Present bias and externalities: Can government intervention raise welfare?

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Abstract. Quasi-hyperbolic discounted preferences imply that consumers overemphasize immediate current rewards and overlook future ones (they have a "bias for the present"). Within this context the literature has emphasized that the misalignment between immediate and future rewards can be rectified by government policy. Importantly, it has also been shown that intervention by a government that shares the same biased intertemporal preferences with consumers does not deliver welfare improvements. Focusing on the latter, this paper identifies conditions under which, in the presence of quasi-hyperbolic preferences and a market imperfection (which takes the form of a negative externality), intervention by a present-biased government *is welfare enhancing*. This is the case if the market imperfection is sufficiently strong or the consumers' bias for the present is weak.

Résumé. Biais en faveur du présent et effets externes : l'intervention du gouvernement peut-elle rehausser le bien-être? Les préférences quasi hyperboliques laissent croire que les consommateurs mettent trop l'accent sur les récompenses actuelles immédiates et qu'ils négligent les récompenses futures (ils ont un « biais en faveur du présent »). Dans ce contexte, la littérature a souligné que le décalage entre les récompenses immédiates et les récompenses futures peut être corrigé par la politique gouvernementale. Fait important, il a également été démontré qu'une intervention par un gouvernement qui partage les mêmes préférences intertemporelles biaisées que les consommateurs n'engendre pas d'améliorations du bien-être. Mettant l'accent sur ce dernier aspect, cet article détermine les conditions dans lesquelles, en présence de préférences quasi hyperboliques et d'une imperfection du marché (qui prend la forme d'un effet externe négatif), une intervention par un gouvernement biaisé en faveur du présent permet de rehausser le bien-être. C'est le cas si l'imperfection du marché est suffisamment puissante ou si le biais des consommateurs en faveur du présent est faible.

JEL classification: H23, D15, D9

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1. Introduction

EXPERIMENTAL AND FIELD research in behavioural economics has shown that consumers have a "bias for the present" in the sense that a consumer's current self action does not agree with the action taken by the future self and her preferences are time inconsistent. Unsurprisingly, understanding the implications of this "lack of self-control" in choices for public policy has been the focus of the academic literature. One of the old results in this literature[1] is that as consumers choose a consumption plan for present and future periods, given their intertemporal budget constraint, their marginal rate of substitution between consumptions in two given future periods depends on the period at which it is computed. In future periods, individuals will save less than what they currently would plan to save for those periods. On an aggregate level this lack of savings translates into insufficient investment and accumulation, measured against a benchmark where agents are not biased for the present, which may call for government intervention to correct the inefficiency.

An extensive literature, briefly surveyed in section 2, has shown that such government intervention can indeed raise savings and hence improve welfare. This conclusion, however, relies on a paternalistic view for the role of government, prevalent in public policy approaches to behavioural failures, which assumes that government corrects the bias of the individuals.[2] This is a valuable normative benchmark, since, as Gruber and Köszegi (2001, p. 1287) observe, individuals would choose this policy if they could postpone implementation to the next period and could bind future decision-makers.[3] From a positive point of view, however, it is unclear whether a real-world government acts in this way. The reason for this is that the political actors are chosen from the set of citizens, or are responsible to them, so that, in a democratic society, the government will likely represent individuals' current preferences.[4] Moreover, in realistic political settings the current government

1 See Strotz (1955), Phelps and Pollak (1968) for early contributions and Frederick et al. (2002) for an insightful survey of the issues.

2 Research has examined which government interventions can induce individuals to behave as if they were unbiased. See for example Gruber and Köszegi (2004), O'Donoghue and Rabin (2003; 2006), Thaler and Sunstein (2003), Aronsson and Thunström (2008).

3 Recently, Feng and Ke (2018) provide a normative foundation for exponential discounting on the part of the planner by showing that this form of discounting is consistent with being intergenerationally Pareto (in the sense that if all individuals from all generations prefer one choice over another, then the planner agrees with this choice) and strongly non-dictatorial (in the sense that no individual from any generation is ignored).

4 Bisin et al. (2015) present a model where office-seeking candidates align policies with the preferences of a biased electorate. In this contribution,

cannot easily bind successors and hence policies must be optimal in every period.

For these reasons, in the present paper we take a contrasting view, where the government shares the biased preferences of individuals, applies the same discounting as the citizens it serves and cannot bind future governments.[5] Within this context, we analyze whether, and under what circumstances, such a government can improve welfare. We address this question in an environment where investment, which is affected by the private agents' bias for the present, both raises future consumption opportunities and causes, via enhanced production, an environmental damage. This makes it possible to highlight the potential of public policy balancing growth and environmental objectives even if executed by a government that is subject to the same behavioural failures as citizens are.

The relevance of taking this line of investigation stems from the fact that public policy need not necessarily improve welfare once one abandons the paternalistic view of government. This has been shown in the important contribution of Krusell et al. (2002), who analyze an economy within which consumers discount the future in a "quasi-hyperbolic" way when choosing how much to save.[6] Since they cannot commit to future actions (though they would like to), the consumers can be viewed as if they are playing a game with their future selves, with whom they disagree about how much to save. Consumers are, however, rational ("sophisticated") about their "internal friction," anticipating the savings decisions of future selves and taking into account how their own current saving, by raising future income, affects future saving. The social planner and consumers share the same time-inconsistent preferences with quasi-hyperbolic discounting. Within this set-up, Krusell et al. (2002) show that not only does a benevolent social planner *not* deliver the same consumption allocation as does a laissez-faire world but also the planner delivers strictly *lower welfare*. The intuition behind this relies on the idea that the social planner sees through the impact of savings on the rate of return thereby choosing to save less than private individuals would under laissez-faire. As more savings and so more capital

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consumers/voters have self-control problems that they can "correct" through investment in illiquid assets that allows them to constrain their future selves' consumption plans. Office-seeking candidates may offer deficit-financed transfers to voters, who, being rational, respond by rebalancing their portfolio by holding more illiquid assets, thereby increasing public debt accumulation.

5 While the paternalistic view dominates the literature, there are some contributions who take a similar view on the government as we do in this paper. See, for example, Karp (2005), Halac and Yared (2014; 2018) and Gerlagh and Liski (2018).

6 As will be seen shortly, this is equivalent to shifting uniformly all future utilities (that is, from the date the decision is taken) by a factor β.

accumulation is preferable from an unbiased perspective, the decentralized allocation brings the economy closer to the full commitment outcome. The implication of this is that intervention from the social planner (who makes the same biased intertemporal choices) will imply more immediate consumption and therefore less welfare relative to the competitive case. This is a powerful result pointing towards the undesirability of government intervention showing that welfare improvements are not warranted by such intervention.

The preceding discussion naturally raises the issue of whether intervention by a biased government is still undesirable in the presence of market imperfections. To keep things tractable, the focus will be on externalities arising as a by-product of output affecting consumer utility. The aim of the paper is to explore two ultimately interrelated questions: (a) How does the bias for the present interact with a negative output-induced externality? (b) Are there circumstances in which government intervention is welfare enhancing?

While we model a particular kind of externality, there are of course many market imperfections one could conceivably consider. For example, one can think of instances where the externality is on the production possibilities of the economy, affecting productivity, or one could think of harmful consumptive activities such as, for example, driving. We choose to focus on output-induced environmental externalities affecting utility since these are a topical issue and examples of such environmental externalities abound: air pollution, congestion and, more broadly, climate change, to name three. Moreover, the precise form of the externality is not the key point here. Instead, we see our approach as having a broader perspective regarding the interaction between behavioural biases and any unintended consequences caused by private actions and the role of a government that shares citizens' biases in correcting inefficiencies.

To address the issues discussed above, we make use of the model of Krusell et al. (2002), introducing a negative externality that arises as a by-product of economic output. In this set-up, we characterize the recursive competitive equilibrium among the sequence of selves of private consumers without any government intervention. Since in this framework the externality emanates from production and ultimately from investment, the bias for the present, by reducing savings, mitigates the damage brought about by the externality. Thus, in response to question (a) above, the model shows that a bias for the present counteracts an output-induced externality.[7] The competitive equilibrium is contrasted with the equilibrium in a game among a sequence of planner's selves who control savings and investment, distinguishing between a planner who shares the individuals' biased preferences and, as a benchmark, a

7 That the present bias might be something desirable, when interacted with another friction, has also been emphasized in the contribution by Bénabou and Tirole (2002).

planner who chooses policies so as to maximize a utility function with exponential discounting. Looking for mnemonic labels, we conveniently call the former "biased" and the latter "unbiased" planner. Within this context, it is shown that in the presence of the externality the biased planner saves even *less* than she would under no externality.[8] The reason for this is intuitive: the planner internalizes the negative externality and therefore has an additional reason to reduce savings further, an incentive that aggravates the effect of controlling the interest rate mentioned above.

We calculate the investment tax rate that implements the biased planner's optimal choice in a competitive equilibrium. The tax rate increases if the present bias becomes stronger or the externality becomes more severe. Interestingly, the two effects reinforce each other in the sense that the tax rate increases faster in the externality if the present bias is stronger. This arises from the desire of the current self to change the behaviour of next period's self, which is a typical feature of the equilibrium among a sequence of sophisticated present-biased selves. In our model, the current selves, both of the private agent and the biased planner, consider the saving of their next selves insufficient. They therefore derive an additional benefit from current saving since this raises next period's saving via an income effect. The marginal propensity to save is, however, smaller for next period's planner than for next period's private agent, since the next period's planner will consider the negative externality his or her saving will cause in the period after the next. Hence, from the planner's perspective, private agents overestimate the additional benefit of savings, justifying a further rise in the investment tax rate.

Normatively, we assess the impact of the biased planner's intervention on welfare, measured as the discounted utility that can be achieved under commitment. In line with the two views of government outlined above, the analysis distinguishes between "unbiased" welfare, where equilibrium utilities are evaluated with exponential discounting, and "biased" welfare, where utilities are evaluated in the same way as the current self would evaluate them, including quasi-hyperbolic discounting. According to the first criterion, the bias for the present is considered a failure and therefore should not count when outcomes are judged normatively. According to the second criterion, outcomes should be judged according to the preferences that determine the agents' actions. It should be emphasized that while use is made, for convenience, of the labels unbiased and biased for these criteria, the analysis does not take a normative stand on which one is to be preferred.[9] Instead,

8 As in Krusell et al. (2002).

9 In the behavioural economics literature, the first view appears to be more popular. See, for example, Thaler and Sunstein (2003, p. 173), who emphasize that revealed preferences do not always equate with welfare. See also O'Donoghue and Rabin (2006, footnote 12, p. 1829). In contrast, Krusell et al. (2002) and Karp (2005) focus on the second welfare criterion.

it analyzes the welfare consequences for both criteria following government intervention.

The results show that introducing an externality into the model does not per se imply that intervention by a biased government is beneficial. We find that government intervention improves welfare according to both criteria if and only if the externality is sufficiently strong or the bias for the present is weak. When the externality is very important, the competitive equilibrium (laissez-faire) leads to over-saving relative to the welfare maximizing choice and hence the downward bias on savings exerted by the biased planner is beneficial. Conversely, when bias for the present is very pronounced, the planner's savings rate is far too low, whereas the market will save a little less, or only somewhat more, than what would be efficient. Moreover, we find that the range of parameters where government intervention is beneficial is larger when one uses the biased than when one uses the unbiased criterion. Altogether, in response to question (b) above, these results show that, on the one hand, one cannot realistically expect that a government that shares the biased objective of current citizens and cannot commit future policy-makers will implement measures that correct for the bias. On the other hand, we see that, in contrast to the finding by Krusell et al. (2002), even when government is biased in this sense, there is room for welfare improving intervention if a market failure, like a negative externality, is present.

While our main analysis assumes sophisticated agents, in an extension, we also briefly consider naïve consumers and planners, who expect their future selves to discount exponentially. We find that naïve present-biased consumers save less than their sophisticated counterparts, and hence the range of parameters that leads to over-saving relative to the chosen welfare criterion is smaller if agents are naïve. The naïve present-biased planner's savings rate equals the one chosen by the sophisticated present-biased planner but is still lower than in competitive equilibrium. Therefore, as in the main analysis, government intervention can be beneficial only if private decisions lead to over-saving. Combining both observations, one finds, quite surprisingly, that government intervention is less likely to enhance welfare when agents are naïve than when they are sophisticated.

The structure of the paper is as follows. In section 2, we review the literature relating to our paper. Section 3 then sets out the model, and section 4 analyzes the competitive equilibrium and the planner's choice. In section 5, we derive welfare maximizing savings rates, compare these to the allocation implemented under laissez-faire and by the biased planner and evaluate whether the planner's intervention improves welfare. Section 6 summarizes, discusses some extensions and concludes. Proofs and longer derivations are relegated to the online appendix.

2. Literature review

In this section, we review the main contributions on the themes addressed in this paper, which are the analysis of savings decisions by present-biased individuals, correcting interventions by government and environmental policy in the context of present-biased preferences.

There is a significant literature that addresses the implication of the bias for the present dating back to the contributions of Strotz (1955) and Phelps and Pollak (1968), who first formalized quasi-hyperbolic discounting and its implications for economic outcomes. A general mathematical analysis of the infinite-horizon decision problem of a quasi-hyperbolically discounting consumer is presented by Harris and Laibson (2001). Further theoretical contributions include, among others, Azfar (1999), who relates declining discount rates to uncertainty, Herings and Rohde (2006), who extend the concept of general competitive equilibrium to economies where agents have time-inconsistent preferences, and Salanié and Treich (2006), who emphasize the distinction between additional discounting and self-control problems.

Particular attention has been paid to the analysis of policies that induce present-biased consumers to save more over their life time. Examples for such intervention policies are encouragement of accumulation of illiquid assets and the introduction of various savings plans and public pension systems. Laibson (1998) shows that several features of observed savings behaviour such as the absence of precautionary saving can be explained by the quasi-hyperbolic discounting model. Similarly, Angeletos et al. (2001) simulate behaviour under exponential and quasi-hyperbolical discounting and conclude that the latter matches data better than the former. Diamond and Köszegi (2003) consider the incentives for earlier selves to affect retirement decisions through changes in savings, Benartzi and Thaler (2007) discuss heuristics in savings decisions for retirement and Gustman and Steinmeier (2012) analyze how structural elements of pension systems affect savings and retirement decisions of individuals with quasi-hyperbolic preferences.

For policy, instruments that aim at overcoming under-saving compared with the unbiased benchmark are of significant interest. In this line of research, Laibson (1997) analyzes the purchase of an illiquid asset as a commitment device and Thaler and Benartzi (2004) report evidence on a program that offers employees the opportunity to commit future pay rises to a retirement savings plan. Malin (2008) shows that a savings floor does not necessarily increase welfare when general equilibrium effects on the interest rate are taken into account and Andersen and Bhattacharya (2011) show that a pay-as-you-go pension system is optimal only if quasi-hyperbolic discounting is sufficiently strong. Integrating savings decisions by consumers with quasi-hyperbolic preferences into a model with internationally mobile capital, Aronsson and Sjögren (2014) show that optimal policies to correct for the bias for the present differ between large open, small open and closed economies.

More recently, Moser and de Souza e Silva (2019) investigate optimal retirement savings policies when there is a paternalistic motive to undo an individual's present bias combined with a redistributive motive. These authors explore the link between savings choices throughout the income distribution, identifying a trade-off between these two motives. They point out that optimal policy enforces high savings rates at low incomes but offers a choice between various subsidized savings options at high incomes. Chan (2017) finds that most individuals are time-inconsistent, exhibiting varying degrees of present bias and perception of the commitment problem. Interestingly, introducing a welfare component to the tax system can make individuals worse off by aggravating the commitment problem.

Consumers who suffer from self-control and have a bias for present consumption may also accumulate excessive private debt (see, for example Shui and Ausubel 2005, Gottlieb 2008, Heidhues and Köszegi 2010). To "correct" for this, the policy recommendation might be to put a constraint on the actions of financial intermediaries.

Some contributions have analyzed quasi-hyperbolic preferences in relation to environmental issues. In the model by Karp (2005), flow emissions that contribute to a stock of pollutant are controlled by a sequence of quasi-hyperbolically discounting regulators. Karp (2005) shows that, for additively separable preferences, a planner with commitment power chooses a trajectory of emissions that eventually leads to a lower stock of pollutant. Moreover, in almost all equilibria, reducing the long-run stock of pollutant would improve welfare.

Gerlagh and Liski (2018) investigate the appropriate discount rate in a model where energy use causes long-run damage. They point out that environmental policy in the form of an energy tax provides a commitment device since its beneficial impact occurs far in the future. As a consequence, the rate of return required for investment in environmental quality falls short of the rate of return required for capital. Like our model, Gerlagh and Liski (2018) build upon the model by Krusell et al. (2002), but our focus differs. Our externality emanates directly from output and hence reducing it requires curbing production. As a consequence, when the planner in Gerlagh and Liski (2018) taxes investment (reflecting the present bias), she reduces welfare, whereas in our model, this tax may be welfare enhancing if the externality is strong enough. Therefore, our set-up makes it possible to emphasize the mitigating impact of the present bias on the environmental damage and to characterize conditions such that government intervention is beneficial in spite of the bias.

The present paper adds to this literature by combining quasi-hyperbolic discounting and an output-induced externality in a unified model. Thereby, we point out that the reduction in savings effectuated by the present bias, which is generally a cause of concern, helps mitigating the environmental externality. Moreover, unlike most of the literature, we model a government that pursues the preferences of current individuals, rather than a paternalistic objective. Our model shows that, in spite of its present bias, such a government will implement welfare-improving environmental policy if market failure is important.

3. The model

The model is familiar from Krusell et al. (2002) appropriately modified to deal with the issue at hand. There is an infinitely lived consumer who derives utility from consumption, denoted by C, and suffers disutility from a non-tradeable negative externality, denoted by D, at different dates. Time begins at 0, is discrete and infinite, and there is no uncertainty.

Utility per period u_t, $t = 0, 1, 2, \ldots$, is additively separable between consumption and externality and given by

$$u_t = \log(C_t) - \gamma \log(D_t) ,$$

where γ, with $0 \leq \gamma < 1$, measures the extent of damage created by the externality.[10] Preferences are time-additive and take the form

$$\begin{aligned} U_o &= u_o + \beta \left(\delta u_1 + \delta^2 u_2 + \delta^3 u_3 + \ldots\right), \\ U_1 &= u_1 + \beta \left(\delta u_2 + \delta^2 u_3 + \ldots\right), \\ U_2 &= u_2 + \beta \left(\delta u_3 + \ldots\right) , \end{aligned} \tag{1}$$

where δ, $0 < \delta < 1$, is a standard discount factor and $0 < \beta \leq 1$, as noted earlier, represents additional discounting between the period of decision making and later periods. If $\beta < 1$, there is a bias towards immediate consumption, and discounting is "quasi-hyperbolic."[11] It is clear that with $\beta < 1$ preferences are time inconsistent: at date $t - 1$, the trade-off between dates t and $t + 1$ is perceived differently than at date t. The point here being that the consumer's self at time t and her self n periods after disagree on the value of consumption in period $t + n$ relative to consumption at date $t + n + 1$. For $\beta = 1$ the model reduces to exponential discounting and so the standard inter-temporal choice model where there is no bias for the present. It will be further assumed that there is no technology consumers can use to commit to future consumption levels.

Following Krusell et al. (2002), we model sophisticated agents, that is, the consumer rationally understands that her preferences will change as time

10 The case $\gamma < 0$ corresponds to a positive externality. We discuss this case in section 6.

11 As suggested in the survey by Cohen et al. (2020), $\beta < 1$ seems to be the empirically relevant case. For example, Laibson et al. (2007, p. 18) report a benchmark estimate of $\beta = 0.70$. In labour market applications, for subgroups of the population, calibration results of $\beta = 0.90$ (DellaVigna and Paserman 2005, p. 568) and estimates of 0.40, 0.89 (Paserman 2008, p. 1433) were found. In real effort experiments, Augenblick et al. (2015, p. 1087) obtain results between $\beta = 0.90$ and $\beta = 0.97$. If $\beta > 1$, as in the point estimate 1.004 by Andreoni and Sprenger (2012, p. 3346), then there is a bias towards future consumption. This is a case that is not analyzed.

goes by. The current decision is therefore made taking this into account. The implication of this is that decision making is modelled as a dynamic game, with the agent's current and future selves as players. The focus is on (first-order) Markov equilibria and so at any moment in time no histories are assumed to matter beyond what is summarized in the current stock of wealth held by the agent.[12]

Production is Cobb-Douglas with full depreciation of capital. The resource constraint simply states that consumption and savings (investment) in a given period must equal output in that period and so

$$C + k' = Ak^{\alpha}, \tag{2}$$

where k' denotes investment, which, due to full depreciation, equals next period's capital stock, k denotes current capital stock, α, with $0 < \alpha < 1$, is the income share of capital and $A > 0$ represents the exogenous stock of knowledge.

To capture the externality in a simple way, it is assumed that one unit of output generates one unit of emissions and thus[13]

$$D = Ak^{\alpha}.$$

Markets are perfectly competitive, which implies

$$r = \alpha A k^{\alpha-1}, \tag{3}$$

$$w = (1-\alpha)Ak^{\alpha}, \tag{4}$$

where r is the price of capital and w is the wage rate.

The next section analyzes the competitive equilibrium and the policy chosen by the planner.

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12 The analysis rules out trigger-strategy equilibria of the type studied in Bernheim et al. (1999). Moreover, Markov equilibria in games among a sequence of quasi-hyperbolically discounting selves are typically not unique (see for example Krusell and Smith, Jr. 2003, Karp 2005, Vieille and Weibull 2009). Krusell et al. (2002, p. 48) focus on equilibria that are limits of equilibria in the finite horizon game. For the parametric formulation, which allows for a closed form solution, this refinement eliminates the multiplicity. We follow this approach and restrict attention to the same kind of equilibrium.

13 That is, we model a flow externality. One interpretation is that this is an externality caused by pollution damage (with the stock being completely dissipated at the end of each period). Since the focus is on the inter-temporal trade-off between current and future consumption, which may be biased by quasi-hyperbolic discounting, and how this is affected by negative externalities generated by investment and production, the analysis abstracts from abatement technologies or the choice between industries that differ in the amount of externality created.

4. Recursive equilibrium

4.1. Competitive equilibrium

In the competitive equilibrium, the consumer makes his or her decision taking as given the prices as functions of the aggregate capital stock, $\overline{k}$, that is, $r(\overline{k})$ and $w(\overline{k})$, the law of motion for the aggregate capital stock $\overline{k}' = G(\overline{k})$ and the decision rule of her future selves, $g(k, \overline{k})$. She also takes as given the level of the externality. The recursive equilibrium requires then two state variables for the consumer; one for the consumer's own saving decision, k, and one for the economy's savings $\overline{k}$, which determine prices according to equations (3) and (4).

In any given time period, the current self chooses investment k', taking prices parametrically, so as to solve the problem

$$V_o(k, \overline{k}) = \max_{k'} \ \{\log\left(r(\overline{k})k + w(\overline{k}) - k'\right) \\ - \gamma \log\left(A\overline{k}^{\alpha}\right) + \beta\delta V\left(k', \overline{k}'\right)\}. \tag{5}$$

This defines the optimal choice rule of the current self given by $k' = \tilde{g}(k, \overline{k})$, which in an equilibrium of the game among the subsequent selves must coincide with expected behaviour, that is, $\tilde{g}(k, \overline{k}) = g(k, \overline{k})$ for all $(k, \overline{k})$. The continuation value function satisfies

$$V(k, \overline{k}) = \log(r(\overline{k})k + w(\overline{k}) - g(k, \overline{k})) \\ - \gamma \log\left(A\overline{k}^{\alpha}\right) + \delta V(g(k, \overline{k}), G(\overline{k})). \tag{6}$$

The laissez-faire solution to the model is given by the definition below.

Definition 1. (See Krusell et al. 2002, definition 1, p. 51.) A recursive competitive equilibrium consists of a decision rule $g(k, \overline{k})$, a value function $V(k, \overline{k})$, pricing functions $r(\overline{k})$ and $w(\overline{k})$ and a law of motion for the aggregate capital stock $G(\overline{k})$ such that:

1. Given $V(k, \overline{k})$, $g(k, \overline{k})$ solves the maximization problem (5).
2. Given $g(k, \overline{k})$ and $G(\overline{k})$, $V(k, \overline{k})$ satisfies equation (6).
3. Firms are price takers and maximize profits, implying that $r(\overline{k})$ and $w(\overline{k})$ satisfy equations (3) and (4).
4. The law of motion for the aggregate capital stock resulting from the current self's decision is consistent with the law of motion of the aggregate capital stock, that is, $g(\overline{k}, \overline{k}) = G(\overline{k})$.

Equipped with the above definition, the following proposition characterizes the laissez-faire equilibrium in the presence of the externality and quasi-hyperbolic discounting.

PROPOSITION 1. *The recursive competitive equilibrium is given by*

1. $V(k,\overline{k}) = a + b\log\overline{k} + c\log(k+\varphi\overline{k})$, *where*

$$c = \frac{1}{(1-\delta)},\quad b = \frac{\alpha-1}{(1-\delta\alpha)(1-\delta)} - \frac{\gamma\alpha}{1-\delta\alpha},\quad \varphi = \frac{(1-\alpha)[1-\delta(1-\beta)]}{\alpha(1-\delta)},$$

2. $g(k,\overline{k}) = \frac{\beta\delta}{1-\delta(1-\beta)} r(\overline{k})k$,
3. $G(\overline{k}) = g(\overline{k},\overline{k}) = \frac{\beta\delta\alpha}{1-\delta(1-\beta)} A\overline{k}^{\alpha}$.

Proof. The proof of the proposition is relegated to online appendix A.I. ■

Proposition 1 reconfirms the result by Krusell et al. (2002): this will be the case if $\gamma = 0$. The savings rate (see no. 3 of proposition 1) is a constant share of aggregate income $A\overline{k}^{\alpha}$ and given by

$$s_l = \frac{\beta\delta\alpha}{1-\delta(1-\beta)}\ . \tag{7}$$

It is unaffected by the presence of the externality (and so it is independent of γ). This is because the externality arises from aggregate production, which is not a choice variable of private agents. Hence, consumers do not take the damage inflicted by their investment into account when making decisions, and, consequently, the savings rate is as in Krusell et al. (2002, p. 52). At the same time, and in a deviation from Krusell et al. (2002), the presence of the externality reduces utility by the term $(\gamma\alpha)/(1-\delta\alpha) > 0$ in the coefficient b (see no. 1 of proposition 1).

4.2. The planner's problem

The analysis now turns to the planner's problem. As discussed in the introductory section, the model distinguishes between two "types" of planner, both of which act in the interest of the representative consumer but differ in what they consider the appropriate objective for the consumer. The first type of planner disregards the bias for the present in the sense that $\beta = 1$ in equation (1), whereas the second type considers the preferences of the current self, where β enters equation (1) with the same value that governs the consumer's choice. For looking for convenient labelling of the two types, and without prejudice regarding the respective normative merits of the two planners' objectives, the first type of planner will be called unbiased and the second one biased.

The analysis now proceeds with the characterization of the biased planner's choices turning to the unbiased shortly after.

Biased planner: The biased planner cannot commit, similar to the consumer, to future actions and anticipates the choices by future planners. Unlike the consumer, however, the planner directly controls aggregate capital $k$and takes the externality into account. So in any given time period, the current self (of the planner) chooses investment k' so as to solve the following problem:

$$V_{op}(k) = \max_{k'} \ \{\log (Ak^{\alpha} - k') - \gamma \log (Ak^{\alpha}) + \beta\delta V_p (k')\}, \tag{8}$$

which defines the biased planner's optimal choice rule $k' = \tilde{h}(k)$, and the value function satisfies

$$V_p(k) = \log (Ak^{\alpha} - h(k)) - \gamma \log (Ak^{\alpha}) + \delta V_p(h(k)). \tag{9}$$

Here, $h(k)$ denotes the anticipated investment rule of future planners, which in an equilibrium must coincide with the current planner's choice, $\tilde{h}(k) = h(k)$.

It can be straightforwardly shown that:

PROPOSITION 2. *The solution to the biased planner's problem is given by*

1. $V_p(k) = a + b\log k$, *where* $b = \frac{(1-\gamma)\alpha}{1-\delta\alpha}$.
2. $h(k) = \frac{\beta\delta\alpha(1-\gamma)}{1-\delta\alpha[1-\beta(1-\gamma)]} Ak^{\alpha}$.

Proof. The proof parallels the proof of proposition 1 and is therefore omitted. ■

Proposition 2 shows that the biased planner also invests a constant share of income given by

$$s_{bp} = \frac{\beta\delta\alpha(1-\gamma)}{1-\delta\alpha[1-\beta(1-\gamma)]}, \tag{10}$$

which, perhaps not surprisingly since this planner internalizes the impact of her decision on the level of the externality, decreases in γ (and so the extent of the externality). As expected the planner, even in the absence of an externality and though she has the same preferences as the consumer, chooses a different savings rate than the consumer since she takes into account the impact on the price of capital.

In proposition 2, the biased planner directly controls investment. The question then that arises is whether the government can induce private agents to save and invest according to equation (10) by the appropriate choice of tax instruments. The answer to this is in the affirmative. Keeping the structure the same, but allowing the planner to have access to proportionate taxes on income τ_y and investment τ_i, it can be shown[14] that:

PROPOSITION 3. *The optimal time consistent tax rates are given by*

$$\tilde{\tau}_i = \frac{\delta(1-\alpha)(1-\beta) + \gamma[1-\delta(1-\beta(1-\alpha))]}{(1-\gamma)[1-\delta(1-\beta(1-\alpha))]}, \tag{11}$$

$$\tilde{\tau}_y = -\frac{\alpha\beta\delta^2(1-\alpha)(1-\beta) + \gamma\alpha\beta\delta[1-\delta(1-\beta(1-\alpha))]}{[1-\delta(1-\beta(1-\alpha))][1-\delta\alpha(1-\beta(1-\gamma))]}. \tag{12}$$

These tax rates implement the savings rate s_{bp} *from equation (10).*

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14 The details of this, being tedious, are relegated to the online appendix.

Proof. See online appendix A.II. ■

Proposition 3 emphasizes that two instruments are needed for the planner to achieve the level of savings in equation (10): one instrument changes the inter-temporal trade-off and the other instrument balances the budget. The investment tax serves the first purpose. It corrects for the inefficiency in the level of savings (through the return to savings). For the second purpose, we use the (negative) income tax. Alternatively, as can be verified by going through the steps of the proof of proposition 3(see online appendix A.II), tax revenues can also be rebated to consumers via a lump-sum transfer.

Denoting the optimal investment tax rate in the absence of externalities[15] by $\tilde{\tau}_i^o$, it is the case that

$$\tilde{\tau}_i - \tilde{\tau}_i^o = \frac{\gamma(1-\delta\alpha)}{(1-\gamma)[1-\delta(1-\beta(1-\alpha))]} > 0, \tag{13}$$

and thus, in the presence of an externality, the government chooses a higher investment tax rate. Since the tax rate in equation (11) is increasing in γ, one deduces that, for any admissible α and δ:

COROLLARY. *For any $0 < \beta \leq 1$, the investment tax rate $\tilde{\tau}_i$ increases in the extent of the externality γ.*

The government taxes investment for two reasons, because of the reduction in the marginal product of capital (as in Krusell et al. 2002) and the additional pollution caused tomorrow by an extra dollar of savings today. These two effects point towards the same direction, so that the resulting tax rate is higher than in the special cases $\beta = 1$ or $\gamma = 0$, where only one of the effects is present. In the benchmark case of exponential discounting, when $\beta = 1$, the first effect disappears, and consequently, $\tilde{\tau}_i^o = 0$: since there is no time inconsistency, there is no reason to distort savings decisions. As can be seen from equation (11), when $\beta = 1$, the resulting tax rate is

$$\tilde{\tau}_i = \gamma/(1-\gamma) > 0.$$

This is the Pigouvian tax rate that internalizes the pollution externality in a standard model without quasi-hyperbolic discounting.

The optimal investment tax rate in equation (11) can be decomposed as

$$\tilde{\tau}_i = \frac{\gamma}{1-\gamma} + \frac{\tilde{\tau}_i^o}{1-\gamma}. \tag{14}$$

The first term, the Pigouvian tax, does not involve β and hence occurs independently of the present bias. The second term is the tax rate τ_i^o that the biased planner sets in the absence of an externality, but enhanced by the factor $1/(1-\gamma) > 1$ if the externality is present. Differentiating the right-hand side of equation (14) with respect to γ and then with respect to β gives

15 When $\gamma = 0$, proposition 4 of Krusell et al. (2002) emerges.

COROLLARY. *The impact of the externality on the investment tax rate is stronger if the present bias is larger:*

$$\frac{\partial^2 \tilde{\tau}_i}{\partial \gamma \partial \beta} = \frac{1}{(1-\gamma^2)} \cdot \frac{\partial \tilde{\tau}_i^o}{\partial \beta} = -\frac{\delta(1-\alpha)(1-\delta\alpha)}{(1-\gamma)^2[1-\delta(1-\beta(1-\alpha))]^2} < 0.$$

Thus, the two motives for taxing capital are not just added together, but rather reinforce each other.

This result can be understood by comparing the generalized Euler equations[16] for the competitive equilibrium

$$\frac{1}{C} = \frac{\beta\delta}{C'} \left[r\left(\overline{k}'\right) + \left(\frac{1}{\beta} - 1\right) \frac{\partial g\left(k', \overline{k}'\right)}{\partial k'} \right] \tag{15}$$

and for the biased planner's solution

$$\frac{1}{C} = \frac{\beta\delta}{C'} \left[\alpha A(k')^{\alpha-1} - \gamma \frac{C'}{D'} \alpha A(k')^{\alpha-1} + \left(\frac{1}{\beta} - 1\right) h'(k') \right]. \tag{16}$$

In these equations, the marginal utility of current consumption on the left-hand side is equated to the discounted marginal benefit of investment on the right-hand side.[17]

The first term in brackets, which is common to equations (15) and (16), is the marginal product of next period's capital stock. The remaining terms differ between the two equations and hence explain why the planner wants to correct the laissez-faire equilibrium. The second term in brackets in equation (16), which is absent in equation (15), is the marginal willingness to pay for avoiding the environmental damage created by an increase in next period's capital stock. Since this damage occurs contemporaneously with the gain in income in the first term, the tax required to correct for it does not depend on the present bias. This effect accounts for the Pigouvian part $\gamma/(1-\gamma)$ in the optimal tax rate given in equation (14).

The last terms in the brackets of equations (15) and (16) arise from the fact that the current selves, both of the private consumer and of the biased planner, value next period's savings higher than their future selves will do and therefore want to incentivize their future selves to invest more. This creates an additional benefit from current investment, since higher capital in the next period raises next period's savings, according to

$$\frac{\partial g\left(k', \overline{k}'\right)}{\partial k'} = \frac{\beta\delta}{1-\delta(1-\beta)} \, r\left(\overline{k}'\right)$$

16 These equations are derived in online appendix A.III.

17 In equations (15) and (16), a prime ($'$) after a variable denotes the value of that variable in the next period.

in the competitive equilibrium and

$$h'(k') = \frac{\beta\delta\alpha(1-\gamma)}{1-\delta\alpha[1-\beta(1-\gamma)]}\ \alpha A(k')^{\alpha-1}$$

for the planner. These reactions differ in two ways. First, as highlighted by Krusell et al. (2002, pp. 54–56), the private agent considers the marginal return of investment to be constant, whereas the planner sees through the diminishing marginal return of capital, expressed by the presence of the curvature parameter $\alpha < 1$ in the planner's reaction $h'(k')$. Second, and this is specific to our model, the planner also anticipates that investment in the next period causes an externality in the period after the next, which is why $(1-\gamma)$ is present in $h'(k)$ but not in $\partial g(k', \overline{k}')/\partial k'$. Both considerations imply that current investment has a smaller impact on the next planner's decision than on the next private consumer's decision, and thus is less valuable for the current planner than for the current private consumer. This discrepancy accounts for the part $\tau_i^o/(1-\gamma)$ in the investment tax rate from equation (14).

The desire to change the next self's decision is specific to a model with sophisticated present-biased agents. Hence, as is apparent from the factor $(1/\beta - 1)$ in equations (15) and (16), the difference between the biased planner's and the consumer's incentives is larger if the present bias is stronger. As a consequence, the externality and the present bias reinforce each other in the second term of the tax rate, $\tau_i^o/(1-\gamma)$.

Unbiased planner: The unbiased planner pursues the long-run preferences of agents. This comes down to analyzing the planner's optimization problem (8) and (9) with β set equal to one. In this case, following from equation (10),

$$s_{up} = \frac{\delta\alpha(1-\gamma)}{1-\delta\alpha\gamma}. \tag{17}$$

We turn now to the welfare analysis of competitive equilibrium and planners' choices.

5. Welfare

5.1. Biased and unbiased welfare

In contrast to the competitive equilibrium and the biased planner's choice, which suffer from time-inconsistency, welfare is defined as the present-value of utility that can be reached by committing to a time path of saving and consumption at date 0. We distinguish two welfare criteria. First, we consider welfare according to the preferences that determine agents' actions. That is, we use the preferences of the current self, which incorporate quasi-hyperbolic discounting, as welfare criterion. Second, we consider welfare according to the long-run preferences of individuals, that is, future utilities are discounted exponentially. Paralleling the labeling of the two types of planners, and again without taking a normative position, we label the former criterion as biased welfare, and the latter as unbiased welfare.

The core decision agents or planners have to take in our model is how much to invest. We therefore define both welfare criteria as functions of the current capital stock k and the savings rate s, which is applied in each period. The capital stock evolves according to $k' = sAk^\alpha$. Making use of this and equation (2), the continuation value function (see equations (6) and (9)) becomes

$$V^*(k;s) = \log\left((1-s)Ak^\alpha\right) - \gamma\log\left(Ak^\alpha\right) + \delta V^*(sAk^\alpha;s), \tag{18}$$

which upon iterating forward[18] gives the unbiased welfare as a function of the savings rate

$$V^*(k;s) = \frac{1}{1-\delta}\log(1-s) + \frac{\delta\alpha(1-\gamma)}{(1-\delta\alpha)(1-\delta)}\log s + \frac{\alpha(1-\gamma)}{1-\delta\alpha}\log k + \frac{1-\gamma}{(1-\delta\alpha)(1-\delta)}\log A. \tag{19}$$

The agent's objective in the current period is (see equations (5) and (8))

$$V_o^*(k;s) = \log\left((1-s)Ak^\alpha\right) - \gamma\log\left(Ak^\alpha\right) + \beta\delta V^*(sAk^\alpha;s), \tag{20}$$

and so, using equation (19) in equation (20), the biased welfare is given by

$$V_o^*(k;s) = \frac{1-\delta(1-\beta)}{1-\delta}\log(1-s) + \frac{\beta\delta\alpha(1-\gamma)}{(1-\delta\alpha)(1-\delta)}\log s + \frac{\alpha(1-\gamma)[1-\delta\alpha(1-\beta)]}{1-\delta\alpha}\log k + (1-\gamma)\left(1+\frac{\beta\delta[1+\alpha(1-\delta)]}{(1-\delta\alpha)(1-\delta)}\right)\log A.$$

The optimal savings rate according to the biased welfare criterion solves the problem max ${}_sV_o^*(k;s)$, with first order condition being

$$\frac{\partial V_o^*(k;s)}{\partial s} = -\frac{1-\delta(1-\beta)}{(1-\delta)(1-s)} + \frac{\beta\delta\alpha(1-\gamma)}{(1-\delta\alpha)(1-\delta)s} = 0. \tag{21}$$

It is straightforward to verify that $\partial^2 V_o^*/\partial s^2 < 0$ and so $V_o^*(k;s)$ is strictly concave in s and the first order condition is sufficient for a global maximum with the welfare-maximizing savings rate, denoted by s_b^*, given by

$$s_b^* = \frac{\beta\delta\alpha(1-\gamma)}{(1-\delta\alpha)[1-\delta(1-\beta)] + \beta\delta\alpha(1-\gamma)}. \tag{22}$$

In the same way, solving max ${}_sV^*(k;s)$ yields the savings rate

$$s_u^* = \frac{\delta\alpha(1-\gamma)}{1-\delta\alpha\gamma}, \tag{23}$$

which maximizes unbiased welfare.

18 This derivation is contained in online appendix A.IV.

5.2. Comparing savings rates

The next result shows how the savings rates chosen by the planners, s_{bp} and s_{up}, and achieved under laissez-faire, s_l, compare with the savings rates s_b^* and s_u^*, which maximize welfare under commitment.

PROPOSITION 4. *For all discount factors and capital shares $0 < \delta, \alpha < 1$, externality parameters $0 \leq \gamma < 1$, and short-run discount factors $0 < \beta \leq 1$:*

1. *The savings rate chosen by the biased planner is at most as large as the savings rate that maximizes the biased consumer's welfare, which in turn is at most as large as the savings rate that maximizes unbiased welfare, which equals the savings rate chosen by the unbiased planner: $s_{bp} \leq s_b^* \leq s_u^* = s_{up}$. The inequalities are strict if and only if $\beta < 1$.*
2. *(a) The savings rate chosen by the biased planner is at most as large as the laissez-faire savings rate: $s_{bp} \leq s_l$. The inequality is strict if and only if $\beta < 1$ or $\gamma > 0$.*
   (b) If $\gamma - \delta\alpha \geq 0$, then the laissez-faire savings rate exceeds the savings rate that maximizes the biased consumer's welfare: $s_l > s_b^*$; if $\gamma - \delta\alpha < 0$, then the laissez-faire savings rate is lower than (is equal to, exceeds) the savings rate that maximizes the biased consumer's welfare, $s_l \gtreqless s_b^*$, if and only if

$$\beta \gtreqless \tilde{\beta}_b(\gamma) := \frac{(\delta\alpha - \gamma)(1 - \delta)}{\delta[\alpha(1 - \delta) + \gamma(1 - \alpha)]}. \tag{24}$$

   (c) The laissez-faire savings rate is lower than (is equal to, exceeds) the savings rate that maximizes unbiased welfare, $s_l \gtreqless s_u^*$, if and only if

$$\beta \gtreqless \tilde{\beta}_u(\gamma) := \frac{(1 - \gamma)(1 - \delta)}{1 - \delta[1 - \gamma(1 - \alpha)]}. \tag{25}$$

Proof. See online appendix A.V. ■

To understand proposition 4, it is useful to consider first two special cases. When individuals discount exponentially, $\beta = 1$, the model reduces to a standard intertemporal equilibrium model where economic activity causes a negative externality. In this case, according to claim 1, the planners' choices and the two welfare maximizing savings rates coincide, $s_{up} = s_{bp} = s_u^* = s_b^*$.

As can be seen from claim 2(a), even when there is no bias towards current consumption the planner's savings rate is lower than in the laissez-faire equilibrium if there is an externality. By saving less, the planner reduces next period's capital stock and, hence, output, which in turn reduces next period's externality. Since, for $\gamma > 0$, we have $\tilde{\beta}_b(\gamma) < 1$ and $\tilde{\beta}_u(\gamma) < 1$, claims 2(b) and 2(c) confirm that this reduction corresponds to a correction of over-saving relative to the welfare maximizing choice, whether one uses the unbiased or biased welfare measure.

In the case $\gamma = 0$ without externality, the model reduces to the one analyzed by Krusell et al. (2002). When there is quasi-hyperbolic discounting, $\beta < 1$, from claim 1, maximizing the current self's welfare requires a lower savings rate than maximizing unbiased welfare, since the biased criterion gives less weight to the future. The biased planner's choice is still lower since she cannot commit future planners and, hence, can implement only a savings rate that reflects quasi-hyperbolic discounting in every period.

Moreover, we see from claim 2(a) that the biased planner's savings rate is lower than the savings rate in competitive equilibrium, $s_{bp} < s_l$, even without externality. As explained in the introduction, this result stems from the fact that the planner considers the impact of additional savings on the marginal product of capital and therefore has less incentive to save. In claims 2(b) and 2(c), we have $\tilde{\beta}_b(0) = \tilde{\beta}_u(0) = 1$. This shows that if there is no externality, the competitive equilibrium necessarily leads to under-saving compared with the welfare maximum. Under-saving is even more severe when the laissez-faire outcome is measured against the unbiased welfare function, which gives future utilities more weight.

We briefly discuss the two opposite extreme cases. If $\gamma \to 1$, then $s_b^* \to 0$, $s_u^* \to 0$, and $s_{bp} \to 0$ but $s_l > 0$. The externality is so dominant that there should not be any savings, and hence hyperbolic discounting does not matter for welfare or the planner. Consequently, savings under laissez-faire, where the externality is ignored, is necessarily excessive.

If $\beta \to 0$, then $s_b^* \to 0$, $s_{bp} \to 0$, and $s_l \to 0$, but $s_u^* > 0$. Here, for the individual and the biased planner, the future has no weight and hence they do not save, in accordance with the biased welfare criterion. In contrast, in the unbiased welfare criterion, the future is only discounted exponentially and hence savings should be positive. Thus, in this case, the externality does not matter and biased planner and market under-save because of quasi-hyperbolic discounting.

The general case, $\beta < 1$ and $\gamma > 0$, shows how present bias and externality together shape savings realized under the various institutions considered. Claim 1 is not affected if an externality is introduced into the model where quasi-hyperbolic discounting is present. Planners and welfare criteria internalize the externality essentially in the same way, implying that the order of savings rates does not change. The biased planner saves less than what would be required by both welfare criteria and, hence, induces lower externalities than the welfare maximizing savings rates.[19]

19 At first sight, this result contrasts with (Karp 2005, proposition 2, p. 272), who shows that under additive separable preferences, the game of quasi-hyperbolically discounting regulators ultimately leads to a higher stock of pollutant than the commitment outcome. To understand this difference, observe that in Karp (2005), the planner trades off current benefits of emissions to future damages of the accumulated stock of pollutant. In our model, the trade-off is between current consumption and future income, which

Turning now to the comparison between the competitive equilibrium and the choice of the biased planner in claim 2(a), one notices—following from equation (10)—that $\partial s_{bp}/\partial\gamma = -\delta\beta\alpha(1-\delta\alpha)/[1-\delta\alpha(1-\beta(1-\gamma))]^2 < 0$ and so s_{bp} is decreasing in γ. Therefore, the presence of the externality reduces the planner's savings rate further. The planner's present bias, combined with his or her power to control the interest rate and his or her desire to internalize the negative externality, point in the same direction, that is, towards a lower savings rate than realized under laissez-faire.

Claim 2(b) compares the laissez-faire outcome with the savings rate that maximizes the biased individual's welfare. It shows that a marginal reduction in the savings rate is welfare improving if and only if the externality is sufficiently strong, that is, if $\gamma > \alpha\delta$, or the present bias is moderate, that is, in equation (24), $\beta > \tilde{\beta}_b(\gamma)$ holds. Conversely, if the externality is weak or if quasi-hyperbolic discounting is strong, the competitive equilibrium leads to under-saving compared with the welfare maximum. For moderate externalities and short-run discount factors, the laissez-faire savings rate is close to or, by coincidence, even equal to the welfare maximizing one. This arises because the reduction in savings required by time consistency is actually welcome for containing the externality.

According to claim 2(c), the laissez-faire savings rate and the savings rate that maximizes unbiased welfare compare in a similar way. Also here the savings rate realized under competition exceeds (falls short of) the welfare maximizing one if the externality is strong (weak) and/or present bias is weak (strong). Relating the critical values for the two welfare criteria, one finds that, for any $\gamma > 0$, it holds that $\tilde{\beta}_b(\gamma) < \tilde{\beta}_u(\gamma)$. Thus, as illustrated in figure 1,[20] the range of parameters that leads to under-saving is larger when welfare is evaluated with exponential discounting than when welfare is evaluated according to the current self's preferences. This reflects the higher weight given to future utilities in the unbiased welfare criterion, which requires higher savings.

5.3. Government intervention and welfare

To judge whether the biased government's optimal policy improves welfare, one has to compare welfare levels. We note first that if the laissez-faire savings rate is smaller than the welfare maximizing savings rate the impact of quasi-hyperbolic discounting and the lack of commitment depress savings already by more than what is required to internalize the externality. In this

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entails an externality as a by-product. Commitment in both cases raises the items that generate future utility, which is clean environment in Karp (2005) and income in our model.

20 Figure 1 and figure 2 of subsection 5.3 have been drawn with the same parameter values as the ones used in Krusell et al. (2002), $\alpha = 0.36$ and $\delta = 0.95$.

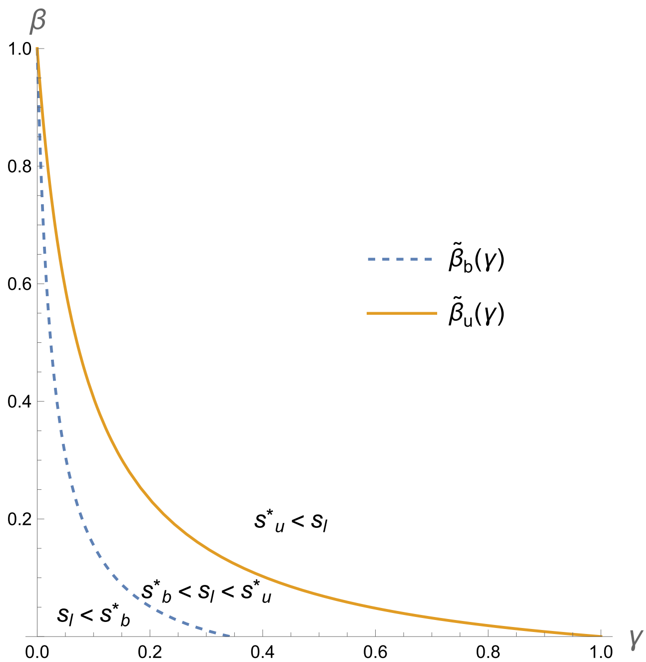

FIGURE 1 Under- and over-saving in competitive equilibrium relative to biased and unbiased welfare, with $\alpha = 0.36$ and $\delta = 0.95$

NOTES: For (β, γ) above (below) the dashed blue line depicting $\tilde{\beta}_b(\gamma)$, the laissez-faire savings rate exceeds (falls short of) the savings rate that maximizes welfare of the current self V_o^*. For (β, γ) above (below) the solid orange line depicting $\tilde{\beta}_u(\gamma)$ the laissez-faire savings rate exceeds (falls short of) the savings rate that maximizes unbiased welfare V^*.

case, government intervention that, according to claim 2(a) in proposition 4, reduces the savings rate further must necessarily be harmful. Thus, the biased planner's choice can increase welfare only if the competitive market leads to over-saving relative to the respective welfare criterion, that is, if $\beta > \tilde{\beta}_b(\gamma)$ in claim 2(b) or, respectively, $\beta > \tilde{\beta}_u(\gamma)$ in claim 2(c) in proposition 4.

This is not sufficient however. Only if over-saving is strong enough will it outweigh the under-saving implied by the biased planner's choice. To see when this is the case, we next compare the level of welfare reached by laissez-faire and the biased government according to both criteria. Naturally, this is daunting analytically given the logs in the utility functions. Therefore, we have taken recourse to numerical simulations.

A typical result is displayed in figure 2. In the figure, we see that, for all $0 < \gamma < 1$, there is a critical $\beta^*(\gamma)$ such that unbiased welfare is increased (decreased) by government intervention, that is, $V^*(k; s_{bp}) > (<) V^*(k; s_l)$, if $\beta > (<)\ \beta^*(\gamma)$. Considering welfare as experienced by the current self, we find that if γ is above a certain threshold government intervention is preferable

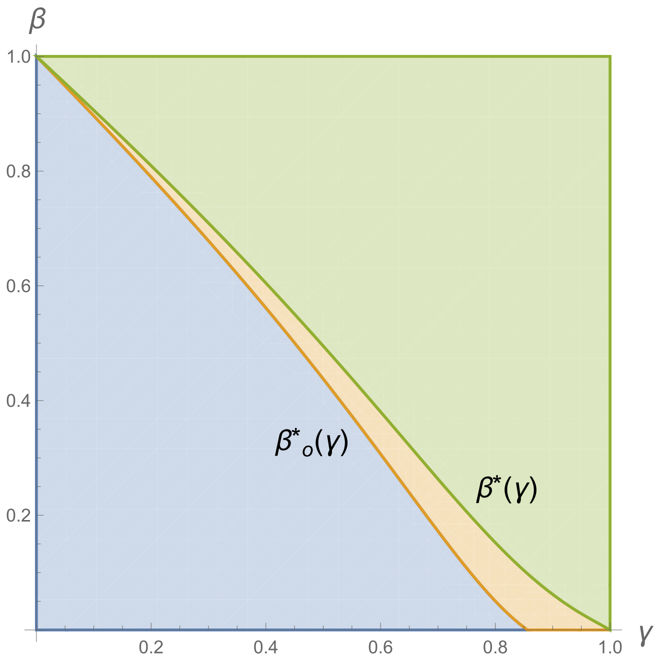

FIGURE 2 Government intervention and welfare, with $\alpha = 0.36$ and $\delta = 0.95$.
NOTES: In the bottom-left area (shaded blue), biased and unbiased welfare are reduced by moving from laissez-faire to the biased planner's decision. In the upper right area (shaded green), welfare improves by the planner's intervention according to both criteria. In the central area (shaded orange), biased welfare increases, but unbiased welfare decreases when the competitive outcome is replaced by the biased planner's choice

to the laissez-faire allocation no matter what β is. For γ below this threshold, there is a function $\beta_o^*(\gamma)$ such that government intervention increases (decreases) welfare, that is, $V_o^*(k; s_{bp}) > (<) V_o^*(k; s_l)$ if $\beta > (<)\ \beta_o^*(\gamma)$.

From the figure, one sees that both cutoff lines $\beta^*(\gamma)$ and $\beta_o^*(\gamma)$ are decreasing. This illustrates that the biased planner's intervention becomes more beneficial when the externality becomes stronger or when hyperbolic discounting is less pronounced. Moreover, the figure shows that $\beta^*(\gamma) > \beta_o^*(\gamma)$. Thus, there is an intermediate range of parameters where government intervention is beneficial if one applies the current self's welfare criterion but detrimental if one values welfare according to unbiased preferences. The difference emerges since unbiased welfare gives more weight to the future than the current self's preferences, and hence the under-saving induced by the planner is more severe when this criterion is used.

This result shows that, on the one hand, a government that represents the same biased preferences as citizens cannot be expected to correct the present bias and implement maximal unbiased welfare. This finding contrasts with the view taken in major contributions to behavioural economics such as O'Donoghue and Rabin (2003; 2006), Thaler and Sunstein (2003) or

Gruber and Köszegi (2004), who argue in favour of government intervention to correct such biases. Quite the contrary, since the biased government has more powers than individuals in a competitive market, it may, and in our model does, use such powers to even reinforce the impact of behavioural biases. Hence, if the present bias is sufficiently strong, the biased government may induce a welfare loss compared with the competitive market if welfare is measured by the current self's preferences, and even more so when one considers unbiased welfare.

On the other hand, we see that even the biased government's action can be beneficial when there is another cause of market failure such as a negative externality. Also the biased government aims at correcting this failure and therefore improves welfare if the externality is strong enough or the bias is weak. This is easier achieved if the welfare criterion allows for quasi-hyperbolic discounting, but even if welfare is measured by unbiased preferences government intervention will increase welfare in a non-negligible subset of parameters. Thus, the negative result of Krusell et al. (2002), where the planner's choice is always detrimental to welfare, does not in general extend to a richer model where, in addition to quasi-hyperbolic discounting, a further motive for public intervention is present.

6. Summary, extensions and concluding remarks

This paper has introduced externalities in a framework where consumers have quasi-hyperbolic preferences and so a bias for present consumption. Within such a framework the paper identified conditions under which government intervention is welfare enhancing. In so doing, it distinguished between a government that is unbiased and a government that has the same quasi-hyperbolic preferences as consumers, and welfare was measured both from the biased viewpoint of the current self and from an unbiased perspective. The results show that even a biased government will improve welfare according to both criteria if the externality is sufficiently important or the bias for the present is not too severe.

Depreciation: Our model is special in that we assume full depreciation of capital. With durable capital, the effect of current savings on output would be stretched over many periods, inducing more involved dynamics. While a full analysis of this extension is beyond the scope of this paper, we conjecture that the main insights remain valid. The fact that a present bias helps to reduce accumulation and hence mitigates an output-induced externality does not seem to depend on the precise time path of accumulation. Consequently, also in such a model, a present-biased government will likely improve welfare if the externality is sufficiently strong.

Naïve present-biased agents: Instead of considering sophisticated agents, who rationally anticipate that their next self will again be present-biased, in online appendix A.VI, we briefly also consider naïve agents. Such agents are themselves present-biased but believe that the next and all future selves discount exponentially. The resulting savings rate of naïve private agents is lower than the savings rate of sophisticated private consumers given in equation (7), but the savings rate of the naïve present-biased planner is the same as the savings rate of a sophisticated planner from equation (10). The Euler equations (15) and (16) illustrate how these results emerge from the naïve current self's disregard of the next self's bias. First, the naïve current self does not feel the need to create incentives for the next self, and hence the corresponding part of the marginal benefit of savings, expressed by the last terms in brackets in equations (15) and (16), is not present for naïve agents. Second, the naïve current self expects the next self to save more, and hence to consume less, than what a current sophisticated self would expect. Thus, from the naïve's perspective, the marginal utility of next period's consumption $1/C'$ is higher than from the sophisticated agent's perspective.

For the planner, both effects exactly cancel out. In the competitive equilibrium, the fact that private agents take the marginal product of capital to be constant matters. As discussed in section 4.2, this perception enhances the sophisticated current self's benefit from creating incentives for the next self. Moreover, the next present-biased self will also save more because of this perception, and hence the difference between consumption of that self and consumption of an unbiased next self, which the naïve current self expects, is reduced. From both considerations, the marginal benefit of savings for a sophisticated agent is raised by an additional amount, and hence moving to a naïve private agent on balance reduces savings.

As a consequence of the lower savings rate, the range of parameters where naïve private agents save more than the welfare maximizing amount is smaller than in the case of sophisticated agents. Moreover, as can be seen in online appendix A.VI, the naïve planner still saves less than the naïve present-biased private agent, and hence government intervention still can possibly enhance welfare only if the competitive equilibrium entails over-saving. Both observations together imply that government intervention is less likely to be beneficial when agents are naïve than when they are sophisticated.

Positive externality: We focus on a negative externality, which, in light of climate change and other environmental damages, is of most pressing concern. By considering $\gamma < 0$, we can, however, apply the model to a positive, output-induced externality.[21] With this modification, the two motivations for the biased planner to correct private decisions point in opposite directions. The present bias, as before, argues for reducing the savings rate, but a positive externality calls for increasing savings relative to laissez-faire. Therefore, the planner saves more than in competitive equilibrium and subsidizes investment if and only if the positive externality is strong enough relative to

21 See online appendix A.VII.

the present bias. At the same time, with present bias and positive externality, the competitive equilibrium necessarily entails under-saving, relative to both the biased and the unbiased welfare criterion. Therefore, government intervention enhances welfare if and only if it raises the savings rate.

Future research: Our approach suggests two avenues for future research. First, it is certainly interesting to study the interaction of quasi-hyperbolic preferences and market failures more generally in contexts different from output-induced externalities. While in the present model a bias for the present mitigates the externality, it is an open question how these elements affect each other in other settings. For example, adverse selection in an insurance market might be more or less pronounced, depending on whether good or bad risks have the stronger bias for the present. Second, a host of public policy issues will likely be decided differently by a government with quasi-hyperbolic preferences than by a benevolent, unbiased planner. For example, a biased government may use its market power to actually reduce the price of a good that is detrimental to health (like cigarettes) rather than raise it as an unbiased planner would do. We hope to have shown that the conclusions derived are instructive and the issues identified merit further investigation.

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Supporting information

Supplementary material accompanies the online version of this article.

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