Three Essays on International Macroeconomics with labor market frictions

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I would like to dedicate this thesis to my family and loved ones . . .

Declaration

Thesis Title: Three Essays on International Macroeconomics with labor market frictions

Submitted by Cholwoo Kim to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Economics, March 2022.

I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

This dissertation contains fewer than 100,000 words including appendices, bibliography, footnotes, tables and equations and has fewer than 150 figures.

(Signature) ....

Cholwoo Kim
September 2022

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Abstract

This thesis consists of three essays on International Macroeconomics with labor market frictions. The first chapter addresses the international co-movement of employment by introducing labor market search frictions along with real wage rigidity into a two-country economy. I show that search and matching frictions in the labor market, combined with wage rigidity account for the positive cross-country correlation of employment as well as labor market activity within a country. With search and matching frictions in the labor market, higher productivity in the home country leads home and foreign employment to rise even at the initial period before productivity shocks spill over. When demand for foreign goods is predicted to rise, foreign firms have an incentive to hire workers in advance in response to the higher expected payoff to a job because hiring takes time and costs.

The second chapter examines a Ramsey-type optimal monetary policy in an open economy with a two-country dynamic general equilibrium model where search and matching frictions exist in labor markets along with the limited participation in financial markets. Monetary policy affects the decision of firms in labor markets because firms finance their wage bills with loans from domestic financial intermediaries in advance. There are two main results associated with optimal monetary policy. The long-run optimal nominal interest rate is zero suggesting deflation because the terms of trade effect on consumption is weaker by search and matching frictions in the labor market. As a result of the Ramsey optimal monetary policy, dynamics of business cycles in both countries show similar patterns in response to productivity shocks and, in turn, higher cross-country correlations of real variables.

In my third chapter, I explore how a country-specific productivity shock generates deviations of the LOP in an open economy by introducing search frictions in labor and goods markets. First, I express the LOP gap by the ratio of marginal utility of aggregate search efforts across countries. Then I show that the LOP gap is expressed in terms of relative aggregate consumption across countries by examining the relationship between the aggregate search efforts and the aggregate consumption. I find that a

country-specific productivity shock generates deviations of LOP through the link between aggregate consumption and aggregate productivity.

If a country-specific productivity shock occurs in the home country, then households in the home country exert more search efforts to consume more goods, which entails the difference of matching probabilities of firms between the domestic and the export markets. Since aggregate productivity and marginal costs of posting vacancies are the same across markets, difference in matching probabilities between markets let firms operating in each market offer different prices. Finally, I study responses of macroeconomic variables to a positive productivity shock in the home economy. I find the two-search model delivers consistent correlations with data, in terms of cross-country correlations of output and consumption, and a negative correlation between the terms of trade and the relative output when taking into account productivity shocks along with preference shocks.

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Introduction

Labor market is one of important factors in international macroeconomics as it affects the propagation of shocks across countries. Moreover, the features of the labor market can have an influence on the international trade, relative prices, and the policy choice in the open economy. However, conventional international macroeconomic models such as Backus et al. (1992) have difficulty to capturing some features of the labor market in the data, including the international co-movement of employment, despite these models account for key empirical characteristics of aggregate output and consumption over time. Since this class of model is based on the frictionless Walrasian labor market, where labor supply is determined by the inter-temporal optimal choice between leisure and working hours, one possible resolution is to incorporate labor market frictions into the open economy model. In this respect, this thesis investigates the role of labor market frictions in the open economy context. In particularly, this thesis focuses on implications of labor market frictions to the optimal policy and international relative prices as well as the role of labor market frictions over business cycles.

To take into account labor market frictions, I introduce the search and matching frictions proposed by Diamond (1982), Mortensen (1982), and Pissarides (1985) into the two-country general equilibrium model. Search and matching frictions in the labor market have excellent microfoundations as the idea that it takes time to find a job reflects an accurate characterization of reality. Moreover, Merz (1995) and Andolfatto (1996) show that this modified general equilibrium model captures the features of the labor market better than the standard RBC model by explicitly considering unemployment in the context of closed economy.

It is useful to consider labor market frictions in an open economy in two aspects. One is the improvement of the ability of the model to explain the behavior of the labor market. The first chapter of this thesis falls into this aspect, focusing on the international co-movement of employment, output, and consumption. There exists literature in this category, such as Hairault (2002) and Cacciatore (2014) which address fluctuations of international business cycles by considering frictional labor markets.

These studies show that introducing search and matching frictions in the labor market into open economy models accounts for the cross-country co-movement of employment. However, Shimer (2005) shows that a conventional labor market search model cannot account for high volatility of employment in a closed economy, which is known as Shimer's puzzle. Thus, I further consider real wage rigidity with labor market frictions jointly to examine the role of labor market frictions over business cycles.

Another aspect is to give an implication on optimal policies and international relative prices through the interaction between labor market frictions and other frictions. The second and the third chapters are associated with this category. Regarding the optimal monetary policy in the open economy, Cooley & Quadrini (2003) suggest that there is a possibility to deviate from the Friedman rule, which states the nominal interest rate should be zero under the optimal policy, with flexible prices and limited financial markets participation, but in the absence of labor market frictions. Thus, I explore how search and matching frictions in the labor market affect the optimal monetary policy in the second chapter. Meanwhile, Drozd & Nosal (2012) highlight search and matching frictions in the goods market to account for the international relative prices. In addition to goods market frictions, I incorporate labor market frictions into the two-country general equilibrium model to address the deviation from the law of one price (LOP) in data in the third chapter.

In the first chapter, entitled *International co-movement of employment with labor market frictions*, I introduce labor market search frictions along with real wage rigidity to analyze dynamics of employment in an open economy. To address the role of labor market frictions for the international transmission of productivity shocks, I consider a standard, two-country and two-good model which introduces search and matching frictions along with real wage rigidity into labor markets as in Gertler & Trigari (2009). In the model, the sluggish wage adjustment is caused by the staggered multi-period wage contracting which implies a fraction of firms cannot negotiate wages in each period.

With the model, I find that a model considering frictional labor markets combined with real wage rigidity resolves the international co-movement puzzle of employment as well as addresses the labor market activity within a country. In particularly, I show that higher productivity in the home country generates an increase in both home and foreign employment, in the absence of spillovers of productivity shocks. Foreign firms have an incentive to hire workers, in advance, in response to the higher expected payoff to a job in the future, because of search and matching frictions in the labor market. When the real wage is rigid, the effect of shocks becomes stronger and more persistent.

This comes from the fact that the effect of positive productivity shock is absorbed into the rise in wage in the case of flexible wage.

My second chapter, entitled *Optimal monetary policy in the open economy with labor market frictions*, is related to the role of labor frictions on the Ramsey-type optimal monetary policy in an open economy. To analyze the Ramsey-optimal monetary policy under labor market frictions, I consider a standard, two-country and two-good model with flexible prices where households' consumption is subject to a cash-in-advance constraint and firms finance wages with working capital before their production as in Christiano et al. (1997).

I find the optimal nominal interest rate in the long run is zero, leading to deflation, different from Cooley & Quadrini (2003). A contractionary monetary policy, i.e. higher interest rates, has an impact on the economy in two ways, financing cost effect and terms of trade effect. A higher interest rate decreases aggregate output by raising costs of production via the first effect, whereas it also increases aggregate output with the improvement of the terms of trade. However, when taking into account labor market frictions, a higher interest rate reduces vacancies and employment, which implies decreased costs of job posting. Thus, the terms of trade effect should be weak under the search and matching friction. Furthermore, as a result of the Ramsey optimal monetary policy, dynamics of business cycles in both countries show similar patterns in response to productivity shocks.

In my final chapter, entitled *Deviations from the LOP with labor and goods market frictions*, I explore how a country-specific productivity shock generates deviations of the LOP in an open economy. To account for the role of labor and goods market frictions for the deviation from the LOP, I consider a standard, two-country and two-good model. While search frictions in labor markets are characterized by specific matching technologies, following Mortensen & Pissarides (1994), directed search frictions are introduced in goods markets as in Moen (1997), recently used in Bai & Ríos-Rull (2015).

I find that a country-specific productivity shock leads to deviations from the LOP because it induces consumption gaps, differing search intensives in goods markets across countries. An increase in home productivity makes foreign firms post more vacancies and hire more workers. However, the increase of the LOP gap of foreign goods induce foreign firms to post less vacancies, because movement of firms across markets leads to a fall in the matching probability for firms. Therefore, employment of the foreign country depends on which effect is stronger.

Chapter 1

International co-movement of employment with labor market frictions

1.1 Introduction

The international transmission mechanism of aggregate economic shocks has been one of the significant issues in the international macroeconomics. However, as first shown by Backus et al. (1993), the standard international real business cycle model predicts that employment is negatively correlated across countries, whereas the correlation is generally positive in the data. One possible reason of this international co-movement puzzle of employment is that the real business cycle model is based on a neoclassical labor market where employment can respond immediately to a shock. In this respect, Hairault (2002) suggests a resolution of the problem by introducing labor market search frictions into an open economy. Shimer (2005), however, shows that a conventional labor market search model cannot account for high volatility of employment in a closed economy, which is known as Shimer's puzzle. This paper introduces labor market search frictions along with real wage rigidity to analyze dynamics of employment in an open economy. The main result of the paper is that a model considering wage rigidity with search frictions in labor markets resolves the international co-movement puzzle of employment as well as addresses the labor market activity within a country.

To address the role of labor market frictions for the international transmission of productivity shocks, I consider a standard, two-country and two-good model with complete asset markets and country-specific productivity shocks. Each country specializes in the production of a single good which is produced by using labor as sole input. The model introduces search and matching frictions along with real wage rigidity into labor markets, following Gertler & Trigari (2009). Gertler & Trigari (2009) show that a labor market search model can account for dynamics of labor market variables in response to a productivity shock in a closed economy when wage rigidity is introduced by the staggered wage bargaining. The paper extends their framework to an open economy. As a result, search frictions in labor markets are captured by the presence of hiring costs reflecting congestion externalities.[1] Furthermore, in the model, the sluggish wage adjustment is caused by the staggered multi-period wage contracting in which a fraction of firms cannot negotiate wages in each period.

With the model, I study responses of macroeconomic variables to a positive country-specific productivity shock to understand the international propagation of productivity shocks under labor market frictions. I show in this analysis that higher productivity in the home country generates a rise in foreign employment, absent spillover of productivity shocks. This transmission of a productivity shock comes from search and matching frictions in labor markets. Since labor inputs can be adjusted instantly in a neoclassical labor market, firms do not need to employ additional workers in the current period when higher demand for their goods is expected in the future. With search and matching frictions, however, foreign firms have an incentive to hire workers, in advance, in response to the higher expected payoff to a job because hiring takes time and costs. Meanwhile, predicted demand for foreign products rises with regard to the shock because an increase in home productivity causes a rise not only in home consumption, but also in foreign consumption due to international risk-sharing through asset markets. Thus, foreign firms also post more vacancies and hire more workers in the current period. As a result, employment in foreign country increases, but the size of changes is larger in the home country because the initial shock happens in the home country.

I then examine quantitatively whether the model generates the international co-movement of output, consumption and employment in the data, comparing with the search friction model without wage rigidity and the standard real business-cycle model suggested by Backus et al. (1993). I find that the international co-movement puzzle of employment disappears in both labor market search models, whereas the real business cycle model still shows a negative correlation of employment across countries. Furthermore, the values of cross-country correlation of employment from sticky and

[1] Increasing job posting makes it harder for other firms to employ new workers. However, each firm does not take into account this effect when it posts a vacancy. Pissarides (2000) refers to this externality as a congestion externality.

flexible wage models are quantitatively quite similar to that in the data. However, the flexible wage model in an open economy has difficulty in addressing high volatility of employment in the data, when there are productivity shocks, as in a closed economy. Thus, Shimer's puzzle still occurs in an open-economy model considering the labor market search frictions without wage rigidity as well.

I analyze the sensitivity of these findings by changing assumptions associated with parameters. As a result, I find consumption is less correlated than output across countries when the elasticity of substitution between home and foreign goods is low enough. This could be a possible explanation to the output-consumption correlation puzzle suggested by Backus et al. (1992).[2] Moreover, cross-country correlations of output, consumption and employment decline considerably as the openness to trade in goods is getting smaller. This implies that strong trade linkages across countries can increase the international co-movements. I also find that in economies with more flexible wages, the cross-country correlation of employment is significantly lower. In addition, employment is less correlated across countries as the spillover effects of productivity shocks are getting smaller. However, these experiments do not change the positive international co-movement of employment.

Since the flexible wage model also exhibits a positive cross-country correlation of employment, I address the role of wage rigidity when there are labor market frictions. I find that it is significant to consider wage rigidity to examine the labor market activity within a country as well as the international co-movement of employment. When the wage is flexible the effect of positive productivity shock is absorbed into the rise in wage, and in turn the change in employment becomes relatively small. As a result, the labor search model along with flexible wages suggests much lower persistence of output and consumption within a country found in the data.

This paper is related to two sets of literature. First, it is associated with the international macroeconomic literature that incorporates labor market frictions into open economy model. Hairault (2002) addresses the observed international fluctuations of business cycles by introducing labor search frictions into the two-country real business cycle model. He suggests a resolution of the international co-movement puzzle of employment by generating a positive cross-country correlation of employment in the model without real wage rigidity. Christiano et al. (2011) account for the effects of a monetary tightening by using a small open economy model with financial and labor market frictions. They find the model considering both financial and labor market frictions expects inflation and nominal interest rate much better than simpler models

[2] Backus et al. (1992) find consumption is more highly correlated across countries than output in their model, which is not consistent with the data.

which take into account either frictions of those. Cacciatore (2014) suggests an open economy model with labor market frictions which does not consider wage rigidity, focusing on the impact of frictions on international trade. He addresses strong trade linkages have a positive effect on welfare even though unemployment can increase temporarily. Furthermore, he suggests that strong trade linkages cause greater co-movement of business cycles. Even though the approach of this paper is similar to literature incorporating labor search frictions into open economy, this study is different from others as it considers wage rigidity. The paper mainly contributes to examine the international co-movement of employment as well as the labor market activity within a country in the data, considering real wage rigidity caused by staggered wage bargaining process.

This paper is also associated with a literature that introduces labor market frictions into general equilibrium models in a closed economy. Merz (1995) and Andolfatto (1996) embed search and matching frictions in labor markets into the RBC model. With exogenous separation rates, these papers address that considering labor frictions leads to persistent unemployment and low volatility of wages relative to the conventional RBC model. Furthermore, Den Haan et al. (2000) find that incorporating endogenous separations along with search and matching frictions into the RBC model generates more persistence and propagation of shocks. There is another literature to develop a general equilibrium model suggested by the New Keynesian approach with search and matching frictions in labor markets. Walsh (2005) shows that the search and matching framework improve the ability of New Keynesian models to explain the joint dynamics of output and inflation in response to monetary shocks, whereas Krause & Lubik (2007) suggest that the search friction does not affect the model dynamics significantly. Gertler et al. (2008) address the nominal wage rigidity by using the staggered wage bargaining as in Gertler & Trigari (2009). Although this paper is based on the same approach about wage rigidity, it contributes to address the role of real wage rigidity for the international business cycles by extending to an open economy.

The remainder of this paper is organized as follows. Section 1.2 introduces a two-country, two-good model with labor market frictions. The calibration procedure is discussed in section 1.3. Section 1.4 reports analytical and quantitative results of the model, comparing with data. The final section concludes.

1.2 Model

There are two countries - home and foreign. The total labor force in each country is normalized to unity. Both countries are comprised of a continuum of households, wholesale firms (henceforth 'firms') and retailers, normalized one. Households supply labor to firms and consume domestic and imported final goods. Perfectly competitive firms, indexed by $i \in [0, 1]$, produce intermediate goods by using labor as their sole input and sell them to retailers who operate in the monopolistically competitive market. The intermediate goods are not traded internationally. Retailers, indexed by $j \in [0, 1]$, sell the final products to either the home or the foreign household. Thus, the imperfect competition of goods markets is considered at the retail level, whereas search frictions in labor markets are assumed to happen in the intermediate goods markets.

Consumption of home and foreign goods of the home family is denoted with a subscript h and f, respectively. A superscript asterisk, $*$, denotes foreign country variables. While prices in this paper denote nominal prices, all prices are flexible. Asset markets in both countries are complete, which implies that households can access to internationally traded Arrow-Debreu securities. In what follows, the home country is focused on in the exposition of the model, whereas analogous expressions hold for the foreign country.

1.2.1 Matching process

Each firm posts vacancies, $v_t(i)$, and the total number of vacancies is $v_t = \int_0^1 v_t(i)di$. Each firm i also employs $n_t(i)$ workers and the aggregate employed worker is $n_t = \int_0^1 n_t(i)di$. All unemployed workers at period t are assumed to look for jobs. Furthermore, it is assumed that newly hired workers go to work immediately, following Blanchard & Galí (2010). Thus, the pool of unemployed workers at period t is given by the difference between the aggregate labor force and the number of employed workers at the end of period $t-1$:

$$u_t = 1 - n_{t-1}. \tag{1.1}$$

The search and matching process in the labor market follows the conventional model presented by Mortensen & Pissarides (1994). Firms post vacancies to hire workers and unemployed workers seek jobs passively. While each firm is assumed to have a job which can either be filled or vacant, workers are considered to be employed or unemployed. The number of new hired workers is expressed as the following

Cobb-Douglas matching function:

$$m_t = \sigma_m u_t^{\sigma} v_t^{1-\sigma}, \qquad 0 < \sigma < 1 \tag{1.2}$$

where σ_m denotes the efficiency of the matching process. The probability that any vacancy is matched, j_t, is expressed as

$$j_t = \frac{m_t}{v_t}. \tag{1.3}$$

Similarly, the probability that an unemployed worker finds a job, s_t, is given by

$$s_t = \frac{m_t}{u_t}. \tag{1.4}$$

Both firms and workers take the vacancy filling probability (j_t) and the job finding probability (s_t) as given.

1.2.2 Households

A representative household has a continuum of members, who are either employed or unemployed. Members currently unemployed are searching for jobs. The number of currently employed members in the representative household is n_t that is determined through the search and matching process. The representative household is assumed as an extended family, following Merz (1995). This assumption provides full consumption insurance between employed and unemployed members since all members gather their income and consume the same amount.

The representative household consumes home and foreign goods and considers all home goods (or foreign goods) as imperfect substitutes with constant elasticity of substitution, $\varepsilon > 1$. Furthermore, workers are assumed to provide the same hours worked in each period once they are employed, consistent with the conventional search and matching literature. Given n_t, the representative household cannot vary labor supply by changing hours worked in the model. The representative household maximizes its expected life-time utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t, \tag{1.5}$$

where $\beta \in (0, 1)$ is the discount factor, c_t is a consumption basket of the representative household at period t. Differentiated final goods from retailers can be sold with monopolistic competition in both countries. Accordingly, consumption of home or foreign goods of the home agent is a CES function over varieties with elasticity of substitution ε. The consumption indexes of home and foreign goods for the home agent is written as

$$c_{h,t}=\left\{\int_0^1 \left[c_{h,t}(j)\right]^{\frac{(\varepsilon-1)}{\varepsilon}} dj\right\}^{\frac{\varepsilon}{(\varepsilon-1)}}, \; c_{f,t}=\left\{\int_0^1 \left[c_{f,t}(j^*)\right]^{\frac{(\varepsilon-1)}{\varepsilon}} dj^*\right\}^{\frac{\varepsilon}{(\varepsilon-1)}}. \tag{1.6}$$

The preference of the representative family in the home country over domestic and foreign goods are expressed by the Armington aggregator. The consumption basket of a home household is given by

$$c_t=\left[\omega^{1/z}c_{h,t}^{(z-1)/z}+(1-\omega)^{1/z}c_{f,t}^{(z-1)/z}\right]^{z/(z-1)}, \tag{1.7}$$

where $\omega \in (0,1)$ denotes measure of openness and $z>0$ is the elasticity of substitution between home and foreign goods. When ω is larger than a half, consumption of the home household has the home-biased property.

Employed members of the representative household earn wages, whereas unemployed workers get unemployment benefits. The representative household obtains an additional income from firms and retailers because the ownership of firms and retailers is attributed it. The budget constraint of the representative household in each period is given by

$$c_t+E_t\left[Q_{t,t+1}B_{t+1}\right]=\int_0^1 w_t(i)n_t(i)di+(1-n_t)b+B_t+\Pi_t-T_t, \tag{1.8}$$

where B_t denotes the real payoff of the state-contingent securities purchased at period t. $Q_{t,t+1}$ indicates the corresponding stochastic real discount factor in units of the consumption good. While w_t means real wage, b and n_t denote unemployment benefits and the employed worker, respectively. While $\Pi_t=\int_0^1 \Pi_t(j)dj$ is aggregate profit from the ownership of retailers, T_t indicates the lump-sum tax used to finance unemployed benefits.[3]

The representative household chooses consumption and asset holdings to maximize its expected life-time utility (Equation (1.5)) subject to the budget constraint (Equation (1.8)). The first-order condition is given as the consumption Euler equation:

$$E_tQ_{t,t+1}=E_t\frac{\beta c_t}{c_{t+1}}. \tag{1.9}$$

[3] In equilibrium,

$$T_t=(1-n_t)b.$$

1.2.3 Firms

There is a continuum of firms that produce intermediate goods in perfectly competitive markets. A firm i employs $n_t(i)$ workers to produce output $y_t(i)$ in every period. The production function which is characterized by constant returns to scale is written as

$$y_t(i) = a_t n_t(i), \tag{1.10}$$

where a_t is a common productivity factor among all firms within a country.

Meanwhile, I define the hiring rate, $x_t(i)$, of a firm i as the ratio of new hires, $j_t v_t(i)$, to the existing workforce, $n_{t-1}(i)$:

$$x_t(i) = \frac{j_t v_t(i)}{n_{t-1}(i)}. \tag{1.11}$$

A firm's workforce can be divided two types of workers who are either employed in the past or hired in the current period. Some of the previously matched jobs are destroyed exogenously and this ratio is denoted by ρ which represents an exogenous separation rate of each period.

The total workforce is the sum of the number of surviving workers, $(1-\rho)n_{t-1}(i)$, and the number of new employed workers, $x_t(i)n_{t-1}(i)$. Thus, employment of a firm i evolves according to

$$n_t(i) = [1-\rho + x_t(i)]\, n_{t-1}(i). \tag{1.12}$$

As the hiring takes time and costs, the costs of posting a vacancy are assumed as quadratic adjustment costs, given by $\frac{\kappa}{2}x_t(i)^2 n_{t-1}(i)$, following Gertler & Trigari (2009).[4]

With considering the hiring costs, the value of a firm, $F_t(i)$, is expressed as

$$F_t(w_t(i), n_{t-1}(i)) = \theta_t y_t(i) - w_t(i)n_t(i) - \frac{\kappa}{2}x_t(i)^2 n_{t-1}(i) + \mathbb{E}_t \beta_{t,t+1} F_{t+1}(w_{t+1}(i), n_t(i)), \tag{1.13}$$

where θ_t and $w_t(i)$ are the real price of an intermediate good and the real wage at period t, respectively. $\beta_{t,t+1}$ denotes the firm's common discount rate between period t and $t+1$, which is given by $\beta_{t,t+1} = \beta c_{t+1}^{-1}/c_t^{-1}$.[5] In each period a firm i chooses $x_t(i)$ by posting vacancies to maximize its value subject to $n_t(i) = [1-\rho + x_t(i)]\, n_{t-1}(i)$.

[4] Quadratic adjustment costs are assumed due to wage dispersion in the model. If the costs were linear as in the conventional search and matching model, then all firms would have the same marginal hiring cost. Labor in turn would shift to the firm with lowest wage. See also Gertler & Trigari (2009).

[5] For consistency, $\beta_{t,t}$ is defined as 1.

The first order condition is given by

$$\kappa x_t(i) = \theta_t a_t - w_t(i) + \mathbb{E}_t \beta_{t,t+1} \frac{\partial F_{t+1}(i)}{\partial n_t(i)}. \tag{1.14}$$

The above result can be rewritten by using the envelope theorem for $\partial F_t(i)/\partial n_{t-1}(i)$:

$$\kappa x_t(i) = \theta_t a_t - w_t(i) + \mathbb{E}_t \beta_{t,t+1} \left[\frac{\kappa}{2} x_{t+1}(i)^2 + (1-\rho)\, \kappa x_{t+1}(i) \right]. \tag{1.15}$$

This is a job creation condition of a firm i. The right-hand side of the equation represents the current marginal profit of the firm, savings on hiring costs and the continuation value of the match. Thus, the job creation condition implies that the hiring rate depends on the discounted earnings from the match and savings on hiring costs in the next period.

1.2.4 Wage bargaining

The model assumes that the wage is determined so that the firm and the marginal worker share the surplus from the marginal match by negotiating. The negotiating rule is the staggered multi-period wage bargaining proposed by Gertler & Trigari (2009). The main difference between this model and the conventional search model proposed by Mortensen & Pissarides (1994) is that only a fraction of firms $(1-\lambda)$ can renegotiate their wages. The hazard rate, λ, is fixed and determined irrespective of the history of the firm.[6] If a firm cannot renegotiate the wage in the current period, all workers in the firm including the existing employees receive the wage in the previous period.

Before analyzing the wage bargaining process, it is necessary to define worker's surplus and firm's surplus from having an additional employment. I define $H_t(w_t(i))$ as a worker's surplus at firm i:

$$H_t(w_t(i)) = V_t(w_t(i)) - U_t, \tag{1.16}$$

where $V_t(w_t(i))$ and U_t denote the value of employment to a worker and the value of unemployment, respectively. The value of employment to a worker, $V_t(w_t(i))$, and the value of unemployment, U_t, are defined as

$$V_t(w_t(i)) = w_t(i) + \mathbb{E}_t \beta_{t,t+1} \left[(1-\rho) V_{t+1}(w_{t+1}(i)) + \rho U_{t+1} \right] \tag{1.17}$$

[6] This is similar to the staggered pricing suggested by Calvo (1983)

$$U_t = b + \mathbb{E}_t \beta_{t,t+1} \left[s_{t+1} V_{x,t+1} + (1 - s_{t+1}) U_{t+1} \right], \tag{1.18}$$

where $V_{x,t} = \int_0^1 V_t(w_t(i)) \frac{x_t(i) n_{t-1}(i)}{x_t n_{t-1}} di$ is the average value of employment conditional on being a new worker at period t. Thus, the worker's surplus, $H_t(w_t(i))$, can be rewritten as

$$H_t(w_t(i)) = w_t(i) - b + \mathbb{E}_t \beta_{t,t+1} \left[(1-\rho) H_{t+1}(w_{t+1}(i)) - s_{t+1} H_{x,t+1} \right] \tag{1.19}$$

where $H_{x,t+1}$ denotes the average worker's surplus conditional on being a new hire.

Let $J_t(w_t(i))$ be the firm's surplus from having an additional worker. The firm's surplus is obtained by differentiating the value of a firm ($F_t(i)$) with respect to an additional worker ($n_t(i)$):

$$J_t(w_t(i)) = \theta_t a_t - w_t(i) + \mathbb{E}_t \beta_{t,t+1} \left[-\frac{\kappa}{2} x_{t+1}(i)^2 + (1 - \rho + x_{t+1}(i)) J_{t+1}(w_{t+1}(i)) \right]. \tag{1.20}$$

For the wage bargaining, let w_t^{con} be the contract wage that the renegotiating firm chooses. The contract wage w_t^{con} of the negotiating firm is chosen to maximize the following Nash product:

$$\begin{aligned} &\max_{w_t^{con}} H_t(w_t(i))^{\eta} J_t(w_t(i))^{1-\eta} \\ &s.t.\ w_{t+1}(i) = w_t(i) \quad \text{with probability} \quad \lambda \\ &\qquad\qquad\quad = w_{t+1}^{con} \quad \text{with probability } 1-\lambda \end{aligned}, \tag{1.21}$$

where $\eta \in (0,1)$ denotes workers' bargaining power. The first order condition of Nash bargaining is given by

$$\eta \left(\frac{\partial H_t(w_t(i))}{\partial w_t(i)} \right) J_t(w_t^{con}) = (1-\eta) \left(-\frac{\partial J_t(w_t(i))}{\partial w_t(i)} \right) H_t(w_t^{con}). \tag{1.22}$$

The term $\partial H_t(w_t(i))/\partial w_t(i)$ and $-\partial J_t(w_t(i))/\partial w_t(i)$ are the impacts of an increase in the contract wage on the worker's surplus and the firm's surplus, respectively.[7] Gertler & Trigari (2009) call the effect of these terms as the horizon effect because firms and workers have different horizons when they renegotiate the contract wage.[8] Following Thomas (2008), however, the horizon effect is excluded in this paper since Gertler &

[7] $H_t(w_t^{con})$ and $J_t(w_t^{con})$ are derived in Appendix.

[8] When a firm sets the contract wage, it considers a longer horizon than a worker. This comes from the fact that newly hired workers in the future also will be affected by the current contract wage until the new contract wage will be set. In contrast to the firm, the worker takes into account the current wage during her tenure at the firm.

Trigari (2009) report this effect is not significant. Thus, the sharing rule is written as

$$\eta J_t(w_t^{con}) = (1-\eta)H_t(w_t^{con}) \tag{1.23}$$

Since the average wage across workers is expressed as $w_t = \int_0^1 w_t(i)\frac{n_t(i)}{n_t}di$, the evolution of the average wage is a linear combination of the contract wage and last period's wages of non-renegotiating firms:

$$w_{t+1} = (1-\lambda)w_{t+1}^{con} + \lambda w_t. \tag{1.24}$$

To better understand how the average wage in each country is determined with labor market frictions, I present a first order approximation of wage dynamics. In what follows, for any variable α_t, α and $\widehat{\alpha}_t$ denote the steady-state value and the log deviation from the steady-state value, respectively.

The log-linearized evolution of the average wage is given by

$$\widehat{w}_{t+1} = (1-\lambda)\widehat{w}_{t+1}^{con} + \lambda \widehat{w}_t. \tag{1.25}$$

To derive the contract wage, let the target wage be the period-by-period bargaining wage given all other firms and workers in the economy are operating on multi-period wage contracts. The following equations for the contract wage ($\widehat{w}_t^{con}$) and the target wage ($\widehat{w}_t^o(w_t^{con})$) can be obtained by log-linearizing the firm's and the worker's surplus and substituting the results into the log-linearized sharing rule of the multi-period wage bargaining. The index i which indicates the individual firm can be eliminated since all renegotiating firms set the same wage.[9]

$$\widehat{w}_t^{con} = (1-\tau)\widehat{w}_t^o(w_t^{con}) + \tau \mathbb{E}_t \widehat{w}_{t+1}^{con} \tag{1.26}$$

$$\widehat{w}_t^o(w_t^{con}) = \widehat{w}_t^o + \frac{\tau_1}{1-\tau}\mathbb{E}_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^{con}\right] \tag{1.27}$$

with

$$\begin{aligned}\widehat{w}_t^o \quad = \quad & \eta\Psi_a(\widehat{\theta}_t + \widehat{a}_t) + \eta\Psi_x\mathbb{E}_t\left[\frac{1}{2}\widehat{\Lambda}_{t,t+1} + \widehat{x}_{t+1}(w_{t+1})\right] \\ & + \eta\Psi_s\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{x}_{t+1}(w_{t+1})\right],\end{aligned} \tag{1.28}$$

[9] The complete derivation is in the Appendix.

where $\widehat{\Lambda}_{t,t+1} \equiv \widehat{c}_t - \widehat{c}_{t+1}$, $\tau \equiv \frac{(1-\eta)\varepsilon+\eta\mu}{1+(1-\eta)\varepsilon+\eta\mu}$, $\tau_1 \equiv \eta\beta x(1+\mu)(1-\tau)+\beta s$, $\mu \equiv \frac{\lambda\beta}{1-\lambda\beta}$, $\varepsilon \equiv \frac{(1-\rho)\lambda\beta}{1-(1-\rho)\lambda\beta}$, $\Psi_a \equiv \theta a/w$, $\Psi_x \equiv \beta xJ/2w$ and $\Psi_s \equiv \beta sJ/w$. $\widehat{w}_t^o$ is defined as the spillover-free target wage as in Gertler & Trigari (2009), which would arise if all firms and workers were negotiating a period-by-period wage contract. The spillover-free target wage, $\widehat{w}_t^o$, is different from the target wage, $\widehat{w}_t^o(w_t^{con})$, since there is no firm which does not renegotiate its wage in each period.

By using the equations related to the contract wage (Equation (1.26)) and the target wage (Equation (1.27)), the above equation can be rewritten as

$$\widehat{w}_t = \gamma_b \widehat{w}_{t-1} + \gamma_o \widehat{w}_t^o + \gamma_f \mathbb{E}_t \widehat{w}_{t+1}, \tag{1.29}$$

where $\gamma_b \equiv \lambda/\phi$, $\gamma_o \equiv (1-\tau)(1-\lambda)/\phi$, $\gamma_f \equiv (\tau - \lambda\tau_1)/\phi$, and $\phi \equiv 1+\lambda(\tau-\tau_1)$. The current average wage is expressed as the linear combination of lagged wage $(\widehat{w}_{t-1})$, the spillover-free target wage $(\widehat{w}_t^o)$ and the future wage $(\widehat{w}_{t+1})$. Because of the staggered multi-period wage contracting, $\widehat{w}_t$ depends on not only the current spillover-target wage, but also the lagged wage and the expected future wage. It is noted that as λ is converged to zero, both γ_b and γ_f go to zero, implying that $\widehat{w}_t$ has the same value of $\widehat{w}_t^o$. Therefore, the model becomes the conventional period-by-period wage bargaining model presented by Mortensen & Pissarides (1994).

1.2.5 Retailers

Retailers buy an homogeneous intermediate good from firms, convert it into a differentiated final good, and resell it to either home or foreign households. Thus, the relative price of intermediate goods, θ_t, is the real marginal cost faced by retailers. The final goods markets in both countries are considered monopolistically competitive.

Let $\Pi_t(j)$ be the real profit of a retailer j in the home country. Each retailer earns profits either from home and foreign consumers. The real profit of each retailer is given by

$$\Pi_t(j) = \Pi_{h,t}(j) + \Pi_{h,t}^*(j) \tag{1.30}$$

with

$$\Pi_{h,t}(j) = \frac{p_{h,t}(j)}{P_t} c_{h,t}(j) - \theta_t c_{h,t}(j), \tag{1.31}$$

$$\Pi_{h,t}^*(j) = \frac{q_t p_{h,t}^*(j)}{P_t^*} c_{h,t}^*(j) - \theta_t c_{h,t}^*(j). \tag{1.32}$$

$p_{h,t}(j)/P_t$ denotes the relative price of a final good made by a retailer j in the home country.[10] q_t denotes the real exchange rate which measures the price of foreign output relative to the price of home output. A price index can be derived from the consumption aggregators (Equation(1.6), Equation (1.7) in each country.

$$P_t = \left[\omega P_{h,t}^{1-z} + (1-\omega) P_{f,t}^{1-z}\right]^{1/(1-z)}, \tag{1.33}$$

$$P_t^* = \left[\omega P_{f,t}^{*1-z} + (1-\omega) P_{h,t}^{*1-z}\right]^{1/(1-z)}, \tag{1.34}$$

where

$$P_{h,t} = \left\{\int_0^1 \left[p_{h,t}(j)\right]^{1-\varepsilon} dj\right\}^{1/(1-\varepsilon)}, P_{f,t} = \left\{\int_0^1 \left[p_{f,t}(j^*)\right]^{1-\varepsilon} dj^*\right\}^{1/(1-\varepsilon)},$$

$$P_{h,t}^* = \left\{\int_0^1 \left[p_{h,t}^*(j)\right]^{1-\varepsilon} dj\right\}^{1/(1-\varepsilon)}, P_{f,t}^* = \left\{\int_0^1 \left[p_{f,t}^*(j^*)\right]^{1-\varepsilon} dj^*\right\}^{1/(1-\varepsilon)}.$$

Given the aggregate price, a retailer j maximizes its profits by choosing the prices in the home and foreign economy. According to the solution of profit maximization, the relative price of home goods should be equal to the markup over the real marginal cost.[11] This is written as

$$\frac{p_{h,t}(j)}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\theta_t, \quad \frac{q_t p_{h,t}^*(j)}{P_t^*} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\theta_t. \tag{1.35}$$

As retailers in home country is identical and all prices are flexible, the optimal prices of retailers are same in both countries. Thus, the following relation holds:

$$\frac{P_{h,t}}{P_t} = q_t\left(\frac{P_{h,t}^*}{P_t^*}\right). \tag{1.36}$$

Considering the definition of the real exchange rate, q_t, this equation implies that the law of one price holds.

[10] Since the output of retailer j is consumed by home and foreign households,the following relations are satisfied.

$$\begin{aligned} y_t(j) &= y_{h,t}(j) + y_{h,t}^*(j) \\ y_{h,t}(j) &= c_{h,t}(j),\ y_{h,t}^*(j) = c_{h,t}^*(j) \end{aligned}$$

[11] For this result, I use the demand curves for $c_{h,t}(j)$ and $c_{h,t}^*(j)$, derived from the cost minimization of obtaining one unit of aggregate consumption.

1.2.6 Resources and Real exchange rate

In each period, output is divided into domestic consumption ($c_{h,t}$), export ($c^*_{h,t}$) and adjustment costs in the labor market .[12] Therefore, the aggregate resource constraint is given by

$$y_t = \int_0^1 c_{h,t}(i)di + \int_0^1 c^*_{h,t}(i)di + \frac{\kappa}{2}\int_0^1 x_t(i)^2 n_{t-1}(i)di \tag{1.37}$$

As state-contingent securities are traded internationally, the real discount factor of securities is written as

$$Q_{t,t+1} = Q^*_{t,t+1}(q_t/q_{t+1}). \tag{1.38}$$

From the inter-temporal efficiency condition (Equation (1.9)), the real exchange rate is expressed the ratio of consumption across countries:[13]

$$q_t = \frac{c_t}{c^*_t}. \tag{1.39}$$

This expression implies that international risk-sharing entails the real exchange rate should be equal to the ratio of marginal utility of consumption between countries with the complete asset market.

Meanwhile, I introduce the terms of trade for the quantitative analysis. The terms of trade for the home country which is a relative price of imports to exports is expressed as

$$TOT_t = P_{f,t}/P_{h,t}. \tag{1.40}$$

1.3 Calibration

In this section, the calibration of the parameters presented in the model is discussed.[14] I assume the home country is the U.S. and the foreign country is the EU for the

[12]As all intermediate goods in each country are used as inputs of final goods, the following equations hold.

$$\begin{aligned} c_{h,t} &= \int_0^1 c_{h,t}(j)di = \int_0^1 c_{h,t}(i)di, \\ c^*_{h,t} &= \int_0^1 c^*_{h,t}(j)dj = \int_0^1 c^*_{h,t}(i)di \end{aligned}$$

[13]When we iterate back to $t = 0$ and normalize the real exchange rate at $t = 0$ (q_0) to 1, the relation can be derived. See appendix in Chari et al. (2002)

[14]The model is solved by the log-linearization around the steady states of variables. The complete log-linearized model is in the Appendix.

calibration of parameters. To compare with the literature, the calibration in this paper is quarterly.

For the parameters related to preferences, I set values used in the literature. The discount factor, β, is assumed 0.99 to adjust the annualized real interest rate to 4.1%. The consumption elasticity between varieties of home and foreign countries are calibrated so that the markups on the marginal cost are 23% and 35%, respectively, following Bayoumi et al. (2004).

The values of parameters for the matching process in both countries follow Mortensen & Pissarides (1994). The elasticity of match (σ) and the scale parameter of match (σ_m) is set to 0.5 and 4, respectively. The worker's bargaining power (η) is assumed to be 0.5. The hazard rate (λ), which is the probability that the firm does not renegotiate its wage, is set to 0.75. This implies that the average time between wage negotiation is one year, which is consistent with Gottschalk (2005). Following Gertler et al. (2008), the job separation rate, ρ, and the job finding rate, s, of home country are set to 0.105 and 0.95 to match the estimates of the U.S. monthly rates suggested by Shimer (2005). This means that a matched job in home country lasts about two and a half year. I choose 0.036 and 0.25 for the job separation rate, ρ^*, and the job finding rate, s^*, of foreign country, respectively, to be consistent with Hobijn & Şahin (2009).[15] From the job separation rates and job finding rates in both countries, steady-state unemployment rates of home and foreign country are derived as 0.10 and 0.13, respectively. This is reasonable as unemployment rate includes those individuals registered as inactive in this model. For the vacancy posting cost (κ) and unemployment benefits (b), I use the replacement ratio which is the ratio of unemployment benefit to average wages. The replacement ratios of the home (the U.S.) and the foreign (the EU) countries are taken as 54% and 66% from OECD (2006).

Turning to the values of parameters associated with open economies, the openness parameter in each country is chosen such that imports are 13% and 18% of aggregate output, respectively, as in Bayoumi et al. (2004). I assume that the consumption elasticity between home and foreign goods (z) is 1.2, following Ruhl (2008). The parameters can be shown in Table 1.1.

[15] These quarterly rates are calculated by using monthly estimates of the EU-15 except for Austria and Luxembourg in Hobijn & Şahin (2009).

Table 1.1 Parameters

Parameters set exogenously				
Statistic	Parameter	Value		Sources
Discount factor	β	0.99		$\left(\beta^{-4}-1\right)\times 100 = 4.102\%$
Elasticity of match	σ	0.5		Mortensen & Pissarides (1994)
Efficiency of match	σ_m	4		Mortensen & Pissarides (1994)
Bargaining power	η	0.5		Mortensen & Pissarides (1994)
Hazard rate	λ	0.75		Gottschalk (2005)
Armington elasticity	z	1.2		Ruhl (2008)
Home transition rates	$[s,\rho]$	$[0.95, 0.105]$		Gertler et al. (2008)
Foreign transition rates	$[s^*,\rho^*]$	$[0.25, 0.036]$		Hobijn & Şahin (2009)
Calibrated Parameters				
Statistic	Parameter	Value	Target(%)	Sources
Import share	$[\omega,\omega^*]$	$[0.87, 0.82]$	$[13, 18]$	Bayoumi et al. (2004)
Mark-up	$[\varepsilon,\varepsilon^*]$	$[5.35, 3.86]$	$[23, 35]$	Bayoumi et al. (2004)
Home replacement rate	$[b,\kappa]$	$[0.48, 3.68]$	54	OECD (2006)
Foreign replacement rate	$[b^*,\kappa^*]$	$[0.58, 28.37]$	66	OECD (2006)

Note: The parameters set exogenously are common across countries except for transition rates. The calibrated parameters are derived from U.S. (home) and EU (foreign) steady-state targets. The parameters with * refer to the foreign country.

1.4 Findings

In this section, I examine whether the international business cycle can be reproduced in the model when a country specific productivity shock occurs, and then verify the usefulness of the model by comparing performances with data.

1.4.1 Responses to shocks

To analyze the effect of a country-specific productivity shock, I assume that the aggregate productivity follows bivariate autoregressive process, following Backus et al. (1992).

$$A_{t+1} = \Omega A_t + \varepsilon_{t+1}, \tag{1.41}$$

where $A_t = [\ln a_t, \ln a_t^*]^T$ and $\varepsilon_{t+1} \sim N(0,V)$. ε_t are considered as serially independent random variables. Thus, the diagonal elements of Ω imply the persistence of country-specific productivity shock, while the off-diagonal elements denote the spill-over effects of a productivity shock across countries. If the off-diagonal elements of Ω are not zeros, it is assumed that a positive productivity shock in a country can affect other country where an innovation does not happen.

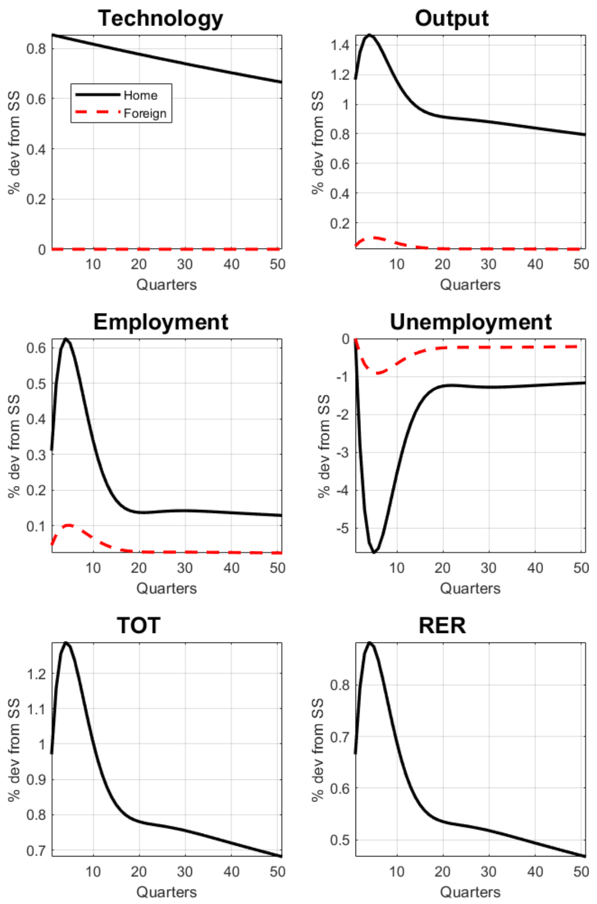

Figure 1.1 Responses to one s.d. productivity shock in the home country

Note: The home (foreign) country is assumed as the U.S. (EU) economy. One period denotes a quarter on the horizontal axis and the percentage deviation from the steady state is depicted on the vertical axis.

To better understand the transmission of a productivity shock through search frictions in labor markets, the spillover parameters are set to 0 as in Baxter (1995), whereas the persistence parameters are set to 0.995. For the variance covariance matrix V, the variances of 0.73 and correlation of shocks of 0.19 are taken from Baxter (1995).

Figure 1.1 shows the percentage changes in output, consumption, employment, real wage, the terms of trade and the real exchange rate to an one-time positive standard deviation shock to home productivity in the benchmark model. An increase in home productivity causes firms in home country post more vacancies inducing more employment as higher productivity leads to an increase in the current and expected future values of jobs. An increase in employment leads to a decrease in unemployment. More employment as well as increasing values of jobs in home country also make the aggregate real wage increase. As a result, output, consumption and employment in home country increases, displaying hump-shaped response due to labor-search frictions and real wage rigidity.

An increase in home productivity leads to a rise in output, consumption and employment in the foreign country although there are not exogenous positive spillovers of a home productivity shock to the foreign economy. The main mechanism on this propagation comes from search and matching frictions in labor markets. Since labor inputs can be adjusted instantly in a neoclassical labor market, firms do not need to employ an additional worker beforehand when increasing demand for foreign products is predicted. With search frictions, however, foreign firms have an incentive to hire more workers in advance as hiring takes time and costs.

Since a positive home productivity shock causes a rise in home consumption, international risk-sharing leads foreign consumption to increase as a result of complete asset markets. Moreover, future consumption in both countries will also increase because the effect of a shock on the home country is persistent as in Figure 1.1. This, in turn, leads expected demand for foreign products to rise. Thus, foreign firms also post more vacancies and hire more workers due to the higher expected returns to jobs.

As a result, output, consumption and employment in foreign country increase, but the size is larger in the home country because the initial shock happens in the home country. Meanwhile, both the real exchange rate and the terms of trade rise in response to a positive productivity shock in the home country. This occurs because the positive innovation in the home country increases consumption in both countries, but less in foreign country.

To better understand why foreign employment increases, the job creation condition of the foreign country can be used.

$$\kappa x_t^*(i^*) = \theta_t^* a_t^* - w_t^*(i^*) + \mathbb{E}_t \beta_{t,t+1} \left[\frac{\kappa}{2} x_{t+1}^*(i^*)^2 + (1-\rho)\, \kappa x_{t+1}^*(i^*) \right]. \tag{1.42}$$

According to the above equation, employment is determined at the level where the marginal cost and the expected payoff of an additional hiring are equal. Left-hand side of the equation indicates the marginal cost of an additional worker to a firm, while the right-hand side of the equation means the expected payoff of a match. The first two terms in the right-hand side are the current earning from hiring an additional worker. The last two terms in brackets represent savings on hiring costs and continuation value of a job which is not separated. When a firm hires one more worker in the current period, the firm potentially hire one less worker in the next period. This makes the firm save hiring costs ($\frac{\kappa}{2}x^*_{t+1}(i^*)^2$) incurred by an additional hire in the next period. $\kappa x^*_{t+1}(i^*)$ is the marginal cost in the next period, which is equal to the expected earnings in the next period under the optimal choice of the firm. Thus, the second term in brackets denotes the expected future earnings of an additional hired worker at present.

If there is no hiring cost in the job creation condition ($\kappa = 0$), employment is determined by the wage equals the marginal revenue product of labor. Thus, employment becomes more sensitive to the current economic environment. However, if there are search frictions ($\kappa \neq 0$), firms take into account the future as well as the present. Moreover, when firms determine how many workers they hire, they are likely to employ more since current employment brings further benefit ($\mathbb{E}_t\beta_{t,t+1}\frac{\kappa}{2}x^*_{t+1}(i^*)^2$) in addition to stream of earnings. Accordingly, firms tend to hire more workers at present with respect to the expected positive situation in the future.

When it is expected that there will be increasing demand of a firm's good in the next period, the expected marginal payoff of an additional worker, which is the right-hand side of the job creation condition, increases due to a rise in the expected future earnings. With real wage rigidity, current hiring rates of foreign firms should be higher to restore balance in the job creation condition. Therefore, the positive productivity shock of the home economy will increase the foreign employment.

This explanation is consistent with the features of the search and matching friction model as pointed out by Pissarides (2000). As employment is an on-going, long-term relationship, and the hiring process is time-consuming and costly, firms consider not only the current value, but also the future value of a job. This feature can play an important role in the open economy model, making more correlated dynamics of employment between countries.

1.4.2 Quantitative results

Table 1.2 shows properties of business cycles both in the data and in the open-economy models considering search and matching frictions in labor market.[16] To analyze business cycles more precisely in the model, I assume the values of parameters associated with spillover and persistence of productivity shocks are 0.088 and 0.906,

[16] The data column are for the period of 1976:1-2015:4, using data for the US and the aggregate of the EU-15. Details are in Appendix.

Table 1.2 Business cycle statistics

Statistic	Data	Model			
		Sticky wage	Flexible wage	No search	BKK
Cross-country Correlation					
GDP(y,y*)	0.55	0.43	0.34	0.27	−0.21
Consumption (c,c*)	0.40	0.76	0.68	0.80	0.88
Employment (n,n*)	0.93	0.56	0.53	−1.00	−0.78
% Standard Deviation(S.D)					
Real exchange rate(q)	3.80	1.07	0.83	0.60	–
Terms of trade(TOT)	2.24	1.56	1.22	1.18	0.48
S.D. relative to GDP					
Consumption (c)	0.60	0.87	0.90	0.90	0.42
Employment (n)	0.88	0.46	0.14	0.02	0.50

Note: The statistics of the data column are for 1970:1 to 2015:4 using U.S. and the aggregate data of the EU-15. Columns of sticky wage, flexible wage, and no search are calibrated with values in Table 1.1, whereas statistics of BKK are given by Backus et al. (1993). All statistics have been HP-filtered with a smoothing parameter of 1,600.

respectively, as in Backus et al. (1992). For the variance covariance matrix V, the variances and correlation of shocks are set to 0.00852^2 and 0.285. All entries in the table are Hodrick-Prescott (henceforth 'HP') filtered values with a smoothing parameter of 1600. While the first column reports characteristics found in the data corresponding the U.S. aggregate, the remaining columns are statistics derived from the models. While labor market frictions are incorporated in the first and second models, the third model denotes as 'No search' is based on the Walrasian labor market.[17] The standard real business-cycle model suggested by Backus et al. (1993)(henceforth 'BKK') is shown in the last column. The difference between those two models with labor market frictions is whether real wage rigidity exists or not. Sticky real wage model, considered as benchmark, assumes a fraction of firms in both countries cannot reset their wages in each period. However, flexible real wage is considered in the second model as firms and workers can renegotiate their wages every period.[18]

The cross-country correlations of output in all models are less than those of consumption, which is inconsistent with the data. However, the output between home

[17] For the analysis of the goods search model, the period utility function is assumed as

$$u(c_t, n_t) = \ln c_t - \iota \frac{n_t^{1+\frac{1}{g}}}{1+\frac{1}{g}},$$

where g set to 0.72. Furthermore, hours worked are targeted at 1/3.

[18] Flexible real wage model is obtained by setting the parameter of wage rigidity (λ) is zero, otherwise same as sticky real wage model.

and foreign countries in BKK are negatively correlated, while the flexible and sticky wage models generate the positive international correlation of output. Comparing with other models, the sticky wage model shows quantitatively higher correlation of output of 0.43, albeit it is still less than that in the data.

Employment in both models with labor market frictions is positively correlated across countries same as in the data. By contrast, No search and BKK models show a negative correlation of employment, which is not consistent with the data. The international co-movement puzzle of employment disappears in both labor market search models.

In Table 1.2, BKK has a problem with accounting for international co-movements of employment and output. Without search frictions in labor markets, wages are always determined to clear the labor market. When there is a positive productivity shock in the home country, labor and wages increase in the home country as labor demand increases following the decease in marginal costs. This leads to an increase in the production of home goods. The increasing output of home country improves the terms of trade. As a result, expenditure switching occurs from foreign goods to home goods as households in both countries prefer goods that become relatively cheaper. Thus, foreign employment and output declines upon the impact, which weakens the co-movement of output and employment between countries.

In the labor market search model, however, employment of the foreign country increases upon the impact as opposed to BKK. With search and matching frictions, productivity shocks influence employment in both countries in the same direction, which leads to the positive international co-movement of employment.

On the other hand, Table 1.2 shows that a significantly high volatile movement of real exchange rate and terms of trade in data is not reproduced by any models, even though values increase significantly in the sticky wage model. Furthermore, the flexible wage model shows too little volatility of employment relative to output of 0.14 within the home country, comparing to the data. This figure significantly increases to 0.46 in the sticky wage model, albeit it is still less than the variability in the data. Shimer (2005) points out that the labor market search model with flexible wage predicts too small volatilities of labor market variables by using the closed economy model. This drawback also occurs in an open-economy model introducing search frictions in labor markets without wage rigidity.

1.4.3 Sensitivity analysis

In this section, I analyze the sensitivity of my results by choosing alternative assumptions about four parameters. First, I examine the results by changing Armington elasticity (z) from 0.05 to 2. Trade elasticity is one of controversial issues in the literature. While Taylor (1993) estimates it as 0.39 by using time series data for the U.S., Backus et al. (1993) set it to 1.5. Furthermore, Heathcote & Perri (2002) and Corsetti et al. (2008) estimate the elasticity as 0.90 and 0.85, respectively. Accordingly, it is significant to address whether the quantitative results in this paper are compatible with lower Armington elasticity. I also change the imports-to-output share between 0% and 20%. As Corsetti et al. (2008) and Chari et al. (2002) suggest much lower import-to-output shares of 5% and 1.6%, it is examined if the results of the benchmark model is robust with the large home-biasedness of consumption.

I then vary the rigidity of real wage by changing λ from 0.5 to 0.85, which implies wage is renegotiated once every 2 quarters or almost every 8 quarters, respectively. Gertler & Trigari (2009) suggest the duration of wage contract might be less than a year as other components such as bonuses could be changed more frequently than wage. Thus, I examine the effect of less rigid wage in line with Gertler & Trigari (2009). Finally, I analyze the cross-country correlations for different values of spillovers of productivity shocks. The value of spillover parameter is varied from 0 to 0.1. To change spillovers with keeping the overall persistence of shocks constant, I adjust the persistence terms of shocks so that the largest eigenvalue of matrix Ω is same as the benchmark value. Figure 2 shows the results of the sensitivity analysis.[19]

In Figure 1.2, the lower Armington elasticity is, the smaller cross-country correlation of consumption is. As a result, consumption is less correlated than output across borders when the Armington elasticity is at about 0.4. This could be a possible explanation to the output-consumption correlation puzzle suggested by Backus et al. (1992). Thus, the sticky wage model accounts for international correlations of output-consumption in the data as well as the positive correlation of employment between countries if the Armington elasticity is low enough. Meanwhile, the cross-country correlations of output, consumption and employment drop significantly when the economy is close to the autarky economy ($\omega = 0$). This implies that strong trade linkages across countries can increase the synchronization of business cycles in lines with Cacciatore (2014).

As the real wage becomes more flexible in both countries, the correlation of output, consumption and employment between countries is relatively low, compared to the

[19]The parameter considered in each experiment is assumed symmetric.

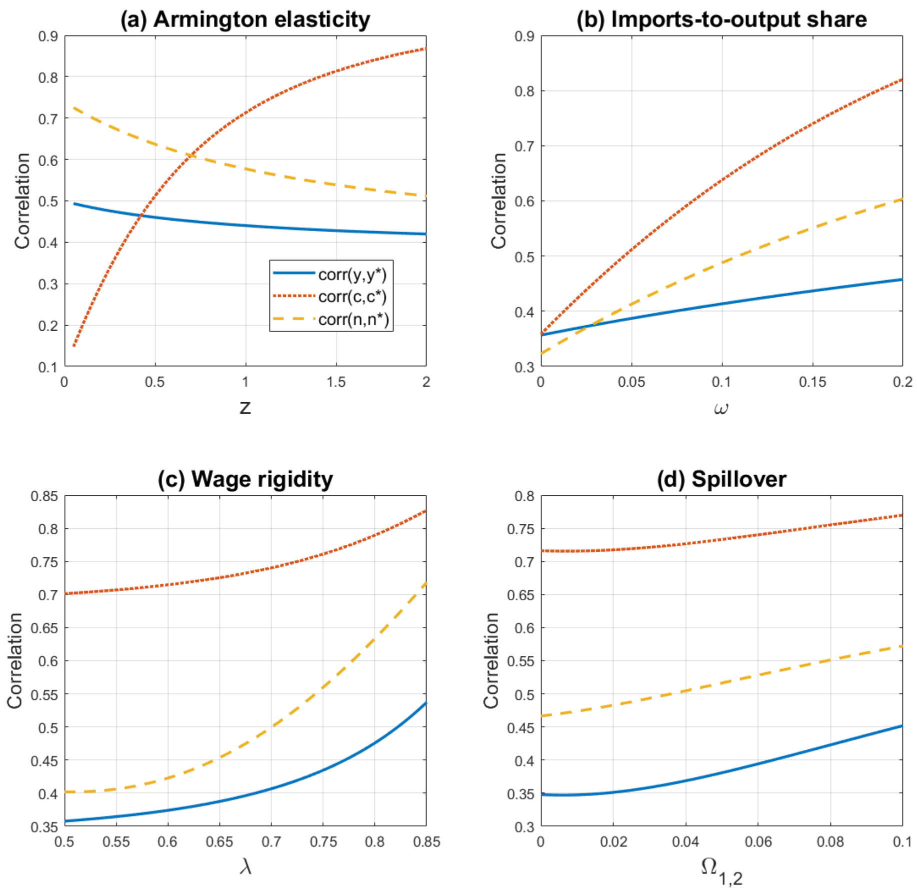

Figure 1.2 Sensitivity analysis

Note: This figure reports cross-country correlations of output, consumption and employment for various values of parameters. (a) Armington elasticity from 0.05 to 2. (b) Imports-to-output share between 0 and 0.2. (c) Hazard rate in labour markets from 0.5 to 0.85. (d) Cross-country spillovers of productivity from 0 to 0.1. Two countries are symmetric in terms of a parameter considered in each experiment.

benchmark model. A flexible real wage can lead to weaker effects of productivity shocks on employment. Intuitively, changes in productivity are absorbed into changes in wages more due to low rigidity of real wage. However, this experiment does not change the positive cross-country correlation of employment.

Finally, the output, consumption and employment are less correlated across countries as the spillover effects of productivity shocks are getting smaller. Due to this change in productivity process, the higher productivity in foreign economy in the future cannot be expected when a positive productivity shock occurs in home country. This, in turn, leads to low international co-movements. Although a positive country-specific productivity shock cannot spill over across countries, it can raise the demand of foreign goods in home country, which increases the foreign employment because of labor market frictions. Thus, the correlation of employment between countries is still positive.

1.4.4 Discussion: Sticky vs. Flexible wage

In this section, I address the importance of wage rigidity in the labor market friction model. This is because the flexible wage model also shows a positive cross-country correlation of employment. However, flexible wage model with search and matching frictions in labor market has a difficult to account for the behavior of wage and labor market activity within a country in terms of volatilities and persistence, which are observed in data. This section focuses on persistence problem in the flexible wage model since the drawback with volatilities is explored in the previous section.

Figure 1.3 shows the dynamics in the sticky and flexible wage model, respectively, when a positive standard deviation shock to productivity occurs in the home country. When wage is flexible the effect of positive productivity shock is offset to some extent by the rise in wage, and in turn the change in employment is relatively small compared to the case where wage is rigid. As a result, the hump-shaped dynamics of output and consumption, which are stylized facts, found in the sticky wage model are weaken due to the low persistence of the effect of the productivity shock. Table 1.3 reports autocorrelation of output, consumption, labor market variables in the data and models.[20] The sticky wage model addresses well the persistence of a shock found in the data, whereas the flexible wage model shows lower persistence of output, consumption and wage.

Table 1.3 Autocorrelation

	Output	Consumption	Employment	Wage
Data	0.88	0.88	0.95	0.87
Sticky wage model	0.81	0.83	0.90	0.95
Flexible wage model	0.73	0.74	0.91	0.73

Note: The statistics of the data row are for 1976:1 to 2015:4 using U.S. data. All statistics have been HP-filtered with a smoothing parameter of 1,600.

Thus, it is significant to assume wage rigidity along with search and matching frictions in the labor market to explain the labor market activity within a country as well as the international co-movement of employment.

[20]The data statistics are for U.S. data from 1976:1 to 2015:4.

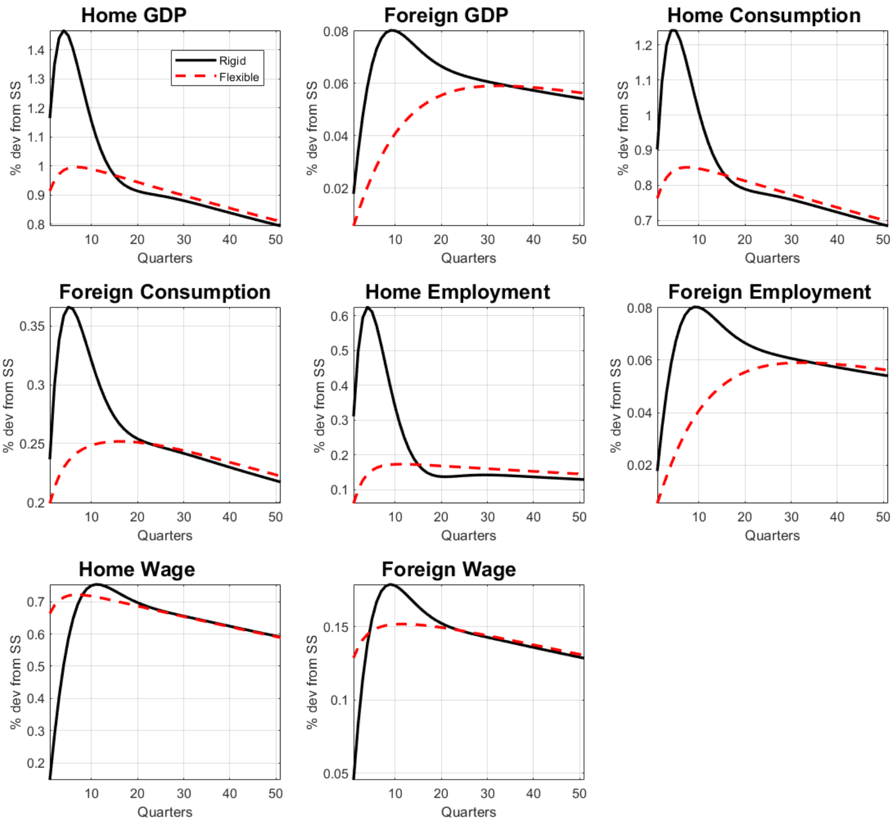

Figure 1.3 Sticky vs. Flexible wage model with a Home productivity shock of one s.d.

Note: The home (foreign) country is assumed as the U.S. (EU) economy. One period denotes a quarter on the horizontal axis and the percentage deviation from the steady state is depicted on the vertical axis.

1.5 Conclusion

In this paper, I incorporate the staggered multi-period wage bargaining proposed by Gertler & Trigari (2009) into a two-country general equilibrium model with labor market frictions. I examine quantitatively whether the model generates not only the international co-movement of employment, but also the volatility and the persistence of employment within a country in the data. I find the cross-country correlation of employment in the model is positive which is consistent with the data when considering productivity shocks. This main result is still robust when changing parameters such as Armington elasticity, imports-to-output share, wage rigidity and spillovers of productivity shocks.

Results of this paper are caused by characteristics of labor markets. With search and matching frictions, productivity shocks influence on employment in both countries in the same direction as the result of matching frictions, which leads to the positive international co-movement of employment.

Chapter 2

Optimal monetary policy in the open economy with labor market frictions

2.1 Introduction

One of traditional research topics in the international macroeconomics is the conduct of optimal monetary policy. However, studies based on a Walrasian labor markets do not give any explicit implication related to unemployment that is one of important economic factors. In this context, this research examines a Ramsey-type optimal monetary policy in an open economy with a two-country dynamic general equilibrium model where search and matching frictions exist in labor markets along with the limited participation in the financial markets. As the limited financial markets participation plays a role of monetary transmission mechanism, monetary policy affects the aggregate real variables such as output, consumption, and employment, by changing the decision of agents. There are two main results associated with optimal monetary policy in this paper. The long-run optimal nominal interest rate is zero suggesting deflation because the terms of trade effect on consumption is weaker by search and matching frictions in the labor market. As a result of the Ramsey optimal monetary policy, dynamics of business cycles in both countries show similar patterns in response to productivity shocks and, in turn, higher cross-country correlations of real variables.

To analyze the Ramsey-optimal monetary policy under labor market frictions, I consider a standard, two-country and two-good model with flexible prices. Each country specializes in the production of a single good which is produced by using labor as sole input. The model introduces search and matching frictions in labor markets, following Mortensen & Pissarides (1994). Furthermore, it is assumed that households' consumption is subject to a cash-in-advance constraint, which is consistent with a limited participation assumption, as well as that firms finance wages with working capital before their production as in Christiano et al. (1997). These specifications suggest a monetary transmission mechanism under the circumstances where prices are flexible because firms' costs could be affected by the monetary policy.

With the model, I derive the Ramsey-optimal monetary policy under commitment. Under the assumption of Ramsey-optimal monetary policy, the monetary authority maximizes the welfare of domestic households, considering the competitive equilibrium conditions as constraints. As the monetary authorities in the home and the foreign countries determine their own policies taking the monetary policy of counterpart as given, the optimal monetary policy is conducted non-cooperatively.

Based on the model, I then explore the long-run optimal monetary policy that the policy maker in each country pursues to achieve. I find a zero nominal interest rate is optimal in long run different from Cooley & Quadrini (2003). They suggest two effects related to monetary transmission mechanism, that is, financing cost effect and terms of trade effect. A monetary contraction leads to a lack of liquidity for working capital, inducing a rise in the nominal interest rate. A higher interest rate decreases aggregate output by increasing costs of production. This is the former effect, implying the Friedman rule is the optimal monetary policy. The terms of trade effect which exists only in an open economy leads to the long-run inflation rate bias, that is, the higher nominal interest rates could improve the terms of trade and in turn, increase consumption. However, when taking into account labor market frictions, a higher interest rate decreases vacancies posted as well as employment, which means decreased costs of job posting. Thus, the terms of trade effect should be weak under search and matching frictions.

Next, I compute responses of macroeconomic variables with respect to a positive country-specific productivity shock with calibrated parameters. An increase in home productivity induces a rise in demand for home goods as well as increased profits of firms. This entails a rise in demand for labor and, in turn, the increased demand of loans for the wage bills in the home country. Under the Ramsey-optimal monetary scheme with labor market frictions, the home monetary authority operates a contractionary monetary policy which in turn induces a rise in inflation. As a result, output, consumption and employment in the home country increases. An increase in home productivity leads to a rise in output, consumption and employment in the foreign country because an increase in home consumption means a higher demand of foreign goods as well as home goods.

Finally, I examine whether the search and matching model under Ramsey optimal monetary policy generates the properties of business cycles across countries when productivity shocks are the source of uncertainty in the economy. The conventional international real business cycle model has a problem with accounting for international co-movements of employment and output, generating negative cross-country correlations. This result is inconsistent with data. However, the model shows positive cross-country correlations of output and employment, which is consistent with data. Moreover, the model seems to reproduce properties of business cycles in data well even though the figures of statistics are less than those in data. However, the model fails to generate enough volatility of employment relative to output within a country.

This paper is associated with two strands of literature. It is related to papers that explore optimal monetary policy with search and matching frictions in labor markets in the closed economy. Thomas (2008) incorporates a New Keynesian model with labor search models and wage rigidity, and studies optimal monetary policy. He shows that optimal policy is deviations from price stability when wages are rigid, whereas stabilizing prices is optimal in the absence of wage rigidity. Faia (2009) also analyzes optimal monetary policy in search frictions along with sticky prices. Instead of imposing the Hosios (1990) efficiency condition as in Thomas (2008), she suggests that optimal inflation volatility increases with workers' bargaining power. Blanchard & Galí (2010) present a simpler framework that integrates New Keynesian approach with search frictions. They demonstrate real wage rigidity generates a case against price stability. While Ravenna & Walsh (2011) derive the objective function of monetary policy by using second-order approximation, Sunakawa (2015) studies optimal monetary policy by introducing right-to-manage bargaining instead of Nash bargaining.

The other set of literature where this study can contribute is open economy studies considering labor market frictions. One of the closest studies to this paper is Cacciatore & Ghironi (2021). They address the effects of trade integration for the conduct of monetary policy in a two-country model with labor market frictions, endogenous entry of firms, and sticky prices and wages. Their analysis shows the need of positive inflation targeting decreases as trade is more integrated, focusing on the cooperative optimal monetary policy. Different from their approch, this paper concentrates on the non-cooperative monetary policy with the limited participation and flexible prices and wages. Hairault (2002) addresses the observed international fluctuations by introducing labor search frictions into the two-country real business cycle model. He suggests a resolution of the international co-movement puzzle by generating a positive cross-country correlation of employment in the model without real wage rigidity.

Christiano et al. (2011) account for the effects of a monetary tightening by using a small open economy model with financial and labor market frictions. They find the model considering both financial and labour market frictions expects inflation and nominal interest rates much better than simpler models which take into account either frictions of those. Campolmi & Faia (2015) also assess the design of the optimal exchange rate and currency regimes in presence of frictional labor markets using a two-country model.

The remainder of this paper is organized as follows. Section 2.2 introduces a two-country, two-good model with labor market frictions, followed by quantitative results as well as the long-run optimal monetary policy in section 2.3. The final section concludes.

2.2 The Model

This section outlines the model economy.[1] The economy consists of two countries, home and foreign country, which are specialized in producing single goods in each country. The aggregate labor force in each country is normalized as one. Each country is comprised of households and firms. Households, uniformly distributed between 0 and 1, supply labor to firms, deposit cash with perfectly competitive financial intermediaries, and consume domestic and imported goods.

Firms, distributed on a unit interval, produce consumption goods by using labor as their sole input and sell them to either home or foreign households. Each market for products is assumed as perfectly competitive. Firms finance wages with loans from financial intermediaries before production occurs. Labor is not allowed to move across borders in this model. The monetary authority is assumed to control the money supply and injects money into the economy with lump-sum transfers, via financial intermediaries.

Consumption, output, and prices of home and foreign goods of the home household is denoted with a subscript h and f, respectively. A superscript asterisk, $*$, denotes foreign country variables. While prices in this paper denote nominal prices, all prices are flexible. In what follows, the home country is focused on in the exposition of the model, whereas analogous expressions hold for the foreign country.

[1] The structure of the model is similar to Chapter 1.

2.2.1 Households

Each home and foreign economy consists of a large number of identical households. The representative household has a continuum of members, who are either employed or unemployed. Members currently unemployed are searching for jobs. The number of currently employed members in the representative household is n_t that is determined through the search and matching process. The representative household is assumed as an extended family, following Merz (1995). This assumption provides full consumption insurance between employed and unemployed members since all members gather their income and consume the same amount.

The representative household consumes home and foreign goods and considers all home goods (or foreign goods) as perfect substitutes. Furthermore, workers are assumed to provide same hours worked in each period once they are employed, as in the conventional search and matching literature. Given n_t unchanged, the representative household cannot vary labour supply by changing hours worked in the model. The representative household maximizes its expected life-time utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln c_t, \tag{2.1}$$

where $\beta \in (0, 1)$ is the discount factor, c_t is a consumption basket of the representative household at period t. The preference of the representative household in the home country over domestic and foreign goods is expressed by the Armington aggregator. The consumption basket of the home household is given by

$$c_t = \left[\omega^{1/z} c_{h,t}^{(z-1)/z} + (1-\omega)^{1/z} c_{f,t}^{(z-1)/z} \right]^{z/(z-1)}, \tag{2.2}$$

where $\omega \in (0,1)$ denotes measure of openness and $z > 0$ is the elasticity of substitution between home and foreign goods. When ω is larger than a half, consumption of the home household has the home-biased property. In contrast, the consumption indexes of domestic and imported products for the home agent is written as

$$c_{h,t} = \int_0^1 \left[c_{h,t}(i) \right] di, \; c_{f,t} = \int_0^1 \left[c_{f,t}(i^*) \right] di^*, \tag{2.3}$$

where $i \in [0,1]$ denotes an individual firm in the home country. Thus, consumption of the representative household in home country can be characterized by the following demand curves.

$$c_{h,t} = \omega \left[\frac{P_{h,t}}{P_t}\right]^{-z} c_t, c_{f,t} = (1-\omega)\left[\frac{P_{f,t}}{P_t}\right]^{-z} c_t \tag{2.4}$$

While the prices of home goods and imported goods in terms of home currency are denoted as $P_{h,t}$ and $P_{f,t}$, respectively, the consumer price index of the home country is P_t. The aggregate consumer price index can be derived from the consumption basket.

$$P_t = \left[\omega P_{h,t}^{1-z} + (1-\omega) P_{f,t}^{1-z}\right]^{1/(1-z)} \tag{2.5}$$

At the beginning of each period, the household holds money stock, M_t, on hand and decides how much cash it deposits with domestic financial intermediaries. The remaining cash is allocated for consumption. Once the decision of deposits is made, the monetary authority injects money into financial intermediaries. Note that the household cannot amend its deposit in response to any circumstances within the period. In each period, thus, the decision of the household is subject to a cash-in-advance constraint

$$P_t c_t \leq M_t - D_t, \tag{2.6}$$

along with the following budget constraint,

$$P_t c_t + M_{t+1} \leq M_t + (R_t - 1) D_t + P_t w_t n_t + (1 - n_t) P_t b + R_t X_t + \Pi_t - T_t, \tag{2.7}$$

where X_t is money injection of the monetary authority into financial intermediaries at the beginning of period t.[2] Employed members of the representative household earn wages, $P_t w_t$, whereas unemployed workers get unemployment benefits, $P_t b$. The household deposits money, D_t, at the start of period t, and then receives $R_t D_t$ at the end of period t, where R_t is the gross interest rate. Furthermore, households obtain an additional income from retailers, Π_t, and financial intermediaries, $R_t X_t$, due to the ownership of them. Finally, T_t denotes the lump-sum taxes used to finance unemployed benefits.

The representative household maximizes its expected life-time utility (Equation (2.1)) subject to the above two constraints. Thus, the first-order condition is given as

$$\mathbb{E}_{t-1}\left[\beta R_t \frac{c_t}{c_{t+1}} \frac{P_t}{P_{t+1}}\right] = 1. \tag{2.8}$$

[2] The timeline of events follows that of the limited participation model in Christiano et al. (1997).

This is the Euler equation of consumption. Note that the expectations operator in the equation is $\mathbb{E}_{t-1}$, instead of $\mathbb{E}_t$, as in Christiano et al. (1997). This is because savings are determined in advance and cannot be readjusted within the period. Thus, this Euler equation also implies the limited participation of households in the financial markets.

2.2.2 Matching process

Each firm posts vacancies, $v_t(i)$, and the total number of vacancies is $v_t = \int_0^1 v_t(i)di$. Each firm i also employs $n_t(i)$ workers and the aggregate employed worker is $n_t = \int_0^1 n_t(i)di$. All unemployed workers at period t are assumed to look for jobs. The pool of searching workers at period t is given by the difference between the aggregate labor force and the number of employed workers at the end of period $t-1$:

$$u_t = 1 - (1-\rho)n_{t-1}, \tag{2.9}$$

where ρ represents an exogenous job separation rate.

The search and matching process in the labour market follows the conventional model presented by Mortensen & Pissarides (1994). Firms post vacancies to hire workers and unemployed workers seek jobs passively. While each firm is assumed to have a job which can either be filled or vacant, workers are considered to be employed or unemployed. The number of new hired workers is expressed as the following Cobb-Douglas matching function:

$$m_t = \sigma_m u_t^{\sigma} v_t^{1-\sigma}, \qquad 0 < \sigma < 1 \tag{2.10}$$

where σ_m denotes the efficiency of the matching process. The probability that any vacancy is matched, j_t, is expressed as

$$j_t = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma}, \tag{2.11}$$

where $\theta_t \equiv v_t/u_t$ denotes the labor market tightness.

Similarly, the probability that an unemployed worker finds a job, s_t, is given by

$$s_t = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma}. \tag{2.12}$$

Both firms and workers take the vacancy filling probability, j_t, and the job finding probability, s_t, as given.

2.2.3 Firms

There is a continuum of competitive firms that produce consumption goods. A firm i employs $n_t(i)$ workers to produce its product $y_t(i)$ in every period. The production function which is characterized by constant returns to scale is written as

$$y_t(i) = a_t n_t(i), \tag{2.13}$$

where a_t is a common productivity factor among all firms within a country.

It is assumed that newly hired workers go to work immediately, following Blanchard & Galí (2010). Accordingly, the workforce of a firm can be divided two types of workers who are either employed in the past or hired in the current period. The total workforce is the sum of the number of surviving workers, $(1-\rho)n_{t-1}(i)$, and the number of new employed workers, $j_t v_t(i)$. Thus, employment of a firm i evolves according to

$$n_t(i) = (1-\rho)n_{t-1}(i) + j_t v_t(i). \tag{2.14}$$

Meanwhile, firms pay additional costs, $P_t\kappa$, for each job posting, $v_t(i)$, as the hiring takes time and costs. Since firms use working capital loans to finance wage bills, real marginal cost of labor should be equal to $R_t w_t(i)$, not $w_t(i)$. With considering the hiring costs, the flow real profits of a firm, $\Pi_t(i)$, are expressed as

$$\Pi_t(i) = \frac{P_{h,t}}{P_t} y_t(i) - R_t w_t(i) n_t(i) - \kappa v_t(i), \tag{2.15}$$

where $w_t(i)$ is the real wage at period t. Note that nominal interest rates, R_t, have an impact on the real profits of the firm directly through the cost channel, as in Ravenna & Walsh (2011).[3] This is because when the nominal interest rate rises the financial costs for wage bills increase.

The firm wants to maximize its real profits by choosing $y_t(i), v_t(i)$ and $n_t(i)$, taking $P_{h,t}, w_t(i), P_t$ and R_t as given:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\frac{\lambda_{t+s}}{\lambda_t}\right) \left[\frac{P_{h,t+s}}{P_{t+s}} y_{t+s}(i) - R_{t+s} w_{t+s}(i) n_{t+s}(i) - \kappa v_{t+s}(i)\right] \tag{2.16}$$

[3] Ravenna & Walsh (2011) address a cost channel occurs when a firm's marginal cost is directly dependent on the interest rate.

subject to

$$\begin{aligned} n_{t+s}(i) &= (1-\rho)n_{t+s-1}(i) + j_{t+s}v_{t+s}(i) \\ y_{t+s}(i) &= a_{t+s}n_{t+s}(i), \end{aligned}$$

where λ_t denotes the marginal utility of consumption. The first-order condition with respect to $v_t(i)$ is given by

$$\frac{\kappa}{j_t} = \left[\frac{P_{h,t}}{P_t}a_t - R_t w_t(i)\right] + (1-\rho)\mathbb{E}_t\beta_{t,t+1}\frac{\partial \Pi_{t+1}(i)}{\partial n_{t+1}(i)}. \tag{2.17}$$

The terms $\beta_{t,t+1}$ denotes the firm's common discount rate between period t and $t+1$, which implies $\beta_{t,t+1} = \beta c_{t+1}^{-1}/c_t^{-1}$.[4] The above result can be rewritten by using the envelope theorem for $\partial \Pi_t(i)/\partial n_t(i)$:

$$\frac{\kappa}{j_t} = \left[\frac{P_{h,t}}{P_t}a_t - R_t w_t(i)\right] + (1-\rho)\mathbb{E}_t\beta_{t,t+1}\frac{\kappa}{j_{t+1}}. \tag{2.18}$$

This is a job creation condition of a firm i. Since all firms are symmetric in the equilibrium, we can drop the subscript (i):

$$\frac{\kappa}{j_t} = \left[\frac{P_{h,t}}{P_t}a_t - R_t w_t\right] + (1-\rho)\mathbb{E}_t\beta_{t,t+1}\frac{\kappa}{j_{t+1}}. \tag{2.19}$$

The job creation condition also can be expressed by the labor market tightness by taking into account the relation between the vacancy filling probability and the labor market tightness:

$$\frac{\kappa}{\sigma_m}\theta_t^{\sigma} = \left[\frac{P_{h,t}}{P_t}a_t - R_t w_t\right] + (1-\rho)\mathbb{E}_t\beta_{t,t+1}\frac{\kappa}{\sigma_m}\theta_{t+1}^{\sigma}. \tag{2.20}$$

While the left-hand side of the Equation (2.20) is the cost of hiring a worker, the right-hand side represents the current marginal profit of the firm (in the bracket) and the continuation value of the match. Thus, the job creation condition implies that the expected cost of hiring a worker should be equal to the expected value of a match. Note that the nominal interest rates affect the job creation by changing current marginal profit of the firm. If the interest rate rises, the expected value of a match declines due to a fall of the current marginal profit. According to the above condition, the firm posts less vacancies and in turn this makes the labor market tightness lower. Therefore, the nominal interest rates have a negative relationship with the job creation.

[4] For consistency, $\beta_{t,t}$ is defined as 1.

2.2.4 Wage bargaining

The model assumes that the real wage is determined so that the firm and the marginal worker share the aggregate surplus from the marginal match by negotiating real wages with Nash bargaining process. The standard Nash bargaining problem is expressed as

$$\max_{w_t} \left(S_t^H\right)^{\eta} \left(S_t^F\right)^{1-\eta}, \tag{2.21}$$

where S_t^H and S_t^F denote worker's surplus and firm's surplus from a match, respectively. $\eta \in (0,1)$ denotes workers' bargaining power. Note that we can drop the subscript (i), with the symmetricity of all firms in the equilibrium and perfect competition. Thus, the optimal sharing rule from the first-order condition with respect to w_t is written as

$$\eta S_t^F = (1-\eta) S_t^H. \tag{2.22}$$

Before analyzing the wage bargaining process, it is necessary to define worker's surplus and firm's surplus from having an additional employment. I define S_t^H as a worker's surplus:

$$S_t^H \equiv V_t - U_t, \tag{2.23}$$

where V_t and U_t denote the value of employment to a worker and the value of unemployment, respectively. The value of employment to a worker, V_t, and the value of unemployment, U_t, are defined as

$$V_t = w_t + \mathbb{E}_t \beta_{t,t+1} \left[(1-\rho)V_{t+1} + \rho s_{t+1} V_{t+1} + \rho (1-s_{t+1}) U_{t+1}\right] \tag{2.24}$$

$$U_t = b + \mathbb{E}_t \beta_{t,t+1} \left[s_{t+1} V_{t+1} + (1-s_{t+1}) U_{t+1}\right]. \tag{2.25}$$

The first term of the Equation (2.24) represents the current wage that an employed worker obtains, whereas the remaining terms stand for the value of employment in the next period. The value of employment in the future is divided into three circumstances: the matched job continues in the next period ($(1-\rho)V_{t+1}$), the worker finds a new job after job separation ($\rho s_{t+1} V_{t+1}$), and the worker remains unemployed due to job severance ($\rho(1-s_{t+1})U_{t+1}$). Similarly, the value of unemployment in the next period is sum of the value of finding a job and the value of remaining unemployed as in Equation (2.25).

Thus, the worker's surplus, S_t^H, can be rewritten as

$$S_t^H = w_t - b + \mathbb{E}_t \beta_{t,t+1} \left[(1-\rho) S_{t+1}^H - (1-\rho) s_{t+1} S_{t+1}^H\right]. \tag{2.26}$$

Let S_t^F be the firm's surplus from having an additional worker. The firm's surplus is obtained by differentiating the value of employment to a firm (Π_t) with respect to an additional worker (n_t):

$$S_t^F \equiv \frac{\partial \Pi_t}{\partial n_t} = \frac{P_{h,t}}{P_t} a_t - R_t w_t + (1-\rho)\mathbb{E}_t \beta_{t,t+1} S_{t+1}^F. \tag{2.27}$$

After substituting (2.27) and (2.28) in the optimal sharing rule, the wage determination condition can be derived:

$$w_t = \frac{1}{\eta R_t + (1-\eta)} \left[\eta \frac{P_{h,t}}{P_t} a_t + (1-\eta) b + \eta \mathbb{E}_t \beta_{t,t+1} (1-\rho) \kappa \theta_{t+1} \right]. \tag{2.28}$$

This equation shows that the real wage depends on the nominal interest rates as well as aggregate productivity, relative price of the product, unemployment benefits, and labor market tightness. It is increasing in aggregate productivity and relative price because demand for labor rises when values of those variables increase. A rise in unemployment benefits leads to higher wages with making the value of unemployment increase. As an increasing future labor market tightness implies a higher expected labor demand, the real wage is also increasing in the labor market tightness. Different from the conventional search and matching model, the real wage is affected by the nominal interest rates as well. This comes from the fact that the firm take into account the cost of an additional labor as $R_t w_t$ instead of w_t. Thus, a higher nominal interest rate causes a lower real wage due to a fall in labor demand.

2.2.5 Financial intermediaries

Financial intermediaries in the home country collect deposits from domestic households and obtain money injection from the monetary authority. They lend them to firms in the home country and take interest revenues. At the end of the period, they give households $R_t D_t$ as the reward of their savings and $R_t X_t$ as dividends.

2.2.6 Monetary Authority and Government

The monetary authority transfers money, X_t, to financial intermediaries at the beginning of period t.

$$X_t = M_{t+1} - M_t \tag{2.29}$$

I define the growth rate of money as the ratio of the stock of money between consecutive periods:

$$x_t \equiv \frac{M_{t+1}}{M_t} - 1. \tag{2.30}$$

Moreover, the monetary authority is assumed to control the growth rate of money, x_t, as a policy instrument. Thus, optimal monetary policy is associated with selecting a sequence of the growth rate of money, $\{x_t\}_{t=0}^{\infty}$. Furthermore, it is possible to derive a specific relation between the nominal interest rate and the growth rate of money that is monetary policy instrument, using the cash-in-advance constraint:

$$R_t = \frac{M_{t+1}}{D_t + X_t} - 1 = \frac{1-d_t}{d_t + x_t}, \tag{2.31}$$

where $d_t \equiv D_t/M_t$.[5] As the deposit is predetermined, the growth rate of money determines the nominal interest rate of the home country.

Meanwhile, the government imposes a lump-sum tax on households to finance the unemployment benefit. The budget constraint of the government is given by

$$T_t = (1-n_t)P_t b. \tag{2.32}$$

This equation implies that there is no additional government spending apart from unemployment benefits in the model.

2.2.7 International relative prices

I assume that there is not international mobility of financial assets, i.e. financial autarky, and the trade account should always be balanced. Thus, the value of exported goods in terms of domestic currency should be equal to the value of imported goods in terms of domestic currency:

$$e_t P_{h,t}^* c_{h,t}^* = P_{f,t} c_{f,t}, \tag{2.33}$$

where e_t denotes the nominal exchange rate which is the price of foreign currency in terms of domestic currency.

I now define three new variables related to international relative prices. First, the terms of trade for the home country which is a relative price of imports to exports is expressed as

$$TOT_t \equiv P_{f,t}/P_{h,t}. \tag{2.34}$$

[5] The complete derivation is in the Appendix.

Second, the ratio of the consumer price index to the domestic price index is defined as

$$\Phi_t \equiv \frac{P_t}{P_{h,t}}, \; \Phi_t^* \equiv \frac{P_t^*}{P_{f,t}^*}. \tag{2.35}$$

Finally, the real exchange rate, q_t, measures the price of foreign output relative to the price of home output:

$$q_t \equiv \frac{e_t P_t^*}{P_t}. \tag{2.36}$$

With these three new variables, it is possible to derive following relations among them.

$$q_t = \Phi_t^* \Phi_t^{-1} TOT_t \tag{2.37}$$

$$\Phi_t^{1-z} = \omega + (1-\omega) TOT_t^{1-z} \tag{2.38}$$

$$\Phi_t^{*1-z} = \omega + (1-\omega) TOT_t^{z-1} \tag{2.39}$$

Furthermore, it is possible to derive the relationship between home and foreign consumption from the balanced trade assumption (Equation (2.33)) using above definitions. Considering the demand of home products and international relative prices, values of exported goods and of imported goods in Equation (2.33) can be rewritten as:

$$\begin{aligned} e_t P_{h,t}^* c_{h,t}^* &= (1-\omega) \left[\frac{P_{h,t}}{P_t}\right]^{-z} q_t^z P_{h,t} c_t^*, \\ P_{f,t} c_{f,t} &= (1-\omega) \left[\frac{P_{f,t}}{P_t}\right]^{-z} P_{f,t} c_t. \end{aligned} \tag{2.40}$$

Under the balanced trade assumption, the above two values should be equal and in turn, the following relation between home and foreign consumption can be derived:

$$c_t^* = \Phi_t^{1-z} \Phi_t^{*z-1} q_t^{1-2z} c_t. \tag{2.41}$$

According to the equation, foreign consumption depends on relative prices and the real exchange rate as well as consumption of the home country.

2.2.8 Market clearing

The loans from the financial intermediaries in each country must be sufficient to cover the borrowing needs of domestic firms:

$$P_t w_t n_t = D_t + X_t. \tag{2.42}$$

Table 2.1 Model Summary

	Home	Foreign
Euler eq.	$\beta E_{t-1} R_t \frac{c_t}{c_{t+1}} \frac{1}{\pi_{t+1}} = 1$	$\beta E_{t-1} R_t^* \frac{c_t^*}{c_{t+1}^*} \frac{1}{\pi_{t+1}^*} = 1$
Resources	$a_t n_t = \omega \Phi_t^z c_t + (1-\omega) \Phi_t^z q_t^z c_t^* + \Phi_t \kappa v_t$	$a_t^* n_t^* = \omega \Phi_t^{*z} c_t^* + (1-\omega) \Phi_t^{*z} q_t^{-z} c_t + \Phi_t^* \kappa v_t^*$
Tightness	$\theta_t = \frac{v_t}{1-(1-\rho) n_{t-1}}$	$\theta_t^* = \frac{v_t^*}{1-(1-\rho) n_{t-1}^*}$
Employment	$n_t = (1-\rho) n_{t-1} + \sigma_m \theta_t^{-\sigma} v_t$	$n_t^* = (1-\rho) n_{t-1}^* + \sigma_m \theta_t^{*-\sigma} v_t^*$
Job creation	$\frac{\kappa}{\sigma_m} \theta_t^{\sigma} = \Phi_t^{-1} a_t - R_t w_t$	$\frac{\kappa}{\sigma_m} \theta_t^{*\sigma} = \Phi_t^{*-1} a_t^* - R_t^* w_t^*$
	$+(1-\rho) E_t \beta_{t,t+1} \left(\frac{\kappa}{\sigma_m} \theta_{t+1}^{\sigma} \right)$	$+(1-\rho) E_t \beta_{t,t+1} \left(\frac{\kappa}{\sigma_m} \theta_{t+1}^{*\sigma} \right)$
Wage	$w_t = \frac{1}{\eta R_t + (1-\eta)} [\eta \Phi_t^{-1} a_t + (1-\eta) b$	$w_t^* = \frac{1}{\eta R_t^* + (1-\eta)} [\eta \Phi_t^{*-1} a_t^* + (1-\eta) b^*$
	$+\eta (1-\rho) E_t \beta_{t,t+1} \kappa \theta_{t+1}]$	$+\eta (1-\rho) E_t \beta_{t,t+1} \kappa \theta_{t+1}^*]$
Rel. prices	$\Phi_t^{1-z} = \omega + (1-\omega) TOT_t^{1-z}$	$\Phi_t^{*1-z} = \omega + (1-\omega) TOT_t^{z-1}$
Real ex. rate	$q_t = \Phi_t^* \Phi_t^{-1} TOT_t$	
Trade	$c_t^* = \Phi_t^{1-z} \Phi_t^{*z-1} q_t^{1-2z} c_t$	

This is the loan market clearing condition. Furthermore, the lending and saving rates are equal in the financial markets because perfect competition is assumed in the loan market.

Meanwhile, goods produced in the home country are consumed by domestic households, foreign households or firms for posting vacancies. The equilibrium in the home goods market requires the following condition:

$$P_{h,t} y_t = P_{h,t} c_{h,t} + e_t P_{h,t}^* c_{h,t}^* + P_t \kappa v_t. \tag{2.43}$$

Using demand curves, the goods market clearing condition can be rewritten as

$$y_t = \omega \Phi_t^z c_t + (1-\omega) \Phi_t^z q_t^z c_t^* + \Phi_t \kappa v_t. \tag{2.44}$$

The equation implies that home production can be affected by the foreign demand as well as the domestic demand.

The main conditions in the model are summarized in Table 2.1. As the deposit is decided at the start of each period, the growth rate of money determines the nominal interest rate in each country. Given nominal interest rates, and exogenous process of aggregate productivity, a_t and a_t^*, the equations in Table 2.1 determine the equilibrium of the model.

2.3 Optimal monetary policy

I describe the specification of Ramsey-optimal monetary policy under commitment in this section. Since I also take into account the non-cooperative optimal monetary policy, the home monetary authority takes the foreign monetary policy as given. The monetary authority maximizes the welfare of domestic households, considering the competitive equilibrium conditions as constraints.

Thus, the Ramsey problem of the home country is choosing control variables

$$\{c_t, c_t^*, n_t, n_t^*, v_t, v_t^*, \theta_t, \theta_t^*, \Phi_t, \Phi_t^*, \pi_{t+1}, \pi_{t+1}^*, q_t, TOT_t, R_t\}_{t=0}^{\infty}$$

along with plans for Lagrangian multipliers related to the equilibrium conditions to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t$$

subject to equilibrium conditions in Table 2.1, taking $\{x_t^*\}_{t=0}^{\infty}$ as given.[6] As the growth rate of money is a one-to-one relationship with the nominal interest rate, it is possible to take $\{R_t^*\}_{t=0}^{\infty}$ as given. The foreign monetary authority solves an analogous maximization problem, taking $\{R_t\}_{t=0}^{\infty}$ as given. Thus, home and foreign countries set the non-cooperative optimal monetary policies each other.

2.3.1 Long-run optimal policy

Before analyzing the optimal monetary policy in response to shocks, I characterize the long-run optimal policy. To develop analytical results with the model, I begin by assuming the elasticity of substitution between home and foreign goods equal to one ($z = 1$).

Returning to the Ramsey problem, let us focus on the following first-order conditions with respect to employment (n_t), the vacancy (v_t), and the labor market tightness (θ_t) which are real variables in the domestic labor market:

$$\begin{aligned} \delta_{2,t} a_t + \beta \mathbb{E}_t \delta_{3,t+1} (1-\rho)\theta_t - \delta_{4,t} + \beta \mathbb{E}_t \delta_{4,t+1} (1-\rho) &= 0 \\ -\delta_{2,t} \Phi_t \kappa + \delta_{3,t} + \delta_{4,t} \sigma_m \theta_t^{-\sigma} &= 0 \\ \delta_{3,t} [1 - (1-\rho) n_{t-1}] + \delta_{4,t} \sigma \sigma_m \theta_t^{-\sigma-1} v_t &= 0, \end{aligned} \tag{2.45}$$

[6] The complete derivation is in the Appendix.

where $\delta_{2,t}$, $\delta_{3,t}$, and $\delta_{4,t}$ denote Lagrangian multipliers for the resource constraint, the definition of labor market tightness, and the evolution of employment, respectively. To obtain the long-run optimal policy, I evaluate above first-order conditions at the steady state. The following equations hold at the steady state:

$$\begin{aligned} \delta_2 + \delta_3\beta(1-\rho)\theta - \delta_4\left[1-\beta(1-\rho)\right] &= 0 \\ -\delta_2\Phi\kappa + \delta_3 + \delta_4\sigma_m\theta^{-\sigma} &= 0 \\ \delta_3\left[1-(1-\rho)n\right] + \delta_4\sigma\sigma_m\theta^{-\sigma-1}v &= 0. \end{aligned} \tag{2.46}$$

When solving the system of equations in terms of Lagrangian multipliers, I write the optimality condition as

$$\left[1-\beta(1-\rho)\right]\frac{\kappa}{\sigma_m}\theta^{\sigma} = (1-\sigma)\Phi^{-1} - \sigma\beta(1-\rho)\kappa\theta. \tag{2.47}$$

Meanwhile, the labor market equilibrium condition at the steady state can be derived by combining the job creation condition (Equation(2.20)) and the wage decision condition (Equation(2.28)):

$$\left[1-\beta(1-\rho)\right]\frac{\kappa}{\sigma_m}\theta^{\sigma} = \Phi^{-1} - \frac{R}{\eta R+(1-\eta)}\left[\eta\Phi^{-1}+(1-\eta)b+\eta\beta(1-\rho)\kappa\theta\right]. \tag{2.48}$$

Comparing this equilibrium condition with the optimality condition, we can find the fact that those two conditions could be identical if 1) a worker's bargaining power (η) is equal to the elasticity of the matching function with respect to unemployment (σ), i.e. $\eta = \sigma$, 2) the unemployment benefit is zero ($b = 0$) and 3) the steady state of the gross nominal interest rate is equal to one ($R = 1$). First two conditions are associated with labor market efficiency condition suggested by Hosios (1990). This means that a worker is fully compensated through the wage for positive externalities that she creates for firms when there is not any policy distortion. Accordingly, the optimal gross nominal interest rate in the long-run could be one when the Hosios (1990) condition is satisfied. This implies the optimal inflation rate in the long-run should be negative but near zero.

Suppose unemployment benefit is not zero ($b > 0$). To make the right-hand sides of both Equation (2.47) and Equation (2.48) equal, the optimal gross interest rate is expressed as

$$R = \frac{1}{1+b/\zeta}, \quad \zeta \equiv \sigma\left[\Phi^{-1}+\beta(1-\rho)\kappa\theta\right]. \tag{2.49}$$

This suggests the optimal gross nominal interest rate is less than one because unemployment benefit, b, is assumed to have a positive value. This is because the positive unemployment benefit leads to a higher real wage and, in turn a lower employment. Thus, the policy maker would like to set a lower interest rate to increase the labor demand. However, this interest rate cannot be achieved due to the zero lower bound for the nominal interest rates. Instead, policy maker would set the long-run interest rate to one.

This result is different from what Cooley & Quadrini (2003) suggest. They derive positive long-run nominal interest and inflation rates with a standard two country model considering limited participation. According to them, a rise in the nominal interest rates has two effects which are the liquidity effect and the terms of trade effect. The former effect is same as in the closed economy. A higher interest rate decreases aggregate consumption of the home country by increasing costs of production. The latter, however, is the effect that exists only in an open economy. In Cooley & Quadrini (2003), the terms of trade effect leads to the long-run inflation rate bias, that is, the higher nominal interest rates could improve the terms of trade and in turn, increase consumption.

The difference between their results and this study could come from search and matching frictions in labor markets. To make it clear, let us express to the terms of trade of the home country at the steady state by using Equation (2.37) and Equation (2.41):

$$TOT = q\frac{\Phi}{\Phi^*} = \frac{\Phi c}{\Phi^* c^*} = \frac{n - \Phi \kappa v}{n^* - \Phi^* \kappa v^*}. \tag{2.50}$$

The last equality is satisfied as the elasticity of substitution between home and foreign goods equal to one. Equation (2.50) shows that terms of trade in the model is decided by employment and vacancies. Without search and matching frictions, an increase in the domestic interest rates causes a fall in employment in the home country under foreign variables unchanged. As a result, the terms of trade would be improved and home consumption would increase. In contrast, when taking into account labor market frictions, a higher interest rate decreases vacancies posted as well as employment. As can be seen from Equation (2.50), this makes not only the first term of the numerator (n) fall, but also the second term, i.e. the costs of job posting, decrease. Thus, the terms of trade effect should be weakened under the search and matching friction. When it is considered that the relative price of home goods is larger than one and falls in response to a rising interest rates, the effect could be much weaker or even reversed. This would depend on the values of parameters. Therefore, since the terms of trade effect is weak or disappears, the liquidity effect dominates the economic effects of nominal interest rates in the model, which induces the monetary authority to set long-run gross nominal interest rates to be one.

2.3.2 Calibration

In this section, the calibration of the parameters presented in the model is discussed. I set the values of parameters to match U.S. data, which is assumed symmetric across countries.[7] A period in this study is a quarter.

For the parameters related to preferences, I set values used in the literature. The discount factor, β, is assumed 0.99 to adjust the annualized interest rate to 4.1%. The values of parameters for the matching process in both countries follow Mortensen & Pissarides (1994). The elasticity of match (σ) and the scale parameter of match (σ_m) is set to 0.5 and 4, respectively. The worker's bargaining power (η) is assumed to be 0.5. Following Gertler et al. (2008), the job separation rate, ρ, and the job finding rate, s, of home country are set to 0.105 and 0.95 to match the estimates of the U.S. monthly rates suggested by Shimer (2005). This means that a matched job in home country lasts about two and a half year. From the job separation rate and job finding rate, steady-state unemployment rate in each country is derived as 0.10. This is reasonable as unemployment rate includes those individuals registered as inactive in this model. For the vacancy posting cost (κ) and unemployment benefits (b), I use the replacement ratio which is the ratio of unemployment benefit to average wages. The replacement ratio is taken as 54% from OECD (2006).

Turning to the values of parameters associated with open economies, the openness parameter in each country is chosen such that imports are 13% of aggregate output, as in Bayoumi et al. (2004). I assume that the consumption elasticity between home and foreign goods (z) is 1.2, following Ruhl (2008). The parameters can be shown in Table 2.2.

2.3.3 Responses to shocks

This section is devoted to examine the dynamics of optimal monetary policy to a country-specific productivity shock. To analyze the effect of a productivity shock, I assume that the aggregate productivity follows bivariate autoregressive process,

[7] The target values of parameters are the same as those of the home country in Chapter 1.

Table 2.2 Parameters

Parameters set exogenously				
Statistic	Parameter	Value		Sources
Discount factor	β	0.99		$\left(\beta^{-4}-1\right)\times 100 = 4.102\%$
Elasticity of match	σ	0.5		Mortensen & Pissarides (1994)
Efficiency of match	σ_m	4		Mortensen & Pissarides (1994)
Bargaining power	η	0.5		Mortensen & Pissarides (1994)
Armington elasticity	z	1.2		Ruhl (2008)
Transition rates	$[s,\rho]$	$[0.95, 0.105]$		Gertler et al. (2008)
Calibrated Parameters				
Statistic	Parameter	Value	Target(%)	Sources
Import share	ω	0.87	13	Bayoumi et al. (2004)
Replace. ratio	$[b,\kappa]$	$[0.48, 3.68]$	54	OECD (2006)

Note: The values of parameters are common across countries. The calibrated parameters are derived from U.S. steady-state targets.

following Backus et al. (1992).

$$A_{t+1} = \Omega A_t + \varepsilon_{t+1}, \tag{2.51}$$

where $A_t = [\ln a_t, \ln a_t^*]^T$ and $\varepsilon_{t+1} \sim N(0,V)$. ε_t are considered as serially independent random variables. Thus, the diagonal elements of Ω imply the persistence of a domestic productivity shock in each country, while the off-diagonal elements denote the spillover effects of a productivity shock across countries. If the off-diagonal elements of Ω are not zeros, it is assumed that a positive productivity shock in a country can affect other country where an innovation does not happen. For the variance covariance matrix V, the variances of 0.00852^2 and correlation of shocks of 0.285 are taken from Backus et al. (1992). Furthermore, the spillover parameters are set to 0.088, whereas the persistence parameters are set to 0.906 as in Backus et al. (1992).

Figure 2.1 reports impulse responses of chosen variables to one positive standard deviation of productivity in the home country under the Ramsey-optimal monetary policy.

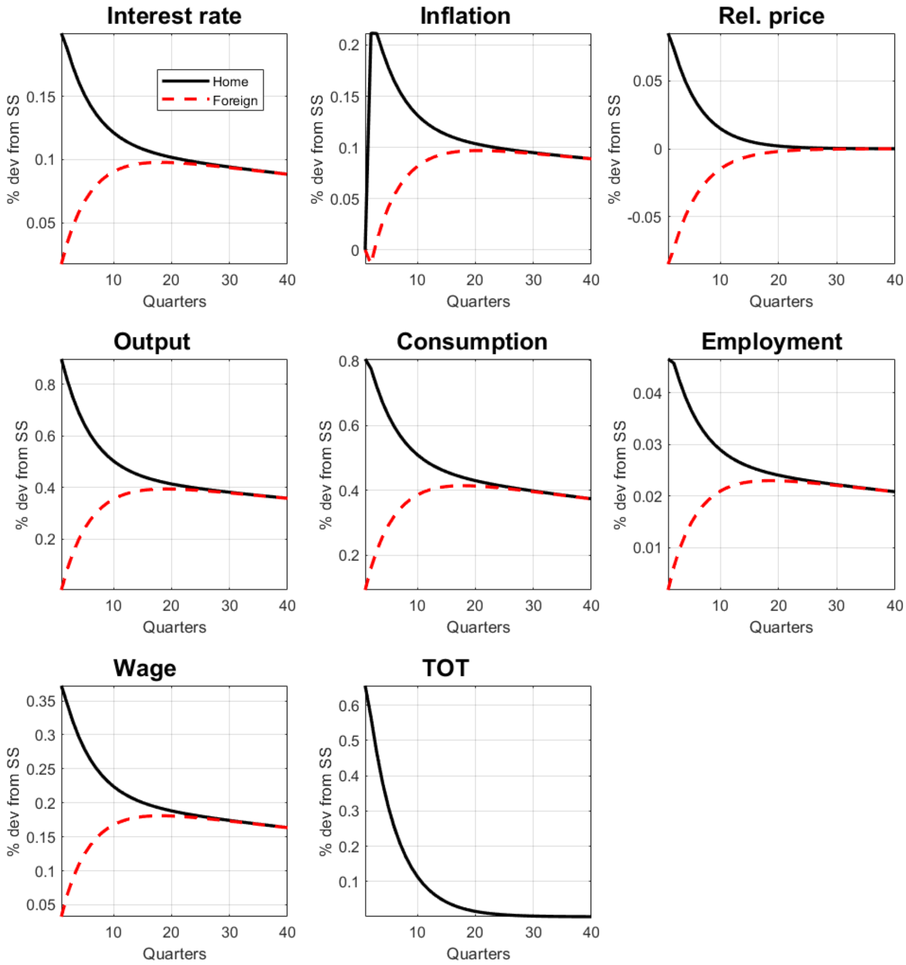

Figure 2.1 Responses to one s.d. productivity shock in the home country under optimal monetary policy

Note: One period denotes a quarter on the horizontal axis and the percentage deviation from the steady state is depicted on the vertical axis. Relative price is defined as the ratio of the consumer price index to the domestic price index.

A rise in home productivity induces an increase in the demand for home goods relative to foreign goods since the relative price of home good would fall instantly due to the assumption of flexible prices. This leads to an increase in the terms of trade and real exchange rates of the home country. Moreover, higher productivity increases firms' profits for given employment level as well as demand for home goods. This entails a rise in the demand for labor and, in turn, the increased demand of loans for the wage bills in the home country. Under the Ramsey optimal monetary scheme, the monetary authority in home country increases interest rates which, in turn, induces a rise in inflation. Accordingly, firms in the home country post more vacancies leading to more employment as well as wages to rise. As a result, output, consumption and employment in the home country increases.

An increase in home productivity leads to a rise in output, consumption and employment in the foreign country because an increase in home consumption means higher demand for foreign goods as well as home goods. Furthermore, the positive spillovers of a home productivity shock to the foreign economy also induces the increased demand for foreign goods. Thus, the positive productivity shock in the home country affects the foreign country similar to the home country, with the analogous mechanism, but the size is larger in the home country.

2.3.4 Cross-country co-movement

Table 2.3 shows properties of business cycles both in the data and in the model considering search and matching frictions in labor market along with limited participation in the financial market under the Ramsey optimal monetary policy regime.[8] To analyze business cycles more precisely in the model, I assume the values of parameters associated with spillover and persistence of technology shocks are 0.088 and 0.906, respectively, as in Backus et al. (1992). All entries in the table are Hodrick-Prescott (HP) filtered values with a smoothing parameter of 1600. While the first column reports characteristics found in the data corresponding the U.S. aggregate, the remaining columns are statistics derived from the model. Furthermore, I compare the results of benchmark model to Walrasian labor market model (No search) and the standard real business-cycle model suggested by Backus et al. (1993) (BKK).[9] is shown in the last column.

The cross-country correlations of consumption in both models are higher than those of output, which is inconsistent with the data. However, the search and matching model generates the positive international correlation of output of 0.32, while the output between home and foreign countries in BKK are negatively correlated which is opposite to the data. Employment in the search model is positively correlated across countries same as in the data. By contrast, BKK shows a negative correlation of employment, which is not matched with the data. As reported in Table 2.3, the standard international real business cycle model has a problem with accounting for

[8] The data column are for the period of 1976:1-2015:4, using data for the US and the aggregate of the EU-15.

[9] No search model is the same as one in the previous chapter.

Table 2.3 Business cycle statistics

Statistic	Data	Model		
		Search	No Search	Standard IRBC
Correlation of Home with Foreign				
GDP(y,y*)	0.55	0.32	0.27	−0.21
Consumption (c,c*)	0.40	0.52	0.80	0.88
Employment (n,n*)	0.93	0.40	−1.00	−0.78
S.D. relative to GDP				
Consumption (c)	0.60	0.83	0.90	0.42
Employment (n)	0.88	0.07	0.02	0.50

Note: The statistics of the data column are for 1970:1 to 2015:4 using U.S. and the aggregate data of the EU-15. The columns of Search and no search models are calibrated with values in Table 2.2, whereas statistics of IRBC are given by Backus et al. (1993). All statistics have been HP-filtered with a smoothing parameter of 1,600.

international co-movements of employment and output. Since wages and employment are always adjusted immediately in BKK, foreign employment and output declines at the period of the shock, which weakens the co-movements of output and employment between countries. In the labor search model, however, employment of the foreign country increases at the period of the shock as opposed to BKK as shown in Figure 2.1. With search and matching frictions, productivity shocks influence on employment in both countries in the same direction as the result of matching frictions, which leads to the positive international co-movement of employment.

Meanwhile, the search model under Ramsey optimal policy seems to reproduce properties of business cycles in data well even though the figures of statistics are less than those in data, comparing to the standard international real business cycle model. However, the model shows too little volatility of employment relative to output of 0.07 within the home country, comparing to the data. As Shimer (2005) points out in the closed economy, search model with flexible wage predicts too small volatilities of employment. This could be improved when considering the wage rigidity as in Gertler & Trigari (2009).

2.4 Conclusion

In this paper, I analyze the Ramsey-optimal monetary policy using an open economy model considering limited participation in the financial market as well as search and matching frictions in labor market. I examine the long-run optimal monetary policy, responses of macroeconomic variables to a positive productivity shock in home economy, and cross-country co-movements over business cycles under Ramsey-optimal monetary policy scheme. I find the long-run optimal nominal interest rate is one when the Hosios condition is satisfied. This implies the optimal inflation rate in the long-run should be negative but near zero. I also find that a country-specific productive shock in the home country induces similar responses of output, consumption and employment in both countries from the impulse response analysis. I also find the cross-country correlations of employment and output in the model is positive which is consistent with the data when the source of uncertainty is productivity shocks.

Chapter 3

Deviations from the LOP with labor and goods market frictions

3.1 Introduction

Standard international macroeconomic models have accounted for many features of the international business cycle. However, this class of model ignores to addressing some features of international macroeconomic data. In particularly, one of these features is the fact that deviations from the law of one price (LOP), which implies export price is not equal to the domestic price for same goods when expressed in a common currency. One leading interpretation of the deviation from the LOP is that firms conduct systematic price discrimination across countries, which is called 'pricing to market' by Krugman (1986). In the context of a single good sold in distinct markets with different prices, what cause price discrimination across markets is one of the issues. This paper introduces search and matching frictions in goods markets to explore how a country-specific productivity shock generates deviations of the LOP, and how goods market frictions interact with employment dynamics in an open economy. The main result of the paper is that a country-specific productivity shock leads to deviations from the LOP because it induces consumption gaps, differing search intensives in goods markets across countries.

To account for the role of labor and goods market frictions for the deviation from the LOP, I consider a standard, two-country and two-good model with complete asset markets. Each country specializes in the production of a single good which is traded internationally and produced by using labor as sole input. The model introduces search and matching frictions in both goods and labor markets. Search frictions in labor markets are characterized by specific matching technologies, following Mortensen & Pissarides (1994). Directed search frictions are introduced in goods markets as in Moen (1997), recently used in Bai & Ríos-Rull (2015). Thus, products supplied by firms are consumed only if firms are matched with consumers in goods markets. As firms are assumed to target either the domestic or export market and search efforts are exerted differently in each market, prices firms post could be different based on the search frictions in each market, which leads to the deviations of the LOP.

With the model, I first define the LOP gap to study the conditions which lead to deviations from the LOP. I show the LOP gap depends on the ratio of marginal utility of aggregate search efforts across countries. Thus, if the aggregate search efforts of the home country are different from search efforts of foreign households, the LOP fails to hold. Furthermore, I find that if the utility function does not have a curvature in search efforts, the LOP holds even if search efforts exerted by home and foreign households are different each other.

I begin by exploring the mechanism which causes deviations from the LOP using a simplified static version of the model. I express the LOP gap in terms of aggregate consumption of the home and the foreign countries, and then suggest conditions where deviations from the LOP occur. Namely, through the link between aggregate consumption and aggregate productivity, I find that a country-specific productivity shock generates deviations of LOP. If a country-specific productivity shock in the home country takes places, then households in the home country exert more search efforts to consume more goods in the domestic and the import markets. Higher search efforts of home households in the domestic market lead to the difference of matching probabilities of firms between the domestic and the export markets, which creates a gap between expected profits of a firm in both markets. Thus, firms move across markets due to disparity of profits. At the same time, firms in the domestic market offer lower prices. Since aggregate productivity and marginal costs of posting vacancies are the same across markets, difference in matching probabilities between markets let firms operating in each market offer different prices.

I also examine responses of macroeconomic variables to a country-specific productivity shock with calibrated values of parameters to understand the international propagation of shocks. An increase in home productivity leads to a rise in demand for home goods as well as increased profits of firms. This entails increased vacancies home firms post, and in turn, more employment. Moreover, increasing income leads to a rise in search efforts in the home country. As home households exert more search efforts in domestic and imported goods markets, the matching probability in the domestic market increases. The LOP gap of the home country increases. As a result, output, consumption, and employment in the home economy rises. This leads to an increasing expected demand for foreign goods. Thus, foreign firms also have an incentive to post more vacancies and hire more workers. However, the increase of the LOP gap of foreign goods induce foreign firms to post less vacancies, because movement of firms across markets leads to a fall in the matching probability for firms. Therefore, employment of the foreign country depends on which effect is stronger.

Finally, I study cross-country correlations of output, consumption, and employment, the correlation for the terms of trade and the relative output, the correlation between the real exchange rate and the relative consumption, and the correlation between output and employment within a country. When productivity shocks are the only source of uncertainty, the model show quantitatively lower correlation of output and higher correlation of consumption than data. However, negative correlation between the terms of trade and the relative output as well as negative correlated employment are not produced by the model. When taking into account productivity shocks along with preference shocks, the model generates a negative cross-country correlation for the terms of trade and the relative output, which is consistent with data.

This paper is related to two strands of literature. One set of literature is open economy studies that focus on the international relative prices such as the LOP, taking into account the role of real rigidities. A feature of this kind of literature is that they allow pricing-to-market which means it is possible to impose different prices for the same commodity in the different markets. Alessandria (2009) and Drozd & Nosal (2012) highlight search frictions in the goods market to account for the international prices. Alessandria (2009) addresses the importance of consumer search in generating persistent real exchange rate movements. Drozd & Nosal (2012) show that pricing-to-market is essential to explaining international price dynamics in the aggregate and product level by introducing marketing frictions in goods market. Besides search frictions, there are also other approaches that examine deviations in the LOP such as distribution costs (Corsetti & Pesenti 2005) market shares (Auer & Schoenle 2016), and deep habits (Jacob & Uusküla 2019). Even though I focus on the role of search frictions in goods markets for deviations from the LOP, this paper also suggests the transmission mechanism behind spillover effects of productivity shocks by considering the interaction between search frictions in labor and goods markets.

The other set of literature where this paper can contribute is papers that explore the propagation of shocks over international business cycles considering search frictions either in labor or goods markets. Hairault (2002) account for the observed fluctuations of international business cycles with a model incorporating search frictions in the labor market. He suggests a resolution for the counterfactual correlation of employment by incorporating conventional search and matching frictions to an open economy. Similar to Hairault (2002), Cacciatore (2014) addresses the strong trade linkages causes the greater co-movement of business cycles introducing labor market frictions along with endogenous entry and exit of firms. Meanwhile, Bai & Ríos-Rull (2015) suggest the role of consumer preference shocks instead of the productivity shock to account for the international business cycles, introducing consumer search in goods markets.

The remainder of this paper is organized as follows. Section 3.2 introduces a two-country, two-good model with labor and goods market frictions. The analytical approach is discussed in section 3.3. Section 3.4 reports quantitative results of the model. The final section concludes.

3.2 Model

The economy is comprised to two countries (home and foreign). Each country is specialized in the production of one good which is traded internationally. Within the home country, there are a measure one of households. Households consume goods and supply labor to domestic firms which sell goods either in the domestic or export markets. There are a measure one of firms in the home country, which consist of $n_{h,t}$ in the domestic market and $n^*_{h,t}$ in the export market. Both the goods and labor markets are subject to search frictions. I assume that each firm posts vacancies, denoted as $v_{h,t}$ by a firm serving in the domestic market and as $v^*_{h,t}$ by a firm serving in the export market, at cost κ in units of domestic goods, to attract unemployed workers. Each household exerts efforts $s_t \in [0,1]$ to search for goods. In what follows, the home country is focused on in the exposition of the model.

3.2.1 Matching process

Search frictions in labor and goods markets are characterized by assuming specific matching technologies. In the labor market, vacancies are filled by a Cobb-Douglas matching function as in the conventional Diamond-Mortensen-Pissarides (hereafter 'DMP') model,

$$H_t = \chi u_t^{\phi} (v_t)^{1-\phi}, \tag{3.1}$$

where $\chi > 0$. u_t denotes the pool of unemployed workers at the beginning of period t. As there is a single labor market in each country, the total vacancies, v_t, should be equal to the sum of the total vacancies posted by firms serving in the domestic market ($n_{h,t} v_{h,t}$) and the total vacancies posted by firms serving in the export market

$(n^*_{h,t} v^*_{h,t})$, where $n_{h,t}$ and $n^*_{h,t}$ is the total mass of firms in the domestic and the export markets, respectively. Thus, $n_{h,t} + n^*_{h,t}$ and $n^*_{f,t} + n_{f,t}$ are the total mass of firms in each economy, measure of one. Defining $\zeta_t \equiv \frac{v_t}{u_t}$ as labor market tightness (vacancies-unemployment ratio), the vacancy filling rate (job finding rate) is $\Phi^v_t \equiv \frac{H_t}{v_t} = \chi \zeta_t^{-\phi}$ $(\Phi^u_t \equiv \frac{H_t}{u_t} = \chi \zeta_t^{1-\phi})$.

Following Blanchard & Galí (2010), I assume workers are immediately productive, such that employment, l_t, evolves according to, $l_t = (1-\rho)\, l_{t-1} + H_t$ where $\rho \in (0,1)$ is the exogenous rate of job separation. The number of searching workers and the number of vacancies, u_t and v_t, are defined as

$$\begin{aligned} u_t &= 1-(1-\delta)l_{t-1} \quad \text{where} \quad l_{t-1} = n_{h,t-1} l_{h,t-1} + n^*_{h,t-1} l^*_{h,t-1}, \\ v_t &= n_{h,t} v_{h,t} + n^*_{h,t} v^*_{h,t}. \end{aligned} \tag{3.2}$$

As in Bai & Ríos-Rull (2015), I assume a directed search friction in goods market.[1] Households exert efforts to search for goods in either the domestic or import market. Matches are formed by the following Cobb-Douglas functions,

$$M_{i,t} = A\,(s_{i,t})^{\varphi}\,(n_{i,t})^{1-\varphi} \quad \text{for } i = \{h, f\} \tag{3.3}$$

where $A > 0$. $n_{h,t}$ $(n_{f,t})$ is the mass of home (foreign) firms serving the home market, whereas $s_{h,t}$ $(s_{f,t})$ is the mass of shoppers search for the home (foreign) goods in the home country. Goods market tightness is source-specific, so $\theta_{h,t} \equiv \frac{n_{h,t}}{s_{h,t}}$ and $\theta_{f,t} \equiv \frac{n_{f,t}}{s_{f,t}}$, are tightness for the domestic and imported goods. In this case, the probability that shoppers are matched with a firm (firms are matched with a shopper) in the domestic market is $\Phi^s_{h,t} \equiv \frac{M_{h,t}}{s_{h,t}} = A\theta_{h,t}^{1-\varphi}$ $(\Phi^n_{h,t} \equiv \frac{M_{h,t}}{n_{h,t}} = A\theta_{h,t}^{-\varphi})$ with similar expression for the import market.

3.2.2 Households

Households are modelled as an extended family, following Merz (1995). This assumption provides full consumption insurance among members since all members gather their income and consume the same amount. Households have the following inter-temporal utility function, $\sum_{t=0}^{\infty} \beta^t u(c_t, s_t)$, where $\beta \in (0,1)$ is the discount factor. The total consumption of final goods, c_t, is defined over the home $(c_{h,t})$ and foreign $(c_{f,t})$ good and utility from total consumption is increasing and strictly concave. The

[1] Under a directed search circumstance, agents select what terms of trade to search for. This implies price is committed ex ante, unlike the undirected search.

total mass of shoppers, s_t, consists of $s_{h,t}$ and $s_{f,t}$:

$$c_t = \left[\omega^{1/z} c_{h,t}^{(z-1)/z} + (1-\omega)^{1/z} c_{f,t}^{(z-1)/z}\right]^{z/(z-1)} \text{ and } s_t = s_{h,t} + s_{f,t}, \tag{3.4}$$

where $\omega \in (0,1)$ denotes measure of openness, and $z > 0$ is the elasticity of substitution between home and foreign goods.

The budget constraint of the households is,

$$\sum_i P_{i,t} c_{i,t} + \mathbb{E}_t Q_{t,t+1} B_{t+1} = W_t l_t + B_t + \Pi_t, \tag{3.5}$$

where Π_t are profits, B_t are domestic currency state-contingent assets ($Q_{t,t} \equiv 1$), and $W_t l_t$ is labor income.

In the goods market, the realized output is different from the amount of goods supplied by firms because of the goods market friction. Furthermore, aggregate realized output is consumed by households and also used by firms to post vacancies due to a labor market friction. Thus, under goods and labor market frictions, the aggregate realized output in each market should be equal to aggregate expenditure which consists of consumption and costs of posting vacancies, at per unit cost $\kappa P_{h,t}$, for home firms, and at per unit cost $\kappa^* P^*_{f,t}$, for foreign firms.

I express the home shopping constraint in domestic and imported goods market as:

$$\begin{aligned} s_{h,t} \Phi^s_{h,t} y_{h,t} &= c_{h,t} + \kappa n_{h,t} v_{h,t} \\ \frac{1}{e_t} P_{f,t} s_{f,t} \Phi^s_{f,t} y_{f,t} &= \frac{1}{e_t} P_{f,t} c_{f,t} + \kappa^* P^*_{f,t} n_{f,t} v_{f,t}, \end{aligned} \tag{3.6}$$

where e_t denotes the nominal exchange rate that means the price of home currency in terms of unit of foreign currency. The shopping constraints imply that the consumption of the home good, $c_{h,t}$, is equal to the mass of shoppers in that market, $s_{h,t}$, multiplied by the probability of a match, $\Phi^s_{h,t}$, and the goods supplied by firms, $y_{h,t}$, net of the cost of posting vacancies, $\kappa n_{h,t} v_{h,t}$, where $\kappa > 0$ is a parameter, and $n_{h,t} v_{h,t}$ is the total mass of vacancies in the domestic market - the number of vacancies multiplied the mass of firms serving the market. Since each export firm of the foreign economy sells foreign products at price $P_{f,t}$ and posts vacancies, $v^*_{h,t}$, at per unit cost $\kappa^* P^*_{f,t}$ to employ workers, they should consider the difference between the domestic and export price of foreign goods.

Households choose consumption, search effort, and state-contingent assets; $\{c_{i,t}, s_{i,t}, B_{t+1}\}$, to maximize expected discounted utility taking price, quantity, and market tightness $\{P_{i,t}, y_{i,t}, \theta_{i,t}\}$, as given:

$$
\begin{aligned}
&\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, s_t) \\
&s.t. \\
&\sum_i P_{i,t} c_{i,t} + \mathbb{E}_t Q_{t,t+1} B_{t+1} \leq W_t l_t + B_t + \Pi_t \\
&c_{h,t} \leq s_{h,t} \Phi^s_{h,t} y_{h,t} - \kappa n_{h,t} v_{h,t} \\
&c_{f,t} \leq s_{f,t} \Phi^s_{f,t} y_{f,t} - \kappa n_{f,t} v_{f,t} \left(\frac{e_t P^*_{f,t}}{P_{f,t}} \right)
\end{aligned}
\tag{3.7}
$$

This leads to the following first-order conditions,

$$
u_{c_i}(t) + \frac{u_{s_i}(t)}{\Phi^s_{i,t} y_{i,t}} = \lambda_t P_{i,t} \quad \text{for } i = \{h, f\} \tag{3.8}
$$

$$
\mathbb{E}_t Q_{t,t+1} = \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t}, \tag{3.9}
$$

where λ_t is the Lagrangian multiplier on the household budget constraint. Note that the optimal condition in Equation (3.8) is different from one in a standard Walrasian model. There is an additional term, $u_{s_i}(t) / (\Phi^s_{i,t} y_{i,t})$, because of the relation of consumption and search efforts.

Given search effort, shoppers choose how to conduct their shopping, i.e., which market to go to either the domestic or imported goods market. With directed search, this means choosing price, quantity, and market tightness; $\{P_{i,t}, y_{i,t}, \theta_{i,t}\}$. When defining the value function, $J(a_t) = \max [u(c_t, s_t) + \beta \mathbb{E}_t J(a_{t+1})]$, I have following conditions that characterize the shopper's choices:

$$
J_{P_i}(a_t) = -\lambda_t c_{i,t} \tag{3.10}
$$

$$
J_{\theta_i}(a_t) \theta_{i,t} = (1 - \varphi) J_{y_i}(a_t) y_{i,t} \tag{3.11}
$$

$$
J_{y_i}(a_t) = [u_{c_i}(t) - \lambda_t P_{i,t}] \Phi^s_{i,t} s_{i,t} \tag{3.12}
$$

for $i = \{h, f\}$ as the remaining equations for the household problem. These conditions account for how households' values change with respect to price, quantity, and market tightness suggested by firms. Thus, firms also consider shoppers' choices to optimize their profits. How these conditions affects on a firm's choice is in the next section.

3.2.3 Firms

A representative firm makes two choices, i.e. how much labor to hire for production and what bundle $\{P_{i,t}, y_{i,t}, \theta_{i,t}\}$ to offer for a match with shoppers. In the domestic (export) market, a firm j, posts $v_{h,t}(j)$ ($v^*_{h,t}(j)$) vacancies, employs $l_{h,t}(j)$ ($l^*_{h,t}(j)$) workers, and produce a final good, $y_{h,t}(j) = a_t l_{h,t}(j)$ ($y^*_{h,t}(j) = a_t l^*_{h,t}(j)$), where a_t is a productivity parameter. Following Bai & Ríos-Rull (2015), firms target either the domestic or export market. Profits of a firm in domestic or export markets are:

$$\begin{aligned}
\pi_{h,t}(j) &= P_{h,t}(j)\Phi^n_{h,t}(j)y_{h,t}(j) - W_t l_{h,t}(j) - \kappa P_{h,t} v_{h,t}(j) \qquad (3.13)\\
\pi^*_{h,t}(j) &= e_t P^*_{h,t}(j)\Phi^{*n}_{h,t}(j)y^*_{h,t}(j) - W_t l^*_{h,t}(j) - \kappa P_{h,t} v^*_{h,t}(j).
\end{aligned}$$

Due to search frictions in the goods market, a firm can sell its goods to households only if the firm is matched with a shopper. Therefore, profits of firms in the domestic (export) market depend on the probability that a firm is matched, $\Phi^n_{h,t}(j)$ ($\Phi^{*n}_{h,t}(j)$). Moreover, labor market frictions cause additional costs to post vacancies.[2]

If a firm targets the domestic market, it chooses $\{v_{h,t}(j), l_{h,t}(j), P_{h,t}(j), y_{h,t}(j), \theta_{h,t}(j)\}$ to maximize its profits, $\pi_{h,t}(j)$, subject to technology, evolution of employment, and a household participation constraint,

$$J\left(a_t; P_{h,t}(j), y_{h,t}(j), \theta_{h,t}(j)\right) \geq \mathscr{J}_h(a_t) \qquad (3.14)$$

where $J\left(a_t; P_{h,t}(j), y_{h,t}(j), \theta_{h,t}(j)\right)$ is the value of the representative household and $\mathscr{J}_h(a_t)$ is the value of the household if it shops optimally in the domestic market. The participation constraint implies that firms must suggest bundles no worse than the most attractive one available in the market to attract shoppers.

[2]Note that the cost per hire for an individual firm is expressed in terms of the bundle of domestic final goods, $\kappa P_{h,t}$, in both markets as in Campolmi & Faia (2015). This is because firms evaluate their profits in terms of domestic price index.

Accordingly, the representative firm's maximization problem in the domestic market is given by:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\frac{\lambda_{t+s}}{\lambda_t}\right) \left\{\pi_{h,t+s}(j)\right\} \tag{3.15}$$

$$s.t.$$

$$y_{h,t+s}(j) \leq a_{t+s} l_{h,t+s}(j)$$

$$l_{h,t+s}(j) \leq (1-\rho)\, l_{h,t+s-1}(j) + \Phi^v_{t+s} v_{h,t+s}(j)$$

$$\mathscr{J}_h(a_{t+s}) \leq J\left[a_{t+s}; P_{h,t+s}(j), y_{h,t+s}(j), \theta_{h,t+s}(j)\right]$$

As all firms in the domestic market are identical, I can eliminate the $'j'$ in the optimization condition. Using the first order conditions of firms serving the domestic market, the job creation condition in the domestic market is

$$\left(\frac{\kappa}{\Phi^v_t}\right) P_{h,t} = \left(\frac{1}{1-\varphi}\right) P_{h,t} \Phi^n_{h,t} a_t - W_t + (1-\rho)\, \mathbb{E}_t \beta_{t,t+1} \left(\frac{\kappa}{\Phi^v_{t+1}}\right) P_{h,t+1}, \tag{3.16}$$

where $\beta_{t,t+1}$ denotes the stochastic discount factor between period t and $t+1$, which is given by $\beta_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$. With an analogous process, I also derive the job creation condition in the export market.[3]

There are two differences between this model and a standard Walrasian model. The domestic price depends on both the vacancy filling rate, Φ^v_t (labor market), and the probability that a firm is matched with a shopper, $\Phi^n_{h,t}$ (goods market). Absent goods market frictions, vacancy costs, $\kappa > 0$, drive a wedge between the price of labor, W_t, and the marginal productivity, a_t. With goods market friction, this wedge also depends on $\Phi^n_{h,t}$.

Since there is a single labor market in each country, an individual firm serving to either domestic or export markets faces the same wage and in turn the same marginal cost. Thus, the only difference of the job creation condition in the export market is the marginal benefit, $\left(\frac{1}{1-\varphi}\right) e_t P^*_{h,t} \Phi^{*n}_{h,t} a_t$, in Equation (3.16). Comparing job creation conditions of domestic and export markets, I have the following equation:

$$e_t P^*_{h,t} = \left(\frac{\Phi^n_{h,t}}{\Phi^{*n}_{h,t}}\right) P_{h,t}, \tag{3.17}$$

which states the export price denoted in terms of home currency depends on the ratio of matching probabilities as well as the relative marginal productivity in the domestic

[3] The details of the derivation are in Appendix.

and export markets. According to Equation (3.17), the relative price of home goods in the domestic and the export markets is linked to the market tightness, $\theta_{h,t}$ and $\theta^*_{h,t}$, in both goods markets.

Using the first order conditions of firms and optimal shopper's choices, the optimal search efforts in the domestic and the export markets are:

$$\begin{aligned} s_{h,t} &= \varphi c_{h,t}\left(\frac{u_{c_h}(t)}{-u_{s_h}(t)}\right)\tau_{h,t} \\ s^*_{h,t} &= \varphi c^*_{h,t}\left(\frac{u_{c^*_h}(t)}{-u_{s^*_h}(t)}\right)\tau^*_{h,t}, \end{aligned} \tag{3.18}$$

where

$$\frac{1}{\tau_{h,t}} \equiv (1-\varphi)+\varphi\frac{c_{h,t}}{s_{h,t}\Phi^s_{h,t}y_{h,t}} \text{ and } \frac{1}{\tau^*_{h,t}} \equiv (1-\varphi)+\varphi\frac{c^*_{h,t}}{s^*_{h,t}\Phi^{*s}_{h,t}y^*_{h,t}},$$

and φ is a matching technology parameter in the goods market.

Note that the optimal search effort in the domestic market is affected by the relative amount of consumption for the home good ($c_{h,t}$) and aggregate realized output ($s_{h,t}\Phi^s_{h,t}y_{h,t}$) in the domestic market. As consumption for the home good ($c_{h,t}$) is equal to aggregate realized output ($s_{h,t}\Phi^s_{h,t}y_{h,t}$) net of aggregate vacancy posting costs ($\kappa n_{h,t}v_{h,t}$) in the domestic market, vacancy costs create a wedge between the consumption and the realized output. Without labor market friction, i.e. $\kappa = 0$, $\tau_{h,t}$ and $\tau^*_{h,t}$ disappear, going back to the goods market friction model as in Bai & Ríos-Rull (2015). However, even though there are search frictions in labor market, $\kappa \neq 0$, I find the following Lemma 1.

[Lemma 1]

The ratio of the realized output and the consumption in each market is equal to each other:

$$\frac{c_{h,t}}{s_{h,t}\Phi^s_{h,t}y_{h,t}} = \frac{c^*_{h,t}}{s^*_{h,t}\Phi^{*s}_{h,t}y^*_{h,t}} = \frac{c^*_{f,t}}{s^*_{f,t}\Phi^{*s}_{f,t}y^*_{f,t}} = \frac{c_{f,t}}{s_{f,t}\Phi^s_{f,t}y_{f,t}}.$$

□ Proof. See the appendix.

According to Lemma 1, the ratio of output provided for job posting to the output is same across markets. Lemma 1 also implies that $\tau_{i,t} = \tau^*_{i,t} = \tau_t$ for $i = \{h, f\}$, which also allows equations such as the international risk sharing condition to be simplified.

3.2.4 Wage Bargaining

I determine the wage by assuming Nash wage bargaining between workers and firms.[4] Although bargaining takes place in each market, in equilibrium, the wage is same in the domestic and the export market because workers are free to move across firms in the home economy. The bargaining power of the household is denoted by α. As a result of Nash bargaining in the domestic market, I have the optimal sharing rule as:

$$\alpha S_t^F = (1-\alpha) S_t^H, \tag{3.19}$$

where S_t^F and S_t^H are the surplus of the household and firm from hiring an additional worker, respectively.

$$\begin{aligned} S_t^H &= W_t^E - W_t^U \\ S_t^F &= \gamma_t, \end{aligned} \tag{3.20}$$

where γ_t is the Lagrangian multiplier for the evolution of employment in the firm's profit maximization problem and thus represents the marginal value of one worker. W_t^E and W_t^U denote the value of employment to a worker and the value of unemployment, respectively. They are defined as:

$$\begin{aligned} W_t^E &= W_t + \mathbb{E}_t \beta_{t,t+1} \left[(1-\rho) W_{t+1}^E + \rho \Phi_{t+1}^u W_{t+1}^E + \rho (1-\Phi_{t+1}^u) W_{t+1}^U \right] \\ W_t^U &= \mathbb{E}_t \beta_{t,t+1} \left[\Phi_{t+1}^u W_{t+1}^E + (1-\Phi_{t+1}^u) W_{t+1}^U \right] \end{aligned} \tag{3.21}$$

The value of employment in the future is divided into three circumstances: the matched job continues in the next period ($(1-\rho)W_{t+1}^E$), the worker finds a new job after job separation ($\rho \Phi_{t+1}^u W_{t+1}^E$), and the worker remains unemployed due to job severance ($\rho(1-\Phi_{t+1}^u)W_{t+1}^U$). Similarly, the value of unemployment in the next period is sum of the value of finding a job and the value of remaining unemployed. Using $S_{t+1}^H = W_{t+1}^E - W_{t+1}^U$, the worker's surplus, S_t^H, can be written as:

$$S_t^H = W_t + \mathbb{E}_t \beta_{t,t+1} \left[(1-\rho) S_{t+1}^H - (1-\rho) \Phi_{t+1}^u S_{t+1}^H \right] \tag{3.22}$$

[4] Since the bargaining over either real wage or nominal wage are same with the flexible prices, we assume the bargaining over the nominal wage to simplify the model.

Meanwhile, the firm's surplus in the domestic market is

$$S_t^F = \left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^n a_t - W_t + (1-\rho)\mathbb{E}_t\beta_{t,t+1}S_{t+1}^F. \tag{3.23}$$

The firm's surplus is the additional profits from hiring one worker net of the real wage. This leads to the following wage determination equation,

$$W_t = \alpha\left[\left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^n a_t + (1-\rho)\mathbb{E}_t\beta_{t,t+1}\kappa\left(\frac{v_{t+1}}{u_{t+1}}\right) P_{h,t+1}\right]. \tag{3.24}$$

Notice that wage depends on not only the labor market tightness in the future, $\frac{v_{t+1}}{u_{t+1}}$, but also tightness in the goods market, via the term $\Phi_{h,t}^n$, which is different from the conventional DMP model. As a firm saves future costs of posting vacancies by maintaining the match, the bargained wage is affected by the labor market tightness in the future. Moreover, if I consider the export market, as above, I find that the difference across market is reflected in the difference in tightness across the two goods markets.

3.2.5 International relative prices and International risk sharing

I now introduce some international relative prices. The real exchange rate is defined as:

$$q_t \equiv \frac{e_t P_t^*}{P_t}. \tag{3.25}$$

The terms of trade of the home country which is a relative price of imports to exports is expressed as

$$TOT_t = \frac{P_{f,t}}{e_t P_{h,t}^*}. \tag{3.26}$$

I also define the home and foreign good the law of one price (LOP) gaps, respectively, as:

$$\Psi_{i,t} \equiv \frac{e_t P_{i,t}^*}{P_{i,t}} \text{ for } i = \{h, f\}, \tag{3.27}$$

which states that the LOP gap should be equal to one if the LOP holds.

With the complete asset market assumption, home and foreign households have access to state-contingent assets which is traded internationally. This implies the following international risk sharing condition holds:[5]

$$\frac{u_{c_f^*}(t)}{u_{c_h}(t)} = \frac{e_t P_{f,t}^*}{P_{h,t}}. \tag{3.28}$$

This condition means that the ratio of marginal utilities between the foreign and the home consumption in domestic markets is associated with the relative price of foreign goods to home goods, expressed in terms of the home currency.

Since I assume that firms are free to target either the domestic or the export market, the expected profits in each market are same in equilibrium ($\pi_{h,t} = \pi_{h,t}^*$). Furthermore, all firms face the identical marginal cost when there is no impediment in the international trade in goods markets, because there is a single labor market in the home economy. Thus, in equilibrium, the mass of vacancies posted by each firm is same, in turn, the employment of a firm is also same in both markets:

$$v_{h,t} = v_{h,t}^* \text{ and } l_{h,t} = l_{h,t}^*. \tag{3.29}$$

In what follows, I denote vacancies (employment) by a firm in both the domestic and the export market by v_t (l_t) instead of $v_{h,t}$ and $v_{h,t}^*$ ($l_{h,t}$ and $l_{h,t}^*$).

Table 3.1 summarized the equilibrium conditions for the world economy in terms of optimal allocations, prices of labor, and prices of goods.

3.3 Analytical results

In this section, I study the conditions which lead to deviation from the law of one price (LOP). First, I take into account specific condition where the LOP holds by deriving the LOP gap in terms of aggregate search efforts. Then, I explain how a country-specific productivity shock generates deviations of the LOP in otherwise symmetric economies.[6]

[5] This expression is equivalent to the below:

$$\frac{u_{c^*}(t)}{u_c(t)} = \left(\frac{e_t P_t^*}{P_t}\right)$$

which expressed as the consumption based real exchange rate.

[6] Under the assumption of symmetric economies, the foreign openness parameter, ω^*, is equal to the home one, ω.

Table 3.1 Model summary

	Home country	Foreign country
Mass of firms	$1 = n_{h,t} + n^*_{h,t}$	$1 = n^*_{f,t} + n_{f,t}$
Unemployment	$u_t = 1 - (1-\rho)\, l_{t-1}$	$u^*_t = 1 - (1-\rho)\, l^*_{t-1}$
Employment	$l_t = (1-\rho)\, l_{t-1} + \Phi^v_t v_t$	$l^*_t = (1-\rho)\, l^*_{t-1} + \Phi^{*v}_t v^*_t$
Production	$y_t = z_t l_t$	$y^*_t = z^*_t l^*_t$
Wage	$W_t = \alpha \left(\frac{1}{1-\varphi}\right) p_{h,t} \Phi^n_{h,t} z_t$	$W^*_t = \alpha \left(\frac{1}{1-\varphi}\right) p^*_{f,t} \Phi^{*n}_{f,t} z^*_t$
	$+\alpha(1-\rho) E_t \beta_{t,t+1} \kappa \left(\frac{v_{t+1}}{u_{t+1}}\right) p_{h,t+1}$	$+\alpha(1-\rho) E_t \beta^*_{t,t+1} \kappa \left(\frac{v^*_{t+1}}{u^*_{t+1}}\right) p^*_{f,t+1}$
Job creation	$\left(\frac{\kappa p_{h,t}}{\Phi^v_t}\right) = \left(\frac{1}{1-\varphi}\right) p_{h,t} \Phi^n_{h,t} z_t$	$\left(\frac{\kappa p^*_{f,t}}{\Phi^{*v}_t}\right) = \left(\frac{1}{1-\varphi}\right) p^*_{f,t} \Phi^{*n}_{f,t} z^*_t$
	$-W_t + (1-\rho) E_t \beta_{t,t+1} \left(\frac{\kappa p_{h,t+1}}{\Phi^v_{t+1}}\right)$	$-W^*_t + (1-\rho) E_t \beta^*_{t,t+1} \left(\frac{\kappa p^*_{f,t+1}}{\Phi^{*v}_{t+1}}\right)$
Export price	$e_t p^*_{h,t} = \left(\frac{\Phi^n_{h,t}}{\Phi^{*n}_{h,t}}\right) p_{h,t}$	$\frac{1}{e_t} p_{f,t} = \left(\frac{\Phi^{*n}_{f,t}}{\Phi^n_{f,t}}\right) p^*_{f,t}$
Int'l prices	$\frac{P_{h,t}}{P_{f,t}} = \left[u_{c_h}(t) + \frac{u_{s_h}(t)}{\Phi^s_{h,t} y_t}\right] / \left[u_{c_f}(t) + \frac{u_{s_f}(t)}{\Phi^s_{f,t} y^*_t}\right]$	$\frac{p^*_{f,t}}{p^*_{h,t}} = \left[u_{c^*_f}(t) + \frac{u_{s^*_f}(t)}{\Phi^{*s}_{f,t} y^*_t}\right] / \left[u_{c^*_h}(t) + \frac{u_{s^*_h}(t)}{\Phi^{*s}_{h,t} y_t}\right]$
Search effort	$s_{h,t} = \varphi c_{h,t} \left[\frac{u_{c_h}(t)}{-u_{s_h}(t)}\right] \tau_t$	$s^*_{f,t} = \varphi c^*_{f,t} \left[\frac{u_{c^*_f}(t)}{-u_{s^*_f}(t)}\right] \tau_t$
	$s_{f,t} = \varphi c_{f,t} \left[\frac{u_{c_f}(t)}{-u_{s_f}(t)}\right] \tau_t$	$s^*_{h,t} = \varphi c^*_{h,t} \left[\frac{u_{c^*_h}(t)}{-u_{s^*_h}(t)}\right] \tau_t$
Shopping	$c_{h,t} = n_{h,t} \Phi^n_{h,t} y_t - n_{h,t} \kappa v_t$	$c^*_{f,t} = n^*_{f,t} \Phi^{*n}_{f,t} y^*_t - n^*_{f,t} \kappa v^*_t$
constraint	$c_{f,t} = n_{f,t} \Phi^n_{f,t} y^*_t - n_{f,t} \kappa v^*_t \left(\frac{e_t P^*_{f,t}}{P_{f,t}}\right)$	$c^*_{h,t} = n^*_{h,t} \Phi^{*n}_{h,t} y_t - n^*_{h,t} \kappa v_t \left(\frac{P_{h,t}}{e_t P^*_{h,t}}\right)$
Risk sharing	$\frac{u_{c^*_f}(t)}{u_{c_h}(t)} = \frac{e_t P^*_{f,t}}{P_{h,t}}$	

3.3.1 LOP gap

I express the LOP gap of the home country as a function of the probability that a firm is matched with a shopper. Using the international risk sharing and optimal search efforts, I derive the following proposition.

[Proposition 1]

The LOP gap depends on the ratio of marginal utility of aggregate search between countries:

$$\Psi_{i,t} = \left[\frac{u_{s^*_i}(t)}{u_{s_i}(t)}\right]^{\varphi} = \left[\frac{u_{s^*}(t)}{u_s(t)}\right]^{\varphi} \text{ for } i = \{h, f\}$$

□ Proof. See the appendix.

Proposition 1 implies that if the marginal utilities of search efforts are equal to each other across countries, then the LOP holds. Furthermore, it is immediate that if the utility function does not have a curvature in search efforts, i.e. the marginal utility of search efforts is constant, the LOP always holds even if search efforts exerted by home and foreign households are different each other.

Note that home and foreign LOP gaps are the same and depend on the marginal utilities of aggregate searches. This is because the aggregate search is the sum of search efforts in the domestic and the import markets, $s_t = s_{h,t} + s_{f,t}$ and $s_t^* = s_{f,t}^* + s_{h,t}^*$, which means that the marginal utility in a sub-market is same as the marginal utility of total search efforts. Moreover, since both countries have the same utility function, equal marginal utilities of searches implies that aggregate search efforts of the home and the foreign countries are equal each other. Thus, if the aggregate search efforts of the home country are different from search efforts of foreign households, i.e. $s_t \neq s_t^*$, the LOP fails to hold.

[Proposition 2]

The ratio of search efforts depends on the ratio of consumption and the ratio of the marginal rate of substitution between consumption and search.

$$\frac{s_t^*}{s_t} = \left[\frac{u_{c^*}(t)/u_{s^*}(t)}{u_c(t)/u_s(t)}\right]\left(\frac{c_t^*}{c_t}\right)$$

□ Proof. This is evident when I take the ratio of the aggregate search efforts. This is because Lemma 1 implies that the aggregate search efforts are expressed by the aggregate consumption:

$$s_t = \varphi\left[\frac{u_c(t)}{-u_s(t)}\right] c_t \tau_t \text{ and } s_t^* = \varphi\left[\frac{u_{c^*}(t)}{-u_{s^*}(t)}\right] c_t^* \tau_t^* \text{ where } \tau_t = \tau_t^* \tag{3.30}$$

Proposition 2 suggests that if the marginal rate of substitution between consumption and search is a function of consumption and search, the ratio of aggregate search efforts only depends on the ratio of aggregate consumption. This implies that the aggregate search efforts of the home and the foreign households are equal to each other if home and foreign consumption is same.

Therefore, Proposition 2 along with Proposition 1 states that the LOP gap depends on the relative aggregate consumption if the marginal rate of substitution between consumption and search is expressed in terms of consumption and search.

3.3.2 Deviations from the LOP

In the previous section, I find when the LOP holds in general terms. In this section, I concentrate on the deviation from the LOP when there is a country-specific productivity shock by considering a functional form.

I focus on Greenwood-Hercowitz-Huffman (GHH) preferences over consumption and search efforts. The period utility function is

$$u(c_t, s_t) = \frac{1}{1-\sigma}\left(c_t - \psi \frac{s_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)^{1-\sigma}, \tag{3.31}$$

where $1/\sigma$ denotes the intertemporal elasticity of substitution. While ψ captures disutility from exerting search efforts, η determines the elasticity of search effort with respect to the return. As the wealth effects in search efforts are eliminated with GHH preferences, shopping efforts are procyclical. This is consistent with empirical research such as Petrosky-Nadeau et al. (2016).

To be able to explore the mechanism which causes deviations from the LOP, I further consider a simplified static version of the model by setting the Armington elasticity, z, to 1 and the separation rate of a job, ρ, to 1. This means a job continues only one period, and in turn, all workers search jobs in every period, i.e. $u_t = 1$. Thus, the job creation condition and the wage equation in the labor market for a firm serving to the domestic market are replaced by the following static conditions:

$$\left(\frac{\kappa}{\Phi_t^v}\right) P_{h,t} = \left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^n a_t - W_t \tag{3.32}$$

$$W_t = \alpha\left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^n a_t. \tag{3.33}$$

Apart from the equilibrium conditions in the labor market, other equations are as reported in Table 3.1.[7]

With the period utility function, I derive the relation between aggregate search and consumption within a country by Equation (3.30):

$$s_t^{1+\frac{1}{\eta}} = \tau_t\left(\frac{\varphi}{\psi}\right) c_t \text{ and } s_t^{*1+\frac{1}{\eta}} = \tau_t\left(\frac{\varphi}{\psi}\right) c_t^*. \tag{3.34}$$

According to Proposition 2, the ratio of search efforts is expressed in terms of ratio of consumption. This implies that the LOP gap is in terms of the ratio of aggregate consumption by Proposition 1:

$$\Psi_{i,t} = \left(\frac{c_t}{c_t^*}\right)^{\frac{\varphi[\sigma(1+\eta)-1]}{1+\eta}} \quad \text{for } i = \{h, f\}. \tag{3.35}$$

[7]The full equations of the static model are in Appendix

This equation states that if (1) there is no search frictions in goods market, i.e. $\varphi = 0$ or (2) σ is equal to $1/(1+\eta)$, the LOP always holds regardless of the fluctuation of consumption. Otherwise, the LOP does not hold when home and foreign consumption is different which implies that the real exchange rate fluctuates. To see why the LOP gap is linked with the consumption gap across countries, I need to use the following Proposition 3.

[Proposition 3]

The different productivity shocks across countries, $a \neq a^*$, lead to consumption gaps ($c \neq c^*$) across countries, in turn deviations from the law of one price.

$$\widehat{c_t} - \widehat{c_t^*} = \frac{(2\omega - 1)(1+\eta)}{\zeta}\left(\widehat{a_t} - \widehat{a_t^*}\right),$$

with

$$\begin{aligned} \zeta &= 2(1-\sigma)(1-\omega)(1+\eta)\left[\varphi(1-\phi) + 2\omega(\phi - \varphi)\right] \\ &\quad + \phi(1+\eta) - \varphi\eta\left[2\omega(1-\phi) + 2\phi - 1\right], \end{aligned}$$

where ˆ denotes the log deviation from the steady state.

□ Proof. See the appendix.

Proposition 3 suggests that a country-specific productivity shock generates consumption gaps, and then deviations from the LOP. If the coefficient of consumption gaps is not equal to 0, different productivity shocks cause a disparity of consumption across countries. Note that if consumption is not home-biased ($\omega = 0.5$), difference of productivity between countries does not link to the consumption gap. This implies that LOP holds even if there is a country-specific productivity shock, in the absence of the consumption home-biasedness. ζ determines whether relative consumption of the home country to the foreign country rises in response to an increase in productivity of home country. Taking into account meaning of each parameter, ζ is positive if consumption is home-biased ($\omega > 0.5$), which states a positive consumption gap ($\widehat{c_t} > \widehat{c_t^*}$) and the positive LOP gap according to Equation (3.35).

To understand the mechanism behind Proposition 3, suppose there is a country-specific productivity shock in the home country. Households in the home country exert more search efforts to consume more in the domestic and the import markets. Given the number of firms operating in each market unchanged, more search efforts of home households in the domestic market, $s_{h,t}$, cause higher matching probability in the domestic market, $\Phi^{n}_{h,t}$, than in the export market, $\Phi^{*n}_{h,t}$. The difference of matching probabilities of firms between the domestic and the export markets entails a gap between expected profits of a firm in both markets:

$$\underbrace{P_{h,t}\Phi^{n}_{h,t}y_t - W_t l_t - \kappa P_{h,t} v_t}_{\pi_{h,t}} \neq \underbrace{e_t P^{*}_{h,t}\Phi^{*n}_{h,t}y_t - W_t l_t - \kappa P_{h,t} v_t}_{\pi^{*}_{h,t}}. \tag{3.36}$$

Thus, firms in the export market move to the domestic market. At the same time, firms in the domestic market offer lower price, $P_{h,t} < e_t P^{*}_{h,t}$. Firms' movement across markets occurs until the expected profits of a firm in both markets are equal to each other.

Intuitively, the mechanism can be explained by the role of matching probability as well. Taking into account the job creation condition (Equation (3.32)) with wage condition (Equation (3.33)) in the domestic and export markets are given by the following expressions:

$$\begin{aligned} P_{h,t} &= \left(\frac{1-\varphi}{1-\alpha}\right)\frac{1}{\Phi^{n}_{h,t}a_t}\left(\frac{\kappa P_{h,t}}{\Phi^{v}_{t}}\right) \\ e_t P^{*}_{h,t} &= \left(\frac{1-\varphi}{1-\alpha}\right)\frac{1}{\Phi^{*n}_{h,t}a_t}\left(\frac{\kappa P_{h,t}}{\Phi^{v}_{t}}\right). \end{aligned} \tag{3.37}$$

Recall that φ and α denote the matching technology parameter in the goods market and the bargaining power of the household, respectively. $\kappa P_{h,t}/\Phi^{v}_{t}$ is the marginal cost of posting an additional job vacancy. According to the above equation, the price which firms offer in goods markets depends on the marginal cost of posting vacancies, matching probability of firms, and aggregate productivity. Since aggregate productivity and marginal costs are the same across markets, difference in matching probabilities between markets let firms operating in each market offer different prices.

Furthermore, Bai & Ríos-Rull (2015) and Bai et al. (2017) highlight the role of matching probability as a productivity shock, measuring aggregate productivity of $\Phi^{n}_{h,t}a_t$. If there is a preference shock affecting the matching probability only, not a_t, firms adjust their price offers in response to the preference shock, because it plays as a productivity shock via the changing of the matching probability.

3.4 Quantitative analysis

In this section, I present a quantitative analysis of the model. I study the responses of aggregate variables to productivity shocks. Then, international correlations of business cycles are also reported.

3.4.1 Calibration

In this section, the calibration of the parameters presented in the model is discussed. I assume the home country is the U.S. and the foreign country is the EU for the calibration of parameters. A period in this paper is set to a quarter. For the parameters related to preferences, I choose standard values used in the literature. The discount factor, β, is assumed 0.99 to adjust the quarterly real interest rate to 1 percent. The CRRA parameter, σ, is set to 2, which implies the inter-temporal elasticity of substitution is 0.5. I assume that the consumption elasticity between home and foreign goods (z) is 1.2, following Ruhl (2008).

The elasticity of match (ϕ) in the labor market is set to 0.5 as in Pissarides (2009). The worker's bargaining power (α) is assumed to be 0.5 so that the Hosios (1990) condition holds. The matching elasticity in goods market, φ, and the parameter η are set to 0.23 and 0.11, respectively, which are calibrated in Bai et al. (2017). Moreover, I set the matching efficiency in goods market, A to 1 as in Bai et al. (2017).

The remaining parameters are calibrated using the steady-state targets. The matching efficiency in labor market, χ and χ^*, is calibrated by setting the steady-state vacancy filling probability to 0.71 suggested by Den Haan et al. (2000). The job separation rate of the home country, ρ, is set to 0.105 as in Gertler et al. (2008), to match the estimates of the U.S. monthly rates suggested by Shimer (2005). I calibrate 0.036 for the job separation rate of the foreign country, ρ^*, using monthly estimates of the EU-15 in Hobijn & Şahin (2009). For parameters of vacancy posting costs, κ and κ^*, I set the targets of unemployment rates at 6% for home and 10% for foreign economy, which is consistent with OECD data.[8] The openness parameter in each country is chosen such that imports are 13% and 18% of aggregate output, respectively, as in Bayoumi et al. (2004). I calibrate the value of disutility parameter for search efforts, ψ, to match the capacity utilization of 81%, based on the series published by the Federal Reserve Board, following Bai & Ríos-Rull (2015). The chosen parameters can be shown in Table 3.2.

[8] Data are taken from OECD (2022) between 1991 and 2005.

Table 3.2 Parameters

Targets	Value	Parameter	Value	Source
Parameters set exogenously				
Risk aversion		σ	2	-
Discount factor		β	0.99	$(\beta^{-4}-1)\times 100 \doteqdot 4\%$
Armington elasticity		z	1.2	Ruhl (2008)
Bargaining power		α	0.5	Hosios (1990)
Matching elas. (labor)		ϕ	0.5	Pissarides (2009)
Matching elas. (goods)		φ	0.23	Bai et al. (2017)
Frisch elas. for search		η	0.11	Bai et al. (2017)
Matching efficiency		A	1	Bai et al. (2017)
Job separation rate		$[\rho,\rho^*]$	$[0.105, 0.036]$	Shimer (2005)
Calibrated Parameters				
Vacancy filling prob.	0.71	$[\chi,\chi^*]$	$[0.66, 0.41]$	Den Haan et al. (2000)
SS employment (L)	$[94\%, 90\%]$	$[\kappa,\kappa^*]$	$[0.94, 2.28]$	OECD (2022)
Imports-to-output	$[13\%, 18\%]$	$[\omega,\omega^*]$	$[0.84, 0.79]$	Bayoumi et al. (2004)
Capacity utilization	81%	$[\psi,\psi^*]$	$[4667, 1175]$	Bai & Ríos-Rull (2015)

Note: The calibrated parameters are derived from U.S. (home) and EU (foreign) steady-state targets. The parameters with * refer to the foreign country.

3.4.2 Responses to shocks

I assume that the aggregate productivity follows bivariate autoregressive process, following Backus et al. (1992).

$$A_{t+1} = \Omega A_t + \varepsilon_{t+1}, \tag{3.38}$$

where $A_t = [\ln a_t, \ln a_t^*]^T$ and $\varepsilon_{t+1} \sim N(0,V)$. ε_t are considered as serially independent random variables. Thus, the diagonal elements of Ω imply the persistence of country-specific productivity shock, while the off-diagonal elements denote the spillover effects of a productivity shock across countries. I set the values of parameters associated with spillover and persistence of productivity shocks are 0.088 and 0.906, while the variance of shock and correlation of shocks are set to 0.00852^2 and 0.258, as in Backus et al. (1992).

Figure 3.1 shows the impulse responses of chosen variables to one standard deviation of productivity in the home country. An increase in home productivity leads firms in home country post more vacancies, inducing more employment. More employment as well as increasing values of jobs in the home country also make wage increase. Furthermore, increasing income (output) leads home households to exert search efforts more because I assume there is a positive relation between income and search efforts with GHH preferences. An increasing search effort of home households in both domestic and imported goods markets causes higher matching probability in

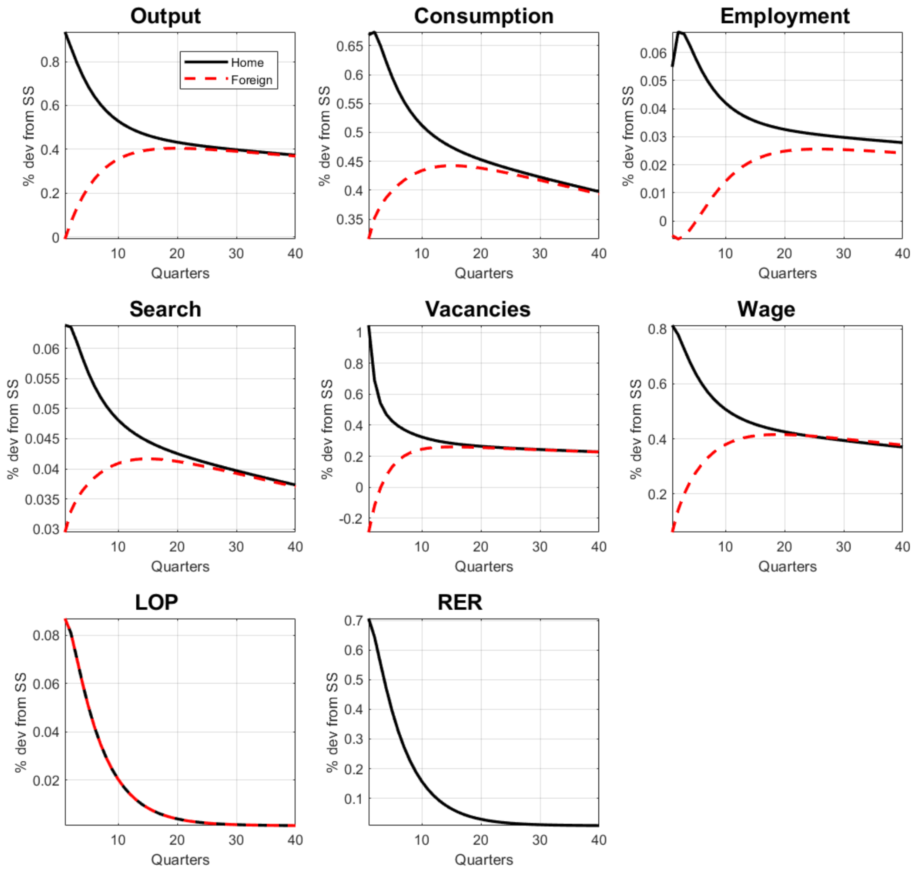

Figure 3.1 Responses to one s.d. productivity shock in the home country

Note: The home (foreign) country is assumed as the U.S. (EU) economy. One period denotes a quarter on the horizontal axis and the percentage deviation from the steady state is depicted on the vertical axis.

the domestic market. Thus, the LOP gap of the home country increases. As a result, output, consumption and employment in the home country increases.

Due to positive spillover effects of productivity shocks, productivity of the foreign country also increases. Moreover, since a positive home productivity shock causes a rise in home consumption, perfect international risk-sharing leads foreign consumption to increase as a result of complete asset markets. Thus, future consumption in both countries will also increase because the effect of a shock on the home country is persistent. This, in turn, leads expected demand for foreign products to rise. Thus, foreign firms also have an incentive to post more vacancies and hire more workers due to the higher expected returns to jobs. However, the increase of the LOP gap over foreign goods gives another incentive for foreign firms to post less vacancies.

To understand the transmission mechanism behind the responses of foreign employment, it is useful to consider the equilibrium condition of labor market in the foreign country. The equilibrium condition is summarized by the equation:

$$\frac{\kappa}{\Phi_t^{*v}} = \left(\frac{1-\alpha}{1-\varphi}\right)\Phi_{f,t}^{*n}a_t^* - (1-\rho)\mathbb{E}_t\beta_{t,t+1}^*\left[\left(\frac{\kappa}{\Phi_{t+1}^{*v}}\right) - \alpha\kappa\left(\frac{v_{t+1}^*}{u_{t+1}^*}\right)\right], \tag{3.39}$$

where

$$\Phi_t^{*v} = \chi\left(\frac{v_t^*}{u_t^*}\right)^{-\phi}.$$

The left-hand side of the equation indicates the marginal cost to a firm, whereas the right-hand side represents the expected marginal benefit. The first term in the right-hand side is the current earning from hiring an additional worker. The terms in the bracket mean discounted continuation values of a job which is not separated in the next period. Thus, the equilibrium condition implies that employment is determined at the level where the marginal cost and the marginal expected profits of an additional worker are equal.

The marginal product of labor in Equation (3.39) is different from the standard DMP model due to an additional variable, $\Phi_{f,t}^{*n}$, which comes from goods market frictions. This is because, except for the own marginal productivity, a_t^*, the condition depends on the matching probability in goods market, $\Phi_{f,t}^{*n}$, affected by movement of firms between markets, as well.

Without goods market frictions, if a positive productivity shock happens in the home country, an increase in foreign productivity due to positive spillover effects of productivity shocks and increasing future value of jobs leads firms to post more vacancies. Considering goods market frictions, however, there is an additional effect via the matching probability. Given the mass of firms in each market unchanged, a rise in the LOP gap in foreign countries induces foreign firms to move from the export market to the domestic market, because the profit in the domestic market is temporarily higher. This causes an increase of the market tightness in the domestic market, inducing a fall in the matching probability. Thus, according to the equilibrium condition, foreign firms also have an incentive to post less vacancies with goods market frictions. Therefore, employment of the foreign country depends on which effect has more impacts than the other.

3.4.3 International correlations

In this section, I calculate cross-country correlations of output, consumption, and employment, the correlation for the terms of trade and the relative output, the correlation between the real exchange rate and the relative consumption, and the correlation between output and employment within a country. Table 3.3 reports correlations both in the data and in the open-economy models.[9] While the first column reports characteristics found in the data corresponding the U.S. aggregate, the remaining columns are statistics derived from the models: search frictions both in labor and goods markets ('Two-search'), search frictions in goods markets ('Goods search'), and search frictions in labor markets ('Labor search').[10] All entries in the table are Hodrick-Prescott filtered values with a smoothing parameter of 1600.

To analyze correlations of selected variables, I examine the effect of productivity shocks introduced in the previous subsection. I also study the implications of productivity shocks along with preference shocks because Bai & Ríos-Rull (2015) emphasize that the role of preference shocks in the consumer search model to explain the business cycles.[11] To introduce preference shocks, I assume that the disutility parameter of search can vary during the given periods, as in Bai et al. (2017):

$$u(c_t, s_t) = \frac{1}{1-\sigma}\left(c_t - d_t \psi \frac{s_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)^{1-\sigma}, \tag{3.40}$$

where d_t follows AR(1) process with respective persistence. I set the persistence and the standard deviation of a shock to 0.99 and 0.61, taking estimates in Bai et al.

[9]The data column are for the period of 1976:1-2015:4, using data for the US and the aggregate of the EU-15.

[10]For the analysis of the goods search model, the period utility function is assumed as

$$u(c_t, s_t, l_t) = \frac{1}{1-\sigma}\left(c_t - d_t \psi \frac{s_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)^{1-\sigma} - \iota \frac{l_t^{1+\frac{1}{g}}}{1+\frac{1}{g}},$$

where g set to 0.72.

The period utility function of the labor search model is assumed as

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}.$$

[11]Since Bai & Ríos-Rull (2015) and Bai et al. (2017) consider endogenous productivity which consists of aggregate productivity and the matching probability in the goods market, preference shocks play a role as productivity shocks.

Table 3.3 Business cycle statistics

Correlations	Data	Productivity			Productivity and preference		
		Two-search	Goods search	Labor search	Two-search	Goods search	Labor search
corr(y,y*)	0.55	0.28	0.03	0.30	0.35	0.19	–
corr(c,c*)	0.40	0.87	0.69	0.88	0.54	0.27	–
corr(n,n*)	0.93	−0.01	0.94	0.12	−0.04	0.61	–
corr(TOT,y/y*)	−0.21	0.99	1.00	0.99	−0.27	−0.35	–
corr(q,c/c*)	0.06	1.00	1.00	1.00	1.00	1.00	–
corr(y,n)	0.83	0.94	−0.85	0.95	0.93	−0.94	–

Note: The statistics of the data column are for 1970:1 to 2015:4 using U.S. and the aggregate data of the EU-15. All statistics have been HP-filtered with a smoothing parameter of 1,600.

(2017).[12] I assume that shocks to preferences have no spillover across countries, and there is no correlation between shocks.

When taking into account productivity shocks only, the cross-country correlations of output in all models are less than those of consumption, which is inconsistent with the data. Comparing with the goods search model, the two-search and the labor search models report relatively higher correlations of output and consumption at 0.28 and 0.87 for the former, at 0.30 and 0.88 for the latter.

Regarding employment, only the two-search model shows a negative international correlation of −0.01, which is not consistent with the data. This is because the negative spillover effects caused by frictions in goods markets, as explored with the impulse responses in the previous subsection. Accordingly, without goods market frictions, the positive spillovers are found in the labor search model. The goods search model shows very highly correlated employment despite the negative spillover effects. This comes from the property that employment of the home country decreases in response to productivity shocks as well, which is opposite with data. According to the correlation between output and employment within the home country, the goods search model reports negative correlations of −0.85 and −0.94, respectively, in both cases, different from data.

Meanwhile, all models do not explain the correlation between the terms of trade and the relative output, and the correlation for the real exchange rate and the relative consumption in data, because of the assumption of complete financial market.

When there are productivity shocks along with preference shocks, both the two-search model and the goods search model report a negative correlation between the

[12] Bai et al. (2017) estimate the process by using U.S. quarterly data from 1967 to 2013 with Bayesian methods.

terms of trade and the relative output, which is consistent with data. Moreover, correlations of output and consumption in the two-search model become close to data quantitatively, as the cross-country correlation of output increases to 0.35, whereas the correlation of consumption decreases to 0.54. Note that the labor search model does not take into account preference shocks due to the property of the model.

Different from Bai & Ríos-Rull (2015), the preference shock, i.e. demand shock, is not sufficient to address the international co-movement over business cycles in this paper. Since the utility function in Bai & Ríos-Rull (2015) does not have a curvature in search efforts, LOP always holds in their model. With the curvature of the utility function in search efforts, however, deviations from LOP can happen as discussion in the previous section, which gives a different intuition on the cross-country co-movement of business cycles.

3.5 Conclusion

In this paper, I analyze deviations from the law of one price and international spillover effects of aggregate productivity and preferences shocks across countries using the two-search model which introduces search and matching frictions in both goods and labor markets. I also examine conditions which lead to deviations from the LOP and the mechanism how productivity shocks make the LOP fail to hold. Finally, I study impulse responses of macroeconomic variables with respect to a positive productivity shock in the home economy, and cross-country correlations. I find the mechanism which causes deviations from the LOP. Since the LOP gap only depends on the ratio of marginal utility of aggregate search across countries and is linked the consumption gap across countries, a country-specific productivity shock entails deviations from the LOP. Moreover, I find the two-search model reports consistent correlations with data, in terms of cross-country correlations of output and consumption, and a negative correlation between the terms of trade and the relative output when productivity and preference shocks considered.

Bibliography

Alessandria, G. (2009), 'Consumer search, price dispersion, and international relative price fluctuations', *International Economic Review* **50**(3), 803–829.

Andolfatto, D. (1996), 'Business cycles and labor-market search', *The American Economic Review* **86**(1), 112–132.

Auer, R. & Schoenle, R. (2016), 'Market structure and exchange rate pass-through', *Journal of International Economics* **98**(C), 60–77.

Backus, D., Kehoe, P. J. & Kydland, F. E. (1992), 'International real business cycles', *Journal of Political Economy* **100**(4), 745–775.

Backus, D., Kehoe, P. J. & Kydland, F. E. (1993), 'International business cycles: Theory and evidence', *National Bureau of Economic Research Working Paper Series* **No. 4493**.

Bai, Y. & Ríos-Rull, J.-V. (2015), 'Demand shocks and open economy puzzles', *American Economic Review* **105**(5), 644–49.

Bai, Y., Ríos-Rull, J.-V. & Storesletten, K. (2017), 'Demand shocks as technology shocks', *Unpublished* .

Baxter, M. (1995), *Chapter 35 International trade and business cycles*, Vol. 3, Elsevier, pp. 1801–1864.

Bayoumi, T., Laxton, D. & Pesenti, P. (2004), 'Benefits and spillovers of greater competition in europe: A macroeconomic assesment', *National Bureau of Economic Research Working Paper Series* **No. 10416**.

Blanchard, O. & Galí, J. (2010), 'Labor markets and monetary policy: A new keynesian model with unemployment', *American Economic Journal: Macroeconomics* **2**(2), 1–30.

Cacciatore, M. (2014), 'International trade and macroeconomic dynamics with labor market frictions', *Journal of International Economics* **93**(1), 17–30.

Cacciatore, M. & Ghironi, F. (2021), 'Trade, unemployment, and monetary policy', *Journal of International Economics* **132**, 103488.

Calvo, G. A. (1983), 'Staggered prices in a utility-maximizing framework', *Journal of Monetary Economics* **12**(3), 383–398.

Campolmi, A. & Faia, E. (2015), ‘Rethinking optimal exchange rate regimes with frictional labor markets’, *Macroeconomic Dynamics* **19**(5), 1116–1147.

Chari, V. V., Kehoe, P. J. & McGrattan, E. R. (2002), ‘Can sticky price models generate volatile and persistent real exchange rates?’, *The Review of Economic Studies* **69**(3), 533–563.

Christiano, L. J., Eichenbaum, M. & Evans, C. L. (1997), ‘Sticky price and limited participation models of money: A comparison’, *European Economic Review* **41**(6), 1201–1249.

Christiano, L. J., Trabandt, M. & Walentin, K. (2011), ‘Introducing financial frictions and unemployment into a small open economy model’, *Journal of Economic Dynamics and Control* **35**(12), 1999–2041.

Cooley, T. F. & Quadrini, V. (2003), ‘Common currencies vs. monetary independence’, *The Review of Economic Studies* **70**(4), 785–806.

Corsetti, G., Dedola, L. & Leduc, S. (2008), ‘International risk sharing and the transmission of productivity shocks’, *The Review of Economic Studies* **75**(2), 443–473.

Corsetti, G. & Pesenti, P. (2005), ‘International dimensions of optimal monetary policy’, *Journal of Monetary Economics* **52**(2), 281–305.

Den Haan, W. J., Ramey, G. & Watson, J. (2000), ‘Job destruction and propagation of shocks’, *American Economic Review* **90**(3), 482–498.

Diamond, P. A. (1982), ‘Aggregate demand management in search equilibrium’, *Journal of Political Economy* **90**(5), 881–894.

Drozd, L. A. & Nosal, J. B. (2012), ‘Understanding international prices: Customers as capital’, *American Economic Review* **102**(1), 364–95.

Faia, E. (2009), ‘Ramsey monetary policy with labor market frictions’, *Journal of Monetary Economics* **56**(4), 570–581.

Gertler, M., Sala, L. & Trigari, A. (2008), ‘An estimated monetary dsge model with unemployment and staggered nominal wage bargaining’, *Journal of Money, Credit and Banking* **40**(8), 1713–1764.

Gertler, M. & Trigari, A. (2009), ‘Unemployment fluctuations with staggered nash wage bargaining’, *Journal of Political Economy* **117**(1), 38–86.

Gottschalk, P. (2005), ‘Downward nominal-wage flexibility: Real or measurement error?’, *The Review of Economics and Statistics* **87**(3), 556–568.

Hairault, J.-O. (2002), ‘Labor-market search and international business cycles’, *Review of Economic Dynamics* **5**(3), 535–558.

Heathcote, J. & Perri, F. (2002), ‘Financial autarky and international business cycles’, *Journal of Monetary Economics* **49**(3), 601–627.

Hobijn, B. & Şahin, A. (2009), ‘Job-finding and separation rates in the oecd’, *Economics Letters* **104**(3), 107–111.

Hosios, A. J. (1990), 'On the efficiency of matching and related models of search and unemployment', *The Review of Economic Studies* **57**(2), 279–298.

Jacob, P. & Uusküla, L. (2019), 'Deep habits and exchange rate pass-through', *Journal of Economic Dynamics and Control* **105**, 67–89.

Krause, M. U. & Lubik, T. A. (2007), 'The (ir)relevance of real wage rigidity in the new keynesian model with search frictions', *Journal of Monetary Economics* **54**(3), 706–727.

Krugman, P. (1986), 'Pricing to market when the exchange rate changes', *National Bureau of Economic Research Working Paper Series* **No. 1926**.

Merz, M. (1995), 'Search in the labor market and the real business cycle', *Journal of Monetary Economics* **36**(2), 269–300.

Moen, E. R. (1997), 'Competitive search equilibrium', *Journal of Political Economy* **105**(2), 385–411.

Mortensen, D. T. (1982), 'Property rights and efficiency in mating, racing, and related games', *The American Economic Review* **72**(5), 968–979.

Mortensen, D. T. & Pissarides, C. A. (1994), 'Job creation and job destruction in the theory of unemployment', *The Review of Economic Studies* **61**(3), 397–415.

OECD (2006), *OECD Employment Outlook 2006*, OECD.

OECD (2022), 'Unemployment rate (indicator)'.

Petrosky-Nadeau, N., Wasmer, E. & Zeng, S. (2016), 'Shopping time', *Economics Letters* **143**(C), 52–60.

Pissarides, C. A. (1985), 'Short-run equilibrium dynamics of unemployment, vacancies, and real wages', *The American Economic Review* **75**(4), 676–690.

Pissarides, C. A. (2000), *Equilibrium unemployment theory*, MIT Press.

Pissarides, C. A. (2009), 'The unemployment volatility puzzle: Is wage stickiness the answer?', *Econometrica* **77**(5), 1339–1369.

Ravenna, F. & Walsh, C. E. (2011), 'Welfare-based optimal monetary policy with unemployment and sticky prices: A linear-quadratic framework', *American Economic Journal: Macroeconomics* **3**(2), 130–62.

Ruhl, K. J. (2008), 'The international elasticity puzzle', *Unpublished* .

Shimer, R. (2005), 'The cyclical behavior of equilibrium unemployment and vacancies', *American Economic Review* **95**(1), 25–49.

Sunakawa, T. (2015), 'Optimal monetary policy with labor market frictions: The role of the wage channel', *Journal of Money, Credit and Banking* **47**(6), 1119–1147.

Taylor, J. B. (1993), *Macroeconomic policy in a world economy: from econometric design to practical operation*, WW Norton.

Thomas, C. (2008), 'Search and matching frictions and optimal monetary policy', *Journal of Monetary Economics* **55**(5), 936–956.

Walsh, C. E. (2005), 'Labor market search, sticky prices, and interest rate policies', *Review of Economic Dynamics* **8**(4), 829–849.

Appendix A

Appendices For Chapter 1

Appendix A.1 Data Sources

I collect the data series of US and EU-15 from OECD Quarterly National Accounts (QNA), OECD Economic Outlook (EO), and Federal Reserve Economic Data (FRED) for the period 1976:1–2015:4. The EU-15 comprises 15 European countries, including Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, and United Kingdom. While data for GDP and consumption are Gross Domestic Product and Private plus Government Final Consumption Expenditure from QNA, respectively, data for employment come from EO. As employment series are not available for all European countries, I compute the series for EU as the aggregate of 12 European countries weighted with populations in 2015, except Greece, Ireland, and Luxembourg. I use the series real effective exchange rate from FRED for the US real exchange rate. Data for the terms of trade are computed by the ratio of import prices over export prices from QNA.

Appendix A.2 Wage Determination

To derive the aggregate wage equation, I first focus on the contract wage between a renegotiating firm and its workers.

A.2.1 Worker's surplus

Given the evolution of average wage, the renegotiating worker's surplus, $H_t(w_t^{con})$, is expressed as

$$\begin{aligned} H_t(w_t^{con}) &= w_t^{con} - b + \mathbb{E}_t\beta_{t,t+1}\left[(1-\rho)H_{t+1}(w_{t+1}^{con}) - s_{t+1}H_{x,t+1}\right] \\ &= w_t^{con} - b + \mathbb{E}_t\beta_{t,t+1}\{(1-\rho)\left[\lambda H_{t+1}(w_t^{con}) + (1-\lambda)H_{t+1}(w_{t+1}^{con})\right] \\ &\quad - s_{t+1}H_{x,t+1}\} \\ &= w_t^{con} - [b + \mathbb{E}_t\beta_{t,t+1}s_{t+1}H_{x,t+1}] \\ &\quad + \mathbb{E}_t\beta_{t,t+1}(1-\rho)H_{t+1}(w_{t+1}^{con}) \\ &\quad + (1-\rho)\lambda\mathbb{E}_t\beta_{t,t+1}\left[H_{t+1}(w_t^{con}) - H_{t+1}(w_{t+1}^{con})\right]. \end{aligned} \tag{A.1}$$

Consider $\mathbb{E}_t\left[H_{t+1}(w_t^{con}) - H_{t+1}(w_{t+1}^{con})\right]$.

$$\begin{aligned} &\mathbb{E}_t\left[H_{t+1}(w_t^{con}) - H_{t+1}(w_{t+1}^{con})\right] \\ = \; &\mathbb{E}_t\left[w_t^{con} - w_{t+1}^{con}\right] + (1-\rho)\lambda\mathbb{E}_t\beta_{t+1,t+2}\left[H_{t+2}(w_t^{con}) - H_{t+2}(w_{t+1}^{con})\right] \end{aligned} \tag{A.2}$$

Log-linearize and iterate forward.

$$\mathbb{E}_t\left[\widehat{H}_{t+1}(w_t^{con}) - \widehat{H}_{t+1}(w_{t+1}^{con})\right] = \frac{1}{1-(1-\rho)\lambda\beta}\left(\frac{w}{H}\right)\mathbb{E}_t\left[\widehat{w}_t^{con} - \widehat{w}_{t+1}^{con}\right] \tag{A.3}$$

Derive the worker's surplus by log-linearizing.

$$\begin{aligned} \widehat{H}_t(w_t^{con}) &= \left(\frac{w}{H}\right)\left[\widehat{w}_t^{con} + \frac{(1-\rho)\lambda\beta}{1-(1-\rho)\lambda\beta}\mathbb{E}_t\left[\widehat{w}_t^{con} - \widehat{w}_{t+1}^{con}\right]\right] \\ &\quad - \beta s\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right] \\ &\quad + (1-\rho)\beta\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}(w_{t+1}^{con})\right], \end{aligned} \tag{A.4}$$

where $\widehat{\Lambda}_{t,t+1} = \widehat{\lambda}_{t+1} - \widehat{\lambda}_t$.

A.2.2 Firm's surplus

Given the evolution of average wage, the renegotiating firm's surplus, $J_t(w_t^{con})$, is expressed as:

$$\begin{aligned} J_t(w_t^{con}) &= \theta_t a_t - w_t^{con} \\ &\quad + \mathbb{E}_t \beta_{t,t+1} \left[-\frac{\kappa}{2} x_{t+1}(w_t^{con})^2 + (1-\rho + x_{t+1}(w_t^{con})) J_{t+1}(w_t^{con}) \right] \\ &= \theta_t a_t - w_t^{con} + \mathbb{E}_t \beta_{t,t+1} \frac{\kappa}{2} x_{t+1}(w_{t+1}^{con})^2 \\ &\quad + \mathbb{E}_t \beta_{t,t+1} \frac{\kappa}{2} \lambda \left[x_{t+1}(w_t^{con})^2 - x_{t+1}(w_{t+1}^{con})^2 \right] \\ &\quad + (1-\rho) \mathbb{E}_t \beta_{t,t+1} J_{t+1}(w_{t+1}^{con}) \\ &\quad + (1-\rho) \mathbb{E}_t \beta_{t,t+1} \lambda \left[J_{t+1}(w_t^{con}) - J_{t+1}(w_{t+1}^{con}) \right] \end{aligned} \tag{A.5}$$

Consider the last term.

$$\begin{aligned} & \mathbb{E}_t \left[J_{t+1}(w_t^{con}) - J_{t+1}(w_{t+1}^{con}) \right] \\ = & \mathbb{E}_t \left[w_t^{con} - w_{t+1}^{con} \right] \\ & + \frac{\kappa}{2} \lambda \mathbb{E}_t \beta_{t+1,t+2} \left[x_{t+2}(w_t^{con})^2 - x_{t+2}(w_{t+1}^{con})^2 \right] \\ & + (1-\rho) \lambda \mathbb{E}_t \beta_{t+1,t+2} \left[J_{t+2}(w_t^{con}) - J_{t+2}(w_{t+1}^{con}) \right] \end{aligned} \tag{A.6}$$

Log-linearize and iterate forward by using $J_t(w_t^{con}) = \kappa x_t(w_t^{con})$ and $x = \rho$.

$$\mathbb{E}_t \left[\widehat{J}_{t+1}(w_t^{con}) - \widehat{J}_{t+1}(w_{t+1}^{con}) \right] = -\frac{1}{1-\lambda\beta} \left(\frac{w}{J} \right) \mathbb{E}_t \left[\widehat{w}_t^{con} - \widehat{w}_{t+1}^{con} \right] \tag{A.7}$$

Derive the firm's surplus by log-linearizing

$$\begin{aligned} \widehat{J}_t(w_t^{con}) &= \left(\frac{\theta a}{J} \right) (\widehat{\theta}_t + \widehat{a}_t) - \left(\frac{w}{J} \right) \left[\widehat{w}_t^{con} + \frac{\lambda\beta}{1-\lambda\beta} \mathbb{E}_t \left[\widehat{w}_t^{con} - \widehat{w}_{t+1}^{con} \right] \right] \\ &\quad + \frac{\beta x}{2} \mathbb{E}_t \left[\widehat{\Lambda}_{t,t+1} + 2\widehat{x}_{t+1}(w_{t+1}^{con}) \right] \\ &\quad + (1-\rho) \beta \mathbb{E}_t \left[\widehat{\Lambda}_{t,t+1} + \widehat{J}_{t+1}(w_{t+1}^{con}) \right] \end{aligned} \tag{A.8}$$

A.2.3 Contract wage

Log-linearize the surplus sharing rule

$$\widehat{J}_t(w_t^{con}) = \widehat{H}_t(w_t^{con}) \tag{A.9}$$

Solve the contract wage by using the worker's and the firm's surplus

$$\begin{aligned}
& \left(\frac{w}{H}\right)\left[\widehat{w}_t^{con} + \varepsilon \mathbb{E}_t\left[\widehat{w}_t^{con} - \widehat{w}_{t+1}^{con}\right]\right] + \left(\frac{w}{J}\right)\left[\widehat{w}_t^{con} + \mu \mathbb{E}_t\left[\widehat{w}_t^{con} - \widehat{w}_{t+1}^{con}\right]\right] \\
= & \left(\frac{\theta a}{J}\right)(\widehat{\theta}_t + \widehat{a}_t) + \frac{\beta x}{2}\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + 2\widehat{x}_{t+1}(w_{t+1}^{con})\right] \\
& + \beta s \mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right],
\end{aligned} \tag{A.10}$$

where $\varepsilon = \frac{(1-\rho)\lambda\beta}{1-(1-\rho)\lambda\beta}$ and $\mu = \frac{\lambda\beta}{1-\lambda\beta}$.

$$\begin{aligned}
& w\left(\frac{1}{H} + \frac{1}{J}\right)\widehat{w}_t^{con} + w\left(\frac{\varepsilon}{H} + \frac{\mu}{J}\right)\mathbb{E}_t\left[\widehat{w}_t^{con} - \widehat{w}_{t+1}^{con}\right] \\
= & \left(\frac{\theta a}{J}\right)(\widehat{\theta}_t + \widehat{a}_t) + \frac{\beta x}{2}\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + 2\widehat{x}_{t+1}(w_{t+1}^{con})\right] \\
& + \beta s \mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right]
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
\widehat{w}_t^{con} = & \left(\frac{(1-\eta)\varepsilon + \eta\mu}{1+(1-\eta)\varepsilon + \eta\mu}\right)\mathbb{E}_t \widehat{w}_{t+1}^{con} + \frac{\eta}{1+(1-\eta)\varepsilon+\eta\mu}\frac{\theta a}{w}(\widehat{\theta}_t + \widehat{a}_t) \\
& + \frac{\beta x}{2}\frac{\eta}{1+(1-\eta)\varepsilon+\eta\mu}\frac{J}{w}\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + 2\widehat{x}_{t+1}(w_{t+1}^{con})\right] \\
& + \beta s \frac{\eta}{1+(1-\eta)\varepsilon+\eta\mu}\frac{J}{w}\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right]
\end{aligned} \tag{A.12}$$

$$\begin{aligned}
\widehat{w}_t^{con} = & \tau \mathbb{E}_t \widehat{w}_{t+1}^{con} \\
& + (1-\tau)\{\eta \frac{\theta a}{w}(\widehat{\theta}_t + \widehat{a}_t) + \eta \frac{\beta x}{2}\frac{J}{w}\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + 2\widehat{x}_{t+1}(w_{t+1}^{con})\right] \\
& + \eta \beta s \frac{J}{w}\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right]\}
\end{aligned} \tag{A.13}$$

where $\tau = \frac{(1-\eta)\varepsilon+\eta\mu}{1+(1-\eta)\varepsilon+\eta\mu}$.

The contract wage can be expressed a linear combination of the target wage, $\widehat{w}_t^o(w_t^{con})$, and the future contract wage, $\mathbb{E}_t\widehat{w}_{t+1}^{con}$:

$$\widehat{w}_t^{con} = (1-\tau)\widehat{w}_t^o(w_t^{con}) + \tau\mathbb{E}_t\widehat{w}_{t+1}^{con} \tag{A.14}$$

with

$$\begin{aligned}\widehat{w}_t^o(w_t^{con}) &= \eta\Psi_a(\widehat{\theta}_t + \widehat{a}_t) + \eta\Psi_x\mathbb{E}_t\left[\frac{1}{2}\widehat{\Lambda}_{t,t+1} + \widehat{x}_{t+1}(w_{t+1}^{con})\right] \\ &\quad + \eta\Psi_s\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{H}_{x,t+1}\right]\end{aligned} \tag{A.15}$$

where $\Psi_a = \theta a/w, \Psi_x = \beta xJ/2w, \Psi_s = \beta sJ/w$.

A.2.4 Target wage

Consider $\mathbb{E}_t\left[\widehat{x}_{t+1}(w_t^{con}) - \widehat{x}_{t+1}(w_{t+1}^{con})\right]$.

$$\mathbb{E}_t\left[\widehat{x}_{t+1}(w_t^{con}) - \widehat{x}_{t+1}(w_{t+1}^{con})\right] = -\frac{1}{1-\lambda\beta}\left(\frac{w}{J}\right)\mathbb{E}_t\left[\widehat{w}_t^{con} - \widehat{w}_{t+1}^{con}\right] \tag{A.16}$$

Derive $\mathbb{E}_t\widehat{x}_{t+1}(w_{t+1}^{con})$.

$$\mathbb{E}_t\widehat{x}_{t+1}(w_{t+1}^{con}) = \mathbb{E}_t\widehat{x}_{t+1}(w_{t+1}) + (1+\mu)\left(\frac{w}{J}\right)\mathbb{E}_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^{con}\right] \tag{A.17}$$

Consider $\mathbb{E}_t\left[\widehat{H}_{t+1}(w_{t+1}^{con}) - \widehat{H}_{t+1}(w_{t+1})\right]$.

$$\mathbb{E}_t\left[\widehat{H}_{t+1}(w_{t+1}^{con}) - \widehat{H}_{t+1}(w_{t+1})\right] = (1+\varepsilon)\left(\frac{w}{H}\right)\mathbb{E}_t\left[\widehat{w}_{t+1}^{con} - \widehat{w}_{t+1}\right] \tag{A.18}$$

Derive $\mathbb{E}_t\widehat{H}_{t+1}(w_{t+1})$ by using above equation.

$$\begin{aligned}\mathbb{E}_t\widehat{H}_{t+1}(w_{t+1}) &= \mathbb{E}_t\widehat{H}_{t+1}(w_{t+1}^{con}) - (1-\varepsilon)\left(\frac{w}{H}\right)\mathbb{E}_t\left[\widehat{w}_{t+1}^{con} - \widehat{w}_{t+1}\right] \\ &= \mathbb{E}_t\widehat{x}_{t+1}(w_{t+1}) + \frac{1}{\eta(1-\tau)}\left(\frac{w}{J}\right)\mathbb{E}_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^{con}\right]\end{aligned} \tag{A.19}$$

Since $\mathbb{E}_t\widehat{H}_{x,t+1} = \mathbb{E}_t\widehat{H}_{t+1}(w_{t+1})$ up to the first order approximation,

$$\mathbb{E}_t\widehat{H}_{t+1} = \mathbb{E}_t\widehat{x}_{t+1}(w_{t+1}) + \frac{1}{\eta(1-\tau)}\left(\frac{w}{J}\right)\mathbb{E}_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^{con}\right]. \tag{A.20}$$

Derive target wage by substituting $\mathbb{E}_t\widehat{x}_{t+1}(w_{t+1}^{con})$ and $\mathbb{E}_t\widehat{H}_{x,t+1}$

$$\begin{aligned}
& \widehat{w}_t^o(w_t^{con}) \\
= \;& \eta\Psi_a(\widehat{\theta}_t+\widehat{a}_t) \\
& +\eta\Psi_x\mathbb{E}_t\left[\frac{1}{2}\widehat{\Lambda}_{t,t+1}+\widehat{x}_{t+1}(w_{t+1})+(1+\mu)\left(\frac{w}{J}\right)\left[\widehat{w}_{t+1}-\widehat{w}_{t+1}^{con}\right]\right] \\
& +\eta\Psi_s\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1}+\widehat{s}_{t+1}+\widehat{x}_{t+1}(w_{t+1})+\frac{1}{\eta(1-\tau)}\left(\frac{w}{J}\right)\left[\widehat{w}_{t+1}-\widehat{w}_{t+1}^{con}\right]\right] \\
= \;& \widehat{w}_t^o+\left[\eta\beta x(1+\mu)+\frac{\beta s}{1-\tau}\right]\mathbb{E}_t\left[\widehat{w}_{t+1}-\widehat{w}_{t+1}^{con}\right] \\
= \;& \widehat{w}_t^o+\frac{\tau_1}{1-\tau}\mathbb{E}_t\left[\widehat{w}_{t+1}-\widehat{w}_{t+1}^{con}\right] \qquad \text{(A.21)}
\end{aligned}$$

with

$$\begin{aligned}
\widehat{w}_t^o \;=\; & \eta\Psi_a(\widehat{\theta}_t+\widehat{a}_t)+\eta\Psi_x\mathbb{E}_t\left[\frac{1}{2}\widehat{\Lambda}_{t,t+1}+\widehat{x}_{t+1}(w_{t+1})\right] \\
& +\eta\Psi_s\mathbb{E}_t\left[\widehat{\Lambda}_{t,t+1}+\widehat{s}_{t+1}+\widehat{x}_{t+1}(w_{t+1})\right] \qquad \text{(A.22)}
\end{aligned}$$

where $\tau_1=\eta\beta x(1+\mu)(1-\tau)+\beta s$.

$\widehat{w}_t^o$ denotes spillover-free target wage which would arise if all firms and workers were negotiating a period-by-period wage contract. This is different from the target wage, $\widehat{w}_t^o(w_t^{con})$, which is computed taking as given that all other firms and workers in the economy are operating on multi-period wage contracts.

A.2.5 Average wage

Log-linearize the evolution of the average wage.

$$\widehat{w}_{t+1}=(1-\lambda)\widehat{w}_{t+1}^{con}+\lambda\widehat{w}_t \qquad \text{(A.23)}$$

Consider the contract wage equation.

$$\begin{aligned}
\widehat{w}_t^{con} \;&=\; (1-\tau)\widehat{w}_t^o(w_t^{con})+\tau\mathbb{E}_t\widehat{w}_{t+1}^{con} \\
&=\; (1-\tau)\widehat{w}_t^o+\tau_1\mathbb{E}_t\widehat{w}_{t+1}+(\tau-\tau_1)\mathbb{E}_t\widehat{w}_{t+1}^{con} \qquad \text{(A.24)}
\end{aligned}$$

$$[1+\lambda(\tau-\tau_1)]\,\widehat{w}_t=\lambda\widehat{w}_{t-1}+(1-\tau)(1-\lambda)\widehat{w}_t^o+(\tau-\lambda\tau_1)\mathbb{E}_t\widehat{w}_{t+1} \qquad \text{(A.25)}$$

Derive the average wage dynamic

$$\widehat{w}_t = \gamma_b \widehat{w}_{t-1} + \gamma_o \widehat{w}_t^o + \gamma_f \mathbb{E}_t \widehat{w}_{t+1}, \tag{A.26}$$

where $\gamma_b \equiv \lambda/\phi$, $\gamma_o \equiv (1-\tau)(1-\lambda)/\phi$, $\gamma_f \equiv (\tau - \lambda\tau_1)/\phi$ and $\phi \equiv 1 + \lambda(\tau - \tau_1)$.

Appendix A.3 Complete log-linearized model

Table A.1 Home Economy

Technology	$\hat{y}_t = \hat{a}_t + \hat{n}_t$
Euler equation	$0 = \mathbb{E}_t \widehat{Q}_{t,t+1} - (\widehat{c}_t - \mathbb{E}_t \widehat{c}_{t+1})$
Marginal utility	$\mathbb{E}_t \widehat{\Lambda}_{t,t+1} = \widehat{c}_t - \mathbb{E}_t \widehat{c}_{t+1}$
Resource constraint	$\widehat{y}_t = \omega \left(\frac{c}{y}\right) \left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\theta\right]^{-z} \left(\widehat{c}_t - z\widehat{\theta}_t\right)$
	$+ (1-\omega)\left(\frac{c}{y}\right) q^{z-1} \left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\theta\right]^{-z} \left[\widehat{c}_t - z\widehat{\theta}_t + (z-1)\widehat{q}_t\right]$
	$+ \frac{\kappa}{2}\left(\frac{x^2 n}{y}\right)(2\widehat{x}_t + \widehat{n}_{t-1})$
Matching	$\widehat{m}_t = \sigma \widehat{u}_t + (1-\sigma)\widehat{v}_t$
Vacancy filling probability	$\widehat{j}_t = \widehat{m}_t - \widehat{v}_t$
Job finding probability	$\widehat{s}_t = \widehat{m}_t - \widehat{u}_t$
Job creation	$\widehat{x}_t = \left(\frac{\theta a}{\kappa x}\right)\left(\widehat{\theta}_t + \widehat{a}_t\right) - \left(\frac{w}{\kappa x}\right)\widehat{w}_t + \beta\left(1 - \frac{\rho}{2}\right)\mathbb{E}_t \widehat{\Lambda}_{t,t+1} + \beta \mathbb{E}_t \widehat{x}_{t+1}$
Average wage	$\widehat{w}_t = \gamma_b \widehat{w}_{t-1} + \gamma_o \widehat{w}_t^o + \gamma_f \mathbb{E}_t \widehat{w}_{t+1}$
Hiring rate	$\widehat{x}_t = \widehat{j}_t + \widehat{v}_t - \widehat{n}_{t-1}$
Employment	$\widehat{n}_t = \widehat{n}_{t-1} + \rho \widehat{x}_t$
Unemployment	$\widehat{u}_t = -\left(\frac{n}{u}\right)\widehat{n}_{t-1}$
Real marginal cost	$\widehat{\theta}_t = -\left[\frac{(1-\omega)T^{1-z}}{\omega + (1-\omega)T^{1-z}}\right]\widehat{T}_t$
Terms of trade	$\widehat{T}_t = \widehat{q}_t + \widehat{\theta}_t^* - \widehat{\theta}_t$
Real exchange rate	$\widehat{q}_t = \widehat{c}_t - \widehat{c}_t^*$

Table A.2 Foreign Economy

Technology	$\widehat{y_t^*}=\widehat{a_t^*}+\widehat{n_t^*}$
Euler equation	$0=\mathbb{E}_t\widehat{Q^*}_{t,t+1}-\left(\widehat{c}_t^*-\mathbb{E}_t\widehat{c}_{t+1}^*\right)$
Marginal utility	$\mathbb{E}_t\hat{\Lambda}_{t,t+1}^*=\hat{c}_t^*-\mathbb{E}_t\hat{c}_{t+1}^*$
Resource constraint	$\hat{y}_t^*=\omega\left(\frac{c^*}{y^*}\right)\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\theta^*\right]^{-z}\left(\widehat{c^*}_t-z\widehat{\theta^*}_t\right)$
	$+(1-\omega)\left(\frac{c^*}{y^*}\right)q^{1-z}\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\theta^*\right]^{-z}\left[\widehat{c^*}_t-z\widehat{\theta^*}_t+(1-z)\widehat{q}_t\right]$
	$+\frac{\kappa}{2}\left(\frac{x^{*2}n^*}{y^*}\right)\left(2\widehat{x^*}_t+\widehat{n^*}_{t-1}\right)$
Matching	$\hat{m}_t^*=\sigma^*\hat{u}_t^*+(1-\sigma^*)\,\hat{v}_t^*$
Vacancy filling probability	$\widehat{j}_t^*=\hat{m}_t^*-\hat{v}_t^*$
Job finding probability	$\widehat{s}_t^*=\hat{m}_t^*-\hat{u}_t^*$
Job creation	$\widehat{x}_t^*=\left(\frac{\theta^*a^*}{J^*}\right)(\widehat{\theta^*}_t+\hat{a}_t^*)-\left(\frac{w^*}{J^*}\right)\hat{w}_t^*+\beta\left(1-\frac{\rho^*}{2}\right)\mathbb{E}_t\hat{\Lambda}_{t,t+1}^*+\beta\mathbb{E}_t\widehat{x^*}_{t+1}$
Average wage	$\hat{w}_t^*=\gamma_b^*\hat{w}_{t-1}^*+\gamma_o^*\hat{w}_t^{o*}+\gamma_f^*\mathbb{E}_t\hat{w}_{t+1}^*$
Hiring rate	$\widehat{x}_t^*=\widehat{j}_t^*+\hat{v}_t^*-\hat{n}_{t-1}^*$
Employment	$\hat{n}_t^*=\hat{n}_{t-1}^*+\rho^*\widehat{x}_t^*$
Unemployment	$\hat{u}_t^*=-(\frac{n^*}{u^*})\hat{n}_{t-1}^*$
Real marginal cost	$\widehat{\theta}_t^*=\left[\frac{(1-\omega)T^{z-1}}{\omega+(1-\omega)T^{z-1}}\right]\widehat{T}_t$

Appendix B

Appendices For Chapter 2

Appendix B.1 Relation between $\mathbf{R}_t$ and $\mathbf{x}_t$

Assume the cash-in-advance constraint is binding

$$P_t c_t = M_t - D_t \tag{B.1}$$

So, households' BC can be rewritten as:

$$M_{t+1} = R_t D_t + P_t w_t n_t + (1 - n_t) P_t b + R_t X_t + \Pi_t - T_t \tag{B.2}$$

In equilibrium, government's fiscal policy is balanced and firms' profits are equal to zero since firms sell their goods in the perfectly competitive markets. Accordingly, the household's BC is given by

$$\begin{aligned} M_{t+1} &= R_t D_t + R_t X_t + P_t w_t n_t \\ &= R_t D_t + R_t X_t + D_t + X_t \end{aligned} \tag{B.3}$$

due to the loan market clearing condition,

$$D_t + X_t = P_t w_t n_t . \tag{B.4}$$

Thus,

$$R_t = \frac{M_{t+1}}{D_t + X_t} - 1 = \frac{1 + x_t}{d_t + x_t} - 1 = \frac{1 - d_t}{d_t + x_t}, \tag{B.5}$$

where $d_t \equiv D_t / M_{t-1}$.

With an analogous process, the following relation for the foreign country is derived:

$$R_t^* = \frac{M_t^*}{D_t^* + X_t^*} = \frac{1 + x_t^*}{d_t^* + x_t^*}. \tag{B.6}$$

Appendix B.2 Ramsey problem

For given the exogenous aggregate productivity, a_t, a_t^*, and R_t^*,

choose $\{c_t, c_t^*, n_t, n_t^*, v_t, v_t^*, \theta_t, \theta_t^*, w_t, w_t^*, \Phi_t, \Phi_t^*, \pi_{t+1}, \pi_{t+1}^*, q_t, TOT_t, R_t\}_{t=0}^{\infty}$ to solve

$$\begin{aligned}
&\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{\ln c_t \\
&+\delta_{1,t} \left[\beta \mathbb{E}_{t-1} R_t \frac{1}{c_{t+1}} \frac{1}{\pi_{t+1}} - \frac{1}{c_t}\right] \\
&+\delta_{2,t} \left[a_t n_t - \omega \Phi_t^z c_t - (1-\omega) \Phi_t^z q_t^z c_t^* - \Phi_t \kappa v_t\right] \\
&+\delta_{3,t} \left[v_t - [1-(1-\rho)n_{t-1}] \theta_t\right] \\
&+\delta_{4,t} \left[(1-\rho)n_{t-1} + \sigma_m \theta_t^{-\sigma} v_t - n_t\right] \\
&+\delta_{5,t} \left[\Phi_t^{-1} a_t - R_t w_t + (1-\rho)\mathbb{E}_t \beta_{t,t+1} \left(\frac{\kappa}{\sigma_m} \theta_{t+1}^{\sigma}\right) - \frac{\kappa}{\sigma_m} \theta_t^{\sigma}\right] \\
&+\delta_{6,t} \left[\frac{1}{\eta R_t + (1-\eta)} \left[\eta \Phi_t^{-1} a_t + (1-\eta) b + \eta (1-\rho) \mathbb{E}_t \beta_{t,t+1} \kappa \theta_{t+1}\right] - w_t\right] \\
&+\delta_{7,t} \left[\beta \mathbb{E}_{t-1} R_t^* \frac{1}{c_{t+1}^*} \frac{1}{\pi_{t+1}^*} - \frac{1}{c_t^*}\right] \\
&+\delta_{8,t} \left[a_t^* n_t^* - \omega \Phi_t^{*z} c_t^* - (1-\omega) \Phi_t^{*z} q_t^{-z} c_t - \Phi_t^* \kappa v_t^*\right] \\
&+\delta_{9,t} \left[v_t^* - \left[1-(1-\rho)n_{t-1}^*\right] \theta_t^*\right] \\
&+\delta_{10,t} \left[(1-\rho)n_{t-1}^* + \sigma_m \theta_t^{*-\sigma} v_t^* - n_t^*\right] \\
&+\delta_{11,t} \left[\Phi_t^{*-1} a_t^* - R_t^* w_t^* + (1-\rho)\mathbb{E}_t \beta_{t,t+1} \left(\frac{\kappa}{\sigma_m} \theta_{t+1}^{*\sigma}\right) - \frac{\kappa}{\sigma_m} \theta_t^{*\sigma}\right] \\
&+\delta_{12,t} \left[\frac{1}{\eta R_t^* + (1-\eta)} \left[\eta \Phi_t^{*-1} a_t^* + (1-\eta) b^* + \eta (1-\rho) \mathbb{E}_t \beta_{t,t+1} \kappa \theta_{t+1}^*\right] - w_t^*\right] \\
&+\delta_{13,t} \left[q_t - \Phi_t^* \Phi_t^{-1} TOT_t\right] \\
&+\delta_{14,t} \left[\Phi_t^{1-z} - \omega - (1-\omega) TOT_t^{1-z}\right] \\
&+\delta_{15,t} \left[\Phi_t^{*1-z} - \omega - (1-\omega) TOT_t^{z-1}\right] \\
&+\delta_{16,t} \left[c_t^* - \Phi_t^{1-z} \Phi_t^{*z-1} q_t^{1-2z} c_t\right]\}.
\end{aligned}$$

Taking into account the unbined constraints ($\delta_{1,t} = \delta_{5,t} = \delta_{6,t} = \delta_{7,t} = 0$) , the problem is reduced to choose $\{c_t, c_t^*, n_t, n_t^*, v_t, v_t^*, \theta_t, \theta_t^*, w_t^*, \Phi_t, \Phi_t^*, q_t, TOT_t\}_{t=0}^{\infty}$.

The first order conditions for each variable are:

1) c_t

$$\frac{1}{c_t} - \delta_{2,t}\omega\Phi_t^z - \delta_{8,t}(1-\omega)\Phi_t^{*z}q_t^{-z} - \delta_{16,t}\Phi_t^{1-z}\Phi_t^{*z-1}q_t^{1-2z} = 0$$

2) n_t

$$\delta_{2,t}a_t + \beta\mathbb{E}_t\delta_{3,t+1}(1-\rho)\theta_t - \delta_{4,t} + \beta\mathbb{E}_t\delta_{4,t+1}(1-\rho) = 0$$

3) v_t

$$-\delta_{2,t}\Phi_t\kappa + \delta_{3,t} + \delta_{4,t}\sigma_m\theta_t^{-\sigma} = 0$$

4) θ_t

$$\delta_{3,t}[1-(1-\rho)n_{t-1}] + \delta_{4,t}\sigma\sigma_m\theta_t^{-\sigma-1}v_t = 0$$

5) c_t^*

$$-\delta_{2,t}(1-\omega)\Phi_t^z q_t^z - \delta_{8,t}\omega\Phi_t^{*z} + \delta_{16,t} = 0$$

6) n_t^*

$$\delta_{8,t}a_t^* + \beta\mathbb{E}_t\delta_{9,t+1}(1-\rho)\theta_t^* - \delta_{10,t} + \beta\mathbb{E}_t\delta_{10,t+1}(1-\rho) = 0$$

7) v_t^*

$$-\delta_{8,t}\Phi_t^*\kappa + \delta_{9,t} + \delta_{10,t}\sigma_m\theta_t^{*-\sigma} = 0$$

8) θ_t^*

$$\begin{aligned}
&-\delta_{9,t}\left[1-(1-\rho)n_{t-1}^*\right] - \delta_{10,t}\sigma\sigma_m\theta_t^{*-\sigma-1}v_t^* - \delta_{11,t}\left(\sigma\frac{\kappa}{\sigma_m}\theta_t^{*\sigma-1}\right)\\
&+\frac{1}{\beta}\delta_{11,t-1}(1-\rho)\mathbb{E}_{t-1}\beta_{t-1,t}\sigma\frac{\kappa}{\sigma_m}\theta_t^{*\sigma-1}\\
&+\frac{1}{\beta}\delta_{12,t-1}\mathbb{E}_{t-1}\beta_{t-1,t}\frac{\eta(1-\rho)\kappa}{\eta R_{t-1}^* + (1-\eta)} = 0
\end{aligned}$$

9) w_t^*

$$\delta_{11,t}R_t^* + \delta_{12,t}\frac{1}{\eta R_t^* + (1-\eta)} = 0$$

10) Φ_t

$$-\delta_{2,t}\left(\omega z\Phi_t^{z-1}c_t+\kappa v_t\right)+\delta_{13,t}\Phi_t^{-2}\Phi_t^{*}TOT_t+\delta_{14,t}\left(1-z\right)\Phi_t^{-z}$$
$$-\delta_{16,t}\left(1-z\right)\Phi_t^{-z}\Phi_t^{*z-1}q_t^{1-2z}c_t=0$$

11) Φ_t^*

$$-\delta_{8,t}\left(\omega z\Phi_t^{*z-1}+\kappa v_t^{*}\right)-\delta_{11,t}a_t^{*}\Phi_t^{*-2}-\delta_{12,t}\frac{\eta a_t^{*}}{\eta R_t^{*}+(1-\eta)}\Phi_t^{*-2}$$
$$-\delta_{13,t}\Phi_t^{-1}TOT_t+\delta_{15,t}\left(1-z\right)\Phi_t^{*-z}-\delta_{16,t}\left(z-1\right)\Phi_t^{1-z}\Phi_t^{*z-2}q_t^{1-2z}c_t=0$$

12) q_t

$$-\delta_{2,t}\left(1-\omega\right)z\Phi_t^{z}q_t^{z-1}c_t^{*}+\delta_{8,t}\left(1-\omega\right)z\Phi_t^{*z}q_t^{-z-1}c_t$$
$$+\delta_{13,t}-\delta_{16,t}\left(1-2z\right)\Phi_t^{1-z}\Phi_t^{*z-1}q_t^{-2z}c_t=0$$

13) TOT_t

$$-\delta_{13,t}\Phi_t^{-1}\Phi_t^{*}-\delta_{14,t}\left(1-z\right)\left(1-\omega\right)T_t^{-z}-\delta_{15,t}\left(z-1\right)\left(1-\omega\right)T_t^{z-2}=0.$$

Appendix C

Appendices For Chapter 3

Appendix C.1 Firm's optimal choice

C.1.1 Domestic market

Following Blanchard & Galí (2010), I assume workers are immediately productive, such that employment, $l_{h,t}$, evolves according to, $l_{h,t} = (1-\rho)\, l_{t-1} + \Phi_t^v v_{h,t}$, and $\rho \in (0,1)$ is the exogenous rate of job destruction.

To derive FOCs of a firm j in the domestic market, I need to solve the following optimization problem taking $P_{h,t}$ as given:

$$
\begin{aligned}
&\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\lambda_t}{\lambda_0}\right) \{P_{h,t}(j)\, \Phi_{h,t}^n(j)\, y_{h,t}(j) - W_t l_{h,t}(j) - \kappa P_{h,t} v_{h,t}(j) \\
&+\xi_t \left[J\left(a_t; P_{h,t}(j), y_{h,t}(j), \theta_{h,t}(j)\right) - \mathscr{J}_h(a_t)\right] \\
&+\mu_t \left[a_t f\left(l_{h,t}(j)\right) - y_{h,t}(j)\right] \\
&+\gamma_t \left[(1-\rho)\, l_{h,t-1}(j) + \Phi_t^v v_{h,t}(j) - l_{h,t}(j)\right]\}
\end{aligned} \tag{C.1}
$$

First, consider the choice of $\left\{v_{h,t}(j), l_{h,t}(j)\right\}$. I find,

$$
\gamma_t = \left(\frac{\kappa}{\Phi_t^v}\right) P_{h,t} \text{ and } \gamma_t = \mu_t a_t f'\left(l_{h,t}(j)\right) - W_t + (1-\rho)\, \mathbb{E}_t \beta_{t,t+1} \gamma_{t+1}
$$

$$\rightarrow \left(\frac{\kappa}{\Phi_t^v}\right) P_{h,t} = \mu_t a_t f'\left(l_{h,t}(j)\right) - W_t + (1-\rho)\mathbb{E}_t \beta_{t,t+1}\left(\frac{\kappa}{\Phi_{t+1}^v}\right) P_{h,t+1} \tag{C.2}$$

where $\beta_{t,t+1}$ denotes the stochastic discount factor between period t and $t+1$, which is given by $\beta_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$.

Now consider the choice of $\left\{P_{h,t}(j), y_{h,t}(j), \theta_{h,t}(j)\right\}$. I find,

$$P_{h,t}(j) \;:\; \Phi_{h,t}^n(j)\, y_{h,t}(j) = -\xi_t J_{P_h} \tag{C.3}$$

$$y_{h,t}(j) \;:\; \mu_t = P_{h,t}(j)\,\Phi_{h,t}^n(j) + \xi_t J_{y_h} \tag{C.4}$$

$$\theta_{h,t}(j) \;:\; \varphi\Phi_{h,t}^n(j)\, P_{h,t}(j)\, y_{h,t}(j) = \xi_t\theta_{h,t}(j)\, J_{\theta_h} \tag{C.5}$$

respectively. As all firms in the domestic market are identical, I can eliminate the $'j'$ in the FOC equations.

$$y_{h,t}\Phi_{h,t}^n = -\xi_t J_{P_h} \tag{C.6}$$

$$\mu_t = P_{h,t}\Phi_{h,t}^n + \xi_t J_{y_h} \tag{C.7}$$

$$\varphi\Phi_{h,t}^n P_{h,t} y_{h,t} = \xi_t\theta_{h,t} J_{\theta_h} \tag{C.8}$$

Recall the participation constraint in the domestic market from the households' problem.

$$J_{P_h} = -\lambda_t c_{h,t} \tag{C.9}$$

$$J_{\theta_h}\theta_{h,t} = (1-\varphi) J_{y_h} y_{h,t} \tag{C.10}$$

$$J_{y_h} = \left[u_{c_h}(t) - \lambda_t P_{h,t}\right]\Phi_{h,t}^s s_{h,t} \tag{C.11}$$

For the price equation in the domestic market, use (C.7), (C.8) and (C.10) to eliminate ξ_t from the firms optimal conditions,

$$\mu_t = P_{h,t}\Phi_{h,t}^n + \xi_t J_{y_h} \quad \text{and} \quad \varphi\Phi_{h,t}^n P_{h,t} y_{h,t} = \xi_t(1-\varphi) J_{y_h} y_{h,t}$$

$$\rightarrow \mu_t = P_{h,t}\Phi_{h,t}^n\left(\frac{1}{1-\varphi}\right) \tag{C.12}$$

Thus, the job creation condition in the domestic market is:

$$\left(\frac{\kappa}{\Phi_t^v}\right) P_{h,t} = \left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^n a_t f'\left(l_{h,t}\right) - W_t + (1-\rho)\mathbb{E}_t\beta_{t,t+1}\left(\frac{\kappa}{\Phi_{t+1}^v}\right) P_{h,t+1} \tag{C.13}$$

For the search effort in the domestic market, use (C.6), (C.8) and (C.10) to eliminate ξ_t:

$$\begin{aligned} y_{h,t}\Phi_{h,t}^n &= -\xi_t J_{P_h} \text{ and } \varphi\Phi_{h,t}^n P_{h,t} y_{h,t} = \xi_t \theta_{h,t} J_{\theta_h} \\ &\rightarrow \; y_{h,t}\Phi_{h,t}^n = -\varphi \frac{\Phi_{h,t}^n P_{h,t} y_{h,t}}{J_{\theta_h}\theta_{h,t}} J_{P_h} \\ &\rightarrow \; -J_{P_h} P_{h,t} = \frac{1}{\varphi} J_{\theta_h}\theta_{h,t} = \frac{1}{\varphi}(1-\varphi) J_{y_h} y_{h,t} \end{aligned} \tag{C.14}$$

Eliminating J_{y_h} by using (C.11), I write:

$$-J_{P_h} P_{h,t} = \frac{1-\varphi}{\varphi}\left[u_{c_h}(t) - \lambda_t P_{h,t}\right] \Phi_{h,t}^s s_{h,t} y_{h,t} \tag{C.15}$$

Finally, use (C.9) and $u_{c_h}(t) + \frac{u_{s_h}(t)}{\Phi_{h,t}^s y_{h,t}} = \lambda_t P_{h,t}$ to eliminate $J_{P_h} P_{h,t}$. I find:

$$\begin{aligned} \varphi c_{h,t}\lambda_t P_{h,t} &= (1-\varphi)\left[u_{c_h}(t) - \lambda_t P_{h,t}\right] \Phi_{h,t}^s s_{h,t} y_{h,t} \\ \varphi c_{h,t}\left[u_{c_h}(t) + \frac{u_{s_h}(t)}{\Phi_{h,t}^s y_{h,t}}\right] &= (1-\varphi)\left[-\frac{u_{s_h}(t)}{\Phi_{h,t}^s y_{h,t}}\right] \Phi_{h,t}^s s_{h,t} y_{h,t} \\ -s_{h,t} u_{s_h}(t) &= \varphi c_{h,t} u_{c_h}(t)\left[\frac{s_{h,t}\Phi_{h,t}^s y_{h,t}}{\varphi c_{h,t} + (1-\varphi) s_{h,t}\Phi_{h,t}^s y_{h,t}}\right] \end{aligned} \tag{C.16}$$

Using the shopping constraint, I can also write the search effort as:

$$s_{h,t} = -\varphi c_{h,t}\frac{u_{c_h}(t)}{u_{s_h}(t)}\left[\frac{c_{h,t} + n_{h,t}\kappa v_{h,t}}{c_{h,t} + (1-\varphi) n_{h,t}\kappa v_{h,t}}\right] \tag{C.17}$$

C.1.2 Export market

To derive FOCs of a firm j in the export market, I need to solve the following optimization problem, taken $P_{h,t}$ as given:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\lambda_t}{\lambda_0}\right) \{ e_t P^*_{h,t}(j) \Phi^{*n}_{h,t}(j) y^*_{h,t}(j) - W_t l^*_{h,t}(j) - \kappa P_{h,t} v^*_{h,t}(j)$$
$$+\xi_t \left[J\left(a_t; P^*_{h,t}(j), y^*_{h,t}(j), \theta^*_{h,t}(j)\right) - \mathscr{J}_{h^*}(a_t) \right]$$
$$+\mu_t \left[a_t f\left(l^*_{h,t}(j)\right) - y^*_{h,t}(j) \right]$$
$$+\gamma_t \left[(1-\rho) l^*_{h,t-1}(j) + v^*_{h,t}(j) \Phi^v_t - l^*_{h,t}(j) \right] \} \tag{C.18}$$

First, consider the choice of $\left\{ v^*_{h,t}(j), l^*_{h,t}(j) \right\}$. I find,

$$\gamma_t = \left(\frac{\kappa}{\Phi^v_t}\right) P_{h,t} \text{ and } \gamma_t = \mu_t a_t f'\left(l^*_{h,t}(j)\right) - W_t + (1-\rho)\mathbb{E}_t \beta_{t,t+1} \gamma_{t+1}$$

$$\rightarrow \left(\frac{\kappa}{\Phi^v_t}\right) P_{h,t} = \mu_t a_t f'\left(l^*_{h,t}(j)\right) - W_t + (1-\rho)\mathbb{E}_t \beta_{t,t+1} \left(\frac{\kappa}{\Phi^v_{t+1}}\right) P_{h,t+1} \tag{C.19}$$

Now, consider the choice of $\left\{ P^*_{h,t}(j), y^*_{h,t}(j), \theta^*_{h,t}(j) \right\}$. I find,

$$P^*_{h,t}(j) : e_t \Phi^{*n}_{h,t}(j) y^*_{h,t}(j) = -\xi_t J_{P^*_h} \tag{C.20}$$
$$y^*_{h,t}(j) : \mu_t = e_t P^*_{h,t}(j) \Phi^{*n}_{h,t}(j) + \xi_t J_{y^*_h} \tag{C.21}$$
$$\theta^*_{h,t}(j) : \varphi e_t P^*_{h,t}(j) \Phi^{*n}_{h,t}(j) y^*_{h,t}(j) = \xi_t \theta^*_{h,t}(j) J_{\theta^*_h} \tag{C.22}$$

respectively. As all firms in the export market are identical, I can eliminate the $'j'$ in the FOC equations.

$$e_t \Phi^{*n}_{h,t} y^*_{h,t} = -\xi_t J_{P^*_h} \tag{C.23}$$
$$\mu_t = e_t P^*_{h,t} \Phi^{*n}_{h,t} + \xi_t J_{y^*_h} \tag{C.24}$$
$$\varphi e_t P^*_{h,t} \Phi^{*n}_{h,t} y^*_{h,t} = \xi_t \theta^*_{h,t} J_{\theta^*_h} \tag{C.25}$$

Recall the participation constraint in the export market from the foreign households' problem.

$$
\begin{aligned}
J_{P_h^*} &= -\lambda_t^* c_{h,t}^* && \text{(C.26)} \\
J_{\theta_h^*} \theta_{h,t}^* &= (1-\varphi) J_{y_h^*} y_{h,t}^* && \text{(C.27)} \\
J_{y_h^*} &= \left[u_{c_h^*}(t) - \lambda_t^* P_{h,t}^* \right] \Phi_{h,t}^{*s} s_{h,t}^* && \text{(C.28)}
\end{aligned}
$$

For the price equation in the export market, use (C.24), (C.25) and (C.27) to eliminate ξ_t from the firms optimal conditions,

$$
\begin{aligned}
\mu_t &= e_t P_{h,t}^* \Phi_{h,t}^{*n} + \xi_t J_{y_h^*} \quad \text{and} \quad \varphi e_t P_{h,t}^* \Phi_{h,t}^{*n} y_{h,t}^* = \xi_t (1-\varphi) J_{y_h^*} y_{h,t}^* \\
&\rightarrow \mu_t = e_t P_{h,t}^* \Phi_{h,t}^{*n} \left(\frac{1}{1-\varphi} \right)
\end{aligned}
$$

The job creation condition in the export market is:

$$
\left(\frac{\kappa}{\Phi_t^v} \right) P_{h,t} = \left(\frac{1}{1-\varphi} \right) e_t P_{h,t}^* \Phi_{h,t}^{*n} a_t f'\left(l_{h,t}^*\right) - W_t + (1-\rho) \mathbb{E}_t \beta_{t,t+1} \left(\frac{\kappa}{\Phi_{t+1}^v} \right) P_{h,t+1} \tag{C.29}
$$

Since wage is common in the domestic and the export markets, I know:

$$
\begin{aligned}
& \left(\frac{1}{1-\varphi} \right) P_{h,t} \Phi_{h,t}^n a_t f'\left(l_{h,t}\right) - \left(\frac{\kappa}{\Phi_t^v} \right) P_{h,t} + (1-\rho) \mathbb{E}_t \beta_{t,t+1} \left(\frac{\kappa}{\Phi_{t+1}^v} \right) P_{h,t+1} \\
= & \left(\frac{1}{1-\varphi} \right) e_t P_{h,t}^* \Phi_{h,t}^{*n} a_t f'\left(l_{h,t}^*\right) - \left(\frac{\kappa}{\Phi_t^v} \right) P_{h,t} + (1-\rho) \mathbb{E}_t \beta_{t,t+1} \left(\frac{\kappa}{\Phi_{t+1}^v} \right) P_{h,t+1} \\
\rightarrow & \; e_t P_{h,t}^* = P_{h,t} \left(\frac{\Phi_{h,t}^n}{\Phi_{h,t}^{*n}} \right) \left[\frac{f'\left(l_{h,t}\right)}{f'\left(l_{h,t}^*\right)} \right] \qquad \text{(C.30)}
\end{aligned}
$$

If employment in each market is same, i.e. $l_{h,t} = l_{h,t}^*$, the price of home goods in the export market is

$$
e_t P_{h,t}^* = P_{h,t} \left(\frac{\Phi_{h,t}^n}{\Phi_{h,t}^{*n}} \right) \tag{C.31}
$$

With an analogous process, I know the search effort in the export market is:

$$
-s_{h,t}^* u_{s_h^*}(t) = \varphi c_{h,t}^* u_{c_h^*}(t) \left[\frac{s_{h,t}^* \Phi_{h,t}^{*s} y_{h,t}^*}{\varphi c_{h,t}^* + (1-\varphi) s_{h,t}^* \Phi_{h,t}^{*s} y_{h,t}^*} \right] \tag{C.32}
$$

Appendix C.2 Proof of Lemma 1

First, focus on the derivation of the Lagrangian multiplier λ_t. I can derive λ_t by using not only the FOCs of a home firm in the domestic market, but also the ones of a foreign export firm.

$$\begin{aligned} \lambda_t &= (1-\varphi)\left(\frac{u_{c_h}(t)}{P_{h,t}}\right)\left[\frac{s_{h,t}\Phi^s_{h,t}y_{h,t}}{\varphi c_{h,t}+(1-\varphi)s_{h,t}\Phi^s_{h,t}y_{h,t}}\right] \\ \lambda_t &= (1-\varphi)\left(\frac{u_{c_f}(t)}{P_{f,t}}\right)\left[\frac{s_{f,t}\Phi^s_{f,t}y_{f,t}}{\varphi c_{f,t}+(1-\varphi)s_{f,t}\Phi^s_{f,t}y_{f,t}}\right] \end{aligned} \tag{C.33}$$

Using $\partial c_t/\partial c_{i,t} = P_{i,t}/P_t$ for $i=\{h,f\}$, I have

$$\begin{aligned} \lambda_t &= (1-\varphi)\left(\frac{u_c(t)}{P_t}\right)\left[\frac{s_{h,t}\Phi^s_{h,t}y_{h,t}}{\varphi c_{h,t}+(1-\varphi)s_{h,t}\Phi^s_{h,t}y_{h,t}}\right] \\ \lambda_t &= (1-\varphi)\left(\frac{u_c(t)}{P_t}\right)\left[\frac{s_{f,t}\Phi^s_{f,t}y_{f,t}}{\varphi c_{f,t}+(1-\varphi)s_{f,t}\Phi^s_{f,t}y_{f,t}}\right] \end{aligned} \tag{C.34}$$

Accordingly, when defining the bracket as τ_t, I know the following relation holds:

$$\tau_t \equiv \frac{s_{h,t}\Phi^s_{h,t}y_{h,t}}{\varphi c_{h,t}+(1-\varphi)s_{h,t}\Phi^s_{h,t}y_{h,t}} = \frac{s_{f,t}\Phi^s_{f,t}y_{f,t}}{\varphi c_{f,t}+(1-\varphi)s_{f,t}\Phi^s_{f,t}y_{f,t}} \tag{C.35}$$

which implies that the ratios of the realized output to the consumption in the domestic and imported markets are equal to each other. With an analogous process, I also derive similar conditions for the foreign economy.

$$\tau^*_t \equiv \frac{s^*_{f,t}\Phi^{*s}_{f,t}y^*_{f,t}}{\varphi c^*_{f,t}+(1-\varphi)s^*_{f,t}\Phi^{*s}_{f,t}y^*_{f,t}} = \frac{s^*_{h,t}\Phi^{*s}_{h,t}y^*_{h,t}}{\varphi c^*_{h,t}+(1-\varphi)s^*_{h,t}\Phi^{*s}_{h,t}y^*_{h,t}} \tag{C.36}$$

Meanwhile, consider the inverses of τ_t and τ^*_t:

$$\frac{1}{\tau_t} = \varphi\frac{c_{h,t}}{s_{h,t}\Phi^s_{h,t}y_{h,t}} + (1-\varphi) \text{ and } \frac{1}{\tau^*_t} = \varphi\frac{c^*_{h,t}}{s^*_{h,t}\Phi^{*s}_{h,t}y^*_{h,t}} + (1-\varphi) \tag{C.37}$$

Using the shopping constraints, I can rewrite the above expressions as:

$$\frac{1}{\tau_t} = \varphi\left[1-\frac{n_{h,t}\kappa v_{h,t}}{n_{h,t}\Phi^n_{h,t}y_{h,t}}\right]+(1-\varphi) \text{ and } \frac{1}{\tau^*_t} = \varphi\left[1-\frac{n^*_{h,t}\kappa v^*_{h,t}}{n^*_{h,t}\Phi^{*n}_{h,t}y^*_{h,t}}\left(\frac{P_{h,t}}{e_tP^*_{h,t}}\right)\right]+(1-\varphi) \tag{C.38}$$

Since there is no impediment in the international trade, I know that $v_t = v_{h,t} = v^*_{h,t}$ and in turn $y_t = y_{h,t} = y^*_{h,t}$. Furthermore, with a single labor market in each country, the home LOP gap $(e_tP^*_{h,t}/P_{h,t})$ is equal to the ratio of probabilities that a firm match with a shopper in the domestic and export markets $(\Phi^n_{h,t}/\Phi^{*n}_{h,t})$. Therefore, I have the following relation:

$$\begin{aligned}\frac{1}{\tau^*_t} &= \varphi\left[1-\frac{\kappa v_t}{\Phi^n_{h,t}y_t}\right]+(1-\varphi) = \frac{1}{\tau_t}\\ &\Rightarrow \tau_t = \tau^*_t\end{aligned} \tag{C.39}$$

This represents that the ratio of the output and the consumption in each market is same.

Appendix C.3 Proof of Proposition 1

With a single labor market assumption, I express the LOP gap of the home country as a function of the probability that a firm is matched with a shopper.

$$\begin{aligned}\frac{e_tP^*_{h,t}}{P_{h,t}} &= \frac{\Phi^n_{h,t}}{\Phi^{*n}_{h,t}}\\ &= \left[\left(\frac{s_{h,t}}{s^*_{h,t}}\right)\left(\frac{n^*_{h,t}}{n_{h,t}}\right)\right]^{\varphi}\end{aligned} \tag{C.40}$$

With Lemma 1, I can simplify the condition of the international risk sharing as:

$$\begin{aligned}\frac{u_{s^*_h}(t)}{u_{s_h}(t)} &= \left(\frac{e_tP^*_{h,t}}{P_{h,t}}\right)\left(\frac{s_{h,t}}{s^*_{h,t}}\right)\left(\frac{c^*_{h,t}}{c_{h,t}}\right)\\ &= \left(\frac{\Phi^n_{h,t}}{\Phi^{*n}_{h,t}}\right)\left(\frac{s_{h,t}}{s^*_{h,t}}\right)\left(\frac{n_{h,t}\Phi^n_{h,t}y_t}{n^*_{h,t}\Phi^{*n}_{h,t}y_t}\right)\\ &= \left(\frac{s_{h,t}}{s^*_{h,t}}\right)\left(\frac{n_{h,t}}{n^*_{h,t}}\right)\end{aligned} \tag{C.41}$$

Plugging the condition of the international risk sharing into the LOP gap, I have the following relation.

$$\frac{e_t P^*_{h,t}}{P_{h,t}} = \left[\frac{u_{s_h^*}(t)}{u_{s_h}(t)}\right]^{\varphi} = \left[\frac{u_{s^*}(t)}{u_s(t)}\right]^{\varphi} \tag{C.42}$$

With an analogous process, I also know the LOP gap for the foreign country as:

$$\frac{e_t P^*_{f,t}}{P_{f,t}} = \left[\frac{u_{s_f^*}(t)}{u_{s_f}(t)}\right]^{\varphi} = \left[\frac{u_{s^*}(t)}{u_s(t)}\right]^{\varphi}. \tag{C.43}$$

Appendix C.4 Proof of Proposition 3

With the static version of the model setting $\rho = 1$, I can derive the equilibrium condition in the labor market.

$$\begin{aligned} \frac{\kappa}{\Phi_t^v} &= \left(\frac{1-\alpha}{1-\varphi}\right)\Phi^n_{h,t} a_t \text{ where } \Phi_t^v = \chi v_t^{-\phi} \\ &\rightarrow \quad v_t = \left[\left(\frac{\chi}{\kappa}\right)\left(\frac{1-\alpha}{1-\varphi}\right)\Phi^n_{h,t} a_t\right]^{1/\phi} \end{aligned} \tag{C.44}$$

By Lemma 1, I can express the consumption for home goods in the domestic market as:

$$c_{h,t} = k n_{h,t} \Phi^n_{h,t} y_t, \tag{C.45}$$

where k is a constant.

Plugging Equation (C.44) into Equation (C.45), I have

$$\begin{aligned} c_{h,t} &= k n_{h,t} \Phi^n_{h,t} y_t \\ &= k n_{h,t} \Phi^n_{h,t} a_t l_t \\ &= k n_{h,t} \Phi^n_{h,t} a_t \left(\chi v_t^{1-\phi}\right) \\ &= k \chi n_{h,t} \Phi^n_{h,t} a_t \left(v_t^{1-\phi}\right) \\ &\rightarrow \quad c_{h,t} = k\chi \left[\left(\frac{\chi}{\kappa}\right)\left(\frac{1-\alpha}{1-\varphi}\right)\right]^{\frac{1-\phi}{\phi}} n_{h,t} \left(\Phi^n_{h,t} a_t\right)^{\frac{1}{\phi}} \end{aligned} \tag{C.46}$$

Log-linearizing Equation (C.46), I derive

$$\widehat{c_{h,t}} = \widehat{n_{h,t}} + \frac{1}{\phi}\widehat{\Phi^n_{h,t}} + \frac{1}{\phi}\widehat{a_t}, \tag{C.47}$$

where ˆ denotes the log deviation from the steady state.

With the GHH preferences and conditions for optimal search efforts, I write the matching probability in terms of consumption and mass of firms:

$$\begin{aligned} \widehat{\Phi^n_{h,t}} &= \varphi\left(\widehat{s_{h,t}} - \widehat{n_{h,t}}\right) \\ &\to \widehat{\Phi^n_{h,t}} = \varphi\left(\frac{1}{1+\eta}\widehat{c_t} - \widehat{n_{h,t}}\right). \end{aligned} \tag{C.48}$$

Plugging Equation (C.48) into Equation (C.47), I calculate

$$\begin{aligned} \widehat{c_{h,t}} &= \widehat{n_{h,t}} + \frac{1}{\phi}\widehat{\Phi^n_{h,t}} + \frac{1}{\phi}\widehat{a_t} \\ &= \widehat{n_{h,t}} + \frac{\varphi}{\phi}\left(\frac{\eta}{1+\eta}\widehat{c_t} - \widehat{n_{h,t}}\right) + \frac{1}{\phi}\widehat{a_t} \\ &\to \widehat{a_t} = (\varphi - \phi)\widehat{n_{h,t}} + \phi\widehat{c_{h,t}} - \frac{\varphi\eta}{1+\eta}\widehat{c_t}. \end{aligned} \tag{C.49}$$

With an analogous process, I obtain

$$\widehat{a^*_t} = (\varphi - \phi)\widehat{n^*_{f,t}} + \phi\widehat{c^*_{f,t}} - \frac{\varphi\eta}{1+\eta}\widehat{c^*_t}, \tag{C.50}$$

for the foreign country.

Subtracting Equation (C.49) into Equation (C.50), I have

$$\widehat{a_t} - \widehat{a^*_t} = (\varphi - \phi)\left(\widehat{n_{h,t}} - \widehat{n^*_{f,t}}\right) + \phi\left(\widehat{c_{h,t}} - \widehat{c^*_{f,t}}\right) - \frac{\varphi\eta}{1+\eta}\left(\widehat{c_t} - \widehat{c^*_t}\right). \tag{C.51}$$

To understand the relationship between productivity shocks and aggregate consumption gaps, I need to express $\left(\widehat{n_{h,t}} - \widehat{n^*_{f,t}}\right)$ and $\left(\widehat{c_{h,t}} - \widehat{c^*_{f,t}}\right)$ in terms of $\left(\widehat{c_t} - \widehat{c^*_t}\right)$.

(1) $\widehat{n_{h,t}} - \widehat{n^*_{f,t}}$

Using the international risk sharing and the property of CES aggregator, I express the LOP gap of home goods as:

$$\begin{aligned}\frac{u_{c_f^*}(t)}{u_{c_h}(t)} &= \frac{e_t P_{f,t}^*}{P_{h,t}} \\ &= \frac{e_t P_{h,t}^*}{P_{h,t}} \frac{P_{f,t}^*}{P_{h,t}^*} \\ &= \frac{e_t P_{h,t}^*}{P_{h,t}} \left(\frac{\partial c_t^* / \partial c_{f,t}^*}{\partial c_t^* / \partial c_{h,t}^*} \right) \\ \rightarrow & \frac{e_t P_{h,t}^*}{P_{h,t}} = \frac{u_{c_h^*}(t)}{u_{c_h}(t)}. \end{aligned} \tag{C.52}$$

With the functional form, I have

$$\begin{aligned}\frac{\Phi_{h,t}^n}{\Phi_{h,t}^{*n}} &= \left(\frac{1-\omega}{\omega}\right)\left(\frac{c_{h,t}}{c_{h,t}^*}\right)\left(\frac{c_t}{c_t^*}\right)^{\sigma-1} \\ \rightarrow & \left(\frac{c_t}{c_t^*}\right)^{1-\sigma} = \left(\frac{1-\omega}{\omega}\right)\frac{n_{h,t}}{n_{h,t}^*}. \end{aligned} \tag{C.53}$$

Log-linearizing Equation (C.53), I express the aggregate consumption in terms of the mass of firms:

$$\begin{aligned}\widehat{c_t} - \widehat{c_t^*} &= \frac{1}{1-\sigma}\left(\widehat{n_{h,t}} - \widehat{n_{h,t}^*}\right) \\ &= \frac{1}{(1-\sigma)(1-n_h)}\widehat{n_{h,t}} \\ &= \frac{1}{(1-\sigma)(1-\omega)}\widehat{n_{h,t}}. \end{aligned} \tag{C.54}$$

where a variable without subscript 't' denotes the steady state value. Note that n_h is equal to ω due to the assumption of symmetric economies.

For the foreign country, I also have

$$\begin{aligned}\widehat{c_t} - \widehat{c_t^*} &= \frac{1}{1-\sigma}\left(\widehat{n_{f,t}} - \widehat{n_{f,t}^*}\right) \\ &= -\frac{1}{(1-\sigma)\left(1-n_f^*\right)}\widehat{n_{f,t}^*} \\ &= -\frac{1}{(1-\sigma)(1-\omega)}\widehat{n_{f,t}^*}. \end{aligned} \tag{C.55}$$

Thus, I derive

$$\widehat{n_{h,t}} - \widehat{n^*_{f,t}} = 2(1-\sigma)(1-\omega)\left(\widehat{c_t} - \widehat{c^*_t}\right). \tag{C.56}$$

(2) $\widehat{c_{h,t}} - \widehat{c^*_{f,t}}$

Log-linearizing the CD aggregator (z=1) for consumption and using the relation of consumptions at the steady state, I calculate

$$\widehat{c_t} = \omega\widehat{c_{h,t}} + (1-\omega)\widehat{c_{f,t}} \tag{C.57}$$
$$\widehat{c^*_t} = \omega\widehat{c^*_{f,t}} + (1-\omega)\widehat{c^*_{h,t}}. \tag{C.58}$$

Meanwhile, by Lemma 1, I express the the relative consumption for home goods in the domestic and the export markets as:

$$\frac{c_{h,t}}{c^*_{h,t}} = \left(\frac{n_{h,t}\Phi^n_{h,t}}{n^*_{h,t}\Phi^{*n}_{h,t}}\right) = \left(\frac{s_{h,t}}{s^*_{h,t}}\right)^{\varphi}\left(\frac{n_{h,t}}{n^*_{h,t}}\right)^{1-\varphi}. \tag{C.59}$$

Log-linearizing Equation (C.59), I derive

$$\widehat{c_{h,t}} - \widehat{c^*_{h,t}} = (1-\varphi)\left(\widehat{n_{h,t}} - \widehat{n^*_{h,t}}\right) + \frac{\varphi\eta}{1+\eta}\left(\widehat{c_t} - \widehat{c^*_t}\right). \tag{C.60}$$

With an analogous process, I obtain

$$\widehat{c^*_{f,t}} - \widehat{c_{f,t}} = (1-\varphi)\left(\widehat{n^*_{f,t}} - \widehat{n_{f,t}}\right) - \frac{\varphi\eta}{1+\eta}\left(\widehat{c_t} - \widehat{c^*_t}\right). \tag{C.61}$$

With Equation (C.57), (C.58), (C.60), and (C.61), I have

$$\begin{aligned}\widehat{c_{h,t}} - \widehat{c^*_{f,t}} &= \frac{(1+\eta) - 2\varphi\eta(1-\omega)}{(2\omega-1)(1+\eta)}\left(\widehat{c_t} - \widehat{c^*_t}\right) \\ &\quad - \frac{(1-\varphi)(1-\omega)}{2\omega-1}\left(\widehat{n_{h,t}} - \widehat{n^*_{h,t}} - \widehat{n^*_{f,t}} + \widehat{n_{f,t}}\right).\end{aligned} \tag{C.62}$$

Using Equation (C.56),

$$\begin{aligned}&\widehat{c_{h,t}} - \widehat{c^*_{f,t}} \\ = \;& \frac{(1+\eta) - 2\varphi\eta(1-\omega) - 2(1-\varphi)(1-\sigma)(1-\omega)(1+\eta)}{(2\omega-1)(1+\eta)}\left(\widehat{c_t} - \widehat{c^*_t}\right).\end{aligned} \tag{C.63}$$

(3) $\widehat{a}_t - \widehat{a}_t^*$

Plug Equation (C.56) and (C.63) into Equation (C.51),

$$\widehat{a}_t - \widehat{a}_t^* = \frac{\zeta}{(2\omega-1)(1+\eta)}\left(\widehat{c}_t - \widehat{c}_t^*\right), \tag{C.64}$$

where

$$\begin{aligned}\zeta &= 2(\sigma-1)(1-\omega)(1+\eta)[\varphi(1-\phi)+2\omega(\phi-\varphi)] \\ &\quad +\phi(1+\eta)-\varphi\eta[2\omega(1-\phi)+2\phi-1].\end{aligned}$$

Appendix C.5 Static model

Table A.3 reports the main equations of static model, assuming $\rho = 1$.

Table C.1 Static Model

	Home country	Foreign country
Mass of firms	$1 = n_{h,t} + n_{h,t}^*$	$1 = n_{f,t}^* + n_{f,t}$
Unemployment	$u_t = 1$	$u_t^* = 1$
Employment	$l_t = \Phi_t^v v_t$	$l_t^* = \Phi_t^{*v} v_t^*$
Production	$y_t = z_t l_t$	$y_t^* = z_t^* l_t^*$
Wage	$W_t = \alpha\left(\frac{1}{1-\varphi}\right) p_{h,t}\Phi_{h,t}^n z_t$	$W_t^* = \alpha\left(\frac{1}{1-\varphi}\right) p_{f,t}^*\Phi_{f,t}^{*n} z_t^*$
Job creation	$\left(\frac{\kappa p_{h,t}}{\Phi_t^v}\right) = \left(\frac{1}{1-\varphi}\right) p_{h,t}\Phi_{h,t}^n z_t - W_t$	$\left(\frac{\kappa p_{f,t}^*}{\Phi_t^{*v}}\right) = \left(\frac{1}{1-\varphi}\right) p_{f,t}^*\Phi_{f,t}^{*n} z_t^* - W_t^*$
Export price	$e_t p_{h,t}^* = \left(\frac{\Phi_{h,t}^n}{\Phi_{h,t}^{*n}}\right) p_{h,t}$	$\frac{1}{e_t} p_{f,t} = \left(\frac{\Phi_{f,t}^{*n}}{\Phi_{f,t}^n}\right) p_{f,t}^*$
Int'l relative prices	$\frac{P_{h,t}}{P_{f,t}} = \left(\frac{\omega}{1-\omega}\right)\frac{c_{f,t}}{c_{h,t}}$	$\frac{P_{f,t}^*}{P_{h,t}^*} = \left(\frac{\omega}{1-\omega}\right)\frac{c_{h,t}^*}{c_{f,t}^*}$
Search effort	$s_{h,t} = \varphi c_{h,t}\left[\frac{u_{c_h}(t)}{-u_{s_h}(t)}\right]\tau_t$	$s_{f,t}^* = \varphi c_{f,t}^*\left[\frac{u_{c_f^*}(t)}{-u_{s_f^*}(t)}\right]\tau_t$
	$s_{f,t} = \varphi c_{f,t}\left[\frac{u_{c_f}(t)}{-u_{s_f}(t)}\right]\tau_t$	$s_{h,t}^* = \varphi c_{h,t}^*\left[\frac{u_{c_h^*}(t)}{-u_{s_h^*}(t)}\right]\tau_t$
Shopping	$c_{h,t} = n_{h,t}\Phi_{h,t}^n y_t - n_{h,t}\kappa v_t$	$c_{f,t}^* = n_{f,t}^*\Phi_{f,t}^{*n} y_t^* - n_{f,t}^*\kappa v_t^*$
	$c_{f,t} = n_{f,t}\Phi_{f,t}^n y_t^* - n_{f,t}\kappa v_t^*\left(\frac{e_t P_{f,t}^*}{P_{f,t}}\right)$	$c_{h,t}^* = n_{h,t}^*\Phi_{h,t}^{*n} y_t - n_{h,t}^*\kappa v_t\left(\frac{P_{h,t}}{e_t P_{h,t}^*}\right)$
Risk sharing	$\frac{u_{c_f^*}(t)}{u_{c_h}(t)} = \frac{e_t P_{f,t}^*}{P_{h,t}}$	