Iterative Reasoning and Markets: Three Experiments

Submitted by Chang Yuan Lawrence Choo to the University of Exeter
as a thesis for the degree of
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DEDICATED TO:
My Dearest Wife (Joyce)
Abstract

We present in this thesis three distinct experiments, studying issues in behavioural economics and financial market. In the first paper, we study the level-k reasoning model in an experimental extension of Arad and Rubinstein (2012) “11-20 Money Request Game”. In the second paper, we introduce an experimental design where traders can buy or sell the rights for performing complex decisional task. The design seeks to test the allocative efficiency of markets. In the third paper, we introduce an Arrow-Debreu market where traders have diverse and partial information about the true state of nature. The design seeks to test the Rational Expectations model’s prediction that all relevant information will be aggregated into market prices.
CHAPTER 1

Introduction to the Thesis

This thesis reports on three papers that uses experiments to study issues pertaining to Microeconomic theory, Behavioural Economics and Financial Markets. In the following, I will present an overview and the central findings of each paper. For the convenience of the reader, the appendix for each chapter will documented in chapter 5.

1.1 Overview Chapter 2

This chapter investigates Arad and Rubinstein (2012) assertion that their "11-20 Money Request" Game could be a useful tool in studying level-k reasoning behaviours. The paper was motivated by concerns that the 11-20 game might have been too simple to capture such behaviours. The level-k model partitions the population of players into level-k types and posits a hierarchy of behavioural rules for each type. With experiment data, the proportion of level-k types are usually inferred or estimated from the aggregated choices of subjects. Since the level-k model also belongs to a class of Theory of the Mind models, this presented an additional challenge as to how one might verify if behaviours were indeed captured by the level-k model.

To address the above concerns, we replicated Arad and Rubinstein 11-20 game in our Baseline treatment, and provided two other extension treatments of the 11-20 game. The extensions treatment retain the same mixed-strategy equilibrium as the Baseline treatment, but result in noticeably different level-k behaviours. This allows us to present our hypothesis test that if the level-k model was indeed the dominant explanation to subjects’ behaviours in the 11-20 game, the model should infer the same proportions of level-k types in all treatments considered in our paper, if subjects were randomly recruited from the same population. We estimated the distribution of $L_k$ types in our data, and we find support for the level-k model.

1.2 Overview Chapter 3

This chapter investigates the conventional wisdom that markets, when properly structured, should allocate the “rights” for performing decisional tasks, to those players who are best suited to perform the tasks. To do so, we embed the decisional tasks in the stylised setting of a game, motivated by Littlewood (1953) “Red Hat Puzzle” and introduce in our TRADE treatment a market design, where players are permitted to trade their participation rights to the game.

In the equilibrium analysis of the game, we show that the optimal choices are Pareto optimal but are also due to a process of logical and epistemological reasonings. For these reasons, players sophistication
(e.g., logical thinking, strategic reasoning, problem solving skills) will be integral in them knowing the optimal choices. This implies that the expected payoffs in the game for a player who has sufficient sophistication to know the optimal choices, will be strictly higher than that of a player with insufficient sophistication to know the optimal choices. Therefore, when presented the opportunity to trade participation rights as in the TRADE treatment, the former players should value the participation rights strictly more than the latter players. Given this difference in valuations, markets should therefore result in the allocation of participation rights to the former players, an outcome that is consistent with conventional wisdom.

To test the conventional wisdom, aggregate performances in the TRADE treatment were contrasted with our control treatments where players were not permitted to trade their participation rights. If the conventional wisdom holds, aggregated performances in TRADE should be higher than those in the control treatments.

However, aggregated performances were found to be significantly lower in TRADE. We show that this phenomenon was primarily driven by price “bubbles” in the markets of TRADE which had resulted in subjects who might have otherwise not known the optimal choices in the game, to be purchasing participation rights.

The paper finds little support for the conventional wisdom. The central contribution of this paper is such that markets might not naturally result in the allocation of rights for performing decisional tasks to those players who are best suited to perform the tasks. More significantly, we show that price bubbles could have severe implications on the allocation outcomes of markets.

1.3 Overview Chapter 4

This chapter uses the Arrow-Debreu market in an experimental design to investigate the performances of the Rational Expectations (e.g., Lucas, 1972; Muth, 1961) model in explaining market behaviours. It focuses on a central consequence of the Rational Expectations (RE) model, such that when traders have partial and diverse information about the true state, they will learn from price movements, the information held by traders. This process of learning will continue infinitely till at equilibrium no trader can learn anymore than what they already know from prices. At this stage prices are predicted to reflect the aggregation of all relevant information in the market.

The experiments employs the information distribution structure of the “Red Hat puzzle” (Littlewood, 1953) to endow subjects with private, diverse and partial information about the true state of nature. Thereafter, the design consist of two stages, the Market stage where subjects trade 4 Arrow-Debreu assets and the Prediction stage where the subjects submit their beliefs about the true state.

To evaluate the RE model, we benchmarked its predictions against the Prior-information (PI) model. Both models make equilibrium predictions about the prices in the market and only differ on the extend to which information is aggregated into prices. In contrast to the RE model, the PI model predicts that relevant information will not be aggregated into prices. In addition, the price predictions of both models can be extended to the beliefs of traders ex-post the market.

In explaining prices, the RE model performed as well as the PI in the earlier rounds, and significantly better than the PI in the later rounds. This suggest that with experienced subjects, the RE model does well in predicting behaviours. In explaining beliefs, the RE model performed significantly worse than the PI in the earlier rounds, and as well as the PI in the later rounds.

These findings lead to an unusual outcome whereby prices reflected the true underlying state even when the beliefs of subjects were no different to an outcome whereby all subjects were unaware of the true underlying state. To further investigate, we studied the trading behaviours of subjects, with the presumption that those subjects who had submitted correct beliefs were influential in driving prices to the RE equilibrium. We found support for this in our econometric analysis.

The central contribution of this paper is such that the RE model could perform well in explaining asset prices, even when there exist a fair amount of traders who behave significantly differently from the prescribed behaviours of the RE model.
Is Behaviour in the 11-20 Consistent with the Level-k Model?

2.1 Introduction

Deviations from equilibrium predictions are often documented in the literature of economic and game theory experiments. The challenge in this field is the provision of better explanatory tools. One behavioural explanation posits that players often avoid the circular concepts e.g., fixed-point arguments, embedded in equilibrium outcomes, and instead make use of rule-of-thumb behaviours (Crawford et al., 2013). The level-k model (Nagel, 1995; Stahl and Wilson, 1994, 1995; Costa-Gomes et al., 2001; Camerer et al., 2004) is a leading candidate in this field.

The general level-k model relaxes the assumption of consistent beliefs (Aumann and Brandenburger, 1995) in the equilibrium predictions, partitioning the population of players into $L_k$ types, where $k \in \mathbb{N}$ denote the structured beliefs of each type. The model anchors upon a non-strategic $L_0$ type, who is assumed to follow some exogenously defined behavioural specification. Such specification is analogous to players’ instinctive reaction in the game and is often taken to be the uniform randomisation over all strategies. Higher $L_k$ types ($k > 0$), are differentiated by their beliefs as to the types of all other players and best-respond to these beliefs via iterative thought-experiments. For example, if the higher types believe that all other players are exactly one type below, then a $L_1$ type will best-respond to the $L_0$ types’ behaviour, a $L_2$ type to a $L_1$ behaviour, a $L_3$ to a $L_2$ and so forth. The level-k model is simple and intuitive, however applications to wider economic settings first requires some prior on the plausible proportions of types and the beliefs of each higher types. This has led to a growing body of literature that examined the level-k model in laboratory experiments and in the field (see Bosch-Domènec et al., 2002; Brown et al., 2013; Östling et al., 2011). However, such investigations naturally leads to concerns as to whether (a) The $L_0$ types’s specification is salient amongst the pool of subjects, (b) The beliefs of higher types are correctly specified and (c) Best responding behaviours driven by iterative thought-experiments are natural. If this is not the case, then any estimated or derived proportions of $L_k$ types may likely be misleading.

To address these concerns, Arad and Rubinstein (2012) - henceforth known as AR - proposed the “11-20”

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1The numeral $k$ is sometimes taken to be the steps of iterative reasoning or “degrees of rationality” (Kawagoe and Takizawa, 2012) that a player is able to perform. Whilst not entirely different, we are of the opinion that $k$ simply denotes the characterisation of a players beliefs. Whether this beliefs are necessarily representative of some cognitive reasoning limits are irrelevant.

2Whether the uniform randomisation is the appropriate specification of the $L_0$ type’s behavioural is by itself a debate. Such specification is attractive since it is context free and easily portable to a variety of games. However, in some games, this also meant that the $L_0$ type player assigns equal weights to strategies that are payoff dominant and dominated.

3In general, the $L_0$ types’ behavioural specification could be extended to include any plausible non-strategic behaviours. For example, Costa-Gomes, Crawford and Broseta (2001) introduced a level-k model with the non-strategic Altruistic, Pessimistic and Optimistic types. Broadly, the $L_0$ type is introduce to model any behavioural specification that does not involve iterative thought-experiments.
game to study the level-k model. The game involves two players simultaneously choosing an integer between 20 to 11, which corresponds to the equivalent amount of payoffs that they would each receive with certainty. In addition, a player receives a bonus payoff of 20, if his chosen integer is 1 less than the other player’s. The game has no pure-strategy equilibrium, but a mixed-strategy equilibrium that assigns positive probabilities to the strategies 20 to 15.

AR found subjects’ behaviour to be significantly different from the mixed-strategy equilibrium and proposed the level-k model to explain such deviations. Their explanation anchors upon the instinctive reaction in the game, to choose 20. In doing so, the \( L_1 \) type would best-respond with 19, the \( L_2 \) type with 18, the \( L_3 \) type with 17 and so forth.\(^4\) Given this, 74% of subjects were identified to be types \( L_1 \)-\( L_3 \), with type \( L_3 \) being the modal. The 11-20 game was subsequently adapted by Lindner and Sutter (2013) to study decision making under time pressure, Alaoui and Penta (2013) to study endogenous iterative reasoning, Dittrich and Leipold (2014) to study gender differences in strategic reasoning and Sousa, Hollard and Terracol (2012) to study steps of reasonings with chess players.

However, does behaviour in the 11-20 experiments necessarily correspond to those predicted by the level-k model? In the same paper, AR provided two other extensions of the 11-20 game, with the same mixed-strategy equilibrium and level-k predictions. However, as also noted by Georganas, Healy and Weber (2014), behaviours in these extensions were significantly different from the original game.\(^5\) Alternatively, were behaviours necessarily inconsistent with some other statistical distortion of the mixed-strategy equilibrium such as in the Quantal Response Equilibrium (McKelvey and Palfrey, 1995, 1996)?\(^6\) Ultimately with experimental data, there could be multiple competing explanations, and the research question in this paper is whether the level-k model is indeed the dominant explanation.

Denoting the 11-20 game as the Baseline game, we propose the following two simple extensions, the Medium and Extreme games. In the Medium game, players choose from following strategies 20.00, 19.50, 19.00,..., 11.00, which they are certain to receive in equivalent payoffs. The bonus of 20 is only awarded if the player’s strategy is 0.50 or 1 less than the other player. In the Extreme game, players choose from the strategies 20.00, 19.75, 19.50, 19.25, 19.00,..., 11.00, which they are again certain to receive in equivalent payoffs. However, the bonus of 20 is now only awarded if the player’s strategy is 0.25, 0.50, 0.75 or 1 less than the other player.

All games - Baseline, Medium and Extreme - share the same decision-making problem. In addition, the games also have equivalent mixed-strategy equilibrium distributions (see Table 2.1). For example, the strategies \{20.00\} in the Baseline game, \{20.00, 19.50\} in the Medium game and \{20.00, 19.75, 19.50, 19.25\} in the Extreme game are all predicted to be chosen with 5% probability. Similarly, the strategies \{19.00\}, \{19.00, 18.50\} and \{19.00, 18.75, 18.50, 18.25\}, in the Baseline, Medium and Extreme games respectively, are predicted by the mixed-strategy equilibrium to be chosen with 10% probability.

When studied with the level-k model, the behavioural specification for the \( L_0 \) type is again identical across all games, to choose 20.00.\(^7\) The \( L_1 \) type will best-respond with 19.00, 19.50 and 19.75, in the Baseline, Medium and Extreme games respectively. The \( L_2 \) type with 18.00, 19.00 and 19.50, the \( L_3 \) type with 17.00, 18.50, and 19.25, and so forth in the respective game.\(^8\)

To summarise, we provide in this paper three games with equivalent mixed-strategy equilibriums but different predicted level-k behaviours. As such, if the level-k model is the dominant explanation to players’

\(^4\)A potential issue is that the design induces cycles in level-k types’ behaviours, where the number 20 will be chosen by \( L_0, L_{10}, L_{20},... \) types. The authors side-step this issue by assuming that numbers 11-16 are rarely associated with higher \( L_k \) types but more likely due to random guess.

\(^5\)In their paper, AR also considered two other extension of the “11-20” game, the costless iteration and cycle versions. Both extensions sought to investigate the saliency of the \( L_0 \) type’s behaviour assuming the level-k model.

\(^6\)It is difficult to see how the rejection of the mixed-strategy equilibrium should immediately imply that behaviours had corresponded to the level-k model, given that the former is a \textit{ex-ante} description of players’ behaviours whilst the latter is an \textit{ex-post} description.

\(^7\)Choosing 20.00 is still the instinctive action in the games as it corresponds to the highest possible payoffs without any considerations for the behaviours of others. One might disagree on this and perhaps take the opinion that the \( L_0 \) type’s behaviour in the Medium and Extreme games is to uniform randomise between all strategies. However, this would imply that \( L_0 \) type’s choice in the Baseline game can no longer be 20.00.

\(^8\)It has been suggested that \( L_4 \) types in the Medium game could either choose 19.50 or 19.50. Whilst this might also be intuitive, it is inconsistent with the best-responding structure of the level-k model.
behaviours in the respective games, we should expect the inferred or estimated proportions of $L_k$ types to be consistent between the differentiated games if players were randomly recruited from the same population.

We believe that our study complements the recent work of Goeree, Louis and Zhang (2013), who perhaps motivated by the same concerns, provided two other extensions of the Baseline game, with different mixed-strategy equilibriums but equivalent predicted level-k behaviours. The authors found the best-responding behaviours of the level-k model in the Baseline game to be “untenable” and showed that the data could be better explained when “common knowledge noise” parameter such as in the Noisy Introspection model (Goeree and Holt, 2004) is introduced to subjects’ behaviours.

The rest of this paper is organised as followed. Section 2.2 describes our experimental procedures, Section 2.3 the overview of our experimental results, Section 2.4 the test of the level-k model and Section 2.5 concludes.

2.2 Experiment Procedure

Four classroom experimental sessions were conducted at the University of Exeter, over two cohorts of Intermediate Microeconomics students. The subjects were mostly economics majors and with no formal training in game theory.\footnote{We choose to conduct the experiments in classroom settings for 2 reasons. The first was for consistency with Arad and Rubinstein (2012) experiments, which were also conducted in the classroom. The second was to recruit a wide number of subjects from the same population pool.} We denote each session by the game which the subjects were enrolled into - Baseline(B), Medium(M) and Extreme(E), followed by the cohort which they were recruited from. For example, session B2012 refers to the Baseline game conducted with subjects from cohort 2012. All sessions were conducted during the first lecture class of the course (approximately 250-300 students in each class) and subjects were informed that their participation was voluntary.\footnote{Subjects were also informed that communication was prohibited during the experimenter session. Although most subjects were observed to have adhered to the experimental rules, we cannot exclude the possibility that some subjects might have discreetly communicated amongst themselves.}

In each cohort, the layout of the lecture class had consisted of three separated seated columns. With cohort 2012 and 2013, subjects in the centre seated column received the instructions for sessions B2012 and M2013 respectively. Subjects in the two other side columns received instructions for sessions M2012 and E2013 respectively. The instructions were as followed:

**Baseline (B) Game:** You and another player will simultaneously request an amount of payoff from the set \{2000, 1900, 1800, 1700, ..., 1100\} denoted in ECU. Each player will receive his chosen amount. In addition, a player will receive a bonus of 2000 if his request amount is 100 ECU less than the other player.

**Medium (M) Game:** You and another player will simultaneously request an amount of payoff from the set \{2000, 1950, 1900, 1850, ..., 1100\} denoted in ECU. Each player will receive his chosen amount. In addition, a player will receive a bonus of 2000 if his request amount is (a) 50 ECU or (b) 100 ECU less than the other player.

**Extreme (E) Game:** You and another player will simultaneously request an amount of payoff from the set \{2000, 1975, 1950, 1925, 1900, ..., 1100\} denoted in ECU. Each player will receive his chosen amount. In addition, a player will receive a bonus of 2000 if his request amount is (a) 25 ECU, (b) 50 ECU, (c) 75 ECU or (d) 100 ECU less than the other player.

Subjects had to circle their choice on a table consisting of all the relevant request amounts. In addition, subjects were to include their contact details and a brief feedback of their behaviour.\footnote{Subjects contact information was important for the administration of payments.} The sessions were completed within 15 minutes and the instruction sheets were thereafter collect by the experimenters. In each cohort, ten pairs of subjects were randomly selected for cash payment (they were privately contacted...
Table 2.1: Summary of Observed Strategy Frequencies and Mixed-Strategy Equilibrium

<table>
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<tbody>
<tr>
<td>2000-1925</td>
<td>.050</td>
<td>.034</td>
<td>.120</td>
<td>.110</td>
<td>.132</td>
</tr>
<tr>
<td>1900-1825</td>
<td>.100</td>
<td>.231</td>
<td>.359</td>
<td>.374</td>
<td>.374</td>
</tr>
<tr>
<td>1800-1725</td>
<td>.150</td>
<td>.265</td>
<td>.188</td>
<td>.154</td>
<td>.088</td>
</tr>
<tr>
<td>1700-1625</td>
<td>.200</td>
<td>.231</td>
<td>.077</td>
<td>.088</td>
<td>.066</td>
</tr>
<tr>
<td>1600-1525</td>
<td>.250</td>
<td>.085</td>
<td>.077</td>
<td>.066</td>
<td>.055</td>
</tr>
<tr>
<td>1500-1425</td>
<td>.250</td>
<td>.026</td>
<td>.085</td>
<td>.044</td>
<td>.121</td>
</tr>
<tr>
<td>1400-1325</td>
<td>.000</td>
<td>.077</td>
<td>.034</td>
<td>.066</td>
<td>.088</td>
</tr>
<tr>
<td>1300-1225</td>
<td>.000</td>
<td>.026</td>
<td>.017</td>
<td>.033</td>
<td>.033</td>
</tr>
<tr>
<td>1200-1125</td>
<td>.000</td>
<td>.009</td>
<td>.009</td>
<td>.011</td>
<td>.011</td>
</tr>
<tr>
<td>1100</td>
<td>.000</td>
<td>.017</td>
<td>.034</td>
<td>.055</td>
<td>.033</td>
</tr>
<tr>
<td>N</td>
<td>(117)</td>
<td>(117)</td>
<td>(91)</td>
<td>(91)</td>
<td></td>
</tr>
</tbody>
</table>

via email) at the exchange rate of 100 ECU to 1 British pound. A total of 130, 140, 114 and 94 subjects participated in sessions B2012, M2012, M2013 and E2013, respectively - see Appendix for overview of data from the respective sessions.

We choose to split the sessions by the seated columns for ease of instructions distribution and to avoid any confusion created by subjects seeing the other instructions. However, the same experimental procedure induce concerns that there might be some natural differences in behaviours due to the seated positions of subjects. For example, anecdotal observations suggest that subjects who were more attentive in the classroom are often more likely to occupy the frontal rows of the centre column.

To address such concerns, we used behaviour of subjects in another game to control for the subject sample pool in each session. The respective sessions were immediately followed up by the Guessing Game (Nagel, 1995). Here each player chooses a number between 0 to 100 and a fixed prized is awarded to the player whose chosen number is closest to 2/3 of the average. Subjects in each cohort competed against each other for a fixed prize of 50 British pound, were informed that the Guessing Game was a different experiment from the previous sessions and that their participation was voluntary. The Guessing Game instructions sheets were distributed and collected within 20 minutes. A total of 274 and 206 subjects participated in the Guessing Game for cohorts 2012 and 2013 respectively - see Appendix for guessing game data from the respective cohort.

Given this control, our data was prepared as followed. We first excluded any session (B2012, M2012, M2013 and E2013), where subjects had not participated in the guessing game. Thereafter, in each cohort, in randomly picked two equal sample from each sessions, such that the distribution of guessing game numbers by the respective sessions of the same cohort were not significantly difference. This resulted in 117 and 91 observations in each session of cohort 2012 and 2013 respectively.

2.3 Overview of Results

The sessions’ results are summarised in Table 2.1. The first and second columns refer to the strategies and mixed-strategy equilibrium predictions respectively, whilst the third column to sixth columns refer to the observed frequency of strategies in the respective session. For completeness, we also include on Figure 2.1 the relevant plots of the observed frequencies and mixed-strategy equilibrium.

As an empirical warm-up, we first investigated if subjects’ behaviours were consistent with the mixed-strategy equilibrium. Here, the two-sided Fisher’s exact test finds all sessions’ data to be significantly different (\( \rho < 0.001 \) for all comparisons). This leads us to our first result:

---

12 We employed the Guessing Game since it was one the most frequently studied game in the level-k literature.

13 Kolmogorov-Smirnov test reports a p-value of 0.242 (0.453) in cohort 2012 (2013) for comparisons of the guessing game numbers between the relevant sessions.

14 For the purposes of our analysis, we choose the Fisher’s Exact test over the conventional \( r \times c \) contingency table chi-square test, since the test statistics in the latter test requires each cell to have an expected value of at least 1 and that 20% of the cells to have an expected value of at least 5 (Sheskin, 2003).
Result 1: Subjects behaviour in B2012, M2012, M2013 and E2013 were significantly different from the mixed-strategy equilibrium.

A prominent difference pertains to the strategies 1600-1425, which although predicted by the mixed-strategy equilibrium to be chosen by 50% of the subjects in each session, were only observed to be chosen by no more than 18% in any session.\(^{15}\)

2.3.1 Are Behaviours consistent across identical Games?

Our experimental design allows for two pairwise comparisons (B2012 vs AR results; M2012 vs. M2013) to investigate if there might be some coherent “structure” in subjects behaviours. To study this, we present on Figures 2.2 and 2.3 the plot of the observed frequencies in the relevant comparisons. Comparing between sessions of the same game, the B2012 session data was not found to be significantly different from AR’s results (two-sided Fisher \(p = 0.323\)).\(^{16}\) Similarly, the M2012 and M2013 sessions’ data were not found to be significantly different (two-sided Fisher \(p = 0.483\)).

Result 2: Behaviours in the B2012 were not found to be significantly different to those in Arad and Rubinstein (2012) experiments and those in M2012 were not found to be significantly different in M2013.

This result suggest there to be some coherent behaviour in the decision-making process of subjects and we know from Result 1, that this process is unlikely to be consistent with the mixed-strategy equilibrium. The question therefore is whether this decision-making process might be consistent with the level-k model as proposed by AR.

---

\(^{15}\)The mixed-strategy equilibrium was derived upon the assumption that players were risk-neutral. If players were instead risk-averse, the equilibrium predictions will higher probabilities to the lower strategies in each game, which is again inconsistent with the observed behaviours.

\(^{16}\)This finding was also shared in replications of the “11-20” game by Lindner and Sutter (2013) and Goeree et al. (2013).
Figure 2.2: Observed Strategy Frequencies: B2012 vs. AR(2012)

Figure 2.3: Observed Strategy Frequencies: M2012 vs M2013
### 2.4 Are Behaviours consistent with the Level-k Model?

To investigate whether behaviours in the respective games are consistent with the level-k model, we report on Table 2.2 the inferred proportions of $L_k$ types (truncated at the $L_8$ type) in the respective sessions.

To test our hypothesis that the level-k model was the dominant explanation to subjects’ behaviours, comparisons were made between sessions of the same cohort. In cohort 2012, the inferred proportions of $L_k$ types were found to be significantly different (two-sided Fisher $\rho < 0.001$). In session B2012, 73% of subjects were classified as types $L_1 - L_3$ whilst the same classification only pertains to 41% of subjects in M2012. In cohort 2013, the inferred proportions of $L_k$ types were again found to be significantly different (two-sided Fisher $\rho < 0.001$). Here, whilst 40% of subjects in session M2013 were classified as types $L_1 - L_3$, only 7% of subjects in session E2013 fall under the same classification. Furthermore, a quarter of all subjects in session E2013 had chosen the amount 1900, which corresponds to the $L_4$ type.

This result could either imply that behaviours in the respective sessions (and consequently the "11-20" game) were inconsistent with the level-k model or that the behaviours were consistent with the level-k model but small changes to the game results in it's own distribution of types.\(^\text{17}\)

To avoid “throwing the baby out with the bathwater” we decided to study the ex-post experiment feedbacks for insights to subjects’ decision-making process. Here 8.5%, 32%, 38% and 30% of the feedbacks from sessions B2012, M2012, M2013 and E2013 respectively were either empty or clearly corresponded to random behaviours.\(^\text{18}\) With the remaining feedbacks, the following two observations were made.

(i) **Iterative thought-experiments anchoring on 2000.** Most subjects in session B2012 described their behaviours as a consequence of an iterative process from 2000 (“I think that a lot of people will choose 1900 because it is 100 lower than the maximum amount. So I have gone for 1800, which is one step lower than that”). Similar descriptions are also observed in session M2012 and M2013 (“I hope that the other person will think that I have ignore the bonus and thus pick 1950. I therefore picked 1900”). In session E2013, the descriptions are less straight forward, but nevertheless involve the discussion of the choice 2000.

(ii) **Subjects expect other subjects to best-respond stochastically.** This is a prominent observation in sessions M2012, M2013 and E2013 - to some extend in session B2012. For example, a typical feedback in E2013 session is as followed “Many people will expect others to choose 2000 and hence themselves choose 1975, 1950, 1925 or 1900. I therefore choose 1875 to get the bonus”.

\(^{17}\)One may disagree with our hypothesis test. More specifically, why should the level-k model imply consistent proportions of $L_k$ types between sessions of the same cohort? In our view, this alternative is merited if the respective sessions involved games that were intrinsically different. However, in the setting of our experiment, this alternative propounds that small modifications to the game results in its own unique proportions of $L_k$ types. Whilst such outcome cannot be exclude, we find it unhelpful, especially if the ambitions of such research is its applicability to wider economic settings.

\(^{18}\)The feedbacks were independently evaluated by a graduate student. Random behaviours were often associated with the following feedbacks “I choose randomly” or “I picked my favourite number”.

---

**Table 2.2: Inferred proportion of $L_k$ types**

<table>
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<tbody>
<tr>
<td>$L_0$</td>
<td>0.034</td>
<td>0.068</td>
<td>0.088</td>
<td>0.066</td>
</tr>
<tr>
<td>$L_1$</td>
<td>0.231</td>
<td>0.051</td>
<td>0.022</td>
<td>0.022</td>
</tr>
<tr>
<td>$L_2$</td>
<td>0.265</td>
<td>0.162</td>
<td>0.209</td>
<td>0.022</td>
</tr>
<tr>
<td>$L_3$</td>
<td>0.231</td>
<td>0.197</td>
<td>0.165</td>
<td>0.222</td>
</tr>
<tr>
<td>$L_4$</td>
<td>0.085</td>
<td>0.128</td>
<td>0.099</td>
<td>0.253</td>
</tr>
<tr>
<td>$L_5$</td>
<td>0.026</td>
<td>0.060</td>
<td>0.055</td>
<td>0.088</td>
</tr>
<tr>
<td>$L_6$</td>
<td>0.077</td>
<td>0.051</td>
<td>0.044</td>
<td>0.011</td>
</tr>
<tr>
<td>$L_7$</td>
<td>0.026</td>
<td>0.026</td>
<td>0.044</td>
<td>0.022</td>
</tr>
<tr>
<td>$\geq L_8$</td>
<td>0.026</td>
<td>0.256</td>
<td>0.275</td>
<td>0.495</td>
</tr>
</tbody>
</table>
Subjects’ feedback seem to hint at some level-k style of reasoning. The question therefore is how one might find support for this in the subjects’ behaviours.

One alternative, is to assume that the $L_0$ type uniformly randomises amongst all strategies. In this case, the $L_1$ type will best-respond with 1900 in all games. The $L_2$ type will best-respond with 1800, 1850 and 1875 in the Baseline, Medium and Extreme games respectively. The $L_3$ type with 1700, 1800 and 1850 in the corresponding games. In cohort 2012, this adaption now suggest that types $L_1$-$L_3$ are most frequently observed in the respective sessions, with $L_2$ being the modal - see Figure 2.1. Furthermore with cohort 2013, this adaption provides some partial explanations to the spike with the strategy 1900, which implies that type $L_1$ are most frequently observed in sessions M2013 and E2013. Corresponding types $L_2$ were also frequently observed in both sessions of cohort 2013. However, this adaptation cannot accord for the spike at 1800 ($L_5$ types prediction) in E2013, since it would require a corresponding spike at 1700 for session M2013.

Another alternative is the implications of the “treatments effects” e.g., the bonus condition, the discretisation of the strategy space, upon the behaviours of each type. For example, if a $L_1$ type in the Medium game believes that the $L_0$ type will choose 2000, he could respond with 1950 or give up 50 and respond with 1900, hedging the risk that the $L_0$ type would “tremble”. This alternative therefore suggest that the $L_1$ type will choose strategies 1975-1900, the $L_2$ type 1875-1800, the $L_3$ type 1775-1700 and so forth, in all games. We report on Table 2.3 the inferred proportions of types, accounting for the aforementioned treatment effects. However, the inferred proportions of $L_k$ types were still found to be significantly different between sessions of cohort 2012 (Fisher exact $\rho = 0.08$) and 2013 (Fisher exact $\rho = 0.06$). Furthermore, thought the modal type in sessions of cohort 2012 are now similar ($L_2$ type), the same phenomenon is not observed with sessions of cohort 2013.

These suggest that the alternative explanations also do not provide adequate explanations to the inferred proportion of types if subjects were behaviours were captured by the level-k model. However, the above discussions are based upon the assumption that the higher $L_k$ type players perfectly best-respond to their beliefs. Most studies of the level-k model (e.g., Stahl and Wilson, 1995; Crawford and Iriberri, 2007; Costa-Gomes and Crawford, 2006) allow for higher types to best-respond stochastically, with a logistic functional form (McFadden, 1976) best-responding function. We hence investigate if support for the level-k model could be achieve with the simple adaption of such a best-responding function.

### 2.4.1 Introducing the Logistic Best-Responding Function

To accomplish the above, we relaxed the best-responding assumption of the level-k model with the introduction of a common knowledge noise $\lambda \geq 0$ parameter. This allows us to consider two types of level-k models, the Logit Level-k (SK) model and the Cognitive Hierarchy (CH) model (Camerer et al., 2004).
The SK and CH models consider a hierarchical of $L_k$ types but differ on their assumed beliefs for each higher types. In applications to our Baseline, Medium and Extreme games, both models involve $i = 1, 2, \ldots$ players, each simultaneously choosing a strategy $a_i \in A$. Denote $\pi_i(a_i, a_{-i}) > 0$ as the payoff to player $i$ for choosing strategy $a_i$ if the other player chooses $a_{-i}$. Both models anchor upon a non-strategic $L_0$ type who is assumed to always choose the strategy $2000$. For any higher $L_k$ type player $i$, let $b_k^i(g) \in [0, 1]$ denote the proportion of $L_g$ type players he believes to exist in the population. We assume that $b_k^i(g) = 0$ for all $g \geq k$, implying that players ignore the possibility that other players might be the same or higher types than himself.\(^{21}\) The SK model assumes that each higher $L_k$ type believes everyone else to be exactly one type below, resulting in beliefs $b_k^i(g) = 1$ if and only if $g = k - 1$ or otherwise 0.

On the other hand, the CH model assumes that each higher $L_k$ type believes everyone else to be a mixture of lower types, distributed accordingly to a normalised Poisson distribution. More specifically, for any population of players, let $f(k) \in [0, 1]$ denote the true proportions of $L_k$ types. The CH model therefore assumes that $f(0), f(1), \ldots, f(k), \ldots$ follows a Poisson distribution with the mean and variance $\tau$, where $f(k) = \tau^k \exp(-\tau)/k!$. The CH model also makes a simplifying assumption that each higher type knows the true relative proportions of lower types, resulting in beliefs

$$b_k^i(g) = \frac{f(g|\tau)}{\sum_{h=0}^{k-1} f(h|\tau)} \quad \forall k > 0, \ g < k$$

If the true proportions of types are clustered around the lower types, then an interesting consequence of the CH model relative to the SK model, is that the beliefs of higher types in the former model become more precise as $k$ increases, whereas the beliefs in latter becomes less precise.

Let $p^k(a_i) \geq 0$ denote the probability of a higher type player $i$ choosing strategy $a_i \in A$

$$p^k(a_i) = \frac{\exp(\lambda \pi_i(a_i, \cdot))}{\sum_{a_i' \in A} \exp(\lambda \pi_i(a_i', \cdot))} \quad \forall k > 0$$

where $\pi_i(a_i, \cdot) = \sum_{a_{-i} \in A} \pi_i(a_i, a_{-i}) \left\{ \sum_{g=0}^{k-1} b_k^i(g) \cdot p^0(a_{-i}) \right\}$ denotes the expected payoff for a higher $L_k$ type player $i$ with choosing strategy $a_i$.\(^{22,23}\) As $\lambda \rightarrow \infty$, each higher type places more weights to the strategy that accords to him the highest payoff. Likewise as $\lambda \rightarrow 0$, each higher type uniformly randomises across all strategies.

With data, the SK and CH models will be fitted through econometric methods. The econometric results make two predictions, the common noise $\lambda$ and the proportions of $L_k$ types. We are primarily interested in the latter predictions. The estimation of the SK model first requires some prior arbitrary specification of $L_K = 2, 3, 4, \ldots$, the highest type one believes to exist in the data. Thereafter, the proportions of types, $L_0$ through to $L_K$, and the noise parameter $\lambda$ are estimated from the data (this results in $K + 1$ free parameters). Since the SK model does not impose any parametric restrictions on the distribution of types, it presents one with certain amount of flexibility in increasing the statistic fit by considering different $L_K$. Estimation of the CH model usually involves setting an arbitrary high $L_K$. Thereafter, the parameters $\tau$ and $\lambda$ are estimated from the data given the restriction that $1 - \sum_{k=0}^{K} f(k) < \epsilon$. One should note that given the parametric assumptions on the distribution of types, the CH model is slightly more restrictive than the SK model. However, is such

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\(^{21}\)Solving a model where $b_k^i(g) \neq 0$ for $g = k$ might also be more complex and involve finding a fixed point at each step of the iterative thought-experiments (Camerer et al., 2004).

\(^{22}\)One could also model the choice probability function with the normalised power function

$$p^k(a_i) = \frac{(\pi_i(a_i, \cdot))^{\lambda}}{\sum_{a_i' \in A} (\pi_i(a_i', \cdot))^{\lambda}} \quad \forall k > 0$$

as in Östling et al. (2011), and the results will most probability be identical. We decided upon the Logistic functional form for natural comparisons against the QRE model.

\(^{23}\)An alternative specification is to assume that the higher $L_k$ types will uniformly randomise with probability $\varepsilon_k \in [0, 1]$ or choose the action which accords the highest expected payoff with probability $1 - \varepsilon$ as in Costa-Gomes et al. (2001). This alternative may not be immediately applicable to the CH model. Since our objective is to restrict any behavioural differences between the SK and CH models to assumptions on higher types’ beliefs, we choose not to adapt this alternative specification.
Strategies | B2012 | M2012
---|---|---
| Obs. | EQ | QRE | SK | CH | Obs. | EQ | QRE | SK | CH |
2000-1950 | .034 | .050 | .128 | .055 | .113 | .120 | .050 | .220 | .146 | .173 |
1900-1850 | .231 | .100 | .197 | .230 | .190 | .359 | .100 | .268 | .341 | .303 |
1800-1750 | .265 | .150 | .220 | .264 | .268 | .188 | .150 | .187 | .179 | .209 |
1700-1650 | .231 | .200 | .188 | .230 | .216 | .077 | .200 | .111 | .086 | .088 |
1600-1550 | .085 | .250 | .120 | .085 | .082 | .077 | .250 | .072 | .065 | .061 |
1500-1450 | .026 | - | .022 | .015 | .025 | .017 | - | .027 | .038 | .034 |
1400-1350 | .077 | - | .034 | .075 | .031 | .009 | - | .020 | .033 | .029 |
1300-1250 | .026 | - | .022 | .016 | .027 | .038 | - | .034 | .034 | .034 |
1200-1150 | .009 | - | .016 | .012 | .020 | .033 | - | .034 | .034 | .034 |
1100 | .017 | - | .012 | .010 | .016 | .034 | - | .008 | .014 | .012 |
λ | .0028 | .0020 | .0021 | .0027 | .0015 | .0017 | .0020 | .038 | .034 | .034 |
τ | 4.09 | 3.90 | - | 401.67 | 228.42 | 217.70 | 225.10 | 442.93 | 308.61 | 302.68 |
-Λ | 4.09 | 3.90 | - | 401.67 | 228.42 | 217.70 | 225.10 | 442.93 | 308.61 | 302.68 |

Table 2.4: Cohort 2012: Observed and Predicted Frequency of Strategies by the Mixed-Strategy Equilibrium, QRE, SK and CH

 restriction tantamount to a significantly worst fit?

### 2.4.2 Econometric Results (SK and CH models)

The estimates from the SK and CH models were derived through maximum likelihood estimation (see Appendix for discussion of MLE procedures in the level-k models). To avoid overfitting the SK model, we first estimated B2012 with highest type $L_k = 3, 4, 5, 6, 7, 8, 9$. At the 1% significance level, the likelihood ratio test prefers the estimates where $L_k = 6$. The remaining sessions were hence estimated with $L_k = 6$. The CH model was estimated by setting $L_k = 16$. However, for the purposes of this presentation, we will only report the estimated proportions of types for $0 \leq k \leq 6$. To do so, we normalised the proportions of types in the same approach demonstrated by Kawagoe and Takizawa (2012).

We report on Tables 2.4 and 2.5 the estimation results for sessions in cohort 2012 and 2013 respectively. We also included the mixed-strategy equilibrium (EQ) and QRE estimates for comparisons. Each table comprises of three panels. The top panel depicts the observed and the predicted frequency of strategies by the mixed-strategy equilibrium, QRE, SK and CH estimates. The middle panel reports the test statistics of Vuong (1989) likelihood ratio test - to be discussed later. The bottom panel reports the proportions of $L_k$ types as estimated by the SK and CH models. We also fitted on Figure 2.4, the predicted frequency of strategies by the QRE (Short-Dash lines), SK (Solid Lines) and CH (Long-Dash Lines) estimates.

As an empirical warm-up, we will first investigate if the SK and CH models are able to explain the data "sufficiently better" than the equilibrium alternatives. To do this, we will employ Vuong (1989) likelihood ratio

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24 The CH model's estimated proportions of $L_k$ types were derived by $f(k)/\sum_{h=0}^{6} f(h)$, where $f(k) = \tau^k \exp(-\tau)/k!$. If the SK and CH models are only able to explain the data as well as the equilibrium alternatives, then there might be no "value" in departing from the rationality assumptions embedded in the equilibrium models.
Table 2.5: Cohort 2013: Observed and Predicted Frequency of Strategies by the Mixed-Strategy Equilibrium, QRE, SK and CH

Figure 2.4: Observed and Predicted Frequency of Strategies: QRE vs SK vs CH
For the ease of interpretation, the test statistics are presented in the following manner: With pairwise comparisons, the model with more (less) favourable log-likelihood value will be positioned in the row (column) - this ensures that the test statistics must be positive. This allows us to conduct a simple one-sided test to evaluate if the row model fits the data significantly better than the column model. In the following, we shall use the terms “out-performed” and “tied” to denote the outcome of the likelihood ratio test between two models. For example, A is said to have out-performed B, if the likelihood ratio test finds A significantly closer to the true model. Similarly, A is said to have tied with B, if one is unable to reject the null hypothesis.

In all comparisons, the QRE, SK and CH were found to have out-performed the mixed-strategy equilibrium. This should not be surprising since the former three were econometrically fitted onto the data. The following discussions will hence focus on the former three.

**B2012:** The SK and CH were found to have out-performed the QRE, but the SK was also found to have out-performed the CH. From top left box of Figure 2.4, these findings become more apparent. The SK model tracks the strategies 2000-1400 much better than the other two models. However, this could also be driven by the fact that such strategies largely correspond to the behaviour profiles of types $L_0 - L_6$, which were by construct free parameters in the SK model. Although the predicted strategies of the QRE and CH were observed to correctly peak at 1800, the QRE was observed to be under-predicting (over-predicting) the strategies 1800 and 1700 (1600 and 1500) relative to the CH model.

**M2012:** The SK and CH were found to have out-performed the QRE and tied with each other. From top right box of Figure 2.4, the SK and CH predicted strategies were observed to correctly peak at 1850 whilst the QRE, at 1800. Furthermore, the QRE under-predicts the three most frequent strategies (1750, 1800 and 1850) relative to the SK and CH.

**M2013:** The SK and CH were found to have out-performed the QRE and tied with each other. From bottom left box of Figure 2.4, the SK was the only model that could account for the sharp drop in strategy frequencies from 2000 to 1950. However, whilst the QRE and CH were observed to correctly peak at 1900, the SK instead peaks at 1850. The QRE was observed to be under-predicting the three most frequent strategies (1800, 1850 and 1900) relative to the CH.

**E2013:** The SK and CH were found to have out-performed the QRE and tied with each other. From bottom right box of Figure 2.4, the performance of the SK and CH over the QRE is obvious. The QRE’s fit was observed to be a small “hump”, with predicted frequencies of around 4% at each strategy 2000-1750 and 3-1% and each strategy 1725-1100. The data exhibits a sharp peak at 1900 (25%) and surprising only CH was able to track this peak, though nearly 2 times lower. The SK model was again found to peak one strategy away from the true peak, at 1875.

We were concerned that the SK model’s statistical fit in all sessions were primarily driven by the $L_K = 6$ specification and hence re-estimated the data with the assumption that $L_K = 3$ - the SK3 Estimates. Employing the same likelihood test, the SK3 estimates were still found to have out-performed the QRE and mixed-strategy equilibrium in all comparisons. However, the SK3 estimates were now found to have tied with the CH in all comparisons. This suggests that the superior performance of the SK estimates over the QRE or mixed-strategy equilibrium cannot be simply attributed to the $L_K$ specification. This also suggests that on average, the CH might have fitted the data as well as the SK.

**Result 3a:** The QRE, SK and CH were found to have fitted the respective sessions’ data significantly better than the mixed-strategy equilibrium.

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26 The test assumes that there exist a true model, and with pairwise comparisons, evaluates which of two models is “closer” to the true model. The Null hypothesis is for both models to be equally close and the test provisions for two one-sided Alternative hypotheses, that one of the two models is significantly closer.

27 The Vuong (1989) test suffers from some logical issues if two fundamentally different models i.e. Rational expectation and Bounded Rationality Models, were found to be equally close to the true model. Without loss of generality, the Null hypothesis can be interpreted as the outcome where we are unable to distinguish between the statistical fit of both models.

28 Corrections for degrees of freedom are often employ in applications of Vuong (1989) likelihood ratio test, penalising estimates with more parameters. Whilst such approach might be sensible with nested models, we do not agree with such premises for the purposes of our study since the models are based on different assumptions of players’ behaviours.
Result 3b: The SK and CH were found to have fitted the respective sessions’ data significantly better than the QRE, but on average, as well as each other

The results suggest that the behavioural models (SK and CH) might be able to explain the data better than the equilibrium alternatives.\textsuperscript{29} We therefore proceed to our main hypothesis test, on the consistency of estimated types, between sessions of the same cohort.

Cohort 2012 (SK Model): The estimated proportions of types are reported on the bottom panel of Table 2.4. The \( \bar{L}_2 \) type was most frequently estimated in both sessions. However, the proportion of types \( L_0 \) to \( L_6 \) in the B2012 and M2012 were found to be significantly different (two-sided Fisher \( \rho < 0.001 \)). Concerned that such findings were primarily driven by the prior specification of \( L_K \), we conducted the same test for \( L_K = 3, 4, 5, 6, 7, 8, 9 \). However, the proportions of types in both sessions were still found to be significantly different (1% significance level) for each \( L_K \) considered.\textsuperscript{30} Returning back to estimates on Table 2.4, the differences were most prominent for the \( L_1 \) type (0.19 and 0.06), \( L_2 \) type (0.26 and 0.51) and \( L_6 \) type (0.21 and 0.08). The estimation procedure of the SK model is of course sensitive to the distribution of data. We hence considered a less restrictive hypothesis test, focusing on the aggregated estimated proportions of \( L_1 - L_3 \) types. Here, the corresponding frequencies in B2012 and M2012 were 0.70 and 0.78 respectively, and were not found to be significantly different (two-sided Fisher \( \rho = 0.295 \)).

Cohort 2012 (CH Model): The estimates of \( \tau \) were found to be 4.09 and 3.90 in sessions B2012 and M2012 respectively, suggesting that types \( L_3 \) and \( L_4 \) to be most frequent in both sessions. Given the Poisson distribution assumption, the reader should naturally expect some formal test on the equality of \( \tau \). There is an extended literature on such test, building on the pioneering works of Przyborowski and Wilenski (1940). However, such test assumes that the data generating process follows a Poisson distribution. This is not the case with the CH model, since the Poisson distribution assumption was instead made on the unobservable distribution of types. We therefore take an alternative approach, comparing the estimated proportions of types \( L_0 - L_6 \) in each session. These were not found to be significantly different (two-sided Fisher \( \rho = 0.998 \)).

Cohort 2013 (SK Model): The estimated proportions of types are reported on the bottom panel of Table 2.5. The \( \bar{L}_2 \) type was again most frequently estimated in both sessions (at least 0.90). Types \( L_3 \) and above were nearly non-existent. Returning to our hypothesis test, the proportions of types \( L_0 \) to \( L_6 \) were now not found to be significantly different (two-sided Fisher \( \rho = 0.797 \)).

Cohort 2013 (CH Model): The estimates \( \tau \) were found to be 3.64 and 3.11 in sessions M2013 and E2013 respectively, suggesting that the \( L_3 \) type was most frequent in both sessions. Given these \( \tau \) estimates, the same hypothesis test did not find the proportions of types in either sessions to be significantly different (two-sided Fisher \( \rho = 0.833 \)).

Result 4a: The SK estimated proportions of \( L_k \) types were not found to be significantly different between sessions of cohort 2013 and in cohort 2012, the aggregated proportions of types \( L_1 - L_3 \) were not found to be significantly different.

Result 4b: The CH estimated proportions of \( L_k \) types were not found to be significantly different between sessions of cohort 2012 and cohort 2013.

One immediate observation with our level-k estimates is the obvious differences in the proportions of types between the SK and CH. Consistent with most other literature on level-k investigations, the \( L_2 \) type was most frequently found in the SK estimations, though this is less that our prior expectation of types given the simplistic nature of the game. On the other hand, the CH estimates were more in line with such prior expectations, where types \( L_3 \) and \( L_4 \) were more frequently found. How does one explain such discrepancy? Are the CH model’s estimates too high?

\textsuperscript{29} Similar results were documented in such comparisons of the level-k models against the equilibrium driven alternatives Costa-Gomes, Crawford and Iriberri (see 2009); Kawagoe and Takizawa (see 2012).

\textsuperscript{30} Even in the most parsimonious case where \( L_K = 3 \) the estimated proportions of \( L_0, L_1, L_2 \) and \( L_3 \) types were found to be 0.00, 0.19, 0.34 and 0.47 in session B(2012) and 0.02, 0.05, 0.63 and 0.30 in session M(2012).
It should be noted that high $\tau$ are not unusual in the literature. For example, in their seven week CH model investigation of the Swedish Lottery LUPI game, Östling et al. (2011) estimated $\tau$ to be above 4 from week 3 onwards. In a recent paper, Kawagoe and Takizawa (2012) estimated a group of level-k models to investigate behaviours in the centipede game. Amongst the models considered, the authors also estimated close variations of the SK and CH models described in this paper. Their SK estimates found types $L_1$ and $L_2$ to be most frequent. However, their CH estimated $\tau$ was found types $L_3$ onwards to be most frequent.

Taken together these results highlight a particular limitation when one attempts to discriminate between types of level-k models. Because the SK and CH models here are differentiated by the beliefs formation of each $L_k$ type, the outcome of any estimation process is simply the consequence of such beliefs formation. Hence it might not be prudent to compare the frequencies of $L_k$ types between the SK and CH models.

**Remark**

We were also interested to investigate the influence of the $L_0$ type behavioural specification on the consistency of the CH model’s estimates. Here we assume that a $L_0$ type player uniformly randomises across all strategies with probability $z \in [0, 1]$ or chooses 2000 with probability $(1 - z)$ - the above estimates were derived with $z = 0$. With the CH model, we estimated the respective sessions for $z = 0, 0.25, 0.50, 0.75, 1$. Employing the same hypothesis test, the estimated proportion of types were not found to be significantly different in all comparisons when $z = 0, 0.25, 0.50$. However, when $z = 0.75, 1.00$, the proportions of types were found to be significantly different.

### 2.5 Conclusions

This paper was motivated by concerns that the behaviours in AR 11-20 game were inconsistent with the level-k model predictions. We devised an experiment design and procedure to test this. The design involved involved three variations of the 11-20 game - Baseline, Medium and Extreme games - that had equivalent mixed-strategy equilibriums but whose strategies corresponded to different $L_k$ type behaviours. Our test is guided by the principle that if players’ behaviours in the respective games were well explained by the level-k model, we should find consistent proportions of $L_k$ types between the games if players were randomly recruited from the same population.\(^{31}\)

Given the above, we find some support for the level-k model, only when subjects are assumed to best-respond stochastically. This result is consistent with that of Goeree et al. (2013), who found that the introduction of common knowledge noise parameters could improve the explanatory power of behavioural models. However, unlike the authors, we disagree that such adaption are inconsistent the level-k model’s prediction.

Nevertheless, whilst our results are suggestive that subjects behaviour a manner that is consistent with the level-k models, there is still much scope to investigate how the model could be better “tuned” to improve it’s explanatory powers. For example, in session E2013, the spike at 1900 is still insufficiently explained by either SK or CH model. This is ambition for further research.

\(^{31}\)This hypothesis test might be viewed by some to be naturally bias against the level-k model.
CHAPTER 3

Do Markets Allocate the Rights for Performing Decision-Making task to those Who are better-abled to Perform them?

3.1 Introduction

Most economies integrate some form of market mechanism where individuals are able to buy or sell the "rights" for performing specific decision-making task. One example is the market for corporate governance where managers compete e.g., buyouts, takeover, for the rights to manage the corporate resources of a targeted firm (Jensen and Ruback, 1983). If individuals' utilities only depended on the payoffs from performing the tasks, and payoffs are strictly increasing with performances, the conventional wisdom in the above and other relevant examples is the idea that when properly structured, markets should result in the allocation of rights for performing decision-making tasks to those who are better abled to perform them. This is simply the consequences of market interaction where better abled individuals should attach higher valuation for such rights.

Whilst the economic intuitions are straightforward and possibly obvious, there is little research as to whether the conventional wisdom necessarily holds. Part of the challenge is the specification of a comparable situation where one is able to measure performances against. That is to test the conventional wisdom, we need a comparative "economy" where such markets for decision-making task do not exist. For these reasons, this paper uses the experimental environment to put the conventional wisdom to the test.

Our main treatment (TRADE) is intuitively simple and involves three stages. In the first stage, players are each endowed with one token and some information about the decision-making task. In the second stage, players enter a market where tokens are traded. In the third stage, only players with at least one token will

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1 Most discussions of corporate governance focus on the incentive compatibility of market governances upon the behaviours of managers (see Shleifer and Vishny, 1993, for a comprehensive survey). The research interest in this paper is slightly different as we focus on the allocation outcomes of markets, namely the managers who purchase other firms.

2 An earlier example from the 17th to 19th centuries, is the British Army's purchase system, where commissioned ranks and responsibilities were sold at pre-determined prices (Bruce, 1980; Brereton, 1986).

3 There is an extensive body of literature that investigate the efficiency and allocative properties of markets in experimental settings (see Duxbury, 1995; Noussair and Tucker, 2013; Sunder, 1995, for comprehensive surveys). In most such experiments, the subjects' preferences for the market security are exogenously defined by the experimenter. This paper is different such that subjects' preferences are endogenously determined by their expected performances in the decision-making task.

4 In the field of robotics and computer science, there is a growing literature that studies market-based system of task allocations (e.g. Kivelevitch et al., 2011; Korsah et al., 2010; Walsh and Wellman, 1998). However, most of such discussion focuses on the algorithm of market allocation, assuming that the task were completed by computerised players.

5 Building upon the seminar work of Hayek (1945), we think of the market mechanism for the allocation of decision-making task as one where "economic planning" is decentralised. In this domain the comparable situation is one where economic planning is centrally managed.
proceed to perform the decision-making task in the framework of a game. Finally, when all players have completed the task (or the game), their tokens will be redeemed at a heterogeneous rate - henceforth known as the redemption rate - depending on their decisions in the game. Notice here that tokens represent the rights for performing the decision-making task in the game. Players who sold their tokens give up their rights for participating in the task and are compensated by the sales revenue. Players who purchased additional tokens, not only buy over the other players’ participation rights but also their potential cash-flow from the redemption of tokens. Given this design, players decisions in the TRADE treatment can therefore be compared against our control treatments (BASE1 and BASE2), where the second stage is omitted and each player go directly into the game with exactly one token. Whilst the experimental design is straightforward, our choice for a suitable game in this study is govern by four points.

1. How does the ability of players influences their decisions in the game?
2. How do we separate decisions in the game from other regarding preferences?
3. How might the decisions of players in TRADE and the control treatments be comparable?
4. Why should the market allocate tokens to players who might perform better in the game?

To jointly address the above points, this paper uses the decision-making tasks in Littlewood’s (1953) “Hats Puzzle” game, a well-known logical reasoning problem. When modelled as a Bayesian game with incomplete information, there exist a set of equilibrium actions for each player in the Hat Puzzle game, which are independent of the treatment variations. Performances in each task is thus taken to be a binary outcome as to whether they had adhered to the equilibrium actions. This implies that decisions of players between treatments are directly comparable. Furthermore, adherence to the equilibrium actions is Pareto optimal for all players in the game, sidestepping issues of other regarding preferences - Pareto optimal token redemption rate.

If the equilibrium predictions in all treatments are ex-ante identical, how would the experimental design investigate the research interest of this paper? For this we refer to two features of the equilibrium actions. The first being that subjects’ ability to know the equilibrium actions in the game is dependent on their sophistication (e.g., logical reasoning abilities, strategic thinking abilities, problem solving skills, counterfactual reasoning). This is the first important feature of our design experimental as it directly links experimental subjects’ abilities to their performances in the game. Previous experimental adaptations of the Hats Puzzle game by Weber (2001) and Bayer and Chan (2007) - variations of our control treatment, highlighted the complexities with regards to the decision-making tasks in the game, pointing towards heterogeneity in subjects' sophistications.

The second important feature of our design is such that subjects who know the equilibrium actions should value each token strictly higher than those who do not. The intuition is as followed. Given that adherence to the equilibrium actions leads to the Pareto optimal token redemption rate, this should implies that the expected token redemption rate for subjects who know the equilibrium actions will be higher than subjects who do not. When presented the opportunity to trade tokens, such as in our TRADE treatment, this difference in expectations should results in incentive compatible trade, with the former subjects purchasing tokens and

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6We choose to use the decision-making tasks in the setting of a game instead of individual decisions as we wanted to create an environment where the performances of a player in his task imposes externalities on the performances of all other players.
7Alternatively, the control treatments can be thought of as a mechanism where players are randomly assigned their decision-making tasks. This is directly contrasted by the TRADE treatment where the market mechanism determines the allocation of tasks.
8Variations of the Hats Puzzle Hat-Puzzle are commonly found in most graduate level Game theory textbooks (e.g., Fudenberg and Tirole, 1993; Myerson, 1997; Osborne and Rubinstein, 1994; Maschler et al., 2013), discussions about common knowledge (Geanakoplos and Polemarchakis, 1982; Geanakoplos, 1994) and epistemological reasoning (Fagin et al., 1995). The puzzle is often told as a scenario whereby three girls each wearing black hat are able to see each other girls hat but their own - they remain uncertain of their own hat. An observer publicly announces that “there is at least one black hat” and asked the first girl if she knew the colour of her hat. The first girl replied with “No”. The same question was poised to the second girl, who once again replied with “No”. However, when the question was poised to the third girl, she now replies with “Yes Black”. The challenge with the puzzle is to explain how the third girl might have learnt about her hat colour.
9Bayer and Renou (2008, 2012) used the decision-making task in the Hat Puzzle game to study logical reasoning abilities.
the latter subjects selling tokens.\textsuperscript{10} Hence our design introduces an environment where markets \textit{should} result in the allocation of tokens to those subjects who might be better able to perform the decision-making task in the Hat Puzzle game.

Of course the above discussions raises the question as to whether subjects’ knowledge and sophistications would be reflected in their pricing behaviours. Kluger and Wyatt (2004) provide some insights in this area in experimental design where the Monty Hall problem was “embedded” into an asset market.\textsuperscript{11} Their design can be summarised with the following thought experiment. Assume that there exist an asset that allows you to switch doors in the Monty Hall problem for a winning prize of $100 - after you had made your initial choice and the non-prize door is opened. A player who is ignorant of the Monty Hall problem would wrongly judge the probability of winning the prize through switching doors at 1/2 and value the asset at $50. A player who knows the Monty Hall problem would realise that the probability of winning the prize through switching doors is in fact 2/3 and value the asset at $67. Focusing on mean prices, the authors found that when all subjects in the market (6 subjects each market) were “ignorant” (judged by their behaviours in the Monty Hall problem) the mean price in the market was close to 50. However, when there were at least two subjects in the market who knew the problem, the mean price was close to 67. Although their research did not focus on the allocation of asset or individual subjects’ pricing behaviours, it does suggest that subjects’ knowledge could influence their pricing behaviours in asset markets.

To make comparisons between treatments, we introduced the \textit{Efficiency-Rate} as a measure of weighted aggregated performances in each treatment. If markets the conventional wisdom is to be supported, we should firstly expect the Efficiency-Rate in TRADE to be higher than those of the control treatments. However, the Efficiency-Rate was found to be significantly lower in TRADE. To provide some explanations, we studied the transaction prices of tokens in markets, and they were often found to be above the equilibrium price (Pareto optimal token redemption rate), a phenomenon sometimes known as price “bubbles”. Distortions due to bubbles could have severe consequences on the allocative outcome of markets, since at such elevated prices, subjects who knew the equilibrium actions would have found it more profitable to sell their tokens and avoid the decision-making task altogether - this is confirmed by our test statistics. This is furthermore supported by our summary statistics which found that subjects with relatively more tokens were proportionally more likely to deviate from the equilibrium actions. Inversely, we found that subjects who had purchased at non-bubble prices were significantly more likely to adhere to the equilibrium actions. Taken together, the findings in this paper provide little support for the conventional wisdom and instead suggest that markets for decision-making task could potentially lead to worse aggregated performances, due to the distortions from prices. However, it is worth to emphasis that this paper examines a specific class of decision-making task where subjects’ ability to employ counterfactual reasoning are integral to their performances. The results could be significantly different in other task where performances depended on other factors such as effort, information, domain of expertise and so forth.

The rest of this paper is organised as followed. Section 3.2 experimental design, Section 3.3 details the equilibrium and behavioural predictions based on the conventional wisdom, Section 3.4 formalises our hypotheses test, Section 3.5 summarises our experimental procedures, Section 3.6 details our experimental results and finally, Section 3.7 concludes.

### 3.2 Experimental Design

To motivate our experimental design, we will first present a generalised framework which is applicable to all treatment variations, and thereafter discuss how the respective treatments are differentiated. The generalised

\textsuperscript{10}The heterogeneity in knowledge about the equilibrium actions side-steps Milgrom and Stokey (1982) no-trade theorem as subjects can have expected gains from trade.

\textsuperscript{11}The Monty Hall problem is from the TV gameshow “Let’s Make A Deal” where the Host, Monty Hall, hides a winning prize behind three closed doors. A contestant is invited to choose one of the doors to open, but before doing so, Monty is committed to opening a non-prize door. Thereafter Monty presents the contestant the opportunity to switch their choice to the other unopened door. The dominant strategy here is for the contestant to always switch since the probability of winning the prize by doing so is 2/3.
framework will consist of three stages, the Information stage, followed by the Market stage and thereafter, the Hat-Puzzle stage. All players first enter the Information stage, where they receive diverse and partial information about the true state of nature. Thereafter, depending on some exogenous parameter $1_G \in \{0, 1\}$, players may or may not enter the Market stage. If $1_G = 0$, players go directly to the Hat-Puzzle stage, where the decision-making task is performed. Otherwise if $1_G = 1$, players’ participation in the Hat-Puzzle stage depends on their decisions in the Market stage.

### 3.2.1 The Generalised Framework

#### The Information Stage

There are $N = \{1, 2, ..., n\}$ set hats with $M = \{1, 2, ..., m\}$ set members under each hat. Let player $i_j$ refer to the $j \in M$ member of hat $i \in N$. Nature chooses the true state $s \in S \equiv \times_{i \in N} H_i \setminus \{R_1, R_2, ..., R_n\}$, where $H_i \in \{B_1, R_1\}$ denotes hat $i$'s colour - Black($B$) and Red($R$). There exist a common prior over $S$ where each state $s' \in S$ is equally likely.

Each player observes all other hats’ colour but his own. Denote $b_{i_j}(s) \in \{0, 1, 2, ..., n - 1\}$ as the total number of $B$ hats that player $i_j$ observes for any $s \in S$.\(^{12}\) In addition, players are also publicly informed that the true state consist of “at least one $B$ hat”. Since players under the same hat must make the same observations, $b_{i_j}(s) = b_{i_j}(s) = b_i(s)$ for any $j, j' \in M, i \in N$ and $s \in S$.

To see the above more clearly, we present on Table 3.1 all possible states of nature when $n = 3$. Here if $s = \{B_1, B_2, B_3\}$, player $1_j$ will observe two other $B$ hats ($b_1(s) = 2$) and assign equal posteriors to the states $\{B_1, B_2, B_3\}$ and $\{R_1, B_2, B_3\}$. However if $s = \{B_1, R_2, R_3\}$, then player $1_j$ now observes no other black hats ($b_1(s) = 0$), and knows the true state to be $\{B_1, R_2, R_3\}$. This implies that players observing $b_i(s) > 0$ will have partial information about the true state of nature and remain uncertain of their hat colour. Finally, each player is endowed with one token and a working capital of $L \gg 0$, issued as an interest-free loan.

#### The Market Stage

Players only enter the Market stage if $1_G = 1$. Here, players are allowed to trade tokens but only with those other players under the same hat, resulting in $n$ markets. In the absence of short-sales, let $p_i \geq 0$ denote the token transaction price in market $i \in N$, $x_{ij} \in \{0, 1, 2, ..., m\}$ denote player $i_j$’s after transaction inventory of tokens and $L_{ij} \geq 0$ denote player $i_j$’s after transaction holding of capital. Assume that token inventories are public information and $L$ is sufficiently large to never be binding. If $x_{ij} = 0$, players’ payoffs are immediately computed - to be discussed later.

#### The Hat-Puzzle Stage

Only players with at least one token ($x_{ij} > 0$) may enter the Hat-Puzzle stage, where they each perform the decision-making task of resolving their hats’ colour. There are $t = 1, 2, ..., n + 1$ discrete periods, where at each period $t < n + 1$, players are simultaneously presented with the question “Do you know your hat

\(^{12}\) Alternatively, one could employ the semantic approach where each player’s knowledge of the true state is represented by the information partition $P_i$ over $S$. Such an approach might be more precise but it makes the discussion more taxing.
where the market clearing conditions require that $L_a = a_r$ had purchased. This prevents players from following some hedging strategy. Each time the player chooses $a_e$ ($e$ colour?) ends the Hat-Puzzle stage at the period $t_j$, whereby the action $e_{ij} \in \{a_r, a_b\}$ was chosen. This implies that players only proceed to the next period if he had chosen $a_n$ in the previous period. To ensure that all players must eventually end the Hat-Puzzle stage, players can only choose from the actions $a_b$ and $a_r$ if they make it to the $n + 1$ period. Finally, any action chosen in period $t$ will be public information in period $t + 1$.

The decision-making structure of the Hat-Puzzle stage thus closely follows Littlewood’s (1953) “Hats Puzzle” game with the exception that all players choose their actions simultaneously instead of sequentially. Also note that the decision-making task for players are independent of the number of tokens own.\textsuperscript{13}

### Player’s Payoffs

Players’ payoffs ($\Pi_{i_j}$) are computed when they have either ended the Market stage with $x_{i_j} = 0$ tokens or ended the Hat-Puzzle stage with choosing $e_{i_j} \in \{a_r, a_b\}$. The loan ($L$) is repaid and their tokens are each redeemed at the heterogeneous rate $\beta(\mu, \delta, \alpha, H_i, t_i, e_{i_j}) \geq 0$ - in a slight abuse of notation we will write $\beta(\mu, \delta, \alpha, H_i, t_i, e_{i_j})$ as $\beta_{i_j}$.

Table 3.2 depicts the generic tokens redemption rate for each player $i_j$, where $\mu > (1/2)\alpha > (n+1)\delta > 0$. The redemption rate can be summarised as followed. Each token has an initial value of $\mu$ that decreases by $\delta$ each time the player chooses $a_n$. In addition, the token’s value decreases by $\alpha$ if he had incorrectly guessed his hat colour - choosing $a_b$ ($a_r$) if $H_i = R_i$ ($H_i = B_i$). The payoffs are therefore determined as\textsuperscript{14,15}

$$\Pi_{i_j} = \begin{cases} 
(L_{i_j} - \bar{L}) + \beta_{i_j}x_{i_j} = \beta_{i_j} & \text{if } 1_G = 0 \& x_{i_j} = 1 \\
(L_{i_j} - \bar{L}) + \beta_{i_j}x_{i_j} = p_t + (\beta_{i_j} - p_t)x_{i_j} & \text{if } 1_G = 1 \& x_{i_j} > 0 \\
(L_{i_j} - \bar{L}) = p_t & \text{if } 1_G = 1 \& x_{i_j} = 0
\end{cases}$$

It is important to emphasis here that all tokens owned by a player will be redeemed at the same rate $\beta_{i_j}$, depending on his decisions in the Hat-Puzzle stage. This of course implies that the rate could different for the different players.

### 3.2.2 How the Treatments are Differentiated

To see how the generalised framework captures much of the research interests in this paper, first notice that players only arrive at the Market stage (where $1_G = 1$), after they have received some information about the decision-making task in the Hat-Puzzle stage ($b_h(s)$). Furthermore, players in each of the $n$ markets will also have the same information. Since only players with at least one token will proceed into the Hat-Puzzle stage, tokens thus represent the player’s “rights” for performing the decision-making task and receiving the cash-flow from the redemption of tokens. Alternatively, players may also give-up their participation rights by selling their tokens, and be compensated by the sales revenue. The design thus induces an environment where each

\textsuperscript{13}A player with three tokens will only make perform the decision-making task for himself and not for the other players’ whose token he had purchased. This prevents players from following some hedging strategy.\textsuperscript{14}When $1_G = 0$, we must have it that $\bar{L} = L_{i_j}$ and $x_{i_j} = 1$ since players are not permitted to enter the Market stage.\textsuperscript{15}Since players are each endowed with one token, their net transactions in the Market stage can be denoted as $v_{i_j} = x_{i_j} - 1$, where the market clearing conditions require that $\sum_i v_{i_j} = 0$ for all $i \in N$. As such, we can rewrite players’ holding of capital as $L_{i_j} = \bar{L} - p_t v_{i_j}$.
player’s valuation or reservation price for tokens depends on their expected token redemption rate. This will be detailed in the later section.

To describe the different treatments, we will first begin with the main focus of the paper, the TRADE treatment, which involves the following parameters \(1_G = 1, n = 3\) and \(m = 6\). Two extreme outcomes could occur in TRADE, one in which a single player purchases all other tokens within his market and the other where no trade occurs. These were the motivation for our control treatments, BASE1 \((1_G = 0, n = 3\) and \(m = 1)\) and BASE2 \((1_G = 0, n = 3\) and \(m = 6)\).

As we will detail in the equilibrium analysis, the optimal action for players at each period \(t\) is the best-response to the actions of each other player in period \(t - 1\). Furthermore, the optimal action will be identical at all periods for players under the same hat. Given that experimental data might often be noisy, subjects in TRADE may enjoy some advantage over those in BASE1 as \(m\) increases, due to their ability to best-respond to the aggregate action of each other subjects. The inclusion of BASE2 thus helps control for such advantage.

### 3.3 Predicted Behaviours

To described the predicted behaviours in all treatments, we shall first begin with the equilibrium predictions, where players are assumed to be risk-neutral, rational and there exist common knowledge of Rationality.

In doing so, we shall derive the Equilibrium Actions for players in the Hat-Puzzle stage, which refers to the sequence of optimal actions at each period \(t\). Working backwards, we will thereafter derive the Equilibrium Price in each market of the Market stage, and consequently the Equilibrium Payoffs.

Through this analysis, we will show that the Equilibrium Price in market \(i \in N\) to be \(p_i^* = \mu - b_i(s)\delta\). We will also show that the Equilibrium Actions in the Hat-Puzzle stage are for players to choose \(a_n\) at all periods \(t < b_i(s) + 1\), and at period \(b_i(s) + 1\), choose \(a_b\) or \(a_r\) if \(H_i = B_i\) or \(H_i = R_i\), respectively. This corresponds to the equilibrium token redemption rate of \(\beta_i^* = \mu - b_i(s)\delta\) and the equilibrium payoff of \(\Pi_i^* = \mu - b_i(s)\delta\).

The emphasis of the equilibrium analysis is to highlight how adherence to the equilibrium actions, although Pareto optimal, will require players to employ sophisticated reasonings. Furthermore, the “intensity” of such reasonings increases with \(b_i(s)\). However, the equilibrium actions and corresponding payoffs will only depend on \(b_i(s)\) and will be independent of the number of tokens owned. This is the central feature of our experimental design, such we are able to present three different treatments with identical equilibrium predictions, providing for a common platform for comparisons.

If this might be the case, then how would our design investigate the research question raised at the introduction of this paper? For this we return to the experimental findings of Weber (2001) and Bayer and Chan (2007) who used the Hat-Puzzle game - in a variation of our BASE1 design - to study level-k (Nagel, 1995; Stahl and Wilson, 1994, 1995; Costa-Gomes et al., 2001) reasoning behaviours. The authors found that the frequencies of equilibrium actions decreases with \(b_i(s)\). In general, their experimental findings suggest that most subjects found the equilibrium actions in the Hat-Puzzle stage non-trivial and cognitively challenging, and we should therefore expect the same fall in frequencies in our BASE1 and BASE2 treatments.

---

19 Physical limitations in our laboratory restrict our design to 18 subjects - 3 hats and 6 subjects under each hat.
20 Since players in BASE1 and BASE2 do not participate in the Market stage, they hence enter the Hat-Puzzle stage with exactly one token.
21 One can think about this as the “Wisdom of the Crowd” phenomenon where though the individuals’ action might be noisy, the aggregated actions smoothen out such noise.
22 Borrowing from the discussion of Rational players by Myerson (1997), we refer to such players as those who seek to maximise their own payoffs, know everything there is to know about the game, and approach the Hat-Puzzle in the same manner as the designer of the puzzle would have done so.
23 Although subjects’ behaviours in this paper may involve elements of level-k reasoning, we will omit such discussions as they divert attention from the main area of interest.
24 Weber (2001) and Bayer and Chan (2007) experimental design employ features that are different from BASE1. For example, subjects their experiment could only choose between \(a_n\) and \(a_b\), and the Hat-Puzzle stage ends for all players at any period which the action \(a_b\) is chosen. Comparisons to their findings will hence not be prudent.
25 Weber’s results could be of independent interest as his subject pool included Caltech undergraduate and graduate students, and Caltech students are often known for their skills in logical reasoning problems (Camerer, 2003).
Alternatively, their findings could also be attributed to the lack of common knowledge of rationality, a central “ingredient” in the equilibrium analysis.23,24

Motivated by the previous experiments, we shall present a behavioural prediction as to why the Market stage in TRADE should result in different outcome in the Hat-Puzzle stage, relative to BASE1 and BASE2. This prediction will be based on the conventional wisdom introduced at the start of the paper, such that markets should allocate the rights for performing the decision-making task in the Hat-Puzzle stage to the player who might be best-abled to perform them. To do so, we will weaken the assumptions in the equilibrium analysis, allowing for heterogeneity in types, which are differentiated by their sophistications, and consequently their valuation for the tokens. Thereafter, we will show how the availability of the Market stage will result in the tokens being purchases by players who know the equilibrium actions. Stated differently, the Market stage should result in the allocation of tokens to types who might be “more rational” and consequently better able to perform the decision-making task in the Hat-Puzzle stage.

3.3.1 Equilibrium Predictions

We will begin our discussion of the equilibrium predictions in the Hat-Puzzle stage and thereafter work backwards to the Market stage (where applicable). To do so, we will first detail the equilibrium actions in BASE1 and thereafter extend the discussions to BASE2 and TRADE. Finally, the equilibrium payoffs will be derived for all treatments.

Equilibrium Predictions in BASE1

Players enter the Hat-Puzzle stage with exactly one token and seek to maximise their token redemption rate since \( \Pi_{ij} = \beta_{ij} \). To show the equilibrium actions, it is useful to first identify the optimal action at each period \( t \) for players who are certain and uncertain of their hats’ colour. In the former case, the optimal action is obvious, choose \( a_s \) or \( a_r \) if they know their hats to be \( B \) or \( R \) respectively - choosing \( a_n \) incurs an additional “cost” of \( \delta \) with no obvious benefits. The optimal action in the latter case is less obvious. By Bayes rule, uncertain players must hold equal posterior to being under either hat colours - this will be clearer in the later discussions. Here, players at period \( t \) face an inter-period tradeoff between (OptionA) Ending the Hat-Puzzle stage with \( c_i \in \{a_0, a_r\} \) and (OptionB) Choosing \( a_n \) and ascertaining their hats’ colour at some later period \( t' = t+1, t+2, \ldots, n+1 \). The expected token redemption rate with OptionA and OptionB are \( \mu - \delta(t-1) - (1/2)\alpha \) and \( \mu - \delta(t'-1) \) respectively. Given that \((1/2)\alpha > (n+1)\delta\), OptionA will always be dominated by OptionB for any \( t' = t+1, t+2, \ldots, n+1 \). Therefore the optimal action for uncertain players should always be \( a_n \).

However, the reader should immediately realise that OptionA is dominated by OptionB only because uncertain players expect to ascertain their true hat colour at some later period. If this is not the case, then OptionA now becomes the dominant action - choosing \( a_n \) incurs the cost of \( \delta \) with no obvious benefits - and the player would be indifferent between actions \( a_0 \) or \( a_r \).

Assuming that all players always expect to ascertain their true hat colour, we are now in the position to describe how they actually do so, and the corresponding equilibrium actions.25 For this we consider the case where \( n = 3 \) and \( s = \{B_1, B_2, B_3\} \). Each player begins period 1 of the Hat-Puzzle stage observing \( b_i(s) = 2 \) and remains uncertain. Since each state in \( S \) is equally likely, players applying Bayes rule must

23In most experiments, the emphasis on subjects’ anonymity, to sidestep other regarding preferences, often meant that subjects have imperfect or little information about the other subjects they have been paired with. Under such circumstances, it is difficult to see how common knowledge of Rationality could easily be established.

24Bayer and Renou (2008) used the Hat Puzzle game to study “depths” of counterfactual reasoning. To sidestep the common knowledge of Rationality issue, the authors paired subjects with computer players. Here adherence to the equilibrium actions were more frequently observed relative to the prior experiments. However, their design also employs the an unique feature where the Hat puzzle game automatically ends at any period which the subject had either correctly determined their hat colour or deviated from the equilibrium action. To introduces the possibility that the higher rates of adherence could also be attributed to subjects “learning” about the equilibrium action through trial and error.

25For a detailed discussion of the equilibrium analysis, we refer the reader to the Maschler, Solan and Zamir (2013) or Geanakoplos and Polemarchakis (1982).
assign equal posterior to being under either hat colours. Given the public announcement, it can only be common knowledge that there is at least one $B$ hat. However, each player privately knows there to be at least two $B$ hats. Uncertain players in period 1 thus choose $a_n$.

At period 2, having observed the public information - the previous period’s actions, each player reasons that if there was only one $B$ hat, then some player must have observed no $B$ hats, ascertained his hat’s colour to be $B$ and choose $a_b$ in period 1. Since no one had done so, there cannot be only one $B$ hat in the true state. Of course each player already knew this and there should be no revisions to their posteriors. However, because it is common knowledge that all players followed the same counterfactual reasoning, it now becomes common knowledge that there is at least two $B$ hats. Again uncertain players choose $a_n$.

Finally at period 3, given the public information, each player reasons that if there were only two $B$ hats, then given the common knowledge facts in period 2, those players who observed two other $B$ hats, should have ascertained their hats’ colour to be $B$, and choose $a_b$ in period 2. Since no player had done done so, there cannot be only two $B$ hat in the true state and given that $b_i(s) = 2$, each player ascertains their own hat to be $B$. Similarly, since it is common knowledge that all players followed the same counterfactual reasonings, it becomes common knowledge that there is exactly three $B$ hats. Players thus choose $a_b$ in period 3 and their tokens are each redeemed at the rate $\mu - 2\delta$.

The above discussions can be extended to any $n \geq 2$ coloured hats and the equilibrium prediction is for players to ascertain their own hat colour at period $b_i(s) + 1$. The equilibrium actions are for players to choose $a_b$ at all periods $t < b_i(s) + 1$, and at period $b_i(s) + 1$, choose $a_b$ or $a_r$ if $H_1 = B_i$ or $H_1 = R_i$ respectively. Also notice that adherence to the equilibrium actions will result in players’ tokens being redeemed at the Pareto optimal rate $\beta^*_i = \mu - b_i(s)\delta$.

The astute reader should immediately realise the centrality of the common knowledge assumptions in the equilibrium predictions. If they are not met, players cannot exclude the possibility that an action chosen by some other player is due to irrational behaviours. However, given that adherence to the equilibrium actions are Pareto optimal for all players, each player should strictly prefer the common knowledge assumption to be met.

**Equilibrium Predictions in BASE2**

BASE2 only differs from BASE1 in the number of players under each hat. However, players under each hat have the same private information ($b_i(s)$), face the same decision-making task and choose their actions both independently and simultaneously. Given that the equilibrium actions only depend on $b_i(s)$, there should be no evidential differences to the equilibrium predictions for players in BASE2.

**Equilibrium Predictions in TRADE**

TRADE only differs from BASE2 on the availability of markets in the Market stage. Players in TRADE enter the Hat-Puzzle stage with $x_{ij} \geq 1$ tokens. The equilibrium discussions in BASE2, suggest that the number of players under each hat has no influence on the equilibrium actions. How about the token ownerships? The answer as it turns out is no. This is because if adherence to the equilibrium actions is Pareto optimal for players with one token (as in BASE1 and BASE2), it must also be Pareto optimal for players with more than one token.

By backward deduction, players in the Market stage observing $b_i(s)$ should expect to ascertaining their hats’ colour in period $b_i(s) + 1$ of the Hat-Puzzle stage. Given the token redemption structure, whatever colour it may be, players should hence expect their tokens to be redeemed at $\beta^*_i = \mu - b_i(s)\delta$ and by this logic, should only purchase additional tokens at prices $p_i \leq \mu - b_i(s)\delta$ or sell tokens at $p_i > \mu - b_i(s)\delta$. Since

\[26\]More precisely, player $1_j$ assigns equal posteriors to the states $\{B_1, B_2, B_3\}$ and $\{R_1, B_2, B_3\}$, player $2_j$ to the states $\{B_1, B_2, B_3\}$ and $\{B_1, R_2, B_3\}$, and player $3_j$ to the states $\{B_1, B_2, B_3\}$ and $\{B_1, B_2, R_1\}$.

\[27\]Alternatively, Aumann (1976) agreement theorem, show that the only event in $S$ which can be commonly knowledge must include the entire states of nature $S$. 

34
players only trade tokens with the other players under the same hat, this establishes the equilibrium price \( p_i^* = \beta_i^* = \mu - b_i(s)\delta \) in each market \( i \in N \), where players are indifferent between buying or selling tokens.\(^\text{28}\)

### Equilibrium Payoffs

Given the equilibrium discussion, the equilibrium payoff can be derived for players in each treatment by substituting \( p_i^* \) and \( \beta_i^* \) where relevant

\[
\Pi_i^* = \begin{cases} 
\beta_i^* = \mu - b_i(s)\delta & \text{if } 1_G = 0 \& x_{i_1} = 1 \\
 p_i^* + (\beta_i^* - p_i^*)x_{i_j} = \mu - b_i(s)\delta & \text{if } 1_G = 1 \& x_{i_j} > 0 \\
 p_i^* = \mu - b_i(s)\delta & \text{if } 1_G = 1 \& x_{i_j} = 0 
\end{cases}
\]

Notice again that the equilibrium payoff (\( \Pi_i^* \)) only depends on \( b_i(s) \) and is independent of the treatment variations.

#### 3.3.2 Behavioural Predictions

To motivate the behavioural predictions, we will begin with the assumption that the population of players consist of both Sophisticated and Unsophisticated types.\(^\text{29}\) Both types are “Rational” in the sense that they always seek to maximise their payoffs, but differ in their level of sophisticated ability to employ counterfactual reasonings. When \( b_i(s) = 0 \), the equilibrium actions are trivial and obvious for both types - choose \( a_o \) - since ascertaining one’s hat does not involve counterfactual reasoning. Where the behavioural differences occur is when they face \( b_i(s) > 0 \).

First consider the optimal behaviour for Unsophisticated types in the Hat-Puzzle stage observing \( b_i(s) > 0 \). Unable to employ counterfactual reasonings, the Unsophisticated types will not ever expect to ascertain their true hat colour, and as such should randomise between \( a_o \) or \( a_s \). The expected token redemption rate is therefore \( \mu - (1/2)\alpha \), which implies that they should only purchase tokens prices \( p_i \leq \mu - (1/2)\alpha \).

Now consider the decision problem for Sophisticated types. If they expect to ascertain their own hat colour, they should only purchase tokens at prices \( p_i \leq \beta_i^* = \mu - b_i(s)\delta \). However, the problem for Sophisticate types in BASE1 and BASE2 is the fact that they are unable to deduce if an action chosen by some other players are due to a randomisation outcome or consistent with the equilibrium actions. Alternatively, Sophisticated types might also hold beliefs about the proportion of Unsophisticated types but how this necessarily manifests itself into the optimal actions for Sophisticated types is unclear, since the equilibrium discussions are build on the notion of common knowledge rather than mutual knowledge. Nevertheless, if Sophisticate types do not expect to ever ascertain their true hat colour, their optimal behaviour would be no different from the Unsophisticated types.

In general the problem in the Hat-Puzzle stage in BASE1 and BASE2 is that of asymmetric information where Sophisticate types do not know the types of all other players. This is where the Market stage in TRADE could sidestep the above issue by “filtering” Unsophisticated players. Let us assume for now that Sophisticated types in TRADE always expect to ascertain their true hat colour. Given that \((1/2)\alpha > (n + 1)\delta\), this implies that Sophisticated types will value each token higher than Unsophisticated types. Therefore at prices \( p_i \in (\mu - (1/2)\alpha, \mu - b_i(s)\delta) \), it becomes incentive compatible for Sophisticated types to purchase tokens and Unsophisticated types to sell tokens.\(^\text{31}\) Anticipating this, Sophisticated players will know that

\(^{28}\)If players are risk-averse, we therefore have a no-trade outcome since Tirole (1982) showed that Rational players will only trade when there is an expected gain from doing so.

\(^{29}\)We can also extend the heterogeneity of types to include levels to the Sophisticated types, much like the level-k models. However, since the focus of this paper is on the market mechanism, such discussions will be omitted.

\(^{30}\)Alternatively, Unsophisticated types may believe that there is no possible “solution” to the Hat-Puzzle stage.

\(^{31}\)The above discussion are in the domain where all players are risk-neutral. If players are risk-adverse trade will occur at prices \( p_i \in (\mu - (1/2)\alpha, \mu - b_i(s)\delta) \), where both risk-adverse Sophisticated and Unsophisticated traders receive expected gains from trade.
the only type of players who will enter the Hat-Puzzle stage must also be Sophisticated types, establishing common knowledge of Rationality. 32 Beside the above rationale, the Hat-Puzzle stage is unique such that Sophisticated types strictly prefers each other players to be Sophisticated and where possible signal their sophistications - adherence to the equilibrium actions is Pareto optimal for all players. Given the payoff structure and the fact that token ownership is public information, Sophisticated players could credibly use the token ownership to signal that they have more “skin in the game” - vested interest to adhere to the equilibrium actions.

The above discussions leads us to the following behavioural predictions (1) Markets should result in the allocation of tokens to those players who know the equilibrium actions, and (2) Tokens in Markets will be transacted at prices \( p_i \in (\mu - (1/2)\alpha, \mu - b_i(s)\delta) \). More generally, the behavioural predictions are based on the premise of the conventional wisdom, such that markets should allocate the rights for performing the decision-making task to those who are better abled to perform them.

### 3.4 Test Hypotheses

This paper seeks to investigate the conventional wisdom that markets would allocate the rights for performing decision-making task to those who might be best abled to perform them. To therefore test the central research question of this paper, comparisons will be made between subjects’ behaviours in the Hat-Puzzle stage of the BASE1 and BASE2 treatments, against those in TRADE, given that the equilibrium actions in the Hat-Puzzle stage were shown to be identical across all treatments. To make such comparisons treatments, two terms are introduced

**Adherence-Rate:** The ratio of players in the Hat-Puzzle stage who had adhered to the equilibrium actions.

**Efficiency-Rate:** The ratio of tokens redeemed at the Pareto optimal equilibrium rate \( \mu - b_i(s)\delta \). 33

Both terms focus on the behaviours of players in Hat-Puzzle stage, whether they had adhered to the equilibrium actions. However, they differ on the weights assigned to the players’ behaviours in respective treatments. The Adherence-Rate assigns uniform weights to the behaviours of all players in the Hat-Puzzle stage, whilst the Efficiency-Rate assigns greater weights to the behaviours of players with more tokens. These differences are irrelevant for BASE1 and BASE2, since players always embark on their decision-making task with exactly one token - the Adherence-Rates and Efficiency-Rates must be identical. However, this will not necessarily be true for players in TRADE due to the introduction of the Market stage. Since comparisons between treatments should account for the activities in the markets, the Efficiency-Rate would be a more suitable measure of aggregated performances in the respective treatments. This brings us to the following test hypotheses

**H1:** The Efficiency-Rate in BASE1 is similar to that of BASE2.

**H2:** The Efficiency-Rate in TRADE is higher than those in BASE1 and BASE2.

**H3:** The likelihood of adherence to the equilibrium actions for subjects in TRADE is increasing with token ownership, at instances where \( b_i(s) = 1, 2 \).

**H4:** The likelihood of adherence to the equilibrium actions for subjects in TRADE is strictly higher for subjects who had purchased tokens at prices \( p_i \in (\mu - (1/2)\alpha, \mu - b_i(s)\delta) \) relative to subjects who had purchased tokens at \( p_i \notin (\mu - (1/2)\alpha, \mu - b_i(s)\delta) \) or had not purchased tokens, at instances where \( b_i(s) = 1, 2 \).

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32 In experimental settings, there is also the possibility that Unsophisticated subjects are able to learn about the equilibrium actions through market prices. However, such learning will be imperfect. For example if prices market price was \( \mu - 2\delta \), then Unsophisticated types might learn that they odd to choose either \( a_x \) or \( a_x \) in period \( t = 3 \). But which action remains unclear.

33 If a player with three tokens had adhered to the equilibrium actions, then three tokens would have been redeemed at the equilibrium rate.
H1 serves as an empirical warm up where we examine the marginal influences of increasing $m$ on the behaviours in the Hat-Puzzle stage. Building on this finding, we can thus proceed to H2, where we examine the main research question of this paper. If the conventional wisdom holds, then markets will allocate relatively more tokens to those subjects who know the equilibrium actions. This should correspond to a higher Efficiency-Rate in TRADE relative to BASE1 and BASE2.

H3 and H4 seeks to provide support for any potential findings from H2. If markets did result in the allocation of tokens to those players who know the equilibrium actions, we should expect the likelihood of adherence to the equilibrium actions to be increasing with token ownership for subjects in TRADE. In addition, since the allocation outcomes of tokens within the markets of TRADE should be due to the difference in valuations amongst subjects, we should therefore expect subjects’ token pricing behaviours to be related to their behaviours in the Hat-Puzzle stage.

As the decisional task are trivial when subjects observe $b_i(s) = 0$, the potential effects of markets in the Market stage should only be evidential when subjects observe $b_i(s) = 1, 2$.

### 3.5 Experimental Procedures

Two experimental sessions, were conducted for each treatment. Each session had involved 18 inexperienced subjects, recruited on a first come basis from the undergraduate cohort at the University of Exeter, through the ORSEE (Greiner, 2004) software. Table 3.3 reports on the subjects’ demography in each treatment, by the schools they were enrolled into - Economic students study at the Business School. Although subjects had no formal training in Game theory, those with stronger background in economics, engineering, mathematic or physics may potentially have some advantage with counterfactual reasoning tasks due to their background training. This will be controlled for in the econometric analysis.

The experiments were conducted with the Z-Tree (Fischbacher, 2007) software and employed non-neutral framing of the Market and Hat-Puzzle stages. Each session had consisted of one practice round and ten paying rounds, where subjects’ payoffs were denoted in the fictitious currency, ECU. The following payoff parameters were employed: $\mu = 950$, $\delta = 50$, $\alpha = 700$ and $L = 6000$. Subjects’ overall payoffs were determined as the average over all ten rounds and converted into cash at the exchange rate of 67ECU/$1 in the BASE1 and BASE2 treatments, and 100ECU/$1 in the TRADE treatments.

The average duration of the BASE1 and BASE2 sessions were 95 minutes, whilst the TRADE sessions were 130 minutes. In addition to their experimental earnings, subjects also received a show-up fee of $5 in the BASE1 and BASE2 sessions, and $8 in the TRADE sessions. Including the show-up fees, the average cash earnings were $16.64, $16.91 and $16.12 in the BASE1, BASE2 and TRADE treatments, respectively. Before collecting their cash payments, subjects were required to complete the Cognitive Reflective Test (Frederick, 2005) and self-declare any prior familiarity with the Hats Puzzle or similar problems.

<table>
<thead>
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<th>BASE1</th>
<th>BASE2</th>
<th>TRADE</th>
</tr>
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<td>23</td>
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</tr>
<tr>
<td>Total</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 3.3: Demographics of Subjects by Schools Enrolled

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34 The difference in exchange rates was introduced to control for any potential income effect due to a higher show-up fee being paid in the TRADE sessions.

35 The Cognitive Reflective Test involves three questions that trigger the wrong “instinctive” answer. (Q1) A bat and a ball cost $1.10 in total. The bat costs $1.00 more than the ball. How much does the ball cost? (Q2) If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets? (Q3) In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take the patch to cover half of the lake?
For efficient comparisons between treatments, two sequences of states \((s \in S)\) were randomly generated prior to the experimental proper. This was introduced to ensure that at each round of the respective treatments, there were the same number of subjects who observed zero, one or two black hats.

Prior to experiment proper, we conducted a pilot test on the software and the instructions. The pilot test had raised some interesting challenges with the experimental design, which prompted us to make minor modifications to the design of BASE1 and BASE2. In the following, we will first detail the modifications made and thereafter the experimental procedures in the respective treatments.

### 3.5.1 Minor Modifications to BASE1 and BASE2

The pilot session was based on the BASE2 treatment design. Here, subjects were sometimes observed to be adhering to the equilibrium actions despite the fact that they were following some randomisation process through their feedbacks.

To overcome the likelihood that observed adherences were purely coincidental, we included an "outside option" for subjects to discretely end the Hat-Puzzle stage in a manner that does not affect the equilibrium predictions. In addition to the actions \(a_b, a_r\) and \(a_n\), subjects could also choose the outside option with the action "Toss a Coin, I will never know (a_c)". If the subject chooses \(a_c\), he ends the Hat-Puzzle stage with a fixed cost of 250 ECU, in addition to any other deductions incurred when choosing \(a_n\). In doing so, he assigns the computer to choose the action \(a_b\) or \(a_r\) on his behalf - with equal probability. The computer’s choice will have no consequence on his payoffs. For example if subject \(A\) choose \(a_n\) in the first period and \(a_c\) in the second period - the computer had chosen \(a_b\) on his behalf, his token will be redeemed at the rate of 950-50-250=650 ECU. All other subjects would have observed that \(A\) had chosen \(a_b\) in the second period. However, only the experimenter would know that subject \(A\) had chosen \(a_c\).

The action \(a_c\) will always be dominated in the equilibrium discussions and does not influence the equilibrium actions. The expected token redemption rate with adhering to the optimal choices is \(950 - 50(b_i(s))\), with choosing \(a_c\) at any period \(t\) is \(700 - 50(t - 1)\), and randomising with either \(a_b\) or \(a_r\) for uncertain players is \(600 - 50(t - 1)\). Thus for subjects who do not expect to ever ascertain their hats’ colour, the action \(a_c\) dominates all other actions. The outside option was omitted from TRADE, since an equivalent outside option already exist, the ability to sell your token and avoid the Hat-Puzzle stage altogether.

### 3.5.2 BASE1

Upon entering the experiment, subjects were allowed 40 minutes to read through the instructions and complete a questionnaire, testing their understanding of the experimental design. Thereafter, subjects were randomly paired with two other subjects into a group and remained within the same group for the duration of the experiment - total of 12 group. At the start each round, subjects were randomly assigned to one of three hats and were presented with the other hats’ colours. Subjects were also informed that there is at least one black hat and proceed directly into the Hat-Puzzle stage. To avoid confusion, the notion of tokens were omitted from the subjects’ instructions. The Hat-Puzzle stage proceed as discussed and each period had lasted a maximum of 240 seconds. At each period \(t > 1\), subjects were presented on their computer screens the period \(t - 1\) actions of all other subjects within their group. A limitation of the software design was such that subjects had to proceed through the periods together. This meant that subjects who had chosen the actions \(a_b, a_r\) or \(a_c\) were facing a blank screen as they waited for other subjects to proceed through the periods. However, subjects were observed to have taken the opportunity to "sketch" their behaviours in the game.

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36See the Appendix for details about the instructions and screenshots.
37We wanted to maintain fixed pairing in BASE1 for consistency with BASE2 and TRADE, where each session formed a group.
3.5.3 BASE2

The sessions differ from the BASE1 sessions in the following: Each group consisted of 18 subjects, with 6 subjects under each hat - total of 2 group. However, subjects were again randomly assign to one of three hats in each round. At each period $t > 1$, subjects were presented on their computer screens a table that depicted the aggregated period $t-1$ actions, by all subjects in the respective hats. For example, subjects under hat 1, will observe the relative frequencies of the actions $a_b$, $a_r$ and $a_n$, chosen by all subjects under hat 2 and 3.

3.5.4 TRADE

Each group again consisted of 18 subjects with 6 subjects under each hat - total of 2 groups. When the round begins, subjects first observed the other hats’ colours. Thereafter, subjects enter the Market stage, where trade was facilitated through a continuous double auction mechanism that lasted for 120 seconds - the market only consisted of the other subjects under the same hat. Here, a price ceiling of 1200 ECU was imposed on the bid and ask prices, to restrict subjects from intentionally making losses. This also ensures that each subject was not capital constrained from purchasing all other tokens within his market.

After the Market stage had ended, only subjects with at least one token entered the Hat-Puzzle stage - subjects without any tokens were able observe the proceedings of the Hat-Puzzle stage on their computer screens but prevented from participating - this was introduce to allow the subjects to “learn” from the behaviours of other subjects. The Hat-Puzzle stage proceeded as described in the BASE1, with the exception that the action $a_r$ was not available and the public information available to subjects at each period $t > 1$. Here, their computerised screens depicted the aggregated period $t-1$ actions, by all subjects under the respective hats ranked by their token ownership. For example, subjects under hat 1, will observe the relative frequencies of the actions $a_b$, $a_r$ and $a_n$, chosen by those subjects under hat 2 and 3 with one, two, three,..., six tokens. Finally as the loan of 6000 ECU had to be repaid at the end of the round, some subjects had incurred negative payoffs - 21 observed bankruptcy out of the 360 instances.

3.6 Experimental Results

In the following discussions, we will omit the suffix $i$ and $j$, and make references to those instances where experimental subjects observed $b = 0, 1, 2$ black hats. To first examine hypotheses H1 and H2, we will focus on the Efficiency-Rates in the respective treatments. Thereafter, we will focus on the behaviour of subjects and the transaction prices of tokens in TRADE, to motivate any potential findings with H3 and H4. Finally, we will revisit H2 in our econometric analysis, where we will also formally examine H3 and H4.

3.6.1 Aggregated Performance: Efficiency-Rate

To evaluate the aggregate performances between treatments, our analysis will first focus on the Efficiency-Rate, which refers to the ratio of tokens redeemed at the equilibrium rate $\beta^* = 950 - 50(b)$. Given the counterfactual nature of the equilibrium actions in the Hat-Puzzle stage, one should naturally expect adherence to the equilibrium actions to be more challenging as $b$ increases. Inversely this suggest that the Efficiency-Rates is expected to decrease with $b$.

With repeated rounds, there is the potentiality that a subject’s behaviours in the Hat-Puzzle stage might be dependent on the sequence of states he is presented with. However, our experimental procedures sidesteps

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38Because the loan of $\bar{L} = 60000$ had to be returned at the end of the round, some subjects would have received negative payoffs. However, we are unable to impose negative cash transfer on experimental subjects, which implies that they enjoy some form of limited liability. Thus the bounds on the bid and asks prices are imposed to minimise the prospects that a subject would intentionally incur losses for the benefit of another subject, at the expense of the experimenter.

39For example, if a subject is presented in the first, second and third round with $b = 0$, $b = 1$ and $b = 2$ respectively, then given that feedback is provided at the end of each round, he is able to extrapolate his behaviours in the first round upon the second, and the second onto the third. The same subject’s behaviours might therefore be very different if he instead presented with $b = 2$, $b = 1$ and $b = 0$, in the first second and third rounds, respectively.
Figure 3.1: Efficiency-Rate at Each Round of BASE1, BASE2 and TRADE

To study the experimental data in greater detail, we present on Table 3.4 the Efficiency-Rate at each round and for each $b$ observation, of the respective treatments.\(^{40}\) Here, each cell depicts the total number of tokens that were redeemed at the Pareto optimal rate $\beta = 950 - 5b(x)$ with the ratio in parenthesis.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
 & $b = 0$ & $b = 1$ & $b = 2$ & $b = 0$ & $b = 1$ & $b = 2$ & $b = 0$ & $b = 1$ & $b = 2$ \\
\hline
I & 12(1.00) & 16(0.67) & - & 12(1.00) & 15(0.63) & - & 12(1.00) & 12(0.50) & - \\
II & 12(1.00) & 21(0.88) & - & 12(1.00) & 17(0.71) & - & 12(1.00) & 12(0.50) & - \\
III & - & 17(0.71) & 1(0.08) & - & 15(0.63) & 1(0.08) & - & 8(0.33) & 5(0.42) \\
IV & - & 7(0.59) & 4(0.17) & - & 8(0.67) & 3(0.15) & - & 4(0.33) & 2(0.08) \\
V & - & 7(0.59) & 2(0.06) & - & 6(0.50) & 5(0.21) & - & 8(0.67) & 5(0.21) \\
\hline
Agg. (I-V) & 24(1.00) & 68(0.71) & 7(0.12) & 24(1.00) & 61(0.64) & 9(0.15) & 24(1.00) & 44(0.46) & 12(0.20) \\
VI & - & 7(0.58) & 5(0.21) & - & 5(0.42) & 3(0.13) & - & 6(0.50) & 8(0.33) \\
VII & 6(1.00) & 15(0.63) & 3(0.50) & 6(1.00) & 15(0.63) & 1(0.17) & 6(1.00) & 11(0.46) & 0(0.00) \\
VIII & - & 18(0.75) & 1(0.08) & - & 18(0.75) & 2(0.17) & - & 14(0.58) & 1(0.08) \\
IX & 12(1.00) & 20(0.83) & - & 12(1.00) & 17(0.71) & - & 12(1.00) & 10(0.42) & - \\
X & - & 6(0.50) & 2(0.08) & - & 7(0.58) & 4(0.17) & - & 10(0.83) & 2(0.08) \\
\hline
Agg. (VI-X) & 18(1.00) & 66(0.69) & 11(0.17) & 18(1.00) & 62(0.65) & 10(0.15) & 24(1.00) & 38(0.40) & 11(0.17) \\
Agg. (I-X) & 42(1.00) & 134(0.70) & 18(0.14) & 42(1.00) & 123(0.64) & 19(0.15) & 42(1.00) & 95(0.49) & 23(0.18) \\
\hline
\end{tabular}
\caption{Efficiency-Rate for Each $b$ in BASE1, BASE2 and TRADE}
\end{table}

This issue by controlling for the sequences to which the states are presented in each treatment. This control also ensures that the number of subjects starting each round observing $b = 0, 1, 2$ will be identical across all treatments. Furthermore, even though at each round, the number of subjects in TRADE who might eventually participate in the Hat-Puzzle stage could be less than those of BASE1 and BASE2, there will still always be the same number of tokens in all treatments, due to be redeemed at $\beta^*$. For these reasons comparisons of the Efficiency-Rate between treatments could help our developer insights to our research question.

To provision for a general overview of our results, we present on Figure 3.1, the Efficiency-Rate at each round of the respective treatments. There is a noticeable fall in the Efficiency-Rates around rounds 4-6 and 10. This was largely due to the sequence of states where most subjects were presented with $b = 2$ in those rounds. Two observations can be immediately made. Firstly, the Efficiency-Rates in BASE1 and BASE2 do not seem different across rounds. Secondly, the Efficiency-Rates in TRADE were often found to be lower that those in the other two treatments.

In general, there are two possible outcomes when a subject sees $b > 0$, one which his own hat is red ($b(R)$) and other which his own hat is black ($b(B)$). In BASE2 and TRADE, we do not find any significant within treatment differences, between the pooled Efficiency-Rate in comparisons of $b(R)$ and $b(B)$ for $b = 1$, 2. However, in BASE1 some differences were observed but only in the case where $b = 1$. Hence for the ease of exposition, our discussions will hence distinguish the outcomes solely on $b$.\(^{40}\)
redeemed at the equilibrium rate $\beta^*$, with the ratio in parenthesis. Interpretation for BASE1 and BASE2 should be straightforward. For example in round 1 of BASE1, there were 24 subjects who began the round observing $b = 1$ black hats - all entered the Hat-Puzzle stage. However, only 16 of those subjects were found to have adhered to the equilibrium actions, and thus only 16 tokens were redeemed at $\beta^* = 900 - 50(1) = 900$ ECU. The Efficiency-Rate was thus computed to be $16/24 \approx 0.67$. Interpretation of TRADE’s data is less straightforward. In the round 1 of TRADE, there were again 24 subjects who began the round observing $b = 1$. However, after trading tokens in the Market stage, only 19 subjects had eventually entered the Hat-Puzzle stage. Out of these 19 subjects, 11 subjects were found to have adhered to the equilibrium actions but 12 tokens were redeemed at the equilibrium rate - this implies that one of the 11 subjects must be owning two tokens. The Efficiency-Rate was thus computed to be $12/24 = 0.50$. To also control for learning, we report on the rows “Agg. (I-V),” “Agg. (VI-X)” and “Agg. (I-X),” the pooled Efficiency-Rates for $b = 0$, $b = 1$, and $b = 2$, over the relevant rounds.

The Efficiency-Rates in all treatments were found to be unity when $b = 0$. This should not be surprising since, seeing $b = 0$ black hats corresponds to the most trivial variation of the Hat-Puzzle stage, where having been publicly informed that there is at least one black hat, subjects immediately ascertained their hats to be black - choose $a_b$ in period 1. Only when $b > 0$ does counterfactual reasoning influence subjects’ behaviours. When $b = 1$, we now notice a fall in the Efficiency-Rates across all treatments, though this fall is larger in TRADE. Finally, when $b = 2$, the Efficiency-Rates was found to be below 0.20 in all treatments.

Our design allows for three pairwise comparisons (BASE1 vs. BASE2; BASE1 vs. TRADE; BASE2 vs. TRADE) for each $b$ and over all $b$ instances. We present on Table 3.5 the p-value for Fisher’s two-tail exact test in each comparisons, where the treatment with the higher Efficiency-Rate is presented in parenthesis. Here, the panel $b = 0, 1, 2$ refers to the pooled Efficiency-Rate over all $b$ instances. This leads us to the following two results.

**Result 1:** Consistent with H1, the Efficiency-Rates in BASE1 and BASE2 were not found to be significantly different.

**Result 2:** Contrary to H2, the Efficiency-Rate in TRADE was found to be significantly lower than those in BASE1 and BASE2. The differences were primarily driven by the lower Efficiency-Rates at instances where subjects in TRADE observed $b = 1$.

Support for the above results are immediate from the test statistics. We do not find any significant differences between Efficiency-Rates in BASE1 and BASE2 in any pairwise comparisons. However, the fall in Efficiency-Rates from $b = 1$ to $b = 2$ is fairly noticeable in the two treatments. A potential explanation is that most

<table>
<thead>
<tr>
<th>$b$</th>
<th>BASE1</th>
<th>BASE2</th>
<th>BASE1</th>
<th>BASE2</th>
<th>BASE1</th>
<th>BASE2</th>
<th>BASE1</th>
<th>BASE2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
</tr>
<tr>
<td>0</td>
<td>1.00(NA) 1.00(NA)</td>
<td>1.00(NA) 1.00(NA)</td>
<td>1.00(NA) 1.00(NA)</td>
<td>1.00(NA) 1.00(NA)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
</tr>
<tr>
<td>2</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
</tr>
<tr>
<td>0, 1, 2</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
<td>BASE2 TRADE</td>
</tr>
</tbody>
</table>

Table 3.5: Comparison of Efficiency-Rates: Fisher’s Two-Tail Exact Test (p-values)
subjects found the decision-making task too complex when observing $b = 2$. Some support for this is evident in their choices, where 50% and 40% of the instances in BASE1 and BASE2, where subjects observed $b = 2$, corresponded to them deviating from the equilibrium actions in the very first period of the Hat-Puzzle stage. To some extent, Result 1 is also convenient since it suggest that increasing the number of subjects under each hat has little or no obvious influences on their behaviours in the Hat-Puzzle stage. Therefore if market mechanism in TRADE had lead to the allocation of tokens to the better abled subjects, we should expect the Efficiency-Rates in TRADES to be higher when $b = 1$ and $b = 2$.

Obviously, this is not the case. The Efficiency-Rate in TRADE pooling over $b = 0, 1, 2$ was found to be significantly lower than those observed in the latter two treatments. Furthermore, this seems to primarily driven by the $b = 1$ instances in TRADE. This is an interesting observation for the following reasons. When $b = 0$, the task was too trivial, and we do not observe any differences between the treatments. When $b = 2$, the task was too complex for most subjects, thus any marginal influence from the ability to trade tokens was minimal. As such, the “tipping point” lies at instances where subjects observed $b = 1$. Taken together, Results 1 and 2 suggest that instead of improving aggregated performances, the market mechanism in TRADE had actually lead to worse performances relative to BASE1 and BASE2. Why this might be so, will be studied in greater detail in the following sub-section.

With repeated games, the reader might be concerned with potential learning over rounds. Table 3.4 suggest there to be little evidence for the learning phenomenon. For example, when $b = 1$, the pooled Efficiency-Rate in BASE1 over rounds I-V and VI-X, were found to be 0.71 and 0.69 respectively.

### 3.6.2 Exploring Subjects’ Behaviours in TRADE

The richness of our data provides for the opportunity to explore many interesting questions. For example, how had subjects best-responded to the behaviours of other subjects or how subjects decisions in a particular round are correlated to their “experiences” in the previous rounds. Whilst many of these questions are certainly interesting, we will however restrict our discussions to the main focus of the paper, the role of the markets in the allocation of tokens.

We begin by presenting on Figure 3.2, the mean and median prices from Market stage of TRADE, where the horizontal lines in each panel refers to the equilibrium price $p^* = 950 - 50(b)$. Here, each observations

\[ p^* = 950 - 50(b) \]

With two experimental sessions, each round in TRADE will therefore comprises of six individual markets, each with their own median and mean prices.
Figure 3.3: Payoffs in TRADE for Subjects who Participated and Did not Participate in the Hat-Puzzle Stage

![Payoff Graph](image)

refers to the median and mean price for a specific market. The mean number of transactions was 3.45 in all markets. Given that each market only consisted of six subjects, there seems to be a robust number of transactions.

We immediately observe mean and median prices to be frequently found above the relevant equilibrium price $p^*$ - 57%, 46% and 71% of observations in markets where subjects observed $b = 0, 1, 2$ respectively. This phenomenon is sometimes known as “Price Bubbles” and is often documented in experimental markets. Given the design of TRADE, price bubbles could have severe implications on the allocative outcomes of markets since at prices above $p^*$, our behavioural predictions is for Sophisticated types to sell their tokens and avoid the Hat-Puzzle stage altogether, since they increase their payoffs for doing so. To see this more clearly, we present on Figure 3.3 the box plot of subjects' payoffs from the relevant $b$ markets, depending on whether they had participated in the Hat-Puzzle stage. Subject who had sold their tokens and avoid the Hat-Puzzle stage were found to have made significantly higher payoffs in markets where $b = 0$ (Mann-Whitney $z = 4.771$), $b = 1$ (Mann-Whitney $z = 3.040$) and $b = 2$ (Mann-Whitney $z = 2.267$). Clearly this raises the question about the sophistications of those subjects who had purchased tokens at such elevated prices. More specifically, could Result 2 (in the previous sub-section) be attributed to the market allocating tokens to subjects who were less likely to adhere to the equilibrium actions?

Another interesting question is whether the market prices of tokens are decreasing with $b$. Though transactions prices in markets where $b = 0$ were found to be significantly higher than those where $b = 1$ (Mann-Whitney $z = 3.487$) and those where $b = 2$ (Mann-Whitney $z = 4.135$), the same was not found in comparisons of markets where $b = 1$ against those of $b = 2$ (Mann-Whitney $z = 1.339$). This suggest that subjects in markets where $b = 2$ may have wrongly anticipate the same equilibrium behaviour as when observing $b = 1$.

If one is to make the case that subjects who had purchased more tokens often had relatively less understanding of the equilibrium actions, what should we naturally expect? Firstly, the Adherence-Rate should be lower for subjects who entered Hat-Puzzle stage with more than one token, relative to those with exactly one token conferring them the rights to participate in the Hat-Puzzle stage. After one of the TRADE session, the experimenter was approached by one of the subjects who posed the following question: “Why do you need three hats, my choice when I see one black hat should be the same as seeing two black hats”.

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42 The mean number of transactions were 4.57, 3.45 and 3.33 in markets where subjects observed $b = 0, b = 1$ and $b = 2$ black hats, respectively.
43 It has been suggested that the price bubble could be associated to some “premium” which subjects are willing to spend to participate in the Hat-Puzzle stage, instead of doing nothing in the experimental round. This is unlikely since subjects already began the round with one token conferring them the rights to participate in the Hat-Puzzle stage.
44 After one of the TRADE session, the experimenter was approached by one of the subjects who posed the following question: “Why do you need three hats, my choice when I see one black hat should be the same as seeing two black hats”.

43
Adherence Rates and Rate of Deviation in Period 1 by Token Ownership (TRADE)

Table 3.6: Adherence-Rates and Rate of Deviation in Period 1 by Token Ownership (TRADE)

<table>
<thead>
<tr>
<th>Tokens Owned</th>
<th>$b = 0$</th>
<th>$b = 1$</th>
<th>$b = 2$</th>
<th>Agg.</th>
<th>$b = 0$</th>
<th>$b = 1$</th>
<th>$b = 2$</th>
<th>Agg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10(1.00)</td>
<td>47(0.67)</td>
<td>12(0.32)</td>
<td>69(0.59)</td>
<td>0(0.00)</td>
<td>13(0.18)</td>
<td>9(0.24)</td>
<td>22(0.22)</td>
</tr>
<tr>
<td>2</td>
<td>9(1.00)</td>
<td>16(0.47)</td>
<td>1(0.05)</td>
<td>26(0.41)</td>
<td>0(0.00)</td>
<td>14(0.41)</td>
<td>10(0.50)</td>
<td>24(0.44)</td>
</tr>
<tr>
<td>3</td>
<td>3(1.00)</td>
<td>5(0.41)</td>
<td>3(0.33)</td>
<td>11(0.45)</td>
<td>0(0.00)</td>
<td>8(0.41)</td>
<td>2(0.22)</td>
<td>7(0.33)</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>1(0.50)</td>
<td>0(0.00)</td>
<td>1(0.17)</td>
<td>-</td>
<td>1(0.50)</td>
<td>4(1.00)</td>
<td>5(0.83)</td>
</tr>
<tr>
<td>5</td>
<td>1(1.00)</td>
<td>0(0.00)</td>
<td>-</td>
<td>1(0.33)</td>
<td>0(0.00)</td>
<td>1(0.50)</td>
<td>-</td>
<td>1(0.50)</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>-</td>
<td>0(0.00)</td>
<td>0(0.00)</td>
<td>-</td>
<td>-</td>
<td>1(1.00)</td>
<td>1(1.00)</td>
</tr>
</tbody>
</table>

Agg. Tokens $> 1$ | 13(1.00) | 22(0.44) | 4(0.12) | 39(0.40) | 0(0.00)| 21(0.42) | 17(0.50) | 38(0.45) |

Agg. Tokens $\geq 1$ | 23(1.00) | 68(0.57) | 16(0.23) | 107(0.50)| 0(0.00) | 34(0.28) | 26(0.37) | 60(0.31) |

token. Secondly, we should expect subjects with more than one token to more frequently deviate from the equilibrium action at the very first period of the Hat-Puzzle stage. The later point is due to the redemption structure of tokens, where it is only dominant for a subject to adhere to the equilibrium action if he expects to ascertain his hat colour with doing so. We thus present on Table 3.6 the Adherence-Rates and the rate of deviation in the very first period of the Hat-Puzzle stage, by token ownership. Here each cell under the header “Adherence-Rates” reports on the total number subjects in the Hat-Puzzle stage who had adhered to the equilibrium action, with the ratio in parenthesis. For example, there were 70 instances where subjects in TRADE observing $b = 1$ had entered the Hat-Puzzle stage with exactly one token, out of which subjects were found to have adhered to the equilibrium actions in 47 instances - the adherence rate was therefore $\frac{47}{70} \approx 0.67$.

Each cell under the header “Rate of Deviation in First Period” reports on the total number of subjects who had deviated from the equilibrium action in the very first period of the Hat-Puzzle stage, with the ratio in parenthesis. The final two rows report on the aggregated statistics for instances where subjects entered the Hat-Puzzle stage with more than one token and at least one token.

At instances where $b = 0$, the ownership of tokens had no influence on the Adherence-Rates. However, at instances where $b = 1$ or $b = 2$, the adherence rates were found to decrease with token ownership. Similarly, at instances where $b = 1$ or $b = 2$, the rate of deviation in the first period of the Hat-Puzzle stage again seems to be increasing with token ownership. These observations are clearly contradictory to H3 and suggest that the Market stage had resulted in tokens being bought by subjects who were less likely to adhere to the equilibrium actions and potentially, less understanding of the equilibrium actions.

An interesting observation that also arises from Table 3.6 is that the aggregated Adherence-Rates (0.59) for subjects with exactly one token is not significantly different from those in BASE1 and BASE2. This suggest that subjects who did not purchase additional tokens had largely behaved in the same manner as those in BASE1 and BASE2. More importantly, this provide some support that the behaviours of subjects (Hat-Puzzle stage) with exactly one token or more than one token are different.

This of course leads to the question as to whether subjects who had adhered to the equilibrium actions actually earned higher payoffs? We thus present of Figure 3.4 the relevant payoffs for those subjects in TRADE who had adhered (numeral 1) or deviated (numeral 0) from the equilibrium actions. Payoffs were found to be significantly higher for the former in instances where $b = 1$ (Mann-Whitney $z = 2.622$) but not in instances where $b = 2$ (Mann-Whitney $z = 0.304$).

As with any markets, the demand for tokens is also conditional on the market prices. To make the argument that prices bubbles had lead to tokens being purchased by those who were less likely to adhere to the equilibrium action, should inversely imply that subjects who had purchased tokens at non-bubble prices were more likely to adhere to the equilibrium actions - if this is not true then price bubbles and token allocation could be independent phenomenons. We hence derived for each subject in TRADE, his average purchase price ($\bar{p}$), which was computed as the sum of all his purchasing expenditure in the market divided by the total number of purchases.
Figure 3.4: Payoffs for Subjects who had Adhered and Deviated from the Equilibrium Actions (TRADE)

![Box plots showing payoffs for subjects who adhered (b=1) and deviated (b=0) vs (b=2) in rounds 1 and 2.]

Figure 3.5: Average Purchased Prices for subjects who Adhered or Deviated from the Equilibrium Action (TRADE)

![Scatter plots showing average purchase prices for adhered and deviated actions across rounds 1 to 10.]

Average Purchase Price (ECU)

Round
of tokens purchased.\footnote{As trade was facilitated through a continuous double auction mechanism, subjects could purchase and sell tokens simultaneously within the trading period. Thus the average purchase price seeks to normalise his overall purchasing activities within the trading period. One could alternatively consider the average sale price, however we prefer to work with the purchasing activities since it may better describe a subject’s expected token redemption rate.} Given this, we present on Figure 3.5 the average purchase prices for subjects who had adhered (hollow circle) or deviated (cross) from the equilibrium actions. The area between the horizontal lines on each panel denotes the range of prices which Sophisticated types in our behavioural discussions are predicted to purchase tokens at. This of course excludes all observation where subjects were inactive - did not purchase tokens in the market - or had sold all their tokens.

In markets where \( b = 1 \), we notice that subjects who had adhered to the equilibrium had more often than not purchased tokens at \( \bar{p} \in (600, 950 - 50(b)] \). The same phenomenon is also observed in markets where \( b = 2 \).\footnote{In markets where \( b = 0 \) we often observe that \( \bar{p} > (600, 950 - 50(b)] \) but all subjects had adhered to the equilibrium action.} To see this more clearly, we present on Table 3.7 the Adherence-Rates at instances where \( \bar{p} \in (600, 950 - 50(b)] \) and \( \bar{p} \notin (600, 950 - 50(b)] \). One immediately observes the rates to be higher in the former relative to the latter condition when \( b = 1 \) - 0.67 and 0.25 respectively - and when \( b = 2 \) - 0.33 and 0.10 respectively.

Taken together, these observations provide some support that price bubbles in the Market stage of TRADE might indeed have contributed to lower Efficiency-Rate in TRADE relative to the control treatments. More specially, the observations are contrary to H3 but seem to support H4. These will be formally tested in our econometric analysis.

### 3.6.3 Econometric Analysis

This section employs econometric methods to study the determinants of individual subject’s adherence to the equilibrium actions. To ensure variation in data, we first omitted all observations where subjects had observed \( b = 0 \) or did not participate in the Hat-Puzzle stage. Thereafter we pooled the BASE1 and BASE2 observations into the BASE sample and introduced the situation dummy “TRADE” to distinguish between the relevant Treatments.\footnote{We pooled the observations from BASE1 and BASE2 as our previous analysis found little to distinguish between the data in both treatments.} The relevant situation dummies were introduced for observations where \( b = 1 \) and \( b = 2 \). We also included controls for the number of tokens owned and whether subjects had purchased tokens at \( \bar{p} \in (600, 950 - 50(b)] \). Finally, to control of subject specific covariates, we included situation dummies for course of studies, gender, prior familiarity with the Hat-Puzzle problem and sequence of states administered, and the subjects’ score in the Cognitive Reflective test.

Given the above, we present on Table 3.8 the estimates of our panel random-effects logistic regressions, where the dependent variable refers to observations where subjects had adhered to the equilibrium action and the relevant standard errors are presented in parenthesis. For completeness, we also conducted the same regression exercise for observations from TRADE only and observations from BASE only.

We will restrict most of our discussions to the estimates with the pooled TRADE and BASE observations. To do so, we will make references to the \textit{average subject} a hypothetical subject where all the coefficients are set to their averages.

As expected the likelihood of adherence for an average subject is significantly lower when \( b = 2 \) relative to \( b = 1 \). Furthermore, there is some evidence that when \( b = 1 \), the likelihood of adherence is significantly lower for an average subject in TRADE relative to an average subject in BASE. However, no significant difference was found when \( b = 2 \).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
& \( b = 0 \) & \( b = 1 \) & \( b = 2 \) \\
\hline
\( \bar{p} \in (600, 950 - 50(b)] \) & 3(1.00) & 27(0.67) & 3(0.33) & 939(0.82) \\
\hline
\( \bar{p} \notin (600, 950 - 50(b)] \) & 13(1.00) & 7(0.25) & 3(0.10) & 23(0.32) \\
\hline
Agg. & 16(1.00) & 34(0.50) & 6(0.15) & 46(0.37) \\
\hline
\end{tabular}
\caption{Adherence-Rates by Average Purchase Price (TRADE)}
\end{table}
Dependent Variable: Adherence to the Equilibrium Actions

<table>
<thead>
<tr>
<th>Sample</th>
<th>(BASE + TRADE)</th>
<th>(TRADE)</th>
<th>(BASE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est.</td>
<td>Est.</td>
<td>Est.</td>
</tr>
<tr>
<td>(b = 2)</td>
<td>(-4.048a)</td>
<td>(-2.144a)</td>
<td>(-3.560a)</td>
</tr>
<tr>
<td></td>
<td>((0.690))</td>
<td>((0.940))</td>
<td>((3.133))</td>
</tr>
<tr>
<td>(b = 1\times\text{TRADE})</td>
<td>(-1.006b)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>((0.548))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b = 2\times\text{TRADE})</td>
<td>0.596</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>((0.645))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b = 1\times\text{Token})</td>
<td>(-0.842a)</td>
<td>(-0.791b)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>((0.385))</td>
<td>((0.372))</td>
<td></td>
</tr>
<tr>
<td>(b = 2\times\text{Token})</td>
<td>(-0.269)</td>
<td>(-0.365)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>((0.514))</td>
<td>((0.493))</td>
<td></td>
</tr>
<tr>
<td>(b = 1\times\text{TRADE} \times p \in (600, 950 - 50(b)])</td>
<td>1.640a</td>
<td>1.527(a)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>((0.606))</td>
<td>((0.584))</td>
<td></td>
</tr>
<tr>
<td>(b = 2\times\text{TRADE} \times p \in (600, 950 - 50(b)])</td>
<td>0.616</td>
<td>0.459</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>((0.998))</td>
<td>((0.962))</td>
<td></td>
</tr>
<tr>
<td><strong>Sequence</strong></td>
<td>0.133</td>
<td>0.149</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td>((0.405))</td>
<td>((0.629))</td>
<td>((0.496))</td>
</tr>
<tr>
<td><strong>Familiarity</strong></td>
<td>(-0.353)</td>
<td>(-1.385)</td>
<td>(-0.065)</td>
</tr>
<tr>
<td></td>
<td>((0.560))</td>
<td>((61.33))</td>
<td>((0.594))</td>
</tr>
<tr>
<td><strong>Gender</strong></td>
<td>0.670</td>
<td>0.780</td>
<td>0.654</td>
</tr>
<tr>
<td></td>
<td>((0.425))</td>
<td>((0.620))</td>
<td>((0.543))</td>
</tr>
<tr>
<td><strong>CRT Score</strong></td>
<td>0.079</td>
<td>0.179</td>
<td>0.560(b)</td>
</tr>
<tr>
<td></td>
<td>((0.138))</td>
<td>((0.169))</td>
<td>((0.227))</td>
</tr>
<tr>
<td><strong>Business School</strong></td>
<td>0.676</td>
<td>0.117</td>
<td>0.237</td>
</tr>
<tr>
<td></td>
<td>((0.873))</td>
<td>((0.862))</td>
<td>((0.907))</td>
</tr>
<tr>
<td><strong>Eng, Math &amp; Phy Science</strong></td>
<td>1.277</td>
<td>2.279</td>
<td>0.584</td>
</tr>
<tr>
<td></td>
<td>((1.015))</td>
<td>((1.094))</td>
<td>((1.396))</td>
</tr>
<tr>
<td><strong>Humanities</strong></td>
<td>0.562</td>
<td>-</td>
<td>0.215</td>
</tr>
<tr>
<td></td>
<td>((0.982))</td>
<td></td>
<td>((1.023))</td>
</tr>
<tr>
<td><strong>Life &amp; Environmental Science</strong></td>
<td>0.729</td>
<td>-17.39</td>
<td>0.948</td>
</tr>
<tr>
<td></td>
<td>((1.064))</td>
<td>((45.06))</td>
<td>((1.131))</td>
</tr>
<tr>
<td><strong>Social Science &amp; Int'l Studies</strong></td>
<td>0.922</td>
<td>-</td>
<td>0.499</td>
</tr>
<tr>
<td></td>
<td>((0.375))</td>
<td></td>
<td>((1.004))</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>0.862</td>
<td>0.839</td>
<td>-0.334</td>
</tr>
<tr>
<td></td>
<td>((0.907))</td>
<td>((0.909))</td>
<td>((0.875))</td>
</tr>
<tr>
<td><strong>Negative Log-Likelihood</strong></td>
<td>463.03</td>
<td>101.44</td>
<td>295.05</td>
</tr>
<tr>
<td>(n)</td>
<td>827</td>
<td>191</td>
<td>636</td>
</tr>
</tbody>
</table>

\(a\): Significant at 1%, \(b\): Significant at 5% and \(c\): Significant at 10% levels

### Table 3.8: Random-Effect Panel Logistic Regression

**Result 2**: Consistent with Result 2, the likelihood of adherence to the equilibrium actions for an average subject in TRADE was significantly lower at instances where \(b = 1\), relative to an average subject in BASE. At instances where \(b = 2\), no significant effect was observed.

For an average subject in TRADE, the likelihood of adherence decreases significantly when \(b = 1\), for each additional token owned. However when \(b = 2\), we find no significant influence of token ownership on adherence likelihood. This result is also consistent with the estimates pooling from TRADE observations only.

**Result 3**: Contrary to H3, the likelihood of adherence to the equilibrium actions was decreasing with token ownership for subjects in TRADE at instances where \(b = 1\). At instances where \(b = 2\), no significant effect was observed.

Finally, for the average subject in TRADE, the likelihood of adherence is higher at instances where \(b = 1\) and \(b = 2\) respectively, when \(\bar{p} \in (600, 950 - 50(b)]\), relative to other instances where subjects were found to be inactive or \(\bar{p} \notin (600, 950 - 50(b)]\). Once again, only the former was found to be significant. To confirm this finding, we also considered an alternative regression where the interactive dummy variables were specified for subjects in TRADE for \(\bar{p} \notin (600, 950 - 50(b)]\). Here the likelihood of adherence was found to be significantly lower for subjects with \(\bar{p} \notin (600, 950 - 50(b)]\) at instances where \(b = 1\) but not significantly different at instances where \(b = 2\). Again this result is consistent with the estimates pooling from the TRADE observations only.

**Result 4**: Consistent with H4, the likelihood of adherence to the equilibrium actions was found to higher for those whose \(\bar{p} \in (600, 950 - 50(b)]\) relative to those who were inactive or whose \(\bar{p} \notin (600, 950 - 50(b)]\), at instances where subjects in TRADE observed \(b = 1\). At instance where \(b = 2\), no such relationship was found to be significant.
The regressions did not find any significant effects due to differences in gender, sequence administered, schools or prior familiarity. The latter point is interesting since the decision-making task for subjects in the Hat-Puzzle stage might be trivial if they had prior familiarity with the problem. However, this finding highlights a central feature of the Hats Puzzle game, such that prior familiarity might only helpful if it was common knowledge.

There is some mild evidence that the likelihood of adherence is increasing with the subjects’ scores in the Cognitive Reflective Test (CRT), but only amongst subjects in the BASE observations. The CRT test involved three questions which required subjects to employ some effort in thought and reasoning before providing the answers. Given that the adherence to the equilibrium actions in the Hat-Puzzle stage also requires logical reasoning, perhaps the CRT test score is capturing some of these abilities.

Once again the results raises the question as to why any differences between the BASE and TRADE treatments or within the TRADE treatment, were only found to be significant at instances where \( b = 1 \). Our prior on this matter is that the task where \( b = 2 \) was too complicated or complex for most subjects to comprehend. By this extension, subjects may have perceived the equilibrium actions at instances where \( b = 1 \) to be similar to those where \( b = 2 \). If such misperception were indeed reflected in the purchase prices of tokens in TRADE, one should expect the average purchase prices at instances where \( b = 1 \) to not be significantly different from those where \( b = 2 \). We hence conducted a panel linear regression on the average purchase price \( \bar{p} \) with the situation dummies \( b_0 \) and \( b_2 \) which refer to these instances where subjects in TRADE observed \( b = 0 \) and \( b = 2 \), with \( b = 1 \) as the reference.

\[
\bar{p} = 878.34 + 94.93b_0 - 18.13b_2; \quad N = 124, \quad R^2 = 0.10
\]

The regression result found \( \bar{p} \) to be significantly higher at instances where \( b = 0 \) relative to instances where \( b = 1 \). However, at instances where \( b = 2 \), \( \bar{p} \) was not found to be significantly different from those at \( b = 1 \). This lends some weight to the possibility that subjects may have misperceived the equilibrium actions at instances where \( b = 2 \) to be similar to those where \( b = 1 \).

### 3.7 Conclusions

This paper was motivated by the question as to whether markets, as suggested by the conventional wisdom, were able to allocate the rights for performing decision-making tasks to those players who are better abled to perform the tasks. To do so we use the decision-making tasks in a Hat-Puzzle motivated by the Hats Puzzle game, and introduced markets where players were able to trade their rights (in the form of tokens) for performing the tasks. Three treatments were considered and we provided an economic intuition, consistent with the conventional wisdom, that aggregated performances in TRADE should be better than those in BASE1 and BASE2.

Aggregated performances in TRADE were found to be significantly worse than BASE1 and BASE2. We show that this was primarily driven by instances in TRADE where subjects had observed \( b = 1 \) black hat. To seek some explanations to this result, we studied the mean prices in the markets of the pre-Hat-Puzzle stage in TRADE. Here, price bubbles were often observed in markets where subjects observed \( b = 0,1,2 \) black hats. We conjectured that price bubbles could have important consequences on the allocative outcomes of markets, as they might result in the tokens being purchased by subjects who might otherwise have not known the equilibrium actions. Our econometric regression provided some support for this conjecture at instance where subjects in TRADE observed \( b = 1 \). The question therefore is why might subjects in TRADE be willing to purchase tokens at prices \( p > 950 - 50b \), especially at instances where \( b = 1 \). Three possible explanations are conjectured.

The first explanation borrows from developments in Prospect Theory (Kahneman and Tversky, 1979). Given that the token redemption structure places subjects in the “loss domain”, subjects who do not know
the equilibrium actions may have an affinity of risk-taking activities and therefore subjectively value the tokens more than the equilibrium price \( \beta^* = 950 - 50(b) \). The second explanation borrows from developments in Behavioural Finances. Subjects may simply be unaware of their limitations and mis-perceive the equilibrium actions in the game. They therefore overvalue the tokens in a phenomenon that is similar to the “overconfidence effect” (Odean, 1998; Shleifer, 2000). The third explanation as suggested by Tor and Bazerman (2003) is that of “focusing failures”. The general idea here is that some subjects in TRADE might focus on the competitive nature of the Market stage and thereafter the logical reasoning aspects of the Hat-Puzzle stage. That is such subjects treat both stages as independent events and behave different across both stages. However, at present our experimental design does not allow us to distinguish between the three explanations.

Alternatively, one can argue that the two stage design in TRADE substantially increases the complexity of the experiment over BASE1 or BASE2. This might be true, but it will also be true for any mechanism design integrating decision-making tasks into security markets. The central question is therefore whether the lower aggregated performances in TRADE is purely due to the complexity of the experiment. Given that the findings in TRADE are indicative of the distortions in market allocation due to price bubbles, we believe that the lower aggregated performance in TRADE is not purely due to the complexity of design.

The results also shed some light on Kluger and Wyatt (2004) experiment findings with the Monty Hall problem, that when there are at least two Sophisticated subjects in the market, prices will converge to the equilibrium price. The authors suggest that their results are due to “Bertrand” like competitions between two Sophisticated subjects. The explanation is contested by Asparouhova, Bossaerts, Eguia and Zame (2012) who question why Unsophisticated subjects do not also participate in the Bertrand competition and why prices do not converge to the incorrect price. The results in this paper suggest that transactions often occur above the equilibrium price, even when the data suggest there to be more than two Sophisticated subjects. The former's results might have been due to the nature of the Monty Hall problem, where the “instinctive” price (Unsophisticated price) is naturally lower than the equilibrium price. When the instinctive price is less obvious, such as in this paper, their results may no longer hold.

To some extend, the results might also shed some light to the empirical literature in corporate governance. In an extensive survey of the corporate takeover literature by Martynova and Renneboog (2008), the authors found little evidence that operating performances of the acquired firms had improved ex-post takeovers. Surveys on behavioural finance also suggest that bidding firms often overpay in corporate takeovers, a phenomenon usually known as the “Winner's Curse” (Kagel and Levin, 1986; Thaler, 1988; Barberis and Thaler, 2003). This paper captures some of the discussions with respect to the Winner’s Curse as most subjects in TRADE who had purchased additional tokens, had done so at elevated prices and were found to have performed worse in the decision-making tasks.

To conclude, this paper provides evidence that introducing a market where rights for performing tasks can be traded, do not naturally lead to the allocation of such rights to those players who are better abled perform them. This is contradictory to the conventional wisdom and has important implications for any economic designer considering the best mechanism to allocate decision-making tasks. Again it is important to emphasis that the task in this paper pertains to those requiring players to employ logical and epistemological reasoning. The conventional wisdom may hold in other circumstances, when the performances in the decision-making tasks depend on other factors such as effort, information, knowledge or expertise. Nevertheless, we see potential for such a market design in other more straightforward games, e.g., Guessing Hat-Puzzle (Nagel, 1995), Centipede Game. This will be a direction for further research.
CHAPTER 4

Information Aggregation in Arrow-Debreu Security Markets: An Experiment based on the “Red Hats Puzzle”

4.1 Introduction

The “information aggregation” properties of markets were first formally noted by Hayek (1945) and subsequently further developed, notably by Grossman (1976, 1981) and Grossman and Stiglitz (1980), and more recently by Ostrovsky (2012). The general intuition is rooted in the Rational Expectations (e.g., Lucas, 1972; Muth, 1961) model, such that if each trader knows how information affects prices, they can infer from the movement of prices, the information held by each other trader. Furthermore, if traders condition their demands for securities on such information, then prices will move till at equilibrium, no trader can learn any more from prices than what the other traders already know.1

Given the central role of markets in economic theory, not least the Efficient Market Hypothesis (Fama, 1970), economists have often used the laboratory settings to investigate the information aggregation properties of markets. Early experiments (e.g., Plott and Sunder, 1982, 1988; Copeland and Friedman, 1987) had demonstrated the markets’ capacity to aggregate information, though the nature of aggregation was found to be dependent on the markets’ design e.g., complete markets, incomplete markets. Subsequent experiments found that the nature of aggregation was dependent on the experience of experimental subjects (Forsythe and Lundholm, 1990), the “complexity” of the securities’ valuation (O’Brien and Srivastava, 1991), the possibility of information mirage or misaligned beliefs (Camerer and Weigelt, 1991; Noeth et al., 1999), the presence of price-manipulators (Hanson et al., 2006; Veiga and Vorsatz, 2010) and so forth.2,3

However, studies by behavioural researchers (e.g., De-Bondt and Thaler, 1985; Hirshleifer, 2001; Odean, 1998; Shleifer, 2000; Shleifer and Vishny, 1993) suggest that the Rational Expectations model places too much demands upon the forecasting abilities of traders, and consequently the capacity of markets to aggregate information.4,5 To provide some resolution to the above concerns, this paper revisits the question about

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1Radner (1979) showed the Rational Expectations price equilibrium to be equivalent to the outcome whereby traders openly communicated their private information.
2Camerer and Weigelt (1991) described information mirage as the situation whereby market prices are seemingly communicating some information, even when no participant in the market has such information. Therefore prices are a “mirage”.
3See Duxbury (1995), Sunder (1995), Noussair and Tucker (2013) and Deck and Porter (2013) for surveys of market experiments that include discussions about information aggregation.
4As noted by Bruguiere, Quartz and Bossaerts (2010), the Rational expectation model has been fairly successful in explaining information aggregation in Plott and Sunder (1988) markets, but failed tremendously in other simple designs such as in Smith, Suchanek and Williams (1988) multi-period single asset market.
5Most of the arguments made by behavioural research broadly fall into the category of psychological biases (see Barberis and Thaler, 2003, for an overview). Generally speaking, it suggest that the failure of traders’ adherence to the prescribed behaviours of the Rational expectation model are largely due to bounds on their mental capacity for doing so.
the capacity of markets to aggregate information in the laboratory setting.

Our experiment uses the information dissemination structure of the “Red Hat Puzzle” (Littlewood, 1953), an often cited logical reasoning problem, to endow traders with private, diverse and partial information about the true state of nature. Thereafter, traders participate in a complete market where 4 types of Arrow-Debreu securities are traded. Building on the previous works of Plott and Sunder (1988) and Plott (2000), who showed that complete markets are successful in aggregating information into prices, we begin with the working hypothesis that markets should be successful at aggregating information into prices, an outcome predicted by the Rational Expectations model. The innovation here are two-fold. Firstly, we are able to show that the process of aggregation information into prices is non-trivial and demanding, requiring traders to employ counterfactual reasonings. Secondly, for some states of nature in our design, the same information aggregation process should lead to a specific group of traders to learn about the true state before others, allowing us to study the process of information aggregation. Finally, ex-post our markets, traders are required to submit their beliefs about the true state of nature by selecting through a menu of possibilities. The experiment therefore allows us to study orthogonal issues on the nature of information aggregation, the prices of securities in the markets and the beliefs of traders ex-post the market.

This paper reports on 7 sessions, each consisting of 10 experimental rounds. To evaluate the notion of information aggregation, we benchmarked the performances of the Rational Expectation model in explaining market prices and subjects beliefs ex-post the market, against two alternative equilibrium-driven models. Whilst the Rational Expectations represents a “fully endogenous” process, where traders continuously infer signals about the true state from prices, the alternative models represent a “fully exogenous” (Forsythe and Lundholm, 1990) process, where traders condition their demands for securities based on their private information and ignore any signals conveyed from prices. Stated differently, the alternative models depict the bounds on the counterfactual reasoning abilities of traders, a feature that is commonly emphasis in behavioural discussions.

Our results can be summarised as followed. Only when subjects were relatively experienced (rounds 5-10), was the Rational Expectations model able to explain security prices significantly better than the alternative models. This result point to both the complexity of the information aggregation task and the ability of Arrow-Debreu markets in aggregation information into prices, albeit only when market participants have some experiences. However, even when subjects were relatively experienced (rounds 5-10), the Rational Expectations model was only able to explain subjects’ ex-post markets beliefs as well as one of the alternative models. This suggest that it is possible for Arrow-Debreu markets to be successfully at aggregation information into security prices even when we cannot conclude that market participants are informed of the true state. Stated differently, success of information aggregation in prices do not necessarily imply the same success in the beliefs of market participants. We are also able to provide some insights to the process of information aggregation that subjects, which were predicted by our discussions to do so, had learnt about the true state of nature before the others.

The rest of the paper is organised as followed. Section 4.2 details our experimental design, Section 4.3 discusses the relevant economic theories, Section 4.4 details our experimental results and Section 4.5 concludes.

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6 Variations of the Red Hat Puzzle are often found in discussions of common knowledge (Geanakoplos, 1994) and reasonings (Fagin et al., 1995). The puzzle requires the decision maker to employ counterfactual reasoning, and has been adapted into experimental settings to study bounded rationality (Weber, 2001; Bayer and Chan, 2007). The puzzle is often told as a scenario whereby three girls each wearing black hat are able to see each other girls hat but their own - they remain uncertain of their own hat. An observer publicly announces that “there is at least one black hat” and asked the first girl if she knew the colour of her hat. The first girl replied with “No”. The same question was posed to the second girl, who once again replied with “No”. However, when the question was posed to the third girl, she now replies with “Yes Black”. The challenge with the puzzle is to explain how the third girl might have learnt about her hat colour.

7 Our results might not be directly comparable to those of Plott and Sunder (1988) and Plott (2000), due to the structural differences in the manner by which information is disseminated.

8 Aggregating across all rounds, the Rational expectations model was only significantly better abled to explain prices than one of the alternative models. Aggregating across the first 5 rounds, the Rational expectations model performed as well as the alternative models.
State \(s \in S\) & \(s_1\) & \(s_2\) & \(s_3\) & \(s_4\) & \(s_5\) & \(s_6\) & \(s_7\) & \(s_8\) \\
Group 1 Type & X & O & X & X & X & O & O & O \\
Group 2 Type & X & X & O & X & O & X & O & O \\
Group 3 Type & X & X & X & O & O & O & X & O \\
x(\(s\)) & 3 & 2 & 2 & 2 & 1 & 1 & 1 & 0 \\

| Table 4.1: States of Nature |

4.2 Experiment Design

This paper reports on 7 experimental sessions, recruiting from the cohort of inexperience undergraduate students at the Ben-Gurion University of the Negev (Israel). All sessions were programmed and conducted with the computerised software z-Tree (Fischbacher, 2007). Each session, involved 12 subjects, who were first allowed 30 minutes to read through the instructions (see Appendix), before participating in 2 non-paying practice rounds followed by 10 experimental rounds. Subjects began each round with 5000 ECU - the fictitious experimental currency used in our experiments, and 6 units for each of the 4 “types” of securities - we denote each security type by the numeral \(j = 0, 1, 2, 3\).

Each round consisted of three stages, the Information stage, where they received some private information about the true state of nature, followed by the Market stage, where they trade securities in a continues double auction market and finally the Prediction stage, where they submit their beliefs about the true state of nature. At the end of the round, subjects’ received feedback on the true state, and their payoffs depended on their decisions in the Market and Prediction stages.

Upon completion of each session, we also conducted two additional non-incentivised surveys, Holt and Laury (2002) risk eliciting task and Frederick (2005) Cognitive Reflective Test (CRT). Finally, one experimental round was randomly chosen for experimental payment, where subjects’ payoffs in that round were converted into cash at the rate of 15 ECU to 1 Israeli Shekel (ILS). Subjects also received a 25 ILS show-up payment. The mean duration of a typical session and experimental earnings were 120 minutes and 73.33 ILS (US$21.27 at time of experiment), respectively. The design for each round will be detailed in the following sub-section, where experimental subjects will be referred to as “traders”.

4.2.1 Information Stage

The information dissemination structure in each round was inspired by Littlewood (1953) “Red Hat Puzzle”. Here traders are evenly distributed into three groups (indexed \(g = 1, 2, 3\)) with 4 members (indexed \(i = 1, 2, 3, 4\)) in each group. Denote trader \(g_i\) as the \(i\)th member of group \(g\).

Nature (the computer) chooses the true state \(s \in S \equiv H_1 \times H_2 \times H_3\), where \(H_g \in \{X, O\}\) denotes group \(g\)’s type. Assume a common prior over \(S\), where each state is equally likely. At any \(s \in S\), denote \(x(s) \in \{0, 1, 2, 3\}\) as the total number of type \(X\) groups - see Table 4.1 for an overview of the states of nature.

Each trader receives private and partial information about the true state of nature, such that they observe each other groups’ type but their own. For example, if \(s = XXX\), then traders 1, will observe that groups 2 and 3 are type \(X\) but remain uncertain of his own group. As formally shown by Aumann (1976), the information distribution structure at any \(s \in S\) induces a “common knowledge problem”, such that the only event in \(S\) that can be common knowledge, must include the entire state of nature.

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9 The working capital is designed to be sufficiently large, to never bind.
10 In our experimental sessions, we used the term “certificates S0, S1, S2 and S3” to refer to the respective securities. This was introduced to avoid confusing subjects and to assist subjects unfamiliar with the term “security”.
11 It thus follows that all traders within the same group must make the same observations.
4.2.2 Market Stage

Trade is facilitated through a continuous double auction (CDA) mechanism for a duration of 300 seconds. Short-sales are prohibited and traders are allowed to buy or sell any security types, subjected to their budget constraints. As we will describe later, subjects in our experiments enjoy some form of limited liability, as they are prevented from making negative cash earnings. To prevent subjects from intentionally making losses, the bids and asks prices for each security were bounded between 0 and 200 ECU.\textsuperscript{12} Denote $L_g \geq 0$ and $e_{g_i}^j \geq 0$ as trader $g_i$’s, inventories of working capital and security $j$ respectively, at the end of the Market stage.

4.2.3 Prediction Stage

Traders are presented with the history of transaction prices for each security type and the following question “How many type X groups are there in this Round?”. Thereafter, traders have up to 60 seconds to respond with an action $b_{g_i} = j$ where $j = 0, 1, 2, 3$. We will refer to this action as the trader’s “belief”.

4.2.4 End of Round Payoffs

At the end of the round, traders receive feedback on the true state and their payoffs are computed as followed:

$$\Pi_{g_i} = \max\{0, (L_{g_i} - \alpha) + \sum_j d(x(s), j)e_{g_i}^j + d(x(s), b_{g_i})\}$$

In general, traders’ payoffs can be decomposed to two components, the Trading-Profit and Prediction-Profit. Here $d(x(s), j)$ refers to the redemption value of security $j$, where $d(x(s), j) = 100$ if and only if $x(s) = j$ or otherwise 0. Similarly, $d(x(s), b_{g_i}) = 200$ if and only if $x(s) = b_{g_i}$ and otherwise -200. Finally, $\alpha = 5000$ refers to some fixed cost that is incurred by all traders. Notice that traders’ payoffs are bounded from below at 0. This was introduced to prevent traders from making negative payoffs - 23% of the observations in our data corresponded to bankruptcies.

In the discussions henceforth, we shall refer to the True Security as the security type, which will be redeemed at 100 ECU - the security that corresponded to the true state of nature. Likewise, we shall denoted a trader to be Correct if his submitted beliefs in the Prediction stage corresponds to the true state of nature. Finally, for states where $x(s) \in \{1, 2\}$, we denote the Minority traders as those who make different observations about the total number of type $X$ groups relative to the other traders - Minority traders do not exist when $x(s) \in \{0, 3\}$. For example if $s = XXO$, traders in group 3 are the Minority traders since they observe two other type $X$ groups, whilst those in group 1 and 2 each observe one other type $X$ group.

Assuming that each trader's utility is increasing in $\Pi_{g_i}$, then each trader should seek to maximise his inventory of the true security, unless he can be sufficiently compensated for selling that security. Also each trader is incentivised to be Correct in the Prediction stage.

4.3 Economic Theory

This paper seeks is to study the information aggregation properties of Arrow-Debreu markets, which we will show to be an onerous and cognitively challenging task. As discussed in the introduction of this paper, the Rational Expectation (RE) model describes a dynamic process where traders, endowed with partial and private information about the true state of nature, learn about the information of other traders through the movements of security prices, and inversely act on their knowledge through the market. This process continues till at

\textsuperscript{12}Given that we cannot impose negative cash transfers to subjects, this introduces a possibility that some subjects might collude to maximise their joint earnings. For example, one subject could intentionally purchase securities from another subject at high prices, making losses on their working capital. The other subjects benefits by increasing the cashflow in his working capital.
equilibrium, no trader can learn any more about the information held by other traders. Radner (1979) showed the RE price equilibrium to be equivalent to an outcome where each trader openly communicates his own private information, which in our design corresponds to perfect information about the true state of nature.\footnote{To formally verify this, we can model each traders’ observations of the other groups’ type as the information partition $\mathcal{P}_g$ over $S$, where $s' \in \mathcal{P}_g(s) \subseteq S$ denotes the states which trader $g$ assigns a positive posterior to at each $s \in S$. Since $\bigcap_g \mathcal{P}_g(s) = \emptyset$ for any $s \in S$, open communication of private information will correspond to perfect information about the true state of nature.}

This implies that the RE equilibrium in Market stage is for prices of the True and non-True securities to be 100 and 0, respectively. Since at equilibrium all traders are informed of the true state, the RE equilibrium in the Prediction stage is for all traders to be Correct. More significantly, the RE model predicts both information aggregation in prices and beliefs of traders.

To evaluate the notion of information aggregation in our experimental sessions, the RE model will be benchmarked against the Prior-Information (PI) and Naive-Information (NI) models.\footnote{Comparisons between the RE and PI models are commonly considered in the relevant literature (e.g., Banks, 1985; Barner et al., 2005; Camerer and Weigelt, 1991; Copeland and Friedman, 1987; Forsythe and Lundholm, 1990; O’Brien and Srivastava, 1991; Plott and Sunder, 1982, 1988). The NI model is included for completeness.} In contrast to the RE model, the PI and NI models describe an “exogenous information” processing behaviour (Forsythe and Lundholm, 1990), where traders condition their demand for securities on their own information and discard any information conveyed by prices. As we will detail in the following, the latter two models only differs on traders’ posterior of the true state.

Whilst the RE equilibrium is well-defined, a detailed process as to how prices converge to the RE equilibrium is less understood.\footnote{Barner, Feri and Plott (2005) provides some insights on this issue with an experimental design involving sequentially arriving information.} Is information aggregation necessarily possible in all states? How do traders infer signals about the true state from prices? Do traders have to learn about the true state at the same time? These are some of the questions that naturally arises in studies of information aggregation.

There is an extensive literature that discusses the dynamic behaviour of traders under Rational Expectations (see amongst others Dubey et al., 1987; Hellwig, 1982; McKelvey and Page, 1986; Ostrovsky, 2012; Radner, 1979). However, such discussions are often complex to study within the context of our experiment. Instead, we will seek to address the above questions through “intuitive” illustrations as to how traders might be learning about the true state from prices. Two issues should be immediately obvious from our illustrations, (a) The process of information aggregation in prices is cognitively challenging as it requires traders to employ counterfactual reasonings, and (b) For states where $x(s) \in \{1, 2\}$, the Minority traders should learn about the true state before the others.

We present on Table 4.2, the equilibrium predictions of the RE, PI and NI models, were $p_j$ refers to the equilibrium price of security $j$ and $B_j$ denotes the equilibrium proportion of traders submitting beliefs $b_{ji} = j$. In the following sub-sections, we will first detail the equilibrium predictions of the PI and NI models before presenting our illustrations of the information aggregation process.

### 4.3.1 Naive-Information Model

The NI model represents an extreme characterisation of risk-neutral traders’ behaviours, where traders are assumed to ignore both the information conveyed in prices and their own private observations. Traders thus condition their pricing behaviours on the common prior over $S$ and the market clears. This implies that for any state $s \in S$, the equilibrium prices in the Market stage must therefore be 12.5, 37.5, 37.5 and 12.5 for securities 0, 1, 2 and 3, respectively. Extending such “ignorance” to the Prediction stage will simply correspond to the predicted proportions 12.5%, 37.5% 37.5% and 12.5% for $B_0$, $B_1$, $B_2$ and $B_3$, respectively.

### 4.3.2 Prior-Information Model

The PI model extends the assumed behaviours in the NI model by allowing for traders to consider their own private observations, use Bayes law to update their posteriors about the true state of nature and the market clears. For example, if $s = XXX$, then traders $i$ will assign equal posterior to the states $XXX$ and $OXX$.
the true state of nature to be equilibrium prices form the basis of the intention here is to provide some 
indirect communication process, as predicted by the RE model, is often complex and beyond the scope of this paper. Instead, 
As we had mentioned at the start of this section, describing a model that fully captures the information aggre-
4.3.3 How information gets Aggregated

traders 2, to the states XXX and XOX, and traders 3, to the states XXX and XXO. This corresponds to the market clearing prices 0, 0, 50 and 50, for securities 0, 1, 2, and 3, respectively. Since by assumption, 
traders are unable to learn about the true state from prices, they remain uninformed. This should be reflected in their submitted beliefs where all traders will uniformly randomise between $b_{g_i} = 2$ and $b_{g_i} = 3$, resulting in the predicted proportions $B_0 = 0\%$, $B_1 = 0\%$, $B_2 = 50\%$ and $B_3 = 50\%$.

4.3.3 How information gets Aggregated

As we had mentioned at the start of this section, describing a model that fully captures the information aggre-
gation process, as predicted by the RE model, is often complex and beyond the scope of this paper. Instead, 
the intention here is to provide some intuitive illustrations as to how diverse and private information will eventually be aggregated into prices. The intuition here builds upon Jordan (1982) idea that prices will first converge to the “temporarily equilibrium” before converging towards the RE equilibrium. Each of these temporarily equilibrium prices form the basis of indirect communication (Geanakoplos and Polemarchakis, 1982) between traders, via the market price mechanism, about the true state of nature.

To do so, we begin by assuming that trade takes place over $t = 1, 2, 3, 4$ hypothetical periods, where traders at each period $t$ update their posteriors on the market prices of period $t - 1$. We also assume that each trader is non-strategic, such that they ignore possible capital gain or losses, and trade in each period as if it was the last. Finally, assume that each trader is not capital constrained. Given the above, we will denote $p^t_j$ as the equilibrium price of security $j$ in period $t$.

First consider the case where $s = XXX$. In period 1, each trader observes all other group’s type and remain uncertain of their own type. If each trader conditions their demand for securities on their private information, the resultant market clearing prices in period 1 will be $p^1_0 = p^1_1 = 0$ and $p^1_2 = p^1_3 = 50$ - the PI equilibrium. In period 2, each trader reasons that the true state of nature can only be XXX. To see why, consider the information held by trader 3, in period 1, he assigns positive and equal posteriors to the states XXX and XXO. In period 2, he observes that $p^1_1 = 0$, which implies to him that no other trader assigns positive posterior to the state XXO. By the same counterfactual reasonings, traders 1, and 2, also deduce the true state of nature to be XXX. This implies that the market clearing price in period 2 can only be

16Jordan (1982) discussion of a dynamic expectation equilibrium is sometimes known as the ‘Jordan Path’. Copeland and Friedman (1987) and Plott and Sunder (1988) have noted that the convergence of security prices to the RE equilibrium in their data seem to follow the Jordan Path. However, as Camerer and Weigelt (1991) have noted, the Jordan’s model cannot be directly applied to experimental data as it assumes that no trade will occur at the temporarily equilibrium prices.

17Given the information distribution structure, trader 3, knows that if the true state is XXO, then trader 2, must assign equal posterior to the states XOO and XXO, whilst trader 1, to the states OXO and XXO. If that is the case then the market clearing prices will result in $p^1_1 > 0$. 

Table 4.2: Equilibrium Predictions of the Rational Expectations, Prior-Information and Naive-Information models

<table>
<thead>
<tr>
<th>Market Stage(^1) ($p_j^t$)</th>
<th>Prediction Stage(^1) ($B_j$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0$</td>
<td>$p_1$</td>
</tr>
<tr>
<td>Rational Expectations Model</td>
<td></td>
</tr>
<tr>
<td>$s = OOO$</td>
<td>100</td>
</tr>
<tr>
<td>$s \in {OOX, OXO, XO0}$</td>
<td>0</td>
</tr>
<tr>
<td>$s \in {OXO, XO0, XXO}$</td>
<td>0</td>
</tr>
<tr>
<td>$s = XXX$</td>
<td>0</td>
</tr>
<tr>
<td>Prior-Information Model</td>
<td></td>
</tr>
<tr>
<td>$s = OOO$</td>
<td>50</td>
</tr>
<tr>
<td>$s \in {OOX, OXO, XO0}$</td>
<td>0</td>
</tr>
<tr>
<td>$s \in {OXO, XO0, XXO}$</td>
<td>0</td>
</tr>
<tr>
<td>$s = XXX$</td>
<td>0</td>
</tr>
<tr>
<td>Naive-Information Model</td>
<td></td>
</tr>
<tr>
<td>$s \in S$</td>
<td>12.5</td>
</tr>
</tbody>
</table>

\(^1\): Equilibrium price of security $j$. \(^1\): Equilibrium percentage of traders submitting beliefs $b_{g_i} = j$. 
\[ p^2_0 = p^2_1 = p^2_2 = 0 \text{ and } p^2_3 = 100 \] - the RE equilibrium. Since there is nothing more to learn from market prices, there should be no evidential revision in prices in period 3 and 4. By symmetry, the intuition is similar for state \( s = OOO \).

Now consider the case where \( s = XXO \). In period 1, each trader remains uncertain and the market clearing prices are \( p^1_0 = p^1_3 = 0 \) and \( p^1_1 = p^1_2 = 50 \) - the PI equilibrium. In period 2, only the Minority traders, those in group 3, will know the true state to be \( XXO \) and value security 2 at 100 ECU - the logic follows from our earlier illustration. However, traders 1, and 2, remain uncertain. As traders act on their posteriors, the market clearing conditions will still correspond to the prices \( p^2_0 = p^2_2 = 0 \) and \( p^2_1 = p^2_3 = 50 \) - there is still excess demand for security 1 by traders 1 and 2. However, security 2 should be allocated to traders 3, given their higher valuation. In period 3, traders 1, and 2, again remain uncertain of the true state. However, as traders act on their posteriors, the market clearing prices will now become \( p^3_0 = p^3_3 = 0 \), \( p^3_1 = 50 \) and \( p^3_2 = 100 \) - trade of security 2 will only occur between traders in group 3. In period 4, traders 1, and 2, will realise that the only reason why \( p^4_2 \gg 50 \) must be because some other traders have ascertained the true state to be \( XXO \). Therefore the market clears at prices \( p^4_0 = p^4_3 = 0 \) and \( p^4_1 = 100 \) - the RE equilibrium.

The same intuitions can be extend to any states where \( x(s) \in \{1, 2\} \), and the Minority traders should always learn about the true state before the other traders. In addition, the information aggregation process will also result in the Minority trader holding greater proportions of the True security relative to the other securities.

We should emphasis that prices in our experimental sessions do not necessarily have to evolve in the same manner as our illustrations. If experimental subjects were instead strategic, we might arrive at a no-trade outcome, where no trader has the incentive to trade until it is possible to ascertain the true state of nature from prices. Also in a fully dynamic CDA market, prices do not necessarily have to converge to any of the aforementioned “temporarily” equilibriums, since traders are able to learn from the unfulfilled market bids and asks prices. Nevertheless, the illustrations reveal that though information aggregation might be possible for all states, it might be a challenging task that requires subjects to employ counterfactual reasonings.

However, for states where \( x(s) \in \{1, 2\} \), the illustrations do make one specific prediction, that Minority traders will learn about the true state before the other traders. If this is indeed true, it should be evidential in the subjects’ trading behaviours in the Market stage and beliefs in the Prediction stages.

### 4.4 Experimental Results

We present chronologically on Figures 4.1 to 4.7, the data from the Market and Prediction stages in the respective sessions. Each figure comprises of \( 4 \times 10 \) panels, with rounds arranged along the columns (rounds 1 and 10 in the left and right most columns, respectively) and security types along the rows (security 0 and security 3 in the top and bottom most rows, respectively). Each figure plots the transaction prices (vertical axis) of a specific security across time (horizontal axis), with the RE and PI equilibriums denoted as solid and dashed horizontal lines, respectively - the NI equilibrium was omitted as it was not state dependent. Finally, the header of each panel refers to the proportion of subjects who had submitted the corresponding beliefs \( B_j \) in that round. For example, 8% of the subjects in round 1 of session 1 (Figure 4.1) had submitted the beliefs \( b_{g1} = 0 \).

This paper is motivated by the question as to whether Arrow-Debreu markets are able to aggregate diverse and private information, an outcome predicted by the RE model. Our design allows us to consider the notion of information aggregation in prices and beliefs of subjects. To evaluate the notion of information aggregation in both stages in both stages, our approach seeks to compare the relative performance of the RE, PI and NI models in explaining prices and submitted beliefs.

#### 4.4.1 Which Model(s) Better Explains the Results?

With the exception of session 4, prices seemed to be “noisier” in the earlier rounds relative to the later rounds. For example, in rounds 1-3, 22% of all transactions had occurred above 100 ECU, relative to 8% in rounds...
Figure 4.1: Session 1 - Market and Prediction Stages

Figure 4.2: Session 2 - Market and Prediction Stages
Figure 4.3: Session 3 - Market and Prediction Stages

Figure 4.4: Session 4 - Market and Prediction Stages
Figure 4.5: Session 5 - Market and Prediction Stages

Figure 4.6: Session 6 - Market and Prediction Stages
8-10. Whilst some of these instances could be attributed to subjects’ “typos”, as they submitted their bids through the computerised system, they are more likely to characterise a learning phenomenon.\textsuperscript{18}

Since information aggregation (if it is evidential) is surely a dynamic process, our analysis will focus on prices towards the end of the trading period and thus computed for each security, the prices \(P_{30}\) and \(P_{60}\), which referred to the average transaction price for that security in the last 30 and 60 seconds of the Market stage, respectively.\textsuperscript{19} As it is common with market experiments, prices \(P_{30}\) and \(P_{60}\) were not perfectly described by any of the candidate models.\textsuperscript{20} To hence index the relative performances of each model, we followed the approach of Plott and Sunder (1982, 1988) and derived the mean absolute deviations (MAD) of security prices - \(P_{30}\) and \(P_{60}\) - at each round, from the respective models’ equilibrium. The relevant MAD with \(P_{30}\) are presented on Figure 4.8 - computing with \(P_{60}\) yields similar observations. The shaded bar denotes the proportion of Correct subjects in the Prediction stage. Given the symmetrical nature of the RE and PI equilibriums, the MAD will often be relatively close and sometimes identical.\textsuperscript{21} This suggest that only when prices are sufficiently close to the RE equilibrium, would the RE model be preferred to the alternative models.

Some immediate observations on the issue of information aggregation can be made from Figure 4.8. Firstly, in the first 5 rounds, there is little to distinguish between the MAD of the respective equilibriums. This implies that neither models were able to provide adequate explanation to prices. However in the last 5 rounds, the MAD from the RE equilibrium is noticeable lower in sessions 2, 6 and 7, and to a lesser, sessions 1, 3 and 5, relative to the PI or NI equilibrium. These observations suggest that the performance of the RE model, more specially the market’s ability to aggregate diverse and private information into prices, may only be evidential

\textsuperscript{18}In addition to the instructions, subjects were also required to complete a computerised questionnaire before they could proceed to the practice rounds, which preceded the experimental proper. Although, admittance into the practice rounds required subject to correctly answer all questions, there may be some subjects who did not fully understand the experiment design or payoff.

\textsuperscript{19}The last observed transaction price was used if no transaction was observed within the stipulated period. The average number of transactions in the last 30 and 60 seconds were 3 and 5 respectively.

\textsuperscript{20}We conducted the linear regression \(p_j = \alpha + \beta p_j^* + \epsilon\) separately for the first 5 rounds, last 5 rounds and over all rounds, where \(p_j^*\) denotes the predicted price of security \(j\) by the RE, PI or NI models, and \(p_j\) refers to the prices \(P_{30}\) or \(P_{60}\). We tested the null hypothesis \(H_0 : \alpha = 0\) and \(\beta = 1\), which implies that prices are perfectly described by the model in consideration. The null hypothesis was comfortably rejected in all comparisons.

\textsuperscript{21}Consider an example where \(x = X\langle 0, p_0 = p_3 = 0, p_1 = 100\) and \(p_2 = 50\). Here, the calculated absolute deviations for both models will be 12.5. Notice here that for any \(p_1 \geq 100\), increasing \(p_2\) whilst keeping the other prices fixed will increase the MAD of both models by the same magnitude.
when subjects had some degree of experiences with the trading mechanism.

The performances of the RE model lead to the more intricate question as to the process by which information gets aggregated into prices. Clearly from Figures 4.1 to 4.7, such a process does not seem immediately consistent with our illustrations in the previous section. For example, prices of those securities which were predicted by the PI model to be non-zero, seem to immediately "jump" towards 100 at the start of the round. This might perhaps reflect some disagreements amongst subjects, about the true state of nature. In some instances, such disagreements are eventually resolved by the market mechanism e.g., Session 7 round 10, as the transaction price of the true security remain close to 100 whilst that of the non-true securities converge towards 0. In other instances, such disagreement is perpetuated by the market mechanism e.g., Session 1 round 10, as the prices of the true and non-true security continue to transact at around 100. However, as we mentioned in the previous section, this might not necessarily be unexpected given that trade is facilitated through a CDA mechanism. Our analysis will hence focus on the behaviours of the Minority subjects, which we will study in later subsections.

Turning our attention now to the Prediction stage, we observe a negative and significant correlation between the proportions of Correct subjects and the MAD of prices from the RE equilibrium - see Figure 4.8. This implies that as prices got closer to the RE equilibrium, more subjects were Correct. The question however is whether the submitted beliefs of subjects in the Prediction stages are different from an outcome where all subjects remain uninformed of the true state - as in the NI and PI equilibrium predictions. To investigate, we again computed the MAD of beliefs from the equilibrium of the respective models - see Figure 4.9. Unlike the observations in the Market stage, we now observe that the RE model was only better able to explain the submitted beliefs in sessions 2 and 7, and only in the later rounds. In all other sessions, the RE models seems to explain the data no better than the PI model, and no worse than the NI model in later rounds.

As shown by Forsythe and Lundholm (1990) and evidential from the above observations, any evaluation of the RE model with respect to information aggregation should take into consideration subjects’ experience with the trading platform. To therefore give the RE model a “better-chance” for succeeding, we will refer to the rounds 1-5 and 6-10 as the EARLY and LATE rounds respectively, and study them separately. The Spearman correlation with $P^{30}$ and $P^{60}$ were found to -0.59 and -0.62, respectively.

The separation of the EARLY and LATE rounds might seem arbitrary. However, we find some support for a structural break between the EARLY and LATE rounds in our linear regressions.

---

22The Spearman correlation with $P^{30}$ and $P^{60}$ were found to -0.59 and -0.62, respectively.

23The separation of the EARLY and LATE rounds might seem arbitrary. However, we find some support for a structural break between the EARLY and LATE rounds in our linear regressions.
To also provide a general overview of the respective models’ performances in explaining prices and submitted beliefs, we also computed the relevant cumulative distribution functions (CDF) of the MAD in both stages on Figures 4.10 and 4.11 - the NMP and PIMP models will be discussed in the later subsections. This allows us to also consider the notion First-Order Stochastic Dominance (FSD) in pairwise comparisons of CDF.

Three statistical test are employed in this paper on the computed MAD from the respective model, (i) The Wilcoxon Signed Ranked Sum test, (ii) The Kolmogorov-Smirnov (KS) two-sided test and (iii) The Davidson and Duclos (2000) or DD test for FSD. The test statistics for each pairwise comparisons in the Market and Prediction stages are reported on Tables 4.3 and 4.4 respectively, where the preferred model by the Wilcoxon and DD test are denoted in parenthesis - the NMP and PIMP models will be discussed in the later subsections. We will hence adopt the criteria that a model is only said to have outperform - explain the data better - than an alternative model if the former model is significantly preferred to by the Wilcoxon and DD test, and the null hypothesis is rejected in the KS test. This leads us to our following two results, which are immediate from our test statistics.

Result 1 (Market Stage): The RE model performs as well as the PI and NI models in the EARLY rounds, and performs outperforms the latter two models in the LATE rounds. When ALL rounds are considered, the RE model outperforms the NI model but performs as well as the PI model.

Result 2 (Prediction Stage): The PI and NI models outperform the RE model in the EARLY rounds. In the LATE round, the RE model performs as well as the PI model but outperforms the NI model. When ALL rounds are considered, the RE model performs better than the NI model but as well as the PI model.

Consider the functions $F$ and $G$ over the real numbers. Formally, the function $F$ is said to exhibit FSD over $G$ if and only if $F(x) \leq G(x)$ for all $x \in \mathbb{R}$.

To apply the Davidson and Duclos (2000) test, we adapted the approached recommended by Heathcote, Brown, Wagenmakers and Eidels (2010), where the joint distributions of MAD from the RE, PI and NI models (each pairwise comparisons) are partitioned into $K = 4$ ‘bins’ as determined by the 20th, 40th, 60th and 80th percentiles. The test statistics are compared to critical values of the ‘Studentized Maximum Modulus Statistics’ as computed by Stoline and Ury (1979). The critical values are 3.143, 2.631 and 2.378 for the significances at the 1%, 5% and 10% levels, respectively.
Figure 4.10: Market Stage - Cumulative Distribution Functions of MAD of Price ($P^{30}$) from Models’ Equilibrium

Figure 4.11: Prediction Stage - Cumulative Distribution Functions of MAD of Submitted Beliefs from Models’ Equilibrium
The results suggest that when the markets had consisted of relatively inexperienced subjects (EARLY rounds), prices were often significantly different from the RE equilibrium. Prices were in fact inadequately explained by any of the candidate models - the PI was also found to perform as well as the NI model. Given the markets’ failure to aggregate diverse and private information in prices, we should naturally expect most subjects to be uninformed of the true state. We indeed observe that both PI and NI models were able to explain the submitted beliefs significantly better than the RE model - the PI model also performs significantly better than the NI.\(^{26}\)

However, when the markets consisted of relatively experienced subjects (LATE rounds), prices were now often found to be significantly closer to the RE equilibrium - the PI also performed significantly better than the NI. This provides some support to the findings of Plott and Sunder (1982, 1988) and Plott (2000), such that markets could function as effective mechanisms to aggregate diverse and private information in prices, even when the process to do so is non-trivial or cognitively challenging. If markets are able to aggregate information, does this necessarily imply that most or all subjects are necessarily informed of the true state? Clearly our findings in the LATE rounds of the Prediction stage suggest otherwise. Even though the proportion of correct subjects was found to have increased from 51% in the EARLY rounds to 71% in the LATE rounds, the RE model was only able to explain the submitted beliefs in the Prediction stage as well as the PI model.\(^{27}\)

This of course does not imply that all subjects had remained uninformed of the true state, but instead suggest that even as markets are sufficiently able to aggregate her information properties, we cannot exclude the possibility that the beliefs of market participants, ex-post the markets, were any different from an outcome where all subjects had remained uninformed about the true state. Stated different, information

\(^{26}\) The relative performances of the PI model over the NI model in the EARLY rounds of the Prediction stage suggest that subjects may have been utilising their private information (their observations of other groups’ types) to update their beliefs on the true state.

\(^{27}\) We also replicated our statistical test in the LATE rounds excluding session 4 - which seems to be an outlier in our sample, and did not find any changes to the statistical conclusions.
Market Stage Prediction Stage

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>Market Stage</th>
<th></th>
<th>Prediction Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAD of Prices ($P_{30}$) from RE Equilibrium</td>
<td>MAD of Prices ($P_{60}$) from RE Equilibrium</td>
<td>MAD of submitted Beliefs from RE Equilibrium</td>
</tr>
<tr>
<td>LATE</td>
<td>−22.45 (a)</td>
<td>−26.55 (b)</td>
<td>−8.66 (c)</td>
</tr>
<tr>
<td>((3.91))</td>
<td>((3.37))</td>
<td>((1.87))</td>
<td></td>
</tr>
<tr>
<td>(X(1,2))</td>
<td>−1.51</td>
<td>−5.23</td>
<td>−5.92</td>
</tr>
<tr>
<td>((4.60))</td>
<td>((3.97))</td>
<td>((1.97))</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>47.36</td>
<td>53.47</td>
<td>27.98</td>
</tr>
<tr>
<td>((4.73))</td>
<td>((4.07))</td>
<td>((4.62))</td>
<td></td>
</tr>
<tr>
<td>(n)</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.26</td>
<td>0.37</td>
<td>0.29</td>
</tr>
</tbody>
</table>

\(a\): Significant at 1% level; \(b\): Significant at 5% level and \(c\): Significant at 10% level.

Table 4.5: Panel Fixed-Effects Regression - Is the Performances of the RE model State Dependent?

aggregation in prices does not necessarily imply information aggregation in beliefs.

Some Comments

A potential explanation to the discrepancies with the RE model’s performances in the Market and Prediction stages of the LATE rounds is the heterogeneity amongst subjects in their ability to use the market prices to infer about the true state. This explanation borrows from the Marginal Trader Hypothesis (Forsythe et al., 1992, 1999), which sought to explain how market prices could be close to reflecting the true underlying state even when most traders demonstrate limitations in the rational information processing. The hypothesis posit that although the average trader may exhibit psychological biases, the marginal traders (potentially a minority within the market) who do not exhibit such biases are able to exert greater influence on the market prices, and thus are instrumental in driving prices to the RE equilibrium.

4.4.2 Are the Performances of the RE model State Dependent?

The above results also raises the question as to whether the performances of the RE model, in both stages, are state dependent. Our design provides for two nature considerations, states with \((s \in \{OOO,XXX\})\) or without \((s \in S \setminus \{OOO,XXX\})\) Minority traders.\(^{28}\) To see this more clearly, we present on Figures 4.12 and 4.14, the aggregated MAD from the RE equilibrium for the Market and Prediction stages, respectively.\(^{29}\)

An immediately observation is the improvement in the performances of the RE model in both stages, across rounds. This is again consistent with the “learning” phenomenon that although subjects do not immediately conform to the RE behaviour upon entering the laboratory, they eventually learn the RE behaviours through repeated interaction.

There seems to be some evidential differences in the performances of the RE model in states with or without Minority traders, but only in the Prediction stage. To formally study this, we conducted a Panel fixed-effect linear regression with the situation dummies “LATE” and “\(X(1,2)\)”, referring to observations in the LATE rounds and states which included Minority Traders, respectively. The estimates are presented on Table 4.5 and the results can be summarised as followed.

**Result 3:** The performances of the RE model in the Market stage improves across rounds and is independent of the state of nature. The performances of the RE model in the Prediction stage also improves across rounds and also improves in states where Minority Traders exist.

A potential explanation for this phenomenon could be the marginal influence of Minority traders’ (they form 1/3 of all subjects) decisions on the performances of the RE model. We present on Figure 4.12 the proportions or Minority and non-Minority subjects who were correct in each round. Here we immediately observe that the Minority subjects are proportionally more often found to be correct than the other subjects. Could this be due

\(^{28}\)Our intuitive illustrations in the previous section suggest that convergence to the RE price equilibrium in the Market stage is seemingly more difficult in states with Minority traders. If the performance of the RE model in the Market stage is state dependent, it is natural to expect the MAD of prices from the RE equilibrium to be higher in states with Minority traders relative to those without.

\(^{29}\)The Aggregated MAD was computed as the average MAD over all sessions for the relevant rounds.
Figure 4.12: Market Stage - Aggregated MAD of Price ($P_{30}$ & $P_{60}$) from the RE equilibrium for States with or without the existence of the Minority Traders.

Figure 4.13: Prediction Stage - Aggregated MAD of submitted Beliefs from the RE equilibrium for States with or without the existence of the Minority Traders.
to the information distribution structure, where the Minority traders are predicted to learn about the true state before the others?

4.4.3 Do Minority Subjects Learn about the True state before the others?

Building on the results and observations made from the previous sub-sections, we now focus on a central predictive outcome of our intuitive illustrations, such that for states where \( x(s) \in \{1, 2\} \), Minority traders should learn about the true state before the other traders. It is difficult to identify a point in time within the Market stage when a subject becomes informed of the true state. However, if Minority subjects had indeed learnt about the true state before the others, we should naturally expect that the former be more likely to be Correct in the Prediction stage relative to the latter. Furthermore, such knowledge should be evidential in their trading behaviours.

The following variables are hence computed for each subject at each round:

1. **PROFIT**: The Trading-Profit of the subject.
2. **TRADE VOL**: The total number of transactions (purchases or sale) performed by the subject.
3. **SHARE TRUE**: The proportion of True securities held by a subject as a percentage of all securities owned.
4. **BUYDEV**: The average deviation from the RE equilibrium for all purchase prices made by the subject.
5. **SELLDEV**: The average deviation from the RE equilibrium for all sale prices made by the subject.

In our illustrations of the price convergence to the RE equilibrium, the Minority traders were able to leverage upon their knowledge by purchasing proportionally more units of the true security. If this were indeed true, we should thus expect SHARETRUE to be higher for Minority subjects relative to the other subjects. Furthermore, being informed of the true state before the others, Minority subjects should on average be purchasing securities at prices closer to the RE equilibrium, implying that BUYDEV should be lower for the Minority subjects. Inversely, an informed subject should demand higher prices to be compensated for the sale of securities, and

---

30 This problem is compounded that the fact that trade is facilitated through a CDA mechanism.
Table 4.6: Panel Random-Effects Regressions: Do Minority Subjects Learn About the True State before the Others?

<table>
<thead>
<tr>
<th>Time</th>
<th>Logit</th>
<th>Linear-Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correct</td>
<td>PROFIT</td>
</tr>
<tr>
<td>LATE</td>
<td>NA (1.67)</td>
<td>.867†</td>
</tr>
<tr>
<td></td>
<td>MINORITY</td>
<td>.669†</td>
</tr>
<tr>
<td></td>
<td>X(1, 2)</td>
<td>.522†</td>
</tr>
<tr>
<td></td>
<td>Risk†</td>
<td>.071†</td>
</tr>
<tr>
<td></td>
<td>CRT Score‡</td>
<td>.033</td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>-.8376</td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>840</td>
</tr>
</tbody>
</table>

Thus implying that SELLSEV should be higher for the Minority subjects. If the above were indeed true for the Minority subjects, we were also interested to study if their might be above to exploit their position with greater trading-profits or trade more frequently than the other subjects.

We present on Table 4.6 the individual level, panel random-effect estimates of the above conjectures.31 The first column describes our covariates, and the subsequently column differ by the econometric functions and dependant variables. For example, the second column considers a Panel-logistic regression, where the dependant variable corresponds to observations where subjects were found to be Correct in the Prediction stage. Column three onwards refer to our Panel-Linear regression estimates. We also computed the relevant variables TRUESHARE, BUYDEV and SELLDEV over the first 150 and 300 seconds of the Market stage. The focus of our econometric exercise pertains to the situation dummy “MINORITY” which refers to observations where the subject was a Minority trader. The estimates leads us to the following result.

Result 4 (Market Stage): Minority subjects held greater proportion of the True security in their security inventory, on average purchased securities at prices closer to the RE equilibrium and on average sold securities at prices further away from the RE equilibrium, relative to the other subjects.

Result 5 (Prediction Stage): Minority subjects were more likely to be Correct, relative to the other subjects.

Again support for the results are immediate from our econometric regressions. More importantly, they provide some support for the conjecture that Minority subjects may be informed about the true state of nature before the other subjects. If this is indeed true, we find no evidence that Minority subjects were able to exploit their knowledge through greater profits.

4.4.4 Can We Better Explain Subjects’ Submitted Beliefs?

The equilibrium of the RE, PI and NI models in the Prediction stage represent extreme positions as to how traders use the market to form beliefs about the true state. With experimental data, subjects may consist of a mixture of the above three models. This begs the question as to how we can better explain, on the aggregated level, the submitted beliefs of subjects in the Prediction stage.

31 A common problem with econometric analysis is whether the fixed-effect or random-effect regression will be more appropriate. The Hausman test fails to reject the null hypothesis that the Random-effects model is preferred to the Fixed-effects model.
their private observations, to infer the prices of securities which are relevant to their beliefs. For example, if
prices. For example, the probability of a trader choosing

their own private observations and their beliefs about the true state are simply the weighted average of all
security

\( j \)

\( B \)

predicted proportion of submitted beliefs (\( b \)) by each model, where \( p_j \) refers to the Market stage prices of
security \( j \).

The intuition for the NMP and PIMP models are as followed. In the NMP model, traders again ignore
their own private observations and their beliefs about the true state are simply the weighted average of all
prices. For example, the probability of a trader choosing \( b_{y_i} = 0 \) is simply \( p_0 / \sum_j p_j \). In the PIMP, traders use
their private observations, to infer the prices of securities which are relevant to their beliefs. For example, if
\( s = XXX \) then trader 3, will know that only states \( XXO \) and \( XXX \) will be possible and hence only prices of
security 2 and 3 are relevant. His probability of submitting beliefs \( b_{y_i} = 2 \) and \( b_{y_i} = 3 \) are hence \( p_2 / (p_2 + p_3) \)
and \( p_3 / (p_2 + p_3) \), respectively.

The price \( P^{30} \) was used to computed the predicted submitted beliefs of the NMP and PIMP models, and
the relevant CDFs are presented on Figure 4.11. From Table 4.3, we find that the NMP and PIMP models both
significantly perform better than the RE model in the Prediction stage in the EARLY, LATE and ALL rounds.
The PIMP was also found to perform better than the PI, NI, NMP is the EARLY, LATE and ALL rounds - test
statistics not presented in this paper. This suggest that when forming beliefs about the true state, subjects
often use a weighted average of the relevant security prices as determined by their own observations.

### 4.5 Conclusion

This paper revisiting the question as to the information aggregation properties of markets, an outcome pre-
dicted by the Rational Expectations (RE) model. The design seeks to create an environment where informa-
tion aggregation is non-trivial and challenging by integrating the information dissemination structure of “Red
Hat Puzzle” into a complete market (4 Arrow-Debreu securities) with homogenous preferences for all traders.
With two stages (Market and Prediction stages), the design also allows us to separately study the notion of
information aggregation with respect to the market prices (Market stage) and the beliefs of subjects ex-post
the market (Prediction stage).

It remains an open question as to how our findings would change if the experimental sessions were
repeated for another 5 or 10 rounds.\(^{32}\) However, our experimental data is sufficient for us to provide some
first results on the natural of information aggregation, namely that markets could indeed be shown to be
successful in aggregating diverse and private information into security prices, even when the task for doing
so is challenging. Furthermore, markets can accomplish the above even when we cannot sufficiently show

\(^{32}\)Given how the performances of the RE model seems to improve across rounds, it is likely that prices and beliefs in the Market and
Prediction stages would eventually converge to the RE equilibrium.
that the beliefs of subjects ex-post the markets are significantly different from an outcome when all subjects remained uninformed of the true state. Stated different, this again implies that information aggregation in prices does not necessarily imply there to be information aggregation in beliefs.

Whilst it is often a “black box” as to how information aggregation occurs in markets, our design also allows us to put forth some intuitive illustrations about the process, which our econometric analysis provide some support for. But more importantly, the econometric analysis suggest that even if the Minority subjects do indeed learn about the true state before the other subjects, they do not necessarily make superior profits.

Given the framework of our design, possible extensions could include changing the structure of the market, e.g., moving from complete to incomplete markets. Using the present study as the benchmark, we can thus investigate how the nature of information aggregation will change as we decrease the number of securities whilst keeping the information dissemination structure fixed. These extensions will be a motivation for future research.

33 Two treatments are immediately obvious. In the first treatment, there are two securities, where each security is redeemed three possible values depending on the state of nature. In the subsequent treatment, there is only one security, which is again redeemed at 4 possible values depending on the state of nature.
5.1 Chapter 2

5.1.1 Data from all Samples in B2012, M2012, M2013 and E2013

We present on Figure 5.1 the observed frequencies from all observations in the relevant sessions. The frequencies were found to be significantly different from the Mixed-Strategy equilibrium (Fisher’s exact $\rho < 0.001$ in all comparisons). Given this data, we also checked for consistency of behaviours between identical games. Comparisons of B2012 and AR(2012) yield the p-value of 0.423 in Fisher’s Exact test. Comparisons of M2012 and M2013 yield the p-value of 0.728.

5.1.2 Data from all Samples in Guessing Game

We present on Figure 5.2 and 5.3 the histogram and cumulative distribution function of the guessing game numbers, respectively. The Kolmogorov-Smirnov test finds the distributions of numbers in cohort 2012 to be significantly different ($\rho = 0.04$). However, we are unable to reject the null hypothesis for the distribution in cohort 2013 ($\rho = 0.147$).

5.1.3 Estimating the Cognitive Hierarchy Model

The model was estimated using the maximum likelihood techniques. Let $p(a)$ denote the probability of observing action $a \in A$ in the game and $y_i$, the $i = 1, 2, ..., N$ observation. Given the model’s construct, one is able to rewrite

$$p(a|\tau, \lambda) = p^0 f(0|\tau) \prod_{k=1}^{K} p^k (a|\lambda, \tau) f(k|\tau)$$

which was optimised given the constraints $1 - \sum_{k=0}^{K} f(k|\tau) < \epsilon$, where $\epsilon = 0.001$, and the boundary conditions $\tau \in [0, K]$ and $\lambda \in [0, 100]$. We were uncertain if the log-likelihood function was concave or kinked and thus employed the direct search, Nelder and Mead (1965) optimisation technique. Cautious of such approach, we explored a fine search termination criteria of 0.0000001 and checked if our estimates ($\tau$ and $\lambda$) were robust for $K = 9, 18, 36$. The estimates were found to be robust and the log-likelihood function was observed to be concave (see Figure 5.4), which suggest that our estimates were indeed the global maximum.
Figure 5.1: Frequencies of Strategies in All Samples (B2012, M2012, M2013 and E2013)

Figure 5.2: Guessing Game Number Histogram
Figure 5.3: Cumulative Distribution Function of Guessing Game

Figure 5.4: Cognitive Hierarchy Model Log-Likelihood Function for Session M2012
5.1.4 Estimating the SK Model

The maximum likelihood technique involves $K + 1$ free parameters. We hence expressed $p(a)$ as

$$p(a|\alpha_0, \alpha_1, ..., \alpha_K, \lambda) = p^0\alpha_0 \prod_{k=1}^{K} p^k(a|\lambda, \tau)\alpha_k$$

where $\alpha_k \in [0, 1]$ denotes the proportion of $L_k$ types in the data, given the constraints that $\alpha_K = 1 - \alpha_0 - \alpha_1 - ... - \alpha_{K-1}$. We again employed the same estimation techniques as in the CH model. To ensure that our estimates are the global maximum, we considered multiple random starting values for the parameters $\alpha_0, \alpha_1, ..., \alpha_{K-1}$. Given this criteria, we repeated the estimation process 10 times for each session and the estimates were found to be identical each time. This suggest that our estimates are also the global maximum.
5.2 Chapter 3

5.2.1 Instruction and Screenshots for BASE1

There will be 10 experiment rounds, where you receive a payoff (denoted in ECU) at the end of each round. Upon completion of all 10 rounds, your earnings for the experiment will be computed as the average over ten rounds and converted to cash at the exchange rate of 67 ECU to $1. In addition, you will also receive a $5 show up fee. We shall now describe each experimental round.

Each round will involve three coloured hats - A, B and C. Each hat can be either Red or Black, and there is exactly one player under each hat.

1. You will not be able to see your own hat colour.
2. You will see the other two hat colours.
3. There will always be one black hat.

To determine the hat colours, the computer randomly picks from 1 of the 7 possible outcomes (see Table A1). For example, in outcome 2, Hat A is red, Hat B is black and Hat C is black. The player under Hat A will see that: Hat B is Black and Hat C is Black. The player under Hat B will see that: Hat A is Red and Hat C is Black. Finally, the player under Hat C will see that: Hat A is Red and Hat B is Black. There is an equal chance for any one of these outcomes. Notice that at each outcome, there will always be at least one black hat. See Figure A1 for an example of what you might observe. Here you are under Hat A and you see that both Hat B and C are black.

Your task in each round is to determine the colour of your hat. You will do this in the decision stage that will consist of 4 period. At each period the computer will present you with the following question, to which you must choose from 4 possible actions (a), (b), (c) or (d).

Computer’s question: “Do You Know your hat colour?”

Your actions (a) My Hat is RED, (b) My Hat is BLACK, (c) No! I will decide in a later period and (d) Toss a Coin, I would never know.

Here are some rules:

RULE 1: At each period, you have a maximum of 4 minutes to choose an action
RULE 2: You will immediately end the decision stage if the actions (a), (b) or (d) were chosen
RULE 3: You will only go to the next period if you had chosen (c) in the previous period
RULE 4: If you arrive at period 4, you can only chose from the actions (a), (b) or (d)
RULE 5: If you had chosen (d) the computer will simulate a coin toss and choose on your BEHALF either option (a) or (b) with equal chances
RULE 6: Any action chosen will be known to all other players in the subsequent period. Note: If you had chosen (d) and the computer chooses (b) on your behalf, the other players will only see that you had chosen (b).

You are said to have “determined your hat colour” when you choose (a), (b) or (d). This is why you will only go to the next period if you have chosen (c). For example, if you had chosen (a) in period 1, the decision stage immediately ends for you. In period 2, all other players will observe that you had chosen (a) in period 1. However, if you had chosen (c) in period 1, you go on to period 2, when you must again choose your action. All players will also observe that you had chosen (c) in period 1. Here, are some screenshots to help you understand the decision stage design (Figure A2 and Figure A3).

Figure A2 presents an illustration of the first period in the decision stage. You are under Hat B and you see the other hat colours. In addition the computer presents you with the question “Do you know you hat colour” to which you must reply with one of the 4 possible actions.

Figure A3 presents an illustration of the second period in the decision stage. You are under Hat B and you see the other hat colours. The computer again presents you with the question “Do you know you hat colour” to which you must reply with one of the 4 possible actions. In addition, you also see on your screen the actions chosen by the other players in the previous period (period 1). You see that the player under Hat...
<table>
<thead>
<tr>
<th>Outcome</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chance</td>
<td>1/7</td>
<td>1/7</td>
<td>1/7</td>
<td>1/7</td>
<td>1/7</td>
<td>1/7</td>
<td>1/7</td>
</tr>
<tr>
<td>Hat A</td>
<td>B</td>
<td>R</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>Hat B</td>
<td>B</td>
<td>B</td>
<td>R</td>
<td>B</td>
<td>R</td>
<td>B</td>
<td>R</td>
</tr>
<tr>
<td>Hat C</td>
<td>B</td>
<td>B</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>B</td>
<td></td>
</tr>
</tbody>
</table>

B=BLACK and R=RED

Table A1: BASE1: The 7 Possible Outcomes

<table>
<thead>
<tr>
<th>Correct</th>
<th>Incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>“determined your hat colour” in Period 1</td>
<td>950 ECU</td>
</tr>
<tr>
<td>“determined your hat colour” in Period 2</td>
<td>900 ECU</td>
</tr>
<tr>
<td>“determined your hat colour” in Period 3</td>
<td>850 ECU</td>
</tr>
<tr>
<td>“determined your hat colour” in Period 4</td>
<td>800 ECU</td>
</tr>
</tbody>
</table>

Table A2: BASE1: Payoffs with Choosing (a) or (b)

A had chosen (b) in period 1. You also see that the player under Hat C had chosen (b) in period 1. Finally, in this illustration you had chosen (c) in period 1.

After all players had ended the decision stage, your hat colour with be made known and your payoffs for the round will be determined. Your Payoffs depends on whether you had correctly determined you hat colour and the period which you had “determined your hat colour”. If you had chosen (a) or (b), then your payoffs will depend on whether you are correct and the period which you had chosen them (see Table A2). If You had chosen (d), then your payoffs will only depend on the period which you had chosen (c) (see Table A3).

The payoffs can be easily summarised as followed. You start the round with 950 ECU. You get 50 ECU deducted for each time you had chosen (c). In addition, you get 700 ECU deducted if you had chosen (a) or (b) and was found to be incorrect, or no deduction if found to be correct. If you had chosen (d), you’ll get a fixed deduction of 250 ECU. Here are some examples to help you understand the payoff

1. You hat is black. In period 1 you choose (c), in period 2 you choose (c) and in period 3 you choose (b). You have correctly determined your hat colour and your payoffs are therefore 850 ECU.

2. You hat is black. In period 1 you choose (c), in period 2 you choose (c) and in period 3 you choose (a). You have incorrectly determined your hat colour and your payoffs are therefore 150 ECU.

3. You hat is black. In period 1 you choose (c), in period 2 you (c) and in period 3 you choose (d). Your payoffs are therefore 600 ECU.

This completes the description of each experimental round. After the completion of 10 experiment rounds, we require you to complete a survey before you receive your cash payments. Please feel free to clarify any question or doubts you might have with regards to the instructions.

<table>
<thead>
<tr>
<th>Correct</th>
<th>Incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>“determined your hat colour” with (d) in Period 1</td>
<td>700ECU</td>
</tr>
<tr>
<td>“determined your hat colour” with (d) in Period 2</td>
<td>650ECU</td>
</tr>
<tr>
<td>“determined your hat colour” with (d) in Period 3</td>
<td>600ECU</td>
</tr>
<tr>
<td>“determined your hat colour” with (d) in Period 4</td>
<td>550ECU</td>
</tr>
</tbody>
</table>
Figure A1: Screen Shot (BASE1) - You see all other Hat colours

<table>
<thead>
<tr>
<th>Hat A (You)</th>
<th>Hat B</th>
<th>Hat C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Black Hat)</td>
<td>(Black Hat)</td>
</tr>
</tbody>
</table>

"there is at least One Black Hat"

Figure A2: Screen Shot (BASE1) - Decision Stage Period 1

<table>
<thead>
<tr>
<th>Hat A</th>
<th>Hat B (You)</th>
<th>Hat C</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Red Hat)</td>
<td></td>
<td>(Black Hat)</td>
</tr>
</tbody>
</table>

Do You know your Hat colour?
- [ ] Red Hat
- [ ] Black Hat
- [ ] I will decide in a later period
- [ ] I think a (client) would know
Figure A3: Screen Shot (BASE1) - Decision Stage Period 2

<table>
<thead>
<tr>
<th>Hat A</th>
<th>Hat B</th>
<th>Hat C</th>
</tr>
</thead>
<tbody>
<tr>
<td>(RED RED)</td>
<td>??</td>
<td>(BLACK BLACK)</td>
</tr>
</tbody>
</table>

### Decision Stage: Period 2

#### What the other subjects had chosen in period 1

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Subject Under Hat A</th>
<th>Subject Under Hat B</th>
<th>Subject Under Hat C</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLACK</td>
<td>MD</td>
<td>BLACK</td>
<td></td>
</tr>
</tbody>
</table>

**Do You know your Hat colour?**

- (a) It's Hat A Red
- (b) It's Hat B Black
- (c) Hat will decide in other period
- (d) Turn a Coin (wouldn't swear)

**NEXT PAGE**
5.2.2 Instruction and Screenshots for BASE2

There will be 10 experiment rounds, where you receive a payoff (denoted in ECU) at the end of each round. Upon completion of all 10 rounds, your earnings for the experiment will be computed as the average over ten rounds and converted to cash at the exchange rate of 67 ECU to $1. In addition, you will also receive a $5 show up fee. We shall now describe each experimental round.

Each round will involve three coloured hats - A, B and C. Each hat can be either Red or Black, and there is exactly six players under each hat.

1. **You will not be able to see your own hat colour.**
2. **You will see the other two hat colours.**
3. **There will always be one black hat.**

To determine the hat colours, the computer randomly picks from 1 of the 7 possible outcomes (see Table B1). For example, in outcome 2, Hat A is red, Hat B is black and Hat C is black. The players under Hat A will see that: Hat B is Black and Hat C is Black. The players under Hat B will see that: Hat A is Red and Hat C is Black. Finally, the players under Hat C will see that: Hat A is Red and Hat B is Black. There is an equal chance for any one of these outcomes. Notice that at each outcome, there will always be at least one black hat. See Figure B1 for an example of what you might observe. Here you are under Hat A and you see that both Hat B and C are black.

Your task in each round is to determine the colour of your hat. You will do this in the decision stage that will consist of 4 periods. At each period the computer will present you with the following question, to which you must choose from 4 possible actions (a), (b), (c) or (d).

**Computer’s question:** “Do You Know your hat colour?”

**Your actions** (a) My Hat is RED, (b) My Hat is BLACK, (c) No! I will decide in a later period and (d) Toss a Coin, I would never know.

Here are some rules:

**RULE 1:** At each period, you have a maximum of 4 minutes to choose an action

**RULE 2:** You will immediately end the decision stage if the actions (a), (b) or (d) were chosen

**RULE 3:** You will only go to the next period if you had chosen (c) in the previous period

**RULE 4:** If you arrive at period 4, you can only chose from the actions (a), (b) or (d)

**RULE 5:** If you had chosen (d) the computer will simulate a coin toss and choose on your BEHALF either option (a) or (b) with equal chances

**RULE 6:** Any action chosen will be known to all other players in the subsequent period. Note: If you had chosen (d) and the computer chooses (b) on your behalf, the other players will only see that you had chosen (b).

You are said to have “determined your hat colour” when you choose (a), (b) or (d). This is why you will only go to the next period if you have chosen (c). For example, if you had chosen (a) in period 1, the decision stage immediately ends for you. In period 2, all other players will observe that you had chosen (a) in period 1. However, if you had chosen (c) in period 1, you go on to period 2, when you must again choose your action. All players will also observe that you had chosen (c) in period 1. Here, are some screenshots to help you understand the decision stage design (Figure B2, Figure B3 and Figure B4).

**Figure B2** presents an illustration of the first period in the decision stage. You are under Hat B and you see the other hat colours. In addition the computer presents you with the question “Do you know you hat colour” to which you must reply with one of the 4 possible actions.

**Figure B3** presents an illustration of the second period in the decision stage. You are under Hat A and you see the other hat colours. The computer again presents you with the question “Do you know you hat colour” to which you must reply with one of the 4 possible actions. In addition, you also see on your screen the actions chosen by the other players in the previous period. For the six players under Hat A, all of them had chosen (c) in period 1. For the six players under Hat B, one of them had chosen (a), one of them had chosen (b) and four of them had chosen (c) in period 1. Finally for the six players under Hat C, two of them had chosen (a),
<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Chance</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hat A</td>
<td>B</td>
<td>B</td>
<td>R</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>Hat B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>R</td>
<td>B</td>
<td>R</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>Hat C</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>B</td>
</tr>
</tbody>
</table>

B=BLACK and R=RED

<table>
<thead>
<tr>
<th>Table B1: BASE2: The 7 Possible Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>“determined your hat colour” in Period 1</td>
</tr>
<tr>
<td>“determined your hat colour” in Period 2</td>
</tr>
<tr>
<td>“determined your hat colour” in Period 3</td>
</tr>
<tr>
<td>“determined your hat colour” in Period 4</td>
</tr>
</tbody>
</table>

Table B2: BASE2: Payoffs with Choosing (a) or (b)

one of them had chosen (b) and three of them had chosen (c) in period 1.

Figure B4 presents an illustration of the third period in the decision stage. You are under Hat A and you see the other hat colours. The computer again presents you with the question “Do you know your hat colour” to which you must reply with one of the 4 possible actions. For the six players under Hat A, all of them had chosen (c) in period 2. For the six players under Hat B, two of them had chosen (b) in period 2, two of them had chosen (c) in period 2 and two of them had not participated in period 2 since they had ended the round in period 1 and are awaiting results. For the six players under Hat C, three of them had chosen (c) in period 2 and three of them had not participated in period 2 as they had ended the round in an earlier period.

After all players had ended the decision stage, your hat colour with be made known and your payoffs for the round will be determined. Your Payoffs depends on whether you had correctly determined you hat colour and the period which you had “determined your hat colour”. If you had chosen (a) or (b), then your payoffs will depend on whether you are correct and the period which you had chosen them (see Table B2). If you had chosen (d), then your payoffs will only depend on the period which you had chosen (c) (see Table B3). Here are some examples to help you understand the payoffs:

- The payoffs can be easily summarised as followed. You start the round with 950 ECU. You get 50 ECU deducted for each time you had chosen (c). In addition, you get 700 ECU deducted if you had chosen (a) or (b) and was found to be incorrect, or no deduction if found to be correct. If you had chosen (d), you’ll get a fixed deduction of 250 ECU. Here are some examples to help you understand the payoff

1. You hat is black. In period 1 you choose (c), in period 2 you choose (c) and in period 3 you choose (b). You have correctly determined your hat colour and your payoffs are therefore 850 ECU.

2. You hat is black. In period 1 you choose (c), in period 2 you choose (c) and in period 3 you choose (a). You have incorrectly determined your hat colour and your payoffs are therefore 150 ECU.

3. You hat is black. In period 1 you choose (c), in period 2 you (c) and in period 3 you choose (d). Your payoffs are therefore 600 ECU.

This completes the description of each experimental round. After the completion of 10 experiment rounds, we require you to complete a survey before you receive your cash payments. Please feel free to clarify any question or doubts you might have with regards to the instructions.
“determined your hat colour” with (d) in Period 1 700ECU
“determined your hat colour” with (d) in Period 2 650ECU
“determined your hat colour” with (d) in Period 3 600ECU
“determined your hat colour” with (d) in Period 4 550ECU

Table B3: BASE2: Payoffs with Choosing (d)

Figure B1: Screen Shot (BASE2) - You see all other Hat colours

Figure B2: Screen Shot (BASE2) - Decision Stage Period 1
Figure B3: Screen Shot (BASE2) - Decision Stage Period 2

Figure B4: Screen Shot (BASE2) - Decision Stage Period 3
5.2.3 Instruction and Screenshots for TRADE

There will be 10 experiment rounds, where you receive a payoff (denoted in ECU) at the end of each round. Upon completion of all 10 rounds, your earnings for the experiment will be computed as the average over ten rounds and converted to cash at the exchange rate of 100 ECU to $1. In addition, you will also receive a $8 show up fee. We shall now describe each experimental round.

Each round will involve three coloured hats - A, B and C. Each hat can be either Red or Black, and there is exactly six players under each hat.

1. You will not be able to see your own hat colour.
2. You will see the other two hat colours.
3. There will always be one black hat.

To determine the hat colours, the computer randomly picks from 1 of the 7 possible outcomes (see Table C1). For example, in outcome 2, Hat A is red, Hat B is black and Hat C is black. The players under Hat A will see that: Hat B is Black and Hat C is Black. The players under Hat B will see that: Hat A is Red and Hat C is Black. Finally, the players under Hat C will see that: Hat A is Red and Hat B is Black. There is an equal chance for any one of these outcomes. Notice that at each outcome, there will always be at least one black hat. See Figure C1 for an example of what you might observe. Here you are under Hat A and you see that both Hat B and C are black.

Overview of the Round

After you have observed the hat colours, the round will proceed to the “trading stage” followed by the “decision stage”. You begin the trading stage with 1 Token and a loan of 6000 ECU cash that must be returned at the end of the round. In the trading stage you have the opportunity to either buy more tokens or sell your token. You will only be trading with the other players under the same hat. After all transaction of tokens are completed, only players with at least one token will proceed to the decision stage - if you do not wish to participate in the decision stage, you should sell your token. In the decision stage, you will perform the task of determining your hat colour. After you have completed the decision stage, you will return the loan of 6000 ECU, and your tokens owned will redeemed by the computer (bought by the computer) at a rate that will depend on your behaviours in the decision stage. In the following, we shall first describe the design of the trading and decision stages. Thereafter, we will describe how you token redemption rate will be determined and finally we will describe your payoffs in the round.

Trading Stage

All players begin the trading stage with One Token and a loan 6000 ECU (Money) that must be paid back at the end of the round. Here you are permitted to buy or sell tokens, but only with the other players under the same hat. This implies that the market will consist of exactly 6 players and will last for 120 seconds. You will buy and sell tokens through a continuous double auction mechanism which we will now explain. See figure C2 for a screenshot of the trading stage.

To buy or sell tokens, you will need to first announce your “Ask” and “Bid” prices to all other players. Your Ask price (between 0 and 1200ECU) tells all other players how much you are willing to sell a token for. Your Bid price (between 0 and 1200ECU) tells all other players how much you are willing to buy a token for. The column “Market Ask Prices” reflects the ask prices of all six players you interact with. The column “Market Bid Prices” reflects the bid prices of all six players you interact with. To buy a token, simply select the price on the “Market Ask Prices” column and click “Buy”. Likewise to sell tokens simply select the price on the “Market Bid Prices” column and click “sell”. The column “Market Price” provides the history of all transaction prices for tokens. After 120 seconds, the trading stage will end and you will see on your screens the amount of money you have and the number of tokens you own. See figure C3 for a screenshot.

Decision Stage
Only players with at least one token can participate in the Decision Stage. If you do not have any tokens, you can observe the decision of all other players participating in the Decision Stage through your computer screens but may not yourself participate. Your task in the decision stage is to determine the colour of your hat. The decision stage will consist of 4 periods. At each period the computer will present you with the following question, to which you must choose from 3 possible actions (a), (b) or (c).

**Computer's question:** “Do You Know your hat colour?”

Your actions (a) My Hat is RED, (b) My Hat is BLACK and (c) No! I will decide in a later period. Here are some rules:

**RULE 1:** At each period, you have a maximum of 4 minutes to choose an action

**RULE 2:** You will immediately end the decision stage if the actions (a) or (b) were chosen

**RULE 3:** You will only go to the next period if you had chosen (c) in the previous period

**RULE 4:** If you arrive at period 4, you can only chose from the actions (a) or (b)

**RULE 5:** Any action chosen will be known to all other players in the subsequent period.

Here, are some screenshots to help you understand the decision stage design (Figure C4, C5 and C6).

You are said to have “determined your hat colour” when you choose (a) or (b). This is why you will only go to the next period if you have chosen (c). To help you understand the experiment design we have include some screen shoots in Figures C4, C5 and C6.

**Figure C4** presents an illustration of the first period in the decision stage. You are under Hat A and you see the other hat colours. In addition the computer presents you with the question “Do you know your hat colour” to which you must reply with one of the 3 possible actions.

**Figure C5** presents an illustration of the second period in the decision stage. You are under Hat A and you see the other hat colours. The computer again presents you with the question “Do you know your hat colour” to which you must reply with one of the 3 possible actions. In addition, you also see on your screen the actions chosen by the other players in the previous period. Here there are only two players under hat A who had participated in the decision stage. One player has 4 tokens and the other player has 2 tokens. You see that the player with 4 tokens had chosen (c) in period 1 and the player with 2 tokens had chosen (c) in period 1. Under hat B, there are three players who had participated in the decisions stage. All three player have 2 tokens and had chosen (c) in period 1. Finally, under Hat C, there are 4 players who had participate in the decision stage, one of them has 3 tokens, whilst the other three have only one token. You see that the 3 token player had chosen (c) in period 1. Two of the players with one token had chosen (c) in period 1 whilst the last player, also with one token, had chosen (b) in period 1.

**Figure C6** presents an illustration of the third period in the decision stage. You are under Hat A and you see the other hat colours. The computer again presents you with the question “Do you know your hat colour” to which you must reply with one of the 3 possible actions. There are two player under hat A. The two token player had chosen (c) in period 2. The four token player had chosen (b) in period 2. There are 3 players under hat B, each of them with two tokens. One of them had chosen (b) in period 2, whilst the other two had chosen (c) in period 2. There are four players under hat C. The three token player had chosen (c) in period 2. Amongst the one token players, one of them did not participate in period 2 as he had chosen either (a) or (b) in the period 1. Thus that player is said to have ended the game. However, the other two players with one tokens had chosen (b) in period 2.

**Token Redemption Rate**

After all players have completed the decision stage, your tokens will be redeemed by the computer. The redemption rate will depend on the period in which he had “determined your hat colour” and whether you were correct. The payoffs can be easily summarised as followed. Each token is initially worth 950 ECU. The token's value decreases by 50 ECU for each time you had chosen (c). In addition, the tokens value decreases by 700 ECU if you had chosen (a) or (b) and was found to be incorrect, or 0 ECU if found to be correct. See Table C2 for an overview of the redemption rate. Here are some examples to help you understand the redemption rate:
Table C1: TRADE: The 7 Possible Outcomes

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chance</td>
<td>1/7</td>
<td>1/7</td>
<td>1/7</td>
<td>1/7</td>
<td>1/7</td>
<td>1/7</td>
<td>1/7</td>
</tr>
<tr>
<td>Hat A</td>
<td>B</td>
<td>R</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>Hat B</td>
<td>B</td>
<td>B</td>
<td>R</td>
<td>B</td>
<td>R</td>
<td>R</td>
<td>B</td>
</tr>
<tr>
<td>Hat C</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>B</td>
</tr>
</tbody>
</table>

B=BLACK and R=RED

Table C1: TRADE: The 7 Possible Outcomes

<table>
<thead>
<tr>
<th></th>
<th>Correct</th>
<th>Incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>“determined your hat colour” in Period 1</td>
<td>950 ECU</td>
<td>250 ECU</td>
</tr>
<tr>
<td>“determined your hat colour” in Period 2</td>
<td>900 ECU</td>
<td>200 ECU</td>
</tr>
<tr>
<td>“determined your hat colour” in Period 3</td>
<td>850 ECU</td>
<td>150 ECU</td>
</tr>
<tr>
<td>“determined your hat colour” in Period 4</td>
<td>800 ECU</td>
<td>100 ECU</td>
</tr>
</tbody>
</table>

Table C2: TRADE: Token Redemption Rate

1. You hat is black. In period 1 you choose (c), in period 2 you choose (c) and in period 3 you choose (b). You have correctly determined your hat colour and your token redemption rate is therefore 850 ECU.

2. You hat is black. In period 1 you choose (c), in period 2 you choose (c) and in period 3 you choose (a). You have incorrectly determined your hat colour and your token redemption rate is 150 ECU.

End of Round Payoff

Your payoffs at the end of each round will be determined as followed:

Payoffs = (Money After Trading Stage - 6000) + (Tokens) x (Redemption Rate)

If your payoffs will found to be negative, we will round it off to 0 ECU. This completes the description of each experimental round. After the completion of 10 experiment rounds, we require you to complete a survey before you receive your cash payments. Please feel free to clarify any question or doubts you might have with regards to the instructions.
Figure C1: Screen Shot (TRADE) - You see all other Hat colours

![Figure C1](image1.png)

**THIS IS A NEW EXPERIMENTAL ROUND**

<table>
<thead>
<tr>
<th>Hat A (You)</th>
<th>Hat B</th>
<th>Hat C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(BLACK HAT)</td>
<td>(BLACK HAT)</td>
</tr>
</tbody>
</table>

"there is at least One Black Hat"

Figure C2: Screen Shot (TRADE) - Trading Stage

![Figure C2](image2.png)

**Trading Stage (You only trade with other Subjects under the same hat)**

<table>
<thead>
<tr>
<th>Year</th>
<th>Hat A (You)</th>
<th>Hat B</th>
<th>Hat C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>(RED HAT)</td>
<td>(BLACK HAT)</td>
</tr>
</tbody>
</table>

Year Tabled: 1

Market Ask Prices Market Bid Prices Year Result

Ask 200 200
Figure C3: Screen Shot (TRADE) - Trading Stage Results

Figure C4: Screen Shot (TRADE) - Decision Stage Period 1
**Figure C5: Screen Shot (TRADE) - Decision Stage Period 2**

<table>
<thead>
<tr>
<th>Hat A (Red)</th>
<th>Hat B (Black)</th>
<th>Hat C (Red)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Decision Stage: Period 2</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Player</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Excluded Game</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Player B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Player C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**What the other subjects choose in period 1**

<table>
<thead>
<tr>
<th>Hat A</th>
<th>Hat B</th>
<th>Hat C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Figure C6: Screen Shot (TRADE) - Decision Stage Period 3**

<table>
<thead>
<tr>
<th>Hat A (Red)</th>
<th>Hat B (Black)</th>
<th>Hat C (Red)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Decision Stage: Period 3</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Player</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Excluded Game</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Player B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Player C</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**What the other subjects choose in period 2**

<table>
<thead>
<tr>
<th>Hat A</th>
<th>Hat B</th>
<th>Hat C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
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</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
5.3 Chapter 4

5.3.1 Instructions - English Version

In this experiment, we are going to simulate a stock market environment where you are able buy and sell certificates. All payoffs in this experiment will be denoted in Experimental Currency (ECU). The experiment will consist of 2 practice rounds (non-paying) followed by 10 experimental rounds. At the end of the experiment, the computer will randomly select one of the 10 experimental rounds, and your earnings in that round will be paid to you in cash at the exchange rate of 15 ECU to 1 Shekels. In addition, we will also pay you a 25 (15 in Session 1) Shekel show up fee.

We will now describe each experimental round. At the beginning of each round, you will be randomly assigned into either Group A, Group B or Group C. Each group will consist of exactly 4 subjects. Thereafter the computer will choose a type for each group, which could either be $X$ or $O$ with equal chances. This implies that there is a 1 in 2 chance of your group type being $X$ and a 1 in 2 chance of your group type being $O$. The table below describes all possible type combinations for the three groups and the chance for each combination (see Table 4.1). You will not know your own group type but will be able to observe the group type of all other groups.

You will first enter the trading stage, where you have the opportunity to buy and sell 4 classes of certificates labelled S0, S1, S2 and S3. Each participant begins the trading stage with 6 units of each class of certificate and a loan of 5000ECU cash, which you will pay back at the end of the round. You buy and sell certificates through a continuous double auction mechanism and the trading period will last for exactly 300 seconds (5 minutes). We shall now describe the mechanism.

The Double Auction Trading Mechanism

Each participant can buy or sell only one class of certificate at each time. Participates will first select the class of certificate they would like to buy or sell and thereafter decide submit the Bid and Ask price to the market.

- **Ask price (between 0 and 200 ECU)** = How much you are willing to sell a unit of Certificate for.
- **Bid price (between 0 and 200 ECU)** = How much you are willing to buy a unit of Certificate for.

The Bid and Ask prices of all participants will be instantaneously reflect in the columns: Market Ask Prices and Market Bid Prices.

- **Buy**: To buy a certificate, simply select the price in the Market Ask Price Column and click the Buy Button
- **Sell**: To Sell a certificate, simply select the price in the Market Bid Price Column and click the Sell Button

The Column Latest Transaction Price describes the history of transaction prices for each class of certificate. Please note that you are able to buy and sell any class of certificates.

At the end of round, your certificates $S_0$, $S_1$, $S_2$ and $S_3$ will be redeemed at a specific value that depends on the total number of type $X$ groups in the round.

- If there was no type $X$ groups, security $S_0$ will be redeemed at 100 ECU and the others at 0 ECU
- If there was only one type $X$ group, security $S_1$ will be redeemed at 100 ECU and the others at 0 ECU
- If there was only two type $X$ group, security $S_2$ will be redeemed at 100 ECU and the others at 0 ECU
- If there was only three type $X$ group, security $S_3$ will be redeemed at 100 ECU and the others at 0 ECU
For example, if Group A & Group B & Group C were all type X, then certificates of class S0, S1 and S2 will be redeemed at 0 ECU, whilst certificate of class S3 will be redeemed at 100 ECU. Alternatively, if Group A & B were type X but Group C was type O, then certificates S0, S1 and S3 will be redeemed at 0 ECU, whilst certificate S2 will be redeemed at 100 ECU.

**Submitting your Decision**

Before you will know the redemption value of your certificates, you will be presented with the following question, which you must answer.

Qns: “How many type X groups are there in this Round?” to which you must answer with either (a) 0 Type X groups, (b) 1 Type X groups, (c) 2 Type X groups or (d) 3 Type X groups

Please note that the question refers to the “total number of type X groups” and not the type of your group. You will be awarded 200 ECU if you are correct and lose 200 ECU if you are wrong.

**End of Round Payoffs**

After you have completed the decision stage, your payoff for the round are computed as:

Payoffs = (Money at the end of Trading Stage) + (Money from Certificate Redemption) + (Payoffs from Decision) - (5000 ECU loan)

Please feel free to clarify any questions or doubts that you might have with regards to the instructions or the experimental design.


