

# The Timing of Mergers along the Production Chain, Capital Structure, and Risk Dynamics

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## Abstract

I demonstrate that the timing of vertical mergers is generally dependent on industry characteristics. My predictions are consistent with empirically observed patterns of vertical mergers. I show that merger activity during economic upturns tends to be motivated by operating efficiencies, while merger activity during economic downturns tends to occur as a means of keeping the production chain operational. Mergers allow firms to capture synergies and improve efficiencies in order to survive economic contractions. The pricing framework implies that a vertical merger decision usually reduces risk during two different economic states.

**JEL Classification:** G13; G12; G34; D23; D24

**Keywords:** Vertical mergers, Real options, Risk

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## 1 Introduction

Mergers tend to occur in waves, where a period of relatively low merger activity is followed by a period of relatively high merger activity. This is a well known pattern of procyclical merger waves as described, for example, in Mitchell and Mulherin (1996). The theoretical literature has thoroughly discussed procyclical patterns of horizontal mergers (i.e., when firms combine operations at the same level of a production chain), and conglomerate mergers (i.e., when firms combine the operations of unrelated activities). However, vertical merger patterns (i.e., when firms combine operations along the same production chain), and the role of debt in this process, are topics that remain largely unexplored.

The primary goal of this paper is to study the timing of vertical mergers when the elimination of market inefficiencies provides incentives to integrate. I contribute to the extant literature by studying the patterns of vertical merger activity. I show that, during economic upturns, merger activity tends to be motivated by synergies that arise due to double markup elimination when firms have some degree of monopoly power. In contrast, during economic downturns, debt plays a role in the decision-making process. Firms may opt to merge as a way to keep their production chains operational. They can capture synergies and improve efficiencies, which offer firms a better chance of surviving during difficult financial times.

As a motivation for this study, I refer to empirical evidence that shows a distinct pattern of mergers along the production chain. For example, Fan and Goyal (2006) claim that vertical merger activity is not completely aligned with the well known pattern of procyclical merger waves found in Mitchell and Mulherin (1996). Their evidence suggests that vertical mergers tend to occur frequently during both economic upturns and downturns. Moreover, a recent empirical study by Netter, Stegemoller, and Wintoki (2011) revises data on mergers and acquisitions, and suggests that, during the recent financial crisis, “Merger activity could at worst be described as merely slowing down” (p. 2318). Bernile, Lyandres, and Zhdanov

(2011) also find that horizontally related firms may merge during economic downturns as a strategy to deter the entry of other firms. Ultimately, however, the reasons why firms merge during downturns are not yet fully understood.

My model's core prediction is that the timing of mergers along the production chain is industry-specific. In particular, procyclical mergers along the production chain that occur during economic upturns are more likely in industries where firms operate with higher markups and end customers are less price sensitive. In contrast, countercyclical mergers along the production chain that occur during economic downturns are more likely in industries where firms operate with lower markups and end customers are more price sensitive. Furthermore, debt can motivate an integration during an economic downturn, and it may affect the timing of an integration during an economic upturn. During economic downturns firms tend to merge as a means of keeping their production chain operational. During economic upturns, on the other hand, equityholders' decisions to postpone (accelerate) their default can delay (speed up) merger decisions. My model shows that vertical mergers may be a successful risk management tool during both economic upturns and downturns. I observe a general decrease in systematic risk at the time of a merger at the procyclical threshold. During the countercyclical threshold, systematic risk also decreases, given that only one firm is financially distressed.

To derive the above predictions, I develop a theoretical model within a real option framework, where firms have an opportunity to merge during either an economic upturn or downturn. The real option framework is a flexible device to model the optimal stopping problems of value-maximising decisions, and it is key to my results. I model the cost of the intermediate production input as a stochastic process.

The underlying assumption of the model is that firms merge vertically as a way to eliminate inefficiencies that arise along the production chain when they have market power in

both upstream and downstream markets.<sup>1</sup> The OECD report (2007, p. 7) summarises this concept as follows:

“Vertical mergers often lead to lower prices because of the elimination of double marginalization when there is market power up and downstream pre-merger. Instead of paying a wholesale price that includes a markup over marginal cost, the integrated firm will be able to access the input at its marginal cost. This gives it an incentive to increase output downstream, to the benefit of consumers.”

In other words, the source of value in a vertical integration takes the form of synergies that can be generated by eliminating the markups that each firm charges over their marginal cost. This decreases the final price and reduces underproduction on the end market, thereby increasing profits.

A vertical merger decision is affected by varying economic conditions in the presence of debt financing. During an economic upturn (i.e., when costs decrease), the synergies resulting from an integration will increase. A firm can therefore exercise an option to merge at a fixed cost. During an economic downturn (i.e., when costs increase), an integration is motivated by a desire to avoid bankruptcy and ensure the smooth operation of the production chain. The stochastic cost is positively related to the likelihood that one of the firms is under financial distress. Both firms that are linked along the production chain tend to be better off if they can keep their production structures operational. The value of becoming integrated during an economic contraction is thus higher than the default risk of one firm.

The pricing framework suggests that vertical mergers reduce risk. First, mergers during economic expansions tend to increase profits and reduce inefficiencies. Second, mergers during economic contractions, when only one firm is in financial distress, tend to reduce bankruptcy risk, while increasing the probability of surviving hard times.

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<sup>1</sup>Tirole (1988), Ch. 4

The remainder of this paper is organised as follows. Section 2 reviews the related literature, while section 3 provides the economic foundations for a theoretical framework of mergers along the production chain. Section 4 presents a baseline model of unlevered firms, and section 5 explores the motivation and timing of merger decisions when firms are levered. Section 6 presents a risk analysis and the asset pricing implications of merger decisions. Section 7 discusses the setting with multiple firms. Section 8 concludes.

## 2 Related literature

There is an extensive literature on industrial organisation, proposing a number of reasons why firms should merge and restructure. For example, consider scale effects in the case of horizontal mergers, diversification in the case of conglomerates, and cost savings in the case of vertical mergers.<sup>2</sup> While horizontal and conglomerate mergers have received a great deal of attention in the financial literature, vertical mergers remain relatively unexplored. In contrast to the industrial organisation literature, which discusses the determinants and welfare consequences of vertical mergers, I focus in this paper on the terms and timing of vertical mergers.

This article studies a vertical merger decision based on neoclassical arguments within a real options framework, where mergers are motivated by eliminating double markup. The analysis is based on a contingent claims model following Lambrecht (2004), Morellec and Zhdanov (2005), Lambrecht and Myers (2007), and Hackbarth and Miao (2012).<sup>3</sup> They

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<sup>2</sup>The cost saving motivation is prevalent in many papers from the industrial organisation literature (Perry (1989), Joskow (2008)). The successive markups are one of the most important issues in the vertical integration literature (Spengler (1950)). This literature primarily analyses the determinants and welfare consequences of vertical mergers. Some examples include the reduction of transaction costs due to incomplete contracting and agency costs (Coase (1937)), holdup problems and asset specificity in combination with uncertainty (Williamson (1971)), property rights (Grossman and Hart (1986), Holmstrom and Roberts (1998)), and price inflexibility (Carlton (1979)). Additionally, Choi (2000) analyse the welfare consequences of vertical mergers where the input cost uncertainty plays a role. They show that asymmetric input cost across firms affects industry structure by increasing rivals' costs at the downstream level.

<sup>3</sup>Margrabe (1978) is an early example of modeling mergers as an exchange option with exogenous timing.

develop real options models to analyse the terms and timing of takeovers when firms are unlevered. Lambrecht (2004) provides a comprehensive theoretical framework of a procyclical merger that is motivated by economies of scale. He explores horizontal mergers motivated by economies of scale. In contrast, the main logic behind my model is the analysis of mergers along the production chain that are motivated by the elimination of double marginalization. Morellec and Zhdanov (2005) extend the behavioural analysis of Shleifer and Vishny (2003) by constructing a two-factor model based on stock market valuations of integrating firms. Lambrecht and Myers (2007) model the disinvestment decision in declining markets. Hackbarth and Miao (2012) analyse horizontal mergers in oligopolistic industries. Furthermore, Bernile, Lyandres, and Zhdanov (2011) analyse strategic incentives for horizontal mergers that explain takeover activity during economic booms and recessions. They find the presence of a U-shaped pattern between demand and merger activity. I contribute to this literature by studying the timing of a vertical merger decision where surplus is derived endogenously from an economic model.

There has been a great deal of discussion in the literature on the effect of debt on firms' investment decisions (see e.g., Lyandres and Zhdanov (2010)). However, despite several recent articles, the relationship between capital structure and merger decisions is still not well understood. Morellec and Zhdanov (2008) focus on the effect of capital structure on winning a bidding contest between two acquirers. Their model supports empirical evidence that the winning bidder exhibits leverage that is below the industry average. Leland (2007), on the other hand, derives a model where only financial synergies motivate the merger decision. He finds that the magnitude of this effect depends on firm characteristics such as default costs, firm size, taxes, and the riskiness of cash flows. Hege and Hennessy (2010) present an analysis where the level of debt plays a strategic role in benefiting from a higher merger share. However, they note a trade-off between a higher surplus and the resulting debt overhang,

which precludes efficient mergers. I contribute to this literature by illustrating how capital structure can affect a merger decision along the production chain, during economic upturns and downturns, where both firms are monopolists.

This article is also related to literature pioneered by Berk, Green, and Naik (1999), which links firms' investment decisions with asset return dynamics. Further papers by Carlson, Fisher, and Giammarino (2004), Hackbarth and Morellec (2008), and Bustamante (2014) present models that explain the dynamics of risk and return by using changes in firm characteristics, such as size and book to market, or firms' strategic interactions in product markets. In particular, firms' levels of systematic risk may differ because their assets and growth options exhibit different sensitivities to market fluctuations. I contribute to this literature by analysing the implications of a vertical merger decision for risk dynamics when firms are financially levered.

### **3 Model of vertical integration**

My model considers two firms operating at different levels of a production chain: a downstream firm (D), and an upstream firm (U). The downstream firm produces a final product that uses an intermediate input supplied by the upstream firm. Firms face a downward-sloping demand and can exercise monopoly power to set the price over the marginal cost. The upstream firm might be regarded as an innovative firm that invests in a new technology. Its invention of a superior know-how means that the upstream firm obtains a perpetual patent (i.e., a certain degree of monopoly power) for an intermediate product. It then sells this product at a monopoly price to the downstream firm. The downstream firm transforms the intermediate input into a final product, and sells it at the monopoly price to the final customer.

Vertical integration is a response to inefficiencies that arise along the production chain

when firms have market power in both the upstream and downstream markets. In particular, those inefficiencies arise from the double markup that producers charge on both markets. Elimination of the double markup typically results in lower prices, higher production, and higher profits.<sup>4</sup>

To illustrate, consider the following examples. In 1987, Microsoft Corporation acquired a company named Forethought that had developed a program called Powerpoint. This allowed Microsoft to integrate Powerpoint into its program suite. Another example is the acquisition of NeXT in 1997 by Apple Inc. NeXT was developing the precursor software to such products as Mac OS X and iOS. Apple ultimately introduced the software into the iPhone and the iPad. Most innovative solutions involve patents that give firms some proprietary power. In both of these examples, the acquirers, possessing unique technology, were able to set pricing to their final customers with some degree of monopoly power.

In the vertical integration model, I abstract from the standard assumptions that the only source of uncertainty is the output price or a shock to the size of the market. I assume that uncertainty is related to the stochastic cost of production of the intermediate input,  $c_t$ , which is a process that is external to the economy. This process can be affected, among other things, by shocks to technology and imperfect indicators of productivity such as innovations, changes in the price of oil imports, or government regulations. Therefore, any shock that affects the upstream firm also affects the production cost of the intermediate product. Hence, the price of the intermediate input will subsequently change, which in turn will affect the price of the final product and the amount of output produced.

This transmission mechanism can be linked to the macroeconomic literature on economic fluctuations that are driven by real supply-side shocks. This strand of the macroeconomic literature is tied to the real business cycles theory.<sup>5</sup> It later evolved into dynamic stochas-

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<sup>4</sup>Double marginalization and related vertical externalities are discussed in Tirole (1988), Ch. 4.

<sup>5</sup>The real business cycles theory (Kydland and Prescott (1982)) is the dominant theory of the dynamics



tic general equilibrium models, which are now widely used by policymakers to explain the aggregate behaviour of an economy. According to theories based on real shocks, economic fluctuations in final output may be driven by changes in the cost of intermediate inputs. Therefore, in this model, a regime of low intermediate input cost can be interpreted as an economic upturn, and a regime of high intermediate input cost can be interpreted as an economic downturn.<sup>6</sup>

In the following subsections, I consider two cases for the downstream firm: i) where it can buy the intermediate input in the market, or ii) where it can make components in-house by merging with the upstream firm. I also specify the conditions necessary for integration along the production chain.

### 3.1 *Non-integrated firms: Buy option*

I first consider the case where the downstream firm buys the intermediate input on the market from the upstream firm, and both firms exercise their monopoly power over the pricing strategy. The downstream firm produces the final output ( $q_d$ ) using a neoclassical Cobb-Douglas production function that combines technology ( $A$ ) with the intermediate input ( $q_u$ ):

$$q_d = Aq_u^\alpha \tag{1}$$

where  $\alpha \in (0, 1)$  is a parameter that implies diminishing returns to scale of the production function<sup>7</sup>. In contrast to horizontal integration, firms in this model cannot scale up capital

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of output fluctuations. It posits that most economic cyclicalities can be explained by shocks to technology and imperfect indicators of productivity such as innovations, changes in oil prices, regulations, and so on.

<sup>6</sup>However, I do not negate theories that explain economic fluctuations as a response to demand-side shocks. My model presents a supply-side explanation of economic fluctuations, which can be considered as a supplementary explanation to our understanding of merger waves. This is further supported by empirical evidence on economic fluctuations that indicate merger waves (associated with economic expansions) are motivated by technology shocks and changes in regulations (Mitchell and Mulherin (1996)).

<sup>7</sup>This is a standard assumption of the neoclassical production function as the first derivative, w.r.t. the input is positive, and the second is negative. This condition will be satisfied if  $\alpha \in (0, 1)$ . For further discussion on the neoclassical production function and its implications, see Barro and Sala-i-Martin (2003).

to create synergies. The demand for the final product requires that the price charged on the downstream market ( $p_d$ ) be:

$$p_d = a q_d^{-\varepsilon} \quad (2)$$

where  $\varepsilon > 0$  ensures the properties of the demand function, and  $a$  is the constant market size parameter. The price elasticity of demand is constant and equals  $1/\varepsilon$ . The profit of the downstream firm when it buys the intermediate input at price ( $p_u$ ) is:

$$\Pi_{dt} = p_d A q_u^\alpha - p_u q_u \quad (3)$$

The downstream firm maximises the profit function with respect to its production input. The quantity of the input demanded by the downstream firm is a function of the price in the upstream market:

$$q_u = \left( \frac{a A^{1-\varepsilon} \alpha (1-\varepsilon)}{p_u} \right)^{\frac{1}{1-\alpha(1-\varepsilon)}} \quad (4)$$

To keep this problem economically meaningful, I assume  $\varepsilon < 1$ .<sup>8</sup> The profit function of the downstream firm with the optimally chosen input is therefore:

$$\Pi_{dt} = \tilde{A}^{\frac{1}{1-\alpha}} p_u^{\frac{\alpha}{\alpha-1}} \left( \tilde{\alpha}^{\frac{\alpha}{1-\alpha}} - \tilde{\alpha}^{\frac{1}{1-\alpha}} \right) \quad (5)$$

where  $\tilde{A} = a A^{1-\varepsilon}$ ,  $\tilde{\alpha} = \alpha(1-\varepsilon)$ .

The upstream firm must determine how to price its product. It exercises monopoly rights over the pricing of a product that has a patent of infinite duration over production and sales. The instantaneous profit flow of the upstream firm is the difference between the price it

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<sup>8</sup>The necessary condition that  $\varepsilon < 1$  implies that the production function has decreasing returns to scale. This assumption is accepted in the literature; see, for example, Barro and Sala-i-Martin (2003) or Lambrecht (2004)

charges ( $p_u$ ) and its marginal cost ( $c_t$ ):

$$\Pi_{ut} = (p_u - c_t)q_u \quad (6)$$

However, the upstream firm does not have control over the production input price. This uncertainty is thus represented by the marginal cost ( $c_t$ ), which in this model is the state variable and evolves as a geometric Brownian motion with drift:

$$dc_t = \mu c_t dt + \sigma c_t dz \quad (7)$$

where  $\mu$  and  $\sigma$  are constant parameters, and  $dz$  is the increment of a Wiener process with a zero mean. The upstream firm maximises its profits with respect to the input price. The first-order condition yields the optimal pricing strategy in the upstream market<sup>9</sup>:

$$p_{ut} = \frac{1}{\tilde{\alpha}} c_t \quad (8)$$

The price in the upstream market is determined as the markup ( $1/\tilde{\alpha}$ ) over the marginal cost. The quantity sold in the upstream market is:

$$q_{ut} = \left( \frac{\tilde{A} \tilde{\alpha}^2}{c_t} \right)^{\frac{1}{1-\tilde{\alpha}}} \quad (9)$$

The profit function of the upstream firm is then:

$$\Pi_{ut} = c_t^\gamma \pi_u(\tilde{A}, \tilde{\alpha}) \quad (10)$$

where:  $\pi_u(\tilde{A}, \tilde{\alpha}) = \tilde{A}^{\frac{1}{1-\tilde{\alpha}}} \left( \tilde{\alpha}^{\frac{1+\tilde{\alpha}}{1-\tilde{\alpha}}} - \tilde{\alpha}^{\frac{2}{1-\tilde{\alpha}}} \right)$  and  $\gamma = \frac{\tilde{\alpha}}{\tilde{\alpha}-1}$

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<sup>9</sup>Solving the model for quantity competition results in the same pricing strategy as for a monopoly.

Note further that the profits of the upstream firm depend negatively on input cost, and positively on a firm's level of technology, measure of returns to scale in the production of final output, and the inverse of the price elasticity of demand in the final goods market. Combining Eq. (5) and (8), I obtain the equilibrium profit of the downstream firm:

$$\Pi_{dt} = c_t^\gamma \pi_d(\tilde{A}, \tilde{\alpha}) \quad (11)$$

where:  $\pi_d(\tilde{A}, \tilde{\alpha}) = \tilde{A}^{\frac{1}{1-\tilde{\alpha}}} \left( \tilde{\alpha}^{\frac{2\tilde{\alpha}}{1-\tilde{\alpha}}} - \tilde{\alpha}^{\frac{1+\tilde{\alpha}}{1-\tilde{\alpha}}} \right)$

The profits of the downstream firm are a decreasing function of input cost. They increase with the returns to scale parameter and with the level of technology (where  $\alpha < \frac{1}{1-\varepsilon}$  is always satisfied).

### 3.2 Integrated firms: Make option

Next, I consider a case where the downstream firm merges with the upstream firm. The high markup in the upstream market provides the incentive for such a merger. The integrated firm can then access the input at a lower price, which is equal to the marginal cost. A decrease in the input price lowers the price of the final good, thereby increasing sales and resulting in higher profits. The integrated firm can eliminate any market power inefficiencies, while increasing production to an optimal level. I show the profit of the integrated firm as follows:

$$\Pi_{mt} = \tilde{A} q_m^{\tilde{\alpha}} - c_t q_m \quad (12)$$

The solution to the integrated firm optimisation problem with respect to its production input results in the following:

$$q_{mt} = \left( \frac{\tilde{A} \tilde{\alpha}}{c_t} \right)^{\frac{1}{1-\tilde{\alpha}}} \quad (13)$$

The profit function of the integrated firm with the optimally chosen input is:

$$\Pi_{mt} = c_t^\gamma \pi_m(\tilde{A}, \tilde{\alpha}) \quad (14)$$

where:  $\pi_m(\tilde{A}, \tilde{\alpha}) = \tilde{A}^{\frac{1}{1-\alpha}} \left( \tilde{\alpha}^{\frac{\alpha}{1-\alpha}} - \tilde{\alpha}^{\frac{1}{1-\alpha}} \right)$ . Note that, because of the elimination of double markup pricing, the integrated firm's profits are higher than the sum of the two non-integrated firms' profits. This result is in line with previous studies, such as, e.g., Motta (2004). In particular, if the firm can obtain the intermediate product at a lower cost, then the price it charges on the final goods market can also decrease. As per the properties of the inverse demand function, I therefore expect that a decrease in price will result in a higher demand for the product, as well as higher sales.

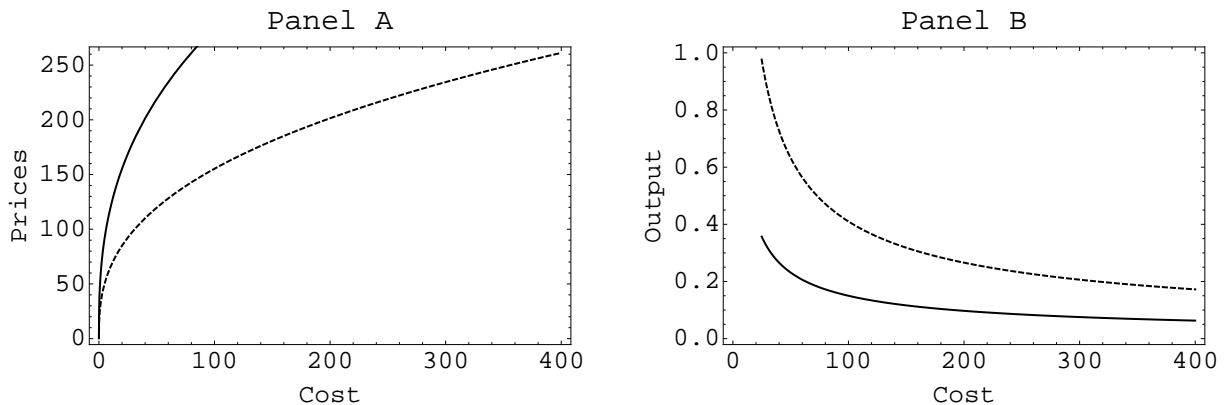


Figure 1: Prices and output of the non-integrated (solid line) and integrated (dashed line) firm. For this figure, the following parameters are fixed:  $A = 1$ ,  $a = 100$ ,  $\alpha = 0.7$ , and  $\varepsilon = 0.3$ .

To confirm that my model satisfies the above arguments, consider the following numerical example. Figure 1 depicts the level of prices on the end market (panel A) and the output sold (panel B) as a function of production cost ( $c_t$ ). The solid line gives the values of the individual firms; the dashed line gives the values of the integrated firms.

As production cost increases, and as a result of the inverse demand function, the output absorbed by the final market decreases. Non-integrated firms produce less and set higher

prices. As the graph shows, integration, despite resulting in lower prices, increases the profits of the merged company because it can expand production. It also increases the social welfare of the end customers.

#### 4 The timing of mergers between unlevered firms

In this section, I derive the value of an unlevered firm when marginal cost follows a stochastic process. I then calculate the realised merger surplus and an optimal exercise threshold.

##### 4.1 Valuation

Investors can borrow and lend at the risk-free rate of interest  $r$ . Managers' incentives are aligned with the maximisation of equityholder wealth. Irreversibility of investment implies that once exercised, the decision cannot be costlessly reversed. The value of the firm is the expected present value of future risky profits. For  $i = \{u, d, m\}$ , the instantaneous profit is  $\Pi_i = c_t^\gamma \pi_i(\tilde{A}, \tilde{\alpha})$ . The state variable ( $c_t$ ) is inversely related to the contingent claim on this asset. Therefore, the value of the firm is negatively related to the price of a production input. In other words, the value of the firm is a put option on the underlying asset:

$$V_i(c_t) = \mathbb{E}^c \left[ \int_0^\infty e^{-rs} \Pi_i(s) ds \right] \quad (15)$$

where  $\mathbb{E}^c$  is a conditional expectation operator, assuming  $c_0 > 0$ . The resulting stand-alone value of the unlevered firm, where the profits are driven by stochastic cost ( $c_t$ ), is:<sup>10</sup>

$$V_i(c_t) = \frac{c_t^\gamma \pi_i(\tilde{A}, \tilde{\alpha})}{r - \xi} \quad (16)$$

where  $\xi = \gamma\mu + 0.5\gamma(\gamma - 1)\sigma^2 < r$ .

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<sup>10</sup>To derive the value of the firm, I follow the procedure of the contingent claims valuation as discussed in Dixit and Pindyck (1994), Ch.3.

I can express the firm value as the discounted present value of cash flows, which can be perceived as the Gordon growth model, where  $\xi$  is the growth rate that depends on the model parameters.

#### 4.2 Merger synergies and the optimal merger threshold

A vertically integrated production structure can create synergies by eliminating the inefficiencies of a successive “monopoly-like” production structure. Monopolistic markups can arise at any stage of a non-integrated production chain when firms have monopoly power over pricing strategy.

Market imperfections result in higher prices and lower production. Therefore, as I noted earlier, non-integrated firms underproduce when compared to integrated firm. Vertical integration eliminates the costs created by the market power at any level of the production chain. More specifically, when two firms merge, the markup on one stage of the production chain is eliminated.

I conduct a detailed comparison of the two production structures next, and I state the benefit of merging resulting from the double markup elimination. I define the merger benefit as the difference between the stand-alone value of the integrated firm, and the sum of the two separate stand-alone entities, which is summarized in Lemma 1.

**Lemma 1** *The merger benefit resulting from the elimination of double markup, when firms have monopoly power over price-setting, is defined as:*

$$\Omega_t = V_{mt} - V_{dt} - V_{ut} = \frac{\Pi_{mt} - \Pi_{dt} - \Pi_{ut}}{r - \xi} = \frac{c_t^\gamma \tilde{A}^{\frac{1}{1-\alpha}} \left( \tilde{\alpha}^{\frac{\alpha}{1-\alpha}} - \tilde{\alpha}^{\frac{1}{1-\alpha}} + \tilde{\alpha}^{\frac{2}{1-\alpha}} - \tilde{\alpha}^{\frac{2\alpha}{1-\alpha}} \right)}{r - \xi} \quad (17)$$

*The vertical merger benefit is always positive, as  $\Pi_{mt} > \Pi_{dt} + \Pi_{ut}$ .*

**Proof.** See A ■

Note that a merger along the production chain results in a positive benefit, given that firms operate on decreasing returns to scale. Hence, the elimination of the intermediate markup boosts production quantity, and ultimately lowers the prices on the end market. The merger benefit stated in Lemma 1 is monotonically decreasing in stochastic cost ( $c_t$ ). The integration is associated with a fixed sunk cost ( $X$ ) to cover restructuring expenses, because firms only consider a merger when the stochastic benefit is higher than the cost of merging. The merger payoff ( $S_t$ ) is defined as follows:

$$S_t = \max_{c_t}[\Omega_t - X, 0] = \max_{c_t} \left[ \frac{c_t^\gamma \tilde{A}^{\frac{1}{1-\alpha}} \left( \tilde{\alpha}^{\frac{\alpha}{1-\alpha}} - \tilde{\alpha}^{\frac{1}{1-\alpha}} + \tilde{\alpha}^{\frac{2}{1-\alpha}} - \tilde{\alpha}^{\frac{2\alpha}{1-\alpha}} \right)}{r - \xi} - X, 0 \right] \quad (18)$$

Firms have an option to merge (denoted as  $OM$ ), but not an obligation, and they can materialise the benefit at sunk cost. There are incentives to postpone a decision to integrate because it cannot be costlessly reversed. However, delaying the exercise also means forgoing future benefits. I demonstrate that there exists a threshold  $\underline{c}$ , at which it is optimal to integrate, as soon as the variable  $c_t$  hits the threshold  $\underline{c}$  from above.

Prior to exercise, the option to merge satisfies an ordinary differential equation (see Appendix C), and can be solved subject to boundary conditions. First, consider the so-called no-bubble condition, which implies that the value of the option converges to zero if  $c_t \rightarrow \infty$ . Second, the option must satisfy the value-matching condition, which stipulates that the value of the option at the optimal threshold must be equal to the realised payoff, i.e.,  $OM(\underline{c}, \underline{c}) = \Omega(\underline{c}) - X$ . These two conditions result in the following expression (see Appendix C for further details):

$$OM(c_t, \underline{c}) = (\Omega_t - X) \left( \frac{c_t}{\underline{c}} \right)^\lambda \quad (19)$$

The expression in the first parentheses in Eq. (19) denotes the benefit created by the integration of the two firms. It increases when the state variable decreases, and it is multiplied



by the probability that the option will be realised at the exercise trigger. It is optimal to exercise the merger option when the state variable hits the merger threshold  $\underline{c}$  from above. This occurs when the marginal cost of the production input decreases. Next, I optimise Eq. (19) with respect to  $\underline{c}$  in order to determine the optimal merger threshold. The result is summarized in the following Lemma:

**Lemma 2** *An optimal merger threshold for firms that operate along the vertical production structure is:*

$$\underline{c} = \left[ \frac{\lambda}{\lambda - \gamma} \frac{X(r - \xi)}{\tilde{A}^{\frac{1}{1-\alpha}} \left( \tilde{\alpha}^{\frac{\alpha}{1-\alpha}} - \tilde{\alpha}^{\frac{1}{1-\alpha}} + \tilde{\alpha}^{\frac{2}{1-\alpha}} - \tilde{\alpha}^{\frac{2\alpha}{1-\alpha}} \right)} \right]^{\frac{1}{\gamma}} \quad (20)$$

where  $\lambda$  is the negative root of the quadratic equation  $z(z - 1)\sigma/2 + z\mu = r$ .

**Proof.** See B ■

Lemma 2 implies that unlevered firms, operating along the production chain, should integrate vertically when the state variable decreases and hits the threshold  $\underline{c}$  from above. According to theories based on real shocks, economic fluctuations in the final output can be caused by variability in the cost of the production input. These theories imply that, within the model, a decrease in the cost of the intermediate input may be associated with an economic upturn. Therefore, I should observe increased vertical merger activity during an economic upturn. This notion is consistent with empirical evidence. In particular, Mitchell and Mulherin (1996) show that increased merger activity is motivated by real changes such as productivity shocks or regulatory changes. They also show that increased merger activity coincides with periods of economic expansion.

My results supplement previous findings in e.g., Lambrecht (2004) and Hackbarth and Miao (2012), that a shock to demand of a nominal nature can affect the benefits of integration. In those models, a positive shock may accelerate the decision to merge, and result in a procyclical merger wave. In my model, a positive shock in the final output price, combined

with a negative shock in the cost of the intermediate input, would magnify the synergies and accelerate integration. The model thus predicts that vertical mergers of unlevered firms motivated by the elimination of double markup are procyclical. The next subsection provides further analysis of the merger threshold.

### 4.3 Model predictions

This subsection explores the characteristics and sensitivity of the merger threshold of unlevered firms to the model parameters: volatility ( $\sigma$ ), merger cost ( $X$ ), the inverse of demand elasticity ( $\varepsilon$ ), and returns to scale ( $\alpha$ ).

The effect of  $\sigma$  is consistent with the predictions of real options theory. It has two potential effects through: (i) the so-called *hysteresis factor*, which delays an investment decision, and is expressed here as  $[\lambda/(\lambda - \gamma)]^{\frac{1}{\gamma}}$ , and (ii) the growth rate of cash flows ( $\xi$ ). Higher volatility increases the growth rate and accelerates the merger decision. In line with the real options literature, I find that the first effect dominates the second. Therefore, the final effect is negative, and the standard outcome that higher volatility delays the investment decision holds. An increase in the merger cost ( $X$ ), on the other hand, delays the acquisition trigger.

It is fairly complex to predict the effect of model parameters such as the inverse of demand elasticity ( $\varepsilon$ ) and returns to scale ( $\alpha$ ) on the timing of vertical mergers. Therefore, I examine numerically the sensitivity of the merger threshold to  $\varepsilon$  and  $\alpha$ .

Figure 2 shows the merger threshold of the unlevered firm ( $\underline{c}$ ) as a function of the inverse of demand elasticity ( $\varepsilon$ ) and the returns to scale parameter ( $\alpha$ ). Panel A of Figure 2 shows that vertical mergers are more likely to occur when end customers are less price sensitive.<sup>11</sup> The intuition behind this finding is as follows. An increase in the inverse of the price elasticity of demand increases the difference between the final output price of non-integrated and

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<sup>11</sup>From previous analysis it can be recalled that  $\varepsilon$  is the component of the price elasticity of demand on the downstream market ( $1/\varepsilon$ ). Therefore, a higher  $\varepsilon$  implies end customers are less price sensitive.

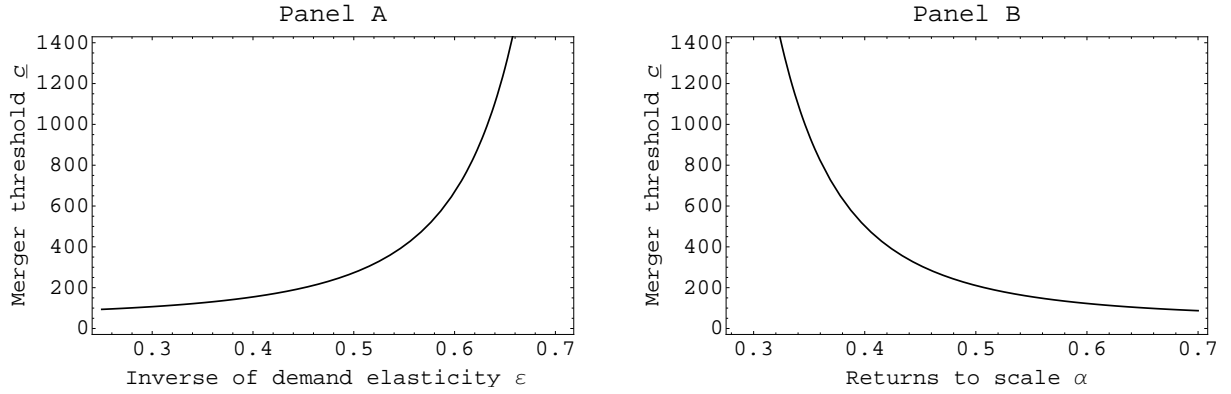


Figure 2: Merger threshold (unlevered firm). The figure illustrates the effect of the inverse of demand elasticity ( $\varepsilon$ ) and returns to scale ( $\alpha$ ) on the merger threshold of the unlevered firm. The following values are set fixed:  $a = 100$ ,  $A = 1$ ,  $\sigma = 0.2$ ,  $\mu = 0.01$ ,  $r = 0.06$ , and  $X = 50$ .

integrated firms due to higher output produced. This results in an increase in the wedge between the profits of integrated and non-integrated firms. Therefore, the incentives to merge increase, as the benefit from integration increases in  $\varepsilon$ .

Panel B of Figure 2 shows that vertical mergers are more likely to occur when firms operate on lower returns to scale ( $\alpha$ ). Note that  $\alpha$  is the component of the markup in the upstream market in Eq. (8). Firms with lower  $\alpha$  are less efficient at transforming the intermediate input. They have a higher markup on the upstream market, resulting in greater differences between the value of the integrated company and the two separate entities. Hence, the lower  $\alpha$ , the higher markup and the higher  $\varepsilon$  are the incentives to merge. Therefore, for firms with low  $\alpha$ , I observe an acceleration in exercising a merger option. The following corollary results from Lemma 2.

**Corollary 1** *Vertical mergers of unlevered firms (along the production chain, motivated by inefficiencies due to monopoly power) are more likely to occur in industries where firms operate with higher markups and where end customers are less price sensitive.*

Due to the model's dynamic features, I provide new empirical predictions about the timing

of merger activity and the patterns of vertical merger waves. In particular, the model implies that mergers along the production chain that occur during economic upturns are specific to certain industries where the end customers are less price sensitive (i.e., entertainment, luxury goods). Furthermore, I find that companies that are more constrained in terms of returns to scale, and thus operate on higher markups are more likely to engage in vertical mergers during economic upturns.

In general, the results imply that vertical mergers of unlevered firms are procyclical, which is similar to previous studies on horizontal mergers by Lambrecht (2004). Yet, the results in this paper show that horizontal and vertical mergers are driven by different incentives. While firms with increasing returns to scale, as in Lambrecht (2004), benefit from integrating by magnifying the scope of their operations; firms with decreasing returns to scale benefit from integrating by eliminating double markup. Moreover, I show that vertical mergers that occur during economic upturns are industry specific.

## 5 The timing of mergers between levered firms

This section discusses the role of debt in a model of mergers between vertically related firms. Each firm is now financed with equity and an infinite maturity risky debt contract with a fixed coupon  $b_i$ , which is necessary to cover part of the firm's expenses. Equityholders default when they are unwilling to inject more capital to cover operating losses and to service debt.

When firms are levered, two equilibrium concepts arise. First, when the cost of the intermediate input decreases, synergies from an integration increase, which implies a greater incentive to merge during an economic upturn. Second, when the cost of the intermediate product increases, the probability that one firm will default increases. The survival of one firm along a production chain depends on the success of another firm operating within that structure. Vertically related firms, therefore, have strategic reasons to merge during economic

downturns. For instance, consider the merger of Oracle and Sun Microsystems. This merger of a software company with a computing systems company allowed Oracle to provide integrated solutions to its clients. The merger, which exhibited vertical features, was announced after Sun Microsystems reported a period of losses, a decline in revenue, and a drop in market capitalisation. The merger ultimately achieved significant gains in efficiency.<sup>12</sup> This strategy is consistent with empirical evidence in Hotchkiss and Mooradian (1998), who suggest that bankrupt companies tend to be acquired by firms with which they have a prior relationship.

In order to specify the terms and timing of vertical integration, I formulate a dynamic programming problem that can be solved by backward induction. In the following subsections, I first solve for the post-merger values of contingent claims. I then solve for the pre-merger values of firms. Lastly, I derive the terms and timing of an optimal merger during different economic states.

### 5.1 *Post-merger valuations of integrated firms*

I derive the value of an integrated firm that combines the assets and liabilities of two non-integrated vertically linked companies. An integration occurs during either a procyclical merger threshold, denoted as  $\underline{c}$ , or a countercyclical merger threshold, denoted as  $\bar{c}$ .

At the time of the merger, a perpetual cash flow of  $\Pi_{mt}$  (defined in section 3.2) begins accruing to the integrated company. Recall that this instantaneous profit is higher than the combined profits of both firms separately because of the elimination of inefficiencies that arise in a monopolistic production structure.<sup>13</sup> Each firm has debt in place. Upon acquisition, the debt is integrated within the combined firm, and equals  $b_m = b_d + b_u$ . Upon bankruptcy, bondholders are entitled to a liquidation value of  $\Phi_m$ . Equityholders then select a bankruptcy threshold at  $\bar{c}_m$ .

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<sup>12</sup>[http : //ec.europa.eu/competition/mergers/cases/decisions/m5529\\_20100121\\_20682\\_en.pdf](http://ec.europa.eu/competition/mergers/cases/decisions/m5529_20100121_20682_en.pdf) (accessed on January 16, 2014)

<sup>13</sup>Proof in A

I denote post-merger equity, debt, and firm value as  $E_m(c_t)$ ,  $B_m(c_t)$ , and  $V_m(c_t)$ , respectively. Given the amount of cash flows for the integrated firm ( $\Pi_m$ ), and the value of the contracted coupon ( $b_m$ ), I solve for the closed-form solutions of the contingent claims values as presented in Lemma 3.

**Lemma 3** *The post-merger value of the equity, given that firms are vertically integrated is:*

$$E_m(c_t) = \frac{c_t^\gamma \pi_m}{r - \xi} - \frac{b_m}{r} + \left( \frac{b_m}{r} - \frac{\bar{c}_m^\gamma \pi_m}{r - \xi} \right) \left( \frac{c_t}{\bar{c}_m} \right)^\vartheta \quad (21)$$

where  $\vartheta$  is the positive root of the quadratic equation  $z(z - 1)\frac{\sigma^2}{2} + z\mu = r$ , and the value of debt is:

$$B_m(c_t) = \frac{b_m}{r} + \left( \Phi_m - \frac{b_m}{r} \right) \left( \frac{c_t}{\bar{c}_m} \right)^\vartheta \quad (22)$$

The value of the firm is thus the sum of its debt and equity:

$$V_m(c_t) = \frac{c_t^\gamma \pi_m}{r - \xi} + \left( \Phi_m - \frac{\bar{c}_m^\gamma \pi_m}{r - \xi} \right) \left( \frac{c_t}{\bar{c}_m} \right)^\vartheta \quad (23)$$

and the default threshold chosen by equityholders is:

$$\bar{c}_m = \left( \frac{\vartheta}{\vartheta - \gamma} \frac{(r - \xi)b_m}{\pi_m r} \right)^{\frac{1}{\gamma}} \quad (24)$$

**Proof.** See C ■

The standard interpretation of Eq. (21) is equity investors' claims to the perpetual cash flows generated by the firm, plus the value of the option to default with a probability of  $(c_t/\bar{c}_m)^\vartheta$ . Eq. (22) is the value of the firm's debt. The perpetual contract for bondholders guarantees a payment of the fixed coupon of  $b_m$  until default. In the case of a bankruptcy, it guarantees a payment of a liquidation value of  $\Phi_m$ . The value of the firm in Eq. (23) is the sum of the equityholders' and debtholders' values. Eq. (24) summarises the default

threshold selected by equityholders.

## 5.2 Pre-merger valuation of non-integrated firms

In this subsection, I determine the pre-merger value of equity for both an upstream and a downstream firm. Prior to a merger, each firm generates a perpetual cash flow of  $\Pi_{it}$ , where now  $i = \{u, d\}$  and pays a coupon  $b_i$ . Each firm can default at the threshold  $\bar{c}_i = \left( \frac{\vartheta}{\vartheta - \gamma} \frac{(r - \xi)b_i}{\pi_i r} \right)^{\frac{1}{\gamma}}$  chosen by the equityholders.<sup>14</sup> Equityholders in each firm obtain additional capital gains in the form of expected future changes in equity over each time interval, tied to a merger option that can be materialised during either an economic upturn or downturn.

Moreover, each firm faces the decision problem of determining the optimal terms and timing for an acquisition. I derive the solution concept as follows. Both firms have perfect information over the stochastic synergies and they negotiate on terms and timing simultaneously, which is a process featuring Cournot game. The decision to merge is associated with a trade-off between the benefits and the costs of executing a merger.<sup>15</sup>

The firms share a new combined entity according to a unique sharing rule  $\Gamma = \{s_i; c\}$ .<sup>16</sup> I denote  $s_i$  as a share in a new integrated entity of  $i$ 's firm, and  $c$  as a merger threshold during an economic upturn ( $\underline{c}$ ) or downturn ( $\bar{c}$ ). The value of an upstream firm's equity at the time of a merger during an economic upturn equals the share in the new entity less the acquisition cost,  $E_u(\underline{c}) = \underline{s}_u(E_m(\underline{c}) - X)$ . The value of a downstream firm's equity at the time of a merger during an economic upturn becomes  $E_d(\underline{c}) = \underline{s}_d(E_m(\underline{c}) - X)$ . The total value of an integrated entity is divided between the two firms, thus satisfying the following condition  $\underline{s}_u + \underline{s}_d = 1$ .

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<sup>14</sup>The derivation of the default threshold is based on logic similar to that discussed in C

<sup>15</sup>Merger costs include transaction expenses (i.e., underwriting and legal fees), the present value of restructuring, and any coordination expenditures. Due to the synergies created, these costs are fully covered by an increase in the value of the merged firm's equity.

<sup>16</sup>If a bidder and target have perfect information over synergies, the payment method does not affect the negotiation process (as discussed in Morellec and Zhdanov (2008)).

An economic downturn is associated with an increase in the cost of an intermediate input. Mergers that take place during an economic downturn are generally motivated by two things: (i) a firm's financial distress, and (ii) an acquirer's desire to capture synergies from a merger. For example, a non-distressed, vertically related firm may desire to maintain an existing production chain, and will thus have an incentive to acquire a financially distressed firm.<sup>17</sup>

Non-distressed firms have some alternatives for keeping their production structure intact, for example, by establishing their own plants. However, the cost of opening a new plant during a recession is expected to be greater than or equal to the cost of buying the assets of a bankrupt company at liquidation value. One explanation for this phenomenon may be found in Tobin's Q theory, which posits that, during a recession, market values tend to be lower than replacement values. The ratio would thus recommend not investing in a new plant. And, if the cost of an intermediate input increases, the non-distressed firm would have an opportunity to merge with the distressed firm at the countercyclical merger threshold ( $\bar{c}$ ).

The roles of the acquirer and acquiree are determined by the relative capital structure of each company. If the bankruptcy of an upstream firm occurs earlier than the bankruptcy of a downstream firm ( $\bar{c}_u < \bar{c}_d$ ), the upstream firm will be acquired at the countercyclical threshold. If  $\bar{c}_u > \bar{c}_d$ , I would observe the opposite effect. The countercyclical merger threshold normally occurs earlier than the default threshold  $\bar{c}_i$ . The reason is that, at  $\bar{c}_i$ , equityholders get nothing for their claim, while at  $\bar{c}$ , they get a share in  $E_m$  and with an embedded option to default at  $\bar{c}_m$ . Therefore, this additional value stipulates the following condition:  $\bar{c} < \bar{c}_i$ .

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<sup>17</sup>In the case of a bankruptcy, the equityholders of a non-distressed firm may still postpone their decision to merge and opt to subsidize the distressed firm. However, they may not be willing to pay such a subsidy forever, because the present value of the instantaneous additional payment may be higher than the lump sum cost of a merger. In fact, it might be optimal for a non-distressed firm to acquire a vertically related firm during or after bankruptcy restructuring, because the price of the company is then equal to liquidation value. Debt is risky in the model, which implies that the value paid at default is lower than the value of perpetual debt. Otherwise, if the cost of an acquisition is too high, the non-distressed firm may opt to set up its own plant.



During an economic downturn, the equityholders of firm  $i$  can opt to either declare bankruptcy, or merge with a vertically related firm. Note, however, that if the equityholders of the distressed company file for bankruptcy, they will be left with nothing; if they remain a part of an integrated firm, their equity value will remain positive. Therefore, if the continuation value of the target firm is positive, the distressed firm's equityholders are essentially agreeing to a share in the integrated firm  $\bar{s}_i$  that is greater than zero.

A non-distressed firm also has an incentive to acquire a distressed firm before bankruptcy restructuring, because, as I noted earlier, survival of the distressed firm may be critical to the continuation of the vertically related firm's operations. Bankruptcy restructuring tends to be a rather long process, and it may result in the suspension of operations of the related firm.

Ultimately, however, it is the firms that decide on the terms and timing of a merger simultaneously. A unique strategy  $\Gamma = \{s_i; c\}$  is determined at the pro- and countercyclical thresholds, and it can be considered as a negotiation game between the bidder and the target. A merger at the procyclical threshold,  $\underline{c}$ , is not feasible if firms already merged at the countercyclical threshold  $\bar{c}$ . Also, merger at the countercyclical threshold,  $\bar{c}$ , is not feasible if firms already merged at the procyclical threshold  $\underline{c}$ . If  $c_0 \in (\underline{c}, \bar{c})$ , I denote  $\mathcal{L}(c_t)$  as the present value of \$1, to be received the first time  $c_t$  reaches  $\bar{c}$ , conditional on reaching  $\bar{c}$  before  $\underline{c}$ . Similarly, I denote as  $\mathcal{H}(c_t)$  the present value of \$1 to be received the first time  $c_t$  reaches  $\underline{c}$ , conditional on reaching  $\underline{c}$  before  $\bar{c}$ . I derive the optimal strategies for the timing of a merger and the share in the merged company for both firms during economic upturns and downturns. I assume that the strategy of each firm is restricted to the optimal exercise of its option. Proposition 1 summarises the results.

**Proposition 1** *If  $i = \{u, d\}$ , a unique negotiation strategy  $\Gamma = \{s_i; c\}$ , which depends on the sharing rule ( $s_i$ ) and timing ( $c_t$ ), is determined between the acquirer and the target. This*

strategy is stated as a Cournot game, where firms decide simultaneously on the terms and timing at each threshold. The procyclical merger threshold  $\underline{c}$  and the sharing rule  $\underline{s}_i$  are the solutions to the following problem:

$$\left. \frac{\partial E_i(c_t)}{\partial c_t} \right|_{c=\underline{c}} = \underline{s}_i \left( \frac{\partial E_m(c_t)}{\partial c_t} \right) \Big|_{c=\underline{c}} \quad (25)$$

The countercyclical merger threshold  $\bar{c}$  and the sharing rule  $\bar{s}_i$  are the solutions to the following problem:

$$\left. \frac{\partial E_i(c_t)}{\partial c_t} \right|_{c=\bar{c}} = \bar{s}_i \left( \frac{\partial E_m(c_t)}{\partial c_t} \right) \Big|_{c=\bar{c}} \quad (26)$$

where:

$$E_i(c_t) = \frac{c_t^\gamma \pi_i}{r - \xi} - \frac{b_i}{r} + \mathcal{L}(c_t) \left[ \bar{s}_i (E_m(\bar{c}) - X) - \frac{\bar{c}^\gamma \pi_i}{r - \xi} + \frac{b_i}{r} \right] + \mathcal{H}(c_t) \left[ \underline{s}_i (E_m(\underline{c}) - X) - \frac{\underline{c}^\gamma \pi_i}{r - \xi} + \frac{b_i}{r} \right] \quad (27)$$

and  $\vartheta$  and  $\lambda$  are the positive and negative roots of the quadratic equation  $z(z-1)\frac{\sigma^2}{2} + z\mu = r$ , respectively, and the stochastic discount factors  $\mathcal{L}(c_t)$  and  $\mathcal{H}(c_t)$  are defined as:

$$\mathcal{L}(c_t) = \frac{c_t^\vartheta \underline{c}^\lambda - c_t^\lambda \underline{c}^\vartheta}{\underline{c}^\lambda \bar{c}^\vartheta - \bar{c}^\lambda \underline{c}^\vartheta} \quad \text{and} \quad \mathcal{H}(c_t) = \frac{c_t^\lambda \bar{c}^\vartheta - c_t^\vartheta \bar{c}^\lambda}{\underline{c}^\lambda \bar{c}^\vartheta - \bar{c}^\lambda \underline{c}^\vartheta}$$

**Proof.** See D ■

The value of each firm before the merger, as defined in Eq. (27), is comprised of three major components. The first is the present value of the instantaneous profit that accrues to each firm before the acquisition, less a fixed coupon. The second is the product of the payoff of the option to merge during an economic downturn and a stochastic discount factor,  $\mathcal{L}(c_t)$ , which represents the present value of \$1 to be received at the time of a countercyclical merger. Finally, the third is a payoff of the option to merge during an economic upturn, and a stochastic discount factor,  $\mathcal{H}(c_t)$ , which represents the present value of \$1 to be received

at the time of a procyclical merger.

Note that the ownership share  $s_i$  in the integrated entity is positively related to the post-integrated value that accrues to firm  $i$ . Firms maximise their share of total merger synergies based on the expectation that their choice of  $s_i$  will have on the other firm's decision. Under perfect information, firms conjecture the optimal threshold when the surplus is maximised, and then divide it equally. No other equilibrium can exist because the merger will not be consummated otherwise. For example, if one firm requires a higher share than it can obtain at the optimal threshold, the merger will be exercised inefficiently, and negotiations will fail.

I derive the basic comparative statics in E. The results suggest that there is a general rule to simultaneously determine the optimal share in the new entity, and the merger threshold. In particular, when the upstream firm's share in the integrated equity increases, which may be associated with higher bargaining power, the integration threshold will decrease as the downstream firm awaits a higher merger surplus in order to exercise its option. This outcome has already been observed in the literature for unlevered firms (e.g., Lambrecht (2004), Morellec and Zhdanov (2005), and Hackbarth and Miao (2012)). I confirm that it holds when firms are levered.

One of the contributions I make here is to derive the effect of a change in the default threshold of the target firm on the merger decision during an economic upturn. I show that the decision of the acquirer depends not only on the share of the merged firm, but also on the target's capital structure. In particular, during an economic expansion, the acquirer is more likely to delay a merger decision if there is any change in the model parameters that affect the target firm equityholders' decision to postpone bankruptcy. Proposition 2 follows.

**Proposition 2** *The decision of the target to postpone (accelerate) bankruptcy delays (speeds up) the decision of the acquirer to integrate during an economic upturn.*

**Proof.** See E ■

Proposition 2 sheds light on the effect that risky debt can have on a merger decision. In essence, if the optimal merger threshold is affected by one firm’s default, then the timing of the merger during an economic upturn could change. Thus, a higher default probability for the target firm increases the value of the option to merge. The timing of the procyclical merger of the levered firm will be accelerated compared to the unlevered firm’s merger threshold.

### 5.3 Model predictions

In this subsection, I examine the effects of industry characteristics on the pro- and countercyclical merger thresholds. Figure 3 illustrates the effects of model parameters on the procyclical merger threshold. I observe that  $\varepsilon$  and  $\alpha$  exhibit similar effects as for an unlevered firm.

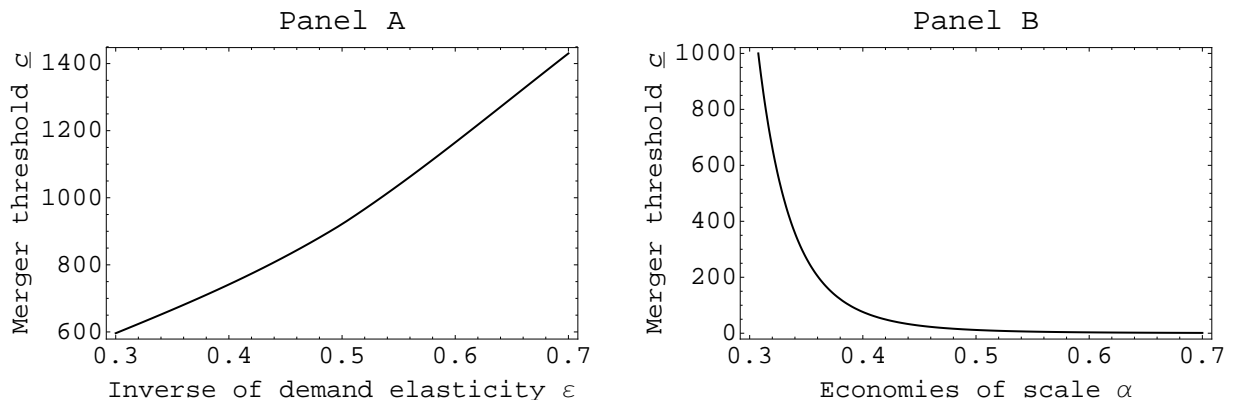


Figure 3: Procyclical merger threshold (levered firm). The figure illustrates the effect of an inverse of demand elasticity ( $\varepsilon$ ), and returns to scale ( $\alpha$ ) on the procyclical merger threshold of a levered firm. The parameters are set as in the base case.

Panel A of Figure 3 shows that vertical mergers that occur during an economic upturn are more likely when end customers are less price sensitive. The results are comparable to those for unlevered firms. In other words, a decrease in the price elasticity of demand increases the difference between the final output price of non-integrated and integrated firms due to an

increase in output supplied. This increases the difference between the profits of integrated and non-integrated firms when  $\varepsilon$  increases. Thus, the benefits from merger increase and an integration is more likely.

Panel B of Figure 3 shows that vertical mergers that occur during economic upturns are more likely when firms operate with higher markup. The effect of returns to scale ( $\alpha$ ) on the merger threshold for levered firms during an economic upturn has properties similar to those for unlevered firms. As  $\alpha$  decreases, markup and merger benefit increase. The intuition is similar to what I previously discussed for unlevered firms. Higher markup results in a greater difference between the value of an integrated firm and the value of two non-integrated firms. Therefore, mergers are more likely to occur. In summary, I state the following corollary for levered firms.

**Corollary 2** *Procyclical vertical mergers of levered firms (along the production chain, motivated by inefficiencies due to monopoly power) are more likely to occur in industries where firms operate with higher markups and end customers are less price sensitive.*

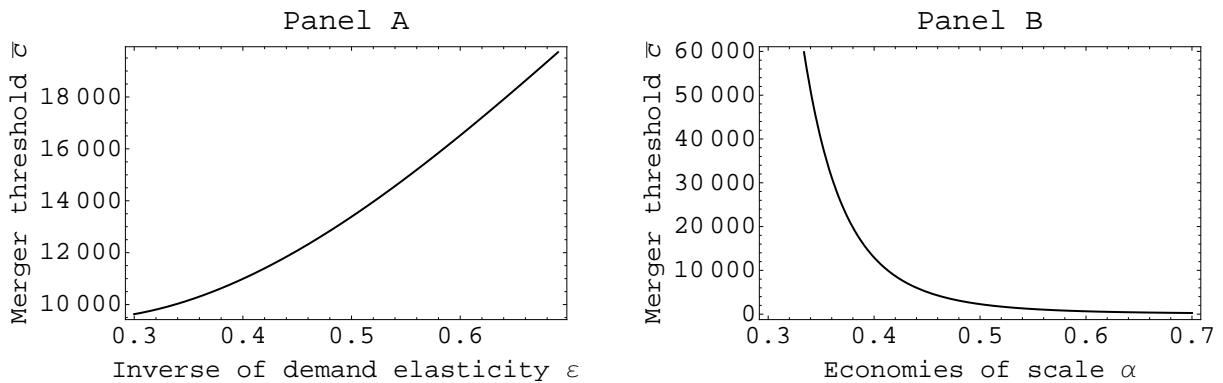


Figure 4: Countercyclical merger threshold (levered firm). The figure illustrates the effect of an inverse of demand elasticity ( $\varepsilon$ ), and returns to scale ( $\alpha$ ) on a countercyclical merger threshold of the levered firm. Parameters are set as in the base case.

Figure 4 illustrates the effects of model parameters on the countercyclical merger threshold.<sup>18</sup> The results in Figure 4 are opposite to the predictions for the procyclical mergers, and identify different conditions for the timing of countercyclical mergers. Panel A of Figure 4 shows that vertical mergers that occur during an economic downturns are more likely when end customers are more price sensitive. A decrease in  $\varepsilon$  increases the price elasticity of demand. Therefore, a price change may result in a greater decrease in profits in industries where end customers are more price sensitive. This results from the fact that firms operating in industries with higher price elasticity might find it difficult to transfer an increase in stochastic cost to their end customers. Therefore, an increase in cost results in lower profits that are associated with a higher probability of default, which accelerates the merger decision during an economic downturn.

Panel B of Figure 4 shows further that an increase in the level of returns to scale ( $\alpha$ ) speeds up the countercyclical merger as it lowers the markup. Firms operating with lower markup are more exposed to increases in the stochastic cost of the intermediate input, and they have more incentives to merge during economic downturns. The following corollary emerges for levered firms.

**Corollary 3** *Countercyclical vertical mergers of levered firms (along the production chain, motivated by inefficiencies due to monopoly power) are more likely to occur in industries where firms operate with lower markups and where end customers are more price sensitive.*

These results are consistent with economic intuition. Countercyclical mergers are more likely to occur when firms have a low cushion in terms of markups and hence lower profits. Similarly, firms profits are lower when end customers are more price sensitive as the increase in the price of the intermediate input cannot be fully passed on the price of the end product.

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<sup>18</sup>Relatively few papers have analysed the effects of capital structure on merger decisions. Exceptions include: Morellec and Zhdanov (2008), Leland (2007), and Hege and Hennessy (2010)

These results provide new insights into the pattern of mergers that is motivated by the elimination of double markups of mergers along the production chain.

The model's predictions for pro- and countercyclical mergers are consistent with empirical evidence on vertical merger patterns reported by Fan and Goyal (2006). They show that vertical merger activity is not fully aligned with the well known pattern of procyclical merger waves found by Mitchell and Mulherin (1996). Fan and Goyal (2006) claim that vertical merger activity "was intense in the oil and gas and the food industries in the beginning of 1980s." The oil, gas, and food industries clearly have a relatively low sensitivity to price changes. These mergers also occurred during a period of generally increased merger activity (i.e., the beginning of the 1980s has been defined as the fourth merger wave).<sup>19</sup> This evidence corresponds to the predictions reported for procyclical mergers.

Whereas, during periods of relatively low economic activity, Fan and Goyal (2006) observe an increase in the number of vertical mergers in "the communication and entertainment industries in the mid to late 1980s." These industries can be classified as having higher price sensitivity and these mergers occurred during periods of relatively lower merger activity. This evidence corresponds to the model's predictions for countercyclical mergers. For example, one of the largest transactions during that time was the merger between Time Inc. (cable television) and Warner Communications Inc. (film producer), which was announced in 1989. The deal had an aspect of survival pressure, and was expected to alleviate financial difficulties while increasing the debt capacity of the combined entity.

## **6 Risk dynamics**

Mergers are among the most important corporate events affecting stock returns because they change firms' systematic risk. Therefore, it is vital to fully understand how the riskiness of

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<sup>19</sup>For a very good recent summary of merger activity, see, e.g., Martynova and Renneboog (2008)

equity changes during the periods surrounding a merger. I can study this issue by referring to the real options literature, which views the merger possibility as an option to invest. The literature posits that exercising an investment option changes firm risk. In particular, when a firm holds assets in place and a growth option to invest at a fixed cost (with call option characteristics) in order to obtain an instantaneous profit flow, its risk will be inflated. When the firm exercises this option, it is expected to see a subsequent decrease in risk.

However, there is an important difference between the investment option and the option to merge. The latter involves two firms with different pre-merger characteristics, such as risk profile, capital structure, and size. It is therefore particularly important to analyse the risk dynamics over the merger episode.

In this section, I perform a risk analysis for integrated and non-integrated firms. I present comparative statics and a numerical example to explore the characteristics of an analytical solution. I first define the risk of a firm as in Carlson, Fisher, and Giammarino (2004), where they trace the risk profile of a firm by its beta. They prove that the beta of the firm can be derived in a form of elasticity that measures how equity value changes with respect to the change in state variable:

$$\beta_i = \frac{\partial E_i(c_t)}{\partial c_t} \frac{c_t}{E_i(c_t)} \quad (28)$$

where  $E_i(c_t)$  represents the equity value of the upstream, downstream, or merged firm.

I then derive the risk dynamics during the pre-merger episode as the sum of the downstream and upstream firm betas. Furthermore, I present post-merger betas for the combined equity. I focus on the effect of systematic risk on equity values. Proposition 3 summarises.

**Proposition 3** *If  $i = \{u, d\}$ , the risk levels of the levered pre-merger downstream and upstream equity, respectively, are stated as:*

$$\beta_i = \gamma + \gamma \frac{F_i}{E_i(c_t)} - \gamma \frac{OD_i(c_t)}{E_i(c_t)} + \tilde{\mathcal{L}}(c_t) \frac{OM_i(\bar{c}, \bar{s}_i)}{E_i(c_t)} + \tilde{\mathcal{H}}(c_t) \frac{OM_i(c, s_i)}{E_i(c_t)} \quad (29)$$



where  $F_i = \frac{b_i}{r}$ ,  $OD_i(c_t) = \left( \frac{b_i}{r} - \frac{\Pi_i(\bar{c}_i)}{r-\xi} \right) \left( \frac{c_t}{\bar{c}_i} \right)^\vartheta$ ,  $\tilde{\mathcal{L}}(c_t) = \frac{\vartheta c_t^\vartheta \bar{c}^\lambda - \lambda c_t^\lambda \bar{c}^\vartheta}{\bar{c}^\lambda \bar{c}^\vartheta - \bar{c}^\lambda \bar{c}^\vartheta}$ ,  $\tilde{\mathcal{H}}(c_t) = \frac{\lambda c_t^\lambda \bar{c}^\vartheta - \vartheta c_t^\vartheta \bar{c}^\lambda}{\bar{c}^\lambda \bar{c}^\vartheta - \bar{c}^\lambda \bar{c}^\vartheta}$ ,  $OM_i(\underline{c}, \underline{s}_i) = \underline{s}_i(E_m(\underline{c}) - X) - \frac{\Pi_i(\underline{c})}{r-\xi} + \frac{b_i}{r}$ , and  $OM_i(\bar{c}, \bar{s}_i) = \bar{s}_i(E_m(\bar{c}) - X) - \frac{\Pi_i(\bar{c})}{r-\xi} + \frac{b_i}{r}$ . If the merger occurs during an economic upturn (downturn), then the risk of the levered post-merger equity is:

$$\beta_m = \gamma + \gamma \frac{F_m}{E_m(c_t)} - (\gamma - \vartheta) \frac{OD_m(c_t)}{E_m(c_t)} \quad (30)$$

where  $F_m = \frac{b_m}{r}$  and  $OD_m(c_t) = \left( \frac{b_m}{r} - \frac{\Pi_m(\bar{c}_m)}{r-\xi} \right) \left( \frac{c_t}{\bar{c}_m} \right)^\vartheta$ .

**Proof.** See F ■

During a merger, the functional form of the equity value changes, and thus the systematic risk of the firm changes as well. Proposition 3 summarises the main factors that affect firm risk for the period preceding the merger in Eq. (29), and in a subsequent phase in Eq. (30).

The first term in both equations is the revenue beta (normalised to gamma), or the risk of the unlevered firm, which consists of the risk of assets in place and that of fixed operating costs.<sup>20</sup> The second term in both equations shows the effect of financial leverage on the riskiness of the equity. Note that, as debt coupons increase, so does the risk of the equity.

The third term in both equations reflects the effect of the option to default. Here I note that, as the probability of default increases, that is,  $(c_t/\bar{c}_m)^\vartheta$ , the value of the option to default as a fraction of firm equity also increases. Debt is risky, therefore  $\frac{\Pi_m(\bar{c}_m)}{r-\xi} < \frac{b_m}{r}$ . Furthermore, the option to default has an opposite effect on firm risk because it decreases the beta of equity.<sup>21</sup>

The pre-merger betas for the downstream and upstream firms are as defined in Eq. (29).

<sup>20</sup>I can prove that the risk of the unlevered firm is  $\gamma$  by taking the derivative of Eq. (16) with respect to the state variable.

<sup>21</sup>The absolute value of beta never crosses zero as  $\gamma \frac{\Pi_m(\bar{c}_m)}{r-\xi} - \vartheta \left( \frac{\Pi_m(\bar{c}_m)}{r-\xi} - \frac{b_m}{r} \right) \left( \frac{c_t}{\bar{c}_m} \right)^\vartheta < 0$

Pre-merger betas capture the aforementioned effects as well as the additional terms that relate to the effect of the options to merge during economic downturns and upturns, respectively. The option to merge during an economic downturn decreases the risk of the equity, as reflected by the term  $\tilde{\mathcal{L}}(c_t)$ . The option to merge during an economic upturn increases the risk of the equity, as reflected by the term  $\tilde{\mathcal{H}}(c_t)$ .

To understand the dynamics of beta at the procyclical merger threshold ( $\underline{c}$ ), consider first a case where the countercyclical exercise threshold converges to infinity  $\bar{c} \rightarrow \infty$ . This means that the derivative of the two-sided probability of a countercyclical merger goes to zero:  $\lim_{\bar{c} \rightarrow \infty} \tilde{\mathcal{L}}(c_t) \rightarrow 0$ , and the derivative of the two-sided probability of a procyclical merger converges to the following value:  $\lim_{\bar{c} \rightarrow \infty} \tilde{\mathcal{H}}(c_t) = \lambda \left( \frac{c_t}{\underline{c}} \right)^\lambda$ . As a result, the pre-merger beta at the procyclical threshold simplifies to the following expression:

$$\beta_i = \gamma + \gamma \frac{F_i}{E_i(c_t)} - \gamma \frac{OD_i(c_t)}{E_i(c_t)} + \lambda \frac{OM_i(\underline{c}, \underline{s}_i)}{E_i(c_t)} \left( \frac{c_t}{\underline{c}} \right)^\lambda \quad (31)$$

And the post-merger beta becomes:

$$\beta_m = \gamma + \gamma \frac{F_m}{E_m(c_t)} - (\gamma - \vartheta) \frac{OD_m(c_t)}{E_m(c_t)} \quad (32)$$

Given that the countercyclical threshold converges to infinity, note that the procyclical merger is associated with a decrease in the level of risk after the option exercise. This decrease in risk is in turn associated with the exercise of the option to merge and an increase in the importance of the option to default, given that the countercyclical merger is unlikely. This case intuitively follows the standard predictions in the literature that the exercise of the investment option decreases firm risk. Therefore, I expect that the risk of two vertically integrated firms will decrease if the merger occurs during an economic upturn.

Next, in order to analyse risk at the countercyclical merger threshold, consider a case

where the procyclical threshold converges to zero,  $\underline{c} \rightarrow 0$ . The derivative of the two-sided probability of a procyclical merger goes to zero:  $\lim_{\underline{c} \rightarrow 0} \tilde{\mathcal{H}}(c_t) \rightarrow 0$ , and the derivative of the two-sided probability of a countercyclical merger goes to:  $\lim_{\underline{c} \rightarrow 0} \tilde{\mathcal{L}}(c_t) = \vartheta \left(\frac{c_t}{\bar{c}}\right)^\vartheta$ . The pre-merger beta at the countercyclical threshold thus simplifies to the following expression:

$$\beta_i = \gamma + \gamma \frac{F_i}{E_i(c_t)} - \gamma \frac{OD_i(c_t)}{E_i(c_t)} + \vartheta \frac{OM_i(\bar{c}, \bar{s}_i)}{E_i(c_t)} \left(\frac{c_t}{\bar{c}}\right)^\vartheta \quad (33)$$

Assuming further that  $\bar{c} \rightarrow \bar{c}_i$ , the pre-merger beta at the countercyclical threshold becomes:

$$\beta_i \approx \gamma + \gamma \frac{F_i}{E_i(c_t)} - (\gamma - \vartheta) \frac{OD_i(c_t)}{E_i(c_t)} + \frac{\vartheta s_i [E_m(\bar{c}, \bar{s}_i) - X] \left(\frac{c_t}{\bar{c}}\right)^\vartheta}{E_i(c_t)} \quad (34)$$

And the post-merger beta is:

$$\beta_m = \gamma + \gamma \frac{F_m}{E_m(c_t)} - (\gamma - \vartheta) \frac{OD_m(c_t)}{E_m(c_t)} \quad (35)$$

Note that the procyclical threshold converges to zero, and the countercyclical threshold is close to the default threshold of firm  $i$ . Therefore, for relatively low level of debt, the countercyclical merger is associated with a decrease in the level of risk after the option exercise. There is also a possibility that the risk may increase after the integration if both vertically related firms are in financial distress.

The above predictions about risk decrease at the pro- and countercyclical thresholds are confirmed further by the following numerical example. In Figure 5, I present the risk dynamics. The dashed line corresponds to the sum of pre-merger betas of the downstream and upstream firms. The solid line corresponds to the beta of the post-merger combined equity value, assuming the merger occurred during an economic upturn ( $\underline{c}$ ) or downturn ( $\bar{c}$ ).

This analysis of risk in Figure 5 shows that beta is a monotonically increasing function

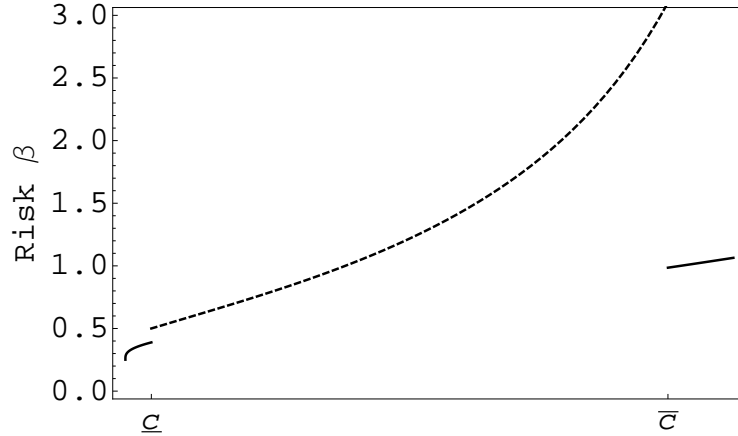


Figure 5: Pre- and post-merger beta. This figure presents betas as a function of the intermediate input cost  $c_t$ . The dashed line corresponds to the sum of pre-merger betas of the downstream and upstream firms. The solid line corresponds to the beta of the post-merger combined equity value, assuming that the merger occurred during an economic upturn ( $\underline{c}$ ) or downturn ( $\bar{c}$ ). The parameters are set as in the base case.

of the input cost. The shape results from the fact that, when the cost increases, the risk driven by the leverage also increases, because default becomes more likely. This numerical example confirms there is a risk reduction at the time of an integration during an economic upturn or downturn. In particular, there is a drop in risk during an economic upturn at the  $\underline{c}$  threshold. This result is based solely on the fact that, when a firm loses its option to merge, the risk associated with this option drops. Furthermore, at the countercyclical threshold  $\bar{c}$ , there is a decrease in risk due to the merger.

## 7 Multiple firms

The monopoly framework of the model allows to capture the effect of eliminating markup inefficiencies in the simplest and most intuitive manner. A more realistic industry structure might be imperfect competition in the upstream and downstream markets. The opportunity to eliminate double markup inefficiencies still remains a plausible motive for an integration,

and the main predictions of the model are still valid. This is because firms operating under conditions of imperfect competition retain some degree of market power, because their pricing strategy includes a markup over marginal cost. However, the markup in an oligopolistic industry is lower than that in a monopoly. Therefore, the level of concentration in an industry is thus positively correlated with the incentives to integrate.

In particular, the strongest incentive to merge, based on the notion of double markup elimination, prevails in the most concentrated industry structures, such as monopoly. The level of competition, however, decreases merger synergies as markups fall (Joskow (2008)). At the procyclical threshold, a fall in markups lowers synergies and therefore delays the merger. At the countercyclical threshold a fall in markups lowers profits and accelerates default. Firms may merge more quickly in order to keep the production chain operational. In the most extreme case, where markups and synergies, due to double markup elimination, vanish would be perfect competition in each market.

At the same time, a more competitive environment may also be accompanied by certain strategic reasons for a merger. The dominant approach in the industrial organisation literature analyses the reasons and welfare implications of vertical integration in competitive industries based on “raising-rival’s-costs” (Riordan (2008)). The raising-rival’s-costs theory suggests that vertical integration may soften competition by enhancing market power. For example, a downstream firm may buy an upstream firm in order to increase rivals’ costs of production, or to increase the costs to potential entrants (e.g., Aghion and Bolton (1987), Hart, Tirole, Carlton, and Williamson (1990)). Non-integrated rivals can become less competitive as a result of the increased cost. This may have several complementary implications for the analysis of incentives and the timing of vertical mergers. However, future research may consider a formal analysis of the timing of strategic vertical mergers in the presence of competition.

## 8 Conclusion

This paper develops a real options framework for mergers along the production chain. I contribute to the existing theoretical literature by analyzing the terms and timing of vertical mergers. In contrast to previous literature, I analyse the pattern of vertical merger waves by providing a rationale for merger activity during not only economic upturns, but also economic downturns. The results of the model are consistent with empirical evidence on vertical merger patterns reported by Fan and Goyal (2006).

I demonstrate that vertical mergers that occur during economic upturns tend to result from a desire to eliminate inefficiencies that are present in the production chain when firms have market power in both upstream and downstream markets. In contrast, during economic downturns, firms merge in order to keep their existing production chain operational.

The model predicts that procyclical mergers along the production chain are more likely to occur in industries where end customers are less sensitive to price changes, and where firms operate with higher markups. In contrast, countercyclical mergers along the production chain are more likely to occur in industries where end customers are more sensitive to price changes, and firms operate with lower markups.

Moreover, a target firm's capital structure also plays an important role in the decision making process. A target's decision to postpone (accelerate) default tends to delay (speed up) the acquirer's decision to merge during economic upturns.

Finally, I analyse the risk dynamics when firms integrate vertically. The model implies that there is a risk reduction when firms merge during an economic upturn, which results from exercising the merger option. During an economic downturn, there is a risk decrease only if one firm has a relatively low level of debt. However, if both firms are financially distressed, the level of risk may increase.

In conclusion, I highlight that the merger activity of firms operating along the same pro-

duction chain depends on industry characteristics and capital structure. The model shows that vertical mergers can reduce risk during both economic upturns and downturns. Future research could address the questions related to strategic interactions between vertically related firms. In particular, it would be interesting to study additional synergies that could arise in other market structures where firms make strategic investments.

## A Proof of Lemma 1

I define a gross merger surplus as:

$$\Omega_t = V_{mt} - V_{dt} - V_{ut} = \frac{c_t^{\frac{\tilde{\alpha}}{\tilde{\alpha}-1}} \tilde{A}^{\frac{1}{1-\tilde{\alpha}}} \left( \tilde{\alpha}^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \tilde{\alpha}^{\frac{1}{1-\tilde{\alpha}}} + \tilde{\alpha}^{\frac{2}{1-\tilde{\alpha}}} - \tilde{\alpha}^{\frac{2\tilde{\alpha}}{1-\tilde{\alpha}}} \right)}{r - \xi} \quad (1)$$

Given that  $c_t$ ,  $A$ , and  $(r - \xi)$  are positive, in order for the proof to be valid, the following expression  $\left( \tilde{\alpha}^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \tilde{\alpha}^{\frac{1}{1-\tilde{\alpha}}} + \tilde{\alpha}^{\frac{2}{1-\tilde{\alpha}}} - \tilde{\alpha}^{\frac{2\tilde{\alpha}}{1-\tilde{\alpha}}} \right)$  must be positive. I can rewrite it as:  $\left( \tilde{\alpha}^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \tilde{\alpha}^{\frac{1}{1-\tilde{\alpha}}} \right) - \tilde{\alpha}^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} \left( \tilde{\alpha}^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \tilde{\alpha}^{\frac{1}{1-\tilde{\alpha}}} \right) - \tilde{\alpha}^{\frac{1}{1-\tilde{\alpha}}} \left( \tilde{\alpha}^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \tilde{\alpha}^{\frac{1}{1-\tilde{\alpha}}} \right) > 0$ . Then, as  $\left( \tilde{\alpha}^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} - \tilde{\alpha}^{\frac{1}{1-\tilde{\alpha}}} \right) > 0$ , the expression simplifies to  $\tilde{\alpha}^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} + \tilde{\alpha}^{\frac{1}{1-\tilde{\alpha}}} < 1$ , which is always satisfied.

## B Proof of Lemma 2

Equityholders have an option to merge (OM) that resembles put option characteristics. Over the continuation region, the value of the merger option satisfies the Bellman equation:

$$rOM(c_t) = \frac{d}{d\Delta} \mathbb{E}[OM_{t+\Delta}] \Big|_{\Delta=0} \quad (1)$$

If  $OM(c_t)$  is a twice continuously differentiable function of the state variable  $c_t$ . By applying Ito's lemma, I can obtain:

$$rOM(c_t) = \mu c_t \frac{\partial OM(c_t)}{\partial c_t} + \frac{\sigma^2}{2} c_t^2 \frac{\partial^2 OM(c_t)}{\partial^2 c_t} \quad (2)$$

And a general solution is:

$$OM(c_t) = A_1 c_t^\vartheta + A_2 c_t^\lambda \quad (3)$$

where  $\vartheta$  and  $\lambda$  are the positive and negative roots of the quadratic equation,  $z(z-1)\frac{\sigma^2}{2} + z\mu = r$ , respectively. This equation can be solved subject to a number of boundary conditions.



First, a no-bubble condition implies that the value of the option converges to zero if  $c_t \rightarrow \infty$ . Since,  $\vartheta > 0$ , it follows that, for  $OM(c_t)$  to remain bounded,  $A_1$  must equal zero. Consequently, for  $c > \underline{c}$ :

$$OM(c_t) = A_2 c_t^\lambda \quad (4)$$

Second, a value-matching condition stipulates that, at the optimal threshold, the value of the option must equal to the realised payoff, i.e.,  $OM(\underline{c}, \underline{c}) = \Omega(\underline{c}) - X$ . Solving for  $A_2$  gives:

$$A_2 = (\Omega(\underline{c}) - X) \underline{c}^{-\lambda} \quad (5)$$

These two conditions determine the value of the option to merge in Eq. 19.

Finally, the optimal integration threshold is the solution to the first-order condition:  $\partial OM(c_t) / \partial c_t |_{c_t = \underline{c}} = 0$ . Lemma 2 follows.

### C Proof of Lemma 3

I assume that an integrated firm is financed with equity and infinite maturity debt, paying a fixed coupon  $b_m$ . Assuming that  $r$  is the risk free rate, the firm's equity  $E_m$  and debt  $B_m$  must satisfy:

$$rE_m(c_t) = \Pi_m - b_m + \frac{d}{d\Delta} \mathbb{E}[E_{m,t+\Delta}] \Big|_{\Delta=0} \quad (1)$$

$$rB_m(c_t) = b_m + \frac{d}{d\Delta} \mathbb{E}[B_{m,t+\Delta}] \Big|_{\Delta=0} \quad (2)$$

If  $E_m$  and  $B_m$  are twice-continuously differentiable functions of the state variable  $c_t$ , then by applying Ito's lemma, I obtain:

$$rE_m(c_t) = \Pi_m(c_t) - b_m + \mu c_t \frac{\partial E_m(c_t)}{\partial c_t} + \frac{\sigma^2}{2} c_t^2 \frac{\partial^2 E_m(c_t)}{\partial c_t^2} \quad (3)$$

$$rB_m(c_t) = b_m + \mu c_t \frac{\partial B_m(c_t)}{\partial c_t} + \frac{\sigma^2}{2} c_t^2 \frac{\partial^2 B_m(c_t)}{\partial^2 c_t} \quad (4)$$

And general solutions are:

$$E_m(c_t) = A_3 + A_4 c_t + A_5 c_t^\vartheta + A_6 c_t^\lambda \quad (5)$$

$$B_m(c_t) = A_7 + A_8 c_t^\vartheta + A_9 c_t^\lambda \quad (6)$$

where  $\vartheta$  and  $\lambda$ , respectively, are the positive and the negative roots of the quadratic equation,  $z(z-1)\sigma/2 + z\mu = r$ .

I solve these equations subject to the following boundary conditions. First, if the firm is closed at  $\bar{c}_m$ , then equityholders can walk due to limited liability, thus:

$$E(\bar{c}_m) = 0 \quad (7)$$

Second, as  $c_t \rightarrow 0$  the possibility of bankruptcy is less likely. Thus,  $E_m$  and  $B_m$  approach the unlimited liability values:

$$\lim_{c_t \rightarrow 0} E_m(c_t) = \frac{\Pi_m(c_t)}{r - \xi} - \frac{b_m}{r} \quad \text{and} \quad \lim_{c_t \rightarrow 0} B_m(c_t) = \frac{b_m}{r}$$

Third, at the bankruptcy threshold  $\bar{c}_m$ , bondholders are liable for the liquidation value of  $\Phi_m$ :

$$B_m(\bar{c}_m) = \Phi_m$$

Fourth, equityholders decide on the closure threshold  $\bar{c}_m$ , as follows:

$$\left. \frac{\partial E_m(\bar{c}_t)}{\partial c_t} \right|_{c_t = \bar{c}_m} = 0$$

Lemma 3 follows.

## D Proof of Proposition 1

I assume that  $c_0 \in (\underline{c}, \bar{c})$ . Over this region the instantaneous change in the value of equity  $E_i$ , for  $i = \{u, d\}$ , satisfies the Bellman equation of the following form:

$$rE_i(c_t) = \Pi_i(c_t) - b_i + \frac{d}{d\Delta} \mathbb{E}[E_{i,t+\Delta}] \Big|_{\Delta=0} \quad (1)$$

Applying Ito's lemma, I can show that the right-hand side of Eq. (1) is equal to:

$$rE_i(c_t) = \Pi_i(c_t) - b_i + \mu c_t \frac{\partial E_i(c_t)}{\partial c_t} + \frac{\sigma^2}{2} c_t^2 \frac{\partial^2 E_i(c_t)}{\partial^2 c_t} \quad (2)$$

The general solution is:

$$E_i(c_t) = \frac{\Pi_i(c_t)}{r - \xi} - \frac{b_i}{r} + A_{i1} c_t^\vartheta + A_{i2} c_t^\lambda \quad \text{for} \quad \underline{c} < c_t < \bar{c} \quad (3)$$

where  $\vartheta$  and  $\lambda$ , respectively, are the positive and negative roots of the quadratic equation,  $z(z - 1)\frac{\sigma^2}{2} + z\mu = r$ .

I calculate the constants  $A_{i1}$  and  $A_{i2}$  from the value-matching conditions at thresholds  $\underline{c}$  and  $\bar{c}$ . At the threshold  $c = \underline{c}$ , each firm exchanges its equity for a share  $\underline{s}_i$  of the integrated firm, thus:

$$E_i(\underline{c}) = \underline{s}_i(E_m(\underline{c}) - X) \quad (4)$$

At the threshold  $c = \bar{c}$ , each firm exchanges its equity for a share  $\bar{s}_i$  of the integrated firm, thus:

$$E_i(\bar{c}) = \bar{s}_i(E_m(\bar{c}) - X) \quad (5)$$

For  $i = \{u, d\}$ , the above conditions, Eqs. (3), (4), (5), give a system of four equations with four unknowns  $A_{u1}$ ,  $A_{d1}$ ,  $A_{u2}$  and  $A_{d2}$ . Solving gives the following expressions:

$$A_{u1} = \frac{\bar{c}^\lambda \left[ \underline{s}_u (E_m(\underline{c}) - X) - \frac{\Pi_u(\underline{c})}{r-\xi} + \frac{b_u}{r} \right] + \underline{c}^\lambda \left[ \bar{s}_u (E_m(\bar{c}) - X) - \frac{\Pi_u(\bar{c})}{r-\xi} + \frac{b_u}{r} \right]}{\underline{c}^\lambda \bar{c}^\vartheta - \bar{c}^\lambda \underline{c}^\vartheta} \quad (6)$$

$$A_{d1} = \frac{\underline{c}^\lambda \left[ \bar{s}_d (E_m(\bar{c}) - X) - \frac{\Pi_d(\bar{c})}{r-\xi} + \frac{b_d}{r} \right] + \bar{c}^\lambda \left[ \underline{s}_d (E_m(\underline{c}) - X) - \frac{\Pi_d(\underline{c})}{r-\xi} + \frac{b_d}{r} \right]}{\underline{c}^\lambda \bar{c}^\vartheta - \bar{c}^\lambda \underline{c}^\vartheta} \quad (7)$$

$$A_{u2} = \frac{\bar{c}^\lambda \left[ \underline{s}_u (E_m(\underline{c}) - X) - \frac{\Pi_u(\underline{c})}{r-\xi} + \frac{b_u}{r} \right] - \underline{c}^\vartheta \left[ \bar{s}_u (E_m(\bar{c}) - X) - \frac{\Pi_u(\bar{c})}{r-\xi} + \frac{b_u}{r} \right]}{\underline{c}^\lambda \bar{c}^\vartheta - \bar{c}^\lambda \underline{c}^\vartheta} \quad (8)$$

$$A_{d2} = \frac{\underline{c}^\vartheta \left[ \bar{s}_d (E_m(\bar{c}) - X) - \frac{\Pi_d(\bar{c})}{r-\xi} + \frac{b_d}{r} \right] - \bar{c}^\lambda \left[ \underline{s}_d (E_m(\underline{c}) - X) - \frac{\Pi_d(\underline{c})}{r-\xi} + \frac{b_d}{r} \right]}{\underline{c}^\lambda \bar{c}^\vartheta - \bar{c}^\lambda \underline{c}^\vartheta} \quad (9)$$

Substituting constants into Eq. (3), for  $i = \{u, d\}$ , I obtain:

$$E_d(c_t) = \frac{\Pi_d(c_t)}{r-\xi} - \frac{b_d}{r} + \mathcal{L}(c_t) \left[ \bar{s}_d (E_m(\bar{c}) - X) - \frac{\Pi_d(\bar{c})}{r-\xi} + \frac{b_d}{r} \right] + \mathcal{H}(c_t) \left[ \underline{s}_d (E_m(\underline{c}) - X) - \frac{\Pi_d(\underline{c})}{r-\xi} + \frac{b_d}{r} \right] \quad (10)$$

and:

$$E_u(c_t) = \frac{\Pi_u(c_t)}{r-\xi} - \frac{b_u}{r} + \mathcal{L}(c_t) \left[ \bar{s}_u (E_m(\bar{c}) - X) - \frac{\Pi_u(\bar{c})}{r-\xi} + \frac{b_u}{r} \right] + \mathcal{H}(c_t) \left[ \underline{s}_u (E_m(\underline{c}) - X) - \frac{\Pi_u(\underline{c})}{r-\xi} + \frac{b_u}{r} \right] \quad (11)$$

At the procyclical merger threshold  $\underline{c}$ , the sharing rule is  $\underline{\Gamma} = \{\underline{s}_i; \underline{c}\}$ . Therefore, the following condition arises:

$$\left. \frac{\partial E_i(c_t)}{\partial c_t} \right|_{c=\underline{c}} = \underline{s}_i \left. \frac{\partial E_m(c_t)}{\partial c_t} \right|_{c=\underline{c}} \quad (12)$$

At at the countercyclical merger threshold  $\bar{c}$ , the sharing rule is  $\bar{\Gamma} = \{\bar{s}_i; \bar{c}\}$ . Therefore, the

following condition arises:

$$\left. \frac{\partial E_i(c_t)}{\partial c_t} \right|_{c=\bar{c}} = \bar{s}_i \left. \frac{\partial E_m(c_t)}{\partial c_t} \right|_{c=\bar{c}} \quad (13)$$

Eqs. (12) and (13) are non-linear in  $c_t$ , and therefore the results are not analytically tractable.

I solve this numerically.

## E Proof of Proposition 2

By calculating the total derivative I aim to obtain the linear dependence of the pro-cyclical merger threshold,  $\underline{c}$ , on the sharing parameter,  $\underline{s}_i$ , and the counter-cyclical merger threshold,  $\bar{c}$ . For simplicity I assume that the roles are given. The the upstream firm is a target and the downstream firm is an acquirer.

I assume that  $\underline{s}_u$  changes by  $d\underline{s}_u$  and I verify how the endogenous variables,  $\underline{c}$ ,  $A_{u1}$ , and  $A_{u2}$ , for the target (i.e, upstream firm) change. I differentiate totally the value-matching condition for the upstream firm  $E_u(c_t) = \underline{s}_u(E_m(c_t) - X)$  at the thresholds  $\underline{c}$  and  $\bar{c}$ .

$$\frac{\partial E_u(\underline{c})}{\partial A_{u1}} dA_{u1} + \frac{\partial E_u(\underline{c})}{\partial A_{u2}} dA_{u2} = d\underline{s}_u (E_m(\underline{c}) - X) \quad (1)$$

$$\frac{\partial E_u(\bar{c})}{\partial A_{u1}} dA_{u1} + \frac{\partial E_u(\bar{c})}{\partial A_{u2}} dA_{u2} = 0 \quad (2)$$

I solve for changes in the coefficients  $A_{u1}$  and  $A_{u2}$ , where  $\underline{s}_u = \underline{c}^\lambda \bar{c}^\theta - \underline{c}^\theta \bar{c}^\lambda$ :

$$dA_{u1} = - \frac{d\underline{s}_u (E_m(\underline{c}) - X) \bar{c}^\lambda}{\Delta} \quad (3)$$

$$dA_{u2} = \frac{d\underline{s}_u (E_m(\underline{c}) - X) \bar{c}^\theta}{\Delta} \quad (4)$$

I then differentiate the smooth-pasting condition for the upstream firm at  $\underline{c}$ . After substitut-

ing, I obtain:

$$d\underline{c} \left( \frac{\partial^2 E_u(\underline{c})}{\partial^2 \underline{c}} - \underline{s}_u \frac{\partial^2 E_m(\underline{c})}{\partial^2 \underline{c}} \right) = \frac{d\underline{s}_u (E_m(\underline{c}) - X) (\theta \underline{c}^{\theta-1} \bar{c}^\lambda - \lambda \underline{c}^{\lambda-1} \bar{c}^\theta)}{\Delta} \quad (5)$$

The expression in brackets on the left-hand side is positive since the equity value is a convex function. The first expression in brackets on the right-hand side is positive since, at the threshold, the claim value should be higher than the exercise price. The second expression in brackets on the right-hand side is positive, and  $\Delta$  is positive, given  $\underline{c} < \bar{c}$ ,  $\lambda < 0$ , and  $\theta > 1$ . These results suggest that the higher the share of the target in the integrated firm the quicker the firm is willing to enter the merger. For the acquirer (i.e., downstream firm), differentiating the smooth-pasting condition at  $\underline{c}$  gives:

$$\begin{aligned} d\underline{c} \left( \frac{\partial^2 E_d(\underline{c})}{\partial^2 \underline{c}} - (1 - \underline{s}_u) \frac{\partial^2 E_m(\underline{c})}{\partial^2 \underline{c}} \right) = \\ \frac{\left[ d\underline{s}_u (E_m(\underline{c}) - X) + d\bar{c} \left( \frac{\partial E_m(\bar{c})}{\partial \bar{c}} - \frac{\partial E_d(\bar{c})}{\partial \bar{c}} \right) \right] (\theta \underline{c}^{\theta-1} \bar{c}^\lambda + \lambda \underline{c}^{\lambda-1} \bar{c}^\theta)}{\Delta} \end{aligned} \quad (6)$$

The expression in brackets on the left-hand side is positive, since the equity value is a convex function. The last expression in brackets on the right-hand side is negative, and  $\Delta$  is positive, given  $\underline{c} < \bar{c}$ ,  $\lambda < 0$ , and  $\theta > 1$ . The expression multiplying  $d\underline{s}_u$  is positive at the exercise threshold. Therefore, the change in sharing parameter ( $d\underline{s}_u$ ) is associated with a negative change in the integration threshold (in other words, it results in a delay). These results suggest that the higher the share of the target in the integrated firm the later the acquirer is willing to merge waiting for a larger merger surplus. The expression multiplying  $d\bar{c}$  is positive as the slope of  $E_m(\bar{c})$  is less negative than the slope of  $E_d(\bar{c})$ . Thus, the positive change in the default threshold of the target (the smaller the coupon) delays integration during an economic upturn. The results hold when firms are given the opposite roles, as

well. Proposition 2 follows.

### F Proof of Proposition 3

Following Carlson2004 the risk of the levered pre-merger equity over the interval  $\underline{c} < c_t < \bar{c}$ , is:

$$\beta_i = \frac{\gamma \frac{\Pi_i(c_t)}{r-\xi} + \tilde{\mathcal{L}}(c_t) OM_i(\bar{c}) + \tilde{\mathcal{H}}(c_t) OM_i(\underline{c})}{E_i(c_t)} \quad (1)$$

Substituting for  $OM_i(\bar{c})$  and  $OM_i(\underline{c})$  and rearranging results in Eq. (29). The risk of the levered post-merger equity, over the interval  $0 < c_t < \underline{c}$  and  $\bar{c} < c_t$ , is:

$$\beta_m = \frac{\gamma \frac{\Pi_m(c_t)}{r-\xi} - \vartheta \left( \frac{\Pi_m(c_m)}{r-\xi} - \frac{b_m}{r} \right) \left( \frac{c_t}{\bar{c}_m} \right)^\vartheta}{E_m(c_t)} \quad (2)$$

After rearranging it results in Eq. (30).

## References

- Aghion, P., and P. Bolton, 1987, Contracts as a barrier to entry, *American Economic Review* 77, 388–401.
- Barro, Robert J., and Xavier Sala-i-Martin, 2003, *Economic Growth, 2nd Edition* . MIT Press Books (The MIT Press).
- Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.
- Bernile, Gennaro, Evgeny Lyandres, and Alexei Zhdanov, 2011, A theory of strategic mergers, *Review of Finance* 16, 517–575.
- Bustamante, M. Cecilia, 2014, Strategic investment and industry risk dynamics, *Review of Financial Studies* p. Forthcoming.
- Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, *Journal of Finance* 59, 2577–2603.
- Carlton, Dennis W, 1979, Vertical integration in competitive markets under uncertainty, *Journal of Industrial Economics* 27, 189–209.
- Coase, R. H., 1937, The Nature of the Firm, *Economica* 4, 386–405.
- Dixit, Avinash K., and Robert S. Pindyck, 1994, *Investment Under Uncertainty* (Princeton University Press).
- Fan, Joseph P. H., and Vidhan K. Goyal, 2006, On the patterns and wealth effects of vertical mergers, *The Journal of Business* 79, 877–902.



- Grossman, Sanford J, and Oliver D Hart, 1986, The costs and benefits of ownership: A theory of vertical and lateral integration, *Journal of Political Economy* 94, 691–719.
- Hackbarth, Dirk, and Jianjun Miao, 2012, The dynamics of mergers and acquisitions in oligopolistic industries, *Journal of Economic Dynamics and Control* 36, 585 – 609.
- Hackbarth, Dirk, and Erwan Morellec, 2008, Stock returns in mergers and acquisitions, *Journal of Finance* 63, 1213–1252.
- Hart, Oliver, Jean Tirole, Dennis W. Carlton, and Oliver E. Williamson, 1990, Vertical integration and market foreclosure, *Brookings Papers on Economic Activity, Microeconomics* pp. 205–286.
- Hege, Ulrich, and Christopher Hennessy, 2010, Acquisition values and optimal financial (in)flexibility, *Review of Financial Studies* 23, 2865–2899.
- Holmstrom, Bengt, and John Roberts, 1998, The boundaries of the firm revisited, *Journal of Economic Perspectives* 12, 73–94.
- Hotchkiss, Edith S, and Robert M Mooradian, 1998, Acquisitions as a means of restructuring firms in chapter 11, *Journal of Financial Intermediation* 7, 240–262.
- Joskow, Paul L., 2008, Vertical integration, in Claude Menard, and Mary M. Shirley, ed.: *Handbook of New Institutional Economics* . pp. 319–348 (Springer Berlin Heidelberg).
- Kydland, Finn E., and Edward C. Prescott, 1982, Time to build and aggregate fluctuations, *Econometrica* 50, pp. 1345–1370.
- Lambrecht, Bart M., 2004, The timing and terms of mergers motivated by economies of scale, *Journal of Financial Economics* 72, 41–62.

- , and Stewart C. Myers, 2007, A theory of takeovers and disinvestment, *Journal of Finance* 62, 809–845.
- Leland, Hayne E., 2007, Financial synergies and the optimal scope of the firm: Implications for mergers, spinoffs, and structured finance, *Journal of Finance* 62, 765–807.
- Lyandres, Evgeny, and Alexei Zhdanov, 2010, Accelerated investment effect of risky debt, *Journal of Banking & Finance* 34, 2587–2599.
- Margrabe, William, 1978, The value of an option to exchange one asset for another, *Journal of Finance* 33, 177–86.
- Martynova, Marina, and Luc Renneboog, 2008, A century of corporate takeovers: What have we learned and where do we stand?, *Journal of Banking & Finance* 32, 2148 – 2177.
- Mitchell, Mark L., and J. Harold Mulherin, 1996, The impact of industry shocks on takeover and restructuring activity, *Journal of Financial Economics* 41, 193–229.
- Morellec, Erwan, and Alexei Zhdanov, 2005, The dynamics of mergers and acquisitions, *Journal of Financial Economics* 77, 649–672.
- , 2008, Financing and takeovers, *Journal of Financial Economics* 87, 556–581.
- Motta, Massimo, 2004, *Competition Policy: Theory and Practice* (Cambridge University Press).
- Netter, Jeffrey, Mike Stegemoller, and M. Babajide Wintoki, 2011, Implications of data screens on merger and acquisition analysis: A large sample study of mergers and acquisitions from 1992 to 2009, *Review of Financial Studies* 24, 2316–2357.

- Perry, Martin K., 1989, Vertical integration: Determinants and effects, in R. Schmalensee, and R. Willig, ed.: *Handbook of Industrial Organization* vol. 1 of *Handbook of Industrial Organization* . chap. 4, pp. 183–255 (Elsevier).
- Riordan, Michael H, 2008, *Competitive effects of vertical integration*. In: *Buccirossi P. (Ed.) Handbook of Antitrust Economics* (The MIT Press).
- Shleifer, Andrei, and Robert W. Vishny, 2003, Stock market driven acquisitions, *Journal of Financial Economics* 70, 295–311.
- Spengler, Joseph J, 1950, Vertical integration and antitrust policy, *The Journal of Political Economy* pp. 347–352.
- Tirole, Jean, 1988, *The Theory of Industrial Organization* (The MIT Press).
- Williamson, Oliver E, 1971, The vertical integration of production: Market failure considerations, *American Economic Review* 61, 112–123.