Three Essays on New Keynesian Macroeconomics

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Submitted in July 2021
for the degree of
Doctor of Philosophy in Economics

February 2022

I would like to dedicate this thesis to my family and loved ones . . .

Declaration

Thesis Title: Three Essays on New Keynesian Macroeconomics

Submitted by Ernil Sabaj to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Economics, July 2021.

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

This dissertation contains fewer than 65,000 words including appendices, bibliography, footnotes, tables and equations and has fewer than 150 figures.

(Signature) ....

Ernil Sabaj
February 2022

Acknowledgements

First and foremost, I would like to express my most profound gratitude to my supervisors. Namely, Dudley Cooke and Hrishikesh "Rish" Singhania for their incredible support and encouragement over the course of my whole Ph.D. Without their support, I could not have completed this thesis and I owe them a considerable debt of gratitude.

I would also like to thank the faculty of the economics department for their continued and unlimited support. Namely, Brit Grosskopf, Rajiv Sarin, Surajeet Chakravarty, Engin Kara, Carlos Cortinhas, Jack Rogers, Julian Neira, Sebastian Kripfganz, Climent Quintana−Domeque, Namhyun Kim. Furthermore, I would like to thank Exeter University for its financial support throughout my Ph.D. A special thanks to Kate Gannon, Jim Milles, and Natalie Bartram.

Furthermore, I am thankful to my colleagues and dear friends at the University of Exeter: Haytem Troug, Rami Chehab, Lorenz Adams, Lutfi Rahimi, Laura Colombo, Elif Soytoprak, Canan Turan, Ali Akaak, Wan Mohd Adzim, Monica Figueira, Yingtong Dai, Yingji Li, Chaoyuan She, Xiao Chen, Jinlin Li, Cholwoo Kim, Qian Li, Ashley McCrea, Melinda Szoc, Cevdet Acu, Mohamed Adel, Nicholas Lacourse, Eleanya Nduka, Nurullah Çiftçi, Hannes Titeca, John Manoul, Josep Pinyol, Cliff Li, and Michael Melidis.

A special thanks to my family for their never-ending support in all my life challenges.

Abstract

This thesis consists of three essays on New Keynesian Macroeconomics. The first essay presents empirical evidence that long-run inflation is important in explaining cross-country differences in the response of private consumption to a government spending shock. Contributing to the debate on the size of fiscal multipliers, I motivate my analysis by documenting, in a quarterly dataset of OECD countries, that countries with high long-run inflation display a relatively higher response of private consumption to an increase in government spending. It is on this basis that I develop a small-scale DSGE model with positive trend inflation and show that the higher the trend inflation in an economy is, the higher the response of private consumption to a government spending shock. If we interpret positive trend inflation as the long-run inflation target, I show, convincingly, that the monetary stance of the central banks has important implications for the effectiveness of short-run fiscal policy interventions. Finally, I calculate consumption multipliers. I find that the consumption multipliers in countries with low trend inflation are below one, while under high trend inflation are higher than 2. These multipliers are consistent with the empirical evidence, which I provide in the paper.

The second and third essays focus on the macro-implications of sectoral heterogeneity. In essay two, I study output dynamics in a closed economy New Keynesian model that allows for heterogeneity in price stickiness across sectors. Whilst it has been shown that heterogeneity in price stickiness is the central force for the real effects of nominal shocks, I present theoretical results that demonstrate the importance of labor mobility across sectors and the intratemporal elasticity of substitution across sectors. Typically, labor is assumed to be reallocated immediately across sectors when shocks occur, and my results show how the reallocation of labor across sectors plays an important role in generating well-known results. The main insight provided by the analytical results is that there is an equivalence between changes in labor mobility and the intratemporal elasticity of substitution across sectors when they are taken into account. These results are driven by the differences in price stickiness across sectors. I then go on to calibrate the model for the US economy based on manufacturing and services and broadens the set of shocks driving the business cycle, verifying his results.

Continuing my work on the macro-implications of sectoral heterogeneity, in my third essay, I use a Bayesian likelihood approach to contribute to the debate on the origins of business cycles. I estimate a multi-sector New Keynesian model for the US economy and provide support for the idea of the importance of studying sectoral shocks. I incorporate real and nominal frictions and focus on sectoral and aggregate structural shocks. In estimating the model, I use data at the sectoral level for price inflation, real wages, and output contrary to the literature choices of using in general only aggregate data or sectoral only for one variable, usually price inflation. In doing so, I address important questions such as: Which shocks drive the fluctuations in price inflation, in real wages, and in output? Do sectoral elements such as the labor mobility and elasticity of intratemporal substitutability across sectors matter in New Keynesian multisector models? The findings from the estimation, suggest that the assumption of price/wage heterogeneity across sectors leads to better estimates on sectoral parameters such as labor mobility in an economy. The version of the model with sectoral data explains more of the variability in output from sectoral shocks compared to the version of the model with aggregate data and aggregate shocks. This result brings forward further evidence in support of using multi-sector models versus one sector model for macroeconomic analysis.

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Chapter 1

Introductory chapter

This thesis consists of three essays on New Keynesian Macroeconomics. "In August 2007, when the first signs emerged of what would come to be the most damaging global financial crisis since the Great Depression, the New Keynesian paradigm was dominant in macroeconomics," Galí (2018), nevertheless, while this true and still the main framework used for macroeconomic research, the New Keynesian paradigm, has faced many criticisms and requires expanding and improvements in many directions. The thesis mainly concerns with the transmission of demand and supply shocks and the effects on output subject to the implementation of trend inflation, price heterogeneity and labor mobility in New Keynesian dynamic stochastic general equilibrium models used for macroeconomic analysis.

While the inflation levels in the last year, 2021, started to jump to high levels 3-5% on average for many countries, in the last 20 years, as it can be seen from Figure 1.1, the inflation levels have not been as high as they used to be. Figure 1.1 shows on the top panel the inflation rates for five advanced economies and the OECD average for the last 60 years. Meanwhile the bottom panel focuses on six countries that have had on average higher inflation rates for the last forty years. Focusing on the values reported in Schmitt-Grohe & Uribe (2004), refereeing to the period 1960-1998, the calibrated trend inflation rate was 4.2% for the US[1], while for Germany and the UK was respectively 3.2% and 9.0%.

Nevertheless, the idea of positive trend inflation in DSGE models has gained additional importance, especially after the recent financial crisis. After the financial crisis of 2008, many economies experienced the zero lower bound constraint on monetary policy and faced its implications. One of the proposals that policymakers and economists did such as Blanchard et al. (2010), was that central banks should have increased their inflation targets. Ball (2014)

[1] Cogley & Sbordone (2008) estimations have shown that allowing positive trend inflation in the purely forward-looking Phillips Curve describes successfully the US inflation dynamics. Adam & Weber (2019) estimations have shown that "the optimal US inflation rate was strictly positive throughout the years 1977–2015".

recommended that a four percent target would have been enough overcome the constraints on monetary policy arising from the zero bound on interest rates[2]. Ascari & Sbordone (2014) argue that, "in light of this debate, it is worth investigating the implications of a higher target rate of inflation for the conduct of policies in normal times".

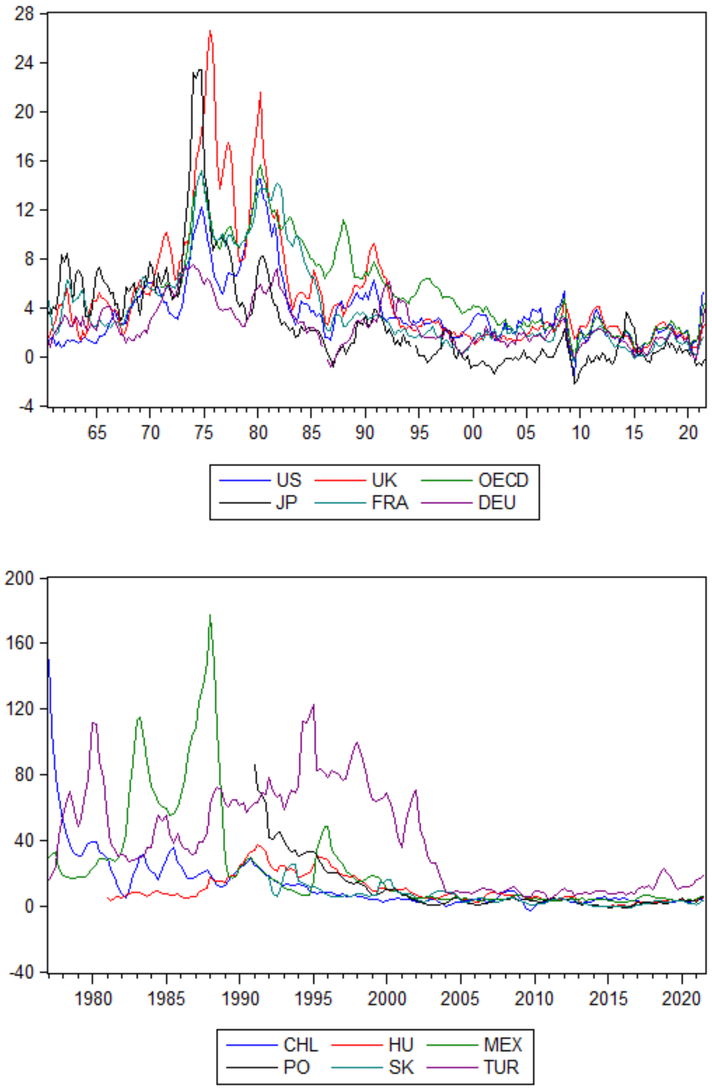

Figure 1.1 Inflation in selected countries

NOTES: The figure presents data on inflation for selected advanced economies (top panel) and selected high inflation economies (bottom panel). Data from OECD.

[2] Coibion et al. (2011) model estimations have shown that if the trend inflation value was 3.5% during the crisis, the nominal interest rate would have been 6%, as consequence the zero lower bound would not have been constraining.

While the implications for monetary[3] and technology shocks are investigated respectively in Ascari & Sbordone (2014) and Cooke & Kara (2019), the implications for fiscal policy are ambiguous and not studied. In the last decade and especially after the 2008 crisis, fiscal policy gained added relevance as a macro-policy. Despite a surge in research on fiscal policy, there is still a lack of consensus on the effect of government spending and thus, the size of the government spending multiplier. Therefore it is important to understand what drives government spending multipliers and how the mechanism in which government spending affects the economy works. In the first chapter, I combine these two important topics that sparked interest after the financial crisis, and contribute to the debate on the government spending multiplier by showing its estimations under various positive values of trend inflation in the economy.

Understanding how demand and supply shocks transmit to the real economy and what drives the business cycles are vital questions for research in macroeconomics and have been the focus of the New Keynesian paradigm. In my second and third chapter I explain aspects of this transmission mechanism by expanding the New Keynesian framework at the sectoral level. The motivation for this research comes from the increasing work done in the last two decades on multi-sector economies from Aoki (2001), Carlstrom et al. (2006), Barsky et al. (2007) and the increasing importance of sectoral price heterogeneity (see Carvalho (2006)) in the transmission of monetary policy shocks.

Carvalho & Lee (2019) regards the assumptions about the nature of price setting as "important drivers of the dynamics of fully-specified macroeconomic models", while Kara (2015), demonstrates that "incorporating recent micro evidence on prices into existing New Keynesian models can significantly improve the performance of these models". The heterogeneity in price stickiness is found to be an important feature of the micro-evidence on prices, as reported in Bils & Klenow (2004), Klenow & Kryvtsov (2008) and Nakamura & Steinsson (2008), while the implications of price heterogeneity on monetary policy shocks transmission are considered key in determining which sectors are the most important contributors, as argued by Pasten et al. (2019), which found, it increases real output effects relative to an economy with homogeneous price stickiness by 70%. At the sectoral level, Carvalho & Nechio (2018) find that "heterogeneous multisector models feature much larger and more persistent real effects than one-sector economies with similar average degrees of nominal rigidities", while Pasten et al. (2018) argue that depending on the interactions with other heterogeneous features of the economy, heterogeneous price rigidity can amplify or mute the aggregate volatility from sectoral shocks.

[3]Historically speaking, Ball (2014) shows that because the US was a high trend inflation country, during the 1981-82 recession, the Federal Reserve was able to cut the nominal federal funds rate by 10 percentage points, from 19% to 9%.

As it can be seen from Figure 1.2, there exists high heterogeneity in price stickiness across sectors for the US economy, regardless of the classification codes used NAICS or ELI.

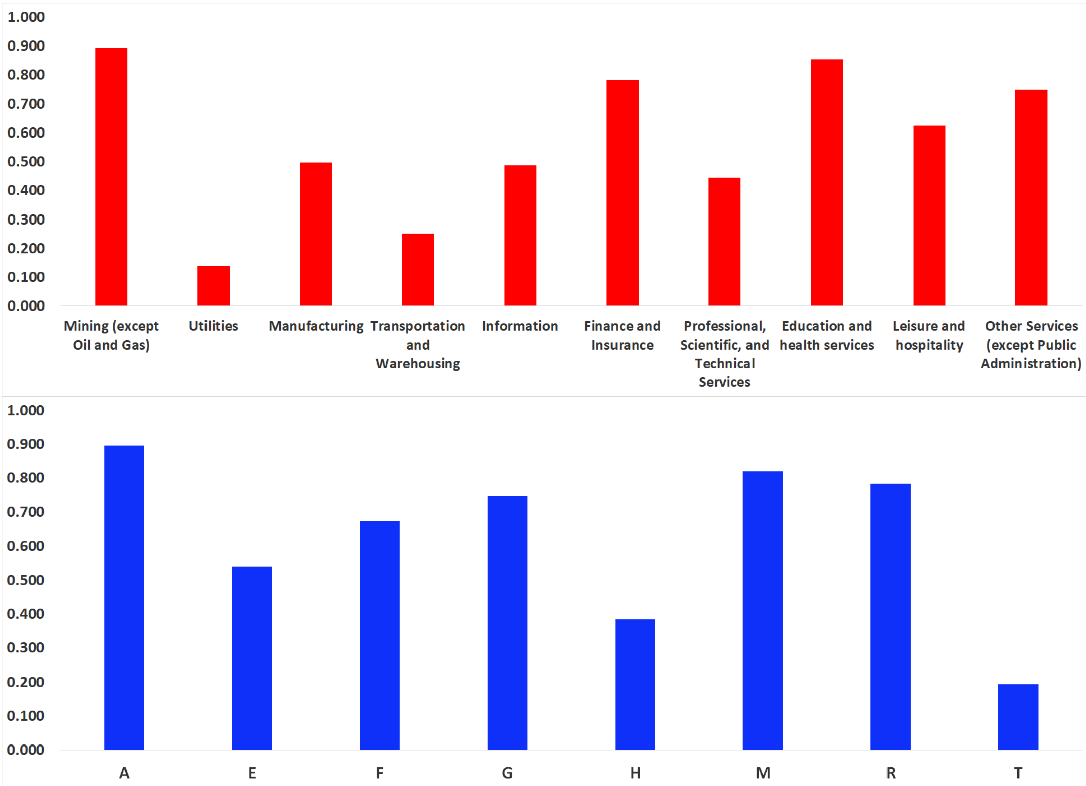

Figure 1.2 Heterogeneity in price stickiness across sectors

NOTES: The figure presents data on price heterogeneity (probability that the firms can not reset prices) for 10 aggregated sectors (Naics classification, top panel) and 8 aggregated sectors (ELI classification, bottom panel). The author's calculation are based on the weighted average (as in Carvalho & Nechio (2011)) from Nakamura & Steinsson (2008) database.

Evidently firms in sectors such as Utilities and Manufacturing tend to change prices more often than firms in sectors such as Education and Health services and Other Services, showing clearly than on average Services firms face stickier prices than Manufacturing firms. Clearly this data supports the notion that there exists heterogeneity in price stickiness across sectors and serves as motivation to conduct further research at the sector level for New Keynesian macroeconomics models.

The increasing research work on multi-sector economies and the increasing importance of sectoral price heterogeneity, raises questions on other aspects that might affect the transmission of shocks when interacting with with price heterogeneity, such as the labor mobility between sectors and the substitutability of goods between sectors. Most multi-sector New Keynesian models of the last generation do not concentrate on explaining output dynamics to various shocks subject to sectoral elasticities, but with the increasing research on price heterogeneity, their role might be underestimated. Further, counting in previous studies, I give some insight regarding the importance of labor mobility and the intratemporal elasticity of substitution between sectors. Airaudo & Zanna (2012)) argues that the intratemporal elasticity of substitution between sectors, matters for determinacy conditions. Caunedo (2019) shows that the effect of sectoral shocks on aggregate depends on the degree of heterogeneity in substitutability, and it is, therefore, independent of the elasticity when this is identical across sectors. Cantelmo & Melina (2017) suggests that the degree of labor mobility between sectors is an aspect of the economy that central banks should not overlook in setting the monetary policy stance. While Cardi et al. (2020) highlight the role of labor mobility in the transmission of government spending shocks.

In terms of data, most New Keynesian models assume either labor is perfectly mobile across sectors (eg. Barsky et al. (2007)) or that labor is sector-specific (eg. Carlstrom et al. (2006)), while the substitutability of goods between sectors is assumed either to be the same as the elasticity of substitution across different varieties of intermediate goods (eg. Carvalho (2006)) or is assumed equal to one (eg. Carvalho & Nechio (2011)). These two approaches as argued by Hobijn & Nechio (2018) have resulted in "in calibrations of the elasticity ranging from 1 to as large as 11..., with little discussion about the level of aggregation of the expenditure categories in the analysis". The evidence about labor mobility across sectors is somehow not clear too, where Davis & Haltiwanger (2001) find limited labor mobility across sectors in response to monetary and oil shocks. Horvath (2000) reports a relatively low estimate for the elasticity of substitution of labor across sectors using the U.S. sectoral labor hours data, while Alvarez & Shimer (2011), show that high labour mobility is inconsistent with U.S wage data. On the other hand Bouakez et al. (2011) report evidence suggesting that perfect labor mobility across sectors when sectoral nominal wages are the same.

Regardless of these studies, as it can be observed in Figure 1.3 there is evidence of increasing labor mobility across industries/sectors for the US economy. Data from Kambourov & Manovskii (2008) reported in the bottom panel show that there is a high level of increase in worker mobility in the US over 1968-97 at various levels of industry aggregation going for values of 5-7% increase for 1969 to 12-14% increase for 1997. Increasing rates of labor mobility are as well confirmed by the data of Park (2019) who reports (on the middle panel) labour transition rates across sectors, calculated as the fraction of switchers from one sector to another sector. The data presented in Figure 1.3 suggest that there exists limited labor mobility across sectors in the US labor market. While no assumption of perfect labour mobility seems to fit the data, it is fair to assume that there exists a limited wage gap between sectors, acting as support to the idea that there exists heterogeneity across sectors in the wage-setting of firms[4].

Figure 1.3 Labor mobility and elasticity of substitution across sectors

NOTES: On the top panel, the figure presents data on the elasticity of substitution across sectors depending on the COICOPs level of aggregations of sectors from Hobijn & Nechio (2018). The three colours represent the interval values. On the middle panel, the figure presents data on the labor transition rate across sectors for 2001-2015 from Park (2019). The acronyms denote sectors: HM high-wage manufacturing, LM low-wage manufacturing, HS high-wage services, LS low-wage services, OI other industries, UN unemployment. On the bottom panel, the figure presents data on industry labor mobility from 1969-97 depending on the level of industry aggregation, by Kambourov & Manovskii (2008).

[4]Keeping in mind that according to the standard neoclassical push and pull theory of labour mobility, if labour was perfectly mobile across sectors, assuming perfect competitive markets, then there would not be any difference in the wages across sectors for eg. see Ngai & Pissarides (2007).

If one is to follow Taylor (1999) and Taylor (2016) arguments that there is heterogeneity in the wage-setting, Klenow & Malin (2010) statement "that price changes are linked to wage changes", would suggest that when studying shocks and frictions, in order to understand what drives the US business cycle using New Keynesian models, the framework should account both the heterogeneity in the price and wage-setting. Turning the focus into the elasticity of substitution across sectors, the data obtained from Hobijn & Nechio (2018) in Figure 1.3, on the top panel, makes a strong case that there exists a positive non-zero degree of elasticity of substitution across sectors such as goods and services for the US economy. These values range from 1 to 5 depending on the level of aggregation in the economy. The evidence presented in Figures 1.2 and 1.3 that show the existence of heterogeneity in price stickiness across sectors, values above one for the elasticity of substitution across sectors, and limited labor mobility across sectors that leads as consequence to wage heterogeneity across sectors, serves as motivation for the third and fourth chapter which focuses on the macro-implications of sectoral heterogeneity.

Chapter 2 Overview.

Research question: Does the long-run inflation level of a country matter for the size of the government spending multiplier?

This chapter has dealt with an important question and one of ongoing interest in macroeconomics "What are the effects of government spending shocks on consumption?". Though there exists considerable theoretical and empirical work, there is still space in investigating further on this matter. This paper focuses on the idea that heterogeneity among countries coming from country characteristics highly matters in the effects of government spending on private consumption. The second chapter answers the research question by presenting empirical evidence that long-run inflation is important in explaining cross-country differences in the response of private consumption to a government spending shock.

I motivate my analysis by documenting, in a quarterly dataset of 34 OECD countries, that countries with high long-run inflation display a relatively higher response of private consumption to an increase in government spending. I estimate a panel VAR and identify government spending shocks using the approach proposed by Blanchard & Perotti (2002). An exogenous increase in government spending causes a rise in output, a positive response to private consumption, a positive hump-shaped of real wage, which follows an initial decline in inflation for the first 10 periods, to be turned positive after, as the government spending shock goes to low levels. I extend the analysis at a second stage, where I investigate whether the countries that have a different long-run rate of inflation have a different response on private consumption. I find that private consumption reacts more to government spending shocks in countries with high long-run inflation than in countries with low long-run inflation due to higher inflation expectations in those countries. On maximum impact, private consumption reacts almost 4 times more in high inflation countries than in low inflation countries. Due to the inflation channel, inflation expectations are higher in countries with higher inflation, this leads to lower real interest rates, and as consequence, this leads to higher private consumption. This result confirms that the country characteristics are important in explaining the response of private consumption when faced with government spending shocks.

On the grounds that I provide strong empirical evidence in favor of long-run inflation being important in determining the effects of government spending on private consumption, I present a theoretical investigation on the implications of trend inflation into it. It is on this basis that I develop a small-scale DSGE model with positive trend inflation and show that the higher the trend inflation in an economy is, the higher the response of private consumption to a government spending shock. If we interpret positive trend inflation as the long-run inflation target, I show, convincingly, that the monetary stance of the central banks has important implications for the effectiveness of short-run fiscal policy interventions. Finally, I calculate consumption multipliers. I find that the consumption multipliers in countries with low trend inflation are below one, while under high trend inflation are higher than 2. These multipliers are consistent with the empirical evidence, which I provide in the paper.

Overall in Chapter 2, I showed that when long-run inflation is used to explain the heterogeneity between countries, the response of private consumption to a government spending shock reflects these differences. The novelty of this chapter is that it treats the level of long-run inflation as an important component of calculating government spending multipliers, and secondly, it provides an explanation of the effects of government spending under trend inflation which is not provided so far by the literature which focuses mainly on monetary policy.

Chapter 3 Overview.

Research question: Is labor mobility and the intratemporal elasticity of substitution between sectors important for the transmission of shocks on output dynamics?

This chapter focuses on the macro-implications of sectoral heterogeneity. In chapter 3, motivated by the empirical evidence presented in Figure 1.2 and 1.3, I study output dynamics subject to demand and supply shocks in a closed economy New Keynesian model that allows for heterogeneity in price stickiness across sectors and highlights the role of labor mobility and the intratemporal elasticity of substitution across sectors. Whilst it has been shown that heterogeneity in price stickiness is the central force for the real effects of nominal shocks, I present theoretical results that demonstrate that the interaction of price heterogeneity, with changes in labor mobility and/or the intratemporal elasticity of substitution between sectors, plays an important role in determining output dynamics.

To determine how labor mobility and the intratemporal elasticity of substitution between sectors is important for the transmission of shocks on output dynamics, I start my analysis by considering a standard closed economy two-sector New Keynesian (NK) model with price heterogeneity across sectors. The firms in one sector have fully flexible prices, while in the other sector firms face standard sticky Calvo (1983) prices. The first step in this analysis of understanding how the labor mobility and the intratemporal elasticity of substitution between sectors affect the dynamics of output subject to demand and supply shocks in a New Keynesian model is by exploring the case where the firms in all sectors face the same price stickiness. I conduct the analysis as a second step under price heterogeneity between the two sectors. At first, I consider only the case of the demand shock and explain how the level of labor mobility across sectors influences the results. Then, I focus on the implications of the intratemporal elasticity of substitution across sectors. In a second step, I explore the implications when the economy is hit by a supply shock.

The main insight provided by the analytical results is that there is an equivalence between changes in labor mobility and the intratemporal elasticity of substitution across sectors when they are taken into account. Furthermore, to confirm these results I take the baseline New Keynesian model with one sector with flexible price firms and one sector with sticky price firms and replace it with a two-sector Calvo pricing calibrated version of the model, with positive trend inflation, that matches the US economy with manufacturing and services sectors for 2017 I show that these results are driven by the differences in price stickiness between sectors. Depending on the type of shock and the differences in adjusting prices fast, the key mechanism behind these results focuses on the firms increased demand of output or labor demand with their ability to hire more workers or shift their production toward the good in higher demand.

Overall, the analytical results and the impulse response functions show that a high labor mobility between sectors (or the intratemporal elasticity of substitution between sectors) can increase the response of output to demand and supply shocks. The novelty of this chapter is that it emphasises the importance of the labor mobility between sectors and the intratemporal elasticity of substitution between sectors on output dynamics subject to supply and demand shocks.

Chapter 4 Overview.

Research question: Which shocks drive the fluctuations in price inflation, in real wages, and in output?

Chapter 4 Overview. Continuing my work on the macro-implications of sectoral heterogeneity motivated again by Figures 1.2 and 1.3, in my fourth chapter, I use a Bayesian likelihood approach to contribute to the debate on the origins of business cycles. This paper attempts to help to answer the transmission of shocks in the US economy at the sectoral level. I estimate a multi-sector medium scale New Keynesian model for the US economy building on the Smets & Wouters (2007) canonical model with sectoral heterogeneity in price and wage rigidities and provide support for the idea of the importance of studying sectoral shocks. I incorporate real and nominal frictions and focus on sectoral and aggregate structural shocks. In estimating the model, I use data at the sectoral level for price inflation, real wages, and output contrary to the literature choices of using in general only aggregate data or sectoral only for one variable, usually price inflation. In doing so, I address important questions such as: Which shocks drive the fluctuations in price inflation, in real wages, and in output? Do sectoral elements such as the labor mobility and elasticity of intratemporal substitutability across sectors matter in New Keynesian multisector models?

I start the analysis with an economy that consists of a "Goods" sector, a "Trade, Transportation, and Utilities" sector, and a "Services" sector, and I estimate it with data from 2006Q3-2019Q4 for the US economy. I focus on showing the differences between the standard one-sector model versus multi-sector versions of the model, focusing on two estimation dimensions such as the number of sectors and the aggregation level of the data. To do so, I proceed in three steps. First, I estimate a one-sector model with aggregated data, second, I estimate the three-sector model with only aggregate data, and third, I estimate the three-sector model with both aggregate and sectors data. I do so because estimating with many sectors and sectoral data leads to results closer to the data values.

The findings from the estimation, suggest that the assumption of price/wage heterogeneity across sectors leads to better estimates on sectoral parameters such as labor mobility in an economy. The heterogeneity in price and wage stickiness has implications for the size of the sectoral price and wage shocks, increasing as well the persistence of the shocks in that sector, while it reduces the persistence of the monetary policy shock. The version of the model with sectoral data explains more of the variability in output from sectoral shocks compared to the version of the model with aggregate data and aggregate shocks. This result brings forward further evidence in support of using multi-sector models versus one sector model for macroeconomic analysis.

Overall these results suggest and provide support that sectoral mark-up shocks have a higher importance in explaining business cycles than aggregate mark-up shocks. The novelty of this chapter is that it combines the specifics of sectoral heterogeneity in price and wage stickiness to answer questions regarding the fluctuations in the business cycle. To my knowledge, this might be one of the few papers that uses in the Bayesian estimation of shocks and frictions in the US business cycle sectors data on three different aspects, contrary to the literature choices of using in general only data on price inflation, real wages or aggregate output, enriching further the results.

This thesis makes a number of contributions to the New Keynesian literature. First, it contributes to the literature on the effects of government spending and the size of fiscal multipliers such as in Ilzetzki et al. (2013). Secondly, the thesis contributes to the literature on the effects of including positive trend inflation in New Keynesian models such as Ascari & Sbordone (2014) and Ascari et al. (2018). Third, it relates to the literature that argues the importance of price heterogeneity in New Keynesian models, eg. Carvalho (2006) and Nakamura & Steinsson (2008), Carvalho & Lee (2019), and Bouakez et al. (2014) which show that sectoral heterogeneity in price stickiness matters for the effects of policy shocks. Fourth, it contributes to the literature on the effects of demand and supply shocks on the economy, such as Christiano et al. (2005) and Gali (2004), and expands the literature that explains the fluctuations in the US Business cycle such as Smets & Wouters (2007), Justiniano et al. (2010), Sims & Wolff (2018) etc. This thesis is related to papers that use sectoral data in the estimation of DSGE models such as Bouakez et al. (2014), Bils et al. (2012), Kara (2015), Kara (2017*b*). And at last, it connects to the papers that estimate or discuss sectoral labor mobility and sectoral elasticity such as Horvath (2000), Carlstrom et al. (2006), Bouakez et al. (2009), Bouakez et al. (2011), Iacoviello & Neri (2010*a*), Petrella & Santoro (2011) and Petrella et al. (2019), Carvalho (2006), Carvalho & Nechio (2011) and Hobijn & Nechio (2018).

The structure of the thesis is as follows. Chapters 2 to 4 present the core research materials. The second chapter begins by empirically estimating the effects of government spending shocks on private consumption using a panel SVAR model. The panel SVAR provides motivation and empirical evidence about the effects of the shocks in countries with high and low long-run inflation levels. Then, it presents the model economy and its features and discusses the dynamics of trend inflation and its implications on the economy. The third chapter develops a closed economy New Keynesian (NK) model with heterogeneity in price stickiness. It presents the main analytical results and relying on impulse response functions, discusses the implications of labor mobility and the intratemporal elasticity of substitution between sectors on output dynamics. Additionally, for realism it presents the results of the model calibrated for the US economy with manufacturing and services and positive trend inflation. Finally, the fourth chapter presents a three-sector medium-scale New Keynesian model with heterogeneity in price and wage stickiness. It discusses the data used in the estimation and the Bayesian methodology. Lastly, it goes on comparing the three-sector model results with other versions of the model, while it discusses how well the macroeconomic data are matched.

Chapter 2

The effects of government spending under trend inflation: theory and empirics

2.1 Introduction

In the last decade and especially after the 2008 crisis, fiscal policy gained added relevance as a macro-policy. Despite a surge in research on fiscal policy, there is still a lack of consensus on the effect of government spending and thus, the size of the government spending multiplier. Therefore it is important to understand what drives government spending multipliers and how the mechanism in which government spending affects the economy works. While doing so, due to the attention received by the literature, in this paper, I focus mainly on the effects of government spending on private consumption as a key element. In this paper, I will provide answers to the following questions: What is the size of fiscal multipliers? Does the magnitude of the multiplier depend on country characteristics? Does the long-run inflation level of the country matter for the size of the multiplier? On the grounds that I provide strong empirical evidence in favor of long-run inflation being important in determining the effects of government spending on private consumption, I present a theoretical investigation on the implications of trend inflation into it.

This paper, focusing on fiscal policy with nominal rigidities, makes two contributions to the New Keynesian literature. It contributes to the empirical literature, about the size of government spending multipliers and in showing country characteristic's importance in explaining government spending shocks reactions. It also contributes to the DSGE model literature by widening more on the effects of trend inflation, and the effects of government spending shocks and fiscal multipliers. The main result of this paper is that both empirically and in a DSGE model, long-run inflation is a significant element in explaining the size of the effect of private consumption from government spending shocks.

The fiscal multiplier has been investigated empirically cross-countries where Favero et al. (2011), argues that there is heterogeneity between countries, and this heterogeneity matters for the size of fiscal multipliers. There is increasing work being done looking at state, or regions multipliers such as in Nakamura & Steinsson (2014), Farhi & Werning (2016), Dupor & Guerrero (2017) and Chodorow-Reich (2017), which show that subnational multipliers differ from aggregate multipliers. Other studies such as Dellas et al. (2005), Beetsma & Giuliodori (2011), Corsetti et al. (2012a), Born et al. (2013), Ilzetzki et al. (2013), Kim (2015), Farhi & Werning (2016) and Koh (2017) suggest that what makes countries heterogeneous are macroeconomic fundamentals, such as capital mobility, trade openness, exchange rate regime, level of debt, economic development, and the business cycle, and because of these elements, the size of the fiscal multiplier changes across countries too[1]. These studies show that the response of private consumption to a government spending shock changes from country to country. None of the previous studies focuses on the differences in long-run inflation between countries as a source of heterogeneity in explaining the differences in the response of private consumption between countries, which is what I do in this paper, empirically and in a DSGE model.

In the first part of the analysis, I explore the evidence on the impact of government spending shocks on output, private consumption, real wages, and inflation in 34 OECD countries for the period 1995−2017 with a quarterly frequency. Specifically, I estimate a panel VAR and identify government spending shocks using the approach proposed by Blanchard & Perotti (2002). An exogenous increase in government spending causes a rise in output, a positive response to private consumption, a positive hump-shaped of real wage, which follows an initial decline in inflation for the first 10 periods, to be turned positive after, as the government spending shock goes to low levels. The positive response of private consumption to government spending shock was also confirmed by Blanchard & Perotti (2002), Bouakez & Rebei (2007), Gali et al. (2007) and Lewis & Winkler (2017) concluding that empirically private consumption is crowded in by government spending[2].

Secondly, I investigate whether the countries that have a different long-run rate of inflation have a different response on private consumption. I find that private consumption reacts more to government spending shocks in countries with high long-run inflation than in countries

[1]For a more detailed review of the literature on fiscal multipliers see Ramey (2011a) and Ramey (2019)

[2]Other empirical studies such as Ramey & Shapiro (1999), Edelberg et al. (1999),Ramey (2011b) suggest that under a narrative identification of the government spending shock, a negative response on private consumption is obtained. While Burnside et al. (2004) and Mountford & Uhlig (2009) argue that the consumption response is insignificant when faced with government spending shock.

with low long-run inflation due to higher inflation expectations in those countries. On maximum impact, private consumption reacts almost 4 times more in high inflation countries than in low inflation countries. Due to the inflation channel, inflation expectations are higher in countries with higher inflation, this leads to lower real interest rates, and as consequence, this leads to higher private consumption. This result confirms that the country characteristics are important in explaining the response of private consumption when faced with government spending shocks. Under different modelling choices for the panel VAR and different lag selections, I test the robustness of the obtained results. All in all, the most important idea from the empirical results is confirmed, that the importance of high long-run inflation in explaining the response of private consumption to a government spending shock is robust.

The evidence on the role of inflation in the response of private consumption to a government spending shock provides the motivation which I use to develop a DSGE model. I introduce a government sector in a New Keynesian model. Government spending is assumed to enter the utility function in a non-separable way, making it complementary with private consumption as suggested by Bouakez & Rebei (2007) [3]. The usefulness of government spending by entering in the utility function allows the households to obtain additional utility. Taking into account the empirical suggestion of having a higher inflation in the economy model, one perspective would be adding trend inflation into a small-scale DSGE model to explain the effects of government spending shocks on private consumption. Trend inflation causes the Philips curve to be flatter and makes inflation less sensitive to current marginal costs. The Philips curve now will depend more on expected future inflation, and less on marginal costs, thus making the firms more forward-looking.

The baseline results from the model show that for a 1% increase in government spending private consumption increases by almost 0.3% on impact effect. The increase in government spending produces a crowding-in effect on private consumption due to the complementarity effect which is strong enough to overcome the standard negative wealth effect.[4] Furthermore, labor increases, while the real interest rate has an initial increase which drops after a few periods as the decline in inflation shows lower magnitude.

[3] They argue that when government spending and private consumption are complementary, a government spending shock is able to produce a crowding in effect on private consumption. There is a wide literature that has shown that the Real Business Cycle model and the New Keynesian model are not able to simulate an increase in private consumption after an increase in government consumption. One solution to this puzzle is to assume that private consumption and government spending are Edgeworth complementary, which I use in the baseline model.

[4] An increase in government spending as argued by Baxter & King (1993), which is expected to be financed by current or future lump-sum taxes, has a negative wealth effect that decreases private consumption. While on the other side, workers want to work more, and this induces a rise in labor supply at any given wage. This will lead to a lower wage in the future, higher employment, and lower output.

Focusing on the importance of trend inflation and its effect, the main result of this paper is that trend inflation amplifies the response of private consumption to the government spending shock. Now, for an increase of 1% in government spending, on maximum impact, the model generates a response of 0.35% in private consumption when trend inflation is 2%, and 0.45% and 0.6% respectively when trend inflation is 4% and 6%. Hence, increasing the persistence in the response of private consumption. At higher rates of trend inflation price-setting firms are more forward-looking, they react less to the increase in private consumption, so that inflation reacts less, becoming more persistent, and the interest rate increases more, by inducing a larger reaction of consumption due to the Euler equation.

As a further step, I look at the behaviour of the results under different parameter specifications. I find that the effects of trend inflation on government spending, are highly dependent on the persistence of the government spending shock, on how strong the complementarity between private consumption and government spending is, and are highly driven by the parameter governing Frisch labour supply elasticity. The choice of the Taylor rule influences the magnitude of the effect of trend inflation, but not the general idea of the response of private consumption.

The consumption multipliers calculated with the standard approaches as suggested by the literature[5], for both the times series results and the DSGE model are in line with previous studies[6] ranging between $0.6-1$ for low inflation countries as defined empirically, and in the case of the DSGE model with low trend inflation. For a higher trend inflation in the model or high long-run inflation empirically, the consumption multipliers are above 1 and go as high as 2. The values of the consumption multipliers confirm empirically and theoretically that the level of inflation in countries matters for the effects of government spending on private consumption.

As a robustness check for the main results of the model, I rely on the assumption that the period utility function of the representative agent is assumed to be non-separable in consumption (C) and labor (N) as in Greenwood et al. (1988). In this model the wealth effect on labor supply is shut off, indicating that government spending can influence positively private consumption as long as labor and consumption are complements. Under GHH preferences, trend inflation, when comparing the model with 0% and 4% trend inflation cases, amplifies the positive response of private consumption to the shock but with a lower magnitude than in the case where I assumed government spending and private consumption complementarity.

[5]See Blanchard & Perotti (2002) and Mountford & Uhlig (2009) for more on this topic.

[6]A review of the empirical and NK models multipliers is provided in Ramey (2019).

The idea of positive trend inflation in DSGE models gained additional importance, especially after the recent financial crisis. After the crisis, many economies experienced the zero lower bound constraint on monetary policy and faced its implications. One of the proposals that policymakers and economists did such as Blanchard et al. (2010), was that central banks should have increased their inflation targets. In line with this development in the policymaking and the economic literature, amongst other studies[7], brings the necessity of looking at the effects of government spending shocks under higher inflation levels in the steady state[8].

This paper is organized as follows: Section 2.3 empirically estimates the effects of government spending shocks on private consumption using a panel SVAR model. The purpose of the panel SVAR is to provide motivation and empirical evidence about the effects of the shocks in countries with high and low long-run inflation levels. Section 2.4 presents the model economy and its features. I discuss the dynamics of trend inflation and its implications on the economy. In section 2.5, I present the results and comment on their importance. Subsequently, it is presented an estimation of the consumption multipliers from both the DSGE model and the Panel SVAR. In section 2.6, I provide an alternative model specification as robustness. Finally, Section 2.7 concludes.

2.2 Related literature

2.2.1 Empirical literature

Other papers related to the effects of government spending shocks on private consumption empirically, such as Ramey & Shapiro (1999), suggest that military spending affects consumption negatively, while Blanchard & Perotti (2002), conclude that empirically private consumption is crowded in by government spending. Similar empirical findings were also found in Ravn et al. (2012) and Corsetti et al. (2012a). Most of the previously mentioned studies have brought results regarding the US. In this study, I focus on a panel of countries, as stronger evidence for support is required to show that inflation matters for the effects of government spending. Monacelli & Perotti (2010) focus on 4 countries, the US, Canada, Australia, and the UK demonstrating that they differ in terms of the response of private consumption. On a panel approach for the same countries Ravn et al. (2012) report positive results on private consumption. Several other studies attempted to explain the empirical

[7] See Ascari & Sbordone (2014) and Cooke & Kara (2018) for the effects of monetary policy and technology shocks under trend inflation.

[8] As argued by Ascari et al. (2018), "Implementing such proposals over a sufficiently long period of time would eventually lead to higher long-run or trend inflation", page 56.

response of consumption using different states/elements of the economy that exhibit heterogeneity among countries. Auerbach & Gorodnichenko (2012), find that in the OECD countries private consumption is crowded out in expansions and appears to be increasing in recessions. Beetsma & Giuliodori (2011), studying government purchases shocks in open and closed economies, conclude that the more open an economy is, the lower the response of consumption will be. Corsetti et al. (2012a) finds out that the impact on consumption is not necessarily different under a pegged regime compared to a flexible one, there is no difference in the response when debt is high or low, while when in a crisis consumption rises almost twice as the increase of government spending. Huidrom et al. (2016) thinks differently than Corsetti et al. (2012a), wherein a panel of advanced and developing economies, they show that when the fiscal position is weak, consumption falls, and when the fiscal position is strong (government debt and deficits are low), the effect on consumption is positive. While Ilzetzki et al. (2013) brings evidence that private consumption is positive in the case of the pegged regime turning negative only at a later stage, while in the case of the flexible exchange regime the response of consumption is always negative. Koh (2017) goes further wherein a large panel countries data set brings evidence that in economies with high capital mobility private consumption increases.

2.2.2 DSGE models literature

This paper is related theoretically mainly to other DSGE papers that study the size of fiscal multipliers under different macroeconomic fundamentals or economy features and DSGE papers that study the effects of government spending shocks in an economy and specifically on private consumption. A few papers to be mentioned that have dealt with generating a positive private consumption response to government spending shocks same as in the empirical literature are: Linnemann (2006) relies on a utility function with not additively separable in consumption and leisure, Bouakez & Rebei (2007), who focus on the fact that private and public spending are Edgeworth complementary, Gali et al. (2007) considers rule of thumb consumers, while Ravn et al. (2012) uses a model with deep habits. Corsetti et al. (2012b), uses a New Keynesian model with expected spending reversals, while Bilbiie (2011) shows that when the utility function shows certain properties and under non-separable preferences over consumption and leisure, the Real Business Cycle model can generate an increase in private consumption in response to government spending shock. Dupor et al. (2017), demonstrates that one does not need any of the previous ingredients to cause an increase in consumption, it can just be done by adding nominal wage rigidity to a standard, closed economy with sticky prices. In this paper, I rely on the solution provided by Bouakez

& Rebei (2007) and for robustness I use the preferences of Greenwood et al. (1988), satisfying Bilbiie (2011) properties.

In terms of fiscal multipliers this paper is close to other papers such as the ones of Christiano et al. (2011), Mertens & Ravn (2014) and Farhi & Werning (2016) that argue that the size of the government spending multiplier can be larger when the nominal interest rate is on the zero lower bound[9]. Born et al. (2013), in a New Keynesian model find that government spending multipliers are larger under fixed exchange rate regimes than in flexible exchange rate regimes. Farhi & Werning (2016) consider the multipliers in a currency union and show that self-financed multipliers are always below unity, while outside-financed multipliers can be larger. Cacciatore & Traum (2018) show that high trade can imply that domestic multipliers are larger than in the case of low trade dynamics. Another related paper, is the one of Sims & Wolff (2018), which examines the effects of changes in government spending highlighting monetary passiveness situations. There is also a growing literature on fiscal multipliers taking into account heterogeneous agents models with incomplete markets featuring that households have different marginal propensities to consume[10].

Table 2.1 Literature overview on the effects of government spending

Government spending study	Focus of the study
Empirical studies	
Corsetti et al. (2012a)	weak/strong fiscal position, high/low debt
Beetsma & Giuliodori (2011)	open/closed economies
Auerbach & Gorodnichenko (2012)	recessions/expansions
Huidrom et al. (2016)	Fixed/flexible exchange rate regime
Ilzetzki et al. (2013)	Fixed/flexible exchange rate regime
Koh (2017)	high/low capital mobility
DSGE models	
Christiano et al. (2011), Mertens & Ravn (2014)	multipliers under lower bound
Born et al. (2013)	Fixed/flexible exchange rate regime
Woodford (2011), Kara & Sin (2018)	Liquidity traps
Farhi & Werning (2016)	multipliers in a currency union/outside
Cacciatore & Traum (2018)	high trade/low trade
Sims & Wolff (2018)	highlighting monetary passiveness situations

2.3 Panel VAR Analysis

In this section, I present the empirical results of the effects of government spending on private consumption using a Panel VAR framework. Initially, I start the analysis using a mean group

[9]Other papers, for example, that have looked at liquidity traps and fiscal multipliers are Woodford (2011), Kara & Sin (2018), etc.

[10]For more on this see Mitman et al. (2017), Auclert et al. (2018) and Bilbiie (2017).

estimator panel on 34 OECD countries. Secondly, I discuss the effects of a government spending shock on private consumption and show how the results are different when I split the country sample according to their level of long-run inflation. Thirdly, I show that the results obtained from this framework are robust under different modelling choices of the Panel VAR.

2.3.1 Specification

a. A mean group estimator panel VAR. In order to obtain the pooled results from the impulse responses I began by using a mean group estimator as described by in Pesaran & Smith (1995)[11]. Using this estimator, which relies on a maximum likelihood framework, I account for the cross-sectional dimension of the data. The benefit of this framework is that it does not require information about the economic structure of countries. The Pesaran & Smith (1995) estimator allows for country heterogeneity and produces parameters that are means of the group of countries used. The panel VAR model that I consider has the following representation:

$$y_{i,t} = A_i^p y_{i,t-p} + C_i x_t + \varepsilon_{i,t} \tag{2.1}$$

where the errors are normally distributed and the residual variance-covariance matrix is heterogeneous across countries, but characterized by a common mean $\varepsilon_{i,t} \sim N(0, \sum_i)$. $y_{i,t}$ denotes a vector comprising the n endogenous variables of unit i at time t, while p shows the lag of the variable. x_t is the vector of exogenous variables. A and C are respectively matrices of coefficients providing the response of unit i to the p^{th} lag of variable m of unit j at period t and the response of the endogenous variables to the exogenous ones. $\varepsilon_{i,t}$ is a vector of residuals for the variables of unit i.

Transposing 2.1 and writing it in a compact form after vectorizing it, gives the following equation:

$$y_i = \bar{X}_i \beta_i + \varepsilon_i \tag{2.2}$$

For each unit of i the mean group estimator assumes:

$$\beta_i = b + b_i \tag{2.3}$$

which shows that the coefficients of the VAR in different units will differ, while the means and variances will be similar. Since the parameter of interest is the mean effect b, the mean

[11] To estimate this panel VAR I use the BEAR toolbox v. 4.2 as documented in Dieppe et al. (2016).

group estimator would be:

$$\hat{b} = \frac{1}{N} \sum_{i=1}^{N} \hat{\beta}_i \tag{2.4}$$

A similar approach is done for the mean-group estimate of the residual variance−co-variance matrix $\sum$.

b. Identification strategy. I adopt the Blanchard & Perotti (2002) approach with government spending ordered first[12]. Government spending is predetermined relative to the other variables, responding with minimum one lag delay to other shocks than to itself. The ordering of the endogenous variables will be as follows:

$$\bar{X}_i = \begin{pmatrix} G \\ Y \\ C \\ W \\ \pi \end{pmatrix} \tag{2.5}$$

Where G denotes government consumption, Y is the GDP, C private consumption, W denotes wages and π is the y−o−y quarterly inflation growth. The choice of identification does matter, but in this paper, I will be focusing on the first method as it allows me to compare results from different samples more clearly.

c. Data. The data are all used in real term dollars[13], in logarithmic form, re-scaled[14], seasonally adjusted and detrended using a linear trend in order to deal with the problem of non-stationarity[15]. I used the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) to choose the number of lags to be included in the panel VARs reported. The tests suggest using 1 or 2 lags rather than 3 or 4 which are often more used in quarterly datasets as in the case of this paper, though the results change only slightly with adding more lags. As results prove to be more consistent on different samples when I use

[12] Identifying government spending shocks has been a challenge in the literature during the last decade, where many influential papers have led the area, deepening the debate on fiscal shocks. Alternative identification methods are summarised by Ramey (2016) for government spending shocks as follows: SVARs with contemporaneous restrictions, sign restrictions, medium horizons restrictions, narrative methods, and using DSGE models. The use of different identification techniques has not always produced consistent results on the response of consumption, as pointed out by Hebous (2011).

[13] A summary statistics of the data is presented in Appendix A.1.

[14] Multiplied by 100.

[15] The Breitung panel unit root test suggests that the panels contain unit roots, while the Im-Pesaran-Shin panel unit root test indicates that all the panels have unit roots in the cases of government spending and wages. The empirical results proved that detrending the variables with a linear trend, except for the case of Inflation which I do not detrend as it is on growth values, proves superior in obtaining the standard hump-shaped response of private consumption, compared to the case of detrending with a hp filter.

2 lags, this choice is kept in all the estimations, making sure that results on the different cases are not driven by a different lag choice. The data set used includes a quarterly balanced panel of 34 OECD countries[16] from 1995Q1 $-$ 2017Q4. I split the countries into low and high inflation countries, depending on the long-run inflation rate, calculated as an average of the period that the panel is estimated. If a country has an average long-run inflation of more than 2.5, the country is considered to be a high inflation country while below this line the country is considered to be a low inflation country. Since typically central banks' inflation target is around 2%, a value of 2.5 is justified. 19 countries from the sample fit into the criteria of being low inflation countries, while 15 of them are considered high inflation countries. A summary of the statistics of the divisions of the sample is given in Table 2.2, while in Appendix A1 in Tables A.1 and A.2 can be found general statistics of the sample and individual country statistics.

Table 2.2 Trend Inflation Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Trend Inflation All Sample	34	3.0	2.0	0.1	9.0
Low Inflation Countries	19	1.7	0.6	0.1	2.3
High Inflation Countries	15	4.7	1.8	2.6	9.0

The quarterly feature in the data is important for the identification of shocks, especially when using the Blanchard & Perotti (2002) identification technique, as it allows ruling out the contemporaneous response of government spending. Ilzetzki et al. (2013) argues that " while fiscal authorities require a quarter to respond to shocks, it is unrealistic to assume that an entire year is necessary" (p.241). Though, the use of yearly frequencies is not ruled out by the empirical evidence of Born & Mueller (2012), the government spending shocks on a yearly basis might be influenced by the anticipated effects suggested by Ramey (2011b). The data for the 5 endogenous variables are taken from the OECD statistics database[17].

2.3.2 Empirical results

In Figure 2.1 I report the impulse responses from the baseline panel VAR model for the 34 OECD countries. The blue (dashed) line displays the point of the estimate, while the

[16] Turkey and the US is left out of the sample as the first has a more volatile inflation rate, while the second is characterized as a large economy.

[17] The notion of private consumption it refers to the notion of "$P31S14_{S1}5$ Private final consumption expenditure " and of government spending it refers to "$P3S13$ General government final consumption expenditure " as used by the OCED statistics.

red lines asides give the 90% confidence bands. On the left side of each graph is found the name of each variable while the vertical axes show the amplitude of the impulse responses in percentages. In the horizontal axes are given the periods of study which amounts to a total of 30 quarters. On Figure 2.1 are shown the impulse responses of 5 variables G, Y, C, W and π, in response to a government spending shock of around 1.2%. On impact output and consumption responds positively to the government spending shock both by around 0.14%.

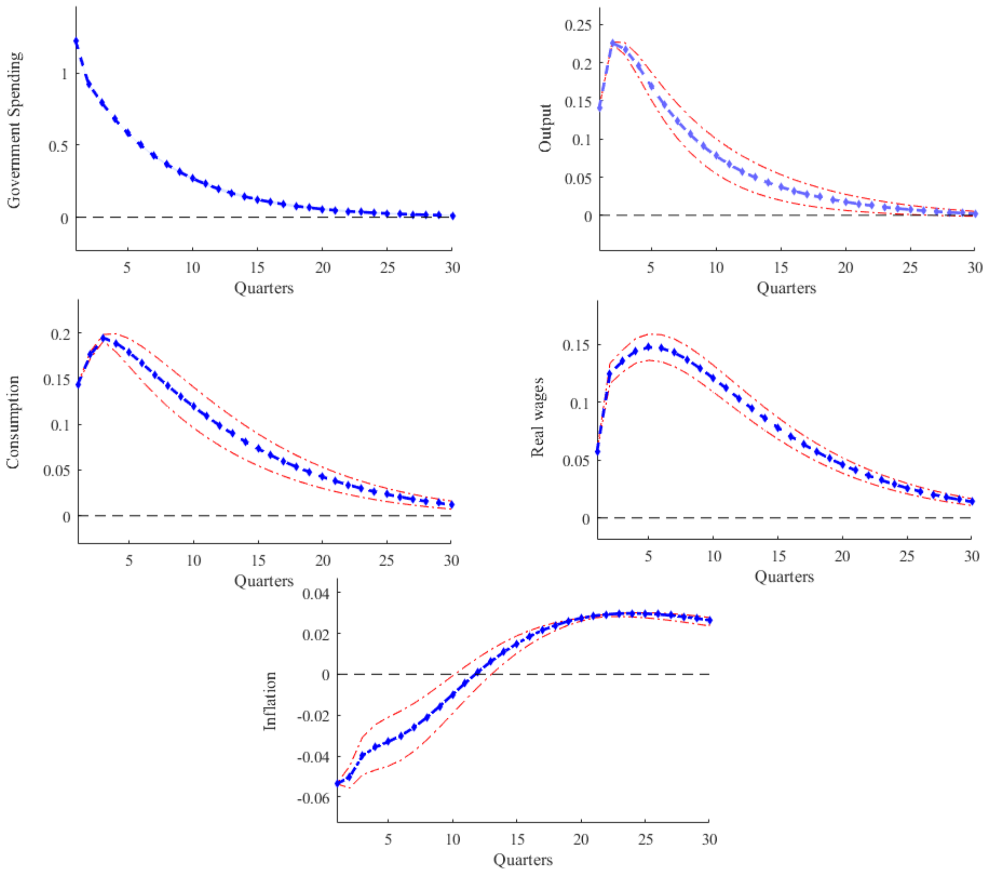

Figure 2.1 Impulse Responses to a Government Spending Shock in a Panel VAR

NOTES: The figure presents the impulse responses of Output, Private Consumption, Real Wages, and the quarterly Inflation y-o-y growth rate to a 1.2% government spending shock. One period corresponds to 1 quarter on the horizontal axis and the response in percentage is reported on the vertical axis. Red lines represent 90% confidence intervals based on Monte Carlo simulations.

In all periods both the responses of consumption and output is always positive, and hump-shaped, though the effect on output ends sooner than in the case of consumption. The positive response of consumption in terms of direction seems to be in line with many other studies such as Blanchard & Perotti (2002), Gali et al. (2007) etc, while the size of the response is similar to the ones seen in Bouakez & Rebei (2007), Ravn et al. (2012) and Lewis & Winkler (2017). The real wage also increases following the shock, displaying a hump-shaped response with a maximum value/peak response at its 5th quarter, an increase also seen in Fatas & Mihov (2001) and Bouakez & Rebei (2007). The response of inflation is almost significant in value, where after the impact period the value of inflation is negative till the 12fth period, becoming positive after but close to zero. Similar response, but less volatile of inflation[18] to a government spending shock is also seen in Corsetti et al. (2012b) and Erceg & Linde (2014).

As discussed in the section of data description I have divided the sample according to the long-run inflation rate[19] and I run the panel VAR on each of the samples. The panel VAR is run on the same variables as on the baseline panel and the same lag choice is kept as in the baseline. In Figure 2.2, I present only the responses of private consumption to a government spending shock on both cases and compare them. On the left-hand side, an increase in government spending induces an initial impact of 0.25% in private consumption corresponding to the response in high inflation countries. The response of private consumption, in this case, is statistically significant and shows a higher response than in the baseline case.

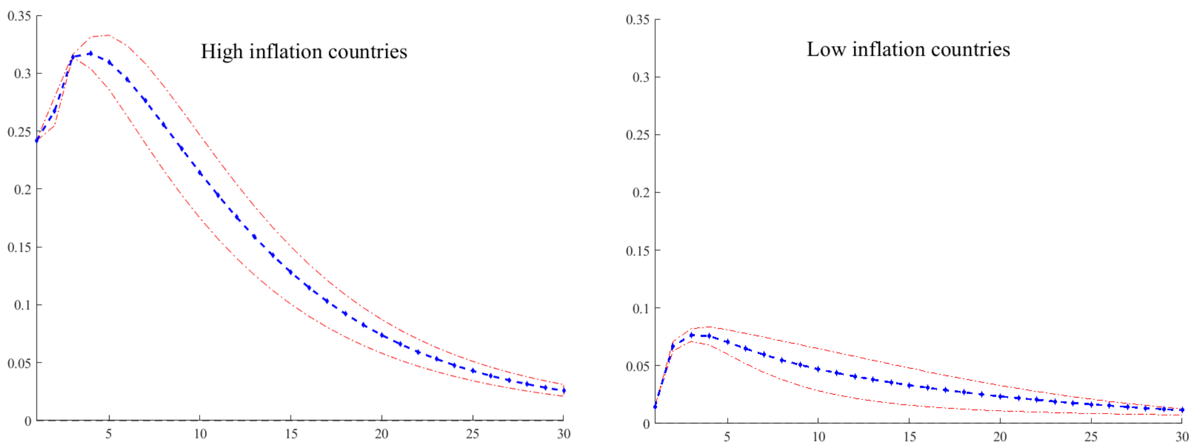

Figure 2.2 Impulse Responses of Private Consumption to a Government Spending Shock in High and Low Inflation Countries

NOTES: The figure presents the impulse responses of private consumption (C) to a 1.2% government spending shock. One period corresponds to 1 quarter on the horizontal axis and the response in percentage is reported on the vertical axis. On the left are presented the High inflation countries and on the right the Low inflation countries. Red lines represent 90% confidence intervals based on Monte Carlo simulations.

[18] A detailed survey of the response of Inflation to government spending shocks is given by Jorgensen & Ravn (2018), which they themselves find that the response of inflation is mainly negative.

[19] The long-run inflation rate is calculated as the average inflation rate for the whole period.

On the right-hand side, a shock in government spending causes an initial effect of 0.015% in private consumption in low inflation countries, being almost 10 times lower than in the case of the increase in private consumption in the baseline results. On the maximum value impact point, there is a response of private consumption to the government consumption shocks of 0.32% and 0.08% respectively in the cases of high and low inflation countries, almost 4 times higher response. The maximum response happens around the 3rd period for both cases while it continues its marginal effects toward zero. The dynamic adjustment is hump-shaped under both country cases, but more strongly in countries with high long-run inflation rates. The results reported here imply that private consumption increases less in low inflation countries than in high inflation countries. The difference in the impulse responses is $3-4$ times between the two cases and continues to persist almost all the period.

These results are explained by the fact that in countries with high long-run inflation, due to higher inflation expectations, the interest rate goes down, and this leads to an increase in private consumption. On the other hand, due to the high effectivity of monetary policy being larger in low inflation countries, fiscal policy becomes less expansionary, and lower fiscal multipliers are expected.

2.3.3 Robustness

In this subsection, I perform robustness checks for the main results obtained in the baseline panel VAR investigating private consumption in a low vs high inflation environment. I focus on robustness related to the modelling choices of the Panel VAR[20], to adding variables to control for the interest rates and the international monetary and real transmission channel's. Next, as robustness I consider changes in how I split the sample across time (pre vs post Great Recession) and if the trend inflation cutoff is slightly higher or lower than in the baseline scenario.

a. Different modelling choices I consider three alternative modelling choices[21] for the Panel VAR.

Countries VARs. Initially, I run separate VARs on each country in the sample, keeping the same number of lags, and other specifications mentioned in the data description. According to the definition of low and high inflation countries, further, I take the averages of the impulses responses of both groups.

A pooled Bayesian estimator. Secondly, I relax all the properties of the Panel VAR, and I use a Bayesian pooled estimator to re-obtain the results from the baseline panel VAR. In this

[20] I already have discussed in the data subsection about the choice of different lags as a measure of robustness.

[21] The VARs and panel VARs in this section are all estimated using the BEAR toolbox v. 4.2 as documented in Dieppe et al. (2016).

model, the data comes all as from many units, and the dynamic coefficients are homogeneous across units. The identification strategy used for the priors relies on a normal−Wishart distribution.

A hierarchical panel VAR. Thirdly, focusing more on the issue of heterogeneity between countries, I estimate a hierarchical panel VAR as proposed by Jarocinski (2010). This model recognizes and uses the heterogeneity among the countries and brings estimations for each of the countries allowing for cross-country comparisons. This methodology relies on Bayesian estimation where the coefficients of the VAR differ across units but are drawn from a distribution with similar mean and variance.

$$\beta_i \sim N(b, \sum_b) \tag{2.6}$$

The distribution of the vectors for the coefficients β_i is still normal, but now with a common mean b and common variance $\sum_b$. As in the case of the single countries' VARs, here as well for comparison reasons, I take the averages of the impulses responses of both groups.

Results. Overall, the key findings from the benchmark specification on the response of private consumption to a government spending shock under a specific environment of inflation, stand and are confirmed. In figure 2.3, I present the results of the impulse responses for all the three alternative modelling choices, grouping the responses of private consumption to a government spending shock on high inflation countries on the left, while on the right the responses in low inflation countries.

In each case, I present the point estimates of the impulse responses obtained under the alternatives, which are statistically significant on a 90% confidence interval. For all the alternative modelling choices, a positive government spending shock is found to be followed by a positive response of private consumption. All in all, the response of private consumption is higher in countries with high long run inflation under all the modelling choices. There does not seem to be high differences between the three alternative approaches and the baseline one, with the exception that the response of private consumption on high inflation countries on average is higher when the results are obtained with a hierarchical panel VAR than with a mean group estimator panel[22].

[22] As an additional robustness measure, I tried different versions of splitting the sample of countries and I was not able to find much action in the impulse response as in the case where I take into account the level of inflation.

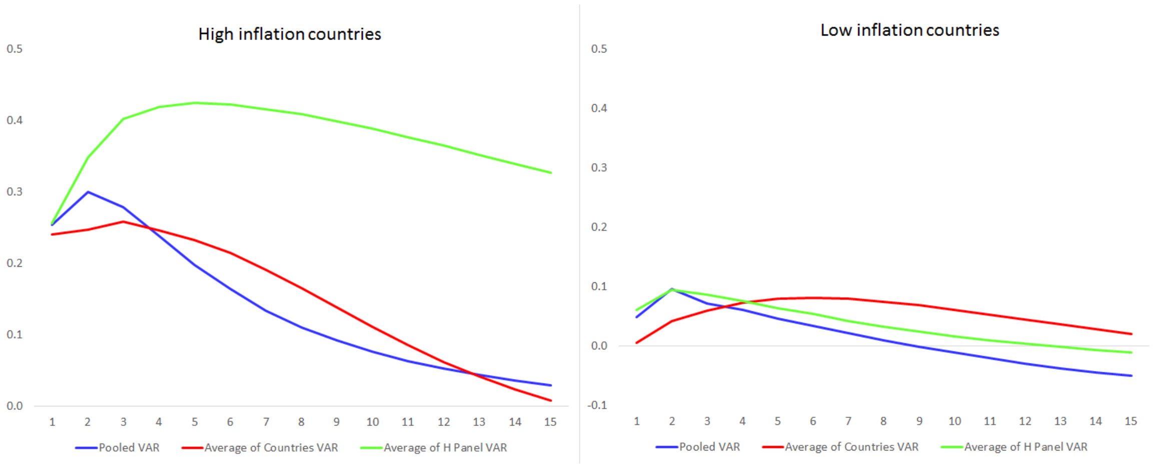

Figure 2.3 Impulse Responses of Private Consumption to a Government Spending Shock in High and Low Inflation Countries under Alternative Modelling Choices

NOTES: The figure presents the impulse responses of private consumption (C) to a 1.2% government spending shock. One period corresponds to 1 quarter on the horizontal axis and the response in percentage is reported on the vertical axis. On the left are presented the High inflation countries and on the right the low inflation countries. Blue lines represent the impulse responses from the Pooled VAR, the red lines represent the impulses from the average of all individual countries VAR and the green lines represent the impulses of the average of the countries from the Hierarchical Panel VAR.

b. Adding control variables.

In this subsection for robustness I focus first on controlling for the interest rates and the international monetary and real transmission channel's. In the first case I lose 6 country observations and add the interest rate variable in the Panel VAR, leaving the sample with 17 low inflation countries and 11 high inflation countries. In the second case, I account for the international monetary and real transmission channel and add in the Panel VAR framework the variable of the Real Effective Exchange Rate REER[23].

In figures 2.4 and 2.5, I present the results of the impulse responses for the cases where I add the interest rate and the REER variable respectively. In both cases, the responses of private consumption to a government spending shock on high inflation countries are given on the left, while on the right the responses in low inflation countries. Adding the interest rate or the REER in the Panel VAR confirms the initial results obtained in 2.2.

[23]The sample under this scenario remains the same as in the baseline scenario.

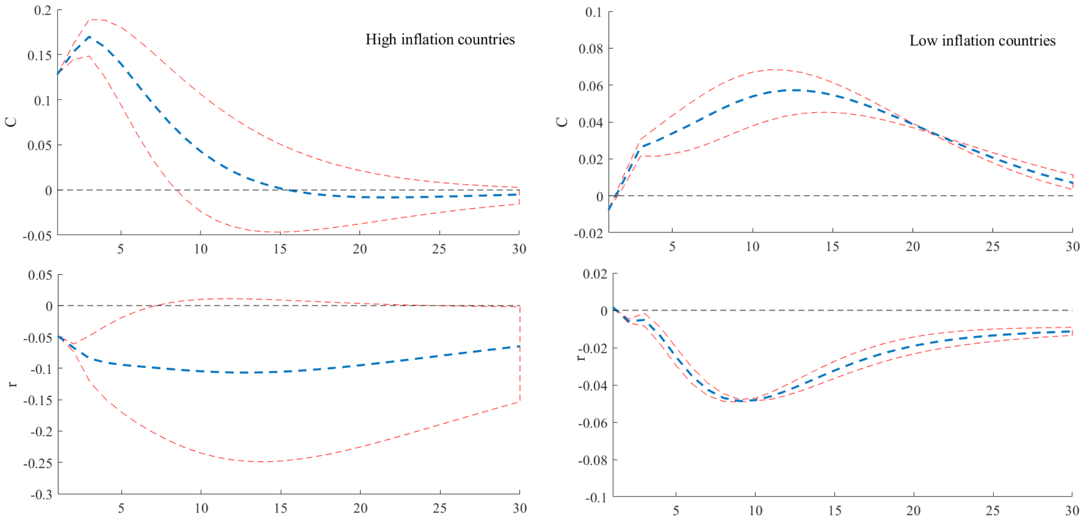

Figure 2.4 Impulse Responses of Private Consumption to a Government Spending Shock in High and Low Inflation Countries when adding interest rates.

NOTES: The figure presents the impulse responses of private consumption and interest rates to a government spending shock. One period corresponds to 1 quarter on the horizontal axis and the response in percentage is reported on the vertical axis. On the left are presented the High inflation countries and on the right the Low inflation countries. Red lines represent 90% confidence intervals based on Monte Carlo simulations.

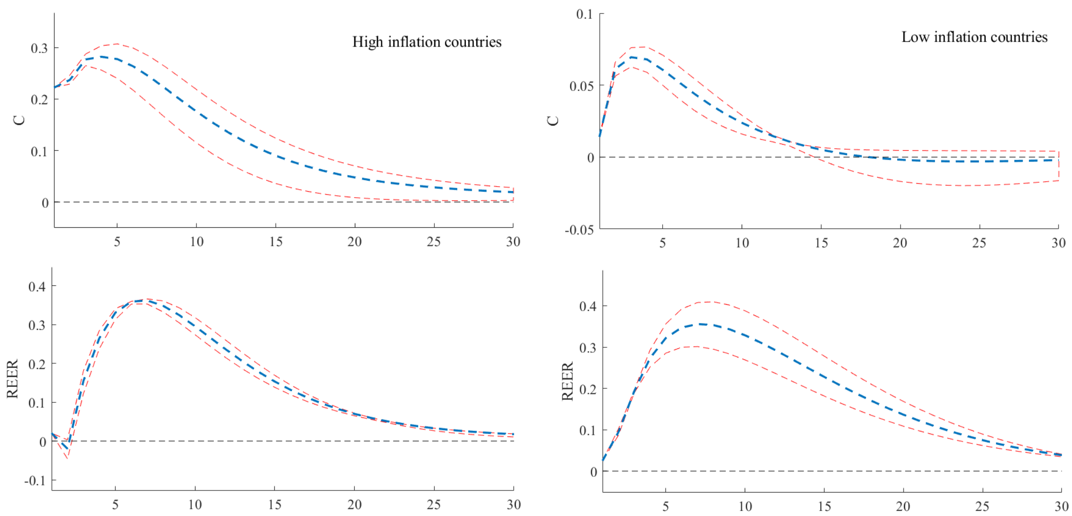

Figure 2.5 Impulse Responses of Private Consumption to a Government Spending Shock in High and Low Inflation Countries when adding REER.

NOTES: The figure presents the impulse responses of private consumption and REER to a government spending shock. One period corresponds to 1 quarter on the horizontal axis and the response in percentage is reported on the vertical axis. On the left are presented the High inflation countries and on the right the Low inflation countries. Red lines represent 90% confidence intervals based on Monte Carlo simulations.

c. Different split of the sample across time and according to different cut off for trend inflation.

In this subsection for robustness I focus first on controlling if splitting the sample to take into account for the 2008 financial crisis influences the results of the government spending shock. therefore, I look at the sample in two periods, 1995Q1-2008Q2 and 2008Q3-2017Q4. Next, I look to see if the choice of splitting the sample according to the long run inflation influences the results. While in the baseline scenario I considered a split point of 2.5, for robustness I look at the cases where the split point is respectively 2 (lower than baseline), 3 and 3.5 (higher than baseline).

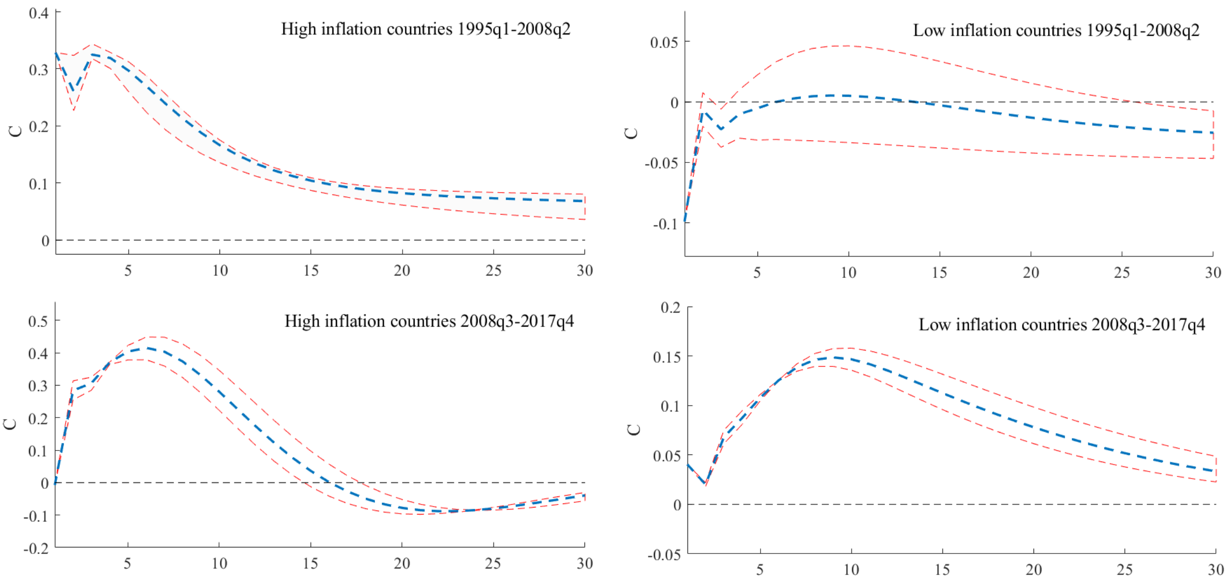

Figure 2.6 Impulse Responses of Private Consumption to a Government Spending Shock in High and Low Inflation Countries before and after the financial crisis.

NOTES: The figure presents the impulse responses of private consumption to a government spending shock. One period corresponds to 1 quarter on the horizontal axis and the response in percentage is reported on the vertical axis. On the left are presented the High inflation countries and on the right the Low inflation countries. On the top panel are presented the results for the period 1995q1-2008q2, while on the bottom panel are presented the results for the period after the financial crisis 2008q3-2017q4. Red lines represent 90% confidence intervals based on Monte Carlo simulations.

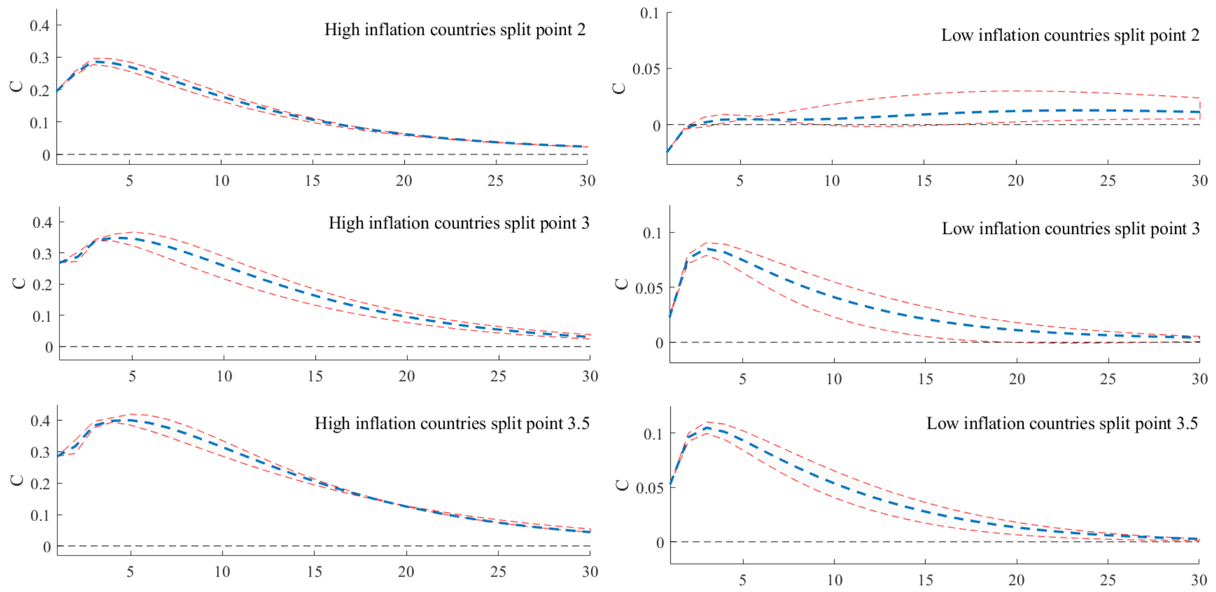

Figure 2.7 Impulse Responses of Private Consumption to a Government Spending Shock in High and Low Inflation Countries for different splits points of the long run inflation.

NOTES: The figure presents the impulse responses of private consumption to a government spending shock. One period corresponds to 1 quarter on the horizontal axis and the response in percentage is reported on the vertical axis. On the left are presented the High inflation countries and on the right the Low inflation countries. On the top panel are presented the results when countries are split following the split point 2 percent for long run inflation, middle panel for split point 3 percent, and bottom panel for 3.5 percent. Red lines represent 90% confidence intervals based on Monte Carlo simulations.

In figures 2.6 and 2.7, it can be seen that the results are robust before and after the 2008 financial crisis, while choosing a lower or higher inflation level to split the sample does not influence the results, on contrary it emphasizes them.

2.4 Small-scale DSGE model with trend inflation

This section describes the features of the model, which I develop to rationalize the empirical evidence presented in the previous section 2.3. The model that I use is a New Keynesian model with trend inflation. I study the effect of a government spending shock on private consumption and determine the size of fiscal multipliers. I explain how an increase in trend inflation amplifies the response of private consumption to a government spending shock. In what follows, I introduce the sectors of the economy, the dynamics of the model, present the pricing equations, discuss the role of trend inflation and the specifications of fiscal policy. A more detailed description of the economy is given in Appendix A.2.

2.4.1 Households

The economy is populated by a representative agent, that is infinitely lived, with a utility function, which is assumed to be non-separable in consumption (C_t) and government spending (G_t). I follow Bouakez & Rebei (2007) in assuming non-separability[24] between private consumption and government spending to get a positive response of private consumption to a government spending shock[25]. The typical representative agent seeks to maximize the following utility function:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j U(\widetilde{C}_t, N_t) \tag{2.7}$$

The utility function is assumed to be continuous and twice differentiable. β is the intertemporal discount factor. $\widetilde{C}_t$ is the aggregate consumption bundle, and $\widetilde{C}_t$ is a constant elasticity of substitution aggregate consisting of private consumption C_t and government consumption G_t:

$$\widetilde{C}_t = \left[\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{2.8}$$

Where δ is the share of private consumption in the aggregate consumption bundle, and χ is the inverse elasticity of substitution between private consumption and government consumption[26]. The utility function is non-decreasing in government consumption G_t.

C_t is the private consumption of goods and this composite consumption good is given by: $C_t = \left[\int_0^1 C_{t,j}^{1-\frac{1}{\varepsilon}}\right]$, where $C_{t,j}$ represents the quantity of good j consumed by the household in period t, and it is assumed that the existence of a continuum of goods is given by the interval $[0,1]$. The household allocates its consumption expenditures among different goods, by maximizing C_t for any given level of expenditures $\int_0^1 P_{t,j} C_{t,j} dj$[27]. The solution of this problem yields the demand for good j, $C_{t,j} = \left(\frac{P_{t,j}}{P_t}\right)^{-\varepsilon} C_t$ for all $j \in [0,1]$, and with ε being the elasticity between the goods in the economy. The aggregate price index is given as $P_t \equiv \left[\int_0^1 P_{t,j}^{1-\varepsilon} dj\right]^{1-\frac{1}{\varepsilon}}$, and the total consumption expenditures can be written as the product of the price index times the quantity index $\int_0^1 P_{t,j} C_{t,j} dj = P_t C_t$.

[24] It is assumed public spending shows Edgeworth complementarity with private consumption. This specification would allow for the usefulness of government spending and would make it possible to generate a crowding-in effect of government spending.

[25] Another way of obtaining a positive response of private consumption to government spending would be as well by having a utility function with GHH preferences, where the labor effect is shut down and consumption (C_t) and labor (N_t) are complements.

[26] A similar modified form to Bouakez & Rebei (2007) is seen also in Troug (2020) and Pieschacon (2012).

[27] The proof of this is shown appendix 3.1 in Gali (2008b).

The maximization of the utility function is subject to the following budget constraint:

$$P_tC_t + (1+i_t)^{-1}B_t = W_tN_t + D_t + B_{t-1} - T_t \tag{2.9}$$

where i_t is the nominal interest rate, B_t is one-period bond holdings, W_t is the nominal wage rate, N_t is the labor input, and D_t is profits (distributed dividends), and the households pays taxes to the government, where T_t is a lump sum tax. Each period the household is endowed with one unit of time, which is divided between work and leisure $N_t + L_t = 1$. Obtaining the first-order condition from the optimization problem of the agent yields the following Euler equation:

$$1 = \beta(1+i_t)\mathbb{E}_t\left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t}\right)^{\chi-\sigma}\left(\frac{C_{t+1}}{C_t}\right)^{-\chi}\right] \tag{2.10}$$

The Euler equation shows the smoothing of consumption over time and depends on the two parameters χ and σ. σ is the intertemporal elasticity of substitution in consumption. The complementarity between government spending and private consumption does not only depend on the value of χ, but also on the interaction with σ. In order for government spending and private consumption to be considered complementary, it is necessary that $\chi > \sigma$. If $\chi = \sigma$, the Euler equation transforms into its standard form. The case where $\chi < \sigma$, makes government spending and private consumption being substitutes of each other. The aggregate consumption bundle variable also appears in the equation, prior to the standard elements of the equation

The labor supply equation is:

$$\frac{W_t}{P_t} = d_n e^{\varsigma_t} N^{\varphi} \overline{C}_t^{\sigma} \left(\frac{C_t}{\widetilde{C}_t}\right)^{\chi} \delta^{-\chi} \tag{2.11}$$

where ς_t is the labor supply shock and φ is the inverse Frisch elasticity of labor supply. While the labor supply equation presented in nominal terms, shows that it depends on the aggregate consumption bundle, additional to the consumption variable, the labor supply which is governed by the inverse Frisch elasticity parameter, and the parameters related to the behaviour of consumption and government spending. A similar interpretation regarding the parameters χ and σ is also done for the labor supply equation. When $\chi > \sigma$, which is crucial for obtaining government spending and private consumption complementarity as discussed above, government spending will have a negative effect on real wages, as it influences positively labour supply.

2.4.2 Firms

The firms in this economy chose prices to maximize their profits, which are subject to three constraints, their production function summarizing the available technology, the constraint given by the demand curve each firm faces, and that in each period some firms are unable to adjust their prices.

a.Technology. In each period t, a final good, Y_t, is produced by perfectly competitive firms, which combine a continuum of intermediate inputs $Y_{j,t}, j \in [0,1]$, via the technology: $Y_t = \left[\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon-1}{\varepsilon}}$ where $\varepsilon > 1$ is the elasticity of substitution among intermediate goods following the standard constant elasticity of substitution (CES) production function. The zero profit condition and profit maximization implies that the price index associated with the final good Y_t which is a CES aggregate of the prices of the intermediate inputs $P_{i,t}$ where: $P_t = \left[\int_0^1 P_{j,t}^{1-\varepsilon} dj\right]^{\frac{\varepsilon-1}{\varepsilon}}$, and the optimal demand for intermediate inputs is $Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} Y_t$.

The production function of intermediate goods and the labor demand of firm j are respectively:

$$Y_{j,t} = A_t N_{j,t} \text{ and } N_{j,t}^d = \left[\frac{Y_{j,t}}{A_t}\right] \tag{2.12}$$

where A_t is an exogenous process for the level of technology. The labor demand of the firm is positive to the production function and is related inversely to the level of technology. On the other hand, intermediate inputs are produced by a continuum of firms with a simple linear technology in labor, which is the only input of production. The total cost and real marginal costs are given by:

$$TC_{j,t}^r = \frac{W_t}{P_t} N_{j,t} \text{ and } MC_{j,t}^r = \frac{W_t}{A_t P_t} \tag{2.13}$$

Nominal wages fluctuate in perfectly competitive markets and are the same across firms. An increase in real wages would increase the marginal costs of the firm, while a better level of technology would decrease marginal costs.

b. Profit maximization. The prices are based on the model of price stickiness used by Calvo (1983), where a fraction of firms re-optimize their nominal price with fixed probability $1-\theta$, while with probability θ it maintains the price charged in the previous period. The parameter θ measures the degree of nominal rigidity; a higher θ means that fewer firms re-optimize their price each period and a longer time is needed for the price changes to happen.

The problem of firm j who re-optimizes prices is to choose the reset price $P^*_{j,t}$ to maximize expected profits, and it can do so by solving the problem:

$$\max_{P^*_{j,t}} \mathbb{E}_t \sum_{l=0}^{\infty} \theta^j D_{t,t+l} \left[\frac{P^*_{j,t}}{P_{t+l}} Y_{i,t+l} - \frac{W_{t+l}}{P_{t+l}} \frac{Y_{j,t+l}}{A_{t+l}} \right] \quad \text{s.t the demand constraint } Y_{j,t+l} = \left(\frac{P^*_{j,t}}{P_{t+l}} \right)^{-\varepsilon} Y_{t+l} \tag{2.14}$$

where $D_{t,t+l} \equiv \beta^l \frac{\lambda_{t+l}}{\lambda_0}$ is the stochastic discount factor, and $\lambda_{t+l} = \overline{C}^{\chi-\sigma}_{t+l} C^{-\chi}_{t+l}$ is the marginal utility of consumption. The firm's first order condition by re-arranging would yield:

$$P^*_{j,t} = \frac{\varepsilon}{(\varepsilon - 1)} \frac{\psi_t}{\phi_t} \tag{2.15}$$

The auxiliary variables ψ_t and ϕ_t can be written recursively following Ascari & Ropele (2009) and Ascari & Sbordone (2014) as:

$$\psi_t = MC_t \overline{C}^{\chi-\sigma}_{t+j} C^{-\chi}_{t+j} Y_{t+j} + \theta\beta \mathbb{E}_t \left[\pi^{\varepsilon}_{t+1} \psi_{t+1} \right] \text{ and } \phi_t = \overline{C}^{\chi-\sigma}_{t+j} C^{-\chi}_{t+j} Y_{t+j} + \theta\beta \mathbb{E}_t \left[\pi^{\varepsilon-1}_{t+1} \phi_{t+1} \right] \tag{2.16}$$

They depend both on output and future expectations of inflation. ψ_t can be interpreted as the present discounted value of the marginal costs when the optimal reset price changes, while ϕ_t can be considered as the marginal revenues. ε and $\varepsilon - 1$ are respectively treated as the weights of the marginal costs and marginal revenues on resetting the optimal price in equation 2.15. As suggested because $\varepsilon > \varepsilon - 1$, the future expected rate of inflation has a higher impact on the marginal costs of setting the price than on the marginal revenues. Returning to the aggregate price level, it evolves as Calvo (1983) prices and it can be expressed as:

$$P_t = \left[\int_0^1 P^{1-\varepsilon}_{j,t} di \right]^{1/(1-\varepsilon)} = \left[\theta P^{1-\varepsilon}_{t-1} + (1-\theta) P^{*\,1-\varepsilon}_{j,t} \right]^{1/(1-\varepsilon)} \tag{2.17}$$

And a $\theta = 0$ would show the expression in the standard case with flexible prices. Reformulating the aggregate price equation, allows me to express the reset price as follows:

$$P^*_{j,t} = \left(\frac{1 - \theta \pi^{\varepsilon-1}_t}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} \tag{2.18}$$

2.4.3 The Government and Monetary policy

The government purchases quantity $G_t(j)$ of good j, for all $j \in [0,1]$:

$$G_t \equiv \left[\int_0^1 G_{t,j}^{1-\frac{1}{\varepsilon}} dj\right]^{\varepsilon/(\varepsilon-1)} \tag{2.19}$$

where the government seeks to maximize for any level of expenditures $\int_0^1 P_{t,j} G_{t,j} dj$. [28] Next, the government expenditures are financed by means of lump-sum taxes $P_{t,j}T_{t,j}$. Government spending evolves exogenously according to the following first-order autoregressive process:

$$G_t = G_{t-1}^{\rho_G} \exp(\mu_{Gt}) \tag{2.20}$$

μ_{Gt} is an independent and identically distributed shock with zero mean and a constant variance.

Regarding monetary policy, the economy has a central bank that follows a conventional Taylor rule, with weight ϕ_π on deviations of inflation from target $\overline{\pi}$ and weight ϕ_y on output deviations from steady state output[29] Y:

$$\left(\frac{1+i_t}{1+\bar{i}}\right) = \left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\rho_i} \left(\left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\overline{Y}}\right)^{\phi_Y}\right)^{1-\rho_i} e^{v_t} \tag{2.21}$$

where v_t is a monetary policy shock which is *iid* with zero mean and a constant variance, and ϕ_π , $\overline{\pi}$ and ϕ_y are non-negative parameters. Some inertia is also added to the Taylor rule. $\bar{i}$ is the steady-state interest rate and ρ_i is the inertia parameter.

2.4.4 Aggregation and price dispersion

The market clearing in the goods market requires: $Y_{t,j} = C_{t,j} + G_{t,j}$ for all $j \in [0,1]$ and all t, or can be written as:

$$Y_t = C_t + G_t \tag{2.22}$$

[28] The government is assumed to allocate expenditures across goods in order to minimize total cost, acting in the same way as the household.

[29] There exists a debate on whether to use output growth or output gap as a measure of economic activity in the interest rules. When the model takes into account for positive trend inflation, it can lead to substantial welfare losses according to Sims (2013). Given that I do not deal with welfare issues on this paper I use the output gap.

From the individual firms production function, and combining it with equation 2.12, by aggregating over j, I derive the aggregate labor demand as:

$$N_t^d = \int_0^1 N_{j,t} dj = \int_0^1 \frac{Y_{j,t}}{A_t} dj = \frac{Y_t}{A_t} \underbrace{\int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} dj}_{s_t} = \frac{Y_t}{A_t} s_t \tag{2.23}$$

As shown by equation 2.23, price dispersion influences the relationship between labor demand and output. I define the relative price dispersion measure as $s_t = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} di$. The higher the dispersion of relative prices, the higher price dispersion s_t is, and therefore a higher amount of labor input is needed for the production of a certain level of output (Ascari & Sbordone (2014). A increase in price dispersion, keeping constant output and the level of technology, from equation 2.12 would mean a higher wage which would convert in higher marginal costs for the firm (equation 2.13) and therefore a higher discounted value of the marginal costs $\widehat{\psi}_t$ (equation 2.16). As in Schmitt-Grohe & Uribe (2007), s_t can be written under the assumption of the Calvo model as follows:

$$s_t = (1-\theta)(p_{j,t}^*)^{-\varepsilon} + \theta \pi_t^{\varepsilon} s_{t-1} \tag{2.24}$$

Itself, price dispersion is dependent on inflation expectations, and increases the higher ε and θ.

2.4.5 The IS and the Generalized New Keynesian Philips Curve

In order to obtain the GNKPC in terms of marginal costs, I log-linearize the firm's equilibrium conditions around a steady state characterized by a positive trend inflation. And then, following the standard approach I would have[30]:

$$\widehat{\pi}_t = k(\overline{\pi})\widehat{mc}_t + b_1(\overline{\pi})\mathbb{E}_t\widehat{\pi}_{t+1} + b_2(\overline{\pi})\left[(\chi-\sigma)\widehat{\widehat{C}}_t - \chi\widehat{C}_t + \widehat{Y}_t - \mathbb{E}_t\widehat{\psi}_{t+1}\right] \tag{2.25}$$

Where $\overline{\pi}$ represents trend inflation and the equation gives the Philips curve with the dynamics of inflation. The parameters on the Philips curve depending on trend inflation are respectively: $k(\overline{\pi}) = \frac{(1-\theta\overline{\pi}^{\varepsilon-1})(1-\theta\beta\overline{\pi}^{\varepsilon})}{\theta\overline{\pi}^{\varepsilon-1}}$, $b_1(\overline{\pi}) = \beta\left[1+\varepsilon(\overline{\pi}-1)\left(1-\theta\overline{\pi}^{\varepsilon-1}\right)\right]$ and $b_2(\overline{\pi}) = \beta\left[1-\overline{\pi}\right]\left(1-\theta\overline{\pi}^{\varepsilon-1}\right)$.

Focusing on the New Keynesian Philips Curve in terms of the marginal costs, one can notice that compared to the standard Philips curve seen in textbooks such as Gali (2008b), it

[30] Variables with hat are expressed in log-linear form.

has some additional terms, which are functions of trend inflation. The extra term $\overline{\pi}$ on the term $k(\overline{\pi})$ which is the parameter governing the slope makes the Philips curve flatter. An increase in trend inflation $\overline{\pi}$ decreases the slope $k(\overline{\pi})$, and makes inflation less sensitive to current marginal costs. A value of trend $\overline{\pi} = 1$, which means that the steady-state of inflation is zero, would cause $k(1) = \frac{(1-\theta)(1-\theta\beta)}{\theta}$ transforming the parameter as in the standard slope of New Keynesian Philips. Trend inflation reduces the weight of determining inflation from current marginal costs by reducing $k(\overline{\pi})$ and on the other side by increasing $b_1(\overline{\pi})$, increases the weight on expected future inflation. Now the curve depends more on expected future inflation, and less on marginal costs[31]. In the standard case $b_1 = \beta$, and in this case the curve would depend less on expected future inflation.

An additional term appears on the Philips curve on the right hand side which shifts at some level the Philips curve depending positively by the level of private consumption and negatively by the discounted value of the marginal costs at period $t+1$ and governed by the parameter $b_2(\overline{\pi}) = \beta\left[1-\overline{\pi}\right]\left(1-\theta\overline{\pi}^{\varepsilon-1}\right)$ dependent on trend inflation, which shows the dynamics changes happening in the marginal costs and private consumption. A value of trend $\overline{\pi} = 1$, means that the steady state of inflation is zero, and this would cause a $b_2 = 0$, $k(\overline{\pi}) = \frac{(1-\theta)(1-\theta\beta)}{\theta}$ and $b_1(\overline{\pi}) = \beta$, transforming the New Keynesian Philips curve in its standard form.

Further the evolution of the present discounted value of future marginal costs and marginal revenues would be:

$$\widehat{\psi}_t = \left(1-\theta\beta\overline{\pi}^{\varepsilon}\right)\left[\widehat{mc}_t + (\chi-\sigma)\widehat{\widehat{C}}_t - \chi\widehat{C}_t + \widehat{Y}_t\right] + \theta\beta\overline{\pi}^{\varepsilon}\left[\varepsilon\mathbb{E}_t\widehat{\pi}_{t+1} + \mathbb{E}_t\widehat{\psi}_{t+1}\right] \tag{2.26}$$

and:

$$\widehat{\phi}_t = \left(1-\theta\beta\overline{\pi}^{\varepsilon-1}\right)\left((\chi-\sigma)\widehat{\widehat{C}}_t - \chi\widehat{C}_t + \widehat{Y}_t\right) + \theta\beta\overline{\pi}^{\varepsilon-1}\left[(\varepsilon-1)\mathbb{E}_t\widehat{\pi}_{t+1} + \mathbb{E}_t\widehat{\phi}_{t+1}\right] \tag{2.27}$$

The second part of both equations on the right-hand side represents the forward-looking part of the equations and they are both dependent on trend inflation.

The Generalized New Keynesian Philips Curve can also be expressed in terms of output and private consumption[32]. To express the GNKPC as an inflation-consumption relationship, I first substitute the marginal costs and using the fact that there is the following relationship between consumption and price dispersion: $\widehat{N}_t = (\widehat{Y}_t - \widehat{A}_t) + \widehat{s}_t$. I substitute $\widehat{N}_t$ into the expression for the real wage $w_t = \varsigma_t + \varphi\widehat{N}_t + \sigma_\delta\widehat{C}_t + (\sigma-\sigma_\delta)\widehat{G}_t$, and following the same

[31] For further discussion see Ascari & Sbordone (2014).

[32] Expressing the Philips curve into this form comes into hand in showing analytically the effects of government spending on private consumption through the method of undetermined coefficients discussed in the next subsection.

procedure as in the case of the GNKPC in terms of marginal costs would bring:

$$\begin{aligned}\widehat{\pi}_t &= \lambda(\overline{\pi})\widehat{Y}_t + k(\overline{\pi})\left[\varsigma_t + \varphi\widehat{s}_t - (\varphi+1)\widehat{A}_t + (\sigma-\sigma_\delta)\widehat{G}\right] + b_1(\overline{\pi})\mathbb{E}_t\widehat{\pi}_{t+1} \\ &+ b_2(\overline{\pi})\left[(\chi-\sigma)\widehat{\widehat{C}}_t - \chi\widehat{C}_t + \widehat{Y}_t - \mathbb{E}_t\widehat{\psi}_{t+1}\right]\end{aligned} \tag{2.28}$$

with one additional parameters $\lambda(\overline{\pi})$ which is the slope of the curve with respect to consumption and is equal to: $\lambda(\overline{\pi}) = \varphi k(\overline{\pi})$. The New Philips Curve expresses the relationship between inflation and consumption and has an additional term compared to the previous curve in terms of marginal costs, including also government spending, and shifts the Philips curve depending on the parameter $k(\overline{\pi})$.

Further log-linearizing the Euler equation 2.10 and substituting there the log-linear form of equation 2.8, I would obtain the dynamic IS curve as follows:

$$\widehat{C}_t = \mathbb{E}_t\widehat{C}_{t+1} - \frac{1}{\sigma_\delta}(i_t - \mathbb{E}_t\{\widehat{\pi}_{t+1}\}) + \frac{(\sigma-\sigma_\delta)}{\sigma_\delta}\mathbb{E}_t\Delta\widehat{G}_{t+1} \tag{2.29}$$

The Euler equation now is free from the aggregate consumption bundle variable and is expressed in terms of $\widehat{C}_t$ and $\widehat{G}_t$. $\sigma_\delta = (\chi(1-\delta)+\sigma\delta)$, and $\frac{1}{\sigma_\delta}$ gives the slope of the IS curve, which in the case that $\sigma_\delta > \sigma$, because $\chi > \sigma$ (government spending and private consumption are complementary) the slope of the IS curve decreases and the IS curve becomes flatter. Adding government spending into this economy adds an extra term to the IS curve and causes a shift of the curve more into the right. When $\sigma_\delta = \sigma$, as a consequence the government spending term would drop, and the slope of the IS curve would the same as in the standard case $\frac{1}{\sigma_\delta} = \frac{1}{\sigma}$. The same result would be obtained if the share of private consumption in the aggregate consumption bundle would be $\delta = 1$, which would mean that there is no government spending in this economy.

2.5 Model results

In this section, I present the results from the New Keynesian model with trend inflation and a government sector. Firstly, I show analytically how government spending affects private consumption. I discuss the implications of the complementarity between private consumption and government spending, on the effect of the government spending shock on private consumption and show how trend inflation changes the effects of fiscal policy and the dynamics of the response to shocks.

Secondly, I present the parameter values and discussion regarding them. Third, I show the impulse responses of private consumption, inflation, real interest rate, price dispersion, and labor to a government spending shock and show the implications of trend inflation in these results. Fourth, I argue that the results are robust to different parameter specifications. Lastly, I calculate and discuss the consumption multipliers from the model and the panel VAR.

2.5.1 Macroeconomic dynamics

In this subsection, I discuss the effects of government spending on private consumption. Initially, I focus on showing analytically the effects that government spending shocks bring on private consumption and I discuss the implications of the complementarity between private consumption and government spending. Secondly, I argue that trend inflation changes the effects of fiscal policy and the macro-dynamics.

Proposition 1. An increase in government spending produces a crowding-in effect on private consumption. An increase in the inverse elasticity of substitution between private consumption and government spending increases the positive response of private consumption to a government spending shock.

Proof. See Appendix A.3.

Assuming that the government spending shock is an independent identically distributed standard normal process as in subsection 2.4.3 the response of private consumption to a government spending shock would be:

$$c_g = \frac{c_1 k(\overline{\pi}) + c_2 b_2(\overline{\pi}) + c_3}{c_4 + c_5 b_2(\overline{\pi})} \tag{2.30}$$

where c_g represents the impact of the government spending shock on private consumption. I have defined the group of parameters[33] as c_i where $i = 1,2,3,4,5$, and are all signed values > 0 or < 0.

In the standard case a trend inflation value of $\overline{\pi} = 1$ corresponds to a steady state value of 0 inflation, $k(\overline{\pi})$ is transformed into $k(1) = \frac{(1-\theta)(1-\theta\beta)}{\theta} > 0$ and represents the standard $k(\overline{\pi})$ in the Philips curve. When $\overline{\pi} > 1$ and $\phi_\pi > 1$[34], $k(\overline{\pi}) > 0$ and $b_2(\overline{\pi}) > 1$, which results in $c_1 k(\overline{\pi}) + c_2 * b_2(\overline{\pi}) + c_3 < 0$ and $c_4 + c_5 b_2(\overline{\pi}) < 0$, so to conclude $c_g > 0$. An increase in g_t, from $\widehat{C}_t = c_g g_t$, suggests that this leads to an increase in $\widehat{C}_t$. And an increase in χ while c_2 and c_5 are equivalent in size with the condition that $c_1 k(\overline{\pi}) + c_2 * b_2(\overline{\pi}) + c_3 > c_4 + c_5 b_2(\overline{\pi}) < 0$, suggests a further increase in c_g.

[33]The detailed form of these parameters can be found in Appendix A.3.

[34]The inflation response parameter ϕ_π can not be lower than 1 in order for the solution of the model to be unique, as shown by Bullard & Mitra (2002).

When the inverse elasticity of substitution between private consumption and government spending is $\chi > 1$, an increase in government spending, ceteris paribus, increases the marginal utility of consumption, suggesting that government spending has a positive effect on private consumption, due to the Edgeworth complementarity between private consumption and government spending. For a high enough value of χ, the complementarity effect overpasses the negative wealth effect, suggesting that on aggregate, private consumption is crowded in by government spending.

On the other hand, a higher χ, leads to an increase in σ_δ keeping in mind that $\sigma_\delta = (\chi(1-\delta)+\sigma\delta)$, which means that $\frac{1}{\sigma_\delta}$ the slope of the IS curve goes down, with the curve becoming more flatter. The increase in government spending, from the labor supply equation 2.11, suggests that government spending influences positively the labor supply even more, which as consequence amplifies the decline in the real wage more as well. Now the complementarity effect is higher than before, widening the difference with the negative wealth effect, leading to a higher effect of government spending on private consumption c_g, and therefore higher $\widehat{C}_t$.

For a value of $\chi = \sigma = 1$, we would have the Euler Equation would take its standard form, Equation 2.30 would turn into:

$$c_g = \frac{\gamma b_2(\overline{\pi})\phi_\pi - \phi_Y\gamma}{(1+b_2(\overline{\pi})\phi_\pi + \phi_Y(1-\gamma))} \tag{2.31}$$

Proposition 2. An increase in trend inflation $\overline{\pi}$ increases the value of the positive response of private consumption to a government spending shock $\frac{\partial c_g}{\partial \overline{\pi}} > 0$.

Proof. See Appendix A.3.

Using c_g simplified definition in 2.31 with the restrictions on the parameters as in Proposition 1, the effect of the trend inflation on the response of private consumption from a government spending shock would be:

$$\frac{\partial c_g}{\partial \overline{\pi}} = \frac{\partial \frac{\gamma b_2(\overline{\pi})\phi_\pi - \phi_Y\gamma}{(1+b_2(\overline{\pi})\phi_\pi+\phi_Y(1-\gamma))}}{\partial \overline{\pi}} = \frac{\gamma\phi_\pi\left[\gamma\phi_Y - 2\phi_Y - 1\right]}{\left[1+b_2(\overline{\pi})\phi_\pi + \phi_Y(1-\gamma)\right]^2}\frac{\partial b_2(\overline{\pi})}{\partial \overline{\pi}} > 0 \tag{2.32}$$

where $\frac{\partial b_2(\overline{\pi})}{\partial \overline{\pi}} < 0$ and $\gamma\phi_\pi\left[\gamma\phi_Y - 2\phi_Y - 1\right] < 0$, as $\phi_\pi > 1$, $\phi_y > 0$ and $0 < \gamma < 1$, as consequence $\frac{\partial c_g}{\partial \overline{\pi}} > 0$. Therefore, trend inflation amplifies the impact of the government spending shock on private consumption. Similar results are also seen in the case of the monetary policy shock discussed in Ascari & Sbordone (2014). The intuition behind these results comes from the fact that with trend inflation, the Philips curve will be flatter, making inflation less sensitive to current marginal costs. The Philips curve now will depend more on expected future inflation, and less on marginal costs, thus making the firms more forward−looking. At higher rates of trend inflation price−setting firms are more forward−looking, they react less to the increase in private consumption happening due to the complementarity effect, so that inflation reacts less, becoming more persistent, as consequence the interest rate increases more, by inducing a larger reaction of consumption due to the Euler equation.

2.5.2 Results from the calibrated version of the model

Parametrization. Table 2.3 below displays the values assigned to the parameters in the baseline model with non-separability between government spending and private consumption. Each period I assume to correspond to a quarter. I chose a value for the discount factor $\beta = 0.99$, which means that the annual interest rate is equal to 4% in the steady-state, as used also in Gali et al. (2007). I keep a value of $\sigma = 1$ for the inverse elasticity of intertemporal substitution of consumption as in Ascari & Sbordone (2014), which implies that the preferences are separable in leisure and consumption. The value of the inverse Frisch labour supply elasticity is set as the standard value used in macro, following the evidence of Domeij & Floden (2006), $\varphi = 3$.

Table 2.3 Parameter values used in baseline model

β	Discount factor	0.99
σ	inverse elasticity of intertemporal substitution	1
φ	Inverse Frisch labour supply elasticity	3
ε	Elasticity of substitution among goods	10
θ	Calvo price stickiness parameter	0.75
ρ_g	AR(1) coefficient of government expenditure	0.9
ρ_v	AR(1) coefficient of monetary policy	0.9
ρ_A	AR(1) coefficient of technology	0.9
ρ_ζ	AR(1) coefficient of labor supply shock	0.9
ϕ_π	Inflation elasticity of the nominal interest rate	2
ϕ_Y	Output gap elasticity of the nominal interest rate	0.125
ρ_i	Inertia parameter with past interest rate	0.8
χ	Inverse elasticity of substitution between C and G	20
δ	Share of private consumption in the aggregate consumption bundle	0.8

I use standard parameter values as in Ascari & Sbordone (2014), for the elasticity of substitution among goods $\varepsilon = 10$, which corresponds to a steady-state mark up of 1.1 and the Calvo parameter of price stickiness $\theta = 0.75$. I do not change either the values related to the

Taylor rule. Though a value of $\phi_\pi = 1.5$ is more standard following Taylor (1993*a*), a value of 2 is needed to allow for determinacy for the 6% trend inflation case. I define the value for the inverse elasticity of substitution between government spending and private consumption $\chi = 20$ following Bouakez & Rebei (2007), Pieschacon (2012) and Troug (2020), while for the size of household's consumption in the aggregate consumption bundle $\delta = 0.8$ as in Bouakez & Rebei (2007) and Sims & Wolff (2018). To capture the persistence of government spending, I set a value $\rho_g = 0.9$ as in Corsetti et al. (2012b) and Gali et al. (2007). I set standard persistence values as well for the productivity shock, labor supply shock, and the monetary policy shock, respectively $\rho_v = 0.9$, $\rho_A = 0.9$, and $\rho_\zeta = 0.9$.

Impulse Responses. Figure 2.8 depicts the impulse responses functions of private consumption, annual inflation, annual real interest rate, price dispersion, and labor to a positive 1 percent government spending shock for four values of trend inflation: 0, 2, 4, and 6 percent.

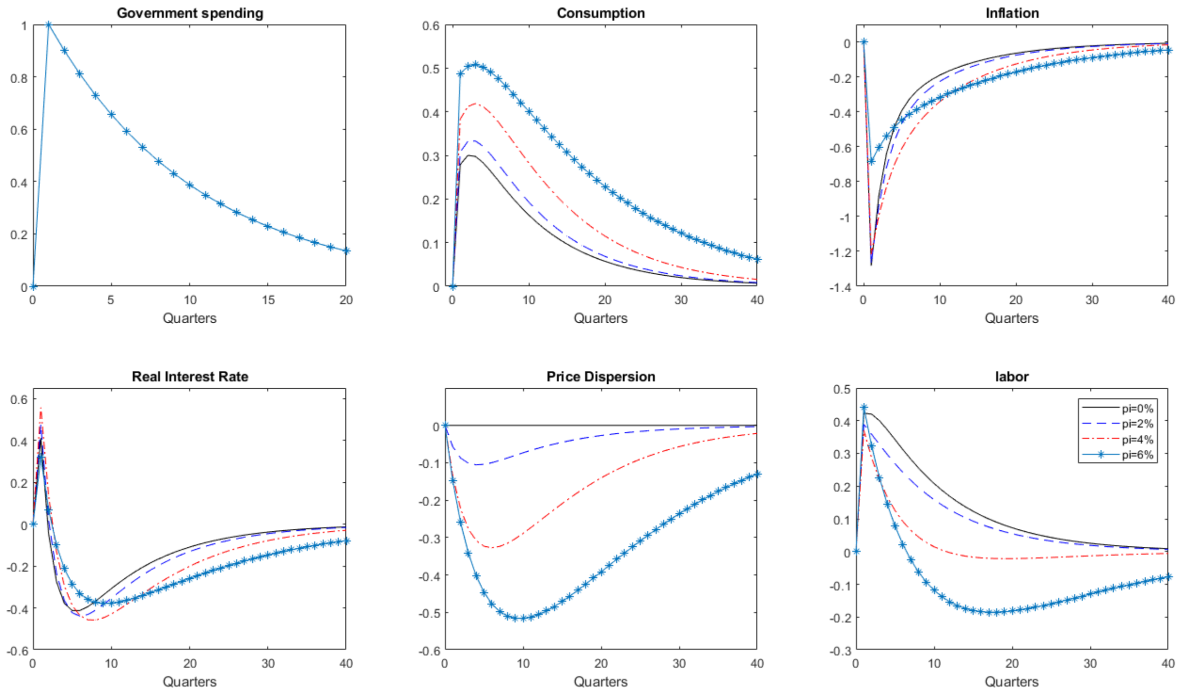

Figure 2.8 Impulse responses from a government spending shock in a small scale DSGE model

NOTES: The figure presents the impulse responses of Consumption, Annual Inflation, Annual Real Interest Rate, Price Dispersion, and Labor to a 1% Government Spending Shock in a Small Scale DSGE model. One period corresponds to 1 year on the horizontal axis and the response in percentage is reported on the vertical axis. The four lines represent respectively the impulse responses under a level of 0, 2, 4, and 6% of trend inflation. In this case, the government spending shock displays high persistence $\rho_g = 0.9$.

Figure 2.8 shows that on average a positive government spending shock increases private consumption, labor and it has an initial positive impact on the real interest rate for the first 2-3 periods. An increase of 1% in government spending produces on maximum impact almost a response of 0.3% in private consumption, displaying a hump-shaped form of the impulse responses. Because private and public spending are treated as Edgeworth complements, as government spending increases it raises the marginal utility of consumption, the complementarity effect is strong enough to overcome the negative wealth effect, and private consumption increases. On the other hand, the shock effects follow a decline in inflation, price dispersion, and the real interest rate after the first three periods. The increase in consumption will be mirrored by output, following the market clearing condition therefore I do not present the response of output in the figure. Labor, because of the linear function of technology, will increase as private consumption increases.

Focusing on the importance of trend inflation and its effect, the higher the trend inflation moving from 0 to 6% values, the higher the response of private consumption to the government spending shock. An increase of 1% in government spending produces on maximum impact almost a response of 0.33% in private consumption, when trend inflation is 2%, and 0.42% and 0.53% respectively when trend inflation is 4% and 6%, increasing the persistence in the results more and more. Trend inflation amplifies the impact of the government spending shock on private consumption because price-setting firms are more forward-looking. Trend inflation reduces the slope of the Philips curve and therefore reduces initially the impact of the government spending shock on inflation by increasing as well its persistence. Because there is mutual feedback between price dispersion and inflation the persistence increases even more, by reducing inflation. The larger increase in the interest rate in the first periods brings a larger reaction of consumption as suggested by the Euler equation.

The main result from this section is that the higher the trend inflation the higher the effects of a government spending shock in this economy. Trend inflation amplifies the increase in private consumption (increasing as well its persistence), the decline in price dispersion and inflation, it amplifies initially the effect on labor supply, to later reducing the increase more than in the previous case. This result supports Ascari & Sbordone (2014) idea that higher trend inflation amplifies economic shocks and increases their persistence. These results not only confirm the analytical results in subsection 2.5.1 but also are consistent with the empirics shown in subsection 2.3.2.

Further on, I discuss some other implications of trend inflation in amplifying the effects of a government spending shock on private consumption by considering some alternative parameter specifications. In figure 2.9 I present four different cases where I change the values of ρ_g, χ, φ and lastly I simplify the Taylor rule by assuming $\phi_Y = 0$, which makes it a CPI Taylor rule.

In the first column, it is presented the case where the persistence of the shock of government spending is initially put to 0 and then increases to 0.4. In the first case, when the shock persistence is equal to zero, in the first periods where the shock happens, trend inflation diminishes the response of private consumption to the shock, and after the shock ends, trend inflation amplifies the increase in private consumption. In the second case where the persistence of the shock is increased to 0.4, trend inflation, as the shock happens, starts in the second period to amplify the effect on private consumption, similar to the response attitude in the baseline case where the persistence of the shock is high 0.9. The persistence of the shock matters as well for the size of the response of private consumption, where the response in the baseline, in the case of zero trend inflation, is almost 3 to 4 times higher than in the cases where the persistence of the shock is low.

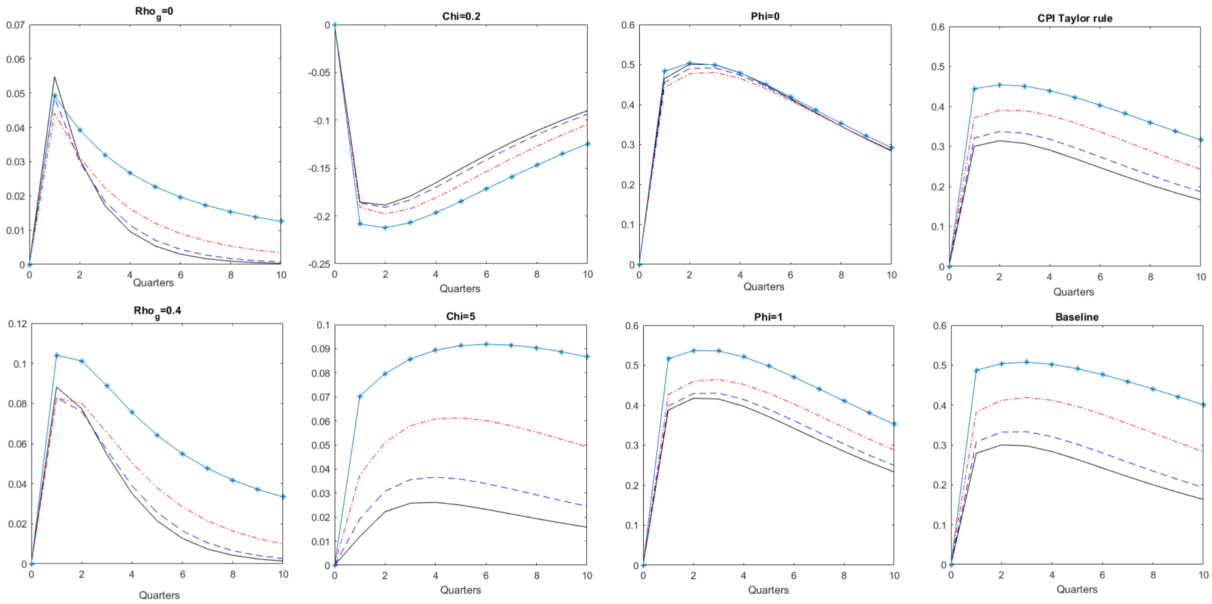

Figure 2.9 Impulse responses from a government spending shock in a small scale DSGE model under different parameter values (1)

NOTES: The figure presents the impulse responses of Consumption to a 1% Government Spending Shock under alternative parameter choices for ρ_g, χ, φ and by assuming $\phi_Y = 0$ in the Taylor rule. The first column presents the case of changing ρ_g from 0 to 0.4, the second column changing χ from 0.2 to 2, the third column changing φ from 0 to 1, and the fourth column presents the case os a simple Taylor rule and then the baseline where $\rho_g = 0.9$, $\chi = 20$, $\varphi = 3$ and $\phi_Y = 0.125$. One period corresponds to 1 year on the horizontal axis and the response in percentage is reported on the vertical axis. The lines four lines represent respectively the impulse responses under a level of 0, 2, 4 and 6% of trend inflation.

In the second column, it is presented the case where inverse elasticity of substitution χ is changed. Initially it is given the value of 0.2, which makes government spending and private consumption slightly substitutes, and then the value of 2 which makes government spending and private consumption slightly complements. In the first case, compared to the baseline the response of private consumption to a government spending shock, is slightly negative, and trend inflation amplifies the negative results of the government spending shock. In the second case, when private consumption and government spending becomes complements, the response of private consumption becomes positive as in the baseline. Trend inflation amplifies the positive response of private consumption moving from 0 to 6 % cases of trend inflation. In both cases the size of the response is relatively low compared to the baseline case where private consumption and government spending are strongly complements, being in the case of 6% trend inflation, $8-10$ times higher.

In the third column, it is presented the case where inverse Frisch labour supply elasticity φ is changed. Initially, it is given the value of 0, and then the value of 1, where the baseline itself has a value of 3. In the first case, the wealth effect of the labor supply is shut down, while in the second case the Frisch elasticity is higher. In the first case, compared to the baseline, the response of private consumption to a government spending shock is still positive, but now, a higher trend inflation brings about a slightly lower positive response of private consumption. To be noticed is that in this case, the importance of trend inflation in affecting the size of the response of private consumption is not that significant in size as before. When the inverse Frisch labour supply elasticity φ takes the value of 1, trend inflation amplifies the response of private consumption as in the baseline though not on the same magnitude. In the trend inflation case of 0 when $\varphi = 1$ the response of private consumption is slightly higher than in the case of $\varphi = 3$.

In the fourth column, it is presented the case where the output elasticity of the nominal interest rate is $\phi_Y = 0$ compared to the baseline where $\phi_Y = 0.125$. This change in the parameter converts the Taylor rule into a CPI rule, where the interest rate is decided only by taking into account the level of inflation in the economy. Simplifying the Taylor rule into a CPI rule does not seem to matter for the results as no big changes are noticed in the response of private consumption to the government spending shock and neither on the influence of trend inflation on the shock. Despite all, some small differences are noticed in the CPI rule case, as when trend inflation is equal to zero, the response of private consumption is slightly higher than in the baseline case. The amplifying effects of trend inflation on the response of private consumption, on the 6% trend inflation case, seem to be slightly lower in size compared to the baseline case.

Next, I discuss some other implications of trend inflation in amplifying the effects of a government spending shock on private consumption by considering different values for the intratemporal elasticity of substitution σ and for habit formation h. As shown in Fuhrer (2000) habit formation in consumption has important implication for the response to demand shocks. In figure 2.10 I present 3 different cases where I change the values of σ and h.

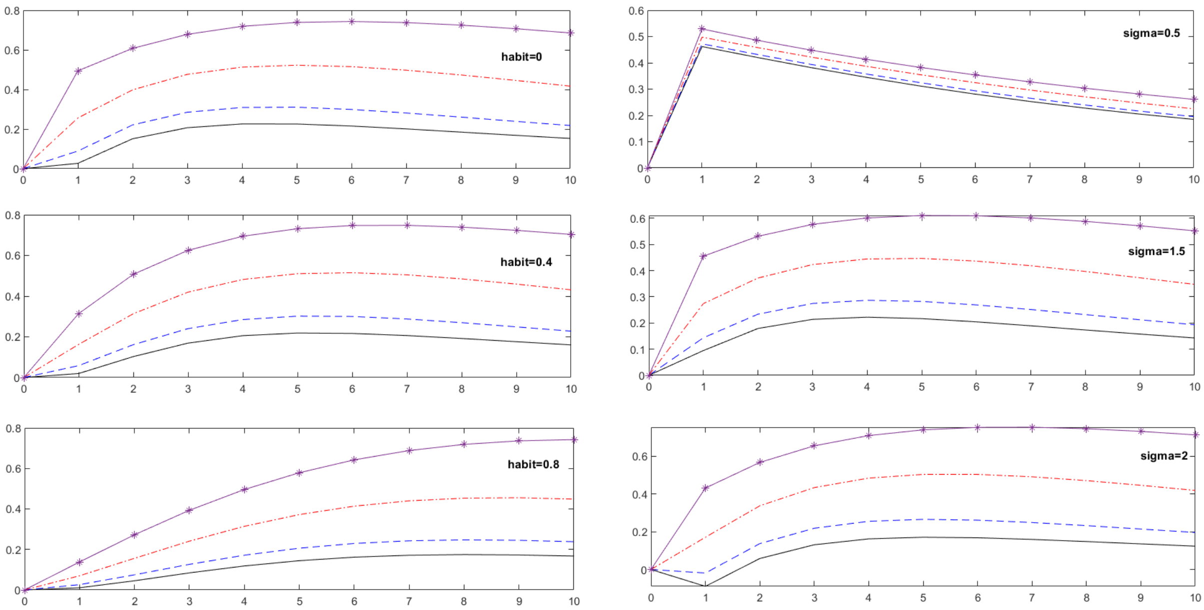

Figure 2.10 Impulse responses from a government spending shock in a small scale DSGE model under different parameter values (2)

NOTES: The figure presents the impulse responses of Consumption to a 1% Government Spending Shock under alternative parameter choices for σ (on the right panel) and h (on the left panel). One period corresponds to 1 year on the horizontal axis and the response in percentage is reported on the vertical axis. The lines four lines represent respectively the impulse responses under a level of 0, 2, 4 and 6% of trend inflation.

The amplifying effects of trend inflation on the response of private consumption to a government spending shock stand under different values for σ and h. The difference in the response of private consumption under higher values of trend inflation is reduced with values of σ under 1, while it increases with values higher than 1. Including positive values for habit formation while it does not change

2.5.3 Consumption multipliers

Consumption multipliers: model vs data−panel VAR. In order to compare the results on private consumption obtained from the baseline GNK model with the empirical results from the benchmark panel VAR, in this section, I calculate the consumption multipliers. A consumption multiplier measures the impact on private consumption due to a change in government spending. Blanchard & Perotti (2002), calculates the multipliers in the case of the output as the ratio of the output response to the initial government spending shock. While Mountford & Uhlig (2009), suggests calculating the multiplier by discounting it to the present value using the long $-$ run average interest rate. As both in the model and in the Panel VAR both government spending and private consumption are taken in log form, to obtain the multipliers in dollars, requires converting the impulse responses into dollars, and for this, I use the standard conversion technique of multiplying the multiplier by the ratio of consumption to government spending:

$$M_{\max\_impact} = \frac{(c_t - c)}{(g_t - g)} \frac{c}{g} \tag{2.33}$$

$$M_{Cumulative} = \frac{\sum_{i=0}^{N}(c_{t+i} - c)}{\sum_{i=0}^{N}(g_{t+i} - g)} \frac{c}{g} \tag{2.34}$$

$$M_{Cumulative\_present\_value} = \frac{\sum_{i=0}^{N}(1+r)^{-i}(c_{t+i} - c)}{\sum_{i=0}^{N}(1+r)^{-i}(g_{t+i} - g)} \frac{c}{g} \tag{2.35}$$

I calculate three types of multipliers that are, the maximum impact multiplier, the cumulative multiplier, and the cumulative multiplier at the present value. The maximum impact multiplier refers to the point where the response of private consumption to government spending shock is the highest. The consumption multiplier results are shown in Table 2.4.

Table 2.4 Consumption multipliers

	Empirical multipliers		Model multipliers	
	Low Inflation	High Inflation	0% Trend Inflation	6% Trend Inflation
Maximum Impact	0.4	0.8	0.57	1.05
Cumulative	0.8	2.1	0.75	2.27
Cumulative present value	0.7	2.0	0.75	2.16

The consumption multipliers coming from the panel VAR are calculated according to the sample division done of the countries in low and high inflation countries, while for the model case I calculate them for the cases of 0% and 6% trend inflation[35]. The maximum impact

[35]For the DSGE model multipliers I chose a value of 0.91 for the parameter δ as this value allows producing from the model, multipliers closer to the empirical multipliers. In the case of a $\delta = 0.8$ as in Sims & Wolff (2018) then the multipliers would be higher than 1 due to estimated complementarity of government spending with private consumption.

multiplier is 0.4 for low inflation, while it is slightly higher for 0% trend inflation with about 0.57. In the case of high inflation and 6% trend inflation, the multipliers are respectively 0.8 and 1.05 being in both cases almost twice higher than in the cases of low inflation and 0% trend inflation. The long-run consumption multipliers represented by the cumulative present value multipliers are respectively 0.7 and 0.75 for low trend inflation case and 0% trend inflation case, while the values for the high inflation case and 6% trend inflation are 2.0 and 2.16 respectively.

All the three empirical consumption multipliers show that they are 2-3 times higher in countries with high long-run inflation compared to countries with low inflation. In the case of the model consumption multipliers, the difference between the cases of low trend inflation vs high trend inflation is 2-4 times higher, under the specific parameters of the model. These multipliers are close to the ones suggested by Ramey (2019), where on a review of the work done on fiscal multipliers she argues that on average government spending multipliers vary from 0.6-1 for both time series and DSGE models estimates, without taking into account country characteristics.

2.6 Robustness model results

In this subsection, the period utility function of the representative agent is assumed to be non-separable in consumption (C) and labor (N) as in Greenwood et al. (1988). In this model the wealth effect on labor supply is shut off, suggesting that government spending can influence positively private consumption. Monacelli & Perotti (2008), also relies on GHH preferences in crowding in private consumption, but with the difference of using GHH preferences on the form introduced by Jaimovich & Rebelo (2009). I consider here the form of the utility function used in Lewis & Winkler (2017).

$$U(C_t, N_t) = \ln(C_t - \frac{\varsigma}{1+\widetilde{\varphi}} N_t^{1+\widetilde{\varphi}}) + h(G_t) \tag{2.36}$$

$h(G_t)$ represents the utility the household gets from government spending, where the wealth effect of labor supply χ is expressed as $\chi = -\frac{U_{cc}C}{U_c} + \frac{U_{c(1-N)}C}{U_{(1-N)}}$ and $\widetilde{\chi} = \frac{\chi}{c_y}$, and c_y is the steady-state share of private consumption to output. Additional definitions would be $\nu = \frac{U_{c(1-N)}N}{U_c}$ and $\varphi = \frac{U_{(1-N)(1-N)}N}{U_{(1-N)}}$. The inverse of the constant-consumption labor supply elasticity is defined as: $\frac{1}{\widetilde{\varphi}} = \frac{1}{\varphi + \nu - \frac{\nu\widetilde{\chi}}{\widetilde{\chi}-\nu}}$. Under this preferences government spending can crowd in private consumption only when Bilbiie (2011, 2018) conditions are fulfilled [36] $\widetilde{\chi} \geq 0$, $\widetilde{\varphi} \geq$

[36] For details see Lewis & Winkler (2017).

0 and $\nu \leq \frac{\widetilde{\varphi}\widetilde{\chi}}{\widetilde{\chi}+\widetilde{\varphi}}$. Dealing with GHH preferences, requires that the wealth effect on labor supply is $\chi = 0$ and $\nu < 0$, so labor and consumption are complements[37].

I chose a value of $\nu = -3.2$, almost an average value compared to the choices of Bilbiie (2011) and Furlanetto & Seneca (2014), respectively -1.29 and -5. Keeping the same value for the inverse Frisch elasticity $\varphi = 3$ as in the baseline model would bring a value for the constant-consumption labor supply elasticity $\widetilde{\varphi} = 0.2$. Regarding the monetary policy rule, I decided to keep a simple one to allow for determinacy in each of the cases of trend inflation, but at the same time, I keep the same inertia. Some inertia in the Taylor rule in this model is determinant for obtaining a crowding-in on private consumption.

In this model with GHH preferences, the negative wealth effect problem which happens in a model with separable preferences is not present, and this allows for government spending to induce a crowding-in effect on private consumption. An increase in government spending in the standard case causes a negative wealth effect because it will be associated with higher taxes, which on the other hand shifts the labor supply down because the household now consumes less and works more, this leads to a rise in working hours, rise in output, and a decline in private consumption and the real wage. As argued by Monacelli & Perotti (2008) in the case of GHH preferences the wealth effect on labor supply is shut down, and the labor supply curve does not shift. On the firm's side, the increase in government spending, cause the firm's product demand to increase, and this leads to a shift out in the labor demand, causing hours, and real wages to increase, followed by an increase in inflation. When the hours and consumption are complementary, the hours and private consumption must increase when government spending increases because of the aggregate resource constraint that I assumed in this model.

Additionally, in this subsection, I discuss the results from the New Keynesian model with non-separable preferences in consumption and labor, and with government spending in the utility function under different values of trend inflation.

Figure 2.11 shows the impulse responses functions of private consumption to a government spending shock for three values of trend inflation: 0, 2, and 5 percent. A government spending shock here induces a positive response of private consumption, an increase in inflation, price dispersion, and labor supply, while it suggests an initial decline in the real interest rate, which turns positive after 1-2 periods. Under a government spending shock, higher trend inflation again, facing more forward-looking firms, under a new Philips curve which depends more on expected inflation, amplifies price dispersion, causes inflation to increase after 2 periods, with a higher persistence. This increase in inflation reflects in

[37]The main log-linearized equations the IS curve, labor supply and Philips curve can be found in Appendix A.2.6.

lower real interest rates, and as consequence, the response of private consumption is higher. Under GHH preferences, trend inflation, when comparing the model with 0% and 4% trend inflation, amplifies the positive response of private consumption to the shock but with a lower magnitude than in the case where I assumed government spending and private consumption complementarity.

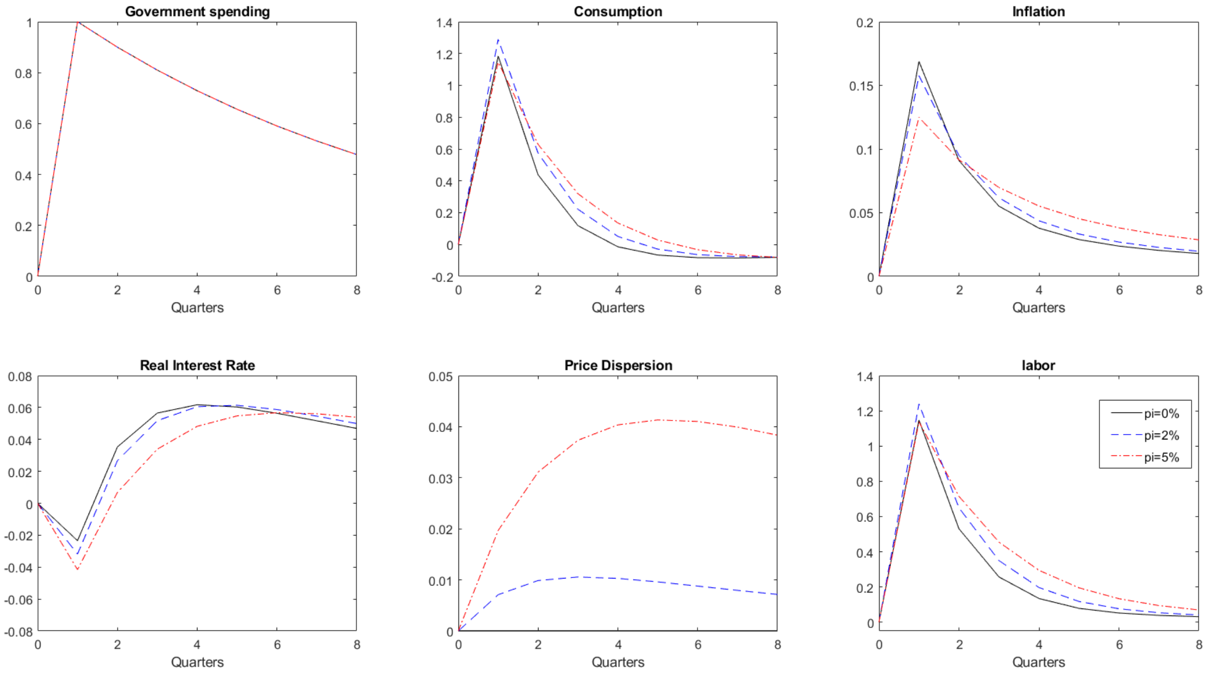

Figure 2.11 Responses of private consumption to a government spending shock from an alternative model

A similar effect, though not of the same size was also seen in the empirical analysis when comparing low vs high inflation countries. Overall, I can argue that the results from the baseline DSGE model are robust when compared to the alternative of a different preference choice.

2.7 Conclusion

This paper has dealt with an important question and one of ongoing interest in macroeconomics "What are the effects of government spending shocks on consumption?". Though there exists considerable theoretical and empirical work, there is still space in investigating further on this matter. This paper focuses on the idea that heterogeneity among countries coming from country characteristics highly matters in the effects of government spending on private consumption. When estimating a panel VAR in a quarterly panel data set of 34 OECD countries, from $1995 - 2017$ I find that in countries with high long-run inflation the response of private consumption is higher than in countries with low long-run inflation. These results turned out to be robust under three different modelling choices. In a second step, I investigated the empirical results in a small$-$scale DSGE New Keynesian model with trend inflation I found that an increase in trend inflation increases the positive response of private consumption. Overall, when long$-$run inflation is used to explain the heterogeneity between countries, the response of private consumption to a government spending shock reflects these differences.

Chapter 3

The Intratemporal Elasticity of Substitution Across Sectors, Labour Mobility and Output Dynamics

3.1 Introduction

What is the response of output to demand and supply shocks? Understanding how demand and supply shocks transmit to the real economy are vital questions for research in macroeconomics. The increasing research work on multi-sector economies (see Aoki (2001) and the increasing importance of sectoral price heterogeneity (see Bils & Klenow (2004), Carvalho (2006) and Nakamura & Steinsson (2008)), raises questions on other aspects that might affect the transmission of shocks such as the labor mobility between sectors and the substitutability of goods between sectors.

New Keynesian models of the last generation answer this question with the assumption that either labor is mobile across sectors (eg. Barsky et al. (2007)) or that labor is sector-specific (eg. Carlstrom et al. (2006)), while the substitutability of goods between sectors is assumed either to be the same as the elasticity of substitution across different varieties of intermediate goods (eg. Carvalho (2006)) or it is just assumed of being equal to one (eg. Carvalho & Nechio (2011)). In this paper I will explore further the dynamics of output to demand and supply shocks in a closed economy New Keynesian (NK) model with price heterogeneity, focusing on labor mobility and the intratemporal elasticity of substitution between sectors.

To determine how labor mobility and the intratemporal elasticity of substitution between sectors is important for the transmission of shocks on output dynamics, I start my analysis by considering a standard closed economy two−sector New Keynesian (NK) model with price heterogeneity across sectors. The firms in one sector have fully flexible prices, while in the other sector firms face standard sticky Calvo (1983) prices. The main findings of the analysis are: First, an increase in the intratemporal elasticity of substitution between sectors increases the response of output to a negative demand shock. Second, an increase in the labor mobility between sectors increases the response of output to a negative demand shock, same as in the case of the increase in the intratemporal elasticity of substitution between sectors. From the first two points, it can be derived that the interaction between the labor mobility parameter and the intratemporal elasticity of substitution between sectors when the model takes into account for both being present, can alternate the points mentioned. Looking at the implications from the supply side, it can be observed that an increase in the labor mobility (or the intratemporal elasticity of substitution between sectors) increases the response of output to a supply shock similar to in the case of demand shocks.

The first step in this analysis of understanding how the labor mobility and the intratemporal elasticity of substitution between sectors affect the dynamics of output subject to demand and supply shocks in a New Keynesian model is by exploring the case where the firms in all sectors face the same price stickiness. Looking at first a negative demand shock[1], what happens is that it generates a decrease in the real interest rate and an increase in inflation and output in both sectors. Under a supply shock[2], output increases (with a better technology, it can be produced more) and prices go down. Employment (hours) goes down, while labor demand and wages go up. When firms in both sectors face the same degree of price stickiness their reaction toward the shock (either demand or supply shock) will be on the same pace (they change prices to adjust to the shock equally), therefore having high labor mobility (intratemporal elasticity of substitution between sectors) or not, does not alter the dynamics of output across sectors.

I conduct the analysis as a second step under price heterogeneity between the two sectors. At first, I consider only the case of the demand shock and explain how the level of labor mobility across sectors influences the results. Then, I focus on the implications of the intratemporal elasticity of substitution across sectors. In the second step, I explore the implications when the economy is hit by a supply shock. In one sector prices are fully flexible, while in the other sector firms face standard sticky prices.

Under a negative demand shock[3], flexible price firms change prices faster to accommodate the shock, while the sticky price firms require some time to do so. Because sticky sector

[1]See Gali (2008) textbook and Christiano et al. (2005) about more regarding the effects of demand shocks on output in the standard one-sector Calvo (1983) model.

[2]See Gali (2004) and Canova et al. (2010) for more regarding the effects of supply shocks on output.

[3]The results are symmetric as well in the case of a positive demand shock.

price firms require more time to accommodate prices, they would be selling with lower prices, and as consequence, their output demand would increase, while output demand of the flexible firms would be lower than before the shock as they managed to change their prices sooner than sticky-price sector firms. When the intratemporal elasticity of substitution is high because the prices that the flexible price firm operates are high, the production of their good would be reduced and they would shift producing more the good of the sticky-price sector because of higher demand for it.

While, under high labor mobility, workers in the flexible sector, would notice the differences in real wages coming due to the increase in the demand for labor in the sticky sector[4] and would shift to offer labor hours in the sticky sector. This shift in labor would cause an increase in output in the sticky sector and a decline in output in the flexible sector, which is subject to the size labor mobility. These differences persist until the sticky price firms are able to adjust prices. In both cases, when the increase in the output in the sticky sector is high enough to overcome the decline of output in the flexible sector, total output increases, and vice−versa.

Next, I look at the response of output to a supply shock. An increase in the labor mobility (intratemporal elasticity of substitution between sectors) parameter increases the response of output to a supply shock. The intuition applied in the case of the demand shock is valid as well in the case of the supply shock. With a higher intratemporal elasticity of substitution, the products between the two sectors will have a higher substitutability, and because the flexible firms adjust prices faster, their product will be more in demand, and as consequence, their output increases even more, while the output in the sticky sector is reduced in size. Considering the case of high labor mobility, lower prices in the flexible sector would result in higher real wages in this sector, making it more attractive for labor, which now can shift between sectors. Sticky price firms require more time to make adjustments in prices due to the shock. More labor in the flexible sector means that now it can be produced more of its good under the current supply shock, and less of the good in the sticky sector, due to the loss of labor to the flexible sector. In order for the total output to increase due to the supply shock, in needs that the extra increase in output due to the high labor mobility (or higher intratemporal elasticity of substitution), in the flexible sector, to be bigger than the change in the sticky sector.

Furthermore, to confirm these results I take the baseline New Keynesian model with one sector with flexible price firms and one sector with sticky price firms and replace it with a version with a two−sector Calvo pricing and positive trend inflation. I calibrate the economy

[4] As a consequence of the reaction of the flexible price firms, real wages will be lower in the flexible sector, than in the sticky sector. Under no labor mobility, these differences would persist, as no reaction would be possible by the workers.

such that it matches the US economy with the manufacturing and services sectors for 2017. The calibrated model for manufacturing and services confirms the importance of labor mobility (or the intratemporal elasticity of substitution between sectors) in the transmission of demand and supply shocks to output. These results are driven by the differences in price stickiness between the two sectors.

Further, counting in previous studies, I give some insight regarding the importance of labor mobility and the intratemporal elasticity of substitution between sectors. Airaudo & Zanna (2012) argues that the intratemporal elasticity of substitution between sectors, matters for determinacy conditions. Caunedo (2019) shows that the effect of sectoral shocks on aggregate depends on the degree of heterogeneity in substitutability, and it is, therefore, independent of the elasticity when this is identical across sectors. Cantelmo & Melina (2017) suggests that the degree of labor mobility between sectors is an aspect of the economy that central banks should not overlook in setting the monetary policy stance. While Cardi et al. (2020) highlight the role of labor mobility in the transmission of government spending shocks. The evidence about labor mobility across sectors is somehow not clear, where Davis & Haltiwanger (2001) find limited labor mobility across sectors in response to monetary and oil shocks, while Bouakez et al. (2011) report evidence suggesting that perfect labor mobility across sectors when sectoral nominal wages are the same. The implications of price heterogeneity on monetary policy shocks are important as argued by Pasten et al. (2019), who see it as key in determining the transmission of monetary policy shocks.

This paper's contribution to the New Keynesian literature goes in several directions. First, it contributes to the literature on the effects of demand shocks and supply shocks on the economy, such as Christiano et al. (2005) and Gali (2004). Secondly, it contributes to the literature that argues the importance of price heterogeneity in New Keynesian models, eg. Carvalho (2006) and Nakamura & Steinsson (2008), Carvalho et al. (2019), and Bouakez et al. (2014) which show that sectoral heterogeneity in price stickiness matters for the effects of demand shocks.

The remainder of this paper is organized as follows. In section 3.2, I develop a closed economy New Keynesian (NK) model with heterogeneity in price stickiness. Section 3.3 solves the model and presents the main analytical results. In section 3.4 relying on impulse response functions, I discuss the implications of labor mobility and the intratemporal elasticity of substitution between sectors on output dynamics. Additionally, in section 3.5, I present the results in a calibrated model for the US economy with manufacturing and services and positive trend inflation. Finally, section 3.6 concludes.

3.2 The Model

This section describes the features of the model. I develop a two−sector New−Keynesian dynamic general equilibrium model of a closed economy with heterogeneity in nominal price rigidities. I will first describe the behaviour of the households, the firms, and then of the monetary authority. There is a continuum of households and firms over the unit interval. The remaining parts of the model are reported in Appendix B.2.

3.2.1 Households

The economy is populated by a representative agent, that is infinitely lived, and consumes the final consumption good C_t and supplies of labour L_t. Households derive their income from supplying labor to the production sectors, investing in bonds, and from the stream of profits generated in the production sectors. The lifetime utility of the household equals:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \Psi \frac{L_t^{1+\varphi}}{1+\varphi}\right) \tag{3.1}$$

where β is the discount factor, and σ is the inverse of the intertemporal elasticity of substitution and φ is the inverse of the Frisch elasticity of labor supply.

The household maximizes lifetime utility (1) subject to the budget constraint:

$$P_{1,t}C_{1,t} + P_{2,t}C_{2,t} + (1+i_t)^{-1} B_t = W_{1,t}L_{1,t} + W_{2,t}L_{2,t} + D_{1,t} + D_{2,t} + B_{t-1}, \tag{3.2}$$

The household buys $C_{1,t}$ from sector 1 at the nominal price $P_{1,t}$ and $C_{2,t}$ from sector 2 at the nominal price $P_{2,t}$, where $P_t C_t = P_{1,t}C_{1,t} + P_{2,t}C_{2,t}$. The agents receive their nominal wage income $(W_{1,t}L_{1,t}; W_{2,t}L_{2,t})$ and lump−sum profits $(D_{1,t}; D_{2,t})$ from firms at the end of the period. The household also invests in a one-period bonds.

The consumption basket C_t is a CES aggregate of sub-baskets of individual goods produced in sectors 1 and 2:

$$C_t = \left[\alpha_1^{\frac{1}{\eta}} (C_{1,t})^{\frac{\eta-1}{\eta}} + \alpha_2^{\frac{1}{\eta}} (C_{2,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{3.3}$$

where α_1 denotes the share of the goods in sector 1, and $\alpha_2 = 1 - \alpha_1$ denotes the share of the goods in sector 2 in the CES aggregator, and η is the elasticity of substitution between sectors 1 and sectors 2 goods. The goods in both sectors can be Edgeworth complements, neutral, or substitutes. The goods are complements when $\eta < \frac{1}{\sigma}$, neutral if $\eta = \frac{1}{\sigma}$, and substitutes when $\eta > \frac{1}{\sigma}$.

Consumption in sector i is $C_{i,t} = \left[\int_0^1 c_{1,t}^{j}{}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ for the differentiated consumption units $c_{1,t}^{j}$. Given the form of preferences in equation 3.3, the household allocates its consumption to individual brands of the goods in each sector according to the demand schedules, $c_{i,t}^{j} = \left(\frac{P_{i,t}^{j}}{P_{i,t}}\right)^{-\varepsilon} C_{i,t}$ and $C_{i,t} = \alpha_i \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} C_t$, where $P_{i,t}^{j}$ denotes the price of individual brand j in sector i where $i = 1,2$. I substitute $C_{i,t}$ to obtain the demand curves, $c_{1,t}^{j} = \left(\frac{X_{1,t}^{j}}{P_{1,t}}\right)^{-\varepsilon} \alpha_1 \left(\frac{P_{1,t}}{P_t}\right)^{-\eta} C_t$ and $c_{2,t}^{j} = \left(\frac{X_{2,t}^{j}}{P_{2,t}}\right)^{-\varepsilon} \alpha_2 \left(\frac{P_{2,t}}{P_t}\right)^{-\eta} C_t$, where the CPI index corresponds to:

$$P_t = \left[\alpha_1 \left(P_{1,t}\right)^{1-\eta} + \alpha_2 \left(P_{2,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{3.4}$$

I assume that labor can be either supplied to sector 1 or sector 2, according to a constant elasticity of substitution (CES) aggregator as in Petrella & Santoro (2011) and Horvath (2000):

$$L_t = \left[b_1^{-\frac{1}{\lambda}} \left(L_{1,t}\right)^{\frac{1+\lambda}{\lambda}} + b_2^{-\frac{1}{\lambda}} \left(L_{2,t}\right)^{\frac{1+\lambda}{\lambda}}\right]^{\frac{\lambda}{\lambda+1}} \tag{3.5}$$

where λ represents the elasticity of substitution in labor supply and b_1 ($b_2 = 1 - b_1$) is the steady-state ratio of labor supply in the goods sector 1 (sector 2) over total labor supply. Depending on λ, this functional form allows us to account for different degrees of labor mobility between sectors. When $\lambda = 0$, labor is immobile across sectors $\Psi_1 \frac{L_{1,t}^{1+\varphi}}{1+\varphi} + \Psi_2 \frac{L_{2,t}^{1+\varphi}}{1+\varphi}$ with the same Frisch elasticity, while when $\lambda < \infty$ hours are not perfectly substitutes. For $\lambda = \infty$ perfect labor mobility is achieved, and all sectors pay the same hourly wage, and we are at the standard case where[5]: $L_t = L_{1,t} + L_{2,t}$. As argued by Petrella et al. (2019), Equation 4, *"...implies that labor market frictions are neutralized in the steady-state, so that the inefficiency associated with sectoral wage discrepancies is only temporary..."*

The first-order conditions of the household optimization problem can be expressed as,

$$1 = \beta(1 + i_t) * \left(\frac{P_t}{P_{t+1}}\right) * \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \tag{3.6}$$

and,

$$\frac{C_{1,t}}{C_{2,t}} = \frac{\alpha_1}{\alpha_2} \left(\frac{\rho_{2,t}}{\rho_{1,t}}\right)^{\eta} \tag{3.7}$$

[5]Most of the literature takes this case for granted and is commonly assumed in DSGE models.

$\frac{\rho_{2,t}}{\rho_{1,t}}$ represents the relative price in sector 1, over the relative price in sector 2, while the labor supply equations,

$$w_{1,t} = \Psi(b_1)^{-\left(\frac{1}{\lambda}\right)} L_t^{\varphi-\frac{1}{\lambda}} (L_{1,t})^{\frac{1}{\lambda}} C_t^{\sigma-\frac{1}{\eta}} (C_{1,t})^{\frac{1}{\eta}} \alpha_1^{-\frac{1}{\eta}} \tag{3.8}$$

and

$$w_{2,t} = \Psi(b_2)^{-\left(\frac{1}{\lambda}\right)} L_t^{\varphi-\frac{1}{\lambda}} (L_{2,t})^{\frac{1}{\lambda}} C_t^{\sigma-\frac{1}{\eta}} (C_{2,t})^{\frac{1}{\eta}} \alpha_2^{-\frac{1}{\eta}} \tag{3.9}$$

$w_{1,t}$ and $w_{2,t}$ are the real wage rate in units of consumption in each sector[6]. Real wages in sectors 1 and 2 may differ because of limited labour mobility[7]. The presence of the parameters The aggregate wage index associated with the labor index is:

$$W_t = \left[b_1 W_{1,t}^{1+\lambda} + b_2 W_{2,t}^{1+\lambda} \right]^{\frac{1}{\lambda+1}} \tag{3.10}$$

where $W_{1,t}$ and $W_{2,t}$ are wages paid in sector 1 and in sector 2, respectively. From the wage index in (10), can be derived the allocation of aggregate labor supply to sector 1 and sector 2: $L_{1,t} = b_1 \left(\frac{W_{1,t}}{W_t}\right)^{-\lambda} L_t$ and $L_{2,t} = b_2 \left(\frac{W_{2,t}}{W_t}\right)^{-\lambda} L_t$, where the parameters are the same as in equation 3.5.

3.2.2 Final Good Firms

The final goods are the same as in the standard New Keynesian models. In each sector, the final goods forms aggregate the different varieties produced by the continuum of intermediate goods firms using the CES function respectively for good in sector 1 and the good in sector 2:

$$Y_{1,t} = \left[\int_0^1 y_{1,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon-1}{\varepsilon}} \text{ and } Y_{2,t} = \left[\int_0^1 y_{2,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon-1}{\varepsilon}} \tag{3.11}$$

The parameter ε denotes the elasticity of substitution across different varieties of intermediate goods and initially is assumed constant across sectors.

3.2.3 Intermediate Good Firms

Firms in each sector produce output according to the following linear technology:

[6] The real wages are defined as $w_{1,t} = \frac{W_{1,t}}{P_{1,t}}$ and $w_{2,t} = \frac{W_{2,t}}{P_{2,t}}$.

[7] On the real wages formulas, the parameters λ and η show in the same form in relation to total labor and total consumption, and same form for labor and consumption in sector one. This detail will come into hand when explaining the effect of labor mobility and sector substitutability, as potential substitutes of each other.

$$Y_{j,t} = A_{j,t}L_{j,t} \text{ and } j = 1,2, \tag{3.12}$$

where A_t for each of the sectors is an exogenous process for the level of technology, which I assume stationary. The labor demand of the firm is positive to the production function and is related inversely to the level of technology. While on the other hand, intermediate inputs are produced by a continuum of firms with a simple linear technology in labor, which is the only input of production. As well to be highlighted is that the resource constraints are as following:

$$Y_{j,t} = C_{j,t} \text{ and } j = 1,2, \tag{3.13}$$

and as consequence $Y_t = C_t$.

Flexible Price firms

I assume that the firms in sector 1 have a representative flexible price firm that adjusts prices every period, and there is an optimal price that equals marginal costs:

$$\max_{X_{1,t}} X_{1,t}Y_{1,j,t} - W_{1,t}L_{1,t} \quad \text{s.t} \quad Y_{1,t} = A_{1,t}L_{1,t} \text{ and } Y_{1,j,t} = \left(\frac{X_{1,t}}{P_{1,t}}\right)^{-\varepsilon} Y_{1,t} \tag{3.14}$$

From the first-order condition we would obtain the following log-linear equation:

$$\rho_{1,t} = \widehat{w}_{1,t} - \widehat{A}_{1,t} \text{ where } \rho_{1,t} = \widehat{P}_{1,t} - \widehat{P}_t \tag{3.15}$$

$\rho_{1,t}$ represents the relative price in sector 1.

Sticky Price firms

The firms in sector 2 have a representative sticky-price firm, where the prices are based on the specific model of price stickiness used by Calvo (1983), where a fraction of firms re-optimize their nominal price with fixed probability $1-\theta_2$, while with probability θ_2 it maintains the price charged in the previous period. The parameter θ_2 measures the degree of nominal rigidity; a higher θ_2 means that fewer firms re-optimize their price each period and a longer time is needed for the price changes to happen.

The problem of firm i, which sets its price at time t, is to choose $P^*_{2,t}(i)$ to maximize expected profits:

$$\max_{P^*_{2,t}(i)} \mathbb{E}_t \sum_{j=0}^{\infty} \theta_2^j D_{t,t+j} \left[P^*_{2,t}(i) Y_{2,t+j} - W_{2,t+j} \frac{Y_{2,t+j}}{A_{2,t+j}} \right] \text{ s.t the demand constraint} \tag{3.16}$$

$$Y_{2,t+j}(i) = \left(\frac{P^*_{2,t}(i)}{P_{2,t+j}} \right)^{-\varepsilon} b \left(\frac{P_{2,t+j}}{P_{t+j}} \right)^{-\eta} Y_t$$

where $D_{t,t+j} \equiv \beta^j \frac{\lambda_{t+j}}{\lambda_0} = \beta^j \frac{u'(Y_{t+j})}{u'(Y_t)} \frac{P_t}{P_{t+j}} = \beta \frac{Y_{t+1}^{-\sigma}}{Y_t^{-\sigma}} \left(\frac{P_t}{P_{t+1}} \right)$ is the discount factor for the nominal payoffs and λ_{t+j}, the marginal utility of consumption. The firm's first order condition by re-arranging would yield:

$$P^*_{2,t}(i) = \frac{\varepsilon}{(\varepsilon - 1)} \frac{\psi_{2,t}}{\phi_{2,t}} \tag{3.17}$$

The auxiliary variables $\psi_{2,t}$ and $\phi_{2,t}$ can be written recursively following as:

$$\psi_{2,t} = \alpha_2 MC_{2,t} Y_t^{-\sigma} + \theta_2 \beta \mathbb{E}_t \left[\pi_{2,t+1}^{\varepsilon-\eta} \pi_{t+1}^{\eta} \psi_{2,t+1} \right] \tag{3.18}$$

and

$$\phi_{2,t} = \alpha_2 Y_t^{-\sigma} + \theta_2 \beta \mathbb{E}_t \left[\pi_{2,t+1}^{\varepsilon-\eta} \pi_{t+1}^{\eta-1} \phi_{2,t+1} \right] \tag{3.19}$$

They depend both on output and future expectations of inflation. $\psi_{2,t}$ can be interpreted as the present discounted value of the marginal costs when the optimal reset price changes, while $\phi_{2,t}$ can be considered as the marginal revenues. ε and $\varepsilon - 1$ are respectively treated as the weights of the marginal costs and marginal revenues on resetting the optimal price in equation 3.17. The real marginal costs for the sticky price firms are expressed log$-$linearized (similar as in Carlstrom et al. (2006)):

$$\widehat{MC}_{2,t} = \widehat{w}_{2,t} - \widehat{P}_t + \widehat{P}_{2,t} - \widehat{A}_{2,t} \text{ and } \rho_{2,t} = \widehat{P}_{2,t} - \widehat{P}_t \tag{3.20}$$

where $\rho_{2,t}$ represents the relative price in sector 2, while $\widehat{w}_{2,t}$ is the same definition as in equation 3.9.

3.2.4 Monetary authority

Regarding the monetary policy, the economy has a central bank that follows a conventional Taylor rule, with weight ϕ_π on deviations of inflation from target $\overline{\pi}$ and weight ϕ_y on output deviations from steady state output. Y:

$$\left(\frac{1+i_t}{1+\bar{i}}\right) = \left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\rho_i} \left(\left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\overline{Y}}\right)^{\phi_Y}\right)^{1-\rho_i} e^{m_t} \tag{3.21}$$

where m_t is a monetary policy shock which is *iid* with zero mean and a constant variance, and ϕ_π , $\overline{\pi}$ and ϕ_y are non-negative parameters. Some inertia is also added to the Taylor rule. $\bar{i}$ is the steady-state interest rate and ρ_i is the inertia parameter.

3.2.5 Inflation and relative prices

In this subsection, I discuss how the heterogeneity in price rigidities affects price dynamics and the representative Philips curve for each of the sectors. In this subsection I will work for simplicity with log−linear forms of the main equations. From the equation 3.4, the aggregate equation inflation would be:

$$\widehat{\pi}_t = \alpha_1 \widehat{\pi}_{1,t} + \alpha_2 \widehat{\pi}_{2,t} \tag{3.22}$$

Keeping in mind the previous definitions that $\rho_{1,t} = \widehat{P}_{1,t} - \widehat{P}_t$ and $\rho_{2,t} = \widehat{P}_{2,t} - \widehat{P}_t$, I can write the equations that relate sectoral inflations[8] to aggregate inflation as in Cooke & Kara (2019) and Carvalho et al. (2019):

$$\widehat{\rho}_{1,t} = \widehat{\rho}_{1,t-1} + \widehat{\pi}_{1,t} - \widehat{\pi}_t \text{ and } \widehat{\rho}_{2,t} = \widehat{\rho}_{2,t-1} + \widehat{\pi}_{2,t} - \widehat{\pi}_t \tag{3.23}$$

I substitute the optimal price setting in sector 1[9], $\rho_{1,t} = \widehat{w}_{1,t} - \widehat{A}_{1,t}$, in the equation 3.23 for the sector 1, just to get the expression for inflation in sector 1:

$$\widehat{\pi}_{1,t} = \widehat{w}_{1,t} - \widehat{w}_{1,t-1} - (\widehat{A}_{1,t} - \widehat{A}_{1,t-1}) + \widehat{\pi}_t \tag{3.24}$$

Since prices adjust every period in this sector, inflation in sector 1, does not have a standard representation of the Philips curve. In sector 2, where the firms have Calvo sticky prices, the Philips curve would be:

$$\widehat{\pi}_{2,t} = k_2 \left[\widehat{w}_{2,t} - \widehat{A}_{2,t} + \widehat{\rho}_{2,t}\right] + (1-\theta_2)\beta\widehat{\pi}_{t+1} + \beta\theta_2\widehat{\pi}_{2,t+1} \tag{3.25}$$

where $k_2 \equiv \frac{(1-\theta_2)(1-\theta_2\beta)}{\theta_2}$. Equation (24) is a representation of the standard New Keynesian Philips curve formulated at sectoral level, where now three terms, one depending on marginal

[8]The implications for this as argued by Cooke & Kara (2019), is that in the flexible sector, inflation $\widehat{\pi}_{1,t}$ depends negatively on the relative sector−specific price in the previous period, such that, if the relative price were high, flexible−price firms would reduce their prices, and this would reduce sector−level inflation.

[9]This reflects the fact that changes in the price are equal to changes in the marginal cost of production.

costs, a term which depends from sectoral future inflation expectations $\widehat{\pi}_{2,t+1}$ and one depending on future total inflation expectations $\widehat{\pi}_{t+1}$. The full log−linear equations of the model, including the Philips curve are presented in Appendix B.2.

3.3 Model analytics

In this section, I make a set of simplifying assumptions to obtain some analytical results. This allows me to characterize the dynamics of the model for different values of the intratemporal elasticity of substitution between sectors (η) and the labor mobility (λ). In the first subsection, I will show the implications that a monetary shock has on aggregate output. In the second subsection, I will concentrate on a TFP shock and output dynamics subject to the mentioned changes. Additional details of the proofs will be presented in Appendix B.3.

3.3.1 Monetary Policy Shock

Initially in this subsection[10] I focus on showing analytically the effects that monetary shocks have on output depending on the intratemporal elasticity of substitution between sectors (η). Secondly, I will focus on the labor mobility (λ) parameter implications. Further, I will discuss and derive conclusions from the obtained analytics. The assumptions that will be made in this subsection are as follows: I assume an infinite Frisch elasticity of labor supply ($\varphi = 0$), $\sigma = 1$, $\phi_y = 0$, $\rho_i = 0$, $\beta = 0$. Whilst the additional restrictions mean that I can not calibrate the model, they do not change the main point of the analysis. Additionally, dropping the technology variable ($Y_{1,t} = L_{1,t}$ and $Y_{2,t} = L_{2,t}$), would result in $\alpha_1 = b_1$ and $\alpha_2 = b_2$. Now, focusing only on the intratemporal elasticity of substitution between sectors (η) change would result in the real wage equations being in the following form:

$$\widehat{w}_{1,t} = (1-\frac{1}{\eta})\widehat{Y}_t + \frac{1}{\eta}\widehat{Y}_{1,t} \text{ and } \widehat{w}_{2,t} = (1-\frac{1}{\eta})\widehat{Y}_t + \frac{1}{\eta}\widehat{Y}_{2,t} \tag{3.26}$$

Further substituting the interest rate from the Taylor rule into the Euler Equation would allow the following representation:

$$\widehat{Y}_t = (\widehat{Y}_{t+1} + \widehat{\pi}_{t+1}) - \phi_\pi \widehat{\pi}_t - \widehat{m}_t \tag{3.27}$$

The equations 3.24 and 3.25 now would be simplified into:

$$\widehat{\pi}_{1,t} = \widehat{w}_{1,t} - \widehat{w}_{1,t-1} + \widehat{\pi}_t \text{ and } \widehat{\pi}_{2,t} = k_2\,[\widehat{w}_{2,t} + \widehat{\rho}_{2,t}] \tag{3.28}$$

[10] In this section I will work with log−linearized versions of the model explained in the previous sections.

As first step, I substitute $\widehat{w}_{2,t}$ from equation (25), into $\widehat{\pi}_{2,t}$, and $\widehat{w}_{1,t}$ into $\widehat{\pi}_{1,t}$, to have respectively $\pi_{2,t} = \left(1-\frac{1}{\eta}\right) k_2 \widehat{Y}_t + \left(\frac{1}{\eta}\right) k_2 Y_{2,t} + k_2 \rho_{2,t}$ and $\widehat{\pi}_{1,t} = \left(1-\frac{1}{\eta}\right) \widehat{Y}_t + \left(\frac{1}{\eta}\right) Y_{1,t} - \left(1-\frac{1}{\eta}\right) \widehat{Y}_{t-1} + \left(\frac{1}{\eta}\right) Y_{1,t-1} + \widehat{\pi}_t$. In the equation for $\widehat{\pi}_{2,t}$ I use the information that $\alpha_1 \widehat{\rho}_{1,t} + \alpha_2 \widehat{\rho}_{2,t} = 0$ to get rid of $\widehat{\rho}_{2,t}$ by expressing it in terms of $\widehat{\rho}_{1,t} = \widehat{w}_{1,t}$. Inserting the newly obtained expressions for $\widehat{\pi}_{1,t}$ and $\widehat{\pi}_{1,t}$ into the aggregate inflation equation $\widehat{\pi}_t = \alpha_1 \widehat{\pi}_{1,t} + \alpha_2 \widehat{\pi}_{2,t}$, I can obtain an expression for $\widehat{\pi}_t$ depending on output, both present and lagged, and sectoral outputs.

$$\begin{aligned}\widehat{\pi}_t &= \left(\frac{\alpha_1}{\alpha_2} + k_2 - \frac{\alpha_1}{\alpha_2} k_2\right)\left(1-\frac{1}{\eta}\right)\widehat{Y}_t - \frac{\alpha_1}{\alpha_2}\left(1-\frac{1}{\eta}\right)\widehat{Y}_{t-1} \\ &\quad + (1-k_2)\frac{\alpha_1}{\alpha_2}\left(\frac{1}{\eta}\right)\widehat{Y}_{1,t} + \frac{\alpha_1}{\alpha_2}\left(\frac{1}{\eta}\right)\widehat{Y}_{1,t-1} + \left(\frac{1}{\eta}\right) k_2 \widehat{Y}_{2,t} \end{aligned} \tag{3.29}$$

Now, combining the Philips curve for this economy, the Euler Equation, the Taylor rule and the monetary policy shock process, I express output in terms of its own lag, leads (as well for the flexible flexible sector) and the monetary shock. So, in the following equation $\widehat{Y}_t - \widehat{Y}_{t+1} + \phi_\pi \widehat{\pi}_t + \widehat{m}_t - \widehat{\pi}_{t+1} = 0$ derived from equation 3.27, I substitute $\widehat{\pi}_t$ and $\widehat{\pi}_{t+1}$ (obtained by moving the equation 3.29 one period forward). I simplify the obtained equation, using the information from the aggregate output equation $\widehat{Y}_t = \alpha_1 \widehat{Y}_{1,t} + \alpha_2 \widehat{Y}_{2,t}$, that $\widehat{Y}_{2,t}$ can be expressed in terms of $\widehat{Y}_{1,t}$. This substitution would allow having a variable less in the final equation to worry about solving. Now output can be written as the following:

$$\widehat{Y}_t = \tau_1^m \widehat{Y}_{t-1} + \tau_2^m \widehat{Y}_{t+1} + \tau_3^m \widehat{m}_t + \tau_4^m \widehat{Y}_{1,t+1} + \tau_5^m \widehat{Y}_{1,t} + \tau_6^m \widehat{Y}_{1,t-1} \tag{3.30}$$

where τ_i^m for $i = 1,2,3,4,5,6$[11] are elasticity parameters that express output dependent on lagged output, future output, the monetary policy shock, future output in the flexible sector, current output in the flexible sector and its lagged value.

To show the implications of the intratemporal elasticity of substitution between sectors (η) on the dynamics of output subject to monetary policy shocks, I solve equation 3.30, relying on the method of undetermined coefficients. This method involves guessing the general functional form of the solution first and then solving the coefficient equations. I guess that $\widehat{Y}_t$ is a linear function of $\widehat{Y}_{t-1}$, $\widehat{Y}_{1,t-1}$, and $\widehat{m}_t$, to have:

$$\widehat{Y}_t = \chi_y \widehat{Y}_{t-1} + \chi_1 \widehat{Y}_{1,t-1} + \chi_m \widehat{m}_t. \tag{3.31}$$

[11] The detailed form of these parameters can be found in Appendix B.3.1. where I have as well assumed that the sectoral shares are equal $\alpha_1 = \alpha_2$.

Where χ_y, χ_1 and χ_m are respectively the effects of $\widehat{Y}_{t-1}$, $\widehat{Y}_{1,t-1}$, and $\widehat{m}_t$, on $\widehat{Y}_t$. Using as well the definitions that $\mathbb{E}_t\widehat{m}_{t+1} = \sigma_m\widehat{m}_t =$ and $\mathbb{E}_t\widehat{Y}_{1,t+1} = \sigma_{y_1}\widehat{Y}_{1,t}$, the coefficients on Equation 3.31 are given by the following system of equations, $\tau_2^m\chi_y^2 - \chi_y + \tau_1^m = 0$, and $\chi_m = -\frac{\tau_3^m}{\left(\tau_1^m\chi_y^{-1}+\tau_2^m\chi_y+\tau_2^m\sigma_m-1\right)}$ and $\chi_1 = \frac{\left(\tau_4^m\sigma_{y,1}^2+\tau_5^m\sigma_{y,1}+\tau_6^m\right)}{\left(1-\tau_1^m\chi_y^{-1}-\tau_2^m\chi_y-\tau_2^m\sigma_{y,1}^2\right)}$. The solution for χ_y, considering that we are dealing with quadratic equation, would be: $\chi_y = \frac{1\pm\sqrt{1-4*\tau_2^m*\tau_1^m}}{2*\tau_2^m}$, with the condition that $1-4*\tau_2^m*\tau_1^m \geq 0$. Finding χ_y, allows further solving easily for χ_1 and χ_m.

Proposition 1. An increase in the intratemporal elasticity of substitution between sectors ($\lim_{\eta\longrightarrow\infty}\frac{1}{\eta} = 0$) increases the impact of output (χ_m) to a negative monetary policy shock[12].

Proof. See Appendix B.3.1.

The main intuition to be emphasized here is that while the intratemporal elasticity of substitution between sectors though it does not seem to matter under the multi−sector model with the same price stickiness, it brings important results under high price heterogeneity. In the two-sector Calvo model (same price stickiness), under a monetary policy shock[13], firms in both sectors change prices at the same time, therefore having the same rate of accommodation toward the shock. As the response to the shock is the same through firms in both sectors, a higher intratemporal elasticity of substitution between sectors would not cause any shifts in output from one sector to the other.

The intratemporal elasticity of substitution between sectors appears to matter for output dynamics under a negative monetary policy shock when price heterogeneity is high (one sector has firms that have flexible prices, and the other sector has a firm that faces standard Calvo prices). What happens, in this case, is that under a negative monetary policy shock, flexible price firms change prices faster to accommodate the shock, while the sticky price firms require some time to do so. Under a high intratemporal elasticity of substitution between sectors, because sticky price sector firms require more time to accommodate prices, they would be selling with lower prices, and as consequence, their output demand would increase, while output demand of the flexible firms would be lower than before the shock as they managed to change their prices sooner than sticky-price sector firms. Now, the prices that the flexible price firm operate are high, suggesting a further reduction in the production of the good $\widehat{Y}_{1,t}$, and because of the high intratemporal elasticity of substitution between sectors, they would shift producing more $\widehat{Y}_{2,t}$ because of higher demand for it. When the

[12] The results are symmetric as well in the case of a positive monetary policy shock.

[13] Under a negative monetary policy shock in the standard one-sector model (see Gali (2008) textbook for the case of a positive monetary policy shock), the shock generates a decrease in the real interest rate and an increase in inflation and output.

increase in the output in the sticky sector is high enough to overcome the decline (or being reduced) of output in the flexible sector for good $\widehat{Y}_{1,t}$, total output increases, and vice−versa.

Proposition 2. An increase in the labor mobility between sectors ($\lim_{\lambda \longrightarrow \infty} \frac{1}{\lambda} = 0$) increases the response of output to a negative monetary policy shock[14], same as in the case of the impact in the intratemporal elasticity of substitution between sectors.

Proof. See Appendix B.3.1.

Modifying the labor supply equations in both sectors in 3.8 and 3.9, to concentrate only on the lambda parameter, I would have similar expression as in equation 3.26, $\widehat{w}_{1,t} = (1 - \frac{1}{\lambda})\widehat{Y}_t + \frac{1}{\lambda}\widehat{Y}_{1,t}$ and $\widehat{w}_{2,t} = (1 - \frac{1}{\lambda})\widehat{Y}_t + \frac{1}{\lambda}\widehat{Y}_{2,t}$ respectively. The rest of the analytics is same as before from equation 3.26−3.31.

To answer how the labor mobility mechanism works for output dynamics, under a monetary policy shock is important to look at the role of price heterogeneity. As previously mentioned, a negative monetary policy shock generates a decrease in the real interest rate, and an increase in inflation, and an increase in output in both sectors. When firms in both sectors face the same degree of price stickiness they reaction toward the shock will be on the same pace, and having high labor mobility or not, does not alter the dynamics of output. Under the same price stickiness, the dynamics of total output would be the same as the one in the one sector Calvo model.

When we deal with an economy with price heterogeneity across sectors, flexible price firms change prices faster (increase them) to accommodate the shock to lower the costs they face, while the sticky sector firms require some time to do so. Because sticky price sector firms require more time to accommodate prices, they would be selling with lower prices, and as consequence, their output demand would increase. The increase in output in the sticky sector increases the demand for labor, which as consequence raises marginal costs and decreases the marginal product of labor. With higher sectoral labor mobility, labor can shift to the sticky sector causing employment to increase in the sticky sector. The increase in marginal costs acts as an adverse supply shock for the producers in the flexible sector leading to less labor employed and less goods being produced in the flexibly sector[15]. This shift in labor would cause an increase in output in sector 2 and a decline in output in sector 1, depending on the size of labor mobility. These differences persist as long as the sticky price firms are able to adjust prices. Again, as in Proposition 1., when the increase in the output in

[14]The results are symmetric as well in the case of a positive monetary policy shock.

[15]The explanation for this case is similar to the case of a monetary expansion shock in Kim & Katayama (2013) where they look sectors with durable and nondurable goods, where one sector has flexible prices and the other one faces sticky prices under the conditions of perfect labor mobility.

the sticky sector for good $\widehat{Y}_{2,t}$ is high enough to overcome the decline (or smaller increase) of output in the flexible sector for good $\widehat{Y}_{1,t}$, total output increases, and vice−versa.

From Proposition 1., and Proposition 2., it can be derived that the interaction between the labor mobility parameter λ and the intratemporal elasticity of substitution between sectors η when the model takes into account for both being present, can alternate the results mentioned in the previous propositions. As both the parameters λ and η, appear in the same form in the solution that takes into account both[16] as $\frac{1}{\eta}+\frac{1}{\lambda}$, they can be interpreted as a single parameter τ^m. A change in the same amount of both λ and η but in opposite direction, would suggest that the effect of a change in labor mobility λ (or the intratemporal elasticity of substitution between sectors η), would result in a net zero effect on τ^m, $\triangle\tau^m = \triangle\left(\frac{1}{\lambda}\right)+\triangle\left(\frac{1}{\eta}\right)$.

Further, when looking at the analytics regarding output response to a monetary policy shock, it is important to investigate as well the shape of this response. This comes important, as while empirical studies show that monetary policy shocks suggest a hump-shaped response of output, this is not the case in standard New Keynesian DSGE models (see Gali (2008) and Christiano et al. (2005)). The equation 3.31, shows that output dynamics depends from the coefficients χ_y, χ_1 and χ_m, and a hump−shaped response of output to a monetary shock requires that $\widehat{Y}_{t+1} > \widehat{Y}_t$. Moving equation 3.31 one period forward, we would have $\widehat{Y}_{t+1} = \chi_y\widehat{Y}_t + \chi_1\widehat{Y}_{1,t} + \chi_m\widehat{m}_{t+1}$, while the total contributions again would be $\widehat{Y}_{t+1}$ is $1 = \chi_y + \chi_1 + \chi_m$. The hump−shaped response of output requires $\chi_y > 1-\chi_1-\chi_m$, as it ensures that the backward−looking behaviour in the price setting is strong enough to generate the hump. As shown in Appendix B.3.1., for Proposition 1., since χ_1 goes to 0, the condition is reduced to:

$$\chi_y > 1-\chi_m \tag{3.32}$$

From equation 3.31, we have $\widehat{Y}_t = \chi_m\widehat{m}_t$ and the monetary shock motion where $\zeta_{m,t+1} = 0$, $\mathbb{E}_t m_{t+1} = \sigma_m m_t$. The hump−shaped response of output condition 3.31, now transforms into $\chi_y > 1-\rho_m$ as shown in Kara & Park (2017). Under this condition, for a level of shock persistence, an increase in the η or λ can replace the need for a higher share of the flexible sector to achieve a hump shaped response of output to a monetary policy shock.

3.3.2 TFP Shock

In this subsection in a similar way as in the previous subsection I study the effects of a technology shock on output subject to λ and η changes. At first, I bring back the technol-

[16] Equations 3.8 and 3.9 by applying the simplifications mentioned at the beginning of this section, would be as following when considering both parameters η and λ, $\widehat{w}_{1,t} = (-\frac{1}{\lambda}-\frac{1}{\eta})\widehat{Y}_t + (\frac{1}{\lambda}+\frac{1}{\eta})\widehat{Y}_{1,t}$ and $\widehat{w}_{2,t} = (1-\frac{1}{\eta})\widehat{Y}_t + \frac{1}{\eta}\widehat{Y}_{2,t}$.

ogy variable, having now $Y_{1,t} = \widehat{A}_t L_{1,t}$ and $Y_{2,t} = \widehat{A}_t L_{2,t}$. From equation 3.24 and 3.25, I replace the sectoral technology shock with the TFP shock to have the new equations, $\widehat{\pi}_{1,t} = \widehat{w}_{1,t} - \widehat{w}_{1,t-1} - (\widehat{A}_t - \widehat{A}_{t-1}) + \widehat{\pi}_t$ and $\widehat{\pi}_{2,t} = k_2 \left[\widehat{w}_{2,t} - \widehat{A}_t + \widehat{\rho}_{2,t}\right] + (1-\theta_2)\beta\widehat{\pi}_{t+1} + \beta\theta_2\widehat{\pi}_{2,t+1}$. Following same steps as before, in the analytics for the monetary policy shock, I can write $\widehat{Y}_t$ as the following[17]:

$$\widehat{Y}_t = \tau_1^A \widehat{Y}_{t-1} + \tau_2^A \widehat{Y}_{t+1} + \tau_3^A \widehat{A}_t + \tau_4^A \widehat{Y}_{1,t} + \tau_5^A \widehat{Y}_{1,t-1} + \tau_6^A \widehat{Y}_{1,t+1} \tag{3.33}$$

where τ_i^A for $i = 1,2,3,4,5,6$[18] are elasticity parameters that express output dependent of lagged output, future output, the technology shock, current output in the flexible sector, its lagged value and the future output in the flexible sector. To show the implications of the labor mobility parameter, I solve equation 3.34, and I guess that $\widehat{Y}_t$ is a linear function of $\widehat{Y}_{t-1}$, $\widehat{Y}_{1,t-1}$, and $\widehat{A}_t$, to have:

$$\widehat{Y}_t = \chi_y \widehat{Y}_{t-1} + \chi_1 \widehat{Y}_{1,t-1} + \chi_A \widehat{A}_t. \tag{3.34}$$

Using as well the definitions that $\mathbb{E}_t \widehat{A}_{t+1} = \sigma_m \widehat{A}_t =$ and $\mathbb{E}_t \widehat{Y}_{1,t+1} = \sigma_{y_1} \widehat{Y}_{1,t}$, the coefficients on Equation 3.34 are given by the same system of equations as in the case of the monetary policy shock, but with different parameter definitions.

Proposition 3. An increase in the labor mobility (or the intratemporal elasticity of substitution between sectors) λ (or η) parameter increases the response of output to a technology shock[19].

Proof. See Appendix B.3.2

Here, under a technology shock[20], output increases and prices go down. Now, with a better technology, more can be produced with given labor input. Employment (hours) goes down, while labor demand and wages go up. Again as in the case of monetary shocks, the importance of labor mobility is determined by heterogeneity in price stickiness. Assuming at first that we face two sectors where firms in both sectors would have the same price stickiness a la Calvo (1983). In this case, the response of the firms in both sectors would not be sensitive

[17] Equation 3.33 is the case where I focus on the labor mobility parameter, and $\widehat{L}_t = b_1\left(\widehat{Y}_{1,t} - \widehat{A}_t\right) + b_2\left(\widehat{Y}_{2,t} - \widehat{A}_t\right)$, and labor supply would be respectively: $\widehat{w}_{1,t} = b_2 \frac{1}{\lambda}\widehat{Y}_{1,t} - \frac{1}{\lambda} b_2 \widehat{Y}_{2,t} + \widehat{Y}_t$ and $\widehat{w}_{2,t} = b_1 \frac{1}{\lambda}\widehat{Y}_{2,t} - \frac{1}{\lambda} b_1 \widehat{Y}_{1,t} + \widehat{Y}_t$. While for the case where i focus on η only the labor supply equations would be as in Equation 3.26.

[18] The detailed form of these parameters that are different from the ones in the monetary shock case, can be found in Appendix B.3.2, where I have as well assumed that the sectoral shares are equal $\alpha_1 = \alpha_2$.

[19] The results are symmetric as well in the case of a negative technology shock.

[20] For more regarding the effects of technology shocks on the economy, see Gali (2004) and Canova et al. (2010).

to changes in labor mobility, as there are no differences in prices between them, and as consequence, the adjustment in prices because of the shock would happen at the same period in time. The output response to a technology shock under the same price stickiness would be the same as in the case of the standard one-sector New Keynesian DSGE model.

If one considers the case where firms face heterogeneity in price stickiness, then, labor mobility becomes relevant. Under no labor mobility, these firms would realize that they would face price changes due to the technology shock and they would adjust to it, but without consequences in total output since labor is immobile between sectors. If labor would be mobile, flexible price firms would adjust prices immediately. Lower prices in the flexible sector price firms would result in higher real wages in this sector, making it more attractive for labor, who now can shift between sectors. Sticky price firms require more time to make adjustments in prices due to the shock. More labor in sector 1, means that now it can be produced more of good $\widehat{Y}_{1,t}$ under the current technology shock, and less of good $\widehat{Y}_{2,t}$, due to the loss of labor to sector 1. In order for the total output to increase to the technology shock, it needs that the extra increase in output due to the high labor mobility in sector 1, to be bigger than the change in sector 2 goods.

Following this logic of flexible and sticky firms response, the effects of a higher intratemporal elasticity of substitution between sectors will influence total output under the same intuition as in the case of the monetary policy shocks.

3.4 Impulse responses analysis

In section 3.3, I analytically demonstrated how the labor mobility parameter and the intratemporal elasticity of substitution between sectors matters and have implications for the dynamics of output to monetary and technology shocks. In this section, I will investigate further these results relying on some exercises with impulse response functions. In the first subsection, I will discuss the choice of parameters, while in the second subsection I will concentrate on output dynamics to a monetary policy shock, and in the third, I will focus on a TFP shock. In both subsections 2 and 3, first I report the IRFs from the two$-$sector Calvo baseline, and at a second step, I allow the variability in the parameters λ and η and discuss the results.

3.4.1 Parameterization

The baseline parameterization of the model presented in table 1, is as follows: I set $\beta = 0.995$, implying an annualized real interest rate of 2% in the steady-state. I keep a value of $\sigma = 1$

for the inverse elasticity of intertemporal substitution of consumption, which implies that the preferences are separable in leisure and consumption. Regarding the (inverse) Frisch elasticity of labor supply, I assume a value of unity[21], $\varphi = 1$, as in Christiano et al. (2005). Turning to the specifications for monetary policy, I set $\phi_\pi = 1.5$, $\phi_Y = 0.2$, which are standard values for Taylor (1993*b*) rules, following Clarida et al. (2000). On the production side, I follow Carvalho & Nechio (2011) and set $\varepsilon = 10$, implying a steady−state markup of 11 percent, while for the Calvo parameter of price stickiness $\theta_2 = 0.75$. I set the share of good 1 in the consumption basket $\alpha_1 = 0.5$, while the share of labor used in sector 1 $b_1 = 0.6$[22]. I consider the same persistence in the shocks $\rho_v = 0.9$ (where $\rho_i = 0$) and $\rho_A = 0.9$.

Table 3.1 Parametrization

β	Discount factor	0.995	Galesi & Rachedi (2019)
σ	inverse elasticity of intertemporal substitution	1	standard
η	intratemporal elasticity of substitution between sectors baseline value	1	Carvalho et al. (2019)
λ	labor mobility parameter baseline value	1	Petrella & Santoro (2011)
φ	inverse Frisch labour supply elasticity	1	Christiano et al. (2005) (Domeij & Floden (2006))
ε_1	elasticity of substitution between goods in sector 1	10	Carvalho & Nechio (2011)
ε_2	elasticity of substitution between goods in sector 2	10	
θ_2	Calvo parameter in sector 2	0.75	standard
α_1	share of good 1 in the consumption basket	0.5	
b_1	share of labor in sector 1	0.6	
ρ_v	AR(1) coefficient of monetary policy	0.9	standard
ρ_A	AR(1) coefficient of TFP shock	0.9	standard
ϕ_π	inflation elasticity of the nominal interest rate	1.5	standard
ϕ_Y	output gap elasticity of the nominal interest rate	0.2	standard
ρ_i	inertia parameter with past interest rate	0	

The empirical evidence and the theoretical papers so far have not been settled regarding the parameter values to use regarding the intratemporal elasticity of substitution between sectors η and the labor mobility λ. Studies such as Bouakez et al. (2009), Monacelli (2009), Carvalho & Nechio (2011) and Carvalho et al. (2019) chose a value for the intratemporal elasticity of substitution between sectors of $\eta = 1$, while Airaudo & Zanna (2012) use values such as 0.5, 1 and 1.5. Low values are used as well from Herrendorf et al. (2013), Atalay (2017), Galesi & Rachedi (2019), Pasten et al. (2018) and Duarte & Restuccia (2010) respectively 0.002, 0.15, 0.4, 0.5, and 0.8. Higher values are used by Carvalho & Lee (2019), 2, while Hobijn & Nechio (2018) estimates suggest values between 1 and 3. Petrella et al. (2019) assumes sectoral elasticities of substitution at a steady state value of 11. Other part of the literature (Carvalho (2006) and Kara (2015)) usually assumes for simplicity that $\eta = \varepsilon$, which in the case of this paper would be 10. Regarding the values for labor mobility between

[21] A value of 3 following the evidence of Domeij & Floden (2006), is applied in the case of the technology shock.

[22] In the case of the monetary policy shock since I assume $\widehat{Y}_{1,t} = \widehat{L}_{1,t}$ and $\widehat{Y}_{2,t} = \widehat{L}_{2,t}$, therefore $\alpha_1 = b_1$.

sectors, most multi sector New Keynesian DSGE models either treat labor as sector specific, so immobility (Erceg & Levin (2006), Carlstrom et al. (2006), Airaudo & Zanna (2012) or they assume labor is perfectly mobile across sectors (Aoki (2001), Barsky et al. (2007), Monacelli (2009), Sterk (2010), Sudo (2012), Kim & Katayama (2013) and Kimball et al. (2016). Other papers consider limited labor mobility, with $\lambda = 1$ such as Petrella & Santoro (2011), Bouakez et al. (2009) and Petrella et al. (2019). Bouakez et al. (2011) explore values between 0.5 and 1.5, while Iacoviello & Neri (2010*b*) and later Cantelmo & Melina (2018) estimate values of 1.51 and 1.03 for savers and borrowers, respectively.

In this section, I chose to simulate impulse responses in both of the shocks with 4 similar values of η and λ, which are good representatives of the values of the studies mentioned before. Initially, I chose a value of η and λ of 0.002[23], then a value of 1[24], 3[25], and at last a value of 10[26]. These identical values for η and λ are chosen to study further the similar implication driven in both by the analytical results in section 3.3. In all the figures while either η or λ varies, a value of 1 will be used as a baseline for the counterpart.

3.4.2 Impulse responses functions to a Monetary Policy shock

Given this baseline parameterization, Figure 3.1 reports the IRFs for a 1% Monetary Policy shock to output in a two−sector NK model considering different values of λ and η. On the left side, I present the IRFs for 4 different values of λ, respectively for values[27] of 0.002, 1, 3 and 10, as explained in the parameters subsection, keeping as baseline $\eta = 1$. While on the right side of the panel are shown the IRFs for 4 different values of η, respectively for values of 0.002, 1, 3, and 10, keeping as baseline $\lambda = 1$. To show the clear differences with the standard Calvo model (either one sector or multi-sector economy) and compare it with this paper's results, I add a line in magenta color in both graphs showing total output.

The impulse responses exercise shown in Figure 3.1, suggest that higher values of λ and higher values of η as we move from 0.002 toward 10, increase the response of output in the sticky sector (second-row graphs) to a monetary policy shock, while it decreases the response of output in the flexible sector (third-row graphs). Under a value of $\eta = 10$ the response of output in the sector with flexible prices becomes negative (third row, right-hand

[23]Which is a value almost zero which corresponds to Herrendorf et al. (2013) value of η, and its close to zero to capture labor immobility.

[24]This value is in line with Petrella & Santoro (2011), Bouakez et al. (2009) and Petrella et al. (2019) for λ, and Bouakez et al. (2009), Monacelli (2009), Carvalho & Nechio (2011) and Carvalho et al. (2019) for η.

[25]A value in line with Hobijn & Nechio (2018) max value for η, and express some limited level of labor mobility.

[26]Which is line with $\eta = \varepsilon$ assumption and close to the value used by Petrella et al. (2019), while for λ is in line with the studies that assume high labor mobility.

[27]With corresponding colors as black, blue, red and green.

side), while under a value of $\lambda = 10$ it becomes almost zero (third row, left-hand side). The size of the effect on the sticky and flexible sector respectively in both cases for λ and for η will determine the impulse shape of total output shown in the first row.

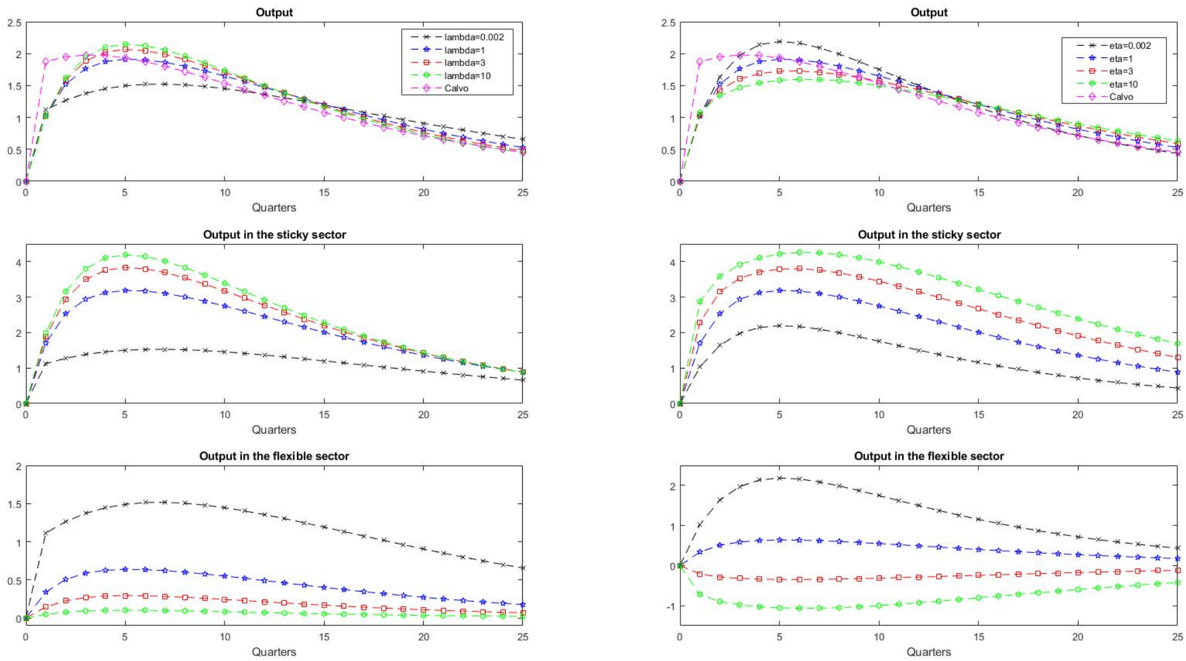

Figure 3.1 IRFs to monetary policy shock

Notes: The figure presents the impulse responses of Output to a 1% Monetary Policy Shock in the model subject to the changes in the parameters of λ (left column) and η (right column). In the first row are presented the results of total output, in the second row, output in the sticky sector, and in the third row, output in the flexible sector. The four lines represent respectively the impulse responses under a level of 0.002, 1, 3, and 10 of λ (η), with corresponding colors as black, blue, red, and green. The magenta color represents the case of the Calvo model

On the first row of the graph on the right-hand side, with a higher value of η, as it moves from 0.002 to 10, decreases the response of output to the monetary policy shock in the first periods, and decreases it after the fourth period. As η moves from 0.002 to 10, the bigger is the hump-shaped response of total output to the monetary policy shock. After the twelfth period, as the sticky sector firms start to accommodate prices more, the intratemporal elasticity of substitution between sectors starts to become less relevant in the differences created between sectors, the increase in the output in the sticky sector is not high enough to compensate the decline of output in the flexible sector, therefore as η goes down, the output will be slightly lower than before.

On the first row, on the left-hand side, with a higher value of λ, as it moves from 0.002 to 10 increases the response of output to the monetary policy shock in the first periods, and decreases it after the twelfth period. As λ moves from 10 to 0.002, the bigger and bigger is the hump-shaped response of total output to the monetary policy shock. After the twelfth period, as the sticky sector firms start to accommodate prices more and more, the increase in the output in the sticky sector is high enough to compensate for the smaller increase in the output in the flexible sector, therefore as λ goes down, the output will be slightly higher than before. Comparing the results about total output with the standard Calvo model, some differences can be noticed: First, the IRFs under the two−sector model with price heterogeneity are lower in size (on average) than in the Calvo model, and second, the IRFs under price heterogeneity can be altered under different values of λ and η, by improving or not the hump−shaped response of output to the monetary shock. In Appendix B.1.1[28], I conduct the same type of exercise in a model with GHH[29] preferences, where the same type of results as in Figure 3.1 are confirmed.

3.4.3 Impulse responses functions to a TFP shock

In Figure 3.2, I report the IRFs for a 1% TFP Policy shock to output in a two−sector NK mode considering different values of λ and η. The figure description is the same as in Figure 3.1.

On the left side, I present the IRFs for 4 different values of λ, keeping as a baseline a value of $\eta = 10$[30], while on the right side of the panel are shown the IRFs for 4 different values of η, keeping $\lambda = 10$ as a baseline, respectively for the values of 0.002, 1, 3, and 10, keeping as baseline $\eta = 10$. As in Figure 3.1, I add the line in magenta color in both graphs, in the first row, showing total output response to the TFP shock in the standard Calvo model (either one sector or multi-sector economy with same price stickiness). The impulse responses exercise shown in Figure 3.2, suggest that higher values of λ and higher values of η as we move from 0.002 toward 10, increase the response of output in the flexible sector (third-row graphs) to a TFP shock, while it decreases the response of output in the sticky sector (second-row graphs). The size of the effect on the sticky and flexible sector

[28]See Figure B.1, and Appendix B.4.1 for the specifications.

[29]According to Kim & Katayama (2013), ..."non-separable preferences between consumption and labor are also necessary to delay the peak response of non-durable and durable spending in the estimated two-sector New Keynesian model"... In the case of the GHH preferences, the utility function would look like the following $U(C_t, L_t) = \ln(C_t - \frac{\varsigma}{1+\varphi} L_t^{1+\varphi})$.

[30]The choice of this value is done purely for emphasizing the results on total output, as the direction on the responses for sectoral outputs does not change.

respectively in both cases for λ and for η will determine the impulse shape of total output shown in the first row.

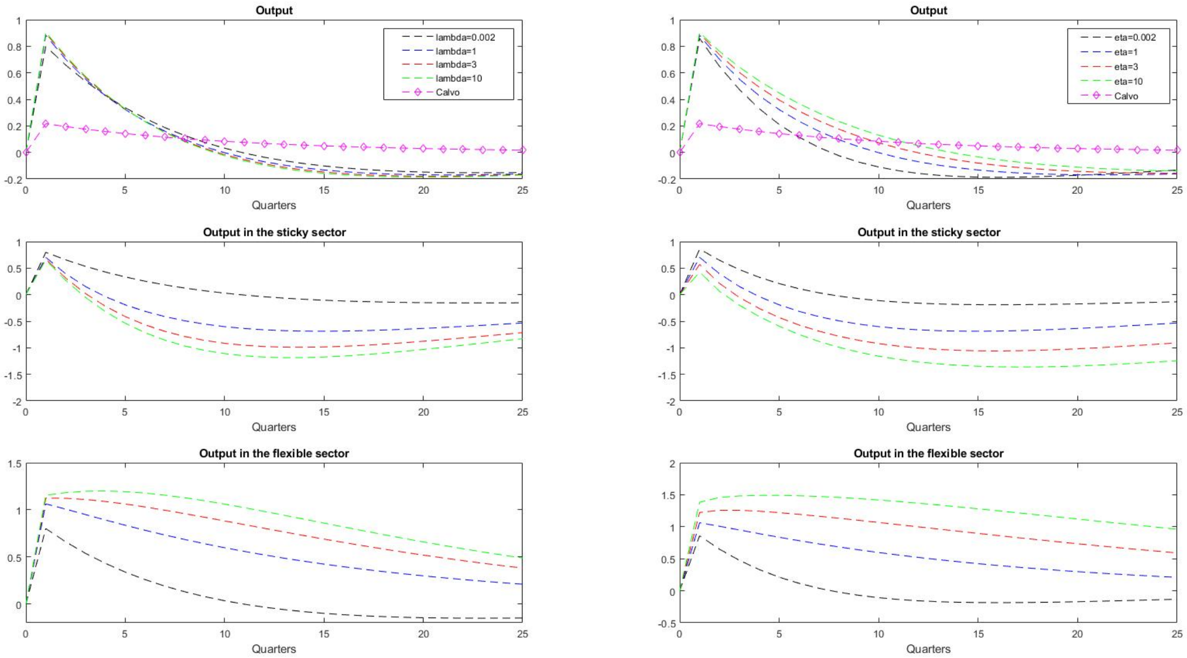

Figure 3.2 IRFs to TFP policy shock

Notes: The figure presents the impulse responses of Output to a 1% TFP Shock in the model subject to the changes in the parameters of λ (on the left) and η (on the right). In the first row are presented the results of total output, in the second row, output in the sticky sector, and in the third row, output in the flexible sector. The four lines represent respectively the impulse responses under a level of 0.002, 1, 3, and 10 of λ (η), with corresponding colors as black, blue, red, and green. The magenta color represents the case of the Calvo model.

On the first row of Figure 3.2, on both graphs, as we face a higher value of a higher η (right-hand side), and a lower λ (left-hand side), the effect of the technology shock on output in the flexible sector surpasses the effect of the technology shock on output in the flexible sector, causing a higher response of total output[31]. What happens, is that a common technology shock, increases output and decreases prices as now can be produced more, but with a better technology. Employment goes down (hours), while labor demand and wages go up. Flexible price firms adjust prices immediately, while the sticky prices firms need some time to adjust their prices. With a higher η, the products between the two sectors will have a higher substitutability, and because the flexible firms adjust prices their product will be more in demand, and as a consequence their output increases even more, while the output in the

[31] As the reasoning behind these results, driven by labor mobility was detailed in Proposition 3., in this subsection, I will focus more on the results driven by the intratemporal elasticity of substitution between sectors.

sticky sector is reduced in size. Comparing the results about total output with the standard Calvo model, some differences can be noticed: First, the IRFs under the two−sector model with price heterogeneity are higher in size than in the Calvo model for the first periods of the shock, and second, the IRFs under price heterogeneity can be increased under higher values of λ and η for the first seven periods of the shock.

3.5 Output dynamics in a calibrated model with manufacturing and services

In this section, to show the implications of labor mobility and the substitutability between the sectors for the response of output dynamics in a two−sector Calvo model with heterogeneity in price rigidities, I calibrate the economy such that it matches the US economy. Additionally, for more realism, I add in the model, trend inflation[32], following Ascari & Sbordone (2014) and Cooke & Kara (2019), who advise that monetary and technology shocks should account for trend inflation. In the first subsection, I will discuss the calibration of the US economy and other parameters choice, while in the second subsection I will discuss the impulse responses to monetary policy shocks, TFP shocks, and sectoral technology shocks. In the third subsection, I discuss the role of different parameter values for the results.

3.5.1 Calibration

The calibration of the economy is presented in table 3.2. I calibrate the economy such as it matches in terms of sectoral shares for manufacturing and services for the US of 2017. For manufacturing the sectoral share value equals[33] $\alpha_m = 0.35$, while the share for services equals $\alpha_s = 0.65$. The share of labor in manufacturing is calculated from the data for "Total hours worked by employees, th", and is $b_m = 0.26$, while the share of labor for services corresponds to the value of $b_s = 0.74$. I set the Calvo parameter for services $\theta_s = 0.75$ and for manufacturing $\theta_m = 0.25$ as in Galesi & Rachedi (2019), following the evidence[34] of Bils & Klenow (2004), Klenow & Kryvtsov (2008) and Nakamura & Steinsson (2008). Further, I set as in Galesi & Rachedi (2019) $\beta = 0.995$, and I keep the (inverse) Frisch elasticity

[32] For a detailed description of how trend inflation changes the NKPC see Appendix B.4.3.

[33] This value is taken from the EU KLEMS database, 2019 release from Stehrer et al. (2019), and the value for Manufacturing is taken from the Value-Added data, code C, while for Services as the sum of the codes $I+J+M+N+R+S$, from NACE2 industry classification.

[34] Bils & Klenow (2004) reports duration of prices 7.8 months for services and 3.2 for manufacturing, while Klenow & Kryvtsov (2008) 9.6 months for services and 3.4 for manufacturing and Nakamura & Steinsson (2008) reports 13.0 months for services and 3.8 for manufacturing.

of labor supply $\varphi = 1$, as in Christiano et al. (2005). For the monetary policy rule, I set $\phi_\pi = 1.5$, $\phi_Y = 0.2$, as in Clarida et al. (2000) and the policy inertia $\rho_i = 0$. As in Smets & Wouters (2007) I consider a monetary policy shock of $\rho_v = 0.15$, and a persistence for the TFP shock $\rho_A = 0.95$. I chose the same persistence for the sectoral technology shocks, respectively $\rho_{A_s} = 0.95$ and $\rho_{A_m} = 0.95$, while I set the size of the standard deviation of the monetary shock equal to 0.24 and the size of the TFP shock equal to 0.45 (including for the sectoral technology shocks) following Smets & Wouters (2007). On the production side, I set the elasticity of substitution for the intermediate goods for manufacturing $\varepsilon_m = 6$ and for services $\varepsilon_s = 6$ as in Galesi & Rachedi (2019), implying a steady−state markup of 20 percent. The elasticity of intertemporal substitution is set at $\sigma = 1$, which is a standard value.

Following the overview of the literature in the subsection 3.4.1, I chose two values for the intratemporal elasticity of substitution between sectors η, respectively 0.002 (as in Herrendorf et al. (2013), and 2 (as in Carvalho & Lee (2019). I set the labor mobility parameter as well within these values, as they are acceptable from the literature observed in subsection 3.4.1[35]. Finally, I set trend inflation at the rate of 3.5 percent considering the US average inflation rate in 2018 (which was 3.46%).

Table 3.2 Calibration

β	Discount factor	0.995	Galesi & Rachedi (2019)
σ	inverse elasticity of intertemporal substitution	1	standard
η	intratemporal elasticity of substitution between sectors	1	Carvalho et al. (2019)
λ	labor mobility parameter	1	Petrella & Santoro (2011)
φ	inverse Frisch labour supply elasticity	1	Christiano et al. (2005)
ε_m	elasticity of substitution between goods in manufacturing	6	Galesi & Rachedi (2019)
ε_s	lasticity of substitution between goods in services	6	Galesi & Rachedi (2019)
θ_m	Calvo parameter in manufacturing sector	0.25	Galesi & Rachedi (2019)
θ_s	Calvo parameter in services sector	0.75	Galesi & Rachedi (2019)
α_m	share of manufacturing in the consumption basket	0.35	EU KLEMS database, (2019)
b_m	share of labor in manufacturing	0.26	EU KLEMS database, (2019)
ρ_v	AR(1) coefficient of monetary policy	0.15	Smets & Wouters (2007)
ρ_A	AR(1) coefficient of TFP shock	0.95	Smets & Wouters (2007)
ρ_{A_m}	AR(1) coefficient of technology shock in the manufacturing sector	0.95	
ρ_{A_s}	AR(1) coefficient of technology shock in the services sector	0.95	
ϕ_π	inflation elasticity of the nominal interest rate	1.5	Clarida et al. (2000)
ϕ_Y	output gap elasticity of the nominal interest rate	0.2	Clarida et al. (2000)
ρ_i	inertia parameter with past interest rate	0	

3.5.2 Impulse responses of the calibrated model

Figure 3.3, presents the impulse responses of the calibrated model for manufacturing and services sectors for a 0.24 sd monetary policy shock. On the left-hand side, I present the

[35] Petrella & Santoro (2011) and Galesi & Rachedi (2019) which calibrate their models directly with manufacturing and services data chose respectively for labor mobility 1 and perfect labor mobility.

IRFs for 2 different values of λ, respectively for values of 0.002 and 2, as explained in the parameters subsection and the results from the multi−Calvo model. On the right-hand side, I present the IRFs for 2 different values of η, respectively for values of 0.002 and 2, and the results from the multi−Calvo model.

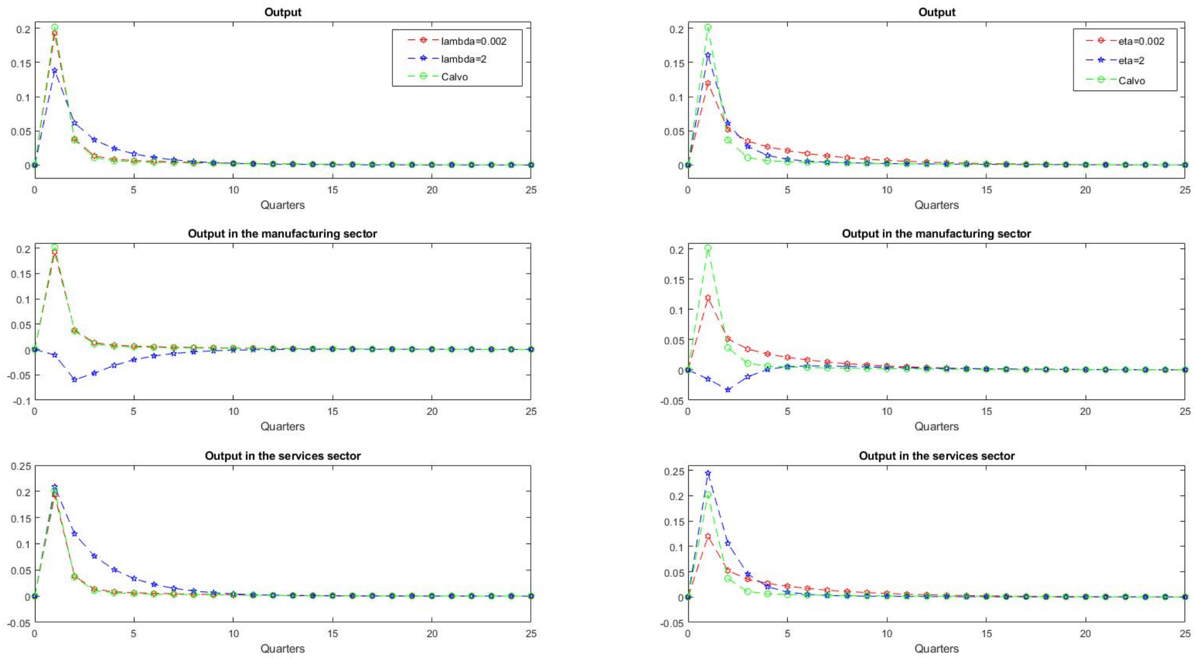

Figure 3.3 IRFs to monetary policy shock for Manufacturing and Services

Notes: The figure presents the impulse responses to a 0.24 sd Monetary Policy Shock in the model with manufacturing and services. In the first row are presented the results of total output, in the second-row output in the manufacturing sector, and in the third-row output in the services sector. The two lines represent respectively the IRF under a level of 0.002 and 2 of λ (left column) and η (right column), with corresponding colors as red and blue. The Calvo model results are represented in the green line.

In Figure 3.3, I show the results only for output and sectoral outputs, while in Appendix B.1.2, Figure B.2, I show the effects on other variables such as annual inflation, real annual interest rate, real wages in the manufacturing sector and real wages in the services sector. The figure shows under higher values of λ and η as we move from a value of 0.002 to 2 output in the services sector increases, while output in the manufacturing sector decreases. On the effect on total output services will have the biggest effect having a higher share than manufacturing. A value of $\eta = 2$, has a bigger impact size effect on total output, but a smaller persistence, while the opposite is noticed under a value of $\lambda = 2$. The strength of price heterogeneity influence in the results is smaller than the ones discussed in Figure 3.1, as in this section manufacturing has some degree of price stickiness rather than being fully flexible. From Figure B.2, in Appendix B.1.2., it can be seen that under higher labor mobility from 0.002 to 2 value of λ real wages in the manufacturing sector decrease, while the opposite happens in the services sector. The increase in labor mobility increases the differences between the wages in both sectors. As labor leaves the manufacturing sector for the services sector, real wages in manufacturing decrease while real wages in services have a bigger increase than before. A higher η has opposite effects. Due to the positive trend inflation allowed in this model as it can be seen from Figure B.6, in Appendix B.1.2, the response of total output is almost 0.1 p.p lower per period. Comparing the results of total output with the standard Calvo model, it can be noticed that in the case of changing labor mobility the IRFs are similar as in the case when labor mobility is low, while reported to be higher than both cases when changing η.

In Figure 3.4, I report the IRFs for a 0.45 sd TFP Policy shock. The figure description is the same as in Figure 3.3.

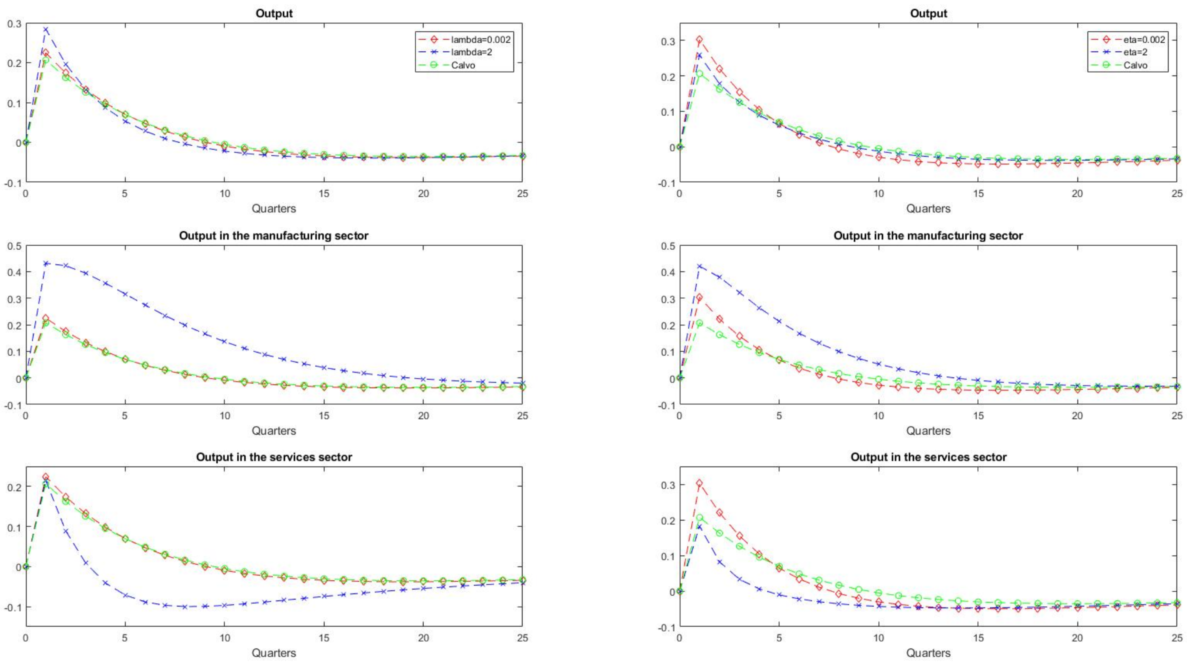

Figure 3.4 IRFs to TFP shock for Manufacturing and Services

Notes: The figure presents the impulse responses to a 0.45 sd TFP Shock in the model with manufacturing and services. In the first row are presented the results of total output, in the second-row output in the manufacturing sector, and in the third-row output in the services sector. The two lines represent respectively the IRF under a level of 0.002 and 2 of λ (left column) and η (right column), with corresponding colors as red and blue. The Calvo model results are represented in the green line.

While in Figure 3.4, I show the results only for output and sectoral outputs, in Appendix B.1.2, Figure B.3, I show the effects on other variables. From the results in Figure 3.4, it can be noticed that: First, under an increase in the value of η and λ, output in the manufacturing sector (less sticky sector) increases more, while it increases less in the services sector (more sticky sector), as discussed in Proposition 3. Second, due to the small effect of the changes in both services and manufacturing, total output changes in both cases are small, having only influence on the peak of the impulse response, with slightly higher under higher labor mobility and slightly lower and higher value of η. The response of the rest of the variables from Figure B.3, in Appendix B.1.2, show that the changes in real wages are smaller in size too, while the changes in the real interest rate happen mostly during the first 4 periods of the shock. From Figure B.6, in Appendix B.1.2, it can be noticed that the response of total output is almost 0.1 p.p lower per period due to allowing for positive trend inflation.

Following Aoki (2001) that argues that a model with sectoral shocks has different optimal monetary policy implications than a model with aggregate shocks, I decide to present the impulse response functions for sectoral technology shocks as well. In Figure 3.5, I report the IRFs for a 0.45 sd technology shock in the manufacturing sector. The rest of the IRFs is shown in Appendix B.1.2, Figure B.4.

From the results in Figure 3.5, it can be noticed that under a technology shock in the manufacturing sector (which has less sticky prices than services), under higher values of λ, output in manufacturing increases less, while output in services increases more than before when λ was 0.002. Real wages in manufacturing increase more than before (as it can be seen from Figure B.4, Appendix B.1.2, where under high labor mobility was slightly negative), while it increases less in the services sector. The same happens under higher values of η, output in manufacturing increases, while output in services increases less (with small negative values) than before when η was 0.002. Real wages become lower in manufacturing and higher in services. What happens is that, when a 0.45 sd sectoral technology shock hits the manufacturing, the output in manufacturing will go up, while this does not happen in the services sector. When η is high, the production would shift more toward manufacturing who would be more attractive, and as consequence, output in manufacturing would be increasing, and output in services would be decreasing.

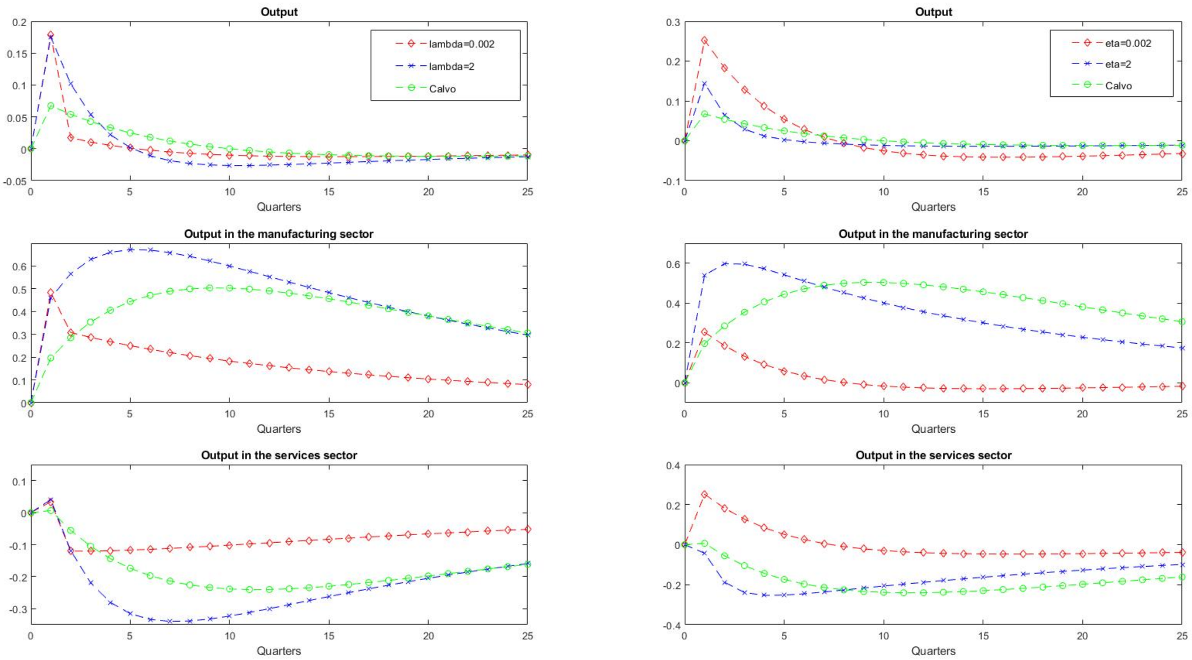

Figure 3.5 IRFs to a technology shock in the manufacturing sector

Notes: The figure presents the impulse responses to a 0.45 sd technology shock in the manufacturing sector. In the first row are presented the results of total output, in the second-row output in the manufacturing sector, and in the third-row output in the services sector. The two lines represent respectively the IRF under a level of 0.002 and 2 of λ (left column) and η (right column), with corresponding colors as red and blue. The Calvo model results are represented in the green line.

While when labor mobility increases from 0.002 to 2, because wages are lower in manufacturing than in services, labor shifts to services, output in services increases while it decreases in manufacturing. Because of this shift in labor, wages start to grow in manufacturing as they are smaller in services. As the impact of manufacturing is higher than that of services, total output is higher in the first periods under higher labor mobility but not as well under high η. From Figure B.6, in Appendix B.1.2, it can be noticed that the response of total output does not change much due to allowing for positive trend inflation.

In Figure 3.6, I report the IRFs for a 0.45 sd technology shock in the services sector. The rest of the IRFs is shown in Appendix B.1.2, Figure B.5. Figure 3.6 shows that when a 0.45 sd technology shock hits the services sector, outputs increases in services, and prices go down. Manufacturing price firms realize they are competing at disadvantage with the services firms, but because they are already operating at zero marginal costs point, they can not adjust prices further. As η increases, more of the goods in the services sector will be required, and as consequence, the output in the services sector will increase. Because the manufacturing sector reacts faster than the services sector, the decline in output in the manufacturing sector is bigger than the increase of output in the services sector, total output is higher at high levels of η in the first periods, and as sticky prices firms adjust the effect ends. When λ goes up, the technology shock in the services sector will increase output more than before when $\lambda = 0.002$ and real wages in that sector will decrease in the first periods. Because wages now are higher in the services sector, as labor shifts to the services sector to produce more services goods, output in the services sector increases more. The increase in output in the services sector is smaller than the decrease in output in the manufacturing sector.

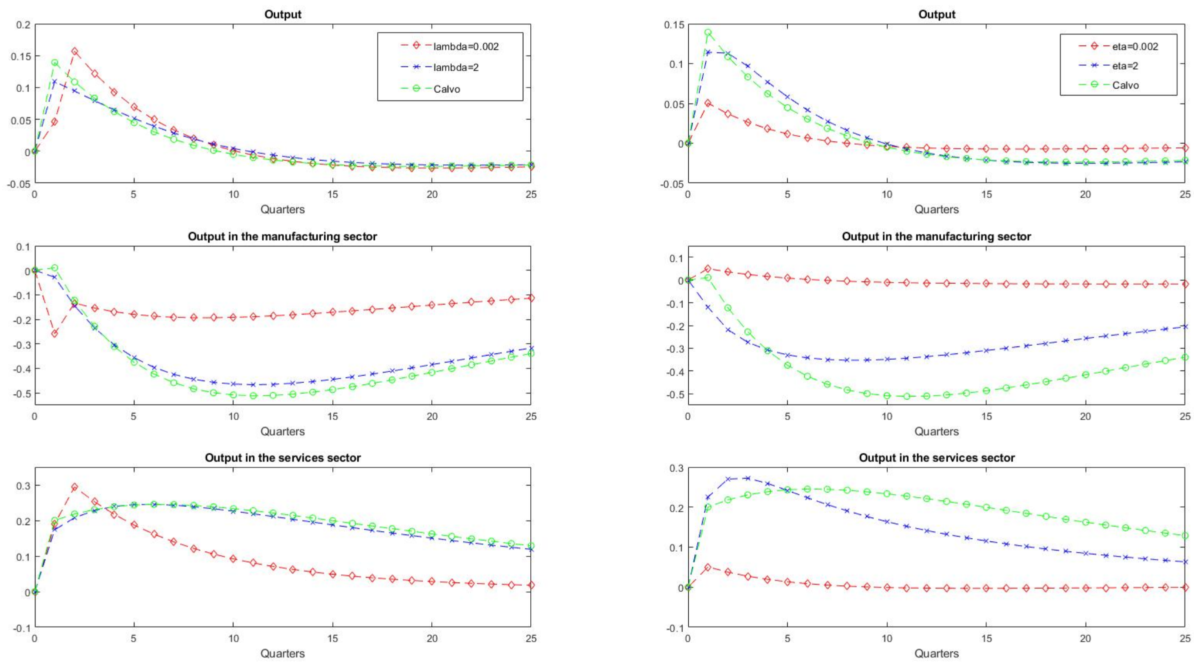

Figure 3.6 IRFs to a technology shock in the services sector

Notes: The figure presents the impulse responses to a 0.45 sd technology shock in the services sector. In the first row are presented the results of total output, in the second-row output in the manufacturing sector, and in the third-row output in the services sector. The two lines represent respectively the IRF under a level of 0.002 and 2 of λ (left column) and η (right column), with corresponding colors as red and blue. The Calvo model results are represented in the green line.

From Figure B.6, in Appendix B.1.2, it can be noticed that the response of total output under a shock in the services sector does not change much due to allowing for positive trend inflation. Overall, figures 3.3 to 3.6 show the importance of η and λ changes in models with different price stickiness.

3.5.3 Persistence measures in the calibrated model for output

After discussing the impulse response functions I focus on showing some persistence measures over the calibrated model results. Table 3.3, reports the half-lives measures and the cumulative impulse responses of the results obtained from Figure 3.3 to 3.6. Both the half-lives measures and the cumulative impulse responses (CIR) are reported over 25 periods same as in the IRFs, and are shown over the same η and λ differences as in the previous subsection.

Table 3.3 Persistence measures in the calibrated model for output

Monetary policy shock	$\eta = 0.002$	Half−life	0.88	$\lambda = 0.002$	Half−life	0.62
		CIR	0.34		CIR	0.29
	$\eta = 2$	Half−life	0.81	$\lambda = 2$	Half−life	0.90
		CIR	0.30		CIR	0.31
TFP shock	$\eta = 0.002$	Half−life	2.76	$\lambda = 0.002$	Half−life	2.89
		CIR	0.18		CIR	0.28
	$\eta = 2$	Half−life	1.97	$\lambda = 2$	Half−life	1.94
		CIR	0.25		CIR	0.21
Technology shock in the manufacturing sector	$\eta = 0.002$	Half−life	2.76	$\lambda = 0.002$	Half−life	0.55
		CIR	0.13		CIR	0.01
	$\eta = 2$	Half−life	0.91	$\lambda = 2$	Half−life	1.72
		CIR	0.02		CIR	-0.04
Technology shock in the services sector	$\eta = 0.002$	Half−life	1.78	$\lambda = 0.002$	Half−life	2.90
		CIR	0.05		CIR	0.27
	$\eta = 2$	Half−life	4.79	$\lambda = 2$	Half−life	3.95
		CIR	0.23		CIR	0.25

Overall, the statistics of table 3.3, confirm the results obtained from the impulse responses functions, that the half-lives and the CIR change due to different values of η and λ, therefore their values matter for output dynamics. These results are as well confirmed by the business cycle properties reported in table B.1 and B.2 in Appendix B.1.2.

3.5.4 Parameters roles and other robustness

This subsection explores the roles of different parameter values on output dynamics focusing on the case for the monetary policy shock. The parameters values which I explore are ϕ_π, φ, the persistence of the shock ρ_v, policy inertia ρ_i, sectoral share changes, and lastly I add habit formation[36] in one of the sectors (the manufacturing sector).

Figure 3.7 displays[37] the impulse responses of the model to a monetary policy shock of 1 percent, subject to testing baseline results to the sensitivity of the share of the manufacturing sector α_m (first row, on the left), φ (first row, on the right), the persistence of the shock ρ_v

[36]The presence habit formation in the consumer's utility function to a standard New Keynesian model as shown in Fuhrer (2000), is important for obtaining the hump-shaped response to the monetary shock.
[37]In this subsection I use as baseline the values $\eta = 1$ (as in Carvalho & Nechio (2011)) and $\lambda = 1$ (as in Petrella & Santoro (2011)).

(second row, on the left), ϕ_π (second row, on the right), ρ_i (third row, on the left) and finally under different values of habit formation (third row, on the right). In terms of the values of η and λ, I use respectively as baseline 1 and 1. In each of the cases, I show three different values of the parameters, where they are all presented with different colors, respectively starting from black, blue, and red (in increasing scale for each of the parameters, except for the changes in the share which is in reverse).

From the results in Figure 3.7, 6 mains points are drawn: First, when a 1% monetary shock hits the economy, the lower is the share of manufacturing[38] (higher the share of services), the bigger the response of total output in the first periods (slightly higher). Second, under a higher inverse Frisch elasticity, the response of total output will be smaller in size, but with a higher persistence.

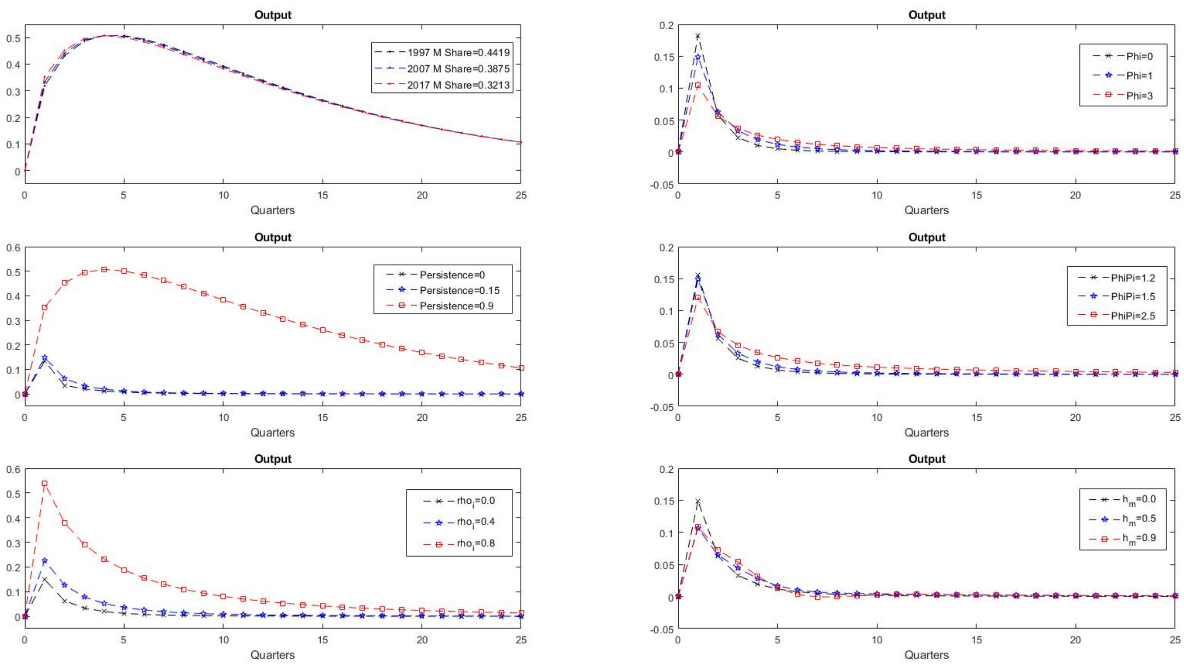

Figure 3.7 IRFs to Monetary policy shock under different parameters

Notes: The figure presents the impulse responses of Output to a 1% Monetary Policy Shock under different values of α_2 (first row, on the left), φ (first row, on the right), ρ_v (second row, on the left), ϕ_π (second row, on the right). The three lines represent respectively higher values of the three parameters (in increasing order, starting from the color black, blue, and red)

[38] I calculate the sectoral shares for the US economy for 1997, 2007 and 2017, and it can be noticed that in the last 20 years the share of manufacturing has decreased while the share of services has increased. The effects of this change in sectoral shares are also treated in Galesi & Rachedi (2019), which deals with the issue of services deepening, and the effects on the transmission of monetary policy. Respectively as the sectoral shares change, labor shares change too, respectively b_m is 0.39 for 1997, 0.30 for 2007, and 0.26 for 2017.

Third, a high persistence of the monetary shock is necessary to obtain the hump-shaped response of output to a monetary policy shock, as it can be seen from the figure the hump-shaped is not obtained when the persistence of the shock is at the levels 0 and 0.15. Fourth, the higher the inflation elasticity of the nominal interest rate, the smaller is in size the output response to the shock, and a bigger persistence is obtained, and vice−versa. Fifth, The output dynamics in a two−sector economy with manufacturing and services is highly sensitive to policy inertia, where under a high value of ρ_i, a high increase in the response of output is obtained. And lastly, the size total output response is reduced almost by fifty percent on impact when I add increasing levels of habit formation in manufacturing (a less sticky sector than services).

3.6 Conclusion

Most multi-sector New Keynesian models of the last generation do not concentrate on explaining output dynamics to various shocks, subject to sectoral elasticities, but with the increasing research on price heterogeneity, their role might be underestimated. This paper studies the response of output to monetary and technology shocks in a closed economy New Keynesian model that allows for heterogeneity in price stickiness across sectors highlighting the role of labor mobility and the intratemporal elasticity of substitution across sectors. I show that the interaction of price heterogeneity, with changes in labor mobility and/or the intratemporal elasticity of substitution between sectors, plays an important role in determining output dynamics. The analytical results and the impulse response functions show that a high labor mobility between sectors (or the intratemporal elasticity of substitution between sectors) increases the response of output to monetary and technology shocks. In a calibrated version of the model, with positive trend inflation, that matches the US economy with manufacturing and services sectors I show that these results are driven by the differences in price stickiness between sectors.

Chapter 4

Sectoral shocks, labor mobility and heterogeneity in price/wage stickiness

4.1 Introduction

One of the interesting questions that have been studied for a long time has been on what drives the US business cycle. The increasing availability of disaggregated data raises important questions at the sectoral level. Which shocks drive the fluctuations in price inflation, real wages, and output? Do sectoral shocks matter for the fluctuations in the US economy? While looking at these issues at the sectoral level, poses the question if sectoral elements such as the labor mobility and elasticity of intratemporal substitutability across sectors matter in New Keynesian multi−sector models? Although these questions have spiked the interest of macroeconomists during the last decades[1], much research needs to be done. This paper attempts to help to answer the transmission of shocks in the US economy at the sectoral level.

Shocks and frictions in the US business cycle have been widely studied by numerous influential papers, for example, Christiano et al. (2005), Smets & Wouters (2007) (hereby referred in the text as SW), Justiniano et al. (2010) etc. With the increasing work on multi−sector models, these models have been complemented with dimensions such as the price heterogeneity across sectors as in Bils et al. (2012) and Kara (2015). Other papers such as Taylor (1999) and Taylor (2016), have argued that there is heterogeneity in the wage-setting and New Keynesian DSGE models should take it into account. Studying shocks and frictions in models with heterogeneity in the price and wage-setting is important, and is necessary in order to understand what drives the US business cycle. Studying shocks

[1]Following, Baqaee & Farhi (2020), the recent COVID-19 event showed that in times of crisis the effects across sectors in an economy are heterogeneous. While many industries closed activity almost immediately, others became innovative and benefited.

transmission at the sectoral level raises questions on whenever labor is mobile across sectors or not, or if the goods are substitutable across sectors or not. Sabaj (2020) argues that the decision on both proves important for output dynamics in an economy.

Most New Keynesian models assume either labor is perfectly mobile across sectors (eg. Barsky et al. (2007)) or that labor is sector-specific (eg. Carlstrom et al. (2006)), while the substitutability of goods between sectors is assumed either to be the same as the elasticity of substitution across different varieties of intermediate goods (eg. Carvalho (2006)) or is assumed equal to one (eg. Carvalho & Nechio (2011)). Alvarez & Shimer (2011), argue that high labour mobility is inconsistent with U.S wage data, while Sabaj (2020), Cantelmo & Melina (2017), Katayama & Kim (2018) argue the importance of measuring correctly the labor mobility across sectors or measuring the elasticity of substitution across sectors (Hobijn & Nechio (2018)).

In this paper, I explore these important gaps in the literature building on the standard medium-scale DSGE model of Smets & Wouters (2007), allowing for both price and wage heterogeneity consistent with microdata. Moreover, I do not make strong assumptions regarding labor mobility and the substitutability of goods across sectors. As suggested by Carvalho et al. (2019) over the number of sectors to be used when approximating aggregate shocks, I conduct my analysis estimating a three−sector NK Keynesian model that allows for the SW shocks, sectoral price shocks as in Kara (2015) and Bils et al. (2012), sectoral wage shocks, sampling errors in sectoral output and nonseparability between private and government consumption as in Bouakez & Rebei (2007) and Sims & Wolff (2018). I use data at the sectoral level for price inflation, real wages, and output in estimating the model, which I obtain from the Bureau of Economic Analysis and Bureau of Labor Statistics. To my knowledge, this might be one of the few papers that use in the Bayesian estimation of shocks and frictions in the US business cycle sectors data on three different aspects, contrary to the literature choices of using in general only data on price inflation, real wages or aggregate output.

I start the analysis with an economy that consists of a "Goods" sector, a "Trade, Transportation, and Utilities" sector, and a "Services" sector, and I estimate it with data from 2006Q3-2019Q4 for the US economy. I focus on showing the differences between the standard one-sector model versus multi-sector versions of the model, focusing on two estimation dimensions such as the number of sectors and the aggregation level of the data. To do so, I proceed in three steps. First, I estimate a one-sector model with aggregated data, second, I estimate the three-sector model with only aggregate data, and third, I estimate the three-sector model with both aggregate and sectors data. I do so because estimating with many sectors and sectoral data leads to results closer to the data values. Later on, for robustness, I count on a six-sector version of the model with both aggregate data and sectors data.

The main findings of the paper are as follows. One-sector models tend to produce overestimated values for price stickiness, while estimating with many sectors and sectoral data, leads to richer results. For the US economy, the estimations with aggregate data tend to mirror the values of price stickiness in the services sector which constitutes the largest sector in the US economy and is estimated to be highly sticky. The sector persistence(s) for the wage and price mark-up shocks are estimated to be for all sectors higher than the persistence of aggregate mark-up shocks in the three-sector model estimated with aggregate data and higher than the one-sector model. The biggest changes are observed in the persistence of the wage shocks. These results clearly show that sector mark-up shocks are more persistent than aggregate mark-up shocks. It is important to note that the magnitude of these results is subject to the degree of sectoral price stickiness heterogeneity, where having a higher price and wage flexibility in one sector, has consequences for the persistence of the price and mark-up shocks in that sector by increasing it, while on the other hand it reduces the persistence of the monetary policy shock.

The standard deviations of the price mark-up shocks are estimated at 0.902, 0.886 and 0.088. From this estimation, it can be observed that the size of sector price shocks is much higher than the aggregate price markup shock estimated in the one sector model or the three-sector model estimated with aggregate data respectively 0.196 and 0.094, where the size of the shock in the "Services" sector, due to its highest share is reflected in the estimations of the shock when estimating models with only aggregate data.

The estimation results show that when the analysis is conducted in an estimated multi-sector model with aggregated and sectoral data has implications as well for sectoral elasticities. The intratemporal elasticity of substitution across sectors, when estimated with aggregate data, is 2.3, close to the range of $1-3$ found in Hobijn & Nechio (2018), while when estimated with sectoral data ranges from $0.29-0.47$. On the other hand, the labor mobility parameter is estimated to be around $1.17-2.62$, close to the range $0.5-1.5$ explored in Bouakez et al. (2011). These results suggest that models estimated with aggregate data tend to overestimate the parameter of intratemporal elasticity of substitution across sectors, while no considerable changes are obtained regarding labor mobility.

Turning into the main contributions in the fluctuations in the US economy over the period $2006-2019$, the following points can be highlighted: First, in the one-sector model, almost 7.12% of the fluctuations in the growth of output is explained by the aggregate price shock, while in the three-sector version of the model estimated with sectoral data sector price shocks explain 18.58% in the growth of output. Similar results are noticed in the growth of consumption in the three-sector versus one-sector where the contributions of the sector price shocks almost double by going to 24.18%. Further, 72.56% of the changes in aggregate inflation is explained by the sector mark-up shocks, with the biggest contribution of 41.24 point percentage of the "Goods" sector. The fluctuations in the growth of wages in sectors "Goods" and "Trade, Transportation, and Utilities", are explained mostly by the sector price mark-up shocks, respectively by 90.05% and 88.38%, while in the "Services" sector, where the prices are more sticky, only 45.66% of the fluctuations are explained by the sector price mark-up shock. The rest is explained by the wage mark-up shock in the "Services" sector. Overall these results suggest that sectoral mark-up shocks have a higher importance in explaining business cycles than aggregate mark-up shocks.

This paper relates to the literature in several directions. First, it contributes to the literature that explains the fluctuations in the US Business cycle such as Smets & Wouters (2007), Justiniano et al. (2010), Sims & Wolff (2018) etc. Second, it relates to the literature that argues the importance of price heterogeneity in New Keynesian models, eg. Carvalho (2006) and Nakamura & Steinsson (2008), Carvalho & Lee (2019), and Bouakez et al. (2014) which show that sectoral heterogeneity in price stickiness matters for the effects of policy shocks. This paper is related to papers that use sectoral data in the estimation of DSGE models such as Bouakez et al. (2014), Bils et al. (2012), Kara (2015), Kara (2017*b*). And at last, it connects to the papers that estimate or discuss sectoral labor mobility and sectoral elasticity such as Horvath (2000), Carlstrom et al. (2006), Bouakez et al. (2009), Bouakez et al. (2011), Iacoviello & Neri (2010*a*), Petrella & Santoro (2011) and Petrella et al. (2019), Carvalho (2006), Carvalho & Nechio (2011) and Hobijn & Nechio (2018).

The remainder of this paper is organized as follows. Section 4.2 presents a three-sector medium-scale New Keynesian model with heterogeneity in price and wage stickiness. Section 4.3 discusses the data used in the estimation and the methodology for the estimation. Section 4.4 presents the estimation results, compares the three-sector model with other versions of the model, while 4.5 discusses how well the macroeconomic data are matched. Section 4.6 summarises and concludes.

4.2 The Model

This section describes the features of the model. I develop a three-sector[2] New-Keynesian medium-scale DSGE model with a number of real and nominal frictions. The model is built

[2] 3 sectors are a good approximation for a multi-sector economy according to Carvalho & Nechio (2018). This model has similarities to the 2 sector model used in Bils et al. (2012), but with the distinction that they do not allow for sectoral wage heterogeneity.

in the spirit of Smets & Wouters (2007) and has features found in Christiano et al. (2005), Justiniano et al. (2010), Kara (2015), Bils et al. (2012), Sims & Wolff (2018) and Carvalho et al. (2019). In addition to the Smets & Wouters (2007), the model is characterized by sectoral heterogeneity in price and wage stickiness, limited labor mobility, and preferences that permit non-separability between private and government consumption as in Bouakez & Rebei (2007).

In this section, I will first describe the behaviour of the households, the firms, and then of the monetary authority. There is a continuum of households and firms over the unit interval. The remaining parts of the model are reported in Appendix C.3.

4.2.1 Households

The economy is populated by a representative agent, that is infinitely lived, and consumes the final consumption good C_t and supplies of labour L_t. Households derive their income from supplying labor to the production sectors, investing in bonds, and from the stream of profits generated in the production sectors. In addition, they rent capital services to firms and accumulate capital given capital adjustment costs. The household can also choose how to utilize its existing stock of physical capital. The lifetime utility of the household equals:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\overline{C}_t^{1-\sigma}}{1-\sigma} - \Psi \frac{L_t^{1+\varphi}}{1+\varphi} \right) \tag{4.1}$$

where β is the discount factor, and σ is the inverse of the intertemporal elasticity of substitution and φ is the inverse of the Frisch elasticity of labor supply. $\overline{C}_t$ is the aggregate consumption bundle, and is a constant elasticity of substitution aggregate consisting of private consumption C_t and government consumption[3] G_t:

$$\overline{C}_t = \left[\omega^{\frac{1}{\chi}} \left(C_t - hC_{t-1} \right)^{\frac{\chi-1}{\chi}} + (1-\omega)^{\frac{1}{\chi}} \left(G_t \right)^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}} \tag{4.2}$$

The parameter ω denotes the weight of aggregate consumption in total consumption services, whereas χ is the elasticity of substitution between aggregate consumption and aggregate government services. When $\chi < 1$, private and government consumption are utility complements, and when $\chi > 1$ they are substitutes. When $\chi = 1$, utility becomes additively separable in private and government consumption. The parameter $0 \leq h \leq 1$ measures the internal habit formation over private consumption.

[3] This relationship between C_t and G_t is also supported by Sims & Wolff (2018), Coenen et al. (2013) and Leeper et al. (2017).

The consumption basket C_t is a CES aggregate of subbaskets of individual goods produced in sectors 1, 2, and 3:

$$C_t = \left[\sum_{j=1}^{3} \alpha_j^{\frac{1}{\eta}} \left(C_{j,t} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{4.3}$$

where α_1 denotes the share of the goods in sector 1, α_2 the share of the goods in sector 2 and $\alpha_3 = 1 - \alpha_1 - \alpha_2$ denotes the share of the goods in sector 3 in the CES aggregator, and η is the elasticity of substitution between sectors 1, 2 and sectors 3 goods. The goods in all sectors can be Edgeworth complements, neutral, or substitutes. The goods are complements when $\eta < 1$, neutral if $\eta = 1$, and substitutes when $\eta > 1$. Consumption in sector j is $C_{j,t} = \left[\int_0^1 C_{j,i,t}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$ for the differentiated consumption units $C_{j,i,t}$. Given the form of preferences in equation 4.3, the household allocates its consumption to individual brands of the goods in each sector according to the demand schedules, $C^i_{j,t} = \left(\frac{P^i_{j,t}}{P_{j,t}} \right)^{-\varepsilon} C_{j,t}$ and $C_{j,t} = \alpha_j \left(\frac{P_{j,t}}{P_t} \right)^{-\eta} C_t$, where $P^i_{j,t}$ denotes the price of individual brand i in sector j. Given the consumption index in 4.3 the CPI index corresponds to:

$$P_t = \left[\sum_{j=1}^{3} \alpha_j \left(P_{j,t} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{4.4}$$

The household maximizes lifetime utility 4.1 subject to the budget constraint:

$$C_{j,t} + I_{j,t} + (1+i_t)^{-1} B_t = \frac{W_{j,t}}{P_{j,t}} L_{j,t} + \frac{R^k_{j,t}}{P_{j,t}} Z_{j,t} K_{j,t-1} - a(Z_{j,t+1})_t K_{j,t-1} + \frac{D_{j,t}}{P_{j,t}} + B_{t-1} \tag{4.5}$$

The household buys $C_{j,t}$ from sector j at the nominal price $P_{j,t}$, where $P_t C_t = P_{1,t} C_{1,t} + P_{2,t} C_{2,t} + P_{3,t} C_{3,t}$. The agents receive their nominal wage income ($W_t L_t = W_{1,t} L_{1,t} + W_{2,t} L_{2,t} + W_{3,t} L_{3,t}$) and lump$-$sum profits ($D_{1,t}$;$D_{2,t}$;$D_{3,t}$) from firms at the end of the period. The household also invests in a one-period bonds. The term $\frac{R^k_{j,t}}{P_{j,t}} Z_{j,t} K_{j,t-1} - a(Z_{j,t+1})_t K_{j,t-1}$, represents the return to owning $K_{j,t}$ units of capital. Households chose the utilization rate of their own capital $Z_{j,t}$, and rent to firms in period t an amount of effective capital equal to:

$$\widetilde{K}_{j,t} = Z_{j,t} K_{j,t-1} \tag{4.6}$$

by getting into return $\frac{R^k_{j,t}}{P_{j,t}} Z_{j,t} K_{j,t-1}$, while the cost of changing capital utilization is $a(Z_{j,t+1})_t K_{j,t-1}$. The households accumulate capital according to the equation:

$$K_{j,t} = e^i_{j,t}\left(1 - S\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right) I_{j,t} + [1-\delta] K_{j,t-1} \tag{4.7}$$

where δ is the rate of depreciation, $e^i_{j,t}$ is the investment-specific technology shock, and $S\left(\frac{I_{j,t}}{I_{j,t-1}}\right) = \frac{k}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2$ is the cost adjustment of investment, where $k > 0$, with $S()$ satisfies $S^{'}() \geq 0$, $S^{''}() = k \geq 0$, $S(1) = 0$, and $S^{'}(1) = 0$.

The households, in equlibrium will make the same choices for consumption, hours worked, bonds, investment and capital utilization. From the first-order conditions of the household optimization problem for consumption $\frac{\partial \mathcal{L}}{\partial C_t}$, $\frac{\partial \mathcal{L}}{\partial C_{j,t}}$, bond holding $\frac{\partial \mathcal{L}}{\partial B_t}$, investment $\frac{\partial \mathcal{L}}{\partial I_{j,t}}$, capital $\frac{\partial \mathcal{L}}{\partial K_{j,t}}$ and capital utilization $\frac{\partial \mathcal{L}}{\partial Z_{jt}}$ we can obtain:

$$1 = \beta(1+i_t)\frac{\overline{C}_{t+1}^{-\sigma+\frac{1}{\chi}}\left(C_{t+1} - hC_t\right)^{\frac{-1}{\chi}}}{\overline{C}_t^{-\sigma+\frac{1}{\chi}}\left(C_t - hC_{t-1}\right)^{\frac{-1}{\chi}}}\frac{P_t}{P_{t+1}} \tag{4.8}$$

$$\lambda_{j,t} = \omega^{\frac{1}{\chi}}\alpha_j^{\frac{1}{\eta}}\overline{C}_t^{-\sigma+\frac{1}{\chi}}\left(C_t - hC_{t-1}\right)^{\frac{-1}{\chi}} C_t^{\frac{1}{\eta}}\left(C_{j,t}\right)^{-\frac{1}{\eta}} \tag{4.9}$$

$$Q_{j,t} = \beta E_t\left(\left(\frac{\lambda_{t+1}}{\lambda_t}R^k_{j,t+1}Z_{j,t+1} - a(Z_{j,t+1})\right) + (1-\delta)Q_{j,t+1}\right) \tag{4.10}$$

$$1 = Q_{j,t}e^i_t\left(1 - \frac{k}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2 - k\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)\frac{I_{j,t}}{I_{j,t-1}}\right) + \beta E_t\frac{\lambda_{t+1}}{\lambda_t}Q_{j,t+1}k\left(\frac{I_{j,t+1}}{I_{j,t}} - 1\right)\left(\frac{I_{j,t+1}}{I_{j,t}}\right)^2 \tag{4.11}$$

$$R^k_{j,t-1} = \frac{R^k_{j,t-1}}{P_{j,t}} = a^{'}(Z_{j,t}) \tag{4.12}$$

where Equation 4.8, represents the Euler equation, while Equation 4.9 represents the multiplier in respect to sectoral consumption. The equations from 4.10 −4.12 are respectively the first order equations for the real value of capital, investment, and the rate of capital utilization[4].

[4]A detailed description of these three equations can be found in Smets & Wouters (2007) Appendix.

4.2.2 Labor Supply Decisions and the wage setting

I assume that labor can be either supplied to sector 1, 2, or sector 3 according to a constant elasticity of substitution (CES) aggregator [5]:

$$L_t = \left[\sum_{j=1}^{3} b_j^{-\frac{1}{\lambda}} (L_{j,t})^{\frac{1+\lambda}{\lambda}} \right]^{\frac{\lambda}{\lambda+1}} \tag{4.13}$$

where λ represents the elasticity of substitution in labor supply and b_1, b_2 and $b_3 = 1 - b_1 - b_2$ are the steady-state ratios of labor supply in the goods sector 1, 2 and 3, over the total labor supply. Depending on λ, this functional form allows us to account for different degrees of labor mobility between sectors. When $\lambda = 0$, labor is immobile across sectors $\Psi_j \frac{L_{j,t}^{1+\varphi}}{1+\varphi}$ for $j = 1,2,3$ with the same Frisch elasticity, while when $\lambda < \infty$ hours are not perfectly substitutes. For $\lambda = \infty$ perfect labor mobility is achieved, and all sectors pay the same hourly wage, and we are at the standard case where[6] $L_t = L_{1,t} + L_{2,t} + L_{3,t}$.

The aggregate wage index associated with the labor index is:

$$W_t = \left[\sum_{j=1}^{3} b_j W_{j,t}^{1+\lambda} \right]^{\frac{1}{\lambda+1}} \tag{4.14}$$

where $W_{1,t}$ and $W_{2,t}$ are wages paid in sector 1 and in sector 2, respectively. From the wage index in 4.14, can be derived the allocation of aggregate labor supply to sector j, $L_{j,t} = b_j \left(\frac{W_{j,t}}{W_t}\right)^{-\lambda} L_t$ where the parameters are the same as in equation 4.13.

As in Erceg et al. (2000) labor input in sector j is equal to $L_{j,t} = \left[\int_0^1 L_{j,i,t}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$, for the differentiated[7] labor inputs i. We can express the relative demand for labor of type i in sector j as a function of its relative wage, with elasticity ε_w, $L_{j,i,t} = \left(\frac{W_{j,i,t}}{W_{j,t}}\right)^{-\varepsilon_w} L_{j,t}$. In a way exactly analogous to intermediate goods, the relative demand for labor of type i in sector j is a function of its relative wage, with elasticity ε_w, $W_{j,t} = \left[\int_0^1 W_{j,i,t}^{1-\varepsilon_w} di \right]^{\frac{1}{\varepsilon_w - 1}}$. As in Smets & Wouters (2007), ε_w follows an exogenous $ARMA(1;1)$ process. In the same form as Calvo (1983) households are not freely able to choose their wage each period. In particular, each period they face the probability $1 - \theta_{w_j}$ of being able to adjust their wage.

[5] Similar forms are used as well in Petrella & Santoro (2011), Horvath (2000), Bouakez et al. (2009), Bouakez et al. (2020).

[6] This represents the standard assumed case in DSGE models.

[7] Alternatively, one might consider the version in which households supply a homogenous labor input that is transformed by monopolistically competitive labor unions into a differentiated labor input as in Schmitt-Grohé & Uribe (2006).

Imposing that labor supply exactly equal demand, allows me to switch notation from choosing $L_{j,i,t}$ to instead choosing $W_{j,i,t}$. The Lagrangian is:

$$\overline{L} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_{w_j})^s \left(\begin{array}{c} -\Psi e^{\varsigma_t} \dfrac{\left(\left[\sum_{j=1}^{3} b_j^{-(\frac{1}{\lambda})} (L_{j,t})^{\frac{1+\lambda}{\lambda}}\right]^{\frac{\lambda}{\lambda+1}}\right)^{1+\varphi}}{1+\varphi} \\ +F_t \left(\Pi_{j,t-1,t+m-1}^{\rho_{j,w}} W_{i,j,t} \left(\left(\dfrac{W_{j,i,t} \Pi_{j,t-1,t+m-1}^{\rho_{j,w}}}{W_{j,t}} \right)^{-\varepsilon_w} L_{j,t} \right) \right) \end{array} \right) \tag{4.15}$$

As in Christiano et al. (2005) $\rho_{j,w}$ indicates the degree of indexation to past inflation for wages. The first order condition is:

$$\frac{\partial \overline{L}}{\partial W_{j,i,t}} = 0 \tag{4.16}$$

Simplifying, all updating households will update to the same wage, reset wage:

$$\left(w_{j,t}^{\#}\right)^{\frac{\varepsilon_w+\lambda}{\lambda}} = \frac{\varepsilon_w}{(\varepsilon_w - 1)} \frac{\mathbb{H}_{1,j,t}}{\mathbb{H}_{2,j,t}} \tag{4.17}$$

Where $H_{1,j,t}$ and $H_{2,j,t}$ are:

$$\mathbb{H}_{1,j,t} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_{w_j})^s \left(\Psi e^{\varsigma_t} L_t^{\frac{\varphi\lambda-1}{\lambda}} \alpha_j^{-(\frac{1}{\lambda})} \Pi_{j,t-1,t+m-1}^{-\rho_{j,w}\varepsilon_w(\frac{1+\lambda}{\lambda})} \Pi_{j,t,t+s}^{\varepsilon_w(\frac{1+\lambda}{\lambda})} w_{j,t+s}^{\varepsilon_w(\frac{1+\lambda}{\lambda})} L_{j,t+s}^{(\frac{1+\lambda}{\lambda})}) \right) \tag{4.18}$$

and

$$\mathbb{H}_{2,j,t} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_{w_j})^s \left(F_{j,t+s} P_{j,t+s} w_{j,t+s}^{\varepsilon_w} \Pi_{j,t-1,t+m-1}^{\rho_{j,w}-\varepsilon_w\rho_{j,w}} \Pi_{j,t,t+s}^{\varepsilon_w-1} L_{j,t+s} \right) \tag{4.19}$$

$\lambda_{j,t+s}$ represents the condition obtained from equation 4.9.

The aggregate real wage index for sector j, where $1-\theta_{w_j}$ of households will update the same reset wage, and θ_{w_j} will be stuck with the last period's wage, is:

$$w_{j,t}^{1-\varepsilon_w} = \theta_{w_j} \left(1+\pi_{j,t}\right)^{\varepsilon_w-1} w_{j,t-1}^{1-\varepsilon_w} \Pi_{j,t-1}^{\rho_{j,w}(1-\varepsilon_w)} + \left(1-\theta_{w_j}\right) \left(w_{j,t}^{\#}\right)^{1-\varepsilon_w} \tag{4.20}$$

4.2.3 Final Good Firms

The final goods are the same as in standard New Keynesian models. In each sector, the final goods forms aggregate the different varieties produced by the continuum of intermediate goods firms using the CES function respectively for good in sector 1, 2 and the good in sector

3:

$$Y_{j,t} = \left[\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon-1}{\varepsilon}} \quad \text{for } j = 1,2,3 \tag{4.21}$$

The parameter ε denotes the elasticity of substitution across different varieties of intermediate goods and initially is assumed constant across sectors.

4.2.4 Intermediate Good Firms

Firms in each sector produce output according to the following technology:

$$Y_{j,i,t} = A_t \widetilde{K}_{j,it}^{\alpha} \left(\gamma^t L_{j,i,t} \right)^{1-\alpha} - \gamma^t \Phi \text{ for } j = 1,2,3 \tag{4.22}$$

where $\widetilde{K}_t$ is capital services used in production, $L_{j,i,t}$ is aggregate labour input and Φ is a fixed cost > 0 used to rule out entry and exit in the steady state and serves as a source of real rigidity. γ^t represents the labour-augmenting deterministic growth rate in the economy. The parameter α captures the share of capital in production. A_t is total factor productivity, which is an exogenous process for the level of technology, which I assume stationary. The firm's profit is given by:

$$\pi_{j,t} = P_{j,t} A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - W_{j,t} L_{j,t} - R_{j,t}^{k} \widetilde{K}_{j,t} \tag{4.23}$$

where $W_{j,t}$ is the aggregate nominal wage rate in sector j and $R_{j,t}^{k}$ is the rental rate on capital in sector j . From here it is possible to find the cost minimization which yields the conditions: $\frac{\partial \pi_{j,t}}{\partial K_{j,t}}$ and $\frac{\partial \pi_{j,t}}{\partial L_{j,t}}$. Using the FOC, and given the constant returns to scale, real marginal costs can be found by finding the amount of factor inputs used in the production of one unit of intermediate good, which would bring:

$$MC_{j,t} = \frac{\partial TC_{j,t}}{\partial Y_{j,t}} = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} W_{j,t}^{1-\alpha} (R_{j,t}^{k})^{\alpha} A_{j,t}^{-1} (\gamma^t)^{-(1-\alpha)} \tag{4.24}$$

The pricing decision of firms

The firms in sector j have a representative sticky price firm, where the prices are based on the model of price stickiness used by Calvo (1983), where a fraction of firms re$-$optimize their nominal price with fixed probability $1-\theta_j$, while with probability θ_j it maintains the price charged in the previous period. The parameter θ_j measures the degree of nominal rigidity; a higher θ_j means that fewer firms re-optimize their price each period and a longer time is needed for the price changes to happen. The firm i in sector j can index its price to the previous period inflation rate: $P_{i,t} = \pi_t^{\rho} P_{i,t-1}$; where inflation is $\pi_t = \frac{P_t}{P_{t-1}}$ and $\rho_{j,p} \in [0,1]$

indicates the degree of indexation (e.g., Christiano et al. (2005)). The problem of firm i, which sets its price at time t, is to choose $P^*_{i,j,t}$ to maximize expected profits:

$$\max_{P^*_{j,i,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \theta^s D_{j,t,t+s} \left[P^*_{j,i,t} \Pi^{\rho}_{j,t-1,t+s-1} Y_{j,i,t+s} - MC_{j,t+s} Y_{j,i_{t+s}} \right] \text{ s.t the demand constraint (4.25)}$$

$$Y_{j,i,t+s} = \left(\frac{P^*_{j,i,t} \Pi^{\rho}_{j,t-1,t+s-1}}{P_{j,t+s}} \right)^{-\varepsilon} Y_{j,t+s}$$

where $D_{j,t,t+s} \equiv \beta^j \frac{F_{j,t+s}}{F_{j,t}} = \beta^s \frac{u'\left(C_{j,t+s}\right)}{u'\left(C_{j,t}\right)} \frac{P_{j,t}}{P_{j,t+s}}$ is the discount factor for the nominal payoffs and λ_{t+j}, the marginal utility of consumption. The firm's first order condition by re−arranging would yield:

$$P^*_{j,i,t} = \frac{\varepsilon}{(\varepsilon - 1)} \frac{\psi_{j,t}}{\phi_{j,t}} \tag{4.26}$$

The auxiliary variables $\psi_{j,t}$ and $\phi_{j,t}$ can be written recursively following as:

$$\begin{aligned} \psi_{j,t} \quad = \quad & \mathbb{E}_t \sum_{s=0}^{\infty} \left(\theta_{j,i}\beta\right)^s \omega^{\frac{1}{\chi}} \alpha_j^{\frac{1}{\eta}} \overline{C}_{t+s}^{-\sigma+\frac{1}{\chi}} \left(C_{t+s} - hC_{t+s-1}\right)^{\frac{-1}{\chi}} C_{t+s}^{\frac{1}{\eta}} \left(C_{j,t+s}\right)^{-\frac{1}{\eta}} \\ & \left(\Pi^{\rho}_{j,t-1,t+m-1}\right)^{-\varepsilon} mc_{j,t+j} \Pi^{\varepsilon}_{j,t,t+s} Y_{j,t+s} \end{aligned} \tag{4.27}$$

and

$$\phi_{j,t} = \mathbb{E}_t \sum_{s=0}^{\infty} \left(\theta_i\beta\right)^s \omega^{\frac{1}{\chi}} \alpha_j^{\frac{1}{\eta}} \overline{C}_{t+s}^{-\sigma+\frac{1}{\chi}} \left(C_{t+s} - hC_{t+s-1}\right)^{\frac{-1}{\chi}} C_{t+s}^{\frac{1}{\eta}} \left(C_{j,t+s}\right)^{-\frac{1}{\eta}} \text{(4.28)}$$

$$\left(\Pi^{\rho}_{t-1,t+m-1}\right)^{1-\varepsilon} \Pi^{\varepsilon-1}_{t,t+s} Y_{j,t+s}]$$

They depend both on output and future expectations of inflation. $\psi_{j,t}$ can be interpreted as the present discounted value of the marginal costs when the optimal reset price changes, while $\phi_{j,t}$ can be considered as the marginal revenues. ε and $\varepsilon - 1$ are respectively treated as the weights of the marginal costs and marginal revenues on resetting the optimal price in equation 4.26.

4.2.5 Monetary and Fiscal authorities

Regarding the monetary policy, the economy has a central bank that follows a conventional Taylor rule, with weight ϕ_π on deviations of inflation from target $\overline{\pi}$ and weight ϕ_Y on output deviations from the output in the economy with flexible prices and wages Y^*_t, and the weight ϕ_{dY} of output growth deviations from the output growth in the economy with flexible prices:

$$\left(\frac{R_t}{R^*}\right) = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_i} \left(\left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_Y}\right)^{1-\rho_i} \left(\frac{\frac{Y_t}{Y_{t-1}}}{\frac{Y_t^*}{Y_{t-1}^*}}\right)^{\phi_{dY}} e^{m_t} \tag{4.29}$$

where e^{m_t} is a monetary policy shock which is *iid* with zero mean and a constant variance and follows an autoregressive process. ϕ_π, ϕ_Y, ϕ_{dY} and $\overline{\pi}$ are non−negative parameters. Some inertia is also added to the Taylor rule. R^* is the steady-state interest rate and ρ_i is the degree of interest rate smoothing.

Government spending is expressed as in Smets & Wouters (2007) and follows a first−order autoregressive process with an IID−Normal error term and is:

$$\widehat{G}_t = \rho_G \widehat{G}_{t-1} + \mu_{G,t} + \rho_{GA}\mu_{A,t} \tag{4.30}$$

where is also affected by the productivity shock.

4.2.6 Resource constraint

The aggregate resource constraint is given by:

$$Y_t = C_t + G_t + I_t + a(Z_t)\widetilde{K}_{t-1} \tag{4.31}$$

where perfect mobility across sectors is assumed for capital and investment. Output Y_t is absorbed by consumption C_t, investment I_t, capital-utilization costs that are a function of the capital utilization rate $a(Z_t)\widehat{K}_{t-1}$, and exogenous spending G_t. In the appendix I express the aggregation of equation 4.31 in more detail terms following Smets & Wouters (2007), steady states of the model.

4.2.7 Main Log−linearised equations

In this subsection, I present some of the main equations in their log−linear form.

Looking at first the inflation Philips curve we observe that now $\widehat{\pi}_{j,t}$ will depend on past and expected future inflation and marginal costs in sector j, while is subject to a price mark−up disturbance which is sector-specific and follows and ARMA(1;1).

$$\widehat{\pi}_{j,t} = \frac{(1-\theta_{j,p})\left(1-\theta_{j,p}\overline{\beta}\gamma\right)}{\theta_{j,p}} \frac{1}{(1+\overline{\beta}\gamma\rho_{j,p})}\widehat{mc}_{j,t} + \frac{\overline{\beta}\gamma}{(1+\overline{\beta}\gamma\rho_{j,p})}\pi_{t+j} + \frac{\rho_{j,p}}{(1+\overline{\beta}\gamma\rho_{j,p})}\pi_{t-j} + \widehat{\varepsilon}^p_{j,t} \tag{4.32}$$

where $\widehat{mc}_{j,t} = (1-\alpha)\widehat{W}_{j,t} + \alpha R^k_{j,t} - \widehat{A}_t$. When the price indexation $\rho_{j,p} = 0$, this turns into the standard Price Philips curve, and when the Calvo parameter $\theta_{j,p} = 0$ we return to the flexible price economy situation.

From equation 4.4, we can the aggregate inflation in each of the sectors to have:

$$\widehat{\pi}_t = \sum_{j=1}^{3} \alpha_j \widehat{\pi}_{j,t} \tag{4.33}$$

Keeping in mind the definitions that $\rho_{1,t} = \widehat{P}_{1,t} - \widehat{P}_t$, $\rho_{2,t} = \widehat{P}_{2,t} - \widehat{P}_t$ and $\rho_{3,t} = \widehat{P}_{3,t} - \widehat{P}_t$, I can write the equations that relate sectoral inflation's to aggregate inflation as in Carvalho et al. (2019) to have:

$$\widehat{\rho}_{j,t} = \widehat{\rho}_{j,t-1} + \widehat{\pi}_{j,t} - \widehat{\pi}_t \tag{4.34}$$

Next, in the wage Philips curve, the real wage is a function of past and expected real wages, expected, past and current inflation, the wage mark−up, and a sector-specific wage mark−up disturbance, which follows an ARMA(1;1) as in Smets & Wouters (2007).

$$\begin{aligned} \widehat{W}_{j,t} \quad = \quad & \frac{\left(1-\overline{\beta}\gamma\theta_{j,w}\right)\left(1-\theta_{j,w}\right)}{\theta_{j,w}\left(1+\overline{\beta}\gamma\right)} \frac{1}{\left(1+\frac{\varepsilon_w}{\lambda}\right)} \widehat{\mu}_{j,t} + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)} \widehat{W}_{j,t+1} + \frac{1}{\left(1+\overline{\beta}\gamma\right)} \widehat{W}_{j,t-1} \\ & + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)} \widehat{\pi}_{j,t+1} - \frac{1+\overline{\beta}\gamma\rho_{j,w}}{\left(1+\overline{\beta}\gamma\right)} \widehat{\pi}_{j,t} + \frac{\rho_{j,w}}{\left(1+\overline{\beta}\gamma\right)} \widehat{\pi}_{j,t-1} + \widehat{\varepsilon}^w_{j,t} \end{aligned} \tag{4.35}$$

where the wage mark−up is expressed as $\widehat{\mu}_{j,t} = \widehat{mrs}_{j,t} - \widehat{W}_{j,t}$ and $\widehat{mrs}_{j,t} = \left(\sigma - \frac{1}{\chi}\right)(1-\omega)\left(\widehat{G}_t\right) + \sigma_\lambda \widehat{C}_t + \sigma_\delta \frac{h}{\gamma}\widehat{C}_{t-1} + \frac{1}{\eta}\widehat{C}_{j,t} + \frac{1}{\lambda}\widehat{L}_{j,t} + \left(\varphi - \frac{1}{\lambda}\right)\widehat{L}_t$, where the parameters σ_L and σ_δ are respectively: $\sigma_L = \left[\left(\sigma\omega - \frac{1}{\chi}\omega + \left(\frac{1}{\chi} - \frac{1}{\eta}\right)\right)\frac{1}{1-\frac{h}{\gamma}} - \frac{1}{\eta}\right]$ and $\sigma_\delta = \left(-\sigma\omega + \frac{1}{\chi}\omega - \frac{1}{\chi}\right)\left(\frac{1}{1-\frac{h}{\gamma}}\right)$. Keeping in mind that h is the habit formation parameter, ω denotes the weight of aggregate consumption in total consumption services, whereas χ is the elasticity of substitution between aggregate consumption and aggregate government services and η and λ are respectively the elasticity of substitution and labor mobility across sectors. $\widehat{mrs}_{j,t}$ can be expressed as well as the sector wage in the economy with flexible wages. When the price indexation in relation to wages $\rho_{j,w} = 0$, this turns into the standard Price Philips curve, and when the Calvo wage parameter $\theta_{j,w} = 0$ we return to the flexible wage economy situation. Interesting to point out that contrary to the version in SW, the labor mobility affects the slope of the wage Philips curve and as well the size of the marginal rate of substitution in the specific sector. The size of the marginal rate of substitution now is affected as well by the elasticity of substitution across sectors η and the substitution elasticity of aggregate consumption with government spending χ. Another equation that is different from SW due to the complementarity between $\widehat{C}_t$ and $\widehat{G}_t$, is the Consumption Euler equation.

$$\begin{aligned}\widehat{C}_t &= \frac{\frac{h}{\gamma}}{\left(1+\frac{h}{\gamma}\right)}\widehat{C}_{t-1} + \frac{1}{\left(1+\frac{h}{\gamma}\right)}\widehat{C}_{t+1} + \frac{\left(\sigma-\frac{1}{\chi}\right)}{\left(1+\frac{h}{\gamma}\right)\sigma_\delta}(1-\omega)\left(\widehat{G}_t-\widehat{G}_{t+1}\right) \\ &\quad + \frac{1}{\left(1+\frac{h}{\gamma}\right)\sigma_\delta}\left(\widehat{R}_t-\widehat{\pi}_{t+1}\right)+\widehat{\varepsilon}_t^b \end{aligned} \tag{4.36}$$

where I write $\sigma_\delta = \left(-\sigma\omega+\frac{1}{\chi}\omega-\frac{1}{\chi}\right)\left(\frac{1}{1-\frac{h}{\gamma}}\right)$. An extra term appears which is related to government spending due to the relationship that it has with private consumption $\frac{\left(\sigma-\frac{1}{\chi}\right)}{\left(1+\frac{h}{\gamma}\right)\sigma_\delta}(1-\omega)\left(\widehat{G}_t-\widehat{G}_{t+1}\right)$. From the coefficients in front of $\left(\widehat{G}_t-\widehat{G}_{t+1}\right)$ it can be observed that the effects of government spending on private consumption are dependent as well on the interaction between the elasticity of substitution between aggregate consumption $\widehat{C}$ and aggregate government services $\widehat{G}$, χ and σ.

The full log−linear equations of the model are presented in Appendix C.3.

4.3 Data, priors and estimation strategy

I estimate the model using Bayesian techniques and quarterly US data (2006 Q3-2019 Q4). I start with updated seven macroeconomic series used by SW, real output growth, consumption growth, investment growth, hours worked, wage inflation, price inflation, and the federal funds rate. At a second stage, I add series with sectoral data on real wages, price inflation, and output. In order to obtain sectoral data for wages, I use the Current Employment Statistics (CES) database from the Bureau of Labor Statistics (BLS). For each sector, I compute the quarterly growth rate for real wages as the log-difference of quarterly average hourly earnings of all employees in 1982-1984 dollars as used in Brinca et al. (2020). The series on sectoral output and sectoral inflation are obtained from the Bureau of Economic Analysis and consist of data on the real value added by industry [Billions of 2012 chain dollars] and chain−type price indexes for value added by industry. All the series are seasonally adjusted.

The sectoral data consist of 13 main aggregate sectors: mining and logging, construction, manufacturing, wholesale trade, retail trade, transportation and warehousing, utilities, information, financial activities, professional and business services, education and health services, leisure and hospitality, and other services. In order to estimate my three-sector model, I group the 13 sectors into 3 main groups[8]: The first sector is named as "Goods sector" and consists of three sectors[9], construction, mining and logging, and manufacturing, with a share of 21% to the total. The second sector named "Trade, Transportation, and Utilities", consists of four main sectors such as wholesale trade, retail trade, transportation and warehousing, and utilities, with a share of 19.2% of the total. The third sector named "Services", consists of six main sectors such as information, financial activities, professional and business services, education, and health services, leisure and hospitality, and other services, with a share of 59.8% to the total.

The following measurement equations link the data to the endogenous variables of the model, where the first seven series are originally used by Smets & Wouters (2007), while the rest are the sectoral series that I add:

$$\Delta\widehat{Y} = \widehat{Y}_t - \widehat{Y}_{t-1} + trend \text{ and } \Delta\widehat{C} = \widehat{C}_t - \widehat{C}_{t-1} + trend \tag{4.37}$$

$$\Delta\widehat{I} = \widehat{I}_t - \widehat{I}_{t-1} + trend \text{ and } \Delta\widehat{W} = \widehat{W}_t - \widehat{W}_{t-1} + trend + \widehat{\varepsilon}_t^W \tag{4.38}$$

$$\widehat{\pi}_t^{obs} = \widehat{\pi}_t + \overline{\pi}_t + \widehat{\varepsilon}_t^P \text{ and } \widehat{R}_t^{obs} = \widehat{R}_t + \overline{R}_t \text{ and } \widehat{L}_t^{obs} = \widehat{L}_t + \overline{L}_t \tag{4.39}$$

$$\Delta\widehat{Y}_1 = \widehat{Y}_{1,t} - \widehat{Y}_{1,t-1} + trend + \widehat{\varepsilon}_{1,t}^y \tag{4.40}$$

$$\Delta\widehat{Y}_2 = \widehat{Y}_{2,t} - \widehat{Y}_{2,t-1} + trend + \widehat{\varepsilon}_{2,t}^y \tag{4.41}$$

$$\Delta\widehat{Y}_3 = \widehat{Y}_{3,t} - \widehat{Y}_{3,t-1} + trend + \widehat{\varepsilon}_{3,t}^y \tag{4.42}$$

$$\Delta\widehat{W}_1 = \widehat{W}_{1,t} - \widehat{W}_{1,t-1} + trend \tag{4.43}$$

$$\Delta\widehat{W}_2 = \widehat{W}_{2,t} - \widehat{W}_{2,t-1} + trend \tag{4.44}$$

$$\Delta\widehat{W}_3 = \widehat{W}_{3,t} - \widehat{W}_{3,t-1} + trend \tag{4.45}$$

$$\widehat{\pi}_{1,t}^{obs} = \widehat{\pi}_{1,t} + \overline{\pi}_t \text{ and } \widehat{\pi}_{2,t}^{obs} = \widehat{\pi}_{2,t} + \overline{\pi}_t \text{ and } \widehat{\pi}_{3,t}^{obs} = \widehat{\pi}_{3,t} + \overline{\pi}_t \tag{4.46}$$

where *trend* is the common quarterly trend growth rate to real GDP, investment, and wages. $\overline{\pi}_t$ is the quarterly steady−state inflation rate. $\overline{L}_t$ is the steady−state hours worked, $\overline{R}_t$ is the steady−state nominal interest rate.

[8]From previous studies there does not seem to be any agreement over which sectors should be used or grouped when studying the effects of economic shocks at sectoral level. This issue seems to be affected heavily by data availability and the scope of each of the studies, focusing mainly on durable vs non−durable goods in Barsky et al. (2007), manufacturing vs services in Galesi & Rachedi (2019) and flexible vs sticky sectors Kara (2017*a*).

[9]Looking at the data, the first sector consists 66.8% of manufacturing goods, 12.7% on mining and logging, and 20.6% on construction.

While we have sixteen observable variables, seven of them are aggregate series and nine of them are the sectoral series. On the other side we face the same number structural shocks, a total factor productivity shock $\widehat{A}_t$, investment−specific technology $\widehat{\varepsilon}_t^i$, risk premium $\widehat{\varepsilon}_t^b$, exogenous spending $\widehat{G}_t$, monetary policy shock $\widehat{\varepsilon}_t^m$, sectoral price mark−up shocks $\widehat{\varepsilon}_{1,t}^p$, $\widehat{\varepsilon}_{2,t}^p$, $\widehat{\varepsilon}_{3,t}^p$ and $\widehat{\varepsilon}_t^p$ which captures sampling errors in inflation as in Bils et al. (2012), wage mark−up shocks $\widehat{\varepsilon}_{1,t}^w$, $\widehat{\varepsilon}_{2,t}^w$, $\widehat{\varepsilon}_{3,t}^w$ and the sampling errors in wages $\widehat{\varepsilon}_t^w$, and $\widehat{\varepsilon}_{1,t}^y$, $\widehat{\varepsilon}_{2,t}^y$, $\widehat{\varepsilon}_{3,t}^y$ which can be interpreted as capturing the errors in output. A list of the shocks and their process equation is given in Appendix C.3.

In estimating the parameters I rely in four prior distributions, respectively the Normal, Beta, Gamma and InvGamma distributions. The choice of the prior distribution is an important part of the Bayesian estimation process as it restricts the parameters outcomes. The Beta distribution provides bounded support for the priors from [0;1], which I will use for fractions or probabilities. The Gamma distribution bounds the parameters from $[0;\infty]$ where non-negativity constraints are necessary, while the InvGamma from $]0;\infty]$ where 0 is not included, which usually are used for variances. Lastly, the Normal prior distribution allows for unbounded support. In choosing the prior distributions for each of the parameters I will rely on the economic intuition and the support provided by the literature. Choosing a higher variance for the priors puts more weight on extreme values of the bounds, why smaller variances means higher weight on the prior mean.

The assumed prior distributions for most of the parameters are the same as those in Smets & Wouters (2007). All of the remaining parameters have been fixed. Parameters that are calibrated include four from SW, $\delta = 0.025$, the share of government spending to output $\gamma_2 = 0.18$, the elasticities of substitution within sectors in the goods and labor market ($\varepsilon_p = 10$ and $\varepsilon_w = 10$), which are both set at 10, corresponding to steady-state markups of 1.1. The weight of aggregate consumption in total consumption services is calibrated at $\omega = 0.7$. Additionally, I calibrate the common quarterly trend growth rate to real GDP, investment, and wages $trend = 0.1$ (calculated as an average of the growth rates for all sectors and total output), while I set the quarterly steady−state inflation rate $\overline{\pi}_t = 0$, the steady−state hours worked $\overline{L}_t = 0$, the relative risk aversion $\sigma = 1$ for simplicity without any loss of generality, and the growth adjusted discount factor in the Euler equation $\overline{\beta} = \beta\gamma^{-\sigma} = 0.1$, which corresponds to $\beta = 0.999$. Following Sims & Wolff (2018), I make an informed guess for the elasticity of substitution between aggregate consumption and aggregate government consumption χ assuming a prior with mean 0.3, standard deviation 0.1, and a *gamma* distribution. Following Hobijn & Nechio (2018), Cantelmo & Melina (2017), and Iacoviello & Neri (2010*a*), I set for λ and η priors with mean 2.0, standard deviation 0.5, and a *gamma* distribution.

Table 4.1 Calibration

$100(\beta^{-1}-1)$	Discount factor	0.1
σ	relative risk aversion	1
γ_2	share of government spending to output	0.18
ε_p	elasticity of substitution in the goods market	10
ε_w	elasticity of substitution in the wage market	10
δ	depreciation rate	0.025
ω	weight of aggregate consumption in total consumption services	0.7
$trend$	quarterly trend growth rate	0.1
$\overline{\pi}_t$	quarterly steady−state inflation rate	0
$\overline{L}_t$	steady−state hours worked	0

Further, I estimate the price indexation in relation to wages and prices respectively assuming a prior with mean 0.4, standard deviation 0.05, following a *beta* distribution.

The posterior distribution of the parameters is estimated using the random walk Metropolis-Hastings algorithm with two parallel chains of 200,000 draws each (after dropping the first 20% of the draws) which are used to generate a sample from the posterior distribution in order to report the mean, the mode, and the 10 and 90 percentiles of the posterior distribution of the estimated parameter[10].

4.4 Estimation Results

In this section, I present and discuss the results from the Bayesian estimation for the US economy. In the first subsection 4.4.1, I have estimated the three-sector model with aggregated data only, and I discuss its differences with the one-sector version of the model. In the second subsection 4.4.2, I go one step further and estimate the three-sector model enriching it with sectoral level data. At the end in 4.4.3 for robustness, I extend the model to six sectors and discuss its implications. The key result is that the three-sector model with sectoral level data not only leads to better estimates for sectoral parameters but as well that sector mark-up shocks are more persistent than aggregate mark−up shocks estimated with aggregate data.

4.4.1 An analysis of a three-sectoral model with aggregate data

Table 4.2 reports the mode, the mean, and the 10 and 90 percentiles of the corresponding posterior distribution of the parameters obtained by the Metropolis−Hastings algorithm. Table 4.3 presents the results of the shock processes. In both tables, the first part shows results from the estimation of the one-sector model (namely model 1), while the second part

[10] The acceptance rate for each of the cases is within suggested limits from 23%-33%.

shows results for the three-sector model calibrated with the shares of an economy consisting of the "Goods sector", "Trade, Transportation, and Utilities sector" and "Services", (namely model 2). On the brackets, after the posterior mean of the results for the one-sector model I present as well the estimation results from the original Smets & Wouters (2007), as it is quite the common practice comparing to it in the literature.

Focusing initially on the one-sector model and referring mostly to the posterior mean value, the price stickiness Calvo parameter $\theta_p = 0.939$ is higher than the ones reported in Smets & Wouters (2007)[11] where $\theta_p = 0.66$, due to different data period, while the wage stickiness Calvo parameter $\theta_w = 0.677$ is reported slightly lower compared to $\theta_w = 0.70$ in SW.

Table 4.2 Prior and posterior distribution of structural parameters for model 1 and 2

	Prior Distribution			**Posterior Distribution**					
				One sector (original SW)			**Three Sector**		
Parameters	Distr.	Mean	St. Dev.	Mode	Mean	90% HPD Interval	Mode	Mean	90% HPD Interval
θ_p	Beta	0.5	0.10	0.950	0.939(0.66)	0.93−0.95	-	-	-
$\theta_{1,p}$	Beta	0.5	0.10	-	-	-	0.727	0.675	0.55−0.81
$\theta_{2,p}$	Beta	0.5	0.10	-	-	-	0.708	0.683	0.55−0.81
$\theta_{3,p}$	Beta	0.5	0.10	-	-	-	0.830	0.857	0.79−0.93
ι_p	Beta	0.5	0.15	0.475	0.542(0.24)	0.29−0.81	-	-	-
$\iota_{1,p}$	Beta	0.4	0.05	-	-	-	0.402	0.401	0.32−0.48
$\iota_{2,p}$	Beta	0.4	0.05	-	-	-	0.404	0.406	0.32−0.49
$\iota_{3,p}$	Beta	0.4	0.05	-	-	-	0.398	0.396	0.31−0.48
θ_w	Beta	0.5	0.15	0.677	0.677(0.70)	0.51−0.84	-	-	-
$\theta_{1,w}$	Beta	0.5	0.15	-	-	-	0.547	0.510	0.30−0.68
$\theta_{2,w}$	Beta	0.5	0.15	-	-	-	0.695	0.624	0.39−0.83
$\theta_{3,w}$	Beta	0.5	0.15	-	-	-	0.840	0.761	0.62−0.90
ι_w	Beta	0.5	0.15	0.410	0.412(0.58)	0.17−0.66	-	-	-
$\iota_{1,w}$	Beta	0.4	0.05	-	-	-	0.396	0.401	0.32−0.48
$\iota_{2,w}$	Beta	0.4	0.05	-	-	-	0.394	0.396	0.31−0.48
$\iota_{3,w}$	Beta	0.4	0.05	-	-	-	0.383	0.395	0.31−0.47
χ	Gamma	0.3	0.10	0.431	0.440	0.29−0.59	0.351	0.353	0.27−0.43
λ	Gamma	2	0.5	-	-	-	1.372	1.535	0.77−2.25
η	Gamma	2	0.5	-	-	-	2.082	2.252	1.35−3.06
k	Normal	4.0	1.50	4.687	5.138(5.74)	3.07−7.33	5.273	6.113	4.04−8.37
Φ	Normal	1.25	0.125	1.473	1.488(1.6)	1.31−1.67	1.547	1.554	1.38−1.74
φ	Normal	2.0	0.75	2.425	2.513(1.83)	1.46−3.49	3.163	2.364	0.75−3.70
α	Normal	0.3	0.05	0.106	0.112(0.19)	0.08−0.14	0.113	0.121	0.10−0.15
ϕ_π	Normal	1.5	0.25	1.464	1.511(2.04)	1.09−1.90	1.000	1.249	1.00−1.58
ρ	Beta	0.75	0.10	0.878	0.880(0.81)	0.83−0.93	0.787	0.834	0.76−0.90
ϕ_Y	Normal	0.12	0.05	0.029	0.059(0.08)	0.01−0.14	0.109	0.093	0.00−0.17
ϕ_{dY}	Normal	0.12	0.05	0.078	0.091(0.22)	0.05−0.13	0.049	0.071	0.01−0.13
h	Beta	0.7	0.10	0.658	0.691(0.71)	0.59−0.80	0.727	0.791	0.64−0.93
Υ	Beta	0.5	0.15	0.725	0.733(0.54)	0.56−0.92	0.428	0.631	0.39−0.87
Log likelihood				-269.64			-261.91		
Log data density				-336.43			-336.63		

Notes: The table shows on the left-hand side the priors used in estimating the model. In the middle are presented the posterior estimated parameters for the one-sector model (model 1), which this paper version of SW, while in brackets are shown the values from the original SW model. On the right-hand side are presented the estimations from the three-sector model (model 2) estimated with aggregated data only(same data as the one-sector model)

[11] Referring to table 1A and 1B in SW.

The degree of price indexation is estimated $\iota_p = 0.542$ which is closer to the prior mean and higher than in SW, while the degree of wage indexation is somewhat closer to the prior compared to SW, but still higher than 0.5 prior mean. The parameters governing the Taylor rule are found to be slightly lower than SW with $\phi_{dY} = 0.091$, $\phi_Y = 0.059$ and $\phi_\pi = 1.511$, compared to $\phi_{dY} = 0.22$, $\phi_Y = 0.08$ and $\phi_\pi = 2.04$, while $\rho = 0.880$ is slightly higher compared to $\rho = 0.81$ in SW. The inverse Frisch elasticity is reported to be higher than in SW 2.513(1.83), while the share of capital in the production process is lower 0.112(0.19). The elasticity of substitution between aggregate consumption and aggregate government consumption is $\chi = 0.44$, a higher value compared to the 0.357 value reported in Sims & Wolff (2018).

In terms of the estimated processes for the exogenous shock variables (Table 3), a few differences with SW are worth mentioning. First, due to the zero lower bound effects, the persistence of the monetary shock process is almost 3 times higher than in SW, being closer to the mean prior, while the standard deviation of the shock is three times lower. Second, due to the financial crisis of 2009, the risk premium shock persistence is almost 3.5 times higher than in SW, while the standard deviation of the shock is almost 3.8 times lower than in SW. Third, the wage mark−up shock displays almost 2.25 times lower persistence than in SW, in the AR term, while in the MA term is almost 0.2 points lower than in SW. These differences are as well reflected in the standard deviation of the shock being 2.8 times higher than in SW, suggesting bigger mark−up shocks. The price mark−up shock while it does not have huge changes in the AR term, it displays a higher value in the coefficient of the MA term. Lastly, the standard deviation of the investment shock is almost 2 times higher than in SW. Overall, the estimation of the one-sector model suggests higher wage mark−up shocks than in the SW.

Moving on to the estimation of the three-sector model with the same aggregate data as in the one-sector model helps identify the traits of estimating multi−sector models without using sectoral data, providing some information about sectoral elasticities. The labor mobility λ parameter is estimated to be 1.535 slightly higher than the 1.51 value estimated in Iacoviello & Neri (2010*a*) for savers, and close to the range 0.5 −1.5 explored in Bouakez et al. (2011), and Cantelmo and Melina (2017) that report a value of almost 1.2. I estimate a value of 2.252 for the elasticity of substitution across the sectors η, which follows within the range of 1−3 suggested by Hobijn & Nechio (2018). Comparing the priors and the posterior of λ and η and the fact that they are rather apart, suggests that the sector's parameters are correctly identified. The elasticity of substitution between aggregate consumption and aggregate government consumption is $\chi = 0.353$, being closer to the estimated value in Sims & Wolff

(2018), rather than in the one-sector model. The cost adjustment of investment goes from 5.138 in the one-sector model to 6.113 in the three-sector model (model 2).

In terms of price and wage stickiness, the price stickiness parameters are lower in each of the sectors, compared to the one-sector estimation, while the sectors seem to be more heterogeneous in terms of wage stickiness. No obvious differences can be seen in terms of wage indexation, while in the price indexation sectors estimations are reported to be lower than in the one-sector model. While the Wage and Philips curves are the same in all sectors what makes them different is only their respective share in the economy[12].

Table 4.3 Prior and posterior distribution of shock processes for model 1 and 2

	Prior Distribution			Posterior Distribution					
				One sector (original SW)			**Three Sector**		
Parameters	Distr.	Mean	St. Dev.	Mode	Mean	90% HPD Interval	Mode	Mean	90% HPD Interval
ρ_a	Beta	0.5	0.2	0.921	0.929(0.95)	0.88−0.99	0.835	0.876	0.75−0.999
ρ_m	Beta	0.5	0.2	0.361	0.472(0.15)	0.25−0.73	0.580	0.588	0.38−0.80
ρ_i	Beta	0.5	0.2	0.639	0.676(0.71)	0.47−0.88	0.833	0.754	0.60−0.91
ρ_b	Beta	0.5	0.2	0.830	0.786(0.22)	0.68−0.90	0.786	0.623	0.39−0.84
ρ_{ga}	Beta	0.5	0.25	0.442	0.401(0.52)	0.21−0.58	0.443	0.387	0.21−0.56
ρ_g	Beta	0.5	0.2	0.997	0.983(0.97)	0.95−0.9996	0.994	0.950	0.92−0.98
ρ_p	Beta	0.5	0.2	0.938	0.879(0.89)	0.79−0.96	0.468	0.842	0.66−0.999
ρ_w	Beta	0.5	0.2	0.546	0.373(0.84)	0.10−0.64	0.190	0.327	0.06−0.62
μ_{p_m}	Beta	0.5	0.2	0.984	0.970(0.69)	0.94−0.998	0.418	0.476	0.21−0.75
μ_{w_m}	Beta	0.5	0.2	0.699	0.670(0.84)	0.40−0.88	0.647	0.637	0.48−0.91
μ_a	Invgamma	0.1	2	0.598	0.593(0.45)	0.46−0.72	0.683	0.599	0.45−0.75
μ_g	Invgamma	0.1	2	0.367	0.383(0.53)	0.30−0.46	0.321	0.340	0.26−0.42
μ_b	Invgamma	0.1	2	0.050	0.060(0.23)	0.04−0.08	0.057	0.086	0.04−0.13
μ_i	Invgamma	0.1	2	0.769	0.783(0.45)	0.50−1.06	0.538	0.683	0.47−0.90
μ_m	Invgamma	0.1	2	0.070	0.081(0.24)	0.06−0.10	0.073	0.084	0.07−0.10
μ_p	Invgamma	0.1	2	0.181	0.196(0.14)	0.15−0.24	0.103	0.094	0.07−0.12
μ_w	Invgamma	0.1	2	0.674	0.681(0.24)	0.54−0.82	0.622	0.539	0.42−0.66

Notes: The table shows on the left-hand side the priors used in estimating the model. In the middle are presented the posterior estimated shocks for the one-sector model (model 1), which this paper version of SW, while in brackets are shown the values from the original SW model. On the right-hand side are presented the estimations from the three-sector model (model 2) estimated with aggregated data only(same data as the one-sector model)

The third sector because it has the highest share or calibrated for the services sector has the highest price stickiness from all sectors, suggesting that when estimated with a one-sector model, the services sector which has the most sticky prices is the one driving the results on the estimation of the Calvo parameter.

In terms of the shock process, the monetary policy shock and investment shock, display higher persistence, while the risk premium shock displays lower persistence. The standard deviations for both of the mark−up shocks are almost lower than the estimations in the one-sector model. The rest of the shock processes do not display noticeable differences,

[12] For comparison purposes the sectors have the same shares as of the three-sector economy mentioned in the data section, where the first sector share is 21%, the second 19.2% and the third sector has a share of 59.8%.

except for the investment shock which is 0.1 points lower. By using the log data density, because the two models are estimated using the same data it is possible to compare them. The identical value of the log data density for the one-sector model suggests that both models do a good job in fitting the data. Nevertheless, estimating with three sectors allows for richer sector information of the economy.

4.4.2 A three-sector model analysis with sectoral data

In this subsection, I present the estimation results for the three-sector model using in addition to the aggregate data in the previous subsection as well as sectors data. Table 4.4 reports the mode, the mean, and the 10 and 90 percentiles of the corresponding posterior distribution of the parameters obtained by the Metropolis−Hastings algorithm. Table 4.5 presents the results of the shock processes. In the first part of tables 4.4 and 4.5 (namely model 3), I present the results assuming the same priors for the wage and price stickiness as in SW. On the second part of the table, I restrict the price stickiness priors for the first sector so it fits relatively to a more flexible price sector[13]. Considering that manufacturing constitutes the largest share in the sector[14], I assume for the first sector price stickiness prior mean of 0.2, and a standard deviation of 0.05. The results with the new estimation priors for the first sector are presented in the second part of the table (named model 4).

I start my analysis of the table by comparing initially the estimation results for model 3, to the ones where I used only aggregate data, model 1 and model 2. Some of the results to be highlighted from model 3 are: The labor mobility parameter λ is estimated at 1.914, 0.3 points higher than the estimated value in model 2. The intratemporal elasticity across sectors is estimated on a value $\eta = 0.378$, significantly lower than the value of 2.252 in model 2, suggesting a low elasticity of substitution across sectors. The inverse Frisch elasticity is estimated lower than in model 2 and to the one-sector value, $\varphi = 1.323$. The cost adjustment of investment which was 5.138 in the one-sector model, 6.113 in the three-sector model with aggregated data (model 2), goes to 4.882 in the three-sector model estimated with sector data (model 3). Comparing with model 2, can be observed looking at the Taylor rule parameters that the interest rate now will depend more on inflation with $\phi_\pi = 1.813$,

[13] Bils & Klenow (2004), Klenow & Kryvtsov (2008), and Nakamura & Steinsson (2008), suggest that the price duration in manufacturing is respectively 3.2, 3.4 and 3.8 months. Galesi & Rachedi (2019), use a Calvo parameter of 0.25 for Manufacturing, while the estimations of Bouakez et al. (2014), indicates a value around 0.436. For the construction sector, they suggest a Calvo parameter of 0.876, while for metal mining, coal mining nonmetallic mining, oil and gas extraction respectively 0.176, 0.872, 0.927, and 0.0.

[14] Transport and utilities, has a Calvo parameter of 0.851, while Trade has a value of 0.906, therefore I do not change the priors assumed in the initial model. Lastly, for Services Bils & Klenow (2004), Klenow & Kryvtsov (2008), and Nakamura & Steinsson (2008), suggest that the price duration in months is respectively 7.8, 9.6 and 13.0.

more on the growth of output $\phi_{dy} = 1.125$ and less from the output itself. The price and wage indexation parameters are relatively lower than estimated in the three-sector model with aggregate data and the one-sector model. In terms of the Calvo parameters, estimating with sector data suggests higher Calvo wage stickiness for the "Goods" sector and "Trade, Transportation, and Utilities" increasing respectively to 0.643 and 0.787. A higher Calvo price stickiness is seen in the first sector while a lower one is observed in the second sector "Trade, Transportation, and Utilities".

In terms of the shock process, the persistence of the monetary policy shock is estimated at $\rho_m = 0.337$, almost 0.25 points lower than when estimated with aggregated data. The persistence of the investment shock is almost 0.2 points lower than when estimated with aggregated data, while the persistence of the risk premium shock is 0.14 points higher. The sector persistence(s) for the wage and price mark−up shocks[15] are estimated to be for all sectors higher than the persistence of aggregate mark−up shocks in model 2, estimated with aggregate data and higher than the one-sector model. The biggest changes are observed in the persistence of the wage shocks. These results clearly show that sector mark−up shocks are more persistent than aggregate mark−up shocks.

The standard deviations of the price mark−up shocks are estimated at $\mu_{1,p} = 0.902$, $\mu_{2,p} = 0.886$ and $\mu_{3,p} = 0.088$, while for the sampling error in inflation is $\mu_p = 0.105$. From this estimation, it can be observed that the size of sector price shocks is much higher than the aggregate price mark−up shock, where the size of the shock in services is reflected in the estimations of the shock when estimated with aggregate data (model 2).

The wage mark−up shocks standard deviations are estimated at $\mu_{1,w} = 0.279$, $\mu_{2,w} = 0.258$ and $\mu_{3,w} = 0.232$. The sector wage mark−up shocks display more than half the size of the standard deviation of the aggregate wage mark−up shock calculated in model 2, and almost 1/3 of the size of the standard deviation of the aggregate wage−mark shock in the one-sector model.

In model 4, where the sectors are subject to a higher price heterogeneity, the Calvo price stickiness parameter in the Goods sector is estimated at around a mean of $\theta_{1,p} = 0.379$, while the price indexation in the goods sector increases to $\iota_{1,p} = 0.416$. This implication affects as well the wage stickiness in this sector which now decreases to $\theta_{1,w} = 0.601$, which is in line with the argumentation given by Klenow & Malin (2010) that price changes are linked to wage changes.

[15]When estimating the model with sectoral data, to facilitate the estimation I calibrate the persistence of the sampling error for aggregate inflation and aggregate wages using the persistence values obtained from the mark−up shocks for wages and inflation in model 2.

Table 4.4 Three-sector: Prior and posterior distribution of structural parameters

	Prior Distribution			Posterior Distribution					
				Model 3			**Model 4**		
Parameters	Distr.	Mean	St. Dev.	Mode	Mean	90% HPD Interval	Mode	Mean	90% HPD Interval
$\iota_{1,p}$	Beta	0.4	0.05	0.391	0.391	[0.31 0.47]	0.416	0.416	[0.33 0.50]
$\iota_{2,p}$	Beta	0.4	0.05	0.385	0.389	[0.31 0.47]	0.354	0.388	[0.30 0.47]
$\iota_{3,p}$	Beta	0.4	0.05	0.383	0.383	[0.30 0.46]	0.382	0.384	[0.31 0.47]
$\iota_{1,w}$	Beta	0.4	0.05	0.320	0.324	[0.25 0.40]	0.328	0.331	[0.25 0.40]
$\iota_{2,w}$	Beta	0.4	0.05	0.390	0.386	[0.32 0.46]	0.387	0.384	[0.31 0.46]
$\iota_{3,w}$	Beta	0.4	0.05	0.390	0.390	[0.31 0.47]	0.390	0.391	[0.31 0.48]
$\theta_{1,p}$	Beta	0.5(0.2)	0.1(0.05)	0.731	0.716	[0.63 0.83]	0.355	0.379	[0.30 0.45]
$\theta_{2,p}$	Beta	0.5	0.1	0.593	0.596	[0.47 0.72]	0.589	0.595	[0.46 0.72]
$\theta_{3,p}$	Beta	0.5	0.1	0.888	0.881	[0.83 0.93]	0.889	0.881	[0.83 0.93]
$\theta_{1,w}$	Beta	0.5	0.15	0.645	0.643	[0.49 0.80]	0.557	0.601	[0.42 0.78]
$\theta_{2,w}$	Beta	0.5	0.15	0.803	0.787	[0.69 0.89]	0.796	0.783	[0.68 0.88]
$\theta_{3,w}$	Beta	0.5	0.15	0.746	0.731	[0.61 0.86]	0.740	0.726	[0.60 0.83]
χ	Gamma	0.3	0.05	0.360	0.361	[0.28 0.44]	0.369	0.369	[0.29 0.45]
λ	Gamma	2	0.5	1.883	1.914	[1.17 2.62]	1.768	1.874	[1.15 2.59]
η	Gamma	2	0.5	0.376	0.378	[0.29 0.47]	0.386	0.384	[0.30 0.47]
k	Normal	4	1.5	4.572	4.882	[2.83 6.83]	4.521	4.844	[2.75 6.82]
Φ	Normal	1.25	0.125	1.562	1.569	[1.40 1.74]	1.581	1.586	[1.41 1.76]
φ	Normal	2	0.75	1.151	1.323	[0.39 2.18]	0.780	1.117	[0.25 1.85]
α	Normal	0.3	0.05	0.110	0.110	[0.08 0.14]	0.111	0.110	[0.08 0.14]
ϕ_π	Normal	1.5	0.25	1.785	1.813	[1.48 2.16]	1.750	1.796	[1.46 2.13]
ρ	Beta	0.75	0.1	0.895	0.895	[0.86 0.93]	0.889	0.894	[0.86 0.93]
ϕ_Y	Normal	0.125	0.05	0.031	0.05	[0.01 0.05]	0.023	0.029	[0.01 0.05]
ϕ_{dY}	Normal	0.125	0.05	0.120	0.125	[0.08 0.17]	0.128	0.126	[0.08 0.17]
h	Beta	0.7	0.1	0.648	0.665	[0.55 0.78]	0.620	0.655	[0.54 0.77]
Υ	Beta	0.5	0.15	0.681	0.665	[0.48 0.87]	0.705	0.672	[0.48 0.87]
Log posterior (or likelihood)				-723.35			-736.53		
Log data density				-847.84			-861.44		

Notes: The table shows on the left-hand side the priors used in estimating the model. In the middle are presented the posterior estimated parameters for the three-sector model (model 3) estimated with sectoral data (using the same priors as in SW). On the right-hand side are presented the estimations from the three-sector model (model 4), with priors for the first sector that indicate flexible prices in sector 1)

Having higher price heterogeneity does not seem to have consequences on the estimation of other structural parameters. A crucial point that can be drawn regarding the shock processes of the model is that when the persistence of the price shock in the goods sector increases to $\rho_{1,p} = 0.969$, the coefficient in the MA part cuts in half going to $\rho_{1,p_m} = 0.224$, while the standard deviation of the shock almost triples going to $\mu_{1,p} = 2.311$. Comparing the log-likelihood of model 3 and model 4, it is obvious that the value is bigger for model 3 (-723.35>-736.53), suggesting that model 3 does a better job overall in terms of fit.

Table 4.5 Three-sector: Prior and posterior distribution of shock processes

	Prior Distribution			Posterior Distribution					
				Model 3			**Model 4**		
Parameters	Distr.	Mean	St. Dev.	Mode	Mean	90% HPD Interval	Mode	Mean	90% HPD Interval
ρ_a	Beta	0.5	0.2	0.923	0.912	[0.87 0.96]	0.924	0.911	[0.86 0.96]
ρ_m	Beta	0.5	0.2	0.338	0.337	[0.17 0.50]	0.312	0.327	[0.17 0.49]
ρ_i	Beta	0.5	0.2	0.596	0.577	[0.40 0.76]	0.635	0.594	[0.40 0.79]
ρ_b	Beta	0.5	0.2	0.786	0.760	[0.66 0.86]	0.801	0.769	[0.67 0.87]
ρ_{ga}	Beta	0.5	0.25	0.448	0.449	[0.30 0.61]	0.443	0.449	[0.29 0.60]
ρ_g	Beta	0.5	0.2	0.997	0.995	[0.99 0.999]	0.997	0.994	[0.99 0.9994]
$\rho_{1,y}$	Beta	0.5	0.1	0.436	0.442	[0.31 0.58]	0.440	0.442	[0.31 0.57]
$\rho_{2,y}$	Beta	0.5	0.1	0.349	0.358	[0.22 0.48]	0.351	0.361	[0.23 0.50]
$\rho_{3,y}$	Beta	0.5	0.1	0.374	0.379	[0.25 0.51]	0.376	0.386	[0.25 0.52]
$\rho_{1,p}$	Beta	0.5	0.2	0.970	0.937	[0.87 0.997]	0.981	0.969	[0.94 0.997]
$\rho_{2,p}$	Beta	0.5	0.2	0.982	0.973	[0.95 0.998]	0.982	0.973	[0.95 0.998]
$\rho_{3,p}$	Beta	0.5	0.2	0.987	0.976	[0.95 0.998]	0.988	0.976	[0.95 0.999]
$\rho_{1,w}$	Beta	0.5	0.2	0.699	0.691	[0.49 0.89]	0.785	0.725	[0.54 0.93]
$\rho_{2,w}$	Beta	0.5	0.2	0.556	0.514	[0.23 0.78]	0.589	0.522	[0.27 0.78]
$\rho_{3,w}$	Beta	0.5	0.2	0.553	0.563	[0.35 0.77]	0.549	0.559	[0.35 0.77]
μ_{1,p_m}	Beta	0.5	0.2	0.650	0.551	[0.31 0.80]	0.198	0.224	[0.05 0.39]
μ_{2,p_m}	Beta	0.5	0.2	0.342	0.343	[0.10 0.59]	0.344	0.350	[0.10 0.58]
μ_{3,p_m}	Beta	0.5	0.2	0.797	0.687	[0.47 0.90]	0.804	0.697	[0.49 0.91]
μ_{1,w_m}	Beta	0.5	0.2	0.230	0.269	[0.06 0.48]	0.243	0.262	[0.05 0.45]
μ_{2,w_m}	Beta	0.5	0.2	0.502	0.470	[0.19 0.87]	0.529	0.483	[0.20 0.87]
μ_{3,w_m}	Beta	0.5	0.2	0.227	0.266	[0.05 0.47]	0.239	0.264	[0.05 0.46]
μ_a	Invgamma	0.1	2	0.578	0.601	[0.48 0.72]	0.572	0.600	[0.48 0.72]
μ_g	Invgamma	0.1	2	0.367	0.391	[0.32 0.46]	0.369	0.390	[0.32 0.46]
μ_b	Invgamma	0.1	2	0.053	0.062	[0.04 0.08]	0.051	0.060	[0.04 0.08]
μ_i	Invgamma	0.1	2	0.806	0.872	[0.57 1.17]	0.747	0.845	[0.54 1.14]
μ_m	Invgamma	0.1	2	0.078	0.083	[0.07 0.10]	0.077	0.082	[0.07 0.10]
$\mu_{1,y}$	Invgamma	0.1	1	1.095	1.127	[0.93 1.32]	1.099	1.126	[0.94 1.31]
$\mu_{2,y}$	Invgamma	0.1	1	0.823	0.845	[0.70 0.98]	0.820	0.847	[0.70 0.99]
$\mu_{3,y}$	Invgamma	0.1	1	0.468	0.482	[0.40 0.56]	0.470	0.482	[0.40 0.56]
μ_p	Invgamma	0.1	2	0.102	0.105	[0.09 0.12]	0.102	0.104	[0.09 0.12]
$\mu_{1,p}$	Invgamma	0.1	2	0.841	0.902	[0.60 1.18]	2.439	2.311	[1.76 3.00]
$\mu_{2,p}$	Invgamma	0.1	2	0.637	0.686	[0.41 0.94]	0.646	0.692	[0.42 0.95]
$\mu_{3,p}$	Invgamma	0.1	2	0.092	0.088	[0.06 0.12]	0.091	0.090	[0.06 0.12]
μ_w	Invgamma	0.1	2	0.923	0.945	[0.79 1.10]	0.924	0.948	[0.80 1.10]
$\mu_{1,w}$	Invgamma	0.1	2	0.244	0.279	[0.17 0.38]	0.249	0.285	[0.17 0.39]
$\mu_{2,w}$	Invgamma	0.1	2	0.249	0.258	[0.18 0.33]	0.245	0.263	[0.19 0.34]
$\mu_{3,w}$	Invgamma	0.1	2	0.210	0.232	[0.15 0.31]	0.213	0.231	[0.15 0.30]

Notes: The table shows on the left-hand side the priors used in estimating the model. In the middle are presented the posterior estimated shocks for the three-sector model (model 3) estimated with sectoral data (using the same priors as in SW). On the right-hand side are presented the estimations from the three-sector model (model 4), with priors for the first sector that indicate flexible prices in sector 1)

4.4.3 Robustness (A six-sector analysis)

In this subsection, I present the estimation results for a six-sector model using sectors data[16]. The six sectors are: "Construction and Mining", "Manufacturing", "Trade, Transportation, and Utilities", "Information and Financial activities", "Professional and Business Services" and "Other Services". Table 4.6 reports the mode, the mean, and the 10 and 90 percentiles of the corresponding posterior distribution of the parameters obtained by the Metropolis−Hastings algorithm. Tables 4.7 and 4.8 present the results of the shock processes. In the first part of tables 4.6−4.8 (namely model 5), I present the results assuming the same priors for the wage and price stickiness as in SW. On the second part of the table, I restrict the price stickiness priors for the first and second sector so they fit relatively, to a more flexible price sector[17], while I restrict the wage stickiness priors so the first, fourth, fifth and six sectors have some degree of wage flexibility[18]. In the estimations for this section, I focus mainly on the sectoral parameters and shocks considering that the rest were quite steady from the estimations for the three-sector models with sector data[19] (model 3 and 4).

[16] I estimate the model using six sectors, which consist in: The first sector is named as "Construction and Mining" and consists in two sectors construction and mining and logging, with a share of 7.2% to the total. The second sector is named "Manufacturing" and consists of one sector only which is manufacturing, with a share of 12.8% of the total. The third sector named "Trade, Transportation, and Utilities", consists of four main sectors which are wholesale trade, retail trade, transportation and warehousing, and utilities, with a share of 21.9% of the total. The fourth sector named "Information and Financial activities", consists of two main sectors such as information and financial activities, with a share of 10.0% to the total. The fifth sector named "Professional and Business Services", consists of one sector only which is professional and business services, with a share of 15.2% of the total. The sixth sector named "Other Services", consists of three main sectors such as education and health services, leisure and hospitality, and other services, with a share of 28.4% to the total.

[17] In the previous subsection as discussed on the assumptions regarding priors for the three sectors, "Construction and Mining" and "Manufacturing" is considered from the literature to have flexible prices. On the other hand, "Trade, Transportation, and Utilities" and "Services" had sticky prices. Taking into consideration that in this subsection I have split "Services" into three sub−sectors respectively, "Information and Financial activities", "Professional and Business Services" and "Other Services", the sub−sectors are assumed to share the same priors as the aggregate "Services" sector used in the three-sector model.

[18] Following Barattieri et al. (2014) which suggests that services and mining are the industries where wages appear more flexible.

[19] In order to avoid that the estimations of the six-sector model in both cases are not affected by the calibration of the parameters, I calibrate the non-estimated parameters of model 5 following the estimations of model 3, and for the calibration of model 6, I follow the estimations of model 4. Nevertheless, even not doing so, the differences are not significant in affecting the results.

Table 4.6 Six sector: Prior and posterior distribution of structural parameters

	Prior Distribution			Posterior Distribution					
				Model 5			**Model 6**		
Parameters	Distr.	Mean	St. Dev.	Mode	Mean	90% HPD Interval	Mode	Mean	90% HPD Interval
$\iota_{1,p}$	Beta	0.4	0.05	0.404	0.406	[0.33 0.49]	0.413	0.417	[0.33 0.50]
$\iota_{2,p}$	Beta	0.4	0.05	0.385	0.388	[0.31 0.47]	0.394	0.393	[0.31 0.47]
$\iota_{3,p}$	Beta	0.4	0.05	0.397	0.403	[0.32 0.48]	0.395	0.396	[0.32 0.47]
$\iota_{4,p}$	Beta	0.4	0.05	0.392	0.400	[0.31 0.48]	0.395	0.399	[0.32 0.48]
$\iota_{5,p}$	Beta	0.4	0.05	0.380	0.378	[0.30 0.46]	0.384	0.384	[0.31 0.46]
$\iota_{6,p}$	Beta	0.4	0.05	0.393	0.389	[0.31 0.47]	0.422	0.429	[0.35 0.51]
$\iota_{1,w}$	Beta	0.4	0.05	0.184	0.208	[0.15 0.26]	0.182	0.199	[0.14 0.25]
$\iota_{2,w}$	Beta	0.4	0.05	0.377	0.381	[0.31 0.46]	0.384	0.385	[0.31 0.46]
$\iota_{3,w}$	Beta	0.4	0.05	0.393	0.394	[0.32 0.47]	0.382	0.392	[0.32 0.46]
$\iota_{4,w}$	Beta	0.4	0.05	0.377	0.382	[0.30 0.46]	0.379	0.384	[0.31 0.47]
$\iota_{5,w}$	Beta	0.4	0.05	0.394	0.392	[0.31 0.47]	0.395	0.394	[0.32 0.48]
$\iota_{6,w}$	Beta	0.4	0.05	0.385	0.394	[0.32 0.48]	0.397	0.401	[0.32 0.49]
$\theta_{1,p}$	Beta	0.5(0.2)	0.1(0.05)	0.623	0.661	[0.56 0.75]	0.543	0.550	[0.52 0.59]
$\theta_{2,p}$	Beta	0.5(0.2)	0.1(0.05)	0.887	0.644	[0.47 0.78]	0.316	0.355	[0.29 0.41]
$\theta_{3,p}$	Beta	0.5	0.1	0.910	0.913	[0.88 0.95]	0.917	0.916	[0.89 0.95]
$\theta_{4,p}$	Beta	0.5	0.1	0.919	0.920	[0.90 0.95]	0.921	0.921	[0.90 0.95]
$\theta_{5,p}$	Beta	0.5	0.1	0.946	0.938	[0.93 0.95]	0.940	0.932	[0.91 0.95]
$\theta_{6,p}$	Beta	0.5	0.1	0.850	0.861	[0.82 0.91]	0.950	0.947	[0.94 0.95]
$\theta_{1,w}$	Beta	0.5(0.4)	0.15(0.1)	0.551	0.587	[0.40 0.77]	0.451	0.473	[0.36 0.60]
$\theta_{2,w}$	Beta	0.5	0.15	0.701	0.720	[0.59 0.85]	0.637	0.656	[0.51 0.80]
$\theta_{3,w}$	Beta	0.5	0.15	0.757	0.732	[0.61 0.85]	0.748	0.720	[0.60 0.85]
$\theta_{4,w}$	Beta	0.5(0.4)	0.15(0.1)	0.756	0.753	[0.63 0.89]	0.553	0.575	[0.44 0.71]
$\theta_{5,w}$	Beta	0.5(0.4)	0.15(0.1)	0.785	0.786	[0.69 0.89]	0.659	0.647	[0.51 0.78]
$\theta_{6,w}$	Beta	0.5(0.3)	0.15(0.1)	0.801	0.713	[0.58 0.84]	0.524	0.533	[0.40 0.66]
λ	Gamma	2	0.5	0.954	1.364	[0.69 2.04]	0.657	0.889	[0.6 1.33]
η	Gamma	2	0.5	0.387	0.377	[0.30 0.45]	0.370	0.371	[0.30 0.45]
Log posterior (or likelihood)				-1424.77		-1472.84			
Log data density				-1604.47		-1659.40			

Notes: The table shows on the left-hand side the priors used in estimating the model with only wages and prices. In the middle are presented the posterior estimated shocks for the six-sector model (model 5) estimated with sectoral data (with priors being the same as SW)). On the right-hand side are presented the estimations from the six-sector model (model 6), with priors for the first, second sector that indicate flexible prices and the first, fourth, fifth, sixth sector that indicate flexible wages). The non-estimated parameters of model 5 are calibrated following the estimations of model 3, and for the calibration of model 6, I follow the estimations of model 8.

The posterior mean of λ is 1.364, and the associated 90% probability interval ranges from 0.69 to 2.04. This result suggests that having more sectors reduces labor mobility across sectors. A reason for this is due to the fact that more sectors translate as well into a bigger heterogeneity between sectors. These values of labor mobility seem to be as well closer to previous estimations in the literature such as Horvath (2000), Bouakez et al. (2009), Petrella & Santoro (2011) and Petrella et al. (2019). The intratemporal elasticity of substitution across sectors is estimated at a posterior mean of 0.377, in line with previous estimations from the three-sector model. Assuming a lower degree of wage and price stickiness for sectors 1, 4,5,6 in model 6, cuts the posterior mean to $\lambda = 0.889$, reducing the labor mobility across sectors. The degree of price stickiness in model 5 is for "Construction and Mining" 0.661 (0.550 for model 6), "Manufacturing" 0.644 (0.355 for model 6), "Trade, Transportation, and Utilities"

0.913, "Information and Financial activities" 0.920, "Professional and Business Services" 0.938, and for "Other Services" is 0.861.

Turning to the estimates on the degree of wage stickiness, in model 5 we have for "Construction and Mining" 0.587 (0.473 for model 6), "Manufacturing" 0.720, "Trade, Transportation, and Utilities" 0.732, "Information and Financial activities" 0.753 (0.575 for model 6), "Professional and Business Services" 0.786 (0.647 for model 6), and for "Other Services" is 0.713 (0.533 for model 6). On average it can be emphasized that the structural parameters in the six-sector model confirm the findings from the estimation in the three-sector model with sector data.

Looking at the estimates of the persistence of the shock processes in Table 4.7, and comparing it to the three-sector model, focusing on model 5 (and comparing it with model 3), it can be seen lower persistence(s) on the technology shock, investment shock, and the monetary policy shock. The persistence in the price mark−up shocks is respectively the highest in the sector "Other services" $\rho_{6,p} = 0.984$ and "Manufacturing", while the lowest in "Professional and Business Services" $\rho_{5,p} = 0.321$. The highest persistence in the wage mark−up shocks is seen in "Construction and Mining" $\rho_{1,w} = 0.959$, while the lowest in "Information and Financial activities" $\rho_{6,w} = 0.543$. This result suggests higher heterogeneity in the persistence of the mark−up shocks than in the three-sector model (model 3).

Finally, focusing on the standard deviation of the shocks in Table 4.8, some key points can be made. On average the standard deviation of wage mark−up shocks are not that different than in the three-sector model, while the price mark−up shocks tend to have a higher standard deviation and be more heterogeneous. The biggest price mark−up shock is noticed in the sector of "Construction and Mining" $\mu_{1,p} = 2.656$, while the smallest in "Other services" $\mu_{6,p} = 0.069$. The biggest standard deviation of the shock in the wage mark−up shocks is seen in "Professional and Business Services" $\mu_{5,p} = 0.415$, while the smallest in "Other services" $\mu_{6,p} = 0.2017$. Looking at model 6 estimates, it can be observed that having higher heterogeneity in price and wage stickiness between the sectors, has more effect on the standard deviation of the price shock in the flexible sector, and less on the standard deviation of the wage shock.

As extra robustness for the three-sector model estimated with sectors data, In Appendix C.2.3. (Tables C.8 and C.9), I present the cases of model 3 and 4, with the distinction that I assume priors for the third sector so it fits more to a flexible wage sector. This way I explore if higher wage heterogeneity has effects on the results, naming the two models 7 and 8. Overall, looking at the estimations from the six-sector model and Appendix C.2.3, it can be said that the results produced by estimating the three-sector model with sectors data are robust and consistent.

Table 4.7 Six sector: Prior / posterior distribution of the persistence of the shock processes (1)

	Prior Distribution			Posterior Distribution					
				Model 5			**Model 6**		
Parameters	Distr.	Mean	St. Dev.	Mode	Mean	90% HPD Interval	Mode	Mean	90% HPD Interval
ρ_a	Beta	0.5	0.2	0.875	0.866	[0.82 0.91]	0.885	0.872	[0.83 0.92]
ρ_m	Beta	0.5	0.2	0.313	0.311	[0.16 0.45]	0.236	0.251	[0.12 0.38]
ρ_i	Beta	0.5	0.2	0.544	0.531	[0.37 0.70]	0.603	0.596	[0.44 0.75]
ρ_b	Beta	0.5	0.2	0.791	0.752	[0.68 0.83]	0.812	0.795	[0.73 0.85]
ρ_{ga}	Beta	0.5	0.25	0.428	0.440	[0.29 0.59]	0.430	0.433	[0.31 0.57]
ρ_g	Beta	0.5	0.2	0.997	0.994	[0.99 0.999]	0.997	0.994	[0.99 0.9995]
$\rho_{1,y}$	Beta	0.5	0.1	0.0.596	0.586	[0.46 0.71]	0.587	0.584	[0.47 0.71]
$\rho_{2,y}$	Beta	0.5	0.1	0.415	0.426	[0.29 0.56]	0.408	0.414	[0.27 0.55]
$\rho_{3,y}$	Beta	0.5	0.1	0.356	0.367	[0.24 0.49]	0.352	0.366	[0.23 0.49]
$\rho_{4,y}$	Beta	0.5	0.1	0.351	0.365	[0.23 0.49]	0.345	0.357	[0.23 0.48]
$\rho_{5,y}$	Beta	0.5	0.1	0.371	0.386	[0.25 0.51]	0.368	0.379	[0.24 0.51]
$\rho_{6,y}$	Beta	0.5	0.1	0.428	0.430	[0.29 0.57]	0.424	0.432	[0.31 0.56]
$\rho_{1,p}$	Beta	0.5	0.2	0.738	0.729	[0.54 0.96]	0.686	0.695	[0.53 0.87]
$\rho_{2,p}$	Beta	0.5	0.2	0.726	0.913	[0.83 0.996]	0.985	0.975	[0.95 0.997]
$\rho_{3,p}$	Beta	0.5	0.2	0.930	0.829	[0.69 0.96]	0.937	0.859	[0.73 0.93]
$\rho_{4,p}$	Beta	0.5	0.2	0.897	0.792	[0.60 0.94]	0.910	0.782	[0.54 0.96]
$\rho_{5,p}$	Beta	0.5	0.2	0.178	0.321	[0.04 0.61]	0.183	0.391	[0.05 0.72]
$\rho_{6,p}$	Beta	0.5	0.2	0.992	0.984	[0.97 0.998]	0.951	0.937	[0.90 0.97]
$\rho_{1,w}$	Beta	0.5	0.2	0.983	0.959	[0.92 0.997]	0.993	0.986	[0.97 0.999]
$\rho_{2,w}$	Beta	0.5	0.2	0.670	0.673	[0.49 0.86]	0.614	0.663	[0.47 0.87]
$\rho_{3,w}$	Beta	0.5	0.2	0.519	0.544	[0.27 0.82]	0.461	0.527	[0.25 0.80]
$\rho_{4,w}$	Beta	0.5	0.2	0.511	0.543	[0.34 0.75]	0.628	0.641	[0.43 0.86]
$\rho_{5,w}$	Beta	0.5	0.2	0.549	0.557	[0.34 0.77]	0.593	0.613	[0.39 0.83]
$\rho_{6,w}$	Beta	0.5	0.2	0.403	0.576	[0.36 0.80]	0.627	0.644	[0.43 0.87]
μ_{1,p_m}	Beta	0.5	0.2	0.369	0.345	[0.10 0.58]	0.254	0.258	[0.07 0.43]
μ_{2,p_m}	Beta	0.5	0.2	0.896	0.540	[0.33 0.76]	0.292	0.318	[0.14 0.49]
μ_{3,p_m}	Beta	0.5	0.2	0.983	0.957	[0.91 0.996]	0.984	0.963	[0.92 0.997]
μ_{4,p_m}	Beta	0.5	0.2	0.970	0.953	[0.91 0.99]	0.975	0.958	[0.92 0.996]
μ_{5,p_m}	Beta	0.5	0.2	0.828	0.854	[0.76 0.94]	0.841	0.827	[0.74 0.98]
μ_{6,p_m}	Beta	0.5	0.2	0.555	0.505	[0.24 0.78]	0.995	0.992	[0.99 0.999]
μ_{1,w_m}	Beta	0.5	0.2	0.281	0.364	[0.09 0.61]	0.261	0.323	[0.10 0.55]
μ_{2,w_m}	Beta	0.5	0.2	0.289	0.313	[0.07 0.54]	0.242	0.316	[0.07 0.56]
μ_{3,w_m}	Beta	0.5	0.2	0.460	0.432	[0.17 0.69]	0.414	0.432	[0.17 0.68]
μ_{4,w_m}	Beta	0.5	0.2	0.246	0.292	[0.06 0.50]	0.241	0.283	[0.07 0.50]
μ_{5,w_m}	Beta	0.5	0.2	0.947	0.905	[0.86 0.99]	0.959	0.760	[0.19 0.996]
μ_{6,w_m}	Beta	0.5	0.2	0.909	0.270	[0.05 0.47]	0.207	0.277	[0.04 0.48]

Notes: The table shows on the left-hand side the priors used in estimating the model with only wages and prices. In the middle are presented the posterior estimated persistence(s) of the shocks for the six-sector model (model 7) estimated with sectoral data (with priors being the same as SW)). On the right-hand side are presented the estimations from the six-sector model (model 6), with priors for the first, second sector that indicate flexible prices and the first, fourth, fifth, sixth sector that indicate flexible wages). The non-estimated parameters of model 5 are calibrated following the estimations of model 3, and for the calibration of model 6, I follow the estimations of model 8.

Table 4.8 Six sector: Prior and posterior distribution of the shock processes (2)

	Prior Distribution			Posterior Distribution					
				Model 5			**Model 6**		
Parameters	Distr.	Mean	St. Dev.	Mode	Mean	90% HPD Interval	Mode	Mean	90% HPD Interval
μ_a	Invgamma	0.1	2	0.603	0.632	[0.50 0.76]	0.603	0.624	[0.50 0.74]
μ_g	Invgamma	0.1	2	0.368	0.385	[0.31 0.45]	0.363	0.381	[0.32 0.45]
μ_b	Invgamma	0.1	2	0.055	0.067	[0.04 0.09]	0.055	0.062	[0.04 0.08]
μ_i	Invgamma	0.1	2	0.884	0.960	[0.64 1.25]	0.798	0.853	[0.58 1.12]
μ_m	Invgamma	0.1	2	0.082	0.085	[0.07 0.10]	0.085	0.089	[0.07 0.10]
$\mu_{1,y}$	Invgamma	0.1	1	2.110	2.266	[1.78 2.54]	2.057	2.157	[1.77 2.55]
$\mu_{2,y}$	Invgamma	0.1	1	1.127	1.169	[0.97 1.36]	1.130	1.174	[0.97 1.37]
$\mu_{3,y}$	Invgamma	0.1	1	0.823	0.849	[0.70 0.98]	0.824	0.860	[0.71 1.01]
$\mu_{4,y}$	Invgamma	0.1	1	1.230	1.272	[1.07 1.49]	1.226	1.267	[1.06 1.46]
$\mu_{5,y}$	Invgamma	0.1	1	0.934	0.962	[0.80 1.11]	0.932	0.969	[0.81 1.12]
$\mu_{6,y}$	Invgamma	0.1	1	0.593	0.611	[0.51 0.71]	0.588	0.609	[0.51 0.71]
μ_p	Invgamma	0.1	2	0.155	0.158	[0.13 0.18]	0.154	0.160	[0.13 0.19]
$\mu_{1,p}$	Invgamma	0.1	2	2.978	2.656	[2.30 3.00]	3.00	2.925	[2.83 3.00]
$\mu_{2,p}$	Invgamma	0.1	2	1.002	0.973	[0.60 1.34]	2.873	2.512	[2.03 3.00]
$\mu_{3,p}$	Invgamma	0.1	2	0.748	0.809	[0.66 0.96]	0.731	0.784	[0.64 0.93]
$\mu_{4,p}$	Invgamma	0.1	2	0.182	0.303	[0.25 0.36]	0.278	0.300	[0.24 0.36]
$\mu_{5,p}$	Invgamma	0.1	2	0.234	0.249	[0.20 0.30]	0.234	0.244	[0.19 0.30]
$\mu_{6,p}$	Invgamma	0.1	2	0.068	0.069	[0.04 0.10]	0.1803	0.195	[0.16 0.23]
μ_w	Invgamma	0.1	2	0.914	0.936	[0.77 1.08]	0.915	0.935	[0.78 1.09]
$\mu_{1,w}$	Invgamma	0.1	2	0.188	0.264	[0.11 0.41]	0.206	0.253	[0.14 0.36]
$\mu_{2,w}$	Invgamma	0.1	2	0.205	0.231	[0.15 0.31]	0.228	0.242	[0.16 0.32]
$\mu_{3,w}$	Invgamma	0.1	2	0.234	0.236	[0.17 0.31]	0.237	0.239	[0.18 0.30]
$\mu_{4,w}$	Invgamma	0.1	2	0.204	0.220	[0.15 0.30]	0.194	0.210	[0.14 0.28]
$\mu_{5,w}$	Invgamma	0.1	2	0.399	0.415	[0.33 0.53]	0.419	0.388	[0.19 0.53]
$\mu_{6,w}$	Invgamma	0.1	2	0.328	0.217	[0.14 0.29]	0.199	0.225	[0.14 0.30]

Notes: The table shows on the left-hand side the priors used in estimating the model with only wages and prices. In the middle are presented the posterior estimated shock standard deviations for the six-sector model (model 7) estimated with sectoral data (with priors being the same as SW)). On the right-hand side are presented the estimations from the six-sector model (model 6), with priors for the first, second sector that indicate flexible prices and the first, fourth, fifth, sixth sector that indicate flexible wages). The non-estimated parameters of model 5 are calibrated following the estimations of model 3, and for the calibration of model 6, I follow the estimations of model 8.

4.5 Variance decomposition and macroeconomic data

In this section, I present the variance decomposition of the forecast errors for the observable variables and explore the macroeconomic matching of the data. I will show that sectoral shocks explain more output fluctuations and other variables compared to a model that faces aggregate mark−up shocks.

4.5.1 Variance decomposition analysis

In this subsection, I present the variance decomposition and examine the sources of business cycle fluctuations in the US economy. Table C.3, C.4 and C.5 in Appendix C.2.1, report the variance decomposition of the forecast errors for all the observable variables. Table C.3 presents the results for the one-sector model, model 1. Table C.4 presents the results for the three-sector model, specifically model version 3. Table C.5 presents the results for the six-sector model, specifically model version 5. As a point of reference, I will use the one-sector model (Table C.3), which represents my version of SW with new data from 2006Q3-2019Q4. Each value on the tables gives the contribution (in percent) of each shock to fluctuations in each variable of the models. First of all, I will start the analysis by explaining the main sources of the business cycle in the first model (one sector). Almost 87.1% of the fluctuations in the growth of output are explained by four shocks, the investment shock, government spending shock, TFP shock, and risk premium shock. The risk premium shock explains 59.17% of the fluctuations in the growth of private consumption, while the investment shock explains 91.21% of the fluctuations in investment. Almost all of the fluctuations in inflation are explained by the aggregate price shock, while wage shocks explain almost 87.62% of the variations in wages. Most of the variations in the changes of hours (labor) are explained by the government spending shock (26.59%), investment shock almost 21.65%, and the price shock 19.47%.

Moving on to the variance decomposition for the three-sector (model 3), and comparing it with the baseline one-sector model, there are some notable findings: First, sector price shocks explain more changes in output growth than the aggregate price shock (7.12%<18.58%), while the contribution of the risk premium shock falls from 18.10 to 15.15 percent. Second, the fluctuations in sector 1 ("Goods sector") are explained by 32.71% by the sampling error in this sector and almost 39.83% the price shock that sector, reflecting the higher importance of price flexible shocks. The contrary happens in sector 3 ("Services"), where the influence of the sticky sector price shock is lower, by only 6.21%. Third, interesting is what happens in the fluctuations of the growth of consumption, which now is explained only 53% by the risk premium shock, while the influence of the sector price shocks doubles to 24.18%. Around 1.87% of the fluctuations in the growth of consumption now is explained by the sector mark−up shocks in wages. Fourth, another interesting finding is that almost 42% of the fluctuations in labor hours is explained by the government spending shock, compared to only 26.59% in the one-sector model. Fifth, 72.56% of the changes in aggregate inflation are explained by the sector mark−up shocks, with the biggest contribution of 41.24 point percentage of sector one. Sixth, The fluctuation in the wage growth is explained 62.86% by the sampling error in aggregate wages and 12.6% by price mark−up shocks in the goods sector. The fluctuations in the growth of wages in sectors 1 and 2 are explained mostly by the sector price mark−up shocks, by 90.05% and 88.38%, while in sector 3, where the prices are more sticky, only 45.66% of the fluctuations are explained by the sector price mark−up shock, the rest is explained by the services sector wage mark−up shock with 50.79%. Finally, I look at the variance decomposition of the six-sector model and observe that the variance decomposition analysis confirms the observations obtained from the analysis of the three-sector model.

To conclude, the variance decomposition analysis showed that sectoral mark−up shocks explain more the dynamics of output and other variables compared to aggregate mark−up shocks.

4.5.2 Matching macroeconomic data

In this subsection, I analyze the quantitative performance of the three-sector model by presenting the second moments of the observable variables and by comparing them with those obtained from the model. Table 4.9 presents the standard deviation and the autocorrelation of the observable variables from the data and generated by the one-sector model (Model 1), three-sector model (Model 3), and the six-sector model (Model 5). Table(s) C.6 and C.7, in Appendix C.2.2, report the Cross−Correlations of the observable variables generated from the one-sector model and the three-sector model, while for data is presented in Table C.2. Examining the standard deviations for the US economy from 2006Q3−2019Q4 and looking at first the standard deviations of the aggregate observable variables it can be observed that does a better job in matching the standard deviations than the one-sector (SW) model for aggregate wages, while the six-sector version does a better job in matching the growth in investment, consumption wages and inflation (comparing again with the one-sector model, but as well with the three-sector model). On the other hand, while the multi−sector versions of the model do not necessarily do a better job than SW in matching the other aggregate variables, it still succeeds in matching quite good the standard deviations. The results suggest that the three-sector model does a good job in producing enough volatility as well for the sector's growth output, inflation, and wages.

The three-sector model also effectively does a good job in capturing the autocorrelations output growth, consumption growth, and investment growth. The autocorrelation of inflation in the three-sector model is 0.71, doing so a better job than the one-sector model (0.84) in matching the autocorrelation for the inflation data (0.27). In terms of the sectors, the multi-sector models make both a relatively good job in matching the autocorrelations coming from the data. We now turn the attention to the correlation between the observable variables and see that the model three-sector model does a good job too. It successfully explains the positive relationship between output and sectoral outputs, consumption, and investment. If I look at the correlations between inflation rates, the model does a good job too. It shows good correlations as well between aggregate inflation and wages.

Table 4.9 Second Moments

	Data		One sector		Three Sector		Six Sector	
Parameters	Autocorrelation	St. Dev.	Autocorrelation	St. Dev.	Autocorrelation	St. Dev.	Autocorrelation	St. Dev.
DC	0.44	0.48	0.54	0.49	0.57	0.54	0.51	0.52
DINVE	0.50	3.35	0.60	4.10	0.58	4.11	0.51	3.90
DW	-0.34	1.14	-0.20	1.06	0.18	1.19	0.10	1.07
DY	0.37	0.58	0.36	0.76	0.42	0.88	0.35	0.82
LABOBS	0.69	0.60	0.94	1.93	0.98	3.59	0.97	3.10
PINFOBS	0.27	0.23	0.84	0.51	0.71	0.63	0.57	0.49
ROBS	0.92	0.40	0.96	0.42	0.95	0.36	0.93	0.30
DW1	0.27	1.81	-	-	0.47	2.06	-	-
DW2	0.14	0.89	-	-	0.40	1.31	-	-
DW3	0.25	0.61	-	-	0.52	0.83	-	-
DY1	0.24	1.42	-	-	0.57	2.20	-	-
DY2	0.19	1.37	-	-	0.40	1.32	-	-
DY3	0.08	0.54	-	-	0.38	0.98	-	-
PINFOBS1	0.21	1.20	-	-	0.66	2.03	-	-
PINFOBS2	0.20	0.88	-	-	0.69	1.48	-	-
PINFOBS3	0.35	0.20	-	-	0.88	0.59	-	-
DW1	0.40	4.29	-	-	-	-	0.38	4.13
DW2	0.01	1.24	-	-	-	-	0.28	1.57
DW3	0.14	0.89	-	-	-	-	0.34	1.24
DW4	0.31	0.66	-	-	-	-	0.33	0.73
DW5	0.28	0.72	-	-	-	-	0.07	0.75
DW6	0.08	0.53	-	-	-	-	0.56	0.82
DY1	0.21	2.39	-	-	-	-	0.58	4.56
DY2	0.29	1.81	-	-	-	-	0.42	1.66
DY3	0.19	1.23	-	-	-	-	0.38	1.28
DY4	-0.10	1.37	-	-	-	-	0.35	1.59
DY5	0.46	1.16	-	-	-	-	0.35	1.32
DY6	0.31	0.46	-	-	-	-	0.38	1.07
PINFOBS1	0.38	3.76	-	-	-	-	0.56	4.43
PINFOBS2	0.17	1.08	-	-	-	-	0.62	1.81
PINFOBS3	0.20	0.88	-	-	-	-	0.60	1.25
PINFOBS4	0.46	0.37	-	-	-	-	0.68	0.52
PINFOBS5	0.47	0.35	-	-	-	-	0.49	0.34
PINFOBS6	0.58	0.27	-	-	-	-	0.89	0.64

Notes: The table shows the autocorrelations and standard deviations for the data, the one-sector model (my version of SW), the three-sector model corresponding to model 3, the six-sector model corresponding to model 5.

4.6 Conclusion

This paper studies the fluctuations of the business cycle in the US economy building on the Smets & Wouters (2007) canonical model with three sectors and heterogeneity in price and wage rigidities. In estimating the model, I use data at the sectoral level for price inflation, real wages, and output contrary to the literature choices of using in general only aggregate data or sectoral only for one variable, usually price inflation. I show that the interaction of heterogeneity in price and wage stickiness plays an important role in determining the value of labor mobility in the US economy while adding sectors has implications for the elasticity of substitution of the goods across sectors. While the heterogeneity in price and wage stickiness has implications for the size of the sectoral price and wage shocks, it increases as well the persistence of the shocks in that sector, while it reduces the persistence of the monetary policy shock. The results provide support for the importance of sectoral shocks over aggregate shocks in DSGE models. Additionally, this paper brought evidence to the idea of Carvalho & Nechio (2018) that three sectors are a good approximation for a multi−sector economy comparing it to a one and six sectors model economy.

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Appendix A

Appendices For Chapter Two

Appendix A.1

Table A.1 Summary Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
G	3128	12.7	1.44	8.9	15.4
Y	3128	11.1	1.42	7.6	13.8
C	3128	12.1	1.48	8.4	14.8
W	3128	10.4	0.67	8.1	11.4
INF	3128	3.0	4.23	−6.1	48.7

Table A.2 Country Inflation Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Average	34	3.0	1.7	0.1	9.0
AU	92	2.6	1.3	-0.4	6.1
AT	92	1.8	0.8	0.0	3.7
BE	92	1.9	1.1	-1.2	5.6
CA	92	1.8	0.8	-0.9	4.5
CL	92	3.8	2.3	-3.0	9.3
CZ	92	3.4	3.2	-0.4	13.3
DN	92	1.8	0.8	0.1	4.2
ET	92	5.9	7.0	-2.0	28.8
FN	92	1.4	1.2	-1.0	4.6
FR	92	1.4	0.8	-0.4	3.3
DE	92	1.4	0.7	-0.2	3.1
GR	92	2.9	2.7	-2.4	10.2
HU	92	7.6	7.3	-1.1	30.4
IS	92	4.3	3.3	1.1	17.1
IR	92	2.0	2.4	-6.1	6.6
IZ	92	3.1	3.5	-2.5	13.1
IT	92	2.1	1.3	-0.4	5.6
JP	92	0.1	1.0	-2.2	3.6
KO	92	3.0	1.7	0.6	8.9
LV	92	5.5	6.4	-3.8	26.4
LT	92	5.4	9.4	-1.7	46.4
LX	92	1.9	1.0	-0.1	4.3
ME	92	9.0	9.9	2.3	48.7
NL	92	1.9	0.9	0.0	4.4
NZ	92	2.1	1.3	-0.5	5.3
NO	92	2.1	1.0	-1.4	4.7
PO	92	5.6	7.1	-1.5	33.0
PR	92	2.2	1.4	-1.5	4.8
SK	92	4.5	3.7	-0.8	15.8
SN	92	4.5	3.8	-0.7	19.1
ES	92	2.3	1.5	-1.1	5.1
SW	92	1.1	1.2	-1.4	4.3
CH	92	0.5	0.9	-1.4	3.0
UK	92	2.1	0.8	0.3	4.5

Appendix A.2

A.2.1 Households optimization problem

$$\max_{C_t,N_t,B_t} \frac{\widetilde{C}_t^{1-\sigma}}{1-\sigma} - d_n e^{\varsigma_t} \frac{N^{1+\varphi}}{1+\varphi} \quad \text{s.t} \quad P_t C_t + (1+i_t)^{-1} B_t = W_t N_t + D_t + B_{t-1} - T_t \tag{A.1}$$

$\widetilde{C}_t$ where the aggregate consumption bundle is:

$$\widetilde{C}_t = \left[\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{A.2}$$

I can form a Lagrangian:

$$L = \frac{C_t^{1-\sigma}}{1-\sigma} - d_n e^{\varsigma_t} \frac{N^{1+\varphi}}{1+\varphi} + \lambda \left[W_t N_t + D_t + B_{t-1} - T_t - P_t C_t - (1+i_t)^{-1} B_t\right] \tag{A.3}$$

The first order conditions are:

$$\frac{\partial L}{\partial C_t} = \delta^{\chi} \overline{C}_t^{-\sigma} \left(\frac{C_t}{\widetilde{C}_t}\right)^{-\chi} - \lambda_t P_t, \ \frac{\partial L}{\partial N_t} = -d_n e^{\varsigma_t} N^{\varphi} + \lambda_t W_t \text{ and } \frac{\partial L}{\partial B_t} = \lambda_t (1+i_t)^{-1} - \beta \lambda_{t+1}$$

where the Labor supply equation would be:

$$\text{from } \lambda_{1t} = \lambda_{2t} \text{ I get } \frac{W_t}{P_t} = d_n e^{\varsigma_t} N^{\varphi} \overline{C}_t^{\sigma} \left(\frac{C_t}{\widetilde{C}_t}\right)^{\chi} \delta^{-\chi} \tag{A.4}$$

I find the Euler equation starting from: $\lambda_t = \beta(1+i_t)\lambda_{t+1}$

$$\text{and then I get } \frac{\delta^{\chi} \widetilde{C}_t^{-\sigma} \left(\frac{C_t}{\widetilde{C}_t}\right)^{-\chi}}{P_t} = \beta(1+i_t)^{-1} \frac{\delta^{\chi} \widetilde{C}_{t+1}^{-\sigma} \left(\frac{C_{t+1}}{\widetilde{C}_{t+1}}\right)^{-\chi}}{P_{t+1}} \tag{A.5}$$

which I express as:

$$1 = \beta(1+i_t)\mathbb{E}_t \left[\left(\frac{P_t}{P_{t+1}}\right) \left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t}\right)^{\chi-\sigma} \left(\frac{C_{t+1}}{C_t}\right)^{-\chi}\right] \tag{A.6}$$

A.2.2 Recursive formulation of the optimal price−setting equation

I use a recursive formulation of the optimal price-setting equation to find ψ_t and ϕ_t as discussed in the text. First, I start from:

$$p^*_{j,t} = \frac{\varepsilon}{(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{l=0}^{\infty} \theta^l D_{t,t+l} Y_{t+l} \Pi^{\varepsilon}_{t,t+l} \frac{W_{t+l}}{A_{t+l}P_{t+l}}}{\mathbb{E}_t \sum_{l=0}^{\infty} \theta^l D_{t,t+l} Y_{t+l} \Pi^{\varepsilon-1}_{t,t+l}} \text{ and I write } p^*_{j,t} = \frac{\varepsilon}{(\varepsilon-1)} \frac{\psi_t}{\phi_t}$$

where, denoting the real wage as $w_t = \frac{W_t}{P_t}$; using the definition of the discount factor $D_{t,t+l} \equiv \beta^l \frac{\lambda_{t+l}}{\lambda_0}$, and the fact that $\lambda_{t+l} = \overline{C}^{\chi-\sigma}_{t+l} C^{-\chi}_{t+l}$, or $D_{t,t+1} = \beta \frac{\overline{C}^{\chi-\sigma}_{t+1} C^{-\chi}_{t+1}}{\overline{C}^{\chi-\sigma}_{t} C^{-\chi}_{t}}$, the auxiliary variables ψ_t and ϕ_t are defined, respectively, as:

$$\psi_t = \mathbb{E}_t \sum_{l=0}^{\infty} (\theta\beta)^l \overline{C}^{\chi-\sigma}_{t+l} C^{-\chi}_{t+l} Y_{t+l} w_{t+l} (A_{t+l})^{-1} \Pi^{\varepsilon}_{t,t+l} \text{ and } \phi_t = \mathbb{E}_t \sum_{l=0}^{\infty} (\theta\beta)^l \overline{C}^{\chi-\sigma}_{t+l} C^{-\chi}_{t+l} Y_{t+l} \Pi^{\varepsilon-1}_{t,t+l} \tag{A.7}$$

I can write the two above infinitive summations recursively, starting first with ψ_t:

$$\begin{aligned} \psi_t &= w_t {A_t}^{-1} \overline{C}^{\chi-\sigma}_t C_t^{-\chi} Y_t + \theta\beta \mathbb{E}_t \left[w_{t+1} {A_{t+1}}^{-1} \overline{C}^{\chi-\sigma}_{t+1} C^{-\chi}_{t+1} Y_{t+1} \Pi^{\varepsilon}_{t,t+1} \right] \\ &\quad + \theta\beta^2 \mathbb{E}_t \left[w_{t+2} {A_{t+2}}^{-1} \overline{C}^{\chi-\sigma}_{t+2} C^{-\chi}_{t+2} Y_{t+2} \Pi^{\varepsilon}_{t,t+2} \right] + \cdots . \end{aligned} \tag{A.8}$$

The above equation can be readjusted as:

$$\begin{aligned} \psi_t &= w_t {A_t}^{-1} \overline{C}^{\chi-\sigma}_t C_t^{-\chi} Y_t + \theta\beta \mathbb{E}_t \pi^{\varepsilon}_{t+1} \left[w_{t+1} {A_{t+1}}^{-1} \overline{C}^{\chi-\sigma}_{t+1} C^{-\chi}_{t+1} Y_{t+1} \right] \\ &\quad + \theta\beta \Pi^{\varepsilon}_{t+1,t+2} w_{t+2} {A_{t+2}}^{-1} \overline{C}^{\chi-\sigma}_{t+2} C^{-\chi}_{t+2} Y_{t+2} + \cdots . \end{aligned} \tag{A.9}$$

The expression in the square brackets is exactly the definition for $\mathbb{E}_t \psi_{t+1}$ and because of this I would have:

$$\psi_t = w_t {A_t}^{-1} \overline{C}^{\chi-\sigma}_t C_t^{-\chi} Y_t + \theta\beta \mathbb{E}_t \left[\pi^{\varepsilon}_{t+1} \psi_{t+1} \right] \tag{A.10}$$

replacing the $MC_t = w_t A_t^{-1}$ the equation would be:

$$\psi_t = MC_t \overline{C}^{\chi-\sigma}_t C_t^{-\chi} Y_t + \theta\beta \mathbb{E}_t \left[\pi^{\varepsilon}_{t+1} \psi_{t+1} \right] \tag{A.11}$$

Now the recursive form of ϕ_t following the same procedure, it is possible to quasi-differentiate the equation $\phi_t = \mathbb{E}_t \sum_{l=0}^{\infty} (\theta\beta)^l \Pi_{t,t+l}^{\varepsilon-1} Y_{t+l}^{1-\sigma}$, would bring:

$$\phi_t = \overline{C}_t^{\chi-\sigma} C_t^{-\chi} Y_t + \theta\beta\mathbb{E}_t \left[\Pi_{t,t+1}^{\varepsilon-1} \overline{C}_{t+1}^{\chi-\sigma} C_{t+1}^{-\chi} Y_{t+1}\right] + \mathbb{E}_t \left[(\theta\beta)^2 \Pi_{t,t+2}^{\varepsilon-1} \overline{C}_{t+2}^{\chi-\sigma} C_{t+2}^{-\chi} Y_{t+2}\right] + \cdots \tag{A.12}$$

$$\phi_t = \overline{C}_t^{\chi-\sigma} C_t^{-\chi} Y_t + \theta\beta\mathbb{E}_t \left\{\pi_{t,t+1}^{\varepsilon-1} \left[\overline{C}_{t+1}^{\chi-\sigma} C_{t+1}^{-\chi} Y_{t+1} + \theta\beta \Pi_{t+1,t+2}^{\varepsilon-1} \overline{C}_{t+2}^{\chi-\sigma} C_{t+2}^{-\chi} Y_{t+2} + \cdots\right]\right\} \tag{A.13}$$

and obtain:

$$\phi_t = \overline{C}_t^{\chi-\sigma} C_t^{-\chi} Y_t + \theta\beta\mathbb{E}_t \left[\pi_{t+1}^{\varepsilon-1} \phi_{t+1}\right] \tag{A.14}$$

These variables ψ_t and ϕ_t can be interpreted as the present discounted value of marginal costs and marginal revenues (for a unit change in the optimal reset price), respectively.

A.2.3 The complete non−linear system

To summarize, the complete non-linear model, which I reproduce here for convenience:

$$Y_t = C_t + G_t \tag{A.15}$$

$$1 = \beta(1+i_t)\mathbb{E}_t \left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t}\right)^{\chi-\sigma}\left(\frac{C_{t+1}}{C_t}\right)^{-\chi}\right] \tag{A.16}$$

$$w_t = d_n e^{\varsigma_t} N^{\varphi} \widetilde{C}_t^{\sigma} \left(\frac{C_t}{\widetilde{C}_t}\right)^{\chi} \delta^{-\chi} \tag{A.17}$$

$$\widetilde{C}_t = \left[\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{A.18}$$

$$p_{j,t}^* = \left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} \tag{A.19}$$

$$p_{j,t}^* = \frac{\varepsilon}{(\varepsilon-1)} \frac{\psi_t}{\phi_t} \tag{A.20}$$

$$\psi_t = MC_t \overline{C}_t^{\chi-\sigma} C_t^{-\chi} Y_t + \theta\beta\mathbb{E}_t \left[\pi_{t+1}^{\varepsilon} \psi_{t+1}\right] \tag{A.21}$$

$$\phi_t = \overline{C}_t^{\chi-\sigma} C_t^{-\chi} Y_t + \theta\beta\mathbb{E}_t \left[\pi_{t+1}^{\varepsilon-1} \phi_{t+1}\right] \tag{A.22}$$

$$MC_t = w_t A_t^{-1} \tag{A.23}$$

$$N_t = s_t \frac{Y_t}{A_t} \tag{A.24}$$

$$s_t = (1-\theta)(p^*_{j,t})^{-\varepsilon} + \theta \pi_t^{\varepsilon} s_{t-1} \tag{A.25}$$

$$\left(\frac{1+i_t}{1+\bar{i}}\right) = \left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\rho_i} \left(\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_Y}\right)^{1-\rho_i} e^{v_t} \tag{A.26}$$

The model has nine endogenous variable: Y_t,C_t, G_t, i_t, π_t, w_t, N_t, $p_{i,t}$, ψ_t, ϕ_t, s_t and four exogenous shocks: A_t, ς_t, v_t, G_t.

A.2.4 The log−linearization of the resource constraint and obtaining the IS curve

Market clearing in the goods market requires: $Y_{t,j} = C_{t,j} + G_{t,j}$, for all $j \in [0,1]$ and all t or $Y_t = C_t + G_t$. The log-linearized Euler equation would be:

$$\log C_t^{-\chi} = \log\beta + \log(1+i_t) + \log \mathbb{E}_t \pi_{t+1}^{-1} + \log\log \mathbb{E}_t \left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t}\right)^{\chi-\sigma} + \log\log \mathbb{E}_t C_{t+1}^{-\chi} \tag{A.27}$$

Where I substitute $\widehat{\widetilde{C}}_t = \delta\widehat{C}_t + (1-\delta)\widehat{G}_t$ which is the log-linear version of

$\widetilde{C}_t = \left[\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right]^{\frac{1}{1-\chi}}$ and this gives:

$$\widehat{C}_t = \mathbb{E}_t \widehat{C}_{t+1} - \frac{1}{\sigma_\delta}(i_t - \mathbb{E}_t\{\widehat{\pi}_{t+1}\}) + \frac{(\sigma-\sigma_\delta)}{\sigma_\delta}\mathbb{E}_t \Delta\widehat{G}_{t+1} \tag{A.28}$$

Integrating the Euler equation over $i \in [0,1]$ and combining the resulting difference equation with $\widehat{Y}_t = (1-\gamma)\widehat{C}_t + \gamma\widehat{G}_t$, yields the union wide dynamic IS equation:

$$\widehat{Y}_t = \widehat{Y}_{t+1} - \frac{(1-\gamma)}{\sigma_\delta}(i_t - \widehat{\pi}_{t+1}) + \left(\frac{(1-\gamma)\sigma - \sigma_\delta}{\sigma_\delta}\right)(\widehat{G}_{t+1} - \widehat{G}_t) \tag{A.29}$$

A.2.5 The complete log−linear equations

$$\widehat{C}_t = \mathbb{E}_t \widehat{C}_{t+1} - \frac{1}{\sigma_\delta}(i_t - \mathbb{E}_t\{\widehat{\pi}_{t+1}\}) + \frac{(\sigma-\sigma_\delta)}{\sigma_\delta}\mathbb{E}_t \Delta\widehat{G}_{t+1} \tag{A.30}$$

or

$$\widehat{Y}_t = \widehat{Y}_{t+1} - \frac{(1-\gamma)}{\sigma_\delta}(i_t - \widehat{\pi}_{t+1}) + \left(\frac{(1-\gamma)\sigma - \sigma_\delta}{\sigma_\delta}\right)(\widehat{G}_{t+1} - \widehat{G}_t) \tag{A.31}$$

$$w_t = \varsigma_t + \varphi\widehat{N}_t + \sigma_\delta\widehat{C}_t + (\sigma - \sigma_\delta)\widehat{G}_t \tag{A.32}$$

$$\widehat{N}_t = (\widehat{Y}_t - \widehat{A}_t) + \widehat{s}_t \tag{A.33}$$

$$\widehat{i}_t = \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi\widehat{\pi}_t + \phi_Y\widehat{Y}_t\right) + v_t \tag{A.34}$$

$$\widehat{p}_{i,t}^{*} = \widehat{\psi}_t - \widehat{\phi}_t \tag{A.35}$$

$$\widehat{\psi}_t = (1-\theta\beta\overline{\pi}^{\varepsilon})\left[\widehat{w}_t - \widehat{A}_t + (\chi-\sigma)\widehat{\widehat{C}}_t - \chi\widehat{C}_t + \widehat{Y}_t\right] + \theta\beta\overline{\pi}^{\varepsilon}\left[\varepsilon\mathbb{E}_t\widehat{\pi}_{t+1} + \mathbb{E}_t\widehat{\psi}_{t+1}\right] \tag{A.36}$$

$$\widehat{\phi}_t = \left(1-\theta\beta\overline{\pi}^{\varepsilon-1}\right)\left((\chi-\sigma)\widehat{\widehat{C}}_t - \chi\widehat{C}_t + \widehat{Y}_t\right) + \theta\beta\overline{\pi}^{\varepsilon-1}\left[(\varepsilon-1)\mathbb{E}_t\widehat{\pi}_{t+1} + \mathbb{E}_t\widehat{\phi}_{t+1}\right] \tag{A.37}$$

$$\widehat{s}_t = -\varepsilon\left(1-\theta\overline{\pi}^{\varepsilon}\right)\widehat{p}_{i,t}^{*} + \theta\overline{\pi}^{\varepsilon}\left(\varepsilon\widehat{\pi}_t + \widehat{s}_{t-1}\right) \tag{A.38}$$

$$\widehat{\pi}_t = k(\overline{\pi})\widehat{mc}_t + b_1(\overline{\pi})\mathbb{E}_t\widehat{\pi}_{t+1} + b_2(\overline{\pi})\left[(\chi-\sigma)\widehat{\widehat{C}}_t - \chi\widehat{C}_t + \widehat{Y}_t - \mathbb{E}_t\widehat{\psi}_{t+1}\right] \tag{A.39}$$

This gives a Philips curve that comprises the dynamics of inflation. Where: $k(\overline{\pi}) = \frac{\left(1-\theta\overline{\pi}^{\varepsilon-1}\right)\left(1-\theta\beta\overline{\pi}^{\varepsilon}\right)}{\theta\overline{\pi}^{\varepsilon-1}}$, $b_1(\overline{\pi}) = \beta\left[1+\varepsilon\left(\overline{\pi}-1\right)\left(1-\theta\overline{\pi}^{\varepsilon-1}\right)\right]$, $b_2(\overline{\pi}) = \beta\left[1-\overline{\pi}\right]\left(1-\theta\overline{\pi}^{\varepsilon-1}\right)$. While price dispersion is written as:

$$\widehat{s}_t = \left(\frac{\varepsilon\theta\overline{\pi}^{\varepsilon-1}}{\left(1-\theta\overline{\pi}^{\varepsilon-1}\right)}\left(\overline{\pi}-1\right)\right)\widehat{\pi}_t + \theta\overline{\pi}^{\varepsilon}\widehat{s}_{t-1} \tag{A.40}$$

The log-linearized aggregate resource constraint would be:

$$\widehat{Y}_t = (1-\gamma)\widehat{C}_t + \gamma\widehat{G}_t \tag{A.41}$$

For the aggregate consumption bundle I have:

$$\widehat{\widehat{C}} = \delta\widehat{C}_t + (1-\delta)\widehat{G}_t \tag{A.42}$$

The shocks of the economy can be expressed as: the technology shock $\widehat{A}_t = \rho_A\widehat{A}_{t-1} + \mu_{At}$, the labor supply shock $\varsigma_t = \rho_\sigma\varsigma_{t-1} + \mu_{\varsigma_t}$, the monetary policy shock in the Taylor rule $\upsilon_t = \rho_\upsilon\upsilon_{t-1} + \mu$ and the government spending shock: $\widehat{G}_t = \rho_G\widehat{G}_{t-1} + \mu_{Gt}$. The innovations μ_{At}, μ_{ς_t}, $\mu_{\upsilon t}$, and μ_{Gt} are assumed to be i.i.d. standard normal processes.

A.2.6 The key log-linear equations for the GHH preferences case

The main log−linearized equations the IS curve, labor supply and Philips curve would be: The IS curve

$$\widehat{Y}_t = E\left\{\widehat{Y}_{t+1}\right\} - \gamma E\left\{\Delta\widehat{G}_{t+1}\right\} + (1-\gamma)\left(\left(\mathbb{E}_t\left(\widehat{\pi}_{t+1}\right) - \widehat{i}_t\right) - (1+\widetilde{\varphi})\Delta\mathbb{E}_t\widehat{N}_{t+1}\right) \tag{A.43}$$

The labor supply curve

$$\widehat{w}_t = \widetilde{\varphi}\widehat{N}_t \tag{A.44}$$

GNKPC for GHH preferences

$$\widehat{\pi}_t = k(\overline{\pi})\widehat{mc}_t + b_1(\overline{\pi})\mathbb{E}_t\widehat{\pi}_{t+1} + b_2(\overline{\pi})\left[\widehat{Y}_t - \widehat{C}_t + (\widetilde{\varphi}+1)\widehat{L}_t - \mathbb{E}_t\widehat{\psi}_{t+1}\right] \tag{A.45}$$

Appendix A.3

Proof: In order to prove that a government spending shock crowds in private consumption and then that an increase in trend inflation amplifies the response of consumption, I start by making some basic assumptions to simplify my calculations, where: log preferences in consumption ($\sigma = 1$), indivisible labor ($\varphi = 0$), and no persistence in the shocks ($\rho_\sigma = 0$ for $i = A,\varsigma,\upsilon$), while I keep the persistence for . For the Taylor rule, I assume no inertia. The log-linearized GNK model now can be described by the following main equations, seen in Appendix B:

$$\widehat{\pi}_t = k(\overline{\pi})\left[\varsigma_t - \widehat{A}_t + (\sigma-\sigma_\delta)\widehat{G}\right] + b_1(\overline{\pi})\mathbb{E}_t\widehat{\pi}_{t+1} + b_2(\overline{\pi})\left[\left((\chi-\sigma)\widehat{\widehat{C}}_t - \chi\widehat{C}_t + \widehat{Y}_t\right) - \mathbb{E}_t\widehat{\psi}_{t+1}\right] \tag{A.46}$$

$$\widehat{C}_t = \mathbb{E}_t\widehat{C}_{t+1} - \frac{1}{\sigma_\delta}(i_t - \mathbb{E}_t\{\widehat{\pi}_{t+1}\}) + \frac{(\sigma-\sigma_\delta)}{\sigma_\delta}\mathbb{E}_t\Delta\widehat{G}_{t+1} \tag{A.47}$$

$$\widehat{\widehat{C}} = \delta\widehat{C}_t + (1-\delta)\widehat{G}_t \tag{A.48}$$

$$\widehat{Y}_t = (1-\gamma)\widehat{C}_t + \gamma\widehat{G}_t \tag{A.49}$$

$$\widehat{\psi}_t = (1-\theta\beta\overline{\pi}^{\varepsilon})\left[\varsigma_t - \widehat{A}_t + (\sigma-\sigma_\delta)\widehat{G} + \left((\chi-\sigma)\widehat{\widehat{C}}_t - \chi\widehat{C}_t + \widehat{Y}_t\right)\right] + \theta\beta\overline{\pi}^{\varepsilon}\left[\varepsilon\mathbb{E}_t\widehat{\pi}_{t+1} + \mathbb{E}_t\widehat{\psi}_{t+1}\right] \tag{A.50}$$

$$\widehat{i}_t = \phi_\pi\widehat{\pi}_t + \phi_Y\widehat{Y}_t + v_t \tag{A.51}$$

Price dispersion, $\widehat{s}_t$, does not affect the dynamics of the above system, because of the assumption made where linear utility in labor is ($\varphi = 0$). In this economy, I specify the following auto-regressive processes for the 4 shocks of the baseline model, technology, labor supply, monetary and government spending shocks, which are specified as AR(1) processes and the innovations μ_{At}, μ_{ς_t}, μ_{Gt} and $\mu_{\upsilon t}$ are assumed to be i.i.d. standard normal processes.

$$\widehat{A}_t = \rho_A\widehat{A}_{t-1} + \mu_{At} \text{ and } \varsigma_t = \rho_\sigma\varsigma_{t-1} + \mu_{\varsigma_t} \text{ and } \upsilon_t = \rho_\upsilon\upsilon_{t-1} + \mu_{\upsilon t} \text{ and } \widehat{G}_t = \rho_G\widehat{G}_{t-1} + \mu_{Gt}$$

Given that the shocks are i.i.d., there are no transitional dynamics and the economy returns to the steady-state in the period after the shock. Focusing on the effect of trend inflation on private consumption, I use the method of undetermined coefficients to solve the system of equations, the impact of the government spending shock is derived as follows: First I write

$z_t = \varsigma_t - \widehat{A}_t$, secondly I substitute the Taylor equation into the Euler Equation, obtaining:

$$\widehat{C}_t = \widehat{C}_{t+1} - \frac{1}{\sigma_\delta}(\phi_\pi \widehat{\pi}_t + \phi_Y \widehat{Y}_t + v_t - \widehat{\pi}_{t+1}) + \frac{(\sigma - \sigma_\delta)}{\sigma_\delta}\Delta\widehat{G}_{t+1} \tag{A.52}$$

and

$$\widehat{\pi}_t = k(\overline{\pi})\left[z_t + (\sigma - \sigma_\delta)\,\widehat{G}\right] + b_1(\overline{\pi})\mathbb{E}_t\widehat{\pi}_{t+1} + b_2(\overline{\pi})\left[\left((\chi - \sigma)\widehat{\widehat{C}}_t - \chi\widehat{C}_t + \widehat{Y}_t\right) - \mathbb{E}_t\widehat{\psi}_{t+1}\right] \tag{A.53}$$

Further, I substitute $\Delta\widehat{G}_{t+1} = \widehat{\widehat{C}}_{t+1} - \frac{\delta}{(1-\delta)}\widehat{C}_{t+1} - \widehat{\widehat{C}}_t + \frac{\delta}{(1-\delta)}\widehat{C}_t$ into the Euler Equation and $\widehat{Y}_t = (1-\gamma)\widehat{C}_t + \gamma\widehat{G}_t$. Then I express the system of equations as:

$$\begin{bmatrix} \widehat{\pi}_t \\ \widehat{C}_t \\ \widehat{\psi}_t \\ \widehat{\widehat{C}}_t \end{bmatrix} = \begin{bmatrix} \widehat{\pi}_z & \widehat{\pi}_v & \widehat{\pi}_g \\ \widehat{C}_z & \widehat{C}_v & \widehat{C}_g \\ \widehat{\psi}_z & \widehat{\psi}_v & \widehat{\psi}_g \\ \widehat{\widehat{C}}_z & \widehat{\widehat{C}}_v & \widehat{\widehat{C}}_g \end{bmatrix} \begin{bmatrix} z_t \\ v_t \\ g_t \end{bmatrix} \tag{A.54}$$

where $z_t = \varsigma_t - \widehat{A}_t$, and:

$$\mathbb{E}_t \begin{bmatrix} \widehat{\pi}_{t+1} \\ \widehat{C}_{t+1} \\ \widehat{\psi}_{t+1} \\ \widehat{\widehat{C}}_{t+1} \\ \widehat{i}_{t+1} \end{bmatrix} = \begin{bmatrix} \widehat{\pi}_z & \widehat{\pi}_v & \widehat{\pi}_g \\ \widehat{C}_z & \widehat{C}_v & \widehat{C}_g \\ \widehat{\psi}_z & \widehat{\psi}_v & \widehat{\psi}_g \\ \widehat{\widehat{C}}_z & \widehat{\widehat{C}}_v & \widehat{\widehat{C}}_g \\ \widehat{i}_z & \widehat{i}_v & \widehat{i}_g \end{bmatrix} \begin{bmatrix} \rho z_t \\ \rho_\upsilon v_t \\ \rho_g g_t \end{bmatrix} \tag{A.55}$$

where $\rho z_t = \rho_\sigma \varsigma_t - \rho_A \widehat{A}_t$.

I would have now the new equations where I keep only the term g_t:

$$\pi_g g_t = k(\overline{\pi})\,(\sigma - \sigma_\delta)\,g_t + b_1(\overline{\pi})\pi_g\rho_g g_t \tag{A.56}$$

$$+ b_2(\overline{\pi})\left[(\chi - \sigma)\,d_g g_t + (1 - \gamma - \chi)\,c_g g_t + \gamma g_t - \psi_g\rho_g g_t\right]$$

$$c_g g_t = c_g\rho_g g_t - \frac{1}{\sigma_\delta}\left[\phi_\pi\pi_g g_t + (1-\gamma)\phi_Y\,(c_g g_t + \gamma g_t) - \pi_g\rho_g g_t\right] \tag{A.57}$$

$$+ \frac{(\sigma - \sigma_\delta)}{\sigma_\delta}\left(d_g\rho_g g_t - d_g g_t - \frac{\delta}{(1-\delta)}c_g\rho_g g_t + \frac{\delta}{(1-\delta)}\,(c_g g_t + \gamma g_t)\right)$$

$$\psi_g g_t = (1 - \theta\beta\overline{\pi}^{\varepsilon})\left[(\sigma - \sigma_\delta)\,g_t + ((\chi - \sigma)\,d_g g_t - \chi c_g g_t + (1-\gamma)c_g g_t + \gamma g_t)\right] \tag{A.58}$$

$$+\theta\beta\overline{\pi}^{\varepsilon}\left[\varepsilon\pi_g\rho_g g_t+\psi_g\rho_g g_t\right]$$

$$d_g g_t=\delta\left(c_g g_t\right)+(1-\delta)g_t \tag{A.59}$$

Dividing on both sides by g I would have the equations once more:

$$\pi_g=k(\overline{\pi})\left(\sigma-\sigma_\delta\right)+b_1(\overline{\pi})\pi_g\rho_g+b_2(\overline{\pi})\left[\left(\chi-\sigma\right)d_g+\left(1-\gamma-\chi\right)c_g+\gamma-\psi_g\rho_g\right] \tag{A.60}$$

$$c_g=c_g\rho_g-\frac{1}{\sigma_\delta}\left[\phi_\pi\pi_g+(1-\gamma)\phi_Y\left(c_g+\gamma\right)-\pi_g\rho_g\right] \tag{A.61}$$

$$+\frac{(\sigma-\sigma_\delta)}{\sigma_\delta}\left(d_g\rho_g-d_g-\frac{\delta}{(1-\delta)}c_g\rho_g+\frac{\delta}{(1-\delta)}\left(c_g+\gamma\right)\right)$$

$$\psi_g=\left(1-\theta\beta\overline{\pi}^{\varepsilon}\right)\left[\left(\sigma-\sigma_\delta\right)+\left(\left(\chi-\sigma\right)d_g-\chi c_g+(1-\gamma)c_g+\gamma\right)\right] \tag{A.62}$$

$$+\theta\beta\overline{\pi}^{\varepsilon}\left[\varepsilon\pi_g\rho_g+\psi_g\rho_g\right]$$

$$d_g=\delta c_g+(1-\delta) \tag{A.63}$$

Next I assume the persistence in the shocks ($\rho_\sigma=0$ for $i=A,\varsigma,\upsilon,G_t$), and I solve for the undetermined coefficients π_g, ψ_g, y_g, c_g, by first substituting d_g to c_g and d_g to π_g, and then π_g into c_g to obtain its the solution for c_g:

I substitute d_g to π_g and rearrange:

$$\begin{aligned}\pi_g \;&=\; k(\overline{\pi})\left(\sigma-\sigma_\delta\right)+\left[\left(\delta-1\right)\chi+1-\gamma-\delta\sigma\right]b_2(\overline{\pi})c_g+\\ &\quad\; b_2(\overline{\pi})\left[\chi\left(1-\delta\right)-\sigma+\sigma\delta+\gamma\right]\end{aligned} \tag{A.64}$$

I substitute d_g to c_g:

$$\begin{aligned}c_g \;&=\; -\frac{1}{\sigma_\delta}\left[\phi_\pi\pi_g+(1-\gamma)\phi_Y\left(c_g+\gamma\right)\right]+\\ &\quad\;\frac{(\sigma-\sigma_\delta)}{\sigma_\delta}\left(-\left(\delta c_g+(1-\delta)\right)+\frac{\delta}{(1-\delta)}\left(c_g+\gamma\right)\right)\end{aligned} \tag{A.65}$$

by substituting π_g into c_g I can rearrange to obtain finally the effect of the government spending shock on private consumption:

$$c_g=\frac{c_1k(\overline{\pi})+c_2b_2(\overline{\pi})+c_3}{c_4+c_5b_2(\overline{\pi})} \tag{A.66}$$

where I have defined as: $c_1 = -\frac{1}{\sigma_\delta}\phi_\pi(1-\sigma_\delta)$ and $c_2 = -\frac{\phi_\pi[\chi(1-\delta)-1+\delta+\gamma]}{\sigma_\delta}$ and $c_3 = \frac{(1-\sigma_\delta)}{\sigma_\delta}\frac{\delta}{(1-\delta)}\gamma - \frac{1}{\sigma_\delta}(1-\gamma)\phi_Y\gamma - \frac{(1-\sigma_\delta)}{\sigma_\delta}(1-\delta)$ and $c_4 = 1+\frac{(1-\gamma)\phi_Y}{\sigma_\delta}+\frac{(1-\sigma_\delta)}{\sigma_\delta}\delta - \frac{(1-\sigma_\delta)}{\sigma_\delta}\frac{\delta}{(1-\delta)}$ and $c_5 = \frac{\phi_\pi[(\delta-1)\chi+1-\gamma-\delta]}{\sigma_\delta}$, and then $\widehat{C}_t = c_g g_t$ shows how government spending affects private consumption. An increase in government spending, as shown in Proposition 1, induces an increase in private consumption.

For a value of $\chi = \sigma = 1$, we would have the Euler Equation would take its standard form, and Equation A.66 would turn into:

$$c_g = \frac{\gamma b_2(\overline{\pi})\phi_\pi - \phi_Y\gamma}{(1+b_2(\overline{\pi})\phi_\pi + \phi_Y(1-\gamma))} \tag{A.67}$$

which for $\overline{\pi} = 1$ and $\beta\phi_\pi > \phi_y$ shows clearly that $c_g > 0$. Now it is easy to look at the effects of trend inflation on the impact of the government spending shocks. What I am looking for is $\frac{\partial c_g}{\partial \overline{\pi}}$, which is

$$\frac{\partial c_g}{\partial \overline{\pi}} = \frac{\partial \frac{\gamma b_2(\overline{\pi})\phi_\pi - \phi_Y\gamma}{(1+b_2(\overline{\pi})\phi_\pi+\phi_Y(1-\gamma))}}{\partial \overline{\pi}} = \frac{\phi_\pi(\gamma b_2(\overline{\pi})\phi_\pi - \phi_Y\gamma) - (1+b_2(\overline{\pi})\phi_\pi + \phi_Y(1-\gamma))\gamma\phi_\pi}{[1+b_2(\overline{\pi})\phi_\pi + \phi_Y(1-\gamma)]^2} \tag{A.68}$$

$$\frac{\partial c_g}{\partial \overline{\pi}} = \frac{\phi_\pi[\gamma b_2(\overline{\pi})\phi_\pi - \phi_Y\gamma - \gamma - \gamma b_2(\overline{\pi})\phi_\pi - \gamma\phi_Y(1-\gamma)]}{[1+b_2(\overline{\pi})\phi_\pi + \phi_Y(1-\gamma)]^2} \tag{A.69}$$

$$\frac{\partial c_g}{\partial \overline{\pi}} = \frac{\gamma\phi_\pi[-\phi_Y - 1 - \phi_Y + \gamma\phi_Y]}{[1+b_2(\overline{\pi})\phi_\pi + \phi_Y(1-\gamma)]^2} \tag{A.70}$$

$$\frac{\partial c_g}{\partial \overline{\pi}} = \frac{\gamma\phi_\pi[\gamma\phi_Y - 2\phi_Y - 1]}{[1+b_2(\overline{\pi})\phi_\pi + \phi_Y(1-\gamma)]^2}\frac{\partial b_2(\overline{\pi})}{\partial \overline{\pi}} > 0 \tag{A.71}$$

$$\frac{\partial b_2(\overline{\pi})}{\partial \overline{\pi}} = \frac{\partial \beta[1-\overline{\pi}]\left(1-\theta\overline{\pi}^{\varepsilon-1}\right)}{\partial \overline{\pi}} = \beta\left[-(\varepsilon-1)\theta\overline{\pi}^{\varepsilon-2} - 1 - \theta\varepsilon\overline{\pi}^{\varepsilon-1}\right] < 0 \tag{A.72}$$

$$b_2(\overline{\pi}) = \beta[1-\overline{\pi}]\left(1-\theta\overline{\pi}^{\varepsilon-1}\right) = \beta\left[1-\theta\overline{\pi}^{\varepsilon-1} - \overline{\pi} - \theta\overline{\pi}^{\varepsilon}\right] \tag{A.73}$$

Since $c_g = \frac{\beta\gamma\phi_\pi - \phi_Y\gamma}{(1+\beta\phi_\pi+\phi_Y(1-\gamma))} >$, $\frac{\partial b_2(\overline{\pi})}{\partial \overline{\pi}} < 0$ and $\gamma\phi_\pi[\gamma\phi_Y - 2\phi_Y - 1] < 0$, as $\phi_\pi > 1$, $\phi_y > 0$ and $0 < \gamma < 1$, as consequence $\frac{\partial c_g}{\partial \overline{\pi}} > 0$, so it follows that an increase in trend inflation increases the absolute value of the response of private consumption.

Appendix B

Appendices For Chapter Three

Appendix B.1 Additional Impulse Response Functions

B.1.1 IRFs in the exercise section

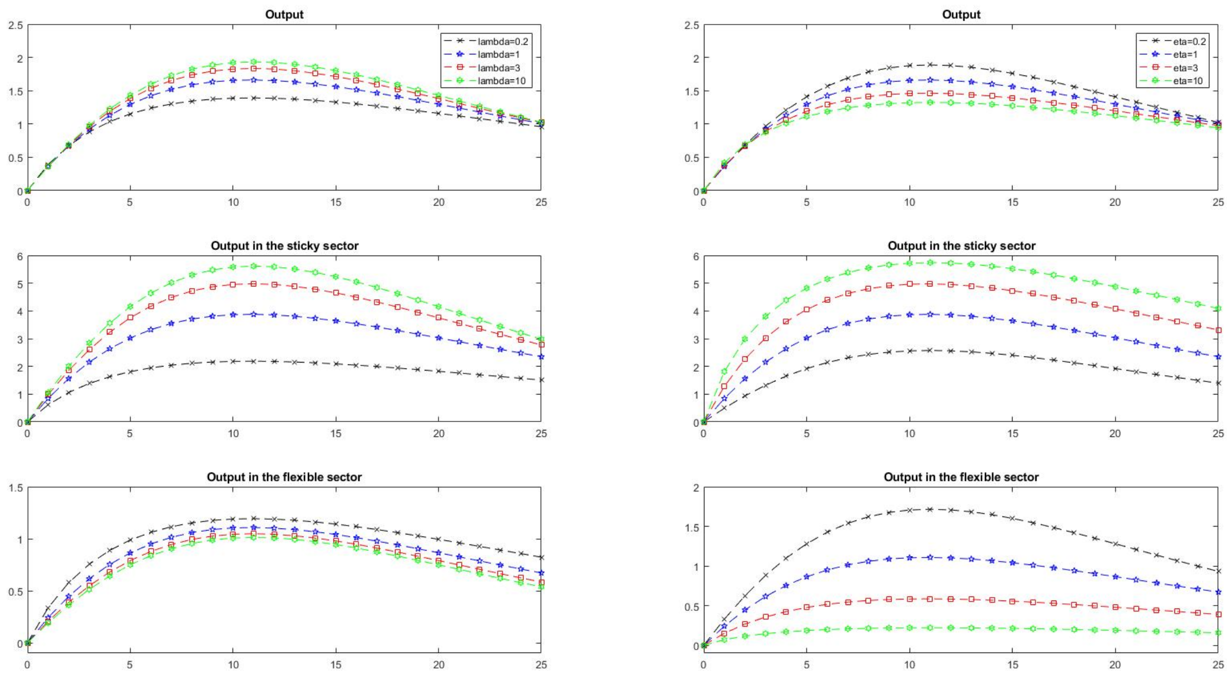


Figure B.1 IRFs to Monetary Policy Shock GHH exercise

Notes: The figure presents the impulse responses of Output to a 1% Monetary Policy Shock in the model subject to the changes in the parameters of λ (left column) and η (right column). In the first row are presented the results of total output, in the second row, output in the sticky sector, and in the third row, output in the flexible sector. The four lines represent respectively the impulse responses under a level of 0.002, 1, 3, and 10 of λ (η), with corresponding colors as black, blue, red, and green. The sectoral shares for this case are $\alpha_1 = 0.8$ and $\alpha_1 = 0.2$.

B.1.2 IRFs and tables in the calibrated model section

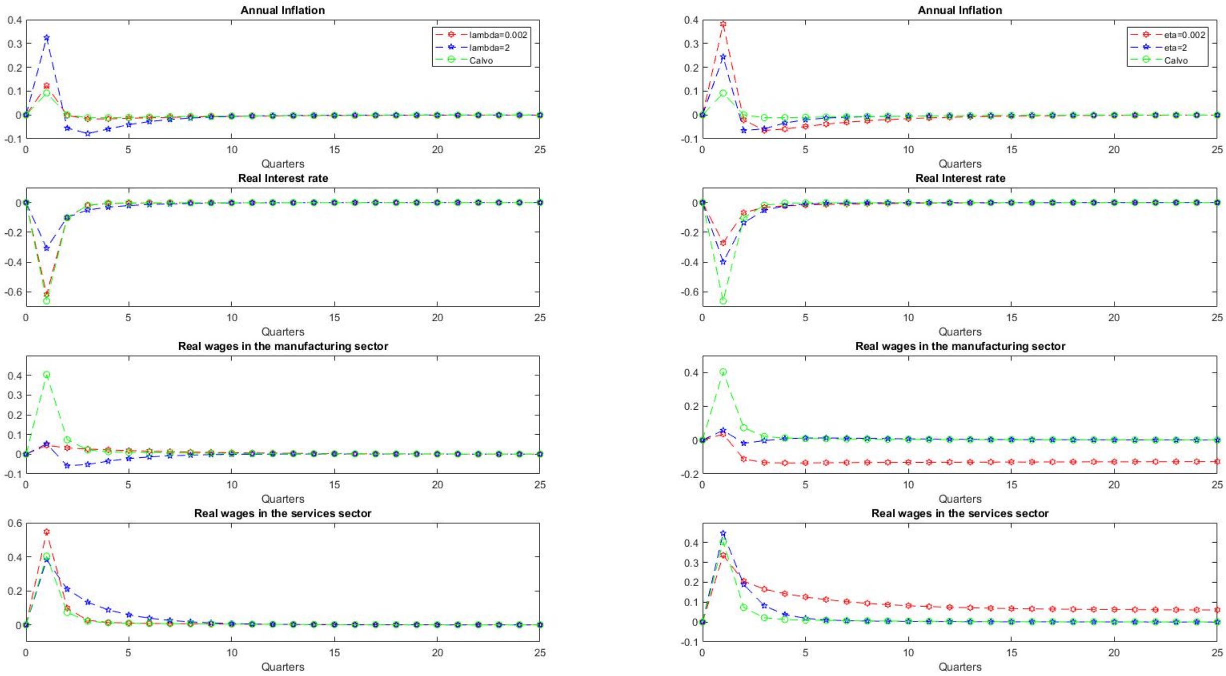


Figure B.2 Other variables IRFs to Monetary Policy Shock for Manufacturing and Services

Notes: The figure presents the impulse responses to Monetary Policy Shock in the model with manufacturing and services. In the first row are presented the results of annual inflation, in the second row, real annual interest rate, in the third row, real wages in the manufacturing sector, and in the fourth row, real wages in the services sector. The two lines represent respectively the IRF under a level of 0.002 and 2 of λ (left column) and η (right column), with corresponding colors as red and blue. The Calvo model results are represented in the green line. The Calvo model results are represented in the green line.

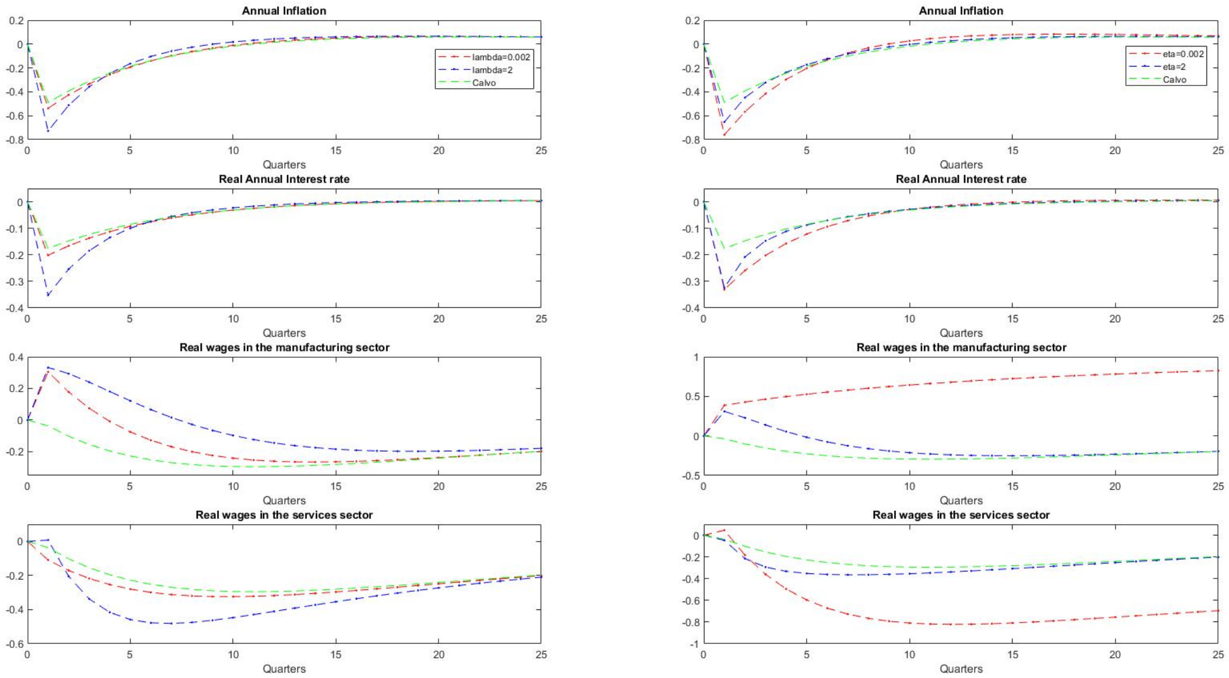


Figure B.3 Other variables IRFs to TFP shock for Manufacturing and Services

Notes: The figure presents the impulse responses to a TFP Shock in the model with manufacturing and services. In the first row are presented the results of annual inflation, in the second row, real annual interest rate, in the third row, real wages in the manufacturing sector, and in the fourth row, real wages in the services sector. The two lines represent respectively the IRF under a level of 0.002 and 2 of λ (left column) and η (right column), with corresponding colors as red and blue. The Calvo model results are represented in the green line.

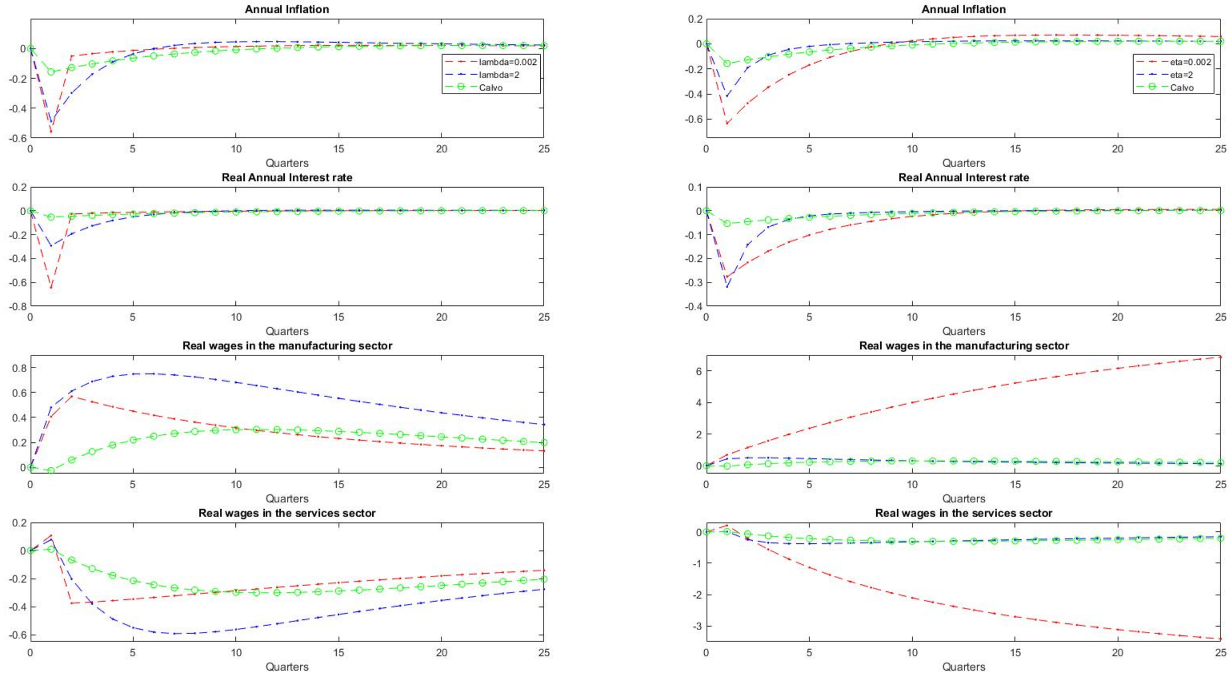


Figure B.4 Other variables IRFs to Technology shock in the manufacturing sector

Notes: The figure presents the impulse responses to a Technology Shock in manufacturing. In the first row are presented the results of annual inflation, in the second row, real annual interest rate, in the third row, real wages in the manufacturing sector, and in the fourth row, real wages in the services sector. The two lines represent respectively the IRF under a level of 0.002 and 2 of λ (left column) and η (right column), with corresponding colors as red and blue. The Calvo model results are represented in the green line.

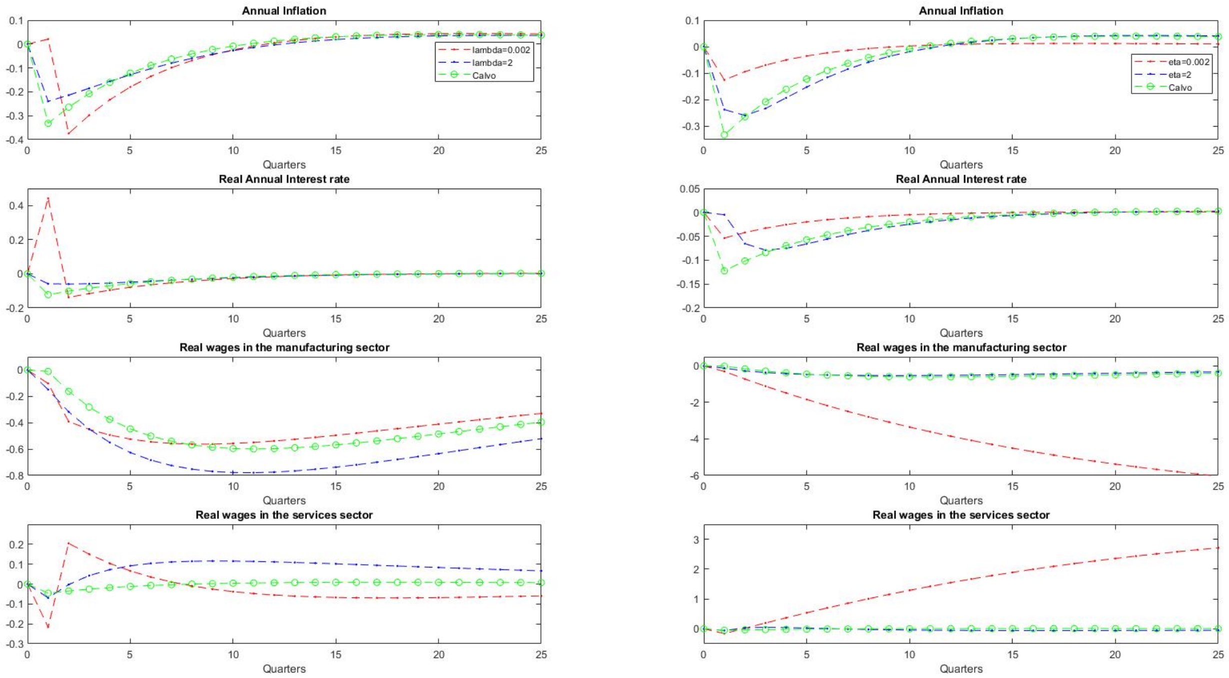


Figure B.5 Other variables IRFs to Technology shock in the services sector

Notes: The figure presents the impulse responses to a Technology Shock in services. In the first row are presented the results of annual inflation, in the second row, real annual interest rate, in the third row, real wages in the manufacturing sector, and in the fourth row, real wages in the services sector. The two lines represent respectively the IRF under a level of 0.002 and 2 of λ (left column) and η (right column), with corresponding colors as red and blue.

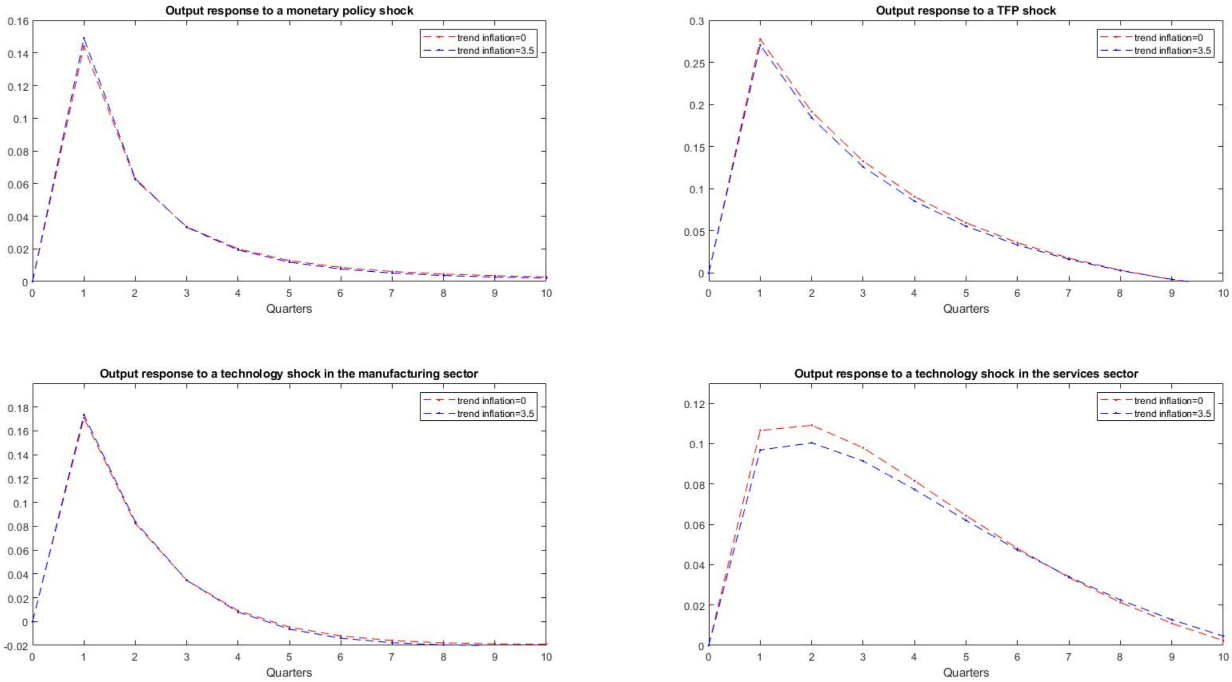


Figure B.6 The IRFs to trend inflation changes

Notes: In the first row (on the left) are presented the results of total output to a monetary policy shock. In the row (on the right) are presented the results of total output to a TFP shock. On the second row (on the left) are presented the results of total output to a technology shock in the manufacturing sector. On the second row (on the right) are presented the results of total output to a technology shock in the services sector. The two lines represent respectively the IRF under a level of 0 and 3.5 of trend inflation, with corresponding colors as red and blue.

Table B.1 Theoretical Moments

A. Under a TFP shock				
	$\lambda = 0.002$	$\lambda = 2$	$\eta = 0.002$	$\eta = 2$
Total output	0.383	0.426	0.475	0.399
Manufacturing output	0.384	1.002	0.472	0.789
Services output	0.382	0.432	0.477	0.298
Annual Inflation	0.896	1.056	1.163	0.964
Real Annual Interest rate	0.353	0.513	0.527	0.451
B. Under a monetary policy shock				
	$\lambda = 0.002$	$\lambda = 2$	$\eta = 0.002$	$\eta = 2$
Total output	0.198	0.159	0.142	0.175
Manufacturing output	0.197	0.087	0.141	0.041
Services output	0.198	0.261	0.143	0.271
Annual Inflation	0.128	0.348	0.400	0.263
Real Annual Interest rate	0.627	0.330	0.284	0.426

Notes: The above figures are the standard deviation of selected macroeconomic variables to a 0.24 and 0.45 sd shock respectively for the monetary policy shock and the TFP shock.

Table B.2 Coefficients of Autocorrelation

A. Under a TFP shock				
	$\lambda = 0.002$	$\lambda = 2$	$\eta = 0.002$	$\eta = 2$
Total output	0.801	0.732	0.759	0.750
Manufacturing output	0.801	0.901	0.755	0.845
Services output	0.801	0.806	0.761	0.736
Annual Inflation	0.796	0.719	0.751	0.730
Real Annual Interest rate	0.720	0.727	0.778	0.694
B. Under a monetary policy shock				
	$\lambda = 0.002$	$\lambda = 2$	$\eta = 0.002$	$\eta = 2$
Total output	0.209	0.489	0.519	0.394
Manufacturing output	0.210	0.792	0.516	0.646
Services output	0.209	0.598	0.521	0.433
Annual Inflation	0.051	-0.037	0.033	-0.130
Real Annual Interest rate	0.163	0.353	0.279	0.347

Notes: The above figures are the coefficients of autocorrelation of order one of selected macroeconomic variables to a 0.24 and 0.45 sd shock respectively for the monetary policy shock and the TFP shock

Appendix B.2 The complete log−linear equations main model

$$\widehat{L}_{2,t} = \widehat{Y}_{2,t} - \widehat{A}_{2,t} \text{ and } \widehat{L}_{1,t} = \widehat{Y}_{1,t} - \widehat{A}_{1,t} \tag{B.1}$$

$$\widehat{L}_t = b_1\widehat{L}_{1,t} + b_2\widehat{L}_{2,t} \tag{B.2}$$

$$\widehat{w}_t^1 = -\frac{1}{\lambda}\widehat{Y}_t + \frac{1}{\lambda}\widehat{Y}_{1,t} + \left(\sigma - \frac{1}{\eta}\right)\widehat{Y}_t + \frac{1}{\eta}\widehat{Y}_{1,t} \text{ and } \widehat{w}_{2,t} = -\frac{1}{\lambda}\widehat{Y}_t + \frac{1}{\lambda}\widehat{Y}_{2,t} + \left(\sigma - \frac{1}{\eta}\right)\widehat{Y}_t + \frac{1}{\eta}\widehat{Y}_{2,t} \tag{B.3}$$

$$\widehat{A}_{2,t} = \sigma_{A^1}\widehat{A}_{2,t-1} + \zeta_{A^2} \text{ and } \widehat{A}_{1,t} = \sigma_{A^1}\widehat{A}_{1,t-1} + \zeta_{A^1} \tag{B.4}$$

$$\widehat{\pi}_t = \alpha_1\widehat{\pi}_{1,t} + \alpha_2\widehat{\pi}_{2,t} \text{ and } \widehat{Y}_t = \alpha_1\widehat{Y}_{1,t} + \alpha_2\widehat{Y}_{2,t} \tag{B.5}$$

$$\widehat{Y}_t = \widehat{Y}_{t+1} + \frac{1}{\sigma}\left(\widehat{\pi}_{t+1} - \widehat{i}_t\right) \text{ and } \widehat{Y}_{2,t} - \widehat{Y}_{1,t} = \eta\left(\widehat{\rho}_{1,t} - \widehat{\rho}_{2,t}\right) \tag{B.6}$$

$$\widehat{m}_t = \sigma_m\widehat{m}_{t-1} + \zeta_m \tag{B.7}$$

$$\widehat{\pi}_{2,t} = k_2\left[\widehat{w}_{2,t} - \widehat{A}_{2,t} + \widehat{\rho}_{2,t}\right] + (1-\theta_2)\beta\widehat{\pi}_{t+1} + \beta\theta_2\widehat{\pi}_{2,t+1} \; ; \; k_2 \equiv \frac{(1-\theta_2)(1-\theta_2\beta)}{\theta_2} \tag{B.8}$$

$$\widehat{\pi}_{1,t} = \widehat{w}_{1,t} - \widehat{w}_{1,t-1} - (\widehat{A}_{1,t} - \widehat{A}_{1,t-1}) + \widehat{\pi}_t \tag{B.9}$$

$$\rho_{1,t} = \widehat{p}_{1,t} - \widehat{p}_t \text{ and } \rho_{2,t} = \widehat{p}_{2,t} - \widehat{p}_t \text{ and } \rho_{1,t} = \widehat{w}_{1,t} - \widehat{A}_{1,t} \tag{B.10}$$

$$\widehat{\rho}_{2,t} = \widehat{\rho}_{2,t-1} + \widehat{\pi}_{2,t} - \widehat{\pi}_t \text{ and } \widehat{\rho}_{1,t} = \widehat{\rho}_{1,t-1} + \widehat{\pi}_{1,t} - \widehat{\pi}_t \tag{B.11}$$

Appendix B.3 Proposition proofs

B.3.1 Proof of Propositions 1−2

In this Appendix I present the proof for propositions 1 and 2. I start with equation 3.30, written as the following:

$$\widehat{Y}_t = \tau_1^m\widehat{Y}_{t-1} + \tau_2^m\widehat{Y}_{t+1} + \tau_3^m\widehat{m}_t + \tau_4^m\widehat{Y}_{1,t+1} + \tau_5^m\widehat{Y}_{1,t} + \tau_6^m\widehat{Y}_{1,t-1} \tag{B.12}$$

where the detailed parameters are as following:

$$\overline{\tau}_1^m = \left(1 + \left(\frac{\alpha_1}{\alpha_2}\phi_\pi + \phi_\pi k_2 - \phi_\pi\frac{\alpha_1}{\alpha_2}k_2 + \frac{\alpha_1}{\alpha_2}\right)\left(1 - \frac{1}{\eta}\right) + \phi_\pi\left(\frac{1}{\eta}\right)k_2\frac{1}{\alpha_2}\right) \text{ and } \tau_1^m = \frac{1}{\overline{\tau}_1}\phi_\pi\frac{\alpha_1}{\alpha_2}\left(1 - \frac{1}{\eta}\right)$$

$$\tau_2^m = \tfrac{1}{\overline{\tau}_1}\left(1 + \left(\tfrac{\alpha_1}{\alpha_2} + k_2 - \tfrac{\alpha_1}{\alpha_2}k_2\right)\left(1 - \tfrac{1}{\eta}\right) + \left(\tfrac{1}{\eta}\right)k_2\tfrac{1}{\alpha_2}\right) \text{ and } \tau_3^m = -\tfrac{1}{\overline{\tau}_1}$$

$\tau_4^m = \frac{1}{\tau_1}(1-2k_2)\frac{\alpha_1}{\alpha_2}\left(\frac{1}{\eta}\right)$ and $\tau_5^m = \frac{1}{\tau_1}((2k_2-1)\phi_\pi+1)\left(\frac{1}{\eta}\right)\frac{\alpha_1}{\alpha_2}$ and $\tau_6^m = -\frac{1}{\tau_1}\frac{\alpha_1}{\alpha_2}\left(\frac{1}{\eta}\right)\phi_\pi$

Moving equation 3.31 one period forward $y_{t+1} = \chi_y y_t + \chi_1 y_{1,t} + \chi_m m_{t+1}$ and using the fact that $\mathbb{E}_t m_{t+1} = \sigma_m m_t$ and $\mathbb{E}_t y_{t+1} = \chi_y y_t$, I can write $y_{t+1} = \chi_y \chi_y y_{t-1} + \sigma_{y,1}\chi_1 y_{1,t-1} + \sigma_m \chi_m m_t$. Substituting in equation 3.30, I would have

$\chi_y y_{t-1} = \tau_1^m y_{t-1} + \tau_2^m(\chi_y\chi_y y_{t-1} + \sigma_{y,1}\chi_1 y_{1,t-1} + \sigma_m\chi_m m_t) + \tau_3^m m_t + \tau_4^m(\sigma_{y,1}\sigma_{y,1}y_{1,t-1}) + \tau_5^m(\sigma_{y,1}y_{1,t-1}) + \tau_6^m y_{1,t-1}$, which transforms into:

$$\chi_y y_{t-1} = \left[\tau_1^m y_{t-1} + \tau_2^m \chi_y^2 y_{t-1}\right] + \left[(\tau_2^m\chi_1 + \tau_4^m)\sigma_{y,1}^2 + \tau_5^m\sigma_{y,1} + \tau_6^m\right] y_{1,t-1} + \left[\tau_2^m\sigma_m\chi_m m_t + \tau_3^m m_t\right] \tag{B.13}$$

Where I need to solve

$$\tau_2^m\chi_y^2 - \chi_y + \tau_1^m = 0 \tag{B.14}$$

to find χ_y. The solutions would be:

$$\chi_y = \frac{1-\sqrt{1-4*\tau_2^m*\tau_1^m}}{2*\tau_2^m} \quad \text{and} \quad \chi_y = \frac{1+\sqrt{1-4*\tau_2^m*\tau_1^m}}{2*\tau_2^m} \quad \text{with the condition } D = 1-4*\tau_2^m*\tau_1^m > 0 \tag{B.15}$$

Further can be written using the relationship between as y_t and m_t, $y_{1,t-1}, \mathbb{E}_t m_{t+1} = \sigma_m m_t$, from equation 3.30 we can write as in 3.31:

$$\chi_m m_t = \left[\tau_1^m y_{t-1} + \tau_2^m \chi_y^2 y_{t-1}\right] + \left[(\tau_2^m\chi_1 + \tau_4^m)\sigma_{y,1}^2 + \tau_5^m\sigma_{y,1} + \tau_6^m\right] y_{1,t-1} + \left[\tau_2^m\sigma_m\chi_m m_t + \tau_3^m m_t\right] \tag{B.16}$$

or focusing only on the effect of the monetary policy shock:

$$\chi_m m_t = \left[\tau_1^m\chi_y^{-1}\chi_m m_t + \tau_2^m\chi_y\chi_m m_t\right] + \left[\tau_2^m\sigma_m\chi_m m_t + \tau_3 m_t\right] \tag{B.17}$$

or can be written as $0 = \left(\tau_1^m\chi_y^{-1} + \tau_2^m\chi_y + \tau_2^m\sigma_m - 1\right)\chi_m + \tau_3^m$ where after finding χ_y, I can find the solution of $\chi_m = -\frac{\tau_3^m}{\left(\tau_1^m\chi_y^{-1}+\tau_2^m\chi_y+\tau_2^m\sigma_m-1\right)}$.

I apply the same procedure to find the solution for χ_1. From Equation 3.31 again I obtain:

$$\chi_1 y_{1,t-1} = \left[\tau_1^m\chi_y^{-1}\chi_1 y_{1,t-1} + \tau_2^m\chi_y\chi_1 y_{1,t-1}\right] + \left[(\tau_2^m\chi_1 + \tau_4^m)\sigma_{y,1}^2 + \tau_5^m\sigma_{y,1} + \tau_6^m\right] y_{1,t-1} \tag{B.18}$$

Now after finding the solution χ_y, I can find the solution of $\chi_1 = \frac{\left(\tau_4^m\sigma_{y,1}^2+\tau_5^m\sigma_{y,1}+\tau_6^m\right)}{\left(1-\tau_1^m\chi_y^{-1}-\tau_2^m\chi_y-\tau_2^m\sigma_{y,1}^2\right)}$.

To provide the proof for proposition 1: I simplify the interpretation of the coefficients on equation 3.29, I assume that $\alpha_1 = \alpha_2$. When the intratemporal elasticity of substitution between sectors (η) is high, and $\lim\eta = \infty$, then goes to $\frac{1}{\eta} = 0$, then $\overline{\tau}_1^m$ decreases,

τ_1^m and τ_2^m increases. As long as the condition $\sqrt{1-4*\tau_2^m*\tau_1^m}$ is satisfied , χ_y is lower than before. Moving next to χ_m, after simplifying for $\alpha_1=\alpha_2$, and $\frac{1}{\eta}=0$, I would have $\chi_m=\frac{1}{\left(\phi_\pi\chi_y^{-1}+((2))\chi_y+2\sigma_m-1\right)}$. When $\left(\phi_\pi\chi_y^{-1}+((2))\chi_y+2\sigma_m-1\right)>\overline{\tau}_1^m\left(\tau_1^m\chi_y^{-1}+\tau_2^m\chi_y+\tau_2^m\sigma_m-1\right)$, χ_m goes down, and vice versa. So, when $\lim\eta=\infty$, χ_m goes down dependent on denominator direction. And as $\lim\eta=\infty$ and $\frac{1}{\eta}=0$ then τ_4^m, τ_5^m,τ_6^m, goes to $=0$, and as consequence $\chi_1=0$.

B.3.2 Proof of Propositions 3

In this Appendix I present the proof for proposition 3, focus on η parameter. I start with equation 3.33, written as the following:

$$\widehat{Y}_t=\tau_1^A\widehat{Y}_{t-1}+\tau_2^A\widehat{Y}_{t+1}+\tau_3^A\widehat{A}_t+\tau_4^A\widehat{Y}_{1,t}+\tau_5^A\widehat{Y}_{1,t-1}+\tau_6^A\widehat{Y}_{1,t+1} \tag{B.19}$$

where now the new coefficients would be respectively

$$\overline{\tau}_1^A=\left[1+(1+\phi_\pi)\frac{\alpha_1}{\alpha_2}+k_2\phi_\pi(1-\frac{\alpha_1}{\alpha_2})-\frac{1}{\eta}\left((\phi_\pi+1)\frac{\alpha_1}{\alpha_2}-\left(\frac{\alpha_1}{\alpha_2}+\frac{1}{\alpha_2}-1\right)\phi_\pi k_2\right)\right]$$

$\tau_1^A=\frac{1}{\overline{\tau}_1^A}\phi_\pi\frac{\alpha_1}{\alpha_2}\left(1-\frac{1}{\eta}\right)$ and $\tau_2^A=\frac{1}{\overline{\tau}_1^A}\left[\left(\frac{\alpha_1}{\alpha_2}+\left(1-\frac{\alpha_1}{\alpha_2}\right)k_2\right)\left(1-\frac{1}{\eta}\right)+k_2\frac{1}{\eta}\frac{1}{\alpha_2}+1\right]$
and $\tau_3^A=\frac{1}{\overline{\tau}_1^A}\left(-\frac{\alpha_1}{\alpha_2}\sigma_A+(1+\phi_\pi)\frac{\alpha_1}{\alpha_2}-\phi_\pi\frac{\alpha_1}{\alpha_2}\sigma_A^{-1}\right)$ and $\tau_4^A=-\frac{1}{\overline{\tau}_1^A}\left(1+\phi_\pi\left(1-2k_2\right)\right)\frac{1}{\eta}\frac{\alpha_1}{\alpha_2}$
and $\tau_5^A=\frac{1}{\overline{\tau}_1^A}\left[\phi_\pi\frac{\alpha_1}{\alpha_2}\frac{1}{\eta}\right]$ and $\tau_6^A=\frac{1}{\overline{\tau}_1^A}\left((1-2k_2)\frac{1}{\eta}\frac{\alpha_1}{\alpha_2}\right)$
Following the same approach as in Appendix B.3.1 (again $\alpha_1=\alpha_2$), I am able to provide the solutions for the coefficients in equation 3.34, to have respectively: $\chi_y=\frac{1+\sqrt{1-4*\tau_2^A\tau_1^A}}{2\tau_2^A}$ and $\chi_A=-\frac{\tau_3^A}{\left(\tau_1^A\chi_y^{-1}+\tau_2^A\chi_y+\tau_2^A\sigma_A-1\right)}$ and $\chi_1=\frac{\left(\tau_4^A\sigma_{y,1}^2+\tau_5^A\right)}{\left(1-\tau_1^A\chi_y^{-1}-\tau_2^A\chi_y-\tau_2^A\sigma_{y,1}^2\right)}$.

Now, for Proposition 3 I would have: When the intratemporal elasticity of substitution between sectors (η) is high, and $\lim\eta=\infty$, then goes to $\frac{1}{\eta}=0$, then $\overline{\tau}_1^A$ decreases, τ_1^A and τ_2^A increases. As long as the condition $\sqrt{1-4\tau_2^A\tau_1^A}$ is satisfied , χ_y is lower than before, and a solution for it exists. So the effect of χ_A is positive as long $\left(\tau_1^A\chi_y^{-1}+\tau_2^A\chi_y+\tau_2^A\sigma_A-1\right)$ goes down. Simplifying χ_A, with $\frac{1}{\eta}=0$ and substituting the coefficients, we would have $\chi_A=\frac{\left(\sigma_A+\phi_\pi\sigma_A^{-1}-(1+\phi_\pi)\right)}{\left(\phi_\pi\chi_y^{-1}+2\chi_y+2\sigma_A-1\right)}$. When $\left(\phi_\pi\chi_y^{-1}+2\chi_y+2\sigma_A-1\right)<\left(\tau_1^A\chi_y^{-1}+\tau_2^A\chi_y+\tau_2^A\sigma_A-1\right)$, where $\left(\sigma_A(1+\phi_\pi\sigma_A^{-2})-(1+\phi_\pi)\right)$ is a positive number, χ_A goes up, and vice versa. So, when $\lim\eta=\infty$, χ_A goes down dependent on denominator direction. And as $\lim\eta=\infty$ and $\frac{1}{\eta}=0$ then τ_4^A, τ_5^A,τ_6^A, goes to $=0$, and as consequence $\chi_1=0$.

Appendix B.4 Additional log−linear equations

B.4.1 The key log−linear equations for the GHH preferences case

$$\widehat{Y}_t = \widehat{Y}_{t+1} + \widehat{\pi}_{t+1} - \widehat{i}_t - (1+\varphi)\left(\widehat{L}_{t+1} - \widehat{L}_t\right) \tag{B.20}$$

$$\widehat{w}_t^1 = -\frac{1}{\lambda}\widehat{Y}_t + \frac{1}{\lambda}\widehat{Y}_{1,t} + \left(\sigma - \frac{1}{\eta}\right)\widehat{Y}_t + \frac{1}{\eta}\widehat{Y}_{1,t} \text{ and } \widehat{w}_{2,t} = -\frac{1}{\lambda}\widehat{Y}_t + \frac{1}{\lambda}\widehat{Y}_{2,t} + \left(\sigma - \frac{1}{\eta}\right)\widehat{Y}_t + \frac{1}{\eta}\widehat{Y}_{2,t} \tag{B.21}$$

B.4.2 The key log−linear equations for the case with habit formation

$$\widehat{Y}_{2,t} - \left(\widehat{Y}_{1,t} - h_1\widehat{Y}_{1,t-1}\right) = \eta\left(\widehat{\rho}_{1,t} - \widehat{\rho}_{2,t}\right) \tag{B.22}$$

$$\widehat{Y}_t = \alpha_1\left(\widehat{Y}_{1,t} - h_1\widehat{Y}_{1,t-1}\right) + \alpha_2\widehat{Y}_{2,t} \tag{B.23}$$

$$\widehat{w}_{1,t} = \left(\varphi - \frac{1}{\lambda}\right)\widehat{L}_t + \frac{1}{\lambda}\widehat{L}_{1,t} + \left(\sigma - \frac{1}{\eta}\right)\widehat{C}_t + \frac{1}{\eta}\left(C_{1,t} - h_1 C_{1,t-1}\right) \tag{B.24}$$

B.4.3 Main log−linear equations for the case of New Keynesian Philips curve with trend inflation

Here, I present the main features of allowing for positive trend inflation in the model. In order to obtain the GNKPC in terms of marginal costs, I log−linearize the firm's equilibrium conditions around a steady state characterized by shifting trend inflation. And then, following the standard approach I would have:

$$\pi_{2,t} = \zeta(\overline{\pi})\widehat{MC}_{2,t} + b_1(\overline{\pi})\left(\sigma\widehat{Y}_t + \widehat{\psi}_{2,t+1}\right) + b_2(\overline{\pi})\widehat{\pi}_{t+1} + b_3(\overline{\pi})\pi_{2,t+1} \tag{B.25}$$

Where $\overline{\pi}$ represents trend inflation and the equation gives the Philips curve with the dynamics of inflation. The parameters on the Philips curve depending on trend inflation are respectively: $\zeta(\overline{\pi}) = \frac{\left(1-\theta_2\overline{\pi}^{\varepsilon-1}\right)\left(1-\theta_2\beta\overline{\pi}^{\varepsilon}\right)}{\theta_2\overline{\pi}^{\varepsilon-1}}$, and $b_1(\overline{\pi}) = \left(1-\theta_2\overline{\pi}^{\varepsilon-1}\right)(\overline{\pi}-1)\beta$ and, $b_2(\overline{\pi}) = \left(1-\theta_2\overline{\pi}^{\varepsilon-1}\right)[\eta(\overline{\pi}-1)+1]\beta$ and $b_3(\overline{\pi}) = \beta\left[\left(1-\theta_2\overline{\pi}^{\varepsilon-1}\right)(\varepsilon-\eta)[\overline{\pi}-1] + \overline{\pi}^{\varepsilon-1}\theta_2\right]$. An increase in trend inflation $\overline{\pi}$ decreases the slope $\zeta(\overline{\pi})$, and makes inflation less sensitive to current marginal costs. Trend inflation reduces the weight of determining inflation from current marginal costs by reducing $\zeta(\overline{\pi})$ and on the other side by increasing $b_2(\overline{\pi})$ and $b_3(\overline{\pi})$, increases the weight on expected future inflation. Now the curve depends more on an expected future aggregate and sector-specific inflation, and less on marginal costs. The η parameter which shows up on the coefficients $b_2(\overline{\pi})$ and $b_3(\overline{\pi})$, indicates that a higher η, increases the weight on expected aggregate inflation and reduces the weight on expected sectoral inflation. Further, the evolution of the present discounted value of future marginal costs and marginal revenues would be:

$\widehat{\psi}_{2,t} = (1-\theta_2\beta\overline{\pi}^{\varepsilon})\left[\widehat{MC}_{2,t} - \sigma\widehat{Y}_t\right] + \theta_2\beta\overline{\pi}^{\varepsilon}\left[(\varepsilon-\eta)\,\mathbb{E}_t\widehat{\pi}_{2,t+1} + \eta\mathbb{E}_t\widehat{\pi}_{t+1} + \mathbb{E}_t\widehat{\psi}_{2,t+1}\right]$ and:

$\widehat{\phi}_{2,t} = -\left(1-\theta_2\beta\overline{\pi}^{\varepsilon-1}\right)\sigma\widehat{Y}_t + \theta_2\beta\overline{\pi}^{\varepsilon-1}\left[(\varepsilon-\eta)\mathbb{E}_t\widehat{\pi}_{2,t+1} + (\eta-1)\mathbb{E}_t\widehat{\pi}_{t+1} + \mathbb{E}_t\widehat{\phi}_{2,t+1}\right].$

The second part of both equations on the right-hand side represents the forward-looking part of the equations and they are both dependent on trend inflation.

Appendix C

Appendices For Chapter Four

Appendix C.1 Data

C.1.1 Data Documentation

Definition of data variables
consumption = LN((PCEC/GDPDEF)/LNSindex)*100
investment = LN((FPI/GDPDEF)/LNSindex)*100
interest rate = Federal Funds Rate/4
hours = LN((PRS85006023*CE16OV/100)/LNSindex)*100
output = LN(GDPC12/LNSindex)*100
output in sector 1 = LN(Output 1/LNSindex)*100
output in sector 2 = LN(Output 2/LNSindex)*100
output in sector 3 = LN(Output 3/LNSindex)*100
inflation = LN(GDPDEF/GDPDEF(-1))*100
inflation in sector 1 = LN(Output DEF 1/Output DEF 1(-1))*100
inflation in sector 2 = LN(Output DEF 2/Output DEF 2(-1))*100
inflation in sector 3 = LN(Output DEF 3/Output DEF 3(-1))*100
real wage = LN(PRS85006103/ GDPDEF) * 100
real wage in sector 1 = LN(PR1 / Output DEF 1) * 100
real wage in sector 2 = LN(PR2 / Output DEF 2) * 100
real wage in sector 3 = LN(PR3/ Output DEF 3) * 100

Source of the original data
GDPC12: Real Gross Domestic Product - Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate, Source: U.S. Department of Commerce, Bureau of Economic Analy-

sis

Output in sector 1 - Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate, Source: U.S. Department of Commerce, Bureau of Economic Analysis

Output in sector 2 - Billions of Chained 2012 - Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate, Source: U.S. Department of Commerce, Bureau of Economic Analysis

Output in sector 3 - Billions of Chained 2012 - Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate, Source: U.S. Department of Commerce, Bureau of Economic Analysis

GDPDEF : Gross Domestic Product - Implicit Price Deflator - 2012=100, Seasonally Adjusted, Source: U.S. Bureau of Economic Analysis

Output Def 1, deflator in sector 1 - 2012=100, Seasonally Adjusted, Source: U.S. Bureau of Economic Analysis

Output Def 2, deflator in sector 2 - 2012=100, Seasonally Adjusted, Source: U.S. Bureau of Economic Analysis

Output Def 3, deflator in sector 3 - 2012=100, Seasonally Adjusted, Source: U.S. Bureau of Economic Analysis

PCEC : Personal Consumption Expenditures - Billions of Dollars, Seasonally Adjusted Annual Rate, Source: Bureau of Economic Analysis

GPDI : Gross private domestic investment - Billions of Dollars, Seasonally Adjusted Annual Rate, Source: Bureau of Economic Analysis

CE16OV : Civilian Employment: Sixteen Years Over, Employment Level, Thousands of Persons, Quarterly, Seasonally Adjusted, Source: U.S. Department of Labor: Bureau of Labor Statistics

CE16OV index : CE16OV (2012:3)=1

Federal Funds Rate : Effective Federal Funds Rate, Percent, Quarterly, Not Seasonally Adjusted, Source: Board of Governors of the Federal Reserve System

CNP16OV: Population level - 16 Years and Older - Thousands of Persons, Quarterly, Not Seasonally Adjusted, Source: U.S. Bureau of Labor Statistics,

LNU00000000 : Population Level : 16 Years and Older - Thousands of Persons, Quarterly, Not Seasonally Adjusted, Source: U.S. Bureau of Labor Statistics,

LNSindex : LNU00000000(2012:3)=1

PRS85006023 - Nonfarm Business Sector, Average Weekly Hours, Index 2012=100, Quarterly, Seasonally Adjusted, Source : U.S. Department of Labor

COMPRNFB - Nonfarm Business Sector, Real Compensation Per Hour, Index 2012=100, Quarterly, Seasonally Adjusted, Source : U.S. Department of Labor

PR1 Hourly earnings in sector 1, Average hourly earnings of all employees, 1982-1984 dollars, seasonally adjusted, Index 2012Q3=100, Quarterly, Seasonally Adjusted, Source : U.S. Department of Labor
PR1 Hourly earnings in sector 1, Average hourly earnings of all employees in sector 1, 1982-1984 dollars, seasonally adjusted, Index 2012Q3=100, Quarterly, Seasonally Adjusted, Source : U.S. Department of Labor
PR2 Hourly earnings in sector 2, Average hourly earnings of all employees in sector 2, 1982-1984 dollars, seasonally adjusted, Index 2012Q3=100, Quarterly, Seasonally Adjusted, Source : U.S. Department of Labor
PR3 Hourly earnings in sector 3, Average hourly earnings of all employees in sector 3, 1982-1984 dollars, seasonally adjusted, Index 2012Q3=100, Quarterly, Seasonally Adjusted, Source : U.S. Department of Labor
Note: When calculating the Average hourly earnings and Price deflators in each of the sectors the weights of each of the subsectors are taken into account.

C.1.2 Data visuals and statistics

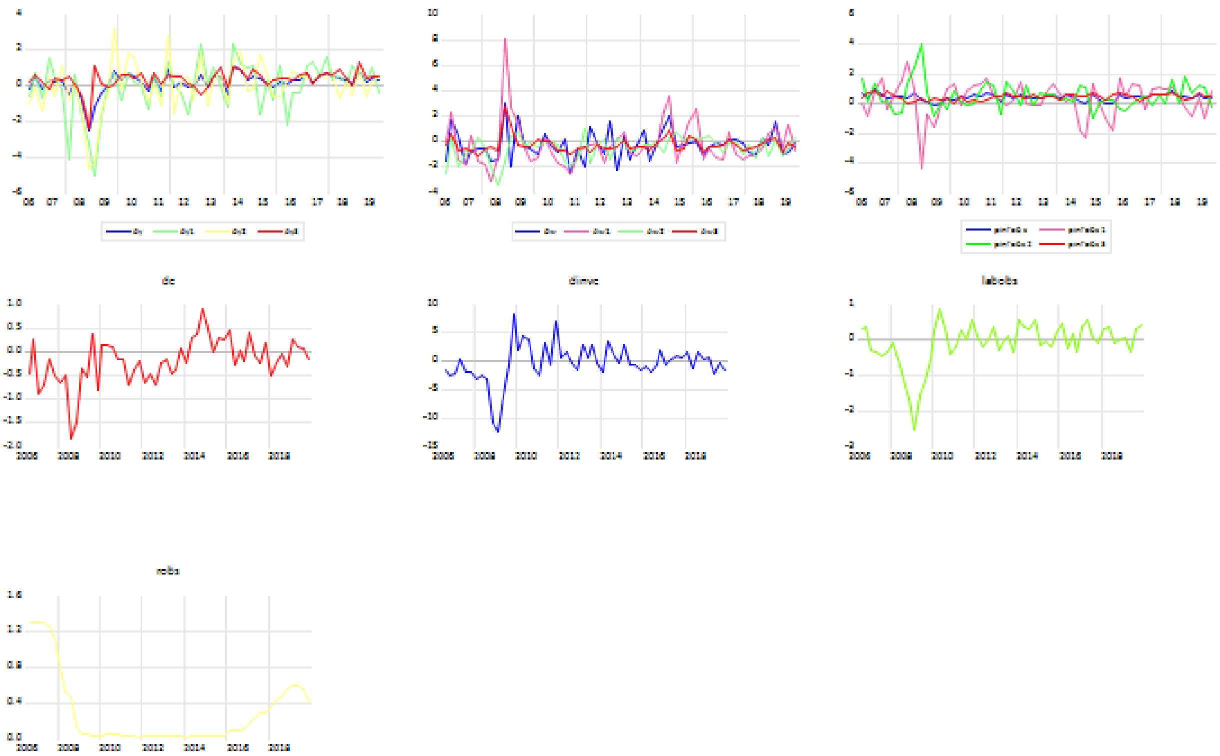


Figure C.1 A visualization of the data (1)

Notes: A graph of all the data series used in the estimation of the three-sector model

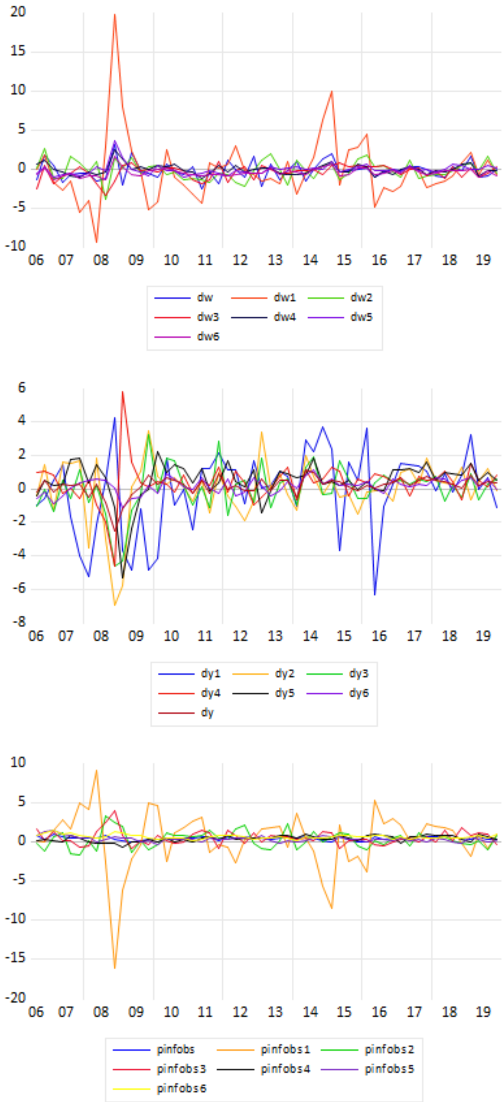


Figure C.2 A visualization of the data (2)

Notes: A graph of the data series used in the estimation of the six-sector model

Table C.1 Descriptive statistics of the data

	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
DC	-0.19	-0.18	0.90	-1.81	0.48	-0.81	4.82
DINVE	-0.28	-0.45	8.19	-12.38	3.35	-0.91	6.72
DW	-0.25	-0.29	3.06	-2.52	1.14	0.54	3.36
DW1	-0.16	-0.46	8.10	-3.14	1.81	2.04	9.52
DW2	-0.45	-0.21	0.97	-3.36	0.89	-1.03	3.85
DW3	-0.23	-0.39	2.74	-1.03	0.61	2.53	12.09
DY	0.19	0.28	1.13	-2.49	0.58	-2.09	10.29
DY1	-0.04	0.22	2.29	-5.04	1.42	-1.34	5.37
DY2	0.02	0.21	3.24	-4.61	1.37	-0.81	5.72
DY3	-0.09	0.01	0.88	-2.50	0.60	-1.76	7.12
LABOBS	-0.09	0.01	0.88	-2.50	0.60	-1.76	7.12
PINFOBS	0.42	0.44	1.02	-0.11	0.23	-0.27	3.29
PINFOBS1	0.29	0.50	2.80	-4.33	1.20	-1.26	5.80
PINFOBS2	0.51	0.32	3.95	-0.92	0.88	1.15	5.82
PINFOBS3	0.45	0.47	0.85	-0.04	0.20	-0.28	2.82
ROBS	0.29	0.05	1.31	0.02	0.40	1.57	4.29

Notes: The table gives a summary of the descriptive statistics of the data used in the three-sector model

Table C.2 Data Cross−Correlations

	DY	DY1	DY2	DY3	DC	DINVE	LABOBS	PINFOBS	PINFOBS1	PINFOBS2	PINFOBS3	DW	DW1	DW2	DW3	ROBS
DY	1.00	0.77	0.86	0.70	0.58	0.79	0.60	0.10	0.37	-0.52	0.23	-0.28	-0.46	0.34	-0.48	-0.03
DY1	0.77	1.00	0.61	0.24	0.46	0.70	0.56	0.03	0.03	-0.09	0.22	-0.09	-0.14	0.05	-0.25	0.03
DY2	0.86	0.61	1.00	0.40	0.35	0.83	0.53	0.09	0.42	-0.64	0.15	-0.30	-0.49	0.43	-0.47	-0.10
DY3	0.70	0.24	0.40	1.00	0.56	0.35	0.35	0.05	0.31	-0.44	0.13	-0.23	-0.35	0.32	-0.32	-0.03
DC	0.58	0.46	0.35	0.56	1.00	0.27	0.45	-0.43	-0.15	-0.35	0.09	0.13	0.10	0.49	0.04	-0.21
DINVE	0.79	0.70	0.83	0.35	0.27	1.00	0.63	0.12	0.30	-0.34	0.10	-0.23	-0.40	0.20	-0.40	-0.20
LABOBS	0.60	0.56	0.53	0.35	0.45	0.63	1.00	0.18	0.18	-0.20	0.22	0.00	-0.25	0.09	-0.29	-0.01
PINFOBS	0.10	0.03	0.09	0.05	-0.43	0.12	0.18	1.00	0.61	0.19	0.34	-0.35	-0.59	-0.56	-0.54	0.36
PINFOBS1	0.37	0.03	0.42	0.31	-0.15	0.30	0.18	0.61	1.00	-0.36	-0.11	-0.60	-0.96	-0.16	-0.74	0.08
PINFOBS2	-0.52	-0.09	-0.64	-0.44	-0.35	-0.34	-0.20	0.19	-0.36	1.00	-0.17	0.20	0.39	-0.76	0.43	0.08
PINFOBS3	0.23	0.22	0.15	0.13	0.09	0.10	0.22	0.34	-0.11	-0.17	1.00	0.10	0.08	0.17	-0.24	0.27
DW	-0.28	-0.09	-0.30	-0.23	0.13	-0.23	0.00	-0.35	-0.60	0.20	0.10	1.00	0.59	0.11	0.49	-0.07
DW1	-0.46	-0.14	-0.49	-0.35	0.10	-0.40	-0.25	-0.59	-0.96	0.39	0.08	0.59	1.00	0.18	0.87	-0.08
DW2	0.34	0.05	0.43	0.32	0.49	0.20	0.09	-0.56	-0.16	-0.76	0.17	0.11	0.18	1.00	0.15	-0.30
DW3	-0.48	-0.25	-0.47	-0.32	0.04	-0.40	-0.29	-0.54	-0.74	0.43	-0.24	0.49	0.87	0.15	1.00	-0.12
ROBS	-0.03	0.03	-0.10	-0.03	-0.21	-0.20	-0.01	0.36	0.08	0.08	0.27	-0.07	-0.08	-0.30	-0.12	1.00

Notes: The table shows the cross-correlations between the data used in the three-sector model

Appendix C.2 Additional estimation results

C.2.1 Variance decomposition(s)

Table C.3 Variance decomposition one sector (in percent)

Parameters	μ_a	μ_b	μ_g	μ_i	μ_m	μ_p	μ_w
DY	22.84	18.10	21.54	24.62	5.64	7.12	0.14
DC	9.20	59.17	2.80	3.55	12.20	13.04	0.03
DINVE	2.35	1.17	0.00	91.21	1.67	3.59	0.01
LABOBS	10.51	13.21	26.59	21.65	8.47	19.47	0.10
PINFOBS	2.88	1.37	0.41	0.11	2.03	92.85	0.351
DW	0.21	0.39	0.00	0.27	0.29	11.22	87.62
ROBS	16.16	33.40	1.74	10.28	8.84	29.31	0.26

Notes: The table gives a summary of the posterior mean decomposition in the one-sector model (in percent)

Table C.4 Variance decomposition three-sector (in percent)

Parameters	μ_a	$\mu_{1,y}$	$\mu_{2,y}$	$\mu_{3,y}$	μ_b	μ_g	μ_i	μ_m	μ_p,	$\mu_{1,p}$,	$\mu_{2,p}$,	$\mu_{3,p}$,	μ_w,	$\mu_{1,w}$,	$\mu_{2,w}$,	$\mu_{3,w}$
DY	22.46	0.00	0.00	0.00	15.15	19.22	18.56	4.87	0.00	8.40	3.21	6.97	0.00	0.63	0.12	0.42
DY1	8.57	32.71	0.00	0.00	2.14	3.01	2.52	0.90	0.00	39.83	3.01	3.94	0.00	2.95	0.12	0.30
DY2	10.49	0.00	47.17	0.00	7.06	8.59	8.47	2.50	0.00	0.78	14.09	0.23	0.00	0.04	0.55	0.03
DY3	12.42	0.00	0.00	28.25	13.93	15.73	15.97	5.13	0.00	1.43	0.53	6.21	0.00	0.07	0.03	0.32
DC	8.69	0.00	0.00	0.00	53.06	0.52	0.41	11.29	0.00	11.07	4.64	8.44	0.00	1.01	0.20	0.66
DINVE	4.52	0.00	0.00	0.00	1.32	0.06	76.11	1.98	0.00	6.45	2.27	6.31	0.00	0.51	0.08	0.39
LABOBS	16.04	0.00	0.00	0.00	3.06	42.01	4.72	1.94	0.00	7.09	3.61	20.19	0.00	0.53	0.09	0.72
PINFOBS	7.72	0.00	0.00	0.00	3.08	0.04	0.42	4.42	2.82	41.24	15.81	19.20	0.00	3.00	0.66	1.58
PINFOBS1	0.95	0.00	0.00	0.00	0.52	0.00	0.10	0.44	0.00	89.89	0.24	1.23	0.00	6.56	0.01	0.05
PINFOBS2	5.36	0.00	0.00	0.00	0.80	0.01	0.10	1.05	0.00	0.24	87.63	1.12	0.00	0.02	3.63	0.03
PINFOBS3	5.24	0.00	0.00	0.00	2.87	0.05	0.31	4.95	0.00	1.44	0.89	77.90	0.00	0.10	0.02	6.24
DW	1.28	0.00	0.00	0.00	0.43	0.00	0.12	0.20	0.00	12.59	4.17	8.09	62.86	1.11	0.31	8.84
DW1	0.91	0.00	0.00	0.00	0.20	0.00	0.03	0.07	0.00	90.05	0.01	0.01	0.00	8.73	0.00	0.00
DW2	4.33	0.00	0.00	0.00	0.01	0.01	0.04	0.00	0.00	0.00	88.38	0.00	0.00	0.00	7.23	0.00
DW3	0.98	0.00	0.00	0.00	1.37	0.00	0.36	0.75	0.00	0.05	0.03	45.66	0.00	0.01	0.00	50.79
ROBS	22.59	0.00	0.00	0.00	27.25	0.26	4.74	7.34	0.00	13.22	5.90	16.12	0.00	1.29	0.25	1.05

Notes: The table gives a summary of the posterior mean decomposition in the three-sector model (model 3, in percent)

Table C.5 Variance decomposition six-sector (in percent)

Parameters	μ_a	$\mu_{1,y}$	$\mu_{2,y}$	$\mu_{3,y}$	$\mu_{4,y}$	$\mu_{5,y}$	$\mu_{6,y}$	μ_b	μ_g	μ_i	μ_m	μ_p	$\mu_{1,p}$,	$\mu_{2,p}$,	$\mu_{3,p}$	$\mu_{4,p}$,	$\mu_{5,p}$,	$\mu_{6,p}$	μ_w,	$\mu_{1,w}$,	$\mu_{2,w}$	$\mu_{3,w}$	$\mu_{4,w}$,	$\mu_{5,w}$,	$\mu_{6,w}$
DY	18.68	0.00	0.00	0.00	0.00	0.00	0.00	20.75	21.73	22.22	6.63	0.00	2.44	0.02	2.60	0.96	0.04	2.53	0.00	1.13	0.00	0.01	0.05	0.01	0.20
DY1	2.80	34.36	0.00	0.00	0.00	0.00	0.00	0.87	0.68	0.58	0.85	0.00	42.63	0.06	5.37	1.87	0.18	2.56	0.00	6.85	0.01	0.01	0.06	0.01	0.26
DY2	5.95	60.55	0.00	0.00	0.00	0.00	4.74	5.27	5.28	1.48	0.00	0.46	13.66	0.33	0.11	0.01	0.04	0.00	0.01	2.08	0.00	0.00	0.00	0.00	
DY3	6.48	0.00	0.00	50.93	0.00	0.00	0.00	9.05	8.97	9.37	3.16	0.00	0.78	0.00	10.86	0.19	0.02	0.10	0.00	0.03	0.00	0.04	0.00	0.00	0.01
DY4	4.15	0.00	0.00	0.00	73.92	0.00	0.00	5.90	5.81	6.08	2.08	0.00	0.51	0.00	0.37	1.02	0.01	0.07	0.00	0.02	0.00	0.00	0.05	0.00	0.01
DY5	5.90	0.00	0.00	0.00	0.00	62.76	0.00	8.66	8.47	8.89	3.14	0.00	0.74	0.00	0.54	0.18	0.52	0.11	0.00	0.03	0.00	0.00	0.00	0.05	0.01
DY6	10.01	0.00	0.00	0.00	0.00	0.00	39.79	12.45	12.71	13.13	4.14	0.00	1.11	0.00	0.80	0.26	0.03	5.10	0.00	0.03	0.00	0.00	0.00	0.00	0.41
DC	3.29	0.00	0.00	0.00	0.00	0.00	0.00	67.89	0.51	0.41	13.80	0.00	4.46	0.03	2.96	1.08	0.07	3.09	0.00	2.01	0.01	0.02	0.08	0.02	0.29
DINVE	1.90	0.00	0.00	0.00	0.00	0.00	0.00	2.23	0.06	85.23	3.08	0.00	0.90	0.01	2.57	0.94	0.03	2.10	0.00	0.70	0.00	0.01	0.04	0.01	0.18
LABOBS	20.77	0.00	0.00	0.00	0.00	0.00	0.00	5.81	43.99	6.18	4.07	0.00	0.70	0.02	3.79	1.35	0.04	9.54	0.00	3.20	0.00	0.03	0.14	0.03	0.35
PINFOBS	3.55	0.00	0.00	0.00	0.00	0.00	0.00	2.35	0.05	0.21	4.08	10.42	34.24	0.20	20.16	7.03	0.66	10.32	0.00	5.32	0.03	0.06	0.25	0.04	1.03
PINFOBS1	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.02	0.08	0.00	85.19	0.00	0.31	0.12	0.00	0.49	0.00	13.36	0.00	0.00	0.01	0.00	0.04
PINFOBS2	2.83	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.01	0.09	0.84	0.00	0.02	82.57	0.07	0.02	0.00	0.29	0.00	0.09	12.45	0.00	0.00	0.00	0.01
PINFOBS3	0.47	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.01	0.03	0.73	0.00	0.03	0.00	97.35	0.03	0.00	0.52	0.00	0.15	0.00	0.29	0.01	0.00	0.01
PINFOBS4	2.32	0.00	0.00	0.00	0.00	0.00	0.00	1.82	0.06	0.11	3.67	0.00	0.15	0.00	0.44	84.44	0.01	2.92	0.00	0.82	0.00	0.01	3.18	0.01	0.06
PINFOBS5	3.49	0.00	0.00	0.00	0.00	0.00	0.00	2.79	0.12	0.12	6.04	0.00	0.27	0.01	0.89	0.33	72.63	6.34	0.00	1.75	0.00	0.01	0.07	5.00	0.13
PINFOBS6	3.92	0.00	0.00	0.00	0.00	0.00	0.00	2.56	0.04	0.25	4.21	0.00	0.14	0.00	0.38	0.13	0.00	79.57	0.00	0.62	0.00	0.00	0.02	0.01	8.14
DW	0.19	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.00	0.16	0.37	0.00	7.07	0.04	4.18	1.42	0.14	2.94	76.48	0.15	0.00	0.76	2.39	0.80	2.32
DW1	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.01	0.01	0.00	97.14	0.00	0.00	0.00	0.00	0.01	0.00	2.51	0.00	0.00	0.00	0.00	0.00
DW2	2.11	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.03	0.01	0.00	0.00	90.19	0.00	0.00	0.00	0.00	0.00	0.00	7.62	0.00	0.00	0.00	0.00
DW3	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.00	0.15	0.39	0.00	0.00	0.00	82.85	0.01	0.00	0.02	0.00	0.02	0.00	15.92	0.00	0.00	0.00
DW4	0.10	0.00	0.00	0.00	0.00	0.00	0.00	1.35	0.00	0.38	0.99	0.00	0.01	0.00	0.04	34.94	0.00	0.06	0.00	0.04	0.00	0.00	62.08	0.00	0.00
DW5	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.94	0.00	0.28	0.85	0.00	0.01	0.00	0.03	0.01	14.49	0.07	0.00	0.03	0.00	0.00	0.00	83.27	0.00
DW6	0.66	0.00	0.00	0.00	0.00	0.00	0.00	1.08	0.00	0.27	0.54	0.00	0.00	0.00	0.02	0.01	0.00	51.00	0.00	0.02	0.00	0.00	0.00	0.00	46.40
ROBS	20.65	0.00	0.00	0.00	0.00	0.00	0.00	34.78	0.37	5.06	13.25	0.00	6.57	0.05	4.04	1.49	0.09	8.78	0.00	4.00	0.01	0.04	0.21	0.05	0.55

Notes: The table gives a summary of the posterior mean decomposition in the six-sector model (model 5, in percent)

C.2.2 Model Cross−Correlations

Table C.6 One-sector model Cross−Correlations

	DY	DC	DINVE	LABOBS	PINFOBS	DW	ROBS
DY	1.00	0.51	0.65	0.03	-0.25	0.17	-0.22
DC	0.51	1.00	0.06	0.00	-0.30	0.14	-0.23
DINVE	0.65	0.06	1.00	0.10	-0.16	0.10	-0.15
LABOBS	0.03	0.00	0.10	1.00	0.02	0.04	0.28
PINFOBS	-0.25	-0.30	-0.16	0.02	1.00	-0.29	0.59
DW	0.17	0.14	0.10	0.04	-0.24	1.00	-0.12
ROBS	-0.22	-0.23	-0.15	0.28	0.59	-0.12	1.00

Notes: The table gives a summary of the cross-correlations obtained from the estimations of model 1

Table C.7 Three-sector model Cross−Correlations

	DY	DY1	DY2	DY3	DC	DINVE	LABOBS	PINFOBS	PINFOBS1	PINFOBS2	PINFOBS3	DW	DW1	DW2	DW3	ROBS
DY	1.00	0.61	0.61	0.76	0.67	0.71	0.02	-0.41	-0.25	-0.21	-0.27	0.28	0.31	0.24	0.29	-0.26
DY1	0.61	1.00	0.26	0.28	0.50	0.43	0.00	-0.68	-0.64	-0.23	-0.25	0.34	0.60	0.21	0.18	-0.34
DY2	0.61	0.26	1.00	0.49	0.38	0.42	0.00	-0.13	0.07	-0.39	0.00	0.12	0.00	0.39	0.10	-0.10
DY3	0.76	0.28	0.49	1.00	0.46	0.54	0.03	-0.03	0.11	0.03	-0.20	0.11	-0.01	0.03	0.26	-0.08
DC	0.67	0.50	0.38	0.46	1.00	0.34	0.03	-0.44	-0.27	-0.22	-0.28	0.30	0.34	0.24	0.31	-0.26
DINVE	0.71	0.43	0.42	0.54	0.34	1.00	0.07	-0.29	-0.18	-0.12	-0.21	0.22	0.23	0.15	0.24	-0.20
LABOBS	0.02	0.00	0.00	0.03	0.03	0.07	1.00	-0.02	0.03	0.02	-0.09	0.04	0.01	-0.01	0.09	-0.11
PINFOBS	-0.41	-0.68	-0.13	-0.03	-0.44	-0.29	-0.02	1.00	0.65	0.48	0.55	-0.40	-0.59	-0.40	-0.25	0.50
PINFOBS1	-0.25	-0.64	0.07	0.11	-0.27	-0.18	0.03	0.65	1.00	0.00	-0.05	-0.29	-0.85	0.01	0.05	0.24
PINFOBS2	-0.21	-0.23	-0.39	0.03	-0.22	-0.12	0.02	0.48	0.00	1.00	0.05	-0.18	-0.02	-0.87	0.01	0.25
PINFOBS3	-0.27	-0.25	0.00	-0.20	-0.28	-0.21	-0.09	0.55	-0.05	0.05	1.00	-0.22	0.00	-0.02	-0.51	0.40
DW	0.28	0.34	0.12	0.11	0.30	0.22	0.04	-0.40	-0.29	-0.18	-0.22	1.00	0.38	0.23	0.44	-0.15
DW1	0.31	0.60	0.00	-0.01	0.34	0.23	0.01	-0.59	-0.85	-0.02	0.00	0.38	1.00	0.03	0.04	-0.21
DW2	0.24	0.21	0.39	0.03	0.24	0.15	-0.01	-0.40	0.01	-0.87	-0.02	0.23	0.03	1.00	0.03	-0.13
DW3	0.29	0.18	0.10	0.26	0.31	0.24	0.09	-0.25	0.05	0.01	-0.51	0.44	0.04	0.03	1.00	-0.12
ROBS	-0.26	-0.34	-0.10	-0.08	-0.26	-0.20	-0.11	0.50	0.24 0.25	0.40	-0.15	-0.21	-0.13	-0.12	1.00	

Notes: The table gives a summary of the cross-correlations obtained from the estimations of the three-sector model (model 3)

C.2.3 Changing wage heterogeneity priors

Table C.8 Three-sector (2): Prior and posterior distribution of structural parameters

	Prior Distribution			Posterior Distribution					
				Model 7			**Model 8**		
Parameters	Distr.	Mean	St. Dev.	Mode	Mean	90% HPD Interval	Mode	Mean	90% HPD Interval
$\iota_{1,p}$	Beta	0.4	0.05	0.392	0.391	[0.31 0.47]	0.414	0.420	[0.34 0.50]
$\iota_{2,p}$	Beta	0.4	0.05	0.384	0.388	[0.31 0.47]	0.384	0.389	[0.31 0.47]
$\iota_{3,p}$	Beta	0.4	0.05	0.389	0.390	[0.31 0.47]	0.387	0.388	[0.30 0.47]
$\iota_{1,w}$	Beta	0.4	0.05	0.322	0.327	[0.26 0.40]	0.328	0.333	[0.26 0.41]
$\iota_{2,w}$	Beta	0.4	0.05	0.386	0.387	[0.31 0.46]	0.384	0.384	[0.31 0.45]
$\iota_{3,w}$	Beta	0.4	0.05	0.392	0.396	[0.31 0.48]	0.392	0.394	[0.31 0.48]
$\theta_{1,p}$	Beta	0.5(0.2)	0.1(0.05)	0.716	0.709	[0.60 0.82]	0.333	0.373	[0.30 0.44]
$\theta_{2,p}$	Beta	0.5	0.1	0.594	0.599	[0.47 0.73]	0.585	0.597	[0.47 0.72]
$\theta_{3,p}$	Beta	0.5	0.1	0.872	0.863	[0.81 0.91]	0.875	0.864	[0.81 0.92]
$\theta_{1,w}$	Beta	0.5	0.15	0.539	0.581	[0.41 0.75]	0.402	0.516	[0.33 0.68]
$\theta_{2,w}$	Beta	0.5	0.15	0.783	0.759	[0.65 0.88]	0.771	0.751	[0.63 0.87]
$\theta_{3,w}$	Beta	0.3	0.05	0.386	0.402	[0.32 0.48]	0.375	0.397	[0.31 0.48]
χ	Gamma	0.3	0.05	0.383	0.385	[0.31 0.46]	0.376	0.387	[0.31 0.46]
λ	Gamma	2	0.5	1.502	1.559	[0.93 2.19]	1.100	1.503	[0.86 2.10]
η	Gamma	2	0.5	0.372	0.370	[0.28 0.46]	0.411	0.381	[0.29 0.47]
k	Normal	4	1.5	4.358	4.724	[2.66 6.68]	4.332	4.683	[2.61 6.62]
Φ	Normal	1.25	0.125	1.570	1.580	[1.41 1.75]	1.560	1.595	[1.42 1.77]
φ	Normal	2	0.75	0.541	0.844	[0.25 1.40]	0.355	0.722	[0.25 1.20]
α	Normal	0.3	0.05	0.113	0.112	[0.09 0.14]	0.113	0.112	[0.09 0.14]
ϕ_π	Normal	1.5	0.25	1.814	1.853	[1.52 2.17]	1.772	1.827	[1.51 2.14]
ρ	Beta	0.75	0.1	0.877	0.883	[0.85 0.92]	0.877	0.882	[0.84 0.92]
ϕ_Y	Normal	0.125	0.05	0.020	0.025	[0.00 0.04]	0.017	0.025	[0.004 0.04]
ϕ_{dY}	Normal	0.125	0.05	0.137	0.135	[0.09 0.18]	0.140	0.137	[0.09 0.18]
h	Beta	0.7	0.1	0.559	0.592	[0.48 0.71]	0.541	0.583	[0.47 0.70]
Υ	Beta	0.5	0.15	0.678	0.659	[0.46 0.86]	0.703	0.663	[0.47 0.88]
Log posterior (or likelihood)				-729.83			-742.032		
Log data density				-856.76			-870.62		

Notes: The table shows on the left-hand side the priors used in estimating the model. In the middle are presented the posterior estimated parameters for the three-sector model (model 7) estimated with sectoral data (with priors for the third sector that indicate flexible wages in sector 3)). On the right-hand side are presented the estimations from the three-sector model (model 8), with priors for the first sector that indicate flexible prices in sector 1)

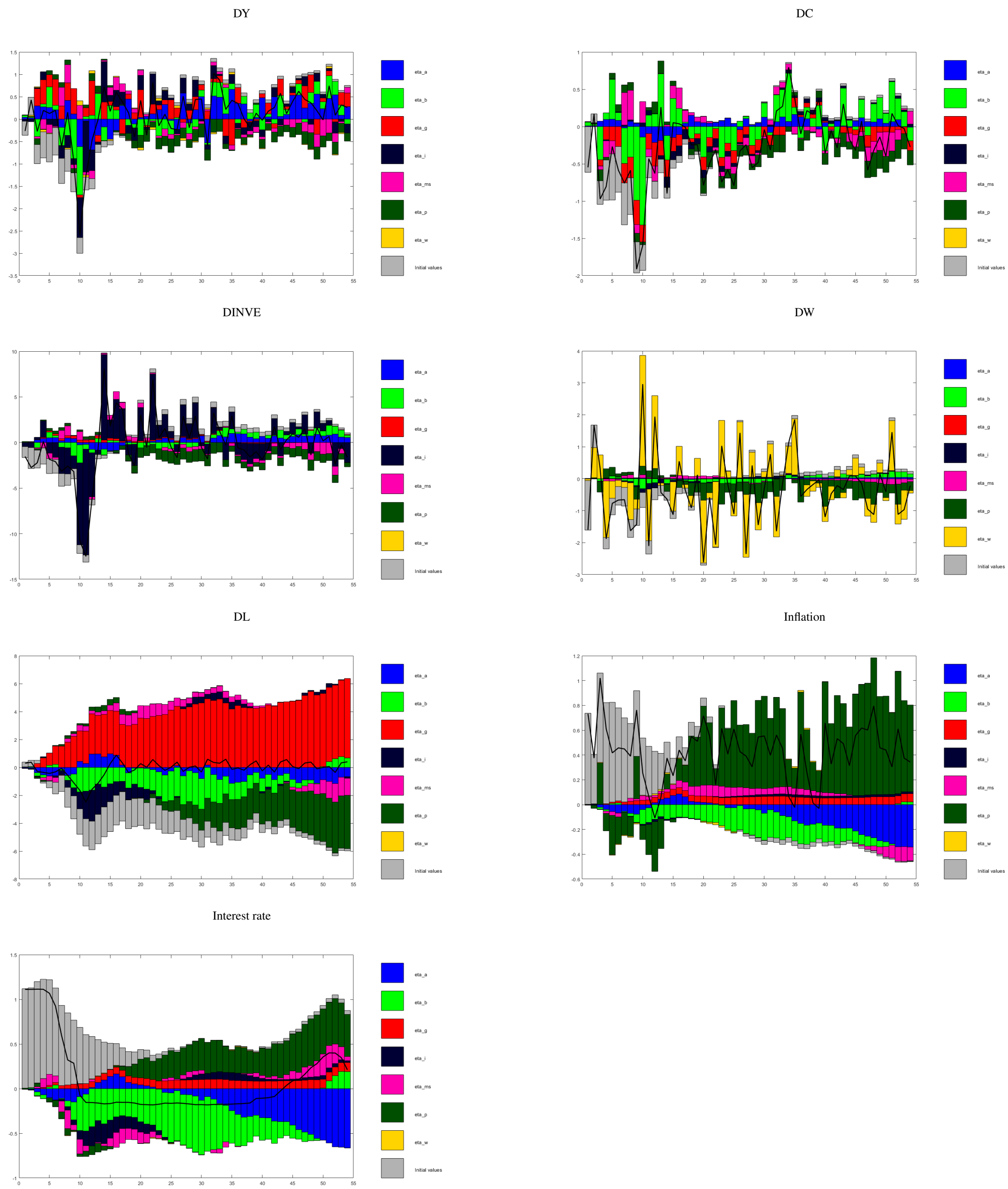
Table C.9 Three sector (2): Prior and posterior distribution of shock processes

Prior Distribution				Posterior Distribution					
				Model 7			**Model 8**		
Parameters	Distr.	Mean	St. Dev.	Mode	Mean	90% HPD Interval	Mode	Mean	90% HPD Interval
ρ_a	Beta	0.5	0.2	0.935	0.919	[0.87 0.97]	0.933	0.918	[0.87 0.97]
ρ_m	Beta	0.5	0.2	0.303	0.315	[0.16 0.47]	0.280	0.307	[0.15 0.46]
ρ_i	Beta	0.5	0.2	0.673	0.632	[0.45 0.82]	0.663	0.645	[0.46 0.82]
ρ_b	Beta	0.5	0.2	0.805	0.777	[0.69 0.87]	0.822	0.787	[0.71 0.87]
ρ_{ga}	Beta	0.5	0.25	0.442	0.446	[0.30 0.61]	0.442	0.453	[0.29 0.61]
ρ_g	Beta	0.5	0.2	0.997	0.994	[0.99 0.9995]	0.997	0.993	[0.99 0.9994]
$\rho_{1,y}$	Beta	0.5	0.1	0.435	0.442	[0.31 0.58]	0.445	0.440	[0.31 0.57]
$\rho_{2,y}$	Beta	0.5	0.1	0.349	0.360	[0.23 0.49]	0.350	0.356	[0.23 0.49]
$\rho_{3,y}$	Beta	0.5	0.1	0.374	0.379	[0.25 0.50]	0.376	0.385	[0.26 0.51]
$\rho_{1,p}$	Beta	0.5	0.2	0.975	0.945	[0.89 0.997]	0.983	0.971	[0.95 0.997]
$\rho_{2,p}$	Beta	0.5	0.2	0.982	0.973	[0.95 0.998]	0.984	0.971	[0.95 0.998]
$\rho_{3,p}$	Beta	0.5	0.2	0.990	0.981	[0.96 0.999]	0.991	0.980	[0.96 0.999]
$\rho_{1,w}$	Beta	0.5	0.2	0.800	0.733	[0.54 0.94]	0.960	0.780	[0.62 0.98]
$\rho_{2,w}$	Beta	0.5	0.2	0.578	0.530	[0.26 0.79]	0.574	0.534	[0.27 0.80]
$\rho_{3,w}$	Beta	0.5	0.2	0.765	0.734	[0.57 0.91]	0.742	0.728	[0.56 0.90]
μ_{1,p_m}	Beta	0.5	0.2	0.643	0.558	[0.32 0.80]	0.190	0.219	[0.05 0.37]
μ_{2,p_m}	Beta	0.5	0.2	0.349	0.341	[0.09 0.57]	0.348	0.345	[0.10 0.58]
μ_{3,p_m}	Beta	0.5	0.2	0.773	0.676	[0.46 0.90]	0.783	0.669	[0.45 0.90]
μ_{1,w_m}	Beta	0.5	0.2	0.266	0.278	[0.06 0.49]	0.231	0.263	[0.05 0.46]
μ_{2,w_m}	Beta	0.5	0.2	0.511	0.434	[0.16 0.70]	0.505	0.446	[0.14 0.73]
μ_{3,w_m}	Beta	0.5	0.2	0.228	0.240	[0.05 0.42]	0.244	0.249	[0.05 0.44]
μ_a	Invgamma	0.1	2	0.565	0.591	[0.47 0.71]	0.570	0.584	[0.46 0.70]
μ_g	Invgamma	0.1	2	0.374	0.394	[0.31 0.47]	0.371	0.400	[0.32 0.48]
μ_b	Invgamma	0.1	2	0.050	0.060	[0.04 0.08]	0.046	0.058	[0.04 0.08]
μ_i	Invgamma	0.1	2	0.698	0.788	[0.51 1.05]	0.707	0.761	[0.48 1.01]
μ_m	Invgamma	0.1	2	0.080	0.085	[0.07 0.10]	0.079	0.084	[0.07 0.10]
$\mu_{1,y}$	Invgamma	0.1	1	1.093	1.125	[0.93 1.32]	1.117	1.129	[0.94 1.33]
$\mu_{2,y}$	Invgamma	0.1	1	0.822	0.856	[0.71 0.999]	0.816	0.844	[0.70 0.99]
$\mu_{3,y}$	Invgamma	0.1	1	0.467	0.479	[0.40 0.56]	0.472	0.483	[0.40 0.56]
μ_p	Invgamma	0.1	2	0.102	0.105	[0.09 0.12]	0.102	0.105	[0.09 0.12]
$\mu_{1,p}$	Invgamma	0.1	2	0.875	0.913	[0.62 1.20]	2.738	2.357	[1.81 3.00]
$\mu_{2,p}$	Invgamma	0.1	2	0.637	0.684	[0.41 0.93]	0.654	0.683	[0.42 0.93]
$\mu_{3,p}$	Invgamma	0.1	2	0.097	0.098	[0.06 0.13]	0.097	0.096	[0.06 0.13]
μ_w	Invgamma	0.1	2	0.923	0.949	[0.79 1.10]	0.924	0.942	[0.79 1.09]
$\mu_{1,w}$	Invgamma	0.1	2	0.233	0.274	[0.17 0.38]	0.225	0.282	[0.17 0.38]
$\mu_{2,w}$	Invgamma	0.1	2	0.245	0.255	[0.19 0.32]	0.242	0.261	[0.19 0.34]
$\mu_{3,w}$	Invgamma	0.1	2	0.261	0.285	[0.20 0.38]	0.249	0.289	[0.20 0.38]

Notes: The table shows on the left-hand side the priors used in estimating the model. In the middle are presented the posterior estimated shocks for the three-sector model (model 7) estimated with sectoral data (with priors for the third sector that indicate flexible wages in sector 3)). On the right-hand side are presented the estimations from the three-sector model (model 8), with priors for the first sector that indicate flexible prices in sector 1)

C.2.4 Impulse responses and variance decomposition graphs for the one sector model

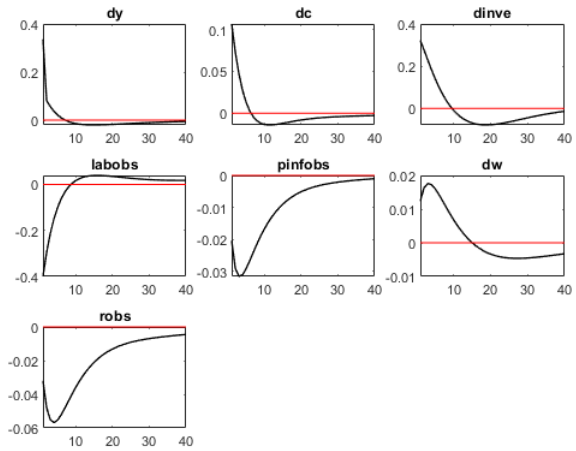
Figure C.3 Variance decomposition graph one sector



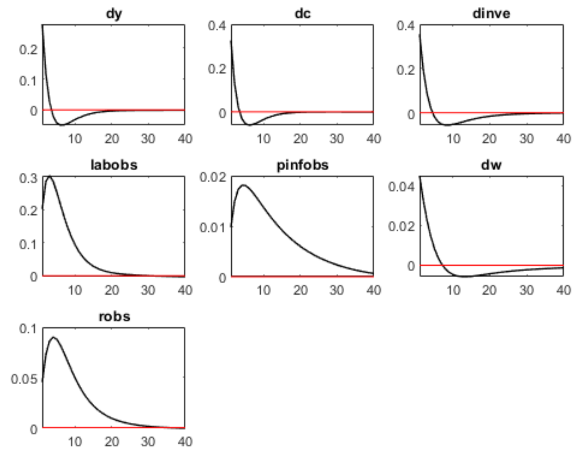
Note: For one sector model.

Figure C.4 Impulse responses one sector (1)

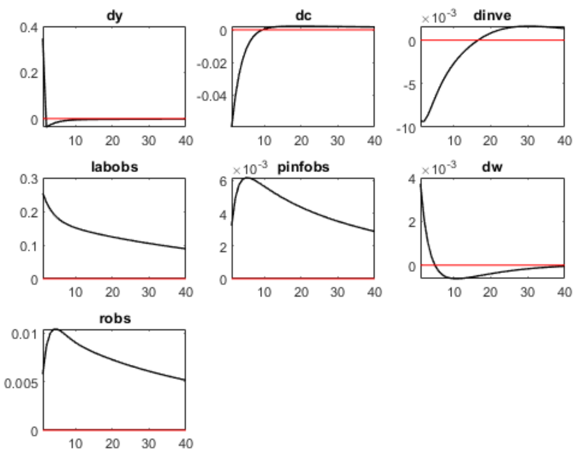
Technology shock



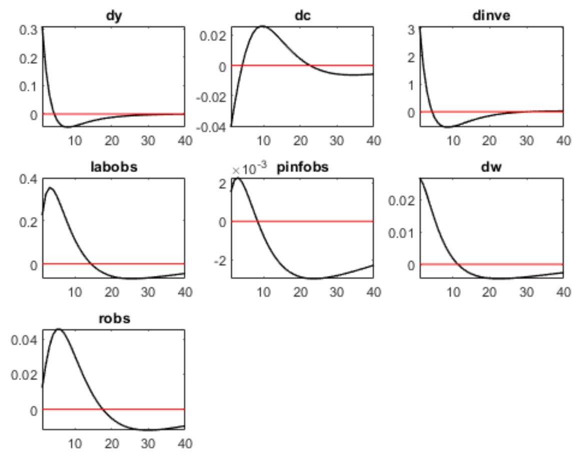
Risk premium shock



Government spending shock



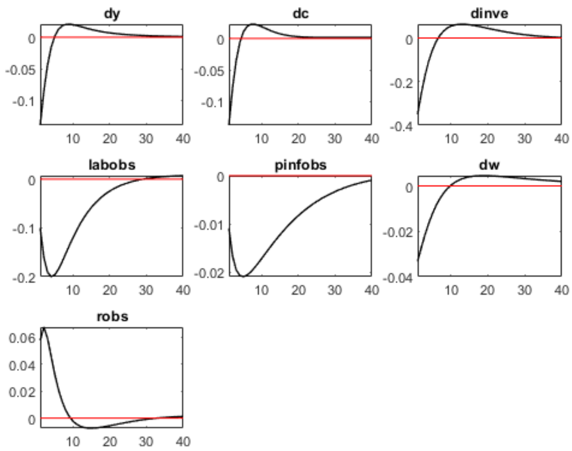
Investment shock



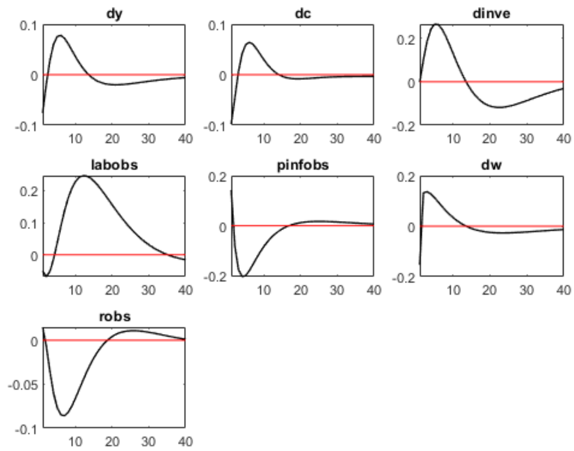
Note: For one sector model.

Figure C.5 Impulse responses one sector (2)

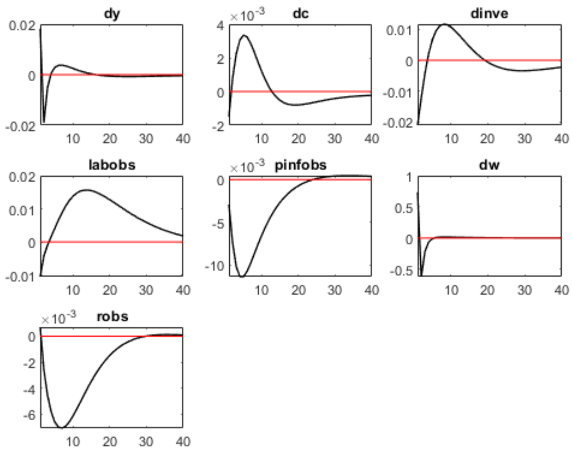
Monetary shock



Price shock



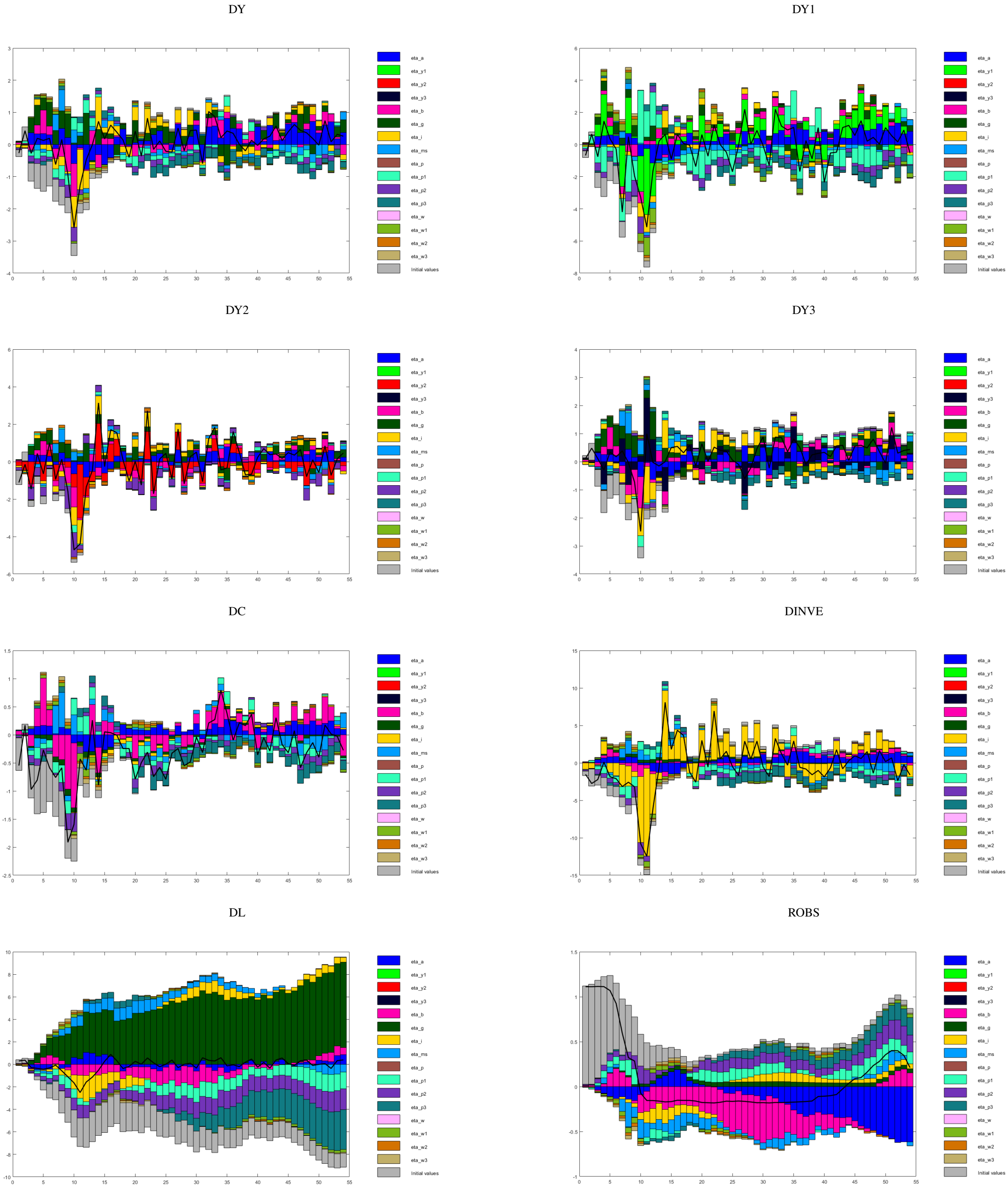
Wage shock



Note: For one sector model.

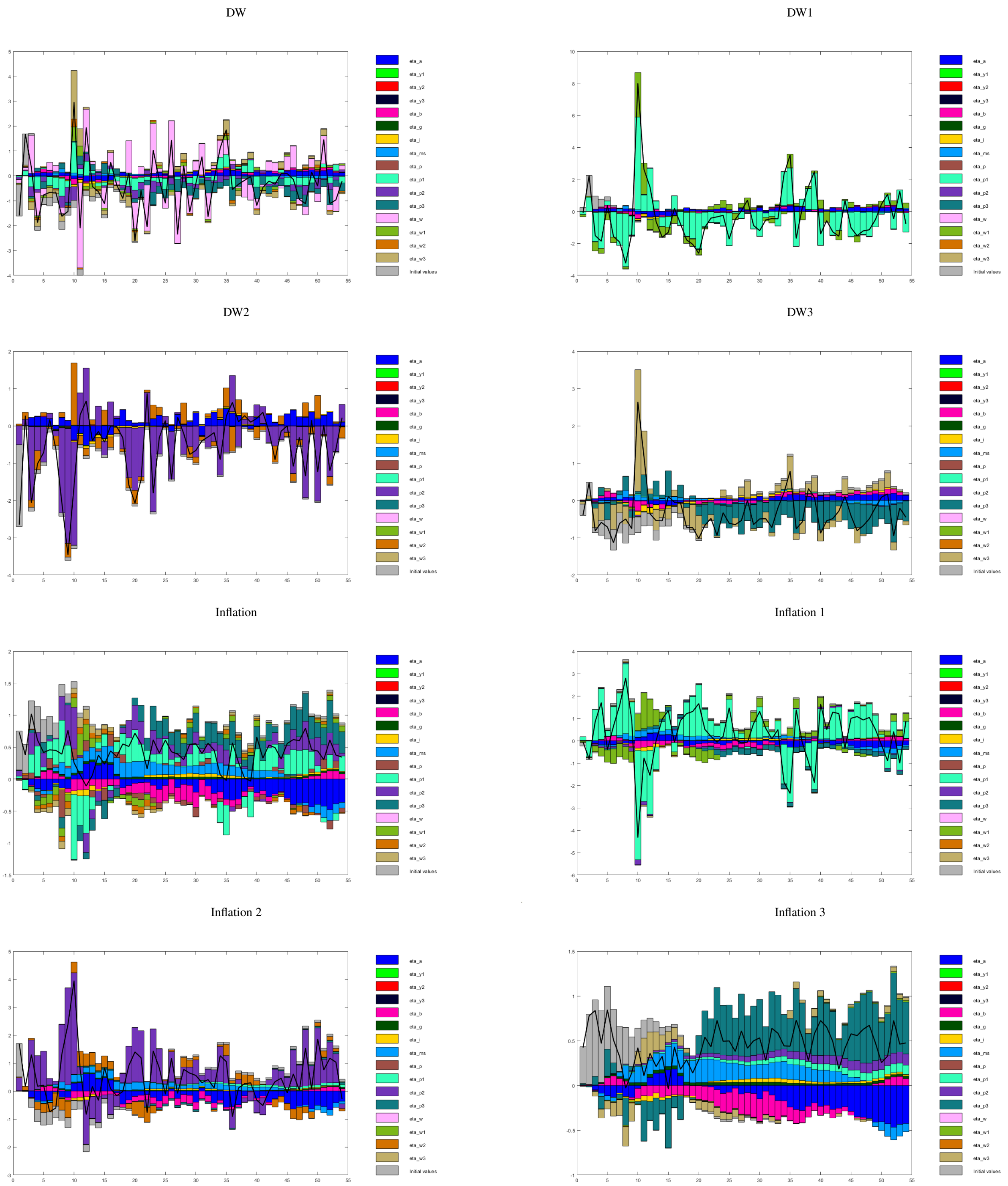
C.2.5 Impulse responses and variance decomposition graphs for the three-sector model

Figure C.6 Variance decomposition graphs three-sector(1)



Note: For three-sector model.

Figure C.7 Variance decomposition graphs three-sector (2)

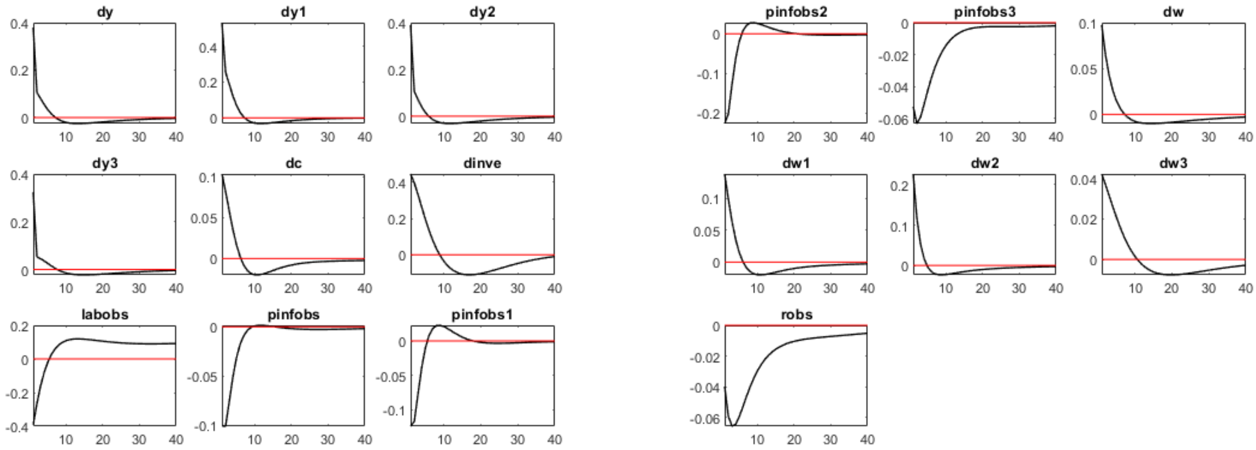


Note: For three-sector model.

Figure C.8 Impulse responses three-sector (1)

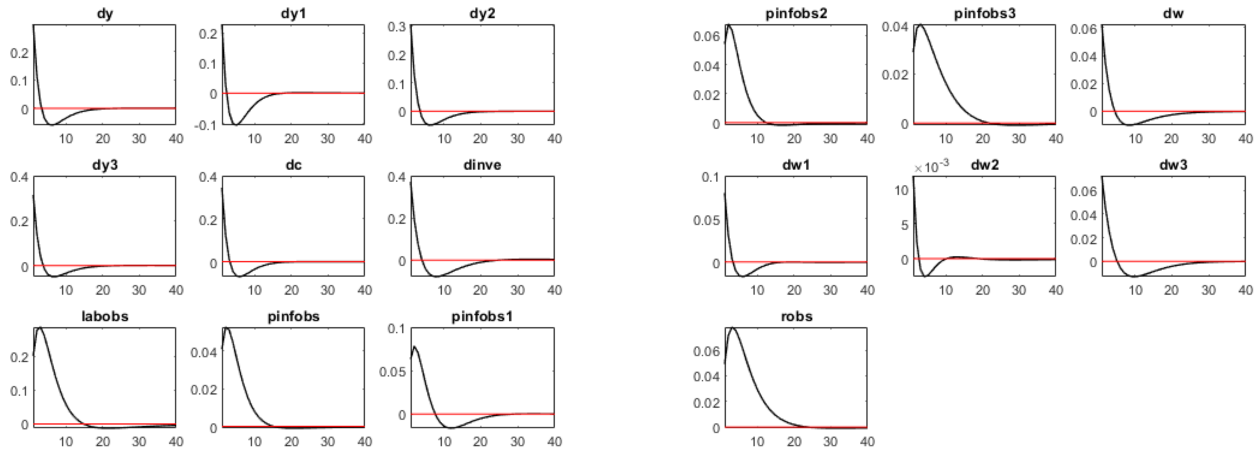
Technology shock

(a) (b)



Risk premium shock

(a) (b)

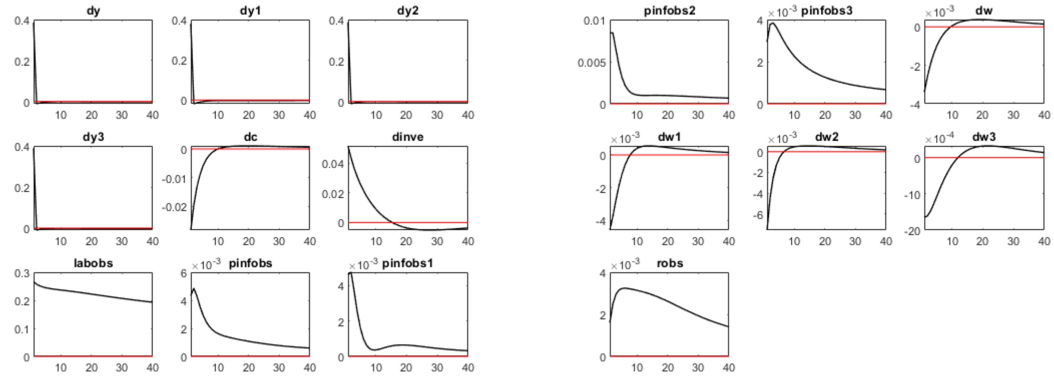


Note: For three-sector model.

Figure C.9 Impulse responses three-sector (2)

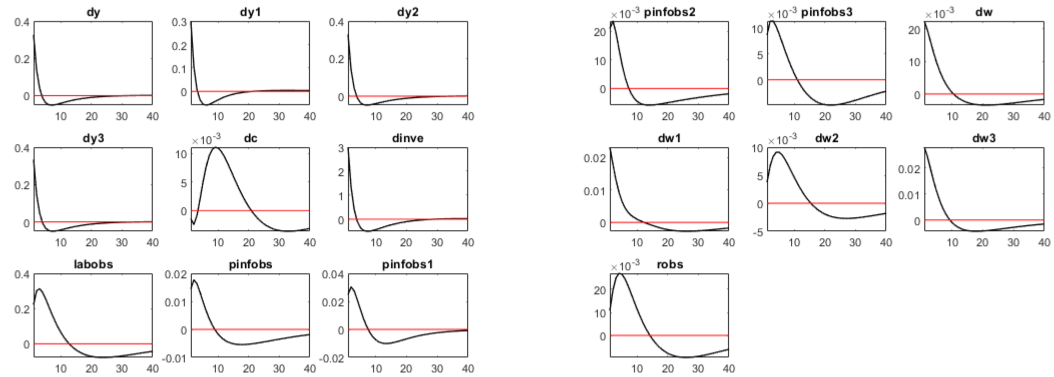
Government spending shock

(a) (b)



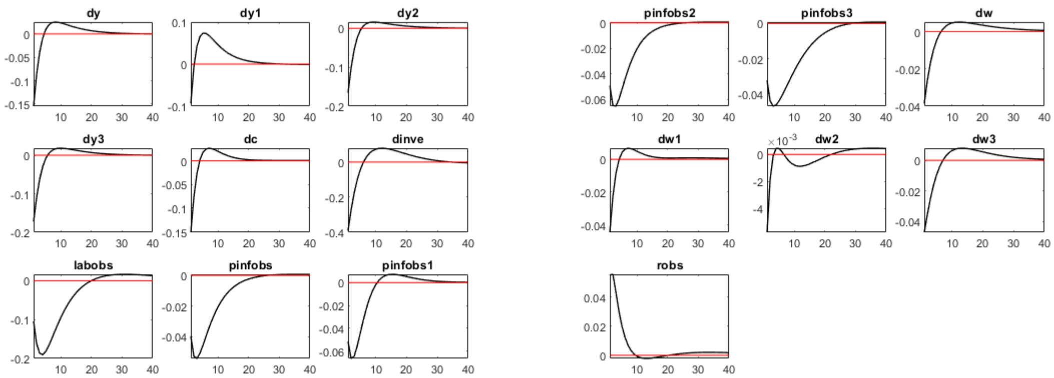
Investment shock

(a) (b)



Monetary shock

(a) (b)

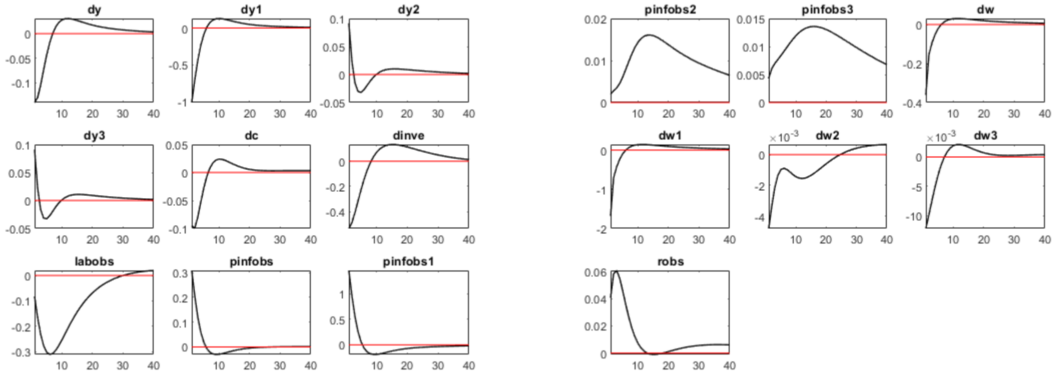


Note: For three-sector model.

Figure C.10 Impulse responses three-sector (3)

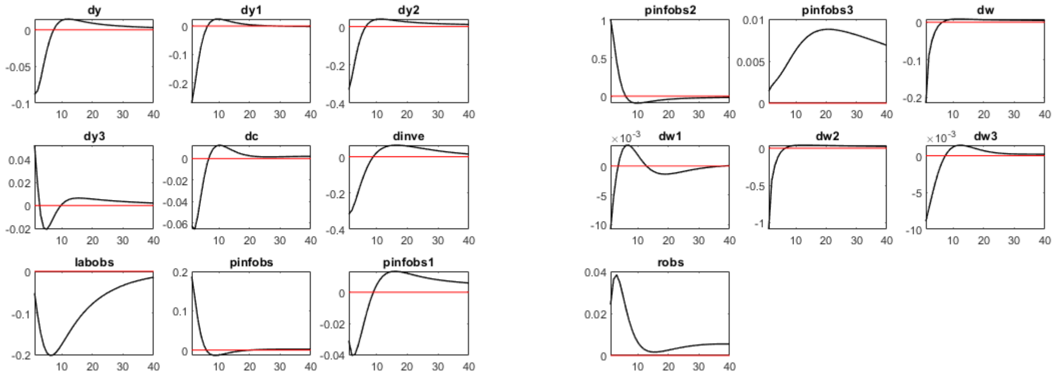
Price shock in sector 1

(a) (b)



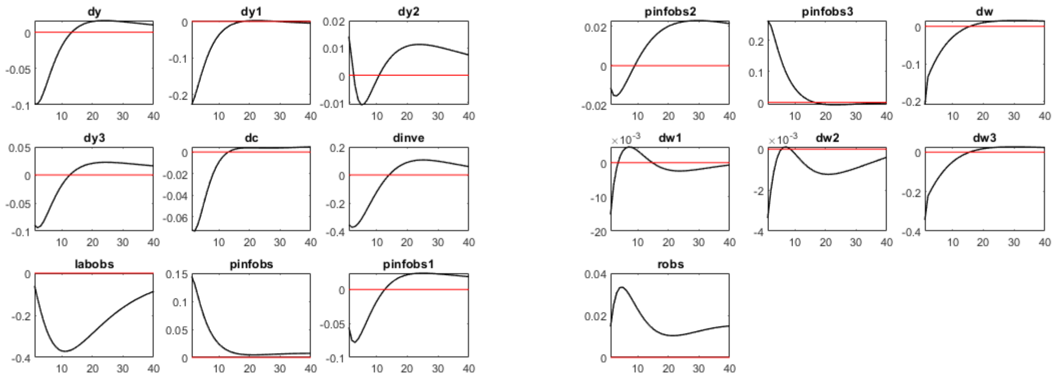
Price shock in sector 2

(a) (b)



Price shock in sector 3

(a) (b)

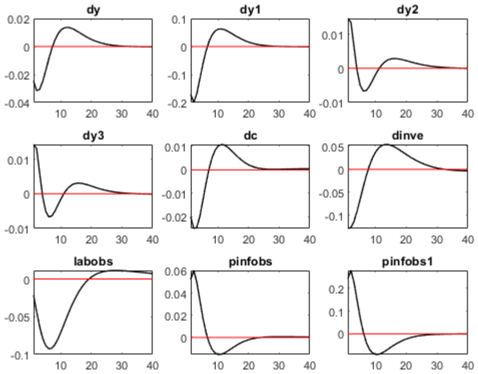


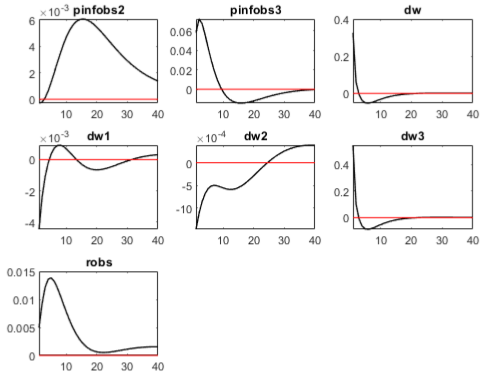
Note: For three-sector model.

Figure C.11 Impulse responses three-sector (4)

Wage shock in sector 1

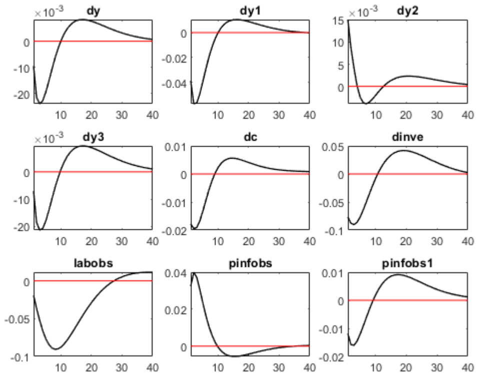
(a) (b)

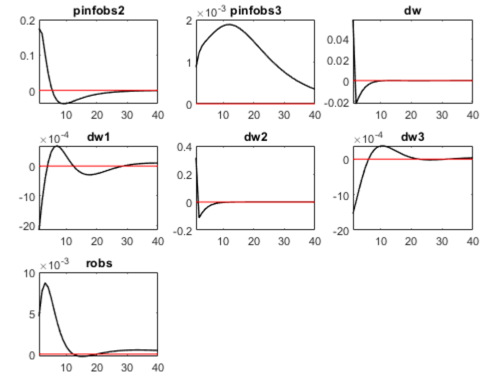




Wage shock in sector 2

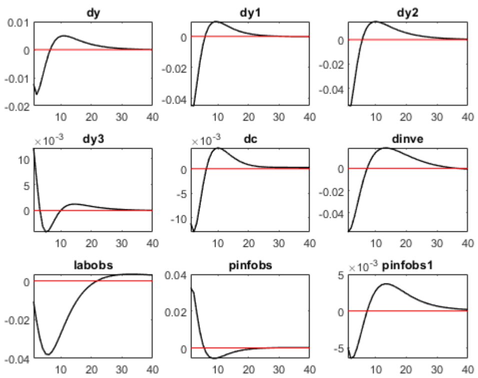
(a) (b)

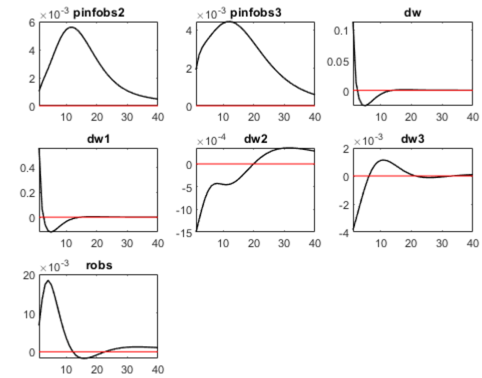




Wage shock in sector 3

(a) (b)





Note: For three-sector model.

Appendix C.3 Log linear equations for dynare

C.3.1 Three-sector sticky economy

Consumption Euler Equation

$$\widehat{C}_t = \frac{\frac{h}{\gamma}}{\left(1+\frac{h}{\gamma}\right)}\widehat{C}_{t-1} + \frac{1}{\left(1+\frac{h}{\gamma}\right)}\widehat{C}_{t+1} + \frac{\left(\sigma - \frac{1}{\chi}\right)}{\left(1+\frac{h}{\gamma}\right)\sigma_\delta}(1-\omega)\left(\widehat{G}_t - \widehat{G}_{t+1}\right) + \frac{1}{\left(1+\frac{h}{\gamma}\right)\sigma_\delta}\left(\widehat{R}_t - \widehat{\pi}_{t+1}\right) + \widehat{\varepsilon}_t^b \tag{C.1}$$

where I write $\sigma_\delta = \left(-\sigma\omega + \frac{1}{\chi}\omega - \frac{1}{\chi}\right)\left(\frac{1}{1-\frac{h}{\gamma}}\right)$

Aggregation of consumption

$$\widehat{C}_t = \sum_{j=1}^{3} \alpha_j \widehat{C}_{j,t} \tag{C.2}$$

Ratio of consumption in sector 1, 2 and 3

$$\widehat{C}_{1,t} - \widehat{C}_{2,t} = \eta\left(\widehat{P}_{1,t} - \widehat{P}_{2,t}\right) \text{ and } \widehat{C}_{2,t} - \widehat{C}_{3,t} = \eta\left(\widehat{P}_{2,t} - \widehat{P}_{3,t}\right) \tag{C.3}$$

Aggregation of Consumption and Government spending

$$\widehat{\widehat{C}}_t = \left[\omega\left(\widehat{C}_t - h\widehat{C}_{t-1}\right) + (1-\omega)\left(\widehat{G}_t\right)\right] \tag{C.4}$$

Marginal costs in sector 1 2 3

$$\widehat{mc}_{1,t} = (1-\alpha)\widehat{W}_{1,t} + \alpha R^k_{1,t} - \widehat{A}_t$$

$$\widehat{mc}_{2,t} = (1-\alpha)\widehat{W}_{2,t} + \alpha \widehat{R}^k_{2,t} - \widehat{A}_t$$

$$\widehat{mc}_{3,t} = (1-\alpha)\widehat{W}_{3,t} + \alpha \widehat{R}^k_{3,t} - \widehat{A}_t$$

Production function in sector 1 2 3

$$\widehat{Y}_{1,t} = \Phi\left(\widehat{A}_t + \alpha\widehat{K}_{1,t} + (1-\alpha)\widehat{L}_{1,t}\right) \tag{C.5}$$

$$\widehat{Y}_{2,t} = \Phi\left(\widehat{A}_t + \alpha\widehat{K}_{2,t} + (1-\alpha)\widehat{L}_{2,t}\right) \tag{C.6}$$

$$\widehat{Y}_{3,t} = \Phi\left(\widehat{A}_t + \alpha\widehat{K}_{3,t} + (1-\alpha)\widehat{L}_{3,t}\right) \tag{C.7}$$

Definition capital services in sector 1 2 3

$$\widehat{\widehat{K}}_{1,t} = \widehat{K}_{1,t-1} + \widehat{Z}_{1,t} \text{ and } \widehat{\widehat{K}}_{2,t} = \widehat{K}_{2,t-1} + \widehat{Z}_{2,t} \text{ and } \widehat{\widehat{K}}_{3,t} = \widehat{K}_{3,t-1} + \widehat{Z}_{3,t} \tag{C.8}$$

Aggregation of capital services

$$\widehat{\widehat{K}}_t = \sum_{j=1}^{3} \alpha_j \widehat{\widehat{K}}_{j,t} \tag{C.9}$$

FOC capacity utilizationin sector 1 2 3

$$\widehat{R}^k_{1,t} = \frac{\chi}{1-\chi}\widehat{Z}_{1,t} \text{ and } \widehat{R}^k_{2,t} = \frac{\chi}{1-\chi}\widehat{Z}_{2,t} \text{ and } \widehat{R}^k_{3,t} = \frac{\chi}{1-\chi}\widehat{Z}_{3,t}$$

Aggregation of capacity utilization

$$\widehat{Z}_t = \sum_{j=1}^{3} \alpha_j \widehat{Z}_{j,t} \tag{C.10}$$

Firm in sector 1 2 3 FOC capital

$$\widehat{R}^k_{1,t} = \widehat{W}_{1,t} + \widehat{L}_{1,t} - \widehat{\widehat{K}}_{1,t} \tag{C.11}$$

$$\widehat{R}^k_{2,t} = \widehat{W}_{2,t} + \widehat{L}_{2,t} - \widehat{\widehat{K}}_{2,t} \tag{C.12}$$

$$\widehat{R}^k_{3,t} = \widehat{W}_{3,t} + \widehat{L}_{3,t} - \widehat{\widehat{K}}_{3,t} \tag{C.13}$$

Aggregation of labor supply

$$\widehat{L}_t = \sum_{j=1}^{3} b_j \widehat{L}_{j,t} \tag{C.14}$$

Investment Euler equation 1 2 3

$$\widehat{I}_{1,t} = \frac{1}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{1,t-1} + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{1,t+1} + \frac{\widehat{Q}_{1,t}}{\left(1+\overline{\beta}\gamma\right)\gamma^2 k} + \widehat{\varepsilon}^i_t \tag{C.15}$$

$$\widehat{I}_{2,t} = \frac{1}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{2,t-1} + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{2,t+1} + \frac{\widehat{Q}_{2,t}}{\left(1+\overline{\beta}\gamma\right)\gamma^2 k} + \widehat{\varepsilon}^i_t \tag{C.16}$$

$$\widehat{I}_{3,t} = \frac{1}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{3,t-1} + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{3,t+1} + \frac{\widehat{Q}_{3,t}}{\left(1+\overline{\beta}\gamma\right)\gamma^2 k} + \widehat{\varepsilon}^i_t \tag{C.17}$$

Aggregation of investment

$$\widehat{I}_t = \sum_{j=1}^{3} \alpha_j \widehat{I}_{j,t} \tag{C.18}$$

Aggregation of output

$$\widehat{Y}_t = \gamma_1 \widehat{C}_t + \widehat{G}_t + \gamma_3 \widehat{I}_t + \gamma_4 \widehat{Z}_t \tag{C.19}$$

$$\widehat{Y}_t = \sum_{j=1}^{3} \alpha_j \widehat{Y}_{j,t} \tag{C.20}$$

$$\widehat{Y}_{2,t} = -\eta \left(\widehat{P}_{2,t} - \widehat{P}_t\right) + \widehat{Y}_t \text{ and } \widehat{Y}_{3,t} = -\eta \left(\widehat{P}_{3,t} - \widehat{P}_t\right) + \widehat{Y}_t \tag{C.21}$$

Q Equation in sector 1 2 3

$$\widehat{Q}_{1,t} = \left(\pi_{t+1} - \widehat{R}_{t+1}\right) + \frac{r_k^*}{r_k^* + (1-\delta)} \widehat{R}^k_{1,t+1} + \frac{(1-\delta)}{r_k^* + (1-\delta)} \widehat{Q}_{1,t+1} + \frac{\left(1+\frac{h}{\gamma}\right)\sigma}{\left(1-\frac{h}{\gamma}\right)} \varepsilon_t^b \tag{C.22}$$

$$\widehat{Q}_{2,t} = \left(\pi_{t+1} - \widehat{R}_{t+1}\right) + \frac{r_k^*}{r_k^* + (1-\delta)} \widehat{R}^k_{2,t+1} + \frac{(1-\delta)}{r_k^* + (1-\delta)} \widehat{Q}_{2,t+1} + \frac{\left(1+\frac{h}{\gamma}\right)\sigma}{\left(1-\frac{h}{\gamma}\right)} \varepsilon_t^b \tag{C.23}$$

$$\widehat{Q}_{3,t} = \left(\pi_{t+1} - \widehat{R}_{t+1}\right) + \frac{r_k^*}{r_k^* + (1-\delta)} \widehat{R}^k_{3,t+1} + \frac{(1-\delta)}{r_k^* + (1-\delta)} \widehat{Q}_{3,t+1} + \frac{\left(1+\frac{h}{\gamma}\right)\sigma}{\left(1-\frac{h}{\gamma}\right)} \varepsilon_t^b \tag{C.24}$$

where we have the steady-state rental rate $r_k^* = \frac{1}{\beta} + \delta - 1$

Wage Philips curve in sectors 1 2 3

$$\begin{aligned}\widehat{W}_{1,t} \;=\;& \frac{\left(1-\overline{\beta}\gamma\theta_{1,w}\right)(1-\theta_{1,w})}{\theta_{1,w}\left(1+\overline{\beta}\gamma\right)} \frac{1}{(1+\frac{\varepsilon_w}{\lambda})} \widehat{\mu}_{1,t} + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)} \widehat{W}_{1,t+1} + \frac{1}{\left(1+\overline{\beta}\gamma\right)} \widehat{W}_{1,t-1} \\ & \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)} \widehat{\pi}_{1,t+1} - \frac{1+\overline{\beta}\gamma\rho_{1,w}}{\left(1+\overline{\beta}\gamma\right)} \widehat{\pi}_{1,t} + \frac{\rho_{1,w}}{\left(1+\overline{\beta}\gamma\right)} \widehat{\pi}_{1,t-1} + \widehat{\varepsilon}^w_{1,t}\end{aligned} \tag{C.25}$$

$$\begin{aligned}\widehat{W}_{2,t} \;=\;& \frac{\left(1-\overline{\beta}\gamma\theta_{2,w}\right)(1-\theta_{2,w})}{\theta_{2,w}\left(1+\overline{\beta}\gamma\right)} \frac{1}{(1+\frac{\varepsilon_w}{\lambda})} \widehat{\mu}_{2,t} + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)} \widehat{W}_{2,t+1} + \frac{1}{\left(1+\overline{\beta}\gamma\right)} \widehat{W}_{2,t-1} \\ & + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)} \widehat{\pi}_{2,t+1} - \frac{1+\overline{\beta}\gamma\rho_{2,w}}{\left(1+\overline{\beta}\gamma\right)} \widehat{\pi}_{2,t} + \frac{\rho_{2,w}}{\left(1+\overline{\beta}\gamma\right)} \widehat{\pi}_{2,t-1} + \widehat{\varepsilon}^w_{2,t}\end{aligned} \tag{C.26}$$

$$\widehat{W}_{3,t} = \frac{\left(1-\overline{\beta}\gamma\theta_{3,w}\right)(1-\theta_{3,w})}{\theta_{3,w}\left(1+\overline{\beta}\gamma\right)}\frac{1}{(1+\frac{\varepsilon_w}{\lambda})}\widehat{\mu}_{3,t}+\frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)}\widehat{W}_{3,t+1}+\frac{1}{\left(1+\overline{\beta}\gamma\right)}\widehat{W}_{3,t-1} \tag{C.27}$$

$$+\frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)}\widehat{\pi}_{3,t+1}-\frac{1+\overline{\beta}\gamma\rho_{3,w}}{\left(1+\overline{\beta}\gamma\right)}\widehat{\pi}_{3,t}+\frac{\rho_{3,w}}{\left(1+\overline{\beta}\gamma\right)}\widehat{\pi}_{3,t-1}+\widehat{\varepsilon}^{w}_{3,t}$$

where $\widehat{\mu}_{1,t}=\widehat{mrs}_{1,t}-\widehat{W}_{1,t}$ and $\widehat{\mu}_{2,t}=\widehat{mrs}_{2,t}-\widehat{W}_{2,t}$ and $\widehat{\mu}_{3,t}=\widehat{mrs}_{3,t}-\widehat{W}_{3,t}$

Marginal rate of substitution in sectors 1 2 3

$$\widehat{mrs}_{1,t}=\left(\sigma-\frac{1}{\chi}\right)(1-\omega)\left(\widehat{G}_t\right)+\sigma_\lambda\widehat{C}_t+\sigma_\delta\frac{h}{\gamma}\widehat{C}_{t-1}+\frac{1}{\eta}\widehat{C}_{1,t}+\frac{1}{\lambda}\widehat{L}_{1,t}+\left(\varphi-\frac{1}{\lambda}\right)\widehat{L}_t$$

$$\widehat{mrs}_{2,t}=\left(\sigma-\frac{1}{\chi}\right)(1-\omega)\left(\widehat{G}_t\right)+\sigma_\lambda\widehat{C}_t+\sigma_\delta\frac{h}{\gamma}\widehat{C}_{t-1}+\frac{1}{\eta}\widehat{C}_{2,t}+\frac{1}{\lambda}\widehat{L}_{2,t}+\left(\varphi-\frac{1}{\lambda}\right)\widehat{L}_t$$

$$\widehat{mrs}_{3,t}=\left(\sigma-\frac{1}{\chi}\right)(1-\omega)\left(\widehat{G}_t\right)+\sigma_\lambda\widehat{C}_t+\sigma_\delta\frac{h}{\gamma}\widehat{C}_{t-1}+\frac{1}{\eta}\widehat{C}_{3,t}+\frac{1}{\lambda}\widehat{L}_{3,t}+\left(\varphi-\frac{1}{\lambda}\right)\widehat{L}_t$$

and $\sigma_L=\left[\left(\sigma\omega-\frac{1}{\chi}\omega+\left(\frac{1}{\chi}-\frac{1}{\eta}\right)\right)\frac{1}{1-\frac{h}{\gamma}}-\frac{1}{\eta}\right]$

Aggregation of wages

$$W_t=\sum_{j=1}^{3}+b_j\widehat{W}_{j,t} \tag{C.28}$$

Accumulation of capital in sectors 1 2 3

$$\widehat{K}_{1,t}=\left[\frac{1-\delta}{\gamma}\right]\widehat{K}_{1,t-1}+\left(1-\frac{1-\delta}{\gamma}\right)I_{1,t}+\left(1-\frac{1-\delta}{\gamma}\right)\gamma^2k\widehat{\varepsilon}^i_t$$

$$\widehat{K}_{2,t}=\left[\frac{1-\delta}{\gamma}\right]\widehat{K}_{2,t-1}+\left(1-\frac{1-\delta}{\gamma}\right)I_{2,t}+\left(1-\frac{1-\delta}{\gamma}\right)\gamma^2k\widehat{\varepsilon}^i_t$$

$$\widehat{K}_{3,t}=\left[\frac{1-\delta}{\gamma}\right]\widehat{K}_{3,t-1}+\left(1-\frac{1-\delta}{\gamma}\right)I_{3,t}+\left(1-\frac{1-\delta}{\gamma}\right)\gamma^2k\widehat{\varepsilon}^i_t$$

Taylor Rule for monetary policy

$$\widehat{R}_t=\rho\widehat{R}_{t-1}+(1-\rho)\left(\phi_\pi\widehat{\pi}_t+\phi_Y\left(\widehat{Y}_t-\widehat{Y}^{flex}_t\right)\right)+\phi_{dY}\left(\left(\widehat{Y}_t-\widehat{Y}_{t-1}\right)-\left(\widehat{Y}^{flex}_t-\widehat{Y}^{flex}_{t-1}\right)\right)+\widehat{\varepsilon}^m_t \tag{C.29}$$

Inflation Philips curve in sectors 1 2 3

$$\pi_{1,t} = \frac{(1-\theta_{1,p})\left(1-\theta_{1,p}\overline{\beta}\gamma\right)}{\theta_{1,p}} \frac{1}{(1+\overline{\beta}\gamma\rho_{1,p})}\widehat{mc}_{1,t} + \frac{\overline{\beta}\gamma}{(1+\overline{\beta}\gamma\rho_{1,p})}\pi_{t+1} + \frac{\rho_{1,p}}{(1+\overline{\beta}\gamma\rho_{1,p})}\pi_{t-1} + \widehat{\varepsilon}^{p}_{1,t} \tag{C.30}$$

$$\pi_{2,t} = \frac{(1-\theta_{2,p})\left(1-\theta_{2,p}\overline{\beta}\gamma\right)}{\theta_{2,p}} \frac{1}{(1+\overline{\beta}\gamma\rho_{2,p})}\widehat{mc}_{2,t} + \frac{\overline{\beta}\gamma}{(1+\overline{\beta}\gamma\rho_{2,p})}\pi_{t+1} + \frac{\rho_{2,p}}{(1+\overline{\beta}\gamma\rho_{2,p})}\pi_{t-1} + \widehat{\varepsilon}^{p}_{2,t} \tag{C.31}$$

$$\pi_{3,t} = \frac{(1-\theta_{3,p})\left(1-\theta_{3,p}\overline{\beta}\gamma\right)}{\theta_{3,p}} \frac{1}{(1+\overline{\beta}\gamma\rho_{3,p})}\widehat{mc}_{3,t} + \frac{\overline{\beta}\gamma}{(1+\overline{\beta}\gamma\rho_{2,p})}\pi_{t+1} + \frac{\rho_{2,p}}{(1+\overline{\beta}\gamma\rho_{2,p})}\pi_{t-1} + \widehat{\varepsilon}^{p}_{3,t} \tag{C.32}$$

Aggregate inflation and relations

$$\widehat{\pi}_t = \sum_{j=1}^{3} \alpha_j \widehat{\pi}_{j,t} \tag{C.33}$$

$$\rho_{1,t} = \rho_{1,t-1} + \widehat{\pi}_{1,t} - \widehat{\pi}_t$$

$$\rho_{2,t} = \rho_{2,t-1} + \widehat{\pi}_{2,t} - \widehat{\pi}_t \tag{C.34}$$

$$\rho_{3,t} = \rho_{3,t-1} + \widehat{\pi}_{3,t} - \widehat{\pi}_t \tag{C.35}$$

where I have $\rho_{1,t} = \widehat{P}_{1,t} - \widehat{P}_t$ and $\rho_{2,t} = \widehat{P}_{2,t} - \widehat{P}_t$ and $\rho_{3,t} = \widehat{P}_{3,t} - \widehat{P}_t$

Supply shock process

$$\widehat{\varepsilon}^{y}_{1,t} = \rho_{1,y}\widehat{\varepsilon}^{y}_{1,t-1} + \mu_{1,y,t} \tag{C.36}$$

$$\widehat{\varepsilon}^{y}_{2,t} = \rho_{2,y}\widehat{\varepsilon}^{y}_{2,t-1} + \mu_{2,y,t} \tag{C.37}$$

$$\widehat{\varepsilon}^{y}_{3,t} = \rho_{3,y}\widehat{\varepsilon}^{y}_{3,t-1} + \mu_{3,y,t} \tag{C.38}$$

Technology process

$$\widehat{A}_t = \rho_A \widehat{A}_{t-1} + \mu_{A,t} \tag{C.39}$$

Intertemporal preference shifter (Financial risk premium process)

$$\widehat{\varepsilon}^{b}_t = \rho_b \widehat{\varepsilon}^{b}_{t-1} + \mu_{b,t} \tag{C.40}$$

Investment relative price process

$$\widehat{\varepsilon}^{i}_t = \rho_i \widehat{\varepsilon}^{i}_{t-1} + \mu_{i,t} \tag{C.41}$$

Monetary Policy Shock

$$\widehat{\varepsilon}_t^m = \rho_m \widehat{\varepsilon}_{t-1}^m + \mu_{m,t} \tag{C.42}$$

Government spending process

$$\widehat{G}_t = \rho_G \widehat{G}_{t-1} + \mu_{G,t} + \rho_{GA}\mu_{A,t} \tag{C.43}$$

Wage Mark-up shock

$$\widehat{\varepsilon}_t^w = \rho_w \widehat{\varepsilon}_{t-1}^w + \mu_{w\_ma,t} + \mu_w \mu_{w\_ma,t-1} \tag{C.44}$$

$$\widehat{\varepsilon}_{1,t}^w = \rho_{1,w} \widehat{\varepsilon}_{1,t-1}^w + \mu_{1,w\_ma,t} + \mu_{1,w} \mu_{1,w\_ma,t-1} \tag{C.45}$$

$$\widehat{\varepsilon}_{2,t}^w = \rho_{2,w} \widehat{\varepsilon}_{2,t-1}^w + \mu_{2,w\_ma,t} + \mu_{2,w} \mu_{2,w\_ma,t-1} \tag{C.46}$$

$$\widehat{\varepsilon}_{3,t}^w = \rho_{3,w} \widehat{\varepsilon}_{3,t-1}^w + \mu_{3,w\_ma,t} + \mu_{3,w} \mu_{3,w\_ma,t-1} \tag{C.47}$$

Price Mark-up shock

$$\widehat{\varepsilon}_t^P = \rho_p \widehat{\varepsilon}_{t-1}^P + \mu_{p\_ma,t} + \mu_p \mu_{p\_ma,t-1} \tag{C.48}$$

$$\widehat{\varepsilon}_{1,t}^P = \rho_{1,p} \widehat{\varepsilon}_{1,t-1}^P + \mu_{1,p\_ma,t} + \mu_{1,p} \mu_{1,p\_ma,t-1} \tag{C.49}$$

$$\widehat{\varepsilon}_{2,t}^P = \rho_{2,p} \widehat{\varepsilon}_{2,t-1}^P + \mu_{2,p\_ma,t} + \mu_{2,p} \mu_{2,p\_ma,t-1} \tag{C.50}$$

$$\widehat{\varepsilon}_{3,t}^P = \rho_{3,p} \widehat{\varepsilon}_{2,t-1}^P + \mu_{3,p\_ma,t} + \mu_{3,p} \mu_{3,p\_ma,t-1} \tag{C.51}$$

C.3.2 The series used from Smets & Wouters (2007) plus the sectoral series added for the three-sector model:

$$\Delta\widehat{Y} = \widehat{Y}_t - \widehat{Y}_{t-1} + trend \tag{C.52}$$

$$\Delta\widehat{Y}_1 = \widehat{Y}_{1,t} - \widehat{Y}_{1,t-1} + trend + \widehat{\varepsilon}_{1,t}^y \tag{C.53}$$

$$\Delta\widehat{Y}_2 = \widehat{Y}_{2,t} - \widehat{Y}_{2,t-1} + trend + \widehat{\varepsilon}_{2,t}^y \tag{C.54}$$

$$\Delta\widehat{Y}_3 = \widehat{Y}_{3,t} - \widehat{Y}_{3,t-1} + trend + \widehat{\varepsilon}_{3,t}^y \tag{C.55}$$

$$\Delta\widehat{C} = \widehat{C}_t - \widehat{C}_{t-1} + trend \tag{C.56}$$

$$\Delta\widehat{I} = \widehat{I}_t - \widehat{I}_{t-1} + trend \tag{C.57}$$

$$\Delta\widehat{W} = \widehat{W}_t - \widehat{W}_{t-1} + trend + \widehat{\varepsilon}_t^w \tag{C.58}$$

$$\Delta\widehat{W}_1 = \widehat{W}_{1,t} - \widehat{W}_{1,t-1} + trend \tag{C.59}$$

$$\Delta\widehat{W}_2 = \widehat{W}_{2,t} - \widehat{W}_{2,t-1} + trend \tag{C.60}$$

$$\Delta\widehat{W}_3 = \widehat{W}_{3,t} - \widehat{W}_{3,t-1} + trend \tag{C.61}$$

$$\widehat{\pi}_t^{obs} = \widehat{\pi}_t + \overline{\pi}_t + \widehat{\varepsilon}_t^p \tag{C.62}$$

$$\widehat{\pi}_{1,t}^{obs} = \widehat{\pi}_{1,t} + \overline{\pi}_t \tag{C.63}$$

$$\widehat{\pi}_{2,t}^{obs} = \widehat{\pi}_{2,t} + \overline{\pi}_t \tag{C.64}$$

$$\widehat{\pi}_{3,t}^{obs} = \widehat{\pi}_{3,t} + \overline{\pi}_t \tag{C.65}$$

$$\widehat{R}_t^{obs} = \widehat{R}_t + \overline{R}_t \tag{C.66}$$

$$\widehat{L}_t^{obs} = \widehat{L}_t + \overline{L}_t \tag{C.67}$$

while $\gamma = 1 + \frac{trend}{100}$ and $P^* = 1 + \frac{\overline{\pi}_t}{100}$ and $\overline{\beta} = \beta\gamma^{-\sigma}$

Where I assume the values: $trend = 0.1$; $\overline{\pi}_t = 0, \overline{L}_t = 0$

$$\overline{R}_t = \left(\frac{P^*}{\beta\gamma^{-\sigma}} - 1\right) * 100 \tag{C.68}$$

Appendix C3. three-sector flexible economy

$$\widehat{C}_t^{flex} = \frac{\frac{h}{\gamma}}{\left(1+\frac{h}{\gamma}\right)}\widehat{C}_{t-1}^{flex} + \frac{1}{\left(1+\frac{h}{\gamma}\right)}\widehat{C}_{t+1}^{flex} + \frac{\left(\sigma - \frac{1}{\chi}\right)}{\left(1+\frac{h}{\gamma}\right)\sigma_\delta}(1-\omega)\left(\widehat{G}_t - \widehat{G}_{t+1}\right) + \frac{1}{\left(1+\frac{h}{\gamma}\right)\sigma_\delta}\left(\widehat{R}_t\right) + \widehat{\varepsilon}_t^b \tag{C.69}$$

where I write $\sigma_\delta = \left(-\sigma\omega + \frac{1}{\chi}\omega - \frac{1}{\chi}\right)\left(\frac{1}{1-\frac{h}{\gamma}}\right)$

Aggregation of consumption

$$\widehat{C}_t^{flex} = \sum_{j=1}^{3} \alpha_j \widehat{C}_{j,t}^{flex} \tag{C.70}$$

Ratio of consumption in sector 1, 2 and 3

$$\widehat{C}_{1,t}^{flex} - \widehat{C}_{2,t}^{flex} = \eta\left(\widehat{P}_{1,t} - \widehat{P}_{2,t}\right) \tag{C.71}$$

$$\widehat{C}_{2,t}^{flex} - \widehat{C}_{3,t}^{flex} = \eta\left(\widehat{P}_{2,t} - \widehat{P}_{3,t}\right) \tag{C.72}$$

Aggregation of Consumption and Government spending

$$\widehat{\widehat{C}}_t^{flex} = \left[\omega\left(\widehat{C}_t^{flex} - h\widehat{C}_{t-1}^{flex}\right) + (1-\omega)\left(\widehat{G}_t\right)\right] \tag{C.73}$$

Marginal costs in sector 1 2 3

$$\widehat{mc}_{1,t}^{flex} = (1-\alpha)\widehat{W}_{1,t}^{flex} + \alpha R_{1,t}^{flex,k} - \widehat{A}_t$$

$$\widehat{mc}_{2,t}^{flex} = (1-\alpha)\widehat{W}_{2,t}^{flex} + \alpha\widehat{R}_{2,t}^{flex,k} - \widehat{A}_t$$

$$\widehat{mc}_{3,t}^{flex} = (1-\alpha)\widehat{W}_{3,t}^{flex} + \alpha\widehat{R}_{3,t}^{flex,k} - \widehat{A}_t$$

$$0 = \alpha_1\widehat{mc}_{1,t}^{flex} + \alpha_2\widehat{mc}_{2,t}^{flex} + \alpha_3\widehat{mc}_{3,t}^{flex} \tag{C.74}$$

Production function in sector 1 2 3

$$\widehat{Y}_{1,t}^{flex} = \Phi\left(\widehat{A}_t + \alpha\widehat{K}_{1,t}^{flex} + (1-\alpha)\widehat{L}_{1,t}^{flex}\right) \tag{C.75}$$

$$\widehat{Y}_{2,t}^{flex} = \Phi\left(\widehat{A}_t + \alpha\widehat{K}_{2,t}^{flex} + (1-\alpha)\widehat{L}_{2,t}^{flex}\right) \tag{C.76}$$

$$\widehat{Y}_{3,t}^{flex} = \Phi\left(\widehat{A}_t + \alpha\widehat{K}_{3,t}^{flex} + (1-\alpha)\widehat{L}_{3,t}^{flex}\right) \tag{C.77}$$

Definition capital services in sector 1 2 3

$$\widehat{\widehat{K}}_{1,t}^{flex} = \widehat{K}_{1,t-1}^{flex} + \widehat{Z}_{1,t}^{flex} \tag{C.78}$$

$$\widehat{\widehat{K}}_{2,t}^{flex} = \widehat{K}_{2,t-1}^{flex} + \widehat{Z}_{2,t}^{flex} \tag{C.79}$$

$$\widehat{\widehat{K}}_{3,t}^{flex} = \widehat{K}_{3,t-1}^{flex} + \widehat{Z}_{3,t}^{flex} \tag{C.80}$$

Aggregation of capital services

$$\widehat{\widehat{K}}_t^{flex} = \sum_{j=1}^{3}\alpha_j\widehat{\widehat{K}}_{j,t}^{flex} \tag{C.81}$$

FOC capacity utilizationin sector 1 2 3

$$\widehat{R}_{1,t}^{flex,k} = \frac{\chi}{1-\chi}\widehat{Z}_{1,t}^{flex}$$

$$\widehat{R}_{2,t}^{flex,k} = \frac{\chi}{1-\chi}\widehat{Z}_{2,t}^{flex}$$

$$\widehat{R}_{3,t}^{flex,k} = \frac{\chi}{1-\chi}\widehat{Z}_{3,t}^{flex}$$

Aggregation of capacity utilization

$$\widehat{Z}_{t}^{flex} = \sum_{j=1}^{3} \alpha_j \widehat{Z}_{j,t}^{flex} \tag{C.82}$$

Firm in sector 1 2 3 FOC capital

$$\widehat{R}_{1,t}^{flex,k} = \widehat{W}_{1,t}^{flex} + \widehat{L}_{1,t}^{flex} - \widehat{\widehat{K}}_{1,t}^{flex} \tag{C.83}$$

$$\widehat{R}_{2,t}^{flex,k} = \widehat{W}_{2,t}^{flex} + \widehat{L}_{2,t}^{flex} - \widehat{\widehat{K}}_{2,t}^{flex} \tag{C.84}$$

$$\widehat{R}_{3,t}^{flex,k} = \widehat{W}_{3,t}^{flex} + \widehat{L}_{3,t}^{flex} - \widehat{\widehat{K}}_{3,t}^{flex} \tag{C.85}$$

Aggregation of labor supply

$$\widehat{L}_{t}^{flex} = \sum_{j=1}^{3} b_j \widehat{L}_{j,t}^{flex} \tag{C.86}$$

Investment Euler equation 1 2 3

$$\widehat{I}_{1,t}^{flex} = \frac{1}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{1,t-1}^{flex} + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{1,t+1}^{flex} + \frac{\widehat{Q}_{1,t}^{flex}}{\left(1+\overline{\beta}\gamma\right)\gamma^2 k} + \widehat{\varepsilon}_t^{i} \tag{C.87}$$

$$\widehat{I}_{2,t}^{flex} = \frac{1}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{2,t-1}^{flex} + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{2,t+1}^{flex} + \frac{\widehat{Q}_{2,t}^{flex}}{\left(1+\overline{\beta}\gamma\right)\gamma^2 k} + \widehat{\varepsilon}_t^{i} \tag{C.88}$$

$$\widehat{I}_{3,t}^{flex} = \frac{1}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{3,t-1}^{flex} + \frac{\overline{\beta}\gamma}{\left(1+\overline{\beta}\gamma\right)}\widehat{I}_{3,t+1}^{flex} + \frac{\widehat{Q}_{3,t}^{flex}}{\left(1+\overline{\beta}\gamma\right)\gamma^2 k} + \widehat{\varepsilon}_t^{i} \tag{C.89}$$

Aggregation of investment

$$\widehat{I}_{t}^{flex} = \sum_{j=1}^{3} \alpha_j \widehat{I}_{j,t}^{flex} \tag{C.90}$$

Aggregation of output

$$\widehat{Y}_{t}^{flex} = \gamma_1 \widehat{C}_{t}^{flex} + \widehat{G}_t + \gamma_3 \widehat{I}_{t}^{flex} + \gamma_4 \widehat{Z}_{t}^{flex} \tag{C.91}$$

$$\widehat{Y}_{t}^{flex} = \alpha_1 \widehat{Y}_{1,t}^{flex} + \alpha_2 \widehat{Y}_{2,t}^{flex} + \alpha_3 \widehat{Y}_{3,t}^{flex} \tag{C.92}$$

$$\widehat{Y}_{2,t}^{flex} = -\eta\left(\widehat{P}_{2,t} - \widehat{P}_t\right) + \widehat{Y}_t^{flex} \tag{C.93}$$

$$\widehat{Y}_{3,t}^{flex} = -\eta\left(\widehat{P}_{3,t} - \widehat{P}_t\right) + \widehat{Y}_t^{flex} \tag{C.94}$$

Q Equation in sector 1 2 3

$$\widehat{Q}_{1,t}^{flex} = \left(-\widehat{R}_{t+1}\right) + \frac{r_k^*}{r_k^* + (1-\delta)}\widehat{R}_{1,t+1}^{flex,k} + \frac{(1-\delta)}{r_k^* + (1-\delta)}\widehat{Q}_{1,t+1}^{flex} + \frac{\left(1+\frac{h}{\gamma}\right)\sigma}{\left(1-\frac{h}{\gamma}\right)}\varepsilon_t^b \tag{C.95}$$

$$\widehat{Q}_{2,t}^{flex} = \left(-\widehat{R}_{t+1}\right) + \frac{r_k^*}{r_k^* + (1-\delta)}\widehat{R}_{2,t+1}^{flex,k} + \frac{(1-\delta)}{r_k^* + (1-\delta)}\widehat{Q}_{2,t+1}^{flex} + \frac{\left(1+\frac{h}{\gamma}\right)\sigma}{\left(1-\frac{h}{\gamma}\right)}\varepsilon_t^b \tag{C.96}$$

$$\widehat{Q}_{3,t}^{flex} = \left(-\widehat{R}_{t+1}\right) + \frac{r_k^*}{r_k^* + (1-\delta)}\widehat{R}_{3,t+1}^{flex,k} + \frac{(1-\delta)}{r_k^* + (1-\delta)}\widehat{Q}_{3,t+1}^{flex} + \frac{\left(1+\frac{h}{\gamma}\right)\sigma}{\left(1-\frac{h}{\gamma}\right)}\varepsilon_t^b \tag{C.97}$$

steady state rental rate $r_k^* = \frac{1}{\beta} + \delta - 1$

Marginal rate of substitution in sector 1 2 3

$$\widehat{w}_{1,t}^{flex} = \left(\sigma - \frac{1}{\chi}\right)(1-\omega)\left(\widehat{G}_t\right) + \sigma_\lambda \widehat{C}_t^{flex} + \sigma_\delta \frac{h}{\gamma}\widehat{C}_{t-1}^{flex} + \frac{1}{\eta}\widehat{C}_{1,t}^{flex} + \frac{1}{\lambda}\widehat{L}_{1,t}^{flex} + \left(\varphi - \frac{1}{\lambda}\right)\widehat{L}_t^{flex}$$

$$\widehat{w}_{2,t}^{flex} = \left(\sigma - \frac{1}{\chi}\right)(1-\omega)\left(\widehat{G}_t\right) + +\sigma_\lambda \widehat{C}_t^{flex} + \sigma_\delta \frac{h}{\gamma}\widehat{C}_{t-1}^{flex} + \frac{1}{\eta}\widehat{C}_{2,t}^{flex} + \frac{1}{\lambda}\widehat{L}_{2,t}^{flex} + \left(\varphi - \frac{1}{\lambda}\right)\widehat{L}_t^{flex}$$

$$\widehat{w}_{3,t}^{flex} = \left(\sigma - \frac{1}{\chi}\right)(1-\omega)\left(\widehat{G}_t\right) + +\sigma_\lambda \widehat{C}_t^{flex} + \sigma_\delta \frac{h}{\gamma}\widehat{C}_{t-1}^{flex} + \frac{1}{\eta}\widehat{C}_{3,t}^{flex} + \frac{1}{\lambda}\widehat{L}_{3,t}^{flex} + \left(\varphi - \frac{1}{\lambda}\right)\widehat{L}_t^{flex}$$

and $\sigma_L = \left[\left(\sigma\omega - \frac{1}{\chi}\omega + \left(\frac{1}{\chi} - \frac{1}{\eta}\right)\right)\frac{1}{1-\frac{h}{\gamma}} - \frac{1}{\eta}\right]$

Aggregation of wages

$$W_t^{flex} = \sum_{j=1}^{3} b_j \widehat{W}_{j,t}^{flex} \tag{C.98}$$

Accumulation of capital in sector 1 2 3

$$\widehat{K}_{1,t}^{flex} = \left[\frac{1-\delta}{\gamma}\right]\widehat{K}_{1,t-1}^{flex} + \left(1 - \frac{1-\delta}{\gamma}\right)I_{1,t}^{flex} + \left(1 - \frac{1-\delta}{\gamma}\right)\gamma^2 k\widehat{\varepsilon}_t^i$$

$$\widehat{K}_{2,t}^{flex} = \left[\frac{1-\delta}{\gamma}\right]\widehat{K}_{2,t-1}^{flex} + \left(1 - \frac{1-\delta}{\gamma}\right)I_{2,t}^{flex} + \left(1 - \frac{1-\delta}{\gamma}\right)\gamma^2 k\widehat{\varepsilon}_t^i$$

$$\widehat{K}_{3,t}^{flex} = \left[\frac{1-\delta}{\gamma}\right]\widehat{K}_{3,t-1}^{flex} + \left(1-\frac{1-\delta}{\gamma}\right)I_{3,t}^{flex} + \left(1-\frac{1-\delta}{\gamma}\right)\gamma^2 k\widehat{\varepsilon}_t^i$$

C.3.3 Steady States Equations as in Smets & Wouters (2007)

$$\widehat{Y}_t = \gamma_1\widehat{C}_t + \widehat{G}_t + \gamma_3\widehat{I}_t + \gamma_4\widehat{Z}_t \tag{C.99}$$

Steady state 1:

$$K_t = \left[\frac{1-\delta}{\gamma}\right]K_{t-1} + \widehat{\varepsilon}_t^i\left(1-\frac{k}{2}*\left(\frac{I_t}{I_{t-1}}-1\right)^2\right)I_t$$

I remove t to have:

$$\begin{aligned}
K &= \left[\frac{1-\delta}{\gamma}\right]K + \left(1-\frac{k}{2}*\left(\frac{I}{I}-1\right)^2\right)I \\
K-\left[\frac{1-\delta}{\gamma}\right]K &= (1-0)\,I \\
\left(1-\frac{1-\delta}{\gamma}\right)K &= I \\
\frac{I}{K^*} &= \left(1-\frac{1-\delta}{\gamma}\right)
\end{aligned}$$

$$\widetilde{K}_t = Z_t\frac{K_{t-1}}{\gamma}$$

Knowing that: $Z^* = 1$

$$IK = \frac{I^*}{K^*} = \left(1-\frac{1-\delta}{\gamma}\right) \tag{C.100}$$

$$\widetilde{K}^* = \gamma K^* \tag{C.101}$$

$$K^* = \frac{\widetilde{K}^*}{\gamma} \tag{C.102}$$

$$IK\_bar = \frac{I^*}{\widetilde{K}^*} = \left(1-\frac{1-\delta}{\gamma}\right)\gamma = \gamma-(1-\delta) \tag{C.103}$$

Steady state 2:

$$1 = Q_t e_t^i\left(1-S\left(\frac{I_t}{I_{t-1}}\right)-S^{'}\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right)+\beta E_t\frac{\lambda_{t+1}}{\lambda_t}Q_{t+1}S\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

I remove t and $S\left(\frac{I_t}{I_{t-1}}\right) = S^{'}\left(\frac{I_t}{I_{t-1}}\right) = 0$ to have:

$$1 = Q_t\,(1-0) + 0$$

$$Q^* = 1 \tag{C.104}$$

Steady state 3:

$$Q_t = \overline{\beta} E_t \left(\left(\frac{\lambda_{t+1}}{\lambda_t} r^k_{t+1} Z_{t+1} - a(Z_{t+1}) \right) + (1-\delta) Q_{t+1} \right) \tag{C.105}$$

where $a^*(Z) = 0$ while $r^k = a^{*'}(Z)$ and $Z^* = 1$ I remove t to have:

$$\begin{aligned}
Q &= \overline{\beta} \left(\left(\frac{\lambda}{\lambda} r^k Z - a(Z) \right) + (1-\delta) Q \right) && \text{(C.106)} \\
Q &= \overline{\beta} \left(\left(r^k * 1 - 0 \right) + (1-\delta) Q \right) && \text{(C.107)} \\
Q &= 1 && \text{(C.108)} \\
1 &= \overline{\beta} \left(r^k + 1 - \delta \right) && \text{(C.109)} \\
r^*_k &= \frac{1}{\overline{\beta}} + \delta - 1 && \text{(C.110)}
\end{aligned}$$

because: $\widehat{\lambda}_t = \lambda_t \gamma^\sigma$ so: $\overline{\beta} = \beta \gamma^\sigma$

$$r^*_k = \frac{1}{\overline{\beta}} + \delta - 1 \tag{C.111}$$

Steady state 4:

$$\widetilde{K}_t = \frac{\alpha}{(1-\alpha)} \frac{W_t}{r^k_t} L_t \tag{C.112}$$

I remove t to have:

$$\frac{L^*}{\widetilde{K}^*} = \frac{(1-\alpha)}{\alpha} \frac{r^*_k}{W} \tag{C.113}$$

Steady state 5:

$$MC_t = \frac{\partial TC_t}{\partial Y_t} = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} R_t^{\alpha} A_t^{-1}$$

I remove t to have:

$$MC^* = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} W^{1-\alpha} R_k^{\alpha}$$

$$MC^* = \Phi \tag{C.114}$$

$$W^* = \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(R^k_*\right)^{-\alpha} MC^*\right)^{\frac{1}{1-\alpha}} \tag{C.115}$$

$$W^* = \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\left(R^k_*\right)^{-\alpha} \Phi\right)^{\frac{1}{1-\alpha}} \tag{C.116}$$

Steady state 6:

$$Y_{i,t} = A_t \widetilde{K}_t^{\alpha}\left(\gamma^t L_t\right)^{1-\alpha} - \gamma^t \Phi \tag{C.117}$$

I remove t to have:

$$Y^* = \widetilde{K_*}^{\alpha} L_*^{1-\alpha} - \Phi \tag{C.118}$$

$$\begin{aligned}
Y^* + \Phi &= \widetilde{K_*}^{\alpha} L_*^{1-\alpha} && \text{(C.119)}\\
\frac{Y^*+\Phi}{Y^*} &= \frac{\widetilde{K_*}^{\alpha}}{Y^*} L_*^{1-\alpha} && \text{(C.120)}\\
\frac{\widetilde{K_*}^{\alpha}}{Y^*} L_*^{1-\alpha} &= \frac{Y^*+\Phi}{Y^*} && \text{(C.121)}\\
\frac{\widetilde{K_*}^{\alpha}}{Y^*} L_*^{1-\alpha} &= \left(\frac{Y^*+\Phi}{Y^*}\right) L_*^{\alpha-1} && \text{(C.122)}\\
\frac{\widetilde{K_*}^{1-\alpha+1}}{Y^*} L_*^{1-\alpha} &= \left(\frac{Y^*+\Phi}{Y^*}\right) L_*^{\alpha-1} && \text{(C.123)}\\
\frac{\overline{K}^*}{Y^*} &= \left(\frac{Y^*+\Phi}{Y^*}\right)\left(\frac{L_*}{K_*}\right)^{\alpha-1} && \text{(C.124)}
\end{aligned}$$

$$\frac{\overline{K}^*}{Y^*} = \left(\frac{Y^*+\Phi}{Y^*}\right)\left(\frac{L_*}{\overline{\overline{K}}_*}\right)^{\alpha-1} \tag{C.125}$$

Knowing from above that: $\frac{L^*}{\widetilde{K}^*} = \frac{(1-\alpha)}{\alpha}\frac{r_k^*}{W^*}$

$$\frac{\overline{K}^*}{Y^*} = \left(\frac{Y^*+\Phi}{Y^*}\right)\left(\frac{(1-\alpha)}{\alpha}\frac{r_k^*}{W}\right)^{\alpha-1} \tag{C.126}$$

Steady state 7:

$$\frac{C^*}{Y^*} = \left(1 - g\_y - \frac{I^*}{Y^*}\right) \tag{C.127}$$

$$\gamma_1 = \left(1 - \gamma_2 - \frac{I^*}{Y^*}\right) \tag{C.128}$$

Steady state 8:

$$\frac{I^*}{Y^*} = \frac{I^*}{\overline{K}^*} * \frac{\overline{K}^*}{Y^*} \tag{C.129}$$

$$\frac{I^*}{Y^*} = \left(1 - \frac{1-\delta}{\gamma}\right)\gamma\left(\frac{Y^*+\Phi}{Y^*}\right)\left(\frac{L_*}{\overline{\overline{K}}_*}\right)^{\alpha-1} \tag{C.130}$$

$$\gamma_3 = \frac{I^*}{Y^*} = [\gamma - (1-\delta)]\left(\frac{Y^*+\Phi}{Y^*}\right)\left(\frac{L_*}{\overline{\overline{K}}_*}\right)^{\alpha-1} \tag{C.131}$$

For dynare $\left(\frac{Y^*+\Phi}{Y^*}\right) = \phi$

Steady state 9:

$$\gamma_4 = Z\_Y = r_k^* \frac{\overline{K}^*}{Y^*} \tag{C.132}$$

Steady state 10:

$$r_{t-1}^k = a^{'}(Z_t) \tag{C.133}$$

$$a^{'} r_{t-1}^k = a^{''}(Z_t) \tag{C.134}$$

$$r_{t-1}^k = \frac{a^{''}}{a^{'}}(Z_t) \tag{C.135}$$

$$r_{t-1}^k = \frac{\Upsilon}{1-\Upsilon}(Z_t) \tag{C.136}$$

For dynare $\frac{a^{''}}{a^{'}} = \frac{\Upsilon}{1-\Upsilon}$.