Essays on Fiscal Policy and Political Economy

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

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Abstract

This thesis consists of three essays concerned with endogenous fiscal policy and its interaction with political economy constraints. The first essay presented in Chapter 2 examines the cyclical behavior of endogenous government consumption over the business cycle absent a commitment mechanism in a neoclassical economy with Total Factor Productivity (TFP) shocks and investment shocks. Tax rates that finance public consumption are chosen in a time consistent way in a dynamic game between the government and a representative agent that values public goods in his utility. It is found that government consumption set without commitment behaves procyclical in response to the mentioned shocks. The government-consumption-output ratio is mildly procyclical or countercyclical depending on the selected calibration. Particularly, the elasticity of substitution between private and public goods plays an important role. The second essay showed in Chapter 3 extends the model studied in Chapter 2 adding agent heterogeneity in wealth and labor productivity. The aim of this study is to identify how policy outcomes are affected by inequality of households, particularly the median voter's choice of tax rates that finance public goods. For a standard RBC calibration to the U.S. economy the result is a strong procyclical comovement of public consumption with output, and a relatively weak procyclical comovement of the output share of public consumption with output, that becomes stronger with rising inequality. The politico-economic channel induces causality from output to lagged tax rates, therefore after a Hicks neutral productivity shock the median voter tries to delay the increase in the tax rate, such that the increment will take place just after the accumulation of more capital. In the case of equal agents the response is to decrease the tax rate in the first year after the shock. Additionally, the model predicts that the size of government consumption decreases with inequality. The last essay in Chapter 4 presents a stylized model of external sovereign debt that incorporates corruption in the form of rent-seeking groups by which the choice to cooperate or non-cooperate in providing public goods, in extracting rents and in issuing debt, is endogenized. More than one rent-seeking group originates a "tragedy of the commons" over fiscal resources that make the borrower economy to show collective fiscal impatience. External creditors envision that impatience and require higher interest rates for buying bonds, exacerbating the problem of high debt. The high level of interest rates decreases the wealth of the country and endangers its ability to repay the debt. We show that bailout plans, defined as temporary loans with lower than market level interest rates, are not effective in such economies.

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Chapter 1

Introduction

The research presented in this thesis deals with endogenous fiscal policy setting in the presence of political economy constraints. Two chapters of this work focus on how government consumption policy is set over the business cycle in an economy where the benevolent government cannot commit to future policy beyond one period of one year length. The government is constrained to have a balanced budget while the heterogenous households that differ in wealth and labor productivity finance public goods with labor and capital income taxes. These public goods provide benefits to the households. Their individual preferences over tax rates translate to the societal level through the political mechanism of majority voting. In order to analyze the tax rates outcomes in expansion and recession periods, the economy is exposed to Hicks neutral productivity shocks and investment shocks. Additionally, the model is extended to incorporate variable capital utilization. The final chapter is devoted to study sovereign external debt in an economy with free access to international capital markets and rent-seeking groups that have the power to set fiscal policy. The number of rent-seeking groups and their choice to cooperate or not in austere debt issuance, public goods provision or in seeking rents is a relevant factor for the debt-to-GDP ratio dynamics. More than one rent seeking group originates a "tragedy of the commons" over fiscal resources that makes the borrower economy to show collective fiscal impatience leading the economy to increase its level of indebtedness over time.

The three essays included in this dissertation are related to several strands of literature that have been blended to enrich the model environments in which fiscal policy can be studied with forward looking agents. The core of the models that I analyze is based on infinite horizon dynamic models of political economy that are time consistent by construction. To these belong the study of Krusell and Rios-Rull (1999) that presented the dynamic version of the seminal work of Meltzer and Richard (1981) extended to include heterogenous agents that differ not just in labor productivity but also in wealth holdings. Meltzer and Richard (1981) studied the size of government in a general equilibrium model where the median voter chose tax rates used to finance transfers while government consumption remained exogenous. Klein et al. (2008) used a similar setup which endogenized public consumption and abstracted from agent heterogeneity. Klein and Rios-Rull (2003) analyzed optimal time consistent fiscal policy where government consumption does not provide utility to the representative agent. The work of Battaglini and Coate (2008) provided a positive theory of endogenous fiscal policy in a stylized dynamic political economy model combined with a neoclassical real business cycle framework. They found government spending behaves procyclical while taxes decreased in booms and increased in recessions. To deal with time consistency in policy setting given the lack of commitment I followed closely the approach of Cohen and Michel (1988) which employs optimal control theory to find time consistent equilibria.

The second strand of the literature on which my work builds is the one of fiscal policy over the business cycle. For the business cycle theory I have followed the survey and exposition of real business cycles models of King and Rebelo (1999). The business cycle of the models presented in Chapter 2 and 3 are generated by Hicks neutral productivity shocks and investment shocks together with capacity utilization that has been included along the lines of Greenwood et al. (1988) and Greenwood et al. (2000).

Consequently, after having included business cycle theory into the analysis of endogenous fiscal policy, the next stream of literature to consider is the one which studies the cyclicality of government consumption. A large part of the literature studied government consumption as an exogenous variable. Theoretical studies include Aiyagari et al. (1992) who investigated the effects of government consumption on output, employment and interest rates in a stochastic neoclassical growth model. Baxter and King (1993) used a neoclassical model to show that permanent changes in government purchases can create multipliers bigger than one, and permanent changes are likely to have larger effects on output than temporal changes. The empirical studies that find procyclical government spending in developing countries include Lane (2003) and Iltzestki and Vegh (2008). The work presented on this thesis relates more to the studies of government spending as an endogenous variable reacting to exogenous shocks to the economy. Few studies have been devoted to this task. One of the first studies that focused on that direction was the one of Ambler and Cardia (1996) which analyzed the business cycle properties of endogenous public spending and non-military current expenditures. Some recent studies pursued the same approach, as the work by Bachmann and Bai (2011) that studied the volatility of government purchases as an endogenous reaction to macroeconomic conditions. Both studies had found procyclical behavior of government consumption as response to output shocks. Chapter 2 and 3 differ from the previous studies by extending the economy to have variable capacity utilization and investment shocks, and particularly Chapter 3 focuses on the heterogenous agents' demand of public goods as a response to exogenous shocks in productivity and investment specific shocks.

The last chapter of this thesis analyzes sovereign external debt in a rather stylized model featuring rent-seeking groups. It relates to the literature in both sovereign debt and endogenous fiscal policy, which interlinked those topics with three relevant aspects: the risk of default, the effects of government debt on growth and the influence of political instability on the level of debt issued. A study with a similar framework to the work presented in the last Chapter is the one of Yared (2010) which followed the model structure of Lucas and Stokey (1983) with rent-seeking politicians that increase macroeconomic volatility and debt levels. Models on sovereign debt and risk of default include the work of Aguiar and Gopinath (2006) and Arellano (2008). They rely on Eaton and Gersovitz (1981) which provided a contribution on the understanding of debt. A new strand of literature on sovereign debt crises in monetary unions, to which this thesis aims to contribute, includes the work of Daniel and Shiamptanis (2010) and Roch and Uhlig (2011).

Fiscal policy gained primary importance given the current challenges that the world economy is facing. Simultaneously, monetary policy became less effective since interest rates reached the zero lower bound and the overhang of the recent financial crises left most of the advanced economies with high levels of indebtedness and deteriorated public finances. The eurozone debt crises demanded fiscal discipline and undesired austerity measures creating political unrest. Therefore, the way fiscal policy is going to be managed in the years to come will have a great impact on economic performance. The following three chapters make a contribution to the subject of endogenous fiscal policy and political economy.

Chapter 2 studies the business cycle properties of endogenous government consumption that yields utility to the representative agent. The model is similar to the one that Klein et al. (2008) used to examine time consistent public policies of a government that does not have a commitment mechanism. In order to be able to analyze the business cycle properties of endogenous public consumption, the model is enriched with Hicks neutral technology shocks, investment shocks and variable capacity utilization as in Greenwood et al. (1988). I solved the model for two kinds of utility functions: CES form and GHH preferences. While the CES form allows to investigate the influence of variations on parameters like the elasticity of substitution between public and private goods on the cyclical properties of the studied variables, the inclusion of the GHH preferences was motivated by the fact that in these preferences the marginal rate of substitution between consumption and leisure does not depend on the consumption level within the period and it hinders the wealth effect on labor supply. Some papers concerned with business cycles in small open economies like in Correia et al. (1995) and Neumeyer and Perri (2005) have used such preferences to better reproduce some of the business cycle properties of emerging economies. Aguiar and Gopinath (2007) mentioned that GHH preferences created larger responses of consumption and labor to productivity shocks, due to the high degree of substitutability between leisure and consumption in the utility function. However, their main results are robust to the choice of GHH preferences or Cobb-Douglas preferences.

For a calibrated model to the U.S. economy, it is shown that government consumption behaves procyclical as a reaction to TFP shocks and investment shocks. The representative agent chooses to increase government spending in good times as he benefits from the public good although tax rates are imposed on labor and capital income which is distortionary. The government-consumption/output ratio is slightly procyclical or countercyclical depending on the parameters selected in the calibration. A relevant parameter is the elasticity of substitution between private and public goods. In the case of public and private consumption being complements, the government-consumption/output ratio is countercyclical. Contrarily, when public and private consumption are substitutes, the correlation turns to be positive. The contribution of this study is the characterization of the business cycle properties of the selection of government consumption policy without commitment in a time consistent environment within a stochastic economy with productivity and investment-specific shocks. The model predicts procyclical government consumption when public goods are included in the agents' utility function, i.e. when government consumption is useful for the agents.

Chapter 3 is joint work with Christos Koulovatianos and João Madeira. It analyzes an extended version of the model described in Chapter 2. We combine a RBC model as presented by King and Rebelo (1999) with a modification of the heterogeneous agents model of Krusell and Rios-Rull (1999). Hence, instead of focusing on voting over fiscal transfers, we study voting over public consumption when agents derive utility from a public good. In order to focus on government consumption, taxes are assumed to finance just the provision of public goods or services. The model just abstracts from transfers and debt. The inclusion of wealth and income heterogeneity is a key element in the definition of the role of the median voter in the demand for government consumption in a democracy. The main contribution of this study is to set up a framework which explains the cyclical behavior of endogenous government consumption in a parsimonious business cycles model that allows heterogeneity of households and the aggregation of preferred policies by means of the political mechanism of majority voting. The versatility of our model allows us to compare the way government consumption is selected in equal and unequal societies and how different macroeconomic variables react to TFP and investment shocks as a result of the pivotal voter's selection of the tax rate that finances public goods.

We find that public consumption is procyclical. The dynamic responses analysis calibrated for the U.S. economy show the way in which individuals, who differ in wealth and labor income, adjust their selection of fiscal variables as a response to changes in productivity. Although we allow for two dimensions of heterogeneity, majority voting is reduced to one-dimensional heterogeneity since the pivotal voter has intermediate values of wealth as well as intermediate labor productivity.[1]

The politico-economic channel induces causality from output to lagged tax rates. Hence, as a consequence of a positive TFP shock the median voter, although he chooses to increase the tax rates, tries to postpone the increase in the tax rate until the time at which the economy has accumulated more capital. This reaction is different in the case of equal agents: the pivotal voter, who behaves like a representative agent in the equal economy, chooses a decrease in the tax rate in the first year after the stochastic stimulus of a positive productivity shock. Five years later, approximately, equal agents will increase the tax rate getting back again rapidly to the initial values. This feedback makes the tax rates behaving acyclical or countercyclical in the model calibrated to equal agents. In the calibration with heterogeneity, tax rates behave procyclical. An additional result originates from the analysis of the cyclical

[1] This is shown in the calibration of the distribution of wealth and earnings in Table 3.2.

behavior of the output share of government consumption. Our model predicts that the standard RBC calibration for the U.S. economy implies a procyclical output share of government consumption in the economy with inequality. The positive correlation between the output share of government consumption and output becomes stronger, the higher the wealth/income inequality. In the case without inequality the public consumption share of output and output is almost acyclical. In some cases it can be countercyclical, namely when the private and public goods tend to be more complements. Another remarkable result is that the size of government consumption decreases with inequality, i.e. the median voter chooses smaller tax rates when he benefits just from public goods and not from transfers. Since in the model there are no transfers and no debt, the choice of tax rates determines directly the size of government consumption. The relationship between the size of government consumption and inequality is derived from the comparison of the size of government consumption when all the agents are the same (equality) and the size of government obtained when agents are heterogenous in initial wealth and labor productivity. In the inequality case tax rates are lower which implies a smaller size of government consumption. The result of an inverse relationship between the government consumption level and inequality is supported by the empirical study of Shelton (2007).

Chapter 4 is the result of joint work with Christos Koulovatianos and John Tsoukalas. It builds on a stylized model of external sovereign debt and endogenous interest rates in an economy that suffers from corruption. Corruption is modelled as rent-seeking groups which have the power to appropriate public resources for their own benefit. We show that in the deterministic underlying model, more than one rent-seeking group creates fiscal impatience. This implies that the government spends more fiscal resources today and delays taxation, which increases sovereign debt. The main consequence of fiscal impatience is that the model exhibits a different rate of time preference of the domestic economy compared to the one of the external creditors. It results in higher country-specific real interest rates which worsen the debt burden of the corrupt country. In the case of a single rent-seeking group, fiscal impatience could be avoided. Nevertheless, even with one or more rent-seeking groups, high levels of the sovereign-debt ratio decrease the utility of non-rent-seekers because the revenue resulting from the tax rates charged will not correspond to the amount of public goods provided due to the appropriation of part of those revenues by the rent seeking groups. Therefore, corruption is considered in any case as a reason for social discontent and of vulnerability for countries that may need to issue excessive debt in emergency situations. In economies free from corruption, an equal rate of time preference for the domestic economy and external creditors produces constant levels of debt over the infinite horizon. However, those countries that show high debt-GDP ratios over a certain threshold may as well end up in a trap of high interest rates because creditors may consider that threshold excessive as to generate social pressures that could induce an unexpected haircut to alleviate the debt burden. Such anticipation of a haircut will trigger an increase in interest rates to compensate investors. This causes debt-GDP ratios to increase even further, leading the country into such a trap. There are two main factors that may trigger such a trap. The first one is high initial debt-GDP ratio as a consequence of, for example, a disaster occurred in the immediate past that required high emergency public spending/borrowing. The second factor can be information asymmetries between external creditors and the indebted country. Since our model is deterministic we do not explicitly model information and we do not study how a country enters such a trap in this context. Nevertheless, we explain the mechanics of such a trap and we show in our model that even under optimal policy, high interest rates increase debt-GDP ratios over time such that a bailout plan is needed. The key contribution of this work is the presentation of a model that puts together elements that allow describing a positive theory on sovereign external debt in a monetary union member that has rent-seeking groups. However, the framework has some weakness as it assumes a deterministic environment, it abstracts from the existence of capital and the closed form solutions rely on the logarithmic utility function. Such simplifications have been properly justified. Nevertheless, the model provides a pillar to study the dynamics of external-sovereign debt with political economy constraints that generate fiscal impatience.

Chapter 2

Endogenous Government Consumption without Commitment and the Business Cycle

2.1 Introduction

This chapter investigates the business cycle properties of endogenous government consumption when it is chosen in a time consistent manner, without commitment or any reputation mechanism, as a result of a dynamic game between the government and a representative agent inside the framework of a neoclassical economy. The cycle is driven by total factor productivity (TFP) shocks and investment shocks. The model captures the optimal response of government consumption policy to random shocks. Individuals value government consumption and it is therefore present in their utility function. Public goods are financed only by labor and capital income taxes and there is no debt. The government is thus restricted to have a balanced budget every period. The government is also unable to commit to future policy and one reason why no debt can be issued is that the lack of commitment would make agents to anticipate the possibility of default.[1] The described setup is similar to the one that Klein et al. (2008) used to analyze time consistent public policy when the government does not have a commitment mechanism. In order to study the business cycle properties of endogenous public consumption I add a Hicks neutral shock, an investment shock and variable capacity utilization as in Greenwood et al. (1988).

[1] See Klein et al. (2008) for more details on the absence of debt in such policy settings.

Most of the literature on fiscal policy has focused on studying how output reacts to shocks in government purchases and attempted to measure the size of the fiscal multiplier since public spending has been used as a tool to generate expansion of the economy. This action known as fiscal stimulus is employed in times of recession as a way to bring the economy back to a good stand. The role of automatic stabilizers in smoothing out the effects of business cycles has been intensively studied particularly during economic slowdowns. However, the lack of good instruments to estimate the value of the multiplier for non-defence purchases has lead to an overestimation of the magnitude of the multiplier. This literature has ignored as well, with some exceptions, the issue of causation from output to government spending.[2] In this study, we are interested rather in the optimal government consumption response to shocks in productivity and shocks to the relative price of investment.

Few papers have studied government expenditures as an endogenous variable. The work of Ambler and Cardia (1996) was one of the first attempts to analyze the business cycle properties of endogenous public spending and non-military current expenditures. They studied the optimal response of endogenous government spending to shocks to technology and military spending. A recent paper by Bachmann and Bai (2011) studied the volatility of government purchases as an endogenous reaction to macroeconomic conditions. To replicate the business cycles properties of government purchases of the U.S. economy they added implementation lags, implementation costs and taste shocks. Both papers found procyclical behavior of endogenous government consumption.

In the model proposed here I add variable capacity utilization and employ different specifications for the utility function that have been widely used in the RBC literature. The benchmark utility function is of the CES form and the alternative utility function is of the form introduced by Greenwood et al. (1988) referred as GHH. GHH preferences allow studying the response of agents to productivity shocks when there is no wealth effect. The CES utility function includes several parameters that help to perform a rigorous sensitivity analysis in order to analyze the effect of the elasticity of substitution between government and private consumption and different degrees of risk aversion and its effect on the election of tax rates that finance government consumption. With public spending in the utility function, the elasticity of substitution between government and public consumption plays an important role in the cyclical movement of government consumption. The latter mentioned

[2] See Barro and Redlick (2011).

preferences have been used for studying business cycle properties of macroeconomic variables in emerging countries, therefore for completeness I explore the behavior of our model with such preferences.[3]

For a model calibrated to the U.S. economy, I find that government consumption behaves procyclical in the presence of TFP shocks and investment shocks.[4] Agents consider optimal to increase government spending in good times as they benefit from the public goods in spite of the distortionary effect of labor and capital income taxes. The government-consumption/output ratio is mildly procyclical or countercyclical depending on the selected calibration, particularly of the parameter for the elasticity of substitution between private and public goods. When public and private consumption are complements, the government-consumption/output ratio is countercyclical. When public and private consumption are substitutes, the correlation becomes positive. An alternative utility function, in this case of the GHH form, shows the same results, namely procyclicality of government consumption after positive Hicks neutral shocks or investment shocks. Given that the specification of the GHH utility function causes private and public consumption being substitutes, the government-consumption/output ratio is always procyclical. However, in an specification of the utility function in GHH form with the private and public goods being neither complements or substitutes, government consumption shows the same behavior as in the CES form. An interesting finding is the government set the tax rates after a TFP shock. As it emerges from the impulse response function, tax rates increase during booms with GHH preferences (where private and public consumption are perfect substitutes). Contrary, agents with CES preferences decide to support that the government decreases tax rates in a boom and increase them in a recession, regardless of the degree of substitutability or complementary between private and public goods. The intuition behind those results, is that in the GHH preferences in the case of private and public consumption being perfect substitutes (notice that in such a case the tax rate is positive only if public goods substitute more than one unit of private consumption), if there is a boom, the tax increment will allow a higher amount of public consumption that agents can substitute for private consumption. According to the sensitivity analysis, the CES utility will show similar results if the

[3]See Correia et al. (1995) and Aguiar and Gopinath (2007).

[4]Procyclicality here means that there is a positive correlation between government consumption and output. Through this Chapter procyclicality refers to positive correlation between two variables and countercyclicality, negative correlation between two variables. Correspondingly, acyclicality means zero correlation between two variables.

parameter determining the amount of the public good preferred increases.

The results of the simulated model are confronted to the data. Time series data for government final consumption expenditure for 33 OECD countries and 46 non-OECD countries were taken from the World Bank Development Indicators database for the period from 1960 to 2010 and were analyzed in a simple econometric exercise.[5] It shows procyclical government consumption for most OECD countries and developing countries.

The contribution of this study is to characterize the business cycle properties of the optimal selection of government consumption policy without commitment in a time consistent environment within a stochastic economy with variable capacity utilization, random fluctuations in productivity and investment-specific shocks. The model predicts procyclical fiscal policy when government consumption is included in the utility function.

The literature that studies exogenous government spending is vast. Several papers documented the impact of government spending shocks on output fluctuations. Barro (1981) distinguished between the effects of temporary and permanent government purchases shocks on output fluctuations. He showed empirically and theoretically that a temporal shift in defence spending has a larger expansionary effect on output than permanent changes in defence purchases. The effect from non-defence government purchases was rather inconclusive. A recent paper by Barro and Redlick (2010) estimated the multiplier for temporary defence spending to be around 0.6-0.7 over a two years period, the multiplier for non-defence expenditure is declared as non reliable due to the lack of good instruments. On the theoretical side, Aiyagari et al. (1992) investigated the effects of government consumption on output, employment and interest rates in a stochastic neoclassical growth model. They showed that a persistent change in government consumption has a bigger impact on output, employment and interest rates than temporal ones. Baxter and King (1993) showed that in a neoclassical model permanent changes in government purchases can create multipliers bigger than 1, and permanent changes are likely to have larger effects on output than temporal changes. This chapter is related as well to the literature on time consistent public policy. As mentioned before, this paper builds on the work of

[5] Government final consumption expenditures include all government current expenditures for purchases of goods and services (including compensation of employees). They also include most expenditures on national defense and security, but exclude government military expenditures that are part of government capital formation (definition of the World Bank).

Klein et al. (2008).

A large literature investigates the cyclicality of government spending in developed and developing countries. Lane (2003) found procyclical government consumption for a sample of 25 OECD countries in the period 1960 to 1998. When he studied the different components of government consumption, he encountered that government consumption in the form of wages is more procyclical than non-wage government consumption. Iltzestki and Vegh (2008) found procyclical government consumption for high income countries and developing countries, their sample used quarterly data for a time span from 1960 to 2006. Kamisnky et al. (2005) found that government spending is procyclical in the majority of developing countries. These papers emphasized on the importance of endogeneity in determining the effect of government purchases on output. Rigobon (2004) confirmed the results of Kamisnky et al. (2005) on procyclicality of real expenditures of developing countries but highlighted that it is important to look at the variances of the shocks and not only at the coefficients of the regression. He found that countries indeed with larger variances of output shocks tend to have countercyclical government expenditures.

The frictionless neoclassical business cycle model with endogenous consumption chosen under no commitment, and given that marginal utilities of private and public goods are contemporaneously aligned, yields procyclical public consumption regardless of the preferences chosen. The cyclical behavior of the government consumption share on output and tax rates depends on the calibration. Perhaps adding some implementation lags or some other political-decision-making frictions can fit the comovement of government purchases and output to the data. Yet, given the high complexity of dynamic games and their demanding numerical implementation, these are extensions for future work in which one may need to introduce some other political frictions (adjustment costs, political ratchet effects, etc). In Chapter 3, the model is extended to include political economy considerations. For that, agents are considered to be heterogeneous in wealth and labor productivity.

The rest of the Chapter is organized in the following way. Section 2 describes the model. Section 3 explains the economic equilibrium and Section 4 presents a detailed explanation of the calibration. Section 5 explains the simulation results, Section 6 displays the sensitivity analysis results and Section 7 analyzes impulse responses. Section 8 presents some empirical evidence and concluding remarks are given in Section 9.

2.2 The model

This setup shares similar characteristics to the one that Klein et al. (2008) used to study time consistent public policy without commitment. In that sense the framework employed in this study follows the same equilibrium concept as Krusell and Rios-Rull (1999) but in a representative agent environment and in a stochastic neoclassical economy. Since I am interested in the business cycles behavior of government consumption chosen without commitment, I include the Real Business Cycle properties as described in King and Rebelo (1999) into the model setup. Hence, I add a Hicks neutral shock, an investment shock and variable capacity utilization as in Greenwood et al. (1988).

The economy is inhabited by a large number of identical infinitely lived households of total mass one. Each household derives utility from a single final consumable good, a public good and leisure. Individuals are infinitesimal and take prices as given. The horizon is infinite and time is discrete. One period corresponds to one year and it is indexed by t = 0, 1, 2, ... All households own the same stock of capital in period 0 and $k_0 > 0$. Agents maximize their lifetime utility function that is time-separable and stationary, meaning that utility at time t is independent of past and future consumption levels. Households obtain income from wages and letting capital. Firms rent capital from the households and produce a consumption good with a technology that has constant returns to scale in labor and capital inputs. When supplying capital, households also decide about the rate of utilization.[6]

2.2.1 Households

All households have the same preferences over the infinite sequence of consumption and discount the future exponentially at rate β. The representative household benefits from the consumption of a private good c_t, from consumption of a public good G_t and from leisure $(1 - l_t)$. The expected lifetime utility is represented by,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t, 1 - l_t, G_t\right) \right] \right\} \quad ; \quad \beta \in (0, 1)\,, \tag{2.1}$$

where E_0 is the expectation operator with respect to the information set available to the representative household at time $t = 0$. The household's sources of income are wages and renting capital.

[6] Through the model individual variables are denoted by lower-case letters and aggregate variables by upper-case letters. The only exception to this rule is the notation of prices.

2.2.2 Firms

There is a large number of small competitive firms. They hire labor and capital from the households. Since households can choose the rate of capital utilization, the production function includes the rate of utilization and a Hicks-neutral shock, such that the production technology which exhibits constant returns to scale is given by,

$$y_t = F(k_t h_t, l_t) \tag{2.2}$$

$$F(k_t h_t, l_t) = z_{1,t} \left[\alpha \left(k_t h_t\right)^{1-\frac{1}{\nu}} + (1-\alpha) \, l_t^{1-\frac{1}{\nu}} \right]^{\frac{\nu}{\nu-1}} , \tag{2.3}$$

where $\alpha \in (0,1)$, $\nu > 0$. Capital is denoted by k_t and labor by l_t. The variable h_t represents capacity utilization, it is a higher speed or higher number of hours on the usage of capital and therefore determines the intensity of utilized capital $k_t h_t$.[7] The TFP shock is denoted by $z_{1,t}$. It follows an AR(1) process,

$$\ln\left(z_{1,t+1}\right) = \rho_1 \ln\left(z_{1,t}\right) + \varepsilon_{1,t+1} ,$$

with $\varepsilon_{1,t+1} \sim N\left(0, \sigma^2_{1,\varepsilon}\right)$, i.e. it is i.i.d. over time. The accumulation of capital is described by the following equation:

$$k_{t+1} = z_{2,t} \cdot i_t + \left[1 - \delta\left(h_t\right)\right] k_t , \tag{2.4}$$

with $z_{2,t}$ representing an investment shock. Investment in time t is denoted by i_t. The investment shock behaves as well as an AR(1) process

$$\ln\left(z_{2,t+1}\right) = \rho_2 \ln\left(z_{2,t}\right) + \varepsilon_{2,t+1} , \tag{2.5}$$

with $\rho_2 \in (-1,1)$, $\varepsilon_{2,t+1} \sim N\left(0, \sigma^2_{2,\varepsilon}\right)$, i.e. it is i.i.d. over time.

Problem of the household

The household maximizes his utility, (equation (2.1)) and chooses the optimal sequence of consumption, labor supply, savings and the rate of capital utilization $\{c_t, l_t, k_t, h_t\}$ subject to the budget constraint given by,

$$e^{-x_{2,t}} k_{t+1} = \left(e^{-x_{2,t}} + \bar{r}_t\right) k_t + \bar{w}_t l_t - c_t . \tag{2.6}$$

where

$$x_{2,t} \equiv \ln\left(z_{2,t}\right) .$$

[7] As defined in Greenwood et.al 1988.

Labor and capital income are taxed at a flat tax rate τ_t. The reason to have a flat tax is the same for not having debt and it is that given the numerical method used here of quadratic approximation, the policy choice space should be of one-dimension in order to guarantee time consistency. Klein et al. (2003) explained that time consistency holds in this environment because there is only one dimensional way in which the government can affect future taxes and it is choosing τ that influences K' which has an effect on the choice of τ' and so on.[8]

The corresponding after tax interest rates and after tax wages respectively given by $\bar{r}_t$ and $\bar{w}_t$, are determined in competitive factor markets and represented by the following equations:

$$\bar{r}_t \equiv (1-\tau_t)\left[R_t h_t - e^{-x_{2,t}}\delta\left(h_t\right)\right] \ , \tag{2.7}$$

and the after tax wage rate is

$$\bar{w}_t \equiv (1-\tau_t)\, w_t \ , \tag{2.8}$$

R_t is the rental rate of capital and the rate of capital utilization is denoted by h_t. The corresponding rate of depreciation is given by $\delta\left(h_t\right)$. Higher utilization of capital causes higher depreciation. Therefore, depreciation is an increasing and concave function of the rate of capital utilization. It is denoted by $\delta\left(h_t\right)$ and has the following form:

$$\delta\left(h_t\right) = \delta_c + \frac{b_\delta}{1+\xi}h_t^{1+\xi} \ ; \qquad \delta_c, b_\delta, \xi > 0 \ . \tag{2.9}$$

The economy is closed and output can be used for private consumption, public goods or investment, such that,

$$Y_t = C_t + I_t + G_t. \tag{2.10}$$

Substituting this into (2.6) we obtain

$$e^{-x_{2,t}}K_{t+1} = Y_t + e^{-x_{2,t}}\left[1-\delta\left(H_t\right)\right]K_t - C_t - G_t \ . \tag{2.11}$$

Problem of the firm

The representative firm rents a capital amount $h_t k_t$ and labor l_t in order to maximize profits,

[8] The simbol ($'$) denotes next period variables.

$$\max_{k_t,h_t,l_t} z_{1,t}\left[\alpha\left(h_t k_t\right)^{1-\frac{1}{\nu}}+(1-\alpha)\, l_t^{1-\frac{1}{\nu}}\right]^{\frac{\nu}{\nu-1}}-R_t h_t k_t-w_t l_t\ , \tag{2.12}$$

where w_t is wage per hour and R_t is the rental cost of capital utilized in period t. First order conditions of the maximization problem of the firm imply,

$$F_1(k_t h_t, l_t) = R_t \tag{2.13}$$

and,

$$F_2(k_t h_t, l_t) = w_t. \tag{2.14}$$

The rental cost of capital R_t is optimally given by the marginal product of utilized capital, given by

$$R_t = \alpha z_{1,t}^{1-\frac{1}{\nu}}\left(\frac{y_t}{k_t h_t}\right)^{\frac{1}{\nu}}\ , \tag{2.15}$$

in the same way, optimal labor demand should equalize the wage rate to the marginal product of labor, such that,

$$w_t = (1-\alpha)\, z_{1,t}^{1-\frac{1}{\nu}}\left(\frac{y_t}{l_t}\right)^{\frac{1}{\nu}}. \tag{2.16}$$

In neoclassical production theory households rent perfectly divisible machines to firms in each period. So, they have the right to dictate the way that owners of machines (households) want their machines to be operated in each period (a different contract each period). The stock k_t has been determined in the previous period, therefore the stock of capital in period t is taken as given. Determining the utilization rate is finding h_t that maximizes the expression in the RHS of equation (2.7). The optimal rate of utilization h_t makes the marginal cost of the user of capital equal to the marginal benefit of capital.

2.2.3 Government

The government is benevolent in the sense that it cares about maximizing the utility of the representative household. The government is not allowed to have debt and therefore it has a balanced budget. The government provides a public good that is financed with labor and capital taxes denoted by τ_t. The budget constraint of the government with tax exempted depreciation is given by,

$$G_t = \tau_t\left[Y_t - \delta\left(H_t\right) e^{-x_{2,t}} K_t\right]\ . \tag{2.17}$$

The government does not have any long term commitment technology and it just can commit to the policy announced for one period. Taxation is distortionary and there are no lump-sum transfers.

2.2.4 Specifications of the utility function

In order to determine if the way that public goods enter the utility function plays a role on the cyclical behavior of government consumption, we present in this Section three different specifications for the preferences of the representative agent.

CES preferences (benchmark)

This form of utility function is the most widely used for business cycle models. There is a CES function between consumption and leisure that also comprises a CES function between private and public goods. This combination of functions is embedded in a function that yields Constant Relative Risk Aversion. The functional form for the preferences chosen as our benchmark model are given by,

$$u\left(c_t, 1-l_t, G_t\right) = \frac{\left\{\phi\left[\theta c_t^{1-\frac{1}{\chi}} + (1-\theta)\left(1-l_t\right)^{1-\frac{1}{\chi}}\right]^{\frac{(\zeta-1)\chi}{\zeta(\chi-1)}} + (1-\phi)\,G_t^{1-\frac{1}{\zeta}}\right\}^{\frac{(\eta-1)\zeta}{(\zeta-1)\eta}} - 1}{1-\frac{1}{\eta}} \tag{2.18}$$

with $\eta > 0, \phi \in (0,1)$, $\zeta > 0$, $\theta \in (0,1)$ and $\chi > 0$. θ is the share of government consumption, χ is the elasticity of substitution between consumption and leisure, ϕ is the share of private goods, ζ is the elasticity of substitution between private and public goods and η is the elasticity of intertemporal substitution.

GHH preferences (c and G as perfect substitutes)

An alternative utility function is specified to assess the robustness of the results of the benchmark model. I use a utility function that does not induce an income effect on the labor supply. This utility function was proposed by Greenwood et al. (1988) and has been labeled as GHH preferences. In this particular specification I followed Iltzestki (2011) who extended the function by including a public good. GHH preferences have the particularity that the marginal rate of substitution between consumption and leisure does not depend on the consumption level within the period.

Therefore, there is no wealth effect on labor supply since it does not depend on current consumption. The specification of the utility function is then

$$u(c_t, 1-l_t, G_t) = \frac{(c_t + \theta G_t - \varphi l_t^{\upsilon})^{1-1/\eta} - 1}{1-1/\eta}, \tag{2.19}$$

where $\theta > 1$, $\varphi > 0, \upsilon > 1$ and $\eta > 0$.

The public good in this utility function enters in a way that makes public and private consumption Edgeworth substitutes, since $u_{cg} < 0$ implies $\theta > 0$. In the model, given that government consumption is endogenous, it requires values of $\theta > 1$ in order to generate positive demand for the public good. It is worth noticing that in this specification a unit of government consumption is equivalent to θ units of private consumption.[9]

GHH preferences

Keeping the property of not having income effect on labor supply, we can also include government consumption as a separable function from private variables as in Cuadra et al. (2010), it is,

$$u(c_t, 1-l_t, G_t) = \pi \frac{(G_t)^{1-1/\eta}}{1-1/\eta} + (1-\pi) \frac{\left(c_t - \frac{\varphi l_t^{1+\upsilon}}{1+\upsilon}\right)^{1-1/\eta} - 1}{1-1/\eta}, \tag{2.20}$$

where π is the share of government consumption, $1/\upsilon$ is the elasticity of labor supply and η is the risk aversion parameter.

2.2.5 Time consistency

A relevant issue in endogenous government policy is the time inconsistency that arises from the absence of a commitment technology affecting the interaction of decision making of the government and private agents. First pointed out by Kydland and Prescott (1977) the government has an incentive to deviate from the committed policy if there is not a commitment mechanism. The interest in tackling this problem and to deliver policies that are time consistent even though the policy maker may not have any commitment technology or not even reputation mechanism has led to a

[9] This specification has been used in Barro (1981). However since the policy choice is endogenous, perfect substitution between public and private goods requires values of $\theta > 1$, otherwise the selected tax is zero.

large number of studies on optimal fiscal policy without commitment. Several methods have been suggested to calculate equilibria that take into account the possibility of the policy maker deviating from committed rules. The method used in this essay is the one developed by Klein et al. (2003) which shares common characteristics with the model that Cohen and Michel (1988) employed to find time consistent policies. According to Cohen and Michel (1988), a time consistent policy obeys the Bellman optimality principle which implies that policy functions set by the government had considered future decisions to be optimal. The solution to the dynamic programming problem stated in the research undertaken here satisfies the Bellman optimality principle and therefore time consistency is guaranteed. Agents internalize the decision process of the government when making their private decisions. That is, at each time t private agents can observe the current decision of the government that has the ability to pre-commit only for one period. By considering the one-period deviation of the proposed tax, agents calculate the equilibrium response of all variables in the future. The policy chosen is therefore time consistent since both the private sector and the government expect it to be implemented at later dates and the policy maker considers it optimal to implement in any point in time.

2.2.6 Variable capacity utilization and investment shocks

Business cycle research has shown that having variable capacity utilization is a more realistic approach to model capital, since machinery and equipment are more used in booms than in recessions. Kydland and Prescott (1988) showed that incorporating variable capacity utilization improved the amplification of the impact of technological shocks on macroeconomic variables. Greenwood et al. (1988) added variable capacity utilization in the real business cycle model in a neoclassical framework with shocks to the productivity of investment to account for the Keynes (1936) statement about the impact of the marginal efficiency of investment on economy fluctuations. They found that direct investment-specific technological shocks can affect labor productivity and employment through variable capacity utilization. In a latter study, Greenwood et al. (2000) argued that investment-specific technological change showed to be a source of economic growth: according to U.S. postwar data, the relative price of equipment and the ratio of equipment to GNP are negatively correlated. In their study, they found that investment-specific technological shocks accounted for almost 60 percent of output fluctuations.[10] To include variable capacity utilization on our model we follow the guidelines of Greenwood et al. (1988) and make depreciation an increasing function on the rate of utilization of capital. For completeness the model showed here includes investment shocks.

2.3 Equilibrium

2.3.1 Dynamic policy game

Dynamic policy games are a common method to analyze the equilibrium of endogenous public policy in infinite horizons settings with forward looking agents. Households' expectations about the future policy setting influence their behavior while at the same time the way governments set present and current policies takes into account the expectations of households. Policies can be chosen once and for all or can be chosen sequentially. Policies chosen sequentially resemble the no commitment approach implying that the timing of chosen policies has different outcomes. Commitment technologies are supposed to ensure that government policies remain as announced at the start of the economy throughout the infinite horizon. That approach seems unrealistic since governments lack commitment mechanisms. Therefore, policies with forward looking rational expectations lack credibility without commitment since private agents can anticipate that the policy makers may not honor their promises. To approach the problem of time consistency in an environment without commitment mechanism or other replacement of commitment mechanism as reputation, I follow the methodology proposed by Klein et al. (2008) where the government chooses policies based on fundamentals only focusing on Markov perfect equilibria which are time consistent.

The equilibrium concept employed here is characterized in Klein et al. (2008) and Krusell and Rios-Rull (1999). The tax rate that finances public consumption next period is determined in the current period by a function Ψ that depends on the aggregate states of the economy as denoted by equation (2.21). Due to the lack of commitment in every point in time private agents form expectations about the policy rules and the government forms expectations about household decisions. Choices are made sequentially as a result of a dynamic game between the policy makers and the

[10] See King and Rebelo (1999) for a complete survey about the literature related to variable capacity utilization in business cycles.

individual agents.

At the beginning of each period the government observes the realization of the shocks and move first choosing the next period policy that the next government should comply. Consequently, the representative agent decides how much to consume and save as well as the level of labor supply and the utilization of capital. Finally, the fiscal budget should clear. The government anticipates the way that current policy will affect future policy and it can reoptimize in every period. This kind of time-consistent fiscal policy was described in Cohen and Michel (1988) and have been dubbed as instantaneous pre-commitment. A policy with instantaneous pre-commitment is expected to be followed later on by the households and the government. The government finds optimal to implement in future periods the chosen policy at time t when it had the chance to choose this policy before the households did their economic decisions. In this sense the instantaneous pre-commitment time consistent policy is related to a feedback Stackelberg equilibrium.

The policy is chosen from a single-dimension policy space. The government commits to pre-announced policies only one year. The policy rule that will be time consistent by construction is described as:

$$\tau' = \Psi\left(x_1, x_2, K, \tau\right). \tag{2.21}$$

where x_1 denotes the TFP shock and x_2 represents the investment shock. Aggregate states for labor, next period capital and capacity utilization are conditional on rule Ψ and evolve according to a law of motion $\mathcal{A}\left(x_1, x_2, K, \tau \mid \Psi\right)$. They are represented by,

$$\begin{aligned} L &= \mathcal{A}_L\left(x_1, x_2, K, \tau \mid \Psi\right), \\ K' &= \mathcal{A}_K\left(x_1, x_2, K, \tau \mid \Psi\right), \\ H &= \mathcal{A}_H\left(x_1, x_2, K, \tau \mid \Psi\right). \end{aligned} \tag{2.22}$$

Timing of the game The timing of the game described above can be summarized on the following stages,

Stage 1	Stage 2	Stage 3
Shocks are revealed and government sets (τ, G) in order to maximize utility	Agents make economic decisions subject to (τ, G)	Fiscal budget clears $G_t = \tau_t\left[Y_t - \delta\left(H_t\right) e^{-x_{2,t}} K_t\right]$

2.3.2 Economic equilibrium

The economic equilibrium involves the finding of a competitive equilibrium for a given policy. Agents take as given the policy function. The law of motion for the state variables of the economy will be given by the function $\mathcal{A}(.)$ that depends on the state variables and the policy rule $\Psi(.)$ capturing the direct mapping from the state variables in period t to the policy variable τ_t and on the future values of the states variables. The representative household solves its economic problem stated by the Bellman equation,

$$V\left(x_1, x_2, \mathbf{K}, k, \tau \mid \Psi\right) = \max_{k' \in \left[\underline{k}, \overline{k}\right],\ l \in [0,1],\ h \geq 0} u\left(c, 1-l,\ G\right) + \beta V\left(x_1', x_2', \mathbf{K}', k', \tau' \mid \Psi\right) \tag{2.23}$$

subject to,

$$c = \left(e^{-x_2} + \bar{r}\right) k + \bar{w} l - e^{-x_2} k' \ , \tag{2.24}$$

$$\begin{aligned} L &= \mathcal{A}_L\left(x_1, x_2, K, \tau \mid \Psi\right), \\ K' &= \mathcal{A}_K\left(x_1, x_2, K, \tau \mid \Psi\right), \\ H &= \mathcal{A}_H\left(x_1, x_2, K, \tau \mid \Psi\right), \end{aligned} \tag{2.25}$$

$$G_t = \tau_t \left[Y_t - \delta\left(H_t\right) e^{-x_{2,t}} K_t\right], \tag{2.26}$$

$$r_t \equiv r(L, K, H), \tag{2.27}$$

$$w_t \equiv w(L, K, H), \tag{2.28}$$

$$\tau' = \Psi\left(x_1, x_2, K, \tau\right). \tag{2.29}$$

The solution of this problem is the fixed point for $V\left(x_1, x_2, K, k, \tau \mid \Psi\right)$, where the aggregate decision rules equal the individual decision rules. The following definitions characterizing the equilibrium resemble the one described in Person and Tabellini (2002) adapted to this particular problem.

Definition 1 *An economic equilibrium or a competitive equilibrium given the policy rule* Ψ*,is a law of motion for the individual consumer* $\mathcal{A}^i\left(x_1, x_2, K, k, \tau \mid \Psi\right)$*, a law of motion for the aggregate variables* $\mathcal{A}\left(x_1, x_2, K, \tau \mid \Psi\right)$ *and a value function* $V\left(x_1, x_2, K, k, \tau \mid \Psi\right)$ *such that a)* $\mathcal{A}^i\left(. \mid \Psi\right)$ *is optimal for the individual consumer and the value function* $V\left(. \mid \Psi\right)$ *solves his dynamic programming problem as described in 2.23 and b) the optimal law of motion of the individual households reproduce the aggregate law of motion the households perceive when solving their decision problems, i.e.* $\mathcal{A}\left(x_1, x_2, K, \tau \mid \Psi\right) = \mathcal{A}^i\left(x_1, x_2, K, K, \tau \mid \Psi\right).$

2.3.3 Economic equilibrium after one-period deviation from policy Ψ

Consider the situation in which the government can reoptimize and choose a policy $\tilde{\tau}'$ that can be different from the policy rule given by Ψ. Therefore, it is an intermediate step where the government sets tax rates in a different way to its promised policy Ψ, namely, there will be a one time period tax deviation. This equilibrium, that is not an outcome, will be called "intermediate equilibrium" denoted as (*IE*), where next period tax rate will be given by $\tilde{\tau}'$ and future taxes are chosen according to function Ψ. The set of laws of motion in the "intermediate equilibrium" for the individual households are denoted as $\mathcal{A}^i_{IE}\left(x_1, x_2, K, k, \tau, \tilde{\tau}' \mid \Psi\right)$.[11] This is the solution to the modified value function (2.30) which takes into account the effects on the welfare of individual households given both a one-period deviation policy and future expectations on the policy rule Ψ held fixed. The modified dynamic problem for a one-period deviation from policy Ψ is described by

$$\hat{V}^i\left(x_1, x_2, K, k, \tau, \tilde{\tau}' \mid \Psi\right) = \max_{k' \in \left[\underline{k}, \overline{k}\right],\ l \in [0,1],\ h \geq 0} \left\{u\left(c, (1-l), G\right) + \beta V\left(x_1', x_2', K', k', \tau' \mid \Psi\right)\right\}, \tag{2.30}$$

subject to

$$c = \left(e^{-x_2} + \bar{r}\right)k + \bar{w}l - e^{-x_2}k', \tag{2.31}$$

$$G_t = \tau_t\left[Y_t - \delta\left(H_t\right)e^{-x_{2,t}}K_t\right], \tag{2.32}$$

$$r_t \equiv r(L, K, H), \tag{2.33}$$

$$w_t \equiv w(L, K, H), \tag{2.34}$$

$$\begin{aligned} L &= \mathcal{A}_{L,IE}\left(x_1, x_2, K, \tau, \tau' \mid \Psi\right), \\ K' &= \mathcal{A}_{K,IE}\left(x_1, x_2, K, \tau, \tau' \mid \Psi\right), \\ H &= \mathcal{A}_{H,IE}\left(x_1, x_2, K, \tau, \tau' \mid \Psi\right). \end{aligned} \tag{2.35}$$

Accordingly, $V\left(x_1', x_2', K', k_i', \tau' \mid \Psi\right)$ is used only on the RHS of (2.30) and iterations are on the rule $\mathcal{A}^i_{IE}\left(x_1, x_2, K, k, \tau, \tau^{\tilde{\prime}} \mid \Psi\right)$. Then, the individual decision rules implied by (2.30) are consistent with the average aggregate decisions rules denoted by $\mathcal{A}_{IE}\left(x_1, x_2, K, \tau, \tau' \mid \Psi\right)$ and then we obtain the value function $\hat{V}^i\left(x_1, x_2, \mathbf{K}, k, \tau, \tilde{\tau}' \mid \Psi\right)$. The law of motion including τ', $\mathcal{A}_{IE}\left(x_1, x_2, K, \tau, \tilde{\tau}' \mid \Psi\right)$ implies that the aggregate economy takes into account the changes in the tax rate

[11] The subscript IE, denotes "intermediate equilibrium" and was initially named this way by Krussel and Rios-Rull (1999).

one period ahead, i.e. changes in τ' as the model assumes commitment for one period only. It is the equivalent to the "instantaneous pre-commitment" policy-setting concept of Cohen and Michel (1988).

Definition 2 *An equilibrium with one-period deviation from policy* Ψ, *is a law of motion for the individual consumer,* $\mathcal{A}^i_{IE}\left(x_1, x_2, K, k, \tau, \tilde{\tau}' \mid \Psi\right)$, *a modified value function* $\hat{V}\left(x_1, x_2, \mathbf{K}, k, \tau, \tilde{\tau}' \mid \Psi\right)$, *a law of motion for the aggregate variables* $\mathcal{A}_{IE}\left(x_1, x_2, K, \tau, \tilde{\tau}' \mid \Psi\right)$, *and a set of functions* $\mathcal{A}^i\left(x_1, x_2, K, k, \tau \mid \Psi\right)$, $\mathcal{A}\left(x_1, x_2, K, \tau \mid \Psi\right)$ *and* $V\left(x_1, x_2, \mathbf{K}, k, \tau \mid \Psi\right)$ *such that: a)* $\mathcal{A}^i(.\mid \Psi)$, $\mathcal{A}\left(.\mid \Psi\right)$ *and* $V\left(.\mid \Psi\right)$ *constitute and economic equilibrium under policy rule* Ψ *according to Definition 1, b) the optimal law of motion of the individual households reproduces the aggregate law of motion the households perceive when solving their decision problems, i.e.* $\mathcal{A}_{IE}\left(x_1, x_2, K, K, \tau, \tau' \mid \Psi\right) = \mathcal{A}_{IE}\left(x_1, x_2, K, \tau, \tau' \mid \Psi\right)$ *and c)* $\hat{V}\left(. \mid \Psi\right)$ *solves the modified dynamic problem of the consumer defined in 2.30 given the policy deviation* $\tilde{\tau}'$ *and the value function* $V\left(.\mid \Psi\right)$.

2.3.4 Equilibrium without commitment

Taking into account the step where there is a one period deviation from the policy rule Ψ defined in the competitive equilibrium, we can find an equilibrium that is time consistent. In this equilibrium the agents can take into account the possibility of the government changing the tax rate of next period τ' chosen on the current period and take into account the response of all future variables given that in all future periods policy is given by Ψ. The equilibrium without commitment can be summarized in the following definition:

Definition 3 *An equilibrium without commitment is a policy rule* Ψ, *a law of motion for the individual consumer* $\mathcal{A}^i\left(x_1, x_2, K, k, \tau \mid \Psi\right)$, *a law of motion for the aggregate variables* $\mathcal{A}\left(x_1, x_2, K, \tau \mid \Psi\right)$, *a value function* $V\left(x_1, x_2, \mathbf{K}, k, \tau \mid \Psi\right)$ *and a set of functions,*
$\mathcal{A}^i_{IE}\left(x_1, x_2, K, k, \tau, \tilde{\tau}' \mid \Psi\right)$, $A_{IE}\left(x_1, x_2, K, \tau, \tilde{\tau}' \mid \Psi\right)$ *and* $\hat{V}\left(x_1, x_2, \mathbf{K}, k, \tau, \tau' \mid \Psi\right)$ *such that a)* $\mathcal{A}^i_{IE}(.\mid \Psi)$, $\mathcal{A}_{IE}\left(.\mid \Psi\right)$ *and* $\hat{V}\left(.\mid \Psi\right)$ *constitute a economic equilibrium after a one-period deviation from policy rule* Ψ *according to Definition 2; b)* $\mathcal{A}^i(.\mid \Psi)$, $\mathcal{A}\left(.\mid \Psi\right)$ *and* $V\left(.\mid \Psi\right)$ *constitute a economic equilibrium under policy rule* Ψ *according to Definition 1 and c) the government has no incentive to deviate from* Ψ *taking into account that* $\Psi(.) = \arg\max_{\tilde{\tau}'} (\hat{V} x_1, x_2, K, k, \tau, \tilde{\tau}' \mid \Psi)$ *for all* $K = k$.

2.3.5 Steady states

Benchmark utility function

Consider the Lagrangian of an individual household's problem. The individual household is price-taker and policy-rule taker. The representative household will choose optimal sequences of consumption $\{c_t\}_{t=0}^{\infty}$, labor supply $\{l_t\}_{t=0}^{\infty}$, capital stock $\{k_{t+1}\}_{t=0}^{\infty}$ and the rate of capital utilization $\{h_t\}_{t=0}^{\infty}$ in order to maximize (2.1) subject to a sequence of budget constraints given by 2.6, such that

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1-l_t, G_t\right) + \sum_{t=0}^{\infty} \lambda_t \left[\left(e^{-x_{2,t}} + \bar{r}_t\right) k_t + \bar{w}_t l_t - c_t - e^{-x_{2,t}} k_{t+1} \right] \right\} \tag{2.36}$$

the first-order conditions for this economy are given by

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \beta^t u_1\left(c_t, 1-l_t, G_t\right) = \lambda_t \Rightarrow \beta^t \{\cdot\}^{\frac{\zeta\left(1-\frac{1}{\eta}\right)}{\zeta-1}-1} \phi\left[\cdot\right]^{\frac{\chi\left(1-\frac{1}{\zeta}\right)}{\chi-1}-1} \theta c_t^{-\frac{1}{\chi}} = \lambda_t \tag{2.37}$$

$$\frac{\partial \mathcal{L}}{\partial l_t} = 0 \Rightarrow \beta^t u_2\left(c_t, 1-l_t, G_t\right) = \lambda_t \bar{w}_t \Rightarrow$$

$$\beta^t \{\cdot\}^{\frac{\zeta\left(1-\frac{1}{\eta}\right)}{\zeta-1}-1} \phi\left[\cdot\right]^{\frac{\chi\left(1-\frac{1}{\zeta}\right)}{\chi-1}-1} \left(1-\theta\right)\left(1-l_t\right)^{-\frac{1}{\chi}} = \lambda_t \bar{w}_t$$

$$\frac{\partial \mathcal{L}}{\partial h_t} = 0 \Rightarrow \frac{\partial \bar{r}_t}{\partial h_t} = 0 \Rightarrow R_t = \delta'\left(h_t\right) \tag{2.38}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \Rightarrow e^{-x_{2,t}} \lambda_t = E_t\left[\lambda_{t+1}\left(e^{-x_{2,t+1}} + \bar{r}_{t+1}\right)\right] \tag{2.39}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \Rightarrow e^{-x_{2,t}} k_{t+1} = \left(e^{-x_{2,t}} + \bar{r}_t\right) k_t + \bar{w}_t - c_t \tag{2.40}$$

for all $t \in \{0, 1, ...\}$, together with the transversality condition and initial conditions $k_0 > 0$. From (2.37) and (2.35) it follows,

$$\frac{1-\theta}{\theta}\left(\frac{c_t}{1-l_t}\right)^{\frac{1}{\chi}} = \bar{w}_t \tag{2.41}$$

and from (2.37) and (2.39), we obtain the Euler equation:

$$\frac{\beta^t u_1\left(c_t, 1-l_t, G_t\right)}{\beta^{t+1} u_1\left(c_{t+1}, 1-l_{t+1}, G_{t+1}\right)} = e^{x_{2,t}} E_t\left[\left(e^{-x_{2,t+1}} + \bar{r}_{t+1}\right)\right] \tag{2.42}$$

In the deterministic steady state, $u\left(c_t^{ss}, 1-l_t^{ss}, G_t^{ss}\right) = u\left(c_{t+1}^{ss}, 1-l_{t+1}^{ss}, G_{t+1}^{ss}\right)$, so (2.37) and (2.39) imply,

$$\frac{1-\beta}{\beta} = \bar{r}^{ss} \ , \tag{2.43}$$

the rental rate of capital depends on the discount factor and the tax rate. While (2.7) implies,

$$\frac{1-\beta}{\beta(1-\tau^{ss})} = R^{ss}H^{ss} - \delta\left(H^{ss}\right) \ . \tag{2.44}$$

Solving for H we get the steady state value for the optimal selection of the utilization of capital, which depends on the rate of time preference, the tax rate and the parameters of depreciation,

$$H^{ss} = \left[\frac{\frac{1-\beta}{\beta(1-\tau^{ss})} + \delta_c}{\left(1-\frac{1}{1+\xi}\right) b_\delta}\right]^{\frac{1}{1+\xi}} \ . \tag{2.45}$$

From equation (2.44) together with (2.15) we can get the utilized-capital/labor ratio,

$$\Lambda^{ss} \equiv \frac{K^{ss}H^{ss}}{L^{ss}} = \left\{\frac{\left[\frac{\alpha}{\frac{1-\beta}{\beta(1-\tau^{ss})}+\delta(H^{ss})}\right]^{1-\nu} - \alpha}{1-\alpha}\right\}^{\frac{\nu}{1-\nu}} \ . \tag{2.46}$$

On the production side of the economy all the steady states values remain the same. Since we have used different utility functions, the steady states of the demand side of the economy differ. In this section we show the solution for the benchmark model and for the other preferences they can be found in the Appendix.

From (2.40) evaluated in the steady state it is,

$$C^{ss} = \frac{\bar{r}^{ss}}{H^{ss}}K^{ss}H^{ss} + \bar{w}^{ss}L^{ss} \Rightarrow \frac{C^{ss}}{L^{ss}} = \frac{\bar{r}^{ss}}{H^{ss}}\Lambda^{ss} + \bar{w}^{ss} \tag{2.47}$$

From (2.41) aggregated, and taken in the steady state,

$$\left(\frac{1-\theta}{\theta}\right)^{\chi} \frac{C^{ss}}{L^{ss}} \frac{L^{ss}}{1-L^{ss}} = \left(\bar{w}^{ss}\right)^{\chi} \tag{2.48}$$

We can solve for L^{ss} and get the steady state of labor supply,

$$L^{ss} = \frac{1}{1 + \frac{\frac{\bar{r}^{ss}}{H^{ss}}\Lambda^{ss} + \bar{w}^{ss}}{\left(\frac{\theta}{1-\theta}\right)^{\chi}(\bar{w}^{ss})^{\chi}}} \ . \tag{2.49}$$

Having found the steady state values for H^{ss} and L^{ss}, we obtain K^{ss} combining (2.49), (2.45) and (2.46).

2.4 Numerical method and calibration

The solution to the equilibrium described cannot be found analytically. It is computed numerically using quadratic approximation techniques. The utility function is approximated by a quadratic function derived from a second order Taylor expansion around the steady state values. Quadratic utility has not been used in the first place since such utility function has the undesirable property of satiation which means that beyond a point more wealth generates less utility. While this kind of utility function might be appropriate in some financial economic models, they are not suitable for the kind of problem studied here.

2.4.1 Calibration of the benchmark case

The parameters values for the benchmark model are summarized in Table 2.1. Most parameters are selected to match the U.S. postwar average values of the first and second moments of the main macroeconomic variables. The length of one period is one year.

- *Preferences*: parameters β, θ, η and ϕ take traditional values assigned in business cycle literature. The discount rate β is set to 0.96 and the share of consumption θ is set to 0.36. The elasticity of intertemporal substitution denoted by η is chosen to be 0.5 to generate a risk aversion value of 2 for the composite good comprised by private consumption and public consumption and ϕ represents the share of private goods, set to 0.8.[12] The remaining preferences parameters, χ, the elasticity of substitution between leisure and consumption and ζ, the elasticity of substitution between private and public goods, were chosen to be close to 1 to keep results around the log utility case (Amano and Wirjanto 1998 find an intratemporal substitution between public and private goods of 0.9 for US data from 1953-1994). Additionally the selection of χ and θ took into account that the labor supply will be approximately 33%.[13] All preference parameters for the benchmark economy are shown in Table 2.1.

- *Technology*: these parameters were calibrated in the same fashion, α, the share of capital which is used as input for production is set to 1/3, ν is set close to

[12] The key reference for the benchmark calibration is King and Rebelo (1999). For the risk aversion parameter I have followed Kydland and Prescott (1982). The value of $\phi = 0.8$ was taken from Klein et al. (2008).

[13] See Krusell and Rios Rull (1999) for similar targets of labor supply.

1 (e.g. 0.95) in order to mimic a Cobb-Douglas production function and δ and all its components (δ_c,b_δ,ξ) were chosen to match a steady state depreciation rate of 9%. It is worth to mention that the selection of ξ was restricted to be bigger than 1 as in Greenwood et al. (2000). The calibration of b_δ and ξ yields a value of 0.15 for $b_\delta h^{1+\xi}$.[14] Calibrated technology parameters are summarized in Table 2.1.

- *Stochastic shocks*: the persistent parameter for the TFP shock follows King et al. (1988). The persistent parameter for the investment shock was calibrated according to the value suggested by Greenwood et al. (2000). The standard deviation of the TFP shock, σ_1 and the standard deviation of the investment shock, σ_2 were set to make the model's first and second moments close to the ones observed in the data.

2.4.2 Calibration of the utility function of GHH form

Since G_t is a public good (non exclusion and non rival properties) it requires $\theta > 1$, such that one unit increase in G_t will substitute more than one unit of private consumption at the margin. When $\theta < 1$ the optimal level of government consumption is zero. The selected value for θ is 1.85, and this yields a size of government consumption of 20%. The other important parameters in the calibration are $\upsilon = 1.4$ and $\varphi = 0.7$ that result in a labour supply of 0.34. This calibration values are similar to the ones used by Iltzestki (2011).

2.5 Simulation results

After solving the model, it was calibrated with the parameters described in the previous section. For the simulation, Monte Carlo methods were used to generate 500 observations for 200 experiments. The results reported in Table 2.3 are the averages of the individual experiments for both the benchmark utility and the utility function in GHH form having public and private consumption as perfect substitutes. The average results are confronted to postwar U.S. data. The calibrated model matches the volatility and persistence of the main macroeconomic variables quite

[14] Greenwood et al. (2000), calibrate this value around 0.20

Table 2.1: Calibration parameters for the benchmark economy and the economy with GHH preferences

Preference parameters	Description	benchmark	GHH	GHH*
β	Discount factor	0.96	0.96	0.96
θ	Share of consumption/efficiency of public good provision	0.36	1.85	NA
χ	Elasticity of substitution between consumption and leisure	0.95	NA	NA
ϕ, π	Share of private goods/public goods	0.8	NA	0.2
ζ	Elasticity of substitution between private and public goods	1.05	NA	NA
η	Elasticity of intertemporal substitution	0.5	0.5	0.5
υ	Labor exponent	NA	1.4	0.7
φ	Labor coefficient	NA	0.75	NA
Technology parameters				
α	Share of capital	0.33	0.33	0.33
ν	Elasticity of substitution between labor and capital	0.99	0.99	0.99
δ_c	Depreciation parameter	0.03	0.03	0.33
δ_b	Depreciation parameter	0.25	0.25	0.25
ξ	Depreciation parameter	1.42	1.42	1.42
Shocks parameters				
ρ_1	TFP shock persistence	0.9	0.8	0.8
ρ_2	Investment shock persistence	0.64	0.64	0.64
σ_1	Standard deviation of TFP shock	0.0045	0.005	0.005
σ_2	Standard deviation of investment shock	0.004	0.005	0.005

GHH* refers to the utility function specification of Cuadra et al.(2010)

well. As usual in RBC models, output volatility is around 2%, investment is more volatile than output and in both cases consumption is less volatile than output. Government consumption, which is endogenous in our model, is more volatile than output, particularly in the case of GHH preferences, however this behavior can be true in developing economies.[15] Capacity utilization is very persistent and its level in steady state is 82%. The interest rate before taxes is around 4%.

Size of government consumption

The size of government consumption was calibrated to be 25% in the benchmark model.[16] The model with GHH preferences generates a smaller government size of 20%. The tax rates associated with the size of government consumption are 32% for

[15]See Statistics of Government Consumption for non-OECD countries Table 2.7 page 51.

[16]For these target levels of public consumption shares see, for example, Angeletos and Panousi (2009).

the benchmark model and 24% for the model with GHH preferences for a calibrated labor supply of around 33%. The size of government consumption, (which here will be the total size of government since we abstract from transfers) changes with the calibration of preferences parameters; more on this is presented in the next section concerned with sensitivity analysis.

Volatility of government consumption

Government consumption for the calibrated benchmark model shows a higher volatility than output and private consumption. The model reproduces quite well the volatility of output, investment and consumption for the postwar values of the U.S. economy, however government consumption shows a higher volatility in the model than in the data taken from Klein and Rios-Rull (2003). The data analyzed in Section 2.8, shows high volatility from government consumption particularly in developing countries.

Correlations

The model replicates closely the comovement of the main macroeconomic variables, i.e. the cyclical behavior of consumption and investment is procyclical and the correlation with output is around 80-90%. Capacity utilization and labor supply are also procyclical. Burnside et.al (1995), documented that electric consumption is procyclical because equipment and machinery are used more intensively in booms. Results are shown in Table 2.3. The comovement of government consumption, the share of government consumption and taxes with output is discussed in more detail in what follows.

Cyclicality of government consumption

The comovement of government consumption with output is procyclical regardless of the set up of the utility function that models the individual preferences. Some authors have found positive correlation of government consumption and output in models where the consumption of government is endogenously determined and affects the utility of individuals, i.e. they benefit from a public good. For example, Battaglini and Coate (2008) extended their model of political economy theory of fiscal policy incorporating persistent productivity shocks and found that government spending increases in booms and decreases during recessions, while tax rates decrease during booms and increase in recessions. Ambler and Cardia (1996) solved a RBC model where government spending is endogenous. The optimal public spending in their model should behave similar to private consumption, and therefore they get highly procyclical government expenditures. Debortoli and Nunes (2010) found procyclical government expenditures in the presence of political turnover analyzed as loose commitment.

Cyclicality of government-consumption/output ratio

The cyclical behavior of government-consumption/output ratio depends on the calibration of the elasticity of substitution of private (including leisure) and public goods and the share of government consumption on the utility function. For the benchmark model the comovement of the share of government consumption on output and output is almost zero. When public and private goods become complements/substitutes, government-consumption/output behaves countercyclical/procyclical. In the case of GHH preferences, private and public good are Edgeworth substitutes, therefore there is always procyclicality between the share of government consumption on output and output.

Cyclicality of taxes

The correlation between tax rates and output is mildly procyclical or countercyclical in the benchmark model. According to the calibration the procyclicality may increase, particularly if there is an increment in the degree of substitution between public and private goods. The model with GHH preferences with public and private goods as perfect substitutes shows a positive correlation. Since there is no wealth effect on labor supply, agents wanting to increase government consumption during a boom will choose to increase taxes to enjoy more of the public good. Nevertheless, the level of taxes has a big effect on the labor supply, therefore in the GHH economy steady state tax rates are lower than in the benchmark model. When the income effect is present as in the benchmark preferences, agents will decrease the tax rate during the boom to be able to enjoy more consumption.

Table 2.2: Steady state values of the baseline economy

	C/Y	G/Y	I/Y	L	K/Y	r	τ	δ	H
benchmark Model	0.55	0.25	0.20	0.33	2.13	0.04	0.32	0.09	0.82
GHH preferences	0.60	0.20	0.20	0.34	2.4	0.04	0.24	0.08	0.76

Table 2.3: Model statistics

	Standard Deviation			First Order Autocorrelation			Contemporaneous correlation with output		
	Data	Benchmark	GHH	Data	Benchmark	GHH	Data	Benchmark	GHH
Y	1.81	2.28	2.78	0.84	0.89	0.85	1.00	1.00	1.00
C	1.35	1.46	1.72	0.80	0.95	0.89	0.88	0.83	0.80
I	5.30	5.70	6.46	0.87	0.81	0.77	0.80	0.89	0.89
G	1.21	2.36	4.06	0.68	0.86	0.92	0.80	0.97	0.98
H	n.a	1.01	1.27	n.a	0.80	0.79	n.a	0.19	0.01
L	1.79	0.96	1.71	0.88	0.77	0.78	0.88	0.71	0.99
τ	n.a	0.19	0.34	n.a	0.83	0.89	n.a	-0.01	0.85
G/Y	n.a	0.12	0.26	n.a	0.68	0.90	0.17	0.05	0.88
w	0.68	1.50	0.97	0.66	0.95	0.92	0.12	0.91	0.98
r	0.30	0.10	0.10	0.60	0.90	0.81	-0.35	-0.04	-0.04

Source:U.S. Data for standard deviation, first order autocorrelation and contemporaneous correlation with output were taken from King and Rebelo (2000) for Y, C, I, L, w and r. Values for G, G/Y were taken from Klein and Rios-Rull (2003). n.a: not available.

2.6 Sensitivity analysis

The sensitivity analysis is a robustness check. It compares the benchmark model results with different calibration values for η, ζ and in some cases different combinations of η and ϕ and ζ and ϕ. The results obtained showed that the comovement of government consumption and output is positive regardless of the calibration of the mentioned parameters and the business cycle properties of the share of government consumption on output and taxes depend mainly on the elasticity of substitution between private and public consumption. Results are shown in Table 2.4.

The elasticity of intertemporal substitution

As mentioned before the strong positive correlation between government consumption and output remains no matter which values are used in the calibration. Nevertheless, when agents become less risk averse, the correlation between the government-consumption/output ratio and output is positive. The same occurs with the comovement between taxes and output. Less risk averse agents increase tax rates in booms. Increasing risk aversion to 4 compared to 2 in the benchmark model, increase the tax rate to 34% and consequently the size of government consumption increases to 27%, since individuals rely more on taxation as an insurance, however the correlation between taxes and output becomes almost zero and at the same time the correlation between the share of government consumption on output and output is as well close to zero.

The elasticity of substitution between private and public consumption

The positive correlation between government consumption and output does not depend on the degree of complementarity or substitution between private and public goods. On the contrary, the government-consumption/output ratio is procyclical when the public and private consumption are substitutes and is countercyclical when private and public goods are complements. The correlation becomes stronger with different values given to the share of government consumption in the utility function. When private and public goods are complements and the share of government consumption on the utility function becomes smaller, the correlation between government-consumption/output ratio and output becomes more negative and the tax rates decrease. The degree of complementarity plays an important role in the size of government consumption, high elasticity of substitution yields smaller governments and obviously lower tax rates. However the share of government consumption on the utility can increase the tax rate even in the case of high elasticity of substitution.

The elasticity of substitution between consumption and leisure

The positive correlation between government consumption and output is very robust and does not change with different values for the elasticity of substitution between consumption and leisure. The tax rate is indeed responsive to the change on the parameter values. An increase in $\chi = 1.5$ increases the tax rate slightly to 32,5% but creates a huge decrease in the labor supply from 33% to 21%. The government-consumption/output becomes highly procyclical. The correlation between taxes and output behaves procyclical as well.

Table 2.4: Sensitivity analysis

	τ	G/Y	$\rho(G,Y)$	$\rho(G/Y,Y)$	$\rho(\tau,Y)$	$\rho(G,Y_{-1})$	$\rho(G,Y_{-2})$	$\rho(G,Y_{-3})$
Baseline	0.32	0.25	0.97	0.05	0.01	0.86	0.81	0.76
$\zeta = 0.7$	0.39	0.31	0.98	0.08	0.02	0.87	0.80	0.74
$\zeta = 1.5$	0.24	0.20	0.97	0.40	0.25	0.86	0.81	0.76
$\zeta = 0.7, \phi = 0.95$	0.21	0.16	0.95	-0.55	-0.52	0.80	0.77	0.74
$\zeta = 1.5, \phi = 0.7$	0.35	0.28	0.98	0.66	0.50	0.88	0.82	0.76
$\zeta = 1.5, \phi = 0.95$	0.04	0.03	0.91	-0.01	-0.06	0.76	0.75	0.73
$\eta = 0.25$	0.34	0.27	0.98	0.00	-0.02	0.88	0.83	0.78
$\eta = 0.9, \zeta = 1.5$	0.24	0.19	0.97	0.46	0.31	0.85	0.80	0.74
$\eta = 0.9$	0.30	0.23	0.98	0.24	0.27	0.87	0.82	0.76
$\chi = 1.5$	0.32	0.25	0.97	0.88	0.54	0.88	0.78	0.70

2.7 Impulse responses

To get a better insight on the propagation mechanism of the Hicks neutral productivity shock and the investment specific shock we plot the impulses-responses. This is the exercise performed in this section for output, consumption, tax rates, investment, government consumption, the share of government consumption, labor, interest rate, and wages for a period of 30 years. The main macroeconomic variables, output, consumption, labor and investment behave procyclical and in compliance with the expected behavior from business cycles corroborated by the stylized facts of the U.S. (see Table 2.3). Government consumption however behaves highly procyclical and interestingly taxes and the share of government consumption behave countercyclical. The government chooses to decrease tax rates in the period immediately after the shock but increase them as soon as there is an increase in the accumulation of capital.

Comparison of benchmark model with the model with GHH preferences (C and G as perfect substitutes)

TFP shocks The impulse responses to a 1% TFP shock of the previously mentioned macroeconomic variables are shown in Figure 2.2. In general, the reaction of the output, investment and government consumption is similar under CES preferences and GHH preferences. Nevertheless GHH preferences generate higher response to TFP shocks particularly for labor, investment and capital. The negative response observed in the impulse responses for consumption with GHH preferences is more related to the way that the public good has been incorporated than to the GHH preferences on its own[17]. In the model with GHH preferences where private and public consumption are perfect substitutes, in order to generate positive tax rates, public goods require a high value on the utility function, therefore the response from agents to a TFP shock is to choose an increase in the tax rate as proposed by the government, in contrast to the benchmark model where the response is a lower tax rate in the period immediately after the shock. With GHH preferences the behavior of taxes is procyclical since the weight of the public goods on the utility function is high enough to create demand for them, this issue affects the cyclical behavior of taxes compared to the case of the benchmark model with CES preferences. Some empirical studies have stated that government consumption is procyclical in developed and developing countries. However the correlation values for government consumption and output in this model seem to be too high under any preferences specification.

Investment shocks The impulse responses to a 1% investment shock of the previously mentioned macroeconomic variables is shown in Figure 2.3. In general the reaction to this shock is milder than to the TFP shock. The main differences compared to the TFP shocks are in consumption, government consumption, labour supply, tax rates and wages. The investment shock makes consumption and investment move in opposite directions. As pointed out in Greenwood et al. (1988) including variable capacity utilization should solve this issue, however in this model (GHH where private and public consumption are perfect substitutes) consumption will move countercyclical until the second or third year after the shock when it becomes positive again. The response of government consumption is less strong than with the TFP shock

[17] Impulse responses for the model with the GHH utility function suggested by Sapriza et.al (2010) generated a positive response from consumption after a 1% productivity shock.

and the response of output is milder as well. Labor supply in the CES function is slightly less responsive to investment shocks than to productivity shocks and wages tend to remain almost unchanged.

2.8 Empirical evidence

Data

The main source of the data is the database of the World Bank Development Indicators for the period 1960 - 2010. Variables are percapita. The series employed are government final consumption expenditure and GDP, in constant values of 2000. The sample consists on time series for 79 countries from which 33 belong to OECD classification and 46 are non-OECD countries. Government consumption is a considerable size of output, in OECD countries it can vary from 6% in Romania, 17% in the USA to 25% in Denmark, or 27% in Sweden and in non-OECD countries can differ from about 6% in Guatemala, 10% in Singapore to 20% in Brazil. It is almost as volatile as output, or can be more volatile than output particularly in non-OECD countries[18]. Therefore the study of its business cycle properties has been a constant subject of research, since it is relevant to shed light on its impact on output and other macroeconomic variables.

Estimation

The cyclicality of government consumption is estimated in two ways. As in Lane (2003) the regression estimated country by country takes the following form

$$d(\log(G_{it})) = \alpha_i + \beta_i d(\log(Y_{it})) + \varepsilon_{it}.$$

We perform OLS estimation with AR(1) correction. The second form is to calculate the correlation of the detrended series with Hodrick-Prescott filter with a smoothing parameter of 100 corresponding to annual data. Results are shown in Table 2.6 for OECD countries and in Table 2.8 for non-OECD countries and denoted by the β coefficient on the second column of the mentioned Tables. The definition of cyclicality through the paper refers to the correlation between the level of output and the level of government consumption. The estimation of the relationship

[18] See Table 2.5 and Table 2.7 in pages 47 and 49 respectively.

between the growth rate of output and the growth rate of government consumption has been done as an alternative empirical way of calculating cyclicality as proposed by Lane (2003) but it is of course not comparable to the simulation results of the model shown in Table 2.3.

The estimation results show that for non-OECD countries 27 countries exhibited significant positive coefficients, confirming as well the procyclicality of government consumption. This coefficient is significant for half of the OECD countries, and for the ones where it is significant, 11 showed a positive coefficient, i.e. confirming procyclical government consumption expenditure, only the U.S.A. has a negative significant coefficient. On the non-OECD countries 26 out of 46 showed a significant β coefficient, from those all were positive, again signalling procyclical government consumption. Interestingly the magnitude of the β significant coefficients is larger on non-OECD countries. These results coincide with the ones reported by Lane (2003) for the period 1960-1998.

Correlations Taking into account the measure of cyclicality as the simple correlation of HP detrended series we found that for the period 1960-2010, 26 out of 33 OECD countries show positive correlation between government consumption and output providing evidence for procyclical behavior of government consumption. All countries have a negative correlation between government consumption/output ratio and output. In non-OECD countries all countries have a positive correlation between government consumption and output except for four countries. Results are shown in Table 2.6 for OECD countries and in Table 2.8 for non- OECD countries and denoted as $\rho(G,Y)$ for the correlation between government consumption and GDP and $\rho(G/Y,Y)$ for the correlation between the share of government consumption on output and output. On the last one, it is worth to notice that all countries show a negative correlation result just with the exception of some developing countries, those were, Bolivia, Philippines and Morocco.

Endogeneity

Empirical studies about the cyclical pattern of government consumption deal with the fact of endogeneity. Iltzestky and Vegh (2008) used instrumental variables to test for endogeneity. The employed instruments for GDP were weighted GDP growth of countries' trading partners, the Real Interest Rate on 6-month U.S. Treasuries, and lagged GDP Growth. Their results support evidence of causality from output to government consumption. They obtained significant results for procyclicality for developing countries and in some cases also significant evidence for procyclical government consumption in developed countries. Jaimovich and Panizza, which have used as well instrumented variables to control for endogeneity, however have found evidence of causality on the direction of fiscal policy to GDP.

2.9 Conclusion

In this Chapter, we have analyzed optimal government consumption, over the business cycle, chosen in a time-consistent way by a successive game between the benevolent government and a representative agent. Households benefit from government consumption and therefore it is included in their utility function. The business cycle is driven by TFP shocks and productivity shocks. From the demand side we have included several different specifications of the utility function in order to study if there is any effect, on the way the public goods enter the preferences, over the cyclicality of government consumption. We employed a CES utility function, GHH preferences where public and public goods are perfect substitutes and a GHH preferences when there is no relation between those goods. Using GHH preferences was motivated by the fact that in these preferences the marginal rate of substitution between consumption and leisure does not depend on the consumption and therefore there is no wealth effect on labor supply. Correia et al. (1995) and Neumeyer and Perri (2005) have used such preferences to better reproduce some of the business cycle properties of small open economies.

The optimal response given lack of commitment is to increase government consumption after a productivity shock or an investment shock. The results suggest procyclical government consumption over the business cycle, i.e., government consumption increases in booms and decreases in recessions. This behavior is persistent and the correlation is very high regardless of the type of preferences or the calibrated parameters. The model provides insights as well for the cyclicality of income tax rates and of the government-consumption/output ratio. The results depend on the specification of the utility function and on the calibration of certain parameters. In a CES utility function tax rates are acyclical or countercyclical when private and public goods tend to be complements. The government-consumption/output ratio behaves countercyclical, but in the case of high substitution between public and private goods, it is likely to become acyclical or procyclical. It is also the case with low risk aversion.

Procyclical government consumption is assured by empirical evidence. We have used the database of the World Bank Developing Indicators for a sample of 33 OECD countries and 46 non-OECD countries for the period 1960-2010 (for some countries the period is shorter). The cyclicality is tested in a simple regression on the way Lane (2003) has done it and also looking at the correlations of HP detrended time series. The estimations results and the correlations of the detrended time series suggest procyclical government consumption.

Nevertheless the model without any frictions fails to offer more realistic correlations values as the ones presented on Table 2.6 and in Table 2.8 from the empirical exercise. The next chapter studies the same problem with heterogenous agents since differences in wealth and labor productivity can generate different responses of agents and therefore distinct cyclical properties for endogenous government consumption. Another interesting extension could be to add implementation lags or some other political-decision-making frictions however this thesis abstracts from that given the high complexity of dynamic games and their demanding numerical implementation.

Table 2.5: Government consumption statistics: OECD countries

Country	$\mu(G/Y)$	$\mu/\sigma(G)$	$\sigma(Y)$	$\sigma(G)$	$\rho(G_t, G_{t-1})$
Australia	0.17	0.31	1.69	1.83	0.50*
Austria	0.20	0.29	1.55	1.31	0.64*
Belgium	0.22	0.28	1.64	1.35	0.47*
Canada	0.22	0.21	2.13	1.95	0.71*
Czech R.	0.21	0.09	3.91	4.13	0.32
Denmark	0.25	0.30	2.04	1.65	0.64*
Estonia	0.18	0.27	9.06	8.98	0.41
Finland	0.21	0.34	3.46	2.17	0.70*
France	0.22	0.32	1.47	1.04	0.61**
Germany	0.19	0.18	1.72	1.66	0.67*
Greece	0.17	0.34	2.95	3.12	0.30*
Hungary	0.20	0.29	3.57	4.46	0.32*
Iceland	0.20	0.45	4.22	2.35	0.65
Ireland	0.19	0.28	3.49	3.36	0.76*
Italy	0.20	0.20	1.77	1.73	0.83*
Japan	0.15	0.41	2.61	1.38	0.41*
Korea	0.16	0.59	3.07	3.19	0.79*
Luxembourg	0.16	0.36	3.49	2.13	0.70*
Mexico	0.10	0.29	3.24	2.66	0.59**
Netherlands	0.24	0.32	1.78	1.76	0.29*
New Zealand	0.18	0.20	2.29	2.15	0.29*
Norway	0.19	0.40	1.76	1.43	0.58*
Poland	0.18	0.21	3.14	1.86	0.54*
Portugal	0.16	0.56	3.56	2.96	0.55*
Romania	0.66	0.18	6.41	9.2	0.19*
Slovakia	0.20	0.15	5.99	4.13	0.12
Slovenia	0.18	0.18	4.49	1.46	0.05*
Spain	0.15	0.51	2.55	2.49	0.73*
Sweden	0.27	0.25	2.37	1.3	0.52
Switzerland	0.11	0.22	1.89	2.95	0.63*
Turkey	0.11	0.18	3.68	2.86	0.30*
U.K.	0.22	0.22	2.31	1.63	0.73**
U.S.A	0.17	0.11	2.06	2.21	0.80*
Mean	0.18	0.29	3.07	2.69	0.52

Source: World Development Indicators. (*, **, ***) significance at the 1, 5 and 10 percent levels, respectively.

Table 2.6: Cyclical properties of government consumption: OECD countries

Country	$\rho(G,Y)$	β	$\rho(G/Y,Y)$	$\rho((G/Y)_t,(G/Y)_{t-1})$
Australia	0.40*	0.15*	-0.52*	0.34*
Austria	0.17**	0.35	-0.72*	0.59*
Belgium	0.18*	0.13	-0.73*	0.35*
Canada	-0.11	-0.21	-0.76*	0.53*
Chezh R.	0.43*	-0.42	-0.47*	0.49
Denmark	0.23**	0.24*	-0.72*	0.55*
Estonia	0.14	0.19	-0.63*	0.66
Findland	0.36*	0.03	-0.80*	0.56*
France	0.03	-0.05	-0.82*	0.57*
Germany	-0.11***	0.02	-0.75*	0.57*
Greece	0.16*	0.44*	-0.62*	0.33*
Hungary	0.02	0.26	-0.59*	0.44
Iceland	0.48*	0.15	-0.82*	0.57*
Ireland	0.54*	0.33*	-0.42*	0.52*
Italy	0.31*	-0.07	-0.61*	0.42*
Japan	-0.02	0.21*	-0.88*	0.59**
Korea	0.46*	-0.01	-0.40*	0.37*
Luxembourgh	0.53*	0.1	-0.79*	0.37*
Mexico	0.49*	0.37*	-0.67*	0.46
Netherlands	0.08	0.16	-0.69*	0.43
New Zealand	0.07	0.17	-0.82*	0.53
Norway	-0.13*	0.4*	-0.81*	0.7*
Poland	-0.42	0.33	-0.92	0.27
Portugal	0.39*	0.48*	-0.69*	0.36*
Romania	0.42*	0.65**	-0.17	-0.01
Slovakia	-0.07	0.13	-0.49	0.21
Slovenia	0.69	0.21*	-0.96	0.35
Spain	0.60*	0.3*	-0.50*	0.65*
Sweden	-0.01	0.1	-0.86*	0.67
Switzerlands	0.46	0.16	-0.39*	0.58*
Turkey	0.26	0.27**	-0.75	0.5
U.K.	0.23*	0.09	-0.75*	0.57*
U.S.A	0.20**	-0.19*	-0.59*	0.46*

Source: World Development Indicators. (*, **, ***) significance at the 1, 5 and 10 percent levels, respectively.

Table 2.7: Government consumption statistics: Non-OECD countries

Country	$\mu(G/Y)$	σ/μ	$\sigma(Y)$	$\sigma(G)$	$\rho(G_t, G_{t-1})$
Belize	0.16	0.13	5.05	7.68	0.48*
Bolivia	0.16	0.17	3.97	5.2	0.61*
Brazil	0.19	0.28	3.99	7.23	0.58*
Chile	0.19	0.18	4.67	3.56	0.42*
China	0.15	1.04	6.25	7.79	0.76*
Cameroon	0.08	0.25	5.38	8.13	0.47*
Colombia	0.10	0.60	2.37	6.31	0.61*
Costa Rica	0.18	0.16	3.48	3.14	0.78*
Cuba	0.29	0.28	6.67	4.85	0.71*
Cyprus	0.13	0.32	3.49	6.38	0.38*
Dominica	0.08	0.40	4.71	12.61	0.53*
Algeria	0.12	0.30	5.37	8.55	0.32*
Ecuador	0.12	0.32	3.16	8.53	0.71*
Ethiopia	0.11	0.26	5.56	19.57	0.57*
Guatemala	0.06	0.28	2.62	5.67	0.70*
Hong Kong	0.09	0.44	3.71	3.1	0.61*
Honduras	0.13	0.20	3.05	6.79	0.56*
Indonesia	0.08	0.49	4.09	6.89	0.60*
India	0.10	0.55	2.85	5.8	0.72*
Iran	0.17	0.43	7.13	10.68	0.74*
Jordan	0.44	0.20	5.81	9.82	0.48*
Kazakhstan	0.13	0.23	8.05	9.86	0.51*
Kenya	0.12	0.32	3.69	7.03	0.69*
Morocco	0.15	0.42	3.13	7.82	0.61*
Mali	0.07	0.43	4.27	10.25	0.29
Malta	0.17	0.53	4.88	5.39	0.50*
Malawi	0.21	0.24	4.42	8.61	0.36*
Malaysia	0.12	0.54	3.49	5.66	0.57*
Namibia	0.23	0.09	2.44	4.9	0.34*
Nicaragua	0.12	0.56	5.74	13.65	0.65*
Pakistan	0.10	0.37	2.17	8.93	-0.02
Panama	0.16	0.11	4.2	5.73	0.40*
Peru	0.09	0.22	5.07	6.92	0.50*
Philippines	0.12	0.17	3.27	5.73	0.76*
Papua New Guinea	0.24	0.28	5.07	5.21	0.35*
Paraguay	0.11	0.30	4.17	12.05	0.70*
Senegal	0.09	0.29	5.03	10.54	0.61*
Singapore	0.15	0.23	2.54	5.97	0.57*
South Africa	0.17	0.23	2.09	2.51	0.58*
Sudan	0.10	0.45	4.06	4.89	0.45*
El Salvador	0.13	0.19	4.54	7.92	0.51*
Thailand	0.12	0.57	4.25	4.66	0.66*
Trinidad	0.09	0.39	5.16	6.07	0.31*
Tunisia	0.16	0.44	2.68	2.77	0.49*
Uruguay	0.12	0.24	5.38	4.9	0.58*
Venezuela	0.14	0.13	5.18	6.23	0.64*

Source: World Development Indicators. (*, **, ***) significance at the 1, 5 and 10 percent levels respectively. .

Table 2.8: Cyclical properties of government consumption: Non-OECD countries

Country	$\rho(G,Y)$	β	$\rho(G/Y,Y)$	$\rho((G/Y)_t,(G/Y)_{t-1})$
Algeria	0.63	0.77*	0.05*	0.39*
Belize	0.07	0.11	-0.54*	0.49*
Bolivia	0.71*	1.28*	0.13*	0.49*
Brazil	0.65*	0.82*	0.11*	0.46*
Chile	0.51*	0.41*	-0.66*	0.58*
China	0.79*	0.87*	-0.03*	0.51*
Cameroon	0.46*	0.51*	-0.23*	0.23
Colombia	0.28*	0.63	-0.21*	0.63*
Costa Rica	0.57*	0.19	-0.53*	0.34*
Cuba	0.89*	0.71*	-0.72*	0.6*
Cyprus	0.15	0.23	-0.38*	0.42*
Dominica	0.17	0.17	-0.19*	0.47*
Ecuador	0.63*	1.12*	0.34*	0.55*
Ethiopia	0.50	1.55*	0.19*	0.63*
Guatemala	0.45**	0.59**	-0.02*	0.54*
Hong Kong	-0.10	-0.01	-0.76*	0.4*
Honduras	0.08*	0.13	-0.34*	0.55*
Indonesia	0.45*	1.03*	-0.11**	0.25**
India	0.50*	0.87*	0.01*	0.58*
Iran	0.65*	0.84*	-0.12*	0.53*
Jordan	0.60*	0.21	-0.54*	0.08
Kazakhstan	0.60*	0.92*	-0.26*	0.22
Kenya	0.04	0.45**	-0.39*	0.65*
Morocco	0.47*	0.41*	0.05*	0.54*
Mali	0.20	0.39	-0.28	0.45*
Malta	0.65*	0.3	-0.13*	0.61*
Malawi	-0.18	-0.41	-0.59*	0.25
Malaysia	0.52*	0.5*	-0.06*	0.44*
Namibia	0.12	0.41	-0.37**	0.36*
Nicaragua	0.26**	0.48	0.16*	0.61*
Pakistan	0.29*	1.13**	0.05	-0.03
Panama	0.33*	0.76**	-0.44*	0.65*
Peru	0.57*	0.76*	-0.20*	0.37*
Philippines	0.63*	0.76*	0.07*	0.7*
Papua New Guinea	0.36	0.8*	-0.49	0.56*
Paraguay	0.54*	0.61	0.20*	0.57*
Senegal	0.37	0.44**	-0.12*	0.45*
Singapore	-0.35*	-0.06	-0.75*	0.58*
South Africa	0.18	0.51*	0.05*	0.58*
Sudan	-0.55	0.29	-0.43*	0.65*
El Salvador	0.16	0.26	-0.37*	0.57*
Thailand	0.57*	0.53*	-0.30**	0.03
Tunisia	0.05	0.08	-0.67*	0.39*
Uruguay	0.66*	0.51*	-0.55*	0.38*
Venezuela	0.68*	0.72*	-0.35*	0.64*

Source: World Development Indicators. (*, **, ***) significance at the 1, 5 and 10 percent levels respectively. .

Figure 2.1: Impulse responses to a one percent TFP shock. CES preferences (dashed line) and GHH preferences for perfect substitutes (solid line)

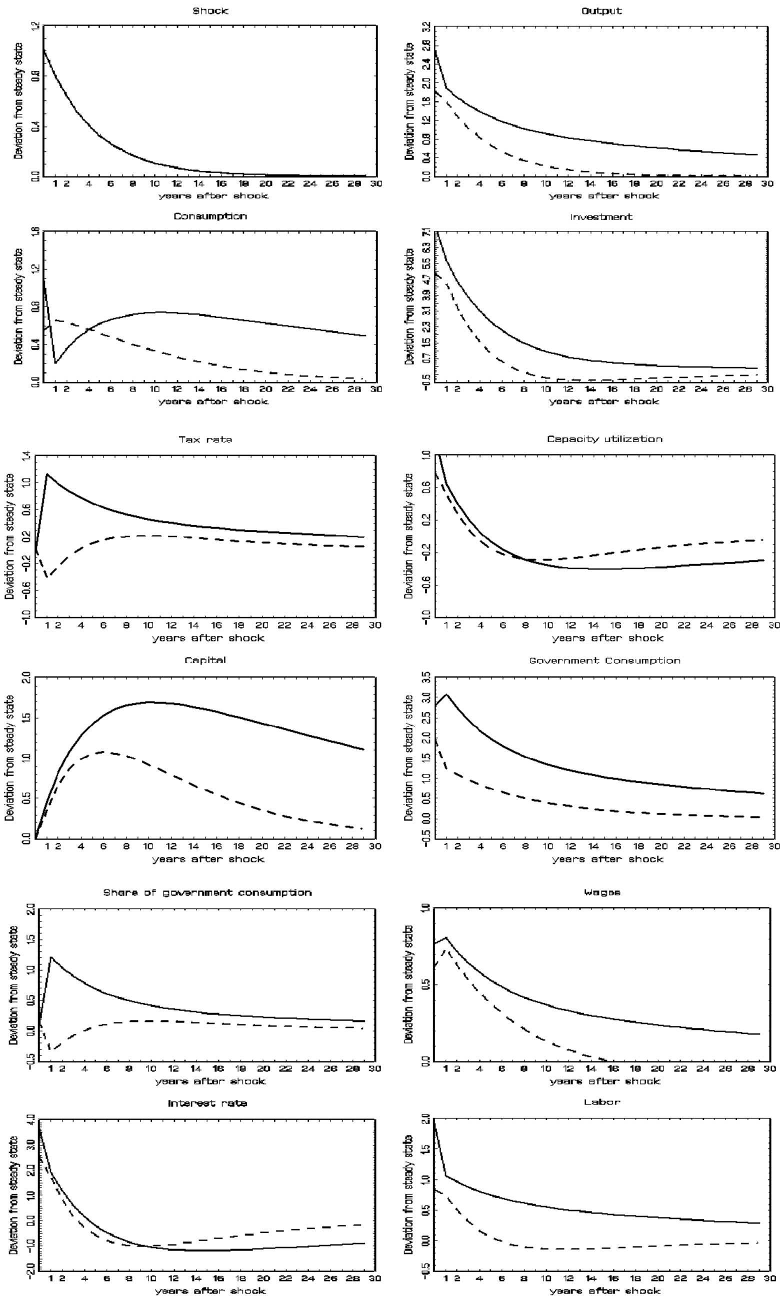


Figure 2.2: Impulse responses to a one percent TFP shock. CES preferences (dashed line) and GHH preferences (solid line)

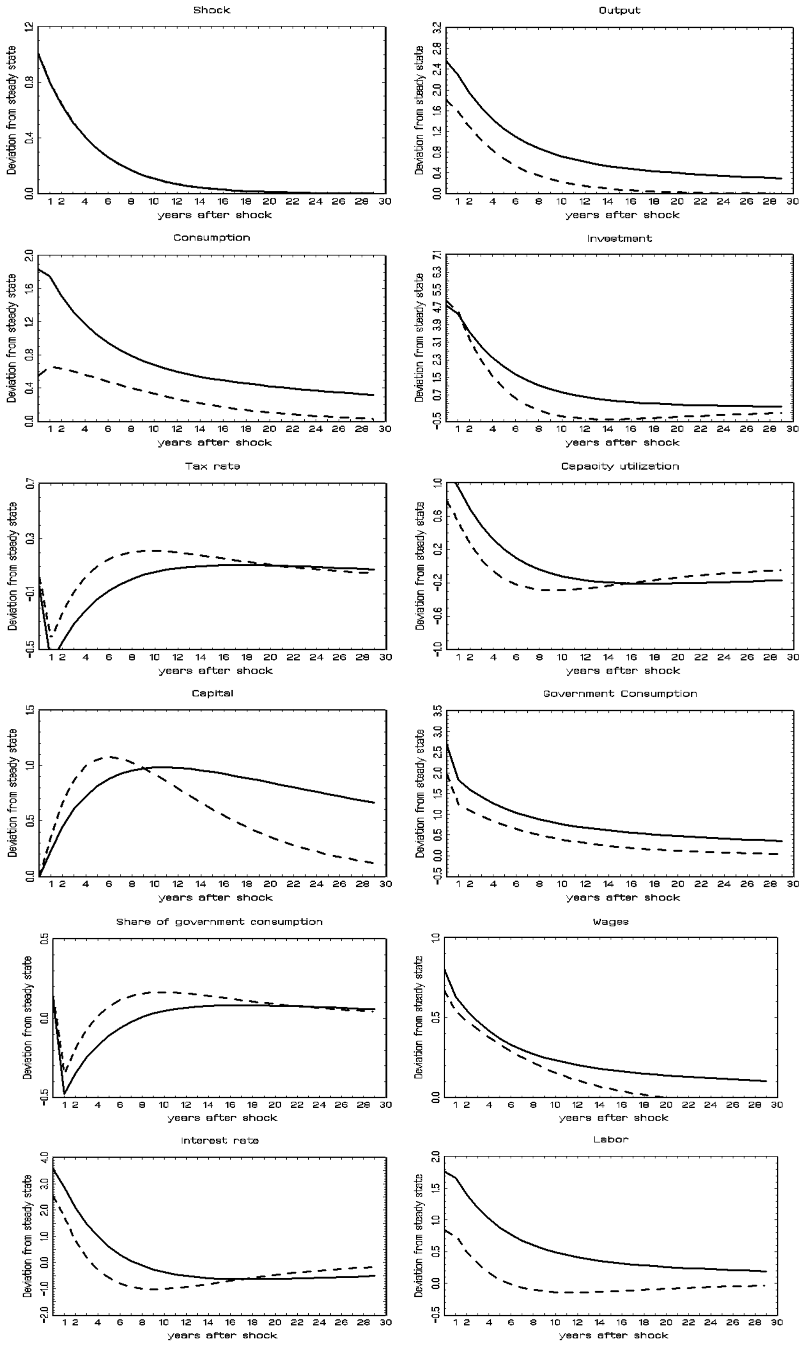
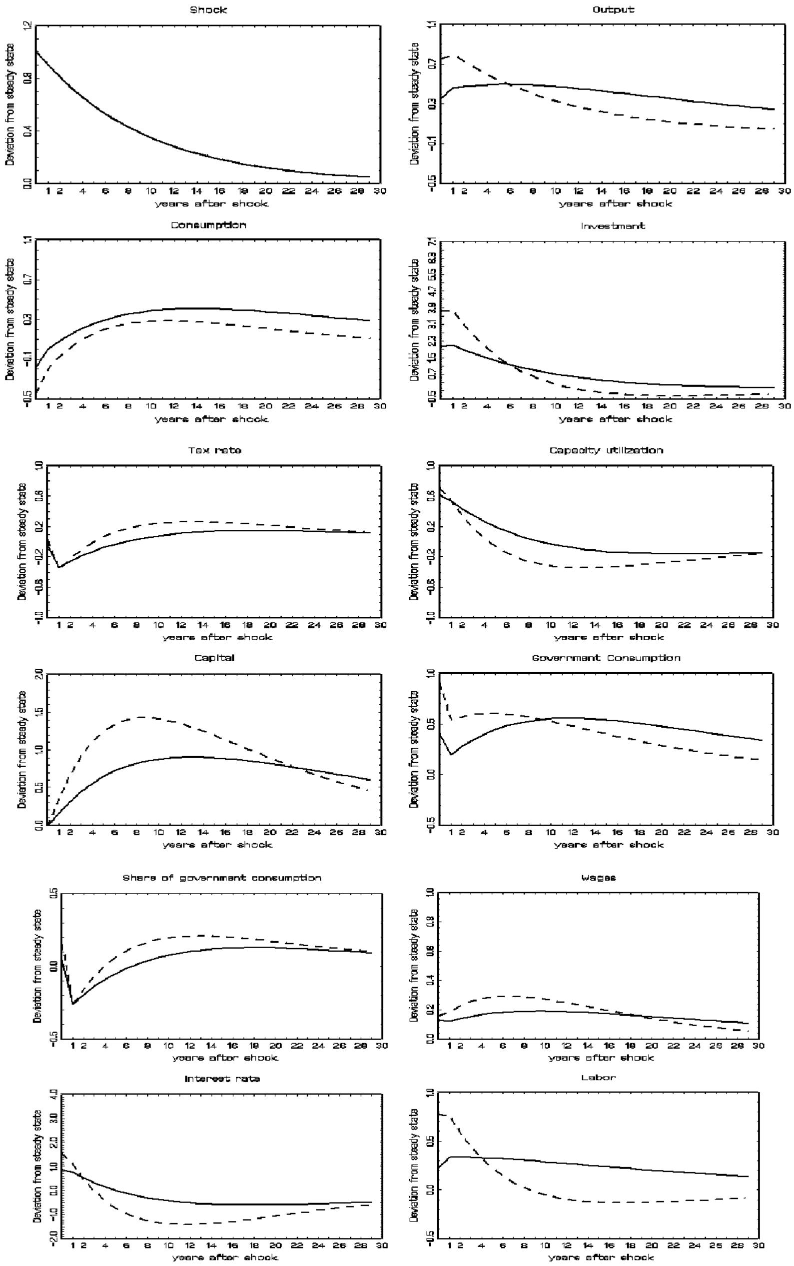


Figure 2.3: Impulse responses to a one percent investment shock. CES preferences (dashed line) and GHH preferences for perfect substitutes (solid line)



Chapter 3

Inequality and Provision of Public Goods over the Business Cycle

3.1 Introduction

The previous Chapter studied the business cycle properties of endogenous government consumption in a representative agent environment. The government chose procyclical government consumption as a response to positive shocks on productivity and investment in the presence of distortionary taxes (in contrast to non-distortionary lump sum taxes) on labor and capital income. In this Chapter, we extend the model to include heterogenous agents that differ in income and asset holdings. The purpose is to investigate whether including income and wealth inequality would lead to a different behavior of endogenous government consumption over the business cycle. Given that the government lacks a commitment technology, the reoptimization in every period can act in favor of more or less taxation according to the decisions of the income groups. The model will also allow us to identify the value of the proportional tax rate on capital and labor income chosen by the median voter and it will be compared with the tax rate chosen by the representative agent in the economy without inequality. This can shed light on how inequality influences the level of provision of public goods as an outcome of a political process developed as a dynamic policy game.

Real Business Cycle (RBC) theory states that exogenous changes in productivity are the main source of fluctuation of the macroeconomy's long-run trend. Extensions on RBC literature have added the effect of exogenous fiscal policy.[1] Nevertheless, a not widely explored issue on this literature has been how political economy considerations affect the cyclical movement of macroeconomic variables, particularly how heterogenous voters can influence the way fiscal variables are chosen over the business cycles in response to productivity shocks. For example, it is not straightforward to describe how the welfare effects of tax rates will affect the winning policy outcome in future periods if in a period of a positive productivity shock, political demand implies an increase in the tax burden. The interaction between resource constraints and the political process may be important for understanding the cyclical fluctuation of both economic and fiscal variables. This study is devoted to exploring the mentioned interactions. Our goal is to understand how individual preferences of three groups of heterogeneous agents, classified as rich, median and poor according to their wealth and consistently with the level of productivity, can be aggregated to the society level given the existence of those disparities and how these collective choices respond to random shocks to the economy. A political mechanism will allow the aggregation from the individual level to the societal level represented by the preferences of a median income group that through majority voting expresses the tax selection that maximizes its utility and resembles the choice of the whole population in a democracy.

The exercise we perform is straightforward. We combine the textbook RBC model as presented by King and Rebelo (1999) with a modification of the heterogeneous-agent model of Krusell and Rios-Rull (1999). Hence, instead of focusing on voting over fiscal transfers, we study voting over public consumption when agents derive utility from a public good. Taxes finance just the provision of public goods or services. A similar model setup is the one of Klein et al. (2008), but we add household heterogeneity, productivity shocks, investment shocks and capacity utilization. Cortinhas et al. (2010) use a similar framework to analyze the cyclical movement of endogenous transfers; our study however focuses on endogenous government consumption and abstracts from transfers.

Adding household heterogeneity increases the burden of the numerical calculation. It implies a number of state variables subject to the curse of dimensionality. First described by Bellman (1955), the curse of dimensionality states that there is an exponential increase on the computational time as the number of dimensions of the

[1] A pioneer study of the effects of exogenous public consumption shocks within the standard RBC framework is Aiyagari et al. (1992).

state variables augment. In this model the level of taxes and the steady state level of capital are endogenous which implies that the steady state level of government consumption and capital needs to be approximated numerically. To approach this problem we employ a quadratic approximation method as in Krusell and Rios-Rull (1999).

In our model, a public consumption good enters the utility function of households and this public consumption externality is sufficient to generate political demand for positive taxes that finance the provision of the public good along the cycle. The inclusion of wealth and income heterogeneity is a key element in the definition of the role of the median voter on the demand for government consumption using the political mechanism of majority voting. The median voter faces a trade off between burdensome taxation and the benefits of public goods, and this decision will determine at the same time the size of government consumption. As in the previous chapter, for a rigorous understanding of the behavior of government consumption over the business cycle, we incorporate in the heterogenous version as well capacity utilization and investment shocks along the lines of Greenwood et al. (1988), Greenwood et al. (2000), and King and Rebelo (1999).

We find that the level of public consumption is procyclical. The dynamic responses analysis calibrated for the U.S. economy explains how individuals who differ in wealth and income adjust their selection of fiscal variables as a response to changes in productivity. Interestingly, the politico-economic channel induces causality from output to delayed future tax rates changes, therefore after a TFP shock the median voter tries to postpone the increase in the tax rate, such that the increment will take place just after the time at which the economy has accumulated more capital. This response is different in the case of equal agents: after a productivity shock, the median voter, who behaves in the equal economy like a representative agent, chooses a decrease in the tax rate in the first year after the shock. Five years later, approximately, equal agents will slightly increase the tax rate getting back again very fast to the initial values. This reaction makes the tax rates being acyclical or countercyclical in the model calibrated to equal agents while in the calibration for heterogeneity tax rates behave procyclical.

An additional key issue is the cyclical behavior of the output share of government consumption. Through our model we find that the standard RBC calibration for the U.S. economy implies a procyclical output share of government consumption with inequality. The positive correlation between the output share of government consumption and output becomes stronger, the higher the wealth/income inequality. In the case without inequality, the share of government consumption on output is almost acyclical. However, when there is a high degree of complementary between the private and public goods, the share of government consumption can be countercyclical. Hercowitz and Strawczynski (2002) who study OECD countries report that only the public consumption component of the fiscal budget as a share of GDP is found to be countercyclical. Rigobon (2004) reported negative correlations between government expenditure shares and output for developed countries with Belgium as an exception and positive correlation between government expenditure shares and output for most developing countries.

Another remarkable result is that the size of government consumption decreases with inequality, i.e. the median voter chooses smaller tax rates when there is inequality given that public consumption is endogenous. This result has been supported in empirical studies (more details are given in the simulation results section in page 70). The following Section about stylized facts shows that countries which are more equal according to their GINI coefficient tend to have a higher size of government consumption than more unequal countries.[2] Whether citizens prefer to have higher or lower tax rates seems to be a question with a straightforward answer, however countries such Sweden or Norway with fairly large welfare states have coincidentally very low levels of income inequality (measured with GINI coefficient), while countries like Colombia, with a small size of government consumption have one of the highest levels of inequality in the world. The path followed to achieve large or small welfare states and how it affects the wealth and income distribution of the citizens is not really understood. Should democratic states choose high tax rates that allow governments to provide public goods and increase infrastructure and education of their population and after some level of equality has been reached decrease tax rates again in order to foster investment and growth? The model developed here attempts to find a part of the answer. We can describe how unequal and equal agents choose tax rates after a Hicks neutral shock or an investment shock. Relevant to this analysis is the assumption that all citizens benefit from the public goods financed with the income taxes they paid.

Our work is related to a fairly new strand of literature that has developed dynamic models to study the business cycle properties of fiscal policy in a political economy

[2] Meltzer and Richard (1981) found that redistribution increases with inequality. The difference to our result is that the size of government is defined as government consumption and not as transfers, i.e. voters focus on demand for redistribution and not for public goods.

environment. For example Battaglini and Coate (2008) extended their own model of political economy theory incorporating persistent productivity shocks to study the behavior of fiscal policy over the business cycle, and Bachmann and Bai (2011) used a heterogeneous agents neoclassical growth model with endogenous government purchases and wealth bias in the political aggregation process. Both models have found procyclical government spending.

To summarize, the main contribution of our paper is to explain the cyclical behavior of government consumption in the framework of a parsimonious business cycle model that includes wealth and productivity heterogeneity of households and endogenous government consumption chosen by the median voter through the political mechanism of majority voting. The versatility of our model allows us to compare the way government consumption is selected in equal and unequal societies and how different macroeconomic variables react to TFP and investment shocks according to the way the pivotal voter chooses the tax rate that finances public goods.

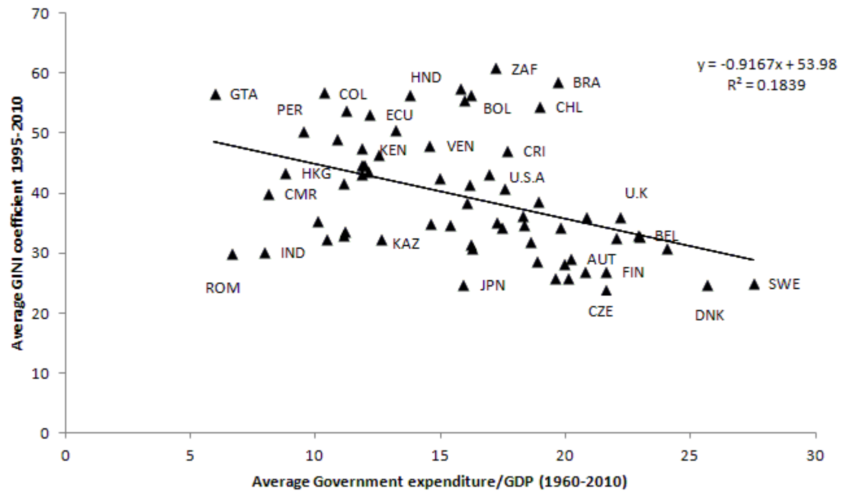
The structure of the Chapter is as follows. In Section 2 we describe the model, in Section 3 we explain the characterization of the politico-economic equilibrium and Section 4 presents a detailed explanation of the calibration. Section 5 explains the simulation results, the sensitivity analysis and data findings. Section 6 analyses impulse responses and the concluding remarks are shown in Section 7.

Stylized facts

Several empirical studies find evidence for procyclical public expenditures in Latin American countries, for example, Gavin et al. (1996), Gavin and Perotti (1997) and Stein et al. (1999). In OECD countries or high income economies, Talvi and Vegh (2005) and Lane (2003) find evidence of procyclicality. Iltzestki and Vegh (2008) find procyclical government consumption in high income economies and developing economies using quarterly data. Other empirical papers have incorporated political issues (for example, corrupt governments) as an explanation for procyclicality in developing countries. Alesina et al. (2008) and Talvi and Vegh (2005) incorporated in their models political pressure for increasing spending to explain the unexpectedly high increase in government revenues as result of, for instance, commodity booms. Few studies have focussed on the impact of inequality on fiscal performance. Larch (2010) uses cross-sectional data from EU countries to prove that fiscal discipline is easier to maintain in more equal societies. Woo (2009) shows evidence that social polarization measured by income inequality and educational inequality is positively correlated with procyclical fiscal policy.

A scatterplot of the share of government consumption over output and GINI coefficient data from the database of World Bank for the period 1995-2010 shows in Figure 3.1 that there is a negative correlation between size of government and income inequality. Additionally the simple scatter plot in Figure B.1 shows a positive correlation between procyclical government consumption and income inequality (Woo 2009 presents more concrete evidence on this).

Figure 3.1: Inequality and government consumption expenditure as a share of output



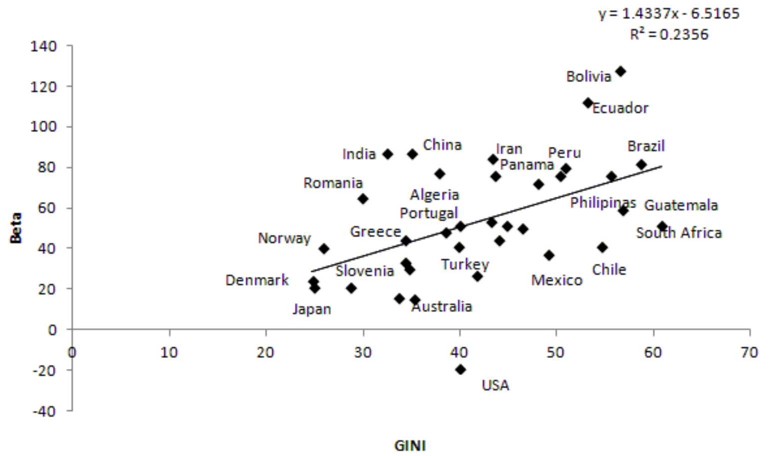
Source: World Bank and own calculations. The R-square of the correlation increases when considering the average of government consumption/GDP in (1995-2010). See Appendix B.

3.2 The model

Our model is built on a combination of the textbook RBC model as presented by King and Rebelo (1999) and the heterogeneous-agent model of Krusell and Rios-Rull (1999). We differ from their model by letting the agents vote over public consumption instead of fiscal transfers. Additionally we assume that agents derive utility from a public good included in their utility function.

Time is discrete and the horizon is infinite, indexed by t. There is a large number of infinitely-lived households with total mass normalized to 1. There are three types of households, namely, rich, median and poor (denoted by the subscript "r”, "m" and "p", respectively) and each type has mass μ_i, $i \in \{r, m, p\}$, $\sum_i \mu_i = 1$. The heterogeneity among these types is due to their initial asset holdings, $a_{i,0} > 0$, and

Figure 3.2: Inequality and correlation between government consumption expenditure and output



Source: GINI coefficients from World Bank and Beta significant coefficients from regression in Chapter 1.

their labor productivity which is constant over time, denoted by ω_i, $i \in \{r, m, p\}$, and normalized so that $\sum_i \mu_i \omega_i = 1$. The motivation to include initial wealth heterogeneity is the same as mentioned in Krusell and Rios Rull (1999), it is, that the wealth distribution is more skewed and taxing wealth accumulation is particularly distortionary.[3]

All households are identical within each class and the number of households of the same type is so large that individual households cannot affect the level of any household-type specific aggregate variables. Households own firms which employ labor and capital in order to maximize profits. Uncertainty is induced by the possibility of productivity shocks and investment shocks. Due to the presence of investment shocks the optimal utilization of capital must generate a positive effect on the marginal productivity of labor that causes an intratemporal substitution effect in favor of consumption creating procyclical movement of consumption and labor. The introduction of variable capacity utilization implies that depreciation will be a convex function of the rate of utilization.

[3]Please see the Simulation Results in Section 3.5 where an additional exercise has been carried out to see the differences in the results when agents are just heterogenous in labor productivity.

3.2.1 Firms

There is a large number of small firms. Production uses two factors, labor and capital. The utilization of capital diverges over the cycle and therefore we incorporate a factor H_t that denotes the rate of capital utilization. The production function which incorporates a Hicks-neutral shock is given by,

$$F(H_tK_t, L_t) = Y_t = z_{1,t}\left[\alpha\left(K_tH_t\right)^{1-\frac{1}{\nu}} + (1-\alpha)L_t)_t^{1-\frac{1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \tag{3.1}$$

where $\alpha \in (0,1)$, $\nu > 0$. K_t, denotes capital and $L_t = \sum_i \mu_i\omega_i l_{i,t}$ is labor. Here, we could be looking at the example of a small firm and use l_t instead of L_t and analyze the case of a small price taking firm. However, since the production function is linear and homogenous and there are no externalities in production, we can impose aggregation in the demand and supply of capital and labor.

The TFP shock is denoted by $z_{1,t}$ and follows an AR(1) process,

$$\ln\left(z_{1,t+1}\right) = \rho_1 \ln\left(z_{1,t}\right) + \varepsilon_{1,t+1}\ , \tag{3.2}$$

with $\rho_1 \in (-1,1)$, $\varepsilon_{1,t+1} \sim N\left(0, \sigma^2_{1,\varepsilon}\right)$, i.e. it is i.i.d. over time.[4] Capital, K is owned by households and they rent it to firms. Owners are responsible for maintenance of these capital goods and when signing leasing contracts, households choose the level of utilization, H_t, because the maintenance is determined according to capital utilization. It makes H_t a control variable for households. As a result, firms maximize profits deciding how much labor to hire and how much is the efficient utilization of capital, i.e. the product H_tK_t. The firm's problem is,

$$\max_{K_t,H_t,L_t} z_{1,t}\left[\alpha\left(H_tK_t\right)^{1-\frac{1}{\nu}} + (1-\alpha)\,L_t^{1-\frac{1}{\nu}}\right]^{\frac{\nu}{\nu-1}} - R_tH_tK_t - w_tL_t\ , \tag{3.3}$$

where R_t is the rental cost per unit of utilized capital, and w_t is the wage per hour. From the first order conditions, $F_1(H_tK_t, L_t) = R_t$ and $F_2(H_tK_t, L_t) = w_t$, we get that the optimal demand for utilized capital implies that the rental cost per unit of utilized capital equals the marginal product of utilized capital,

$$R_t = \alpha z_{1,t}^{1-\frac{1}{\nu}}\left(\frac{Y_t}{K_tH_t}\right)^{\frac{1}{\nu}}, \tag{3.4}$$

[4] Individual variables are denoted by lower-case letters and aggregate variables by upper-case letters (some exceptions to this rule are for the notation of prices).

and optimal labor demand is determined by making salaries equal to the marginal product of labor,

$$w_t = (1-\alpha)\, z_{1,t}^{1-\frac{1}{\nu}} \left(\frac{Y_t}{L_t}\right)^{\frac{1}{\nu}} \ . \tag{3.5}$$

The second shock, $z_{2,t}$, is on investment (denoted by I_t). The investment shock leads to the increase of new capital and to the fastest utilization and consequently rapid depreciation of the existing capital. The law of motion of capital in its aggregated form is,

$$K_{t+1} = z_{2,t} \cdot I_t + [1-\delta(H_t)]\, K_t \ , \tag{3.6}$$

where the investment shock is modeled as AR(1) process,

$$\ln(z_{2,t+1}) = \rho_2 \ln(z_{2,t}) + \varepsilon_{2,t+1} \ , \tag{3.7}$$

with $\rho_2 \in (-1, 1)$, $\varepsilon_{2,t+1} \sim N\left(0, \sigma^2_{2,\varepsilon}\right)$, i.e. it is i.i.d. over time. The depreciation rate of capital is given by $\delta(H_t)$, which is an increasing and strictly convex function of H_t, as suggested by Greenwood et al. (1988), Greenwood et al. (2000), and King and Rebelo (1999). Rearranging (3.6) we obtain and expression for investment, I_t, such that,

$$I_t = e^{-x_{2,t}} \left\{K_{t+1} - [1-\delta(H_t)]\, K_t\right\} \ , \tag{3.8}$$

where $x_{2,t} \equiv \ln(z_{2,t})$. We assume here that the TFP shock and the investment shock are uncorrelated.

3.2.2 Households

Each household is endowed with one unit of time in every period, and supplies $l_{i,t}$ fraction of its time for work and (1-$l_{i,t}$) is dedicated for its leisure activities. Its private consumption is denoted by $c_{i,t}$. The household benefits from public consumption, denoted G_t. For any household $i \in \{r, m, p\}$ expected lifetime utility is given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left[u\left(f\left(c_{i,t}, 1-l_{i,t}\right), G_t\right)\right]^{1-\frac{1}{\eta}} - 1}{1-\frac{1}{\eta}} \right\}, \tag{3.9}$$

where $\beta \in (0, 1)$ is the discount factor and $\eta > 0$ is elasticity of intertemporal substitution and its inverse is the risk aversion parameter. The utility function is represented as,

$$u\left(f\left(c_{i,t}, 1-l_{i,t}\right), G_t\right) \equiv \left\{ \phi \left[f\left(c_{i,t}, 1-l_{i,t}\right)\right]^{1-\frac{1}{\zeta}} + (1-\phi)\, G_t^{1-\frac{1}{\zeta}} \right\}^{\frac{\zeta}{\zeta-1}} \quad ;$$

with $\phi \in (0,1)$ as the fraction of private goods: $\zeta > 0$, is the elasticity of intratemporal substitution between private and public consumption where the relation between those two goods is represented by a CES function,

$$f(c_{i,t}, 1-l_{i,t}) \equiv \left[\theta c_{i,t}^{1-\frac{1}{\chi}} + (1-\theta)(1-l_{i,t})^{1-\frac{1}{\chi}}\right]^{\frac{\chi}{\chi-1}} \quad ;$$

with $\theta \in (0,1)$ being the fraction of household consumption of goods; and $\chi > 0$, the elasticity of substitution between consumption of goods and leisure. CES preferences are chosen since they are of the Gorman form and facilitate aggregation.

The budget constraint of the household in each period is given by

$$e^{-x_{2,t}} a_{i,t+1} = \left(e^{-x_{2,t}} + \bar{r}_t\right) a_{i,t} + \bar{w}_t \omega_i l_{i,t} - c_{i,t} \ , \tag{3.10}$$

where the after tax interest rate is described by,

$$\bar{r}_t \equiv (1-\tau_t)\left[R_t h_{i,t} - e^{-x_{2,t}} \delta(h_{i,t})\right] \ , \tag{3.11}$$

and the after tax wage is represented as,

$$\bar{w}_t \equiv (1-\tau_t) w_t. \tag{3.12}$$

Since in a closed economy, $Y_t = C_t + I_t + G_t$, (Y_t is GDP) equation (3.8) implies,

$$e^{-x_{2,t}} K_{t+1} = Y_t + e^{-x_{2,t}} \left[1 - \delta(H_t)\right] K_t - C_t - G_t \ . \tag{3.13}$$

The effect of shock x_2 is to change the replacement cost of capital today and tomorrow (if x_2 increases, the utility cost of the current period of an additional unit of capital decreases).

3.2.3 Government

The government is benevolent because its objective is to maximize the utility of all agents. It levies a flat proportional tax rate denoted by τ_t on labor and capital income in each period in order to finance government consumption. The government is constrained to have a balanced budget and there are no transfers. We do not allow for debt since in an environment of time-consistent policy making co-determining debt and the size of government consumption requires different and more advanced methods to linear-quadratic approximation. Klein and Rios-Rull (2003, p. 1218) discuss the technical requirements of introducing debt to such a model of time-consistent policy setting. Including the investment shock, x_2, and deducting the depreciation of capital that varies depending on the rate of utilization, the balanced fiscal budget can be written as,

$$G_t = \tau_t \left[Y_t - \delta \left(H_t \right) e^{-x_{2,t}} K_t \right] \ . \tag{3.14}$$

3.3 Politico-economic equilibrium

The political mechanism employed to reach the politico-economic equilibrium is majority voting. We follow the characterization of politico-economic equilibrium of Krusell and Rios-Rull (1999). We employ here the concept of *sincere voting* meaning that every citizen votes for the alternative that yields the highest utility.

The politico-economic equilibrium implies three steps. First, the calculation of a competitive equilibrium given a law of motion for the policy, it means in the competitive equilibrium the policy rule Ψ is exogenous. Subsequently, there is an intermediate equilibrium which will account for one-time policy deviation after which the economy returns to the given policy Ψ. Agents set their preferences according to the currently chosen policy since there is no commitment and they evaluate the situation in which the government may deviate. After considering the possibility of the government deviating from the initial policy for one time, the politico-economic equilibrium can be finally calculated by means of the political mechanism of majority voting. It is found making the law of motion of the median voter's result from the political outcome equal to the aggregate law of motion found in the competitive equilibrium and we obtain the fixed point that generates Ψ. The sequence of the game implies that in the political equilibrium the policy makers choose the same policy that they expect the future governments to follow as well.

The only voting issue each period is the current level of income tax rate due to the assumption of balanced budget. Elections take place every period and tax rates are chosen through majority voting where the median voter is the winner. The fiscal budget is announced at the beginning of each period and the government commits to it only for that period (we assume a period is one year and elections are held every period). This is the concept of instantaneous pre-commitment that is time-consistent as described by Cohen and Michel (1988). Our politico-economic equilibrium concept assumes elections are held every year and voter preferences are reflected into the pre-announced fiscal budget at the beginning of each fiscal year. Policies are time consistent since the government set the tax rates with a perfect foresight of the way that future governments will set the stream of subsequent tax rates. At the same time the government looking at the current state of the economy anticipates how households decisions, in particular the capital stock, will affect the selection of those tax rates. The solution of the dynamic game is a Markov Perfect equilibrium without history dependence of the strategies, since the tax rate chosen in period t depends solely in the state variables available in period t. The timing of the dynamic game between the government and the private sector is the same as the one described in Chapter 2. It is, each period the government inherits a certain tax rate and after observing the realization of the Hicks neutral shock and the investment shock, move first choosing the tax rate the next government should follow. Successively, the private sector moves, choosing how much to work, how much to agree on the utilization of capital and how much to save, finally the government budget should clear.

The policies are chosen sequentially and policy preferences are forward looking. The politico-economic equilibrium result is represented by the following time-consistent rule that aggregates the induced policy preferences and gives as outcome the optimal tax rate for the next period, such that,

$$\tau' = \Psi\left(x_1, x_2, \mathbf{A}, \tau\right) \ ,$$

where the vector $\mathbf{A} \equiv [A_r \ \ A_m \ \ A_p]^T$, contains the aggregate wealth holdings of each population class denoted by A_i with class $i \in \{r, m, p\}$, x_1 denotes the TFP shock and x_2 represents the investment shock. The law of motion for labor, wealth and capital utilization of these three classes is described by $\mathcal{A}\left(x_1, x_2, \mathbf{A}, \tau \mid \Psi\right)$ conditional on rule Ψ and denoted like, where $\mathcal{A}$ is a 3x7 matrix,

$$\begin{bmatrix} \mathbf{L} \\ \mathbf{A}' \\ \mathbf{H} \end{bmatrix} = \mathcal{A}\left(x_1, x_2, \mathbf{A}, \tau \mid \Psi\right),$$

with $\mathbf{L}$ being the vector that represents the aggregate labor supply of each class, $\mathbf{L} \equiv [L_r \ \ L_m \ \ L_p]^T$, and correspondingly $\mathbf{H}$ the vector containing the agreed capital utilization of capital for each class, $\mathbf{H} \equiv [H_r \ \ H_m \ \ H_p]^T$ with class $i \in \{r, m, p\}$.).

Competitive equilibrium In the competitive equilibrium we assume that the policy is set by an arbitrary policy rule Ψ, therefore the maximization problem of the household is conditional on this policy rule. The Bellman equation of the individual household is,

$$V^i(x_1, x_2, \mathbf{A}, a_i, \tau \mid \Psi) =$$

$$= \max_{a_i' \in [\underline{a}_i, \bar{a}_i], l_i \in [0,1], h_i \geq 0} \left\{ \frac{\left[u\left(f\left(\left(e^{-x_2} + \bar{r}\right) a_i + \bar{w}\omega_i l_i - e^{-x_2} a_i' \ , \ 1 - l_i\right) \ , \ G\right)\right]^{1-\frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} + \right.$$

$$\left. + \beta V^i\left(x_1', x_2', , \mathbf{A}', a_i', \tau' \mid \Psi\right) \right\}$$

subject to,

$$\begin{bmatrix} \mathbf{L} \\ \mathbf{A}' \\ \mathbf{H} \end{bmatrix} = \mathcal{A}(x_1, x_2, \mathbf{A}, \tau \mid \Psi) \ , G_t = \tau_t \left[Y_t - \delta(H_t) e^{-x_{2,t}} K_t\right]$$

and

$$\tau' = \Psi(x_1, x_2, \mathbf{A}, \tau) \ .$$

The solution of this problem is the fixed point for $V^i(x_1, x_2, \mathbf{A}, a_i, \tau \mid \Psi)$, where the aggregate decision rules equal the individual decision rules.

Intermediate equilibrium To calculate the political equilibrium, we need to consider equilibria where taxes are set in a different way, it means there will be a one time period tax deviation and this is done by means of the "intermediate equilibrium" where next period tax rate will be given by τ' and after that all taxes are chosen according to function Ψ. The "intermediate equilibrium" rule is denoted as, $\mathcal{A}_{IE}(x_1, x_2, \mathbf{A}, \tau, \tau' \mid \Psi)$ and it is calculated by solving the non-Bellman equation given below,

$$\hat{V}^i(x_1, x_2, \mathbf{A}, a_i, \tau, \tau' \mid \Psi) =$$

$$= \max_{a_i' \in [\underline{a}_i, \bar{a}_i], \ l_i \in [0,1], \ h_i \geq 0} \left\{ \frac{\left[u\left(f\left(\left(e^{-x_2} + \bar{r}\right) a_i + \bar{w}\omega_i l_i - e^{-x_2} a_i' \ , \ 1 - l_i\right) \ , \ G\right)\right]^{1-\frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} \right.$$

$$\left. + \beta V^i\left(x_1', x_2', \mathbf{A}', a_i', \tau' \mid \Psi\right) \right\} \tag{3.15}$$

subject to,

$$\begin{bmatrix} \mathbf{L} \\ \mathbf{A}' \\ \mathbf{H} \end{bmatrix} = \mathcal{A}_{IE}(x_1, x_2, \mathbf{A}, \tau, \tau' \mid \Psi) \ , \tag{3.16}$$

and

$$G_t = \tau_t \left[Y_t - \delta \left(H_t \right) e^{-x_{2,t}} K_t \right],$$

according to which $V^i \left(x_1', x_2', \mathbf{A}', a_i', \tau' \mid \Psi \right)$ is used only on the RHS of (3.15) and iterations are on the rule $\mathcal{A}_{IE} \left(x_1, x_2, \mathbf{A}, \tau, \tau' \mid \Psi \right)$ so that the individual decision rules implied by (3.15) are consistent with $\mathcal{A}_{IE} \left(x_1, x_2, \mathbf{A}, \tau, \tau' \mid \Psi \right)$. When this happens, we obtain the value function $\hat{V}^i \left(x_1, x_2, \mathbf{A}, a_i, \tau, \tau' \mid \Psi \right)$. The intuition behind $\mathcal{A}_{IE} \left(x_1, x_2, \mathbf{A}, \tau, \tau' \mid \Psi \right)$ is that it captures the responsiveness of the aggregate economy to changes in the tax rate one period ahead, i.e. changes in τ' only, as the model assumes commitment for one period only (the equivalent to the "instantaneous pre-commitment" policy-setting concept of Cohen and Michel (1988)). The value function $\hat{V}^i \left(x_1, x_2, \mathbf{A}, a_i, \tau, \tau' \mid \Psi \right)$ is able to reveal the political preference of individuals belonging to class $i \in \{r, m, p\}$, given the current state of the world $\left(x_1, x_2, \mathbf{A}, a_i, \tau \right)$ and conditional upon the fact that future policies will be set according to the policy-making rule Ψ.

Politico-economic equilibrium We assume the existence of politico-economic equilibrium but perform the numerical cross-check that Krusell and Rios-Rull (1999) have used as well, it is to check for single-peakedness of political preferences around the computed steady-state equilibrium.[5] No formal proof exists for the median voter theorem in general dynamic settings. In static environments this proof has been done using arguments such as single peakedness or single crossingness. To the best of our knowledge, the only model where the median voter result from a dynamic setting defined as voting equilibrium in the way explained in Krusell and Rios-Rull (1999), is the one of Koulovatianos and Mirman (2010). They use a tractable parametric model that permits them to prove the single-crossingness property. Nevertheless, since their model has a setting close to ours, to assume the median-voter result makes sense in the current context.

The median voter is the winner and we then maximize his value function with respect to the tax rate of next period,

$$\max_{\tau'} \hat{V}^m \left(x_1, x_2, \mathbf{A}, a_m, \tau, \tau' \mid \Psi \right)$$

and the policy-making rule Ψ must satisfy that,

$$\Psi \left(x_1, x_2, \mathbf{A}, \tau \right) = \psi \left(x_1, x_2, \mathbf{A}, A_m, \tau \mid \Psi \right) ,$$

[5] A graph for different tax rate values is depicted on Appendix B.

where

$$\psi\left(x_1, x_2, \mathbf{A}, a_m, \tau \mid \Psi\right) \equiv \arg\max_{\tau'} \hat{V}^m\left(x_1, x_2, \mathbf{A}, a_m, \tau, \tau' \mid \Psi\right) \ .$$

and finally $\Psi\left(x_1, x_2, \mathbf{A}, \tau\right) = \psi\left(x_1, x_2, \mathbf{A}, a_m, \tau \mid \Psi\right)$ for all $\mathbf{A}$ and τ

This results yields the fixed point that determined function Ψ. This is the politico-economic equilibrium given by the median voter.

3.4 Calibration

The parameters values are chosen to match the benchmark economy to the first moments, volatility and cyclical behavior of the main U.S. macroeconomic variables. The time period is one year, i.e. agents choose the optimal tax rate every year. Notice that the function we use for depreciation is,

$$\delta\left(h\right) = \delta_c + \frac{b_\delta}{1+\xi} h^{1+\xi} \ ; \qquad \delta_c, b_\delta, \xi > 0 \ .$$

- *Preferences, technology and stochastic shocks:* calibration for preferences and technology parameters and parameters concerning the stochastic shocks were selected in the same way as in Chapter 2, please Section 2.4.1 for details.

- *Distribution of wealth*: wealth inequality and relative productivity values were taken from Krussel and Rios-Rull (1999). We have chosen the corresponding values for the population between 41-65 years old and the corresponding parameters sorted by wealth. The distributional statistics for the U.S. economy (Table 3.2) are divided in three groups where 49% of the households are rich, 2% belong to the middle group and 49% to the group with the lowest wealth, as Krussel and Rios-Rull (1999) explained, this partition makes the median group small, in order to have an exact identification of the political preferences of the pivotal voter.

3.5 Simulation results

The model was simulated using Monte Carlo methods generating 200 experiments with 500 observations. The analysis consists of a comparison of several statistics

Table 3.1: Calibration parameters for the benchmark economy

Preference parameters	Description	Value
β	Discount factor	0.96
θ	Share of consumption	0.36
χ	Elasticity of substitution between consumption and leisure	0.95
ϕ	Share of private goods	0.8
ζ	Elasticity of substitution between private and public goods	1.05
η	Elasticity of intertemporal substitution	0.5
Technology parameters		
α	Share of capital	0.33
ν	Elasticity of substitution between labor and capital	0.99
δ_c	Depreciation parameter	0.03
δ_b	Depreciation parameter	0.25
ξ	Depreciation parameter	1.42
Shocks parameters		
ρ_1	TFP shock persistence	0.9
ρ_2	Investment shock persistence	0.64
σ_1	Standard deviation of TFP shock	0.0045
σ_2	Standard deviation of investment shock	0.004

Table 3.2: Calibration of distribution of wealth and earnings for the benchmark economy

Agents distribution parameters			
	Rich	Median	Poor
Percentiles	0-49	49-51	51-100
Fraction	0.49	0.02	0.49
Wealth	4.97	1	0.34
Earnings	2.15	1	0.60

Source: Krusell and Rios-Rull (1999).

between the model with equality, inequality, and U.S. data. The statistics reported are the average of the individual experiments. The calibrated model matches the data quite well. After calibrating the model to have a labor supply of 33%, we obtain a capital-output ratio of 2.12 in equality and 2.18 in inequality. The steady state of consumption is 54% in inequality since the size of government consumption is 26%, with a two percentage points difference in inequality where the size of government consumption is 24% and the share of consumption over GDP is 56%. The share of investment on output remains unchanged on 20% either in equality or inequality. The model is simulated just taking into account heterogeneity in income, to have a more clear insight of the role of wealth heterogeneity.

As usual in RBC models, output volatility is around 2%, investment is more volatile than output and around 5% on both cases. Consumption is less volatile than output in the calibration for equality and as well on inequality. However government consumption, which is endogenous in our model, is more volatile than output. The value obtained in the data for the standard deviation of government consumption is around 1.21% while in our model it is around 2%. The empirical exercise performed in Chapter 2 delivered a standard deviation of 2.21% for government consumption on the data for the U.S. economy over the period 1960-2010. This volatility is higher than the standard deviation of output that was 2.06%. Capacity utilization is very persistent and the utilization rate in steady state is 82%, the capital-output ratio is 2.12 and 2.18 for equality and inequality respectively. These results are standard in the literature. The interest rate before taxes is around 6%. All values are reported in Table 3.3 and Table 3.4.

Size of government consumption

Table 3.3 reports the government consumption share for the benchmark model with equality to be 26% and 24% for the model with inequality (for these target levels of public consumption shares see, for example, Angeletos and Panousi 2009). The steady state tax rate for the model with equality is 32% and in the model for inequality the taxes are slightly smaller, around 29%. It shows that with inequality the median voter prefers always less taxes when he votes over taxes that will finance government consumption and therefore the size of government consumption decreases with inequality. This finding coincides with Shelton (2007) who asserts that demand for redistribution is met with the most transparent redistributive type of public spending, like direct transfers. He finds zero or negative correlation between inequality measured by GINI coefficient and other forms of government spending different to transfers, therefore he concludes that Meltzer and Richard's theory most clearly applies to vertically redistributive transfers like social protection. Koulovatianos and Mirman (2010) find that the level (and output share) of public consumption can as well decrease with rising inequality depending mostly on the choice of preference parameters.

Correlations Contemporaneous correlations with output of consumption and investment are in line with the observed correlation in U.S. postwar data. The correlation of consumption and output is 83% and the correlation of investment and output is around 89%. Capacity utilization and labor supply are also procyclical. Burnside, Eichenbaum and Rebelo (1995), documented that electric consumption is procyclical because of equipment and machinery are used more intensively in booms. The correlation found for electricity use and output for the period 1977:1-1992:4 was 0.72. Results are shown in Table 3.4.

Cyclicality of government consumption The contemporaneous correlation between government consumption and output is positive and near 1, therefore government consumption is strongly procyclical in the model with equality and in the model with inequality. The representative agent and the median voter support the government choice of increasing government consumption in booms as a reaction to a positive TFP shock or a positive investment shock.

Cyclicality of the share of government consumption on output The correlation between the fraction of output spent by the government and output is positive. In the benchmark calibration it is about 16% for the model with equality and increases to 72% with inequality. Some authors have found positive correlation of government consumption and output in models where the consumption of government is endogenously determined and affects the utility of individuals, i.e. they benefit from a public good. Battaglini and Coate (2008) extended their model of political economy theory of fiscal policy incorporating persistent productivity shocks and found that government spending increases in booms and decreases during recessions, while tax rates decrease during booms and increase in recessions. Ambler and Cardia (1996) solve a RBC model where government spending is endogenous. The optimal public spending in their model should behave similar to private consumption, and therefore they obtain highly procyclical government expenditures. Debortoli and Nunes (2010) find procyclical government expenditures in the presence of political turnover analyzed as loose commitment. However, these models did not have heterogeneity in wealth or income. The inclusion of heterogeneity in wealth and labor productivity confirm the procyclicality of government consumption in models that abstracted from it.

Cyclicality of tax rates and output The correlation between tax rates and output is mildly procyclical in the benchmark model with equality. In the model with inequality there is a positive correlation between tax rates and output of 58%, therefore, the model predicts that for such calibrated economies, societies with high inequality, will tend to increase tax rates in booms and decrease them in recessions.

The cyclical properties of taxes over the business cycle depend in our model on the elasticity of substitution between private and public goods. When private and public goods are substitutes, the correlation between taxes and output is positive while in the case of public consumption and private goods being complements, the taxes tend to behave countercyclical.

Heterogeneity in wealth Ignoring heterogeneity in wealth and having just heterogeneity in income, does not change the cyclicality of government consumption. However, there is a higher size of government consumption demanded by the median voter. These results confirm the fact that taxation of wealth accumulation is more distortionary and then the median voter chooses a lower tax when there is wealth heterogeneity. Results are shown in Appendix B.

Table 3.3: Steady state values of the benchmark economy

	Equality	Inequality
C/Y	0.54	0.56
G/Y	0.26	0.24
I/Y	0.20	0.20
L	0.33	0.33
K/Y	2.12	2.18
r	0.06	0.06
τ	0.32	0.29
δ	0.09	0.09
H	0.82	0.82

3.6 Sensitivity analysis

The sensitivity analysis exercise compares the benchmark model with some variations on the calibration for arbitrary values given to η (elasticity of intertemporal substitution or the inverse value of risk aversion) and ζ (elasticity of substitution between private and public goods). The performed exercise confirms that the procyclical behavior of government consumption and output share of government consumption is

Table 3.4: Model statistics of the benchmark economy

	Standard Deviation			First Order Autocorrelation			Contemporaneous correlation with output		
	Data	Equality	Inequality	Data	Equality	Inequality	Data	Equality	Inequality
Y	1.81	1.97	1.90	0.84	0.90	0.88	1.00	1.00	1.00
C	1.35	1.26	1.08	0.80	0.96	0.95	0.88	0.84	0.83
I	5.30	4.80	4.59	0.87	0.81	0.78	0.80	0.90	0.89
G	1.21	2.10	2.46	0.68	0.89	0.91	0.80	0.97	0.98
H	n.a	0.85	0.79	n.a	0.80	0.76	n.a	0.23	0.27
L	1.79	0.79	0.76	0.88	0.77	0.70	0.88	0.69	0.69
$\mathcal{T}$	n.a	0.19	0.22	n.a	0.89	0.98	n.a	0.10	0.58
G/Y	n.a	0.12	0.16	n.a	0.82	0.98	0.17	0.16	0.72
w	0.68	1.36	1.32	0.66	0.95	0.95	0.12	0.93	0.93
r	0.30	0.18	0.16	0.60	0.85	0.81	-0.35	0.14	0.22

Source:U.S. data for standard deviations, first order autocorrelation and contemporaneous correlations with output were taken from King and Rebelo (1999) for Y, C, I, L, w and r; values for G, G/Y were taken from Klein and Rios-Rull (2003). n.a: not available

robust in the model with inequality regardless of the level of elasticity of intratemporal substitution between public and private consumption and regardless of the level of risk aversion. Results are summarized in Table 3.5. Capacity utilization is procyclical in the simulation. An empirical exercise of Burnside et al. (1995) showed that capacity utilization is procyclical. They estimated procyclicality as a positive correlation between the growth rates of aggregated electricity consumption (proxy for capacity utilization) and output. The high correlation between wages and labor is a common characteristic of business cycles models.

The elasticity of intertemporal substitution This parameter was set to 0.5 in the benchmark model. For the sensitivity analysis we compare two different values, one is to set $\eta = 0.25$ that implies a risk aversion of 4, and the second was to set $\eta = 0.9$, decreasing the risk aversion parameter to 1.11. For the case of $\eta = 0.9$, that implies less risk averse agents, we observe that the correlation between government consumption and output continues being positive and high, while the correlation between the share of government consumption on output and output is around 44% with equality and increasing to 75% with inequality. The size of government consumption decreases as well with inequality as in the benchmark case. In the case of $\eta = 0.25$, government consumption remains procyclical, but in a smaller magnitude than in the benchmark model for the case with inequality and

Table 3.5: Sensitivity analysis

	$\mathcal{T}$		corr(G,Y)		corr(G/Y,Y)		corr($\mathcal{T}$,Y)	
	Equality	Inequality	Equality	Inequality	Equality	Inequality	Equality	Inequality
Benchmark	0.32	0.29	0.97	0.98	0.16	0.72	0.10	0.58
$\eta = 0.9$	0.32	0.28	0.98	0.98	0.44	0.75	0.27	0.62
$\eta = 0.25$	0.34	0.32	0.98	0.94	-0.08	0.20	-0.06	0.12
$\zeta = 0.7$	0.39	0.35	0.95	0.93	-0.16	0.15	-0.18	0.07
$\zeta = 1.5$	0.25	0.22	0.97	0.93	0.46	0.60	0.32	0.54

it becomes slightly countercyclical in the case with equality; a possible interpretation is that more risk averse agents prefer to smooth consumption and save in the good times, however a decrease in the intertemporal rate of substitution leads to more taxation. Similar results are obtained for the correlation between taxes and output, i.e., with inequality the correlation between taxes and output is positive and the tax rate behaves mildly countercyclical in the equality case.

The elasticity of substitution between private and public consumption, ζ It was chosen in the benchmark model as 1.05. We have changed it in the sensitivity analysis to 1.5, making the private and public goods more substitutes. As a result we get a smaller size of government consumption, since taxes decreases from 29% (benchmark tax) to 22% in the case with inequality. However the cyclical behavior of government consumption and the share of public consumption over output remains highly procyclical. When $\zeta = 0.7$, private and public consumption become more complementary and the output share of public consumption and output become slightly procyclical and countercyclical for equal agents. The tax rate behaves almost acyclical with inequality and the correlation between taxes and output changes to -18% with equality. It means equal societies that consider public and private goods as complements, will vote for high taxes and will demand an increase in the size of government consumption during bad times and a decrease in the tax rates during good times.

Overall the sensitivity analysis shows that with inequality the tax rates tend to be procyclical, it means the median voter, which in the calibration is poor in relative terms, chooses to increase taxes in booms. This causes as well the output share of government consumption behaving procyclical, since increasing tax rates in booms will raise as well the government consumption share on output more than in the equality case, at the same time, because of higher taxation, output reacts less to the positive shocks causing the boom (this becomes more clear in the impulse responses analysis, particularly see Figure 3.5).

3.7 Impulse responses

In this section we describe the behavior of impulse responses of the main macroeconomic variables to a one percent TFP shock and one percent investment shock. Results are shown in Figure 3.3 with both shocks plotted for the inequality case, Figure 3.4 presents the effects of a TFP shock on macroeconomic variables in the presence of inequality and equality and Figure 3.5 contains the impulses responses of an investment shock for the inequality and equality case. Time is in the horizontal axis and deviation from the steady state values is in the vertical axis.

The most remarkable result of the TFP shock is observed in the response of the tax rate. In the inequality case agents will increase taxes slowly, trying to delay the positive response of taxes to productivity shocks. Equal agents instead decrease the tax rate in the next year after the shock leading afterwards to increases in taxes (less than 0.2%) going back rapidly to the steady state level. It implies that taxes are less procyclical with equality in the benchmark model. Unequal agents increase taxes after increasing the accumulation of capital. The response of tax rates has also an impact on capacity utilization which shows a faster decline in the inequality case where the tax rate remains higher than the equality case.

An interesting result is the response over consumption in the equality case. One year after the TFP shock consumption increases more than in the inequality case due to the immediate reduction in taxes.

Government consumption is very responsive to a TFP shock, increasing almost by 2%, in the equality case in spite of the reduction of taxes, the effect on government consumption remains and vanishes slowly after year 30.

Another significant result is the strong reaction of investment to the TFP shock in the inequality case, it increases by almost 7% whereas in the equality case, investment increases by less than 6%. Results are shown in Figure 3.5.

Figure 3.6 shows the importance of the parameter determining the elasticity of substitution between public and private goods in the inequality case. When the two goods are more substitutes, i.e., $\zeta = 1.5$, the response of agents is to decrease taxation after a positive TFP shock, it also coincides with a almost acyclical behavior of the share of government consumption on output. Interestingly given the opposite relationship between private and public goods, i.e., when they are more complements and $\zeta = 0.7$, the response after a positive TFP shock is to increase taxes immediately and then persistently as far as capital increases as well as a consequence of the positive effect of the productivity shock on output. Consequently, the share of government consumption on output behaves highly procyclical. The other macroeconomic variables keep their cyclical behavior regardless of calibration of the elasticity of substitution between public and private goods.

Figure 3.3 shows the main differences of the impact of TFP shocks and investment shock just in the inequality case. In this panel of graphs, it is noteworthy that the positive response of the unequal agents in the tax rate seems to be driven particularly by the TFP shock, when the investment shock is the only factor driving the impulse responses, the tax rate decreases in the first year and increases afterwards but in a lower magnitude than with the TFP shock.

Output increases rapidly almost by 2% with the TFP shock and the effect dies slowly and it returns to his initial value after 30 years. In the case of the investment shock, output increases almost by 1% but the effect vanishes rapidly. The effect on government consumption is similar, but it takes longer than 30 years for the effect to totally die out. In the case of consumption, it increases by less than 1% with the TFP shock and it decreases slowly and returns to its normal value after 30 years, in the case of the investment shock, consumption becomes negative but in the second year it returns to the steady state increasing slowly until a maximum point in the sixth year and decreases again until reaching the steady state. In general the effect of the TFP shock over the macroeconomic variables studied is of greater magnitude than the investment shock.

3.8 Conclusion

We have studied the interaction between resource constraints and political conflict to understand the cyclical fluctuation of both economic and fiscal variables. The economic environment combined the textbook RBC model as presented by King and Rebelo (1999) with some modifications of the heterogeneous-agent model of Krusell and Rios-Rull (1999), i.e. instead of focusing on voting over fiscal transfers as they do, we studied voting over public consumption that enters the utility function of agents.

We found that the level of public consumption is procyclical with inequality on the model calibrated to the U.S economy with agents being heterogenous on initial wealth and labor productivity. Several empirical studies and the data used in this study support this finding. Yet, evidence suggests that the output share of government consumption is countercyclical in most cases. These results are associated to the fact that government consumption is determined endogenously and it makes this variable highly persistent.

The standard RBC calibration implies mildly procyclical output share of government consumption. This result is robust in the inequality case as shown in the sensitivity analysis. Interestingly, the positive correlation between the output share of government consumption and output becomes stronger the higher the wealth/income inequality. However these results do not hold in the equality case and depend on the parameters chosen, i.e., increasing risk aversion and making the public and private good more complements make the share of government consumption over output countercyclical. Interestingly, the size of government consumption decreases with inequality. Unequal agents choose a lower tax rate than equal agents on wealth and labor productivity ceteris paribus. This finding fits the empirical exercise of Shelton (2007) who finds zero or negative correlation between inequality measured by GINI coefficient and other components of government spending with exception of transfers contradicting theoretical studies that have predicted that the size of government consumption increases with inequality.

Another remarkable issue observed in this study is the voting over tax rates in the presence of productivity shocks where heterogeneity of wealth and income of agents play an important role. In the equality case the productivity shocks lead to reduction in taxes while unequal agents prefer increasing taxes slowly according with the accumulation of capital. Therefore the level of inequality across agents has an impact on the response of the main macroeconomic variables to productivity shocks in the kind of economies we have modeled here.

Figure 3.3: Impulse responses to a one percent TFP shock (solid line) and one percent investment shock (dashed line) in inequality.

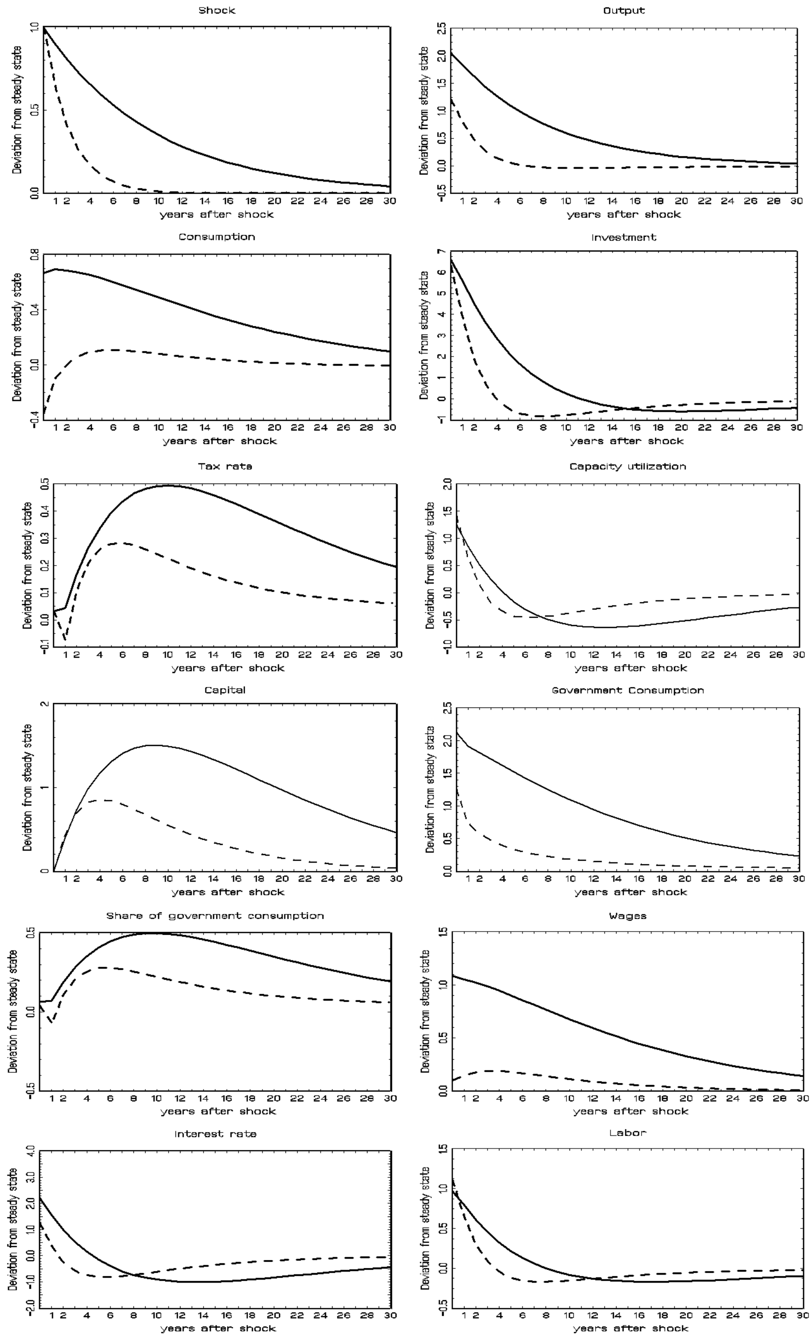


Figure 3.4: Impulse responses to a one percent TFP shock in equality (solid line) and inequality (dashed line).

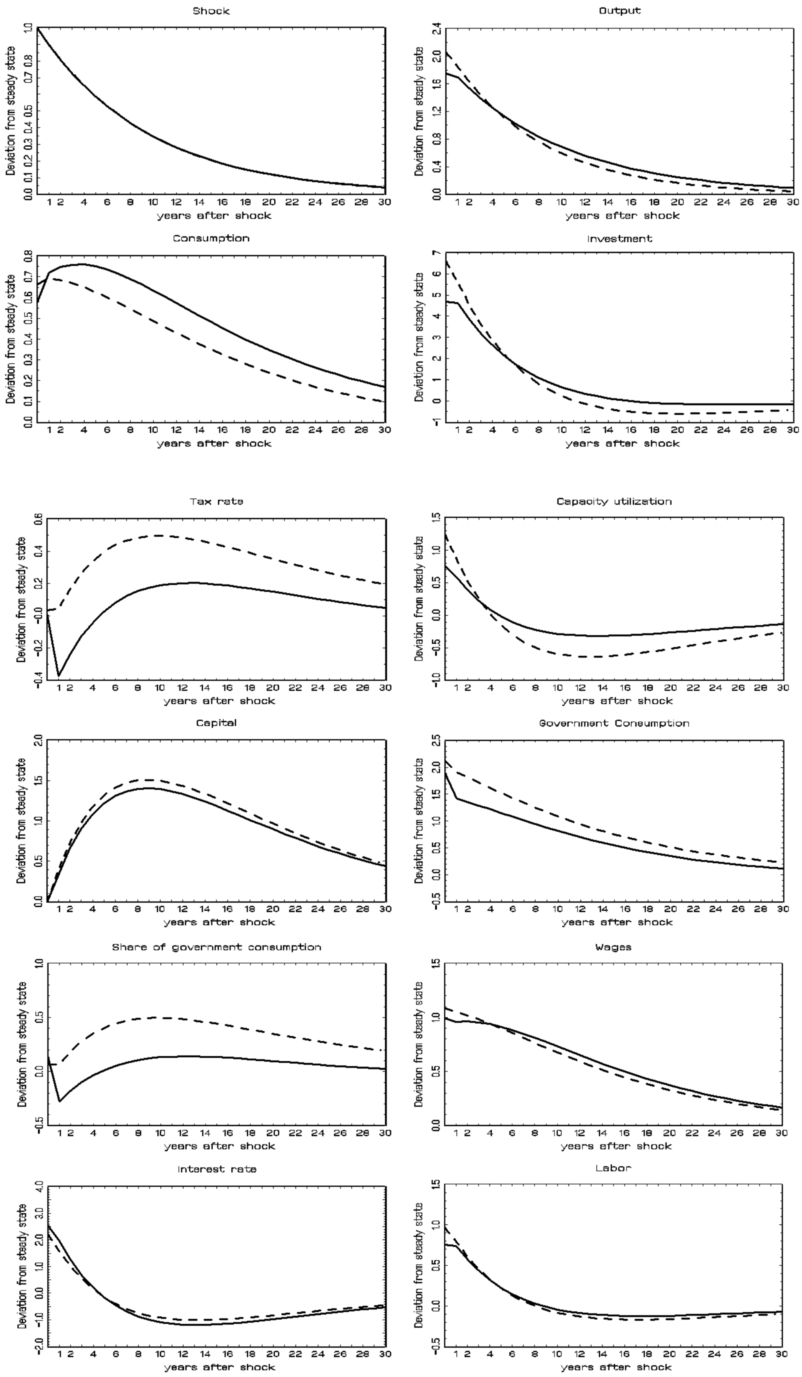


Figure 3.5: Impulse responses to a one percent investment shock in equality (solid line) and inequality (dashed line).

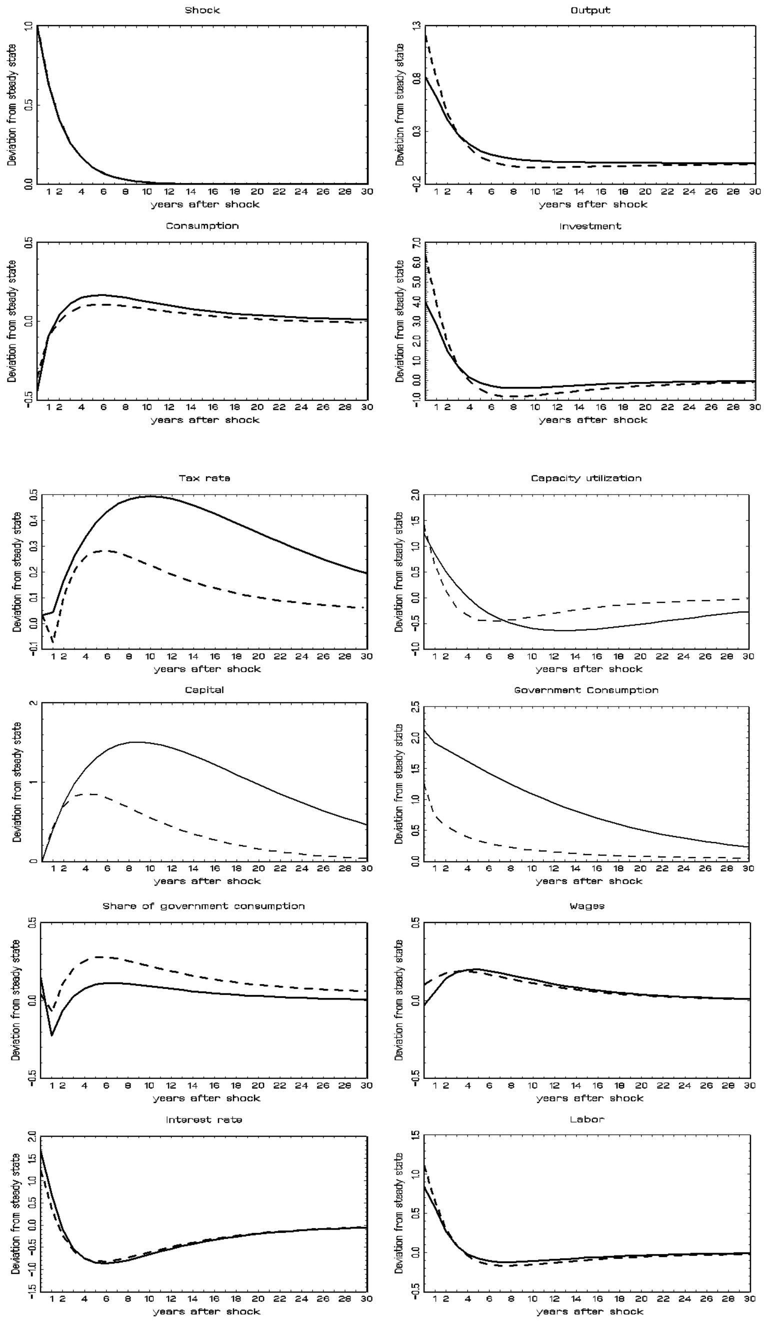
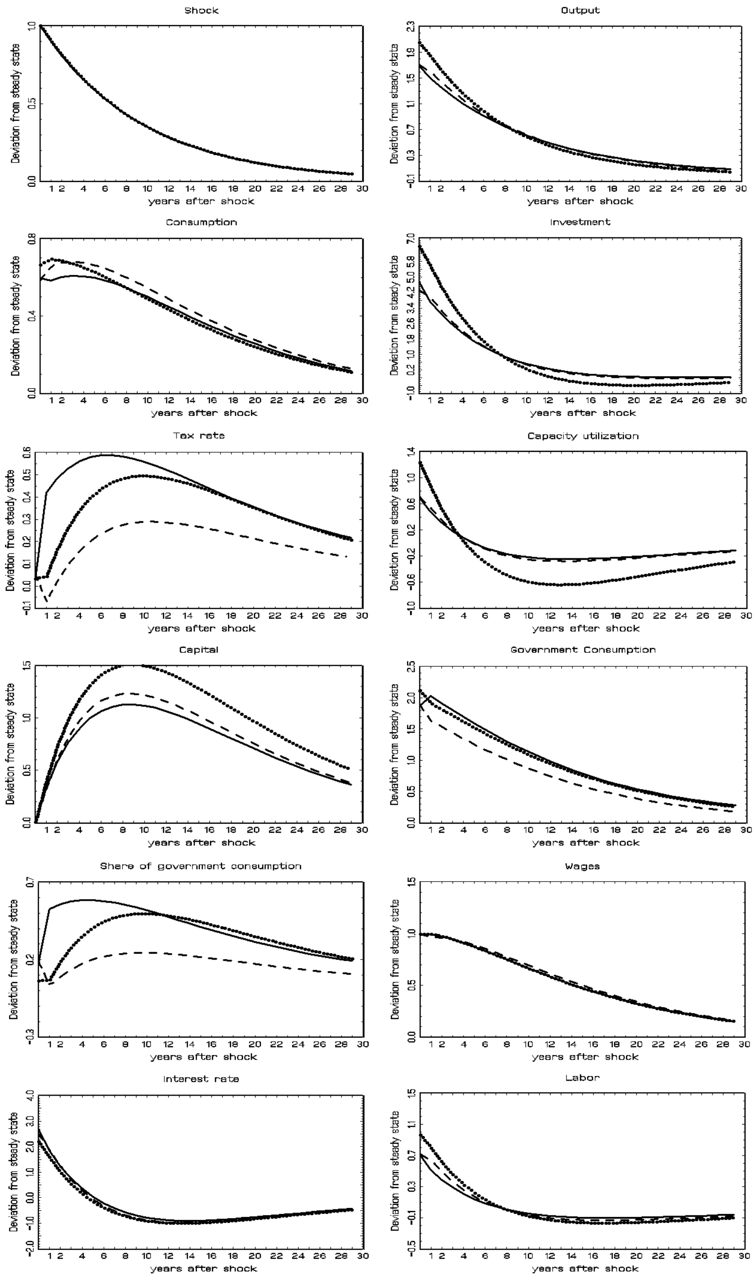


Figure 3.6: Impulse responses to a one percent TFP shock in inequality for different values of the elasticity of substitution between public and private goods. Benchmark (dotted line), complements (solid line), substitutes (dashed line).



Chapter 4

External Sovereign Debt in a Monetary Union: Bailouts and the role of Corruption

4.1 Introduction

This paper is motivated by the recent sovereign debt crises in the eurozone and the following facts: eurozone countries have some of the largest sovereign external debt-to-GDP ratios in the world and those ratios have increased remarkably over the last few years, as depicted in Figure 4.1. The ten-year maturity bond yields of these countries with highest external debt increased after the Lehman Brothers collapse in 2008 as shown Figure 4.2. Interestingly, those countries used to have higher bond yield spreads already before the introduction of the euro and also have high levels of corruption as measured by the Corruption Perception Index (CPI) as it is shown in Figure 4.3.[1] These data facts could unveil that countries with high levels of corruption entering a monetary union may end up having high levels of external debt, particularly, when accessing international capital markets as a monetary union member and not as an individual country which gave them by time the advantage of benefiting of historically low bond yields spreads.

Our study seeks to bring together the mentioned facts by means of a theoretical model that endogenizes the degree of corruption in terms of the number of rent seek-

[1] The empirical relationship between high fiscal debt and corruption has been studied by Grechyna (2012). She presents evidence on correlations between fiscal-debt/GDP ratios and corruption-perceptions measures in high-income countries.

ing groups which have influence on the level of external-debt issuing. An additional component of our model is the endogeneity of interest rates. Those are included since high country-specific interest rates set in international markets are considered a factor that exacerbates fears on external creditors. Kohlscheen (2010) reported that the likelihood of external-debt defaults is higher than domestic-debt defaults since there is strong pressure for default given the high level of interest rates formed in international markets.

Fears of default triggered by high level of indebtedness and high country-specific interest rates brought the need for bailouts. Bailouts are used as a mechanism to prevent the contagion which could be generated by government default of any member of the eurozone affecting the banking system of the other members whose banks may have losses through the non-repayment of bonds.[2]

Countries borrowing at high interest rates increase the cost of servicing new debt and therefore the total debt burden. This endangers sustainability and reduces the welfare of the society since higher taxes will be charged and/or public consumption will be reduced. Given the negative impact of high international interest rates in countries that belong to a monetary union, bailouts are supposed to control these interest rates and bring the country back to levels of debt-GDP ratio that are socially acceptable and politically feasible. However, successful implementation of bailouts can be distorted if economies have some structural deficiencies like corruption.

The interaction between international interest rates and the political economy of external sovereign debt has not been deeply studied. This creates several debates about bailout plans implementation and effectiveness. Those debates are understandable since advanced economies in the EU were perceived free of sovereign default risk.[3] With the framework proposed here we intend to fill this gap.

Our model focuses on the interplay between the political economy of external-sovereign debt and international capital markets. It allows studying countries that end up in sovereign-debt traps because of structural problems like corruption and countries that can get in such traps but are free from corruption.

We show that fiscal impatience is created if rent-seeking groups have the power of expending public resources for their own benefit. Fiscal impatience means that it

[2]Bolton and Jeanne (2011) compiled foreign debt exposures from European commercial banks as of 2010. They developed as well a sovereign debt model linking government debt to banking crises because government bonds serve as collateral for interbank loans.

[3]Reinhart and Rogoff (2008) report that the latest incidences of sovereign default in Europe are Germany in 1939, Austria in 1940 and Hungary in 1941.

is decided to spend more today and to delay the increase of taxation for tomorrow, hence increasing sovereign debt today. Fiscal impatience therefore implies in the model that the rate of time preference of the domestic economy is higher than the one of the external creditors within a monetary union. It produces as a result higher country-specific real interest rates worsening the debt burden of the corrupt country.

A relevant fact of countries suffering from corruption is the number of non-cooperative rent-seeking groups. In the case of more than one rent-seeking group, fiscal impatience is created by a "tragedy of the commons" problem increasing the voracity effect over public resources.[4] If there is a single rent-seeking group or "big mob", fiscal impatience could be avoided. Nevertheless, even with one or more rent-seeking groups, high levels of the sovereign-debt ratio decrease the utility of non-rent-seekers because of a gap between tax rates paid and the public goods received, therefore corruption is considered in any case as a reason for social discontent and of vulnerability for countries that may need to issue excessive debt in emergency situations.

The absence of corruption benefits optimal fiscal policy in the sense of equalizing the rate of time preference from the domestic economy and external creditors, avoiding the perverse effect of fiscal impatience. It does not mean, however, that those countries would be exempt of difficulties if they show high debt-GDP ratios because external creditors may also consider a certain outstanding debt-GDP ratio to be so excessive that it may generate social pressure. Social pressure could be caused because government spending is reduced to pay back the debt. As a solution, the government may choose to do an unexpected haircut to alleviate the debt burden. Such an anticipation of a haircut by the creditors will trigger an increase in interest rates to compensate investors, making debt-GDP ratios to increase even further, leading the country into a trap. There are two main factors that may trigger such a trap. The first one is high initial debt-GDP ratio as a consequence of, for example, a disaster occurred in the immediate past that required high emergency public spending/borrowing. The second factor can be information asymmetries between external creditors and the indebted country. Since our model is deterministic we do not explicitly model information and we do not study how a country enters such a trap in this context. Nevertheless, we explain the mechanics of such a trap and we show in our model that even under optimal policy, high interest rates increase

[4]For an introduction to the "voracity effect" caused by the presence of rent-seeking groups see, for example, Lane and Tornell (1996) and Tornell and Lane (1999).

debt-GDP ratios over time such that a bailout plan is needed.

We study the efficiency of a bailout of the type implemented in Greece, that is, loans with lower than international markets interest rates.[5] The key factor of the designed bailout is the interest rate level determined for a specific type of economy. We distinguish the implementation of bailouts in countries without corruption and countries with it. In our model, in countries without corruption the domestic borrowers and external creditors have the same rate of time preference and this leads to an equilibrium of interest rates that we called "normal". For countries with high initial levels of debt-to-GDP ratio that end up in a trap of high interest rates, there are two kinds of bailout interest rates. The first bailout rate is for debt-to-GDP ratios below a certain threshold. In this case a short-duration bailout plan can offer above "normal" interest rates and still be successful. It means that the external-debt-GDP ratio is reduced to a level socially sustainable and the country returns to the free international bond market. The second option is for debt-to-GDP ratios above the threshold. The bailout plan should offer lower than "normal" interest rates, which implies a transfer of resources from the monetary union to the indebted country. The bailout plan will be successful in this context if debt levels are reduced to a sustainable level that allows the country to return to the free international bond market.

Bailouts in the presence of corruption are likely to fail, due to the fact that more than one rent-seeking group creates structural fiscal impatience pulling out the government from international markets for a long period of time, even when the bailout plan offers lower than "normal" interest rates. If left alone in free markets for some time, excessive debt-GDP ratios will return, therefore a bailout of the form described here will not work. A more promising strategy would be to eradicate rent-seeking groups.

We construct a stylized model in which there is no physical capital and no uncertainty. The reason to study bailouts in a deterministic setting is to exclude technical complexities particularly related to optimal debt-maturity and state contingency. Those simplifications allow us to have endogenous international interest rates and to analyze the commons problem resulting from the Markovian-Nash strategic behavior of rent-seeking groups. An additional simplifying assumption is that productivity growth is exogenous as we focus on the impact of exogenous growth rates on the dynamics of sovereign debt and on international-market interest rates of government

[5] The bailout of Greece is very well described in Buiter and Rahbari (2010) pages 9-10.

bonds. A study about the relationship between external sovereign debt and growth is the one of Cohen and Sachs (1986). Cohen and Sachs (1986) showed in a growth model for a small open economy with access to international capital markets that the equilibrium strategy of a competitive lender is to make dependent the growth of the foreign debt to the growth of the borrowing country.

An extensive body of literature studied sovereign debt and risk of default making simplifying assumptions in order to avoid complexities related to optimal debt-maturity and state contingency. Most of these studies were built on the model of Eaton and Gersovitz (1981) with a debt maturity of one period and financial autarky as a default penalty. Using this framework several papers try to explain the factors involved in government default, particularly the role of interest rates and output fluctuations. These papers include Aguiar and Gopinath (2006), Arellano (2008) and Guimaraes (2011). Two key studies explaining the optimal maturity structure in economies without capital are Stokey and Lucas (1983) and Angeletos (2002). Cuadra et al. (2010) analyzed fiscal policy in the presence of sovereign default risk in emerging economies in a dynamic stochastic small open economy with constant interest rates. Another recent work studying fiscal policy in the presence of external debt under constant world interest rates is the one by Aguiar et al. (2009), they investigated optimal taxation of foreign capital and optimal sovereign debt policy without commitment where governments are more impatient than the private sector.

Our model is related to the literature that studies the relationship between macroeconomic policy and political economy. One recent study focusing on the impact of rent-seeking politics and fiscal debt is Yared (2010). In his model rent-seeking governments do not provide the insurance that a benevolent government is expected to supply and this increases macroeconomic volatility and taxes respond with high persistence to shocks and at the same time debt increases. Some other studies focus on political instability and sovereign default in emerging countries. Examples are Cuadra and Sapriza (2008) and Alesina and Tabellini (1989). Cuadra and Sapriza (2008) found that economies that experience more political instability and polarization have rather high default rates and more volatile sovereign debt interest rate spreads. Alesina and Tabellini (1989) showed that in a general equilibrium model, where governments alternate randomly, the uncertainty about fiscal policy increases the over-accumulation of external debt.

A new strand of literature studies sovereign debt crises in monetary unions. Daniel and Shiamptanis (2010) considered as a main cause of the sovereign debt crises in the

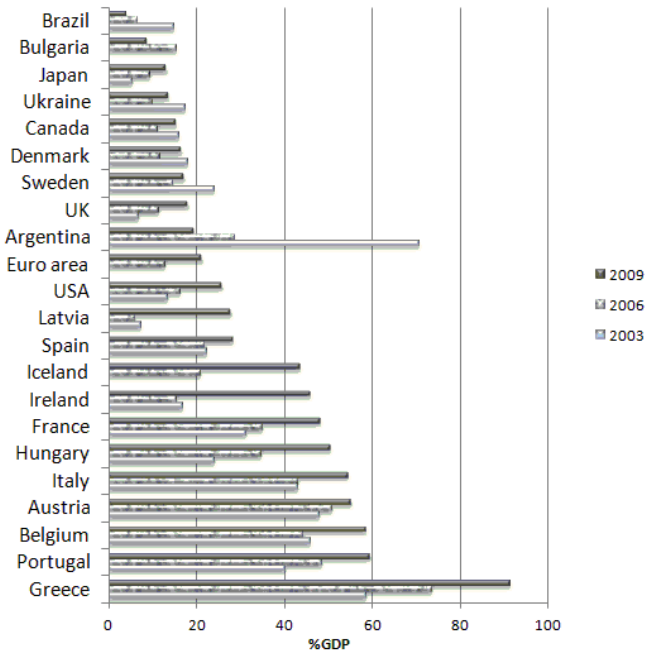
EU a discrepancy between the present value of fiscal surpluses and sustainable long run debt trajectory unable to maintain fiscal solvency. Therefore, partial sovereign default is considered as a policy response in order to restore fiscal solvency. However, this can be an ineffective response and fiscal surpluses are still required through fiscal reforms. Roch and Uhlig (2011) showed that the effect of existing EU bailout plans is just postponing default if strong fiscal disciple is not imposed, because the bailout allows governments to keep borrowing to maintain the same level of public consumption, offering just a temporary relief.

The outline of the chapter is as follows. In Section 2 we describe the benchmark model of optimal fiscal policy. In Section 3 we incorporate political economy considerations adding rent-seeking groups to the benchmark. In Section 4 we study sovereign bailouts focusing on the role of interest rates and also consider bailouts in the presence of corruption, offering some comments to the bailout of Ireland, Greece, Portugal and to the sovereign-debt prospects of Spain and Italy. We make some concluding remarks in Section 5.

Facts As mentioned in the introduction, some empirical facts have motivated this study. First, the high and increasing sovereign external debt-to GDP ratios particularly in the eurozone countries. Figure 4.1 shows gross external general government debt-to-GDP ratios for some selected countries for the years, 2003, 2006 and 2009. While some countries like Brazil, Canada, and the U.K. remain with a sovereign external debt-to-GDP ratio below 20%, Ireland, Italy and Portugal surpassed 40% and Greece surpassed 90% in 2009. Second, the main countries involved in the European sovereign debt crises used to have higher spreads over German bunds prior to joining the eurozone as is cleared represented in Figure 4.2. It seemed like if markets had ignored the factors that made countries like Greece, Portugal, Spain and Italy have higher bond yields spreads before becoming part of the common currency zone. And third, the eurozone countries with high levels of external debt-to-GDP ratio and high ratios of cyclically adjusted budget balance over GDP are as well the countries with higher levels of corruption as measured by the Corruption Perception Index (CPI) from Transparency International. The CPI ranks countries according to the perceptions of public sector corruption. These perceptions are collected in a composite index that combines polls and corruption-related data delivered by several reputable institutions. The CPI Index expresses the views of internal and external observers of the respective countries evaluated. Transparency International has defined corruption as the abuse of entrusted power for private gain in public sector, including public officials, civil servants or politicians.

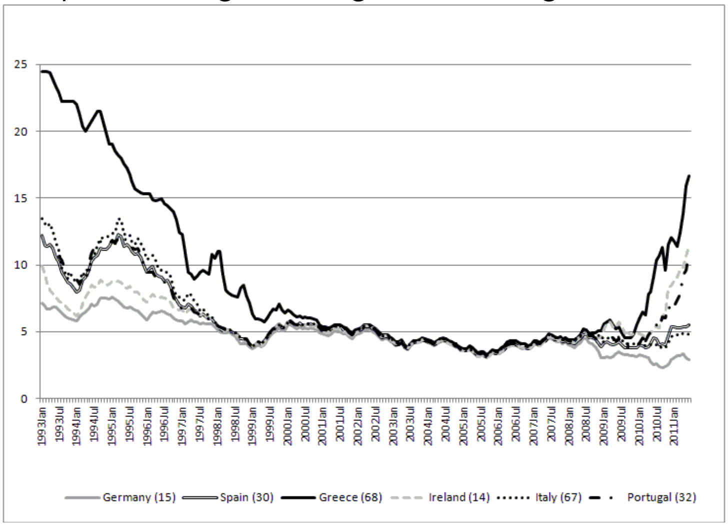
The correlations between cyclically adjusted budget balance over GDP and the CPI of eurozone countries are depicted in Figure 4.3. A simple linear regression seems to suggest a positive correlation between government deficits and high corruption. These data facts brought us to the need of studying an external debt model that would link high levels of indebtedness to the prevalence of structural problems as corruption.

Figure 4.1: Gross external debt of the general government of selected countries



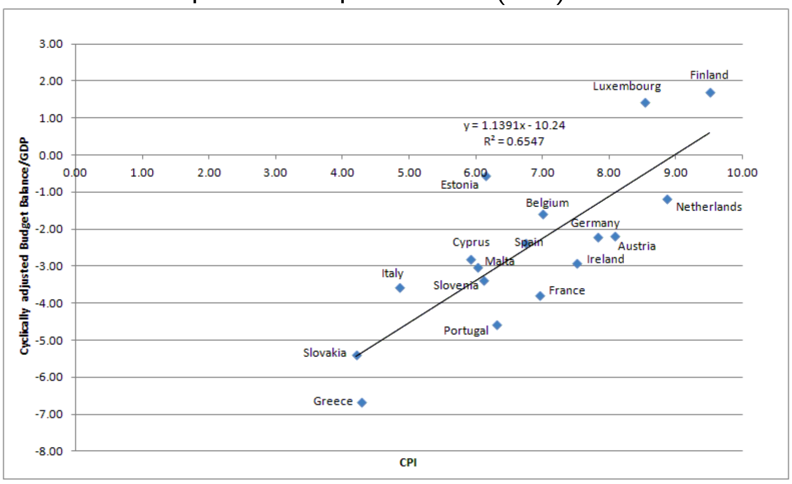
Source: Jorge Diz Dias, ECB.

Figure 4.2: Secondary markets yields of government bonds with maturities close to 10 years and corruption ranking according to CPI average 2007-2009



Source: European Commission.

Figure 4.3: Cyclically adjusted net lending or net borrowing of general government and averages of the Corruption Perception Index (CPI) 1996-2010



Source: European Commission. Cyclical adjustment of budget balances autumn 2011. Table 9A, page 82. Corruption perception index from Transparency International and authors' own calculations.

4.2 Benchmark model of optimal fiscal policy

4.2.1 The domestic economy

Competitive equilibrium

The domestic economy is populated by a large number of identical infinitely-lived agents whose total mass equals 1. There is no uncertainty and there is one consumption good produced under perfect competition, the technology is linear and the only input is labor, therefore the production function has the following form,

$$y_t = z_t \cdot l_t \ , \tag{4.1}$$

where l is labor hours, y is units of output, and z is productivity that in $t = 0$ is equal to 1, i.e, $z_0 = 1$ and that it grows at the exogenous rate γ, such that,

$$z_t = (1 + \gamma)^t \ . \tag{4.2}$$

The representative agent derives utility from private consumption (c), leisure $(1 - l)$, and the consumption of a public good, G. The utility function is logarithmic and can be expressed as

$$\sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) + \theta_l \ln(1 - l_t) + \theta_G \ln(G_t)\right] \tag{4.3}$$

where the discount factor is $\beta \in (0, 1)$ and θ_l, $\theta_G > 0$ represent the weight that the agent places on leisure and public consumption respectively. Public goods are financed with income taxes and fiscal debt such that the government budget constraint is

$$G_t = B_{t+1} - (1 + r_t)B_t + \tau_t Y_t \ , \tag{4.4}$$

in which Y_t is aggregate production in period t and B_t is fiscal debt in period t. We assume the government issues only one period zero coupon bonds and r_t is the interest rate for servicing the debt in period t. We further assume that agents in this economy cannot hold any government bonds, so B_t is external debt in all periods. Finally, we assume that agents cannot have access to domestic government bonds in the future, and that the consumable good is not storable, i.e., that there is no other form of capital. Under these assumptions, the budget constraint of an individual household is,

$$c_t = (1 - \tau_t) z_t l_t \; . \tag{4.5}$$

The problem of the representative household is to maximize its lifetime utility given by (4.3), subject to the budget constraint given by (4.5), choosing optimally the stream of consumption and labor supply throughout the infinite horizon, $(\{(c_t, l_t)\}_{t=0}^{\infty})$, given a sequence of tax rates and the quantities of the public good, $\{(G_t, \tau_t)\}_{t=0}^{\infty}$. The solution to this problem is the intra-temporal condition for labor supply given by

$$l_t = \frac{1}{1+\theta_l} = L \; . \tag{4.6}$$

The market clearing conditions imply that aggregate labor supply is equal to L as well. Labor supply is then a constant value and there is no response to changes in marginal tax rates because the logarithmic utility makes the income and substitution effects of taxation on leisure cancel out each other. After combining L with (4.1) and (4.2) we obtain the competitive-equilibrium GDP level,

$$Y_t = (1+\gamma)^t L \; . \tag{4.7}$$

The absence of any marginal tax rates in equation (4.7) demonstrates that our logarithmic-utility setup neutralizes the impact of taxes on GDP performance but taxes do reduce consumption and utility (see equation 4.5). Therefore taxes have profound impact on welfare. However our economy reflects its ability/inability to repay fiscal debt from the effect that interest rates have on the economy's wealth. In what follows we demonstrate that international interest rates at which a country borrows externally influence its ability to repay fiscal debt in the future.

Policy Setting

The government chooses the optimal sequence of taxes and debt $(\{(\tau_t, B_{t+1})\}_{t=0}^{\infty})$ in order to maximize the lifetime utility of the representative household (or a utilitarian social welfare since everyone is the same in this economy), subject to the fiscal-budget constraint given by (4.4), and for a given sequence of interest rates, $\{r_t\}_{t=0}^{\infty}$, determined in international markets. Substituting the competitive-equilibrium solution given by equations (4.6) and (4.7) into (4.3), gives an indirect-utility function of the form,

$$V\left(\{(\tau_t, B_t)\}_{t=0}^{\infty} \mid \{r_t\}_{t=0}^{\infty}\right) = \kappa_V +$$

$$+\sum_{t=0}^{\infty}\beta^{t}\left\{\ln\left(1-\tau_{t}\right)+\theta_{G}\ln\left[B_{t+1}-(1+r_{t})B_{t}+\tau_{t}(1+\gamma)^{t}L\right]\right\} \ , \tag{4.8}$$

where κ_V is a constant that does not affect optimization regarding the determination of the optimal sequence $\{(\tau_t, B_{t+1})\}_{t=0}^{\infty}$. In Appendix 4A we show that first-order conditions $\partial V/\partial\tau_t = 0$ and $\partial V/\partial B_{t+1} = 0$, lead to the price-dependent ($\{r_t\}_{t=0}^{\infty}$) solutions,

$$G_t = \theta_G\left(1-\tau_t\right)Y_t \ , \tag{4.9}$$

$$B_1-\left(1+r_0\right)B_0=\phi\left(\tau_0\right)-L \ , \tag{4.10}$$

and

$$B_t = B_0\prod_{j=0}^{t-1}(1+r_j)+\phi\left(\tau_0\right)\frac{1-\beta^t}{1-\beta}\prod_{j=1}^{t-1}(1+r_j)-$$

$$-\frac{L}{1+\gamma}\left[\left(1+\gamma\right)^{t}+\sum_{s=1}^{t-1}\left(1+\gamma\right)^{s}\prod_{i=s}^{t-1}\left(1+r_i\right)\right] \ , \quad \text{for all } t\geq 2, \tag{4.11}$$

in which

$$\phi\left(\tau_0\right)\equiv\left(1+\theta_G\right)\left(1-\tau_0\right)L \ . \tag{4.12}$$

The coefficient $\phi(\tau_0)$ is still undetermined and therefore equations (4.11) and (4.10) are only the general solution to the system of optimality conditions (shown in Appendix 4A). Solving for $\phi(\tau_0)$ we find the level of τ_0, the level of government consumption in period 0 and the level of outstanding fiscal debt in period 1, B_1 (see equations 4.9 and 4.10). The following section explains how the government chooses τ_0 for a given stream of interest rates.

Fiscal Solvency Considerations and Optimal Government Size

To ensure that external debt will be paid back the fiscal-debt transversality condition must be fulfilled,

$$\lim_{t\to\infty}\frac{B_t}{\prod_{j=0}^{t-1}\left(1+r_j\right)}=0 \ . \tag{4.13}$$

In order to determine the coefficient $\phi(\tau_0)$ we combine the transversality condition given by equation (4.13) with equation (4.11) and we obtain the following equation,

$$\phi\left(\tau_0\right)=\left(1-\beta\right)\left[W_1-\left(1+r_0\right)B_0\right] \ , \tag{4.14}$$

where W_1 is the present value of all future levels of GDP conditional on a given stream of interest rates $\{r_t\}_{t=1}^{\infty}$. In other words, W_1 is the total worth of the private sector of the economy, with[6]

$$W_1 = \lim_{t\to\infty} \frac{\frac{L}{1+\gamma}\left[(1+\gamma)^t + \sum_{s=1}^{t-1}(1+\gamma)^s \prod_{i=s}^{t-1}(1+r_i)\right]}{\prod_{j=1}^{t-1}(1+r_j)} \tag{4.16}$$

Combining equations (4.9) and (4.14) we obtain the optimal government size of fiscal spending as a percentage of GDP in period 0,

$$\frac{G_0}{Y_0} = \frac{\theta_G}{1+\theta_G}(1-\beta)\left[\frac{W_1}{Y_0} - (1+r_0)\frac{B_0}{Y_0}\right] , \tag{4.17}$$

or, more generally, for all t,

$$\frac{G_t}{Y_t} = \frac{\theta_G}{1+\theta_G}(1-\beta)\left(\underbrace{\frac{z_t W_{t+1}}{Y_t}}_{\text{Economy's worth/GDP}} - \underbrace{\frac{B_t}{Y_t}}_{\text{Fiscal debt/GDP}} - \underbrace{r_t\frac{B_t}{Y_t}}_{\text{Fiscal-debt interest/GDP}}\right). \tag{4.18}$$

The equation (4.18) provides insight over the effect of the debt burden and the economy's worth over the optimal size of the government. The term B_t/Y_t or debt burden reveals that future taxes will be employed to pay back the outstanding sovereign debt-GDP ratio, reducing G_t/Y_t in the current period and the term r_tB_t/Y_t shows how the current cost of servicing the debt decreases G_t/Y_t. Finally, the term $z_t\mathbb{W}\left(\{r_s\}_{s=t+1}^{\infty}\right)/Y_t$ contains all future interest rates, $\{r_s\}_{s=t+1}^{\infty}$ and therefore equation (4.16)) means that higher future interest rates decrease the current economy's worth, decreasing G_t/Y_t as well.

The next step is to determine the level of interest rates in any future period. In the following two sections we describe the external creditors' model and determine the interest rates.

[6] The general form of (4.16) is,

$$W_{t+1} = \lim_{k\to\infty} \frac{\frac{L}{1+\gamma}\left[(1+\gamma)^k + \sum_{s=t+1}^{k-1}(1+\gamma)^s \prod_{i=s}^{k-1}(1+r_i)\right]}{\prod_{j=t+1}^{k-1}(1+r_j)} . \tag{4.15}$$

4.2.2 International interest rates and the ability to repay external-sovereign debt

Equation (4.17) allows to deduct a relationship between the size of government consumption, the debt-to-GDP ratio and the economy's wealth. The term B_t/Y_t reveals that future taxes must pay back the outstanding sovereign debt-GDP ratio, which also contributes to reduce G_t/Y_t in the current period. Equation (4.16) indicates that $\partial\mathbb{W}\left(\{r_s\}_{s=t+1}^{\infty}\right)/\partial r_s < 0$ for all $s \geq t+1$ and all $t \in \{0, 1, ...\}$, i.e., higher interest rates in any future period reduce the current economy's worth $\left(z_t\mathbb{W}\left(\{r_{t+1}\}_{s=0}^{\infty}\right)/Y_t\right)$ through increasing the future interest burden for servicing sovereign debt. In turn, such a reduction in economy's worth makes the optimal G_t/Y_t to decrease as well.[7]

4.2.3 The external creditors

Assume that external creditors only hold bonds, and maximize their total life-time utility derived only from consumption. All external-creditor variables will be denoted using a star. Life utility of the external creditors is given by,

$$\sum_{t=0}^{\infty} \beta^t \ln\left(c_t^*\right) \tag{4.19}$$

subject to the budget constraint,

$$B_{t+1}^* = \left(1 + r_t\right) B_t^* - c_t^* \ . \tag{4.20}$$

We assume that the rate of time preference of external creditors, $\left(1-\beta\right)/\beta$, is equal to the rate of time preference of domestic households.

Maximizing the utility function of creditors given by equation (4.19) subject to the budget constraint, (4.20) gives us the optimal consumption of creditors,

$$c_t^* = \left(1-\beta\right)\beta^t \prod_{i=0}^{t}\left(1+r_i\right) B_0^* \ ,$$

from which we can derive the following equation that shows the demand for bonds in period $t+1$,

$$B_1^* = \beta\left(1+r_0\right) B_0^*. \tag{4.21}$$

[7] Note that (4.16) can be written as: $\mathbb{W}\left\{r_s\right\}_{s=t+1}^{\infty} = \left[\prod_{s=t+1}^{\infty}\frac{1}{1+\tilde{r}_s} + 1 + \sum_{s=t+1}^{\infty}\frac{1}{\prod_{j=t+1}^{s}\left(1+\tilde{r}_s\right)}\right]\cdot L,$ where $1+\tilde{r}_s \equiv \frac{1+r_s}{1+\gamma}$.

External creditors' demand for bonds maturing in period 1 depends only on the return of bonds in period 0. This easy algebraic solution for bonds demand is a result of using logarithmic preferences.

Determining interest-rate levels

To determine the interest rates we need to equalize demand and supply of government bonds. The demand for bonds in period 0 maturing in period 1, B_1^*, is given by equation (4.21) and the supply of bonds in period 0 maturing in period 1 is derived from combining equation (4.10) with equation (4.14), we obtain,

$$B_1 = \beta\left(1 + r_0\right) B_0 + \left(1 - \beta\right) W_1 - L. \tag{4.22}$$

Equalizing (4.21) and (4.22) and applying the equilibrium condition $B_1 = B_1^*$ together with $B_0 = B_0^*$, we get,

$$W_1 = \frac{L}{1 - \beta} \ . \tag{4.23}$$

Generalized to all $t \in \{0, 1, ...\}$, equation (4.23) can also be written as,

$$\mathbb{W}\left(\{r_s\}_{s=t+1}^{\infty}\right) = \frac{L}{1 - \beta} \ . \tag{4.24}$$

Notice from equation (4.16) that the sequence $\{W_t\}_{t=1}^{\infty}$ satisfies the recursion,

$$W_{t+1} = \frac{1 + \gamma}{1 + r_{t+1}} W_{t+2} + L. \tag{4.25}$$

And finally substituting (4.24) into (4.25) we obtain the level of international interest rates in steady state denoted by the symbol r^{ss},

$$r_{t+1} = r^{ss} = \frac{1 + \gamma}{\beta} - 1 \quad . \tag{4.26}$$

Equation (4.26) determines the entire sequence of interest rates. The level of interest rates with $\beta\left(1 + r^{ss}\right) / \left(1 + \gamma\right) = 1$, for $r_{t+1} = r^{ss}$ and for all $t \in \{0, 1, ...\}$ allows the domestic economy to have a balanced growth steady state with a constant tax rate, debt-GDP ratio, and public-consumption-GDP ratio throughout the whole horizon.

To determine the debt-GDP ratio in period $t+1$ we can simply combine equations (4.22) and (4.23) to get,

$$\frac{B_1}{Y_1} = \frac{\beta\left(1 + r_0\right)}{1 + \gamma} \frac{B_0}{Y_0}, \tag{4.27}$$

or in general form

$$\frac{B_{t+1}}{Y_{t+1}} = \frac{\beta\,(1+r_t)}{1+\gamma}\frac{B_t}{Y_t}. \tag{4.28}$$

The initial conditions, B_0 and r_0 are given and determined in period $t-1$. In the same fashion in each period t $(B_t,\, r_t)$ are determined in period $t-1$ while the pair $(B_{t+1},\, r_{t+1})$ is determined in period t in the bond market by demand and supply. Assuming r_0 had taken the following values and taking into account equation (4.28),

$$r_0 = \begin{cases} r_{ss}, \text{ the economy starts and remains in steady state } B_{t+1}/Y_{t+1} = B_0/Y_0 \\ -1,\ B_{t+1}/Y_{t+1} = 0, \text{ there is a full default} \\ -1 < r_0 < r_{ss}, \text{ there is a partial default or agreed bailout} \\ r_0 > r_{ss}, \text{ this case is only valid for the economy with rent seeking groups} \end{cases}. \tag{4.29}$$

If the economy starts with an interest rate $r_0 = r^{ss}$, the solution of the model implies that the economy would have a constant level of debt-to-GDP ratio equal to the initial ratio B_0/L, and from (4.28) we can infer that $B_{t+1}/Y_{t+1} = B_0/Y_0$. However, if $r_0 = -1$, the external creditors will not buy any future debt since it would have implied that in period 0 there will be a full default. Again, replacing $r_0 = -1$ in (4.28) we obtain $B_{t+1}/Y_{t+1} = 0$ for all $t \in \{0, 1, ...\}$. If the economy will pay back in period 0 an interest rate $r_0 < r^{ss}$, it means that creditors did not keep their promises and the international investors would have gotten a return lower than the optimal $r^{ss} = \frac{1+\gamma}{\beta} - 1$, and therefore the debt-to-GDP ratio in future periods is lower than the starting ratio B_0/Y_0. We assume that cases $-1 < r_0 < r_{ss}$ or $r_0 = -1$ can be caused by structural problems of rent-seeking groups, therefore more detail on that is explained in the following sections. The case $-1 < r_0 < r_{ss}$ could arise as well because in period 0 the economy finds itself very indebted and does not manage to pay back the promised interest rate $r_0 = r_{ss}$.A bailout might be agreed with the creditors as a solution to keep the economy on the bond markets and not end up in a full default. The purpose of the bailout in this case is to avoid full default or partial default that leads to a permanent lower debt level. Agreeing on a bailout plan may offer the chance to the domestic economy to pay less for the time of the duration of the bailout and then borrow more again than the amount borrowed in the situation of partial default but never so high again than the level of debt that triggered the option of partial default.

The initial conditions of the model then determine the future debt-to-GDP ratios, however the model does not determine which ratios are optimal. However in a later

Section it is shown that certain threshold, also not quantified can lead to increasing interest rates and therefore explosive debt-to-GDP ratios.

4.2.4 Debt-GDP ratio dynamics in the benchmark model

In the previous section we have shown that in international capital-markets equilibrium in the benchmark model the interest rates remain constant over the whole horizon. In order to simplify notation we denote debt-GDP ratio and the public-consumption-GDP ratio in the following form,

$$b_t \equiv \frac{B_t}{Y_t} \quad \text{and} \quad g_t \equiv \frac{G_t}{Y_t} \ , \qquad t = 0, 1, \dots \ .$$

Remember from equations (4.28) that if $r_0 = r_{t+1} = r^{ss} = \frac{1+\gamma}{\beta} - 1$, $t = 0, 1, \dots$, we can conclude that,

$$b_t = b_0 \ , \qquad t = 0, 1, 2, \dots \ . \tag{4.30}$$

Equation (4.30) implies that the debt-GDP ratio stays at the value that it was in period 0. This expression guarantees that the debt-to-GDP ratio remains constant throughout the infinite horizon in the domestic economy.

Balanced growth steady state tax rates are determined from the optimality condition $\tau_t = 1 - [\beta / (1+\gamma)]^t \, \Pi_{j=1}^{t-1} (1 + r_j) \cdot (1 - \tau_0)$ (derived in Appendix 4A), and since $r_{t+1} = r^{ss}$ for all $t \in \{0, 1, 2, \dots\}$ we obtain,

$$\tau_t = \tau_0 \ , \qquad t = 0, 1, \dots \ . \tag{4.31}$$

Additionally, public-consumption-GDP ratio throughout the whole horizon is derived from combining (4.9) and (4.31) implying that

$$g_t = g_0 \ , \qquad t = 0, 1, \dots \ . \tag{4.32}$$

We conclude from equations (4.31) and (4.32) that in the international equilibrium taxes and public-consumption-GDP ratios remain constant over time. We can also determine the following public-consumption-GDP and tax rates in period 0, as follows from equations (4.17) and (4.9),

$$g_0 = \frac{\theta_G}{1+\theta_G} \left[1 - \frac{1-\beta}{\beta} (1+\gamma) \, b_0 \right] \quad \text{and} \quad \tau_0 = 1 - \frac{g_0}{\theta_G} \ . \tag{4.33}$$

4.3 Corruption: Rent-seeking groups and external sovereign debt

In this section we enrich our setup to allow for political economy considerations incorporating rent-seeking groups in the society of the domestic economy which influence the setting of the fiscal budget and, in a form of corruption, deviate resources for their own consumption. We link the ability of rent-seeking groups to consume resources to the "voracity effect" on public spending which leads to higher tax rates, lower provision of public goods and excess borrowing in the long run. The term "voracity effect" was introduced by Lane and Tornell (1996) and Tornell and Lane (1999). Tornell and Lane (1999) argued that if some economies lack strong legal political institutions, they may experience a more than proportional redistribution of fiscal resources after a positive shock to their income, decreasing growth. Their mechanism relies upon rent-seeking groups stealing public resources. We analyze the effect of rent-seeking groups on fiscal policy over the time horizon finding the Markov-Nash equilibrium of the new setup.

4.3.1 Model with rent-seeking groups

We take the benchmark model presented in Section 2 and add to the domestic economy, N rent-seeking groups with power to extract resources from the public budget. There will be N rent-seeking groups indexed by $j \in \{1, ..., N\}$ that in every period $t \in \{0, 1, ...\}$ consume a total rent of size $C^R_{j,t}$. All households participating in a rent-seeking group j are identical within the group for all $j \in \{1, ..., N\}$ and have equal shares of private individual rents, $c^R_{j,t}$. Each rent-seeking group has a population mass μ_j with $\sum_{j=1}^{N} \mu_j < 1$, such that the total rent expropriated by group j in the form of private consumption is,

$$C^R_{j,t} = \mu_j \cdot c^R_{j,t} \ . \tag{4.34}$$

The total rent is distributed in an equal amount between individual members of each rent-seeking group in the form of various in-kind private benefits common to rent-seeking unions, for example, subsidies for gasoline, housing benefits, etc. Therefore, the government budget constraint takes into account the total consumption of the rent seeking groups in the following way,

$$B_{t+1} = (1 + r_t) B_t + G_t + \sum_{j=1}^{N} C_{j,t}^{R} - \tau_t Y_t. \qquad (4.35)$$

For simplicity we assume that taking part on rent-seeking activities is exogenous and costless. Additionally we assume that private consumption of an individual household, c_t, is a different good from the consumption or rent $c_{j,t}^{R}$ of a rent-seeking group member, because the last one is represented in the form of private benefits that are not related to the typical consumer basket. However, we assume that the price per unit of $c_{j,t}^{R}$ is equal to the cost of the consumer basket a priory, for all $j \in \{1, ..., N\}$ and for all $t \in \{0, 1, ...\}$.

We assume that even if rent-seeking groups have to lobby, this is a costless collective action that does not require any individual effort. Due to the fact that a rent-seeker member of group $j \in \{1, ..., N\}$ does not control the level of consumption $c_{j,t}^{R}$ that she obtains from the group through any form of private effort or cost, this individual rent-seeker of group j maximizes her utility function,

$$\sum_{t=0}^{\infty} \beta^t \left[\ln(c_{j,t}) + \theta_l \ln(1 - l_{j,t}) + \theta_G \ln(G_t) + \theta_{R,j} \ln\left(c_{j,t}^{R}\right) \right] , \qquad (4.36)$$

with $\theta_{R,j} > 0$, subject to the budget constraint

$$c_{j,t} = (1 - \tau_t) z_t l_{j,t} . \qquad (4.37)$$

The problem of any non-rent-seeker is to maximize (4.3) subject to (4.5), exactly as in the benchmark model of Section 2. Optimal consumption and labor choices are given by,

$$l_{j,t} = l_t = \frac{1}{1 + \theta_l} = L , \qquad (4.38)$$

while

$$c_{j,t} = c_t = (1 - \tau_t) z_t L . \qquad (4.39)$$

The fact that labor supply is identical across rent seekers and non rent seekers give us the same optimal GDP level, $Y_t = z_t \cdot L$, as in the benchmark model presented in Section 2.

4.3.2 Influence of rent-seeking groups in politics: Symmetric Markov-Nash equilibrium

Policy is now set through rent-seeking groups that compete non-cooperatively between them for rents, while ensuring that they have the support of the broader rest of the society. The level of government spending G_t, the tax rate, τ_t, and the level of debt one period ahead B_{t+1}, are the results of the Nash equilibrium of a dynamic game among rent-seeking groups, which also determines its rents in each period. We do not model explicit majority voting on these policy variables, assuming that all existing rent-seeking groups actively and simultaneously influence policy making in each period. We abstract as well of alternating political parties with rent-seeking groups in power because of the increasing complication of the derivation of equilibrium without adding additional insights to the model.

Instead, we have all rent-seeking groups acting simultaneously which illustrates the mechanics of a commons problem adequately: a rent-seeking group tends to expropriate as much rent as possible before other groups do so as well.

Government policy in an economy with rent-seeking groups is determined by the Markovian Nash equilibrium of the rent-seeking game among these groups. Whenever there is a common-pool resource as a state variable (debt in our model), each player decides about an exploitation variable, but at the same time each player also manages the resource and cares about its preservation, like in for example, the resource-exploitation setup in Koulovatianos and Mirman (2007, Sections 2.2 and 2.3, pp. 203-204) in which next period's capital stock is jointly determined after exploitation and strategic supply. The existence of a common-pool resource implies that the only source of disagreement among players would be heterogeneity in the structural features of each player (usually preferences) about the management of the resource (which is next-period's debt stock and also the current tax rates). The only space for such heterogeneity would be different θ_R among groups. Still, in this case one would be able to find an equilibrium with rents representing different shares of the GDP, but with agreement among players about next-period's debt stock and also the current tax rates, even if one needs to use Kuhn-Tucker conditions in order to derive such an equilibrium.

Nevertheless, complications such as heterogeneous preferences among rent-seeking groups are beyond the scope of this thesis. We therefore focus on a symmetric equilibrium which conveys sufficiently the commons problem that arises if rent seeking groups do not cooperate.

Some requirements should be met by rent-seeking groups in order to gain society support, that is, each rent-seeking group $j \in \{1, ..., N\}$ should maximize a convex combination of, (i) the sum of individual utilities of non-rent seekers and, (ii) the group's utility derived by the stream of the group's consumption $\left\{C_{j,t}^{R}\right\}_{t=0}^{\infty}$. So, rent-seeking group $j \in \{1, ..., N\}$ chooses the stream $\left\{\left(C_{j,t}^{R}, \tau_t, B_{t+1}\right)\right\}_{t=0}^{\infty}$ that maximizes the utility function

$$\hat{V}^{j}\left(\left\{\left(C_{j,t}^{R}, \tau_t, B_t\right)\right\}_{t=0}^{\infty} \mid \left\{\mathbb{C}^{i,R}\right\}_{\substack{i=1 \\ i\neq j}}^{N}, \{r_s\}_{s=t+1}^{\infty}\right) = \kappa_{V_j} +$$

$$+\sum_{t=0}^{\infty}\beta^{t}\left[\ln\left(1-\tau_t\right)+\theta_G \ln\left(G_t\right)+\theta_{R,j}\ln\left(C_{j,t}^{R}\right)\right] \tag{4.40}$$

subject to,

$$G_t = B_{t+1} - (1+r_t)B_t - \sum_{\substack{i=1 \\ i\neq j}}^{N}\mathbb{C}^{i,R}\left(B_t, Y_t \mid \{r_s\}_{s=t}^{\infty}\right) - C_{j,t}^{R} + \tau_t Y_t \ , \tag{4.41}$$

and

$$Y_t = z_t L = (1+\gamma)^t L \ , \tag{4.42}$$

in which κ_{V_j} is a constant which does not affect optimization, $\theta_{R,j} > 0$ is the weight placed on utility derived from the group's extracted rent and notice that $\ln(1-\tau_t) + \theta_G \ln(G_t)$ is the variable component of non-rent-seeker momentary indirect utility. The set $\left\{\mathbb{C}^{i,R}\right\}_{i\neq j}$ is the Markov-Nash strategies of type $C_{i,t}^{R} = \mathbb{C}^{i,R}\left(B_t, z_t \mid \{r_s\}_{s=t}^{\infty}\right)$ by all other rent-seeking groups. We assume symmetry in political influence, it means that all rent-seeking groups have the same power to extract rents and all groups have the same size, namely:

$$\theta_{R,j} = \theta_R \quad \text{and} \quad \mu_j = \mu \quad \text{for all } j \in \{1, ..., N\} \ . \tag{4.43}$$

In the following we explain in detail the concept of domestic political equilibrium with rent-seeking groups.

Definition 1 *Given a stream of interest rates, $\{r_t\}_{t=0}^{\infty}$, a Markov-Perfect Nash Political Equilibrium (MPNPE) is a set of strategies, $\left\{\mathbb{C}^{i,R}\right\}_{i=1}^{N}$ of the form $C_{i,t}^{R} = \mathbb{C}^{i,R}\left(B_t, z_t \mid \{r_s\}_{s=t}^{\infty}\right)$ and a set of policy decision rules $\{\mathbb{T}, \mathbb{B}\}$ of the form $\tau_t = \mathbb{T}\left(B_t, z_t \mid \{r_s\}_{s=t}^{\infty}\right)$ and $B_{t+1} = \mathbb{B}\left(B_t, z_t \mid \{r_s\}_{s=t}^{\infty}\right)$, such that each and every rent seeking group $j \in \{1, ..., N\}$ maximizes (4.40) subject to (4.35), (4.42), $\{\mathbb{T}, \mathbb{B}\}$, and $\left\{\mathbb{C}^{i,R}\right\}_{i\neq j}$.*

Markov-perfect games can be expressed in a recursive format through the use of Bellman equations.[8] Proposition 1 provides a MPNPE in linear strategies.

Proposition 1 *Given a stream of interest rates, $\{r_t\}_{t=0}^{\infty}$, there is a symmetric MPNPE equilibrium given by,*

$$\mathbb{C}^{i,R}\left(B_t, z_t \mid \{r_s\}_{s=t}^{\infty}\right) = \mathbb{C}^{R}\left(B_t, z_t \mid \{r_s\}_{s=t}^{\infty}\right) = \xi_R \cdot \left[z_t \mathbb{W}\left(\{r_s\}_{s=t}^{\infty}\right) - B_t\right] \tag{4.44}$$

for all $i \in \{1, ..., N\}$, in which W_t is given by (4.15) and,

$$\xi_R = \frac{(1-\beta)\,\theta_R}{1+\theta_G+\theta_R+(N-1)(1-\beta)\,\theta_R}\ , \tag{4.45}$$

also

$$B_{t+1} = \mathbb{B}\left(B_t, z_t \mid \{r_s\}_{s=t}^{\infty}\right) = \beta_N\,(1+r_t)\,B_t + (1-\beta_N)\,z_t W_{t+1} - Y_t\ , \tag{4.46}$$

and

$$\beta_N = \frac{1+\theta_G+\theta_R}{1+\theta_G+\theta_R+(N-1)(1-\beta)\,\theta_R}\beta\ , \tag{4.47}$$

while

$$\tau_t = \mathbb{T}\left(B_t, z_t \mid \{r_s\}_{s=t}^{\infty}\right) = 1 - \frac{1}{\theta_G}\frac{G_t}{Y_t}\ , \tag{4.48}$$

in which

$$\frac{G_t}{Y_t} = \frac{\theta_G}{1+\theta_G+N\theta_R}\,(1-\beta_N)\left[\frac{z_t W_{t+1}}{Y_t} - (1+r_t)\,\frac{B_t}{Y_t}\right]\ . \tag{4.49}$$

Proof See Appendix 4B.

In what follows we explain the implications of Proposition 1 and describe how the governments can enter into sovereign traps due to the constant increment of international interest rates.

4.3.3 Determination of international interest rates and debt traps

The outstanding result of Proposition 1 is the introduction of a new discount factor, β_N, that reflects the degree of patience of the domestic economy in the presence

[8] In order to formulate Bellman equations, the value function of each player would be of the form $V^j\left(B_t, z_t \mid \left\{\mathbb{C}^{i,R}\right\}_{i\neq j}, \mathbb{T}, \mathbb{B}, \{r_s\}_{s=t}^{\infty}\right)$. In the Proof of Proposition 1, which appears in Appendix B, we use Lagrangians and not Bellman equations.

of corrupt politicians. The discount factor β_N is related to β through equation (4.47). This equation shows that when $N \geq 2$, then $\beta_N < \beta$, meaning that the domestic economy is always more impatient than the external creditors due to the voracity effect exerted by the rent-seeking groups. This voracity effect originates a need of excessive bond supply compared to the case when the domestic economy is free of corruption, or when there is just one rent-seeking group, $N = 1$. In the next section we present the differences between the case of $N = 1$ from the one of $N \geq 2$, and demonstrate how the voracity effect leads to high interest rates and explosive debt-GDP ratios as a consequence of excessive sovereign bonds issuance. Strulik (2012) showed that an equilibrium with voracity effect can be dominated by an equilibrium without voracity effect, and therefore the voracity effect disappears under certain conditions. This result is not relevant to our analysis since there is no informal economy in our model.

One big "mob"

When there is just one rent-seeking group, $N = 1$ and the discount factor $\beta_N = \beta$, making equal demand for bonds given by equation (4.21) to domestic supply of bonds given by (4.46) but with β_N instead of β that in this case are the same value, we get again equation (4.24) and consequently to (4.26), which leads us to $r_{t+1} = r^{ss}$ for all $t \in \{0, 1, ...\}$. Again we obtain equation (4.27) of Section 2, namely that, $b_t = b_0$ for all $t \in \{0, 1, ...\}$. However, we get different expressions for g_0 and τ_0 compared to the ones in Section 2; equations (4.49) and (4.48) imply that

$$g_0 = \frac{\theta_G}{1 + \theta_G + \boldsymbol{\theta_R}} \left[1 - \frac{1-\beta}{\beta} (1+\gamma)\, b_0\right] \quad \text{and} \quad \tau_0 = 1 - \frac{g_0}{\theta_G} \,, \tag{4.50}$$

therefore $g_t = g_0$ and $\tau_t = \tau_0$ for all for all $t \in \{0, 1, ...\}$. Equation (4.50) implies that one rent-seeking group makes the steady-state public consumption as a share of GDP lower compared to the one in the benchmark model and the steady state tax rate becomes higher.

The existence of one rent-seeking group may avoid fiscal impatience but however, the economy gets a lower provision of public goods, higher tax rates, making the country more vulnerable to the situation of high initial levels of external-sovereign debt.

More than one non-cooperative rent-seeking groups

In the case with more than one rent-seeking group, i.e, with $N \geq 2$ the solution is not so straightforward, therefore all the procedure is on Appendix 4B and the main results of the dynamics of interest rates are summarized in Proposition 2.

Proposition 2 *If* $N \geq 2$*, then*

$$b_{t+1} = \frac{\beta(1+r_t)}{1+\gamma} b_t \ , \quad \text{for all } t \in \{0, 1, ...\} \ , \tag{4.51}$$

$$r_t > r^{ss} \text{ and } \ b_{t+1} > b_t \ , \ \text{ for all } t \in \{1, 2, ...\} \ , \tag{4.52}$$

with

$$\lim_{t\to\infty} r_t = r^{ss} \text{ and } \lim_{t\to\infty} b_t = \frac{1}{(1-\beta)(1+r^{ss})} = \frac{\beta}{(1-\beta)(1+\gamma)}, \tag{4.53}$$

and

$$\frac{1-\beta}{\beta}(\beta - \beta_N) b_t + \frac{\beta_N}{1+\gamma} = \frac{1}{1+r_t} \ , \ \text{ for all } t \in \{1, 2, ...\} \ , \tag{4.54}$$

while (b_0, r_0) *satisfy,*

$$(1+r_0) b_0 = \frac{1+\gamma}{\beta} b_1 < \frac{1}{1-\beta} \ .$$

Proof See Appendix 4B.

The main result provided by Proposition 2 is that when the domestic economy remains forever in free markets, the rent-seeking groups continue increasing the debt-GDP perpetually, the limiting level of the debt-GDP ratio corresponds to exhausting the domestic economy's total wealth-to-GDP ratio, when the latter is evaluated at the asymptotic level of interest rates, r^{ss}, i.e. $b_t \to 1/[(1-\beta)(1+r^{ss})]$. This result holds for all initial debt-GDP levels b_0 with $b_0 < 1/[(1-\beta)(1+r^{ss})]$.

The existence of more than one non-cooperative rent-seeking group and its influence in setting fiscal policy generates a "tragedy of the commons" problem, that is, the overexploitation of public resources. The propensity to overexploit creates collective impatience reflected in the domestic government's discount factor β_N. Because $\beta_N < \beta$ when $N \geq 2$, the effect of this impatience is that the interest rates remain above the steady state value for a while and then converge to the steady state value on the infinite horizon. As an obvious consequence, high interest rates lead to a perpetual increment of the debt-GDP ratio. The tax rates however do not depend directly from the number of rent-seeking groups as shown by equation (4.48) Nevertheless, they depend on the size of government, which as shown by equation

(4.49) a higher number of rent seeking groups implies lower government spending share on output and higher tax rates.

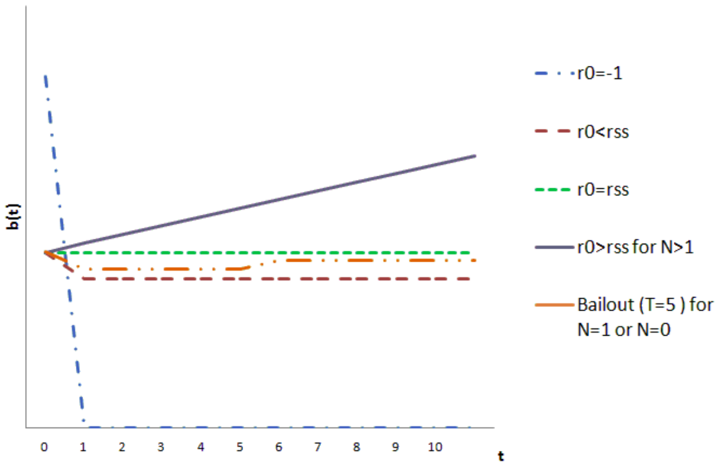
Debt-GDP-ratio dynamics in the model with rent-seeking groups

In the case of corruption and more than one rent-seeking groups arising at the beginning of the economy in period 0, the interest rate agreed in period $t-1$ might be altered and the promises made before (such as $r_0 = r_{ss}$) may not be kept. It is assumed that in period -1 the economy was free of corruption and that at the bond market supply and demand agreed on $r_0 = r_{ss}$. In period 0 corruption suddenly arises and there can be the following possibilities:

- There is a partial default, meaning that the domestic economy (borrowers) do not keep their promise to pay $r_0 = r_{ss}$, and instead actually pay $r_0 < r_{ss}$, therefore the lenders receive in period 0 $(1 + r_0)B_0 < (1 + r_{ss})B_0$. Therefore, $B_{t+1}/Y_{t+1} < B_0/Y_0$. An agreement between borrowers and lenders on a partial default cannot occur if the number of rent-seeking groups N is bigger than 1. In order to achieve a partial-default agreement with $-1 < r_0 < r_{ss}$ borrowers and lenders must agree simultaneously (in period 0) to reduce the number of rent-seeking groups to $N = 1$, or to $N = 0$. But in a real situation this cannot be achieved in period 0. Therefore the efficacy of bailouts would be reflected on given the domestic economy the time to implement reforms that aim to eliminate rent seeking groups. The bailout in this case would have another definition different to the one proposed in this paper, i.e. loans with temporary lower than markets interest rates, but rather some monitoring from an international institution to impose fiscal austerity that may lead rent-seeking groups to abandon their selfish purpose.
- There is a full default. This is the extreme case of borrowers paying $r_0 = -1$. Then, the domestic economy will never borrow from the international bond market again, but it will also have no debt burden, since replacing $r_0 = -1$ in equation (4.28) leads to $B_{t+1}/Y_{t+1} = 0$.
- The case $r_0 > r_{ss}$ arises only in the case of multi-group corruption because the rent-seeking groups want to borrow more that the B_1 obtained at r_{ss}. In order for the domestic borrowers to convince external creditors to roll over the previous outstanding debt B_0 and to lend more in the future with $B_1 > B_0$,

creditors will demand increasing interest rates in further periods $r_t > r_{ss}$ for all t as shown in Proposition 2: $B_{t+1}/Y_{t+1} > B_0/Y_0$. Figure 4.4 summarizes the situation described above for a random tuple (B_0, r_0) agreed between borrowers and lenders in period $t-1$.

Figure 4.4: Debt dynamics for a random debt-to-GDP ratio.



4.4 Debt-GDP thresholds and sovereign bailouts

The study of Manasse and Roubini (2009) identified explicit external debt-to-GDP thresholds using data for emerging economies that are associated with default. More specifically, using a novel methodology, Manasse and Roubini (2009) determined several important indicators that predicted future sovereign defaults. Most notably their analysis found the external debt-to-GDP ratio (exceeding 50%) as one of the most important predictors of the majority of sovereign defaults episodes in their sample.

In 2009, Greece and Portugal reached sovereign external-debt-to-GDP ratios of 91.4% and 59.6%[9] respectively. Those levels seemed to have triggered high country-specific interest rates putting pressure on the newly issued debt that brought the need of bailouts in 2010 and 2011.

Our model can isolate the effect of international-market pressures on the cost of financing new debt and their effect on optimal sovereign debt policy. While a more

[9]See Figure 4.1.

rich theory explaining bond spreads should include modeling of default risk, we focus on a deterministic environment avoiding complexities such as the determination of the optimal maturity structure of bonds, or the incorporation of expectations asymmetries in a dynamic game between the domestic economy and external creditors. Avoiding this complexities allow us to evaluate bailouts by paper and pencil methods.

A trap of high international sovereign interest rates

During the analysis of this section we use the benchmark model presented in Section 2, therefore we leave aside the rent-seeking groups as we will retake them in a later extension which examines bailouts in the presence of corruption. We start with the scenario in which a country, in period 0, has a high outstanding external-debt-GDP ratio as a result of, for example, a natural disaster in its immediately recent past. External creditors observe the initial debt-GDP ratio, b_0, of the domestic economy and compare it to a threshold κ, if $b_0 > \kappa$, then the foreign lenders think of this country as being constrained. It means that external creditors consider that even if interest rates from period 1 onwards are equal to r^{ss}, the difference between taxes and spending ($\tau_t - g_t = r^{ss} b_t = r^{ss} b_0$) for all $t \in \{0, 1, 2, ...\}$, is making both private and public consumption low, creating a decrease of the utility of the citizens of the debtor country. Those circumstances make the external creditors believe that there is social pressure for partial default, in this case, a haircut equal to $(b_0 - \bar{\kappa})$, as a way to get back to a socially acceptable debt-GDP ratio, κ.[10]

In such scenario the external-creditors demand of domestic bonds should take into account the anticipation of a haircut for any anticipated stream of interest rates from period 0 and on, $\{r_t\}_{t=0}^{\infty}$, as a result equation (4.21) implies that,

$$B_1^* = \beta (1 + r_0) \lambda_0 B_0^* \ , \tag{4.55}$$

in which λ is the fraction of outstanding debt after the haircut, given by,

$$\lambda = \begin{cases} 1 - (b_0 - \bar{\kappa}) & \text{if} \quad b_0 \geq \kappa \\ 1 & \text{else} \end{cases} . \tag{4.56}$$

with $\bar{\kappa} \geq \kappa$. We also assume that $b_0 - \bar{\kappa} < 1$. External-creditor's demand for bonds that incorporates the anticipation of haircuts is now denoted by $\widetilde{B}_1^*$. Domestic supply for bonds is still given by $B_1 = \beta (1 + r_0) B_0 + (1 - \beta) W_1 - L$.

[10] Notice that equation (4.17) implies lower public goods provision with increases in the debt to GDP ratio.

The anticipation of a haircut and the presence of λ must be also reflected on the price of the bonds and even if the domestic government does not intend to make a haircut the interest rate in period 1 must increase. If the anticipated interest rate was $r_0 = r^{ss}$, the price of 1-period zero-coupon bonds, P^{ss}, has to vary according to $\widetilde{P}/P^{ss} = \lambda$, in which $\widetilde{P}$ is the price that incorporates the anticipation of a haircut, while P^{ss} corresponds to an intrinsic bond return equal to r^{ss}. Therefore $P^{ss} = 1/\left(1 + r^{ss}\right)$ and $\widetilde{P} = 1/\left(1 + \widetilde{r}_0\right)$ which combined give us the new gross-effective interest rate,

$$1 + \widetilde{r}_0 = \frac{1 + r^{ss}}{\lambda} \; . \tag{4.57}$$

In the case that the domestic government starts with $b_0 > \kappa$ and mistakenly foresees that $\{r_t = r^{ss}\}_{t=1}^{\infty}$, then equation (4.23) implies that $W_1 = L/\left(1 - \beta\right)$. Such an error may be due to an asymmetry in disaster-risk expectations between the domestic government and external creditors. An example of an early study that models informational asymmetries between governments and external creditors explicitly is the one of Cole et al. (1995).

Since the strategies are Markovian, each player's strategy depends only on the current value of the relevant state variable that summarize the history of the game, therefore the past actions have no influence. This a characteristic of memory less closed-loop Nash Equilibria (see Basar and Olsder 1982 Ch.6). Additionally, any deviation from the equilibrium path is rule-out and the government finds optimal to follow the equilibrium strategies at any point in time.

At the same time, equation (4.22) implies that $B_1 = \beta\left(1 + \widetilde{r}_0\right) B_0$. So, $b_1 = \beta\left(1 + \widetilde{r}_0\right) b_0/\left(1 + \gamma\right)$, and after this equation is combined with (4.57) and (4.26), it is,

$$b_1 = \frac{1}{\lambda} b_0 > b_0 > \kappa \; . \tag{4.58}$$

Equation (4.58) shows that the domestic government worsens its problem in period 1 since the debt-GDP ratio continues increasing further from the threshold κ, and additionally $\widetilde{r}_1$, is expected to rise to an even higher value than $\widetilde{r}_0$ due to the fact that external creditors fear a bigger haircut of size $b_1 - \kappa > b_0 - \kappa$. This sequence of events demonstrates in a simple way the trap of high international sovereign interest rates which trigger default from the domestic government within a short period of time in a similar way to which the literature has called self-fulfilling sovereign default. Cole and Kehoe (2000) formally illustrated how self-fulfilling default expectations can lead to sovereign debt crises. The crisis ensues when creditors lose confidence in the government's ability to roll over debt. In their model market beliefs can trigger a default when a country's fundamentals such as the level, maturity structure of government debt and capital stock are within a crisis zone. One of the important insights generated by their model is the role played by the maturity structure of debt, with longer maturities shrinking the region of self-fulfilling defaults.

A solution for the countries in a monetary union trapped with high international interest rates is the one of sovereign-debt bailouts in order to avoid the risk of bank runs and instability of the financial system. The following section studies whether such bailouts have the potential to be successful given an initial level of external sovereign debt and subject to bailout-plan duration and to commonly agreed interest rates.

4.4.1 Sovereign bailout plans

The main objective of a bailout plan or rescue package is to help a country to reach a lower sovereign debt-GDP ratio, below a threshold level κ, which should bring the country to free international capital markets again and be able to borrow again. Countries that might end up in a high-interest-rate trap as the one detailed depicted in the previous section can benefit from sovereign bailout plans that offer financing with controlled interest rates over a certain period of time. A very controversial aspect of bailouts is the ability of the country to pay back the financial aid since normally it requires fiscal discipline from the country that is receiving help. Sovereign states are independent democracies and set their policy in an endogenous way making the evaluation of success of bailout plans a highly demanding task. Our model overcomes this difficulty and can be used as a tool to carry out further analysis of the possible success of a bailout plan. Our model abstracts from the endogeneity of growth rates, and especially from the connection between tax burdens and growth prospects. These elements may not be the most crucial in evaluating the success of potential bailouts. Nevertheless, the explicit modeling of growth rates and default risk are useful extensions for future work.

Let us start by considering the duration of a bailout plan to be T years with a fixed and controlled interest rate, r_b. The bailout plan is effective until period $T-1$, starting from period 0, and its goal is to achieve

$$b_T \leq \kappa \; , \tag{4.59}$$

to allow the country returning to free capital markets from period T and on. The parameters to specify in the bailout plan are then T and r_b (in practice, the IMF-EU plan for eurozone countries specifies a more explicit plan of monitoring the progress of policy-making in the bailed-out country).

Appendix C shows that for all bailout periods, $t \in \{0, ..., T-1\}$, the law of motion for the debt-GDP ratio is

$$b_{t+1} = \alpha b_t + \underbrace{(1-\alpha)\left[1-\left(\frac{1+\gamma}{1+r_b}\right)^{T-t}\right]\frac{1}{r_b-\gamma}}_{\kappa_b} , \tag{4.60}$$

in which,

$$\alpha \equiv \frac{\beta\left(1+r_b\right)}{1+\gamma} . \tag{4.61}$$

We want to find first the value of the debt-GDP ratio where $b_t = b_{t+1}$ and it is given by equation (4.60) where for each $t \in \{0, ..., T-1\}$, the intersection of the line implied by (4.60) with the 45-degree line in the (b_t, b_{t+1}) space is,

$$\bar{b}_t = \left[1-\left(\frac{1+\gamma}{1+r_b}\right)^{T-t}\right]\frac{1}{r_b-\gamma} . \tag{4.62}$$

Given equation (4.62), we make a distinction whether r_b is greater than r^{ss} or not. In the case $r_b > r^{ss}$, the bailout plan must guarantee that $b_t < \bar{b}_t$ for all $t \in \{0, ..., T-1\}$ such that the domestic government's optimal debt-GDP ratio, b_t, decreases over time. A geometric analysis as depicted in Figure 4.5 demonstrates that, in case $r_b > r^{ss}$, the requirement for a decreasing optimal debt-GDP ratio over time is having

$$b_t < \bar{b}_t , \tag{4.63}$$

for all $t \in \{0, ..., T-1\}$.

In the case of providing a low interest rate, $\gamma < r_b < r^{ss}$, $b_t < \bar{b}_t$ as depicted in Figure 4.6 it will eventually lead to a final success, conditional on duration, T, not being too long such that there will be no incentives for issuing more debt, given low costs of sovereign borrowing. In Appendix C we demonstrate that the explicit dynamics of b_t under the bailout plan are given by,

$$b_t = \frac{1}{r_b-\gamma} - \alpha^t\left(\frac{1}{r_b-\gamma} - b_0\right) - \frac{1-\alpha}{r_b-\gamma}\left(\frac{1+\gamma}{1+r_b}\right)^{T-t+1}\frac{1-\beta^t}{1-\beta} , \tag{4.64}$$

for all $t \in \{0, ..., T-1\}$, which is the solution of equation (4.60).

The duration of the bailout is relevant and to make clear that it should not last so long with $\gamma < r_b < r^{ss}$, notice that equation (4.64) can be re-written as,

$$b_t = \frac{1}{r_b - \gamma} - \frac{(1-\alpha)\left(\frac{1+\gamma}{1+r_b}\right)^T}{(r_b - \gamma)(1-\beta)}\left(\frac{1+r_b}{1+\gamma}\right)^t + \left\{b_0 - \frac{\frac{1+r_b}{1+\gamma} - \beta}{1-\beta}\left(\frac{1+\gamma}{1+r_b}\right)^T\right\}\alpha^t \ . \tag{4.65}$$

When T is too big, the factor multiplying α^t in the last term of equation (4.65) can become negative and since $\alpha < 1$ when $\gamma < r_b < r^{ss}$, b_t may start increasing after some time. Avoiding this possibility in the bailout plan with low interest rates $\gamma < r_b < r^{ss}$ makes this solution very effective since there is a faster transition of b_t to a level below κ, compared to the case of $r_b > r^{ss}$.

If the bailout plan reaches the target of $b_T \leq \kappa$, then, from period T and on, the country returns to international capital markets with interest rate r^{ss}, as shown in Section 2, and $b_t = b_T \leq \kappa$ for all $t \geq T$. Proposition 1 describes the features of successful bailout plans.

Proposition 3 *Given an initial sovereign-debt-GDP ratio $b_0 > \kappa$, a bailout plan characterized by a combination (r_b, T) with $r_b > \gamma$, is successful if T is the smallest integer such that*

$$\frac{1}{r_b - \gamma} - \alpha^T\left(\frac{1}{r_b - \gamma} - b_0\right) - \frac{1-\alpha}{r_b - \gamma}\frac{1+\gamma}{1+r_b}\frac{1-\beta^T}{1-\beta} \leq \kappa \ , \tag{4.66}$$

and in the special case of $r_b > r^{ss}$, the tuple (r_b, T) should also satisfy

$$\frac{1}{r_b - \gamma} - b_0 > \left(\frac{1+\gamma}{1+r_b}\right)^T \frac{1}{(1-\beta)(1+r_b)}\left[\frac{1}{\beta^T} - \frac{\beta(1+r_b) - (1+\gamma)}{r_b - \gamma}\right] \ . \tag{4.67}$$

Proof See Appendix 4C. □

Proposition 3 shows that designing a bailout plan with a choice of (r_b, T) where $r_b > r^{ss}$ needs to take into account very specific constraints. However the most important constraint we wanted to emphasize through our model is the political feasibility of a bailout plan in countries that belong to a monetary union. In the following section we study bailout plans in the presence of corruption through rent-seeking groups.

Figure 4.5: Bailout Case 1: rb>rss

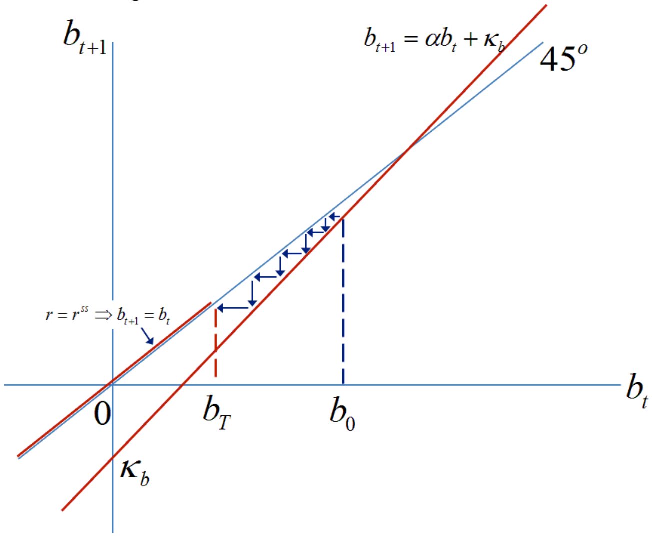
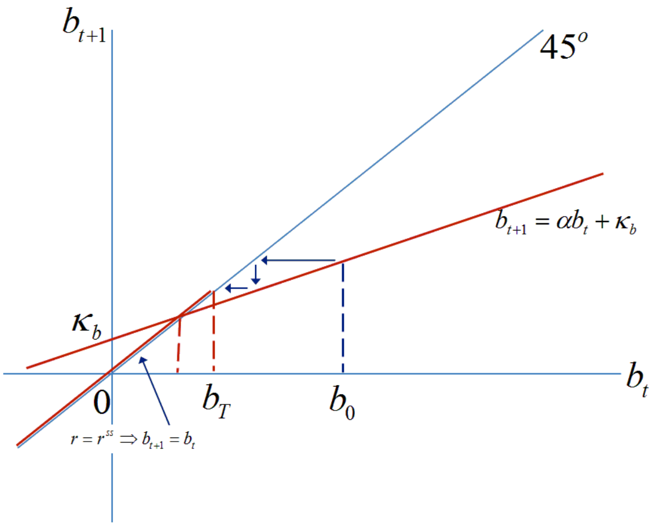


Figure 4.6: Bailout Case 2: rb<rss



4.4.2 Corruption and bailouts in the eurozone

Fiscal imbalances may have as a cause corruption and the recent sovereign debt crisis has awakened the concern about corruption particularly in periphery EU countries. Eichengreen (2010) found positive correlation between Intra-Euro-Area Imbalances and corruption for the period 1999-2009. Country studies as the "IMF country reports" International Monetary Fund (2011a, b, d) for Greece, Italy, and Portugal, make explicit reference to the need for reducing rent-seeking activities in the first three countries, while the country report for Ireland (International Monetary

Fund (2011c)) focuses on Ireland's banking problems. Indeed, corruption indicators constructed from various institutions (World Bank, OECD and the Transparency International Organization) consistently rank the corruption levels in Greece, Italy and Portugal as the highest of the euro area countries. Some studies, as the one from Arghyrou (2010) reported the three-year average (2007, 2008 and 2009) of the Corruption Perception Index from Transparency International and ranks Greece as 50th and Italy as 45th in a sample of 115 countries (with lower ranking equivalent to higher corruption), near the ranking of many developing countries with weak political and social institutions. Spain and Portugal rank closely behind. All three indices of corruption correlate very strongly with each other and aim at measuring the overall extent of corruption in the public sector (e.g. bribes, kickbacks, public funds appropriation, etc.). It is also worth noting that credit ratings agencies (e.g. Standard and Poor's), take into account corruption as a factor in the political dimension that determines sovereign ratings and consequently affects country specific interest rates.[11]

Interest rates have been the basis for analyzing sovereign-debt problems, therefore in Figure 4.2 we related interest rates and corruption. For the period before the introduction of the Euro and up to recent months, it comprehends the period (1993-2011), we plot the 10-year-maturity yield in the countries of the south periphery of the eurozone, in Ireland and in Germany. Before the introduction of the euro in 1999, Greece, Italy, Portugal and Spain had high spreads compared to the yields of German bonds, coincidentally the mentioned countries appear to have a high ranking in corruption. The 10-year-maturity yields in Figure 4.2 are nominal. For the pre-Euro period in the 1990s, even if we had controlled for inflation expectations, (a) the picture would have been similar, and (b) in that case we would have lost the inflation premium, which still reflects a credibility concern. One reason for that credibility concern is corruption, but other concerns must be at play as well (such as growth prospects of particular economies, etc.).

Interestingly Ireland has the lowest corruption ranking and its yields in the 1990s are close to that of the German bonds. The high spreads of Ireland start after the fall of 2008. Ireland's sovereign debt is increasing uncontrollably due to the problems of its banking sector.[12] Ireland is the example that according to our model, is trapped

[11] According to the methodology described on S&P's website for sovereings rating, Sovereign Government Rating Methodology and Assumptions, June 30 2011.

[12] See IMF report, International Monetary Fund (2011c, p. 21), in which it is mentioned: "Debt (in Ireland) is projected to peak at 120 percent of GDP in 2013, and to then decline gradually."

due to its debt burden and not because of rent-seeking. A bailout plan based on our analysis above in a country as Ireland is most likely to succeed, for Greece and Portugal we should look at the sections analyzing bailouts and corruption.

"One big mob" and bailouts

The existence of even one rent-seeking group decreases the welfare of non-rent seekers, it is easy to see substituting equation (4.50) into the value function of a non-rent seeker, as there is a distortion caused by the resources spent by the rent-seeking group, i.e., the presence of parameter $\boldsymbol{\theta}_{R.}$. Nevertheless, rent-seeking households have higher welfare compared to non-rent seekers because of the additional consumption of the rent.

If a country's sovereign external debt is high enough to create concerns among external creditors about a haircut as stated by equation (4.56), the bailout plan described in Section 3.4 may be implemented. If there is just one rent-seeking group or one big "mob" ($N = 1$), a bailout-plan analysis is the same as in Section 3. However depending in the degree of disutility that can generate social dissatisfaction of non rent seekers, the threshold κ can be required to be a lower value compared to that of the benchmark model.

In International Monetary Fund (2011b, p. 28, footnote 5), it is mentioned: "Italy ranks 67th on the Transparency International Corruption Perception Index 2010, among the lowest in the EU. The Council of Europe's Group of States against Corruption in its last evaluation of Italy (2008) noted that "...corruption in public administration is widely diffused" and recognized the "connections (that) exist between corruption and organized crime." " According to our analysis, if Italy enters a bailout plan, the possibility that its rent-seeking groups play a cooperative strategy may give to Italy an advantage. For Portugal, in International Monetary Fund (2011d, p. 66), among the stated objectives is mentioned: "Ensure a level playing field and minimize rent-seeking behavior by strengthening competition and sectoral regulators; eliminate special rights of the state in private companies (golden shares); [...]." Perhaps the request by the European Commission for consensus among political parties in Portugal aims at placing Portugal in the category of one-rent-seeking group related to politics at a first stage.

More than one rent-seeking groups and bailouts

When there is more than one rent-seeking group, as we have shown in Section 4.3.3, the debt-GDP ratio always increases over time.[13] Consequently, there is no bailout plan able to bring the country to a desired debt-GDP ratio target asymptotically, unless the bailout plan controls interest rates forever. Under these circumstances, the main target of a bailout plan in a corrupt economy should be to eradicate rent-seeking groups.

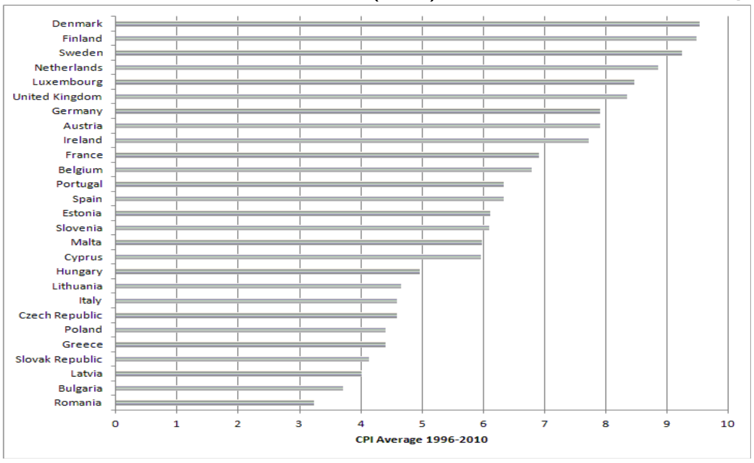
Countries that suffer from corruption with $N \geq 2$, have a structural problem when they are bailed-out, then if for example, rent-seeking groups are present in the form of a strong bipartisan system, then the monetary union should require guarantees of transparency by the two dominant political parties of those countries; hopefully those transparency requirements towards the external lenders may eliminate rent-seeking groups and then the bailout could be implemented following the features for getting the appropriate tuple (r_b, T) as presented in Proposition 3.

4.4.3 Corruption in the European Union states

A recent report from Transparency International has warned on the risk of corruption in Europe. Particularly, an assessment of 25 European countries in 2011 revealed that southern Europe countries, i.e.,Greece, Italy, Portugal and Spain, showed a high degree of inefficiency in the public sector and corrupt behavior that is not properly controlled or punished. Corruption has been so far considered as a structural problem in developing countries, however this new results from Transparency International provide evidence that some that EU countries have high levels of corruption even compared to some Latin-American or African countries. Since the literature on corruption and growth has not agreed neither on the impact of causality or the relationship between those two variables, this Section presents scatterplots on the relationship between the CPI and various macroeconomic indicators in EU countries. Correlations are shown in Figures 4.8 to 4.10. These data facts provide some examples that could relate corruption to the behaviour of important macroeconomic variables like inflation, gross national saving and current account deficits. These point out to consider corruption as a key structural problem in southern Europe. More accurate statements require a rigorous econometric analysis. Therefore, we

[13] In particular we have shown that $b_t \to 1/\left[(1-\beta)\left(1+r^{ss}\right)\right]$, i.e. b_t asymptotically converges to exhausting the domestic economy's total wealth-to-GDP ratio, with the latter being evaluated at the asymptotic level of interest rates, r^{ss}.

Figure 4.7: Corruption Perception Index (CPI) in EU members average 1995-2010.



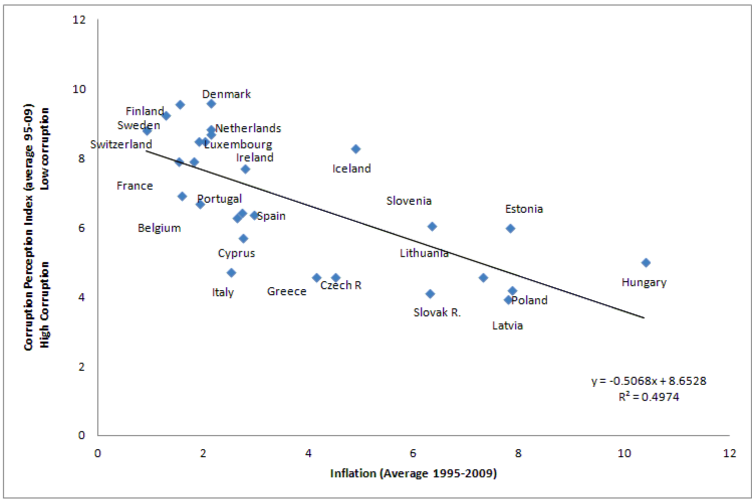
Source: Transparency International and authors' own calculations.

consider important to take into account in economic analysis not only fundamentals but such kind of institutional failures. Figure 4.7 presents the CPI for all EU members, the index goes from 1 to 10, the higher the number of the index, the lower the level of corruption.

4.5 Conclusion

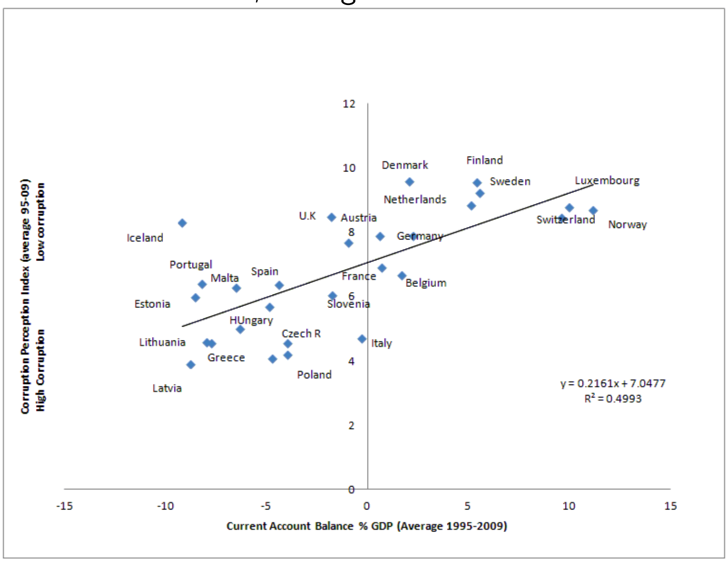
We have constructed a stylized model where an economy has free access to international capital markets to issue sovereign external debt. The domestic economy has rent-seeking groups that can cooperate or not in austere debt issuing, the provision of public goods or in seeking rents. When there is more than one rent-seeking group, a "tragedy of the commons" problem arises that leads to fiscal impatience. If rent-seeking groups cooperate fiscal impatience may be avoided. Noncooperation results in excess of debt issuance because rent-seeking groups want to expropriate more rents before other groups do so as well and because they need the support of the broad society to win elections, they do not increase tax rates but rather issue more debt. Because the result of the noncooperative game is a time-consistent Markov-perfect equilibrium, there are not incentives from the rent seekers to renege their promises. However, when the event of default occurs, the government will have to run a balance budget, which may imply increase in taxes or decrease in government spending.

Figure 4.8: Corruption Perception Index (CPI) and inflation in EU members average 1995-2009



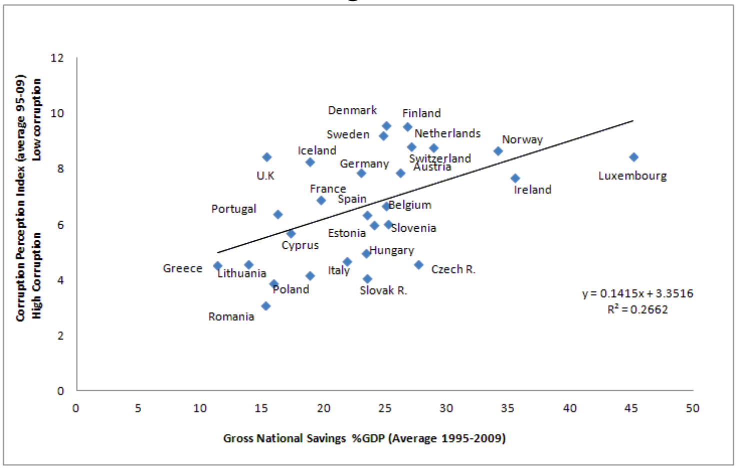
Source: Transparency International, World Economic Outlook and authors' own calculations.

Figure 4.9: Corruption Perception Index (CPI) and current account balance as percentage of GDP for EU members, average 1995-2009



Source: Transparency International, World Bank and authors' own calculations.

Figure 4.10: Corruption Perception Index (CPI) and gross national savings as percentage of GDP for EU members, average 1995-2009



Source: Transparency International, World Bank and authors' own calculations.

We demonstrate that noncooperation triggers an external-debt default vicious circle. Default occurs because international credit markets charge very high interest rates once they foresee the arising rent-seeking commons problem which leads to excess future debt issuing and exhaustion of the country's wealth. If rent-seeking groups cooperate default is avoided. However, such cooperation is an equilibrium outcome only if the initial outstanding debt-GDP ratio is below a certain threshold.

To alleviate the high debt burden, sovereign bailouts, as the ones implemented in Greece, try to help countries to return to free capital markets bringing them into lower outstanding debt-GDP-ratios. However, in countries suffering from corruption in the form of rent-seeking groups bailouts may not work. This is due to the fact that rent-seeking groups increase collective impatience for public goods and rents, leading to excessive bond issuance and an explosive debt-GDP ratio over time. Our model is a starting point to describe sovereign external debt in an environment with endogenous international interest rates. Some extensions to enrich the setup may include: taxation that affects GDP, inclusion of uncertainty, imperfect information or asymmetries and political economy of voters and governments of countries financing bailouts.

Appendix A

Appendix to Chapter 2

Computation of the tax rate The numerical method employed is linear quadratic approximation.

1. The outer loop starts with a guess for τ^{ss}. The initial guess is denoted by $\tau_0^{ss,}$, and the further iterations are denoted by subscript n, such that the symbol τ_n^{ss} is the tax rate in period n. The initial guess allows the calculation of K^{ss}, L^{ss} and H^{ss}. In this way we find the steady state values relevant to constitute the vector around which the quadratic form of the momentary utility function of each problem is approximated.

2. The next loop implies the construction of the policy rule Ψ and a new guess is necessary, for this problem it corresponds to a row vector of 5 variables. The initial guess is denoted by Ψ_0 and as before, the iteration index of this loop will be n, and therefore the policy rule Ψ in time n is given by Ψ_n. The policy rules is then defined as,

$$\tau^{t+1} = \bar{\Psi}_n \cdot \begin{bmatrix} 1 \\ x_{1,t} \\ x_{2,t} \\ K_t \\ \tau_t \end{bmatrix} = \begin{bmatrix} \psi_{n,c} & \psi_{n,x_1} & \psi n,_{x_2} & \psi_{n,\mathbf{K}} & \psi_{n,\tau} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ x_{1,t} \\ x_{2,t} \\ K_t \\ \tau_t \end{bmatrix} \tag{A.1}$$

3. This step comprises two stages, a) find out how the economy will react to the guessed rule and b) find out the reaction of the economy to changes in one-period's committed tax rate. The first part of this step involves the calculation of the decision rule of our competitive equilibrium (CE) and the definition the representative household's value function subject to the policy rule Ψ_n. In the second part we calculate the decision rule of the *"intermediate equilibrium"* (IE) where the representative household has a value function subject to Ψ_n and incorporates as well a one time deviation in the way tax rates are set, that we define as in $\tilde{\tau}_{t+1}$.

4. Find the competitive equilibrium subject to Ψ_n. Start a loop making a guess for the law of motion of the aggregated labor, capital and capacity utilization. Each

iteration is denoted with the index κ. The following vectors describe the described law of motions,

$$(L_t, K_{t+1}, H_t)^{T1} = K_\kappa \left(x_{1,t}, x_{2,t},\ K_t, \tau_t \mid \Psi_k\right) = \bar{\kappa}_{\kappa,|\Psi_n} \cdot \left(1, x_{1,t}, x_{2,t}, K_t, \tau_t\right)^T, \tag{A.2}$$

where $\bar{\kappa}_{\kappa,|\Psi_n}$ is a 3×5 matrix.

5. Make a guess for the quadratic form of the value function, V^Q_{0,Ψ_n}, and use the RHS of the Bellman equation as an updating mapping. This will gives us as a result a fixed point for solving the above equation using quadratic approximation. A common guess is a negative definite matrix, for example $V^Q_{0,\Psi_n} = -\mathbf{I}_6$, where $\mathbf{I}_6$ is the 6×6 identity matrix. This matrix guarantees the existence of a global maximum.

The Bellman equation as a contraction mapping can be written as follows:

$$s^T V^Q_{0,\Psi_n} s = \max_{l,k',h} \left\{ r^T Q_u r + \beta \left(s'\right)^T V^Q_{0,\Psi_n} s' \right\}, \tag{A.3}$$

subject to (A.1) and (A.2), with $s \equiv [1\ x_1\ x_2\ K\ \ k\ \tau]$, and $r \equiv [1\ \ x_1\ \ x_2\ \ K\ \ \ k\ \tau\ \ l\ \ k'\ \ h\ \ L\ \ H]$

The following step is to impose the constraints (A.1) and (A.2) into the RHS of (A.3). With help of a 17×9 matrix we incorporate the respective constraints, in the following way

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hat{\kappa}_{L,1} & \hat{\kappa}_{L,2} & \hat{\kappa}_{L,3} & \hat{\kappa}_{L,4} & 0 & \hat{\kappa}_{L,5} & 0 & 0 & 0 \\ \hat{\kappa}_{H,1} & \hat{\kappa}_{H,2} & \hat{\kappa}_{H,3} & \hat{\kappa}_{H,4} & 0 & \hat{\kappa}_{H,5} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hat{\kappa}_{K,1} & \hat{\kappa}_{K,2} & \hat{\kappa}_{K,3} & \hat{\kappa}_{K,4} & 0 & \hat{\kappa}_{K,5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \psi_c & \psi x_1 & \psi x_2 & \psi_K & 0 & \psi_\tau & 0 & 0 & 0 \end{bmatrix}$$

and form the 9×9 matrix

$$W_{n,CE} = M^T \cdot \begin{bmatrix} Q_u & 0 \\ 0 & \beta V^Q_{n,CE,\Psi_n} \end{bmatrix} \cdot M .$$

Finally, the RHS of the Bellman equation has been transformed into an unconstrained optimization problem, adopting the following form:

$$\max_{l,k',h} s_u^T \cdot W_{n,CE} \cdot s_u.$$

where $s_u = [1\ x_1\ x_2\ K\ k\ \tau\ l\ k'\ h]$

[1]Whenever there is a superscript T it denotes transposed vector or matrix.

A 3×6 matrix should contain the first-order conditions, and it can be denoted as,

$$B_{n,_{CE}} \equiv -\left(W_{n,_{CE}}(7:9,7:9)\right)^{-1} W_{n,_{CE}}(7:9,1:6) \ ,$$

such that quadratic form of the value function is updated and denoted as,

$$V^{Q}_{n+1,\Psi_n,CE} = M_2^T \cdot W_{n,_{CE}} \cdot M_2 \ , \text{ where } M_2 \equiv \begin{bmatrix} I_6 \\ B_{n,_{CE}} \end{bmatrix}$$

The iterations continue until the fixed point of $V^{Q*}_{n+1,\Psi_n,CE}$ is found and we get as well corresponding individual decision rules denoted B^*.

6. We need an external loop containing the previous loop of iteration for the fixed point. This loop is helping to iterate as to find the fixed point for the aggregate law of motion for capital, $\mathcal{K}_{\kappa_n}$. So, the update for $\mathcal{K}_{\kappa_{n+1}}$. The loop stops when $\underline{\kappa}_{\Psi,n}$ and $\bar{\kappa}_{\Psi,n}$ are arbitrarily close, otherwise the procedures goes back to step 4. When convergence is reached we get the optimal decision rules $\bar{\kappa}^*_{\Psi,n}$.

7. Calculate the *intermediate equilibrium (IE)* subject to the policy rule Ψ_n. This step allow us to determine how the economy responds to one-period deviation from the committed tax rate τ'. This step does not involve iteration, since this is not a Bellman equation.

$$\hat{V}\left(x_1, x_2, K, k, \tau, \tau' \mid \Psi_n\right) = \max_{k' \in [\underline{k}, \bar{k}]} u\left(c, 1-l, \ G\right) + \beta V^{**}\left(x_1', x_2', K', k', \tau' \mid \Psi_n\right)\}$$

subject to,

$$\begin{bmatrix} L \\ K' \\ H \end{bmatrix} = K_{n.IE}\left(x_1, x_2, K, \tau, \tau' \mid \Psi_n\right) \qquad \text{(A.4)}$$

where $K_{n.IE}$ is the decision rule responding to changes in τ'. The important thing to note here is that V^{**} denotes the fixed point that complies with both $\bar{\kappa}^*_{\Psi,n}$, and the solution to the individual problem. This quadratic form is denoted by $V^{Q^{**}}_{\Psi,n}$, and therefore the quadratic approximation of the above non-Bellman equation is given by,

$$\hat{s}^T V^{Q}_{n+1,\Psi_n,CE} \hat{s} = \max_{k'} \left\{ r^T Q_u r + \beta \left(s'\right)^T V^{Q^{**}}_{\Psi,n} s' \right\} \ , \qquad \text{(A.5)}$$

subject to (A.4), with $s^T \equiv [1 \ x_1 \ x_2 \ K \ \ k \ \tau]$, $\hat{s}^T \equiv [1 \ x_1 \ x_2 \ K \ \ k \ \tau \ \tau']$

and $r^T \equiv [1 \ \ x_1 \ x_2 \ \ K \ \ k \ \tau \ \ l \ \ k' \ h \ \ L \ \ H]$

To impose the constraint (A.4) into the RHS of (A.5), express the RHS of (A.5) in a matrix form as, define the 17×10 matrix

$$\hat{\mathbf{M}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hat{\kappa}_{L,1} & \hat{\kappa}_{L,2} & \hat{\kappa}_{L,3} & \hat{\kappa}_{L,4} & 0 & \hat{\kappa}_{L,5} & \hat{\kappa}_{L,6} & 0 & 0 & 0 \\ \hat{\kappa}_{H,1} & \hat{\kappa}_{H,2} & \hat{\kappa}_{H,3} & \hat{\kappa}_{H,4} & 0 & \hat{\kappa}_{H,5} & \hat{\kappa}_{H,6} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hat{\kappa}_{K,1} & \hat{\kappa}_{K,2} & \hat{\kappa}_{K,3} & \hat{\kappa}_{K,4} & 0 & \hat{\kappa}_{K,5} & \hat{\kappa}_{K,6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and form the 10×10 matrix

$$\hat{W}^{(n_{IE})} = \hat{M}_1^T \cdot \begin{bmatrix} Q_u & 0 \\ 0 & \beta V_{\Psi,n}^{Q^{**}} \end{bmatrix} \cdot \hat{M}_1 \ .$$

So, the RHS of the Bellman equation has been transformed into an unconstrained optimization problem, namely,

$$\max_{l,k',h} s_u^T \cdot \hat{W}_{n,IE} \cdot s_u$$

where $s_u^T \equiv [1 \ x_1 \ x_2 \ K \quad k \ \tau \ \tau' l \ k' h]$ .And the first-order conditions are summarized by the 3×7 matrix,

$$\hat{B}_{n,IE} \equiv -\left(\hat{W}_{n,IE}\left[8:10, 8:10\right]\right)^{-1} \hat{W}_{n,IE}\left[8:10, 1:7\right] \ .$$

So, the update for $\underline{\kappa}_{\Psi,n,IE}$ and $\bar{\kappa}_{\Psi,n,IE}$ are arbitrarily close, the loop stops, otherwise we go back to step **6.**

8. Only when $\underline{\hat{\kappa}}^*_{|\Psi^{(n_\psi)}}$ has been reached, use the final $\hat{B}^*$ which is consistent with $\hat{\kappa}^*_{|\Psi^{(n_\psi)}}$, namely,

$$\hat{B}^* \equiv -\left(\hat{W}^*_{[8:10,8:10]}\right)^{-1} \hat{W}^*_{[8:10,1:7]} \ ,$$

in order to construct the quadratic approximation of $\hat{V}\left(x_1, x_2, K, k, \tau, \tau' \mid \Psi^{(n_\psi)}\right)$, namely calculate the quadratic form $V_{\Psi,n}^{Q^{**}}$, where

$$\hat{\mathbf{s}}^T Q^*_{\hat{V}|\Psi^{(n_\psi)}} \hat{\mathbf{s}} = \max_{l,k',h} \left\{ \mathbf{r}^T Q_u \mathbf{r} + \beta \left(\mathbf{s}'\right)^T V_{\Psi,n}^{Q^{**}} \mathbf{s}' \right\} \ , \tag{A.6}$$

subject to,

$$\begin{bmatrix} L \\ K' \\ H \end{bmatrix} = \hat{\kappa}^*_{|\Psi^{(n_\psi)}} \cdot \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ K \\ \tau \\ \tau' \end{bmatrix} \ .$$

It is done on the following way,

$$V_{\Psi,n}^{Q^*} = \hat{M}_2^T \cdot \hat{W}^* \cdot \hat{M}_2 \ ,$$

with

$$\hat{M}_2 \equiv \begin{bmatrix} \mathbf{I}_7 \\ \hat{B}^* \end{bmatrix} \ .$$

With matrix $Q^*_{\hat{V}|\Psi\left(n_\psi\right)}$ we can find the individual representative household's policy preference, by solving the unconstrained problem,,

$$\max_{\tau'} \hat{s}^T \cdot Q^*_{\hat{V}|\Psi\left(n_\psi\right)} \cdot \hat{s}$$

and the first-order conditions are summarized by the 1×6 vector,

$$\psi^{\cdot} \equiv -\left(V_{\Psi,n}^{Q^*}(7:7,7:7)\right)^{-1} V_{\Psi,n}^{Q^*}(7:7,1:6) = \begin{bmatrix} \psi_c & \psi_{x_1} & \psi_{x_2} & \psi_K & \psi^{\cdot}_k & \psi^{\cdot}_\tau \end{bmatrix} \ .$$

The update for Ψ is accomplished by,

$$\Psi_{n+1} = \lambda_\psi \Psi_n + (1-\lambda_\psi)\, \underline{\Psi}_n \ ,$$

where λ_ψ is an overshooting parameter and

$$\underline{\Psi}_n \equiv \begin{bmatrix} \psi_c & \psi^{\cdot}_{x_1} & \psi_{x_2} & (\psi^{\cdot}_K + \psi^{\cdot}_k) & \psi^{\cdot}_\tau \end{bmatrix} \ .$$

As before, if $\underline{\Psi}_n$ and Ψ_n are as close as an arbitrary small number, then go back to Step 3 and continue until convergence leading to Ψ^*.

9. Finally calculate,

$$\underline{\tau}^{ss,(n_\tau)} = \mathbf{\Psi}^* \cdot \begin{bmatrix} 1 \\ x_1^{ss} \\ x_2^{ss} \\ K^{ss} \\ \tau^{ss} \end{bmatrix} \ ,$$

and check whether $\underline{\tau}^{ss,(n_\tau)}$ and $\tau^{ss,(n_\tau)}$ are close enough according to any arbitrary small number, otherwise update with and overshooting parameter and go back to Step 2. When the convergence is reached, the $\tau^{ss,*}$ is obtained, together with the $\Psi^*$and optimal steady value for capital K^{ss}.

Elasticity of labor supply From the first order condition, we get the equation (2.49), and applying the definition of the elasticity of labor supply given by $\varepsilon = \frac{w}{l} \cdot \frac{\partial l}{\partial w}$ we get

$\varepsilon = \chi(1-l)/l.$

Labor supply with GHH preferences The Lagrangian is written as

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1-l_t, G_t\right) + \sum_{t=0}^{\infty} \lambda_t \left[\left(e^{-x_{2,t}} + \bar{r}_t\right) k_t + \bar{w}_t l_t - c_t - e^{-x_{2,t}} k_{t+1} \right] \right\}, \tag{A.7}$$

taking the first-order conditions for this economy we get,

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Rightarrow \beta^t u_1\left(c_t, 1-l_t, G_t\right) = \lambda_t \Rightarrow \beta^t \left(c_t + \theta G_t - \varphi l_t^{\upsilon}\right)^{-1/\eta} = \lambda_t, \tag{A.8}$$

$$\frac{\partial \mathcal{L}}{\partial l_t} = 0 \Rightarrow \beta^t u_2\left(c_t, 1-l_t, G_t\right) = \lambda_t \bar{w}_t \Rightarrow \beta^t \beta^t \left(c_t + \theta G_t - \varphi l_t^{\upsilon}\right)^{-1/\eta} \varphi \upsilon l_t^{\upsilon-1} = \lambda_t \bar{w}_t, \tag{A.9}$$

$$\frac{\partial \mathcal{L}}{\partial h_t} = 0 \Rightarrow \frac{\partial \bar{r}_t}{\partial h_t} = 0 \Rightarrow R_t = \delta'\left(h_t\right), \tag{A.10}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \Rightarrow e^{-x_{2,t}} \lambda_t = E_t \left[\lambda_{t+1} \left(e^{-x_{2,t+1}} + \bar{r}_{t+1} \right) \right], \tag{A.11}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \Rightarrow e^{-x_{2,t}} k_{t+1} = \left(e^{-x_{2,t}} + \bar{r}_t\right) k_t + \bar{w}_t - c_t, \tag{A.12}$$

combining equation (A.9) and equation (A.8) we get,

$$L^{ss} = \left(\frac{\bar{w}^{ss}}{\varphi} \right)^{1/\upsilon}.$$

Following the same procedure with the utility function where G and C are perfect substitutes we obtain:

$$L^{ss} = \left(\frac{\bar{w}^{ss}}{\upsilon \varphi} \right)^{1/\upsilon - 1}.$$

Data

The following data series we used on the empirical exercise shown in Chapter 2. The source of the data is the database of the World Bank Development Indicators for the period 1960 - 2010. Variables are percapita. The series employed are government final consumption expenditure denoted as GC in the Table A. 1 and GDP, in constant values of 2000. The sample consists on time series for 79 countries from which 33 belong to OECD classification and 46 are non-OECD countries.

Country	Code	GC	GDP
Algeria	DZA	1960-2009	1960-2011
Australia	AUS	1960-2011	1960-2011
Austria	AUT	1960-2011	1960-2011
Belgium	BEL	1960-2011	1960-2011
Belize	BLZ	1980-2008	1960-2011
Bolivia	BOL	1970-2011	1960-2011
Brazil	BRA	1960-2011	1960-2011
Bulgaria	BGR	1970-2011	1960-2011
Cameroon	CMR	1960-2011	1960-2011
Canada	CAN	1960-2011	1960-2011
Chile	CHL	1960-2011	1960-2011
China	CHN	1960-2011	1960-2011
Colombia	COL	1960-2011	1960-2011
Costa Rica	CRI	1960-2011	1960-2011
Denmark	DNK	1960-2011	1960-2011
Dominican Republic	DOM	1960-2011	1960-2011
Ecuador	ECU	1960-2011	1960-2011
El Salvador	SLV	1960-2011	1960-2011
Finland	FIN	1960-2011	1960-2011
France	FRA	1960-2011	1960-2011
Greece	GRC	1960-2011	1960-2011
Guatemala	GTM	1960-2011	1960-2011
Honduras	HND	1960-2011	1960-2011
Hungary	HUN	1960-2010	1960-2011
Iceland	ISL	1960-2011	1960-2011
India	IND	1960-2011	1960-2011
Indonesia	IDN	1960-2011	1960-2011
Italy	ITA	1960-2011	1960-2011
Japan	JPN	1960-2011	1960-2011
Korea, Rep.	KOR	1960-2011	1960-2011
Latvia	LVA	1980-2010	1960-2011
Luxembourg	LUX	1960-2011	1960-2011
Malaysia	MYS	1960-2011	1960-2011
Mali	MLI	1967-2002	1960-2011
Malta	MLT	1970-2010	1960-2011
Mexico	MEX	1960-2011	1960-2011
Morocco	MAR	1960-2011	1960-2011
Namibia	NAM	1980-2011	1960-2011
Netherlands	NLD	1960-2011	1960-2011
Nicaragua	NIC	1960-2011	1960-2011
Norway	NOR	1960-2011	1960-2011
Pakistan	PAK	1960-2011	1960-2011
Panama	PAN	1980-2011	1960-2011
Papua New Guinea	PNG	1961-2004	1960-2011
Paraguay	PRY	1960-2011	1960-2011
Peru	PER	1960-2011	1960-2011
Philippines	PHL	1960-2011	1960-2011
Poland	POL	1990-2009	1960-2011
Portugal	PRT	1960-2011	1960-2011
Senegal	SEN	1960-2011	1960-2011
Singapore	SGP	1975-2011	1960-2011
Spain	ESP	1960-2011	1960-2011
Sudan	SDN	1960-2008	1960-2011
Sweden	SWE	1960-2011	1960-2011
Switzerland	CHE	1960-2011	1960-2011
Thailand	THA	1960-2011	1960-2011
Trinidad	TTO	1960-2008	1960-2011
Tunisia	TUN	1961-2011	1960-2011
Turkey	TUR	1987-2011	1960-2011
United Kingdom	GBR	1960-2011	1960-2011
United States	USA	1970-2011	1960-2011
Uruguay	URY	1960-2011	1960-2011
Venezuela	VEN	1974-2011	1960-2011

Table A.1: Time series length

Appendix B

Appendix to Chapter 3

Steady-state calculations

Maximization problem of the household.

$$L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, 1 - l_{i,t}, G_t\right) + \sum_{t=0}^{\infty} \lambda_t \left[\left(e^{-x_{2,t}} + \bar{r}_t\right) a_{i,t} + \bar{w}_t \omega_i l_{i,t} - c_{i,t} - e^{-x_{2,t}} a_{i,t+1} \right] \right\}$$

First-order conditions

$$\frac{\partial L}{\partial c_{i,t}} = 0 \Rightarrow \beta^t u_1\left(c_{i,t}, 1 - l_{i,t}, G_t\right) = \lambda_t \tag{B.1}$$

$$\frac{\partial L}{\partial l_{i,t}} = 0 \Rightarrow \beta^t u_2\left(c_{i,t}, 1 - l_{i,t}, G_t\right) = \lambda_t \bar{w}_t \omega_i \tag{B.2}$$

$$\frac{\partial L}{\partial h_{i,t}} = 0 \Rightarrow \frac{\partial \bar{r}_t}{\partial h_{i,t}} = 0 \Rightarrow R_t = \delta'\left(h_{i,t}\right) \tag{B.3}$$

$$\frac{\partial L}{\partial a_{i,t+1}} = 0 \Rightarrow e^{-x_{2,t}} \lambda_t = E_t\left[\lambda_{t+1}\left(e^{-x_{2,t+1}} + \bar{r}_{t+1}\right)\right] \tag{B.4}$$

$$\frac{\partial L}{\partial \lambda_t} = 0 \Rightarrow e^{-x_{2,t}} a_{i,t+1} = \left(e^{-x_{2,t}} + \bar{r}_t\right) a_{i,t} + \bar{w}_t \omega_i l_i - c_{i,t} \tag{B.5}$$

for all $t \in \{0, 1, ...\}$, together with the transversality condition and initial conditions $a_{i,0} > 0$.

Household optimality conditions are:

$$a_{i,t+1} = \left(1 + \bar{r}_t\right) a_{i,t} + \bar{w}_t \omega_i l_{i,t} - c_{i,t} \tag{B.6}$$

$$\frac{1-\theta}{\theta}\left(\frac{c_{i,t}}{1 - l_{i,t}}\right)^{\frac{1}{\chi}} = \bar{w}_t \omega_i \tag{B.7}$$

Considering (B.6) in the steady state it is,

$$c_i^{ss} = \bar{r}^{ss} a_i^{ss} + \bar{w}^{ss} \omega_i l_i^{ss} \tag{B.8}$$

Moreover,

$$\Rightarrow c_i^{ss} = \left(\frac{\theta}{1-\theta}\right)^{\chi} (\bar{w}^{ss}\omega_i)^{\chi} (1 - l_i^{ss}) \Longrightarrow$$

$$\bar{r}^{ss} a_i^{ss} + \bar{w}^{ss}\omega_i l_i^{ss} = \left(\frac{\theta}{1-\theta}\right)^{\chi} (\bar{w}^{ss}\omega_i)^{\chi} (1 - l_i^{ss}) \Rightarrow$$

$$\Rightarrow \bar{w}^{ss}\omega_i l_i^{ss} \left[1 + \left(\frac{\theta}{1-\theta}\right)^{\chi} (\bar{w}^{ss}\omega_i)^{\chi-1}\right] = \left(\frac{\theta}{1-\theta}\right)^{\chi} (\bar{w}^{ss}\omega_i)^{\chi} - \bar{r}^{ss} a_i^{ss} \quad \text{(B.9)}$$

Equation (B.9) can be solved for l_i^{ss}:

$$l_i^{ss} = \frac{\left(\frac{\theta}{1-\theta}\right)^{\chi} (\bar{w}^{ss}\omega_i)^{\chi}}{\bar{w}^{ss}\omega_i + \left(\frac{\theta}{1-\theta}\right)^{\chi} (\bar{w}^{ss}\omega_i)^{\chi}} - \frac{\bar{r}^{ss} a_i^{ss}}{\bar{w}^{ss}\omega_i + \left(\frac{\theta}{1-\theta}\right)^{\chi} (\bar{w}^{ss}\omega_i)^{\chi}} \ . \quad \text{(B.10)}$$

The deterministic steady state implies, $u\left(c_{i,t}^{ss}, 1 - l_{i,t}^{ss}, G_t^{ss}\right) = u\left(c_{i,t+1}^{ss}, 1 - l_{i,t+1}^{ss}, G_{t+1}^{ss}\right)$, so (B.1) and (B.4) imply,

$$\frac{1-\beta}{\beta} = \bar{r}^{ss} \ ,$$

while (3.11) implies,

$$\frac{1-\beta}{\beta\left(1-\tau^{ss}\right)} = R^{ss}H^{ss} - \delta\left(H^{ss}\right) \ , \quad \text{(B.11)}$$

and (B.3) gives,

$$\frac{1-\beta}{\beta\left(1-\tau^{ss}\right)} = \delta'\left(H^{ss}\right) H^{ss} - \delta\left(H^{ss}\right) \ , \quad \text{(B.12)}$$

which is expressed in its aggregated form, equation (B.12) gives

$$H^{ss} = \left[\frac{\frac{1-\beta}{\beta(1-\tau^{ss})} + \delta_c}{\left(1 - \frac{1}{1+\xi}\right) b_\delta}\right]^{\frac{1}{1+\xi}} \ . \quad \text{(B.13)}$$

From (B.13) we can see that

$$\delta\left(H^{ss}\right) = \delta_c + \frac{b_\delta}{1+\xi}\left(H^{ss}\right)^{1+\xi} = \delta_c\left(1 + \frac{1}{\xi}\right) + \frac{1-\beta}{\beta\xi} \ . \quad \text{(B.14)}$$

Equation (B.11) together with (3.4) give,

$$\frac{Y^{ss}}{K^{ss}H^{ss}} = \left[\frac{\alpha H^{ss}}{\frac{1-\beta}{\beta(1-\tau^{ss})} + \delta\left(H^{ss}\right)}\right]^{-\nu} \Rightarrow \quad \text{(B.15)}$$

$$\Rightarrow \Lambda^{ss} \equiv \frac{K^{ss}H^{ss}}{L^{ss}} = \left\{ \frac{\left[\frac{\alpha H^{ss}}{\frac{1-\beta}{\beta(1-\tau^{ss})}+\delta(H^{ss})} \right]^{1-\nu} - \alpha}{1-\alpha} \right\}^{\frac{\nu}{1-\nu}} , \tag{B.16}$$

given that the unconditional mean of z_1 and z_2 are both equal to 1. Using quadratic approximation techniques we can apply the certainty equivalence property and we can make $E(z_1) = E(z_2) = 1$ at all times.

We still need to obtain the values of $[\omega_r \;\; \omega_m \;\; \omega_p]^T$ and with K^{ss} we can obtain l_i^{ss} from (B.10). Using the data on Krusell and Rios-Rull (1999, Table 2.), where $\left[A_r^{rel} \;\; A_m^{rel} \;\; A_p^{rel}\right]^T$ we can calculate the vectors,

$$\begin{bmatrix} A_r^{norm} \\ A_m^{norm} \\ A_p^{norm} \end{bmatrix} = \begin{bmatrix} A_r^{rel} \\ A_m^{rel} \\ A_p^{rel} \end{bmatrix} \cdot \frac{1}{\left[\mu_r \;\; \mu_m \;\; \mu_p\right] \cdot \begin{bmatrix} A_r^{rel} \\ A_m^{rel} \\ A_p^{rel} \end{bmatrix}}$$

By construction,

$$\left[\mu_r \;\; \mu_m \;\; \mu_p\right] \cdot \begin{bmatrix} A_r^{norm} \\ A_m^{norm} \\ A_p^{norm} \end{bmatrix} = 1 \; ,$$

and

$$\begin{bmatrix} A_r^{norm} \\ A_m^{norm} \\ A_p^{norm} \end{bmatrix} \cdot K^{ss} = \begin{bmatrix} A_r^{ss} \\ A_m^{ss} \\ A_p^{ss} \end{bmatrix} . \tag{B.17}$$

Using the same way for productivity,

$$\begin{bmatrix} Y_{L,r}^{norm} \\ Y_{L,m}^{norm} \\ Y_{L,p}^{norm} \end{bmatrix} = \begin{bmatrix} Y_{L,r}^{rel} \\ Y_{L,m}^{rel} \\ Y_{L,p}^{rel} \end{bmatrix} \cdot \frac{1}{\left[\mu_r \;\; \mu_m \;\; \mu_p\right] \cdot \begin{bmatrix} Y_{L,r}^{rel} \\ Y_{L,m}^{rel} \\ Y_{L,p}^{rel} \end{bmatrix}} .$$

$$\begin{bmatrix} A_r^{norm} \\ A_m^{norm} \\ A_p^{norm} \end{bmatrix} \cdot K^{ss} = \begin{bmatrix} A_r^{ss} \\ A_m^{ss} \\ A_p^{ss} \end{bmatrix} . \tag{B.18}$$

In the same fashion,

$$\begin{bmatrix} Y_{L,r}^{norm} \\ Y_{L,m}^{norm} \\ Y_{L,p}^{norm} \end{bmatrix} = \begin{bmatrix} Y_{L,r}^{rel} \\ Y_{L,m}^{rel} \\ Y_{L,p}^{rel} \end{bmatrix} \cdot \frac{1}{\left[\mu_r \;\; \mu_m \;\; \mu_p\right] \cdot \begin{bmatrix} Y_{L,r}^{rel} \\ Y_{L,m}^{rel} \\ Y_{L,p}^{rel} \end{bmatrix}} .$$

Since $Y_{L,i}^{relative}$ depends on both ω_i and l_i^{ss}, we use equation (B.9) to calculate $[\omega_r \;\; \omega_m \;\; \omega_p]$ numerically. Finally, since l_i^{ss} depends on A_i^{ss},

$$\begin{bmatrix} Y_{L,r}^{norm} \\ Y_{L,m}^{norm} \\ Y_{L,p}^{norm} \end{bmatrix} = \begin{bmatrix} \omega_r L_r^{ss} \\ \omega_m L_m^{ss} \\ \omega_p L_p^{ss} \end{bmatrix} \cdot \frac{1}{L^{ss}} \, . \tag{B.19}$$

So, from (B.10) it is,

$$\omega_i l_i^{ss} = \frac{\left(\frac{\theta}{1-\theta}\right)^{\chi} \left(\bar{w}^{ss}\right)^{\chi} \omega_i^{\chi+1}}{\bar{w}^{ss}\omega_i + \left(\frac{\theta}{1-\theta}\right)^{\chi} \left(\bar{w}^{ss}\omega_i\right)^{\chi}} - \frac{\omega_i \bar{r}^{ss} A_i^{norm}}{\bar{w}^{ss}\omega_i + \left(\frac{\theta}{1-\theta}\right)^{\chi} \left(\bar{w}^{ss}\omega_i\right)^{\chi}} K^{ss} \tag{B.20}$$

and

$$L^{ss} = \sum_i \mu_i \omega_i l_i^{ss} = \sum_i \mu_i \frac{\left(\frac{\theta}{1-\theta}\right)^{\chi} \left(\bar{w}^{ss}\right)^{\chi} \omega_i^{\chi+1}}{\bar{w}^{ss}\omega_i + \left(\frac{\theta}{1-\theta}\right)^{\chi} \left(\bar{w}^{ss}\omega_i\right)^{\chi}} - K^{ss} \sum_i \mu_i \frac{\omega_i \bar{r}^{ss} A_i^{norm}}{\bar{w}^{ss}\omega_i + \left(\frac{\theta}{1-\theta}\right)^{\chi} \left(\bar{w}^{ss}\omega_i\right)^{\chi}} \tag{B.21}$$

Solve the last equation for L^{ss} we obtain,

$$L^{ss(n)} = \frac{\sum_i \mu_i \frac{\left(\frac{\theta}{1-\theta}\right)^{\chi} \left(\bar{w}^{ss}\right)^{\chi} \left[\omega_i^{(n)}\right]^{\chi+1}}{\bar{w}^{ss}\omega_i^{(n)} + \left(\frac{\theta}{1-\theta}\right)^{\chi} \left[\bar{w}^{ss}\omega_i^{(n)}\right]^{\chi}}}{1 + \frac{\Lambda^{ss}}{H^{ss}} \sum_i \mu_i \frac{\omega_i^{(n)} \bar{r}^{ss} A_i^{norm}}{\bar{w}^{ss}\omega_i^{(n)} + \left(\frac{\theta}{1-\theta}\right)^{\chi} \left[\bar{w}^{ss}\omega_i^{(n)}\right]^{\chi}}} \, . \tag{B.22}$$

With this result we can solve for $K^{ss(n)} = \Lambda^{ss} \cdot L^{ss(n)}/H^{ss}$. The level of labor hours is,

$$l_i^{ss(n)} = \frac{\left(\frac{\theta}{1-\theta}\right)^{\chi} \left[\bar{w}^{ss}\omega_i^{(n)}\right]^{\chi}}{\bar{w}^{ss}\omega_i^{(n)} + \left(\frac{\theta}{1-\theta}\right)^{\chi} \left[\bar{w}^{ss}\omega_i^{(n)}\right]^{\chi}} - \frac{\bar{r}^{ss} A_i^{norm}}{\bar{w}^{ss}\omega_i + \left(\frac{\theta}{1-\theta}\right)^{\chi} \left[\bar{w}^{ss}\omega_i^{(n)}\right]^{\chi}} K^{ss(n)} \tag{B.23}$$

Algorithm

The algorithm is very similar to the one exposed on the previous chapter, with the main difference that instead of having one variable for capital we have a vector $\mathbf{A}$ that contains the asset holding selection of every income group. Additionally we have labor decisions for the three groups as well as law of motions for the utilization of capital of the three different groups.

The policy rule is the represented by, $\tau_{t+1} = \Psi\left(x_{1,t}, x_{2,t}, \mathbf{A}_t, \tau_t\right)$.

1. Make a guess for τ^{ss}. The initial guess is denoted as τ_0^{ss}, and the tax rate at the iteration in will denoted τ_n^{ss}. Use this guess in order to calculate K^{ss}. Lets define,

$$\mathbf{A} \equiv \begin{bmatrix} A_r \\ A_m \\ A_p \end{bmatrix} \quad \mathbf{H} \equiv \begin{bmatrix} H_r \\ H_m \\ H_p \end{bmatrix}, \; \boldsymbol{L} \equiv \begin{bmatrix} L_r \\ L_m \\ L_p \end{bmatrix}$$

2. Make a guess for the policy rule Ψ. After making this initial guess denoted as Ψ_0, the n iteration will be denoted Ψ_n. Here, the policy rule $\Psi_n\left(\tau' = \Psi_n\left(x_1, x_2, \mathbf{A}, \tau\right)\right)$ takes the form,

$$\tau_{t+1} = \bar{\Psi}_n \cdot \begin{bmatrix} 1 \\ x_{1,t} \\ x_{2,t} \\ A_t \\ \tau_t \end{bmatrix} = \begin{bmatrix} \psi_{c,n} & \psi_{x_1,n} & \psi_{x_{2.n}} & \boldsymbol{\psi}_{\mathbf{A,n}} & \psi_{\tau,n} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ x_{1,t} \\ x_{2,t} \\ A_t \\ \tau_t \end{bmatrix}, \quad \text{(B.24)}$$

3. Construct the matrix P and start the loop for the competitive equilibrium.

$$\mathbf{P} = \begin{bmatrix}
1 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I_{3*3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{0} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 \\
\hat{\kappa}_{L,1} & \hat{\kappa}_{L,2} & \hat{\kappa}_{L,3} & \hat{\kappa}_{L,4:6} & 0 & \hat{\kappa}_{L,7} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\hat{\kappa}_{H,1} & \hat{\kappa}_{H,2} & \hat{\kappa}_{H,3} & \hat{\kappa}_{H,4:6} & 0 & \hat{\kappa}_{H,7} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
1 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 \\
0 & \rho_1 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_2 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 \\
\hat{\kappa}_{A,1} & \hat{\kappa}_{A,2} & \hat{\kappa}_{A,3} & \hat{\kappa}_{A,4:6} & 0 & \hat{\kappa}_{A,7} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 1 \\
\psi_c & \psi x_1 & \psi x_2 & \psi_A & 0 & \psi_\tau & 0 & 0 & 0
\end{bmatrix}$$

4. Take a guess for the value function of the rich agents and repeat the same procedure for the median group and the poor households.

5. Aggregate decision rules in the economy to obtain the new aggregate law of motion for capital.

6. Calculate the intermediate equilibrium to identify the response of the economy to a one-year deviation from the announced tax.

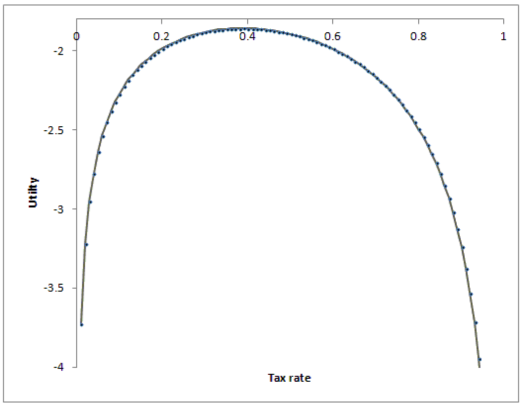
7. Construct the matrix $\widehat{P}$, and update the rule Ψ.

$$\hat{\mathbf{P}} = \begin{bmatrix}
1 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I_{3*3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{0} & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 1 \\
\hat{\kappa}_{L,1} & \hat{\kappa}_{L,2} & \hat{\kappa}_{L,3} & \hat{\kappa}_{L,4:6} & 0 & \hat{\kappa}_{L,7} & \hat{\kappa}_{L,8} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\hat{\kappa}_{H,1} & \hat{\kappa}_{H,2} & \hat{\kappa}_{H,3} & \hat{\kappa}_{H,4:6} & 0 & \hat{\kappa}_{H,7} & \hat{\kappa}_{H,8} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
1 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \rho_1 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_2 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 \\
\hat{\kappa}_{A,1} & \hat{\kappa}_{A,2} & \hat{\kappa}_{A,3} & \hat{\kappa}_{A,4:6} & 0 & \hat{\kappa}_{A,7} & \hat{\kappa}_{A,8} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
0 & 0 & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & \mathbf{0} & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}$$

8. Update $\tau_{new} = \Psi * (1, x_1, x_2, \mathbf{A}, \tau)$;

9. Check the difference between the updated τ_{new} and the guess, and according to a convergence criteria, in this case, .0001, end up the loop for the tax rate.

Single peakedness Since single peakedness was not proved mathematically we have used the approach of Krusell and Rios-Rull (1999) to depict the utility function calibrated to the benchmark parameters and we find that it is single peaked on the tax rate.



Simulation results of just heterogeneity in income

Table B.1: Steady state values of the model just with income inequality

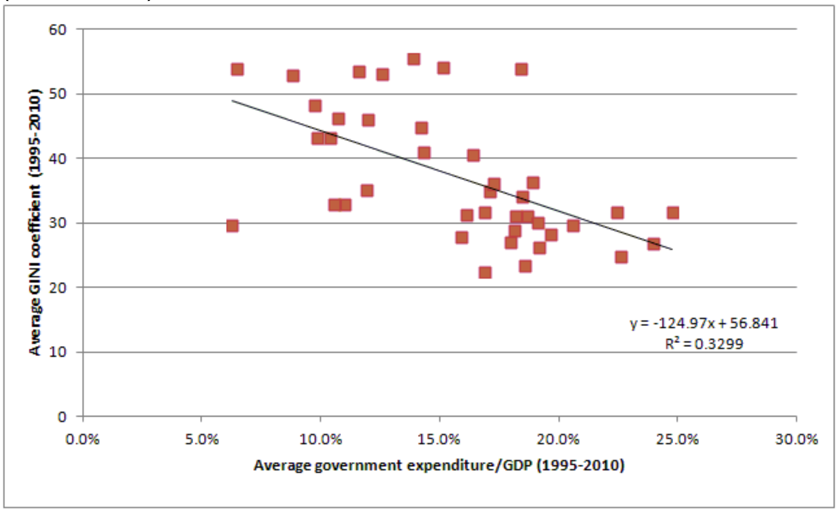
	Equality	Inequality	Just income inequality
C/Y	0.54	0.56	0.53
G/Y	0.26	0.24	0.27
I/Y	0.20	0.20	0.20
L	0.33	0.33	0.33
K/Y	2.12	2.18	2.15
r	0.06	0.06	0.06
τ	0.32	0.29	0.34
δ	0.09	0.09	0.09
H	0.82	0.82	0.82

Table B.2: Model statistics just with income inequality

	Standard Deviation			Contemporaneous correlation with output		
	Data	Benchmark	Just income inequality	Data	Benchmark	Just income inequality
Y	1.81	2.28	1.98	1.00	1.00	1.00
C	1.35	1.46	1.47	0.88	0.83	0.80
I	5.30	5.70	4.79	0.80	0.89	0.89
G	1.21	2.36	1.97	0.80	0.90	0.90
H	n.a	1.01	0.88	n.a	0.19	0.10
L	1.79	0.96	0.80	0.88	0.71	0.99
$\mathcal{T}$	n.a	0.19	0.15	n.a	-0.01	-0.28
G/Y	n.a	0.12	0.10	0.17	0.05	-0.26
w	0.68	1.50	1.34	0.12	0.91	0.92
r	0.30	0.10	0.09	-0.35	-0.04	0.10

Source:U.S. Data for standard deviation, first order autocorrelation and contemporaneous correlation with output were taken from King and Rebelo (2000) for Y, C, I, L, w and r. Values for G, G/Y were taken from Klein and Rios-Rull (2003). n.a: not available.

Figure B.1: Inequality and government consumption expenditure as a share of output averages (1995-2010)



Source: World Bank and own calculations.

Appendix C

Appendix to Chapter 4

C.1 Appendix 4A

C.1.1 Optimal policy setting in the benchmark model

First-order conditions $\partial V/\partial \tau_t = 0$ and $\partial V/\partial B_{t+1} = 0$, imply equation (4.9) and also,

$$\frac{G_{t+1}}{G_t} = \beta(1 + r_{t+1}) \ , \tag{C.1}$$

and

$$G_t = B_{t+1} - (1 + r_t)B_t + \tau_t Y_t \ . \tag{C.2}$$

Combining equations (4.9) and (C.2) leads to,

$$B_{t+1} - (1 + r_t)B_t = [\theta_G - (1 + \theta_G)\,\tau_t]\,Y_t \ . \tag{C.3}$$

Moreover, solving (C.1) forward gives $G_t = \beta^t \Pi_{j=1}^t (1 + r_j) \cdot G_0$, which can be combined with (4.9) to obtain,

$$\tau_t = 1 - \left(\frac{\beta}{1+\gamma}\right)^t \cdot \prod_{j=1}^{t} (1 + r_j) \cdot (1 - \tau_0) \ . \tag{C.4}$$

Equation (C.3) together with (C.4) and (4.7) yield the recursion,

$$B_{t+1} - (1 + r_t)B_t = \phi(\tau_0) \cdot \beta^t \prod_{j=1}^{t} (1 + r_j) - (1 + \gamma)^t L \ , \quad \text{for all } t \geq 1 \ , \tag{C.5}$$

together with (4.10). Solving equation (C.5) forward, leads to equation (4.11).

C.2 Appendix 4B

C.2.1 Proof of Proposition 1

We take a guess on the functional form of strategies,

$$\mathbb{C}^{i,R}\left(B_t, z_t \mid \{r_s\}_{s=t}^{\infty}\right) = \xi_i\left(z_t W_{t+1} - B_t\right) \quad \text{for all } i \in \{1, ..., N\} \ , \tag{C.6}$$

in which $\{\xi_i\}_{i=1}^{N}$ is a set of undetermined coefficients. The Lagrangian of group j's problem, this of maximizing (4.40) subject to (4.35), (4.42) given the guess given by (C.6) for all $i \neq j$ is,

$$\begin{aligned}\mathcal{L}_j \;=\;& \kappa_{V_j} + \sum_{t=0}^{\infty} \beta^t \left[\ln(1-\tau_t) + \theta_G \ln(G_t) + \theta_{R,j} \ln\left(C_{j,t}^R\right)\right] + \\ & + \lambda_{j,t}\left[B_{t+1}/(1+r_{t+1}) - B_t - G_t - \sum_{\substack{i=1 \\ i\neq j}}^{N} \xi_i\left(z_t W_{t+1} - B_t\right) - C_{j,t}^R + \tau_t Y_t\right]\end{aligned}$$

First-order conditions lead to,

$$G_t = \theta_G(1-\tau_t)Y_t \ , \tag{C.7}$$

$$C_{j,t}^R = \frac{\theta_{R,j}}{\theta_G} G_t = \frac{\theta_R}{\theta_G} G_t = C_t^R \ , \tag{C.8}$$

due to that $\theta_{R,j} = \theta_R$ for all $j \in \{1, ..., N\}$, and

$$\frac{G_{t+1}}{G_t} = \beta\left(1 - \sum_{\substack{i=1 \\ i\neq j}}^{N} \xi_i\right)(1+r_{t+1}) = \underbrace{\beta\left[1-(N-1)\,\xi_R\right]}_{\beta_N}(1+r_{t+1}) \, , \tag{C.9}$$

due to the symmetry of the problem, which allows us to consider that $\xi_i = \xi_R$ for all $i \in \{1, ..., N\}$. Combining (C.7) and (C.8), and substituting them into (4.35) we obtain,

$$B_{t+1} = (1+r_{t+1})\left[B_t + \frac{1 + \overbrace{\theta_G + N\theta_R}^{\theta_H}}{\theta_G} G_t - Y_t\right] \ . \tag{C.10}$$

Using equations (C.7) through (C.10), the rest of the analysis follows this in Section 2, in which, after imposing the transversality condition $\lim_{t\to\infty} B_t / \prod_{j=0}^{t-1}(1+r_j) = 0$, we arrive at,

$$G_t = \frac{\theta_G}{1+\theta_H}(1-\beta_N)\left[z_t W_{t+1} - (1+r_t)\,B_t\right] \ . \tag{C.11}$$

From (C.11) and (C.9) it is,

$$C_t^R = \frac{\theta_R}{1+\theta_H}\left(1-\beta_N\right)\left[z_t W_{t+1} - \left(1+r_t\right)B_t\right] \ . \tag{C.12}$$

Equations (C.6) and (C.12) imply that

$$\xi_R = \frac{\theta_R}{1+\theta_H}\left(1-\beta_N\right) \ ,$$

and since $\beta_N = \beta\left[1-\left(N-1\right)\xi_R\right]$, we can prove (4.45) and (4.47). Proving (4.46) and (4.48) follows from direct substitution, completing the proof of the proposition. □

C.2.2 Proof of Proposition 2

Equating demand for bonds (equation (4.21)) and supply of bonds (equation (4.46)) leads to,

$$\left(\beta-\beta_N\right)\left(1+r_t\right)b_t = \left(1-\beta_N\right)\left[\frac{\left(1+\gamma\right)^t W_{t+1}}{Y_t} - \frac{1}{1-\beta_N}\right] \ . \tag{C.13}$$

Substituting (C.13) into (4.46) leads to equation (4.51).

From (4.25) it is,

$$\frac{W_{t+2}}{L} = \frac{1+r_{t+1}}{1+\gamma}\left(\frac{W_{t+1}}{L}-1\right) \ . \tag{C.14}$$

After considering equation (C.13) one period ahead and substituting (C.14) into it, we combine the result with (C.13) referring to period t, and, after some algebra, the result is

$$\left[\left(1-\beta\right)\left(1-\beta_N\right)\frac{W_{t+1}}{L} - \left(1-\beta-\beta_N\right)\right]\frac{1+r_{t+1}}{1+\gamma} = 1 \tag{C.15}$$

After rearranging terms in (C.13) it is,

$$\left(1-\beta_N\right)\frac{W_{t+1}}{L} = 1 + \left(\beta-\beta_N\right)\left(1+r_t\right)b_t \ , \tag{C.16}$$

and after substituting (C.16) into (C.15) we arrive at

$$\left[\left(1-\beta\right)\left(\beta-\beta_N\right)\left(1+r_{t+1}\right)b_t + \beta_N\right]\frac{1+r_{t+1}}{1+\gamma} = 1 \ . \tag{C.17}$$

Combining (C.17) with (4.51) proves (4.54).

For proving the dynamics given by (4.52), equation (C.17) implies that

$$(1-\beta)(\beta-\beta_N)(1+r_t)\,b_t+\beta_N<\beta \Leftrightarrow \frac{\beta(1+r_{t+1})}{1+\gamma}>1 \Leftrightarrow r_{t+1}>r^{ss} \ ,$$

or

$$b_t<\frac{1}{(1-\beta)(1+r_t)} \Leftrightarrow r_{t+1}>r^{ss} \quad \text{for all } t\in\{0,1,...\} \ . \tag{C.18}$$

Optimization requires that $G_t > 0$ for all $t \in \{0,1,...\}$, and so equation (4.49) implies

$$G_t>0 \Leftrightarrow \frac{W_t}{L}>b_t(1+r_{t+1}) \ , \quad \text{for all } t\in\{0,1,...\} \ . \tag{C.19}$$

Combining (C.16) with (C.19) leads to

$$G_t>0 \Leftrightarrow b_t<\frac{1}{(1-\beta)(1+r_{t+1})} \quad \text{for all } t\in\{0,1,...\} \ , \tag{C.20}$$

and given (C.18) proves that $r_t > r^{ss}$ for all $t \in \{0,1,...\}$ in (4.52).

Combining (4.54) and (4.51) directly and using the recursion

$$\frac{1-\beta}{\beta}(\beta-\beta_N)\,b_t+\frac{\beta_N}{\beta}=\frac{b_t}{b_{t+1}} \ ,$$

or,

$$\frac{1}{b_{t+1}}=\frac{\beta_N}{\beta}\cdot\frac{1}{b_t}+\frac{1-\beta}{\beta}(\beta-\beta_N) \ ,$$

with solution,

$$\frac{1}{b_t}-\frac{1-\beta}{\beta}(\beta-\beta_N)=\left(\frac{\beta_N}{\beta}\right)^t\left[\frac{1}{b_1}-\frac{1-\beta}{\beta}(\beta-\beta_N)\right] \ , \tag{C.21}$$

since (4.54) holds for $t \in \{0,1,2,...\}$. since $1/b_1-(\beta-\beta_N)(1-\beta)/\beta$ in (C.21) is equivalent to $b_1 < 1/(1-\beta)$, (C.20) implies that $b_1 < 1/(1-\beta)(1+r_{ss})$ holds as well. In addition, since for $N \geq 2$ equation (4.47) implies that $\beta_N < \beta$, after taking the limit $t \to \infty$, equation (C.21) proves that $b_t \to 1/(1-\beta)(1+r_{ss})$ in (4.53), and substituting $1/(1-\beta)(1+r_{t+1})$ for b_t in (4.54) proves that $r_{t+1} \to r^{ss}$. Finally, for all b_0, there exists an r_0 such that, according to (4.54),

$$(1+r_1)\,b_0=\frac{1+\gamma}{\beta}b_1<\frac{1}{1-\beta} \ ,$$

proving the proposition. □

C.3 Appendix 4C

To calculate the law of motion given by (4.60), notice that, under a proposed bailout plan (r_b, T), equation (4.16) implies,

$$W_1 = \left\{ \left[1 - \left(\frac{1+\gamma}{1+r_b} \right)^T \right] \frac{1+r_b}{r_b - \gamma} + \left(\frac{1+\gamma}{1+r_b} \right)^T \frac{1}{1-\beta} \right\} L \ . \tag{C.22}$$

Combining (C.22) with (4.22) leads to (4.60), which is the generalized version for all $t \in \{0, ..., T-1\}$. For the solution of (4.60) given by (4.64), express (4.60) as,

$$b_{t+1} = \alpha b_t + \zeta - \eta \omega^t \ , \tag{C.23}$$

in which $\zeta \equiv (1-\alpha) / (r_b - \gamma)$, $\eta \equiv [(1+\gamma) / (1+r_b)]^T$, and $\omega \equiv (1+r_b) / (1+\gamma)$. Solving (C.23) forward leads to,

$$b_t = \alpha^t b_0 + \zeta \frac{1-\alpha^t}{1-\alpha} - \eta \omega^{t-1} \sum_{j=0}^{t-1} \left(\frac{\alpha}{\omega} \right)^j \ ,$$

which is equivalent to (4.64).

C.3.1 Proof of Proposition 3

Inequality (4.66) reflects the condition $b_T \leq \kappa$ and it is derived directly from (4.64) after setting $t = T$. Regarding inequality (4.67), in the case of $r_b > r^{ss}$, we need to guarantee that $b_t < \bar{b}_t$, for all $t \in \{0, ..., T-1\}$, as required by (4.63) for stability. Fix any $t \in \{0, ..., T-2\}$ and observe that (4.62) implies,

$$\bar{b}_{t+1} = \left[1 - \left(\frac{1+\gamma}{1+r_b} \right)^{T-t-1} \right] \frac{1}{r_b - \gamma} \ . \tag{C.24}$$

In order to meet the requirement $b_{t+1} < \bar{b}_{t+1}$, combine (4.60) with (C.24) to obtain, after some algebra,

$$\bar{b}_t - b_t > \left(\frac{1+\gamma}{1+r_b} \right)^{T-t} \frac{1}{\beta (1+r_b)} \ ,$$

which can be expressed as,

$$\frac{1}{r_b - \gamma} - b_t > \left(\frac{1+\gamma}{1+r_b} \right)^{T-t} \left[\frac{1}{\beta (1+r_b)} + \frac{1}{r_b - \gamma} \right] \ . \tag{C.25}$$

After substituting (4.64) into (C.25) and after some algebra we arrive at,

$$\frac{1}{r_b - \gamma} - b_0 > \left(\frac{1+\gamma}{1+r_b} \right)^T \frac{1}{(1+r_b)(1-\beta)} \left[\frac{1}{\beta^{t+1}} - \frac{\beta (1+r_b) - (1+\gamma)}{r_b - \gamma} \right] \ . \tag{C.26}$$

Inequality (C.26) must hold for all $t \in \{0, ..., T-1\}$. Since $1/\beta < 1/\beta^T$, substituting $t = T-1$ into (C.26) guarantees that (C.26) holds for all $t \in \{0, ..., T-1\}$, and doing so leads to inequality (4.67). $\square$

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