

Figure 9: HGP run of artificial ant with population size 500, showing the evolution of SHD frequencies (left) and the evolution of diversity and average SHD (right). Legend as in Figure 8.

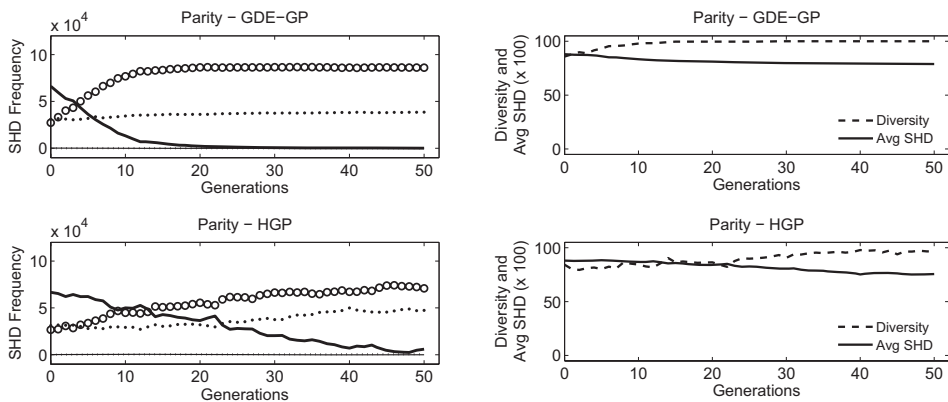


Figure 10: GDE1 and HGP runs of parity with population size 500, showing the evolution of SHD frequencies (left) and the evolution of diversity and average SHD (right). Legend as in Figure 8.

Figure 10 reports examples of the typical behavior of GDE-GP (top) and HGP (bottom) in the parity problem, in terms of the evolution of distance frequencies (left) and average distance along with diversity (right). As expected, the behavior of GDE-GP is different from the one observed in the artificial ant examples (see Figure 8). Also as expected, given the similarities in diversity evolution between GDE-GP and HGP, the behaviors of the two approaches are not so dissimilar as in the artificial ant problem.

## 9 Conclusions

Geometric differential evolution is a formal generalization of DE on continuous spaces that retains the original geometric interpretation and that applies to generic combinatorial spaces. GDE can be formally specified to specific spaces associated, in principle, to any solution representation. In this article, we have illustrated that this is indeed possible in practice by deriving the specific GDEs for the Hamming space associated with binary strings, for the space of permutations endowed with the swap distance, for the space of vectors of permutations endowed with the row-swap distance, and for the space of genetic programs endowed with the structural Hamming distance. These are quite different spaces based on nontrivial solution representations. The derived representation-specific GDEs are, in a strong mathematical sense, the same algorithm doing the same type of search on different spaces.



We have analyzed the behavior of specific GDE algorithms experimentally, tested them on standard benchmarks, and compared them against a set of classic evolutionary algorithms defined on the same search space and representation. The binary GDE and the GDE-GP outperformed the other algorithms in the comparison. The GDE based on permutations did well on the TSP, but only for short tour lengths; for long tours, it performed very badly. Finally, GDE on vectors of permutations on Sudoku did almost as well as a very finely tuned GA. We believe these are very promising initial results. This is a rather interesting achievement, as the present work is one of the very rare examples in which theory has been able to inform the practice of search operator design successfully. GDE is a very recent algorithm and further analysis is required to gain an in-depth understanding of its behavior on different representations. Also, further experimentation is needed to more thoroughly explore its potential to effectively solve combinatorial problems and problems naturally formulated as search in spaces of programs. This constitutes an important piece of future work.

The formal generalization methodology employed to generalize differential evolution, which is the same that was used to generalize PSO, can be applied in principle to generalize virtually any search algorithm for continuous optimization to combinatorial spaces. Interestingly, this generalization methodology is rigorous, conceptually simple, and promising as both GDE and GPSO seem to be quite good algorithms in practice. In future work, we will generalize, using this methodology, other classical derivation-free methods for continuous optimization that make use of geometric constructions of points to determine the next candidate solution (e.g., the Nelder and Mead method and the controlled random search method).

## Acknowledgments

We would like to thank Riccardo Poli for passing us the code of the homologous crossover for genetic programs, and Paulo Fonseca and João Carriço for ideas on visualizing distances. The first author acknowledges EPSRC grant EP/I010297/1 that partially supported this work. The third author acknowledges the support by national funds through FCT under contract Pest-OE/EEI/LA0021/2011 and project PTDC/EIA-CCO/103363/2008, Portugal.

## References

- Caprara, A. (1997). Sorting by reversals is difficult. In *Proceedings of the 1st Annual International Conference on Computational Molecular Biology*, pp. 75–83.
- Ekart, A., and Nemeth, S. Z. (2000). A metric for genetic programs and fitness sharing. In *Proceedings of the European Conference on Genetic Programming, EuroGP'2000*, pp. 259–270.
- Gong, T., and Tuson, A. L. (2007). Differential evolution for binary encoding. In *Soft Computing in Industrial Applications* (pp. 251–262). Berlin: Springer-Verlag.
- Kauffman, S. (1993). *Origins of order: Self-organization and selection in evolution*. Oxford, UK: Oxford University Press.
- Kennedy, J., and Eberhart, R. C. (1997). A discrete binary version of the particle swarm algorithm. In *Proceedings of the IEEE International Conference on Systems, Man, and Cybernetics, Computational Cybernetics and Simulation*, pp. 4104–4108.
- Kennedy, J., and Eberhart, R. C. (2001). *Swarm intelligence*. San Mateo, CA: Morgan Kaufmann.
- Koza, J. R. (1992). *Genetic programming: On the programming of computers by means of natural selection*. Cambridge, MA: MIT Press.

- Langdon, W., and Poli, R. (2002). *Foundations of genetic programming*. Berlin: Springer-Verlag.
- Moraglio, A. (2007). Towards a geometric unification of evolutionary algorithms. Ph.D. thesis, University of Essex, UK.
- Moraglio, A., Di Chio, C., and Poli, R. (2007). Geometric particle swarm optimization. In *Proceedings of the European Conference on Genetic Programming*, pp. 125–136.
- Moraglio, A., Di Chio, C., Togelius, J., and Poli, R. (2008). Geometric particle swarm optimization. *Journal of Artificial Evolution and Applications*, doi: 10.1155/2008/143624.
- Moraglio, A., and Poli, R. (2004). Topological interpretation of crossover. In *Proceedings of the Genetic and Evolutionary Computation Conference*, pp. 1377–1388.
- Moraglio, A., and Poli, R. (2005). Geometric landscape of homologous crossover for syntactic trees. In *Proceedings of IEEE Congress on Evolutionary Computation*, pp. 427–434.
- Moraglio, A., and Poli, R. (2006a). Product geometric crossover. In *Proceedings of Parallel Problem Solving from Nature Conference*, pp. 1018–1027.
- Moraglio, A., and Poli, R. (2006b). Inbreeding properties of geometric crossover and non-geometric recombinations. In *Proceedings of the Workshop on the Foundations of Genetic Algorithms*, pp. 1–14.
- Moraglio, A., and Poli, R. (2011). Topological crossover for the permutation representation. *Intelligenza Artificiale* 5(1):49–70.
- Moraglio, A., and Togelius, J. (2007). Geometric PSO for the Sudoku puzzle. In *Proceedings of the Genetic and Evolutionary Computation Conference*, pp. 118–125.
- Moraglio, A., and Togelius, J. (2009). Geometric differential evolution. In *Proceedings of the 11th Annual Conference on Genetic and Evolutionary Computation*, pp. 1705–1712.
- Moraglio, A., Togelius, J., and Lucas, S. (2006). Product geometric crossover and the Sudoku puzzle. In *Proceedings of IEEE Congress on Evolutionary Computation*, pp. 470–476.
- O’Neill, M., and Brabazon, A. (2006). Grammatical differential evolution. In *Proceedings of the 2006 International Conference on Artificial Intelligence*, pp. 231–236.
- Onwubolu, G. C., and Davendra, D. (Eds.). (2009). *Differential evolution: A handbook for global permutation-based combinatorial optimization*. Berlin: Springer.
- Pampara, G., Engelbrecht, A. P., and Franken, N. (2006). Binary differential evolution. In *Proceedings of the IEEE Congress on Evolutionary Computation*, pp. 1873–1879.
- Price, K. V., Storm, R. M., and Lampinen, J. A. (2005). *Differential evolution: A practical approach to global optimization*. Berlin: Springer.
- Storn, R., and Price, K. (1997). Differential evolution a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, 11(4):341–359.
- Syswerda, G. (1989). Uniform crossover in genetic algorithms. In *Proceedings of the Third International Conference on Genetic Algorithms*, pp. 2–9.
- Togelius, J., De Nardi, R., and Moraglio, A. (2008). Geometric PSO+GP = particle swarm programming. In *Proceedings of the Congress on Evolutionary Computation (CEC)*, pp. 3594–3600.
- Whitley, D., Starkweather, T., and Shaner, D. (1991). Travelling salesman and sequence scheduling: Quality solutions using genetic edge recombination. In L. Davis (Ed.) , *Handbook of Genetic Algorithms* (pp. 350–372). New York: Van Nostrand Reinhold.
- Yato, T. (2003). Complexity and completeness of finding another solution and its application to puzzles. Master’s thesis, University of Tokyo, Japan.