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In periodic systems of low-symmetry, the Bragg condition for the complete interference of waves along the contour of the Brillouin zone (BZ) boundary is not generally satisfied. As a result, band-gaps do not necessarily occur at this boundary. This letter demonstrates this experimentally by recording the iso-frequency contours for surface plasmon polaritons (SPPs) supported on a diffraction grating with an underlying 2D oblique Bravais lattice. It is shown that these contours do not intersect the BZ boundary perpendicularly, as the symmetry operations of the lattice place no conditions on the surface wave interference at this boundary. © 2015 AIP Publishing LLC.

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From polaritons in photonic thin-films and phonons in acoustic metamaterials to spin-waves in magnetic media and electrons in crystals, the interaction and formation of stationary waves at the Brillouin zone (BZ) boundaries are a phenomenon that permeate many active areas of modern physics.

Recently, there has been huge interest in the formation of standing waves in 2D systems, spurred on by the isolation of graphene in 2004 and its associated electron transport characteristic. The unique properties of wave propagation in the honeycomb 2D lattice of graphene have inspired work with electron waves at Brillouin zone (BZ) boundaries which is a consequence of the lattice symmetry. The reciprocal lattice set of all the two-dimensional Bravais lattices. This reciprocal lattice is itself oblique, with the reciprocal lattice vectors defined as \( \mathbf{k}_{\perp} \) and \( \mathbf{k}_{\parallel} \), oriented at an angle with respect to each other (\( \alpha \)), such that \( \alpha \neq 90\)°. To realise this symmetry using surface-relief gratings, two diffraction gratings of different pitches are “crossed” at an angle \( \alpha \) such that \( \alpha \neq 90\)°, forming an oblique bigrating. Such a grating is illustrated in Figure 1(a).

The wave used for this investigation is a surface plasmon polariton (SPP) supported on a diffraction grating that provides the underlying oblique lattice. An oblique lattice is formed of an infinite array of lattice points separated by two lattice vectors of different magnitudes, oriented at an angle with respect to each other (\( \alpha \)), such that \( \alpha \neq 90\)°. To realise this symmetry using surface-relief gratings, two diffraction gratings of different pitches are “crossed” at an angle \( \alpha \) such that \( \alpha \neq 90\)°, forming an oblique bigrating. Such a grating is illustrated in Figure 1(a).

The reciprocal space map of the corresponding lattice is shown in Figure 1(b) and constitutes the lowest symmetry lattice set of all the two-dimensional Bravais lattices. This reciprocal lattice is itself oblique, with the reciprocal lattice vectors defined as \( \mathbf{k}_{\perp} \) and \( \mathbf{k}_{\parallel} \), oriented at an angle \( \alpha \) with respect to each other. The only symmetry operation possible for this oblique lattice is a rotation around a lattice point of 180°, which for 2D lattices is the equivalent of an inversion operation. Centred about each lattice point in Figure 1(b) are circles representing various scattered iso-frequency contours. The circles formed with solid lines show the contours for a grazing photon, and the dashed lines represent the iso-frequency contours for scattered SPPs which lie outside their respective grazing-photon lines (diffraction lines) due to the greater momentum of SPPs compared to light. In this simple cartoon, the SPPs do not interact and cross through each other unperturbed. However, if the SPPs interact to form band-gaps these iso-frequency contours will deform. The black circle is the case of the un-scattered zero-order light, which is the region of \( k \)-space accessible for mapping using the experimental method of imaging scatterometry.

To realise an underlying oblique lattice along which the surface waves propagate, SPP supporting bigratings were fabricated. The coordinate system for this type of grating is illustrated in Figure 1(a).

![Image](image.png)

**FIG. 1.** (a) A schematic of the oblique bigrating and the coordinate system. (b) The corresponding reciprocal space map of the oblique lattice with solid/dotted line circles indicating scattered light/SPP momentum states at a fixed frequency. (c) Modeled and (d) experimentally mapped iso-frequency contours via reflection (colorscale) of SPPs on an oblique lattice. The mode minima from (a) have been included in (d) as yellow circles for comparison.

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shown in Figure 1(a). The plane of incidence is defined at an azimuthal angle of $\phi$ so that when $\phi = 0^\circ$ the wavevector of incidence light lies along the $x$-direction. When the electric field vector of the impinging radiation is contained within the plane of incidence, the light is said to be TM polarised, and when the electric vector lies orthogonal to the plane, it is TE polarised. The $x$-direction is collinear with the grating vector $k_{gx} = 2\pi x / \lambda_{gx}$ for the longer-pitch grating, which possesses a periodicity of $\lambda_{gx}$. This period, for all the gratings presented in this letter, is $\lambda_{gx} = 600$ nm. The second, shorter-pitch grating lies at an angle of $x = 75^\circ$ along the $v$ axis (defined lying in the $xy$ plane at an angle $x$ to $x$) and for the grating presented in this letter has a period $\lambda_{gy} = 400$ nm. The grating vector of this short-pitch grating is defined as $k_{gy} = 2\pi y / \lambda_{gy}$. An angle of $x = 75^\circ$ was chosen to lie midway between the high symmetry cases of $x = 60^\circ$ (hexagonal-like) and $x = 90^\circ$ (square/rectangular). The ratio of grooves to pitch of the gratings is designed as $\Gamma_x = \Gamma_y = 0.5$. The depths of the gratings are $d_1$ and $d_2$, and are designed as $d_1 = d_2 \approx 40$ nm. The gratings for this letter were fabricated using electron beam lithography (EBL) and a template stripping method.\(^5\)

The iso-frequency contours of SPPs are mapped using imaging scatterometry.\(^4\) In this technique, reflectivity anomalies map the allowed momentum contours of the SPP modes. Figure 1(c) shows theoretical iso-frequency contours for an oblique bgrating at $\lambda_0 = 700$ nm. The corresponding experimentally obtained iso-frequency surface is shown in Figure 1(d). In the experimental plot at the illuminating wavelength of 700 nm, the contrast of the entire SPP contour must identify how plasmonic band-gaps are illustrated in the surface waves on a periodic lattice. To show this clearly, we must identify how plasmonic band-gaps are illustrated in the iso-frequency contours recorded with imaging scatterometry. The iso-frequency image obtained using scatterometry maps $k$-space at a single frequency, with SPP bands seen as an anomaly in the reflected light. The group velocity of a general propagating wave is defined as, $v_g = \nabla_k \omega(k)$, where $v_g$ is the group velocity, $\omega(k)$ is the angular frequency of the wave as a function of wavevector, $k$, and $\nabla_k$ is the gradient operator with respect to $k$. For a small change in frequency $\Delta \omega$, the corresponding small movement in $k$-space, $\Delta k$, is related to this group velocity simply by $\Delta \omega = \nabla_k \omega(k) \cdot \Delta k$. For an iso-frequency contour, there must be no change in frequency along the contour ($\Delta \omega = 0$). Setting $\Delta \omega = 0$ restricts the values of $\Delta k$ to those values that move along a contour of equal frequency. It is then apparent since $v_g \cdot \Delta k = 0$, $v_g$ must lie perpendicular to $\Delta k$. This is true for any general contour of constant frequency. If the group velocity in one direction falls to zero at a boundary, such as at the BZ boundary, the iso-frequency SPP contour will intersect that boundary perpendicularly.

Figure 2 shows the mapped iso-frequency contours of SPPs for the oblique grating at a wavelength of $\lambda_0 = 650$ nm. The position of the SPP contours are found to present as bands of low reflectivity, with the polariser of the experiment chosen in this case to best couple light to the $(-1, 0)$ modes. Also annotated on the figure are the diffracted light circles (blue lines) and the BZ boundary formed using the Wigner-Seitz method\(^11\) (green line).

It is observed that the SPP contour passes through the BZ boundary seemingly unperturbed. In Figure 2, the SPP contour following the $(-1, 0)$ diffracted light circle is shown
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FIG. 2. Experimentally mapped surface plasmon iso-frequency contour via reflectivity (colorscale) at \( \lambda_0 = 650\) nm passing through the first BZ boundary. This contour neither meets the boundary perpendicularly nor is perturbed as it passes through, showing that the group velocity of the SP mode is still finite across the boundary.

FIG. 3. (Left) The BZ boundary of a (a) rectangular lattice and (b) and oblique lattice. The symmetry operations labelled allow the deduction of the shown vectors (arrows) from a single unit vector, \( \mathbf{r} \). (Right) Measured SPP iso-frequency contours around the BZ boundary (indicated by the red squares, (left) for (a) a rectangular lattice at \( \lambda_0 = 550\) nm and (b) the oblique lattice at \( \lambda_0 = 650\) nm.

to pass through the BZ at an angle which is not perpendicular to the boundary. This means that at the boundary, the SPP’s group velocity in the \((0, 1)\) direction is not zero, and no standing-wave states in this direction have formed. The \((0, 1)\) scattered SPP is not observed in this figure as the polarisation of the illuminating light has been chosen to only couple strongly to the \((1, 0)\) SPP. Additionally, the \((-1, 0)\) and \((0, 1)\) SPPs are not seen to interact, separated as they are by a weak multiple scattering process.

An example of SPP contours intersecting the BZ boundary on a rectangular lattice is shown for comparison in Figure 3(c). This scattergram is taken from a plasmonic grating with rectangular symmetry previously published in Ref. 4, at a wavelength of 550 nm. The visible dark bands correspond to the momentum states of the \((\pm 1, 0)\) scattered SPPs, and are highly perturbed from their corresponding \((\pm 1, 0)\) light lines, constrained as they are to meet this boundary perpendicularly. Such perturbations of SPP contours at the BZ boundary have also been observed for hexagonal \(^{12}\) and square \(^{13}\) symmetry lattices.

A Brillouin zone boundary in reciprocal space outlines a primitive unit cell in the reciprocal lattice and contains on the boundary points of high-symmetry. To determine this boundary that contains the maximum amount of high-symmetry points, the perpendicular bisectors of the vectors connecting the nearest neighbours to one lattice point are drawn, a method known as the Wigner-Seitz method.\(^{11}\) For the highly symmetric cases of square, rectangular, or hexagonal lattices, the boundary is a constant contour of high symmetry. Neumann’s principle with respect to our system requires that the physical properties of phenomena associated with the grating possess the same symmetry as the point symmetry group of the grating.\(^{14}\) Figure 3(a) shows an arbitrary vector, \( \mathbf{r} \) lying on the boundary of a rectangular unit cell. In the case of a rectangular grating, the mirror and translational symmetry allows the deduction of the other shown vectors through various reflections in the \( \sigma_v \) and \( \sigma_h \) planes or rotations about the \( C \) point. These vectors sum to give a magnitude of zero in the direction perpendicular to the zone boundary. Whether this vector field represents the SPP’s momentum, group velocity, or Poynting vector, the conclusion is the same: a standing wave forms perpendicular to the BZ boundary. These are observed experimentally as discontinuities of the SPP curves at the BZ boundary.

Using the same approach, we apply the symmetry operations of the oblique lattice to an arbitrary vector field in Figure 3(b). With no mirror symmetry, the oblique lattice possesses only translational and a two-fold rotation symmetry operations. As shown in the figure, there are no special conditions on the vectors lying along the BZ boundary formed using the Weigner-Seitz method, and no condition for the vectors to cancel perfectly. Standing waves do not necessarily occur at the BZ boundary. Notice that where SPPs meet at other locations inside the unit cell (see Figure 2) they do form band-gaps, but there is no requirement from symmetry that these intersections occur at the BZ boundary. This condition has recently been found in a theoretical study of acoustic waves in 2D phononic crystals.\(^{15}\)

In conclusion, SPPs propagating on an oblique bigrating have been investigated. The dispersion of these surface modes has been mapped experimentally and the SPP interactions discussed in terms of the available scattering amplitudes of the grating. Using imaging scatterometry, it is observed that the SPP contours are not perturbed as they pass through the conventional BZ boundary. A generalized discussion on the symmetry of the BZ is presented, concluding that this is because the BZ boundary on an oblique grating is not a contour of high symmetry, and only contains...
isolated points around which the symmetry conditions may be met for the formation of SPP standing waves. The experimental results are supported by excellent agreement with numerical predictions.

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