Impact of correlated price sensitive demand on the dynamics and economics of supply chains

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Abstract
We investigate the dynamics of a supply chain with a price-sensitive, correlated, stochastic, linear demand model. We assume the exogenous market price follows a first order auto-regressive AR(1) process. The demand process is a weighted function (w) of the current and previous market price, the market potential (a), and the positive demand sensitivity coefficient (b). We assume that a supplier faces five different types of customers in the market: responsive, selective, naïve, speculative, and slow customers. A weighting factor w determines how each of the customers react to period-to-period price changes.

Keywords: supply chain dynamics, price sensitive demand, ARMA model

Introduction
Most studies on supply chain dynamics assumes a constant sales price and subsequently a constant market size (or mean demand) is present. In this paper we adapt a stochastic linear demand model that allows for auto-correlation in the market price. This innovation allows us to develop a stochastic supply chain model where a weighting factor w is used to determine how customers react to period-to-period prices changes. For example, some customers may reduce their consumption of gasoline during periods of inflated prices. Conversely, others (farmers, truck drivers, and people living in rural areas) may be unable to do so. Those price-insensitive customers, have no choice but to pay higher prices. Another class of customers may even buy forward - stockpiling - in anticipation of even higher prices in the future.

Literature review
The AR(1) price model that we consider herein extends the demand model of Wang et al. (2014), who considered i.i.d. prices. Giri & Glock (2021) consider a remanufacturing supply chain setting with ARMA(1,1) prices and a stochastic linear demand process. They derive expressions for the demand process and the bullwhip effect in their setting. They find the demand process is a non-standard ARMA(1,1) demand process. The presence of bullwhip effect was found to depend upon the ARMA parameters of the price process and that b, the price sensitivity coefficient, plays an important role in decreasing the bullwhip effect. Giri & Glock (2021) recommend a stable pricing regime in order to reduce the bullwhip effect.

Another approach is to assume the price to be decision variable and demand to be dependent upon that price. This leads to research streams on inventory-based dynamic pricing, Shen et al. (2018), and coordinating supply chain contracts, Gong et al. (2022). When consumers are strategic, a centralized supply chain may perform strictly worse than a decentralized supply chain, Su & Zhang (2008). A similar demand model was used in Zhu et al. (2011). Cohen et al. (2022) investigate the consumer surplus under a deterministic price-sensitive demand model using price-sensitive uncertain (stochastic) demand and supply. The demand models used are a general function of the market price. Two different types of stochastic demand functions are considered, with an additive and a multiplicative noise component. They show uncertainty in the demand and the supply has significant impact on the consumer surplus.

Demand model and market size

We assume the market is a perfectly competitive market; the supplier is a price taker and has no control over the market price. We also assumed the market price follows the first order auto-regressive process (Box et al., 2008), and the market demand is a linear function of current and previous prices. In our model, demand at time period t, d_t, depends on the current and previous prices in the market, p_t and p_{t-1}:

$$d_t = a - b((1-w)p_t + wp_{t-1}) = a - bp_t + wb(p_t - p_{t-1}), \tag{1}$$

where a is the market potential at time period t; a has no impact on the stability and stationary, as its value is a constant over time. a represents the maximum market size, which is achieved when the price is zero. b is a positive demand sensitivity coefficient. The weighting factor $0 \leq w \leq 1$ determines how the customers react to period-to-period price changes:

- *Responsive customers:* When $w = 0$, customers consider only the current market price. In this case, the demand model is identical to the classical linear demand model.
- *Selective customers:* When $0 < w < 0.5$, customers tend to be price sensitive and place more emphasis on the current price, p_t. With selective customers, recent price increases ($p_t > p_{t-1}$) create a smaller demand than with naïve consumers (and vice versa).
- *Naïve customers:* When $w = 0.5$, customers are unknowing of, or indifferent to, the change in the market price.
- *Speculative customers:* When $0.5 < w < 1$, customers place less emphasis on the current market price, p_t. For example, if an expected future price is higher than the current price, they may order in the current period.
- *Slow customers:* When $w = 1$, customers determine their order based on the previous market price. This could happen when the decision-making process is time consuming (for example, due to a bureaucratic runaround).

We assume the market price follows the first order auto-regressive, AR(1), process. This is similar to the model used in Wang et al. (2014) where p_t follows an independently and identically distributed (i.i.d.) random process. Notice, unlike Giri & Glock (2021), the demand process does not contain its own source of noise; it only contains noise from the price process p_t. Another demand model that incorporates the most recent consecutive prices can be seen in Tai et al. (2019).

The first order auto-regressive process (Box et al., 2008)is given by,

$$p_t = \mu_p + \phi(p_{t-1} - \mu_p) + \epsilon_t, \tag{2}$$

where μ_p is the mean market price, ϕ is the auto-regressive coefficient, and ϵ_t is an i.i.d. random error term with zero mean and a constant standard deviation σ_ϵ. Notice, the price p_t can become negative, which obviously presents conceptual issues as products are not often sold at negative prices. To avoid this problem we assume $\mathbb{V}[p] >> 4\mu_p$ such that the probability of negative prices is negligible when the noise process ϵ_t is normally distributed. The variance of an AR(1) process, (such as p_t), is well known to be

$$\mathbb{V}[p_t] = \frac{\sigma_\epsilon^2}{1-\phi^2}, \quad (3)$$

Box et al. (2008). We assume $0 \le \phi < 1$ to simplify the exposition of this short conference paper, but acknowledge that $|\phi| < 1$ is stationary, invertible, and stable. In what follows, some important characteristics of our demand model are presented. Due to the page limitation, we omit some of the details to proofs but they are available upon request.

Lemma 1. (Demand process) *With AR(1) prices and our linear demand function, the demand process is an ARMA(1,1) process with a mean of* $\mu_d = \mathbb{E}[d] = a - b\mu_p$ *and a variance of* $\mathbb{V}[d] = b^2\sigma^2(1 + 2w(w-1)(1-\phi))(1-\phi^2)^{-1}$.

Proof. The expected demand is given by:

$$\mu_d = \mathbb{E}[d] = \mathbb{E}[a - bp_t + wb(p_t - p_{t-1})] = a - b\mu_p + wb(\mu_p - \mu_p) = a - b\mu_p.$$

By using (2) inside (1) and simplifying provides the following expression for d_t:

$$\begin{aligned} d_t &= a - b(\mu_p + \phi(p_{t-1} - \mu_p) + \epsilon_t \\ &= a - b\mu_p + \phi(b\mu_p + wb(p_{t-1} - p_{t-2})) + b(w-1)\epsilon_t - wb\epsilon_{t-1} \end{aligned}$$

Using $b\mu_p + wb(p_{t-1} - p_{t-2}) = d_{t-1} - a$, further simplification yields,

$$d_t = \mu_d + \phi(d_{t-1} - \mu_d) + b(w-1)\epsilon_t - wb\epsilon_{t-1}. \quad (4)$$

Eq. (4) is an ARMA(1,1) process with the mean of μ_d. An alternative formulation of the demand process found by substituting of (2) into (1) and simplifying:

$$d_t = \mu_d + b(\phi(w-1) - w)(p_{t-1} - \mu_p) + b(w-1)\epsilon_t. \quad (5)$$

The variance of the demand, $\mathbb{V}[d]$, can then be found from (5):

$$\begin{aligned} \mathbb{V}[d] &= \mathbb{E}[(d_t - \mathbb{E}[d])^2 = \mathbb{E}[\big(b(\phi(w-1) - w)\big(p_{t-1} - \mu_p\big) + b(w-1)\epsilon_t)^2\big] \\ &= b^2(\phi(w-1) - w)^2\mathbb{V}[\mathrm{p}] + b^2(w-1)^2\sigma_\epsilon^2 = b^2\sigma_\epsilon^2\frac{1+2w(1-\phi)(w-1)}{1-\phi^2}. \ \square \end{aligned} \quad (6)$$

Remark 1. It is surprising that both p_t and p_{t-1} disappear from the demand expression in (6). A similar result, in a different setting, can be seen in Giri & Glock's (2021) bullwhip study of a closed loop supply chain with ARMA(1,1) prices. □

Remark 2. The customer weighting factor *w* has no influence on the mean demand level, however it does affect the variance of the demand. We also note when *w* = 0, the classical linear demand model is present and the demand process becomes an AR(1) process. □

Lemma 1 shows that even though we assume a linear demand model and the price follows an AR(1) process, the demand model becomes to an ARMA(1,1) process (which degenerates into an AR(1) when *w* = 0).

Differentiating (6), the demand variance, w.r.t. *w* provides:

$$\frac{d\mathbb{V}[d]}{dw} = \phi_\epsilon^2\frac{2b^2(2w-1)}{1+\phi} \text{ and } \frac{d^2\mathbb{V}[d]}{dw^2} = \phi_\epsilon^2\frac{4b^2\mathbb{V}[d]}{1+\phi} > 0, \quad (7)$$

which leads to the following theorem:

Theorem 1. (Naïve customers) *Naïve customers (i.e. w = 0.5) generate minimal demand variance. The minimized demand variance is* $\mathbb{V}[d] = b^2\sigma_\epsilon^2(2(1-\phi))^{-1}$.

Proof. The first order condition, (7), identifies a stationary point at w = 0.5. The second order condition in (7), reveals it is a minimum. The second statement is proved by placing w = 0.5 into (6) and simplifying. □

Figure 1 plots the demand variance as a function of the weighting parameter w and the demand correlation ϕ, verifying the above results.

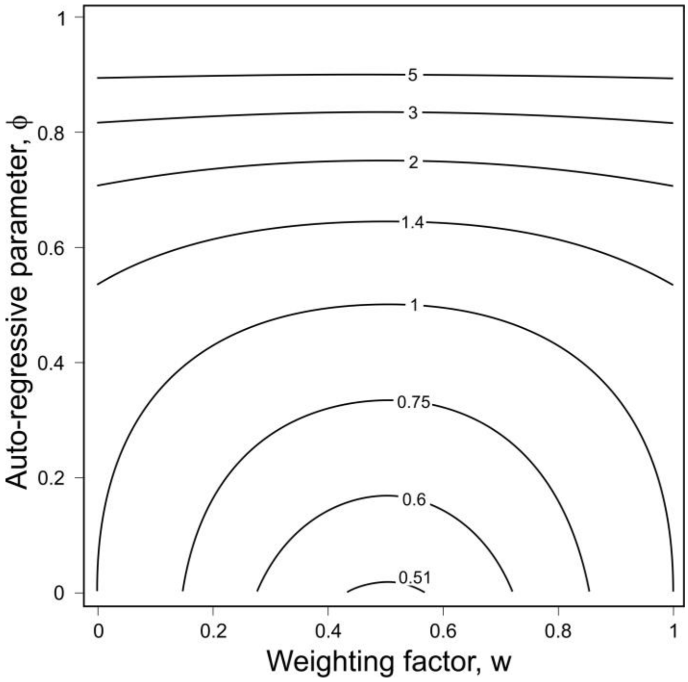

Figure 1: The demand variance as a function of the weighting parameter w and the auto-regressive parameter ϕ, *when* $b = \sigma_\epsilon = 1$.

Theorem 2. (Market size) *The expected value of the market size per time period,*

$$\mathbb{E}[d_t \cdot p_t] = \mu_p\mu_d - b\mathbb{V}[p](1-w+w\phi) = wb\mathbb{V}[p](1-\phi) + \mu_p\mu_d - b\mathbb{V}[p],$$

is a linear function of w.

Proof. The variance of the sum of two correlated random variables, $\mathbb{V}[d_t + p_t] = \mathbb{V}[p] + \mathbb{V}[D] + 2(\mathbb{E}[d_t \cdot p_t] - \mathbb{E}[p_t]\mathbb{E}[d])$, can be re-arranged to yield the required relation:

$$\mathbb{E}[d_t \cdot p_t] = \frac{\mathbb{V}[d_t+p_t]}{2} - \frac{\mathbb{V}[p]}{2} - \frac{\mathbb{V}[D]}{2} + \mathbb{E}[p_t]\mathbb{E}[d]. \quad (9)$$

The variance of the sum of price and demand is $\mathbb{V}[d_t + p_t] = \sigma_\epsilon^2(b(b(2(w-1)w(\phi-1)-1) + 2w(\phi-1)+2)-1)(\phi^2-1)^{-1}$, the variance of the price was given by (3), and the variance of the demand was given by (6). Substituting these expressions into (9) and simplifying yields the stated relation. □

Remark 3. The expected market size is less than the multiplication of the expected values of the demand (μ_d) and the price (μ_p), when $(1 - w + w\phi) \neq 0$. The reduction in the market size is driven by half the sum of the variance of the prices and the variance of the demand. □
Remark 4. With slow customers (i.e. $w = 1$) and the price follows an i.i.d. process (i.e. $\phi = 0$), the expected market size is maximized (at $\mathbb{E}[d_t \cdot p_t] = \mu_p \mu_d$). □
Remark 5. The market size reduces in the variance of the price $\mathbb{V}[p]$. □
Remark 6. As

$$\frac{d\mathbb{E}[d_t \cdot p_t]}{dw} = (1 - \phi) b \mathbb{V}[p] > 0,$$

coercing responsive customers to become slow customers increases the expected market size. That is, customer's purchasing behavior influences the market size; larger *w*'s create larger markets. □

Figure 2 verifies these remarks.

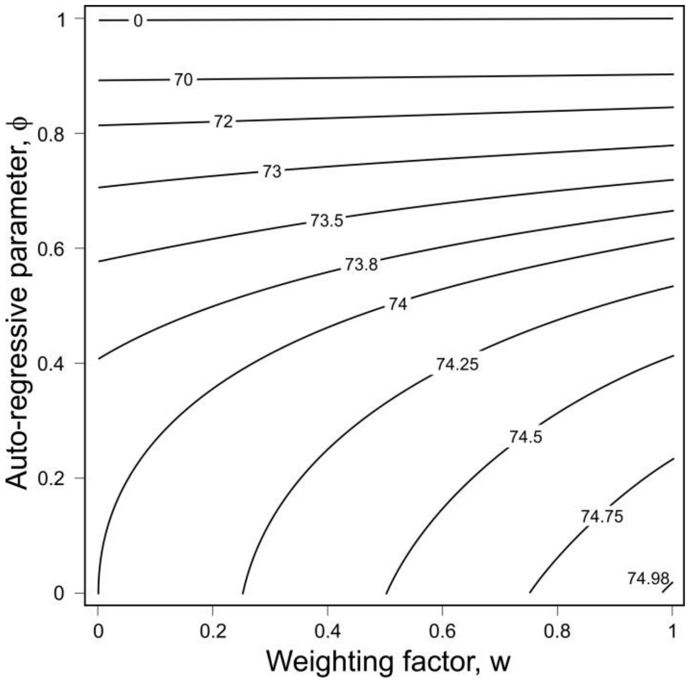

Figure 2: The market size as a function of the weighting parameter w and the auto-regressive parameter ϕ, when a = 20, $b = 1, \sigma_\epsilon = 1$ and $\mu_d = 5$.

The order-up-to replenishment policy

We assume the order-up-to (OUT) policy with the minimum mean square error (MMSE) forecasts, Box et al. (2008), is used to make replenishment orders. The OUT policy contains the following inventory balance equation:

$$ns_t = ns_{t-1} + o_{t-T_p-1} - d_t.$$

Here ns_t is the net stock level at time *t*, o_t is the order quantity placed by the supplier at time *t*, and T_p is the replenishment lead-time. It is known there are three different formulations of the OUT policy. The first formulation (Li et al., 2014) uses the inventory position, s_t,

$$o_t = s_t - \left(\sum_{i=1}^{T_p} o_{t-i} + ns_t\right); \text{ where } s_t = \mu_{ns} + \mathbb{E}\left[\sum_{i=1}^{T_p+1} d_{t+i} | t\right]. \quad (10)$$

Here μ_{ns} represents the time-invariant target net stock (or, safety stock) level. The expected demand *i* periods ahead, conditional upon the information available at time *t*, is given by,

$$\mathbb{E}[d_{t+i}|t] = \mu_d + b(\phi(w-1)-1)-w)\phi^{i-1}(p_t-\mu_p),$$

which can be obtained by recursion and knowing that the conditional expectation of future realizations of the noise term has an expected value of zero. This can be used to determine the conditional, or MMSE, forecast of the demand over the lead-time and review period:

$$\mathbb{E}\left[\sum_{i=1}^{T_p+1} d_{t+i}\,\middle|\,t\right] = \mu_d\left(T_p+1\right) + b(\phi(w-1)-w)\frac{1-\phi^{T_p+1}}{1-\phi}\left(p_t-\mu_p\right).$$

The second formulation of the OUT policy (Hosoda & Disney, 2006) is:

$$o_t = d_t + s_t - s_{t-1}. \tag{11}$$

This formulation is useful as it contains only feed-forward paths, facilitating the analysis of the OUT policy.

The third formulation (Disney et al., 2015) is:

$$o_t = \mathbb{E}\left[d_{t+T_p+1}\middle|t\right] + \underbrace{\mu_{ns}-ns_t}_{Inventory\ feeback} + \underbrace{\mathbb{E}\left[\sum_{i=1}^{T_p} d_{t+i}\,\middle|\,t\right] - \sum_{i=1}^{T_p} o_{t-i}}_{WIP\ feedback}. \tag{12}$$

In this formulation we can see the OUT policy contains two interacting feedback loops; one for the inventory levels and one for the work-in-process (WIP) levels. Note, all three OUT representations, (10), (11), and (12), produce the same order and inventory dynamics.

Lemma 2. (Variance of the net stock level) *The variance of the net stock levels generated by the OUT policy is given by the following expression:*

$$\mathbb{V}[ns] = b^2\sigma_\epsilon^2 \frac{\begin{pmatrix} w^2(\phi-1)^2(\phi^{2T_p+2}-1)+\phi^{2T_p+4}+T_p(\phi^2-1)- \\ 2w(\phi-1)(\phi^{T_p+1}-1)(\phi^{T_p+2}-1)-2\phi^{T_p+2}(\phi+1)+2\phi(\phi+1)-1 \end{pmatrix}}{(\phi-1)^3(\phi+1)}$$

Proof. The proof is omitted due to its length in the short conference paper; but it is available upon request. □

The net stock variance expression leads to the following theorem:

Theorem 3. (Slow customers) *Slow customers (i.e. w = 1) minimize the variance of the net stock levels.*
Proof. The first- and second-order derivatives of $\mathbb{V}[ns]$ w.r.t. *w* are:

$$\frac{d\mathbb{V}[ns]}{dw} = \frac{2b^2\phi_\epsilon^2(\phi^{T_p+1}-1)\left(1-\phi^{T_p+2}+w(\phi-1)(\phi^{T_p+1}+1)\right)}{(\phi-1)^2(\phi+1)} \text{ and } \frac{d^2\mathbb{V}[ns]}{dw^2} = \frac{2b^2\phi_\epsilon^2(\phi^{T_p+1}-1)(\phi^{T_p+1}+1)}{\phi^2-1} > 0.$$

Since the second-order derivative is always positive, the optimum value of *w*, $w^\star$which minimizes $\mathbb{V}[ns]$ satisfies

$$1-\phi^{T_p+2}+w(\phi-1)(\phi^{T_p+1}+1)=0.$$

$w^\star$ is equal to, or greater than, unity as

$$w^{\star}=\frac{1-\phi^{T_p+2}}{(1-\phi)(1+\phi^{T_p+1})}\geq\frac{1-\phi^2}{(1-\phi)(1+\phi^{T_p+1})}\geq\left(\frac{1-\phi^2}{(1-\phi)(1+\phi)}=1\right).$$

As the value of w is restricted to $0\leq w\leq 1$, setting $w^{\star}=1$ minimizes $\mathbb{V}[ns]$. □

Order process

We now turn our attention to the replenishment orders generated by the OUT policy under AR(1) price sensitive demand.

Lemma 3. (Order process) *Under AR(1) prices and a linear demand function, the OUT policy with conditional expectation forecasts generates an ARMA(1,1) replenishment order process with a mean of* μ_d *and a variance of:*

$$\mathbb{V}[o]=b^2\sigma_\epsilon^2\frac{1+\phi-2\phi^{T_p+1}(w+\phi-w\phi)(1+\phi-\phi^{T_p+1}(w+\phi-w\phi))}{(\phi-1)^2(\phi+1)}. \tag{13}$$

Proof. Departing from (11), the orders o_t can be rewritten as follows:

$$\begin{aligned}o_t&=(a-bp_t)+wb(p_t-p_{t-1})+b(\phi(w-1)-w)(p_t-p_{t-1})\frac{1-\phi^{T_p+1}}{1-\phi},\\&=(a-bp_t)+(p_t-p_{t-1})\left(wb+b(\phi(w-1)-w)\frac{1-\phi^{T_p+1}}{1-\phi}\right).\end{aligned} \tag{14}$$

Using $b\phi(1+(w(\phi-1)-\phi)\phi^{T_p})(\phi-1)^{-1}=A$, (14) can be written as

$$o_t=(a-bp_t)+A(p_t-p_{t-1}). \tag{15}$$

Substituting (2) into (15) provides

$$o_t=a-b(\mu_p+\phi(p_{t-1}-\mu_p)+\epsilon_t))+A(\mu_p+\phi(p_{t-1}-\mu_p)+\epsilon_t-\mu_p-\phi(p_{t-2}-\mu_p)-\epsilon_{t-1}).$$

Noting that $A(p_{t-1}-p_{t-2})-bp_{t-1}=o_{t-1}-a$ leads to

$$\begin{aligned}o_t&=(a-b\mu_p)+\phi(b\mu_p-a+o_{t-1})+(A-b)\epsilon_t-A\epsilon_{t-1}\\&=\mu_p+(\phi(o_{t-1}-\mu_d)+(A-b)\epsilon_t-A\epsilon_{t-1},\end{aligned}$$

which is an ARMA(1, 1) process with the mean of μ_d. To obtain the variance of the orders, we use (2) in (15) to yield

$$\begin{aligned}o_t&=a-b(\mu_p+\phi(p_{t-1}-\mu_p)+\epsilon_t)+A(\mu_p+\phi(p_{t-1}-\mu_p)+\epsilon_t-p_{t-1})\\&=\mu_p+(\phi(A-b)-A)(p_{t-1}-\mu_p)+(A-b)\epsilon_t\end{aligned}$$

from which variance of the orders can be easily obtained,

$$\begin{aligned}\mathbb{V}[o]&=\mathbb{E}[(o_t-\mathbb{E}[o])^2]=\mathbb{E}[((\phi(A-b)-A)(p_{t-1}-\mu_p)+(A-b)\epsilon_t)^2]\\&=(\phi(A-b)-A)^2\mathbb{V}[p]+(A-b)^2\sigma_\epsilon^2\end{aligned}$$

Further simplification provides (13). □

Theorem 4. (Influence of the lead time) *The variance of the orders,* $\mathbb{V}[o]$, *is strictly increasing in* T_p *when* $0<\phi<1$.

Proof. As $0 \leq w \leq 1$ and $0 < \phi < 1$, then $(w + \phi - w\phi) > 0$. As $0 \leq \phi < 1$ and $T_p \in \mathbb{Z}_0$ then both ϕ^{T_p+1} and $2\phi^{T_p+1}$are decreasing in T_p. Furthermore, the denominator of (13) is positive; and as (13) is a variance, the numerator of (13) must also be positive. These facts imply the numerator is increasing in T_p. □

Theorem 5. (**Customer behavior)** *Slow customers (i.e. w = 1) minimize the variance of orders, and responsive customers (i.e. w = 0) maximize the variance of the orders.*
Proof. The first-order derivative of $\mathbb{V}[o]$ w.r.t. *w* is

$$\frac{d\mathbb{V}[o]}{dw} = \left(\frac{2b^2\phi^{T_p+1}(1+\phi+2(w\phi-w-\phi)\phi^{T_p+1}}{\phi^2-1}\right)\sigma_\epsilon^2$$

which is always non-positive since:

$$\begin{aligned} 1 + \phi + 2(w\phi - w - \phi)\phi^{T_p+1} &= 1 + \phi + 2(w(\phi - 1) - \phi)\phi^{T_p+1} \\ &\geq 1 + \phi + 2(\phi - 1 - \phi)\phi^{T_p+1} = 1 + \phi - 2\phi^{T_p+1} \\ &\geq 1 + \phi - 2\phi = 1 - \phi > 0. \end{aligned}$$

The second-order derivative of $\mathbb{V}[o]$ w.r.t. *w* is:

$$\frac{d^2\mathbb{V}[o]}{dw^2} = \sigma_\epsilon^2 \frac{(2b\phi^{T_p+1})^2}{\phi + 1} \geq 0.$$

Therefore, the order variance is a decreasing-convex function in *w*, which indicates that $\mathbb{V}[o]$ is minimized when *w* = 1 and maximized when *w = 0.* □

Economic analysis
As the market size changes depending on the system settings, it would be interesting to conduct an economic analysis. We assume we wish to maximize profit *π,*

$$\pi = \mathbb{E}[d_t \cdot p_t] - v\mathbb{E}[d_t] - h\mathbb{E}[[ns_t]^+] - b\mathbb{E}[[-ns_t]^+] - uk - um\mathbb{E}[[o_t - k]^+].$$

Here $\mathbb{E}[d_t \cdot p_t]$ is the expected market size as given in (8), the expected demand $\mathbb{E}[d_t]$ was given by Lemma 1, *v* is the variable cost for each unit of production (material and direct energy costs), *h* is the per period per unit inventory holding costs, *p* the per period per unit backlog costs, *u* is the per unit production (labor) cost within the per period capacity of *k* and *um*, (where $m \geq 1$ is the over-time multiplier for production above the capacity of normal working hours) is the per unit production (labor) cost in overtime above the nominal capacity of *k*. Notice, labor are guaranteed their nominal per period (perhaps weekly) wage of *uk*, but that overtime has quantity flexibility.

The inventory costs are minimized by setting the safety stock target, μ_{ns} to the critical newsvendor factor,

$$\mu_{ns}^\star = \sqrt{\mathbb{V}[ns]}\Phi^{-1}\left[\frac{p}{p+h}\right].$$

When $\mu_{ns}^\star$ is present, the minimised inventory costs are given by

$$h\mathbb{E}[[ns_t]^+] + b\mathbb{E}[[-ns_t]^+] = (h + p)\sqrt{\mathbb{V}[ns]}\varphi\left[\Phi^{-1}\left[\frac{p}{p+h}\right]\right],$$

Churchman et al. (1957), where $\varphi[\cdot]$ and $\Phi^{-1}[\cdot]$ is the pdf and inverse cdf of the standard normal distribution. Using the same newsvendor techniques, the capacity costs are minimized by setting the nominal capacity k to

$$k^{\star} = \mu_d \sqrt{\mathbb{V}[o]} \Phi^{-1}\left[\frac{m-1}{m}\right].$$

When $k^{\star}$ is present, the minimized capacity costs are

$$uk + um\mathbb{E}[[o_t - k]^+] = u\mu_d + um\sqrt{\mathbb{V}[o]}\varphi\left[\Phi^{-1}\left[\frac{m-1}{m}\right]\right],$$

Boute et al. (2021). Figure 3 shows the highest profit is found with i.i.d. demand and slow customers, i.e. with $\phi = 0$ and w = 1. Though the demand variance near w = 1 is not minimal, both the inventory and order variances are minimized by the OUT policy with as $w \to 1$. The highest profit also concurs with the largest market sizes.

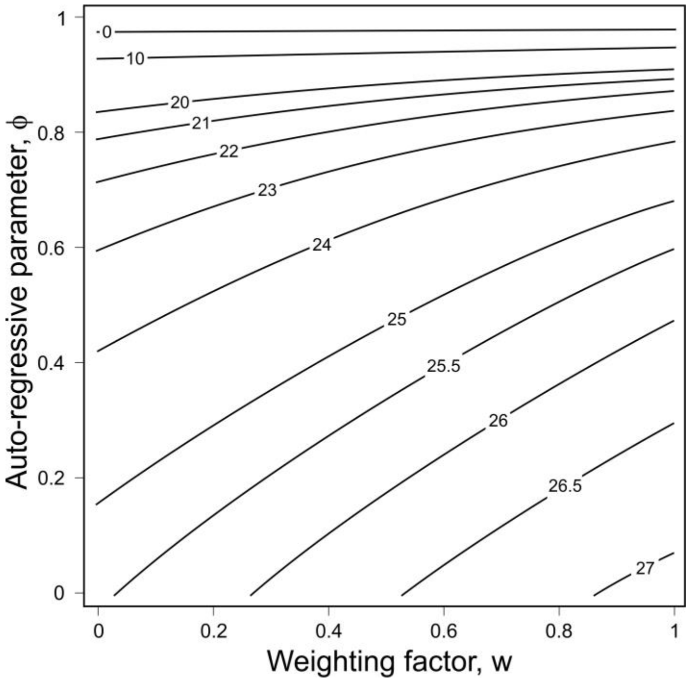

Figure 3: Profit π as a function of the weighting parameter w and the auto-regressive parameter ϕ, when $a = 20, b = 1, \sigma_\epsilon = 1, \mu_p = 5, h = 1, p = 9, u = 2,$ and $m = 1.5$.

Conclusions

We have investigated the impact of auto-correlated price sensitive demand on the supply chain dynamics maintained by the OUT replenishment policy. A weighting factor w was used to represent different customer responses to changes in the market prices. The demand variance was found to be minimized with naïve customers, w=0.5; whereas slow customers, w=1, lead to minimized inventory and order variances maintained by the OUT policy. The potential market size, $\mathbb{E}[p_t \cdot d_t]$, reduces in the variance of the prices. Interestingly, this can be observed from simply re-arranging the textbook expression for the variance of the sum of two correlated random variables. We also found companies should strive for i.i.d. market prices to maximize their potential market size, regardless of the nature of their customers. The supply chain profit, as a function of the market size, variable costs, inventory costs and capacity costs was investigated. We found that supply chain profit was maximized with i.i.d. demand and slow customers ($\phi = 0$ and $w = 1$), concurring with the maximum market size and minimum inventory and order variances. We also revealed a traditional linear price-sensitive demand model with an AR(1) price process is identical to a first-order auto-regressive moving average, ARMA(1,1), demand model.

Future work could directed to: a) empirically verifying that smaller price fluctuations and i.i.d. prices lead to a larger market sizes. b) modelling multi-product price sensitive demand settings, perhaps by using the vector auto-regressive demand modelling approach advocated by Boute et al. (2013) and Ma et al. (2015).

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