A cross-inefficiency approach based on the deviation variables framework

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Abstract

This paper presents a solution to the problem of ranking efficient decision-making units (DMUs) in data envelopment analysis (DEA). We develop a cross-inefficiency approach for the deviation variables framework based on a pair of epsilon-based benevolent and aggressive models for both constant and variable returns-to-scale technologies. The new method improves the discrimination power in DEA, solves the non-uniqueness of ranking solutions, and avoids negative efficiency scores that are facing present models in the deviation variables framework. We demonstrate the performance of the approach with a real-life case study. The research not only improves the discrimination power but also encourages the first step towards integrating the deviation variables framework in the context of decision-making uncertainty.

Keywords: Data envelopment analysis; Deviation variables; Cross-inefficiency; Ranking; Discrimination power; Negative efficiency score.

1. Introduction

Data Envelopment Analysis (DEA) is a popular data-enabled performance evaluation technique that has proven useful in various fields, supporting decision-making worldwide (Charles, Gherman, & Zhu, 2021; Charles, Tsolas, & Gherman, 2018). Conventional DEA models (Charnes, Cooper, & Rhodes, 1978; Banker, Charnes, & Cooper, 1984) classify a set of observations termed as decision-making units (DMUs) into efficient and inefficient sets. It is known, however, that traditional DEA models often yield solutions that identify too many DMUs as efficient, leading to low discrimination power between DMUs (Charles, Aparicio, & Zhu, 2019). Various streams of literature support a full ranking of efficient DMUs but bear certain limitations in the attempt to improve the discrimination power. The main methods are super-efficiency (Andersen & Petersen, 1993); cross-efficiency (Sexton, Silkman, & Hogan, 1986); imposing weight restrictions (Thompson, Langemeier, Lee, Lee, & Thrall, 1990); using a common set of weights (CSWs) (Karsak & Ahiska, 2008); applying an MCDM approach (Li & Reeves, 1999); and the deviation variables framework (Ghasemi, Ignatius, & Rezaee, 2019). More recently, Charles, Aparicio, and Zhu (2019) provided a simple approach using the well-known pure DEA model to increase the discriminatory power between efficient and inefficient DMUs. For more details about ranking methods in DEA, we refer the readers to Aldamak and Zolfaghari (2017); by considering the structure of the ranking methods, this paper categorises them into 10 groups and describes their benefits and properties.

An approach to resolving the discrimination power problem in DEA is the multi-objective (multi-criteria) optimisation method using the deviation variables framework. Li and Reeves (1999) developed an interactive DEA approach with three objective functions, each relating to a different way of handling deviation variables. The method empowers the decision-maker (DM) to decide which of the three solutions are acceptable. The objective function that generates a solution which discriminates among the efficiency scores is the most preferred solution. Bal, Örkcü, and Çelebioğlu (2010) tried to solve the multi-criteria DEA (MCDEA) model of Li and Reeves (1999) by means of using the goal programming approach for solving all three objectives of the MCDEA model simultaneously, for both constant returns-to-scale (CRS) and variable returns-to-scale (VRS) technologies. Ghasemi, Ignatius, and Emrouznejad (2014), nevertheless, discovered critical issues in Bal *et al*. (2010)'s approach in relation to claims to improve the dispersion of weights and discrimination power in a MCDEA framework, and instead proffered a bi-objective weighted MCDEA (Bio-MCDEA) model to remedy such flaws. Ghasemi *et al.* (2014)'s model aimed to provide better weight dispersion and discrimination power while also allowing for multiple criteria to be optimised simultaneously. Rubem, Soares de Mello, and Angulo Meza (2017) also revisited Li and Reeves (1999)'s approach and pointed out five inconsistencies in Bal *et al*.'s (2010) GP-DEA approach. Ghasemi *et al*. (2019) demonstrated that the Bio-MCDEA approach might actually fail to rank the efficient DMUs fully. To tackle this issue, the authors proposed a novel algorithm that can be applied in any type of returns-to-scale (RTS) assumption. Da Silva, Marins, and Dias (2020) presented a new MCDEA approach to improve the discrimination power of DEA based on goal programming and super efficiency method.

The above-mentioned methods have different drawbacks to rank the efficient DMUs. For example, Mahdiloo *et al*. (2021) claimed that the deviation variables approach proposed by Ghasemi *et al*. (2019) may lead to an unreasonable ranking of efficient units. However, they have not presented a solution to eliminate the problem. Moreover, in section 2.2, we show that Ghasemi *et al.* (2019)'s method may produce negative efficiency scores. We also analyse the proposed method by Rubem *et al*. (2017) and demonstrate that their model has some redundant variables and constraints. In addition, we show that the approach may produce different efficiency scores for the efficient units, and consequently, produce different ranking scores. To address the shortcomings of the existing approaches in the deviation variables framework, we present a novel cross-inefficiency approach to the full ranking of DMUs.

The present paper improves two aspects of the deviation variables framework. First, we provide a unique ranking solution as opposed to multiple optimal solutions. This reduces the need for DMs to weigh the utilities manually across various optimal solutions before arriving at a final ranking solution. Hence, the proposed method is useful for decision support systems when unique ranking solutions are needed from processing large sets of data. Second, our method does not encounter negative efficiency scores, which are commonly present when using the deviation variables framework under the VRS technology. Our approach extends a pair of benevolent and aggressive models for both VRS and CRS technologies. To the best of our knowledge, this is the first attempt to develop an aggregate cross-inefficiency approach for the deviation variables framework.

The rest of the paper is organised as follows: Section 2 reviews the existing major approaches to improving the discrimination power of DEA. Also, we explain the important properties and drawbacks of the deviation variables framework methods. Section 3 explains our approach of introducing secondary goals to rank DMUs under the CRS and VRS assumptions and how this improves the discrimination power and avoids the issue of negative efficiency values. Section 4 introduces a new case study in ranking business schools by the ex-post value of pursuing MBA programmes. This section practically validates our proposed models by comparing the findings against the results of some deviation variables' approaches. Section 5 concludes the study by offering some suggestions for future research.

2. Background

This section includes two sub-sections. Sub-section 2.1 reviews some main approaches to improving the discrimination power of DEA models, e.g., the cross-efficiency methods. Sub-section 2.2 highlights the main properties and drawbacks of key models based on the deviation variables framework in the DEA literature, which will help us to further develop our new DEA models in Section 3 to deal with the issues.

2.1. Major approaches to improving the discrimination power of DEA

There are two types of evaluation in DEA: self-evaluation and peer-evaluation (Sexton *et al*., 1986). The former assesses each DMU in light of its most favourable weights (multipliers) whereas the latter uses the most favourable weights of its peers. The discrimination power of the peer-evaluation approach is significantly more discernible than the self-evaluation approach (see Angulo-Meza & Estellita Lins, 2002; Despotis, 2002). Sexton *et al*. (1986) ranked the DMUs based on the peer-evaluation technique, which is termed as the cross-efficiency (CE) method. Doyle and Green (1994) argued that the CE method might produce different ranking scores due to the existence of multiple optimal solutions. The authors proposed a pair of secondary models termed as *benevolent* and *aggressive* models and provided three alternative ways to formulate the solutions for the said models. Over the years, various cross-efficiency models and applications have emerged. From a theoretical perspective, for example, under CRS, Liang, Wu, Cook, and Zhu (2008) extended the method by using a deviation variables framework. They proposed three different objective functions to minimise the inefficiency scores with different scenarios, i.e., minimising the sum of deviation variables; minimising the maximum of deviation variables, and minimising the mean absolute deviation of the variables while retaining the efficiency score of the evaluated DMU at a precalculated level. Wu (2009) introduced a revised benevolent cross-efficiency model and used it to construct a fuzzy preference relation to better rank DMUs; the preference relation can be directly constructed from the original sample data instead of based on the average cross-efficiency. Jahanshahloo *et al*. (2011) proposed a method for applying the symmetric weight assignment technique for cross-efficiency evaluation that rewards decision-makers (DMs) that make a symmetric selection of weights. For more details about the cross-efficiency methods, we refer the readers to Balk *et al*. (2021), who investigated the performance of different cross-efficiency approaches from a productivity perspective.

As the weights used in the cross-efficiency evaluation may sometimes differ significantly among the inputs and outputs, different methods have been proposed to select suitable weights that are neither aggressive nor benevolent towards the other DMUs. For example, Wang and Chin (2010a) developed some alternative DEA models in order to calculate the cross-efficiency scores by maximising\minimising the total deviation variables. Wang and Chin (2010b) developed a neutral DEA model to determine a set of optimal weights for each unit such that the calculated cross-efficiency scores are acceptable. The reason is that the obtained cross-efficiency scores are neither benevolent nor aggressive. Lam and Bai (2011) presented an approach to obtain more reasonable weights in the cross-efficiency method by minimising the deviations of weights from their means. Wang, Chin, and Wang (2012) and Wu, Sun, and Liang (2012) proposed the setting of lower bounds and Wang and Chin (2011) proposed the use of ordered weighted averaging operators. Lim (2012) proposed new aggressive and benevolent formulations of cross-efficiency in DEA where a minimax or a maximin type secondary objective was incorporated. Jeong and Ok (2013) modified the cross-efficiency matrix by replacing the diagonal elements with the super efficiency scores. Wu *et al*. (2016) incorporated a target identification model to get reachable targets for all DMUs, as well as proposed several secondary goal models for weights selection considering both desirable and undesirable targets of all the DMUs. Davtalab-Olyaie (2019) proposed models for cross-efficiency evaluation based on the cardinality of the set of "satisfied DMUs", *i.e.*, the DMUs that achieve their maximum efficiencies. More recently, Aparicio *et al.* (2020) extended the cross-efficiency approach to cross-productivity in order to provide a dynamic peer-evaluation based on the standard Luenberger indicator. With regards to relevant concerns about the interpretation of the cross-efficiency method in terms of production theory, the interested readers can refer to the studies by Førsund (2018) and Olesen (2018).

Under VRS, Wu, Liang, and Chen (2009) and Soares De Mello, Angulo Meza, Da Silveira, and Gomes (2013) illustrated that the cross-efficiency matrix might derive negative efficiency scores, which was rectified by adding some nonnegative constraints to the model by Banker, Charnes, and Cooper (1984) (hereinafter BCC model). Lim and Zhu (2015) proposed that the VRS cross-efficiency evaluation should be done via a series of CRS models under translated Cartesian coordinate systems, which would allow negative efficiencies to become positive. Lin (2019) adopted a directional distance function-based approach to calculating the efficiencies. Kao and Liu (2020) developed a slacks-based DEA model to calculating the cross-efficiencies; their approach prevents negative efficiency scores under the VRS assumption. Aparicio and Zofío (2021) developed an approach to connecting the concepts of cross-efficiency and economic efficiency; they showed that their method solves the negative efficiency score problem under the VRS condition.

Likewise, cross-efficiency models and their extensions have been applied across a wide range of empirical contexts. Chen (2002) performed a cross-efficiency assessment to identify the overall efficient and 'false standard' efficient electricity distribution sectors in Taiwan. Sun (2002) used cross-efficiency to evaluate computer numerical control machines in terms of system specifications and cost and to differentiate between good and bad systems. Ertay and Ruan (2005) used a cross-efficiency formulation to determine the most efficient number of operators and the efficient measurement of labour assignment in a cellular manufacturing system. Lu and Lo (2007a) applied a cross-efficiency measure to China's regional development by examining the economic performance of China's 31 regions while taking into account various environmental factors, while Lu and Lo (2007b) further integrated the cross-efficiency measure with cluster analysis to construct a benchmark-learning roadmap for those inefficient regions to improve their efficiency progressively. Wu, Liang, and Chen (2009) presented a new and modified DEA game cross-efficiency model to evaluate the performance of the countries participating in the Summer Olympic Games. Falagario *et al*. (2012) applied cross-efficiency in the context of supplier selection in public procurement to select the best supplier among the eligible candidates. Liu *et al*. (2017) introduced a DEA cross-efficiency evaluation considering undesirable outputs, which they applied to study the eco-efficiency of 23 major coal-fired power plants in China. Most recently, Navas *et al.* (2020)

extended a cross-efficiency DEA approach to evaluate the efficiency of the Colombian Higher Education institutions, Aparicio *et al*. (2020) introduced a cross-productivity approach applied in the context of 28 national innovation systems in Europe, and Aparicio and Zofio (2021) applied an economic cross-efficiency method to study the efficiency of European warehouses.

The conventional DEA models derive the most favourable weights for each DMU from the raw data of inputs and outputs. This approach may return zero values for some input or output weights, implying disregard for some factors in the computation of efficiency scores (Khalili, Camanho, Portela, & Alirezaee, 2010). In other words, there may be a more significant number of efficient DMUs when some factors are ignored in the performance evaluation process. Hence, by applying a weight restriction (WR) one can limit the range of weight values, thus leading to a reduction in the number of efficient units. In short, methods from WR can improve the discrimination power of DEA. Thompson, Singleton, Thrall, and Smith (1986) and Thompson *et al.* (1990) are among the first authors who employed WRs to improve the discrimination power of DEA models. There are various WR methods, with the most popular being one that imposes linear constraints. There are three groups of WR methods with linear constraints: (i) assurance region type I (ARI), (ii) assurance region type II (ARII), and (iii) absolute weight restrictions (see Allen, Athanassopoulos, Dyson, & Thanassoulis, 1997; and Thanassoulis, Portela, & Allen, 2005). The challenge of WR methods is the need for prior information on the importance of the factors, which may be difficult to obtain. Another method to improve the discriminatory power of DEA is by using the concept of bootstrap (Simar & Wilson 1998). This method can be used to analyse the sensitivity of the calculated efficiencies (Song *et al*., 2013). Also, the super-efficiency method of Andersen and Petersen (1993) can be used to rank the efficient units. The readers interested in the super-efficiency approach may refer to the studies by Lee and Zhu (2012) and Lin and Chen (2018).

2.2. Deviation variables framework

Consider the interest in evaluating the relative efficiency of n DMUs, which use m inputs to produce s outputs. The m-input-s-output data can be expressed as $(x_{ij}, i = 1, \dots, m, j =$

$1, \ldots, n$) and ($y_{rj}, r = 1, \ldots, s, j = 1, \ldots, n$). Li and Reeves (1999) proposed model (1) to calculate the efficiency score of $\mathrm{DMU_p}$ using deviation variables.

$$
\begin{array}{ll}
\min\limits_{d_{pj}, v_{ip}, u_{rp}} \; d_{pp} & \\
\min\limits_{M, d_{pj}, v_{ip}, u_{rp}} \; M & \\
\min\limits_{d_{pj}, v_{ip}, u_{rp}} \; \sum_{j=1}^{n} d_{pj} & \\
\text{s.t.} & \\
\sum_{i=1}^{m} v_{ip} x_{ip} = 1 & \\
\sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} + d_{pj} = 0 & j = 1, \ldots, n \\
M - d_{pj} \geq 0 & j = 1, \ldots, n \\
u_{rp} \geq 0 & r = 1, \ldots, s \\
v_{ip} \geq 0 & i = 1, \ldots, m \\
d_{pj} \geq 0 & j = 1, \ldots, n
\end{array}
\tag{1}
$$

where u_{rp} ($r = 1, \ldots, s$) and v_{ip} ($i = 1, \ldots, m$) are the input and output weights associated with input i and output r, respectively. d_{pj} is a deviation variable for $\mathrm{DMU_j}$, $M = \max\{d_{p1}, \ldots, d_{pn}\}$ is the decision variable that should be minimised and d_{pp}^{*} is the inefficiency score of $\mathrm{DMU_p}$. Therefore, $1 - d_{pp}^{*}$ is the efficiency score of $\mathrm{DMU_p}$.

Remark 1. By considering only the first objective function, *i.e.*, $min\, d_{pp}$, model (1) is equal to the traditional CCR model. It should be noted that in this condition the constraints $M - d_{pj} \geq 0, j = 1, \ldots, n$ are redundant. We prove this matter in lemma 3 under the VRS situation. The proof under the CRS condition can be done in the same way.

Liang *et al.* (2008) modified model (1) and presented model (2) by minimising the maximum value of inefficiencies while rendering the inefficiency score of the unit under evaluation to be unchanged.

$$
\begin{aligned}
&\min_{M,d_{pj}:j\neq p,v_{ip},u_{rp}} M \\
&\text{s.t.} \\
&\sum_{i=1}^{m} v_{ip}x_{ip} = 1 \\
&\sum_{r=1}^{s} u_{rp}y_{rp} = 1 - d_{pp}^{*} \\
&\sum_{r=1}^{s} u_{rp}y_{rj} - \sum_{i=1}^{m} v_{ip}x_{ij} + d_{pj} = 0 \qquad && j = 1,\dots,n, j \neq p \\
&M - d_{pj} \geq 0 && j = 1,\dots,n \\
&u_{rp} \geq 0 && r = 1,\dots,s \\
&v_{ip} \geq 0 && i = 1,\dots,m \\
&d_{pj} \geq 0 && j = 1,\dots,n
\end{aligned}
\tag{2}
$$

where d_{pp}^{*} is the inefficiency score of DMU_p, which can be obtained by solving model (1) with the first objective function. It is easy to verify that by removing the constraint $\sum_{r=1}^{s} u_{rp}y_{rp} = 1 - d_{pp}^{*}$, model (2) is equal to model (1) with the second objective function. The constraint $\sum_{r=1}^{s} u_{rp}y_{rp} = 1 - d_{pp}^{*}$ is used to keep the efficiency of DMU_p unchanged. It is evident that minimising M is related to maximising the minimum of the efficiency scores. It should be noted that they have developed some similar models by defining different objective functions using deviation variables.

Liang *et al*. (2008) used the optimal input and output weights of model (2) to evaluate the peer-appraisal efficiency of DMUs. It should be noted that the initial aim of model (2) is minimising the inefficiency scores, and so the peer-appraisal should be based on the inefficiency scores. Nevertheless, Liang *et al*. (2008) used model (2) to construct the cross-efficiency matrix in order to calculate the cross-efficiency scores. Albeit, in model (2), we have $\frac{\sum_{r=1}^{s} u_{rp}^{*}y_{rj}}{\sum_{i=1}^{m} v_{ip}^{*}x_{ij}} = 1 - d_{pp}^{*}$, this equation is not necessarily true for other $DMU_j, j \neq p$. As a result, there is no specific relation between $1 - d_{pj}^{*}, \forall j \neq p$, and the efficiency score of DMU_j. It should be noted that in contrast to the efficiency score, $1 - d_{pj}^{*}, \forall j \neq p$, can take negative values. Therefore, it is not reasonable to construct the cross-efficiency matrix using the optimal weights of DMU_p.

Rubem *et al*. (2017) used the concept of goal programming and considered three aspiration levels for three objective functions in model (1) and formulated the following model (3) in order to improve the discrimination power of the DEA model (DMU_p is the DMU under evaluation).

$$
\begin{aligned}
&\min_{d_1^-,d_2^-,d_3^-,d_1^+,d_2^+,d_3^+,d_{pj},v_{ip},u_{rp}} \lambda_1 d_1^+ + \lambda_2 d_2^+ + \lambda_3 d_3^+ \\
&\text{s.t.} \\
&\textstyle\sum_{i=1}^m v_{ip} x_{ip} = 1 \\
&\textstyle\sum_{r=1}^s u_{rp} y_{rj} - \sum_{i=1}^m v_{ip} x_{ij} + d_{pj} = 0 && j = 1,\dots,n \\
&M - d_{pj} \geq 0 && j = 1,\dots,n \\
&d_{pp} + d_1^- - d_1^+ \leq g_1 \\
&M + d_2^- - d_2^+ \leq g_2 \\
&\textstyle\sum_{j=1}^n d_{pj} + d_3^- - d_3^+ \leq g_3 \\
&u_{rp} \geq 0 && r = 1,\dots,s \\
&v_{ip} \geq 0 && i = 1,\dots,m \\
&d_{pj} \geq 0 && j = 1,\dots,n \\
&d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0
\end{aligned}
\tag{3}
$$

where g_1, g_2, and g_3 are the aspiration levels for three goals of d_{pp}, M, and $\sum_{j=1}^n d_{pj}$, respectively, whose values should be determined by the DM in the performance analysis process. A trivial verification shows that $0 \leq d_{pp} \leq 1$; hence, Rubem *et al.* (2017) assigned $g_1 = 1$. d_b^- and d_b^+ are the wanted and unwanted deviation variables from the aspiration level of the b^{th} goal, respectively, $b = 1,2,3$. λ_b is the weight of the b^{th} goal ($b = 1,2,3$) whose value should be determined by the DM such that $\lambda_1 + \lambda_2 + \lambda_3 = 1$. It is evident that the efficiency scores obtained by model (3) are dependent on the values of $\lambda_b, b = 1,2,3$.

Model (3) has some redundant constraint and variable, which is demonstrated in Lemmas 1 and 2.

Lemma 1. In model (3), we have $d_1^{+*} = 0$. In other words, the term $\lambda_1 d_1^+$ is redundant and the objective function can be replaced with $\lambda_2 d_2^+ + \lambda_3 d_3^+$.

Proof: The proof is in Appendix A.

Lemma 2. The fourth constraint of model (3), *i.e.*, $d_{pp} + d_1^- - d_1^+ \leq g_1$, is redundant.

Proof: The proof is in Appendix A.

We further explain that Rubem *et al.* (2017)'s approach to computing the efficiency and ranking score of DMU_p is not unique.

Remark 2: For large enough values of g_2 and g_3 the fifth and sixth constraints, *i.e.,* $M + d_2^- - d_2^+ \leq g_2$ and $\sum_{j=1}^n d_{pj} + d_3^- - d_3^+ \leq g_3$, are redundant. In other words, in the optimal solution we have $d_2^{+*} = d_3^{+*} = 0$. From Lemma 1, we know that $d_1^{+*} = 0$. As a result, the optimal objective function value is equal to zero, which means any feasible solution with different values of d_{pp} is an optimal solution. Consequently, model (3) may produce different efficiency scores for DMU$_p$, which is not acceptable.

Remark 3: The efficiency scores and consequently the ranking scores obtained by Rubem *et al*. (2017)'s approach depend on the values of g_2 and g_3. On the other hand, the values of g_2 and g_3 should be estimated by the DM. That is, the ranking scores in this approach are DM-oriented. We will validate this by a numerical case in section 4.

It can be noted that the approaches of Liang *et al.* (2008) and Rubem *et al*. (2017) were proposed to improve the discriminating power under the CRS environment. Ghasemi *et al.* (2019) further proposed a ranking procedure by using the deviation variables framework to provide a full ranking of the efficient DMUs under both CRS and VRS assumptions.

Ghasemi *et al*. (2019) extended the proposed model by Li and Rees (1999) to the VRS situation by considering the first objective function of model (1). Indeed, they proposed model (4) to measure the relative efficiency score of DMU$_p$ under VRS, as follows:

$$
\begin{array}{ll}
\min\limits_{w_p, d_{pj}, v_{ip}, u_{rp}} d_{pp} & \\
\text{s.t.} & \\
\sum_{i=1}^{m} v_{ip} x_{ip} = 1 & \\
\sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} + d_{pj} - w_p = 0 & j = 1, \dots, n \\
u_{rp} \geq 0 & r = 1, \dots, s \\
v_{ip} \geq 0 & i = 1, \dots, m \\
d_{pj} \geq 0 & j = 1, \dots, n \\
w_p \text{ free in sign} &
\end{array}
\qquad (4)
$$

where w_p is the RTS-free variable, $d_{pj}, j = 1,2, \dots, n$ is the deviation variable of DMU$_j$ from efficiency when DMU$_p$ is being evaluated. Therefore, $1 - d_{pp}^*$ is the efficiency score of DMU$_p$ and the unit is BCC-efficient if and only if $d_{pp}^* = 0$. Model (4) is formulated for the VRS technology, which can be easily adapted to the CRS technology by removing w_p.

Ghasemi *et al.* (2019) obtained the ranking of the efficient DMUs as follows:

Step 1. Solve model (4) for $p = 1,2,\ldots,n$, to obtain the BCC efficiency score of all DMUs, *i.e.*, $1 - d_{pp}^*$. If there exists only one efficient DMU, then rank all units based on their efficiency scores and stop.

Step 2. For the efficient units $\mathrm{DMU_{k_1}}, \mathrm{DMU_{k_2}}, \ldots, \mathrm{DMU_{k_T}}$, calculate $(d_{k_1 1}^*, \ldots, d_{k_1 n}^*)$, $(d_{k_2 1}^*, \ldots, d_{k_2 n}^*)$, ..., and $(d_{k_T 1}^*, \ldots, d_{k_T n}^*)$.

Step 3. Let $D_{k_t} = \frac{(d_{k_t 1}^* + \ldots + d_{k_t n}^*)}{n}$ for $t = 1,2,\ldots,T$, which is associated with each efficient DMU. Rank the efficient DMUs according to the values of D_{k_t} for $t = 1,2,\ldots,T$, from smallest to largest. In other words, the efficient unit $\mathrm{DMU_b}$ is identified as the first unit in the ranking from the top if $D_b = \min\{D_{k_t} \mid t = 1,\ldots,T\}$ and the efficient unit $\mathrm{DMU_a}$ is at the bottom of the ranking if $D_a = \max\{D_{k_t} \mid t = 1,\ldots,T\}$.

We explain some important properties and shortcomings of the proposed approach by Ghasemi *et al.* (2019) in ranking the efficient DMUs through two lemmas and a remark.

It should be noted that Mahdiloo *et al.* (2021) completely discredited Ghasemi *et al.* (2019)'s method. In other words, they showed that the approach produces incorrect ranking scores. Moreover, Mahdiloo *et al.* (2021) mentioned that model (4) is equal to the standard input-oriented BCC model. We prove this matter in Lemma 3.

Lemma 3. Model (4) is *equal* to the standard input-oriented BCC model.

Proof: The proof is in Appendix A.

Lemma 4. The ranking procedure proposed by Ghasemi *et al.* (2019) may produce different ranking scores for the efficient DMUs.

Proof: The proof is in Appendix A.

The approach of Ghasemi *et al.* (2019) may face negative efficiency scores in ranking efficient units (please refer to Appendix B for proof).

Remark 4. As shown by Førsund (2018) and Mahdiloo *et al*. (2021), calculating cross-efficiency scores following Ghasemi *et al*. (2019)'s method is flawed. The reason is that model (4) is solved for $\mathrm{DMU_p}$, and so d_{pj}^* is the inefficiency score of $\mathrm{DMU_j}$ using the optimal input and output weights of DMU_p. Therefore, $\sum_{\substack{j=1\\j\neq p}}^{n} d_{pj}^*$ is the sum of inefficiency scores of all units except DMU_p, using the optimal weights of DMU_p. As a result, $D_p = \frac{\sum_{j=1}^{n} d_{pj}^*}{n}$ is not the average inefficiency score of efficient DMU_p.

In the next section, we develop a cross-inefficiency approach to overcome the drawbacks in this sub-section.

3. New approach

As demonstrated in **Lemma 4**, model (4) has multiple optimal solutions that lead to $D_{k_t}, t = 1,2,\dots,T$, taking different values. In this section, we contribute to the deviation variables framework by first obtaining the possible minimum and maximum value of $D_{k_t}, t = 1,2,\dots,T$. Hence, we propose a pair of secondary goals (*i.e.*, aggressive and benevolent) to rank the DMUs similar to the cross-efficiency method.

In the basic cross-efficiency method, each unit is evaluated by considering both the self-efficiency score and the $n-1$ peer-evaluated efficiency scores obtained from the optimal weights of other units. As a result, the final efficiency score for each DMU is obtained by aggregating n efficiency scores. This approach may obtain different rank orders from the same efficiency scores due to non-uniqueness of the optimal weights, which is also present in the deviation variables framework. To overcome the problem, similar to the cross-efficiency context, we begin by defining two secondary goals as benevolent and aggressive for the context of the deviation variables framework as follows:

Definition 1: In *benevolent inefficiency* (*aggressive inefficiency*), optimal weights are derived by minimising (maximising) the cross-inefficiencies of other units while rendering the inefficiency score of the unit under evaluation to be unchanged.

We incorporate the benevolent and aggressive concepts into the deviation variables framework for both CRS and VRS technology below.

Under the CRS assumption, Model (4) is equal to the CCR model, indicating multiple optimal solutions when DMU_p is efficient. Therefore, similar to the cross-efficiency method, consider the matrix of deviation variables $\boldsymbol{D} = \left[d^*_{pj}\right]_{n\times n}$ (cross-inefficiencies) for all DMUs, as shown in **Figure 1**. The matrix was firstly developed by Doyle and Greene (1994) for cross-efficiencies, and then extended by Mahdiloo *et al*. (2021) to deviation variables. The elements of matrix $\boldsymbol{D}$ can be calculated by solving model (4). In this matrix, d^*_{pj} is the deviation variable assigned to DMU_j using the weights of DMU_p. For instance, d^*_{31} is the inefficiency score of DMU_1 using the optimal weights of DMU_3, d^*_{13} is the inefficiency score of DMU_3 using DMU_1's optimal weights. The elements on the main diagonal, *i.e.*, $d^*_{jj}, j = 1, \dots, n$, indicate the inefficiency scores of DMU_j obtained from the best possible set of weights.

Rated DMUs

$$
\text{Rating DMUs}\quad
\begin{array}{c|cccc|c}
 & 1 & 2 & \cdots & n & \\
\hline
1 & d^*_{11} & d^*_{12} & \cdots & d^*_{1n} & \Delta_1 \\
2 & d^*_{21} & d^*_{22} & \cdots & d^*_{2n} & \Delta_2 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
n & d^*_{n1} & d^*_{n2} & \cdots & d^*_{nn} & \Delta_n \\
\hline
 & \bar{\boldsymbol{d}}_1 & \bar{\boldsymbol{d}}_2 & \cdots & \bar{\boldsymbol{d}}_n &
\end{array}
\quad\text{Averaged peer evaluation}
$$

Averaged peer evaluation

Figure 1. The matrix of cross-inefficiencies.

The elements of the cross-inefficiency matrix $\boldsymbol{D}$ are not unique (except $d^*_{jj}, j = 1, \dots, n$). According to definition 1, we introduce the benevolent and aggressive DEA models for each DMU, respectively, to deal with the non-uniqueness issue.

We propose the following epsilon-based benevolent model (5) to calculate the minimum of the sum of inefficiency scores.

$$
\begin{array}{ll}
\underline{\theta}_p = \min\limits_{w_p, d_{pj}: j\neq p, v_{ip}, u_{rp}} \sum_{j=1}^{n} d_{pj} & \\
\text{s.t.} & \\
\sum_{i=1}^{m} v_{ip} x_{ip} = 1 & \\
\sum_{r=1}^{s} u_{rp} y_{rp} - w_p - \sum_{i=1}^{m} v_{ip} x_{ip} + d^*_{pp} = 0 & \\
\sum_{r=1}^{s} u_{rp} y_{rj} - w_p - \sum_{i=1}^{m} v_{ip} x_{ij} + d_{pj} = 0 & j = 1, \dots, n, j \neq p \\
u_{rp} \geq \varepsilon & r = 1, \dots, s \\
v_{ip} \geq \varepsilon & i = 1, \dots, m \\
d_{pj} \geq 0 & j = 1, \dots, n \\
w_p \ is\ free\ in\ sign &
\end{array}
\qquad (5)
$$

where d_{pp}^* is the inefficiency score of DMU_p (correspondingly, $1 - d_{pp}^*$ is the efficiency score of DMU_p). ε is the non-Archimedean epsilon that forestalls the input and output weights from taking a value of zero. In the DEA literature, different methods have been proposed to prevent zero input and output weights, *e.g.*, by using weight restrictions and imposing a small positive number as the lower bound of the weights (Toloo *et al.*, 2021). In this paper, we present an approach to determine a proper value for epsilon in the Appendix C. The first two constraints of model (5) ensure that the efficiency score of DMU_p remains unchanged in the cross-inefficiency evaluation process. Therefore, the objective function $\sum_{j=1}^{n} d_{pj}$ can be interpreted as the total inefficiency scores of all DMUs with the optimal weights of DMU_p. That is, model (5) minimises the total inefficiency or equivalently maximises the total efficiency scores when the optimal weights of DMU_p are used. Model (5) is developed for the VRS technology, which can be converted to the CRS technology by removing w_p, (*i.e.*, $w_p = 0$).

We develop the following aggressive model for DMU_p which, in contrast to the benevolent model (5), maximises the sum of the inefficiency scores when the optimal weights of DMU_p are used.

$$
\begin{array}{ll}
\bar{\theta}_p = \max\limits_{w_p, d_{pj}: j \neq p, v_{ip}, u_{rp}} \sum_{j=1}^{n} d_{pj} & \\
\text{s.t.} & \\
\sum_{i=1}^{m} v_{ip} x_{ip} = 1 & \\
\sum_{r=1}^{s} u_{rp} y_{rp} - w_p - \sum_{i=1}^{m} v_{ip} x_{ip} + d_{pp}^* = 0 & \\
\sum_{r=1}^{s} u_{rp} y_{rj} - w_p - \sum_{i=1}^{m} v_{ip} x_{ij} + d_{pj} = 0 & j = 1, \dots, n, j \neq p \\
u_{rp} \geq \varepsilon & r = 1, \dots, s \\
v_{ip} \geq \varepsilon & i = 1, \dots, m \\
d_{pj} \geq 0 & j = 1, \dots, n \\
w_p \ is\ free\ in\ sign &
\end{array} \qquad (6)
$$

Note that the optimal objective value of model (5) is always bounded, whereas model (6) may have an unbounded optimal objective value under certain situations (see theorem 1).

Theorem 1. Under the CRS technology, model (6) has an optimal solution if and only if $x_{ip} > 0, \forall i$.

Proof: Let $x_{ip} > 0, \forall i$. In this case, from the constraint $\sum_{i=1}^{m} v_{ip}x_{ip} = 1$ we obtain the following upper bound for $v_{ip}, \forall i$:

$$v_{ip} \le \frac{1}{\min\{x_{ip}, i = 1, \dots, m\}}, \forall i$$

The second constraint of model (6) implies that:

$$d_{pj} = \sum_{i=1}^{m} v_{ip}x_{ij} - \sum_{r=1}^{s} u_{rp}y_{rj} \le \sum_{i=1}^{m} v_{ip}x_{ij} \le \frac{\sum_{i=1}^{m} x_{ij}}{\min\{x_{ip}, i=1,\dots,m\}}, j = 1, \dots, n$$

Therefore, $\sum_{j=1}^{n} d_{pj}$ is bounded and model (6) has an optimal solution.

Without loss of generality, let $x_{1p} = 0$. In this case, we show that the optimal objective value of model (6) is unbounded. Suppose that, contrary to our claim, $(\boldsymbol{v}_p^*, \boldsymbol{u}_p^*, \boldsymbol{d}_p^*)$ is the optimal solution of the aggressive model (6). Let

$$\hat{v}_{ip} = \begin{cases} v_{1p}^* + \gamma, & \text{if } i = 1 \\ v_{ip}^*, & \text{if } i \ge 2 \end{cases} \quad \& \ \hat{d}_{pj} = d_{pj}^* + \gamma x_{1j}, j = 1, \dots, n$$

where γ is an arbitrary strictly positive number. It is easy to verify that $(\hat{\boldsymbol{v}}_p, \boldsymbol{u}_p^*, \hat{\boldsymbol{d}}_p)$ is also a feasible solution of model (6). However, its objective value $\sum_{j=1}^{n} d_{pj}^* + \gamma \sum_{j=1}^{n} x_{1j}$ is strongly greater than the optimal objective value $\sum_{j=1}^{n} d_{pj}^*$ which is not possible. It should be noted that $\sum_{j=1}^{n} x_{1j} > 0$. □

Theorem 1 clarifies that the necessary and sufficient condition for unboundedness for the aggressive model (6) under the CRS technology depends only on the inputs of $\mathrm{DMU_p}$. As a result, the aggressive approach is applicable if and only if the inputs of all DMUs are strictly positive. Under the VRS technology, this model may have an unbounded optimal objective value. In this case, model (5) can be used to rank the efficient units based on the benevolent scenario.

Theorem 2. Model (5) has a unique optimal solution.

Proof. An alternative optimal solution exists if the vector of the objective function has the same direction with a binding constraint. In other words, the vector of the objective function is parallel to a binding constraint (see Taha, 2011, p. 116). However, it is noted that the coefficient vector of the objective function is not a multiplier of any coefficient vectors of the constraints set (either redundant or nonredundant). As a result, we conclude that the model has a unique optimal solution. ☐

We proved that model (5) has a unique optimal solution, and the same proof can be provided for the suggested model (6).

It should be noted that models (5) and (6) may produce negative cross-efficiency scores (Lim & Zhu, 2015; Kao & Liu, 2020). There are several approaches to overcome the negative efficiencies under the VRS condition. Wu *et al.* (2009) and Soares de Mello *et al.* (2013) added some additional constraints to the standard BCC model to ensure that the obtained cross-efficiency scores are non-negative. Lim and Zhu (2015) demonstrated that there is a geometric relationship between the standard VRS and CRS DEA models. Therefore, they proposed a procedure to obtain the cross-efficiency scores under the VRS assumption by solving a series of basic CCR-DEA models under translated Cartesian coordinate systems. As the literature shows, the existing approaches to eliminating the negative efficiencies have different problems. Therefore, we apply the proposed approach by Aparicio and Zofío's (2021, 2020) to develop the allocative inefficiency to prevent the negative efficiency score problem.

The cross-efficiency methods based on standard DEA methods have been criticised by Førsund (2018) and Olesen (2018). Average cross-efficiency cannot be interpreted in terms of comparable productivity measures, because the weights used in each bilateral cross-efficiency are different (Førsund, 2018). Furthermore, from a geometrical perspective, a DMU is evaluated with projections outside the production possibility set by using the weights of the DMU under evaluation, which is non-sense (Olesen, 2018). By using the concept of cross-efficiency concept under the deviation variables framework (Ghasemi *et al*., 2019), it would suffer from the said problems. Aparicio and Zofío's (2021, 2020) approach was recently proposed to solve these two problems. The authors showed that bilateral and averaged cross-efficiency can be reinterpreted in terms of Farrell's (1957) overall productive (economic) efficiency. We therefore adopt Aparicio and Zofío's (2021) approach to solve the same issues and the negative efficiency score problem in cross-inefficiency method. To connect their approach with the cross-inefficiency technique based on the deviation variables framework, each $1 - d_{kj}^*$ can be decomposed into its own technical efficiency $1 - d_{jj}^*$

times a residual that can be interpreted as allocative efficiency (AE). The AE of DMU_j with respect to unit k $(k = 1, \dots, n)$ based on deviation variables is defined as:

$$AE_{kj} = \frac{1-d^*_{kj}}{1-d^*_{jj}}, j = 1, \dots, n; k = 1, \dots, n$$

As a result, the allocative inefficiency (AI) of DMU_j with respect to unit k $(k = 1, \dots, n)$ based on deviation variables can be stated as

$$AI_{kj} = 1 - AE_{kj} = 1 - \frac{1-d^*_{kj}}{1-d^*_{jj}} = \frac{d^*_{kj} - d^*_{jj}}{1-d^*_{jj}}, j = 1, \dots, n; k = 1, \dots, n$$

The aggregate Farrell cross-inefficiency of DMU_j can be defined as follows:

$$FCI_j = (1 - d^*_{jj}) \sum_{k=1}^{n} \frac{AI_{kj}}{n-1}, j = 1, \dots, n \qquad (7)$$

where the first term, i.e., $(1 - d^*_{jj})$, is the technical inefficiency and the second term is the allocative inefficiency. It is easy to verify that $d^*_{jj} \le d^*_{kj}$. The reason is that the inefficiency score of DMU_j with its own (most favourable) weights is smaller than that obtained with the weights of other DMUs, e.g., DMU_k. As a result, the value of $AE_{kj} = \frac{1-d^*_{kj}}{1-d^*_{jj}} \ge 0$ is less than or equal to one, which means $0 \le \sum_{k=1}^{n} \frac{AI_{kj}}{n-1} \le 1$. Now, by considering $0 \le 1 - d^*_{jj} \le 1$, we obtain $0 \le FCI_j = (1 - d^*_{jj}) \sum_{k=1}^{n} \frac{AI_{kj}}{n-1} \le 1$.

Our proposed models (5) and (6) produce two different ranking scores, benevolent and aggressive. In contrast to the method of Ghasemi *et al.* (2019), which uses $\Delta_j, j = 1, \dots, n$, (see Fig. 1) to rank the DMUs, we use the values of $FCI_j, j = 1, \dots, n$, to obtain the correct ranking scores. As explained in Remark 4, using the values of Δ_j is not suitable for ranking the DMUs. Therefore, we propose the following procedure to rank all units. The procedure implements the benevolent model under the CRS technology.

Step 1. Solve model (4) with the additional constraints of $w_p = 0, u_{rp} \ge \varepsilon, v_{ip} \ge \varepsilon, \forall\, i, r, n$ times, one time for each $DMU_p, p = 1, \dots, n$, to obtain the values of $d^*_{pp}, p = 1, .., n$. The proper value for ε can be obtained in line with Appendix C.

Step 2. Solve model (5) for each $\mathrm{DMU_p}, p = 1, \dots, n$, and obtain $d^*_{pj}, j = 1,2, \dots, n$.

Step 3. Construct the cross-inefficiency matrix presented in Fig. 1 and calculate the values of $FCI_j = (1 - d^*_{jj}) \sum_{k=1}^{n} \frac{AI_{kj}}{n-1}, j = 1, \dots, n$.

Step 4. Rank the DMUs according to the values of $FCI_j, j = 1, \dots, n$, from smallest to largest. $\mathrm{DMU_p}$ is identified as the first unit in the ranking from the top if $FCI_p = \min\{FCI_j | j = 1, \dots, n\}$ and $\mathrm{DMU_q}$ is at the bottom of the ranking if $FCI_q = \max\{FCI_j | j = 1, \dots, n\}$.

One can implement the procedure under the VRS technology by removing the restriction $w_p = 0$ in Step 1. The aggressive ranking can be obtained by using model (6) instead of model (5) in Step 2.

Our developed approach differs from the explained approaches in sub-section 2.2 in several major aspects:

I. Our developed models are epsilon-based. Hence, in contrast to the existing approaches, the input and output factors cannot be ignored in the performance evaluation process. We proposed a model to obtain a suitable positive value for epsilon (please refer to Appendix C).

II. The new approach provides the full ranking under both CRS and VRS assumptions.

III. Our developed models (5) and (6) find the minimum and maximum value of the sum of inefficiencies by considering the non-negative efficiency score for all DMUs. The existing approaches, *e.g.*, Ghasemi *et al.* (2019), may produce negative efficiency scores under the VRS situation.

IV. Our developed aggressive model (6) considers the maximum value of the sum of inefficiencies as an aggressive secondary goal. The existing approaches, *e.g.*, Liang *et al.* (2008) and Ghasemi *et al.* (2014), focus only on the benevolent scenario. This is because the aggressive model may produce unbounded solutions as we have described in **Theorem 1**.

V. Our proposed models have a unique optimal solution as proved in **Theorem 2**. Therefore, in contrast to Ghasemi *et al.* (2019)'s approach, our models provide a unique ranking in each benevolent and aggressive scenario.

The next section provides a case study to validate the new approach and to explain the illustrated drawbacks in sub-section 2.2.

4. Case study

In this section, we further illustrate the performance differences between the new approach and the deviation variables' methods by evaluating 25 business schools offering MBA programmes. The performance evaluation is based on a 2-input-2-output model, which would help a DM who is seeking to enroll in an MBA programme with the purpose of maximising the *ex-post* value of his or her education. The explanations of the input and output factors are summarised in Table 1.

Table 1. The description of input and output factors.

Input factors	Applications per Seat	The number of applications received for the programme normalised against the number of seats available for the programme.
	Accepted Applications	The number of candidates accepted into the programme.
Output factors	Average Pay	The average salary of graduates upon successful completion of the programme.
	Employment Rate	The rate of graduates who managed to attain a new position upon successful completion of the programme.

The data are presented in Table 2. We use LINDO 6.0 to solve our developed LP models. There are 4 and 10 efficient DMUs under the CRS and VRS environments, respectively.

Table 2. The dataset and efficiency scores of 25 DMUs.

No.	School	Input1	Input2	Output1	Output2
		Applications per Seat	Accepted Applications	Average Pay	Employment Rate
1	**Stanford**	17.9	78.70	142,834	92.10
2	**Harvard**	10.2	88.80	144,750	89.40
3	**MIT**	11.7	62.30	142,936	92.80
4	**Berkeley**	14.4	52.50	140,935	86.70
5	**Wharton**	7.1	68.00	142,574	95.60

6	**Columbia**	7.8	70.40	139,006	91.10
7	**NYU**	11.3	48.70	135,933	90.40
8	**Chicago**	7.2	59.40	137,615	97.20
9	**Tuck**	8.7	52.20	142,489	93.80
10	**UCLA**	11.7	48.20	127,535	88.60
11	**Kellogg**	6.7	63.90	136,357	88.60
12	**Foster**	9.8	44.70	125,367	95.80
13	**Darden**	8.4	45.80	136,474	93.40
14	**Duke**	7.8	50.90	137,154	89.80
15	**Yale**	8.5	49.50	126,871	88.90
16	**Olin**	12.1	30.90	111,974	96.90
17	**Cornell**	6.3	52.60	132,316	89.80
18	**Emory**	7.5	43.50	128,347	94.80
19	**Michigan**	5.5	50.90	140,497	89.70
20	**Texas**	7.9	44.40	126,160	91.30
21	**Tepper**	6.9	46.60	131,865	88.30
22	**Kelley**	6.6	45.60	119,581	88.10
23	**UNC**	6.8	37.90	124,641	89.00
24	**Owen**	5.3	44.70	113,830	90.80
25	**Georgetown**	6.1	34.50	118,938	88.50

Note: Dataset from https://poetsandquants.com/2015/04/07/a-new-and-better-way-to-rank-the-best-business-schools/5/

Solving model (3) for UCLA gives the following optimal solution ($\lambda_1 = \lambda_2 = \lambda_3 = {}^1/_3, g_1 = g_2 = 1, g_3 = 25$).

$d_1^{+*} = d_2^{+*} = d_3^{+*} = 0, d_{10}^* = 0.36$.

As expected from Lemma 1, $d_1^{+*} = 0$. This optimal solution implies that the efficiency score of UCLA is equal to $1 - d_{10}^* = 0.64$, which differs from the true efficiency score (0.76) reported in Table 2. Solving the model by removing the fourth constraint, *i.e.*, $d_{pp} + d_1^- - d_1^+ \leq g_1$, gives the same optimal solution, which means this constraint is redundant as proved in Lemma 2. It should be noted that the efficiency score obtained by model (2) depends on the values of g_2 and g_3. For example, using $g_2 = 0.5$ and $g_3 = 25$ gives $d_1^{+*} = d_3^{+*} = 0, d_2^{+*} = 0.30, d_{10}^* = 0.28$, which means the efficiency score of UCLA is equal to 0.72. As a result, in Rubem *et al*. (2017)'s approach, the efficiency scores and, consequently, the ranking scores are dependent on the values of g_2 and g_3 as explained in Remarks 2 and 3.

As mentioned in Mahdiloo *et al*. (2021) and explained in Lemma 4, the proposed model by Ghasemi *et al*. (2019) generates alternative optima for efficient DMUs. Therefore, an easy inspection shows that their approach produces different ranking scores for the efficient units.

We now turn to our developed benevolent and aggressive models under the CRS and VRS technologies to rank the DMUs. Since the input data of all CCR- and BCC-efficient DMUs are strictly positive, according to **Theorem 1**, the optimal objective value of the aggressive models (6) is bounded.

Solving the epsilon models (please refer to Appendix C) gives $\varepsilon_{CCR} = 0.3 * 10^{-5}$, $\varepsilon_{BCC} = 0.2 * 10^{-4}$. Now, we solve the CCR models (5-6) to calculate the values of $\bar{d}_k$ in both the benevolent and aggressive scenarios. The following Table 3 summarises the results of the new approach to obtain the ranking of 25 DMUs under the CRS and VRS assumptions. The column FCI_j shows the aggregate Farrell cross-inefficiency scores. The ranking scores are obtained using the proposed algorithm in section 3.

Table 3. Technical efficiency and the results of the new approach under the CRS and VRS assumptions.

DMUs	CRS						VRS					
	Technical eff.	Rank	Benevolent Model (5)		Aggressive Model (6)		Technical eff.	Rank	Benevolent Model (5)		Aggressive Model (6)	
			FCI_j	Rank	FCI_j	Rank			FCI_j	Rank	FCI_j	Rank
Stanford	0.522	25	0.958	25	0.998	25	0.730	24	0.771	25	0.998	25
Harvard	0.580	24	0.667	24	0.688	24	0.660	25	0.344	23	0.771	24
MIT	0.661	23	0.396	22	0.417	22	0.950	12	0.271	20	0.417	19
Berkeley	0.768	19	0.427	23	0.469	23	0.960	11	0.313	21	0.604	22
Wharton	0.823	13	0.208	18	0.229	18	1.000	1	0.087	8	0.188	9
Columbia	0.725	22	0.313	21	0.313	20	0.740	23	0.656	24	0.727	23
NYU	0.801	18	0.229	19	0.260	19	0.930	15	0.167	16	0.375	16
Chicago	0.830	12	0.125	12	0.146	14	1.000	1	0.093	10	0.115	6
Tuck	0.822	14	0.094	10	0.135	12	1.000	1	0.092	9	0.094	2
UCLA	0.763	20	0.292	20	0.323	21	0.830	20	0.323	22	0.563	21
Kellogg	0.812	16	0.177	16	0.198	17	0.820	21	0.260	19	0.375	17
Foster	0.810	17	0.188	17	0.189	16	0.950	13	0.156	15	0.271	15
Darden	0.860	10	0.083	9	0.104	9	1.000	1	0.083	7	0.094	3
Duke	0.849	11	0.135	14	0.136	13	0.930	16	0.094	11	0.208	11
Yale	0.760	21	0.167	15	0.168	15	0.800	22	0.229	17	0.417	20
Olin	1.000	1	0.046	4	0.047	3	1.000	1	0.081	6	0.185	8
Cornell	0.891	7	0.104	11	0.108	10	0.890	18	0.115	13	0.229	13
Emory	0.875	8	0.052	6	0.063	6	1.000	1	0.063	2	0.104	4
Michigan	1.000	1	0.031	2	0.042	2	1.000	1	0.021	1	0.042	1
Texas	0.820	15	0.125	13	0.125	11	0.900	17	0.125	14	0.260	14
Tepper	0.900	6	0.063	7	0.072	7	0.940	14	0.095	12	0.219	12
Kelley	0.872	9	0.073	8	0.083	8	0.870	19	0.240	18	0.385	18
UNC	0.953	5	0.051	5	0.054	5	1.000	1	0.079	5	0.188	10
Owen	1.000	1	0.042	3	0.052	4	1.000	1	0.075	4	0.133	7

Georgetown	1.000	1	0.010	1	0.010	1	1.000	1	0.073	3	0.108	5

In contrast to the deviation variables framework proposed by Ghasemi *et al*. (2019) that generates multiple optima solutions with different ranking order between them for both CRS and VRS technologies, our method (see Table 3) gives a unique ranking for the CRS and VRS technologies, respectively.

In order to measure the strength and direction of the ranking scores produced by benevolent and aggressive scenarios, we calculate Spearman's rank correlation coefficient under both CRS and VRS conditions. This coefficient for the pair of benevolent and aggressive CCR and BCC models are 0.96 and 0.92, respectively. As a result, there is a strong positive association between the ranking scores produced by models (5) and (6).

Our proposed approach produces a unique ranking order for the benevolent and aggressive model, respectively. What will be the implications of having two ranking solutions for a DM?

Suppose that a DM needs to rely on our results for a decision regarding the MBA programme that will maximise his or her value upon graduation. The first step is to clarify what the solutions of benevolent and aggressive models mean to the DM. Since the former seeks to minimise the efficiencies, the benevolent solution is seen as an optimistic solution under positive states of nature, one that implicitly expects that the underlying process that generates the raw data will be fairly certain. On the other hand, the aggressive solution is seen as a pessimistic solution under negative states of nature or outlook, implying that the underlying process that generates the frontier may be merely transitionary in process. Second, we suggest DMs to observe rank positions that remain unchanged between the benevolent and aggressive models. That is, when minimising and maximising the efficiencies does not alter the rank positions for certain DMUs, there is more reason to trust that such solutions are less amenable to externalities affecting the underlying process of generating the frontier. For instance, taking the VRS solution of our proposed method, we observe that several units, *e.g*., Michigan (ranked 1st), Darden (ranked 2nd), and Tuck (ranked 3rd) possess the same ranking in both benevolent and aggressive models. Decision-makers who wish a stable choice with no fluctuation between models would be more comfortable selecting the MBA programme in this manner. The same observation can also be made for the case of CRS.

5. Conclusions

The lack of discriminatory power of conventional DEA models is a fundamentally challenging issue in ranking efficient DMUs. There has been significant progress in the literature in consolidating the field of discrimination power and weight dispersion under an umbrella termed the deviation variables framework. We revisited the major existing approaches and further improved the approaches especially in avoiding negative efficiency scores under the VRS technology, while producing a unique ranking solution. Our proposed pair of benevolent and aggressive approaches would strengthen the promise of the deviation variables framework. The new models use a strong positive value for the epsilon to prevent the input and output weights from taking a value of zero.

Nonetheless, we believe there is still further scope to scale up the deviation variables framework through our pair of benevolent and aggressive models. Interested researchers could use our approach to advance the perspective of the deviation variables framework by resolving it in the context of decision-making under uncertainty. We have begun the journey by showing that one may rely on the ranking solutions that remain unchanged between the two models (benevolent and aggressive) if the decision-maker is looking for a more certain outcome. As for ranking positions that differ significantly between the two models, future researchers may explore options within value judgements of the decision-makers so that context would be provided in selecting the results from either the benevolent or the aggressive model. We briefly mention in this paper that since both models can be perceived as anchoring in the respective continuum of decision-making, more work has to be done to uncover what would be the decision-making concepts that could be applied within this continuum. For instance, the difference between the solutions of benevolent and aggressive models can be seen as one that minimises efficiencies, while the other maximises efficiencies. Thus, the range between their solutions needs further development in order to relate to the range between an optimistic and pessimistic decision-maker. In conclusion, we anticipate the integration between the deviation variables framework and decision-making under uncertainty in uncovering the underlying process affecting the ranking solution such that not only there would be improved discrimination power, but also a clearer insight into the ranking choice and order of the decision-makers.

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Appendix A. Proofs of the Lemmas.

A.1. Proof of Lemma 1

Let $(v_p^*, u_p^*, d_p^*, M^*, d_1^{-*}, d_1^{+*}, d_2^{-*}, d_2^{+*}, d_3^{-*}, d_3^{+*})$ be the optimal solution of model (3). We show that $d_1^{+*} = 0$. Suppose that contrary to our claim, in the optimal solution, we have $d_1^{+*} > 0$. Consider that the fourth constraint, *i.e.*, $d_{pp} + d_1^- - d_1^+ \leq g_1$, reveals that the difference associated with the coefficient of the variables d_1^- and d_1^+ is only in their sign, *i.e.*, 1 and -1, which means these variables are linearly dependent. Therefore, they cannot be in the same basic solution, *i.e.*, $d_1^{-*} \times d_1^{+*} = 0$. Since $d_1^{+*} > 0$, we conclude that $d_1^{-*} = 0$. Now, we find a feasible solution such that its objective optimal value is strongly less than to $\lambda_1 d_1^{+*} + \lambda_2 d_2^{+*} + \lambda_3\, d_3^{+*}$ that is a contradiction. Let $\hat{d}_1^+ = 0$, in this case it is easy to verify that $(v_p^*, u_p^*, d_p^*, M^*, d_1^{-*}, \hat{d}_1^+, d_2^{-*}, d_2^{+*}, d_3^{-*}, d_3^{+*})$ is also a feasible solution to model (3); however, its objective value $\lambda_2 d_2^{+*} + \lambda_3\, d_3^{+*}$ is strictly less than the optimal objective value $\lambda_1 d_1^{+*} + \lambda_2 d_2^{+*} + \lambda_3\, d_3^{+*}$, which completes the proof. □

A.2. Proof of Lemma 2

As explained in sub-section 2.2, $g_1 = 1$. Therefore, from Lemma 1, the constraint $d_{pp} + d_1^- - d_1^+ \leq g_1$ can be converted to $d_{pp} + d_1^- \leq 1$. Since $0 \leq d_{pp} \leq 1$, then the variable d_1^- is redundant and it can be replaced with $d_1^- = 0$. □

A.3. Proof of Lemma 3

From the second constraint of model (4), when $j = p$, we obtain $d_{pp} = \sum_{i=1}^{m} v_{ip} x_{ip} + w_p - \sum_{r=1}^{s} u_{rp} y_{rp}$. The first constraint of model (4), *i.e.*, $\sum_{i=1}^{m} v_{ip} x_{ip} = 1$, leads to $d_{pp} = 1 + w_p - \sum_{r=1}^{s} u_{rp} y_{rp}$. Thus, minimising d_{pp} is equal into maximising $\sum_{r=1}^{s} u_{rp} y_{rp} - w_p$, ensuring that model (4) is *equal* to the following standard BCC model.

$$\max_{w_p, v_{ip}, u_{rp}} \sum_{r=1}^{s} u_{rp} y_{rp} - w_p$$
$$s.t.$$
$$\sum_{i=1}^{m} v_{ip} x_{ip} = 1,$$
$$\sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} - w_p \leq 0, \quad j = 1, \dots, n,$$
$$u_{rp} \geq 0, \quad r = 1, \dots, s,$$
$$v_{ip} \geq 0, \quad i = 1, \dots, m,$$
$$w_p \text{ free in sign,}$$

A.4. Proof of Lemma 4

Doyle and Green (1994) showed that standard DEA models possess multiple optimal solutions. Therefore, according to **Lemma 3**, model (4) also has multiple optimal solutions. The multiple optimal solutions may lead to different values for D_{k_t} such that if $\mathrm{DMU_A}$ has a higher rank than $\mathrm{DMU_B}$ with an optimal solution, then it may have a lower ranking score than $\mathrm{DMU_B}$ with another alternative solution. ☐

Appendix B. Proof of negative efficiency scores.

Model (4) can produce negative efficiency scores. The reason is that the efficiency scores obtained by the traditional BCC model may take negative values (see Soares De Mello *et al.*, 2013). Thus, from **Lemma 3**, model (4) may also produce negative efficiency scores. More precisely, let (v^*, u^*, w_p^*) be an optimal solution to model (4). Although the deviation variable of $\mathrm{DMU_j}$ from the efficiency is non-negative, $d_{pj}^* \geq 0$, the efficiency score of $\mathrm{DMU_j}$ $(j \neq p)$ with the optimal weights of $\mathrm{DMU_p}$, *i.e.*, $\frac{\sum_{r=1}^{s} u_{rp}^* y_{rj} - w_p^*}{\sum_{i=1}^{m} v_{ip}^* x_{ij}}$, can take negative values.

Appendix C. The process of determining a suitable value for epsilon.

Determining a proper value for epsilon in the epsilon-based DEA models is a challenging issue. Charnes, Rousseau, and Semple (1993) showed that using an unsuitable value for epsilon could lead to some drawbacks, *e.g.*, infeasibility and unboundedness in DEA models:

> *". . .if one uses a small number in place of the infinitesimal epsilon, one is caught between Scylla and Charybdis, i.e., for decent convergence to an optimum, the numerical zero tolerance should be as large as possible, whereas the numerical value approximating the infinitesimal should be as small as possible!"*

Whilst using a large epsilon value may lead to infeasibility, a very small value may lead to computational inaccuracies and let the input and output weights get zero values (for more detail see Ali & Seiford, 1993 and Podinovski & Bouzdine-Chameeva, 2017). Therefore, we formulate model (A) to find the maximum value for epsilon. Model (A) is feasible, and its optimal objective value is positive and bounded (see Mehrabian *et al*., 2000 and Amin & Toloo, 2004).

$$
\begin{array}{lll}
\varepsilon = \max\limits_{w_p, d_{pj}, v_{ip}, u_{rp}, \varepsilon_1} \varepsilon_1 & & \\
s.t. & & \\
\sum_{i=1}^{m} v_{ip} x_{ij} \leq 1 & j = 1, \ldots, n & \\
\sum_{r=1}^{s} u_{rp} y_{rj} - w_p - \sum_{i=1}^{m} v_{ip} x_{ij} + d_{pj} = 0 & j = 1, \ldots, n & \quad (A) \\
u_{rp} \geq \varepsilon_1 & r = 1, \ldots, s & \\
v_{ip} \geq \varepsilon_1 & i = 1, \ldots, m & \\
d_{pj} \geq 0 & j = 1, \ldots, n & \\
w_p \ is \ free \ in \ sign & &
\end{array}
$$

By considering $w_p = 0$, we could obtain a suitable value for epsilon under the CRS technology. □

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