An Adaptive Sequence-Based Selection Hyper-Heuristic for Application to Electric Bus Scheduling

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ABSTRACT

Buses are important for public transportation and beneficial for the environment. However, diesel buses are significant polluters emitting greenhouse gases and particulates. Consequently, with the advent of electric vehicles there has been a drive to transition to electric buses. Key to this transition is to optimise electric bus fleets to reduce distance travelled whilst maintaining service levels. This is complex due to the added constraint of the limited range of electric buses. This paper considers the use of a Sequence-based Selection Hyper Heuristic (SSHH) method to solve this problem. Moreover, an adaptive SSHH (A-SSHH) technique is introduced which significantly improves upon SSHH. Indeed, bus fleet non-service distances and sizes are reduced by as much as 10% using A-SSHH over SSHH. Comparing with an optimised diesel bus fleet electric buses reduce carbon dioxide emissions by over 60% and importantly for fleet operators, energy costs are similarly reduced.

CCS CONCEPTS

• **Computing methodologies → Search methodologies**.

KEYWORDS

hyper-heuristics, electric bus scheduling

1 INTRODUCTION

The world is facing a climate emergency to ensure that global temperature rises remain below 1.5C from preindustrial levels. A primary cause is fossil fuel use with transportation a key driver. Internal combustion engine vehicles (ICEVs) emit significant levels of CO_2 with diesel vehicles the worst polluters also emitting considerable particulates which reduce urban air quality. These particulates can cause breathing problems in the young and the elderly and are linked to increased rates of cardiovascular disease.

Buses provide a significant public service and can be seen as an environmental option in reducing road traffic. Buses though are predominantly diesel ICEVs emitting significant levels of CO_2 and particlates. However, recently a transition to electric vehicles has begun which emit much less CO_2 and no particulates. This transition is also occurring within bus transportation although using electric buses adds the additional problem constraint of a range limit in terms of energy consumption and in most cases, a lack of accessible charging points and time to recharge in service. A key methodology to reduce energy consumption within bus transportation networks is to optimise the bus use such that the timetable is serviced but the total distance the bus fleet travels is minimised.

The goal of optimising a fleet of buses is to maintain the timetable service level without tardiness whilst minimising the bus fleet distance traversed. This problem is similar to the Vehicle Routing Problem (VRP) [6] to assign customers and routes to vehicles minimising total distance travelled with the added constraint of the timetable, the VRP with time windows (VRPTW). Electric buses add a further constraint with a limited range due to a battery. An electric bus must be able to perform its assigned routes and return to the depot without running out of charge. The energy required to complete a set of assigned trips is the demand and the capacity is the amount of available battery power of the electric bus.

Electric bus scheduling can be described as a graph $G = (V, E)$, whereby V is the vertex set and E the edges between vertices. The vertex set V is further partitioned, $V_T = V_1, ..., V_n$ representing n timetabled trips and $V_B = V_{n+1}, ...V_{n+p}$ representing p electric buses. Each trip in V has a service time to be met and start finish bus stops. Each bus $v_i \in V_B$ has a capacity in terms of energy. Each edge in E has a cost of traversing it represented by matrix c_{ij}. Edge e_{ij} in c between trips represents the distance from the end bus stop of V_i and the start bus stop of V_j. If V_j is a depot then the distance is from the end bus stop of V_i to depot V_j. The objective is to find a hamiltonian cycle in G of minimal length with all trips serviced on time and the energy capacities of buses not exceeded.

Meta-heuristics such as the Genetic Algorithm (GA) [8] are commonly used to solve bus scheduling problems. Janovec and Kohani [9] used a grouping GA with bus routes as the groups to minimise bus fleet energy use. Wang et al. [16] considered a multi-depot three line electric bus routing problem from Qingdao China solving with a column generation GA whereby columns representing allocated routes are recombined. Zhang et al. [17] optimise the costs of operating electric buses using a GA to schedule multiple vehicle types for a single line in Nanjing, China.

Whereas meta-heuristics search within the space of problem solutions, hyper-heuristics search within the space of low-level heuristics that operate within the solution space [2]. Hyper-heuristics can be categorised into two groups, selection which apply a low-level heuristic at each iteration and generational which attempt to generate novel low-level heuristics. Early work by Cowling et al. [5] considered a range of simple hyper-heuristic methods to select heuristics such as Simple Random (SR) and Random Descent (RD). Advanced hyper-heuristics consider the application of *sequences* of heuristics. Iterated Local Search (ILS) [12] was an early attempt to create sequences of low-level mutation heuristics and rebuild with

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local search heuristics [13]. AdapHH pairs destructive and constructive low-level heuristics [15]. An Evolutionary Programming Hyper-heuristic (EPH) is population-based whereby a population of sequences is maintained using diversification and intensification operations [14]. Drake et al. provide a recent overview of advances in selection hyper-heuristics [7]. Regarding bus scheduling, some hyper-heuristic methods have been applied. Liu et al. [11] compare GAs and a hyper-heuristic for the dynamic bus scheduling which proved superior to the GA. Hyper-heuristics have been also used to optimise the bus routes themselves [1].

This paper proposes to optimise electric bus schedules using the Sequence-based Selection Hyper-Heuristic (SSHH) which generates sequences of heuristics by learning relationships between heuristics. [10]. This is achieved by using a hidden Markov model (HMM) to represent the probability of using one heuristic after another has been previously applied. Moreover, this paper presents an adaptive modification to SSHH for electric bus scheduling.

2 AN ADAPTIVE SEQUENCE-BASED SELECTION HYPER-HEURISTIC

The Sequence-based Selection Hyper-Heuristic (SSHH) [10] constructs sequences using a hidden Markov model (HMM) whereby heuristics are states and the HMM provides a probability of transitioning from one to the next. Given a set of *n* low-level heuristics $[llh_0, llh1, ...llh_{n-1}]$ the transition probabilities of moving from one heuristic to the next is defined by an *n* by *n* matrix. The transition probabilities are simply the success counts of transitions having occurred in sequences which improved the current best solution. SSHH also uses a sequence HMM to decide after a heuristic selection if the sequence should be accepted, an *n* by 2 matrix. Algorithm 1 provides an overview of the SSHH process. The selection of a low-level heuristic occurs on line 7 whereby a roulette wheel selection is made using the HMM transition probabilities *Trans* from the last used heuristic *curr*, $Tran[curr][next]/\sum_{\forall j} Tran[curr][j]$.

Algorithm 1 SSHH

1: S, S', S_b = candidate, new and best solutions respectively
2: $Tran$, Seq = the transition and sequence matrices
3: $[llh_0, llh1_1, llh_2, \ldots, llh_{n-1}]$ = the low-level heuristics
4: $HeuristicSequence$ is the current heuristic sequence
5: $curr$ = select random low-level heuristic
6: **while** iteration less than max iterations **do**
7: $next$=SelectNext($Tran$,$curr$)
8: Add to$HeuristicSequence$ $next$
9: $AcceptStatus$=ComputeStatus(Seq,$next$)
10: **if** $Status$ = complete sequence **then**
11: S' = application of $HeuristicSequence$
12: **if** S' better than S_b **then**
13: S_b=S'
14: Update $Tran$ and Seq success counts
15: **end if**
16: S=S'
17: Clear $HeuristicSequence$
18: **end if**
19: $curr$=$next$
20: **end while**

However, it is hypothesised that this HMM transition probability has a flaw. If a single heuristic transition has large early success this can dominate the other heuristics. Indeed, a transitional state occurs between each heuristic and itself such that it is plausible that a successful heuristic can be used repeatedly further boosting its probability of use. The success counts are retained even when the heuristic is no longer successful. The probabilities are not relaxed, the heuristic will continue to be overused and since for other transitional scores to increase transitions must at least occur, the transitional probabilities can become effectively *locked in*.

To remedy this potential flaw an alternative transitional probability model is proposed whereby the unsuccessful transitions of a sequence are also accounted for. If a heuristic becomes unsuccessful this reduces the probability of its future selection. This probability *relaxation* is achieved by calculating a probability of a heuristic being used based on its success over its usage. The HMM will count both the successes and occurrences of heuristic transitions. The probability of a heuristic transition becomes the success rate divided by the occurrence rate. This *adaptive* HMM approach for SSHH will be referred to as Adaptive-SSHH (A-SSHH).

Algorithm 2 Adaptive-SSHH

1: S, S', S_b = candidate, new and best solutions respectively
2: $Tran$, Seq, $TranOccur$, $SeqOccur$ = the transition, sequence and transition and sequence occurrence matrices
3: $[llh_0, llh1_1, llh_2, \ldots, llh_{n-1}]$ = the low-level heuristics
4: $HeuristicSequence$ is the current heuristic sequence
5: $curr$ = select random low-level heuristic
6: **while** iteration less than max iterations **do**
7: $next$=SelectNext($Tran$, $TranOccur$, $curr$)
8: Add to$HeuristicSequence$ $next$
9: $AcceptStatus$=ComputeStatus(Seq, $SeqOccur$, $next$)
10: **if** $Status$ = complete sequence **then**
11: S' = application of $HeuristicSequence$
12: **if** S' better than S_b **then**
13: S_b=S'
14: Update $Tran$ and Seq success counts
15: **end if**
16: S=S'
17: Clear $HeuristicSequence$
18: **end if**
19: Update $TranOccur$ and $SeqOccur$ occurrence counts
20: $curr$=$next$
21: **end while**

Algorithm 2 shows the Adaptive-SSHH method. Note the additional function at line 19 which always increments the transitional and sequence occurrence matrices (*TranOccur*, *SeqOccur*) for all generated sequences. Also, note the new function calls at line 7 and line 9 which use both the success count and occurrence matrices to select the next low-level heuristic and acceptance. A random proportional rule is now used to select the next heuristic with the probability of selecting heuristic *j* after heuristic *i* defined as:

$$P_j = \frac{Tran_{i,j}}{TranOccur_{i,j}} R_j \quad (1)$$

where R_j is a random value in the range $[0, 1]$ and the heuristic with the largest P value is selected.

3 EXPERIMENTAL RESULTS

To measure the effectiveness of A-SSHH vs. SSHH for solving the electric bus scheduling problem both hyper-heuristics will be tested using a real-world problem based on actual timetables. A UK bus operator operates buses throughout a large area with a radius of 50 km. A fleet of electric buses are considered equipped with a 450kWh battery providing a range of 185 km using 2.42kWh of

Table 1: Real-world electric bus routing problems

Problem	Geographic Lines	Total Line Trips	Available Buses	Total Trip Distance (km)
Scenario A	20	253	150	3112.58
Scenario B	20	223	150	2353.76
Scenario C	20	890	150	13555.99
Scenario D	20	518	150	12145.38
Scenario E	20	676	150	12474.52
Scenario F	24	670	150	7244.83
Scenario G	60	1456	300	21050.11
Scenario H	64	1774	300	29836.56
Scenario I	124	3230	500	50886.67

energy per km. The objective is to assign timetabled trips to electric buses such that the number of buses required are minimised with minimum fleet traversal and no tardiness or violating the range constraint of the electric bus. Hence, a solution with fewer buses required is considered an improvement over a solution requiring more buses but with a lower traversal distance. A candidate solution will consist of a set of unique values representing buses each followed by unique values representing its assigned timetabled trips. An electric bus performs these trips in their assigned order. A set of routing scenarios have been created from the UK bus operator varying in size and are described in Table 1. To apply the SSHH hyper-heuristics to electric bus scheduling a set of low-level heuristics are used as described in Table 2.

The SSHH and A-SSHH methods are compared using the electric bus routing problem for the given set of scenarios. Due to the high degree of complexity of the electric bus scheduling problem a large degree of 5 million sequences of up to a maximum length of 10 heuristics are generated with improvements greedily accepted. An additional simulated annealing acceptance strategy is used with both methods enabling non-improved solutions to be accepted as the current solution with a given probability. In these cases HMM transition success counts are not incremented. Experiments were conducted over 25 random runs.

The results from SSHH and A-SSHH are shown in Table 3 in terms of the solution quality obtained, non-service distance and bus fleet sizes. From these results it can be clearly observed that regarding solution quality A-SSHH achieves significantly superior average non-service distances and bus fleet sizes for all scenarios. Note the differences between SSHH and A-SSHH are more pronounced for larger scenarios, A-SSHH achieves an average non-service distance 15% less for the largest scenario. Also,note there is considerable variance in the results for SSHH especially in terms of buses used which provides an underlying reason for the poorer performance from SSHH.

It was hypothesised that on occasion SSHH could achieve such high transition counts for a given heuristic transition that all others are dominated. SSHH by only considering successes cannot redress this imbalance but A-SSHH by considering heuristic usage and successes can. Examination of the best and worst obtained results reinforces this theory. With best found solutions there is little to choose between SSHH and A-SSHH. In fact, in two scenarios SSHH finds lower non-service distances than A-SSHH over the 25 random runs and is equal or better in terms of bus use over all scenarios. However, in terms of the worst case solution qualities SSHH finds considerably poorer solutions than A-SSHH in all scenarios. This explains the considerable variance in the results from SSHH

Table 2: Available low-level heuristics.

Heuristic	Description
Swap	Selects two random trips assigned to electric buses and exchanges the two trips
Insert	Selects random trip assigned to a bus and and inserts into a random position in a second electric bus schedule
Invert	Randomly selects two points within a bus fleet schedule and reverses all the trips between the two points
Reconstruction	Selects up to 30 buses operating in similar geographical area and rebuilds their schedules using a probabilistic model based on minimising non-service time lost [3, 4]
Local Search Swap	Two electric buses are selected and every bus trip in each iteratively swapped with improvements retained
Local Search Insert	Two electric buses are selected and each trip in second bus schedule is iteratively inserted into every slot in the first bus schedule with improvements retained

which will *skew* the averages in favour of A-SSHH. It should be noted that whilst A-SSHH is more consistent over a set of random runs, SSHH can generate solutions similar to A-SSHH.

3.1 Electric vs. Diesel Bus Fleets

To ascertain the benefits of an electric bus fleet a comparison must be made to a similarly A-SSHH optimised diesel bus fleet. Clearly, a diesel bus will not have a range constraint. In addition to considering the total non-service fleet distance and buses required, the CO_2 emissions and energy costs will be reported. For the diesel fleet it is considered that a typical bus can achieve 1.78 km per litre of diesel and CO_2 emissions are typically 1.35kg[1] per km. CO_2 emissions for electric buses are slightly harder to ascertain. However, the UK energy regulator quoted a figure of 0.181KgCO_2/kWh[2]. In terms of cost, the current UK price per litre of diesel of £1.75 will be used. For electric, a cost of £0.19 per kWh is used.

The results from optimised electric and diesel fleets are shown in Table 4 whereby it can be observed that in all cases the diesel bus fleet traverses fewer non-service km than an electric fleet. This is because the diesel bus fleet requires fewer buses as each bus can operate all day. Electric buses will run out of charge potentially midway through the day meaning more buses and trips to and from the depot. In many instances a diesel bus fleet uses 30% fewer buses. However, even with the greater traversal distance, the CO_2 emissions are 67% lower for the electric bus fleet and zero particulates. Note, the CO_2 emissions include the full route distances described in Table 1 and non-service distances. In terms of energy costs, for an electric bus fleet there is a 50% reduction. However, note that significantly more buses are required by an electric bus fleet incurring additional infrastructure costs, manufacturing carbon emissions and driver man hours.

4 CONCLUSIONS

This paper considered a hyper-heuristic methodology to optimise an electric bus fleet, an important problem as electric buses are better for the environment but pose problems in terms of range limitations. A Sequence-Based Selection Hyper-Heuristic (SSHH) methodology was applied to the optimisation of real-world electric bus problems. Moreover, an adaptive SSHH (A-SSHH) method was introduced which accounts for both successes and failures in

[1] www.carbonindependent.org
[2] http://www.nationalgrideso.com/news/record-breaking-2020-becomes-greenest-year-britains-electricity

Table 3: Non-service distances and fleet sizes when applying SSHH and A-SSHH to each electric bus scheduling scenario.

Scenario	Non-Service Distance (km)						Bus Fleet Size					
	SSHH			A-SSHH			SSHH			A-SSHH		
	Average	Best	Worst	Average	Best	Worst	Average	Best	Worst	Average	Best	Worst
A	1187.26±77.48	1118.99	1471.97	**1137.56±27.57**†	**1100.45**	**1191.79**	25.44±1.26	25	30	**25.20±0.41**	25	**26**
B	1066.99±80.53	**1009.36**	1362.47	**1033.50±13.91**	1011.22	**1059.08**	22.00±1.87	21	28	**21.40±0.50**	21	**22**
C	3067.74±360.07	2722.95	4324.23	**2759.73±41.87**†	**2673.37**	**2854.32**	98.48±10.50	92	143	**92.72±0.46**†	92	**93**
D	1922.67±140.16	**1746.92**	2329.61	**1810.33±40.76**†	1756.61	**1891.91**	80.56±3.78	78	94	**78.96±0.35**†	78	**80**
E	2039.42±176.63	1798.67	2445.15	**1835.03±35.94**†	**1775.45**	**1949.60**	82.96±2.79	80	91	**80.12±0.33**†	80	**81**
F	1138.78±93.06	1032.66	1429.51	**1059.09±36.95**†	**992.90**	**1147.05**	49.36±3.33	47	63	**47.80±0.50**†	47	**49**
G	5655.50±516.00	4971.55	7298.06	**5104.39±66.03**†	**4980.73**	**5216.60**	158.92±12.51	**147**	209	**149.60±0.76**†	148	**151**
H	5185.54±307.82	4623.28	5766.66	**4615.05±82.52**†	**4419.20**	**4770.28**	210.92±20.61	194	268	**195.24±0.60**†	194	**196**
I	11 673.08±944.97	9999.61	13534.06	**9886.87±136.71**†	**9684.96**	**10330.09**	397.00±48.52	**342**	499	**346.76±1.16**†	345	**349**

†Statistically significant improvement of A-SSHH over SSHH with a $p < 0.05$ t-test, a two-sided significance level and 24 degrees of freedom

Table 4: Comparison of diesel and electric bus fleets in terms of distance, bus fleet size, CO_2 emissions and energy costs.

Scenario	Distance (km)		Buses		Cost (£)		CO_2 Emissions (kg)	
	Diesel	Electric	Diesel	Electric	Diesel	Electric	Diesel	Electric
A	**1008.65±9.63**	1137.56±27.57	**22.00±0.00**	25.20±0.41	4051.77±9.47	**1954.22±12.68**	5563.66±13.00	**1851.36±12.01**
B	**1053.66±21.74**	1033.50±13.91	**19.28±0.46**	21.40±0.50	3349.61±21.37	**1557.28±6.40**	4599.50±29.34	**1475.32±6.06**
C	**2210.81±23.45**	2759.73±41.87	**59.56±1.08**	92.72±0.46	15 501.07±23.06	**7501.97±19.25**	21 285.18±31.66	**7107.13±18.24**
D	**1448.27±20.42**	1810.33±40.76	**49.04±0.20**	78.96±0.35	13 364.54±20.08	**6416.83±18.74**	18 351.42±27.57	**6079.10±17.75**
E	**1167.22±34.73**	1835.03±35.94	**44.20±0.65**	80.12±0.33	13 411.82±34.14	**6579.53±16.53**	18 416.34±46.88	**6233.24±15.66**
F	**997.67±27.87**	1059.09±36.95	**36.08±0.49**	47.80±0.50	8103.58±27.40	**3818.14±16.99**	11 127.37±37.62	**3617.19±16.10**
G	**4463.90±61.47**	5104.39±66.03	**108.24±0.93**	149.60±0.76	25 084.00±60.43	**12 025.84±30.36**	34 443.92±82.98	**11 392.90±28.76**
H	**3610.26±67.10**	4615.05±82.52	**122.72±1.14**	195.24±0.60	32 883.11±65.97	**15 840.85±37.94**	45 153.20±90.59	**15 007.12±35.94**
I	**8242.83±95.38**	9886.87±136.71	**240.28±2.03**	346.76±1.16	58 132.94±93.77	**27 943.67±62.86**	79 824.83±128.76	**26 472.95±59.55**

low-level heuristics selection. A-SSHH improved the optimisation results for the electric bus routing scenarios considerably over standard SSHH. Regarding environmental benefits, optimisation demonstrated reductions of over 60% in terms of CO_2 emissions can be achieved over diesel bus fleets. Moreover, the energy costs of operating an electric bus fleet are half those from a diesel bus fleet. However, due to the range limit of electric buses, significantly more electric buses are required to service a current bus timetable.

Future work will integrate A-SSHH within the HyFlex framework to measure it against other hyper-heuristics. Regards electric bus fleet optimisation further research will consider mixed fleets of diesel and electric buses and also modification of the timetables and routes themselves to better accommodate electric buses.

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