decisions. From Equation 9 we have the expected value (to an aggregated university) of a student of quality v_s admitted to m disaggregated universities and $n-m$ aggregated universities.

Consider what happens when a portion μ of universities are disaggregated.[53] By multiplying the expected value by $\frac{n}{n}$ and taking the limit as n goes to infinity, we can write the limit of the expected value conditional on enrollment to an aggregated university of students admitted only to their skill type at disaggregated universities as

$$\begin{aligned} EV[n,m,m,v_s] \to & \frac{(1-r)/(1-\mu)}{(1-r)/(1-\mu)+r(1-\alpha)/(1-\mu)+r\alpha}(\alpha v_s+(1-\alpha)\delta(v_s)) \\ & + \frac{r(1-\alpha)/(1-\mu)}{(1-r)/(1-\mu)+r(1-\alpha)/(1-\mu)+r\alpha}\delta(v_s) \\ & + \frac{r\alpha}{(1-r)/(1-\mu)+r(1-\alpha)/(1-\mu)+r\alpha}v_s \end{aligned} \tag{29}$$

As discussed in Lemma 2, the expected value conditional on enrollment to an aggregated university of students admitted to both colleges at disaggregated universities is

$$EV[n,m,0,v_s] = \alpha v_s + (1-\alpha)\delta(v_s) \tag{30}$$

The expected value for students admitted only to aggregated universities is (as always)

$$EV[n,0,0,v_s] = \alpha v_s + (1-\alpha)\delta(v_s) \tag{31}$$

We are showing that we can implement the US system, so the only disaggregating universities will be those making mistakes. By the law of large numbers, $\mu \overset{\text{prob}}{\to} p$ as $n \to \infty$. Lemma 2 provides an algorithm mapping the expected value conditional on enrollment for each student (here given by Equations 29-31) to the utility of each university. This mapping is continuous. As Equation 29 is a continuous function of μ, the continuous mapping theorem states that the utility of an aggregated university goes in probability to the output of algorithm provided in Lemma 2 with $\mu = p$ in Equation 29 as $n \to \infty$.

By the definition of convergence in probability, we can make the probability that the realized μ differs from p by more than ϵ' arbitrarily small by choosing n large enough. We can bound the worst payoff a university gets when the fraction of universities that disaggregate differs by more than ϵ' from p by 0. Likewise, by the continuity of the universities' utilities in μ, we can choose ϵ' small enough that the difference between its value at p and its infimimum over $[p-\epsilon', p+\epsilon']$ can be made arbitrarily small. Thus, we can focus on the case of comparing

[53] We ignore integer constraints as we can bound all relevant calculations by $\lceil \mu \cdot n \rceil$ and $\lfloor \mu \cdot n \rfloor$ and the difference between these bounds approaches 0 as $n \to \infty$.

utilities to aggregation and disaggregation when a fraction p of universities disaggregate because we can pick n large enough that a university's expected value to aggregating or disaggregating is arbitrarily close to the value when exactly p proportion of universities disaggregate. Call the n needed for this difference to be no greater than ϵ for either the aggregated or disaggregated utility $n_1(r, p, \epsilon)$.

Using the algorithm from Lemma 2, we can make v_2 (the quality such that the expected value conditional on enrollment of a student admitted to all universities but only to the college of her skill type at disaggregated universities is equal to $\alpha(1-q)+(1-\alpha)\delta(1-q)$) arbitrarily close to $1-q$ by taking (r, p) sufficiently small. We can do this because $EV[n, m, m, v_s]$ is increasing in v_s and $\lim_{r\to 0^+}\lim_{n\to\infty} EV[n, m, m, v_s] = \lim_{p\to 0^+}\lim_{n\to\infty} EV[n, m, m, v_s] = \alpha v_s + (1-\alpha)\delta(v_s)$.

As in the proof of Theorem 1, we can make the portion of a disaggregated university's seats taken by unresolved students arbitrarily close to 0 by picking n sufficiently large. Using the notation of the proofs of Lemma 2 and Proposition 6, we can thus get a disaggregated university's utility arbitrarily close to

$$\begin{aligned}\frac{r(1-\delta^{-1}(v_3))}{q}\left(\alpha\frac{1+\delta^{-1}(v_3)}{2}+(1-\alpha)Avg[\delta, \delta^{-1}(v_3), 1]\right)+\\ \frac{r\alpha(\delta^{-1}(v_3)-v_2)}{q}\left(\frac{v_2+\delta^{-1}(v_3)}{2}\right)+\\ \frac{q-r(1-\delta^{-1}(v_3))-r\alpha(\delta^{-1}(v_3)-v_2)}{q}\frac{v_3+v_2}{2}\end{aligned}$$

where the first line is the utility coming from students admitted to both colleges at all disaggregated universities and to all aggregated universities, the second line is the utility coming from resolved, consistent students admitted only to their skill type college at disaggregated universities and to aggregated universities, and the third line is the utility coming from students admitted only to their skill type college at disaggregated universities. Note that as $v_3 \in [1-q, v_2]$, $v_2 \to 1-q$ implies $v_3 \to 1-q$. By first taking r sufficiently small (which both sends v_3 and v_2 to $1-q$ and reduces the contributions of the first two lines) and then by taking n sufficiently large, we can make the utility to deviating to disaggregation arbitrarily close to $1-q$. Let the n large to ensure that the difference between the disaggregated university's utility and the approximation of that utility with it receiving only students admitted to disaggregated universities is less than ϵ be $n_2(r, p, \epsilon)$.

Likewise, we can approximate the value to aggregating by considering only students with quality above v_2. Again, as in the proof of Theorem 1, the fraction of unresolved students with qualities greater than v_2 that enroll in aggregated universities goes to 1 as $n \to \infty$.

Define $n_3(r,p,\epsilon)$ to be the n large enough to ensure that the difference between the value of admitting this set of students (who are also admitted to disaggregated universities) and their value when all but the resolved, consistent students and the resolved, inconsistent students with quality greater than $\delta^{-1}(v_3)$ are admitted only to aggregated universities is less than ϵ. As discussed above, we can pick r and p sufficiently small so that v_2 is arbitrarily close to $1-q$. We can also make the contribution of resolved students to an aggregated university's utility arbitrarily small by taking r sufficiently small. Then, we can pick p small enough that the utility contributed to aggregated universities by students with quality below v_2 is arbitrarily small. Combined, this means that we can first take (r,p) small and then $n_3(r,p,\epsilon)$ to make the difference between the LHS of Inequality 28 and the expected utility to choosing to aggregate less than ϵ.

Finally, we can first pick (r^*,p^*) small enough that the two differences mentioned at the end of the previous two paragraphs are both less than $\frac{\Delta}{8}$ and pick

$$n(r^*,p^*) = \max\{n_1(r^*,p^*,\frac{\Delta}{8}), n_2(r^*,p^*,\frac{\Delta}{8}), n_3(r^*,p^*,\frac{\Delta}{8})\}.$$

As a reminder, n_1 ensures that the difference between the actual expected value and the approximation assuming exactly p proportion of universities is small. n_2 ensures that the difference between the utility to disaggregating and the approximation of $1-q$ is small. n_3 ensures that the difference between the utility to aggregating and the approximation of admitting only unresolved students with quality above v_2 is small. Thus, we can first take (r^*,p^*) small and then take n large to ensure that the expected utilities to aggregating and disaggregating are sufficiently close to the LHS and RHS of Inequality 28 to ensure that the former is greater than the latter and that choosing to aggregate is optimal.

English equilibrium

We show that for any fixed $p>0$, we can find an r sufficiently close to 0 such that the English equilibrium does not hold for sufficiently large n, regardless of other parameters. This then implies the proposition.

For contradiction, suppose that the English equilibrium is stochastically stable. This first implies that all universities who do not make mistakes continue to choose to disaggregate, so that the only aggregated universities are those universities making mistakes. The aggregating universities have $pq+o(\frac{1}{n})$ seats to fill (where the $o(\frac{1}{n})$ term comes from the uncertainty over the number of universities which make a mistake, the variance of which goes to 0 as $n\to\infty$ by the LLN). Sharing the logic of the proof of the stochastic stability of the US equilibrium, the mass of students who are uncertain about their preferred major and most prefer an aggregating university is $r - o(\frac{1}{n})$ (where the $o(\frac{1}{n})$ comes from $\omega^a_{n,m}$ in Lemma 2, such that the probability that a student with unresolved preference uncertainty who is admitted to all colleges will enroll in an aggregated university goes to 1 when $\frac{m}{n}$ is held fixed). The students who learn their major preferences and are admitted to all colleges are equally likely to most prefer any university, so a $p + o(\frac{1}{n})$ fraction of them prefer an aggregated university over any disaggregated university. From this, it follows that the aggregated universities fail to fill all their seats with students of quality $v_s \geq \bar{v}$ with probability converging to 1 in n if

$$(r + (1 - r)p)(1 - \bar{v}) < pq \tag{32}$$

Rearranging, this becomes

$$1 + \frac{r - rp}{p} < \frac{q}{1 - \bar{v}} \tag{33}$$

For any fixed p, this inequality will hold for a sufficiently small r since the right hand side is strictly greater than 1 and the left hand side approaches 1 as $r \to 0$. Then the equilibrium breaks down because each student with quality above $\bar{v}$ can enroll in an aggregated university for which her expected utility draw is arbitrarily close to 1, meaning the benefit to learning her major preferences goes to zero but the cost is fixed at $c > 0$, so no student will choose to resolve.

■

A.2 Proofs of transfers results

We begin with a lemma underpinning our focus on the marginally unadmitted student in large markets. The lemma states that as the number of universities increases, the contributions of all students to a university's utility converges to the value the university would get from the marginally unadmitted student.

We write $R_u(v_s, \theta_s, f, d, \sigma_{-u}, p_s)$ to denote the (infinitesimal) expected equilibrium value that student s gives to aggregated university u conditional on attending, net of expected payment, where p reflects the vector of payments offered to s. Note that R_u implicitly depends on n through its terms and in the calculation of the expected value.

From previous lemmas, we know that exactly students with quality greater than or equal to $1 - q$ are admitted to a university, meaning that the marginally unadmitted student has quality $1 - q$ and expected value $R_u(1 - q, \theta_s, f, d, \sigma_{-u})$.

LEMMA **4.** *Let* $S'(\epsilon) = \{s | R(v_s, \theta_s, f, d, \sigma_{-u}, p_s) > R_u(1-q, \theta_s, f, d, \sigma_{-u}, p_s) + \epsilon\}$. *In any equilibrium in which all universities play the same pre-match action and for any* $\epsilon > 0$ *and* $\gamma > 0$*, there exists* N^* *such that for all* $n > N^*$,

$$|S'(\epsilon)| < \gamma.$$

Proof:

Suppose not. Clearly, no positive measure of students with $v_s \geq 1-q$ provides $R(v_s, \theta_s, r, d, \sigma_{-u}, p_s) < R_u(1-q, \theta_s, r, d, \sigma_{-u}, \sigma)$ since each such student would then be valued below replacement level. Therefore, no student with $v_s < 1-q+\epsilon$ is a member of $S'(\epsilon)$. Consider any university which admits students with quality $v_s < 1-q+\epsilon$ in the proposed equilibrium. Fix $\lambda < \epsilon$. Instead, this university make payments to some $s \in S'(\epsilon)$ such that $R(v_s, \theta_s, r, d, \sigma_{-u}, p_s) > R_u(1-q, \theta_s, r, d, \sigma_{-u}, p_s) - \epsilon + \lambda$, and withhold admissions offers from some students with qualities $v_s < 1-q+\epsilon$. Note that, similarly to Proposition 2 this university will enroll at least $\gamma + \frac{1}{n}(1-\gamma^n)$ of such students, yielding a profitable deviation. Since the capacity of each university is $\frac{q}{n}$, the university can fill an arbitrarily large portion of its seats through this method. Contradiction.

■

We pin down the limit of the payments used by universities as n grows in the following proof by using this lemma to know that the value from each student approaches the value of the marginally unadmitted student.

Proof of Proposition 7:

English equilibrium

Consider the on-path payments discussed in the text:

$$p_u^*(v_s, \theta_s, x_m) = \begin{cases} v_s - (1-q) & \text{if } x_m = \theta_s \\ \delta(v_s) - (1-q) & \text{if } x_m \neq \theta_s \text{ and } v_s \geq \bar{v} \end{cases}$$

To show that students resolve uncertainty if and only if

$$(1-\bar{v})(1-\alpha)\left(b - E\left[\min\{b, v_s - \delta(v_s)\} \,|v_s \geq \bar{v}\right]\right) \geq c$$

we follow the same logic of Proposition 2 and note that the benefit to a student s (with $v_s \geq \bar{v}$) by studying her preferred major when she is inconsistent is $b - (p^n_s(v_s, \theta_s, x_m = \theta_s) - p^n_s(v_s, \theta_s, x_m \neq \theta_s))$ as she loses $p^n_s(v_s, \theta_s, x_m = \theta_s) - p^n_s(v_s, \theta_s, x_m \neq \theta_s)$ in transfers by doing so. When $b < p^n_s(v_s, \theta_s, x_m = \theta_s) - p^n_s(v_s, \theta_s, x_m \neq \theta_s)$, s will be "bribed" to study her skill-type major θ_s, and so she derives no value from resolving her preferences over majors. Therefore, the expected benefit she receives from resolving her uncertainty over majors before enrolling in university is $b - E[\min\{b, p^n_s(v_s, \theta_s, x_m = \theta_s) - p^n_s(v_s, \theta_s, x_m \neq \theta_s)\} \,|v_s \geq \bar{v}]$. Noting that $p^n_s(v_s, \theta_s, x_m = \theta_s) - p^n_s(v_s, \theta_s, x_m \neq \theta_s) = v_s - \delta(v_s)$ verifies the desired condition.

Also note that no alternative payment scheme can weaken this condition. Lemma 4 implies that for all u and almost all s

$$p^n_s(v_s, \theta_s, x_m) \to \begin{cases} v_s - (1-q) & \text{if } x_m = \theta_s \\ \delta(v_s) - (1-q) & \text{if } x_m \neq \theta_s \text{ and } v_s \geq \bar{v} \end{cases}$$

Therefore, any payment scheme will fail to give students the incentive to resolve their major uncertainty if $(1-\bar{v})(1-\alpha)\left(b - E\left[\min\{b, v_s - \delta(v_s)\}\,|v_s \geq \bar{v}\right]\right) < c$.

It remains only to pin down equilibrium strategies following any history such that a single university switches to aggregation, and show that the deviator is worse off. We show this for two exhaustive cases. To do so, we define $v_b := b + 1 - q$, where as we discuss below, all inconsistent students with qualities $v_s < v_b$ will strictly prefer to study their preferred major conditional on attending the lone aggregating university.

Case 1: $v_b \geq \bar{v}$

We hypothesize that on the path of play following a lone aggregator, each disaggregated college will set an application quality threshold of $1 - q$. Under this hypothesis, let the payments offered by disaggregated universities in any history following a lone aggregator, $p^d(v_s, \theta_s, x_m)$ satisfy

$$p^d(v_s, \theta_s, x_m) = \begin{cases} v_s - (1-q) & \text{if } x_m = \theta_s \\ \delta(v_s) - (1-q) & \text{if } x_m \neq \theta_s \text{ and } v_s \geq \bar{v} \end{cases}$$

which, for sufficiently large n, satisfies sequential rationality as discussed in the main text. The lone aggregator selects $T_A \leq \bar{v}$ and admits all students with qualities weakly greater than $T_A(n)$ and makes payments $p^a(v_s, \theta_s, x_m)$ to all admitted students satisfying

$$p^a(v_s,\theta_s,x_m) = \begin{cases} v_s - (1-q) & \text{if } x_m = \theta_s \\ \delta(v_s) - (1-q) & \text{if } x_m \neq \theta_s \text{ and } v_s \geq \bar{v} \\ 0 & \text{if } x_m \neq \theta_s \text{ and } v_s < \bar{v} \end{cases}$$

$p^a(v_s,\theta_s,x_m)$ and $T_A < \bar{v}$ are sequentially rational; given $p^d(v_s,\theta_s,x_m)$ and the assumption that the admission threshold for disaggregated universities is $1-q$, the constrained maximization argument given in the text pins down $p^a(v_s,\theta_s,x_m)$ for all students s who study their skill types, and those who study against their skill types but have $v_s \geq \bar{v}$. A 0 payment to students s who study against their skill type and have $v_s < \bar{v}$ is also pinned down as these students are below replacement level; the aggregator suffers a "loss" from these students relative to the other students it enrolls, and so it will minimize payments to avoid attracting them.

Since all students have learned their preferred majors before enrolling, all students s satisfying one of the following conditions will select the same university, given the same choice set, as she does following the path of play (in which all universities disaggregate):

- $\rho_s = \theta_s$: such students receive the same payments from studying their preferred major, and
- $\rho_s \neq \theta_s$ and $v_s \geq \bar{v}$: such students receive the same payments from studying their preferred major.

The lone aggregator is more popular among students s for whom $\rho_s \neq \theta_s$ and $v_s < \bar{v}$. But note that each of these students gives the aggregated university a utility no greater than $1-q$. Therefore, there is no incentive to deviate.

To complete the argument, we must show that our assumption that the admission threshold of $1-q$ for disaggregated universities satisfies sequential rationality following a history where a single university aggregates. Because $\bar{v} < v_b$, the payment $p^a(v_s,\theta_s,x_m=\theta_s)$ for any s with $v_s < \bar{v} < v_b$ is strictly smaller than b, which follows from $p^a(v_s,\theta_s,x_m) = v_s-(1-q)$ being strictly decreasing in v_s. This implies that all inconsistent students with qualities $v_s \leq \bar{v}$ who enroll in the aggregated university study their preferred major. As argued above, these are the only students who are more likely to enroll in the deviating university compared to on the path of play.

As $n \to \infty$, $T_A(n) \to \bar{v}$, that is, the aggregating university fills its (remaining) seats disproportionately with students who have quality $\approx \bar{v}$ and are inconsistent. This follows by considering Equation 11 and sending $n \to \infty$.[54] By continuity of $\delta(\cdot)$ these students yield expected value converging in n to $\delta(\bar{v}) = 1-q$. Marginal students who are not admitted to any college or university have quality $1-q$, and therefore yield expected value $\alpha(1-q)+(1-\alpha)\delta(1-q)$ to the aggregating university. Because $\delta(1-q) < 1-q$, the aggregating university has no incentive to enroll the marginal students. Therefore, it must admit all students with quality weakly exceeding a cutoff $T_A(n)$.

Case 2: $\bar{v} > v_b$

Let N_1 be the smallest n such that the constraint in Equation 6 binds, that is, $N_1 = \left\lceil 1 - \frac{1}{2(1-q)} \right\rceil$. We partition universities into three groups: the lone aggregated university, N_1 disaggregated universities, and all remaining disaggregated universities. We call these sets A, D_1, and D_2, respectively.

We hypothesize that on the path of play following a lone aggregator, the lone aggregator A admits all students with $v_s \geq T_A$, where T_A solves

$$\frac{q}{n} = \frac{1-\bar{v}}{n} + \frac{\bar{v} - T_A}{N_1 + 1} \tag{34}$$

and offers the following transfers to all admitted students

$$p^A(v_s, \theta_s, x_m) = \begin{cases} v_s - (1-q) & \text{if } x_m = \theta_s \\ \delta(v_s) - (1-q) & \text{if } x_m \neq \theta_s \text{ and } v_s \geq \bar{v} \\ 0 & \text{if } x_m \neq \theta_s \text{ and } v_s < \bar{v} \end{cases}$$

All N_1 universities in the second partition element $u \in D_1$ admit all students s with $v_s \geq \bar{v}$ to both colleges, and all students s with $v_s \in [T_A, \bar{v})$ to her skill type college only, and offer the following transfers to all admitted students

$$p^{D_1}(v_s, \theta_s, x_m) = \begin{cases} v_s - (1-q) & \text{if } x_m = \theta_s \\ \delta(v_s) - (1-q) & \text{if } x_m \neq \theta_s \text{ and } v_s \geq \bar{v} \end{cases}$$

All $n - N_1 - 1$ universities $u \in D_2$ admit all students s with $v_s \geq \bar{v}$ to both colleges, and all students s with $v_s \in [1-q, T_A)$ to her skill type college only, ad offer the following transfers to all admitted students

[54] The exact condition replaces b with $b - (v_s - (1-q))$ in Equation 11 to account for payments students receive at disaggregating universities for studying their skill types. Because $b - (v_s - (1-q)) > 0$ for all $v_s \leq \bar{v}$ by assumption, the limit of $T_A(n)$ as $n \to \infty$ is $\bar{v}$.

$$p^{D_2}(v_s, \theta_s, x_m) = \begin{cases} v_s - (1-q) & \text{if } x_m = \theta_s \\ \delta(v_s) - (1-q) & \text{if } x_m \neq \theta_s \text{ and } v_s \geq \bar{v} \end{cases}$$

We note several important points. First, for sufficiently large n it must be that $T_A > v_b$. This follows from Equation 34 because $1 - \bar{v}$ and N_1 do not grow in n. Second, all colleges fill all of their seats according to the proposed equilibrium strategies. All colleges fill $1 - \bar{v}$ fraction of their seats with students of quality $v_s \geq \bar{v}$. T_A is defined to fill the remaining seats of all (colleges of) universities $u \in \{A\} \cup D_1$, and by the condition that there are q total measure of seats in the market, there are precisely the measure of students s with quality $v_s \in [1-q, T_A)$ to fill remaining seats at (colleges of) universities $u \in D_3$. Third, for $n \geq 2(N_1 + 1)$, $|D_3| \geq N_1 + 1$, that is, the constraint on payments in Equation 6 binds for all universities $u \in D_3$ (since there are strictly more than N_1 of them) and for universities $u \in \{A\} \cup D_1$ since $|\{A\} \cup D_1| > N_1$ and all such universities admit the same set of students. Fourth, all universities receive utility of exactly $1 - q$ from each admitted student. This is clear for all disaggregated universities. It is also true for the aggregated university because of the fact that $T_A > v_b$ for sufficiently large n; all students s admitted to the aggregating university with $v_s \in [T_A, \bar{v})$ are bribed to study their skill types.

The notes above show, in conjunction with our argument in the text of what actions are specified on-path in the English equilibrium, that these actions satisfy sequential rationality: there is not an alternative payment scheme, admitting any subset of students with qualities $v_s \geq 1-q$, that can make any university better off. Since each university receives utility $1-q$ from each admitted student, there is no unadmitted student with quality $v_s \geq 1 - q$ who is more attractive than any admitted student. But note that the utility of each university following this deviation is no higher (it is the same) as the utility of each university on path. Therefore, there is no incentive to deviate.

US equilibrium

By Lemma 4, on the path of play in the proposed equilibrium, the utility each university receives from almost every admitted student converges to $\max\{1 - q - b, \alpha(1-q) + (1-\alpha)\delta(1-q)\}$, where the admitting university bribes the marginal student to study her skill type by paying b contingent on her studying her skill type if and only if $1 - q - b > \alpha(1-q) + (1-\alpha)\delta(1-q)$.

Now suppose university u is the lone disaggregator. To satisfy sequential rationality, following this history for any n there must be some set $\hat{S}$ of students not admitted to any of the aggregated universities such that $|\hat{S}| = \frac{q}{n}$ and $v_s \geq 1 - q - \frac{q}{n}$ for all $s \in S$. Suppose the lone disaggregator admits all students $s \in \hat{S}$ under their skill type, and only students s with $v_s \geq \delta^{-1}(1 - q - \frac{q}{n})$ to both colleges and offers payments

$$p_n^d(v_s, \theta_s, x_m) = \begin{cases} v_s - (1 - q - \frac{q}{n}) & \text{if } x_m = \theta_s \\ \delta(v_s) - (1 - q - \frac{q}{n}) & \text{if } x_m \neq \theta_s \text{ and } v_s \geq \bar{v} \end{cases}$$

All students admitted only to the disaggregated university will enroll. As $n \to \infty$ this implies that the disaggregated university fills all of its seats (since $|\hat{S}| = \frac{q}{n}$) and it receives a limiting utility of at least $1 - q$ from each student it enrolls. Recall that each university receives limiting utility $\max\{1 - q - b, \alpha(1 - q) + (1 - \alpha)\delta(1 - q)\} < 1 - q$ from each student on the path of play in the proposed equilibrium. Therefore, there is a profitable deviation to disaggregation.

■

We now provide a corollary to Lemma 4, which gives corresponding results when caps are introduced. The proof is omitted.

Corollary **3.**

- *Suppose the US equilibrium exists with an unconditional transfer cap. Then the expected utility from almost every admitted student must either converge to $\alpha(1-q)+(1-\alpha)\delta(1-q)$ or the cap must bind regardless of which major is selected.*

- *Suppose the English equilibrium exists with unconditional transfer cap T_u. Then*

$$p_u^*(v_s, \theta_s, x_m) \to \begin{cases} \max\{T,\, v_s - (1 - q)\} & \text{if } x_m = \theta_s \\ \max\{T,\, \delta(v_s) - (1 - q)\} & \text{if } x_m \neq \theta_s \end{cases}$$

- *Suppose the US Equilibrium exists with a conditional transfer cap. Then the expected utility from almost every admitted student must either converge to $\alpha(1 - q) + (1 - \alpha)\delta(1 - q)$ or the cap must bind if the student studies her skill type.*

- *Suppose the English equilibrium exists with conditional transfer cap T_c. Then $p_u^*(v_s, \theta_s, x_m = \theta_s) \to \max\{T,\, v_s - (1 - q)\}$.*

Proof of Theorem 2:

The first claim follows from the first point of Proposition 8. We prove the second claim only for the US equilibrium, as the corresponding proof for the English equilibrium follows similar logic. Let $T^{'} = \epsilon + \max\{b, T + \frac{1-\alpha}{\alpha} \cdot b\}$ for some small $\epsilon > 0$. We first show that students are ex-ante better off under a conditional cap $T_c^{'}$ than an unconditional cap T_u. Note that an uncertain student weakly prefers receiving a transfer of $T^{'}$ if she studies her skill type to receiving a transfer of T regardless of the major she studies. By Corollary 3 students for whom an unconditional cap T_u does not bind will be indifferent in equilibrium between the two schemes, since the cap of $T^{'}$will also not bind. Those for whom the unconditional cap does bind (who recieve T regardless of major studied) will prefer the second scheme since $T^{'} \geq \max\{b, T + \frac{1-\alpha}{\alpha} \cdot b\}$. Therefore, every student is ex-ante better off. To show that universities are better off, note that since the US equilibrium exists, the average university utility from each student is weakly greater than $1 - q$ by Proposition 7. Therefore, the unconditional cap of T_u must bind for some positive measure of students S^T. Therefore, as $b \to 0$, $T^{'} \to T + \epsilon$. As in Part 1. of this Theorem, the total additional payments under the second scheme than the first is less that ϵq for sufficiently small b, while the benefit to the universities is bounded below by $(1 - \alpha) \int_{v \in S^T} v_s - \delta(v_s) dv$. Clearly for sufficiently small ϵ, $(1 - \alpha) \int_{v \in S^T} v_s - \delta(v_s) dv > \epsilon q$ which completes the proof.

■

Appendix: For Online Publication

B Specifics of the university admissions market

B.1 Stylized Facts of Admissions Systems

We now list several key differences in the US and English college admission systems. Although this list is by no means comprehensive, we believe that these are the key facts that drive the differences we explore in this paper.

Stylized Fact **1.** *English universities are* disaggregated *while US universities are* aggregated.

That is, as discussed above, students applying to English universities apply directly to a major and are required to study that major upon matriculation. In contrast, US universities allow students to wait until their junior year to select a major.

Stylized Fact **2.** *Entering English university students are more certain about their preferences over majors than US students.*

While we have been unable to find a survey getting exactly at this question,[55] we offer the following statistics.

- Every year since 2005, English survey (NSS) results show that over 80% of students in the final year of university are satisfied with their major. The most recent survey, in 2015, places this number at 85%.[56]
- US students
  - Only 42% of Illinois students who took the ACT reported that they were very sure about planned major (George-Jackson and Lichtenberger 2012).
  - One major US university reports that 80% of its first year students are uncertain about major.[57]

[55] Perhaps because the question of being happy with major choice does not seem relevant to surveyors in either country - in England, since students must select a major upon application, it would be odd to ask incoming students how uncertain they are over which major they are going to pursue; likewise, in the US, since students are often free to switch majors throughout their college careers, it would be odd to ask graduating students whether they would have liked to switch majors.

[56] http://www.hefce.ac.uk/lt/nss/results/2016/, 13 March 2017.

[57] https://dus.psu.edu/enrollment-dus, 13 March 2017.

- When asked at the beginning of college education, students only place probability .44 on graduating from the major they eventually receive a degree in (Stinebrickner and Stinebrickner 2011).
- Arcidiancono et al. (2012) find "Over 30% of [Duke students] who switched majors in their sophomore year did so in part because of their academic background."

The first statistic illustrates that English students are, by and large, content with the decision of major they made at application. The second set of statistics highlight that students in the US have not spent the time and resources to precisely know their exact preferences over majors.

Stylized Fact **3.** *English "high school" students specialize in a narrow area of study while US students do not.*

- English students have two years of subject specific study to prepare for A level college entrance exams, often at a "sixth form college." Admission is conditioned on the results of 3 tests that correspond to the proposed program of study. The exams that must be taken for each major are determined by the university.[58]
- US students applying to top universities have studied a broad range of subjects. Advanced Placement (AP) courses offered at high school culminate in national exams. Students report results to colleges as parts of application, and are often awarded college credit for good performances on the exams. However, US students take many AP courses and in many more areas. Nearly 60% of students take 4 or more exams, with some students taking 9 or more exams (Espenshade et al. 2005).

Stylized Fact **4.** *English university students rarely switch majors and US students frequently study a major other than their originally intended major.*

- 40.5% of US students switch majors from their original intentions (Adelman 2004). Deferring the selection of majors is of considerable value to students.[59]
- Only 7% of English students switch majors (Malamud 2010).

[58]For example, see http://www.ox.ac.uk/admissions/undergraduate/courses/entrance-requirements, accessed 13 March 2017.

[59]From Strange (2012): "I estimate that option value accounts for 14 percent of the total value of the opportunity to attend college for the average high school graduate and is greatest for moderate-aptitude students." This claim that the option value of switching majors is highest for moderate-aptitude students is supported in our theoretical model.

B.2 Brief historical discussion

What follows is not meant to be a definitive historical account of the development of the admissions programs in the United States and the England. Rather, it is meant to shed light on the plausibility of the market structures we model and to provide the reader insight into our understanding of the historical differences between the two systems.

Prior to the 1800s, the curricula of universities in the US and England were roughly the same - both countries' universities stressed a "classical education," focusing on classical languages (Latin and Greek) and theology. Over the course of the 19^{th} century, however, both systems faced shocks to demand and ideology. There was an increased desire for a more practical education (driven, in part, by the advent of the Industrial Revolution, as discussed in Moberley (2009)), as well as the new model of the modern research university pioneered by Wilhelm von Humboldt in the University of Berlin (Ashby 2009). The universities of the two countries differed in how they responded to these shocks, leading to the systems in place today - the US system with more of an emphasis on breadth and general knowledge versus the English system with an emphasis on earlier specialization. There are a multitude of possible (and non-exclusive) explanations for this difference in response, some of which we will discuss.

The transformation of US universities to the current major system can be ascribed to Harvard and University of Virginia, with the president of Harvard, Charles Eliot, playing a prominent role (much of the following is shaped by the discussion in Rothblatt (2009) and Kerr (2009)). The conceptualization of the liberal arts in the US as a breadth of knowledge directed by individual interests can be traced back to Thomas Jefferson, the Enlightenment, and the Scottish university system.[60] The University of Virginia, founded by Jefferson, experimented with allowing students to choose their courses, the beginnings of the "elective" system, though ultimately abandoned the system due to a lack of preparation on the part of the students (Rothblatt 2009). Eliot, as president of Harvard and in light of the declining importance of the university in American life, drew upon his experiences with various European university systems, particularly the German system developed by Wilhelm von Humboldt, and pushed to introduce electives at Harvard, eventually reaching a point where almost all of a Harvard undergraduate's courses were electives. As at the University of Virginia, it eventually became apparent that students were lost in this new world of complete academic freedom, due in part to the explosion in the number and variety of classes that were offered once professors were allowed to design their own courses. As a result, the system was slowly refined to become the major system of today (a mixture of

[60] A short introduction to the history of the Scottish system can be found in Walker (2009).

elective and prescribed classes).[61]

Of course, it is imaginable that the completely free elective system could have been refined to a more strictly specialized system. However, lack of specialization on the part of students (Rothblatt (2009) discusses lax and different standards at US high schools) likely prevented universities from effectively forcing students to focus their studies while remaining in competition for the best students. The governance structure of universities and high schools did little to change this dynamic. Government direction of universities was minimal following the Supreme Court case of Dartmouth College v. Woodward (Rothblatt 2009). The leadership of universities was usually delegated to a president elected by a Board of Trustees (and in some cases, such as Eliot as Harvard, the president would sometimes then act on his own judgment against the wishes of the Board). A president given autonomy in the direction of the university and given the objective of maximizing the overall competitiveness of the school (that is, to get the highest quality students), might well care less about perfectly satisfying exact major-by-major quotas and override department concerns about the presence of lower quality students switching into a major.[62] There was no central organization certifying universities to issue degrees; instead US universities grouped into region associations to handle such business. High schools were similarly decentralized, and remained relatively broad in terms of educational content. So, US high school students largely did not closely investigate potential majors, and universities competed to get the overall best of these students.

Compare this to England. As explained in Anderson (2009), the universities of England were much more centralized, and the government maintained more control over higher education in the 19^{th} century. For example, state grants and royal charters were needed to issue degrees. There was perceived competition between the England and Germany/Prussia. This national competition led to the National Efficiency movement, the belief that English society must organize itself to be as efficient as possible. It is easy to see why early specialization would appeal to planners in such a mindset. In such a case, the English government would favor students studying what they were good at rather than what they enjoyed most. Along with this cultural leaning, the top tier of English high schools focused secondary education on only a few topics. This system was underpinned by thinking such as that by Matthew Arnold. As a result of his work and the discussion around it, England was able to culturally "detach the idea of liberal education from its previous association with the classics: now it could be embodied by any subject if taught in a 'liberal'. This fitted in well with the

[61] For a much more detailed and thorough account of the evolution of the elective and major system in the United States, interested readers are directed to Rudolph and Thelin (1991).

[62] Further discussion on the extent of the power of the president in United States universities can be found in (Cohen and March 1986).

specialized, single-subject degree and the research ideal..." (Anderson 2009). The internal governance of English universities placed relatively more power with academic departments than the US colleges did (Shattock 2002). These academics likely cared more about knowing with certainty that they would be receiving the best students in their program, and ensuring that capacity constraints were respected across programs. Therefore, English universities had both the desire and an existing narrowly focused student body from which to fill their seats. In line with our model, it is likely there was little cost to requiring student specialization, and significant protection from adverse selection issues.

Nevertheless, governmental control over the English education system has since abated, and universities may have a high degree of autonomy in setting admissions policies. The Education and Reform Act of 1988 removed direct governmental control over secondary schools, instead placing control of the curriculum in the hands of the school. The Further and Higher Education Act of 1992 created the Higher Education Funding Council for England (HEFCE), a quasi-autonomous non-governmental organization, to oversee funding for English universities. The HEFCE provides a similar service as the National Science Foundation and National Institutes of Health in the US, although it is worth noting that the latter two are governmental organizations. Since 1993, admissions to English universities have been managed by the Universities and College Admissions Service, a private organization (although its predecessors have existed since 1961).[63] Perhaps the strongest piece of evidence to support this claim, however, is that three English universities have recently (or soon will) begin enrolling certain students in flexible degree programs; UCL, Kings, and Exeter have announced experimental aggregated degree programs.[64] Therefore, while the legal landscape of education has remained generally unchanged in England for more than 20 years, secondary schools are still creating specialized students, and (almost all) universities are still disaggregated. This suggests that while the English education system may have been created and enforced through governmental control at its onset, it has remained the same as the government has given up control because the system is in equilibrium.

[63] http://www.ucas.com/about-us/inside-ucas, 13 March 2017

[64] http://www.nytimes.com/2013/05/13/world/europe/in-britain-a-return-to-the-idea-of-the-liberal-arts.html, 13 March 2017

C Theoretical Appendix and Extensions of Model

C.1 Example of Market in which US and English Equilibria Exist

The following example gives parameter values for which both the US and English equilibria exist for $n = 2$ universities. In light of Theorem 1, the following parameters support both equilibria for any $n \geq 2$.

Example **2.** : *There are two universities. Let* $\alpha = \frac{3}{4}$, $b = \frac{1}{2}$, $q = \frac{1}{2}$, $\delta(v_s) = v_s^2$, *and* $c = \frac{3}{100}$.

We need to show that the utility of a university in the US equilibrium is higher than from disaggregating and that the benefit to students from resolving uncertainty is greater than the cost in the English equilibrium. From Proposition 1, the former boils down to showing that

$$\frac{1}{2}\int_{\frac{1}{2}}^{1}\left(\frac{3}{4}v + \frac{1}{4}v^2\right)dv \geq (1-(1-\alpha)\,b)^2\left(\frac{1}{2}\right)\int_{t_{u_1}}^{1} v dv + \int_{1-q}^{t_{u_1}} v\,dv$$

where $t_{u_1} = 1 - \frac{\frac{1}{2}}{2\left(1-\left[\left(1-\frac{1}{4}\cdot\frac{1}{2}\right)^2\left(\frac{1}{2}\right)\right]\right)} \approx .589$. Evaluating, the left hand side $\approx .17708$ and the right hand side $\approx .17636$. Therefore, these parameters support the equilibrium.

From Proposition 2, the latter is equivalent to showing $(1-\bar{v})(1-\frac{3}{4})\frac{1}{2} \geq \frac{3}{100}$ where $\bar{v} = \delta^{-1}(\frac{1}{2}) = \sqrt{\frac{1}{2}} \approx .707$. Evaluating, the left hand side $\approx .037$ so the inequality holds, meaning that these parameters support the English equilibrium.

$\blacksquare$

C.2 Example of Parameters with Mixed Aggregation Equilibrium for Small n but not Large n

Example **3.** : *Consider the parameters* $b = 1/(1-\alpha)$, $q = 0.3$, $\alpha = 0.5$,[65] $c > 1$, *and*[66]

[65] To fully comply with the assumptions in the paper, one can consider $\alpha = 0.5 + \epsilon$ for ϵ close to 0 with no lose.

[66] As in Example 1 one can consider absolutely continuous functions that in the limit approach the step function given here. Step functions allow for clearer examples.

$$\delta(v) = \begin{cases} 0 & v \in [0, .7), \\ 0.6 & v \in [0.7, 0.8), \\ 0.61 & v \in [0.8, 0.9), \\ 0.76 & v \in [0.9, 1] \end{cases}$$

There exists $n^* > 0$ such that for all $n > n^*$, the unique equilibrium implements the US outcome but there is an $n < n^*$ for which both the US outcome is implemented as an equilibrium and a mixed aggregation equilibrium exists.

The "small" n we will focus on is $n = 3$. For the "large" n, we will consider the limit case as discussed in the proofs of Corollary 1, Theorem 1, and Proposition 6.

$b = 1/(1-\alpha)$ implies that no student admitted to an aggregated university would rather attend a disaggregated university. Thus, the aggregated universities use a threshold of 0.8 when 1 university is disaggregated and the aggregated university uses a threshold of 0.9 when 2 universities are disaggregated.

$c > 1$ ensures students do not learn their preference types.

Using the notation of Proposition 4, we have

$$U^d_{3,1} = 0.75,\ U^d_{3,2} = 0.8,\ U^d_{3,3} = 0.85$$

and (with slightly more calculations)

$$U^a_{3,0} = 0.753333,\ U^a_{3,1} = 0.7925,\ U^a_{3,2} = 0.855$$

As $U^d_{3,1} < U^a_{3,0}$, the US outcome is supported by an equilibrium. As $U^d_{3,2} > U^a_{3,1}$ but $U^d_{3,3} < U^a_{3,2}$, so is the mixed aggregation equilibrium with 2 universities disaggregating.

However, as $n \to \infty$, the decision of any one university to aggregate or disaggregate has a vanishing influence on the threshold used by the aggregated universities, so it suffices to check that, for all $t \in [0.7, 1]$,

$$\frac{1}{t-0.7}\int_{0.7}^{t} v\, dv \geq \frac{1}{1-t}\int_{t}^{1} \alpha v + (1-\alpha)\delta(v)\, dv$$

It suffices to check at the jump points of δ, which is verified by the utilities given above along with the facts that the left hand side limits to 0.7 as $t \to 0.7$ and the right hand side limits to 0.88 as $t \to 1$.

■

C.3 Yield management

One area that our model abstracts away from is the question of yield management. Due to the imposed symmetry the student type distribution and colleges sizes, all universities set equilibrium admissions cutoffs that do not depend on the field of study and skill type of the student and are able to fill all seats at both colleges. However, without this symmetry, it may be more difficult for universities to fill the desired proportion of seats across its colleges. Here, we consider a minimal modification to our model which illustrates the strategic impacts of asymmetry, and the effects asymmetry has on equilibrium efficiency and enrollment.

We modify our model to remove the symmetry between majors. That is, we assume $\gamma \geq \frac{1}{2}$ measure of students have skill type M and $1-\gamma$ measure of students have skill type L. The case in which $\gamma = \frac{1}{2}$ reduces to our base model. As before, each college has equal capacity, and there is a prohibitively large cost for over-enrollment.

The other change to our model is that aggregated universities observe the skill types of students when making admissions decisions. That is, in the third stage of the game, each aggregated university u also observes θ_s for each student s and selects an admissions function $a^u(d, v_s, \theta_s) \in \{\emptyset, \{x_M(s,u), x_L(s,u)\}\}$, which maps (d, v_s, θ_s) to either no contract involving s and u or both contracts involving s and u. Note that if we did not allow aggregated universities to alter their admissions decisions according to skill type, the key findings below would not be meaningfully altered. There are no other changes to the model.

Existence of equilibrium

We first show that $\gamma > \frac{1}{2}$ does not make it easier (or harder) to support either the US or English equilibria than in our base model. That is, as in the text of the paper, the set of parameters that supports either equilibria does not shrink (grow) in the presence of this asymmetry.

We begin with the US equilibrium and hypothesize that there is an outcome in which each university fills all of its seats on path. We show that depending on parameters, it may be easier or more difficult to sustain the US equilibrium. We later verify that our condition for the existence of equilibrium does not conflict with the hypothesis that each university fills all of its seats on path.

On path, each university u sets thresholds T_u^M and T_U^L and admits all students s with $\theta_s = M$ iff $v_s \geq T_u^M$ and admits all students s with $\theta_s = L$ iff $v_s \geq T_u^L$. Following the logic of Lemma 1, it will be the case that for all $u, u' \in N$, $T_u^M = T_{u'}^M$ and $T_u^L = T_{u'}^L$. Therefore, we will refer to these values as T^M and T^L, respectively

As before, each student is consistent with probability α and each admitted student studies her most-preferred major. The hypothesis that each university fills all of its seats implies that

$$\gamma\alpha(1-T^M) + (1-\gamma)(1-\alpha)(1-T^L) = \frac{q}{2}$$

$$\gamma(1-\alpha)(1-T^M) + (1-\gamma)\alpha(1-T^L) = \frac{q}{2}$$

where the first equation equates the measure of admitted M skill type students who are consistent and the admitted L skill type students who are inconsistent to the measure of seats at M colleges, and the second equation equates the measure of admitted M skill type students who are inconsistent and the admitted L skill type students who are consistent to the measure of seats at L colleges.

Solving these equations simultaneously yields

$$T^M = 1 - \frac{q}{2\gamma}, \quad T^L = 1 - \frac{q}{2(1-\gamma)} \tag{35}$$

From this it can be seen that $T^M \geq 1-q$ and $T^L \leq 1-q$; the admission threshold is higher for M skill type students than in the model without asymmetry and lower for L skill type students.

Using these cutoffs, we see that each university receives utility

$$\frac{1}{2q}\int_{T^M}^{1} [\alpha v + (1-\alpha)\delta(v)]\, dv + \frac{1}{2q}\int_{T^L}^{1} [\alpha v + (1-\alpha)\delta(v)]\, dv$$

on equilibrium path.

Now consider a lone disaggregating university u. Following the logic of Theorem 1, the deviator fills a vanishing portion of its seats with students who are admitted to aggregated universities as $n \to \infty$. That is, the average quality of students enrolling in college M_u converges to T^M and the average quality of students enrolling in college L_u converges in n to T^L. Therefore, there exists n^* such that the US equilibrium (with full enrollment) exists if and only if

$$\frac{1}{q}\int_{T^M}^{1}[\alpha v+(1-\alpha)\delta(v)]\,dv+\frac{1}{q}\int_{T^L}^{1}[\alpha v+(1-\alpha)\delta(v)]\,dv>T^M+T^L \tag{36}$$

This condition is analogous to that in our base model, Inequality 28. Noting that $T^L < 1-q < T^M$ for $\gamma > \frac{1}{2}$, by inspection we can see that for the same parameter values it can either be the case that Inequality 28 is satisfied but Inequality 36 is not, or vice versa. Therefore, the US equilibrium is not always easier or harder to support in the presence of asymmetry.

One assumption we have maintained throughout this asymmetry analysis is that all colleges fill all of their seats on path. We now provide a necessary condition for this to be true. For $\gamma > \frac{1}{2}$ it must be the case that $T^M > T^L$.

To see this, suppose not. Then there is a larger measure of students studying L than M, which means either both colleges are underenrolled (which cannot occur in equilibrium, as each university could admit additional students to improve its utility) or there are exactly $\frac{q}{2}$ students studying M and strictly fewer studying L. In the latter case, consider the following deviation: one university u sets $T_u^M = T^M + \epsilon$ and sets $T_u^L = T^L - \beta$ such that the M college fills all of its seats. Compared to other universities, university u admits a $\frac{\epsilon\gamma}{n}$ smaller measure of M skill type students, and a $\beta(1-\gamma)$ larger measure of L skill type students. Each admitted student studies her skill type with probability α. Therefore, ϵ and β must satisfy

$$\frac{\epsilon\gamma}{n}\alpha = \beta(1-\gamma)(1-\alpha)$$

which yields $\beta = \frac{\epsilon}{n}\frac{\gamma}{1-\gamma}\frac{\alpha}{1-\alpha}$. Note that under this deviation, $\beta(1-\gamma)\alpha - \frac{\epsilon\gamma}{n}(1-\alpha) > 0$ L skill type students are admitted. Because the marginal L skill type student has a weakly higher expected value than the marginal M skill type student (under the assumption that $T^M \leq T^L$) for sufficiently small ϵ, this deviation strictly increases u's utility. Contradiction.

Having now shown that $T^M > T^L$ in equilibrium, we consider whether the full-enrollment cutoffs in Equation 35 are satisfied in equilibrium. It must be the case that the M college is filling all of its seats, otherwise a university could admit more M skill type students and (if necessary) reject more L skill type students while increasing its utility, as in the preceding argument. Therefore, the remaining case is one in which the L colleges are not full. Further, it must be that $T^M \leq 1 - \frac{q}{2\gamma}$ and $T^L \geq 1 - \frac{q}{2(1-\gamma)}$, the respective admissions full-enrollment thresholds, because if $T^M > 1 - \frac{q}{2\gamma}$, the T^L required to fill all seats in the M college will overfill the L college.

However, it is the case that $T^M > T^L$, so the marginal M skill type student has a higher expected value than the marginal L skill type student. Consider $T^M = 1 - \frac{q}{2\gamma} - \epsilon$. Compared to the full-enrollment thresholds, an extra $\alpha\gamma\epsilon$ measure of M preference type students are enrolled, and an extra $(1-\alpha)\gamma\epsilon$ measure of L preference type students are enrolled. To balance out capacity in M colleges, it must be that $T^L = 1 - \frac{q}{2(1-\gamma)} + \beta$ where β solves $(1-\alpha)(1-\gamma)\beta = \alpha\gamma\epsilon$, that is

$$\beta = \frac{\alpha\gamma}{(1-\alpha)(1-\gamma)}\epsilon$$

We claim that the American equilibrium will feature under enrollment if and only if

$$\frac{\alpha(1-\frac{q}{2\gamma}) + (1-\alpha)\delta(1-\frac{q}{2\gamma})}{\alpha(1-\frac{q}{2(1-\gamma)}) + (1-\alpha)\delta(1-\frac{q}{2(1-\gamma)})} < \frac{\alpha}{1-\alpha} \tag{37}$$

To see this, let $\epsilon \to 0$. Universities gain utility

$$\gamma\epsilon\left[\alpha(1-\frac{q}{2\gamma}) + (1-\alpha)\delta(1-\frac{q}{2\gamma})\right]$$

from marginally admitted M skill type students, and lose utility

$$\beta(1-\gamma)\left[\alpha(1-\frac{q}{2(1-\gamma)}) + (1-\alpha)\delta(1-\frac{q}{2(1-\gamma)})\right]$$

from marginally unadmitted L skill type students. The larger is former than the latter if Inequality 37 is satisfied, implying a profitable deviation. Note that we have not explicitly calculated the admission thresholds for a single deviating university. To do so, we would need to scale β up by a factor of n to account for the fact that students admitted to all university will select the deviating university with probability $\frac{1}{n}$. As we are considering the limiting case $\epsilon \to 0$, the arguments made here are unchanged as we are considering the expected value of the marginal student.

If Inequality 37 is satisfied, it implies that universities maximize their utility with cutoffs given in Equation 35. Note also that the difference between the expected value of the marginally admitted M skill type student and the marginally admitted L skill type student is maximized under the full enrollment thresholds. Therefore, if $T^M < 1-\frac{q}{2\gamma}$ then universities would be better off setting cutoffs of $T^M + \epsilon$ and $T^L - \beta$ as defined above, for some small ϵ.

We summarize these findings in the following propositions.

Remark **1.** *There exists $\gamma > \frac{1}{2}$ and other parameters such that the US outcome is supported in equilibrium. Let $\gamma > \gamma' \geq \frac{1}{2}$. The set of other parameters that supports the US outcome* *is neither always larger or always smaller for γ than γ'.*

Remark **2.** *Suppose the American equilibrium exists. Then all colleges fill all seats if and only if Inequality 37 is satisfied.*

We now consider the English equilibrium. On path, the admissions cutoffs for each college need not be the same as in the US equilibrium; because students all study at their admitting college, every college fills all of its seats when $\delta(\cdot)$ is strictly increasing, as any college that does not fill all of its seats can slightly decrease its admissions threshold and admit more students without violating its capacity. In general, we know that $T^M > T^L$ when $\gamma > \frac{1}{2}$. If not then either M colleges do not fill all of their seats (in which case they can profitably admit more students without violating capacity) or all colleges are overfilled (in which case they can profitably reject students to remain under the capacity).

Moreover, $T^M > 1 - q > T^L$ with at least one inequality strict when $\gamma > \frac{1}{2}$.To see this, if $T^L \geq 1 - q$ then fewer than q measure of students are admitted to colleges, meaning that either M colleges or L colleges have not filled their seats. If $1 - q \geq T^M$ then more than q measure of students are admitted to colleges, meaning that either M colleges or L colleges overfill their seats.

We show that the English outcome is neither easier nor harder to sustain in equilibrium for $\gamma > \frac{1}{2}$ compared to $\gamma' = \frac{1}{2}$ by way of two examples.

First, suppose that $\delta(v) = (1 - q)v$ for all $v \in [0, 1]$. Then when $\gamma = 0$, $\delta^{-1}(1 - q) = 1$, that is, a zero measure of students can study against their skill type. For any $c > 0$ there is therefore no English equilibrium. For any $\gamma > \frac{1}{2}$ we have already argued that $T^M > 1 - q > T^L$. This means that no L skill type student will be able to study against her skill type (since $\delta^{-1}(T^M) > \delta^{-1}(1 - q) = 1$, where the inequality follows from monotonicity of $\delta(\cdot)$), however, some M skill type students with qualities sufficiently close to 1 will be admitted to the L college. Therefore, each student will be willing to resolve her uncertainty over majors before matching if and only if

$$\gamma(1 - \delta^{-1}(T^L))(1 - \alpha)b \geq c$$

Note that this is analogous to the condition in the base model where $\gamma = \frac{1}{2}$, where the multiplicative γ term arises because only M skill type students (of sufficiently high score) are admitted to the college that does not match their skill type. Send $c \to 0$. This does not affect T^M or T^L on path, but for sufficiently low c this condition will hold, implying existence of the English equilibrium.

Now suppose that $q > \frac{1}{2}$, and $c \leq (1 - \delta^{-1}(1 - q))(1 - \alpha)b$, that is, the English equilibrium exists when $\gamma = \frac{1}{2}$. Furthermore, $\delta(v) = 1 - q$ for all $v > 1 - q$, that is $\delta^{-1}(t) = 1 - q$ for all

$t > 1-q$.[67] When $\gamma = \frac{1}{2}$, this implies that all students admitted to any college are admitted to all colleges, and will study their favorite major on equilibrium path.

With $\gamma > \frac{1}{2}$ it must be the case that $T^M > 1-q$, if not then, $T^L < T^M$ implies that more than q students are admitted to colleges, implying at least one college is overenrolled. Therefore, $\delta^{-1}(T^M) = 1$. For sufficiently small γ it must be the case that $T^L > 1-q$. To see this, note that if $T^L \leq 1-q$ then $\delta^{-1}(T^L) \leq 1-q$ implying that all inconsistent M skill type students admitted to M colleges are admitted to the L colleges. Therefore, $\gamma\alpha(1-T^M) = \frac{q}{2}$, i.e.

$$T^M = 1 - \frac{q}{2\gamma\alpha}$$

This implies that T^L solves

$$\gamma(1-\alpha)(1-T^M) + \gamma(T^M - T^L) + (1-\gamma)(1-T^L) = \frac{q}{2}$$

As $\gamma \to \frac{1}{2}$, $T^M \to 1-q$ from above. Therefore, the first term converges to $(1-\alpha)\frac{q}{2}$. If $T^L \leq 1-q$ in the limit then the third term converges to $\frac{q}{2}$. Therefore, L colleges enroll strictly greater than $\frac{q}{2}$ measure of students, since $(2-\alpha)\frac{q}{2} > \frac{q}{2}$. Contradiction. Therefore, it must be the case that $T^L > 1-q$ for sufficiently small γ.

Therefore, no student can study against her skill type, regardless of her quality. This implies that for any $c > 0$, the English equilibrium does not exist (for sufficiently small $\gamma > \frac{1}{2}$.

Remark **3.** *There exists $\gamma > \frac{1}{2}$ and other parameters such that the English outcome is supported in equilibrium. Let $\gamma > \gamma' \geq \frac{1}{2}$. The set of other parameters that supports the English outcome is neither always larger or always smaller for γ than γ'.*

We end this section by making a small note. An important finding in the results for the English equilibrium are that students do not know their skill types before deciding whether or not to resolve their uncertainty over majors. If students know their skill types before this decision in our base model where $\gamma = \frac{1}{2}$, nothing changes as all colleges set a threshold of $1-q$ on path. However, with $\gamma > \frac{1}{2}$, students who know their skill type is L before deciding to resolve their major uncertainty would be less inclined to do so than when $\gamma = \frac{1}{2}$ because $T^M \geq 1-q$. Therefore, if skill types are known ex-ante by students, the English equilibrium

[67] This example serves two purposes. It shows that when $\delta(\cdot)$ is not strictly increasing, it need not be the case that all colleges fill all of their seats in the English equilibrium. Second, it shows that the English equilibrium may be unsustainable when $\gamma > \frac{1}{2}$ but it is sustainable when $\gamma = \frac{1}{2}$. A very slight modification to this example shows that the latter conclusion holds even when $\delta(\cdot)$ is strictly increasing.

becomes strictly harder to support as γ increases. However, this would also potentially create equilibria in which only M students resolve major uncertainty on equilibrium path.

Welfare

In general, students no longer Pareto prefer the US equilibrium to the English one when $\gamma > \frac{1}{2}$. This is because it is no longer the case that the same set of students are enroll at universities. For example, when the US equilibrium involves the L college having seats open, some students are better off in the English equilibrium as it results in all seats being filled.

Indeed, if the US equilibrium involves empty seats, then (assuming $\delta(\cdot)$ is strictly increasing so that all colleges fill all of their seats in the English equilibrium) students will ex-ante prefer the English outcome to the US equilibrium for sufficiently small c.

Remark **4.** *If Inequality 37 is satisfied then there exists $C > 0$ such that for all $c < C$, students ex-ante prefer the English equilibrium outcome to the US equilibrium outcome.*

Robustness of symmetric model

This section has highlighted differences caused by asymmetry in the university admissions game. There are few reasons to believe that the symmetry we assume is an exact depiction of the strategic environment. Nevertheless, symmetry yields a cleaner model and illuminates important strategic considerations in this game. We now show that such a modeling choice may be justified; assuming the level of asymmetry is not large, the (expected) utility each agent receives in our base model is approximately equal to that in the model with asymmetry.

As in the Proof of Lemma 1, let r be the proportion of students who have resolved uncertainty, $d_u \in \{agg, disagg\}$ represent whether university u has disaggregated or not, and d represent the vector of such decisions for every university.

Remark **5.** *On equilibrium path given r and d, each agent's expected utility is continuous in γ.*

This result holds by similar arguments as presented above: each disaggregated college fills all of its seats, and each aggregated university fills all of its seats at the M college. As $\gamma \to 0$, the prohibitively large cost implies that admissions cutoffs for each college and university converge to that under $\gamma = \frac{1}{2}$. This implies that given any r and d arising on equilibrium path when $\gamma = \frac{1}{2}$ such that 1) each prescribed has a strict incentive to follow its prescribed aggregation action, and 2) each student has a strict incentive to follow her prescribed resolution action, there exists $\frac{1}{2} < \Gamma$ such that for all $\gamma \in \left(\frac{1}{2}, \Gamma\right)$ the same outcome is sustainable by an equilibrium.

C.4 Additional Results with Monetary Transfers

We next consider the equilibrium effects of holding the cap level fixed, but changing from a conditional cap to an unconditional one. Students are more likely to be bribed to study their skill types with a conditional cap. Recalling that the English equilibrium can be sustained if students are willing to pay the cost to resolve their uncertainty, it thus becomes more difficult to maintain the English equilibrium with a conditional cap. On the other hand, with a conditional transfer cap, universities in the US equilibrium are better off as they no longer provide spillover payments to students who study against their skill types. Therefore, it becomes easier to maintain the US equilibrium with a conditional cap than an unconditional one.

PROPOSITION **8.**

1. *In both the US and English equilibria, more students study their skill types with a conditional cap of T than an unconditional cap of T.*

2. *The English equilibrium is easier to sustain with an unconditional cap of T than a conditional cap of T.*

3. *The US equilibrium is easier to sustain with a conditional cap of T than an unconditional cap of T.*

Proof of Proposition 8:

1. Inconsistent students will be bribed to study against their skill types when $p_u(v_s, \theta_s, x_m = \theta_s) - p_u(v_s, \theta_s, x_m \neq \theta_s) \geq b$. The fact that more students study their skill types with a conditional transfer cap in the English equilibrium follows from point 2. In the US equilibrium, it follows from Corollary 3 that $p_u^{T_c}(v_s, \theta_s, x_m = \theta_s) \geq p_u^{T_u}(v_s, \theta_s, x_m = \theta_s)$ and by non-negativity of transfers $0 = p_u^{T_c}(v_s, \theta_s, x_m \neq \theta_s) \leq p_u^{T_u}(v_s, \theta_s, x_m \neq \theta_s)$.

2. The set of parameters that support the English equilibrium is decreasing in $p_u(v_s, \theta_s, x_m = \theta_s) - p_u v_s, \theta_s, x_m \neq \theta_s)$. Note that $p_u^{T_c}(v_s, \theta_s, x_m \neq \theta_s) = 0$ by definition, and $p_u^{T_u}(v_s, \theta_s, x_m = \theta_s) = p_u^{T_c}(v_s, \theta_s, x_m = \theta_s)$ by Corollary 3. Therefore, $p_u^{T_c}(v_s, \theta_s, x_m = \theta_s) - p_u^{T_c}(v_s, \theta_s, x_m \neq \theta_s) \geq p_u^{T_u}(v_s, \theta_s, x_m = \theta_s) - p_u^{T_u}(v_s, \theta_s, x_m \neq \theta_s)$ for all s with strict inequality for some positive measure of students.

3. In the US equilibrium, by Corollary 3 universities make lower payments under a conditional cap as they cannot pay students to study against their skill types, meaning

that the cap binds more often. Therefore, with a conditional cap universities are made better off. As universities must receive average utility of at least $1-q$ from enrolled students to support equilibrium (Proposition 7), the set of parameters that admits the US equilibrium is larger under an unconditional cap than a conditional cap.

■

The following proposition discusses the effects of changing the dollar value of a transfer cap. All of the comparative static results in this section are contingent on the (continued) existence of equilibria. With unconditional transfers, US university utility is (weakly) decreasing in the level of the cap, because the transfer spill over effect results in higher equilibrium payments. Payments increase in the level of the cap in the English equilibrium as well. However, English universities may be able to profitably bribe more inconsistent students to study their skill types making them better off. Switching to conditional transfers, it is easy to see that a cap of 0 or b is optimal for universities in either equilibrium–a cap of 0 will lead to zero payments, while a cap of b is the cheapest cap that will allow universities to bribe any sufficiently valuable inconsistent student to study her skill type.

Proposition **9.**

1. *In the English equilibrium with an unconditional transfer cap, university utility is non-monotonic in the level of the cap.*

2. *In the US equilibrium with an unconditional transfer cap, university utility is non-increasing in the level of the cap.*

3. *The university optimal conditional transfer cap in either equilibrium is either* 0 *or* b.

4. *Student utility is non-decreasing in the level of either type of cap in both the US and English equilibria.*

Proof of Proposition 9:

1. Suppose that $\delta(v_s)-(1-q)<\epsilon$ for $v_s \in [\bar{v}, v^{'})$ and $v_s-(1-q)>b+\epsilon$ for all $v_s>1-q$. Suppose the transfer cap is initially $T=b+1-q-\epsilon$ and it changes to $T^{'}=b+1-q+\epsilon$. Note that the total additional payments that each university must make is bounded above by $\int_{1-q}^{1} 2\epsilon dv = 2\epsilon q$, which occurs if the university has to pay every student 2ϵ more in transfers. But note that with the higher cap universities are able to bribe all students with $v_s \in [\bar{v}, v^{'})$ to study their skill types since $v_s-\delta(v_s)>b$, whereas they

were not allowed to make sufficiently high payments under the lower cap to bribe these students. Therefore, each university gains at least utility $(1-\alpha)\int_{\bar{v}}^{v'} v_s - \delta(v_s)dv > (1-\alpha)(v' - \bar{v})b$ when the cap increases by bribing inconsistent students to study their skill types. Choosing $\epsilon < \frac{(1-\alpha)(v'-\bar{v})b}{2}$ means that the benefit of increasing the cap outweighs the cost, and for sufficiently small c, the English equilibrium will continue to exist (see Proposition 7). On the other hand, it is easy to see that increasing the cap can harm universities. Simply let $b > 1$, meaning that universities will never be able to bribe students to study against their skill types. As a result, payments weakly increase to all students as the cap increases, making universities no better off.

2. Again, suppose the unconditional transfer cap increases from T to T'. There are three types of students to consider:

    (a) Those for whom the cap of T did not bind, and the cap of T' does not bind. Then the expected payments to these students is unchanged, and their major choice is unchanged.

    (b) Those for whom the cap of T binds, and the cap of T'does not bind. Then expected payments rise. Since cap T binds, each of these students are worth more than $(1-\alpha)(1-q) + (1-\alpha)\delta(1-q)$ to universities in equilibrium. Since cap T' does not bind, it must be the case that these students are worth exactly $(1-\alpha)(1-q) + (1-\alpha)\delta(1-q)$ to universities in equilibrium. Therefore, the universities receive less utility from these students as T rises to T'.

    (c) Those for whom the cap of T binds, and the cap of T'binds. Then clearly, payments increase. Since the cap binds, these students are paid T regardless of chosen major under the first cap, and $T' > T$ regardless of chosen major under the second cap. In both cases, students study the same major, since the payments are identical across majors, and the universities must pay more, and therefore, receive less utility from these students.

    These three cases are exhaustive, showing that universities cannot be made better off in the US equilibrium when an unconditional cap increases.

3. Note that when a conditional transfer cap of b binds, students will study their skill type in equilbrium. Therefore, for any cap $T \geq b$ if a university wishes to bribe a student to study her skill type, she is able to do so. Clearly, utility is dropping as the cap increases over b. Similarly, it is clear that the utility of universities is decreasing

over $[0, b)$ as students cannot be bribed, but payments are increasing. Therefore, the optimal cap for universities is either $T = 0$ or $T = b$.

4. Students receive larger transfers as the cap increases, and they are made better off, including their choice of majors, by revealed preference.

■

English equilibrium with (potentially) negative transfers

We start by considering the universities' strategies. Bertrand price competition will force universities to require payments from students precisely equal to their outside option. Suppose that an equilibrium exists where students with $v \geq t$ for some $t \in (0, 1)$ are admitted to all universities, though potentially only under their skill type major. What is the outside option for a university considering deviating in this scenario?

One deviation is for the university to not compete for any student with $v \geq t$ and instead to fill with students of quality $v = t$ (marginally unadmitted students) who value the deviating university at 1 and who are consistent. The deviating university can then charge these students $1 + b$. Through the combination of the payment and the direct utility from the students, the deviating university gets $1 + b + t$.[68]

This payoff is each universities' outside option, leading to the payment function:

$$-p_u(v_s, \theta_s, x_m) = \begin{cases} 1 + b + t - v_s & x_m = \theta_s \\ 1 + b + t - \delta(v_s) & x_m \neq \theta_s \end{cases}$$

However, under this payment scheme, some students will not find it profitable to attend university. Consider inconsistent students with $v < \min\{\delta^{-1}(t), t + b\} \equiv k(t)$. If they study their skill type, they receive utility at most 1 but pay $1+b+t-v > 1$ (since $v < t+b$). If they study their preference type, they receive utility at most $1 + b$ but pay $1 + b + t - \delta(v) > 1 + b$ (since $v < \delta^{-1}(t)$). Regardless of which major they study, they pay more than the utility they get from studying and so they will not attend.

But this means that fewer than $1 - t$ students are enrolled at any university, so it cannot be the case that $t = 1 - q$, since then universities would be under-enrolled and would lower their admissions threshold to fill. To determine the value of t, note that all students with $v \geq k(t)$ enroll, while only consistent students with $v \in [t, k(t)]$ enroll, so total enrollment is

[68] For a finite number of universities, each of strictly positive size, the deviating university would get some payoff $\epsilon > 0$ less than this, with $\epsilon \to 0$ as $n \to \infty$.

$\alpha(k(t)-t)+1-k(t)$. The university-filling threshold is then the t that sets this value equal to q. Such a t exists since, as $t \to 0$, $k(t) \to 0$, $k(t)-t \to 0$, and $k(t)$ is continuous (so we can apply the intermediate value theorem).

To complete the English equilibrium, we need to ensure that students find it worthwhile to resolve their uncertainty about their preference type. There are now two benefits to knowing one's preferences over majors. First, there is a benefit similar to the one in the non-transfers case of being able to study your preference type when inconsistent. In the transfers case, students will want to study their preference type when inconsistent whenever v is such that $v-\delta(v) < b$, though the gain in studying their preference is only $b-(v-\delta(v))$.

Second, there is the benefit of knowing whether to attend university when your v is sufficiently close to t. Assuming that students are risk neutral, an uninformed student won't attend university if she has low enough v since her expected payoff is negative (as her expected payoff conditional on being consistent goes to 0 as v approaches t from above and her expected payoff conditional on being inconsistent is strictly negative in the same case). Moreover, there is a range of v below $k(t)$ such that an uninformed student with quality in this range receives positive expected utility and so applies, but ex-post realizes she is inconsistent and regrets her decision. If she had been informed, she would have known not to apply. Thus, the second benefit to becoming informed is composed of the two parts of improving her payoff from 0 to $1+b+t-v$ when she is consistent and has v close to t and of improving her payoff from a negative payoff to 0 when she is inconsistent and has v close to $k(t)$. Let the threshold at which the benefit switches from the one to the other (the point at which the expected benefit to an uninformed student is precisely 0) be denoted $q(t)$.

Combining these two types of benefits, one can write down the condition of the expected increase in a student's payoff from being informed and compare it to the cost c, to determine values for which an English style equilibrium exists.

$$c < \int_{k(t)}^{1} \max\{0, b-(v-\delta(v))\}\, dv + \int_{t}^{q(t)} \alpha(v-t)\, dv + \int_{q(t)}^{k(t)} (1-\alpha) \min\{|v-b-t|, |\delta(v)-t|\}\, dv$$

To summarize, adding transfers changes the English equilibrium in the following ways. The lower bound on the quality of students admitted decreases. Some students choose not to attend college even though they could because they view the fees as too high. The extent of this distortion increases with larger b, smaller α and smaller $\delta(\cdot)$. The full description of the equilibrium is given in the appendix.

US equilibrium with (potentially) negative transfers

To begin looking for an equilibrium where universities aggregate (at least nominally, as transfers can lead students to study against their preferences) and students do not learn their preferences before applying, first note that students, at the point of applying and accepting offers, cannot have their strategy depend on anything other their quality. So, there will be no issue with the admission threshold as in the English-with-transfers case, and the admission threshold will be $1-q$.

Let us begin with the university side. Transfers will again be Bertrand competed down to the universities' outside option. What is this outside option in this case? As in the English case, any university can stop competing for the top q students and instead focus on the marginally unadmitted students. Should the deviating university disaggregate or aggregate in admitting these students? If the deviating university disaggregates, it can only charge the students $1+\alpha\beta$ (for studying their skill type, and some sufficiently large amount to studying against skill type to dissuade any such study) since the students do not know their preferences and so assign only an α probability to their being consistent. If the deviating university stays aggregated, it can then charge the students $1+b$ for either major, though it will suffer a $1-\alpha$ percentage of the students studying against their skill type.

The deviating university will remain aggregated iff:

$$1+b+\alpha(1-q)+(1-\alpha)\delta(1-q) > 1+\alpha b+(1-q) \iff b > (1-q)-\delta(1-q)$$

Suppose first that this condition holds so that the deviating university will allow (in both name and consequence[69]) students to study their preference, so then the outside option for each university is $1+b+\alpha(1-q)+(1-\alpha)\delta(1-q)$. One possible transfer scheme is then:

$$-p_u(v_s,\theta_s,x_m) = \begin{cases} 1+b+\alpha(1-q)+(1-\alpha)\delta(1-q)-v_s & x_m=\theta_s \\ 1+b+\alpha(1-q)+(1-\alpha)\delta(1-q)-\delta(v_s) & x_m\neq\theta_s \end{cases}$$

One undesirable aspect of this transfer scheme is that some students will ex post regret their decision to attend college.[70] An alternate transfer scheme that avoids this ex post regret is:

The transfer that universities will charge enrolling students are slightly more complicated than in the English equilibrium case. Namely, for students of quality v such that $v-\delta(v) < b$,

[69] That is, it won't set a prohibitively high fee to studying against one's skill type.

[70] Of course, even in the no transfers case, the English equilibrium features students who ex post regret paying the cost to learn their preferences.

the transfers will be:

$$p(x_m, \theta, v) = \begin{cases} 1 + b + (1-q) - v & x_m = \theta \text{ and } v - \delta(v) < b \\ 1 + b + \delta(1-q) - \delta(v) & x_m \neq \theta \text{ and } v - \delta(v) < b \\ 1 + b + \alpha(1-q) + (1-\alpha)\delta(1-q) - v_s & x_m = \theta_s \text{ and } v - \delta(v) > b \\ \infty & x_m \neq \theta_s \text{ and } v - \delta(v) > b \end{cases} \tag{38}$$

While not suffering from ex post regret, this transfer scheme can have issues with the true reporting of v_s, as a consistent student with v_s such that $v_s - \delta(v_s) < b$ but for a small ϵ, $v_s - \epsilon - \delta(v_s - \epsilon) > b$ could want to report her quality as $v_s - \epsilon$ so as to pay a smaller transfer.

However, when $v - \delta(v) > b$, inconsistent students would (once they realized they were inconsistent) prefer the discount in transfer of studying their skill type than the benefit b of following their preferences. This would lead all students to choose to study their skill type and for universities to earn "excess" profits from these students. Thus, it must be that for these values of v, the payment decreases (discontinuously) so that:

When $b < (1-q) - \delta(1-q)$, the optimal deviation for a university is to fill with marginally unadmitted students but to disaggregate and force them to study their skill type. One transfer scheme that works in this case is:

$$-p_u(v_s, \theta_s, x_m) = \begin{cases} 2 + \alpha b - q - v_s & x_m = \theta_s \\ 2 + \alpha b - q - \delta(v_s) & x_m \neq \theta_s \end{cases}$$

There is no advantage to students to knowing their preferences over majors early, so there is no requirement on the student side for the US equilibrium with unrestricted transfers.

C.5 Matching Invariance

The model presented in the body of the paper relies on a decentralized matching procedure to assign students to university-major pairs, and studies the resulting equilibria. This section shows that replacing a the decentralized matching procedure with any "well-behaved" centralized matching mechanism, roughly one that matches a student more often to contracts listed earlier in her preferences, does not change the set of equilibria of the game. This robustness highlights the central premise of the paper–the strategic interactions before and after matching can be important to study relative to the way the match itself it conducted.

Let η be a measure over S induced by the assumed distributions with respect to $\sum^S$, the Borel $\sigma-$algebra of the standard topology over [0,1]. Let $\varnothing$ represent the student's outside option of being unmatched to any university. Let Y denote the set of ex-interim contracts, that is $Y = S \times N \times \{M, L, A\}$ where each ex-interim contract y is a triplet specifying a student, a university, and either a major (M or L) or aggregation (A). An *ex-interim matching* is a correspondence $\mu : S \cup N \rightrightarrows Y$ such that:

1. For all $s \in S$, $\mu(s) = \varnothing$ or $\mu(s) = y$ such that $y_s = s$,

2. if $\mu(s) = y$ then $y \in \mu(u)$ for university y_u and if $y \in \mu(u)$ then $y = \mu(s)$ for student y_s, and

3. If u is aggregated then $\mu(u) \subseteq \{y|y_u = u\}$, the set $\{y_s|y \in \mu(u)\}$ is measurable, $\eta(\{y_s|y \in \mu(u)\}) \leq \frac{q}{n}$, and $y_t = A$ for all $y \in \mu(u)$,

4. If u is disaggregated then $\mu(u) \subseteq \{y|y_u = u\}$, the sets $\{y_s|y \in \mu(u), y_t = M\}$ and $\{y_s|y \in \mu(u), y_t = L\}$ are measurable, $\eta(\{y_s|y \in \mu(u), y_t = M\}) \leq \frac{q}{2n}$, $\eta(\{y_s|y \in \mu(u), y_t = L\}) \leq \frac{q}{2n}$.

Let $\mathcal{M}$ denote the set of all ex-interim matchings. In words, a matching is a correspondence that 1. assigns a student to either her outside option or a single contract naming the student, 2. does not assign a contract to a student without assigning the same contract to a university, and vice versa, 3. assigns an aggregated university to a set of contracts naming itself, the measure of these contracts does not exceed the university's capacity constraint, and each contract comports with the aggregation decision 4. assigns a disaggregated university to a set of contracts naming itself, the measure of these contracts does not exceed each college's capacity constraint. We refer to such contacts comporting with the aggregation decision of the university as *feasible*.

The timing of a game with a centralized matching mechanism is similar to the main model. First, Nature selects student types. Second, each university makes its aggregation decision (without observing Nature's selections). Third, each student makes her resolution decision (without observing Nature's selections or the aggregation decision of universities). Fourth, each student submits an ordered list of preferences over contracts to a centralized mechanism (each student s observes her preferences over universities $w_s(\cdot)$, her skill type θ_s. s observes her preference type ρ_s if and only if she has resolved her major preferences). Fifth, the centralized mechanism uses an ex-interim matching mechanism to output an ex-interim matching. We formalize this concept immediately after finishing our description of the timing. The final matching is given, as before by assigned students matched to aggregated universities their preferred major at that universities, and students at disaggregated universities study the major corresponding to the contract they are assigned.

Our equilibrium concept is pure-strategy PBE. As in the main paper, we remove equilibria sustained by unrealistic off-path beliefs by assuming that each student has the same beliefs at each information set in which she is called upon to report preferences to the centralized mechanism.

To define an ex-interim matching mechanism, we begin with several preliminaries. Let $\succ = (\succ_s)_{s \in S}$ represent a vector of submitted linear orders in which $\succ_s$ represents student s's complete, transitive, antisymmetric, and asymmetric report of preferences over $Y(s) \cup \{\varnothing\}$. An ex-interim contract $y \in Y(s)$ is reported to be *acceptable* to student s if $y \succ_s \varnothing$. Let $Q^{r,d}$ be a mapping from $X \times N$ to $[0,1]$, specifying the expected value of each contract to each university, depending on the aggregation profile d, and the share of students resolving their major uncertainty r.

An *ex-interim matching mechanism* is a function $\varphi : \succ \cup Q^{r,d} \to \mathcal{M}$.

Definition **2.** *A mechanism φ is said to be* well-behaved *if it satisfies the following properties:*

1. *(Locally respects qualities) Let y^1 and y^2 be two feasible ex-interim contracts where $u = y^1_u = y^2_u$, and let y^1 have (weakly) higher expected value to university (or college) u. If y^1 is ranked no lower than y^2 according to the submitted preferences, $\succ_{y^1_s}$ and $\succ_{y^2_s}$, then if φ assigns contract y^2 in the matching, then student y^1_s is matched to a contract ranked weakly higher according to $\succ_{y^1_s}$ than contract y^1,*

2. *(Individually rational) No student s is matched to an ex-interim contract that is not reported to be acceptable, and*

3. *(Acceptant) Student s^3 is never unmatched if there exists university u^3 such that $\eta(\{y_s | y \in \mu(u)\}) < \frac{q}{n}$ and there exists feasible contract y^3 such that $y^3_u = u^3$, $y^3_s = s^3$ and $y^3 \succ_{s^3} \varnothing$.*

Proposition **10.** *Let $Q^{r,d}$ denote expected values in an equilibrium of the university admissions game with a decentralized match phase (as described in the main body). Then the set of equilibrium outcomes of the university admissions game with a decentralized match phase is the same as the set of equilibrium outcomes of the university admissions game with a centralized, well-behaved matching mechanism φ and expected values specified by $Q^{r,d}$.*

Proof: We show that essentially unique equilibrium outcome (but for a zero measure set of students who are indifferent between contracts) for the decentralized game is the same as that with a well-behaved centralized mechanism, *holding fixed the pre-match decisions of universities and students.* Since the pre-match actions of agents are not determined by the matching protocol, this will complete the claim.

We say that student s *envies* ex-interim contract $\bar{y} \in \{y|y_s = s\}$ if $\bar{y}$ offers strictly higher expected utility than $\mu(s)$, where the expectation is taken at the time preferences are submitted to the centralized mechanism (that is, it depends on the aggregation decisions of universities and the student's own resolution decision). We say that there is no *justified envy* if no student s envies $\mu(s^{'})$ for any $s^{'}$ if there exists a contract with higher expected value than $\mu(s^{'})$ at university $\mu(s^{'})_u$, $y^* = (s, \mu(s^{'})_u, \mu(s^{'})_t),$. In the decentralized matching model there is no justified envy as otherwise student s would have been admitted to contract y^* by Lemma 1. In the centralized game, there can similarly be no justified envy because s could have just submitted preferences $\tilde{\succ}_s$ listing y^* as most preferred and $\varnothing$ as second, leading to y^* as the ex-interim matching due to the assumption that φ is well-behaved.

We now show that any equilibrium outcome of the decentralized game can be supported by the centralized game. Suppose each matched student s reports $\hat{\succ}_s$ in which she lists the contract assigned in the decentralized game as her first choice and $\varnothing$ as her second choice, and each unmatched student lists $\varnothing$ as her first choice. Since φ is well-behaved, φ yields the same ex-interim matching as the decentralized game. Moreover, by the justified envyfreeness property, no student can do better by deviating to any other preference submission. Therefore, the equilibrium outcome of the decentralized game can be supported by the centralized game.

We now show that any equilibrium outcome of the centralized game must also be an equilibrium outcome of the decentralized game. Let μ be the equilibrium outcome of the centralized game. It must be that each university and each disaggregated college fills all of its seats by the acceptant property. In an abuse of notation, let k be an arbitrary aggregated university or disaggregated college. Let $\bar{S}_k := \{s|x \succ_s \mu(s), x \in X_k\}$, that is, $\bar{s}$ is the set of students who prefer a contract at k to their assigned matching according to their true preferences. By the equilibrium hypothesis, no student $s \in \bar{S}_k$ can submit an alternative preference list and be matched to k under the more-desired contract. Let $v{*}_k$ be supremum expected value for k of any student in $\bar{S}_k$. Our distribution assumptions imply that for any $\epsilon > 0$ there is a positive measure students of expected value (to k) $v \in [v{*}_k, v *_k + \epsilon)$ who are matched to k under their expected favorite contract. Therefore, the fact that φ locally respects qualities implies that any student s with expected value at k weakly higher than $v{*}_k$

weakly prefers their final matching to x where $x_s = s$ and $x_u = k$. For each s let K be the collection of all u such that s's expected value at u weakly exceeds $v*_k$. Then in equilibrium, each s must be matched to their favorite contract among those offered by K. As noted, each k must fill all of its seats by the acceptant property. Therefore, each K has an admissions threshold that fills all of its seats. By Lemma 2 there is a unique such threshold for each k given $Q^{r,d}$. Therefore, the centralized outcome must correspond to the decentralized one.

■

C.6 Alternative preference learning model

We study a different model of student preferences for majors and how they learn these preferences. Whereas in our main model, students get a utility benefit of b from one of the two majors (and no utility benefit from the other) and are able to pay a cost c to learn which major this is, here we consider a model of learning that we believe is more realistic but gives qualitatively similar results to our main model. Specifically, assume that the utility boosts to a student from the two majors are random variables. The benefit from to the student's utility from studying her skill type is $b_s \sim F$ and the benefit to the student's utility from studying against her skill type is $b_{-s} \sim G$, for some distributions F and G, where we will assume that F first order stochastically dominates G (to capture the idea that a student is more likely to like what she is good at). When a student chooses to specialize in a certain subject, she learns the realization of the utility benefit for that subject (and that subject only). Then, when making the decision of which major to apply under, she knows the realization of the utility benefit of her specialized subject and the distribution of the utility benefit from the other subject.

While this formulation has a nice real-world interpretation (namely, that a student in England who specializes in math in high school learns whether or not she likes math and not the degree to which she might or might not like literature), the mathematical analysis of it becomes significantly messier. To start, it is not even clear that she will choose to specialize in her skill type in high school.[71] To gain some tractability, we will specialize to the case of G being a uniform $[0, 1]$ variable and F a uniform $[0, \alpha]$ variable, for $\alpha > 1$ (where

[71] This raises an interesting question in probability: Under what conditions on the distributions F and G does $E[\max\{b_s, E[b_{-s}]\}] > E[\max\{E[b_s], b_{-s}\}]$? The inequality corresponds to the choice of a student of high quality who, under the English equilibrium, can choose which major to study, and so will choose the major associated with the higher of the realized utility benefit and the expectation of the unspecialized utility benefit, so that when this inequality holds it will be in her interest to specialize in her skill type. This is related to the multi-armed bandit problem, but we leave further consideration of this connection to later work.

the naming of α is meant to suggest the relationship to the main model greater likelihood for preferring one's skill type). With these distributions, if students choose to specialize, they will specialize in their skill type. We will also only consider the large market case for simplicity of formulae.

In order to sustain the English equilibrium, students must receive an additive bonus κ to their utility when they enroll in a disaggregated university. This reflects the additional necessary year of schooling at aggregated universities due to broad curricula (especially in the first two years before a major is selected). This is necessary because, unlike in the standard model, students are never sure of their major preferences, and so they always value aggregation. Without this assumption, a university could aggregate in the English equilibrium and fill all of its seats with students arbitrarily close in quality to 1.

The analysis of the US equilibrium is omitted, as it is almost exactly the same as in the main model, just with the exact formula specifying the likelihood of a student liking the deviating university most changed slightly. Again, the US equilibrium exists with restrictions on the parameter space. One can derive an expression similar to Proposition 1, but with "α" now equal to the ex ante probability that $b_s > b_{-s}$, which is equal to $\alpha - \frac{1}{2}$ in our current context.

Proposition 11. *In the large market, the US equilibrium exists when* $\kappa \leq E[\max\{b_s, b_{-s}\}] - E[b_s]$ *and* $\int_{1-q}^{1} \delta(v)\, dv > q - q^2 \left(\frac{\frac{5}{4} - \frac{\alpha}{2}}{\frac{3}{2} - \alpha} \right)$.

Now we turn to the analysis of the English equilibrium. For ease of notation, let $\hat{b}_s = E[b_s]$ and $\hat{b}_{-s} = E[b_{-s}]$.

Then, for a student with quality above $\delta^{-1}(1-q)$ (call these "high quality"), the expected contribution of her major to her utility is (taking into account that if she has a low draw for her type major preference, she'll apply to all universities under the other major):

$$E[\max\{b_s, \hat{b}_{-s}\}|b_s] = \hat{b}_{-s} 1_{b_s < \hat{b}_{-s}} + b_s 1_{b_s \geq \hat{b}_{-s}}$$

and the expected contribution of her major to her utility at a deviating university is:

$$E[\max\{b_s, b_{-s}\}|b_s] = \frac{1}{2} + \frac{b^2}{2}$$

so that a high quality student gets a larger contribution from her major to her utility at the deviating university if :

$$E[\max\{b_s, b_{-s}\}|b_s] - E[\max\{b_s, \hat{b}_{-s}\}|b_s] \geq \kappa$$

Students will prefer specialization to non-specialization so long as the ex-ante benefits of specialization outweigh the costs:

$$E[\max\{b_s, \hat{b}_{-s}\}|b_s] - \hat{b}_s \geq c$$

We now consider the perpective of a deviating university. If $\kappa > \frac{1}{8}$, then no high quality student will find it beneficial to attend the deviating university, while for $\kappa < \frac{1}{8}$, students with b_s draws in the interval $(\sqrt{2\kappa}, 1-\sqrt{2\kappa})$. The utility to the deviating university is then:

$$\int_{\sqrt{2\kappa}}^{1-\sqrt{2\kappa}} ((1-b)\delta(1)+b)\frac{1}{1-2\sqrt{\kappa}}\,db = \frac{1}{1-2\sqrt{2\kappa}}(\delta(1)(1-2\sqrt{2\kappa}) + \frac{1-\delta(1)}{2}(1-2\sqrt{2\kappa}))$$

This simplifies to:

$$(\delta(1)+1)\frac{1}{2}$$

If $\kappa > \frac{1}{8}$ but $\kappa < \frac{1}{2}$, the university will fill up on students with quality between $1-q$ and $\delta^{-1}(1-q)$ (call these "low quality" students). These students don't have the option of switching to their non-type major at the non-deviating university, so they prefer the deviating university if:

$$E[\max\{b_s, b_{-s}\}|b_s] - b_s \geq \kappa$$

Low quality students will switch if $b_s < 1-\sqrt{2\kappa}$. The utility of the deviating university is then:

$$\int_{0}^{1-\sqrt{2\kappa}} ((1-b)(1-q) + b\delta^{-1}(1-q))\frac{1}{1-\sqrt{2\kappa}}\,db$$

$$= \frac{1}{1-\sqrt{2c}}((1-q)(1-\sqrt{2c}) + \frac{\delta^{-1}(1-q)-(1-q)}{2}(1-2\sqrt{2c}+2c))$$

This "simplifies" to:

$$\frac{1}{1-\sqrt{2\kappa}}(\frac{1}{2}(\delta^{-1}(1-q)+1-q) - \sqrt{2\kappa}\delta^{-1}(1-q) + \frac{1}{2}\delta^{-1}(1-q) + \kappa(\delta^{-1}(1-q)-1+q))$$

Under equilibrium, a university gets utility:

$$\frac{1}{4\alpha}\delta^{-1}(1-q)^2 - \frac{1}{2}(1-q)^2 + \frac{1}{2}(1-\frac{1}{2\alpha}) + \frac{1}{2\alpha}\int_{\delta^{-1}(1-q)}^{1} \delta(v)\, dv$$

The English equilibrium will exist when the utility under equilibrium is higher than the pertinent utility under deviation (where the pertinent utility is determined by the value of κ). While these equations are far messier than in the main model, the basic intuition is the same: the universities have a desire to disaggregate so as to ensure that students study exactly the major they apply under, but they are tempted to deviate to aggregate and become more attractive to students. In the main model, they become more attractive only to students who will switch majors and this adverse selection makes the deviation undesirable. Under this model, the university becomes more attractive to students who have a higher likelihood to switch than the average student, but the adverse selection effect is attenuated by the possibility that the student might discover that she strongly dislikes her non-skill type major and so stays with her skill type major and by the possibility that very high quality students (students with $v = 1$) might prefer the option value of finding out the values of the benefit from both majors over committing to one of the majors, and so the university fills up with very high quality students.

For completeness, we state the above as a proposition:

Proposition **12.** *The English equilibrium exists when the equilibrium utility of the universities is larger than the utility under deviation to disaggregating and when students find it profitable to specialize given universities are all disaggregated.*

C.7 Non-identical university preferences

Now suppose that universities have correlated, but non-identical preferences over students. Let each university i receive a student specific shock, that is, each university i values student s with quality v_s as $v_s + \epsilon_i^s$ if s applies under her type, and $\delta(v_s) + \epsilon_i^s$ if s applies against her type, where $\epsilon_i^s \sim U(-\gamma, \gamma)$. These shocks are independently and identically distributed across students and universities. Throughout, we will maintain certain interior assumptions, namely that $1 - q - \gamma > 0$ and $\delta^{-1}(1 - q + \gamma) + 2\gamma < 1$.

We first consider the US framework.

Proposition **13.** *As $n \to \infty$ the equilibrium threshold of each university $t(n) \to 1 - q + \gamma$.*

Proof of Proposition 13: We first show that as $n \to \infty$, the proportion of students with $v_s \geq t(n) - \gamma$ who are admitted to at least 1 university converges to 1. Consider students with $v_s \in (t(n) - \gamma, t(n) + \gamma)$. These are precisely the set of students who have a non-zero and non-unity probability of being admitted to any particular university. Break this interval up into J equal and continuous sub-intervals. Define $p(j)$ and $\theta(j)$ (here, θ is not referring to the student's type as discussed in the main body) such that every student with v_s in the j^{th} sub-interval has a probability of admission to each university between $p(j)$ and $p(j)+\theta(j)$. The mass of these students who get admitted to even a single university is greater than $\theta(j) - \theta(j)(1-p(j))^n$ which converges to $\theta(j)$ as $n \to \infty$. Now let $J \to \infty$. Then the 2γ mass of students with $v_s \in (t(n)-\gamma, t(n)+\gamma)$ who are admitted to at least one university is no less than $\lim_{J\to\infty}\sum_{j=1}^{J}\theta(j) = 2\gamma$. Therefore, in the limit as $n \to \infty$, almost every student who has positive probability of being admitted to a university attends a university. Therefore, since the total mass of students who enroll in universities has to equal $1-q$ in equilibrium, the limiting threshold t^* must solve $t^* - \gamma = 1 - q$, which gives the desired result.

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PROPOSITION **14.** *As $n \to \infty$ the US equilibrium can be sustained under certain parameter values.*

Proof of Proposition 14:

The probability that exactly h other universities have admitted a student of quality u to university 1 is:

$$\binom{n-1}{h}\left(\frac{1}{2}+\frac{1}{2\gamma}(u-t)-\frac{1}{8\gamma^2}(u-t)^2\right)^h\left(\frac{1}{2}-\frac{1}{2\gamma}(u-t)+\frac{1}{8\gamma^2}(u-t)^2\right)^{n-1-h}$$

So, the solution to the question of threshold in the equilibrium is the t solving:

$$\int_t^{t+2\gamma}\sum_{h=0}^{n-1}\frac{1}{h+1}\binom{n-1}{h}\left(\frac{1}{2}+\frac{1}{2\gamma}(u-t)-\frac{1}{8\gamma^2}(u-t)^2\right)^h\left(\frac{1}{2}-\frac{1}{2\gamma}(u-t)+\frac{1}{8\gamma^2}(u-t)^2\right)^{n-1-h}du =$$

$$\frac{q-1+t+2\gamma}{n}$$

Let the solution be denoted t^*. Then, in equilibrium, university 1 expects to get utility (normalized by $\frac{q}{n}$):

$$\frac{n}{q}\int_{t^*}^{t^*+2\gamma}\alpha u + (1-\alpha)\int_{-\gamma}^{\gamma}\delta(u-\epsilon)\,d\epsilon\sum_{h=0}^{n-1}\frac{1}{h+1}\binom{n-1}{h}\left(\frac{1}{2}+\frac{1}{2\gamma}(u-t^*)-\frac{1}{8\gamma^2}(u-t^*)^2\right)^h$$

$$\cdot\left(\frac{1}{2}-\frac{1}{2\gamma}(u-t^{*})+\frac{1}{8\gamma^{2}}(u-t^{*})^{2}\right)^{n-1-h}du$$

$$+\frac{1}{q}\int_{1-\gamma}^{1+\gamma}(-\frac{u}{2\gamma}+\frac{1}{2}+\frac{1}{2\gamma})(\alpha u+(1-\alpha)\int_{-\gamma}^{\gamma}\delta(u-\epsilon)\,d\epsilon)\,du+\frac{1}{q}\int_{c^{*}+2\gamma}^{1-\gamma}\alpha u+(1-\alpha)\int_{-\gamma}^{\gamma}\delta(u-\epsilon)\,d\epsilon\,du$$

where we are using the fact that if $u=v+\epsilon$, then $v=u-\epsilon$, so that if the student switches to the major against his type, the university gets $\delta(u-\epsilon)+\epsilon$, and the expectation of this over ϵ is $\int_{-\gamma}^{\gamma}\delta(u-\epsilon)\,d\epsilon$. Denote this utility by R^{A}.

Now, let's consider a deviation from the equilibrium. Assume u_1 disaggregates. As before, the probability that s prefers another university u_i to u_1 is given by $1-\left[(1-(1-\alpha)\,b)^{n}\left(\frac{1}{n}\right)\right]$. Since $u_1$is now less desirable to students, it will have to have a lower threshold quality level than the aggregated universities, who will in turn be able to have a (slightly) higher threshold than they would have otherwise, so that, letting $t_{u_1}(n)$ be u_1's threshold and $t'(n)$ be the threshold of one of the other (still) aggregating universities. Clearly, $t'(n)$ is bounded above and below by $t(n)$ and $t(n-1)$, the thresholds in the proposed equilibrium. Since both $t(n)$ and $t(n-1)$ converge to $1-q+\gamma$ as $k\to\infty$,we see that $t'(n)\to 1-q+\gamma$. We will now argue that $t_{u_1}(n)\to 1-q+\gamma$ and u_1 will, in the limit, get only students of quality $1-q+\gamma$. Consider a student of worth $u_{u_1}>1-q+\gamma$ to u_1. This means that there is an $\epsilon<\gamma$ such that $u_{u_1}=v_s+\epsilon$, there v_s is this student's underlying quality. The probability that this student is admitted to another university u_i is no less than the probability that the student gets a shock at least as great as ϵ at u_i, which is $\frac{\gamma-\epsilon}{2\gamma}>0$. The number of other universities the student can be expected to be admitted to is then $\frac{\gamma-\epsilon}{2\gamma}(n-1)=n'$. Taking this n' as the n in the probability calculation that u_1 would not be this student's favorite choice gives the result that the mass of students u_1 is enrolling of quality greater than $1-q+\gamma$ is going to 0 exponentially fast, while the total mass of students university 1 is enrolling, $\frac{q}{n}$, is going to 0 more slowly, so that as $n\to\infty$, u_1 is admitting an arbitrarily large portion of its seats to students of quality below $1-q+\gamma$. Thus, u_1 gets average utility of $1-q+\gamma$ in the limit.

The US equilibrium will exist when $R^{A}\geq 1-q+\gamma$. Note that this is similar to the case of the no-shocks case, but assuming convexity of δ, then $\int_{-\gamma}^{\gamma}\delta(u+\epsilon)\,d\epsilon<\delta(u)$, so that for the equilibrium utility to be greater than the deviation utility, we require a stricter requirement on δ (that it is closer to linear).

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Next, we consider the English framework.

Remark **6.** *Let t^* be the limiting threshold in equilibrium. Then as $n \to \infty$ the probability that a student with $v_s > \beta^{-1}(t^*) - \gamma$ studies her favorite major converges to 1.*

Proof of Remark 6

Consider an inconsistent student s of quality $v_s > \delta^{-1}(t^*) - \gamma$. Let $p(v_s) > 0$ be the limiting probability that student s is admitted to a university when applying to her favorite major. As $n \to \infty$ the expected number of universities she is admitted to goes to $p(v_s) \cdot n \to \infty$. The probability that she likes one of these universities more under her favorite major than any other university under her type is no greater than $1 - b^{p(v_s) \cdot n} \to 1$.

■

Proposition **15.** *As $n \to \infty$ the English equilibrium can be sustained if and only if $(1 - \bar{v})(1 - \alpha)b > c$.*

Proof of Proposition 15

Given Proposition 6, we see that the proportion of students with $v_s > t^* - \gamma$ that are admitted to a university under their favorite major converges to 1 as $n \to \infty$. As in Proposition 13, we see that as $n \to \infty$ the equilibrium threshold of each university $t(n) \to 1 - q + \gamma$. Define $\bar{v} = \delta^{-1}(1 - q + \gamma)$. From Lemma 2 we know that as $n \to \infty$ the proportion of students who enroll in a disaggregated university with $v_s < \bar{v}$ who are inconsistent converges to 1. Since $t(n) \to 1 - q + \gamma$, in the limit, each of these students gives a utility of $1 - q + \gamma$ to the aggregating university, which is less than what almost every student enrolled would give the university under equilibrium, therefore, no university wishes to deviate. The condition to satisfy equilibrium on the part of students is almost identical to that found in Proposition 2. The one change is that we now require a strict inequality. The reason for this is given by Proposition 6; for any n there is a non-zero probability that a student s with $v_s > \delta^{-1}(t^*) - \gamma$ will be unable to obtain admission at a satisfactory university under her favorite major. Therefore, this student will be worse off than had she not resolved her uncertainty. However, since this probability is going to zero as $n \to \infty$, for any $\gamma > 0$ there exists N_γ such that for any $n > N_\gamma$,as long as $(1 - \bar{v})(1 - \alpha)b + \gamma \geq c$ every student will resolve her uncertainty at the beginning of the game. Therefore, so long as $(1 - \bar{v})(1 - \alpha)b > c$, the English equilibrium can be sustained in the limit as $n \to \infty$.

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C.8 Interdependent preferences

Now suppose that universities gain more utility from happier students. In particular, we model a "bonus" to the utility of a university if an enrolled student studies her favorite major. The assumption here is that students who are more interested in their course of study are more likely to be better students, are more likely to have significant career achievements, or more likely to donate money to the university after graduating. Therefore, the universities are incentivized to allow students to study their favorite major.

In terms of modeling, suppose that a university receives an additive component to their utility from a student of quality v of $\beta(v) > 0$ if the student studies her favorite major (a boost to the university). Furthermore, assume that $\beta(v)$ is non-decreasing in v, so that the university (weakly) prefers making its best students happier than its worst students.

Proposition **16.** *The US equilibrium is always easier to support with interdependent preferences. As $n \to \infty$ the US equilibrium can be sustained under a wider set of parameter values.*

Proof of Proposition 16 In the US equilibrium, every student enrolled at a university gives that university the "boost" that comes from studying her favorite major. Therefore the US equilibrium is easier to support with interdependent preferences as it is costlier for a university to deviate, as they will necessarily forfeit the interdependency boost from some mass of its enrolled students. Following Proposition 1, as $n \to \infty$ the US equilibrium can be sustained if $\frac{1}{q}\int_{1-q}^{1} [\alpha v + (1-\alpha)\delta(v) + \beta(v)]\, dv > 1 - q + \alpha\beta(1-q)$.

■

Proposition **17.** *The English equilibrium is always harder to support with interdependent preferences. As $n \to \infty$ the English equilibrium can be sustained under certain parameter values.*

Proof of Proposition 17 In the English equilibrium, not every student enrolled at a university gives that university the "boost" that comes from studying her favorite major. Therefore the English equilibrium is harder to support with interdependent preferences as it is less costly for a university to deviate, as they will necessarily gain the interdependency from all of its enrolled students. Following Lemma 2, as $n \to \infty$ the average utility from a student with $v_s < \bar{v}$ converges to $1 - q + \delta(1-q)$. Therefore, the English equilibrium can be sustained if $\frac{1}{q}\int_{1-q}^{\bar{v}} [v + \alpha\beta(v)]\, dv > 1 - q + \beta(1-q)$ and $(1-\bar{v})(1-\alpha)b > c$.

■

C.9 Many majors

Thus far, the analysis has proceeded with the assumption that universities can only admit students to one of two major programs. This section is devoted to enriching the model to contain any finite number of majors, and show that the US and English equilibria are robust to this extension in the large market. Suppose there are J majors at each university. Let $\alpha = (\alpha_1, \alpha_{2,} \ldots, \alpha_J)$ be a probability distribution over the J majors such that:

- $\alpha_j > 0$ for all $j \in J$,
- $\alpha_j \neq \alpha_\ell$ for all $j \neq \ell$,and
- $\sum_{i=1}^{J} \alpha_i = 1$.

Let each student be equally likely to be one of the $J!$ permutations of α. We denote a student of permutation t with the distribution α^t. Each student of type t has probability α_j^t of most preferring major j. We write $\alpha_{max} = \{\alpha_j : j \text{ is the largest element of } \alpha^t\}$, and refer to a student's highest major $j_s^* = \underset{j}{\operatorname{argmax}}\, \alpha_j^t$. Assume that for each student s with quality v_s and highest major j_s^*, a university who enrolls s gets utility $v_s$if s studies $j_s^*$and $\delta(v_s)$ otherwise. As before, student s's preferences over contracts are separable in university and major, so that for a given contract $x \in X$, the utility of s is given by $U^s(x) = w_s(x_u) + b_s(x_m)$. $b_s(\cdot)$ can take on two values, $b(1) = b > 0$ the utility from studying her ex-post desired major and 0 for any other major.[72]

US equilibrium

In the analysis before, it was straightforward to consider a university's deviation from equilibrium. Indeed, the only action that could be considered as a profitable deviation was whether to aggregate or not. Now, however, even this decision is extremely complicated, as there are a large number of ways in which a university can partition its J programs into *colleges*.[73] However, we show that in the limiting market as $n \to \infty$ there is an optimal

[72] Although we have assumed that both students and universities have an "all or nothing" utility function when it comes to major preferences, we make these assumptions without loss of generality. Since we are considering a large market case of many universities, the specific formulation of the $b(\cdot)$ function does not play into the final results. As for the $\delta(\cdot)$ function, we note that this assumption gives us the strongest restrictions to ensure the US equilibrium compared to other formulations, allowing, for example, a university to strictly prefer a math student to study engineering over literature.

[73] The number of ways a university can partition its J majors is given by the J'th Bell number $B_J = \frac{1}{e}\sum_{i=0}^{\infty} \frac{i^J}{i!}$. To see how unmanageable this problem is, there are 21,147 ways to sort 10 majors into colleges at a single university.

deviation for any deviating university to full disaggregate, i.e. sort its J different programs into J different colleges. Given this, we see that the condition for sustaining the US equilibrium in Proposition 1is remarkably similar to the condition required to maintain the US equilibrium with many majors.

PROPOSITION **18.** *As $n \to \infty$ with J majors at each university, it is an optimal deviation from the US equilibrium for a university to sort its J programs into J colleges.*

Proof of Proposition 18: We first note that any program at a disaggregated university is ex-ante less popular to uncertain students than the same program at aggregated universities, as students are not given the option of switching to any desired major once their uncertainty is resolved. As before, student s always applies under major j_s^*. This means that any university that disaggregates gets students of quality $1-q$ in the limit as $n \to \infty$, that is, regardless of the level of disaggregation, a deviating university will fill an arbitrarily large proportion of its seats with students of quality just below $1-q$. Now consider any level of disaggregation besides full disaggregation. Then there is at least one college C with at least two majors in it. By the assumption that $\alpha_j > 0$ for all j (each student has a positive probability of most preferring any other major ex-post) a positive mass of these students who are admitted into college C will switch majors away from j_s^*. Therefore, these students will give a utility of $\approx \delta(1-q) < 1-q$, whereas each of these students would give the university a utility of $\approx 1-q$ if college C had been completely broken up. Similar logic applies to each non-singleton college, proving the result.

■

PROPOSITION **19.** *As $n \to \infty$ the US equilibrium can be sustained under certain parameter values.*

Proof of Proposition 19: The logic of Proposition 1 applies here. Indeed, we only need to replace "α" with "α_{max}" to get the necessary and sufficient condition for maintaining equilibrium, that is $\int\limits_{1-q}^{1} \delta(v)dv > q - q^2 \left(\frac{1-\frac{\alpha_{max}}{2}}{1-\alpha_{max}}\right)$.

■

English equilibrium

Universities are still able to deviate from equilibrium in many ways. However, since students are certain of their preferences in the English equilibrium, as $n \to \infty$ every non-singleton college will fill all of its seats with liars. Therefore, any non-singleton college will do strictly better by fully breaking up. Applying this logic to the proof of Lemma 2 yields the following result.

PROPOSITION **20.** *As $n \to \infty$ the English equilibrium is sustainable if and only if* $(1-\bar{v})(1-\alpha_{max})b > c$.

C.10 Non-homogeneous sizes and popularities

The analysis thus far has focused on completely homogeneous universities. However, this is not an entirely accurate view of real world college education–both the sizes and relative popularities of universities vary. This section is devoted to enriching the model presented to allow for universities of different sizes and different popularities, and to show that both US and English equilibria can exist in this setting.

Suppose there are G different *selectivity types* of universities. Let $\frac{\tau_g \cdot q}{n}$ be the mass of seats at a university of type g, where τ_g represents the relative size of a type g university. Let students draw utility for a university of type g uniformly from $[0, j_g]$. To simplify certain calculations, we assume that $j_g \neq j_m$ for $g \neq m$. We rank the types such that $j_1 < j_2 < \ldots < j_G$.

Letting $A_g(n)$ represent the proportion of type g universities out of a total of n universities, we require $\forall n \ \frac{A_g(n)}{A_m(n)} < A$ for all g and m and some finite A, requiring that every type of university has a significant presence in the market, regardless of the total number of universities present. We assume that $n \cdot \sum_{g \in G} A_g(n) \cdot \frac{\tau_g \cdot q}{n} = q$ for all n, so that there are always q seats available for students in the market. We are again interested in the behavior and equilibria of the university admission markets as $n \to \infty$. We assume $A_g(n) \to A_g$ for some $A_g \in (0,1)$ as $n \to \infty$.

US equilibrium

We show here that under certain conditions, the US equilibrium is sustainable in the large market. We first show that, in equilibrium, the best students go to the "most selective" type of university (type G universities), the second tier of students go to the second most selective type of university, and so on. Let t_g represent the limiting equilibrium threshold of a type g university.

LEMMA **5.** *As $n \to \infty$,*

1. *$j_m > j_g$ if and only if $t_m > t_g$,*

2. *A type g university will enroll an arbitrary large proportion of its students from those of quality $v \in [t_g, t_{g+1}]$, and*

3. $t_g \to 1 - \sum\limits_{m \geq g} A_g \tau_g$.

Proof of Lemma 5:

1. To prove the first part of the proposition, it is easy to see that as $n \to \infty$ the probability that a student prefers at least one of the more selective universities to all less selective universities goes to 1. Therefore, if $j_m > j_g$ and $t_m \leq t_g$ then either the type g universities are filling an arbitrarily small proportion of their seats, or type m universities are over filling.

2. This follows from the proof of 1.

3. The requirement that there is always a q mass of seats available for students tells us that $n \cdot \sum\limits_{g \in G} A_g(n) \cdot \frac{\tau_g \cdot q}{n} = q$ for all n. Rearranging yields $\sum\limits_{g \in G} A_g(n) \cdot \tau_g = 1$ for all n. Therefore, the limiting proportion of seats taken up by type g universities $\frac{A_g \cdot \tau_g}{\sum\limits_{g \in G} A_g \cdot \tau_g} = A_g \cdot \tau_g$. From 1. and 2. we know that type G universities enroll the top $A_G \cdot \tau_G$ students, the type $G-1$ students enroll the next $A_{G-1} \cdot \tau_{G-1}$ students and so on. This completes the proof.

■

We now show that a more selective university's deviation from equilibrium may force that university to compete for lower tier students, but will also force students to study their type. Therefore, equilibrium is sustainable if the difference between a student's productivity under different majors is small compared to the quality drop in students. Intuitively, it is easier to support equilibrium in this environment, as a deviating university loses a competitive edge against universities in its own class, but it may also be less competitive than lower-tier universities as well.

Proposition **21.** *As $n \to \infty$, the US equilibrium is sustainable under certain parameter values.*

Proof of Proposition 21: Suppose all universities aggregate except for u_1 of type g. As before, student s will prefer $u_1$to some other university u_j if $w_s^{u_1} \geq w_s^{u_j} + (1-\alpha)b$. Therefore, as $n \to \infty$, u_1 will not effectively be able to compete with any universities of type m with $j_m > j_g - (1-\alpha)b$. In other words, letting $\bar{t}_g = \max_{\substack{m \\ s.t.\, j_g-(1-\alpha)b>j_m}} t_m$, $u_1$will fill up its seats almost entirely with students of quality arbitrarily close to $\bar{t}_g$. Therefore, from Lemma 5, the equilibrium sustaining condition becomes $\frac{1}{A_g \cdot n} \int_{t_g}^{t_{g+1}} [\alpha v + (1-\alpha)\delta(v)]\, dv > \frac{\bar{t}_g}{\frac{\tau_g \cdot q}{n}}$

$\Leftrightarrow \int_{t_g}^{t_{g+1}} [\alpha v + (1-\alpha)\delta(v)]\, dv > \frac{A_g . \bar{t}_g}{\tau_g \cdot q}$ for all $g \in G$.

■

English equilibrium

An initial guess may be that the English equilibrium will allow an aggregated university to poach students from higher tier universities. However, as this section will prove, while this university may be able to poach some of the less competitive students at higher tier universities, it will only poach students who wish to study against their type. We will show that these inconsistent students are not of sufficiently higher quality, and that by lying, are worth less to the university than every student it would admit in equilibrium in the limit as $n \to \infty$.

We note that the first part of Lemma 5 carries over to the English market. The second and third parts, however, do not. To see this, let $\bar{v}_g = \delta^{-1}(t_g)$. Then the set of students that a university of type g gets depends on how $\bar{v}_g$ relates to t_{g+1}. Moreover, it may also depend on how t_g relates to $\bar{v}_{g-1}, \bar{v}_{g-2}, \ldots, \bar{v}_1$. However, we are able to make an analogous statement to the second part of Lemma 5 for the English equilibrium:

LEMMA **6.** *As $n \to \infty$, a type g university in the English scheme will enroll an arbitrary small proportion of its students from those of quality $v_s \leq \bar{v}_{g+1}$.*

We now state the relevant proposition.

PROPOSITION **22.** *As $n \to \infty$, the English equilibrium is sustainable under certain parameter values.*

Proof of Proposition 22: Consider a type g university u_1 which decides to aggregate. Let $t'_g(n)$ be the threshold for a university of type g if it is the lone aggregator, where t'_g is the limit of $t'_g(n)$. We first note that students with $v_s > \bar{v}_g$ find u_1 no more attractive than under equilibrium as they are always able to study their favorite major at u_1. Therefore, they are no more likely to attend under aggregation. Therefore, as in Proposition 2, $t'_g(n) \leq \bar{v}_g$ for all n. Now we consider three types of students with $v_s \in (t'_g, \bar{v}_g)$:

1. Students with $v_s \geq t_{g+1}$ who are inconsistent,
2. Students with $v_s \geq t_{g+1}$ who are consistent, and
3. Students with $v_s < t_{g+1}$.

Since type 1 students are still of sufficiently low quality ($v_s < \bar{v}_g$) each of these students, should they enroll in u_1 will switch majors and give utility of less than t_g to university u_1. Since almost every enrolling student gives utility strictly greater utility than t_g, u_1 is strictly worse off from enrolling these type 1 students. By similar logic as Lemma 5, almost no type 2 students will enroll at u_1 regardless of the aggregation decision. From Proposition 2, we see that u_1 is worse off from type 3 students by aggregating than it would had it disaggregated. Therefore, no university wishes to aggregate.

We now consider the decisions of the students. As before, a student only benefits from paying cost c to learn her preferred major if she actually prefers the major that is not her type. However, only a subset of students in [0,1] are able to effectively take advantage of this information. In the basic model, this was the set of students with $v_s > \bar{v}$. However, this set is more complicated with heterogeneous universities. In particular, this set is now a collection of intervals, not a single interval.

Suppose that there are two tiers of universities. A student with sufficiently high v_s may be able to study her favorite major at a top tier university, while a slightly weaker student may be able to study at a top tier university, but only under his type. However, even if he is able to study his favorite major at a tier two university, he may instead elect to study his least favorite major at a top tier university if the top tier universities are sufficiently better than the second tier universities. There could also be a third even weaker student who is able to study her favorite major at a second tier university, but is not of sufficient quality to be admitted to a top tier university under either major.

In this example, students 1 and 3 can benefit from paying the cost to learn their type, while the student 2 cannot. To state the exact set of students which can benefit from learning their preferred majors in equilibrium, we require some more notation. Let

$$f_g = \begin{cases} t_{g+1} & \text{if } j_g < j_{g+1} - b \\ \bar{v}_{g+1} & \text{otherwise} \end{cases}$$

and let $f_G = 1$. Define $F = \bigcup_{g \in G} [\bar{v}_g, f_g]$.

We claim that a necessary condition for a student with quality v_s to study the major that is not her type in equilibrium (with probability approaching 1 as $n \to \infty$) is that $v_s \in F$. To see this, recall that as $n \to \infty$ every student will find many universities in each tier for which she draws utilities arbitrarily close to the upper bound of the tier in question. Therefore, if $j_g < j_{g+1} - b$ then a student would rather (with probability approaching 1) study at her favorite tier $g+1$ university under either major than at any tier g university. Therefore, students with $v_s \in [\bar{v}_g, t_{g+1}]$ are the only students who enroll in a type g university with non-vanishing probability and can benefit from paying the cost to discover their favorite major. On the other hand if $j_g < j_{g+1} - b$ then a student would rather (with probability approaching 1) study at her favorite tier g university under her favorite major than at any tier g university under her least favorite major. Therefore from Lemma 6, students with $v_s \in [\bar{v}_g, \bar{v}_{g+1}]$ are the only students who enroll in a type g university with non-vanishing probability and can benefit from paying the cost to discover their favorite major. Taking the union of all such intervals for the different tiers of universities yields the set F.

Let $|F| = \sum_{g \in G} f_g - \bar{v}_g$. Then the probability of a student being in region F is precisely $|F|$. Therefore, analogously to the proof of Proposition 2, students will pay the cost to discover their favorite major if $c < |F|(1-\alpha)b$.

■

C.11 Markets without qualities and capacities

In certain markets, "students" do not have a quality that "universities" care about, and "students" may not have ex-ante preferences about the "university" to which they match. For example, sellers may not care (ex-ante) which buyers purchase their products, while buyers are (ex-ante) indifferent about the seller from whom they purchase. We will present this section using the language of sets of a unit mass of buyers B and $n \geq 2$ sellers $S = \{s_1, ..., s_n\}$. The set of contracts is $X = B \times S \times \{K, R\}$ where each contract is a triplet specifying a buyer, a seller, and whether the item is kept (K) or returned (R). For a given contract x, let x_b be the associated buyer, x_s the associated seller, and x_t the associated term. Furthermore, there are no capacity constraints. (Equivalently, there are $q > 1$ mass of objects, and each seller can sell as many of the q goods as demanded. The assumption that $q > 1$ is necessary to ensure that there is some benefit to sellers for being "more popular" than their competitors. If $q \leq 1$ there would be a unique equilibrium in which all sellers disaggregate and still sell all their objects.)

Each seller derives utility $v > 0$ from each non-returned sale, and $\delta < 0 < v$ from each item which is sold and then returned. We normalize the value of an unsold item to 0. Each seller can aggregate (accept returns) or disaggregate (sales are final).

Each buyer b has a type ρ_b of either K or R. K type buyers are consistent and wish to keep their purchase while R type buyers are inconsistent and only want the item for a short period of time. For a contract $x \in X$, the utility of buyer x_b is given by $\beta_b(x_t)$ where $\beta_b(\cdot)$ can take on two values, $\beta_b(x_t) = \beta > 0$ if $x_t = \rho_b$, and $\beta_b(x_t) = -\beta$ otherwise. This means that buyers are unwilling to purchase the good if they know they will not receive favorable terms. Buyers resolve indifferences of to whom to match independently and uniformly at random. Initially, all buyers know that with probability $\alpha \in (\frac{1}{2}, 1)$ they are type K meaning that they wish to keep the item. A buyer can pay cost $c > 0$ before matching with a seller to learn her type ρ_b.

US equilibrium

As before, the US equilibrium features all sellers aggregating (allowing returns) and none of the buyers learning their types (whether or not they want to keep the item). Note that since $q > 1$ a deviating seller who disaggregates will fail to match with any buyers, and will receive utility of 0. By remaining aggregated, a seller will receive expected utility of $\alpha \cdot v + (1-\alpha)\delta$ from each item sold. Therefore, the US equilibrium exists if $\alpha \cdot v + (1-\alpha)\delta > 0$.

English equilibrium

As before, the English equilibrium features all sellers disaggregating (refusing to accept returns) and all of the buyers learning their types (whether or not they want to keep the item). Again, no seller wishes to aggregate because doing so would only make it more attractive to buyers who wish to return their purchases. Since these buyers are worth less than going unmatched, sellers never want to aggregate. On the other hand, buyers are willing to pay the cost if doing so helps them avoid bad purchases often enough. Without paying the cost, a buyer would buy a good since $\alpha > \frac{1}{2}$, and her expected utility would be $\beta(2\alpha - 1)$. By paying the cost, the buyer will receive expected utility $\alpha\beta$ since she will only buy the good if she wishes to keep it. Therefore, a student will pay the cost to learn her type if and only if $c < b(1-\alpha)$.

Stochastic stability

We also study stochastic stability in this special case of our model a la Proposition 6, as we discuss in the main paper the example of return policies after internet retailers became prevalent. The analysis of the stability of the US equilibrium is nearly unchanged. The English equilibrium, however, is even less stable in this context. As long as there exists a single aggregated seller, buyers will no longer resolve uncertainty, as they are ex-ante indifferent between vendors, and the single aggregated seller can service the whole market. In other words, when there is a single aggregated seller of a product without capacity constraints (an online shoe vendor can sell to the whole market as they can increase their own order from the producer directly). Therefore, the English equilibrium cannot exist when there is a high probability of even a single aggregated seller in any sized market.

This leads to an interesting observation. When there is even a single aggregated seller (with high probability) no buyers will pay the cost to resolve uncertainty in equilibrium. For non-generic parameters (when there is positive benefit to matching with buyers) there cannot exist equilibria with some aggregated and some disaggregated buyers, as the disaggregated buyers would match with none of the buyers. Therefore, the only possible equilibria of this model is one in which all sellers aggregate or all sellers disaggregate. This implies that, when the US and English equilibria exist, they are the unique equilibria.

PROPOSITION **23.** *In markets without qualities and capacities, the US and English equilibria are generically unique when they exist.*

Equilibrium with monetary transfers

One perhaps unexpected result in the paper is that the US equilibrium cannot be sustained in large markets with unbounded monetary transfers. Of course, in situations with buyers and sellers, prices are often competitively set, and transfers can be thought of as the amount of the surplus of sale returned to the buyer. We argue that the the US (and English) equilibrium can be retained in the current setting of markets without qualities and capacity constraints. In both the US and English schemes, a (Bertrand) zero profit condition pins down the transfers that must be made. The English equilibrium can be sustained with payments of v to each buyer. The US equilibrium can be sustained with a payment of v to each buyer if she retains the good, and a payment of δ if she returns the good. Realistically, the US transfer scheme requires a "restocking fee" (difference between transfers if item is retained rather than returned) to sustain. Since $\delta < 0$, this assumes that negative transfers are allowed. If not, then the US equilibrium can be sustained with a transfer scheme of

$$t_b = \begin{cases} \alpha \cdot v + (1-\alpha)\delta & \text{if the item is retained and } \alpha \cdot v + (1-\alpha)\delta < 2\beta \\ v & \text{if the item is retained and } \alpha \cdot v + (1-\alpha)\delta > 2\beta \\ 0 & \text{otherwise} \end{cases}$$

where the second case arises because buyers will be bribed to retain the item regardless of their preferences when the "restocking fee" is sufficiently high. Sellers will not deviate to disaggregation as in the main model as doing so would lead to zero sales.