Understanding mathematics in depth’: an investigation into the conceptions of secondary mathematics teachers on two UK subject knowledge enhancement courses

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Signature:
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Abstract

This thesis is an investigation into conceptions of ‘understanding mathematics in depth’, as articulated by two specific groups of novice secondary mathematics teachers in the UK. Most participants in the sample interviewed have completed one of two government funded mathematics subject knowledge enhancement courses, which were devised with an aim of strengthening students’ understanding of fundamental mathematics. Qualitative data was drawn from semi-structured interviews with 21 subjects and more in-depth case studies of two of the sample. The data reveals some key themes common to both groups, and also some clear differences. The data also brings to light some new emergent theory which is particularly relevant in novice teachers’ contexts.

To provide background context to this study, quantitative data on pre-service mathematics Postgraduate Certificate in Education (PGCE) students is also presented, and it is shown that, at the university in the study, there is no relationship between degree classification on entry to PGCE, and effectiveness as a teacher as measured on exit from the course. The data also shows that there are no significant differences in subject knowledge and overall performance on exit from PGCE, between students who have previously followed a subject knowledge enhancement course, and those who have followed more traditional degree routes.
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Definitions and abbreviations

AMET  Association of Mathematics Education Teachers
BERA  British Educational Research Association
BEd    Bachelor of Education
BSRLM  British Society for Research in Learning Mathematics
COACTIV  Professional Competence of Teachers: Cognitively Activating Instruction and the Development of Students’ Mathematical Literacy
GTP    Graduate Teacher Programme
MDPT   Mathematics Development Programme for Teachers
MEC    Mathematics Enhancement Course
OFSTED Office for Standards in Education
PGCE   Postgraduate Certificate in Education
QTS    Qualified Teacher Status
QUANTUM Qualifications for Teachers Underqualified in Mathematics
SKE    Subject Knowledge Enhancement (course)
SKIMA  Subject Knowledge in Mathematics
TDA    Training and Development Agency
TEDS-M Teacher Education and Development Study in Mathematics
TELT   Teacher Education and Learning to Teach
TIMSS  Trends in International Mathematics and Science Study
TTA    Teacher Training Agency (precursor to TDA)

Shulman’s categories of teacher knowledge
PCK    Pedagogical content knowledge
SMK    Subject matter knowledge

Ball’s refinements
PCK further subdivided into:
   KCS    Knowledge of content and students
   KCT    Knowledge of content and teaching
   KC     Knowledge of curriculum
SMK further subdivided into:
CCK  Common content knowledge
SCK  Specialised content knowledge
Chapter 1

Personal and professional motivations

1.1 The researcher’s position: myself as tutor-researcher

I have been a tutor on the Postgraduate Certificate in Education (PGCE) secondary mathematics course for a number of years. My early experience of interviewing mathematics graduates for PGCE places, and teaching them, indicated that whilst these students certainly had experience of rigorous and high level mathematics, often they lacked knowledge of important concepts in school-level mathematics, the links between them, and why certain familiar procedures actually worked. For example, interviewees could demonstrate a method for adding fractions with different denominators, but when the interviewer, perhaps taking the role of a child, asked why this method worked, interviewees often floundered. It was clear that there were deficiencies in students’ knowledge of school mathematics, in that they lacked insight into how certain concepts linked to each other. Probably they themselves had been taught how to execute various procedures without knowing why. This made me start to think about the nature of the mathematics that they had studied, and the extent to which this prepared them for secondary school teaching.

Subsequently I became involved as a tutor on two programmes specifically designed as subject knowledge enhancement courses (SKEs) for secondary mathematics teachers – the Mathematics Enhancement Course (MEC) and the Mathematics Development Programme for Teachers (MDPT). These courses are intended to extend and strengthen the mathematical knowledge for teaching of non-maths-specialist teachers moving into secondary school mathematics. My work on these two programmes included developing the curriculum and the approaches used, as well as leadership of the tutor team. It was a fascinating opportunity to develop ‘bespoke’ mathematics courses for teachers, and challenged myself and the team to think carefully about the mathematics that teachers need to know, and how they need to use this knowledge. I shall return shortly to the questions that began to form in my mind as a result of this ongoing professional experience, but first of all I give brief background details about the two subject knowledge enhancement courses.
1.2 The Mathematics Enhancement Course

The Mathematics Enhancement Course (MEC) forms one strand of the initiatives developed by the UK government in recent years to enhance subject knowledge preparation for entry to secondary teaching. The MEC sits within the wider framework of Subject Knowledge Enhancement (SKE) courses which have also included programmes, of varying length, in Physics, Chemistry, Religious Education, Music, Information and Communication Technology, Design Technology and Modern Languages (Training and Development Agency, 2010). These programmes aim to enhance the subject knowledge of initial teacher education (ITE) applicants so that they are better prepared for those courses. Courses have been funded by the government via the Teaching Agency (TA) and bursaries are available for students registered on the courses. Longer SKE programmes (typically of 3 months, 6 months, or a year) enable students to progress to teacher training routes who would otherwise have been unable to do so because of a deficit in their subject knowledge. The standard teacher training route is the Postgraduate Certificate in Education (PGCE). Also, a small number of students follow an alternative route, the Graduate Teacher Programme (GTP). Therefore, SKE courses are a means of attracting more applicants to the teaching profession in shortage subjects.

The format of SKE courses has broadened since their inception in 2004, and duration and scope of these programmes is now very varied. However, at the start, programmes known as Mathematics Enhancement Courses (MECs) ran for six months from January to June and were taught intensively (the original MEC specification was for 550 teaching hours, although this has since been relaxed). This remains a popular model for MECs at the time of writing, and is the model in use at the university at which this study is based.

A significant proportion of mathematics PGCE students are now entering the course from a MEC / SKE background. A snapshot of SKE data (TDA, 2008) indicated that in 2008 there were roughly 400 six-month mathematics SKE places available nationally, across about 20 providers. Additionally there were a further 140 places available on longer SKE routes. These numbers are probably underestimates, as there are data missing from the database. In 2006, roughly 1770 students were accepted onto mathematics PGCE courses (Royal Society,
2007). So it is now possible that up to one third of mathematics PGCE students enter the course from a MEC / SKE course. This is in contrast to the situation explored a decade ago by Goulding, Hatch & Rodd (2003) in which the “vast majority” of people who trained as secondary mathematics teachers followed a degree plus PGCE route.

The MEC is aimed at graduates who wish to train as secondary mathematics teachers, whose mathematics background is insufficient for entry to PGCE or other routes to Qualified Teacher Status, but who otherwise are suitable candidates for initial teacher education programmes. It has a strong focus upon the development of subject knowledge (Teacher Training Agency, 2003). Universities have considerable freedom to interpret this, and perhaps unsurprisingly there is in many courses some focus on what understanding is needed to teach mathematics, as well as pure subject knowledge. The MECs were piloted in 2004-6 at two universities - one of which was mine - and since then have been offered by providers across the country.

The inception of the pilot MEC in 2004 provided an opportunity for tutors in my team to try to meet the needs of a specific group of students, and to engage them in the mathematics they would need as prospective teachers. The MEC curriculum as developed by the team included a broad range of high-level mathematics topics covering aspects of pure mathematics, statistics, mechanics and decision mathematics, and also ‘fundamentals’, in which key areas of school mathematics were discussed. More detail about the course is given in Chapter 3. Critically, the way in which students learned on the MEC became significant. Much of the MEC teaching was carried out by experienced mathematics education tutors, and the modelling of good practice in learning and teaching was noted as a strength by HMI Peter Seabourne (2006) who noted students’ “exposure to a variety of high quality teaching and the opportunity to experience new approaches to learning mathematics”, describing this as a “significant incidental legacy” of MEC (p. 3).

1.3 The Mathematics Development Programme for Teachers

The Mathematics Development Programme for Teachers (MDPT) is a part-time course for serving teachers. It arose following the recommendations of the Smith Report (2004) that
significant opportunities for professional development should be made available to serving mathematics teachers, in particular those who are non-subject specialists. Thus it is aimed at teachers who are already teaching mathematics at secondary level, but who did not originally qualify in the subject. It is primarily a subject knowledge enhancement course, but, as in the MEC, there is an inevitable overlap of what might be considered conventional mathematical subject knowledge with what understanding is needed to teach mathematics. Indeed, with regard to pedagogical subject knowledge, it can be remarked anecdotally that course participants have been observed to be keen to pick up new ideas for approaches to school mathematics topics. The MDPT course has been fully funded by the government via the Training and Development Agency (TDA), with supply cover costs available for schools to enable them to release teachers to attend. At the time this research was carried out, the specification of course duration was a maximum of 30 taught days and 10 school-based development days, taking place over four terms. To be eligible to join the course, teachers must not have studied the subject to degree level or have trained to teach it at secondary level. Thus, eligibility for the course is based on teachers’ lack of formal background in mathematics.

The MDPT was piloted in 2007-09 in three universities, and was then offered by 12 providers in the following two years. The existence of this course gives formal recognition to a problem reported for a long time, at least as far back as Cockcroft (1982): that much school mathematics teaching is being carried out by non-specialists. Smith (2004) identified a shortage of around 3,400 specialist mathematics teachers. Government figures indicate that around 27% of those teaching secondary mathematics do not have a post A-level qualification in the subject (Department for Education, 2012). ‘Upskilling’ non-specialist mathematics teachers, many of whom have significant teaching experience, enhances the expertise of the existing workforce in teaching. It is more cost-effective than training new mathematics teachers. It also breaks the vicious circle of a poor supply of teachers at a different point (Smith, 2004).

There are interesting questions to discuss regarding how one might define specialist mathematics teachers. The Royal Society (2007) calls for a consensus on such a definition, to facilitate clearer information gathering in surveys and workforce planning. The authors claim that it is currently not possible to state accurately how many specialist mathematics teachers there are in the UK, that quality of available data is variable, and meaningful
comparisons are difficult to make. They recommend that more systematic and meaningful records are kept. From 2011 the MDPT was subsumed into the Subject Knowledge Enhancement (SKE) category, so that the Teaching Agency was funding what were termed as pre-ITE and post-ITE SKE courses. Currently there are approximately 12 providers running MDPT courses, with about 15 participants on each course; thus about 180 places per year are available. Experience at my university suggests that having made the initial commitment to engage with the course, the majority of participants successfully complete it: completion rates are about 95%.

My university was one of the three involved in the pilot MDPT. Although the needs of this group of people were obviously different from those of MEC students, there were clear areas of commonality. The opportunity to pilot this course provided another opportunity for curriculum development. Teachers on the MDPT are characterised by their diversity: many are experienced practitioners and they are all specialists in curriculum areas other than mathematics, and so they bring this dimension to their mathematics teaching. Some originally trained as primary teachers before moving into the secondary sector. Some are employed outside of mainstream schools, in pupil referral units or special schools. In my role as MDPT course leader, I learned that, as these teachers lacked formal preparation as mathematics teachers, at the start of the course many reported low confidence in the subject, and relied heavily upon published texts to enable them to teach. These teachers recognised that, lacking understanding themselves, they tended to rely on teaching their pupils skills and routines rather than dealing with the more difficult and probing questions about why such approaches worked.

The development of these courses has taken place against a backdrop in the UK in which the predominant model of preparation of secondary mathematics teachers had been via degree-level study of the subject followed by a one-year PGCE, with PGCE applicants being required to have a substantial component of mathematics in their degrees. The introduction of alternative routes such as Mathematics Enhancement Courses (MECs) has stimulated debate about the nature of mathematical understanding, and what constitutes appropriate preparation of teachers. In the context of teacher preparation, are MECs a real alternative to degree level study? What are the differences and similarities between the two? What subject knowledge is needed for mathematics teaching?
I thus became interested in these questions about preparation for secondary mathematics teaching. In the next chapter, I will explore ideas from the literature to develop a language with which to focus research questions in this area, and to identify useful frameworks with which to analyse and make sense of research findings.
Chapter 2
Review of Literature: theory uncovered through practice

Introduction and structure of literature review

In Chapter 1, I introduced my personal and professional motivations for the research presented in this thesis, and the early questions that I will pursue. I begin this chapter with a brief discussion of the social background and context to this research, showing how my early experience, and emerging questions about the mathematics that teachers need, were located in a wider context. I then comment upon the methods used for conducting my literature search.

From here I proceed to explore the growing body of research into what constitutes subject knowledge for mathematics teaching, and how it might be identified. I draw out key themes from major writers in the field, and show how their work has helped me to build my own concepts of key themes. I indicate how these concepts are relevant and important in this study, and suggest how my own research in this study adds to this knowledge.

The discussion starts with the work of Shulman (1986) on knowledge for teaching. Development of these ideas in the context of knowledge for mathematics teaching and how it might be measured, is then reported, with reference to Ball and Bass (2003); Ball, Hill and Bass (2005); Ball, Thames and Phelps, (2008); Hill, Blunk, Charalambos, Lewis, Phelps, Sleep and Ball (2008); Rowan, Schilling, Ball, Miller, Atkins-Burnett, Camburn, Harrison, and Phelps (2001); and Baumert, Kunter, Blum, Brunner, Voss, Klusman, Krauss, Neubrand and Tsai (2010). The work of Rowland, Thwaites and Huckstep, (2003, 2004, 2005) and Rowland, Turner, Thwaites, and Huckstep (2009), around the Knowledge Quartet is considered, linking to current discourses about the active, special and situated nature of mathematics knowledge for teaching (Hodgen, 2011). The relevance of research to the mathematical learning of students on the Mathematics Enhancement Course is discussed, with reference to Artzt, , Sutan, Curcio and Gurl (2012).

Limitations of categorisation models, and alternative conceptions of teacher knowledge as developing through active engagement in the processes of mathematics (Watson, 2008;
Barton, 2009; Davis and Simmt, 2006; Askew, 2008) are then discussed. This leads to recent work on teachers enacting mathematics (Watson & Barton, 2011) and of mathematics knowledge for teaching as a holistic single construct (Beswick, 2012) and a learnable disposition (Davis, 2011).

The review then moves to consideration of work on ‘profound understanding of fundamental mathematics’ (Ma, 1999) and ‘understanding mathematics in depth’, (Adler, 1998; Adler, Hossain, Stevenson, Grantham, Clarke and Archer (2009); Adler & Davis, 2006, 2011), which have inspired the empirical research presented in this thesis. The importance of this work in understanding the mathematical learning of participants on the Mathematics Development Programme for Teachers is discussed, with reference to Vale, McAndrew, and Krishnan (2011) and Silver, Clark, Ghouseini, Charalambos, and Sealy (2007).

2.1 Context to research on teacher knowledge

Over the past ten to fifteen years there has been much debate and concern at national and international level about the preparation and supply of mathematics teachers (e.g. Williams, 2008; Smith, 2004; Tickly & Wolf, 2000). In Britain, this is linked to a perceived under-performance in mathematics of school students in comparison to their counterparts in other countries. This has raised questions about teacher knowledge, especially as it is perceived to be linked to student outcomes. In fact, a recent TIMSS study shows that the performance of English children compares favourably with some other countries (Sturman, Ruddock, Burge, Styles, Lin, and Vappula, 2008). Whether or not British children are actually underachieving is open to discussion, but there is evidence to suggest that dissatisfaction with current levels of education, especially in the high-stakes core subjects of mathematics and English, has been a recurrent theme in the British government and media (Ernest, 2007; Bell, Costello and Kuchemann, 1983) since Victorian times. If governments view the mathematics curriculum from a technological pragmatist viewpoint (Ernest, 1991) that prioritises servicing business, employment and utilitarian societal needs, then the mathematics curriculum, and indeed the school curriculum in general, is destined to always be one step behind, since it can respond to rapid advances in technologies and the needs of industry but cannot anticipate them.
There has been a growth in interest in the knowledge and competence of mathematics teachers. In particular, there is an expanding literature on what constitutes subject knowledge for mathematics teaching, and how it is developed. Much of the research has focused upon primary teachers (e.g. Ball, 1988; Ball and Bass, 2003; Rowland, Huckstep, and Thwaites, 2003b; Golding and Suggate, 2001; Murphy, 2006). It can be argued that the peak of the prescriptive approach of UK government in the late 1990s and early 2000s with regard to what teachers should know and do, was found in Circular 10/97 Teaching: High Status, High Standards (DFEE, 1997), which laid down a mathematics curriculum for primary teacher trainees (Ernest, 1999). The National Numeracy Strategy, a major initiative in primary and lower secondary schools, followed quickly after this, and was widely interpreted as prescribing teaching methods. Campbell, McNamara, and Gilroy (2006) suggest that during this period of time battles have emerged between opposing groups in government and in education circles, with one side trying to further professionalise teaching and linking this to the raising of standards in schools, and the other side “highlighting the lack of connection between teachers’ (formal academic) qualifications and pupil achievement” and arguing for deregulation of teacher preparation (p. 14). The ‘raising standards’ agenda during the Labour government of 1997-2010 had a profound effect upon views of teacher professionalism, with formalised and centralised approaches to initial and continuing professional development put in place. One might argue that working with accountability but within a climate of autonomy and trust are hallmarks of a true profession. However in my view this is not the atmosphere that currently pervades the teaching profession in the UK. There remains a lack of trust in the profession, and this is apparent both in media comments and in government pronouncements.

The nature of subject knowledge required by secondary mathematics teachers is relatively under-researched. However there is an emerging literature and debate with an international focus, e.g. Baumert et al. (2010); Davis and Simmt (2006); Silver et al., (2007). It is often assumed that such teachers should, for example, have a degree-level qualification in mathematics, and that this is sufficient. Many in mathematics education would disagree, and a growing body of research points to the fact that subject knowledge for teaching is indeed far more complex than this. It is argued that the particular knowledge needed for effective teaching, and the way in which this knowledge is held, is quite specific, and that this knowledge should form part of teacher education courses at all levels.
Ball and Bass (2003) state that, “even expert personal knowledge of mathematics often may be inadequate for teaching” (p. 4). They argue that knowledge of mathematics for teaching requires that the teacher develop an explicit awareness of subject matter that goes beyond the tacit understanding normally sufficient for personal knowledge of the domain. Perks and Prestage (2001) contend that experience of the rigour and structures of high-level mathematics is necessary but not sufficient for effective classroom teaching. Being able to do the mathematics oneself is not the same as being able to enable others to do it. Hill et al. (2008) refer to data such as the number of mathematics courses studied by teachers as ‘proxy variables’ (p. 432) for teacher knowledge. Such proxy variables have not proved to be adequate predictors of the effectiveness of mathematics teachers (Monk, 1994). Other indicators of mathematical knowledge for teaching and its impact on teaching effectiveness are needed. In recent years there has been a growth of interest in the ideas of ‘profound understanding of fundamental mathematics’ (Ma, 1999) or ‘understanding mathematics in depth’ (Adler et al., 2009; Adler and Davis, 2011) as they apply to mathematics teachers in both primary and secondary schools. Such ideas appear more fruitful as indicators than the above mentioned proxy variables, and are explored later in this chapter.

2.2 Approaches to the literature search

A variety of approaches were used in the construction of this literature review. My growing awareness of the literature relevant to my area of interest has moved forward in parallel with my development as a researcher myself, and has greatly impacted on my own professional role. My own regular reading, including regularly checking certain websites for recent additions, such as British Society for Research in Learning Mathematics (BSRLM), has enabled me to stay in touch with relevant research. Reading that I have done to inform my teaching in my professional role has uncovered new and interesting sources relevant to my research.

My regular attendance at research and professional conferences over the past several years, both to hear other speakers and to present my own work, has helped to keep me informed of recent developments in the field, and has enabled me to network with other researchers. I
have regularly attended conferences of BSRLM, Association of Mathematics Education Teachers (AMET), and have also presented at BSRLM and British Educational Research Association (BERA). At these fora, academic discourse is wide-ranging and I have gained an awareness of the areas of specialism of key writers on mathematics education. I have also gained an awareness of the important writers in the field of mathematics knowledge for teaching.

My ongoing work with Adler and the QUANTUM research project (details given in Chapter 4) has also helped me to stay abreast of important work in the field, as the QUANTUM team has moved in the last two years from the data collection phase of the project into the writing of a suite of papers.

For the purposes of this literature review, I conducted systematic electronic searches using various combinations of key words and phrases. These elicited a wide range of sources, which were then checked for relevance and currency. In general I attached greater weight to more recent sources, judging these to be more current, although I was aware of the importance of including some earlier seminal works, e.g. Shulman (1986). I attached greater weight to the work of well-known and widely published authors than to newer writers, although occasionally the work of a new researcher was particularly relevant, e.g. Clarke (2008). I was able to identify well-known and widely published authors through the citation patterns that emerged from my searches, as I would find other writers citing them. This was an important factor in selecting work that was authoritative. On reading a paper, I would become interested in sources cited by the author, and would then follow these up, evaluate them, and if appropriate add them to my collection of relevant documents. After conducting systematic searches for some time, I found the same articles re-appearing, and at this stage I became confident that my search was comprehensive.

Greater significance was given to articles from international and/or peer-reviewed journals than to those published more informally. I attached greater weight to larger scale research, judging that findings made from rigorous large-scale studies, often conducted by a team over time, would usually be more important and authoritative than, for example, smaller pieces of work carried out by individuals. However I did not discount small-scale research, as these projects could illuminate particular concepts productively and of course can still be rigorous.
On evaluation, I found some sources not to be useful, and chose not to include them. Reasons for discounting papers varied. For example, I found some work, although linked to my area of interest, moved away from my focus and into other realms which I did not judge to be relevant and which would have been a distraction away from my focus.

In her discussion on the role of the researcher in bringing rigour and validity to the research process, Jaworski (1997) comments upon the issue of ‘significance’. She states (p. 115) that “all observation is selective”, the implication being that the researcher makes decisions about significance (of episodes in fieldwork selected for analysis), based upon his/her own viewpoint. That viewpoint may be informed by a researcher’s professional background and experience within the context being researched. Professional knowledge and experience bring validity and insight to the process. Similarly, I note that the decisions that I made regarding selection of literature to inform my research were based upon my own judgements about what was, and was not, significant. This in turn affected my decisions about what to include in my literature review. My judgements were informed by my own position as an experienced mathematics teacher-educator. My professional background meant that I was able to engage with literature quickly. Knowing my own professional arena well, and being engaged in research within it, meant that I was readily able to recognise features of mathematics education research which informed my own investigation. I was also able to discard research that did not inform my work.

I approach this literature review for research as suggested by Maxwell (2006), endeavouring to present a selection of research chosen because of its relevance to this study, rather than to present a comprehensive overview of the whole field. I will discuss mathematics subject knowledge for teaching, and the concept of understanding mathematics in depth. I will identify current gaps in understanding, and thereby show how this study contributes to knowledge.
2.3 Models of teacher knowledge

The distinction between pedagogical content knowledge and subject matter knowledge

We need to start by asking what is meant by subject knowledge for teaching. Historically it had been assumed that advanced study in a subject was sufficient preparation for teaching the subject, and research activity in education tended to focus upon generic, pedagogical and skills-based aspects of teaching such as questioning, planning and classroom management. There was little attention paid to the preparation of teachers with regard to content and subject matter. A move subsequently described by Ball et al. (2008) as a “major breakthrough in the conceptualisation of teacher content knowledge” (p. 1) took place in the mid-1980s, following the seminal work of Shulman (1986). Shulman posed the question “Why this sharp distinction between content and pedagogical process?” (p. 6) and asked how these are related in the process of teaching. He discussed the connections between knowing and teaching, probing how learning for teaching might occur, noting that “mere content knowledge is likely to be as useless pedagogically as content-free skill” (p. 8), and that we need to think about the blend of these aspects of a teacher’s repertoire.

Shulman then proposed his now well-known three part model of content knowledge: subject matter content knowledge, pedagogical subject knowledge and curricular knowledge. His major contribution was the idea of the existence of pedagogical subject knowledge, i.e. subject matter knowledge for teaching, since this implied that there is knowledge that teachers need to have, and there are ways in which they need to hold that knowledge, which are specific to the requirements of teaching the subject and which other mathematicians do not necessarily have. Alongside his categories of content knowledge he also proposed other key categories of teacher knowledge: general pedagogical knowledge, knowledge of learners, knowledge of educational contexts, and knowledge of educational purposes (see Fig. 1).
Figure 1: Major Categories of teacher knowledge (Shulman, 1987, p. 8, cited in Ball et al., 2008, p. 390)

- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organisation that appear to transcend subject matter
- Knowledge of learners and their characteristics
- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, or the character of communities and cultures
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds
- Content knowledge
- Curriculum knowledge, with particular grasp of the materials and programs that serve as “tools of the trade” for teachers
- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

Shulman’s model thus provides a rich interconnected picture of different sets of knowledge and dispositions. It is not implied that there is clear delineation between his categories; rather that teacher knowledge moves between the categories, and may be characterised by different categories in different times and contexts. Many researchers have used Shulman’s model as a springboard to begin their own investigations into the nature of knowledge for teaching, e.g. Ball, Hill, and Rowland (all discussed later). Shulman’s model provides a framework and a vocabulary for discussing teacher knowledge, and is the starting point for my own understanding of these concepts. Importantly for the ongoing discussion in this chapter, we can now distinguish between subject matter knowledge and pedagogical content knowledge.
2.4 Models of knowledge for mathematics teaching

Refining Shulman’s model

A key contributor to this field in the context of mathematics teaching over the past two decades is Ball, whose work, often in collaboration with other researchers, can be traced through from the late 1980s to the present day. In 1988 she noted that it was “only recently” that researchers had “begun to think about teaching as subject-matter specific” (Ball, 1988, p.1). It is important to note that subject knowledge for mathematics teaching was being developed by teachers in training, through, for example, their study of the work of Piaget and of Hart (1981). Perhaps Ball’s experience was that the area was still relatively under-researched. In her 1988 paper, Ball reviewed the current literature on what characterises effective teachers, and found that the results were completely inconclusive – no specific emergent traits or characteristics had been found that seemed to determine effective teaching. She then went on to discuss subject matter knowledge in mathematics, using case studies from her own research to illuminate teachers’ “ideas of” and “ideas about” the subject (op.cit., p. 6).

In the last twenty years Ball and others have been involved in research on mathematical knowledge for teaching. Ball and Bass (2003) discuss teaching as serious mathematical work, requiring a particular type of knowledge: “teaching as mathematically-intensive work, involving significant and challenging mathematical reasoning and problem solving” (p. 11). Over time the work of Ball and others has resulted in a development of the Shulman model. Ball et al. (2008) refine Shulman’s model in the context of knowledge for mathematics teaching (see Fig. 2), introducing some new categories (e.g. common content knowledge) and reconceptualising old categories (e.g. knowledge of content and students). Thus Shulman’s subject matter knowledge is divided into common content knowledge (CCK) and specialised content knowledge (SCK). CCK is general mathematical knowledge that most educated adults would have. SCK, an important addition to the model, is that mathematical knowledge needed for teaching which is detailed in a way that goes beyond what is needed in everyday life, and moreover which is not necessarily known to other mathematicians. The authors argue that,
“the demands of the work of teaching mathematics create the need for a body of mathematical knowledge that is specialized to teaching” (p 11).

An example would be the knowledge and ability to analyse a piece of student’s work such as a written calculation, and understand the student’s mathematical understandings and/or misconceptions that are revealed therein.

Shulman’s pedagogical subject knowledge is divided into knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of curriculum. These categories can be seen in the original Shulman model, but are refined by Ball et al. (2008), so that pedagogical content knowledge is presented as two differing facets, one interacting and overlapping with knowledge of students, and one interacting and overlapping with knowledge about teaching. Thanheiser, Browning, Moss, Watanabe, and Garza-Kling (2010) note that distinctions between the categories of knowledge may sometimes be blurred, giving “fuzzy boundaries” (p. 2): for example, what could be seen as common content knowledge in one context may be regarded as specialised in another.

Delaney (2010) comments that at the time of writing, Ball et al. (2008) also included “horizon knowledge” as a provisional category in their model. This can be understood as knowledge of where the mathematics one is teaching now will lead to later. I would also see this as an understanding of the progression of the mathematics curriculum and of the connections within it. As such, one could argue that this sits within knowledge of curriculum. However, I believe that the notion of “horizon” is a powerful one, and allocating a new category to this type of knowledge emphasises its importance.
Shulman’s model forms the basis of much recent and current research into teacher knowledge, and can certainly be said to have had a significant impact on the field. Ball et al. (2008) state that,

“the continuing appeal of the notion of pedagogical content knowledge is that it bridges content knowledge and the practice of teaching, assuring that discussions of content are relevant to teaching and that discussions of teaching retain attention to content.” (p. 3)

However they argue that much literature concerning teacher knowledge is conceptual and lacks an empirical basis, thereby justifying approaches to research in the area undertaken by teams in which they are involved. These include the development of survey or questionnaire items designed to measure the extent of teachers’ pedagogical content knowledge (Rowan et al., 2001) as well as in-depth case studies and direct observation of teachers at work (Hill et al., 2008), discussed below.
Rowan, Ball et al. (2001) report on work carried out to develop survey items to measure three facets of teacher knowledge: content knowledge, knowledge of students’ thinking, and pedagogical content knowledge, applied to the areas of mathematics and language at elementary school level. Essentially the items used are scenarios that could occur in a classroom, followed by a ‘multiple choice’ list of possible teacher responses, some of which would be appropriate and some inappropriate. I found these instruments informative and useful, as to me they represent a productive start in exploring teachers’ subject knowledge. The multiple choice design means that the items can be used for large-scale studies, making it possible to gather a significant amount of data, although an important limitation of such multiple choice instruments is that they do not elicit the detail and nuanced differences between individual respondents. In this thesis I do not attempt to measure or capture teachers’ pedagogical subject knowledge directly; I explore teachers’ own conceptions of understanding mathematics in depth. However I would be interested to pursue a more direct investigation into subject knowledge in future work, and these test items developed by Ball and team for the Teacher Education and Learning to Teach (TELT) project would provide a useful place to start.

Rowan, Ball et al. (op. cit.) argue that there is a need for researchers to develop meaningful measures of teachers’ professional knowledge and not to rely only upon measures of general cognitive ability or proxy measures of knowledge. They provide a detailed breakdown of methods, and report on the reliability of the scales used, and they discuss the limitations of the work. A key limitation is that, although reliability was good overall, items used did not discriminate finely between teachers of different abilities – many questions were simply very easy for most respondents, or too hard for most. This paper demonstrates that it is possible to develop survey items to measure aspects of teachers’ pedagogical content knowledge in areas of the curriculum, but it also highlights the complexity and difficulty of this task.

Development of this work is reported in Ball et al. (2005) in which the authors suggest that more research is needed into the nature of the links between teachers’ mathematical preparation and students’ achievement, and report on their work to achieve this. A programme of research is described, wherein carefully constructed questionnaire items (e.g. Fig 3 below), designed to draw out aspects of respondents’ mathematics knowledge for teaching, were used within a large-scale survey. The example given in Fig. 3 was devised for use with primary school teachers, but would be relevant to lower secondary teachers too.
Results for individual respondents were subsequently compared with their own students’ scores in an existing test. The team found that teachers’ performance on the knowledge for teaching questions “significantly predicted the size of student gain scores” (p. 44). Not surprisingly, there are links to be found between teachers’ mathematical knowledge for teaching and their students’ success in the subject. But these links are not generally between the superficial or proxy measures of subject knowledge such as level of academic qualification. When the research probes what I believe is genuine knowledge for teaching, the relationships emerge.
Imagine that you are working with your class on multiplying large numbers. Among your students’ papers, you notice that some have displayed their work in the following ways:

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
<th>Student C</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 \times 25</td>
<td>35 \times 25</td>
<td>35 \times 25</td>
</tr>
<tr>
<td>125</td>
<td>175</td>
<td>25</td>
</tr>
<tr>
<td>+ 75</td>
<td>+ 700</td>
<td>150</td>
</tr>
<tr>
<td>875</td>
<td>875</td>
<td>100 + 600</td>
</tr>
<tr>
<td></td>
<td></td>
<td>875</td>
</tr>
</tbody>
</table>

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

<table>
<thead>
<tr>
<th></th>
<th>Method would work for all whole numbers</th>
<th>Method would NOT work for all whole numbers</th>
<th>I’m not sure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method C</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Hill et al. (2008) note that there is widespread support for the idea that strong teacher knowledge results in benefits for quality of teaching and for student achievement. However, they argue that there is a lack of understanding of how this teacher knowledge has its effects (p. 431). They set out to examine the relationship between teachers’ mathematical knowledge for teaching and mathematical quality of instruction. Referring to the model above (Fig 3), they use four categories of mathematical knowledge for teaching - knowledge of content and students, knowledge of content and teaching, common content knowledge and specialised content knowledge - as a basis for design of their research instruments. This research team gathered four types of data from a sample of ten teachers: firstly, responses to pre-designed items as developed by the team and discussed by Hill et al. (e.g. Fig 3 above), then also videotapes of classroom practice, post-observation debriefings, and interviews. They developed a system to score aspects of the teachers’ responses. Their results show that there is a significant, strong and positive association between mathematical knowledge for teaching, and quality of instruction. A key limitation to the study is that it does not cover student achievement data, so that variation in teachers’ results is not shown to result in variation in student performance. However, there is a strong suggestion that this is the case, and other studies have made this link. The study does not investigate teacher’s knowledge of the curriculum, which is a key component of both the Shulman model (Fig 1 above) and the later model of Ball et al. (Fig 2). Also the authors note that they did not try to find out about teachers’ beliefs about mathematics, and they acknowledge that this is also an important factor to consider.

The COACTIV project in Germany (Professional Competence of Teachers: Cognitively Activating Instruction and the Development of Students’ Mathematical Literacy; Baumert et al., 2010; Krauss et al., 2008) used newly constructed knowledge tests to assess knowledge of secondary-level teachers. They exploited the highly differentiated / segregated nature of German secondary school teacher education to make direct comparisons between different groups of teachers. In Germany, teachers preparing to teach in the academic ‘gymnasium’ schools follow a more in-depth and academic course than teachers preparing to work in other secondary schools. Therefore by comparing teachers who had followed different training routes it was possible to distinguish between different facets of teacher knowledge. The research team hypothesised that content knowledge (CK) and pedagogical content knowledge (PCK) are distinct, that PCK is directly associated with the quality of instruction, and that its
effect on student learning is mediated by the quality of instruction (Baumert et al., 2010, p. 136). To maintain consistency with the Ball categories and avoid confusion with the terms already introduced in this thesis, I shall refer hereafter to the COACTIV content knowledge (CK) as subject matter knowledge (SMK).

The COACTIV team recognise the importance of SMK, noting that “an insufficient understanding of mathematical content limits teachers’ capacity to explain and represent that content to students... a deficit that cannot be offset by pedagogical skills” (op.cit., p. 138). However they argue the vital role of PCK in activating SMK: SMK “remains inert in the classroom unless accompanied by a rich repertoire of mathematical knowledge and skill relating directly to the curriculum, instruction, and student learning” (p. 139).

The COACTIV group developed separate tests for SMK and PCK. The SMK test was designed to assess teachers’ ‘deep understanding’ (p. 143) of mathematical content in the school curriculum. I shall return to the concept of ‘deep understanding’ later. The PCK test was designed to assess teachers’ knowledge of representations, explanations, and student tasks. The project found that it was possible to make an empirical distinction between, and to measure, teachers’ pedagogical and content knowledge. Furthermore they found that teachers’ pedagogical content knowledge had a positive effect upon students’ learning gains. However they argue that SMK is essential for PCK; specifically, SMK “defines the possible scope for the development of PCK” (p. 166). Thus teachers who follow preparation programmes in which subject matter training is limited will find their own PCK development, and thus their students’ progress, hindered.

This extends the findings of Ball and the University of Michigan group discussed above, who found that different aspects of teacher knowledge were not distinguishable, but that elementary teachers’ mathematical knowledge for teaching did predict student gains. The COACTIV project also found that the teachers who had followed the academic-track preparation had stronger subject knowledge, and that their knowledge displayed a higher degree of connectivity between different topic areas, thus concluding that expert knowledge is more ‘connected’. This has resonance with the ideas of “teacher-knowledge” as expounded by Perks and Prestage (2001, p110) as “fluid and connected knowledge of mathematics”.

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The work of Ball and others discussed above, extends and refines the framework and vocabulary introduced by Shulman, relating it directly to mathematics knowledge for teaching. The notion of pedagogical content knowledge is further explored, as is the concept of specialist content knowledge (Ball’s SCK). Ideas about progression, ‘horizon’ and ‘connectedness’ in mathematical knowledge emerge from the literature and will be important in ongoing discussions.

The Knowledge Quartet

Rowland et al. (2003b, 2004, 2005, 2009), in work observing beginning teachers, have developed a stage model of teacher content knowledge. This has emerged from the work of the UK SKIMA project (Subject Knowledge in Mathematics). They identify four broad areas which they define as ‘The Knowledge Quartet’: Foundation, Transformation, Connection and Contingency. Foundation refers to the knowledge, conceptions and beliefs that student teachers bring with them from their own background in the subject. This can be compared with Shulman’s subject matter knowledge (SMK). Transformation is the process whereby the teacher’s own subject knowledge is transformed during the teaching process for the benefit of their students’ learning; this is where Shulman’s pedagogical content knowledge PCK may be seen. Connection is concerned with ordering of topics and tasks, level of challenge, and links made with other areas of the curriculum – Shulman’s curricular knowledge. Finally, Contingency reflects the necessity for the teacher to respond to the needs of students and to work flexibly. The notion of ‘contingency’ takes the model beyond what we might think of as ‘knowledge’, and into the domain of professional skills. However, it requires a thorough and connected level of knowledge on the part of the teacher to be able to do this effectively. Indeed, flexibility of response to learners is often cited as the mark of an experienced teacher. Novice teachers are much more likely to need to adhere to a pre-prepared plan.

The Knowledge Quartet is a different type of model from those developed by Shulman and Ball, described above. Rather than suggesting categories of knowledge, the Rowland model is a stage model concerning teacher acquisition and application of knowledge. Rowland et al. (2009) developed the ‘knowledge quartet’ concept as a framework for observing and/or reflecting upon teaching, both for individual teachers and for those mentoring or supporting
teachers. This device has the advantage of simplicity as well as being in my view comprehensive in covering key aspects of teacher knowledge. Rowland et al. (2009) cite research by Ball et al. (2005). However, in discussing ways of researching teachers’ knowledge, Rowland et al. claim that to gain a full picture of a teacher’s knowledge, it is not sufficient merely to ask the teacher to complete questionnaire items, but researchers must also observe the teacher in action. They suggest that different features of a teacher’s knowledge come together “in the teaching moment” (p. 24) and are not fully accessible outside of this context. In other words, teaching must actually be observed. I agree that observation provides a much fuller picture, and this would be supported by the work of Watson (2008) on mathematical knowledge as something which is active. Thus Rowland et al. point out some limitations to Ball’s (2005) research. However Hill, Ball et al. (2008) did indeed adopt a more holistic approach and used a variety of methods to assess teacher knowledge which led to a deeper understanding of the results.

Hodgen (2011) argues that knowledge of mathematics for teaching is not simply applied within the teaching context, but is ‘situated within the complex and social world of mathematics classrooms’ (p. 27). Thus teacher knowledge and student knowledge are located in social practice and discourse, and it may be more meaningful to think about the active processes of knowing, or coming-to-know, rather than the more static idea of ‘knowledge’. Hodgen asserts that,

“teacher knowledge is embedded in the practices of teaching and any attempt to describe this knowledge abstractly is likely to fail to capture its dynamic nature”. (p 29)

This echoes the approach of Rowland et al. (2009) in seeking to capture knowledge made manifest “in the teaching moment” (p. 24). Thus we start to understand ‘knowledge’ as something that is not static, but comes into being through actions – specifically, we start to see ‘teacher knowledge’ as something that comes into being through interaction with learners, in other words, through the action of teaching.
Other perspectives on mathematical knowledge for teaching: key developmental understandings

Silverman and Thompson (2008) claim that “teaching for understanding is predicated on coherent and generative understandings of the big mathematical ideas that make up the curriculum” (p. 5). They propose a framework for studying the development of mathematical knowledge for teaching which focuses holistically on mathematical understandings that move through the teaching sequence and connect into a network of other ideas. Their discussion of a transformative model of pedagogical content knowledge, after Gess-Newsome (1999), has clear resonance with the work of Rowland et al. (2004, 2009, discussed above), in which transformation of one’s personal ‘foundation’ knowledge is necessary to enable one to teach it to another person.

Silverman and Thompson (2008) comment on the concept of “key developmental understandings” in mathematics as introduced by Simon (2002), as “a way to think about understandings that are powerful springboards for learning” (Silverman and Thompson 2008, p. 7). Key developmental understandings (KDU’s) are the ‘big ideas’ that underpin and connect areas of mathematics. Such ideas might include, for example, symmetry, limits, continuity and infinity. Silverman and Thompson further argue that developing mathematical knowledge for teaching depends upon transforming one’s own personal KDUs into an understanding of how these could enable students’ learning, and actions that one might carry out as a teacher to facilitate this. In other words, it is necessary to separate one’s own understanding from the potential understanding of the learner.

The idea of key developmental understandings seems to me to be very similar to that of ‘threshold concepts’ (Atherton, 2008; Meyer and Land, 2003), an area of research that emerged from the field of economics and now generates lively debate within undergraduate education across various subject disciplines. Threshold concepts can be seen as the ‘big ideas’ of a subject – without grasping these, one cannot fully understand the subject. Atherton (2008) discusses the need for courses to develop “ways of thinking and practising” (online, no page ref.) appropriate to the subject. Cousins (2006) gives some key characteristics of threshold concepts (applicable to any discipline):
1) Understanding a threshold concept is transformative because it involves “an ontological as well as a conceptual shift” (p. 4)

2) A threshold concept is often irreversible, in that once it is understood, the learner has difficulty putting themselves back into the situation before they understood it. They cannot imagine not knowing it. A good example is learning to read: once one has learned to read, it is almost impossible to look at words and not read them. However for teachers, it is vital that they do develop the ability to put themselves back into the pre-learning position, in order that they can understand the needs of new learners.

3) Threshold concepts are integrative, i.e. they reveal the interconnectedness of ideas that were formerly understood by the learner as discrete. Thus they lead to fuller, more holistic understanding of the subject.

4) Threshold concepts can be troublesome, in that they may involve difficult or counter-intuitive ideas. For this reason the learner must enter a liminal space (p. 4) whilst they are grappling with the new concept, and this can be emotionally as well as intellectually challenging.

The idea of liminality (Cousins, 2006) has resonance with constructivist Piagetan notions of accommodation and assimilation of new ideas; the learner must either assimilate new knowledge by linking it to his/her pre-existing schemas, or accommodate by adapting the schemas in order to cope with the new knowledge. This accommodation can be uncomfortable. Cousins reminds us that learning is both affective and cognitive, and often involves identity shifts. Teachers must go through the process of accommodation in their own learning, and also challenge, nurture and support their own students as they go through it, via tasks and opportunities designed by the teacher for this very purpose.

2.5 Critiquing categorisation models – mathematics knowledge as an active process

Watson (2008) offers a holistic critique of the models of Ball and Shulman. She adopts a socio-cultural view of learning based on the idea that teachers learn through participation in practice. She conceptualises mathematical knowledge as “a way of being and acting” (p. 1),
and she suggests that development and deepening of knowledge takes place through “doing mathematics and being mathematical...” (p. 1). She critiques the categorical approach to defining types of knowledge as risking a loss of the overall picture. This seems to be similar to the approach of Silverman and Thompson (2008). Of particular interest to me in the context of my own research are her comments about non-specialist mathematics teachers, discussed below.

It is useful at this stage to consider what might be meant by ‘deepening’ of knowledge as mentioned by Watson (2008). I interpret this as understanding key concepts or ‘big ideas’ and how they inter-relate, rather than just being able to perform procedures; as seeing connections between different areas of mathematics; and as the development of willingness to engage with the subject. This will happen at various levels. For example, the deepening of mathematical knowledge for a primary school teacher will take a different form from that of a graduate mathematics student. They are working with different areas of mathematics, at different levels and for different purposes, but in both cases I believe the characteristics listed above will apply.

I agree with Watson that induction into a community of practice has a strong effect in developing teachers’ knowledge and skills. However, frameworks and models in the categorisation of teacher knowledge are useful, helping teachers and researchers to develop different ways of thinking. They provide a vocabulary and a discourse to use when discussing the many varied facets of knowledge and work of a teacher. Also these models lend an important validity to specific forms of knowledge that are implicitly understood and used within teaching, but *poorly understood and recognised outside the profession*. In this sense, they can be seen to be empowering.

Watson (2008) explores the relationship between personal knowledge and teaching through observation of teachers’ practice, and asks the question “What is it that non-specialist [mathematics] teachers who have good teaching skills do not do?” (p. 4). This is an interesting angle, as it mirrors the very questions asked by the tutor team at the university where this study is based. In setting up the MDPT course, specifically for non-specialist teachers, the team tried to pinpoint what was needed by this particular group. Watson’s research suggests that non-specialist teachers move competently through the factual and technical parts of lessons, but omit the higher level analysis, discussion and synthesis stages.
seen in the cases of stronger, specialist teachers. Her conclusion is that non-specialist teachers need “more personal experience of the mathematical canon” (p. 4). In other words, they need to experience and do mathematics for themselves, (as well as thinking about how their students learn mathematics). This is the message given by Ma (1999): “Address teacher knowledge and student learning at the same time.” (p. 146).

Jaworski, (1997, p. 115) argues that “engagement of self” is critical to the role of the researcher. From my perspective as a knowledgeable expert I believe that Watson’s approach is not incompatible with the models of teacher knowledge discussed above. In my view, Watson widens the scope of the debate, in suggesting that we should focus not (only) on what teachers know, but how they know it and what they actually do with it. That does not suggest that the content, or the ‘what’ is unimportant – but simply that we cannot consider ‘what’ in isolation from ‘how’. Barton (2009) supports Watson’s (2008) ideas of knowledge as a way of being, and that being mathematical is essential. He recognises that what teachers know is important, and that analysis of these areas is successful in the well-known models developed by Ball, Rowland and others, (discussed previously). However he contends that equally important is how teachers hold that knowledge. He argues that “teachers must embody modes of mathematical enquiry themselves… Teachers must be mathematicians” (p. 5). He suggests that a key to effective teaching is in the teacher's attitudes and orientation towards mathematics.

Watson and Barton (2011) further assert that models of acquisition of types or categories of content knowledge overlook an important aspect – that “teachers enact mathematics” (p. 67). In other words, it is the process of doing mathematics, using it, and exemplifying it, that is at the heart of teaching mathematics, and a teacher’s knowledge is manifest through engagement in these processes. Watson and Barton refer to the teacher’s capacity for “knowing-to” act in the moment (Mason & Johnston-Wilder, 2004, p. 289). Referring to Watson and Barton’s work, Ruthven (2011) suggests that their conception is one of “subject knowledge mathematised” (p. 90) wherein a teacher’s ongoing personal involvement and experience with mathematics is critical to the teaching and learning process. Citing Bromme (1994), he further suggests that for experienced teachers, an explicit mathematical narrative provides the blueprint for a tacit pedagogical one. This can be seen for example in a teacher’s selection and sequencing of mathematical tasks or problems to be considered. Thus
the mathematics processes lead the pedagogical processes and the demands of the mathematics provide the context for the pedagogical approaches taken.

Stacey (2008) argues that four components of mathematical knowledge are needed for (secondary) mathematics teaching: knowing mathematics, experiencing mathematics in action, knowing about mathematics and knowing how to learn mathematics. Davis and Simmt (2006) argue that mathematics for teaching is not simply more content, or greater depth, than knowledge needed by students, but is qualitatively different, and can be thought of as a distinct branch of mathematics (as in Ball’s (2008) ‘specialised content knowledge’). They note that the work of Monk (1994) and others demonstrates that there is generally no link between teachers studying more mathematics at higher levels, and their students’ performance. (More details of Monk’s research follow in Chapter 4). They cite Freudenthal (1973) in suggesting that teachers do not necessarily need more mathematics per se, but more nuanced understanding of topics in the mathematics curriculum. They suggest that,

“a key competence of mathematics teachers is the ability to move among underlying images and metaphors – that is, to translate notions from one symbolic system to another.” (Davis and Simmt, 2006, p. 303)

Discussing teacher knowledge from a perspective of complexity science, Davis and Simmt (op. cit.) consider mathematics as a learning system that grows within the individual mind, the collective consciousness (classroom) and wider society. They argue that it is important to consider both teachers’ knowledge of mathematics (Ball’s SMK) and their knowledge of how that mathematics is established (PCK), as “inextricably intertwined” (p. 300). They support Ball’s contention that mathematics for teaching is “not a watered-down version of formal mathematics but a serious and demanding area of mathematical work” (p. 295), and add that much mathematics-for-teaching is tacit. They call for a closer integration within teacher education programmes, of disciplinary knowledge and instructional knowledge. Askew (2008) supports this view, arguing that distinctions between SMK and PCK are constructed within the discourse of research literature and are not independently existing objects. He discusses the distributed nature of discipline knowledge within the classroom collective as distinct from the notion of individual cognition. This is echoed in part by Hodgen’s (2011) findings about the situated nature of mathematics-for-teaching. Askew suggests that rather than the acquisition of a body of knowledge, (primary) teachers need to develop a
“mathematical sensibility” (p. 22) to enable them to handle the demands of the curriculum and to embrace change. This is similar to Stacey’s argument that teachers need to know how to learn mathematics. Beswick (2012), in a study on middle school mathematics teachers, calls for a holistic view of teacher knowledge: a single construct versus multiple categories. Davis (2011) suggests that teachers’ mathematics could be seen as “a learnable disposition rather than an explicit body of knowledge” (p1507). By this I understand that the teacher has a positive disposition and confidence towards engagement in and learning of mathematics. S/he is aware that it is not necessary - or possible - to ‘know everything’, but that having the right tools and attitude one can learn what is needed or desired. This implies a flexibility of approach and a view of mathematics as an active and ongoing process rather than a static body of knowledge.

Silver, Clark, Ghouseini, Charalambos and Sealy (2007) discuss the value of practice-based professional development tasks in programmes for inservice teachers in the US. The BIFOCAL project (Beyond Implementation: Focusing on Challenge and Learning), makes use of carefully designed cognitively demanding tasks in the classroom to integrate several domains of knowledge: mathematics, pedagogy and student thinking. The authors argue that participation in such tasks enables teachers to “rethink and reorganise” their mathematics, to make their knowledge more “useful and usable” (p. 276). Again, this underlines the importance of the integration of different areas of teacher knowledge, as advocated by many other researchers. Davis and Simmt (2006) suggest that courses in mathematics-for-teaching should involve mathematics that is “new to the do-ers” (p. 316), and that this is likely to be more effective than separating questions of mathematics from questions of learning / pedagogy. Davis and Brown (2009) pick up this point, noting that teachers’ mathematics background, in-service opportunities and teaching experience are “inextricably intertwined” (p. 153), and together will influence teachers’ mathematical knowledge for teaching.

Summary

We have a well developed categorisation model of teacher knowledge for mathematics teaching (Ball) sitting alongside a more dynamic stage model (Rowland). Both of these offer us frameworks and vocabulary for discussion of the nature of teacher knowledge. Also,
although they are not part of the category or stage models, I believe the notions of key developmental understandings or threshold concepts add a useful angle to our understanding of the nature of both subject matter knowledge and pedagogical content knowledge. They remind us of the learning process, of what happens to the student at challenging thresholds in learning. They also remind us that awareness of key concepts, or the ‘big ideas’ in a subject, is at the heart of subject knowledge. One can easily observe when this knowledge or awareness is lacking. For example, I have on many occasions read lesson plans, or observed lessons of novice mathematics teachers, in which the teacher has become caught up in other details and has failed to emphasise and draw out the key mathematical ideas germane to the topic being taught. By contrast, an experienced teacher is usually able to get his/her students to focus on the big ideas and thereby increase their own understanding.

Views of mathematics knowledge for teaching as an active process (Watson), or a disposition (Askew), or sensibility (Davis), strengthen and enrich the understanding we gain from categorical and stage models of knowledge. Combined with them, the result is a rich and illuminating set of understandings that most mathematics educators would see are borne out in practice. However, the key literature discussed above does not highlight the perceptions of teachers or trainee teachers about their own knowledge, understanding and learning. This is a gap in research that I seek to address in this thesis.

2.6 Understanding mathematics in depth

Ma’s ‘profound understanding of fundamental mathematics’

I now move on to a concept that is at the heart of this study – the notion of understanding mathematics in depth, particularly as articulated by Ma (1999) and Adler et al. (2009). Ma’s seminal work (1999) arose in part from her experience working with Ball and her team at Michigan University, and so can be seen as a natural development or offshoot from the work of Ball. She had the benefit of a cross-cultural perspective, which I believe enabled her to put into clear focus the characteristics and limitations of USA teachers’ mathematical subject knowledge. Ma moves the discussion forward to what she suggests is essential for effective
teachers: a ‘profound understanding of fundamental mathematics’ (PUFM). In her articulation of this concept, Ma makes a significant contribution to the development of theory, which builds upon earlier work, especially that of Ball and Shulman. Ma has had a significant impact on thinking about knowledge for mathematics teaching. In some ways her findings about the importance of profound understanding of mathematics crossed national boundaries, and boundaries between primary, secondary and higher level mathematics education, and her findings are also relevant to policy-makers.

My distillation of Ma’s PUFM is as follows.

A teacher who has PUFM:

- can make explicit connections between mathematical topics which remain tacit for other people,
- can make clear links between concepts and their associated symbolism,
- can draw upon a secure grasp of mathematical concepts relevant to the school curriculum, to enable him/her to generate and exploit useful examples and tasks that lead pupils to develop their understanding,
- has a grasp of the underlying structure of the subject,
- can make appropriate choices of manipulatives and visual aids to underpin learning,
- does not simply teach pupils to learn procedures, but explains why and how procedures work,
- makes conscious decisions about what and how to teach, using text books and other materials to support his/her own ideas,
- is not limited to one method but can see various differing approaches to the same task, and can therefore understand children’s different responses and identify misconceptions.

Ma contends that a teacher develops PUFM over time and through the practice of teaching, with regular opportunities for professional discussion and sharing of ideas with colleagues. Profound understanding of fundamental mathematics as expounded by Ma primarily concerns SMK (subject matter knowledge), but is situated in and developed through the practice of teaching and so also spans PCK (pedagogical content knowledge) and Ball’s SCK (specialist content knowledge).
This can be contrasted to the findings of the COACTIV project (see p. 31) who conceptualise SMK (their CK) and PCK as distinct knowledge categories, with possession of SMK a necessary precursor for the development of PCK. Later I will explain how my own understanding of PUFM is aligned to Ball’s SMK and SCK, with PCK as a distinct construct. Thus my own perceptions of these facets of knowledge, developed over many years during a career in mathematics education, is closer to the COACTIV model than to Ma’s theory.

Ma investigated possible reasons for USA high school children’s relative underachievement in mathematics when compared with those in some Asian countries such as Japan and China, as revealed by the Third International Mathematics and Science Study (TIMSS) results of the 1990s. She had worked with USA teachers, and her background as an elementary school teacher in China meant she had knowledge of teacher preparation in China. She therefore undertook a comparative study of teachers in the USA and in China. Ma hypothesised that elementary school teachers in the two countries possessed “differently structured bodies of mathematical knowledge” (Ma, 1999, p xx) and that Chinese teachers might have a better understanding of mathematics than US teachers. Drawing on work by Shulman (1986) on pedagogical content knowledge, Ball (1989, 1996) on teachers’ subject matter knowledge, and others, she hypothesised that the difference in pupil performance was linked to the nature of subject matter knowledge held by teachers in the two countries.

Ma used the research instruments developed by Ball and others for the Teacher Education and Learning to Teach (TELT) study as the basis for her work (e.g. Fig. 3 above). These questions were designed to investigate teachers’ knowledge of mathematics in the context of common tasks and scenarios they deal with in the course of their teaching. Ma justified her choice of instruments based on appropriateness for purpose, the fact that they were already tried and tested, and their perceived likely usefulness to the mathematics education community. Rowan et al. (2001) provide a critical commentary upon the TELT instruments, claiming that it is possible to reliably measure certain facets of teachers’ pedagogical subject knowledge. An important limitation to Ma’s work is discussed by Delaney, Ball, Hill, Schilling, and Zopf, (2008), in which the authors report on how the TELT items for measures of teacher knowledge were tested to ascertain suitability for use in a study in Ireland. The authors argue that it cannot be assumed that test items developed for use in one country are easily transferable to another. There may be wide variations in educational aims, history,
context and practice across different countries. Researchers who undertake comparative studies must avoid the trap of assuming that there exists an “idealized international curriculum, defined by a common set of performance tasks” (Keitel and Kilpatrick 1999, cited in Delaney et al., 2008). Delaney et al. (2008) point out that Ma did not discuss how the items and tasks she used were adapted for use with Chinese teachers.

From her detailed study of samples of US and Chinese teachers, Ma develops some key theory and offers recommendations for practice in US education. Two important recommendations are:

“Address teacher knowledge and student learning at the same time.” (p. 146)

“Enhance the interaction between teachers’ study of school mathematics and how to teach it.” (p. 147)

These suggestions are based upon a belief that teacher knowledge and student learning are inextricably linked.

Ma explores rich examples of exactly what profound understanding can mean for teachers, and the ways in which teachers gain it. Ma’s study is qualitative and particular, and although the samples are of a reasonable size one must exercise caution in seeking to generalise too widely from her results. Nevertheless the detailed insights which she gains and the meaningful theory that she develops are important, and various writers have noted that they do have implications for mathematics education beyond the original scope of the work. An interesting effect that emerged from Ma’s book was reaction to her work, and the way in which the views of professional mathematicians and professional educators came together in a new way in a convergence of those on ‘both sides of the math wars’ (Shoenfeld, quoted in Lambdin, 1999, p. 482). This has implications for our understanding of the work in terms of different research paradigms and their associated epistemologies. Shulman, in his foreword to the book, suggests why this happens: “This book appears to be about understanding the content of mathematics, rather than its pedagogy, but its conception of content is profoundly pedagogical.” (p. ix) Mathematicians were pleased with the book because of the message that subject knowledge is important in teaching, whereas mathematics educators welcomed its recommendations on initial teacher preparation and ongoing professional development. Ma’s
work was explicitly cited in the specification for the original Mathematics Enhancement Courses (Teacher Training Agency, 2003) and thus MEC course tutors were aware of the work from an early stage. The group that drew up the MEC specification on behalf of the government had consisted of representatives from both the academic mathematics and mathematics education communities. Perhaps they, like their counterparts described by Schoenfeld above, found common ground in this important work.

What is meant by understanding? Relational and instrumental understanding

Within the specification for MECs there was an emphasis on the nature of subject knowledge that should be developed by the courses, and this drew explicitly on the work of Ma (1999), in particular the idea of “profound understanding of fundamental mathematics, emphasising deep and broad understanding of concepts, as against surface procedural knowledge” (Teacher Training Agency, 2003, p. 3). Skemp (1976) discussed two conceptions of understanding of mathematics: instrumental understanding and relational understanding. Instrumental understanding can be described as knowing how (to perform a mathematical procedure for example) without understanding why. This approach is associated with a focus upon mathematical performance and procedures. Procedural, or instrumental understanding, may be described as the adherence to systematic rules and routines without awareness or reason (Skemp, 1976). In contrast when someone has relational (or conceptual) understanding, they know the bigger picture, see connections and relations between mathematical concepts, and understand why certain steps are being carried out. Although conceptual understanding enables students to gain a comprehensive understanding, it can often take time to be achieved. The distinction between procedural/instrumental and relational/conceptual understanding may be seen as the difference between surface and deep understanding.

The relationship between the two types of understanding is not simple. Hiebert and Lefevre (1986) argue that learners are not fully competent in mathematics if they are deficient in either conceptual or procedural knowledge or if the two exist as separate entities. This view is supported by Rittle-Johnson, Siegler & Alibali (2001) in their research on students’ understanding of decimal fractions. They conclude that conceptual and procedural knowledge
appear to develop in a continuous cycle, the gains in one supporting the increases in the other, which in turn supports the increases in the first.

This distinction relates back to my own early experience of interviewing PGCE applicants, discussed in Chapter 1 (p. 11) above. I noticed that although many were proficient in applying common algorithms found in school mathematics, they struggled to explain why these worked. Skemp suggested that these two types of understanding were so different that they resulted in two different types of mathematics being produced in schools. Ma’s PUFM theory has resonance with the idea of relational understanding as expounded by Skemp. I can add to my list of characteristics of a teacher with PUFM that s/he would have a relational understanding of relevant mathematics.

Drawing from Skemp’s ideas, Beswick (2005) reports on a study into pre-service primary teachers’ beliefs about understanding mathematics, in which she found that many held beliefs that were likely to result in them teaching instrumentally. She notes that some difficulty stems from the fact that there are differing conceptions of what actually constitutes ‘understanding’, and that conceptions of understanding are closely linked to beliefs. A constructivist view of learning is consistent with a view of understanding as ‘complex, non-linear and unpredictable’ (Beswick, p. 162). Beswick notes that there is a strong tendency for novice teachers to teach the way that they were taught (which in many cases was for instrumental understanding). She adds that some primary pre-service teachers were,

“pleasantly surprised by their initial experience of learning mathematics for teaching and in particular enjoy achieving …relational understanding of various topic for the first time” (p. 166)

This is supported by the view of Hodgen (2011) discussed by Ruthven (2011, p. 87) that “restructuring existing knowledge and experience may play a more important role in learning to teach...than acquiring wholly new knowledge”. Certainly, as a mathematics teacher educator, I have encountered this phenomenon, within the context of teaching PGCE, MEC and MDPT courses. In my experience this re-learning is for many teachers a key element in their development of deep understanding of mathematics. Beswick (2005) concludes by noting that her findings support the argument for “increasing the integration of teacher education in on-campus settings and in schools” (p. 167). This echoes the call from Ma
(1999) to “Address teacher knowledge and student learning at the same time.” (p. 146). Beswick’s paper, although focussing upon primary pre-service teachers, adds a useful insight into the process which many new secondary mathematics teachers go through.

Adler: Unpacking mathematical ideas

Adler (1998) explores the extent to which the mathematics classroom may be viewed as a community of practice, wherein teachers need knowledge of teaching (PCK) and knowledge of school mathematics (SMK, curricular knowledge). The central claim is that school mathematics is different from other mathematics, that learning of mathematics is situated within the classroom and within the school mathematics curriculum. In similar vein, Ernest (2000) comments that “school mathematics is not the same as academic or research mathematics, but a re-contextualised selection from the parent discipline…” (online, no page ref.), and Hodgen (2011) comments upon the situated nature of mathematics for teaching.

Adler and Davis (2006), building on the work of Ball and Bass (2000) argue that there is an emerging discourse about, and growing support for the idea that “there is specificity to the way that teachers need to hold and use mathematics in order to teach [it] and that this way…differs from the way mathematicians hold and use mathematics” (p. 272). They develop ideas discussed earlier by Ball and Bass (2003) concerning the ‘unpacking’ versus ‘compression’ of mathematical ideas during the process of teaching. By ‘unpacking’, I understand an approach to teaching in which mathematical concepts are broken down into parts to help learners to understand them, exemplified carefully, and linked to other relevant concepts. In contrast, formal academic mathematical discourse, both written and spoken, is often characterised by a concise use of symbols and terminology which, although efficient and elegant, can sometimes obscure key concepts for the learner.

A methodologically interesting aspect of the Adler and Davis (2006) paper is the coding system developed for the description of tasks and activities that took place within the mathematics teacher education sessions that were observed. The researchers investigated secondary mathematics in-service teacher education courses at several institutions in South Africa, and they developed a coding system for the characteristics of the mathematical tasks.
in which the teachers engaged on these courses. The primary object of a task is indicated by a capital M or T, denoting Mathematics or Teaching, and the secondary object by lower case m or t. Tasks that demand a display of understanding or unpacking of the mathematics are demoted by U+ and others by U− (principled elaboration, versus procedural elaboration, Dowling, 1998). For example, the teacher educators present a task for the teachers in which a range of solutions to a quadratic equation is given (as might happen in a typical South African upper secondary class). The teachers have to

(a) explain which is correct/incorrect and why
(b) explain how they would communicate the strengths and limitations of errors in the solutions to the students
(c) devise questions to lead one of the respondents to formulate a more general response

(adapted from Adler and Davis, 2006, p. 273)

This task is coded MU+tU+ (p. 286), as there is a clear primary mathematical object: solving a quadratic equation, and a secondary teaching object: analysing student responses. Both require explicit reasoning of solutions.

Adler and Davis (2006) find that, in the context they studied, compression or abbreviation of mathematical ideas (in contrast to unpacking) is dominant practice: “there is a limited presence of interesting instances of unpacking or decompression of mathematical ideas as valued mathematical practice” (p. 271). It is their contention that this is a problem. As the work of a mathematics teacher must necessarily involve the regular decompression of mathematical concepts and procedures to enable students to learn them, this practice really should feature in teacher education and CPD programmes. It is interesting to reflect on the fact that UK mathematics degree courses do of course contain much ‘compressed’ content. Indeed, it is seen as the mark of a good mathematician that s/he can express mathematical ideas efficiently and elegantly; this is an important feature of academic mathematics. However, for that subset of mathematics graduates who move into teaching, this level of understanding is not sufficient, and they meet challenges early on in their professional training when they need to break down and decompress mathematical ideas to render them accessible to learners. As Hodgen (2011) notes, some aspects of mathematics knowledge for teaching “run counter to the habits and norms of mathematics as a discipline” (p. 35).
Ruthven (2011) further points out that if a teacher’s knowledge of elementary mathematics has taken on a “curtailed and automated” (p. 88) (one could say compressed) character, then this can actually obstruct effective teaching. He notes that the tension between the preferred ways of operating at advanced level within mathematics, and the nature of understanding needed to decompress concepts in order to teach them effectively at elementary level can actually create “expert blind spots” (p. 88) for teachers.

Although the context of Adler and Davis’ (2006) work, the post-apartheid South African education system, is different from that of this thesis, nevertheless the methods and ideas used and the information sought are relevant to research into subject knowledge for teaching elsewhere. Adler and Davis (2011) comment upon the debate around opportunities in mathematics teacher education to integrate or separate opportunities to learn about mathematics and teaching. This echoes Ma’s (1999) recommendation to “address teacher knowledge and student learning at the same time” (p. 146), suggesting a view that it is not only possible but desirable to provide opportunities for teachers to simultaneously enhance their own subject knowledge and develop their pedagogical skills.

**Adler: Understanding mathematics in depth, connected knowledge**

Adler’s recent work includes a study (the QUANTUM project) based partly in South Africa and partly in the UK, in which the focus is on ‘understanding mathematics in depth’. More details about this work follow in Chapter 3 and Chapter 5, as I show how my own research has been stimulated by the QUANTUM study. I interpret ‘understanding mathematics in depth’ and ‘deep understanding of mathematics’ as being fundamentally the same as Ma’s PUFM, discussed above. Adler’s research focuses upon secondary mathematics teachers, whereas Ma investigated primary teachers; however the principal concepts as I understand them are the same, and it is this interpretation of understanding mathematics in depth that underpins my research in this thesis. ‘Understanding mathematics in depth’ can be conceptualised both as a capability and as ‘deep subject knowledge’ in its reified form, and I see it as both. ‘Understanding mathematics in depth’ is about having a relational and connected knowledge of mathematics which is borne out in the process of doing and teaching mathematics. Whereas Ma suggests that PUFM is developed through the practice of teaching, I believe that understanding mathematics in depth (UMID) can begin to be
developed within pre-teaching courses such as the Mathematics Enhancement Course (MEC). Indeed, as noted in Chapter 1, this is a clear aim of the MEC. Adler et al. (2013) find that this is the case. As noted earlier (p. 42), it is clear that Ma’s conceptualisation of PUFM / UMID as something that straddles both subject matter knowledge and pedagogical content knowledge differs from that of the COACTIV team, who see subject matter knowledge and pedagogical content knowledge as distinct entities. My own conception of UMID is aligned to the COACTIV perspective: I see UMID as subject matter knowledge, not pedagogical content knowledge. I see UMID as a preparation for the development of pedagogical content knowledge.

The UK part of the QUANTUM project focuses particularly on MECs in England, and course tutors’ and students’ conceptions of ‘understanding mathematics in depth’ (UMID). This context provides an interesting site for investigation, since MECs have been devised specifically as subject knowledge courses for intending mathematics teachers. Therefore comparisons can be made with, for example, in-service CPD sessions for South African mathematics teachers.

“Despite differences between the UK and SA, programmes share the phenomenon of providing mathematical education specifically geared to the profession of teaching.” (Adler et al., 2009, p. 2)

Focusing on MEC students’ experiences, QUANTUM research shows clearly how the process of re-learning or re-engaging during the MEC with topics previously encountered only in an instrumental fashion, can transform students’ views and knowledge. The following are three excerpts from interviews with MEC students carried out by Adler and team:

“When we are taught on the MEC we are not just taught to” pass an exam on it…We are taught so that we understand it and they tell us where it is all coming from and then you can organise it all a lot better in your head”.

“I just remember the topic at school, I don’t actually remember doing it, despite doing A Level Maths… I knew things through rote learning; I knew all the formulae but I didn’t know why.”
“All I had done [before] was apply the rules…when I was taught before we didn’t draw the graphs. [On MEC] we could see how things increase and everything – that was something that I couldn’t see before…here we went on first principles, plotting the graphs and seeing how things change”.

(Adler et. al., unpublished)

Findings from this research project show that for MEC students, UMID is characterised as “a discourse of mathematics interwoven with discourses of teaching and learning” (Adler et al., 2009, p. 2). MEC students talk of ‘understanding mathematics in depth’ as connected knowledge, as reasoning and proof, and of acquiring a positive and confident disposition towards the subject. (Adler et al., under review). It is interesting that this research is probing orientations to the subject, as some researchers believe that this is a key to effective teaching, e.g. Barton (2009), Watson (2008) discussed above (p. 36-37).

**Challenging the UMID orthodoxy**

I have mentioned above that in learning to teach, the restructuring - or decompressing - of existing mathematical knowledge may have a more important part to play than the acquisition of wholly new concepts. This can often also include a challenge to teachers’ beliefs and thus some reconstruction of identity, ideas which are discussed by Ruthven (2011) and Hodgen (2011). The comments from MEC students noted above go some way to illustrating this phenomenon. More evidence of this emerges later in this thesis, as I discuss the experiences of re-learning as articulated by experienced teachers (who are new to mathematics teaching) on the Mathematics Development Programme for Teachers (MDPT).

It is appropriate to add that criticisms of the importance of the UMID concept are now emerging, e.g. Hossain, Mendick and Adler (in press), which pose a challenge to some of the assumptions and opinions discussed above. Hossain et al., taking a poststructural approach, discuss the effect of UMID discourse upon the positions available to MEC students in terms of their own identities, i.e. what effect the discourse has. The authors suggest that UMID has become an accepted orthodoxy in mathematics education, and yet many teachers do
experience difficulty and challenge with the concept, especially where it comes into conflict with their own experience of what it means to be successful in mathematics. Most MEC students take on the UMID mantle and work it into their own identity, but for some this may not be possible (or desirable). These ideas are illustrated by Hossain et al. by comparing case studies of two MEC students from different cultural and ethnic backgrounds.

Chapter conclusion and research questions

In this chapter I have given an overview of some key research in the area of knowledge for teaching, and knowledge for mathematics teaching in particular. Categorisation models such as those developed by Shulman (1986) and Ball et al. (2008) provide a useful framework and vocabulary for discourse about the different forms of teacher knowledge and how they overlap and interact. Work by Hill et al. (2008), Baumert et al. (2010), and others in developing ways of measuring mathematical knowledge for teaching are starting to bear fruit, and thus we see links between teachers’ mathematical knowledge for teaching and their students’ achievements.

Stage models such as the ‘Knowledge Quartet’ developed by Rowland et al. (2009) give an insight into the dynamic nature of knowledge for teaching. The discourse of ‘big ideas’ or ‘key developmental understandings’ (KDUs) presented by Silverman and Thompson (2008), with its links to ‘threshold concepts’ (Meyer and Land, 2003) offers another helpful way of conceptualising knowledge for teaching. Ruthven (2011) notes that in teaching carried out by experienced teachers, an explicit mathematical narrative provides the organising structure for a tacit pedagogical one. This I believe must be influenced by the fact that such teachers are secure and strong in the relevant mathematical KDUs, and thus can allow the mathematics to lead the pedagogy.

The idea of mathematics as a process, and mathematical knowledge as a way of being or acting, is espoused by Watson (2008). From this perspective, growth of knowledge takes
place through being involved in doing mathematics and ‘being mathematical’. This view is an important part of the research, since it emphasises the view of mathematics as an active and ongoing process rather than a static body of knowledge. In discussions on the nature of mathematical knowledge, we must also consider views about the nature of mathematics. Watson’s perspectives also highlight the importance of the mathematics teaching community as a community of practice, and the importance for new mathematics teachers of induction into this community. This links to Hodgen’s (2011) comments on knowledge for teaching as situated in the classroom context, which in turn echoes Rowland et al. (2009) assertions that aspects of knowledge come together ‘in the teaching moment’. Beswick’s (2005) position on knowledge as a single construct, Davis’s (2011) “learnable disposition” and Askew’s (2008) “mathematical sensibility” all advance the debate along similar lines.

Ma’s (1999) work has developed some key theory stressing the importance for teachers of ‘profound understanding of fundamental mathematics’ (PUFM). Her work with Ball’s team, and use of the TELT instruments to measure teachers’ PUFM, gives some indication of how PUFM might be developed. Ma offers clear recommendations for mathematics teacher education courses in terms of the integration of subject and pedagogical knowledge. Adler explores a similar theme in a different way - understanding mathematics in depth (UMID) - in both the South African and UK contexts.

I have located in the literature key concepts that inform my own research. The language of mathematics subject knowledge for teaching has been explored, including what is meant by subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Distinctions between instrumental and relational understanding have been discussed, and the ideas of ‘profound understanding of fundamental mathematics’ and of ‘understanding mathematics in depth’ have been introduced and discussed. All the literature discussed above informs our understanding of the nature of mathematical knowledge for teaching, and informs this study on perceptions of ‘understanding mathematics in depth’ as espoused by new mathematics teachers. My engagement with this literature has deepened my own appreciation of the nature of mathematical knowledge for teaching, and enabled me to attach words to my experience, to formalise and better understand concepts, with which I was already familiar from a professional perspective, but at a more informal, experiential level. The MEC and MDPT courses studied in this thesis were designed and were being taught prior to this engagement with literature taking place. However it is clear that research supports the
approaches to developing mathematical subject knowledge for teaching that were adopted by the tutor team and which are manifest in the MEC and MDPT. The concepts of mathematics knowledge for teaching, deep understanding, and understanding mathematics in depth, and the ways of designing teacher preparation courses so that participants may achieve these, are fundamental to the work of the tutor team at the university in this study. The literature reported above provides illuminating insights into, and understandings of, these concepts.

**Role of this study in addressing the knowledge gap**

Very little research has been done on MEC and MDPT teachers’ knowledge development, or indeed any aspect of their experience. The majority of the significant models of teacher knowledge in the literature reviewed above have a focus upon the mathematical knowledge of primary not secondary teachers. Indeed the literature base in this area is rich and extensive. Ball et al. (2005, 2008) and Ma (1999) investigated primary practitioners. The Knowledge Quartet developed by Rowland et al. (2005) was explicitly designed as a model to support the knowledge development of primary teachers, although there is now work ongoing to extend the model to the secondary domain. Hodgen (2011), Askew (2008), Beswick (2005) all report on work carried out with primary teachers.

These studies do of course offer useful insights into the development of knowledge for teaching for secondary mathematics teachers. As a teacher-educator with a secondary specialism, I see clear links that can be made between knowledge development of primary and secondary teachers, and I find the theories and frameworks posited by those researchers helpful in understanding my own professional domain. They clearly speak to the wider field. If this were not the case, I would not have chosen to include them in a review of significant literature. Nevertheless there is a notable lack of specific research into secondary teachers, and in this study I seek to address this gap.

Researchers who explicitly discuss secondary mathematics teachers’ knowledge include Baumert et al. (2010) in Germany, Zaskis and Leikin (2010) in Canada, and Artzt et al. (2012) in the U.S. Zaskis and Leikin (op.cit.) interview teachers about their use of Advanced Mathematical Knowledge (AMK) in their teaching. Teachers report that they do not use AMK explicitly in their teaching, but perceive its usefulness more in their general confidence in mathematics and their ability to make connections across topics. Artzt et al. (2012) argue
for the integration of mathematics subject knowledge and pedagogical knowledge, stressing the importance of the study of school mathematics curriculum “from an advanced perspective” (p. 261). These studies have usefully informed my own work, although of course the national contexts are different from my own.

There is also a lack of research into the knowledge development of non-specialist secondary mathematics teachers. Graven (2004) in South Africa, Vale, McAndrew and Krishnan (2011) in Australia, and Crisan and Rodd (2011) in England address this area, but there is certainly room for more contribution here. Given the lack of suitable qualified secondary mathematics teachers in England (see p. 14) and the important role of inservice development for non-specialist teachers (Smith, 2004) in meeting this need, it is perhaps surprising that there has not yet been much research interest in this field. Again, this is a gap which I seek to address.

Thirdly, there is a lack of research on novice mathematics teachers’ own perceptions about mathematical knowledge for teaching. With the exception of work from Adler and team, and the voices of the teachers themselves are not heard in the research. Zazkis and Leikin (2010) interview practicing mathematics teachers; the study provides useful insights but their focus is not upon novice mathematics teachers. As mathematics education tutors place a high value on understanding mathematics in depth, it is relevant to investigate how course participants, especially those on programmes with an explicit aim of developing deep understanding of mathematics, view this idea. In this study I set out to explore this.

The research questions addressed in this study are:

1. How is ‘understanding mathematics in depth’ conceptualised by two particular groups of novice mathematics teachers?

2. What are novice mathematics teachers’ beliefs about how ‘understanding mathematics in depth’ is attained?

3. What themes are privileged in the discourse of novice mathematics teachers in relation to their preparation for, and experience of, mathematics teaching?

I now discuss what is already known and what is not known about these research questions.
Research question 1
The only study of which I am aware which addresses Research question 1 is Adler at al. (in press). Adler at al. report on how MEC students conceptualise understanding mathematics in depth. They find that MEC students discuss UMID as connected knowledge, as reasoning and proof, and in terms of disposition to the subject (being mathematical). These results are drawn from a sample of 18 MEC students, 6 each from three universities in England, a sample which is sufficiently broad to yield useful results. The Adler study does not consider MDPT teachers.

Research question 2
I am not aware of any literature which addresses Research question 2. I know from my involvement in the QUANTUM project (Adler et al, in press), that the question about how UMID is attained was asked of the 18 MEC participants in the study. However no results are reported; this question was not followed up in the paper.

Research question 3
Graven (2004) sheds some light on this question in her analysis of the response of non-specialist mathematics teachers in South Africa to an in-service training course. She highlights the importance of confidence growth in the teachers’ discourse, and also the importance attached to the idea of playing a fuller part in a mathematics education community of practice. Crisan and Rodd (2011) interview a small sample of teachers on an MDPT course in England. The teachers’ discourse highlights an evolving identity (as mathematics teachers) within a community of practice. Vale et al. (2011) interview ‘out-of-field’ teachers on a mathematics in-service training course and find that length of programme, and relationships established with colleagues are important factors in teachers’ successful development. These three studies all involve serving teachers on in-service training courses. They do not specifically include pre-service / recently qualified teachers. My own research spans both of these groups.

The QUANTUM project is not extended to follow up other themes that emerge from MEC students’ discourse.
In the next chapter, I provide further information about the MEC and MDPT courses at the university under study, including some details about the curricula and key features of the courses. I thus aim to explain the contexts in which teachers’ deep understanding of mathematics may be developed through participation in these courses.
Chapter 3

Developing subject knowledge for mathematics teaching: the higher education landscape

Chapter introduction

The research presented in this thesis is located in the context of two specific subject knowledge enhancement courses, both of which were funded by the UK government and available nationally at the time of writing. Both were run at the university where I work. This chapter sets out the political and societal background to these subject knowledge enhancement courses. A description and analysis of key features of the courses is given. I also give details of my involvement in Adler’s QUANTUM research project in which MECs were investigated, as this was a key influence on my own research.

3.1 National context and government initiatives

I remember being encouraged in my study of mathematics, and later in preparation for mathematics teaching, by well-wishers who assured me that mathematicians were in short supply and my skills would always be in demand. That was a quarter century ago, and the situation remains the same today. There seems to be an endemic problem with the supply of appropriately trained teachers of secondary mathematics in Britain. Various writers have analysed the situation and proposed remedies. For example, Smith (2004) supported the recently formed subject enhancement courses, and recommended both increased funding to encourage the expansion of mathematics teacher training places, and enhanced financial incentives for mathematics teachers. Ticky & Wolf (2000) noted that mathematics teachers needed to be recruited from a wide range of disciplines, and in a comment that preceded subject enhancement courses, suggested that subject knowledge should be built into PGCE courses.
Teacher shortages are linked to pupil under-attainment in the subject (HMI report 2001/2, cited by Smith, 2004, p. 21). Shortages of trained specialist mathematics teachers have been reported consistently since the Cockcroft Report (1982). Since that time growth in the banking, insurance and commerce sector has drawn more mathematics graduates away from teaching. Additionally some commentators suggest that a lack of representation of people from STEM (science, technology, engineering, mathematics) backgrounds in government and the media underpins a general problem of widespread public misunderstanding of science and mathematics (Goldacre, 2009).

The UK government has responded to the challenge of teacher supply by funding various schemes such as extra bursaries for those training to teach in ‘shortage subjects’, financial incentives in the form of premium payments (colloquially, the ‘golden hello’) for teachers of some shortage subjects, and a widening of routes to achieve UK Qualified Teacher Status such as the Graduate Teacher Programme (GTP), the ‘Teach First’ programme (Royal Society, 2007) and School Direct (Teaching Agency, 2013a). The Mathematics Enhancement Course (MEC) and the Mathematics Development Programme for Teachers (MDPT) have been developed in this context. More details about these courses are given in sections 3.2 and 3.3 below.

3.2 The Mathematics Enhancement Course - developing a curriculum

The university where this study is based was one of the two where the MEC was piloted. The opportunity to design a bespoke subject knowledge course for beginning mathematics teachers was an exciting challenge for the tutor team. As noted in Chapter 1 above, there was awareness in the team of the limitations of conceptual knowledge held by some graduates coming to PGCE from traditional mathematics degree courses, and the fact that the timescale of the PGCE itself left little opportunity to tackle this explicitly. Clearly, mathematics degree courses are designed to suit the needs of a wide variety of people and to prepare graduates to enter a range of professions. Some people might think that university or academic mathematics is the same as school mathematics but harder and more in depth, and that
therefore studying university mathematics prepared one well for teaching school mathematics. However, as Lerman points out, “school woodwork is not carpentry… and school mathematics is not (academic) mathematics” (Lerman, 1998, in Watson, p. 34). Many teachers report that they do not use their knowledge of degree-level mathematics explicitly in their teaching, but perceive its usefulness more in their confidence and ability to make connections (Zazkis and Leikin, 2010). Davis (2011) conceptualises teachers’ mathematics as a learnable disposition rather than an explicit body of knowledge.

**Related initiatives and prior knowledge**

The MEC presented a chance to broaden and improve the knowledge base of graduate students as beginning teachers, and thus to do something which had not been done in this way before. The main precursor to MECs was the two-year PGCE, offered by a few universities at the time, in which the first year was mainly subject-knowledge based and the second was the professional training year. One may also consider that experience within the UK mathematics education community of the delivery of 20-day and 40-day CPD courses for in-service primary teachers during the 1980s and 1990s, whilst aimed at a different target group of teachers, was a related initiative which foregrounded the importance of subject knowledge for teaching. Taking an international perspective, it is not until 2011 that we see reports of what are termed ‘capstone’ courses for prospective secondary mathematics teachers in the US (Artzt et al., 2012) which appear to be similar in nature to the MEC.

Government funding, including bursaries to students, meant that there was a good rate of take-up of MEC courses. However there was not the same level of financial support from government for the pre-existing two-year PGCE programmes, and so although they may have offered an effective learning experience for beginning teachers, take-up was limited. (Subsequently the first year of the two-year PGCEs became subsumed in the overall SKE model, and subject to appropriate financial support).

In line with the idea of “profound understanding of fundamental mathematics” as developed in Chapter 2, the tutor team developed a curriculum for the MEC in which various strands were interwoven and which included rigorous undergraduate level mathematics topics as well as an in-depth approach to school curriculum topics. An ethos of questioning, understanding
why as well as how - in short an emphasis on deep as opposed to surface learning - was encouraged throughout. This was consistent with the team’s constructivist philosophy of learning mathematics.

**Key aspects of the course**

In order to establish what this particular Mathematics Enhancement Course might offer that is qualitatively different from a degree course in mathematics, it is helpful to consider some key features of the course and its organisation.

*Establishing a learning community and handling strengths and weaknesses in subject knowledge*

The course begins with a 24-hour residential experience at the university’s outdoor education centre in rural Wales. Student evaluations have consistently reported that this intensive start to the course enables them to bond rapidly within the group. It is seen by tutors and students as a highly effective strategy for course induction. During induction, students participate in group tasks, some of a general problem solving nature and others involving mathematical investigation. They also undertake individual needs analysis exercises. The shared experience of living, working and socialising together for a short time, away from the distractions of their everyday lives, is seen as a key component in the effectiveness of the induction programme.

From the outset of the course, tutors make explicit reference to their expectations that individual students’ strengths and weaknesses in terms of subject knowledge will vary according to their own particular backgrounds and experiences. It is acknowledged that a unit containing material familiar for some may present new ideas for others. Students are encouraged to work supportively in the class environment, whilst also tracking and recording their individual progress in subject knowledge development. Hodgen (2011) notes the importance of affective aspects of mathematics learning, and suggests that teachers need “space to question and enjoy mathematics and mathematics teaching” (p. 39). It is acknowledged that there is already a wide range of expertise within the MEC group, and students are encouraged to work collaboratively and share their expertise. The tutor team places a high value upon the benefits of collaborative work in mathematics, as recommended
for example by Swan (2006) and Lee (2006). Swan argues that collaborative approaches to learning are more effective than traditional ‘transmission’ methods. Lee discusses the importance of discourse in mathematics learning, and the need for learners to express their ideas and share meanings.

**Structure of course, breadth and depth of content**

Students perceive the MEC as an intense mathematical experience (Adler et al., under review). To embark upon the course, students must have studied mathematics to A or AS Level or equivalent, and familiarity with the content of the A Level syllabus is assumed. The course is taught over 20 weeks, 4 days per week. Additionally there are four private study weeks. The course includes a broad range of mathematics units of a level and content broadly commensurate with early first degree mathematics and A Level / Further Mathematics. These topics arose from consideration of the original MEC specification, mediated by the tutor team’s own judgements of what constitutes an appropriate curriculum for these students. Topics include Calculus, Group Theory, Matrix Algebra, Complex Numbers, Coordinate Geometry, Mechanics, Statistics and Discrete Mathematics. Units are shaped into series of usually 5 or 6 half-day taught sessions. In most sessions, an interactive lecture-presentation from the tutor is followed by or interspersed with workshop time, during which students tackle questions with tutor support.

Additionally there are some parts of the course that sit outside of the model described above. During the first half of the course, students engage in an extended open-ended investigation in which mathematical processes and communication feature highly. In the second half of the course, students pursue an individual enquiry of an area of mathematics of their own interest and choice. Initial stimulus for choice of the individual enquiry may arise from aspects of the taught course, or previous professional experience, or elsewhere; however their brief is to extend the material beyond the scope of the taught course. The intention is that these course components should be particularly helpful in enhancing students’ appreciation of the processes of thinking mathematically and making connections within mathematics, and of collaborative approaches.

**Peer teaching, school mathematics topics**

An important feature of the course is a unit on ‘misconceptions and fundamental mathematics’ in which key topics from the school mathematics curriculum are explicitly
‘unpacked’ (Adler and Davis, 2006; Ball and Bass, 2003). Students learn early in the course that it will not be sufficient for them, as mathematics teachers, to operate at an instrumental level of understanding (Skemp, 1976). They are encouraged to think carefully about concepts and processes located in elementary mathematics, which they may previously have taken for granted. The tutor team thereby hopes that these students will avoid developing the “expert blind spots” (Ruthven, 2011, p. 88) which can for some teachers obstruct effective mathematics teaching. Teachers need to be able to deconstruct or unpack mathematical ideas and processes for the benefit of their own students; they may not take for granted even simple ideas.

In the ‘peer teaching’ strand, students are put into pairs and each pair is allocated a mathematics topic from the school curriculum at Key Stage 4. Pairs of students are asked to prepare some material from their topic, and explain it to the rest of the group. In this way, over time a wide variety of school mathematics topics are covered, and the group benefit from each others’ preparation and research. Careful preparation by a tutor supports this activity, and the focus is upon accurate and meaningful communication of key ideas in mathematics. This aspect of the course perhaps most closely reflects Ma’s (1999) recommendation to strengthen the interaction between teachers’ study of subject and pedagogy. However the MEC is not an initial teacher education course, and to focus too heavily upon pedagogy would be inappropriate. In the MEC, we can begin to address Ma’s ideas, leaving more of this to follow later in the PGCE year.

There is a strong emphasis through the whole course upon students’ active engagement with mathematics. However the fundamental mathematics, peer teaching, and open investigation strands of the course probably provide the strongest links in terms of students restructuring their existing knowledge (Ruthven, 2011) and understanding how teachers enact mathematics (Watson & Barton, 2011).

School attachment
Unlike many other MECs, this course includes an element of school attachment. Pairs of students are placed in schools to observe mathematics classes. Each student has four days’ attachment in each of two schools. Their brief is to observe children’s learning of mathematics and look out for any misconceptions they might have. The emphasis is upon the learner: MEC students are not expected to analyse pedagogy, as this is takes place later
within the PGCE. Another feature of the MEC is the ‘misconceptions intervention’ project, in which MEC students work with groups of children identified by their schools, who visit the university for some focused teaching and support on specified areas of the curriculum. The taught elements of these sessions are led by Local Authority teacher advisors, and the MEC students work intensively in a support role, each with one or two children. The school attachment and the intervention project lead into a piece of written work, in which MEC students explore misconceptions in the learning of mathematics. The tutor team was clear from the outset that school attachment should be an important part of the MEC. School is where all the MEC students aspire to be, and an attachment provides a context and motivation for their course of study, whilst simultaneously allowing them to begin to think more deeply about how children learn.

Assessment

Assessment of elements of the MEC takes many forms and includes extended project work, tutorial sheets, unseen ‘open-book’ assessments taken under test conditions, and both group and individual presentations. Assessment is part of the learning process and an aim is for students to be able to demonstrate, in a variety of ways, their growth in knowledge and understanding of areas of mathematics and their ability to make connections.

Links between research literature and the Mathematics Enhancement Course

The ideas that discussions about content should be relevant to teaching, and discussions about teaching should be mindful of content (Ball et al., 2008), have resonance with the philosophy of the Mathematics Enhancement Course. I believe that the ideas of Ball and Shulman are highly relevant to the course, and are readily seen in the learning and teaching that takes place therein. Decisions made by the tutor team about the types of tasks that students undertake, and about what course content is appropriate for MEC and what properly belongs later within the PGCE remit, are all illuminated by the categorisation models of Ball and Shulman. For example, short placements in school and the intervention project working with children in the university setting, both provide opportunities for students on the MEC to start to learn about children’s common misconceptions in the learning of algebra. When they move on to the PGCE course later, they will learn explicitly about effective approaches in the teaching of algebra, and ways to draw out and overcome children’s misconceptions. In this
instance the MEC is offering some of Ball’s ‘specialised content knowledge’ (SCK), and PGCE is offering pedagogical content knowledge in the form of Ball’s ‘knowledge of content and teaching’ (KCT). So students’ learning on the PGCE will be underpinned by earlier experience and understanding gained through the MEC. Artzt et al. (2012) comment upon a programme similar to the MEC, for prospective secondary mathematics teachers in the US, in which higher level ‘college’ mathematics is linked with school mathematics and pedagogy in an integrated way. They argue that opportunities to study the school mathematics curriculum “from an advanced perspective” are “essential for teachers’ preparation to teach mathematics meaningfully” (p. 261). Also relevant in the design of the MEC are the arguments put forward by Thanheiser et al. (2010) about the importance of using our understanding of mathematical knowledge for teaching as a framework for course design. The authors promote a series of principles to underpin course design in the mathematical preparation of prospective teachers. One of these is “We teach our preservice teachers in the same way we want them to teach their classes” (p. 4). From my contact with other teacher educators, I am aware that this subtle modelling with an adult group of best practice in teaching younger learners is a common feature of preservice teacher education in the UK. It is seen within the MEC and other courses.

I note that when the MEC was designed, the tutor team at the university in this study were not familiar with much of this research. The course was designed in response to government demand, and through the ideas and knowledge of a team of tutors who were experienced in both teaching mathematics as a subject, and teaching mathematics education courses. Interest in associated research really began as a result of the stimulus of designing and teaching this course. It was interesting subsequently to discover that many elements of the course that, from their professional experience the team judged to be successful were in fact clearly supported by research.

**Feedback from external evaluations**

The Mathematics Enhancement Course has been subject to periodic external evaluation, most recently by OFSTED in 2008. Some of the early feedback from external evaluation is worth noting here, specifically because the focus was upon what was then a brand new concept for a course, and the evaluator was considering this in comparison to existing provision. I describe the MEC as a ‘brand new concept for a course’ because although ‘bespoke’ mathematics
provision for intending teachers existed prior to this, for example within BEd degrees, these were aimed at undergraduate students and often taught along with other subjects and over an extended period of time, two or three years, in contrast to the intensive six-month mathematical experience offered to graduate students who follow the MEC.

In his evaluation of the first year of the pilot MECs at two universities, Seabourne (2004) found that, with regard to course content, “there is a delicate balance to achieve between depth and breadth, which both courses were largely successful in achieving” (p. 2). Here we can interpret ‘depth’, as discussed in Chapter 2 (p. 41), as a profound understanding of the mathematics studied, and ‘breadth’ as a variety of interconnected mathematical topics taught so that students are able to understand the links between them.

Seabourne also commented on the quality of the provision, especially since ‘nobody had prior experience of providing such an intensive programme of subject knowledge development’ (p. 3, my emphasis) and of the benefits gained by students from being taught by teams of tutors who were both good mathematicians and also completely familiar with the requirements of the school mathematics curriculum. He commented that offering MEC students elements from courses designed for other target groups could not achieve the same benefits as this approach. It was the experience of the team that the opportunity to design a ‘bespoke’ mathematics course was, and remains, an exciting challenge. Recognition that courses taken ‘off the peg’ could not match this particular design in fitness for purpose was an encouraging spur to maintain and develop a course culture with strong interactive values.

In his interim report, Seabourne (2005) followed his 2004 sample into their PGCE year. He noted that several former MEC students reported that they were envied by fellow PGCE students because they ‘hit the ground running’ at the start of the PGCE course. He also noted that schools were impressed by the ex-MEC students’ “attitude to their subject knowledge and their drive to understand a topic fully before teaching it” (p. 5). In his final report, Seabourne (2006) tracked his 2004 sample through to their first year of teaching. He comments upon MEC students’ “exposure to a variety of high quality teaching and the opportunity to experience new approaches to learning mathematics”, describing this as a “significant incidental legacy” of MEC (p. 3). Many of the MEC students commented to him on the significance of the way in which they learned mathematics on MEC as well as what they had learned.
Thus Seabourne’s view was clearly that the pilot MECs were meeting the requirements of the course specification in making a significant contribution to students’ development of subject knowledge. It was also apparent that a course that was originally intended to be a straightforward subject knowledge enhancement programme was in fact delivering more, through the modelling of good practice by experienced tutors as well as by other more explicit means such as the analysis of school mathematics topics. Seabourne’s tracking of a sample group of students also showed that the positive effects of the MEC experience remain into the teacher training year and beyond, as the teachers begin their professional careers. With the exception of a few updates, the MEC curriculum at this university has remained largely the same since the pilot; therefore I believe it is reasonable to apply these comments to the course as it is today.

3.3 The Mathematics Development Programme for Teachers

The UK government invested heavily in the MDPT between 2007 and 2011. The course was provided free to participants, and funds to meet the costs of supply teacher cover were available to the schools at which course participants were employed. This meant that it was possible for the course to run on weekdays in school time, so that teachers did not have to do the course in their own time and could come ‘fresh’ to taught sessions (i.e. not tired after a full teaching day). Additionally, a personal financial incentive of £5000 was available to all participants who achieved appropriate academic accreditation associated with the course. Much of this funding was removed for the new 2011-12 course, and at the time of writing only course fees are now funded by the government. This has had implications for course design, which has changed significantly.

A key part of the specification for the MDPT was that course providers had to link progression on the course to academic accreditation at Level 6 (undergraduate Honours degree) or Level 7 (Masters degree), or a combination of both. This immediately presented a challenge to the universities involved. For the majority of course participants, the highest
mathematics qualification that they held was GCSE level C or equivalent (Level 2). Just a few had studied mathematics to A Level (Level 3). Clearly the mathematics subject knowledge encountered on the course was not going to approach the required accreditation level. The course participants, all serving teachers, could however be expected to be able to write about subject pedagogy at the level required. Accordingly, formal course assessment tasks linked to accreditation were developed in the form of critical reflections upon approaches to teaching and learning certain aspects of the mathematics curriculum, i.e. on pedagogical content knowledge. Thus the levels aimed at (6 or 7) were interpreted in terms of pedagogical subject knowledge of mathematics rather than academic mathematical knowledge.

**Key aspects of the course**

In developing an ethos and curriculum for the course, the team was able to draw on the experience of running the MEC successfully. Establishing a strong learning community is regarded as important, and the course begins with a residential induction in a local hotel. Although the overnight stay is optional, participants are encouraged to take part and almost all do so. As with the MEC, an explicit supportive and collaborative approach is established from the start of the course, with openness to variety in peoples’ strengths and weaknesses. As an ‘immersion’ model is believed to be effective, the taught course was structured in the form of two-day (Thursday and Friday) events roughly once per month for twelve months. This enables participants to leave behind the concerns of their working week for two days and immerse themselves in mathematical activity.

Key aspects of the school mathematics curriculum are explored and discussed in taught sessions, with explicit links being made between the strengthening of participants’ personal subject knowledge, and the development of their knowledge and understanding of effective pedagogical approaches. Common misconceptions in mathematics topics are also part of the content taught. Not only should this enhance teachers’ understanding of their own students’ learning and the barriers to it, but it is also intended to provide a ‘safe’ (i.e. personally unthreatening) context for their own mathematical misconceptions and partial understandings to emerge. Thus they should be able to move on and develop their knowledge and understanding.
Another important strand of the MDPT is to encourage participants to identify with the wider mathematics community, and opportunities are provided for them to take part in local subject association activities such as children’s masterclasses and subject meetings. Course funds are made available to support teachers who wish to take part in relevant conferences such as the ATM (Association of Teachers of Mathematics) or BETT (British Educational Training and Technology). All course participants register with the NCETM (National Centre for Excellence in Teaching Mathematics), and all use the NCETM online self-evaluation tools to help them to establish their subject knowledge training needs and to track their progress in development of subject and pedagogical knowledge.

**Links between research literature and the Mathematics Development Programme for Teachers**

The concepts of ‘key developmental understandings’ (Silverman and Thompson, 2008) or ‘threshold concepts’ (Atherton, 2008; Meyer and Land, 2003), are particularly interesting when applied to the learning of mathematics. A substantial amount of mathematics can be thought of as hierarchical, so that lack of full understanding of one key idea can prevent one from grasping another which is built upon it. For example, limited understanding of how to manipulate and solve linear equations will often prevent a student from being able to tackle quadratic equations. There are many such examples: I choose this one because it is something that I have encountered with teachers on the Mathematics Development Programme (MDPT). Working with these teachers and supporting them while they deepen their own understanding and confidence with algebra, I have observed the uncomfortable process of accommodation, followed by the satisfaction and ‘light bulb’ moment when ideas start to come together and make sense. I have also noted, when their own understanding has been deepened, how quickly the teachers make links in terms of the changes they want to make in their own teaching for the benefits of their own students.
In-service training for mathematics teachers: positioning teachers as learners

The Mathematics Development Programme for Teachers (MDPT) is a relatively new addition to the UK mathematics education field, and as yet there has been little research on this programme. Work in this study makes a contribution to addressing that gap. Croisan and Rodd (2011) discuss the effects of the MDPT course in developing participants’ identities as mathematics teachers within a community of practice. Vale, et al. (2011) report on a course in Australia, designed for ‘out-of-field’ (i.e. non-specialist) mathematics teachers. Drawing upon theoretical foundations provided by Shulman, Ball, Ma and others, they discuss the design of a programme for these teachers and they report some of the teachers’ responses to the programme. They conclude that,

“Positioning practicing teachers of secondary mathematics as learners of mathematics affords the opportunity for them to make connections with more complex concepts and to appreciate structure” (p. 209).

They also note that a reasonable length of programme is important for teachers’ mathematical development, as is the quality of relationships established with colleagues whilst they are on the course. All of these findings are in common with my own observations of successful aspects of the MDPT course at the university in this study.

3.4 My background with the QUANTUM project

My active involvement in mathematics education research really began in 2008 when I joined the QUANTUM project (Qualifications for Teachers Underqualified in Mathematics) led by Jill Adler. The aims of the QUANTUM project are:

1) to probe what is constituted as ‘mathematics for teaching’ in contexts of mathematics teacher preparation,

2) in the UK context, to investigate how the MEC as an example of a mathematics course in the context of teaching, can illuminate the discussion on ‘mathematics for teaching’.
In order to gain access to MEC students, Adler was actively seeking to work with current MEC tutors. I was one of these tutors who joined the group. Thus, four MEC tutors based at three universities, all of whom were novice mathematics education researchers, had the opportunity to take part in a collaborative research project and hence to develop their research skills. This was a formative learning experience for me.

At the outset of the project, Adler and Hossein conducted interviews with the MEC tutors who make up the rest of the research group (Archer, Clarke, Grantham and Stevenson). They sought the tutors’ conceptions of what understanding mathematics in depth (UMID) meant, and how it developed. Tutors were asked open questions about this. Later these statements were used in the ranking exercises in student interviews.

In-depth interviews were then held with 18 MEC students, 6 from each of the three different institutions, towards the end of their course in summer 2009. Interviews were carried out by pairs of researchers, with one experienced researcher and one novice researcher (MEC tutor) in each pair. MEC tutors interviewed students from other institutions and not from their own. Students were asked to describe the MEC as if to a prospective student, and to discuss mathematics they had studied on the MEC and activities in which they had taken part. Students were also asked about their conceptions of ‘understanding mathematics in depth’ (UMID), and asked to rank statements describing UMID and how UMID might be attained. In analysis of student interview data, particular attention was given to the ideas that students foregrounded when discussing the course and when discussing UMID. I have discussed some of the findings from this research in Chapter 2 (p. 50).

Further interviews were carried out with the same students one year later as they approached the end of their PGCE course. This data has not yet been analysed.

I had decided that I wanted to research the idea of Understanding Mathematics in Depth as it was played out in the MEC and MDPT courses. Hence my own research moved on in parallel with the UK QUANTUM project, and I found the support of the QUANTUM team very helpful.
Chapter conclusion

This chapter has provided background and context to the thesis in the form of details about the two courses which feature in my research, touching on the political and societal climate in which these courses have developed. I have also described my involvement in the QUANTUM project. Reflecting upon the higher education landscape and the government initiatives that have framed a large part of my professional work in recent years, there is a tension in that these opportunities for the development of ambitious and divergent programmes have taken place against a backdrop of increasing centralised regulation and prescription of the work of teachers. Davis (2011) notes that sequenced and linear curriculum structures may be “incompatible with the goal of deep understanding” (p.1507).

More recently, since the inception of the Coalition government in 2010, the political landscape has changed again. Prescription of the work of teachers is being reduced, and curriculum guidance stripped away to a minimal level of information. Schools are being given more freedom over what is taught, and how. At the same time, a climate of economic austerity is bringing about severe cuts to provision of many services dependent on public money. Thus the future of SKEs is, at the time of writing, insecure.

In the next chapter I will present the results of some quantitative data analysis, which will inform my research design and approach.
Chapter 4  Contextual quantitative data

Chapter introduction

In this chapter, I discuss literature informing current debates about the interpretation of teachers’ qualifications, and what they can indicate about subject knowledge for teaching. I present quantitative data relating to qualifications of PGCE Mathematics students at the university in this study. Discussion of admissions policy to mathematics PGCE courses is also included, as this is relevant to the debate around preparation for PGCE.

In the context of a fragmented and diverse international landscape of mathematics teacher education (Tatto, Schwille, Senk, Ingvarson, Rowley, Peck, Bankov, Rodriguez and Reckase, 2012), I consider the contributions of Monk (1994), Prestage and Perks (2001) and Tennant (2006) to the debate about meaning and interpretation of teachers’ formal academic qualifications, and how these interact with discourses on mathematics knowledge for teaching (Zaskis and Leikin, 2010; Davis, 2011).

To give context to the interpretative research presented later in this thesis, quantitative data on groups of pre-service mathematics PGCE students is also presented here. The aims are firstly to investigate the nature of the relationship (if any) between degree classification and effectiveness as a teacher, and secondly to investigate whether there are any differences in subject knowledge and overall performance, between former Mathematics Enhancement Course (MEC) students and others. This data is presented as background contextual data which informs the main qualitative study that follows.

4.1 Research on teachers’ formal academic qualifications: meaning and interpretation

From an international perspective, routes into mathematics teaching, content and organisation of courses, and means of assessment of prospective teachers’ success, vary widely (Tatto et
al., 2009; Stacey, 2008). When researchers probe the way that teacher preparation is organised, they find wide variations in site, context and level of regulation; the range of approaches used is diverse. The question of concurrent (subject and pedagogy) versus consecutive preparation is raised. The recently published Teacher Education and Development Study in Mathematics (TEDS-M) report (Tatto et al., 2012) provides a detailed analysis of approaches to preparation of primary and secondary mathematics teachers in 17 countries, revealing major differences in mathematics knowledge outcomes of prospective teachers, both within and across countries, and a great variety of approaches to the organisation of teacher preparation.

One might expect there to be a link between teachers’ study of higher level mathematics and their students’ achievement. However there is little evidence to support this. Begle (1979) concluded that beyond a certain threshold of mathematical understanding, further formal study of mathematics did not lead to increased student achievement. Monk (1994) conducted a study in the U.S.A. investigating the relationship between teacher preparation and student achievement. Variables used for measuring teacher preparation included the number and nature of mathematics subject courses studied by the teacher at university (or the number of credits). He found that university courses in mathematics pedagogy taken by teachers contributed more to their pupils’ performance than university courses in mathematics. He also found that a teacher having a mathematics major had no effect upon pupil performance, and the classification of degree had a zero or negative effect. An exception to this was in case of teachers teaching advanced mathematical courses, where the number of mathematics courses the teacher had followed did have an effect upon their students’ performance. These results are consistent with other research (Shulman, 1986; Ball & Bass, 2003; Adler & Davis, 2006) discussed in Chapter 2, (p. 23, p. 20, p. 47) which highlights the need for teachers to be able to unpack or decompress knowledge, to have well-developed pedagogical knowledge including knowledge of their students, and to have knowledge enabling them to make suitable choices of examples in their teaching. These characteristics are generally not acquired through following typical university mathematics courses, but should be developed through participation in courses on pedagogy.

Monk (1994) concludes that

“a good grasp of one’s subject area is a necessary but not a sufficient condition for effective teaching” (p. 142)
and that

“gross measures of teacher preparation (such as degree levels, undifferentiated credit
counts, or years of teacher experience) offer little useful information for those
interested in improving pupil performance”. (p. 142)

I find the phrase ‘gross measures’ in this context helpful. The suggestion is that researchers
have easy access to these measures, and that some people might think they are useful
predictors of effectiveness in teaching. But research clearly indicates that this is not the case
– the measures are too crude, they do not get to the heart of what it means to be ready to
teach. As we have seen, subsequent work devising items that can test pedagogical subject
knowledge, has since moved the research on.

Zaskis and Leikin (2010) define Advanced Mathematical Knowledge (AMK) as knowledge
acquired during undergraduate study. Their study finds that most secondary school
mathematics teachers claim not to use their AMK directly in their teaching. Rather, they see
the advantage of it in terms of their own personal confidence, ability to make connections
between different areas of mathematics, and to deal with students’ questions. There is a
perceived discontinuity between mathematics learned at school and that studied later at
university level. Zaskis and Leikin recommend that university courses integrate more closely
AMK and mathematics for teaching.

Goulding et al. (2003) report the views of mathematics PGCE students about their prior
experience of undergraduate mathematics, and discuss how this might productively be used
in their training as mathematics teachers. The study reports clearly the views of students in
three areas. The first area is the transition between school and university mathematics, which
was often perceived to be difficult. Secondly, the struggle faced by students required to
tackle challenging mathematics - perhaps for the first time in their mathematical careers.
Thirdly, students comment on the style of teaching and assessing which was seen by many as
ineffective and unresponsive. This paper suggests again a discontinuity between approaches
to mathematics learning between school and university, and adds weight to the claim that
more careful integration of the two would be beneficial.

Research evidence suggests that it is not necessarily those teachers with the highest academic
qualifications who are the most effective classroom practitioners: the situation is more
complex than this. Hill et al. (2008) discuss the fact that in early work on subject knowledge
for teaching, scholars used ‘proxy variables’ such as the number of mathematics courses studied, “to stand in for direct measures of teacher knowledge…” (p. 432). They comment that this approach found few stable effects, and that there was then a move to utilise more descriptive studies to investigate teachers’ mathematical knowledge. Davis (2011) argues that mathematics teachers’ knowledge is subtle and tacit - like playing the piano - “learned but not readily available to consciousness” (p. 1506). Taking this view of knowledge for teaching, he suggests that there is therefore little reason to expect a correlation between courses in formal mathematics taken by teachers, and their students’ gains. He describes university courses in mathematics as focusing on “completed ideas, wrung free of the messiness involved in coming to a new insight” (p. 1507). This is in contrast to the complex and unpredictable world of the classroom, where teachers enable the learning of young students tackling new and emergent ideas in a variety of ways. His suggestion (p. 1507) that teachers’ mathematics could be seen as disposition rather than a body of knowledge has resonance with the views of Watson and Barton (2011) and Askew (2008) (see Chapter 2, p. 38, p. 39).

Harries and Barrington (2001) claim that “a high level of subject content knowledge is not a pre-requisite for becoming a successful [primary] mathematics teacher” (p. 30), but that having a holistic view of the subject and its inter-connections is. This work supports the findings of Askew and the King’s College team (1997). This view is supported by Goulding and Suggate (2001) who state “it seems obvious that how teachers know their mathematics is important” (p. 42, my emphasis). They suggest that the connected view of mathematics that the King’s College team (Askew, Brown, Rhodes, Johnson and William, 1997a) found in effective teachers may have developed over time during their career, as they planned work and interacted with pupils, and as they attended in-service training courses. The King’s College study found that highly effective (primary) mathematics teachers were more likely than other teachers to have taken part in mathematics in-service courses. This suggests a link between participation in continuous professional development (CPD) activities and teacher effectiveness, but we cannot assume this is a simple causal link. The link may reflect the fact that enthusiastic and successful teachers are more likely than others to seek opportunities to improve their understanding and practice. Although the authors quoted above were studying primary teachers, I believe their work also informs my study of secondary student teachers, as there are some key trends and ideas common to both groups.
Prestage and Perks (2001, p. 101) distinguish between “knowledge needed to pass an exam” (denoted “learner-knowledge”) and “knowledge needed to help someone else come to know that knowledge” (“teacher-knowledge”), stating that “the first is necessary but not sufficient for the latter” (p. 101). They propose a model that assists in thinking about the distinctions between these types of knowledge, and they stress the importance of ongoing teacher reflection if the process of transformation of learner-knowledge into teacher-knowledge is to continue over time. This is an important point: teachers are not ‘created’ in the space of a year’s course; this is just the beginning, and continual development and reflection are necessary for growth of expertise.

Further, Prestage and Perks extend their model to another level and create a framework for conceptualising mathematics teacher education, and a pedagogy for mathematics teacher education:

“... just as [mathematics teachers] need fluid and connected knowledge of mathematics (teacher-knowledge) so too mathematics educators need an articulated, fluid and connected understanding of teaching mathematics education – the teacher-knowledge of mathematics education” (op. cit., p. 110)

This argument has interesting connotations for the knowledge and expertise needed by tutors on both mathematics PGCE courses and on mathematics enhancement courses. The fact that PGCE tutors need this meta-level of teacher-knowledge is probably uncontentious, given the wide and complex range of ideas encountered in such programmes. But what about MEC tutors? Although MECs were, and are, essentially set up to be subject knowledge courses, in fact universities have considerable freedom to interpret this and to devise appropriate curricula as they see fit. It is therefore not surprising that there is in many courses a focus upon pedagogical knowledge as well as pure subject knowledge, and that tutors who are leading in these areas of the curriculum are those with mathematics education teacher-knowledge. MECs are precisely aimed at developing mathematical knowledge for teaching, whereas mathematics degree courses cover mathematics for many possible ends; mathematics graduates may go on to pursue careers in areas such as accountancy, finance and economics.

Tennant (2006) studied students on a secondary mathematics PGCE. He investigated the relationship between formal academic qualifications, as measured by classification of first degree, and effectiveness in initial teacher training as measured by level of performance in the Standards for Qualified Teacher Status. Tennant found there was no correlation between
his students’ degree classification and their level of success in initial teacher training. His study was subject to a number of limitations, which are discussed later. However this is an important result with implications for PGCE mathematics admissions criteria and selection processes. Tennant suggests that students with a degree in mathematics may actually have gained a narrow understanding of some areas of the subject, with little sense of the overview and connections between areas. Interestingly, this is where today’s bespoke subject knowledge enhancement courses may become relevant. Tennant contends that in mathematics, degree results cannot reliably be used as indicators of subject knowledge for teaching. This begs the question of what indicators could or should be used, and how admissions tutors might best make decisions about entry to PGCE. Tennant’s findings have great resonance for me, as years of selecting and teaching PGCE students has led me to believe that it is far from clear that academic qualifications in mathematics can predict success as a teacher of the subject, and that the picture is much more complex.

In contrast, Parkes (1989) found degree classification, along with professional commitment, and strong motivation to enter teaching as a career, to be strong indicators of overall performance on a PGCE course under study. I assume this to be a secondary PGCE course including various subjects, but this is not explicitly stated. Parkes notes that it might not be a surprise that degree class predicts success in PGCE coursework tasks. However she adds that it is surprising that degree class also appeared to predict teaching ability - an aspect of the PGCE course that is dependent upon good interpersonal skills. Parkes suggests that it may be the case that high academic ability gave students greater confidence in managing classroom learning; alternatively she points out that the interview selection process may already have screened out those applicants deemed to be temperamentally unsuitable for teaching.

Clarke (2008) states that Enhancement Courses are part of the ITE landscape today, and are under-researched. In some early exploratory work (Stevenson, 2008), I used various measures to explore the effectiveness of the MEC at one university, to try to gauge its success in strengthening and deepening students’ subject knowledge. There were two strands to the investigation: a (qualitative) review of external evaluations and student evaluations, and analysis of data. The data analysis focused upon ‘entry’ and ‘exit’ scores for PGCE mathematics students, comparing MEC with non-MEC students. I recognised the limitations inherent in the use of these gradings of student teachers’ performance; however, much rests upon these judgments and so I felt it was justifiable to use them. I also recognised the problematic nature of degree classification as a proxy for subject knowledge. My review of
evaluations gave evidence of the course’s success in meeting the aim above, although I was cautious about this, bearing in mind the subjectivity of students’ responses in particular. Analysis of grades on exit from PGCE showed no significant difference between the two groups of students, overall or in terms of subject knowledge only. Also, there was no relationship between entry and exit grades. Thus my research supported Tennant (2006). I was interested in both of these results and decided that they would be worth further study; this led me to collect further data. The data I presented (Stevenson, 2008) was from a 2006-7 cohort of PGCE students. This data is re-presented in this chapter, as part of a larger sample, spanning three cohorts of students between 2006 and 2009.

I believe that issues about the nature of mathematics knowledge for teaching are fundamental to the processes of decision-making in the selection of potential candidates for initial teacher education programmes such as PGCE. This is why it was relevant for me in this thesis to examine quantitative data on PGCE students’ entry and exit grades, looking at both MEC and non-MEC students, and to explore further what information this reveals about the relationship (or not) between academic qualifications and success in initial teacher education. Research shows that formal academic qualifications alone have limited use in predicting or measuring knowledge for teaching. It is clear to me that in making judgements about graduates’ potential for success in teaching, reliance upon applicants’ academic qualifications alone is insufficient. A number of criteria need to be probed at interview. Those directly relevant to mathematical knowledge for teaching include applicants’ ability to take a discursive approach to mathematical ideas, to decompress simple mathematical processes, to communicate clearly, and to recognise the importance of the role of the learner. All these aspects are present in the literature discussed above. By exploring these areas, admissions tutors can make judgements about applicants’ potential for developing an understanding of mathematics in depth, which is so important for successful teaching.

4.2 PGCE recruitment and selection

In this chapter, I investigate the entry and exit grades of mathematics PGCE students, some of whom have followed the MEC. Therefore it is relevant to comment here about the context and the way in which decisions are made for admission to graduate teacher training courses.
Some universities screen out PGCE applicants with lower degree classifications, so that for these candidates, their other experience or merits are not considered and they do not proceed to interview. Currently the government is offering variable financial incentives to PGCE mathematics students dependent upon degree classification (Teaching Agency 2013b). For me, these decisions made by some universities, to reject an applicant early in the selection process on the grounds of degree classification, bring into sharp focus the very essence of my enquiry, since these decisions are based upon implicit assumptions about the nature of knowledge required for teaching, and how it is measured. Traditional views of teacher knowledge would have held that mathematics teachers’ subject knowledge was reflected in the amount of undergraduate and graduate study of advanced mathematics they had undertaken. But over the past two decades there has been a question over the extent that academic qualifications are good predictors of ability to teach. Clearly as a society we want our teachers to be well educated – however, there is growing research evidence that teachers with the highest academic qualifications are not necessarily the most effective classroom practitioners (Hill et al., 2008; Askew et al., 1997a; Tennant, 2006). Something more than straightforward knowledge of the subject seems to be needed. Askew et al., (1997b) suggest that teachers’ beliefs about the nature of mathematics are more important to their pupils’ gains than are their formal academic qualifications, and they link this to participation in in-service training: “Participation in extended courses of professional development in mathematics was strongly related to belief orientation and with pupil gains” (op.cit., p 335). What it is that constitutes a teacher’s deep knowledge and understanding of mathematics is something that is vigorously discussed (e.g. Ruthven, 2011; Hodgen, 2011) and forms the focus of this study (see Chapter 2). Developing appropriate tools to measure this understanding is a further challenge; the work of Ball and team (also discussed in Chapter 2), is significant here.

The current climate in the UK, in line with other European nations, is for teaching to move towards becoming a Masters profession, and many teachers currently enrolled on Masters degrees in Education, or the Masters in Teaching and Learning (which was funded for a short time by government), are starting to research their own practice and develop as reflective practitioners. It can be argued therefore that it is important to recruit teachers with strong academic qualifications in order that they can progress to higher level study later. However, pursuing further study and research in service is not the same as being required to have, for example, a 2.1 degree classification for admission to a PGCE programme. Some universities,
encouraged by government, continue to operate admissions procedures based on class of degree. There is a linked question to be raised here about the place of subject knowledge enhancement courses (SKE’s) in the admissions landscape. Possession of an SKE is accepted as an alternative to degree qualification in a subject. In this situation, to what extent is the candidate’s former degree classification relevant?

The MEC was devised to provide a route into teaching different from but equivalent to traditional degree routes, for graduates of disciplines other than mathematics. In the first pilot year, MEC students naturally raised the question of whether they would be at a disadvantage when applying for jobs, compared with other applicants who had followed more traditional routes. The tutors’ belief then was that MEC students would be at least as well prepared for teaching as those from other routes. Since that time, the experience of the tutor team at the university where this study is undertaken is that ex-MEC students have had no difficulty securing employment as mathematics teachers in most contexts. There are a few exceptions: applicants to some schools in Northern Ireland and some private schools in England have found that a degree in mathematics was a requirement. This evidence might suggest that there may remain a degree of elitism, or perhaps ignorance, in some contexts. The contextual quantitative data analysis given below shows that there is no evidence to suggest that, among students at the university under study, a person with a degree in mathematics is any better prepared for secondary school mathematics teaching than one who has followed the MEC. An exception to this might be recruitment to jobs requiring the teaching of advanced level mathematics such as academic sixth form colleges, a view that is supported by Monk (1994).

4.3 Quantitative data: analysis of entry and exit grades

Data was explored, to investigate,

1) the nature of the relationship (if any) between degree classification and success at PGCE, at my institution,
2) whether there are any differences in subject knowledge and in overall achievement upon completion of PGCE, between former Mathematics Enhancement Course (MEC) students and others.

Data was collected from three cohorts of mathematics PGCE students at one university, between 2006 and 2009. All cohorts comprised students from both MEC and degree mathematics backgrounds. The total number in the sample is around 106 (there are slight variations in sample sizes due to the availability of data for some students). Notwithstanding the limitations regarding the nature of this data (discussed below), the data used in this part of the study essentially mimics the admissions process at the university under study, and thereby is intended to shed light upon it.

**Nature of the data: exit grades**

At the university under study, PGCE student teachers’ performance across aspects of professional knowledge, teaching, planning and assessment are graded on a five point scale, consisting of three pass grades (1, 2, 3) and two fail grades (4, 5). This grading structure mirrors that used by the Office for Standards in Education (OFSTED) in its inspections of teaching, and is a framework that is widely understood in the profession. The grading structure is used both formatively during the course, and summatively to give an indication of achievement at the end of the course. Grades are awarded both by university tutors and mentors (experienced teachers in school).

At the time the data was collected, the standards for Qualified Teacher Status (QTS) were organised by this university into ten groups, as follows:

1. The Professional Context
2. The Reflective Practitioner
3. Subject Knowledge
4. Children’s Agenda
5. Managing Learning
6. Planning and Teaching
7. Assessment and Planning
8. Literacy, Numeracy and ICT
9. Personalised Learning
10. Remodelling Agenda
The grades awarded at the end of the course and reported here appear in two forms for each individual – a grade representing level of subject knowledge, (no. 3 above) and an overall mean grade of all ten groups, representing achievement across all the standards for Qualified Teacher Status (QTS).

Grades awarded to student teachers during the course by experienced teachers are moderated regularly by tutors from the awarding university. I believe that the system is robust and meaningful insofar as it is not in the interest of schools or universities to grade weak students too highly. Schools usually grade students quite strictly, since within the OFSTED framework, grading of experienced teachers is now a common phenomenon and there is a natural trend to compare new teachers with their experienced colleagues. Universities are also answerable to OFSTED, who inspect accuracy of grading by observing current and former student teachers teaching in school. The university in this study is mindful of its regional reputation; partnerships have been developed over many years and with a large number of schools in the area, and the university wishes to be known to be upholding the highest standards of rigour in assessment.

Therefore accuracy of grading, as far as this is possible to achieve, is encouraged. However I am aware of the need to exercise caution here. I do not know of any formal research findings into the reliability and validity of this grading system. In fact, given the variety of different persons assigning grades, within a system that requires subjective judgements, it would be very difficult to carry out such research. This is a limitation in this aspect of the inquiry, and to that of Tennant (2006) discussed above.

**Nature of the data: entry grades**

In order to compare entry and exit grades for the PGCE mathematics students, a decision needs to be taken about how to represent degree classification. Here the study uses a simple framework similar to that used by Tennant (2006) in seeking to determine whether any relationship can be established between formal academic qualifications prior to PGCE, and success in initial teacher training as measured by the grading system outlined above. To enable meaningful comparisons to be drawn up between scores on entry and exit to the course, it was necessary to convert degree classifications into an interval scale. This was done by scoring a first class degree (or higher degree) as 1, a 2:1 as 2, a 2:2 as 3, a 3rd as 4 and a pass degree as 5. This provides data on an interval scale which has the advantage of
being easy to analyse. However it is not intended by this analysis to suggest that, for example, a 2:1 degree is somehow twice as valuable as a 3rd. Degree classification data is ordinal and does not truly meet the equal-interval criterion (Preece, 1994). Degree grade standards and the level and scope of competence, understanding and knowledge indicated by them are variable and problematic for trustworthy comparisons. However, these data are used all the time in decision making for recruitment for employment and for higher level study. This is not because they are the only data available (employers and others might also interview, or probe other qualities of the applicant) but there is a strong belief that degree classification tells us something about the intellectual quality of the applicant.

There are therefore limitations to this aspect of the enquiry. Judgements or expectations about individuals’ potential for success in mathematics teaching are made on the basis of much broader criteria than merely degree classification. Cohen et al. (2007) note that ‘restricting, simplifying and controlling variables’ (p. 19) risks a reduction in the relevance of the results of an enquiry. However, this aspect of the enquiry deliberately mirrors and therefore probes the university’s admissions process, which can itself be described as exhibiting ‘positivism’s concern for control’ (op cit., p. 18).

**Ethical considerations**

All the data is confidential and anonymous; it is not possible to identify individuals from the data sets. It was not deemed necessary to obtain permission from individual students for this strand of the research. This is because all the data is held by the university, is available to university employees, and is presented simply as a set of numbers in much the same way as would be done for standard internal university quality assurance procedures and for external reporting to OFSTED.

**Comparison of PGCE entry and exit scores**

Paired data scores for entry and exit for 95 PGCE mathematics students were analysed. These 95 students were from the 2006-7, 2007-8 and 2008-9 cohorts at the university in this study. Students’ degree classifications on entry to PGCE were converted into an entry score of 1, 2, 3, 4 or 5, where 1 indicates the highest degree classification, as described above. All students who pass the PGCE course exited the course with a discrete grade of 1, 2 or 3 recorded against each group of ten QTS (Qualified Teacher Status) standards listed above,
where 1 is the highest grade. The mean of these grades constituted the exit score for each student in this study, thus generating exit scores in a continuous range between 1 and 3. Appendix 6 shows a scatter-graph of these paired entry and exit scores, showing entry score (discrete integers between 1 and 5) on the horizontal axis, and exit score (continuous data between 1 and 3) on the vertical axis. The relationship between entry and exit scores was investigated using Kendall’s tau test. Analysis shows that there was no discernible relationship between students’ level of academic qualification on entry to the PGCE course, and their level of success on the PGCE course, as measured by QTS score ($r = 0.129$, $n = 95$, $p = 0.1$). A few data items on degree classification on entry were unavailable, and hence this data set is slightly smaller than the one used below for analysis of exit scores.

The analysis above shows that there was no discernible trend and no correlation between students’ level of academic qualification on entry to the PGCE course, and their level of competence as a teacher on exit from the PGCE course, as measured by QTS score ($n=95$). Therefore, from this data sample, there is no evidence to suggest that a high scoring degree classification may be a predictor of success in initial teacher education. This is in line with the findings of Tennant (2006), and is supported by Monk (1994), Prestage and Perks (2001) and others. There are of course important limitations in the use of this data (discussed earlier). However in the absence of other available measures, it seems to me to be sensible to use what we have and to proceed, albeit with caution.

These findings are highly relevant to the debate about the nature of mathematics subject knowledge for teaching, and what constitutes deep understanding of mathematics. This data challenges the notion that degree classification alone is a reliable indicator of subject knowledge for teaching. The point has been well made that subject knowledge for teaching is more accurately measured by other means, and the quantitative data presented here supports that assertion.

**Comparison of QTS scores on exit from PGCE: MEC and non-MEC students**

**Overall QTS grades**

Analysis of overall QTS grades (i.e. the mean of scores from ten groups of QTS standards) on exit from PGCE yielded a mean grade of 1.77 for the MEC students as against a mean of 1.68 for the non-MEC students. On a grading scale of 1 to 3 (pass grades) where 1 is the highest,
this represents a slightly inferior score for MEC students, but is not statistically significant. Assuming that the samples used were representative of the general population of PGCE mathematics students at the university, and that population scores were normally distributed, an independent-samples t-test was conducted to compare the scores. The difference between the scores for MEC students (M = 1.77, SD = 0.541) and non-MEC students (M = 1.68, SD = 0.479) was not statistically significant; t(104) = 0.937, p = 0.35 (two-tailed).

This data is displayed in a frequency diagram in Appendix 7. The calculation of a mean of ten integer scores of 1, 2 and 3 generates continuous data between 1 and 3. The overall grades of MEC students are plotted above the horizontal axis, and for comparison the overall grades for non-MEC students are plotted below the horizontal axis (seemingly ‘negative’ frequencies below the axis are just a spurious outcome of the software used and should be treated as positive).

**Subject knowledge grades**

Analysis of subject knowledge QTS grades on exit from PGCE yields a mean grade of 1.71 for the MEC students as against a mean of 1.55 for the non-MEC students. Again this represents a slightly inferior score for MEC students, but this is not statistically significant. Assuming that the samples used were representative of the general population of PGCE mathematics students at the university, and that population scores were normally distributed, an independent-samples t-test was conducted to compare the scores. The difference between the scores for MEC students (M = 1.71, SD = 0.638) and non-MEC students (M = 1.55, SD = 0.544) was not statistically significant; t(104) = 1.400, p = 0.165 (two-tailed).

This data is displayed in a frequency diagram in Appendix 8. Subject knowledge grades awarded were integers between 1 and 3. The subject knowledge grades of MEC students are plotted above the horizontal axis, and for comparison the subject knowledge grades for non-MEC students are plotted below the horizontal axis (again, ‘negative’ frequencies below the axis should be treated as positive).

This evidence suggests that although marginally inferior, there is no statistical difference between MEC students and others in terms of their outcomes upon completion of PGCE as measured by QTS scores, so the differences can be disregarded. This suggests that MEC students and degree maths students at the start of their teaching careers are more or less
equivalent. I am not aware of any other studies comparing grades of MEC and non-MEC students on exit from PGCE mathematics, and therefore I suggest that the data presented here is new. This lack of difference suggests that MEC stands up to scrutiny as a preparation for teacher education when compared with traditional degree pathways, and is doing its job as a successful alternative route to PGCE. This is an interesting incidental outcome of this study, which is supported by evaluations of the MEC (Seabourne, 2004, OFSTED, 2008).

Chapter conclusion

In this chapter I have discussed literature pertaining to the meaning and interpretation of teachers’ academic qualifications. I have presented quantitative data which adds a further dimension to the background of MEC and PGCE students at the university under study, suggesting firstly that for these PGCE mathematics students there is no relationship between degree classification on entry to the course and success on the course, and secondly that there is no significant difference in outcome on PGCE between students who have taken the MEC and those who have followed mathematics degree routes. This data usefully informs the debate about subject knowledge for teaching, and what the appropriate preparation of secondary mathematics teachers might entail. Questions about what advanced mathematics for teaching is, and how it can be offered within a programme that also has a distinct pedagogical focus, are informed by the particular response which is the Mathematics Enhancement Course. The MEC can be said to challenge existing ideas about what is appropriate subject knowledge preparation for teachers, and to loosen the boundaries between subject knowledge courses and subject pedagogy courses.

In the next chapter I will give details and justification of the methods used for the main qualitative strand of research in this thesis. Most of the teachers who I interviewed for this research have followed either the MEC or the MDPT. As the mathematics subject knowledge courses that they have followed have been devised with specific aims with regard to integration of subject and pedagogical knowledge, these teachers form a valuable and unique sample group to study. I am not aware of any other study in which these two groups
of teachers are investigated and compared in this way. These teachers’ views and ideas about the nature of subject knowledge for teaching are at the heart of the research in this thesis.
Chapter 5 Research Methods and Methodology

Chapter introduction

This report is an investigation into what characterises ‘understanding mathematics in depth’, as understood by two particular groups of secondary pre-service and serving teachers. A range of literature, discussed in Chapter 2, informs this investigation. Distinctions between different types of mathematics knowledge for teaching (Shulman, 1986; Ball, 2008) frame the discourse, enabling us productively to identify different categories of teacher knowledge. Ruthven (2011, p. 83) calls this ‘subject knowledge differentiated’. The stage model developed by Rowland et al. (2009) introduces a more active element to conceptions of mathematics knowledge for teaching, stressing the need to observe this ‘in the teaching moment’ (p. 24). This is supported by Hodgen’s (2011) work on the situated nature of teacher knowledge, described by Ruthven (2011, p. 86) as ‘subject knowledge contextualised’. Watson and Barton see knowledge as an active process – the teacher must enact mathematics. All of this research provides a discourse for conceptions of mathematics knowledge for and in teaching, and supports the approaches to subject knowledge development which are in use in the two courses under study in this thesis – the Mathematics Enhancement Course (MEC) and the Mathematics Development Programme for Teachers (MDPT).

Ma’s (1999) work on the need for profound understanding of fundamental mathematics (PUFM) has been directly linked with the MEC since its inception, and underpins further work exploring what MEC students’ and tutors’ conceptions of ‘understanding mathematics in depth’ (Adler et al., 2009). In this thesis, I add to the picture by investigating and comparing the conceptions of understanding mathematics in depth given by two groups of people – those who have followed a mathematics PGCE, some of whom studied the MEC, and those who have converted to mathematics teaching via the MDPT.

The research questions addressed in this study are:
1. How is ‘understanding mathematics in depth’ conceptualised by two particular groups of novice mathematics teachers?

2. What are novice mathematics teachers’ beliefs about how ‘understanding mathematics in depth’ is attained?

3. What themes are privileged in the discourse of novice mathematics teachers in relation to their preparation for, and experience of, mathematics teaching?

### 5.1 Research design and methods

Research for this study falls clearly within the qualitative or interpretive paradigm. The main method of data collection is a series of semi-structured interviews with a sample of 21 teachers drawn from the MEC/PGCE and MDPT courses which feature in this study. From this data, new insights into teachers’ conceptions of ‘understanding mathematics in depth’ and other emerging information are obtained. The small-scale quantitative analysis of PGCE entry and exit grades which was reported in chapter 4 provides some context to this study. It demonstrates that, for the sample investigated, degree classification is not an indicator of subject knowledge for teaching, and that notwithstanding the differences in their subject knowledge preparation, MEC and degree students perform equally well on the PGCE course.

There seems to be a clear trend in modern educational research for a combination of approaches as against reliance on one set of methods. Many writers now call for an integrative view of paradigms in educational research, with the use of mixed methods and mixed model studies. The ‘paradigm wars’ of the 1980s are seen as an unproductive debate about the possible supremacy of one paradigm over another, whereas all may have a place in meaningful research. Gage (1989), in a so-called ‘historical’ article which is in fact forward-looking, states that researchers realised that the “oppositional component of paradigm was invalid” (p. 7), i.e. the idea that any one paradigm is incompatible with others. However, current political thinking may act against this. Lather (2006) argues that “grand narratives and one-best-way thinking are being reasserted” and researchers’ efforts “need to be situated
in a context of historical time marked by multiplicity and competing discourses that do not map tidily onto one another…” (p. 47).

The paradigm within which a researcher works should follow from the research questions that are being asked, i.e., the researcher should choose an approach that is fit for purpose. Whatever the approach chosen, the researcher must be aware of the limitations of the results. In probing the nature of understanding of mathematics for teaching via a contextual consideration of PGCE entry and exit grades (chapter 4), it was appropriate to examine those grades as quantitative data and perform simple analysis upon them. However it is important to acknowledge the limitations of the data, and these limitations were discussed above. One would not want to make the mistake of assuming that working within the scientific paradigm meant that one was more likely to achieve objectivity than in other paradigms. Cohen et al. (2007) note that positivism is limited in its usefulness to the study of human behaviour, due to the complexity of human nature and social phenomena.

Moving on to the heart of this study, if one wishes to explore teachers’ conceptions of deep understanding of mathematics, then it is necessary to hear their voices, and an appropriate way to achieve this is through interviews and subsequent analysis of the data thus gathered. The role of the researcher is complex. It is important to recognise that from an interpretative research paradigm perspective one cannot ‘stand outside’ the domain of enquiry and peer within; one is a part of that domain. The researcher interacts with the subject matter, for instance, through choice of questions to be asked and through the way responses are interpreted. In the interactive context of interviews, the researcher is in relationship with the interviewee and this cannot be neutral. There are shared understandings and interests, there is an underlying context and, in some cases, a history to that specific interaction. In the case of this study, all interviewees were ex-students of the researcher. This itself provides a context that must be acknowledged. Wellington (2004) states that in social and educational research, “the researcher is the key instrument” and also that “the researcher influences, disturbs and affects what is being researched…” (p. 41). Silverman (2006) takes this further in suggesting that the interviewer and interviewee co-construct a new reality. Lee (2006) suggests that the practitioner-researcher is “in a position to create and view data with a depth of insight given by [one’s] intimate involvement in it” (p. 9). This is a helpful comment, highlighting as it does firstly the privileged and particular place of the insider-researcher, and secondly the fact that the researcher is not outside of the data, but plays a part in creating it.
The importance of hearing participants’ voices

In probing ideas about understanding mathematics in depth, there are clearly various approaches that could be taken, and for the purposes of this thesis it was necessary for me to be selective and focused in my methods. I did not observe teachers in practice, nor ask them to tackle mathematics related questions. I restricted my investigations to listening to the teachers’ discourse about their experiences on the course and their work as teachers of mathematics. Lerman (2009) suggests that one can learn most from research that draws together both teachers’ practice and their views on that practice. In my construction and application of interview questions, mostly open-ended, I endeavour to achieve this to some extent. However my main strategy is to hear the voices of the teachers, to attend to their meanings and exemplifications, and to draw out my own interpretations from this. I recognise that an interview transcript is a social construct, situated in a particular time and place, and my awareness of this will be critical in my data interpretation. The idea of understanding mathematics in depth is valued highly by mathematics teacher educators, to the extent that it may have become a new orthodoxy or an unquestioned good (Hossain et al., in press). Understanding mathematics in depth (UMID) was an explicit goal of the courses attended by the teachers in this study. In exploring conceptions of understanding mathematics in depth as voiced by participants on two specific courses, I develop the work being undertaken by Adler and her team (Chapter 2, p 48, Chapter 3, p 66-67). I seek to find out how these course participants understand a concept that is highly valued in mathematics education, and what it means to them.

5.2 Theoretical perspective

Reflexivity

I recognise that my position as both tutor and researcher is a particular one, and in my research I seek to exploit the advantages of this position whilst remaining aware of limitations and possible pitfalls. Ball (1990, p. 159) discusses ethnography as a “self-conscious engagement with the world” and suggests that the linking of the social process of engagement in the field with technical processes of data collection - reflexivity - provides the basis for rigour. Jaworski (1997) comments upon the importance of interrogation of her own
previous knowledge and experience, what she terms an ‘engagement of self’ (p. 115) - being crucial to the reflexive process. Graven (2004) notes that she had expected to encounter a tension in her role as both tutor and researcher in a mathematics in-service training course for teachers. However, she

“discovered a powerful praxis in the duality of being both ...worker and researcher” and that the potential tension was “turned into a research advantage by continually addressing and reflecting on the duality explicitly and openly in the broader study” (p. 190).

My own professional experience and practice provided context and knowledge which stimulated my research. As my research proceeded, new insights and knowledge gained then informed my ongoing professional practice, thus setting up a reflexive cycle of knowledge growth linked to practice.

This is an interpretivist study into teachers’ perspectives of the concept of ‘deep understanding of mathematics’ with data obtained via interviews. The interpretative research paradigm develops from a fallibilist epistemological view, that is, that knowledge is contingent and always changing, and is dependent upon the perspective of the knower. Therefore, the researcher may investigate people’s understandings and meanings, but these will remain diverse: there is not a search for some external universal ‘truth’. The researcher hopes instead to gain a rich and meaningful understanding of the perspectives of the subjects. Ernest (2008) links this approach with a postmodern perspective which is “polycentric” and “pluralistic”, stating that “fallibilist approaches to research…do not regard the world as something that can be [objectively] known with any certainty” (p65-66).

A constructionist view sees discourse not as a reflection of reality, but as a product of or artefact of human interaction (Gergen, 1985). Silverman (2006) suggests that from a constructionist perspective, an account or interview becomes not just a representation of reality, but a part of the world that is discussed or described. In other words, through their interaction, interviewers and interviewees co-construct their world and produce something new. Therefore, for the researcher, accessing an authentic understanding of the experience of another person is not at all straightforward, for

“experience is never ‘raw’ but is embedded in a social web of interpretation and re-interpretation.” (Kitzinger, 2004, cited in Silverman, op. cit., p. 129)
Crotty (1998) focuses on the search for meaning in research. He asserts that truth and meaning come into existence through people’s engagement with realities in their world, and that meaning is not discovered but constructed, for there can be “no meaning without a mind” (p. 8).

In this study, minds are working together to construct a new reality through their interaction. An interview dialogue takes place and is recorded and transcribed. The ‘new reality’ finds its form in a written transcript of the interview which the researcher then re-interprets. This has obvious links to the Vygotskian idea of language and talk as a process not merely of representing and communicating thought, but of actually forming thought and meaning. The research process in this study is strongly influenced and underpinned by a constructivist epistemological approach. Therefore it is clear that the researcher is making interpretations of participants’ meaning and furthermore that the researcher and participants are co-constructing a new entity in the form of an interview, recorded and transcribed.

5.3 Interview and ranking exercise with pre-service and serving teachers

This investigation is carried out by means of semi-structured interviews, drawing upon ongoing work by Adler et al. (2009) into understanding mathematics in depth, and with interpretation of participants’ responses informed by, but not purely adhering to, grounded theory (Charmaz, 2006). Use of grounded theory approaches is discussed in section 5.4.

Samples

A sample of twelve current and former PGCE mathematics students were interviewed, with participants drawn from each of the 2007-08, 2008-09 and 2009-10 cohorts. At the time of the interviews, (summer and autumn 2010) the 2009-10 participants were approaching or at the end of their PGCE course, whilst the other participants had been qualified and teaching for one or two years respectively. Both PGCE students who had formerly taken the MEC and
those who had degree mathematics were included in the sample. Similarly, a sample of nine current and former MDPT teachers were interviewed, with participants drawn from each of the 2007-08, 2008-09 and 2009-10 cohorts. Interviews had been planned to take around 30 minutes each, and the total time actually taken for the 21 interviews was 11 hours and 40 minutes. Summary information about the teachers in the sample is given in Tables 1 and 2 below. In Table 1, information about participants is presented simply in the order in which the interviews took place. Participant information is reorganised in Table 2 to show more clearly the breakdown of the sample by course and year of study.
<table>
<thead>
<tr>
<th>Interview number</th>
<th>Course followed</th>
<th>Job at time of interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MDPT 09/10</td>
<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>2</td>
<td>MDPT 09/10</td>
<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>3</td>
<td>PGCE 09/10 non-MEC</td>
<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>4</td>
<td>MDPT 09/10</td>
<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>5</td>
<td>MDPT 09/10</td>
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</tr>
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<td>PGCE 09/10 MEC</td>
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</tr>
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<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>8</td>
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<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>9</td>
<td>PGCE 07/08 MEC</td>
<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>10</td>
<td>PGCE 07/08 non-MEC</td>
<td>Academic sixth form college</td>
</tr>
<tr>
<td>11</td>
<td>PGCE 08/09 MEC</td>
<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>12</td>
<td>MDPT 08/09</td>
<td>Special school</td>
</tr>
<tr>
<td>13</td>
<td>PGCE 07/08 non-MEC</td>
<td>Independent secondary school</td>
</tr>
<tr>
<td>14</td>
<td>MDPT 07/08</td>
<td>Further Education college</td>
</tr>
<tr>
<td>15</td>
<td>MDPT 08/09</td>
<td>Home tutor – local authority outreach team</td>
</tr>
<tr>
<td>16</td>
<td>MDPT 07/08</td>
<td>Special school (EBD)</td>
</tr>
<tr>
<td>17</td>
<td>PGCE 07/08 MEC</td>
<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>18</td>
<td>MDPT 07/08</td>
<td>Children’s secure unit</td>
</tr>
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<td>19</td>
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<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>20</td>
<td>PGCE 08/09 non-MEC</td>
<td>Maintained comprehensive school</td>
</tr>
<tr>
<td>21</td>
<td>PGCE 07/08 non-MEC</td>
<td>Independent secondary school</td>
</tr>
</tbody>
</table>
Table 2  Summary information, interview participants, by course and year of study

<table>
<thead>
<tr>
<th>Int no</th>
<th>Course followed</th>
<th>Year of study</th>
<th>Former subject specialism if not maths</th>
<th>Job at time of interview</th>
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<tbody>
<tr>
<td>1</td>
<td>MDPT</td>
<td>09/10</td>
<td>Physical Education</td>
<td>Maintained comp school</td>
</tr>
<tr>
<td>2</td>
<td>MDPT</td>
<td>09/10</td>
<td>Science</td>
<td>Maintained comp school</td>
</tr>
<tr>
<td>4</td>
<td>MDPT</td>
<td>09/10</td>
<td>Business Studies</td>
<td>Maintained comp school</td>
</tr>
<tr>
<td>5</td>
<td>MDPT</td>
<td>09/10</td>
<td>Geography</td>
<td>Maintained comp school</td>
</tr>
<tr>
<td>12</td>
<td>MDPT</td>
<td>08/09</td>
<td>Primary</td>
<td>Special school</td>
</tr>
<tr>
<td>15</td>
<td>MDPT</td>
<td>08/09</td>
<td>Geography</td>
<td>Home tutor – LA outreach team</td>
</tr>
<tr>
<td>14</td>
<td>MDPT</td>
<td>07/08</td>
<td>Economics</td>
<td>Further Education college</td>
</tr>
<tr>
<td>16</td>
<td>MDPT</td>
<td>07/08</td>
<td>History</td>
<td>Special school (EBD)</td>
</tr>
<tr>
<td>18</td>
<td>MDPT</td>
<td>07/08</td>
<td>Construction (FE)</td>
<td>Children’s secure unit</td>
</tr>
<tr>
<td>6</td>
<td>PGCE MEC</td>
<td>09/10</td>
<td>Psychology</td>
<td>Maintained comp school</td>
</tr>
<tr>
<td>3</td>
<td>PGCE non-MEC</td>
<td>09/10</td>
<td></td>
<td>Maintained comp school</td>
</tr>
<tr>
<td>7</td>
<td>PGCE non-MEC</td>
<td>09/10</td>
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<td>Maintained comp school</td>
</tr>
<tr>
<td>11</td>
<td>PGCE MEC</td>
<td>08/09</td>
<td>Psychology</td>
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</tr>
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<td>19</td>
<td>PGCE non-MEC</td>
<td>08/09</td>
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</tr>
<tr>
<td>20</td>
<td>PGCE non-MEC</td>
<td>08/09</td>
<td></td>
<td>Maintained comp school</td>
</tr>
<tr>
<td>9</td>
<td>PGCE MEC</td>
<td>07/08</td>
<td>Plant engineering</td>
<td>Maintained comp school</td>
</tr>
<tr>
<td>17</td>
<td>PGCE MEC</td>
<td>07/08</td>
<td>Civil engineering</td>
<td>Maintained comp school</td>
</tr>
<tr>
<td>8</td>
<td>PGCE non-MEC</td>
<td>07/08</td>
<td></td>
<td>Maintained comp school</td>
</tr>
<tr>
<td>10</td>
<td>PGCE non-MEC</td>
<td>07/08</td>
<td></td>
<td>Academic sixth form college</td>
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<td>13</td>
<td>PGCE non-MEC</td>
<td>07/08</td>
<td></td>
<td>Independent secondary school</td>
</tr>
<tr>
<td>21</td>
<td>PGCE non-MEC</td>
<td>07/08</td>
<td></td>
<td>Independent secondary school</td>
</tr>
</tbody>
</table>

This sample is made up of a group of people who are at an early stage in developing their expertise as teachers of mathematics. I suggest that they are all, for various reasons, at an early stage in their encounter with the importance of understanding mathematics in depth for teaching. All have recently completed initial teacher training and/or subject knowledge enhancement courses, and this experience is still fairly fresh with them.

Sampling was not done randomly within each stratum. For practical reasons it was convenient to invite those teachers known to be in the local area to take part. At the time of
the interviews, all the participants were employed (or about to commence employment) in
schools, colleges or other settings in the Merseyside region. Most participants were at urban
or suburban maintained comprehensive schools, although there were some interesting
exceptions to this, as detailed in Table 1 above. I also made a decision in some cases to
invite teachers who had been my former personal students, i.e., those with whom I had
already established a relationship of trust. It was felt that these people might regard taking
part in an interview for no external reward as less of an inconvenience than would otherwise
be the case, and indeed that they might feel positive about the opportunity to talk about their
work with an interested former tutor, that they may find this beneficial. Thus there was an
element of an ‘opportunity sampling’.

Given that a prior relationship existed between the interviewer and participants it was
therefore important that for the purposes of this study, I tried to ensure that data used was
taken only from the interviews, and not from prior knowledge of the participants. Steps taken
to ensure this are noted below and included anonymising transcripts, and cutting and pasting
text during analysis. The existence of a prior relationship also introduces limitations to the
objectivity of the process in that data might be distorted due to participants’ desire to please
their former tutor. However, prior knowledge of participants by the researcher should not be
seen only as a limitation to the study. This context enabled a positive rapport and
environment of mutual trust to be established from the start of each interview. Given that
interviews as conversations are necessarily situated and can to some extent feel artificial, this
was a bonus in overcoming some of the awkwardness or unease that might be experienced by
participants and/or interviewer, and enabling the conversation to progress smoothly and
naturally.

The stratification of samples by MEC/non-MEC and by year group was used to enable any
differences in response from ex-MEC and non-MEC students to emerge and secondly to
allow for differences over time. Ma (1999) suggests that when profound understanding of
fundamental mathematics (PUFM) is observed in teachers, it has developed over time during
their professional career. In analysing the interview transcripts, one might anticipate some
differences in responses across years as the teachers develop within their professional role;
this is why it is important to consider teachers at different stages of their careers.
In terms of development of this research, it would be interesting to interview the same teachers again in the future, perhaps one year and then two years on. A longitudinal study would reveal developments in individual teachers’ responses over time. Unfortunately it was not possible to use this approach for the purposes of the current study, but this is a possibility for a future project.

Use of interviews

Wellington (2004) states that interviews ‘can reach the parts which other methods cannot reach’ (p. 71). An interview can probe people’s ideas and feelings – aspects which are difficult to observe by other means. Above all else an interview must give the respondent a ‘voice’, and thus an interview is not the same as an ordinary conversation, no matter how informal or unstructured it may be (Wellington, op. cit.). The interviewer may interact, build rapport, clarify, even express his/her own views, but should not play a lead role in the conversation. It is important that a researcher, whilst keeping the purpose of the research clearly focused, does not see his/her interviewees as ‘objects’ (Fontana & Frey, 1994) and one must be aware of perceived power differentials, and the way the interviewer is seen by the interviewee.

Cohen, Manion and Morrison (2007) comment on the strengths and weaknesses of different types of interviews. In standardised open-ended interviews, the sequence of questions and the exact wording are determined in advance. In the ‘interview-guide’ approach (op. cit. p. 353), topics are determined in advance but the interviewer decides the sequence and wording of questions during the interview itself. The standard open-ended interview provides for comparability of responses (participants are all asked the same questions) and ease of organisation of the data, but lack of flexibility may constrain the interview and mean that some relevant data is not picked up. The interview-guide approach retains some systematic characteristics within a looser, more natural conversation. However, comparability of responses becomes more difficult, and some important topics may be missed out if the conversation moves in a less structured way. The interviews I carried out for this study fall somewhere between the two categories described above, leaning more towards the standard open-ended interview. I refer to the form I used as a semi-structured interview.
The use of semi-structured interviews allows for a degree of informality and empathy which I believe is important in trying to probe complex ideas with participants. It is also worth noting that all the participants knew the interviewer (myself) as their ex-tutor. Therefore it was important to be mindful of a pre-existing tutor-student relationship, even for those teachers who have left the university and are settled into their careers. The interview is a social construct, and is situated. An interview is necessarily a specially arranged and therefore somewhat artificial conversation, and the participants (interviewer and interviewee) each know they have a specific role to play. This is not naturalistic research, the observation of and co-participation with the subject in the (teaching) context. Interviewees are asked to distil from their teaching and professional experience their views and conceptions on a variety of topics. Interviewees’ responses will be heavily influenced by their recent experiences and the matters that are on their mind at the time the interview happens to take place. It is important to recognise this; however if the questioning and subsequent analysis are effective, general ideas should emerge through respondents’ discussion of the more immediate.

I have experience of interviewing applicants for PGCE and MEC courses for many years, and in that respect I am an experienced interviewer. However, with the exception of my involvement in the QUANTUM project led by Adler (2009), the interviews carried out for this study were the first time that I had carried out interviews for the purposes of research. As such, it was a learning process for me, and limitations are acknowledged.

**Rationale for interview items**

Each interview was planned to last about thirty minutes and was digitally audio-recorded and later transcribed. Questions probed, *inter alia*:

- participants’ experience of mathematics during their training/course and how this prepared them for their own teaching,
- topics which the respondent understands well, and where this understanding came from,
- what ‘understanding maths in depth’ means to the participant.
The interview schedule is given in Appendix 1.

The questions asked were as follows.

**Qu 1** Tell me about the route you took to prepare you for maths teaching (degree, MEC, PGCE, other training)

The first question invited the participant to relate their own background and preparation for mathematics teaching. This question is on one level a ‘warm-up’ question, to relax the interviewee by giving them something they can easily talk about. However in some instances the question yields interesting responses on a variety of issues, including the process of becoming a mathematics teacher, which could be understood in terms of identity. As the study is informed by a grounded theory approach, these have been considered with the rest of the data.

**Qu 2** Your education and training will have prepared you in many ways for the complex role of the teacher, but let’s focus on the maths…In what key ways did your education and training prepare you for dealing with mathematical concepts in the classroom?

**Qu 3** What sort of maths did you learn while you were training? Can you give me some examples?

Questions 2 and 3, and responses to them, tended to become merged in the interview process, but this did not really present a problem. The initial plan was to elicit responses about their mathematics from respondents. This proved to be something of a challenge, as there was a tendency for many to want to discuss teaching strategies at this point. It was important for the interviewer to keep aims clearly in mind; however with appeals to the grounded theory approach, the data that was presented was analysed.

**Qu 4** Can you give examples of maths ideas you encountered in training and then used in your own teaching? (What happens to your understanding of the maths, when
you are adapting and transforming ideas to make them appropriate for children to learn?)

Question 4 explicitly probes the transformation of personal knowledge into knowledge for teaching, as proposed by Rowland et al. (2003b). Respondents are invited to discuss particular scenarios. The key information for analysis is not the scenarios that are presented, but responses to the follow-up questions in each case. However it is fair to say that this question did not go as planned. Probably because of the quantity and nature of the responses to earlier questions, I tended to merge this question with others, and the concept of transformation of personal knowledge into knowledge-for-teaching was not elicited.

**Qu 5** Can you tell me about a lesson/topic that you have taught recently where you were confident about the mathematical content you were teaching, and you felt secure going in to teach the topic.

*Why were you confident / how did you gain this knowledge?*

**Qu 6** Can you tell me about a lesson/topic where you felt less confident and secure with the mathematics, or where you were challenged by difficult questions from students.

*What happened next / how did you follow this up?*

Qus 5 and 6 seek explicitly to explore participants’ mathematics knowledge for teaching as located within their own classroom situations. Hodgen (2011) found that teachers’ knowledge differed in interviews and in practice. I understand this as implying that the situated nature of mathematics knowledge means that a focus upon knowledge without the classroom context is not likely to be very helpful. In Qu 5 the follow-up question is intended to locate confidence and preparation for the scenario described, i.e., where confidence and knowledge came from and how they developed. In Qu 6, the follow-up question is intended to probe how respondents handled a challenging situation in terms of their own knowledge, and how they moved beyond this point. Qu 6 is not there to ‘catch out’ respondents or locate gaps in their knowledge. It is part of the interview design because finding out how and where knowledge and confidence is lacking can sometimes highlight where it is present. Qus 5 and
6 are therefore designed to probe the same area, firstly from a positive standpoint and secondly from a negative or deficit standpoint.

In line with the approaches of Adler et al. (2009), interviewees were sent questions 5 and 6 (but not their follow-up questions) by email in advance of the interview, so that they could have a little thinking time.

**Qu 7** In some teacher education courses, emphasis is placed on the importance of ‘understanding maths in depth’. What does this mean for you?

**Qu 8** Here are five statements related to how ‘understanding (fundamental) maths in depth’ may be interpreted. Please arrange the statements in order of importance for you, with the most important first.

Understanding mathematics in depth means being able to justify your mathematical thinking. (A)

Understanding mathematics in depth means being able to explain and/or communicate mathematical ideas and thinking to others. (B)

Understanding mathematics in depth means being able to understand why and how procedures work. (C)

Understanding mathematics in depth means being able to make the connections between concepts and between procedures. (D)

Understanding mathematics in depth means being able to see structure, patterns and general results. (E)

Why is the first one the most important?
Qu 9 Here are three statements related to how ‘understanding of maths in depth’ may develop. Please arrange the statements in order of importance for you, with the most important first.

<table>
<thead>
<tr>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>An understanding of mathematics in depth develops by investigation and/or working out difficult and taxing problems. (F)</td>
</tr>
<tr>
<td>An understanding of mathematics in depth develops by living with and/or spending time thinking about mathematical ideas. (G)</td>
</tr>
<tr>
<td>An understanding of mathematics in depth develops from the understanding that this takes time and it is about being on a journey. (H)</td>
</tr>
</tbody>
</table>

Why is the first one the most important?

Qu 7 gets to the heart of the interview focus, and interviewees were prepared for the question by virtue of the nature of the documentation they had seen (see for example sample email communication and consent forms, Appendices 3 and 4). Qu 8 and 9 use pre-designed statements about what understanding in depth might mean, and how understanding in depth is developed. Participants were asked to rank these in order of importance. The exercise of ranking statements was part of the interview schedule as much to provoke ideas and discussion as to provide data. The task of handling pre-prepared statements offered respondents a vocabulary and some ideas that they might not have spontaneously generated themselves. One can ask if this is a good idea and whether it is appropriate for the interviewer to put ideas into the interviewees’ minds. If the theoretical stance of the interviewer is purely ‘emotionalist’ (Silverman, 2006) concerned with ‘eliciting authentic accounts of subjective experience’ (op.cit, p123) then this strategy would run counter to the desired approach. However, in the spirit of constructionism, both interviewer and interviewee jointly construct the conversation. Therefore learning may take place for both, and offering pre-set statements for consideration need not be problematic.
These ranking exercises also generated some simple quantitative data that may shed some light on participants’ concepts of deep understanding of mathematics.

**Choice of statements in ranking exercise**

The statements used for the ranking exercises in questions 8 and 9 here were the same as those used in the QUANTUM project, and their origin is worth noting. At the outset of the project, and before any interviews were carried out with MEC students, Adler and Hossein conducted interviews with the MEC tutors who make up the rest of the research group (Archer, Clarke, Grantham and Stevenson). They sought the tutors’ conceptions of what understanding mathematics in depth (UMID) meant, and how it developed. Tutors were asked open questions about this. The statements used in the ranking exercises were all statements actually made by MEC tutors in those interviews. As such, they represent the views and conceptions of expert practitioners. In my view, the statements cover quite succinctly the key areas that an expert practitioner might identify. Therefore they are a useful tool for promoting discussion and generating understanding in the context of a research interview.

**Qu 10** Can you give an example of a small bit of maths that you feel that you know in depth? How did you gain this knowledge and understanding?

**Qu 11** Is there anything else that you would like to add?

To conclude the interview, in Qu 10 participants are invited to discuss their knowledge of mathematics in depth with reference to an example, and to discuss how this knowledge developed. This item is probing essentially the same information as Qu 5, but is phrased in a different way. Thus it is hoped there is an improved chance of getting at the data sought. Finally there is an opportunity in Qu 11 for respondents to add in anything else that they may wish to say.

It is important to note that at no stage in the interview process is there an attempt to measure or otherwise investigate respondents’ understanding of mathematics in depth. The work of researchers who are investigating this is detailed in Chapter 2 above. Use of carefully
constructed items such as those designed by Hill, Ball, et al. (2008) might feature in this sort of work, as might direct observation of participants in their teaching context. These methods are beyond the scope of this study, but might be foci for future research. The main thrust of the interviews is to determine participants’ views of understanding of mathematics in depth, and how it is developed in their own experience.

Acknowledgements

The items used for questions 7 to 10 were developed by Adler et al. (2009) for use in the first phase of interviews of MEC students within the QUANTUM project, which took place in summer 2009. Subsequently, Adler and her team developed the items used here as questions 5 and 6, for the second phase of their work, which took place in summer 2010. I am grateful to Jill Adler for her permission for me to use these interview items in my research.

Pilot study

Janesick (2004) uses dance as a metaphor for qualitative research design, and makes comparisons between various stages of the dance / research project. The pilot study is likened to a part of the warm-up or exercises before the dance is performed. For the dancer it is necessary to stretch the muscles and try out various moves before the dance commences. For the researcher using interviews, the pilot study provides an opportunity to test the actual questions to be used. The researcher needs to establish whether the questions actually elicit the information that is being sought. The researcher also needs to experiment with questioning style, use of any feedback or prompts and of course practical arrangements such as seating and the use of an audio-recorder. After the pilot there is then an opportunity to make some changes before the main part of the research is carried out.

In advance of the main series of interviews, three interviews (Interviews 1, 2 and 3) were carried out and analysed as a pilot study. Some key amendments were made as a result of the pilot, as follows. Firstly, the pilot interview schedule (Appendix 2) did not include those items denoted Qu 5 and Qu 6 on the later interview schedule (Appendix 1). The ideas for these items arose from the author’s experience of participating in the second phase (2010) of
Adler’s ongoing research study at the time, and indications of their potential value in eliciting the data sought, as explained above. Secondly, the decision to release these two questions to interviewees in advance by email, was also taken in the light of experience both of the Adler project and the fact that one of the pilot respondents suggested that this would have been helpful. A third area of learning that arose from the pilot study was that respondents needed to be steered very clearly to discuss their subject mathematics knowledge (SMK) as opposed to their knowledge of teaching strategies or pedagogical content knowledge (PCK).

Ethical considerations, practical arrangements

Invitations to take part in this study were sent by email. Since all participants were former students of the mine, I had access to their email addresses. In the initial invitation, it was made very clear that there was no expectation or obligation to take part, and that if an individual chose to do so, arrangements would be made at their convenience (see sample email communications, Appendix 3). All participants were full-time teachers, and therefore busy, and so it was necessary to make arrangements as straightforward as possible for them. Details of exact times and places were not recorded, but most participants chose to come to the university at the end of the school day for their interview, whilst a few opted for the interview to be carried out at their place of work during the day. Interviews were carried out in quiet, private rooms.

As noted above, a small amount of prior information was given within the email communications before the interviews. The intention was to give participants some context and a little advance thinking time, without going into too much detail that might be onerous and off-putting to them (see sample email communications, Appendix 3). Thus participants were aware that the interview would probe their conceptions of ‘deep understanding of mathematics’. It was felt that Qus 5 and 6 would elicit better quality responses if participants had a little advance notice of them, since these items required participants to recount and reflect upon specific episodes in their own teaching – something that would be difficult to do ‘on the spot’. It is worth noting that in every case the final advance email was sent only a day or two before the interview. This is because although it was seen as important to give some prior information, the researcher did not want participants to spend an undue amount of time thinking about the questions as this would have been an unnecessary burden.
Before the start of each interview, and line with the university’s research ethics procedures, participants were given an information sheet to read and a consent form to sign (see information and consent forms, appendix 4). Cohen et al. (2007) stress the importance of obtaining informed consent from participants in social research. They discuss an explanation of ‘informed consent’ by Diener and Crandall (1978) as involving four elements, namely competence, voluntarism, full information and comprehension.

It is the researcher’s responsibility to ensure that participants are competent to take part in the research and able to make reasonable decisions related to it. Voluntarism means that participants freely choose to take part or not to take part, and are free to withdraw at any stage if they wish. It is often not possible to give participants full information (e.g., about how research data will be analysed and used) but in practice a code of “reasonably informed consent” is sufficient (Cohen et al., 2007). Comprehension means that participants fully understand the nature of the project.

I believe that in this study the principle of informed consent was adhered to. Participants were given as much information as possible about the project, without this becoming burdensome. At the interview stage, I did not fully know myself how the data was going to be analysed; it is inherent in the hermeneutic cycle that new themes and categories emerge as data is studied and re-interpreted. Thus as much information as was reasonable was shared with participants.

Interviews proceeded as detailed in the interview schedule (Appendix 1) but with a degree of informality such that the exact wording of the questions was not always used, and such that opportunities to pursue other areas of interest were taken from time to time (see sample annotated transcript, Appendix 5). This loose format afforded by the semi-structured interview arrangement was viewed as likely to be the most productive in terms of gaining insights into participants’ thinking in as natural a manner as possible. Also, I wanted to make the interview experience helpful and interesting for the interviewees, and was genuinely interested in the progress of my ex-students; these interviews provided rare opportunities to meet. Therefore time was allowed before and after the recorded interviews to chat informally with each participant.
Ethics clearance was obtained from the university where I work, as all participants were former students of the university and interviews were generally carried out at the university. Ethics clearance was also obtained from Exeter University under whose auspices this research has been conducted.

5.4 Interpreting interview data, use of grounded theory approaches

Before going on to outline how the interview data was interpreted, and the different stages of this interpretation, I should note that at the outset of this research it was my early intention to adopt the coding structure used by Adler et al. (2009), as discussed in Chapter 2 (p. 47-48). This approach was to attempt to locate responses broadly as discourses of mathematics, of learning, of teaching, of self and of environment. An advantage of this approach would have been that it had been tried and tested by experts, and thus as a novice researcher I could learn from this. Also this would have made it possible later to make direct comparisons between the different studies. However, initial reading and analysis of interview transcripts quickly revealed several other emergent ideas, and a decision was made early on to adopt a more open approach and see what the data revealed through successive levels of analysis. Restricting the analysis to predetermined categories would I believe have closed down avenues of thought prematurely. So I discarded Adler’s coding structure and decided to read the interview transcripts without any pre-existing framework, and see what key themes and ideas emerged from the data. This demanded that I find out more about the generation of theory from data within grounded theory approaches.

Charmaz (2006) explains grounded theory methods as consisting of

“systematic yet flexible guidelines for collecting and analyzing qualitative data to construct theories ‘grounded’ in the data themselves.” (p. 2)

She discusses the processes of making initial codings of data and then successively refining, comparing and analysing them in a series of iterative steps until results and perhaps theory
emerge. Campbell et al. (2006), in an analysis of practitioner research, discuss the situation of the ‘native’ researcher who has a “profound and extensive knowledge of the context” (p. 126) being researched, but needs to learn how to appraise in a new way what they may see as everyday situations, thus learning how to see the familiar as unfamiliar.

It is critical to a grounded theory approach that analytic codes and categories are constructed from the data itself, not from predetermined analyses or from external theoretical frameworks such as those that might have been devised by other researchers. Some researchers involved in grounded theory approaches would conduct a literature review after developing their independent analysis of data, so that their understanding of the data is not tainted by any preconceptions, or as Charmaz puts it, “to avoid seeing the world through the lens of extant ideas” (p. 6). This marks a clear departure from more traditional forms of qualitative data analysis. It is worth noting that one can only see the world through the lens of one’s own experiences, prejudices, and expectations. No matter what lengths are taken to strengthen objectivity (in the scientific paradigm) or to achieve an open, untainted approach (in grounded theory in the interpretative paradigm), no research, or researcher, is value-free. Radnor (1994) points out that the researcher cannot truly be outside of what it is he/she is researching: “I do not stand apart from society as an observer but actively construct the world in which I live…I can only know social reality through my subjective understanding.” (p. 8), and this is a view shared by others who write about this paradigm. Campbell et al. (2006) suggest that the process of open coding requires the researcher to be “simultaneously systematic and creative” (p. 131) in the examination of data. Charmaz (op. cit.) discusses the importance of attending to respondents’ language, noting that this makes it possible to refine questioning and to “learn about their meanings, rather than make assumptions about what they mean” (p. 35). With all this in mind, I began my data interpretation.

Levels of analysis, and ‘making the familiar strange’

Stage 1: identifying significant data

There were four stages of analysis. At the first stage, the initial reading of transcripts, different question items were delineated, and passages of text where the respondent seemed to be saying something interesting or significant were underlined. It is reasonable to question by what means the researcher decides at this stage what is ‘significant’ or ‘interesting’.
Making a start on tentative coding relies upon professional judgement or hunches at this stage; however Charmaz (op. cit.) notes that this is a part of the process. I believe that the researcher, having thought deeply about her questions, having read widely literature related to her inquiry, and having significant first-hand experience of the professional domain she is investigating, may indeed be permitted to exercise tentative professional judgement in identifying interesting or significant passages of text in the dialogue.

Stage 2: initial codes
Then, on the second reading, annotations of initial codes were added. This was a first attempt to classify what the interviewee was saying (see sample annotated transcript, Appendix 5), and to better understand each interviewee and his/her perspectives. At this stage the process enters a realm described by Wellington (2004) when he notes that “the role of the researcher… is to make the familiar strange” (p. 44, attributed to Delamont). Initially during interviews, the researcher may not be taken by surprise by much of what is said by respondents; they are, after all, talking about a domain known very well to both parties. However on close analysis of interview transcripts, specific and important ideas emerge, and familiar territory does indeed start to become more strange as the researcher endeavours to shift away from her own perspective and really attend to what is being said by the respondent.

Stage 3: tabulation of responses to selected questions, reading responses question by question
Following this, a third reading of the transcripts was undertaken, in which responses of all interviewees to questions 5, 6, 7, 8, 9, and 10 were dealt with systematically item by item. All the responses to Qu 5 were tabulated, then all the responses to Qu 6, etc. (see Appendix 12). Responses were anonymised, and text from transcripts was electronically cut and pasted into new documents and tables. This was done in order to construct a comparison between the respondents and to see if this comparison revealed new perspectives on the research questions. These questions were selected as the researcher took the view that they form the heart of the interview. Responses to other questions are also of interest, but it was necessary at that stage to focus the analysis. The ideas that emerged from these readings and analyses are outlined in Chapter 6 below.
Stage 4: back to reading interviews as a whole; finding the main story
At the fourth and final stage of analysis, each interview was re-read in full and treated in a holistic manner. In response to the questions, ‘What is this person telling me?’ and ‘What is this person’s story?’ the researcher jotted down one or two key overall themes and ideas emerging from each interview. These were kept brief by using a single sticky ‘post-it’ note for each interview, in a deliberate attempt to summarise and find any emerging themes.

Charmaz (2006) argues that grounded theorists can add new pieces to the “research puzzle” (p. 14) at any stage while gathering data and even late in the analysis, and can follow leads that emerge during the process. This idea is supported by Wellington (2004) who notes that analysis of (qualitative) data is part of the research cycle.

“It must begin early, in order to influence emerging research design and future data collection, i.e. it is formative not summative.” (p. 134).

I believe that this dynamic approach to interpretative research fits well with this study. As a novice researcher, there is much for me to learn by attending to the language and meanings of interview participants, and this learning process can improve the quality of later work. Initial analysis of the first 13 transcripts took place before the remaining interviews were carried out. This enabled me to gain confidence in the method. After conducting the other 8 interviews, the same methods of analysis were used on the remaining data.

Chapter conclusion
In this chapter, I have given details of the research methodology and methods used in this study, with justification for choice of approaches supported by literature. This is a study in the interpretivist paradigm. The investigation is into the perspectives of ‘deep understanding of mathematics’ offered by two specific groups of teachers. To access this data, I devised a series of interview items and held semi-structured interviews with a sample of 21 teachers, with adherence to appropriate ethical procedures. Justification for choice of interview items,
together with the four stages in reading and analysis of transcripts that I adopted, are reported and explained.

The outcomes of the research described above are given in Chapter 6.
Chapter 6  New courses and new voices: results of enquiry

Chapter introduction

In this chapter, the outcomes of analysis of 21 interviews carried out with former PGCE and MDPT students are reported. The interviewees comprised twelve current and former PGCE mathematics students, and nine current and former MDPT teachers. Interviews were planned to last about 30 minutes each, but there was some variation in the times actually taken. The total recorded interview time was 11 hours and 40 minutes.

Four stages of analysis of interview transcripts were undertaken, as described in Chapter 5. At the first stage, comments that appeared to be interesting or significant were identified, in a fairly unstructured way, by underlining the text. A complete sample interview transcript can be seen in Appendix 5. This initial identification of significant comments can be seen where text is underlined.

At the second stage, annotations relating to emerging codes were added. Annotations can be seen on the sample transcript as handwritten notes in the margin. Outcomes from this are reported below, and a full set of notes taken from this stage of analysis can be seen in Appendix 9. Appendix 9a provides an example analysis memo, showing how one of the codes emerged.

At the third stage of analysis I tabulated and analysed interviewees’ responses to particular questions. Analysis began with responses to questions 5, 6 and 10 of the interview, and then moved on to responses to questions 7, 8 and 9. (These responses are shown in Appendix 12). This cross-participant question-by-question analysis revealed some interesting differences and similarities between the two groups of participants, as discussed below.

Finally I re-read the interviews holistically at the fourth and final stage of analysis. Key themes emerging from the interviews when viewed holistically were identified, and are discussed below. This corresponds to stage 4 of the data analysis as described in Chapter 5 above.
6.1 Stage 1: Gaining familiarity with the data, identification of significant comments

See Appendix 5 sample transcript and underlining for an example of how work progressed at Stage 1.

6.2 Stage 2: Annotation and identification of codes

To show how codes were extracted from the data, Appendix 9 demonstrates, for each individual case, the main codes that were identified in interview responses, together with exemplification. There is a focus upon responses to the earlier interview questions (questions 1-4), as the later questions are analysed in Stage 3 below. The PGCE group are considered first, followed by the MDPT group. The codes that were identified are listed in Figure 4 and Figure 5, including repetition where it occurred.

To illustrate how the Stage 2 codes developed, Appendix 9a (p. 191) is an analysis memo for one of the codes. The memo shows how the particular code was identified in the transcript texts, its relationship to other codes and its significance within the study as a whole.

I do not present a discussion of these outcomes here since this will be incorporated in the overall discussion later in the chapter.
C\textsuperscript{om}pares own schooling with modern approaches, refers to variety of teaching strategies
Notes the change in own teaching approach
Development of PCK while on PGCE course
MEC gave more depth and context to maths
Confidence came from preparation
Developing SMK through preparing to teach
Degree maths was very procedural, not very meaningful. Wants to do differently.
Development of PCK; misjudged an ‘easy’ topic
Through PGCE learned about sequencing of topics (KC), different teaching techniques (KCT) and learners’ common misconceptions (KCS).
Enjoys challenging children’s attitudes to algebra and helping them to enjoy it.
UMID in the MEC, seeing connections
Learning in school context
Development of confidence, professional identity
Relevance of degree maths
Restructuring knowledge, development of PCK
Learning and teaching maths at the same time
Proof is very important in understanding
UMID means being able to explain
Preparation
Identifies what PGCE and degree did for her
School maths as building blocks for later applications
Uses own career experience to present maths
Mathematical knowledge as connected networks / jigsaw
You teach differently when you have recently learned something
Living the dream, always wanted to be a maths teacher
Teaching challenging topics, weaknesses become strengths
Figure 5: Codes identified in MDPT interview transcripts at Stage 2 of analysis

<table>
<thead>
<tr>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own positive experience of school maths</td>
</tr>
<tr>
<td>Explains how she prepared to teach maths</td>
</tr>
<tr>
<td>Appreciation of colleagues’ support - community</td>
</tr>
<tr>
<td>Confidence</td>
</tr>
<tr>
<td>Identity</td>
</tr>
<tr>
<td>Compares own (maths) schooling to modern methods.</td>
</tr>
<tr>
<td>Awareness of how children learn, and how technologies can aid this.</td>
</tr>
<tr>
<td>Comments on own learning</td>
</tr>
<tr>
<td>Distinguishes between areas of confidence and not.</td>
</tr>
<tr>
<td>Compares earlier maths teaching approach with current one and attributes influences.</td>
</tr>
<tr>
<td>Identity and transformation of teaching style</td>
</tr>
<tr>
<td>Promotes advantages of collaborative working</td>
</tr>
<tr>
<td>Discusses effect of MDPT course on his understanding of maths and on his teaching approaches</td>
</tr>
<tr>
<td>Shows self-awareness</td>
</tr>
<tr>
<td>Identifies what UMID is in pupils</td>
</tr>
<tr>
<td>Discusses benefits of being on course - working with other people</td>
</tr>
<tr>
<td>Transformation of SMK to PCK</td>
</tr>
<tr>
<td>Benefits of investigative approaches for developing her own SMK</td>
</tr>
<tr>
<td>Identity, confidence</td>
</tr>
<tr>
<td>Confidence, identity</td>
</tr>
<tr>
<td>Relates own positive experience of maths at school</td>
</tr>
<tr>
<td>Identifies key influences for handling mathematical ideas – the NNS</td>
</tr>
<tr>
<td>Compares own schooling in maths with methods he uses today</td>
</tr>
<tr>
<td>Restructuring existing knowledge</td>
</tr>
<tr>
<td>Distinguishes between SMK and PCK</td>
</tr>
<tr>
<td>Importance of understanding why in teaching</td>
</tr>
<tr>
<td>Making connections - course enabled her to make links. Ongoing development since finishing the course</td>
</tr>
<tr>
<td>Identity, confidence</td>
</tr>
<tr>
<td>Concepts are important</td>
</tr>
<tr>
<td>Reconstructing mathematics as an adult</td>
</tr>
</tbody>
</table>
6.3 Stage 3: Cross-participant question-by-question analysis

In this section, I shall give the results of analysis of interview questions, starting with questions 5, 6 and 10, which were designed to give opportunities for participants to exemplify their understanding or lack of understanding of topics in mathematics, and then moving to questions 7, 8 and 9 which form the heart of the interview.

Confidence / lack of confidence in mathematics and how this arises, questions 5 and 6

Questions 5 and 6 are intended to locate confidence and preparation for the scenario described, i.e., where confidence and knowledge came from, and how they developed. These two questions are designed to probe the same area, firstly from a positive standpoint and secondly from a negative or deficit standpoint.

**Qu 5** Can you tell me about a lesson/topic that you have taught recently where you were confident about the mathematical content you were teaching, and you felt secure going in to teach the topic.

*Why were you confident / how did you gain this knowledge?*

**Qu 6** Can you tell me about a lesson/topic where you felt less confident and secure with the mathematics, or where you were challenged by difficult questions from students.

*What happened next / how did you follow this up?*

A broad range of sources of knowledge about the topics discussed were given. Some respondents traced their knowledge back to their own schooling or university. Others mentioned that they had gained the foundations at school but that MEC or PGCE had given more depth and context to mathematics they already knew. Others clearly indicated the importance of their recent experience of MEC or MDPT as being important in the
development of their knowledge. A number of respondents described gaining knowledge through having to prepare to teach it.

In questions 5 and 6, differences between responses given by the two groups (former mathematics PGCE and former non-specialist MDPT) emerge. The former PGCE students are in general less experienced as teachers, and we see some of them discussing the importance of thorough planning, of being well prepared:

“I did angle identity… That lesson went really well because I had obviously prepared, probably prepared a little bit more because I was thinking I’m going to forget these names of these angles. That probably worked in my favour just because I had actually put a lot more thought into what I was going to do. So yeah that was a good lesson. The confidence probably came from preparation” (Int 6, PGCE MEC, qu 5)

“Because I had taught that year before with a set 2 year 11 and I hadn’t taught it well and I knew I hadn’t…So this year having done that in my NQT year and not been happy with it, I really sat and thought about it and I used Autograph and I had pre-prepared little booklets with loads of blank graphs on for sine and cosine and… we looked at the actual coordinates and what was happening to the coordinates when I was changing the graph. So we looked at it, we linked it into the actual Autograph and to what it was doing. We drew the graphs and then came up with a quick method.

“Yeah, because I think my own understanding of it, I’ve never liked graphs, I’ve never like drawing them in uni and I didn’t like it at A Level so it was kind of one of them areas that because I didn’t like it I might have betrayed that but because I had to sit down and planned it properly then I like teaching it now.” (Int 13, PGCE non-MEC, qu 5)

We also see them learning that knowing a topic oneself is not sufficient for teaching it to others:

“I taught year 7 top set algebra, solving equations, rearranging, getting the variable on one side. Know how to do it perfectly in my head … I think because I went in there thinking yeah I know this like the back of my hand, one of my stronger points,
algebra, it threw me when they didn’t understand… and they were a really able class
so I knew it was my teaching because they are really able. And they just didn’t grasp
it and I was trying to explain it but it just weren’t getting through to them and because
I think I was less prepared for it to go wrong because I think I just assumed that
because I was good at it they would pick it up quickly”. (Int 6, PGCE MEC, qu 6)

The above experience provides a good example of how easy it can be for inexperienced
teachers to fall into Ruthven’s (2011) mathematical “expert blind spots” (p. 88) when they
assume that knowledge they themselves have, and maybe take for granted, will be easily
learned by their students.

The following comments are clearly about PCK, although the respondent does not use this
term herself, and indeed the Shulman / Ball categorisation of subject knowledge for teaching
is not one with which the interview participants are familiar.

“I could do it myself but it’s just about the teaching stuff. It’s about knowing when to
stop, knowing what questions they’re going to ask and the pace to pitch it at. I
haven’t had time to sit down and really think about what questions they’re going to
struggle with.

“When I’m confident then it’s a completely different kind of atmosphere in the
class…it’s just the confidence that it gives you, it’s just that knowledge that you know
it inside out and they could ask you anything and it just changes your… well my
whole appearance at the board really”. (Int 13, PGCE non-MEC, qu 6)

Another PGCE respondent commented about the challenges she faced when teaching a young
class about questionnaire/survey design. She was expecting them to enjoy this and find it
easy; however they struggled, not least because they lacked the communication and literacy
skills needed for this type of task. She concludes:

“It’s things that you see, as what you think is commonsense that you’re not in the
mind of that child. You know, and it’s not common sense to them and that was the
learning curve for me.” (Int 7, PGCE non-MEC, qu 6)
Once again, this is an inexperienced teacher gaining important PCK through the practice of teaching.

Meanwhile some of the non-specialist MDPT teachers focus upon the security or insecurity of having to teach topics that they do not know well:

“… I found that it became... because I wasn’t confident with it, it became very scripted in many ways. And anything that went off the script I really wasn’t comfortable with, you know. I think the best maths teaching (sic) I have seen are the people who are confident enough that if something comes up they can stop what they are doing and go off on a tangent a little bit... and with the basic number... well I didn’t have that confidence, I didn’t have enough tools, I didn’t have enough ideas to kind of go off on those tangents to develop ideas”. (Int 4, MDPT, qu 6)

Some of them also note that their knowledge and confidence have improved as a result of participation in the MDPT. Interviewee 16 uses the following example:

“Some simple algebra...that’s always been my weakness, because I always had problems with it at school...”

then he describes a successful lesson,

“And that was where I felt confident about going in and doing it. See that’s another thing preparation now, whereas in the past I would have probably not prepared and I’d have just gone in and done it. I do think more about what I’m going in to teach and how to teach it.

It’s got to come from that course we did... And it is quite interesting now because if people come to me within school and they ask me things, kids ask me things, I even have other Heads of Department now contacting me and saying ‘...have you got anything that would suit this?’ But it has to come down to doing the (MDPT) course.” (Int 16, MDPT, qu 5)
An area of mathematics that you know in depth, and how you gained this knowledge, question 10

**Qu 10** Can you give an example of a small bit of maths that you feel that you know in depth? How did you gain this knowledge and understanding?

This was the last formal question on the interview schedule, and offered respondents another opportunity to discuss their understanding of chosen topics. Some repetition of themes raised earlier in the interview happened here, but this was not a problem as it served to reinforce consistency of responses. A wide variety of topics were offered by respondents, including volume of prisms, differentiation, data handling, algebraic equations, Pythagoras’ Theorem, fractions, straight line graphs, subtraction, cubic polynomials, and circle theorems.

Interpretations of ‘understanding mathematics in depth’, questions 7, 8, 9

Qu 7 was an open question, and it was necessary that this item preceded the ‘ranking’ exercises that constituted qu 8 and qu 9 to avoid responses to the open question being influenced by the statements in the ranking exercises. However, I shall deal with responses to qu 8 first.

Qu 8 of the interview schedule was a task in which participants were offered five different interpretations of UMID and invited to rank them in order of importance.

**Qu 8** Here are five statements related to how ‘understanding (fundamental) maths in depth’ may be interpreted. Please arrange the statements in order of importance for you, with the most important first.

| Understanding mathematics in depth means being able to justify your mathematical thinking. (A) |  |  |

122
Understanding mathematics in depth means being able to explain and/or communicate mathematical ideas and thinking to others. (B)

Understanding mathematics in depth means being able to understand why and how procedures work. (C)

Understanding mathematics in depth means being able to make the connections between concepts and between procedures. (D)

Understanding mathematics in depth means being able to see structure, patterns and general results. (E)

Why is the first one the most important?

As noted in Chapter 5 above, the exercise of ranking statements was used within the interview more to provoke discussion than to provide hard data. The task of handling pre-prepared statements offered participants a vocabulary and some ideas that they might not have spontaneously generated themselves. It is however worth making a brief analysis of the responses here. The exercise generated results which were quite widely dispersed, and it is not possible to draw firm conclusions from them. However, an analysis of first-choice items reveals that items B and C were clear favourites, with 14 respondents selecting either B or C as their first choice. Only one respondent selected A as first choice (see Tables 3 and 4 below). The majority of respondents conceived ‘understanding mathematics in depth’ as ‘knowing why’ and also in terms of being able to communicate ideas to others. This is perhaps unsurprising, as respondents approached the tasks from a teacher’s point of view. Very similar responses were given to the open question (qu 7) earlier in the interview in which participants were asked what UMID meant to them.

Qu 7 In some teacher education courses, emphasis is placed on the importance of ‘understanding maths in depth’. What does this mean for you?
Individual participants’ responses to questions 7 and 8 were checked for consistency and a fairly high degree of correspondence was found when individuals’ responses to the open question (qu 7) were compared to their responses to the pre-set statements. Of the 21 respondents, 11 showed a clear match in emphasis in their responses to qu 7 and 8. Evidence of this consistency is given in Appendix 12a and 12b, where all responses to these questions are tabulated. A summary of the consistent responses is given in Table 6 below.

**Example responses to Qus 7 and 8**

Interviewee 3 chose statement B as first choice in qu 8, and in her response to qu 7 we see clear consistency; this teacher views understanding mathematics in depth as being able to communicate mathematics:

“you have got to kind of get in there and explain why… and I suppose it’s just going that little bit deeper. If you have got a deeper knowledge then they will trust you as well because they can sense that you do know what you’re talking about” (Int 3, PGCE non-MEC, qu 7)

We also see consistency in Interviewee 6 who chose statement C as first choice in qu 8. Her response to qu 7 indicates that this teacher views understanding mathematics in depth as understanding why and how procedures work:

“For me I’d say the main thing... is understanding why you’re doing it. And understanding where it applies and actually being able to apply what you have taught to some real-life situation where they can see that’s where it’s from. I think that’s a depth in maths. And on a mathematical point of view, I’d say understanding where every single bit comes from. Proofs, seeing that… but obviously that’s higher level as in for pupils but for me just understanding exactly where that formula came from and how it was derived and you know, where every bit of it came from. I’d say that’s a depth. But in teaching and I’d say for my personal benefit I’d say it’s understanding why you’re doing it and where it’s from.” (Int. 6, PGCE MEC, qu 7)
Similarly Interviewee 10 who chose statement D as first choice in qu 8 shows consistency in qu 7, with a view of understanding mathematics as being about making connections:

“Possibly someone that had a deep understanding of mathematics would be someone that was erm… able to see the links between them. Something that someone else might see as a stand-alone topic.

A deep understanding would be when you’re talking about someone who really could see patterns in the way things work and the way the topics are inter-linked with each other”. (Int 10, PGCE non-MEC, qu 7)

We also see some interesting contrasts of views when interviewees are asked to justify their first choice of statement in qu 8. Interviewee 10 (a sixth form college teacher who teaches A Level) puts D as first choice and explains as follows:

“Because in terms of understanding mathematics I think that the highest level of understanding of mathematics, that’s what maths is about. Making connections and you know, yeah pretty much making connections... It’s quite difficult. But the reason I have put these two (B,A) at the bottom, I mean obviously these are the most important to teaching but I don’t think they’re the most important to understanding” (Int 10, PGCE non-MEC, qu 8)

In contrast, interviewee 11 almost reverses the causality in her assertion that in order to understand a concept, she must be able to explain it first:

“I am taking that from my point of view as well, in order for me to understand something properly I need to make sure I explain it properly. If I don’t explain it properly then the element is I don’t understand it basically. So I think in order for you to understand it you need to be able to explain it correctly.” (Int 11, PGCE MEC, qu 8)

This offers us an interesting example of the Vygotskian idea of importance of talk and discourse in the formation of thought and the generation of ideas. Clearly this teacher uses talk not only to explain ideas already formed, but in the formation of her ideas, to the extent
that her test of whether she does indeed understand a concept is whether or not she can explain it.

Some teachers offer useful mathematical examples, such as Interviewee 17 who chose B as first choice in qu 8. In his justification however, he seems to me to be discussing ways of making connections in mathematics as much as communicating:

“...sometimes I like to see and be able to show how one area of maths or similar areas within the same topic area, how it all comes together. For example, maybe if you're teaching them how to draw straight line graphs and then you're teaching them how to solve simultaneous equations and then you move onto show them how, you know, the intersection of the two lines is actually the solution to that. So that type of thing, I like trying to bring that in where you can”. (Int 17, PGCE MEC, qu 8)

Example responses to Qu 9

In the second task, qu 9 of the interview, participants were shown statements about how UMID might develop, and invited to rank them in order of importance.

Qu 9 Here are three statements related to how ‘understanding of maths in depth’ may develop. Please arrange the statements in order of importance for you, with the most important first.

An understanding of mathematics in depth develops by investigation and/or working out difficult and taxing problems. (F)

An understanding of mathematics in depth develops by living with and/or spending time thinking about mathematical ideas. (G)

An understanding of mathematics in depth develops from the understanding that this takes time and it is about being on a journey. (H)
Why is the first one the most important?

Items F and G were first choices for the majority of participants, with 18 respondents selecting either F or G as their first choice (see Tables 3 and 5 below). A number of respondents talked about the importance of working through problems themselves in order to develop a good understanding of the mathematics involved; there were also various comments made about the importance of spending time on mathematics.
Table 3: table of ranked responses to statements in Question 8 and Question 9

Responses ranked highest to lowest from left to right

Merged cells indicate where respondents gave statements equal priority

<table>
<thead>
<tr>
<th>Interview no.</th>
<th>Ranked responses to Qu. 8</th>
<th>Ranked responses to Qu. 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B C D A E F H G</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>B E, A D C G F H</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>B A D, C, E F H G</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>D C E B A F, G H</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>C E D, B A F H G</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>C D A B E G H F</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>A, B C D E F G H</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>B E D C A H G F</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>C A D E B H F G</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>D E C B, A G F H</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>B C A E D G F H</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>E B A D C F G H</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>C D B A E F G H</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>C D E A B F G H</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>D B C E A G F H</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>B D A E, C F, G H</td>
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</tr>
<tr>
<td>17</td>
<td>B E C D A F H G</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>B E A C D G H F</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>D E B C A G H F</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>C, E B A D G, F H</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>E C, A D, B H F G</td>
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Table 4: Frequency of 1st, 2nd etc choices in Question 8

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Table 5: Frequency of 1st, 2nd, 3rd choices in Question 9

<table>
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Table 6: Consistency of responses between qu 7 and qu 8

<table>
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<th>Interview number</th>
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<td>D (connections)</td>
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<tr>
<td>5</td>
<td>yes</td>
<td>C (understand why)</td>
</tr>
<tr>
<td>6</td>
<td>yes</td>
<td>C (understand why)</td>
</tr>
<tr>
<td>7</td>
<td>yes</td>
<td>B (communication)</td>
</tr>
<tr>
<td>9</td>
<td>yes</td>
<td>C (understand why)</td>
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<td>C (understand why)</td>
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<tr>
<td>15</td>
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<tr>
<td>18</td>
<td>yes</td>
<td>B (communication)</td>
</tr>
<tr>
<td>19</td>
<td>yes</td>
<td>D (connections)</td>
</tr>
</tbody>
</table>
Some interesting responses to qu 9 gave insights into participants’ own mathematical learning. For example, Interviewee 18 gave G as his first choice and offered this comment in support of his selection:

“I get deeper and deeper knowledge of it anyway because you read the books and you talk to people and you live in it.
Because maths is a foreign language isn’t it? And the best way to learn a foreign language is to go and live in the country and that’s what I do”. (Int 18, PGCE non-MEC, qu 9)

Interviewee 20 gives a similar impression of immersion in the subject:

“Because when you are… when I was learning my maths and I’m still learning now… I’m learning about my teaching now, I live and breathe it almost” (Int 20, PGCE non-MEC, qu 9),

as does Interviewee 10:

“I think mainly the word ‘living’ there swung it for me because we’re talking about understanding in depth and something that a real understanding that not many people have, I think you have really got to live with it. You know, it sort of takes over your life” (Int 10, PGCE non-MEC, qu 9).

Interviewee 8 relates to the idea of understanding mathematics as being a journey:

“...and I think you can always learn and there's always more you can learn and whoever you're with and whoever is teaching the maths you can learn and take tips off them. So it is a journey and I don't know where the journey ends really because it carries on and on and on and on.” (Int 8, PGCE non-MEC, qu 9)

**Other points emerging from Qus 7, 8 and 9**

Some teachers responded by relating the questions about UMID (qu 7, 8) to themselves, e.g.
“Explaining it to others, I just think that's probably one of the harder things to do. You understand it in your head but if you can actually articulate it, explain it, demonstrate it, lead someone through a process, however it works, the depth is really there” (Int 7, PGCE non-MEC, qu 8),

but others responded by discussing how they identify UMID in their students, e.g.,

“So, rather than just regurgitating things that they have been... tools and tricks that they have been taught, they actually deeply understand every question and where it is coming from. And they can also transfer skills. They don’t pigeonhole this is an algebra question, this is a coordinates question, you know, they have got the ability to transfer the skills to right across all the subject areas”. (Int 4, MDPT, qu 7)

and one identified how she knows if students do not exhibit UMID:

“I have taught a few of those groups… and I feel like I've got them through and they will get their C’s by just learning things, algorithms you do step one, you do step two, then you do step three and there's your answer. And they know how to do it but they actually don't really understand what they're doing”. (Int 8, PGCE non-MEC, qu 7)

One respondent gave an example of what deep understanding would mean for himself, and then for his pupils, suggesting an interesting analogy of jigsaw pieces fitting together:

“I suppose in a very simple way it’s seeing graphs everywhere. I think it’s seeing functions. I mean for me that’s what it really is, it’s that deep understanding is you know, for me the pinnacle of what I would call my mathematical understanding is a functional thinking. You know, it’s seeing inputs and outputs and seeing that translated under graphs and it’s that visual… for me it’s visualising the physical world into graphs.

Now that’s a very top level answer. In terms of pupils and understanding I think there’s two key things. It’s being able to connect the different ideas and I personally put in quite a lot about the jigsaw effect in mathematics and you find yourself putting
in pieces and you have got to… and then at some stage the piece goes in that joins that to that.

And I think when you can start to get enough of the pieces of that jigsaw that you’ve got that… I mean what you end up with is you end up with different routes to the same answer and I think, I suppose if I was to write down an answer that the understanding is being able to find different routes to the same place and to be able to understand how those different routes were”. (Int.19, PGCE non-MEC, qu 7)

This respondent developed his ‘jigsaw’ analogy in response to qu. 8:

“...and taking with us this idea of this networking. The jigsaw...three dimensions and you get a network and all these ideas are tied onto each other by little strings and you know, the more strings you’ve got, the less chance one little bit’s going to fall off at the end”. (Int 19, PGCE non-MEC, qu 8)

6.4 Stage 4: Identification of other key emerging themes

Having analysed the interviews question by question, I then re-read each interview as a whole and jotted down a brief overall ‘message’ that I received from it – in other words, what I thought this person was telling me, their key ideas. This revealed some interesting themes.

1) **Areas of weakness / insecurity can become transformed into areas of strength when one spends time thinking through them and preparing to teach them**

Three respondents gave spontaneous examples of this: interviewee 5 (MDPT, response to qu. 10, Appendix 12b, p. 210-211) discussing preparation and teaching a topic in statistics; interviewee 16 (MDPT, response to qu. 5, Appendix 12a, p. 206), discussing preparation and teaching of algebra; interviewee 13 (PGCE non-MEC, response to qu. 5, Appendix 12a, p. 203-204) discussing preparation and teaching of graph transformations; and interviewee 21 (PGCE non-MEC, response to qu.6, Appendix 12a, p. 209) discussing weaknesses becoming
strengths. In each case, the teacher commented that the topic identified was a perceived area of weakness for them, and that they made extra efforts to prepare the material thoroughly because of this. A closely related issue that emerges is that

2) **Style of teaching is different when the material is new to the teacher.**

One respondent states this explicitly:

> “You teach in a different way when you have recently learned something...you pass on your learning process much more.” (Int 19, PGCE non-MEC, qu 6, Appendix 12a, p. 208)

Another respondent speaks of moving outside his comfort zone (Int 16, MDPT, response to qu. 3, Appendix 9, p. 190). Theme 2 is implicit in the discourse of many respondents.

Several respondents explained that they

3) **developed deep understanding of a topic through preparing to teach it.**

See for example: interviewee 5 (MDPT, response to qu. 10, Appendix 12b, p.212); interviewee 10 (PGCE non-MEC, response to qu. 10, Appendix 12b, p. 214), interviewee 21 (PGCE non-MEC, response to qu. 10, Appendix 12b, p. 219) and interviewee 11 (PGCE MEC, case study, response to qu. 2, p. 145). In other words, this is learning and teaching mathematics at the same time – a phenomenon identified by Ma (1999) as discussed in Chapter 2, p. 44. Interviewee 10 (a sixth form college teacher) stated that his mathematics degree prepared him well for A Level teaching, but not so well for GCSE teaching. He notes that he it was hard to take a step back and think about simple concepts from the point of view of a learner. Interviewee 7 found the mathematics she encountered as a student at degree level and A level were very procedural and not very meaningful, and that she wants to do differently as a teacher herself.

Some other themes emerge as a result of teachers commenting on their experience of mathematics during their training/subject knowledge course, as follows:
4) *Growth in knowledge and confidence can alter one’s perceived identity:*

Several respondents commented in this area, the MDPT interviewees 2, 5, 15, 16 and 18 in particular. For example,

“I am a much more confident teacher from being here and doing all the different things we have done, taking them back with me and I actually feel like part of the mathematics department…and I am comfortable with it, whereas two years ago if someone had said you will be teaching statistics in year 11, I would have looked at them and laughed..” (Int 5, MDPT, qu 11)

This teacher also commented on the benefits of collaboration with peers in deepening her own understanding. She clearly felt a strong sense of being as a maths teacher now. A similar effect can be seen in Interviewee 16, who works in special education and who comments upon the effects upon him of following the MDPT:

“In the classroom [the course has] helped tremendously. And what it has done as well, can I say, is when I go to Heads of Departments meetings... I’m kind of treated as an equal even though I’m not in the true sense of the word, a mathematician...I think if I had not done that course...I don’t think I’d have had the same respect.”

“...what it’s [the course] done is it has given me the confidence to try new things.”

“I now read stuff in the bath about mathematics and stuff like that which I wouldn’t have done.” (Int 16, MDPT, qu 3)

See also interviewee 2 (response to qu. 4, Appendix 9, p. 188, “the old (name) would have given them a rule”); interviewee 15 (response to qu. 10, Appendix 5, p. 1781); and interviewee 18 (Appendix 12a, response to qu. 7, p. 207, “I am born again maths”, “Marcus du Sautoy for a day”).

The final overarching theme that emerges is:
5) *Reconstructing existing knowledge can result in transformation of teaching approach*

This theme is exemplified by interviewee 4, discussed as a case study in Chapter 7:

“…that session that we did last time gave me a clear understanding of why things work and it is actually things I often didn’t understand… and I know how to think about those things now, and I know how to draw it out of pupils rather than going in and saying, ‘this is how it works because it does’.” (Int 4, MDPT, qu 4, p.141)

This theme is supported by Ruthven (2011) (see Chapter 2, p.46) who suggests that for teachers, reconstructing existing knowledge is often more important than acquiring wholly new knowledge. Interviewee 4 reports that he now feels able to use an investigational approach to teaching and learning. As a result of growth of his own confidence in the subject, he has the confidence to allow pupils to explore, and does not feel he has to control everything. Graven (2004), in her study of non-specialist mathematics teachers’ learning on an in-service course, also finds that confidence is pivotal in teachers’ understanding and explaining of mathematics.

Another respondent who began his career as a primary school teacher and now works in special education, comments on his experience of reconstructing knowledge when he encountered guidance on the method of partitioning in arithmetic:

“…the numeracy strategy. I keep going back to that because that actually taught me so when I was using this folder in primary schools I’d open it at the page of partition and I would read through it and I would say, ah, I didn’t really realise that you could do that. Even as an adult because I would be in my 30’s then and it was this kind of hidden thing that I hadn’t discovered until I was grown up. (Int 12, MDPT, qu 5, p. 203)

One teacher admits that as a student she used to just accept what she was taught, but as a result of training to teach, she now questions, and asks why.

“I have started to see things perhaps deeper and try to anticipate questions of why really - because I was never the why person” (Int 3, PGCE non-MEC, qu 3, p. 184)
This indicates a change in approach to teaching, and also could be interpreted in terms of identity.

Interviewee 14 reports a change of teaching approach as a result of her experience on the MDPT – she notes that the course ‘stripped back’ basic ideas of maths (c.f. Adler and Davis (2006) ‘decompression’, see Chapter 2, p.48) so that now she understands learners’ misconceptions better. She reports that she was already confident in the mathematics herself, but now she opens up more contexts and applies it differently, and is more enthusiastic. Similarly, Interviewee 15 states that MDPT gave her the links. She already knew certain mathematical techniques but did not know how they all linked together. She was teaching various techniques but did not have a full understanding; now she has a fuller picture and sees the connections between concepts (Appendix 5, p. 178a).

Chapter conclusion

In this chapter we see the outcomes of a diverse set of interview responses, and bringing these together to form an integrative whole is not straightforward. However it is possible to detect various common threads.

PGCE students’ growth in PCK

Analysis of responses to qus 5 and 6, where respondents talk about topics that they have taught, shows that former PGCE students who are quite new to teaching highlight the importance of thorough planning and preparation. They also cite some clear examples of their own learning in terms of PCK, i.e., asking the right questions, pacing lessons, understanding the needs of the learners. On the whole, their growth in knowledge seems to be more located in PCK, whereas their mathematical content knowledge is already secure. This is to be expected, as they have all studied mathematics at degree level or MEC prior to commencing PGCE.
MDPT teachers’ growth in SMK and shifts in view of themselves as becoming maths teachers

MDPT teachers focus more upon a growing confidence in teaching mathematical material that they previously were not confident with. Thus we see a growth in mathematical content knowledge and specialised content knowledge more so than a growth in general teaching strategies. This is not surprising since most of this sample were already experienced teachers. MDPT teachers also comment more upon alterations to their perceptions of their professional selves, seeing themselves in a different way – as maths teachers – as a result of following the course. This can be understood in terms of identity, and is echoed in the findings of Crisan and Rodd (2011) in their study of MDPT teachers. Graven (2004) also comments upon the shift in teachers’ perceived identities, from ‘teacher of mathematics’ to ‘mathematics teacher’. I note that my findings in relation to the concept of identity arose relatively late in this study. Therefore I have not attempted to locate these ideas in the context of the literature on identity. However I realise that this may be necessary for the development of my work in the future.

There are interesting parallels to be drawn between the results of this study and those of Graven (op. cit.) who highlights the emergence of confidence as a key factor in successful teacher learning, suggesting that confidence is both a product and process of learning. Graven notes that confidence develops over time and so many short-term studies may have overlooked it as an issue. She asserts that “mastery involves the insight to know when you do not know, the confidence to admit to this, and the ability to access the necessary information...” (p. 207). Both Graven and Crisan and Rodd (op. cit.) discuss teachers’ learning on these CPD programmes as induction into a community of practice (Wenger, 1998).

UMID as ‘knowing why’ and ‘being able to communicate’

Analysis of responses to qu 7 and 8 show a high level of consistency between open descriptions of UMID in qu 7 and choice of statements in qu 8. In qu 8, the majority of respondents conceived ‘understanding mathematics in depth’ as ‘knowing why’ and also in terms of being able to communicate ideas to others, choosing statements B or C as most important. This is supported by work by Adler et al. (under review) whose study of MEC students reveals that their perceptions of ‘understanding mathematics in depth’ strongly feature mathematical reasoning or knowing why. Adler et al. also detect a positive
mathematical disposition threaded through much of the students’ discourse, and this is also evident in my study - in the case of both MEC and MDPT teachers.

**Development of UMID through investigation of problems and spending time**
Statements F and G were the most popular first choices in qu 9, and various comments were made relating to immersion in the subject, and spending time working on mathematics. This supports Watson and Barton’s (2011) ideas of teachers enacting mathematics, being involved in the process of mathematics (see Chapter 2, p.38). Mathematics subject knowledge as a ‘learnable disposition’ (Davis, 2011) and Askew’s (2008) ‘mathematical sensibility’ are also visible here (see Chapter 2, p. 40).

**Other key emerging themes**
Additionally when reading the interviews holistically some key overarching themes emerge.

1) *Areas of weakness / insecurity can become transformed into areas of strength when one spends time thinking through them and preparing to teach them*

2) *Style of teaching is different when the material is new to the teacher.*

3) *Teachers can develop deep understanding of a topic through preparing to teach it*

4) *Growth in knowledge and confidence can alter one’s perceived identity*

5) *Reconstructing existing knowledge can result in transformation of teaching approach*

The data presented here is evidence from discussions about ‘deep understanding of mathematics’ with two specific groups of new mathematics teachers. This study, whilst informed by Adler et al., (2009 and ongoing) and also focusing upon students on MECs, moves the research into a new direction by also investigating the ideas of participants on the Mathematics Development Programme for Teachers (MDPT).
It is significant that the teachers in this sample have all followed mathematics courses which were specifically designed to be mindful of Ma’s recommendations to:

1) Address teacher knowledge and student learning at the same time,
2) Enhance the interaction between teachers’ study of school mathematics and how to teach it. (Ma, 1999, p. 146)

It is evident from the discourse that Ma’s themes are important; indeed, I would suggest that they underpin responses throughout the interviews. In the next chapter I shall consider in more detail the interviews of two of the teachers. This will enable me to examine more closely the influence of Ma’s ideas in the articulated experience of the interviewees, and to show more clearly how the above five emergent themes are located in their discourse. I shall consider case studies of two teachers, one a former PGCE student and one a former MDPT participant.
Chapter 7  Case Studies

Chapter introduction

Previously (Chapter 6) I have reported in detail interviewees’ responses to interview questions. Analysis of this data has generated some interesting findings. However, in order to probe and understand more deeply what respondents are saying, it is helpful to consider a small number of cases in greater depth. In this chapter I re-examine the responses of two interviewees, one a former MDPT participant and one a former PGCE student, in order to shed further light upon the experiences and ideas of the teachers involved in this study. Cohen et al. (2007) suggest that case studies can highlight significant behaviour or experiences, with an emphasis upon quality of information rather than quantity. They note that case studies can serve to “separate the significant few from the insignificant many” in terms of instances of behaviour / experience (p. 258). I have selected the particular individuals concerned as I believe the experiences that they report are significant, but also typical of their group, and they have expressed themselves particularly clearly and in depth in their interviews, therefore offering a rich source of material to explore. Referring to Jaworski (1997), I have selected these two case studies because I believe that these examples are “sufficiently generic to represent the validity of the theory they support” (p. 118).

I also discuss which of the key emergent themes that were introduced in Chapter 6 are most prominent in the discourse for these individuals. The themes are as follows:

1) Areas of weakness / insecurity can become transformed into areas of strength when one spends time thinking through them and preparing to teach them

2) Style of teaching is different when the material is new to the teacher.

3) Teachers can develop deep understanding of a topic through preparing to teach it

4) Growth in knowledge and confidence can alter one’s perceived identity

5) Reconstructing existing knowledge can result in transformation of teaching approach
7.1 John – igniting the passion

The first participant, Interviewee 4, I shall refer to as John. At the time of the interview, John was in his late twenties and had been teaching for four or five years. He followed the MDPT course in 2009-10. He had originally trained and worked as a Business Studies teacher, but at the time of the interviews was teaching mathematics at an urban comprehensive school in an economically deprived area of Merseyside.

John clearly articulates the change in his teaching approach that took place as a result of his experience on the MDPT. He describes how formerly because of his own limited subject knowledge he lacked confidence and adopted a prescriptive approach to teaching, but that now he is able to use more open, investigative approaches to teaching and learning (my italics):

“And it is about them investigating it and slowly but surely being able to pull ideas out of kids and I think what that session that we did last time gave me a clear understanding of why things work and it is actually things I often didn’t understand and didn’t have a clear understanding of I now know how to think about those things now and I know how to draw it out of pupils rather than going in and saying you know, this is how it works because it does.

You know it just gives you a lot more confidence to allow the pupils to... I think what a lot of these sessions have done is it gives you the confidence to be a bit freer in the classroom to allow the pupils to investigate things themselves rather than come in and say this is how you do it, it is just because it is and that’s the big difference in my teaching.” (Int.4, qu 3)

John’s comments about being ‘a bit freer in the classroom’ indicate a change – possibly a transformation - in teaching approach (Theme 5).

John describes how when he began teaching mathematics, he was nervous about being asked questions, and used Ten Ticks (2011), a resource which provides multiple pupil worksheets containing hundreds of short procedural questions and exercises:
“There are often things that I had never come across. You know, when... because my training was business studies and to suddenly be thrown into a classroom and say right today you are now a maths teacher without understanding the courses, anything like that and initially for your first 3-6 months it is just making sure that every lesson you go to you have got something for the kids to sit down and do in a way that they don’t ask you many questions, Ten Ticks to me in the first three months was the greatest invention of all time because I could go in and hand them out and I could have a pile of Ten Ticks ready for them. I didn’t want to be asked any questions, you know...”

Interviewer: “Kind of like defensive teaching”.

“Completely, yeah. Completely. Whereas now, even though I wouldn’t say I was an amazingly better mathematician, I have got a lot more confidence in allowing the pupils to investigate and it being a more investigative subject, erm... maths than just a teaching subject, you know and actually having the ability... and you know it has made me plan much better.”

John gives an interesting example of a probability lesson he recently gave, using the ‘horse race’ activity. ‘Horses’, numbered 1 to 12, line up at the start of a race on the game board. Pupils choose a horse to support. Two dice are thrown, and the sum of scores each time indicates which horse moves forward one space. Some horses seem to progress faster than others...

“And I started the lesson off just by playing bingo, you know where the pupils pick six numbers and they really enjoyed that but they couldn’t work out why I kept winning and why I kept putting number sevens on the board and eventually a few of them started copying me. But they couldn’t understand why they just thought it was just... I was setting it up.

And then a couple of them started to look at the dice and think about it and you know, a couple of them were asking questions but I didn’t at that stage didn’t draw it out of them. Then they did the horseracing in groups and I printed off big plastic boards and
they did a horserace where they were moving the counters up erm... and then they could physically see the pattern, you know there was a physical pattern and they could clearly see that there was something going on here and slowly but surely, you know, one or two of them started realising that you know, why it was happening. Then I got a couple of them out to the board to draw what was actually happening and what was the... the chart... so to say a one and a one, a two and a two, erm... and they were fascinated by it, you know, they really were. It was quite interesting because after that lesson you would hear them around school you know betting, saying ‘I’ll bet you’. Which is great” (Int.4, qu 3)

There is clear significance for John in the fact that his pupils leave this lesson talking about the activity and interested in it. This is something he comments on shortly afterwards in the interview:

“And to then hear them talking about maths outside the room, you know that’s ultimately quite erm... and it’s a noticeable change around the school and that is why we’re doing these things like the crystal maze thing that we are going to do in a few weeks and things like that.
And it’s getting this complete (sic) different attitude to maths which is the big challenge, and getting a bit of a buzz about maths which is the big challenge.”

Towards the end of interview, in discussion about qu 9 (how is understanding of mathematics in depth developed?), John comments on his preferred choices F (by investigation and/or working out difficult and taxing problems) and G (living with and/or spending time thinking about mathematical ideas). He then moves on to talk about the importance of passion for the subject, and how he is trying to nurture this in his teaching.

“...understanding maths in depth develops in living with or spending time thinking about mathematical ideas, I think again because once you start enjoying something then you want to do more of it. Someone like [names one of the MDPT tutors] is a great example, you know, everything he does is a mathematical... it becomes part of your life doesn’t it.
...which I think for your subject that is such an important thing, you know, I think if you look at outstanding teachers from OFSTED you know one of the underlying
things is a passion for your subject...and the only way you’re going to get passion for your subject is if that subject becomes a part of you.

... But to take someone like myself who is a business studies teacher with no maths I think actually igniting that passion is the challenge. I think that is something that I’ve, you know, the way I have ignited it personally is... because I have got responsibilities in key stage 3 is to try and create this buzz around maths and an enjoyment of maths, and I have started to find that if you start doing what it says there in F you know, investigating and allowing pupils to investigate, they get that sense of achievement...” (Int 4, qu 9).

In the case of John, then, we see clearly that his teaching approach has been transformed as a result of his engagement with the MDPT and the reconstruction and development of his own knowledge that this has entailed. Theme 5 (Reconstructing existing knowledge can result in transformation of teaching approach) clearly emerges from his discourse as he talks with enthusiasm about his role and his work.

John says that although he does not think of himself as ‘an amazing better mathematician’, he has a lot more confidence teaching the subject and allowing pupils to investigate. This and his other remarks suggest growth in his confidence and knowledge, implying he is a better mathematician at the school level and that his perception of himself as a mathematics teacher has developed. Thus theme 4 (Growth in knowledge and confidence can alter one’s perceived identity) is implied by his discourse.

7.2 Lucy – teaching and learning maths at the same time

For the second case study, I shall consider Interviewee 11, here referred to as Lucy. Lucy completed MEC in 2008 and PGCE in 2009. At the time of the interview she was in her early twenties and had been teaching for one year at a suburban comprehensive school.

From early on in her interview, Lucy discusses the importance for her of fully understanding the mathematics she is learning or teaching. She also indicates that for her, the processes of
learning and teaching mathematics are happening simultaneously – an echo of Ma’s (1999) recommendations (my italics):

“I think the MEC course definitely prepared me because it allowed me to look at a lot of proof work which I had looked at really vaguely but it wasn’t really until the MEC course where I actually gathered a better appreciation for what I was actually learning and teaching at the same time. So the idea of proof is actually real important element of teaching mathematics and learning it at the same time. (Int 11, qu 2)

In response to qu 3 she gives an example of a concept she now understands better – Pythagoras’ Theorem:

“Pythagoras Theorem for example. I always understood it in school, I always got the work that I did with it but I don’t feel like to a true effect I had the actual correct understanding of what it actually all was. And since I started the PGCE it’s gradually progressed and I have a full understanding of it now and it’s been a lot easier to teach this year. It’s been a lot easier to get forwards, the students having that complete understanding basically and I have actually taught the proof to a lot of my classes which has made them understand it more. I think that was something that I wasn’t taught when I was in school and it was just like a given.

You know, C squared equals A squared plus B squared. It was just a given in school, it was never, I don’t think, explained for me. And I was never given any kind of proof towards it.

Yeah just all the different ways you can actually show Pythagoras Theorem and it was never shown to me in school, so it was just as a given really. So that’s probably the one that’s sticking in my mind more than anything that I am able to teach that now with a full visual representation and a proof of where you get it all from. I am able to do that now, which has made a major impact”. (Int 11, qu 3)

In response to qu 8 (what does understanding mathematics in depth mean to you?), Lucy is clear that top of her choices is B (being able to explain/communicate mathematical ideas to others):
“...in order for me to understand something properly I need to make sure I explain it properly. If I don’t explain it properly then the element is I don’t understand it basically. So I think in order for you to understand it you need to be able to explain it correctly.”

She is very emphatic about the importance of clear explanations, continuing:

“It makes a massive difference as well if you don’t explain something properly then it has a massive effect on the students you’re teaching”. (Int 11, qu 8)

As a new teacher, she has had experience of lessons where her explanations have not been as good as she might have liked (she discusses an example in the interview), and so she can compare the effects. She also notices her progression and development in teaching:

“I have noticed that I have improved, the more I have taught something, I have noticed improvement which I am happy about...The more I explain something, the amount of times I teach it, the better I am in explaining it...And it’s just finding different ways to teach it as well which again comes under being able to explain it properly, finding different ways to explain it”. (Int 11, qu 8)

As an example of an area of mathematics that she understands well, Lucy chooses straight line graphs, and again we see her commenting on her own progress in teaching this concept:

“So I think definitely straight line graphs, that’s something that I feel, I just think, as I say the more I’m teaching it, the more I’m picking up new things and something I have never noticed before on this, which is really good and again that just alters the way that I am explaining it to each class which is good. It’s something that I have taught quite a bit this year”. (Int 11, qu 10)

Another key aspect of Lucy’s experience that emerges from her interview is her developing KCS (Knowledge of Content and Students, Ball et al., 2008) – a key component of pedagogical content knowledge. She discusses how she initially found it difficult when students used different methods from her own, to tackle mathematical questions:
“I had answered the question and gone through the question but the way they kind of answered it, which was a right way of answering it, the way they had gone about it… but I hadn’t thought of it, so it created a bit of a problem… I think that’s something that has caused me a few problems in certain lessons, the fact that they have found another way to do it and because I find it difficult sometimes thinking on my feet…

And that’s obviously what I have found out this year, you know from teaching different students. They have different ways of solving it and their way’s not necessarily wrong but it’s just a different way from the way I have taught it, or learnt it...And I think that’s also something that has helped me develop in teaching things differently, taking on board what other students do as well and remembering what they’ve done”. (Int 11, qu 6)

Towards the end of the interview Lucy makes links between her confidence as a teacher and her growth in knowledge and understanding of mathematics. Again, she connects the learning and teaching of mathematics.

“it was something I was conscious of, my subject knowledge, right from the start but I just think it’s improved dramatically and I have noticed, you know, I feel a lot more competent when I am teaching now and I feel a lot more confident when I am answering questions and things like that so… it’s, I have noticed a massive improvement…since I have started really, just over the whole process of it so it’s just, I don’t know, I have just noticed my confidence has really come on and that was a massive thing for me to be honest. But I think the reason that my confidence has come on is because I have got a better understanding of it now.

...I do feel a lot more confident and I feel the reason for that is because I have got a much better understanding of what I am actually teaching and what I’m learning at the same time”. (Int 11, qu 11)

Throughout her interview, Lucy relates her own understanding of mathematics concepts directly to her ability to explain them to learners. She sets this scene early on: “If I don’t explain it properly then... I don’t understand it”. “In order to understand it you need to be able to explain it correctly”. In the case of Lucy, we see clearly how a teacher can develop
deep understanding of a topic through preparing to teach it (Theme 3). For example, she discusses how her teaching improves the more she teaches a particular topic: “the more I explain something...the better I am in explaining it.” This also connects to Theme 2 (Style of teaching is different when the material is new to the teacher), as Lucy discusses her own development during her first year of teaching.

Theme 1 (Areas of weakness / insecurity can become transformed into areas of strength when one spends time thinking through them and preparing to teach them) is also evident in Lucy’s discourse, for example in her discussion of the change in her understanding of Pythagoras’ Theorem and consequent way of teaching this concept.

Chapter conclusion

These case studies are helpful in bringing alive the interview data and making it more meaningful. In each case I have considered in some depth and detail the response of a particular individual. This is not so much because these individuals are especially interesting and worthy of study (although they are), but because a case study can illuminate the general through the particular. In other words, in the cases of John and Lucy it is possible to see facets of experience which are shared by their colleagues in the MDPT and PGCE samples respectively. It is possible to detect themes 1 to 5 across the discourses of all 21 of the interviews, but for each individual the emphasis and weighting between the themes differs. As I have discussed above, Theme 3 is significant in Lucy’s discourse and Theme 5 in John’s.

In Chapter 8 I will provide an overview and discussion of the findings of this investigation.
Chapter 8 Discussion

Chapter introduction

In this chapter, the results of the qualitative data which were reported in Chapters 6 and 7 above will be discussed. Whereas in Chapters 6 and 7 the data was analysed in some detail, in this chapter I will summarise the findings in a more compressed and overarching manner, with the aim of drawing out key findings of interest, taking ideas forward and discussing these in the light of relevant literature. I will discuss the limitations of the study and areas for future research. I will also comment on the impact that this work has had upon my own professional practice and its relevance for mathematics education more widely.

The research questions addressed in this study are:

1. How is ‘understanding mathematics in depth’ conceptualised by two particular groups of novice mathematics teachers?

2. What are novice mathematics teachers’ beliefs about how ‘understanding mathematics in depth’ is attained?

3. What themes are privileged in the discourse of novice mathematics teachers in relation to their preparation for, and experience of, mathematics teaching?

8.1 Novice mathematics teachers’ knowledge growth

Responses to these interview questions reveal differing perceptions from the PGCE and MDPT sample groups. Several PGCE interviewees discuss the importance of planning and preparation – aspects of the role which feature particularly highly for new teachers. With reference to the Shulman / Ball models of teacher knowledge, these teachers’ recent and
ongoing growth in PCK is clear from the discourse, particularly the aspect of PCK which Ball et al. (2008) describe as Knowledge of Content and Students (KCS). Interviewee 7 and her discussion of an early attempt to teach questionnaire/survey design to a Year 7 class (Chapter 6, p. 120) provides a good example of this. We can also see here clearly the enactment of the Knowledge Quartet (Rowland et al., 2009; see Chapter 2, p. 33), as these teachers, with their own basis of knowledge (foundation), then work hard on their planning to effect a transformation and build connections which will make the knowledge accessible to others. Then they find that it is necessary to think on one’s feet - that even when planning is carefully done, learners can still find concepts difficult to understand, and so flexibility or contingency is needed. The case study of Lucy (Chapter 7) illuminates these phenomena clearly as she discusses her own experience during her first year of teaching.

MDPT respondents focus more upon growth in subject matter knowledge (SMK) and confidence, with several making comments that relate to a perceived professional shift as they begin to see themselves as mathematics teachers (as a result of following the MDPT). This links to Cousins’ (2006) suggestion that learning is both affective and cognitive, and often involves identity shifts (see Chapter 2, p. 36). It is also supported by the work of Graven (2004) and Crisan and Rodd (2011) on evolving identity within a community of practice. Emergent from the discourse at interview is the idea that the experience of MDPT has for some teachers been very powerful in terms of a transformation not only of their subject knowledge and confidence, but also their teaching approach and their own views of themselves. In the case of John (Chapter 7), we see a profound effect upon teaching approach as a result of growth of teacher knowledge and confidence.

8.2 Research questions 1 and 2: how ‘understanding mathematics in depth’ is conceptualised and attained

Responses revealed an emphasis upon understanding mathematics in depth as ‘knowing why’ and as being able to explain/communicate ideas to others, with some respondents clearly indicating that a good test of one’s own knowledge is whether one can explain it to someone else. An appreciation of the importance of language as a means to develop and build understanding (Vygotsky, 1962; Lee, 2006) was evident in the discourse. Respondents were
also able to identify where understanding mathematics in depth (UMID) was absent, and gave various examples including observation of some of their students, and descriptions of their own learning in earlier contexts (e.g. school, degree) where an instrumental or procedural approach was followed. This relates clearly to Skemp’s (1976) work on relational and instrumental understanding.

In response to the question about how understanding mathematics in depth might be achieved, there were clear indications of the need to spend time tackling mathematical tasks, and to immerse oneself in mathematics over a period of time. This is supported by Watson (2008) and Barton (2009) (see Chapter 2 above, p. 37-38) in their work on mathematics knowledge as a way of being, and in their argument for the necessity for teachers to have a mathematical disposition. It is also supported by Davis (2011) in his idea of mathematics knowledge as a ‘learnable disposition’ (p. 1507) rather than an explicit body of knowledge (see Chapter 2 above, p. 40). In Chapter 7, John comments upon this in terms of his own mathematical experience and explains how he extends this approach in his teaching through using methods that allow his pupils ‘be mathematical’.

8.3 Research question 3: other themes privileged in the discourse of novice mathematics teachers

When reading the interviews holistically some key overarching themes emerge.

1) Areas of weakness / insecurity can become transformed into areas of strength when one spends time thinking through them and preparing to teach them

2) Style of teaching is different when the material is new to the teacher.

3) Teachers can develop deep understanding of a topic through preparing to teach it

4) Growth in knowledge and confidence can alter one’s perceived identity

5) Reconstructing existing knowledge can result in transformation of teaching approach
I now briefly consider each of these themes.

1) *Areas of weakness / insecurity can become transformed into areas of strength when one spends time thinking through them and preparing to teach them*

This theme was evident in several interviews, from both PGCE and MDPT respondents. It is an articulation of the process of coming to understand concepts that one had previously misunderstood or partially understood, in the context of having to prepare to teach them. Given sufficient time and appropriate support, the teachers in question were able to tackle ideas that they admitted they had previously found difficult. The need and desire to offer a good learning experience to their own students provided the motivation to deepen and extend their own understanding. Their focus upon a topic previously regarded as a ‘weakness’ then resulted in transformation of their view of the topic, such that it became enjoyable, a strength.

2) *Style of teaching is different when the material is new to the teacher.*

This theme was visible in the responses of several interviewees, and one interviewee explicitly stated it. Teaching a topic that is new (to the teacher) demands a high degree of attention to the material. The process of attending to the concepts carefully as is necessary in these circumstances results in a teaching style that is different from that seen when the teacher is handling familiar material. We can think of the teacher needing to be more alert, ‘on their toes’, and experiencing a greater intellectual challenge in these contexts. This in itself is a stimulating experience for the teacher, and thus affects teaching style. One teacher commented that in these circumstances he was more likely to teach in a way in which he passed on his own learning processes.

3) *Teachers can develop deep understanding of a topic through preparing to teach it*

This is a key finding from the research, and is closely related to the idea articulated by many respondents, that understanding mathematics in depth *means* being able to explain it to others. The necessity to break down or decompress mathematical concepts to render them
accessible to young learners demands a deep level of understanding from the teacher. The processes that teachers go through when preparing to teach a topic therefore deepen their own understanding of the topic.

4) *Growth in knowledge and confidence can alter one’s perceived identity*

This theme is evident in responses from MDPT teachers. They describe their movement towards identifying themselves as mathematics teachers, having previously had a clear identity as a teacher of another subject. This is supported by Graven (2004) in her study of teachers on an in-service mathematics course. The teachers’ growing familiarity and confidence with mathematical ideas (gained through their practice and their engagement with the MDPT course) enables them to take on this mantle which previously some had considered daunting.

5) *Reconstructing existing knowledge can result in transformation of teaching approach*

Again, this theme emerges from the responses of MDPT teachers. Lacking appropriate preparation in subject knowledge and pedagogy, many had originally adopted a procedural approach to teaching mathematics. Engagement with the MDPT course provided opportunities for challenges to their understanding of key concepts, and a subsequent re-structuring and deepening of their knowledge. This then enabled them to develop their teaching approaches significantly.

Reflecting upon these findings, I can appreciate the power of research to ‘make the familiar strange’ (Delamont, in Wellington, 2004). All of these five themes have emerged from analysis of the interviews. Viewed on one level, they are not surprising; over a 25 year career in mathematics teaching and teacher education I have been exposed to, and implicitly aware of, all of these phenomena. However, I have not formulated or articulated any of them explicitly – and this is the important missing link which research of, and immersion in, the material has provided for my own understanding.

Themes 1, 2 and 3 above can be linked together, as can themes 4 and 5.
Themes 1, 2 and 3

Themes 1, 2 and 3 speak of the early stages of knowledge growth and development that occur when a teacher prepares new material to teach, perhaps for the first time, or early in their experience. Several respondents spoke about making an effort to prepare material that previously they had not felt comfortable with, and then emerging with a new confidence in those concepts, to the extent that they would now regard them as areas of strength, not weakness.

These findings clearly link to Ma’s recommendations to

3) Address teacher knowledge and student learning at the same time,
4) Enhance the interaction between teachers’ study of school mathematics and how to teach it. (Ma, 1999, p 146; discussed above in Chapter 2, p. 41-42)

Especially during the first few years of their careers, new mathematics teachers are engaging in this simultaneous development of their own knowledge (both SMK and PCK) and that of their students, as they prepare material to teach. In the case of Lucy we see this process clearly articulated and well understood by the teacher involved. I would suggest that in the UK, opportunities for this knowledge development to continue as teachers’ careers progress, are not currently prominent or structured into teachers’ working environment. This is supported by Smith (2004). Many teachers do of course take advantage of ongoing professional development opportunities that are available to them, but this uptake is patchy.

Ma (op. cit.) noticed that opportunities for ongoing subject knowledge development were present generally in the working environment for teachers in China but not in the US. The way in which teachers worked together and planned together made this an integral part of working life.

Themes 4 and 5

Themes 4 and 5 relate strongly to changes or transformation that can take place when an established teacher encounters new concepts, or familiar concepts presented in a new way,
and is offered the opportunity to try out new ways of thinking and working. These themes, whilst present in the background to the PGCE respondents’ discourses, were most prominent in the responses for the MDPT teachers. Several commented on changes to their knowledge and confidence, linking this to an altered self-image. This is also apparent in work done by Crisan and Rodd (2011). All of those on the MDPT are serving teachers, some with many years experience of teaching but in subjects other than mathematics. They have moved into teaching mathematics relatively recently. Many admit that before the MDPT course the only strategies for teaching mathematics that they knew about were those that they had previously encountered as school students themselves, which were often very procedural in approach. The case study of John is a good illustration of transformation of teaching approach brought about by a reconstruction and development of his own knowledge.

Ruthven’s (2011) comments on the importance of reconstruction of existing knowledge in the process of learning to teach mathematics are relevant here. Several of the MDPT respondents comment that they have changed their teaching approaches as a result of their improved or altered understanding of the mathematics involved. In particular, they comment upon making more links between concepts, using more varied applications and contexts, and in particular about giving their students opportunities to explore, investigate and think mathematically, rather than attempting to direct and control their thought. Several make comments about not having thought about it this way before, did not previously see the links, didn’t realise you could do it that way, etc. There is clear evidence of a reconstruction of knowledge happening here. These teachers are not starting from the beginning – they have a knowledge base, but it is incomplete and may involve misconceptions.

Watson’s (2008) assertion that non-specialist teachers need “more personal experience of the mathematical canon” (p. 4) is relevant here, as are Davis and Simmt’s (2006) comments about teacher education courses needing to involve mathematics that is “new to the do-ers” (p. 316; discussed above in Chapter 2, p. 40). It is evident from the interviews in this study that having been given this experience on the MDPT, non-specialist teachers can then go on to change their teaching approach quite significantly.

Looking back to the way in which the MEC and MDPT courses were devised and planned, reported in Chapter 3 above, it is interesting to reflect on these findings. Both courses, being formulated especially for teachers, were devised to incorporate an integrative view of subject
knowledge and pedagogical knowledge, with an emphasis upon depth of understanding rather than procedural knowledge. The closeness of the ideas of subject and pedagogical knowledge in the world of the teacher is evident from the outcomes of this study. It seems to be evident from this study that the courses have, at least for this sample of teachers, achieved what was intended. However this is an incidental outcome. My aim was to explore these teachers’ conceptions of deep understanding of mathematics, and to elicit other key emergent themes.

It is significant that these themes have emerged from discourse of new mathematics teachers; this was my specific sample. Interviews with experienced mathematics teachers would probably elicit different ideas and perspectives, and it would be interesting to compare their responses to those of the new teachers. Subject knowledge concepts are probably more transparent to experienced teachers. It is also the case that after the first few years of teaching, the UK school system often encourages teachers to focus away from their subject, for example to gain promotion and responsibility for whole-school areas.
8.4 Link between findings from quantitative data analysis and main study, and implications for the field

My findings from quantitative data analysis (see Chapter 4) inform and support my main study insofar as they shed light on the outcomes for students of following the MEC course in comparison to a degree course in mathematics. To further explore this idea, I would suggest a visualisation of the content of MEC and degree courses using a Venn diagram, as follows:

*Figure 6: Diagram to illustrate typical content areas of mathematics degree and mathematics subject enhancement course*

- Mathematics degree
  - Core of key developmental understandings (KDU’s) in mathematics e.g. calculus, limits, algebraic structure, mechanics, statistics, discrete.
  - Level of technical proficiency.
  - Opportunities for extended investigation.

- Mathematics Enhancement Course
  - Deep understanding of fundamental mathematics; decompression of school curriculum concepts. Early awareness of variety of pedagogical approaches.

Further development of concepts, greater abstraction, compression of mathematical ideas & communication. Higher level applications.
Figure 6 shows that there is a clear overlap in the content of the two routes, which contains key areas of fairly high-level and in-depth mathematics concepts, techniques and experiences. From my knowledge and experience of both routes, I believe it is not contentious to suggest that in the UK most mathematics degree courses and subject enhancement courses contain these areas in common. Examining the regions in the diagram outside the intersection, we firstly see that degree courses develop these concepts to a significantly higher level than MECs, and thus it is exposure to and engagement with such ideas that makes a degree mathematician’s experience distinctive. There are clearly things that a degree course does which a MEC does not.

Similarly, there are aspects of the MEC which provide a distinctive experience for the student. The MEC region outside the intersection includes in-depth study of aspects of school curriculum and an implicit focus upon pedagogical approaches – concepts which are not generally found in degree courses. It is also important to note that MEC tutors are often former school teachers and so their teaching approaches may differ from those of other academic mathematicians.

My analysis of quantitative data suggests that the two routes are preparing students equally well to move into mathematics teaching as a career. Figure 6 above can help us to understand where the commonality between these two routes lies, and also the distinctive ‘extra’ areas that each route provides. In the debate about approaches to recruitment and selection of potential mathematics teachers, issues about the nature of mathematics knowledge for teaching are critical, since decisions to recruit (or not) are made on the basis of judgements about subject knowledge. This is why it was relevant to examine quantitative data on PGCE students’ entry and exit grades, looking at both MEC and non-MEC students. The qualitative data in this study provides insights into the conceptions and experience of former PGCE students, some of whom have followed the MEC, and some who have followed degree courses. All their interviews shed light on the processes of becoming a mathematics teacher, and contribute to the overall emerging themes that I have identified. All their discourses offer us new ways of understanding the nature of mathematics knowledge for teaching and the nature of understanding mathematics in depth.
The above analysis of the common and distinct areas of the degree and MEC routes raises questions about mathematics teacher preparation more widely. If it is the case that students on MECs experience a curriculum that privileges deep understanding of fundamental mathematics and the decompression of mathematics concepts, and degree students do not, then how and where will degree students learn about these things? The simple answer is that they will pick them up (or not) where they have in the past: during school experience in the PGCE year and early years of teaching. However there are risks inherent in this path. Some novice teachers may spend time handling mathematical concepts in a way which is inappropriately compressed, prior to learning the approach that is needed. During this time student learning is inevitably compromised. Some novice teachers may take time to attain UMID themselves, having to encounter hurdles in student learning before being able to challenge and reconstruct their own understandings and from that point to adapt their teaching approach. Again, student learning is compromised.

It therefore appears that the knowledge growth within the mathematics education community which has taken place through and in the development of MECs now exposes deficits in more traditional mathematics teacher preparation routes. One could argue that all mathematics teachers should have the opportunity to do a MEC. Alternatively, all mathematics degree courses could or should contain units on UMID and decompression of school mathematics. This would provide an intellectually stimulating - and enjoyable - addition to the higher level topics and rigorous approaches taught within other sections of the undergraduate curriculum. Shulman endorses this idea and argues that university mathematics departments have a responsibility to address this priority:

“Current undergraduate mathematics programs seem to have no place for teaching fundamental mathematics for profound understanding... such knowledge is misconstrued as remedial instead of recognizing that it is rigorous and deserving of university-level instruction”

(Shulman, in Ma, 1999, p. xii)
8.5 Limitations of study and areas for future research

There are a number of limitations to this study which must be acknowledged, and which may provide impetus for future projects.

Sample
At 21 participants, the sample size is fairly small. More data, gained from a larger sample, could certainly strengthen the research. However, I am confident that the sample is of a sufficient size to produce meaningful results for the purpose of this thesis. Wellington (2004) discusses novice researchers’ “tendency to over-collect and under-analyse” (p.133). I do not think that I have fallen into this trap; I believe that I have collected sufficient data, and conducted a productive and meaningful analysis of it.

The sample consists only of course participants from one university. Whilst being useful in illuminating the practices at that university, this limits the extent to which wider generalisations can be made regarding MEC and MDPT participants in general in England. To enable more general conclusions to be reached, and to make a more significant contribution to knowledge in the field, future studies could investigate participants from courses at several universities. I note however, that investigation into the perceptions of course participants at one university can inform the field in general, since the case of one university can serve as an example providing insight into similar courses at other universities. The case of one university is also productive as a theme for an EdD in which one researches one’s own practice and its context. I was really interested to find out about the understandings of these students, and this desire drove my research.

Design
There is a limitation to the research in the design. I was the interviewer asking the questions of my current and former students. As discussed in Chapter 5, p 94, the existence of a tutor-student relationship will have introduced an element of subjectivity to the research. However I believe it also created the conditions for the open and extended responses which were given. If I had asked another researcher, not known beforehand to the participants, to conduct the interviews, then I may have obtained more objective responses. However the important
element of the pre-knowledge of each others’ contexts and background would have been lost. I wanted to interview my students myself; I found the whole process fascinating.

Also it would have been interesting and helpful to involve another suitable person in the research to read some of the interview transcripts and offer their own interpretation. This could have provided a triangulation of my own interpretations.

To extend this research further, it would be interesting to observe participants teaching, and to give participants mathematical tasks and discuss their responses. For practical reasons it was not possible at the time the research was carried out to also arrange observation of teaching. Also, it was not my intention to try to measure participants’ UMID, only to gather their conceptions of this. But in a future study, both of these investigations could enrich the data.

**Cultural context**

It is important to note that the questions underpinning this research are culturally situated, emerging from a UK mathematics education context in which ‘learning for understanding’ is valued highly. Other cultures may not attach the same importance to UMID. Indeed, for some MEC students, the level of importance attached to UMID can create challenges as it conflicts with their own prior experience.

Finally, it would be interesting to conduct a longitudinal study, to investigate teachers’ knowledge growth over time. One could also compare novice mathematics teachers with expert mathematics teachers in how they talk about UMID - their own, and that of their students.

**8.6 Impact of study upon my own professional practice**

The research in this thesis has stemmed from my involvement over a number of years as a tutor on two programmes specifically designed as subject knowledge enhancement courses for secondary mathematics teachers – the Mathematics Enhancement Course (MEC) and the Mathematics Development Programme for Teachers (MDPT). These courses are intended to
extend and deepen the mathematical knowledge for teaching of non-mathematics-specialists. I also have extensive experience as a tutor on the PGCE Mathematics course, to which MEC students progress. The opportunity to develop and run ‘bespoke’ mathematics courses for these specific groups of teachers challenged me to think deeply about the mathematics that teachers need to know, in what form this knowledge needs to be held, as well as approaches to teaching that content and creating a supportive and stimulating community of practice. My own professional experience and practice provided context and knowledge which stimulated my research. Conversely, engaging in research enabled me to “make the familiar strange” (Wellington, 2004, p. 44) and view my teacher education practices in a new light.

As my research proceeded, new insights and knowledge gained then informed my ongoing professional practice, setting up a reflexive cycle of knowledge growth. I found that I was engaging regularly in dialogue with other researchers and practitioners on a wide range of issues relevant to my ongoing research. These included, for example: outcomes of MEC programmes, employability and career prospects for former MEC students, content of MEC and MDPT programmes, and needs of MEC and MDPT students. My involvement in the QUANTUM research project has progressed simultaneously with my own research, and my learning about approaches to data collection, application of research methods, and data interpretation, has been very helpful in enabling me to design a structure and approach to my own investigation. I have also had the opportunity to discuss with others literature relevant to my research.

My understanding of subject knowledge for and in mathematics teaching has been deepened and extended as a result of the reading, thinking and investigations I have carried out for this study. In particular, my own awareness of what may be understood as understanding mathematics in depth (UMID), how it develops, and what it means to teachers, has grown. The reading I have done has opened windows for me onto the work of many researchers in the field, enabled me to make links and comparisons with the work of others, and helped to develop and shape my own views and knowledge about subject knowledge for teaching. The time I have spent listening to teachers’ accounts and experiences, then reading and re-reading those accounts in different ways, has given me new insights and understandings into areas that I thought I already knew well. This has been very motivating and rewarding for me. I have never considered that I should have been investigating something else - my interest in
my chosen area has been sustained and strengthened through the process of study and research.

My research has influenced how and what I teach, how I design mathematics subject enhancement and teacher education courses, and what I emphasise as important. I continue to be heavily involved professionally in the MEC, MDPT and PGCE programmes. As a result of my reading and research I am now more confident and explicit about the curriculum and the approaches to learning used in these courses - both with students and with other colleagues. I know that what we do is borne out by research. I constantly look for ways to improve and strengthen students’ learning, and this is now always in the light of research. I recognise that I do now work in a different way to the way I used to work before becoming involved in research. This is a permanent change for me - of the ‘threshold concept’ variety (Meyer and Land, 2003; discussed in Chapter 2 above, p. 36). I cannot now revert back to the way I used to work. I make links to research regularly in my teaching, and this is a natural thing to do because I know my field and I see the links.

As leader of the Mathematics Education team at my university I have oversight of primary mathematics education programmes as well as secondary mathematics education, CPD and subject based programmes including MEC. My growth of knowledge through working for this thesis is also having a wider influence for me and the team. With other colleagues I am now developing new approaches to the observation and feedback of primary ITE students’ mathematics lessons, centred around subject knowledge discussions and based upon research.

I should also add that I have enjoyed being a student again myself - a salutary experience for any teacher! It is good to be reminded of how it feels to grapple with difficult concepts, to trawl through data; to check and re-check references; to bump into conceptual ‘dead ends’; to anxiously await feedback from more expert others - as well as the more positive experiences of reading an article that seems to jump out of the page and enlighten; of seeing links and ideas emerging from one’s own work; to feel the words come rushing out after a period of ‘writer’s block’. These experiences have enabled me to empathise more closely with my own students as they make their own academic journeys and produce literature reviews, small-scale research or other written assignments with my support.
So now I can turn my theory onto myself as a learner and a teacher. I am learning and teaching (about research) at the same time. I used to be insecure and lacking in knowledge about research relevant to my work; now I am developing areas of strength and expertise in this area. I can see aspects of Theme 1 in my own progression through this thesis, and my doctorate as a whole. (Areas of weakness / insecurity can become transformed into areas of strength when one spends time thinking through them and preparing to teach them). I have come to see my professional self in a different way. I am still primarily a university teacher (and manager), but now I am also a researcher, and research informs my practice. My teaching is different because my knowledge and understanding have been partly reconstructed. My leadership is also different. These points are reflected in Themes 4 (Growth in knowledge and confidence can alter one’s perceived identity) and 5 (Reconstructing existing knowledge can result in transformation of teaching approach).

8.7 Implications for the mathematics education field

Through a focused investigation of participants on two courses at one university in England, this study illuminates practice elsewhere. The interview data obtained for this thesis reveals new and interesting perspectives about mathematics subject enhancement courses for teachers, and about the nature of mathematics knowledge for teaching. Bringing together the discourses of former PGCE and MDPT teachers highlights novice teachers’ experiences of learning, re-conceptualising, and teaching mathematics, of changes in their own self-image as mathematics teachers, and of a developing relationship with and disposition to the subject.

Development of UMID during training, and separation of UMID/SMK from PCK

In this study I find that novice mathematics teachers can begin to develop UMID during their training courses, i.e. for MEC’s before they start their teaching careers. This is in contrast to the position of Ma (1999) who argues that UMID (her PUFM) is developed over several years of teaching experience. I suggest that it is likely that other MEC and MDPT courses in England have similar outcomes. In this study we see that development of SMK / UMID can be separated from development of PCK, as was found by the COATIV project (Baumert et al., 2010)
The MEC occupies a key position in the mathematics education landscape and redefines boundaries

The position of the Mathematics Enhancement Course as an established and viable alternative to degree mathematics underpins this study. In the light of current political discussion about what is appropriate teacher preparation, this underscores the important role and value of such courses as part of the teacher education landscape. Access to MECs provides a route to mathematics teaching for a wide range of people who for various reasons did not choose to study mathematics as a degree, and in the national context of ongoing shortages of mathematics teachers, this remains a priority. The existence and success of MECs redefines the boundary between subject matter knowledge courses and pedagogical content knowledge courses, and, as noted above, arguably exposes deficits in the traditional degree route to teacher training. MECs are primarily subject knowledge courses. The approaches to learning and the underlying philosophy which characterise many MECs offer students a real opportunity to gain a deep understanding of the mathematics studied (UMID). The MEC curriculum also ‘overlaps’ into pedagogical content knowledge (PCK), because aspects of the curriculum are close to the school context, and because teaching and learning styles are often modelled by tutors who are themselves former school teachers.

Degree and MEC teachers’ outcomes are equivalent

The new quantitative data presented in this thesis (chapter 4) shows that there is no significant difference in the outcomes for former degree maths and MEC students on completion of the PGCE. This was a minor part of this study, and limited to one university. The variables considered were limited. Further research in this area would certainly be beneficial to the mathematics education community: a sample drawn from a number of institutions could investigate the outcomes for PGCE students from a wider range of degree and MEC backgrounds, and could probe in more detail, resulting in more nuanced research outcomes.

Novice teachers’ knowledge growth

Outcomes of this study show that development of PCK is foregrounded in the discourse of PGCE students, whereas development of SMK/UMID is more prominent in the discourse of MDPT participants, and I would expect similar results to be obtained if research was extended more widely to other universities offering these courses. Development of PCK
through the PGCE is perhaps not surprising, since this is what PGCE courses set out to do. The PGCE is a well-established route into teaching. There is an assumption that on entry to the PGCE, student teachers already have the necessary subject knowledge or SMK, and that they are therefore ready to develop their PCK.

The effects of the MDPT upon teachers’ SMK are interesting, especially since the MDPT is a new - and perhaps temporary - course. Mathematics educators can learn from this study about potentially productive ways forward in the design and implementation of both MEC and MDPT programmes - or programmes like these - and their likely effects upon the participants. Very little research has previously been carried out on MEC and MDPT participants, and undoubtedly these are areas that would benefit from more research.

**Interpretations of understanding mathematics in depth**

There was a clear emphasis in this study upon UMID as ‘knowing why’ and as being able to communicate/explain. Research by the QUANTUM project shows that in some MEC courses, UMID is also seen as connected knowledge, as reasoning and proof, and in terms of disposition to the subject (Adler et al., in press). There thus appear to be a variety of co-existing interpretations of UMID. Perhaps this reflects a diversity in approaches between providers, and the varying degrees to which they privilege different aspects of UMID. Other MEC/MDPT course providers might consider what sort of responses their own students would give to the questions posed in this study - this could shed light on the effects of their own courses.

**New theory: themes privileged in discourse**

The new theory that emerged from my research was unexpected. I had not set out to seek anything specific other than responses to Research Questions 1 and 2 (p. 149). However I was open to the possibility that other themes might emerge from the data. In the light of ongoing education change, particularly current moves by central government to reduce the participation of universities in teacher education programmes, it is worth considering what these themes might say to the wider mathematics education community and to policy makers.

**Themes 1, 2 and 3: Integrate the development of SMK and PCK**

I have already commented above on the finding that UMID can be developed through preparing to teach (Theme 3). Taken together, Themes 1, 2 and 3 underline the importance
for teachers of keeping study of the subject close to the study of how to teach the subject. This was explicit in the original formation of both the MEC and the MDPT as subject enhancement courses, and remains a key characteristic today. I would argue very strongly for a close integration in teacher preparation programmes of SMK development and PCK development, and an explicit awareness on the part of course providers of the characteristics of these aspects of knowledge. My findings in Themes 1, 2 and 3 provide evidence of the power of such an approach.

Course participants would also benefit from an understanding of these different types of teacher knowledge and how they interrelate. Mathematics teacher education courses, whether for specialist secondary teachers or generalist primary teachers, inevitably address aspects of both types of knowledge, to varying degrees. It is important, for example, that primary pre-service teachers appreciate the need to develop their own subject knowledge alongside their knowledge of how to teach. Effective courses will challenge and extend their own understandings as well as helping them to learn about effective teaching strategies. They may find this uncomfortable at times, and they will need to have the confidence and disposition to handle it, reconstruct their own knowledge, and move on in a positive manner.

Themes 4 and 5: The value of substantial in-service training programmes
Themes 4 and 5 underscore the potentially transforming effect of well-designed and substantial in-service training programmes. Much of teachers’ CPD provision in England is in the form of short, one-day courses, which although helpful are necessarily limited in scope. My findings in this thesis show that engagement over a period of time in an in-depth course, can produce transformative results. The message to policy-makers is that this can be achieved at relatively low cost, meeting a national need to provide more trained mathematics teachers. It is possible for non-specialist teachers to develop into mathematics teachers. Key factors in the success of individuals are the extent to which they develop a positive disposition to and confidence in the subject, and start to feel part of a community of practice. Graven (2004) argues that “teachers can [simultaneously] state their confidence as mathematics teachers, and their confidence to admit to what they do not know and still need to learn” (p.177), and that this is a key condition for ongoing learning.
Conclusion and contribution to knowledge

In this thesis I have explored the perceptions of two groups of novice mathematics teachers about how ‘understanding mathematics in depth’ is conceptualised and attained. Other key themes have also emerged from the responses of the teachers. I examine the discourse of novice mathematics teachers on two new and under-researched subject knowledge enhancement courses, finding links and similarities between the two groups, and also differences. My findings address areas which are under-researched, namely: secondary mathematics teachers; non-specialist mathematics teachers; and novice teachers’ perceptions of mathematical knowledge for teaching. Therefore this work extends the knowledge base of the mathematics education community.

My findings reveal a clear perspective on Understanding Mathematics in Depth (UMID) and its relationship to Pedagogical Content Knowledge (PCK), which extends the debate about these theoretical constructs.

My findings are presented in the context of a relationship between mathematics degrees, MECs and the PGCE which I unpick and discuss. The findings have clear implications for practice and for policy with regard to the preparation and training of secondary mathematics teachers.

In this study, teachers’ ideas about how deep understanding of mathematics is developed show clearly the importance of active involvement in the subject over a period of time. At a time when external scrutiny of schools has never been greater, and with huge pressure on teachers to achieve ‘results’ due to publication of exam results league tables, it is sobering to consider the possible effects upon learning of this excessive degree of external accountability. Understanding mathematics in depth requires active engagement and takes time, and there is a temptation for teachers to sacrifice their students’ long-term understanding of mathematics in favour of short-term gains. One can only hope that through their own experience as mathematics learners on subject enhancement courses specifically designed to develop deep understanding of the subject, these teachers will appreciate the importance of enabling their own students to gain a deep and lasting understanding of mathematics.
Appendices
Appendix 1: Main interview schedule

Interview schedule

Introduction, ethics forms, audio recording, reminder about focus of research

- Thank you for agreeing to take part
- Any questions about ethics forms?
- Remind we are recording interview
- Introduce research

I am undertaking this research as part of my doctorate in Mathematics Education. I am investigating the concept of maths for teaching, as this is interpreted by different people.

To teach maths effectively, it is necessary to have subject knowledge of a particular kind, and what maths teachers know, and how they hold and use that knowledge, is what I would like to explore in these interviews with you and with other maths teachers and trainee teachers.

This interview is carried out as part of an academic research project. It is not part of any inspection or evaluation programme. Your comments will not be made available to any third party. Your interview will be anonymous, and you will not be identifiable in the written research.

You may have a copy of your interview transcript if you wish.

1. Tell me about the route you took to prepare you for maths teaching (degree, MEC, PGCE, other training)

2. Your education and training will have prepared you in many ways for the complex role of the teacher, but let’s focus on the maths…In what key ways did your education and training prepare you for dealing with mathematical concepts in the classroom?

3. What sort of maths did you learn while you were training? Can you give me some examples?

4. Can you give examples of maths ideas you encountered in training and then used in your own teaching? (What happens to your understanding of the maths, when you are adapting and transforming ideas to make them appropriate for children to learn?)

5. Can you tell me about a lesson/topic that you have taught recently where you were confident about the mathematical content you were teaching, and you felt secure going in to teach the topic.
6. Can you tell me about a lesson/topic where you felt less confident and secure with the mathematics, or where you were challenged by difficult questions from students.

7. In some teacher education courses, emphasis is placed on the importance of ‘understanding maths in depth’. What does this mean for you?

8. Here are five statements related to how ‘understanding (fundamental) maths in depth’ may be interpreted. Please arrange the statements in order of importance for you, with the most important first.

   Why is the first one the most important?
   
   *(Read letters back into recorder)*

9. Here are three statements related to how ‘understanding of maths in depth’ may develop. Please arrange the statements in order of importance for you, with the most important first.

   Why is the first one the most important?
   
   *(Read letters back into recorder)*

10. Can you give an example of a small bit of maths that you feel that you know in depth? How did you gain this knowledge and understanding?

11. Is there anything else that you would like to add?

End of interview. Thank you for contribution
Appendix 2: Pilot interview schedule

Interview schedule

Introduction, ethics forms, audio recording, reminder about focus of research

- Thank you for agreeing to take part
- Any questions about ethics forms?
- Remind we are recording interview
- Introduce research

I am undertaking this research as part of my doctorate in Mathematics Education. I am investigating the concept of maths for teaching, as this is interpreted by different people.

To teach maths effectively, it is necessary to have subject knowledge of a particular kind, and what maths teachers know, and how they hold and use that knowledge, is what I would like to explore in these interviews with you and with other maths teachers and trainee teachers.

This interview is carried out as part of an academic research project. It is not part of any inspection or evaluation programme. Your comments will not be made available to any third party. Your interview will be anonymous, and you will not be identifiable in the written research.

You may have a copy of your interview transcript if you wish.

1. Tell me about the route you took to prepare you for maths teaching (degree, MEC, PGCE, other training)

2. Your education and training will have prepared you in many ways for the complex role of the teacher, but let’s focus on the maths…In what key ways did your education and training prepare you for dealing with mathematical concepts in the classroom?

3. What sort of maths did you learn while you were training? Can you give me some examples?

4. Can you give examples of maths ideas you encountered in training and then used in your own teaching? (What happens to your understanding of the maths, when you are adapting and transforming ideas to make them appropriate for children to learn?)
5. In some teacher education courses, emphasis is placed on the importance of ‘understanding maths in depth’. What does this mean for you?

6. Here are five statements related to how ‘understanding (fundamental) maths in depth’ may be interpreted. Please arrange the statements in order of importance for you, with the most important first.

Why is the first one the most important?

(Read letters back into recorder)

7. Here are three statements related to how ‘understanding of maths in depth’ may develop. Please arrange the statements in order of importance for you, with the most important first.

Why is the first one the most important?

(Read letters back into recorder)

8. Can you give an example of a small bit of maths that you feel that you know in depth? How did you gain this knowledge and understanding?

9. Is there anything else that you would like to add?

End of interview. Thank you for contribution
Appendix 3: Sample advance email communications

Dear N,

I hope all is well with you. I am doing some research for my doctorate, and I wondered if you would be willing to take part? I am interviewing past and present teachers from our PGCE course about their ideas about deep understanding of mathematics for teaching.

All that is involved is an interview lasting about 30 minutes, which I record and then later transcribe. This could take place at any time/place convenient to you. I would like to do interviews over the next couple of months.

Please let me know if you might be interested in taking part, or if you want to find out more about it. If you don't want to take part that's fine, but please let me know so I can ask other people!

thanks and best wishes,
Mary

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Then typically a series of emails followed in which we arranged time and place to meet.

Hi again N,

Looking forward to seeing you on Monday! A few things for you to think about...

I am trying to find out what people think 'deep understanding of maths' means.

One of the questions I am asking in my research is about how people's degree and teacher training prepared them for handling mathematical concepts in the classroom.

I am also asking people to give me an example of a lesson or topic they have taught where they felt confident in the mathematical content they were teaching, and secure going in to teach the topic.
Also I'm asking people to tell me about a lesson or topic where they felt less confident and secure with the mathematics, or were challenged in the lesson (eg by a difficult question).

thanks again for your help,

Mary
Appendix 4: Information and consent forms

Participant Information Sheet

An investigation into what characterises ‘deep understanding of mathematics’, as understood by particular groups of secondary pre-service and serving mathematics teachers.

You are invited to take part in a research study; your participation is entirely voluntary. The purpose of the study is to investigate the nature of subject knowledge for maths teaching, and in particular what characterises ‘deep understanding’ of mathematics.

If you consent to taking part, you will be interviewed by one researcher for around 30 minutes, and the interview will be audio-recorded.

You are free to withdraw from the research at any time if you wish.

Information collected as part of this study, including digital recordings and transcripts, will be securely retained for 10 years, and stored electronically. Any records containing personal information will remain confidential, and no information which could lead to the identification of any individual participant will be released.

You will be provided with a copy of the final research report or summary of the research findings upon publication of the EdD thesis.

M Stevenson
15 March 2010
CONSENT FORM

Title of research project:  
An investigation into what characterises ‘deep understanding of mathematics’, as understood by particular groups of secondary pre-service and serving mathematics teachers.

Name of lead researcher and institutional affiliation:  
Mary Stevenson, Liverpool Hope University

1. I confirm that I have read and understand the information sheet dated 15 March 2010 for the above study and have had the opportunity to ask questions.
   
   Yes  No

2. I understand that my participation is voluntary and that I am free to withdraw at any time, without giving any reason.

   Yes  No

3. I agree to take part in the above study.

   Yes  No

Name of participant:

Signature:

Date:

Signature of lead researcher:

Date:
Appendix 5: Sample annotated transcript
Mary Stevenson

INT Right, okay the light's on so hopefully we are recording and I think the time is ticking through. Yes it is. Okay, so thanks very much for agreeing to take part. I'm going to ask you a few questions and see what you want to tell me about. So the first thing is could you just give me a bit of your background, about the route that you have taken to become the maths teacher that you are now. So your own degree, PGCE etc.

RES Yeah. I came into teaching quite late, sort of late 40's and I studied at the Open University Geography. I became a geography teacher. Then I was a geography teacher for a year and then I was asked to move into special needs where I taught maths, English and science, mostly maths. And then a friend told me about a maths course and because I didn't feel very confident about my knowledge of maths, although I was teaching it, I asked could I come onto the maths course. So I did the maths course for a year.

INT And that's the MDP. Just saying...

RES Yeah that is the order. That is the MDP. Yeah.

INT And you did that... is it two years ago now.

RES I did it two years ago, yeah. I have done two years of a masters in education study and then took a year off that and came on the maths course.

INT Right, brilliant. Okay thanks. So obviously your background, your training, your degree and all the rest of it will have prepared you in lots of ways for teaching, but if you think about the maths in particular, can you think of ways in which your training prepared you for dealing with maths concepts in the classroom.

RES Are you talking about the training on the MDP.

INT Yes and elsewhere maybe as well.

RES Yeah. Well the MDP, I did have some knowledge of maths but it was mostly knowing how to do things. So I knew how to calculate volume and I knew how to do long division and long multiplication. What I didn't know is how it all linked together and I felt that the MDP course gave me those links. Now I feel that when I left the MDP course I wasn't anything like as good as I am now. And I feel that it gave me that ability to... because I understood the links and as I have carried on teaching they have been strengthened. The links have been strengthened. And I really feel I have got a better understanding. I still think I've got a long way to go but, you know, I do feel I have strengthened the links and I understand what I'm doing now. So when I come across a problem I know how to solve it.
Mary Stevenson – File No. 19.

INT Okay, well that's really fascinating because I think what you're saying there is that the MDP sort of started something which you have continued to learn to develop or whatever since finishing the MDP.

RES Yeah. And I really didn't know that maths was so umm... I wouldn't say easy. I found it, the MDP made it really interesting and just being able to understand that it's such a diverse subject but in a way everything links together. You know, with maths. So once you learn about one thing, it feeds into another. So it's almost like a stream with little water falls. For me now I just feel it's fantastic.

INT Yeah, so that's really useful to know that it helps you make connections. So that's something we want to try to achieve.

RES Yes. If I wasn't teaching special needs, which I am teaching special needs, up to GCSE level and I had to go back into the classroom and do it across the board and the boss said to me 'do you want to back to geography or maths' I would prefer to go to maths. Okay, I would say now maths would be my preference.

INT Yeah. Okay you have already mentioned a couple of examples, can you just elaborate on that or give me some other examples of maths that you remember learning about on the course.

RES Well the biggest thing I remember was the shape and space. And it's really, really helped me in my teaching. So, for me the understanding, you know, and I have sort of drawn little diagrams because that's what I do. Is not knowing how to calculate volume is the length times the height times the width. It's knowing what to do with this and what volume is, you know, and how you do it. And then the units that volume is measured in. You know, you understand then why it's cubed. Which I never really understood before.

INT Yeah.

RES But also things like, then it comes into... when you look at algebra, it sort of feeds into it where x plus x is 2x and x times x is x squared, and x times x times x times x is x cubed. The reason for that is 3 plus 3 is 6 and 3 times 3 is 9. So they are different things whether you're doing them in algebra or you're doing them in number. I just feel that once you learn a rule in maths, you can widen that rule into all the topics, you know. So I see, for instance if you look at... my children found this quite fascinating. If I tell them that a quarter is 1 divided by 4 they have no idea that a quarter is 1 divided by 4. But x over 2 is x divided by 2. So it's half of x, you know. And by the same token mass over volume, is mass divided by the volume. And so their sort of not aware of little things like that. They just look at a quarter and it's a quarter and that's what I used to do.

INT Yes.
Mary Stevenson – File No. 19.

RES I had no idea or any understanding of what it was. And I feel that the MDP course gave me that and then the ability to leak it and take it wider in my teaching.

INT Okay, well that's really good examples. Right thanks for that. Okay, if you... let me think now what I want to ask you next... yeah, I mean have you used any of these ideas in your own teaching then.

RES Yeah, yeah. It's building... for me again I go back to the shape and space, but it then diversifies into the number and the algebra as well. It's building confidence and one of the things that I really use a lot is developing the visual awareness because I remember all of the course but that bit really sort of hit home. So for instance at the minute, coming up to unit 3 of maths it's more than 50% shape and space. You know, so we will start with tessellations, we'll talk about tiling floors with boys who are on construction courses about tiling floors and how the tiles have got to tessellate and how you can't just make a pretty pattern and think it's all going to fit.

INT Yeah, yeah.

RES And then you would go onto isometric drawings. So you have got, you know, the type that are in cubes. Yeah and then you have got more complex shapes. And then shapes without cubes and then you're on surface area then and back to volume. Then you can lead into surface area and then go back to volume, that you learned earlier. Because you're using them same shapes that they were just drawing.

INT Yeah.

RES To get the surface area and the volume. I'm finding that's really good, that's really working really well. And then of course you move onto plans and elevations and then we go back to isometric drawings. So everything is linking in. I just find that's real confidence building. Not only for me as I do year on year, I know it so well, but for the young people as well.

INT Oh fabulous. Okay, that's great. Okay well I asked you a couple of things in advance. If you can think about an example of a topic where you have gone in to teach, could be anything at all, where you have gone in and you felt confident about going in to teach that topic so what...

RES The example I would give was the shape and space and how they lead on. It's a whole scheme of work.

INT Right and it all links in.

RES It links but it also links back to things you have done earlier.
Mary Stevenson – File No. 19.

INT Okay and if I ask you then okay, I know you have already partly told me this but I'll ask again. If I ask you when going to teach that topic now and you're feeling confident about it because you know it well, how did you get that knowledge and confidence. Where do you trace that back to.

RES Well I trace it back to the MDP and the shape and space thing there. I think I've developed it in my own way, in a way that I like to teach it.

INT Yeah, because you have moved on since. Yeah, okay. I thought that's what you were going to say but I wanted to be clear because I have got these different questions and asking everybody the same questions so, yeah. But often people will start answering a question before I've asked it.

RES Absolutely yeah. Because I do. Well you have made notes as well.

INT Yeah that's fine, that's fine. Okay, the other thing I asked you to have a think about was if you can think of a topic, a lesson where you went in and you were having to teach something but you didn't feel that confident about it.

RES I can think of it... I'm really not happy with order of operations. Mostly I'm not happy because I think of the way it sometimes come up. With young people, you know, and particularly special needs young people, you have to be very, very clear and it doesn't always do it that way, they can be thrown. And you can actually get some aggression if they can be thrown. I find that quite hard. So for instance Bodmass, yeah. Okay so I teach them (other) division, multiplication, addition, subtraction) this is the order of operations, this is how you do it, when it's all mixed up. But then someone will write that...

INT Okay.

RES And they'll say well we've got to divide first because division comes first. And then I have to say well actually maybe that should have brackets around it.

INT Well actually it doesn't matter does it. Because you have got 24 times 2 divided by 3 and it doesn't matter what order you do that in I guess.

RES I suppose no because they... yeah.

INT Yeah because division and multiplication are the same.

RES You can move them any way, yeah.

INT The same priority.
And maybe I just don't feel so confident about all that. And I know you can move them around and they're transposable.

But it's a difficult thing to explain isn't it.

Absolutely, because you told me it was this rule. Yeah, you know, and then I found myself then saying well maybe that should have a bracket around it. Well why wasn't there a bracket drawn round it then. And particularly those ones that come on the maths papers, I don't know if you've seen them. They're supposed to use the calculator anyway to confer. But what they are is they have got like a big long complicated thing and it might have some squares and whatever's in them. You're supposed to feed it into your calculator and then there will be a line and then there will be something underneath and they will be going well I have got to do my division before I do... you know, and you're thinking oh no. Do you know what I mean, and I say well you can't do the division, you have got to work out the top and work out the bottom and then do your division. So for people who need things very structured and very simple it's not...

It's difficult. And the actual acronym Bodmass, although it's intended to be helpful, and it is very helpful, it can be misleading can't it.

Yeah.

Because it's not quite that strict rule. Because addition and subtraction aren't equal priority. Multiplication and division are equal priority but we want to make it into a word that we can say easily in English so...

And if I'm given it like that well then division comes before multiplication.

Yeah, yeah. Okay, so how do you... obviously from time to time this topic crops up, you have to teach it, it's on the syllabus, so how do you sort of manage that in terms of...

Well I make that the resources and take it out aren't confusing.

Right, okay.

And I mean I will tell them they've got equal value but they still come above the other and I can't change them round. So I would make sure, well I would try and make sure the brackets were drawn around before I took them out.

Right. So you try to make little things to make it clearer.

Clearer, yeah. Take a variety of examples.
INT Yeah, having good resources, good materials, good examples, will help you to prepare well.

RES And it won't throw me either because I don't like it when... well I like it when they don't understand if there are several ways to... but when it really throws them and you become quite anxious about it, I find that's quite hard for me simply because of the cohorts of young people at each.

INT Yeah and it's quite difficult isn't it, because the brackets thing, you know, if you got a complicated expression in the top of the fraction and another complicated in the bottom, really, you and I know we would have to work out the top and the bottom separately and then do the maths. And it's as though there were brackets round the top and the bottom, and that again is where there is limitation on something like this.

RES So I find that quite hard and I find it very hard to sort of move it into other things.

INT Yeah, dangerous stuff. Okay, that's a really interesting example. Thank you for that. Okay, so umm... you know that my research is about what people understand by what it means to understand maths in depth, okay. And quite a lot of teacher education courses will emphasise that. So what would you say that understanding maths in depth means to you.

RES To me it means that not just knowing how to do it, it's how it works, what it's about and how it links in with other things. And it can help and be useful in other things. That's what maths is to me. But I think it's also making it part of your everyday life, in a way. You see symmetry all around you, don't you. I mean one of the things we have in school now is we have... have you heard of Ed Modo.

INT No.

RES It's like a Facebook for teachers and so the whole group goes on Ed Modo and the teachers can set tasks. So I ask them to take pictures of symmetry that they see around, you know. I think that's knowing it in depth and applying it to your life. So, for instance, some of the shape and space I'm doing and things like Pythagoras theorem, if I'm doing... the younger joiners lads who want to do joinery or bricklaying or anything, I try and take them out and about in the community or get them to measure their television screen, you know and say well your television screen, you know it's from corner to corner. You can use Pythagoras theorem to do that.

INT Yeah.

RES Go and do it on your TV. First of all, do you know what it's supposed to be, right. Well go and check it is by measuring the two sides and getting the hypotenuse and then must be what your television screen says it is on the box.
INT Yeah.

RES So I think that it's not just understanding that volume isn't just the length times the height times the width because it comes in different shapes. I think it's about them putting... and knowing that it's actually all around you. It's in every part of you, wherever you look.

INT Okay.

RES Because symmetry is and your bus timetables are.

INT Mathematics everywhere.

RES Absolutely.

INT Especially with your construction folk, you know, none of these buildings would go up without...

RES Without the mathematics.

INT Yeah. Okay, good. Well what I've got here is five statements about what understanding maths in depth might mean, okay.

RES Okay.

INT Now what I would like you to do if you can is put them in order of importance as far as you can see. Okay, so I'll just give you a few moments to do that.

[PAUSE 12:55]

RES Okay, I think that's probably...

INT Alright thanks. So you've got D, B, C, E and A. Alright, so you have put D at the top. Understanding maths in depth means being able to make connections between concepts and procedures so why do you think that's the most important for you.

RES Because they're important for your understanding of mathematics. Because without that, you can't even have any of these, for me.

INT Yeah, yeah. I mean there's lots of different ways of seeing it, you know there isn't a right answer here of course. It depends how you see it and that's what I'm trying to get at, how different people see it.

RES Yeah.
Okay, anything you want to say about the...

Yeah, I chose this one because I think well as a teacher I have got to be able to communicate what I understand.

So that's B.

So that's B for me. How procedures work. Yeah, it is important. And you have got to know that... umm... but I think that you usually know that in order to get to that, if you like, so that sort of should be there anyway if you like.

Yeah. Okay.

Just structure patterns in general. And again with this one... and this one I suppose really it's a little bit like me, it's the language, justify your mathematical thinking. And that sort of says to me I am justifying it to someone who isn't the people I teach and so I don't feel that's that important.

Okay.

Because I don't have to justify it to the young people, I have to sort of inspire them to understand it.

Okay.

Does that make sense.

Yeah, that's fine. So anything that's not in that last that should be there do you think. Have I missed anything out.

Umm... no because I would call them links, you've called them connections.

Yeah.

So no, I can't see anything that you have actually missed out Mary to be honest. Not off the top of my head.

That's fine. Yeah. You were talking a lot about connections.

Yeah I think it is and how each, you know, this mathematic connects to others.

Yeah, okay. Thank you. I'm going to give you just three more. And these are about how somebody could gain an understanding of maths in depth. How do you get it, how does that happen. So again if you do the same thing, just put them in order of importance for you.
[PAUSE 09:29]

RES Yeah.

INT Okay. So you have got G, then F, then H. So the top one is G, understanding maths in depth develops by living with and spending time thinking about mathematical ideas.

RES Yeah.

INT So why do you think that’s the most important.

RES Well if I take myself when I came on the MDP course, time was actually put aside for doing that. So if time wasn’t put aside for doing that I wouldn’t be able to understand mathematics by investigating and working out difficult taxing problems. Because without this I wouldn’t have even got there, so it was on the MDP actually putting aside the time to be able to do this. Yeah.

INT Yeah.

RES And yeah it takes time, as I said before, I don’t think I’m anywhere near where I’m going to be in the end, you know. Because I am trying to develop it. I mean one of the things I’m doing this year, not just in maths, I’m going to do it for English and science as well, but mainly for maths, is I actually go to develop a set of resources that colleagues can use, the learning mentors and the people in other units can come down and use. So talking about if they’ve got a GCSE student coming in, they’ve got no sort of learning for him or her to do and so they’re going to have a filing cabinet that pulls it out. So he’s at GCSE what level is he on, you know.

INT Yeah.

RES Asking what topic he’s doing and I would start here. So other people can know what I have developed and what I have worked on and how I go about it, whether I meet my young people at the end of year ten and I’ve no idea, some of them haven’t even been in school for two years. So it’s to give everyone a mental resource then to build up on that.

INT That’s really great.

RES You know, I’m nowhere near where... you see everything I know is in my head and it’s all jumbled up and if I went off sick no one would be able to come over and just take over so I’m going to try and get them to some sort of order. Catching it for the benefit of colleagues to be able to... who don’t teach maths or aren’t even teachers. You know, for a stop gap.
INT Yeah, alright. Well that’s really good. Okay so I’ll put those away, thank you. Umm... right we’ve not got much more to do now. Umm... you have talked quite a lot about maths topics already. If I was to ask you to give me an example of a small bit of mathematics that you think you understand in depth, what might that be.

RES  Oooh.

INT  There will be lots I’m sure.

RES  Yeah. There’s quite a lot really. Umm... I love algebra, you know, I really feel I understand the algebra very well now. Certainly up to the level I teach it, which is grade B to C GCSE. And probably up to Grade A GCSE. I understand it really well and I actually love it and I think it’s a really good way of condensing knowledge and making things fit and I just think it’s really good.

INT  Why do you love it then. Is it because it’s such a powerful thing.

RES  Yeah. It is, it’s very powerful isn’t it really, knowing that. And it’s a funny thing but it’s hard to explain when somebody says why will I need this. It’s hard to explain why but you say because you will. You know, and maybe even doing... some of the things we did on the maths course, like with the paper bags and things like that. You get to realise... oh do you remember the triangle where there was... you gave us like a... I’m trying to think because I used it an awful lot last year.

INT  Keep talking.

RES  There was a triangle that you did and it had circles on each corner and circles in the middle.

INT  Yes.

RES  Do you remember that one.

INT  I think so.

RES  And you made the links. So you said, I think...

INT  Okay, did we say if A has a value of 5 and we’ve got some of 4A and A squared and that sort of thing.

RES  Yeah. We didn’t do the value of A, we just give A. So if that’s A, what’s this.

INT  Okay.
They choose three consecutive numbers

Right, right.

Okay, yeah. And so umm... and then, yeah they chose three consecutive numbers on the triangle and then... it might have been he called them x so if they chose 3, 4, 5, in here you had 3 and 4. So that was 7x. And you got the value of x by that. You made an equation out of it.

Right, okay, yeah.

And solved it like an equation.

Yeah.

And they see it then, because they think well actually this is something where I can find out what we started with.

Yeah, it breaks it down. Okay so that’s interesting. So you know, you like algebra, you like teaching it, umm... you feel you understand it well and again where would you trace that understanding back to?

Well I didn’t even know... well I say I didn’t know but that’s not fair because I was still teaching GCSE algebra, but I was teaching it like as though it was, this is how you do it. Look you’ve got this expression, this is... but I didn’t really know what it meant. Now you have got an equation so what you do is you move this to the other side of the bracket. But I didn’t know, for me yeah... so I would trace it again back to the MDP where I began to see well how that all works out.

So you kind of had a, you were able to follow procedures before then. You had obviously learnt that from where...

Absolutely, yeah.

So you had that but then having done the MDP course it just deepened it and you can see where it came from.

Absolutely.

Yeah. Obviously a lot of learning comes from earlier and it’s sort of tracing where it comes from and then built on it. Sometimes it’s quite difficult to think about where we learned something because once you have learned it, you know, you can’t remember what it was like not to know it.
RES I remember being taught GCSE maths and I can remember there being a very good teacher, where he had all these middle aged women in his class, I was one of them you know, and he... I'm 55 and so if we go back, you know, we weren't encouraged to like maths as girls, we were encouraged to type and do shorthand which are the things that I did in my first career. Or I wouldn't call it career really, it was a succession of jobs. Umm... and I can remember then because I wanted to teach having to go an do GCSE maths and I can remember the teacher being faced with all these women and he came in one day and he said 'I've got a terrible hangover, awful hangover' and he said 'we're going to play a game because I can't teach' so we all thought oh that's alright then, because he always taught, it was fine. It was finding the missing number, so when we came in after break he said you have all been doing algebra and we were all like... So I was one of those people that had a real phobia, you know, and thought that I couldn't do maths, so I think the MDP course taught me. Well actually yeah you can do maths.

INT Okay, and presumably he taught you that as well didn't he.

RES Well he started it. I knew I could get to GCSE and when I was teaching it I knew I could pass on what I'd learned at GCSE. But as for knowing really in any depth what I was talking about, you know, I really probably didn't.

INT Yeah, okay. Right.

RES And certainly you can see by the maths results that my young people get, you know, they get much better results.

INT That's really interesting that you can trace that, because again that's sometimes hard to see that sort of direct... you know, you don't always...

RES Yes I can trace it, in fact I can trace it because I have got graphs because I have to do reports, because I sort of have my own little department so I have to report on it every year.

INT Yeah.

RES I'm never happy really because my last results I think there were four C's and six D's but I wanted seven C's. I had seven people in that exam that were capable of C's and only four of them got them.

INT Yeah.

RES That's the truth I can't.
Mary Stevenson – File No. 19.

INT Okay. Well that's fabulous. Okay, that's pretty much all I wanted to ask you. Is there anything you want to add that you think you might want to tell me about, or might be useful for my research.

RES No, I'm quite happy, yeah. I mean I don't know why you're doing it or anything like that but I know that you're doing research and I'm interested in it and I'm glad to have taken part.

INT Okay, well thank you very much. Well I will turn this off...

(Stage 4)

HLP gave me the links.
I knew how to do certain techniques but I didn't know how they all linked together, still learning since finishing the course.
Commentaries for Appendices 6, 7 and 8

Appendix 6: comparison of PGCE entry and exit scores

Paired data scores for entry and exit for 95 PGCE mathematics students were analysed. Students’ degree classifications on entry to PGCE were converted into an entry score of 1, 2, 3, 4 or 5, where 1 indicates the highest degree classification. This was done by scoring a first class degree (or higher degree) as 1, a 2:1 as 2, a 2:2 as 3, a 3rd as 4 and a pass degree as 5.

All students who pass the PGCE course exit the course with a discrete grade of 1, 2 or 3 recorded against each group of ten QTS (Qualified Teacher Status) standards, where 1 is the highest grade. The mean of these grades constituted the exit score for each student in this study, thus generating exit scores in a continuous range between 1 and 3.

Appendix 6 shows a scatter-graph of these paired entry and exit scores, showing entry score (discrete integers between 1 and 5) on the horizontal axis, and exit score (continuous data between 1 and 3) on the vertical axis.

Appendix 7: Overall QTS grades

Analysis of overall QTS grades (i.e. the mean of scores from ten groups of QTS standards) on exit from PGCE yielded a mean grade of 1.77 for the MEC students as against a mean of 1.68 for the non-MEC students.

This data is displayed in a frequency diagram in Appendix 7. The calculation of a mean of ten integer scores of 1, 2 and 3 generates continuous data between 1 and 3. The overall grades of MEC students are plotted above the horizontal axis, and for comparison the overall grades for non-MEC students are plotted below the horizontal axis (seemingly ‘negative’ frequencies below the axis are just a spurious outcome of
the software used and should be treated as positive). To enable comparisons to be drawn, each data set is displayed as a simple dot plot.

Appendix 8: Subject knowledge grades

Analysis of subject knowledge QTS grades on exit from PGCE yields a mean grade of 1.71 for the MEC students as against a mean of 1.55 for the non-MEC students.

This data is displayed in a frequency diagram in Appendix 8. Subject knowledge grades awarded were integers between 1 and 3. The subject knowledge grades of MEC students are plotted above the horizontal axis, and for comparison the subject knowledge grades for non-MEC students are plotted below the horizontal axis (again, ‘negative’ frequencies below the axis should be treated as positive. To enable comparisons to be drawn, each data set is displayed as a simple dot plot.
Appendix 6: Comparison of Entry score and Exit score

Horizontal axis: 1st or higher degree is coded 1, 2:1 is coded 2, 2:2 is coded 3, 3rd is coded 4, pass degree is coded 5

Vertical axis: Three passing grades for QTS. 1 is ‘v. good with outstanding’, 2 is ‘good’, 3 is ‘satisfactory’

Number of points n: 95
Appendix 7: Comparison of overall exit grades of MEC students and non-MEC students

Statistics for MEC students: Number in sample n:43, Mean: 1.77, Standard Deviation: 0.541

Statistics for non-MEC students: Number in sample n:63, Mean: 1.68, Standard Deviation: 0.479
Appendix 8: Comparison of Subject Knowledge grades of MEC students and non-MEC students

Statistics for MEC students: Number in sample n: 43, Mean: 1.71, Standard Deviation: 0.638
Statistics for non-MEC students: Number in sample n: 63, Mean: 1.55, Standard Deviation: 0.544
Appendix 9: Transcript annotation and identification of codes at

Stage two of data analysis

This appendix demonstrates, for each individual case, the main codes that were identified in interview responses, together with exemplification. This illustrates the results of stage 2 of data analysis. There is a focus here upon responses to the earlier interview questions (qus 1-4), since the later questions are analysed elsewhere. The PGCE group are considered first, followed by the MDPT group.

PGCE interviews

Int 3 PGCE

Compares own schooling with modern approaches, refers to variety of teaching strategies (qu 2)
Identity: “I have started to see things perhaps deeper and try to anticipate questions of why really because I was never the why person.”
Notes the change in her own teaching approach: “You have to be prepared for these deeper questions I think, which makes me ask more questions why is it like that, which I never used to do”. (qu 3)

Int 6 PGCE

Development of PCK while on PGCE course: “With teaching, knowing how to do something and teaching something are completely different. Where I thought I had clear knowledge in some things, it became apparent that certain things when I tried teaching them I thought, well actually I don’t really know why that’s the way it is”. (qu 2)
MEC gave her more depth and context to her maths: “Everything we were taught on the enhancement course...we were always told where it comes from, why it’s there, what it was used for, which I never got in school. It was just this is the formula. So just gave me a bit more of a context of maths I think”. (qu 2)
Confidence came from preparation (qu 5)

Int 7 PGCE

Developing SMK through preparing to teach:
“So, from doing [revision before the course] and the PGCE tasks during subject sessions...and then going into school to teach and preparing for each lesson, that’s how I have come back up to speed with subject knowledge.” (qu 2)
Degree maths was very procedural, not very meaningful. Wants to do differently as a teacher herself: “It was this is how you do it and you might have to churn out a proof in an exam and here’s the proof. It doesn’t really explain how do you go from here to here. What does it mean, where was it used, how will this be adapted and anything like that. So it wasn’t very meaningful. So I think I probably struggled with the concepts” (qu 4)

“...which is an excellent lesson for somebody going to be a teacher on why it needs to be meaningful and pupils need to engage with it and understand what it is for, where it’s used and where it’s from...” (qu 4)

Development of PCK, data handling example. Thought it would be an easy topic, but demanding for year 7 as they lacked common-sense skills, wider knowledge (qu 6)

Int 8 PGCE
Through PGCE learned about sequencing of topics (KC), different teaching techniques (KCT) and learners’ common misconceptions (KCS). (qu 4)

Enjoys challenging children’s attitudes to algebra and helping them to enjoy it. (qu 5)

Int 9 PGCE
UMID in the MEC: “With the MEC, I think it was really delving deep into spending two or three weeks perhaps on a certain topic, and really delving deep and grabbing it...” (qu 2)

“The MEC course...just made something click again in me” (qu 3)

Seeing connections [MEC] seemed to bring links together and certain maths topics just seemed to come together as one...(qu 3)

Learning in school context: “I learned the most actually being in school, being around teachers, being in five days a week...”(qu 2)

Development of confidence, professional identity: “on your NQT year from what you have learned on PGCE, I feel that it does give you quite a lot of confidence to think okay it’s me now, I’m here, it’s me, there’s no-one else.” (qu2)

Int 10 (PGCE)
Relevance of degree maths: “I think my degree definitely prepared me well for teaching A Level... I think that maybe my degree didn’t prepare me well for [teaching GCSE] because some of the stuff that I see as just a given, and those students who have not passed their GCSE, and you know, trying to explain this to them, they obviously will not see that” (qu 2)

Restructuring knowledge, development of PCK: “The more simple things I have had to really take a step back and try to explain them, not from how I think about them, but how somebody who didn’t understand it will think about them.”(qu 4)
Int 11 (PGCE)  (Note this participant is discussed in detail in chapter 5)

Learning and teaching maths at the same time
Proof is very important in understanding (quote qu 2 p2 if needed)

UMID means being able to explain: “In order for me to understand something properly I need to make sure I explain it properly” (qu 8)

Int 13 (PGCE)

Preparation: “...preparation for A Level I wanted to that really well starting from the NQT year so I did the whole of the book, each question in the text book and I could just, things were starting to click into place once I remembered them from uni.” (qu 2)

Identifies what PGCE and degree did for her: “The degree just gives you the confidence and recall really. If you don’t remember a certain A Level module or whatever you can pick it up quickly whereas the PGCE training gave me that basis and how to teach things because it’s a completely different skill than being able to do it yourself.” (qu 3)

Int. 17 (PGCE)

School maths as building blocks for later applications: “my understanding is it's between being able to explain why they do a particular area and then... saying it's the building blocks. So doing more complex mathematics which will help them do something you know practical with it at the end of the day” (qu 7).

Int 19 (PGCE)

Uses own career experience to present maths: “Some of the things that I find particularly useful in the classroom is the fact that I can use maths throughout my working career at different time and elements of what I have done right through my life crop up in maths all the time. So that helps me a lot because you know, it’s not an academic exercise, it’s something that I have used. So I find that very helpful in terms of the way that I present it and the way that I present problems”.(qu 2)

Mathematical knowledge as connected networks / jigsaw: “The jigsaw in three dimensions and you get a network and all these ideas are tied onto each other by little strings and you know, the more strings you’ve got, the less chance one little bits going to fall off at the end”.(qu 8)

You teach differently when you have recently learned something: “I hadn’t ever in my life done the completion of the square...so that was quite… discovering wow I liked that, that’s clever. Why haven’t I learned to do that before. And I have learnt things, the box method for factorising. I had not come across that before, I’d just done it by trial and error which works great if your brain works that way but not very good for teaching so that helped. That was interesting. It’s interesting that there are little gaps and the other gap that I had never at any stage come across was circle theorems. I hadn’t done that much geometry. And realised I’d played with it from a sort of problem solving point of view from time to time... So that was
quite fun to learn and then to pass on... And it’s interesting because I think you teach it a different way when you have recently learnt something...yes I think you pass on your learning process much more”. (qu 6)

Int 20  (PGCE)

Living the dream, always wanted to be a maths teacher: “I was lucky enough to be able to pack in work and look after my kids and do my maths degree and with secret hope that perhaps I could teach...Because I didn't want to fail really because it meant a lot to me...

And I wanted to believe that I could do it but I didn't quite know whether I could so I didn't voice it to anybody...I didn't voice it to my husband or anyone and I was doing the maths because I wanted to prove to myself that I could do that, only with one step at a time and see what happens and I'm living the dream. This is it. I mean seriously I'm living a dream that I had when I was 10 years old... And I'm one of the luckiest people on the planet to be doing that”. (qu 5)

Int 21 (PGCE)

Teaching challenging topics, weaknesses become strengths: “I’d said to my Head of Department I’d like to teach mechanics, I’ve not seen it since A-Level, I’d like to teach it and he was happy to give it to me. Umm… but at the same time it’s benefiting me because it’s re-teaching me so I have to re-teach myself it first in order to teach the pupils...That was my challenge. And I also teach further pure two now. Umm… that is much more like what I’ve tackled at University...

So it sort of… these two areas that I’m talking about are my weakness but now they’re becoming my strengths if that makes any sense?”(qu 6)

MDPT interviews

Int 1 MDPT

Own positive experience of school maths

Explains how she prepared to teach maths

Appreciation of colleagues’ support - community

Confidence: “I know what I’m doing and if anyone does ask me I know the answer” (qu 2)

Confidence: “ it gives you the confidence - especially as a non-specialist - even if the kids never ask you, it gives you the confidence to know that if they did ask you then you would know what you were talking about” (qu 7)

Int 2 MDPT
Identity - moving from Science to interest in mathematics teaching. “I always liked ...to have a good argument with them. I always enjoyed solving mathematical problems that [the maths dept.] couldn’t do”. (qu 1)

Motivation, enjoyment. “I found I had recaptured my love for teaching, and that’s where I came to teaching maths”

Compares his own (maths) schooling to modern methods.

Awareness of how children learn, and how technologies can aid this.

Comments on his own learning

Distinguishes between areas of confidence and not.

Compares his earlier maths teaching approach with current one and attributes influences.

Identity and transformation of teaching style: The [own name] of maybe 18 months ago would have given them a rule. Now [name] is probably giving them a visual representation to go with the rule”. (qu 4)

Philosophy: “I think as an educator it is important that we have a bigger picture than perhaps we have teaching the children. That’s why I’m re-doing my A Level. I got a D grade at A Level when I was 18”. (qu 5)

Identity: will read A Level maths textbook while wife is watching TV. Can’t share it with her.

Int 4 MDPT  (Note this participant is discussed in detail in chapter 7)

Promotes advantages of collaborative working

Discusses effect of MDPT course on his understanding of maths and on his teaching approaches

Shows self-awareness

Identifies what UMID is in pupils

Int 5 MDPT

Discusses benefits of being on course - working with other people

Transformation of SMK to PCK: “You know I can do the maths bit but how can I get the pupils to understand it...so that was a big things for me... and I definitely think I’m getting there” (qu 2)

Benefits of investigative approaches for developing her own SMK (qu 9)

Identity, confidence: “Just coming on this course has helped me an awful lot. I am much more of a confident teacher...and I actually feel like part of the mathematics department” (qu 11)

Int 12 (MDPT)
Confidence, identity: “I’m the main maths person (special school) and in completing the MDPT course last year I feel I have got that knowledge now to be able to, you know, give a little authority to some of the knowledge that I am trying to pass on to other staff”. (qu 1)

Relates own positive experience of maths at school

Identifies key influences for handling mathematical ideas – the NNS

Compares own schooling in maths with methods he uses today

Restructuring existing knowledge: “[The NNS] actually taught me so when I was using this folder in primary schools I’d open at the page on partitioning and I would read through it and I would say, ah, I didn’t realise you could do that. Even as an adult in my thirties then and it was this kind of hidden thing that I hadn’t discovered until I was grown up.” (qu 5)

Distinguishes between SMK and PCK: “I think the main benefit or the main training I have got is all the stuff that I learnt coming to your classes, because all the stuff with misconceptions and the groundwork and of where do very simple concepts come from like... even something like multiplication and all the different ways of doing it because if you come straight through school and your maths is fine and your degree is fine but you really, you're really taught how to do things but you're not taught the whole broader base and all the different options and why people struggle with... all the misconceptions. You don't have any of that and so you take for granted about things. So that course for me stripped back quite a lot and helped me to see it not just how to do it but why... there were a lot of whys I think”. (qu 2)

Importance of understanding why in teaching: “you can't just say because it is you know and you have got to be able to understand concepts and that was the benefit of that. Just being able to communicate on a deeper level why it is, where it has come from” (qu 3).

Making connections - course enabled her to make links. Ongoing development since finishing the course: “I did have some knowledge of maths but it was mostly knowing how to do things. So I knew how to calculate volume and I knew how to do long division and long multiplication. What I didn't know is how it all linked together and I felt that the MDP course gave me those links. Now I feel that when I left the MDP course I wasn't anything like as good as I am now. And I feel that it gave me that ability to... because I understood the links and as I have carried on teaching they have been strengthened. The links have been strengthened. And I really feel I have got a better understanding. I still think I've got a long way to go but, you know, I do feel I have strengthened the links and I understand what I'm doing now. So when I come across a problem I know how to solve it” (qu 2)
“And I really didn't know that maths was so umm... I wouldn't say easy. I found it, the MDP made it really interesting and just being able to understand that it's such a diverse subject but in a way everything links together. You know, with maths. So once you learn about one thing, it feeds into another. So it's almost like a stream with little water falls. For me now I just feel it's fantastic” (qu 2).

Int 16 (MDPT)

Identity: “what [the course] has done for me is because now I am responsible for maths across the curriculum as part of my role, it has helped me to look at other areas, and how we can encourage other members of staff to make sure they’re aware that what they’re actually doing is mathematics, whether it be in science, PE, geography map skills, etc, and so it really has enhanced my overall knowledge” (qu 2)

Confidence: It wasn’t just one specific thing, it’s the whole gambit really. What it’s done is it has given me the confidence to try new things and... because if we’re trying to tell children you’ve got to come outside your comport zone, if I can prove I have it helps. So you can do it that way, or you can do it that way. Whereas when I was at school it was just that’s the way you did it.” (qu 3 / 4)

Confidence/ identity: “What it’s done as well is, when I go to Heads of Departments meetings...I’m kind of treated as an equal even though I’m not in the true sense of the word, a mathematician...” (qu 3 / 4)

Int 18 (MDPT)

Concepts are important: “whenever I approach a subject with these young people I try and go in at the concept level if I can. And that means I have got to understand it”. (qu 2)

Reconstructing mathematics as an adult: “Oh the big one was Pythagoras. I had never seen a right-angled triangle with squares actually drawn on it. Never seen that. And I went to a grammar school, I mean we did good maths, we had a really good maths teacher, but he never did that. All I got was a squared plus b squared equals c squared, put the numbers in and off you go. And he drew that and at the time I just thought… to be honest, to be absolutely frank I never realised all through school that that’s why it was squared...

“...And in fact I find myself lots of times when I’m teaching these kids and I start with, listen I went all the way through school and I never got this, and I’m going to tell you. And it spurs me on. I’m evangelical. So I’m a born again mathematician”. (qu 2)
Appendix 9a: Example analysis memo

Code: Developing SMK (UMID) through preparing to teach

This memo sets out the development of one of the codes discussed in Chapters 6 and 7. The code selected is ‘Developing SMK (UMID) through preparing to teach’, since this is central to the work in this thesis, and it feeds in to later theory in the form of emerging themes.

See Figure 4, p. 112 and Figure 5, p. 113 for a list of all the codes identified in the interview transcripts, and Appendix 9 (p. 175) for the list with exemplification.

Developing SMK (UMID) through preparing to teach

Code is seen explicitly in several transcripts, as follows.

Int. 7 (PGCE, non-MEC, response to qu.2, Appendix 9, p.175)

“So, from doing [revision before the course] and the PGCE tasks during subject sessions... and then going into school to teach and preparing for each lesson, that’s how I have come back up to speed with subject knowledge.”

Interviewee 5 (MDPT, response to qu. 10, Appendix 12b, p.201)

“Partly through teaching it, you know, and having to go and do… remind myself initially how to do everything when I was teaching it... I think it’s through having to teach it. You know, I don’t think and from coming on the course but from having to go and teach it. I think there is some saying isn’t there that the best way to know if you understand something is to teach it or to tell somebody else about it”.

Interviewee 21 (PGCE non-MEC, response to qu. 10, Appendix 12b, p. 208)

“I think it’s only mainly from my teaching... Yeah because I’d always have to revisit it. I don’t know whether it goes for every mathematician but if I’ve not been around that particular area, so like I said before with the mechanics, umm... you have to refresh your mind... Yeah but you have to be doing it sort of regularly. Because I can only give it from a teaching point of view of the material that I teach”.

Interviewee 11 (PGCE MEC, case study, ch 7, response to qu. 2, p. 140).

“...it wasn’t really until the MEC course where I actually gathered a better appreciation for what I was actually learning and teaching at the same time”.

Note - this ‘learning and teaching mathematics at the same time’ – is identified by Ma (1999). This is key in the discourse of Lucy (case study, ch7):
Interviewee 10 (PGCE non-MEC, response to qu. 10, Appendix 12b, p. 203)

Interviewee 10 (a sixth form college teacher) stated that his mathematics degree prepared him well for A Level teaching, but not so well for GCSE teaching. He notes that he it was hard to take a step back and think about simple concepts from the point of view of a learner.

“now that I have come back to teach it I think that’s where my understanding has come from”.

Code is also implicit elsewhere in interview transcripts.

**Significance of this code**

This is evidence that it is possible to develop UMID through preparing to teach. It is not only through the experience of teaching - possibly over a number of years - as suggested by Ma (1999) that teachers can develop UMID.

**Relationship to other codes**

**Appreciation of colleagues’ support - community**

Several respondents spoke of the importance of mathematics teacher colleagues / tutors as a supportive community. The development of UMID generally takes place within the context of a community of practice.

**Comparison of own maths schooling to modern methods**

A key motivation for respondents to improve their own subject knowledge was to provide a better learning experience for their pupils. Several made comparisons between their own experience of school maths, and the more active and meaningful experience that they were now - as teachers - trying to offer to pupils.

**Restructuring mathematics as an adult**

Developing UMID may involve re-learning mathematics previously encountered, but in a new way, thereby restructuring knowledge. Several respondents referred to seeing mathematics concepts in different/ new ways as a result of their professional learning.

**Learning and teaching mathematics at the same time**

A key linked theme - see comments above.

**Weaknesses become strengths**

If we accept that it is possible to develop (UMID) through preparing to teach, then it follows that learning may progress so that areas that the teacher previously regarded as their weaknesses could become strengths, through the actions of preparation to teach and then actual teaching.
**Appendix 10: Ranking exercises – table of results**

Table of ranked responses to statements in Question 8 and Question 9

*Responses ranked highest to lowest from left to right*

*Merged cells indicate where respondents gave statements equal priority*

<table>
<thead>
<tr>
<th>Interview no.</th>
<th>Ranked responses to Qu. 8</th>
<th>Ranked responses to Qu. 9</th>
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<tbody>
<tr>
<td>1</td>
<td>B, C, D, A, E</td>
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Appendix 11: Information about interviewees

Summary information, interview participants

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<thead>
<tr>
<th>Interviewee</th>
<th>Course followed</th>
<th>Job at time of interview</th>
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<tr>
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</tr>
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<td>2</td>
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<td>MDPT 09/10</td>
<td>Maintained comprehensive school</td>
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<td>Academic sixth form college</td>
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<tr>
<td>11</td>
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<tr>
<td>12</td>
<td>MDPT 08/09</td>
<td>Special school</td>
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<td>14</td>
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<td>Further Education college</td>
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<td>15</td>
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<td>Home tutor – LA outreach team</td>
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<td>16</td>
<td>MDPT 07/08</td>
<td>Special school (EBD)</td>
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<td>Independent secondary school</td>
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### Summary information, interview participants, by course and year of study

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<thead>
<tr>
<th>Int no</th>
<th>Course followed</th>
<th>Year of study</th>
<th>Former subject specialism if not maths</th>
<th>Job at time of interview</th>
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Appendix 12a : Tabulated responses to interview questions 5, 6, 7

<table>
<thead>
<tr>
<th></th>
<th>Qu 5, Topic confident, where it came from</th>
<th>Qu 6 Topic not confident, why?</th>
<th>Qu 7 What does UMID mean to you?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Int 1</td>
<td>n/a</td>
<td></td>
<td>I think if I can understand how to prove something...then I would say that was understanding it in depth. So kind of the example I used before about the volume or the powers. If I can understand that then I think that’s understanding it in depth. Whether the kids ask you to prove it or not.</td>
</tr>
<tr>
<td>Int 2</td>
<td>n/a</td>
<td>n/a</td>
<td>I would definitely say for me an understanding of maths in depth would definitely mean beyond A Level... up to and beyond A Level in terms of your knowledge... I think as an educator it is important that we have a bigger picture than perhaps we have been teaching the children. That’s why I’m redoing my A Level. I got grade D at A level when I was 18 and I haven’t looked at most of it.</td>
</tr>
<tr>
<td>Int 3</td>
<td>n/a</td>
<td>n/a</td>
<td>...it is not so much just accepting you know, that A equals B, equals C. It’s a case of understanding why it equals that... like when you’re teaching I suppose you will have pupils who just go oh right, okay and write it down and accept that’s right, but you are going to have pupils who don’t understand, and you have got to kind of get in there and explain why... and I suppose it’s just going that little bit deeper. If you have got a deeper knowledge then they will trust you as well because they can sense that you do know what you’re talking about... and I suppose I do worry sometimes whether my knowledge is that deep... as I have said before I have had to ask myself why, which I never used to do...</td>
</tr>
<tr>
<td>Int 4</td>
<td>I think if I had been asked that question six months or so ago I think it would be more the data handling stuff because there is a big link to business studies I was extremely comfortable with it, you know right up to the higher level GCSE stuff really and I could</td>
<td>The basic things like lowest common multiples and highest common factors. Erm... prime numbers. Because I am not secure with my times tables. I know it sounds something very straightforward but I am not... I can’t do times tables very quickly. I tried using</td>
<td>Erm... for me... I look at it purely still as a teaching tool. You know, I think it is confidence. I think confidence to express to pupils and enjoyment of maths but also the ability to look beyond what is on the piece of paper, i.e. to look...</td>
</tr>
</tbody>
</table>

Consistent with Qu 8 response B (communication)
make those lessons interesting because I had a deeper understanding...the lessons were clearly enjoyed by pupils but that came because I understood it. But now I feel more confident to take something like probability and I can understand that. And other things as well, even ratios and things like that, you know, things that I have probably not done since I was 15 at school, you know, I am starting to look at that in different ways and it is confidence, definitely.

some of the things we were using, you know, hundred squares and things like that but I found that it became... because I wasn’t confident with it, it became very scripted in many ways.

And anything that went off the script I really wasn’t comfortable with, you know. I think the best maths teaching I have seen are the people who are confident enough that if something comes up they can stop what they are doing and go off on a tangent a little bit... and with the basic number, the very basic number I just haven’t got that confidence... well I didn’t have that confidence, I didn’t have enough tools, I didn’t have enough ideas to kind of go off on those tangents to develop ideas.

I think this course has allowed me to realise that, that is, you know, you can highlight those areas. It is not that I don’t have a grasp in number, it is the fact that I need to develop... I need to be able to go off on tangents and grow in confidence. It just makes you think a bit more, plan a bit more, which can only be a good thing.

I took over a statistics group so foundation statistics and I had only ever taught averages and things like that and I had to teach Spearman’s Rank. Okay. Now I had never done that, I had not even done it in my geography degree which is something that a lot of geographers do. I know about it but I didn’t know how to calculate it. So I went off and taught myself from a textbook, how to do Spearman’s Rank and what I do is I write out the lesson, how I am going to do it and the examples of how I am going to do it with the pupils and then you know, go in and, you know, as long as I have got my notes I am quite happy. So they are quite happy seeing me with a sheet of paper and you know, sort of doing an example on the board. But then sometimes, sort of part way... I think the pupils are quite a good class and they got it quite quickly but beyond, just having the basic tools to be able to do a sum. I think to have an understanding of why that’s happening and what’s going on in that, you know, and conversations I have had with pupils in higher sets, you know, when I have done some after school revision leading up to these exams it has been quite interesting the way that they approach a question and it doesn’t matter what the numbers are or what is going on in the page they have, what I feel, is a deep understanding of the concept of what is going on in that situation.

So rather than just regurgitating things that they have been... tools and tricks that they have been taught, they actually deeply understand every question and where it is coming from. And they can also transfer skills. They don’t pigeonhole this is an algebra question, this is a coordinates question, you know, they have got the ability to transfer the skills to right across all the subject areas.

Consistent with Qu 8 response D (make connections)

I interpret it to mean a lot of it, why are they doing something. You know, so they often say to you why are we having to do algebra or why are we doing that. So the deep understanding is sort of being able to explain why we’re doing something but also how it relates to everything else as well because I think maths is, you see maths and think I can’t do maths, you know. Then if you look at it in real life and try and put it into that situation so a deeper understanding is sort of understanding it, why we’re doing something, and what it relates to. I think, that’s how I see it. Yeah making the links and not just seeing it as a... and why it’s useful so things like we have done today, the standard deviation, you know, why might that be useful, why would somebody want to go and look at that information and work it out...

Consistent with Qu 8 response C (understand why)
**Int 6**  
I’d say I did angle identity... name and angles corresponding and I had... because I actually didn’t ask to know the names when I was at GCSE, it was F and Z, so I had to learn them and make sure I knew what they were doing. I had puzzles, matching games, ... so they could see it all visual. I got them to draw some in their books and they enjoyed that. That lesson went really well because I had obviously prepared, probably prepared a little bit more because I was thinking I’m going to forget these names of these angles. That probably worked in my favour just because I was thinking I’m going to forget these names of these angles. That probably worked in my favour just because I had actually put a lot more thought into what I was going to do. So yeah that was a good lesson.

The confidence probably came from preparation because I knew I knew all of the how you find the angles and the angle chases but it just wasn’t... I started to question whether I knew to justify it. It says explain why this is this angle...

So I worked through a few past paper questions and things like that and made sure I could give them a like specific, just reasons why this is this type of angle and just showed that this straight line and the triangle proof at the end of that, which they liked, where you rip the angle in three and it makes a straight line and stuff. So I think just probably because I was really well prepared for it and it was quite visual and they enjoyed it and I could see that they were going to enjoy it because it was all matching games really.

I taught year 7 top set algebra, solving equations, rearranging, getting the variable on one side. Know how to do it perfectly in my head.... obviously the teaching’s changed of it where you can’t move things over the equals sign anymore and stuff like that. So I knew how I was taking one from both sides and stuff like that and when I got in there and it was formally observed, I just couldn’t.... the children... I think because I went in there thinking yeah I know this like the back of my hand, one of my stronger points, algebra, it threw me when they didn’t understand... some of them didn’t understand it and they were a really able class so I knew it was my teaching because they are really able. And they just didn’t grasp it and I was trying to explain it but it just weren’t getting through to them and because I think I was less prepared for it to go wrong because I think I just assumed that because I was good at it they would pick it up quickly. And I didn’t really have another... I always try to have a second way of explaining something ready for more or less everything so that if they say I don’t really understand why you’re doing that I can say well have a think about it this way. And I didn’t have it, I just couldn’t do it for them and I ended up messing it up basically.

For me I’d say the main things for me is understanding why you’re doing it. And understanding where it applies and actually being able to apply what you have taught to some real life situation where they can see that’s where it’s from. I think that’s a depth in maths. And on a mathematical point of view, I’d say understanding where every single bit comes from. Proofs, seeing that... but obviously that’s higher level as in for pupils but for me just understanding exactly where that formula came from and how it was derived and you know, where every bit of it came from. I’d say that’s a depth. But in teaching and I’d say for my personal benefit I’d say it’s understanding why you’re doing it and where it’s from.

**Consistent with Qu 8 response C (understand why)**

**Int 7**  
The conversion between fractions, decimals and percentages was a good one. Again the topic probably there was a bit more visual, you know. You...
were able to put up your pizzas and your cakes and think, they're like... and there was a better understanding. I think also in primary school they'd gone through some conversion. Obviously we were taking it to the next step and dealing with numbers that they haven't seen before but there was a confidence and everyone had a go and offered an answer, or came up to the board. Erm... and just a bigger feeling of success really.

Yeah a bit more interactive you know. Starting with an activity they are doing on their own then moving to a more game type scenario on the board and matching cards around to you know, pair them up, pair up the fraction and the decimal or whatever and they enjoy that and are keen to come out and demonstrate their prowess.

For me personally, well... I mean it would just be an inherent knowledge from my own schooling. That's something that just sits with me. I don't remember learning it or how I learnt it but it all makes sense to me so I am confident in that and then it's a case of thinking about what are the possible pitfalls here, where are the children not going to understand the relationship and the idea of things being the same value but the number being represented in a different way. Erm... having had ideas as well from the university sessions that we have had here.

Better equipments and strategies.

beginning of the topic where we’re talking about organising data, planning to collect data, critiquing a survey, why does this not make a good question, the likes of that felt that this was a doddle of a topic. You know, what a nice topic to do and then we’re going to move onto bar charts and all the sort of more mathsy stuff and really, really struggled but a lot of that I think is down to the pupils lack of ability to work independently and also literacy skills.

So they weren’t able to confidently put together a survey or a questionnaire or to be able to critique one that was already there for them and to be able to adapt ideas or even to think how could I go about that, you know, if I want a survey what computer games do you like, who should I ask, where should I go and what should I ask them. And it was very much, some of the problems were around that.

Everybody struggled. So you needed, I felt like they needed a one on one discussion about it. And as well because it wasn’t necessarily a numerical formula or algorithm it wasn’t a case of oh well have a look at this on the board, now you can do it. It needed thinking skills and sort of wider knowledge.

It's things that you see, as what you think is commonsense that you're not in the mind of that child. You know, and it's not common sense to them and that was the learning curve for me.

doesn't mean doing something in your head, you know I'm fine with getting a piece of paper and looking at it that way. It is obviously having the confidence to do it... and being able to explain it to another person. I think probably shows an in depth understanding. If you can explain it to somebody else and put it in a context as well, a real life context hopefully.

Consistent with Qu 8 response B (communication)

| Int 8 | I think probably the thing I am most confident in is algebra. That’s what I enjoyed the most at school and I think that for me is the basics of maths in the puzzling and solving things. And being able to use algebra to sort things out. I really enjoy that and I really enjoy explaining to them why it’s useful because I know everyone always goes algebra what is this going to be used for. So actually covering that... | I have just been teaching top set year 11 GCSE and the very last topic, and I knew I wasn’t confident about it because I kept leaving it to the very end, was to do with graphs and tree graphs and transforming a tree graph and I did leave it to the very end and really particularly didn’t want to do it and couldn’t find many exam questions on it but knew I had to cover it just in case. We have the maths watch disk. So I used the woman doesn't mean doing something in your head, you know I'm fine with getting a piece of paper and looking at it that way. It is obviously having the confidence to do it... and being able to explain it to another person. I think probably shows an in depth understanding. If you can explain it to somebody else and put it in a context as well, a real life context hopefully. Consistent with Qu 8 response B (communication) |
question with them as well.

And I felt very confident doing it because I had been in it, everyday life and then had real jobs in the outside world so it was really good that I was able to use those examples and show them. I think that algebra for me is something I love teaching.

Yeah so it is still my thing that I am most confident about and enjoy passing that love onto the children because they naturally, as soon as you have the heading algebra, they moan. So I enjoy switching that around. Yeah, I always ask them who doesn’t like algebra and they all put their hands up. And then when I taught it for say three or four lessons on that certain subject I say now who’s good at algebra and who likes it now and the difference is great. So I think if you really love a subject you can pass that through.

I can’t say I was 100% happy about teaching it and even though I had done as much work as I could do on it, it still wasn’t something I enjoyed particularly. And thankfully there wasn’t a question at GCSE on the paper with it. So I was quite relieved with that. Because I wasn’t 100% confident I taught it as I should have done really.

I’d say A Level statistics probably. There was a topic, the normal distribution where to be honest at the start of the year I didn’t have a clue. We did it on the MEC course and it was… it didn’t go… so I thought right okay I need to understand this, why do we do the normal distribution and I started to teach myself. The first couple of lessons on it boys were asking me questions and I couldn’t really answer the questions or wasn’t 100% sure so we had a two week break half term and thought right okay let’s get down to it and try to understand it. So again it’s reading through the notes and going to the my maths site and doing a bit of investigation and again something just clicked. You used a formula to standardise the score and ever since that it just seemed to click, I understand it. Personally it seems to be, it just seems to be one little piece of the jigsaw not quite right but once that’s in it all just comes together.

To me, understanding maths in depth is, I feel, if you’re being taught it, it’s you asking questions why do we do this, you know, it’s wanting to really break the topic apart to know why we do things. Why is it like this, why do we do that… and as a teacher as well I find the deep learning is to, you bring the misconceptions out as well.

Consistent with Qu 8 response C (understand why)
I think also it’s wanting the kids to understand it, you know, as a teacher you want the kids to do the best they can. You want them to leave school actually remembering something, understanding it as opposed to parrot-fashion, getting them into we do this, this way, this way, this way.

Int 10  I have been teaching Core 3 we just started with our lower sixth and numerical methods, so actually because it is a bit more relaxed on time because the exams have finished and you know, the summer’s coming up I have been able to go a bit more in depth into that and show them actually… you know a typical exam question would be erm… here’s say a cubic equals zero, can you rearrange it to get this into a formula and I showed them how to do that which they were okay with but then I was confident in understanding enough for me to be able to say but what about another formula, how many more can we make from this. And then showed them, I suppose from using my own understanding of the topic, erm… how they could make more of them and then we plotted them on a graph and then showed where they intercepted and y equals x and then we thought well why is the intersection between y equals x and And because we had more time I think we took a lot more understanding from that

Well, possibly not from my degree really because I don’t think I even did that in my degree. I think it just comes from me being more comfortable in my own mathematical understanding and maybe just understanding the topics better. Essentially it’s just a bit of simultaneous equations and also, I mean the method of understanding about limits but in a sense I can’t put my finger on exactly where it’s come from. It has just come from having all that understanding and then coming back to it and looking at it again.

it was actually a mistake that I had made on the board, the way I had taught quality transformations, I hadn’t really gathered the specific problem and the method that I had used was not correct. And a really bright lad who does further maths as well, who’s studied transformations from a different length, sort of not just this is how you do it and then that happens, he had a better understanding of transformations than I did even. So he said but are you sure you’re not supposed to do it this way instead. And obviously he was right and as soon as he mentioned that I understood where I had gone wrong so I was able to then say to the class, sorry, my mistake, Paul’s right, this is how we do it. But it was interesting because it was actually y equals 2 to the power of 4x I think and then how does that transform to y equals 2 to the power 4x minus 3… And I just said well we’re taking away 3 from the x. So it’s a translation (3,0). But it wasn’t because you have actually replaced the x, you had to factorise is and take that out. So it was 2 to the 4 (x minus three quarters) so a translation of three quarters zero. But then on coming back to that I was obviously not happy with the fact that a student had actually said this is how you do it. I thought about it in more depth afterwards and then in the next lesson I was able to show them another example and there was actually two possible answers to it because you could have also interpreted it as you multiplied it by 2 to the power minus 3, which I a stretch in the y direction, scale factor. So it was really positive, you know even

Possibly someone that had a deep understanding of mathematics would be someone that was erm… able to see the links between them. Something that someone else might see as a stand alone topic.

A deep understanding would be when you’re talking about someone who really could see patterns in the way things work and the way the topics are inter-linked with each other.

Consistent with Qu 8 response D (make connections)
though I had made a mistake it was a positive thing because I was able to come back to it and say, you know, obviously I understand that I make mistakes... but I turned it into a learning process for everyone else and for myself as well. I was able to say there's two possible solutions and then I think that will effect me next year and obviously even though it never come up before I will show it to them.

| Int 11 | Pythagoras Theorem is one. I have set up like a little tutor group for them so that's kind of been something that I have started and feel actually really secure teaching. And again it's just establishing a proof beforehand of where it comes from. It's something that I have picked up and I have probably developed quite a bit on. Erm... I think the other one is looking at straight line graphs. Erm... looking at the equations of them and working out the gradient and the intercept, something that I have felt a lot more confident teaching this year and I have actually had some really good lessons out of it
I didn't like teaching loci, I didn't like teaching it and that was probably... it's not the one I'm thinking of in my head that I said before but it's one that I didn't really enjoy teaching to be honest. It was quite a busy subject and it was just, I don't know, it was really hard to explain and get over to students. It was a lesson that didn't go as well as I would have liked it to, to be honest.
Yeah, they were just getting themselves into like... a lot of them were getting themselves worked up because they were getting a bit frustrated with some of it. Erm... so no I think that one was a topic that I didn't really like teaching and found it created a bit of a problem.
I think just extending the visual learning more maybe. So maybe taking like a step by step process to it rather than just throw them in at the deep end. Maybe that's what I didn't do with that particular lesson.

| Int 12 | I was thinking about place value really. How to introduce that and the way that if you use partitioning cards, you know, the ones with like the arrows at the end and you can overlap and separate out the hundreds, tens and units into five hundred.
How you try and explain... when you're trying to do minus one, subtract minus one... and the course last year gave a good example of a visual one of reversing a car forwards and backwards. I can't remember exactly what it was though but it was Barry and on the day I

| 202 | Well I think for me personally I have got to understand why it is and I know obviously at maths is the right or wrong answer but I have got to understand why it is that right answer. Now whereas when I was in school I was probably never, to a point it didn't always matter because a lot of the time I just wanted to take it as a given. But I have found since I have started teaching maths and learning maths that I have got to understand to a point why it's an answer. So if I have got the wrong answer I have got to know why it's wrong and I have got to know what the right answer is. Just something stupid. I mean I had a question on using BODMAS once and it was a really basic straightforward calculation and I actually got the question right but there was another teacher in the room at the time who said a different answer. Their way was wrong and mine was right but the way they had worked it out, they could have got the same answer as me as well but I was adamant I was right and I needed to know, I needed to have clarification that I was right basically. So it's just like that was a little example but I need to know if I'm not right about something why I am not right and if I am right I need to make sure I know why I'm right.
Understanding maths in depth. I think... I would hope that maths can be used from day to day to help you solve problems really. Real life problems. Because unless you want to take on your studies further in maths. I think maths should be really about the day to day trials and
You can actually see the five zero zero.

Five hundred and sixty seven and then it’s a visual way of being able to do that sum and you can transfer that to doing it mentally in your head... and that was another thing that the national numeracy strategy taught me. But that’s something that when children see numbers split up like that into different elements it does make it easier for them... and also to use a hundreds, tens and units board for addition and subtraction so they can see how the exchange works and then you get ten cubes together, you pick them up, you take them away and you replace it for a ten rod.

I think it’s a mixture really of the numeracy strategy. I keep going back to that because that actually taught me so when I was using this folder in primary schools I’d open it at the page of partition and I would read through it and I would say, ah, I didn’t really realise that you could do that. Even as an adult because I would be in my 30’s then and it was this kind of hidden thing that I hadn’t discovered until I was grown up.

I understand it. I thought, right I’ll have to try and remember that one.

And I think I’m more of a visual learner myself and I find that trying to... like drawing things down or trying to give an image of something like a car going forward and backwards often helps to understand concepts of something quite difficult.

Also at the start of the MDP course I put down algebra was, I felt was a bit of a weakness and when I mention algebra to adults in school, to some of the teaching assistants, they always take a step back and oh I used to hate that at school. Erm... and I felt that the MDP course gave me an opportunity to first of all research about the misconceptions about it... And I’ve got that confidence within myself now... if any child or any adult comes to me with a problem I can feel that I can attempt to try and answer that you know, quite confidently.

I think it means understanding the background of where things are starting. Because you can understand the method, for example the solving equations when I was taught to throw something over to the other side, you can understand that inside out. But as a child you don’t actually know why it’s happening.

And that’s where the depth comes into it. Once you realise that actually you don’t... if you’re timesing one side by 2, you’re timesing the other side by 2 to keep a balance. It’s knowing why you’re doing a method other than simply applying it.
coordinates and what was happening to the coordinates when I was changing the graph and from that then came up with a little quick method that I’d been told by our Head of Department that basically if it effects the x, so if what’s changed is within the brackets you do the opposite. So if it’s times by 3 within the brackets you would actually divide by 3. And if it’s outside the brackets it affects the y coordinates so then you do what it says. So we looked at, we linked it into the actual autograph and to what it was doing. We drew the graphs and then came up with a quick method.

Because I had taught that year before with a set 2 year 11 and I hadn’t taught it well and I knew I hadn’t. They came out and they didn’t fully understand it and this way with like a set 4 year 11, they got it. The class before was so much cleverer but I’d just taught it better this year.

Yeah, because I think my own understanding of it. I’ve never liked graphs, I’ve never like drawing them in uni and I didn’t like it at A Level so it was kind of one of them areas that because I didn’t like it I might have betrayed that but because I had to sit down and planned it properly then I like teaching it now.

## Int 14
It was Steve Paget was coming over to watch me at Upton and, I mean, I was really quite excited about the class I was going to do anyway but he just happened to, you know he was coming, but it was a really good class and it was about umm... using algebra to solve everyday problems and I felt really confident with that. A. Because I was obviously well prepared but I felt really confident because umm... I was very, I was really quite sure about A. the interest value to the students but also the fact that, I mean what I did was brought in a picture frame... the mount of a frame

kind of atmosphere in the class

{But} and if I’m like stopping and starting and doing little examples and then getting them to work while I’m quickly looking through a book to check the next example that we’re going to do, erm... then yeah it does have an affect, it just takes away the friendly atmosphere really because I’m more focused on my own work rather than theirs.

I don’t even know if it [preparation] helps you in teaching that much but it’s just the confidence that it gives you, it’s just that knowledge that you know it inside out and they could ask you anything and it just changes your... well my whole appearance at the board really.

Consistent with Qu 8 response C (understand why)

Int 14 It was Steve Paget was coming over to watch me at Upton and, I mean, I was really quite excited about the class I was going to do anyway but he just happened to, you know he was coming, but it was a really good class and it was about umm... using algebra to solve everyday problems and I felt really confident with that. A. Because I was obviously well prepared but I felt really confident because umm... I was very, I was really quite sure about A. the interest value to the students but also the fact that, I mean what I did was brought in a picture frame... the mount of a frame

Consistent with Qu 8 response C (understand why)

Consistent with Qu 8 response C (understand why)

it was on histograms and the way I taught histograms before was quite sort of separate in the sense that I would teach it sort of equal widths and then unequal. But their teacher had asked me to teach it with equal widths and then they weren't going to do unequal until next year

And I just really made a mess of it and umm... the students started asking questions and I at the back of my mind the whole time I kept thinking I would not have done it this way. I wouldn't introduce it until I can follow the whole thing through and she had said to
Which was from home so I said this is... and I gave them the area of the photograph in the middle but I didn't know the size of frame to buy and I gave them a couple of measurements. They had to use algebra to figure out really the size of the frame to buy. So it was a very practical problem. And so yeah, I felt really confident, but that was... a lot of that was because of, yeah a lot more secure knowledge of algebra.

I think it came from the way you guys taught the course to make algebra into an everyday thing. So it wasn't just this is how you do this equation but this... I had the confidence to try it out in the real world.

I'm really not happy with order of operations. Mostly I'm not happy because I think of the way it sometimes come up. With young people, you know, and particularly special needs young people, you have to be very, very clear and it doesn't always do it that way, they can be thrown. And you can actually get some aggression if they can be thrown. I find that quite hard. So for instance Bodmass, yeah. Okay so I teach them (other) division, multiplication, addition, subtraction) this is the order of operations, this is how you do it, when it's all mixed up. But then someone will write that... And they'll say well we've got to divide first because division comes first. And then I have to say well actually maybe that should have brackets around it.

Shape and space. It's building confidence and one of the things that I really use a lot is developing the visual special awareness because I remember all of the course but that bit really sort of hit home. So for instance at the minute, coming up to unit 3 of maths it's more than 50% shape and space. You know, so we will start with tessellation, we'll talk about tiling floors with boys who are on construction courses about tiling floors and how the tiles have got to tessellate and how you can't just make a pretty pattern and think it's all going to fit. And then you would go onto isometric drawings. So you have got, you know, the type that are in cubes. Yeah and then you have got more complex shapes. And then shapes without cubes and then you're on surface area then and back to volume. Then you can lead into surface area and then go back to volume, that you learned earlier. Because you're using them same shapes that they were just drawing to get the surface area and the volume. I'm finding that's really good, that's really working really well. And then of course you move onto plans and elevations and then we go back to asymmetric drawings. So everything is linking in. I just find that's real confidence building. Not only for me as I do year on year, I know it so well, but for the young people as well.

To me it means that not just knowing how to do it, it's how it works, what its about and how it links in with other things. And it can help and be useful in other things. That's what maths is to me. But I think it's also making it part of your everyday life, in a way.

Consistent with Qu 8 response D (make connections)
<table>
<thead>
<tr>
<th>Int 16</th>
<th>Some simple algebra...that’s always been my weakness, algebra because I always had problems with it at school. But something to do with the algebra and we showed them a little clip from maths watch and gave the worksheets and working thought them together. The kids understood it as well and that was the nice thing... the whole class all sat and worked independently. And that was where I felt confident about going in and doing it. See that’s another thing preparation now, whereas in the past I would have probably not prepared and I’d have just gone in and done it. I do think more about what I’m going in to teach and how to teach it. It’s got to come from that course we did... And it is quite interesting now because if people come to me within school and they ask me things, kids ask me things, I even have other Heads of Department now contacting me and saying ‘Chris have you got anything that would suit this?’ But it has to come down to doing the TDA course.</th>
<th>Algebra... because I never understood it at school. I still struggle with it, you know... I will struggle with it for a while but my confidence has gone from ground floor to first floor and hopefully now it will go to second, third and fourth floor.</th>
<th>For my job a deeper understanding of maths would be for the maths I have to teach at the level to be a level where I know the topic inside out. So I can give the kids the confidence to do it. But na personal basis... having a deeper understanding of maths is going to be an understanding of lots of aspects of maths far beyond what I’m doing or what I need to do now. So that’s going to expand my knowledge of maths even further.</th>
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<tr>
<td>Int 17</td>
<td>Right well, I mean with my sixth form group it's years since I have done the remainder theorem and the factor theorem, algebra, long division. And what I have done in particular with my sixth form is the questions I am going to set I do them myself. And I write out all the examples I am going to do with the board and I just keep them there. So I enjoyed that, I went in, I had done the work, the preparation was good, and I felt very confident. It was similar on the previous unit, things that I'm not even sure I did it in school. I remember doing the quadratic formula, I don't remember doing completing the square but as soon as I saw it I thought what a wonderful method. In some ways it's much more easier than using the formula and how it sort of slips into other questions and other areas of math’s. In particular when they ... I was teaching curve sketching. And like if it's a F of X then it's a stretch vertically and it's what FX plus A, then it's going to the left... and the one that I struggled to get my head round was I knew that F of A is your compressing things, you divide by A don't you, divide the X coordinates by A. But there was a question that I hadn't done but it was there on a past paper and it was sketch the curve of 3 over X. And then it was sketch the curve of 3 over X plus 2. And I must admit I bragged it and said it was shift to the left of 2, and it is but I wasn't 100% prepared because I had not done that one... So I was just feeling not quite comfortable and I need to go away before I teach it next year, I just need to</td>
<td>what is the formula of a box of Kellogg’s or how many can you stick in a wagon, how can you move them around, can he do more than one trip because they only need 2,000 boxes there. And I was trying to bring it into a real life situation. But if you're trying to say why do you solve maybe a quadratic equation or maybe do the difference of two squares, what I've tried to say is that because I can't think of a real life situation is that these are the building blocks of mathematics. That if they go on to study mathematics in greater detail then you use these skills to do more detailed problems and I say it was the case with me when I did Civil Engineering. I had to know the mathematics to be able to do the proofs and the theories behind why an stand might stand up and why a tall building, how much it will sway in a strong wind. But I said the mathematics I'm using to do that, you can't just go</td>
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start doing sixth form, you know, it lends itself to a lot of solutions. And it’s just a nice little technique and I thoroughly enjoyed teaching that. Probably I looked at it on my math’s, I went on Youtube and brought up a lesson so I would just sit there and look at it. And I got the textbook out and wrote all the examples out and then I did the questions in the textbook and then... I photocopied and copied out all the OCR past papers and all the Edexcel past papers and then I cut them all up so they were by topic and I did all the exam questions as well.

I have been caught out by doing things that I didn’t even for a million years know was wrong. I never realised there was a difference between a histogram and a bar chart.

Umm… well for me it’s that being able to explain to somebody else, you know. If I can say to you it works because of this, that means I must understand it. Because like I say I am born again maths. So I am kind of evangelical about it and I am noticing people, while I was waiting for you to come today, I got the girl in the office looking at my power point about measurements and she was like, oh really. I should come to your maths lessons. And loads of times I have had the LSA and learning support system and say you know if you had taught us like that at school… and it goes back to, his name was George Allan, the guy who came to see us when I did the Cert Ed and he had been on Channel 4 and you know, Johnny Ball and all that, so he made it accessible. He gave us the understanding, he drew the pictures and that’s how I want to be. So I kind of want to be like that, I want to be able to say people look... Marcus Du Sautoy. You know what I mean. If you can be Marcus Du Sautoy for a day.

Consistent with Qu 8 response B (communication)

I hadn’t ever in my life done the completion of the square. I hadn’t completed the square ever in my life so that was quite… discovering wow I liked that, that’s I suppose in a very simple way it’s seeing graphs everywhere. I think it’s seeing functions. I mean for me that’s what it really is, it’s that deep understanding is you
conditional probability in that, you know, I find that’s one that I’m using the maths, because you talk about menial mathematical understanding and that’s, you know, I can’t remember the equations, I’ve not necessarily looked them up so I’m going back to the sort of fundamental understanding and with the pupils I am working it out. So in terms of using mathematical understanding I am not relying on a memory and not relying on preparation, I’m relying on the fact that I am very confident in this area and I know that I can apply the understanding to bring out the mathematics if you understand what I’m saying.

...really a lot of my experiences in mathematics are having to do the maths as a scientist and having to work out what maths I need to use and not really, I suppose, only lately becoming an academic mathematician, you know, somebody who has studied it. So that sense of having had to work at it and had to almost develop my own mathematics first and then coming to ahh that’s the right answer.

That’s the way they do it. So I think, I’m not sure but I think that having tried to do it before was taught it in many cases has probably enhanced that quite well.

Rounding numbers to one decimal place, two decimal places and the method I tried today umm... I mean I tried explaining it with a nice little tree at one end and a tree at the other and it's raining and I'm walking along with my dancing umbrella, and if I get sort of half way, before half way then I may as well go back to the tree before and if I get to the middle I may as well go on to the next tree so I try to get my rounding up with that. Umm... and they look at you as if you've gone mental.

I think you need to have studied maths to quite a high level. Umm... because I feel as I said earlier, I feel that it gives you a confidence. You've got to know your subjects

So from my point of view as a teacher I feel that I've got that strong background. I've got a strong knowledge that takes me above and beyond A-Level maths which means that I am equipped to explain and to understand the different ways that the pupils I'm teaching are actually thinking and going round and sometimes seeing the way the things that they say... and even though you know they're not quite right... I mean the thinking, the mathematical thinking that some of these kids have and they don't realise they've got it, it's beautiful to see.

But from a pupils point of view umm... they too need to have a strong mathematical background. And the background that the kids need is they need to be able to do...
Int 21: I can think of one that I did this morning for the sole reason that it encompassed a lot of, like I said, solving quadratic equations, that’s why it came up. Umm… this morning I linked it to umm… showing them… getting them to plot the graph, tell me what the curve looks like so it gets them investigating it and then… so we had already factorised them by this [unclear 36:24] quadratics. I said okay well what if I said if this is equal to zero and said well what would that mean? And they sort of look and say okay well do we know what this graph looks like? And they say no. I said well okay you plot it, see what’s going on. So they went away, they plotted it and I said well we factorise it with $x + 3x + 4$. I said what can you tell me about the graph? And they said well it crosses at $x - 3$ and $x - 4$. And I said okay, you’ve plotted the graph $y$ equals but I’ve got this quadratic equals zero, but can you see the link. And it’s just sort of letting them investigate it rather than this is the method, this is how you do it. Funnily enough, okay, this might sound quite strange, it would be from teaching C1. I can’t say… I can remember solving them at GCSE. I can remember solving them at A-Level and they were there a little bit in my degree but when I say a little bit, they probably were but everything seemed so much harder than that area. Umm… and probably teaching them at the maths centre. But I’d say mainly from teaching them in C1 here. So having to take it that step farther but then at C1 I wouldn’t have got them to investigate it so maybe that’s just my stance on how I should have taught the lesson if that makes sense.

… I would say, I’ve started teaching Mechanics one. Now this is strange because my dissertation majored in mechanics but obviously of a different nature. Umm… I panic when I go into every lesson for mechanics at the moment. And that’s because, a) I have not looked at, you know I haven’t done it for so long and last time I really looked at that module was when I was in school. Umm… and that I’d say, it’s that whole module to be perfectly honest. So I sort of always at the moment feel one step ahead of the children.

And I also teach umm… FPT further pure two now. Umm… that is much more like what I’ve tackled at University. And it’s all coming flooding back and I don’t really panic with certain topics in there. The only one that I could hand on heart say where I felt I had failed in terms of teaching, umm… this young lady was complex transformations but that was something that I struggled with at University as well. Umm… but now I have got that down to a T. So it sort of… these two areas that I’m talking about are my weakness but now they’re becoming my strength if that makes any sense? I’d say without you know the University and if I didn’t have that background I think I’d find teaching difficult because you still need to have all the concepts, strong concepts and strong knowing relationships between certain things as well.

… their times tables, they need to know their number bonds, the really basic stuff.

To me it’s something that I didn’t get at school. Is having everything from first principles.

Because if you spout rules then they just have more rules to learn and it feels overwhelming at that point. Understanding the process. It’s a really difficult question to answer. Understanding the processes rather than just being fed a rule and… if that makes sense.
Appendix 12b: Tabulated responses to interview questions 8, 9, 10

<table>
<thead>
<tr>
<th>Qu 8, What is UMID, justify first choice</th>
<th>Qu 9, How is UMID developed, justify first choice</th>
<th>Qu 10, Topic you understand well and where understanding came from</th>
</tr>
</thead>
<tbody>
<tr>
<td>Int 1 [B]</td>
<td>[F]</td>
<td>Volume of prisms. I think because I’ve spent… I don’t know if I know it in depth, I think I feel like I do because I have spent a lot of time over the last month or so getting different resources on it, watching videos on it, em… so I have taught it and everyone seems to understand it….and be able to get the right answers. I feel like I understand it because when I teach it, the kids get it.</td>
</tr>
<tr>
<td>Int 2 [B] Because I feel that actually shows true understanding if you can actually explain it to others. Because in a way that’s reinforcing it for yourself as well as being able to show that you know your ideas. [E and B joint second] I think you can’t do one without the other. You find out how they work, then you’re making the connections and then you’re seeing the patterns between things and then be able to justify your thinking which then leads on from the communication</td>
<td>[G] I have definitely gone for again another hierarchical approach that first of all…an understanding of maths in depth it takes time, it is like being on a mathematical train. Understanding develops by investigation and working out taxing problems</td>
<td>I would say differentiation... I think it was something that we did at O Level...something that explained gradients to me. For some reason, you know working out the gradient of a curve it was just like the pieces fell into placeYes it is a tangent. Yes the tangent just touches...yes of course it’s the change in the y and x has to be so tiny. It was just like a light opening at the particular moment...</td>
</tr>
<tr>
<td>Int 3 [B]</td>
<td>[F] The first one is about investigation, working out difficult or taxing problems. Because I think investigation-wise that is how I have been approaching at the moment sorting out, in depth. And the second one was about taking time, being on a journey I suppose that’s true as well because when I was sort of 13/14 and very accepting, the teacher says it’s this then it’s this, whereas at university there was perhaps a little bit more thinking involved and now I probably am developing more of an in depth understanding... just because I have had to. ... develops by living with and/or spending time thinking about mathematical ideas... well again I suppose they’re very linked aren’t they.</td>
<td>I do actually remember looking at a proof for differentiation all to do with rectangles underneath, that was integration wasn’t it… rectangles underneath a curve and how if you make a width of them ten towards zero how… so that was how integration kind of developed and also with the differentiation it was the gradient and how if you make the gradient… the triangle, ten towards zero then you know, it tended towards the differentiation rather than just different gradients. I do vaguely remember doing that and I think if I had to teach that I think I would be happy that I did know that to a reasonable depth, but again I probably just accepted it. I suppose you’re not just saying like this is the formula and get on with it.</td>
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<td>Int 4 [D] Well I think that’s what I take as being a deeper understanding, being able... for pupils and staff to have the ability and the confidence and I think confidence</td>
<td>[F, G] I think on F, you know working out difficult and taxing problems. I think... as educators you know we like to be challenged but we also like to see</td>
<td>I would like to say elements of the data handling. ...You know, because there is an overlap with the business studies it is something I feel, you know, when</td>
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from a pupils point of view is extremely important you
know, to have that confidence and you only get
confidence from having a deeper understanding of
something and to have that confidence to be able to
look at a mathematical concept and say well there is
actually a link to this mathematical concept as well is...
if I was having that conversation with a learner I think
that was the ultimate...

results from those challenges so if you take anything
in education that is challenging... and you investigate
it and work out that difficult or taxing problem then
there is a sense of achievement there and anything
that you get that sense of achievement from you
know, you want to carry on doing it, you want to
carry on challenging yourself and it is a sense of
enjoyment and I think that’s a characteristic in all
educators without doubt... and understanding in
maths in depths develops in living with or spending
time thinking about mathematical ideas, I think
again because once you start enjoying something
then you want to do more of it. Someone like Barry
is a great example, you know, everything he does is a
mathematical... it becomes part of your life doesn’t
it.

I think for your subject that is such an important
thing, you know, I think if you look at outstanding
teachers from Ofsted, you know one of the
underlying things is a passion for your subject.

But to take someone like myself who is a business
studies teacher with no maths I think actually
igniting that passion is the challenge. I think that is
something that I’ve, you know, the way I have
ignited it personally is... because I have got
responsibilities in key stage 3 is to try and create this
buzz around maths and an enjoyment of maths, and
I have started to find that if you start doing what it
says there in F you know, investigating and allowing
pupils to investigate, they get that sense of
achievement and that’s the way that you create your
career that was around maths from my point of view
for them.

I’m stood in front of a classroom I feel quite confident
that I could make them understand it and make them
have that similar deeper understanding of.

I think the links and being able to take a focus on the
mathematical side of that understanding I already had
which is quite a qualitative understanding in many
ways, and being able to apply the number side to it and
just having the confidence to plan a lesson where
knew I could get to those objectives in an interesting
way...

Int 5 [C] Well because I feel that I need to have that
understanding so that I can communicate it then to the
pupils and if I don’t understand it then how am I going
to get that across to the pupils. So that would be top

[F] because again for me you know, we have done
something today. So right, we had all this
information, so the data that you gave us and you
know that was, in a way it was the problem so what

I do feel I have got a better understanding of statistics.
Erm... you know averages and mean, median, mode
and all those sorts of things. I think I am developing
that as well so you know, at a lower level I had that
for me because I need to know it before I can do it for them.

are you going to do to sort of work out this problem mathematically so if you investigate and look into it and then ask people and all that. You know, then I will get a better understanding of it, of why I am doing something. Because I have got something to work with. Okay and it makes sense because it was real. It makes more sense to me that way.

understanding but now I feel I am getting even better at it. So yeah that would be one area I think. Partly through teaching it, you know, and having to go and do… remind myself initially how to do everything when I was teaching it. And then again from doing the statistics course, teaching it to year 11’s, you know, so now okay, I would have to look up again how to do Speaman’s Rank for example but I do feel now I am more secure in it. So that has been quite a good thing really for me. I think it’s through having to teach it. You know, I don’t think and from coming on the course but from having to go and teach it. I think there is some saying isn’t there that the best way to know if you understand something is to teach it or to tell somebody else about it.

Int 6

[C] Why the procedure works and how it works. I think that’s probably the most important thing to me.

[G] because it is real life again. It’s living with maths and seeing its relevance and I think to have a depth of understanding it would have to arrive in your day to day life type of thing where you see where it is in real life. I think that’s when you get a deeper understanding

I’d say my strength, probably contradict myself, is algebra. Erm… equations and I particularly enjoy that bit. That’s probably a bit sad but, I like formulae and And yeah I’d say it is and I am more quite intrigued with that type of thing. The visual maths I’d say I’m weaker, I wouldn’t say I’m weak but I think I am weaker and I couldn’t rest with an algebraic problem in front of me until it was solved, whereas I could give up a bit more readily on anything visual so I would say algebra is my strength.

Int 7

[A, B] Gosh it’s difficult because I think when you read each one independently you think to yourself, that’s definitely important. So … Justify your mathematical thinking, so you are articulating the maths…. bringing prior knowledge of concepts or whatever into the four as to why something is the case, so justification is

[F] I think the word investigation attracted me there, the idea of investigating and you know, doing sort of hands on work and working things out for yourself. You know, why is this the case, let’s try it out. It gives you the understanding, it gives you a picture of why the maths is like that.

Pythagoras.

Do you know, funnily enough I think at school I just gained the algorithm of how to do it and how I properly understood it was re-visiting it this year. Erm…and looking at new resources with the visual of having
important. Explaining it to others, I just think that's probably one of the harder things to do. You understand it in your head but if you can actually articulate it, explain it, demonstrate it, lead someone through a process, however it works, the depth is really there. And it's not about the words that you use or you know the maths language necessarily but the explanation of how it works, it's tangible. More tangible... and then understanding why and how procedures work. I mean I suppose for me, you know is the last three certainly became less important.

Int 8 [B] I think that's the whole crux of my job really as a maths teacher is being able to communicate and explain what's going on with the maths... and why it's going on to others really. So I feel that's very important. If you can't communicate to... you might understand how to do it and you might be able to work at home and do it on a piece of paper but you have to be able to communicate that to everybody else. So that's a different skill to being able to do the maths I think

[H] But because that's what's happened with me really, ... and I think you can always learn and there's always more you can learn and whoever you're with and whoever is teaching the maths you can learn and take tips off them. So it is a journey and I don't know where the journey ends really because it carries on and on and on.

I would say my number work and my algebra work are my strongest... areas and, yeah any aspect of those really I would say are the ones that I would know in depth and can also equally explain in depth as well. I think the learning that I had in number work was really, really strong when I was at school. I think it was a lot of good basics and I think a lot of the pupils have missed some of those basics in their primary schools or their early years and it doesn't half help when you get to teaching GCSE's. And I think if you can teach that deeper understanding of, for example, if they are dividing one decimal by another decimal, a lot of them don't understand why you get this big number. They go oh no, if you're dividing point something point something by something point something, it's going to be a really small number. They don't understand but they're asking you, so then to give them an easy example and say we're going to do one whole one and we're dividing it by 0.1, what's 0.1 as a fraction, how many tenths in a whole one, can you understand why you get that answer now.

Int 9 [C] it's all down to this, I feel, the students asking why, wanting to understand it, why we do these things... breaking the topic down instead of just this shallow learning that we have been talking about. Actually going into it, getting the students to understand it and why we do things as opposed to just copy this, this is how we do it, don't forget it for your exam in four years time.

[H] Well personally since I started the MEC course, ... it's taken time. And okay I did the, for example, the M-Stats course. I did two lots of stats with you and Steve... since then things have progressed. I have taught stats this year so I have had to really re-teach myself again and it's only now that the deeper understanding has come into it... probably the easiest then would be the fractions work up to I would say now the normal distribution. Erm... it's really... saying that I would say that there are a lot topics which are linked you know, you could use fractions, you could use the percentages as well. So the main two I would say from the fractions up to normal distribution.

Through the PGCE, through the MEC course and from
<table>
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<th>Int 10</th>
<th>[D] Because in terms of understanding mathematics I think that the highest level of understanding of mathematics, that’s what maths is about. Making connections and you know, yeah pretty much making connections. I was tempted between putting E at the top as well or may be at a level par because you know being able to see structure patterns in terms of a mathematical understanding ... and just behind those two understanding why they actual work. It’s quite difficult. But the reason I have put these two [B,A] at the bottom, I mean obviously these are the most important to teaching but I don’t think they’re the most important to understanding my actual in house teaching as well. Wanting to get a deeper understanding of it and going away and reading old notes, going on the internet. Get the test base..and go through the textbook. Make sure you can answer the very last questions on the topic and…</th>
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<td>[G] I think mainly the word living there swung it for me because we’re talking about understanding in depth and something that a real understanding that not many people have, I think you have really got to live with it. You know, it sort of takes over your life. Cubic polynomials. Because we do a lot on the Core 1, you know you have got to plot it and then moving onto differentiations and look at roots. Also in Core 3 we have looked at numerical methods for… you know ways to approximate and I would say that yeah, I felt as if I quite well understood that. Okay, so I mean again I would not really see much of that at degree level, I mean that was A Level but my understanding has come from, you know, at A Level had a better mathematical ability and I progressed as a mathematician and now that I have come back to teach it I think that’s where my understanding has come from.</td>
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| Int 11 | [B] Because I think again in order for them to understand it they need to be able to explain it properly. I am taking that from my point of view as well, in order for me to understand something properly I need to make sure I explain it properly. If I don’t explain it properly then the element is I don’t understand it basically. So I think in order for you to understand it you need to be able to explain it correctly. Erm… straight line graphs I think I know more in depth now. And I think to a point trigonometry as well although it’s something that I do want to progress further next year. It’s something that I do know more in depth about but it is something that I do want to target myself to progress for next year. So I think definitely straight line graphs, that’s something that I feel, I just think, as I say the more I’m teaching it, the more I’m picking up new things and something I have never noticed before on this, which is really good and again that just alters the way that I am explaining it to each class which is good. It’s something that I have taught quite a bit this year. Erm… I mean, trigonometry was something that I really liked in school. Erm… but again it is just something that has been taken further doing the A Level and the MEC course as well because obviously it has been extended so I think that again really it’s learning from the MEC and the A Level. |
| [G] because again I think it’s important that you are thinking about what you’re actually doing rather than just writing the answers down, thinking about the process you’re taking along the way. I think, so I think out of the three of them that would be the most important one. It was between the first and second one [F] really because I think the fact that you know, it develops by investigating or working out difficult and taxing problems, I mean, that can be quite rewarding which gives you a better understanding if you like. So I mean I like investigating things as I say and I like working out difficult problems because it gives me a sense of reward at the end when I have got the correct answer or whatever. So I don’t know, that’s kind of like split. I think it is that one that’s top but that one was kind of another one that came close to it. But I think in order for you to go through that you need to spend time thinking about maths ideas |

<p>| 214 |  |</p>
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<tr>
<th>Int 12</th>
<th><strong>[E]</strong> I think because well E says, it going on about structure patterns and general results. You don’t have to necessarily be really good at number to use this.... and I think to be able to look at structure and patterns would help you in other aspects of life rather than just dealing with number.</th>
<th><strong>F</strong>’s on about working out problems and investigating difficulties and the taxing problems and I think that’s most important because it’s more relevant to day to day living of the general people. This would be more theoretical the G and the H. G saying about spending time thinking about maths ideas. Professor type thoughts come to mind there. And this one about maths being about it takes time and is about being on a journey. That’s like a romantic type of statement there. I think that was more relevant, practical day to day nitty gritty.</th>
<th>I don’t think there is any one thing that stands out. Like I say I have got the confidence to grab a textbook and just have a little whizz through before the lesson so I have got an insight into what difficulties lie ahead. And on the course as well it was just meeting with other members of staff who were in exactly the same position, they were teaching groups of kids up to GCSE but they didn’t have a qualification to do it. It was speaking to them, it was having two days out every month last year and just given that time and flexibility to ask staff how they approached things in different ways, getting websites off people and they’d say oh yeah you can try this, it’s great. A lot of free resources as well and the CD’s we were given on the course have been great as well.</th>
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| Int 13 | **[C]** because I think understanding it in depth rather than just understanding it, I think you have to understand why and the procedure that. 
[A] I have put second to last but I’m struggling with whether to actually put it last because I think people can justify their mathematical thinking and it doesn’t necessarily mean that they understand it in depth. But at the same time if you understand something in depth you can explain it, you can justify it in a way so... the implication goes one way but not the other. So the being able to justify. Yeah, okay it might mean you can understand in depth but it doesn’t necessarily. I think the communicate ones are really vital for a maths teacher... you’re not going to communicate things clearly unless you do understand something in depth. But it doesn’t mean, you know, it’s not the only thing... it’s a good test of whether you understand something in depth but it’s not the only thing. | **[F]** because I was thinking about how my pupils... how I try to make them understand maths in depth and it is through things like investigations. For Pythagoras... about drawing the actual squares on the side of the triangle and thinking about problems with that, that’s how I try to teach it. Just spending time thinking about mathematical ideas. I think that’s important because you do have to spend time on it but quite a few people... probably wouldn’t just spend time, I think doing things is probably better exercise than just spending time thinking about it. | Erm... if we’re thinking low level something like Pythagoras would be something that I now, from teaching, understand where it’s came from. Because I don’t know whether it’s just that I can’t remember it from school or... I can just remember the formula and that formulas got me right through university and then it was only when I came to teach it that I realised that it was actually the squares on the side of a triangle, the area of the squares. So something like that is where I’ve really gone back to basics. And you feel guilty just simply saying here’s the formula, learn it. Because some pupils would and they’d do it quite happily but a lot wouldn’t and there would be no point in that as an exercise and it makes it more interesting doing the colouring the squares, cutting them out, seeing if they all fit into the same... Into the square and the hypotenuse so erm... it makes it more interesting and it makes them understand it quicker. |
| Int 14 | **[C]** Because if I can’t explain or communicate then that’s it. But to be able to explain and communicate I | **[F]** I think this one [H] is the sort of the umbrella really | Subtraction |
have also got to be able to justify it and I have got to understand it,... so maybe understanding really should be first. To understand it and then to be able to communicate it and then... but understanding it... so communicating maybe should be the end really. So maybe I should understand it and then be able to make connections and see patterns and justify it... and then communicate it. I mean, that's how I would do it but you cannot do that until you have got that.

isn't it. I think this takes away the panic and this takes away the I can or I can't, or I'm maths or I'm not maths...and also understanding that you're not born good at maths or not born good at maths because I think unless you have that, unless kids understand, or children or anyone understands this, you know, it doesn't have to be an instant thing. I get it, I don't get it. That you can take time to figure it out. I think that's... there has to be the underneath or the umbrella. That has to be the basis of maths because if, you know... then... well, living with or spending time thinking about mathematical ideas. I mean I think people don't generally do that consciously. So I think an investigation sometimes brings this out. But then I think from my point of view umm... do I understand maths better when I do a difficult investigation of problem... yes. It definitely develops... and that leads to that so I don't generally think a lot about mathematical ideas unless there is a focus of it

Int 15

[D] Because they're important for your understanding of mathematics. Because without that, you can't even have any of these, for me.

Yeah, I chose this one because I think well as a teacher I have got to be able to communicate what I understand ... So that's B for me. How procedures work. Yeah it is important.

[G] Well if I take myself when I came on the MDP course, time was actually put aside for doing that. So if time wasn't put aside for doing that I wouldn't be able to understand mathematics by investigating and working out difficult taxing problems. Because without this I wouldn't have even got there, so it was on the MDP actually putting aside the time to be able to do this. Yeah. And yeah it takes time, as I said before, I don’t think I’m anywhere near where I’m going to be in the end, you know. Because I am

I love algebra, you know, I really feel I understand the algebra very well now. Certainly up to the level I teach it, which is grade B to C GCSE. And probably up to Grade A GCSE. I understand it really well and I actually love it and I think it’s a really good way of condensing knowledge and making things fit and I just think it’s really good. Well I didn’t even know... well I say I didn’t know but that’s not fair because I was still teaching GCSE algebra, but I was teaching it like as though it was, this
trying to develop it.

is how you do it. Look you’ve got this expression, this
is… but I didn’t really know what it meant. Now you
have got an equation so what you do is you move this
to the other side of the bracket. But I didn’t know, for
me yeah… so I would trace it again back to the MDP
where I began to see well how that all works out.

[Int 16] [B] being able to explain or communicate
mathematical ideas and thinking to others. Because
that is my job at the end of the day.

The course has been the best thing for me maths-wise
because it’s given me the confidence to have a bash
and it’s given me the confidence when talking to… the
Heads of Department, you know the fact that they now
treat me as one of them.

[Qu 11] It’s about confidence, confidence to have a
bash at something else...

[Int 17] [B] Well as a teacher, the students I've got sometimes
there's quite a bit of what we teach and they can't see
the reason why they've got to learn it because they say
it has no meaning right now. Very often you can
actually... even simple things you can show them why
it is necessary.

Understanding math’s structure patterns and general
results. I like... sometimes I like to see and be able to
show how one area of math’s or similar areas within
the same topic area, how it all comes together. For
example, maybe if you're teaching them how to draw
straight line graphs and then you're teaching them how
to solve simultaneous equations and then you move
onto show them how, you know, the intersection of the
two lines is actually the solution to that. So that type of
thing, I like trying to bring that in where you can.

[Int 18] [B] That for me... you see my concept of a
mathematician would somebody who for their
knowledge they would just want to know for
themselves but for me and because of what I do... You
know, that’s why I need to understand.

[G] Because I enjoy the maths because it’s what I
do... And the stuff I do at work is a desperate attempt
to make people realise that maths is a cool thing to do
and it can be interesting and it’s nothing something to
fear. So… and because I’m doing that I get deeper
and deeper knowledge of it anyway because you read
the books and you talk to people and you live in it.
Because maths is a foreign language isn’t it? And the
best way to learn a foreign language is to go and live

Umm… basic geometry. You know up to circle
proofs and stuff like that. I am really there with that.
I think because I used to be in the building trade and I
used to do joinery so I knew about squares and I knew
about, you know, having to cut angles so I had that
concept anyway at that level of, you know, get the
angle right. So it had to be right.
And I know a few different ways of creating an angle.
I didn’t know how or why it worked I just knew you
in the country and that’s what I do.

could do that. You needed to get a 80 degree, you could do something just with a straight edge and you can get a 90 degree on a bench, something to work on. So I did that practical level. And then to go and find out oh there is a reason why it’s that and the other and there is a proof of that. So the whole measuring and angles and all of that, that was just… Yeah and what I was learning to do was present it so that I could tell you I suppose. But circle proof, I love the circle proof. It’s just so pure isn’t it.

Int 19

[D] because I think that’s when the mathematics becomes strongest. And you know, and taking with us this idea of this networking. The jigsaw proving three dimensions and you get a network and all these ideas are tied onto each other by little strings and you know, the more strings you’ve got, the less chance one little bits going to fall off at the end.

Yeah, I think this being able to see, I think it’s giving a structure to understanding all sorts of processes and thought patterns. I think being able to put those into a mathematical structure is a tool that gives you a wonderful vision in the world. And I think that’s almost a philosophical one. It’s about giving you a vision, a way of seeing the world. So I think that’s a tremendous thing for anybody to take away with them. So it’s a little bit less about you know, what you might call functional maths, I think deep understanding takes you beyond the functional, being able to use a calculator, being able to work out your tax return. That’s a function. I think deeper understanding is really past that

[G] I think time, practice, using it, teaching it… I’m not sure it’s something that comes quickly. I think it takes time. That becomes the second point. It is a journey and you know, and really now I’m finding, like completing the square being the classic part. I knew loads and loads of maths. I didn’t have any problem factorising and equation at any stage as long as I can remember but here was this little bit that sat in the middle that I didn’t know about. I’d never recognised. And I just thought lovely, that fits in. I’m still slotting little bits into that jigsaw.

And I think you know, every time you get that it just deepens the understanding that little bit more

So if you don’t have this third one. If you’re not working with problems. Maybe not difficult and taxing but certainly genuine problems. Things that you can’t answer easily so things that are just outside your… well I know exactly how to do that… Yes a bit more taxing. Different contexts. Something where you have to maybe just re-jig the mathematics that you know or use two different pieces of mathematics that you haven’t linked before and that starts to build those connections…

you’ve got to spend time and you’ve got to live with mathematics so umm… so I think to get that deeper understanding you have got to be doing mathematics for quite a while… You can’t write a syllabus for it… But you need that education, you need all those tools there. But that deeper understanding, I think it does come with just playing with it.

Probability might be quite a reasonable one. Which I know fairly well, I think, you know, I’ve used it quite a lot, I’ve taught it recently. It was the last part of my degree that I did.

Yeah well it’s come from years, and it came first of all probably with a fascination with roulette to be honest.

I’m just trying to think what goes before that because I can remember the fascination with roulette and with the idea of was it possible to beat a roulette table, and I suppose from that, you know, I’ve been a simulation gambler all my life.
| Int 20 | [C,E] Umm… because understanding maths in depth is actually… well understanding how procedures work. I mean it’s actually understanding something. It’s actually being able to do something umm… and again the other one – structure, patterns and general results. You’re actually having to do and understand and think and process. Whereas explaining to others, although you have to have those skills you don’t have to have the depth to be able to explain… you don’t know do you. That’s why I’ve put that third. It’s a close third. Well it’s difficult because to communicate… yeah to communicate maths umm… I can get one of the kids in my class to explain to the kids how to do something, how to communicate it. And they’re just explaining how to do something. They don’t necessarily have to have that great understanding of what they’re doing to be able to tell somebody how to do it. So that’s how I read that one. |
| Int 21 | [E] Umm… just thinking about it for statistical data, you know, your general results first will then help you analyse the data and interpret the data then justify why you’ve got this relationship. So for instance, why have you got this relationship and then usually once you’ve got your results and your data and umm… you might have to communicate to justify… so I’m thinking that way. Okay? | [G,F] Because when you are… when I was learning my maths and I’m still learning now… I’m learning about my teaching now, I live and breathe it almost. Umm… even during… I don’t know, all the time, you’re just sort of coming up with different ideas and you just end up sitting there staring into space but you’re not, you’re really going through different ways. I think solving equations. I love solving equations. Every time no matter how easy it is or how complex it is, not that it gets that complex at GCSE but… even that, it just gives me pleasure. Because it is very… because it works every time. And that’s what I like about maths. Umm… and that’s why I suppose I would choose that over anything. Well I can pick up a book umm… one of the kids books and I can mark them and I can put a tick or I can put a cross. And I can look at it and I can find out where they’ve gone wrong. If I pick up a piece of pros that they’ve written I can’t do that. That’s what I like about maths. So it’s cut and dry. It’s more cut and dried at this level, it’s very cut and dried… Yeah, which the majority of maths is that. And always the thinking isn’t necessarily that and there’s a journey that you have to go through to get there with exploring different ways. But at the end of the day you are looking for an answer really. When I did my degree. Yeah. Back to that. Pages and pages of little things that you’ve done Night after night, yeah. Oh yeah. [laughter] Oh it’s great. I loved it, yeah. And I think in a way it… because I did it over, I did it slower than you would normally if you did a full time degree. I think I probably gained quite a lot more out of it in that respect because there’s no point rushing things is there? It’s a shame that you have to rush things. |
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