

Developing the Mathematical Beliefs of Second-level Students: An Intervention
Study

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DECLARATION

I certify that all material in this thesis which is not my own work has been properly identified, and that no material has previously been submitted and approved for the award of a degree by this or any other university.

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September 26th, 2014

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Dedication

I dedicate this thesis to my mother Marguerita and my late father Andrew for their wonderful love, kindness, generosity and support.

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I would like to thank the following people:

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Abstract

This study examined the effects of a learning environment (embodying many of De Corte et al.'s, (2004) CLIA-model components) on secondary students' mathematical beliefs. Such mathematical beliefs have been of interest to the research community due to their expected impact on students' willingness to engage in mathematical problem-solving.

This research adopted an action research methodology using a quasi-experimental sequential explanatory mixed methods design. Data was collected using the Mathematics Related Beliefs Questionnaire (MRBQ) and a number of focus groups and individual interviews were undertaken. The sample selected (age 13-14) was from a population of convenience. There was one treatment class (N=22) and three control classes (N=45). The classroom intervention was of six months duration and was carried out by the researcher teacher in a secondary community school.

Findings revealed no significant positive effects on students' beliefs from the new learning environment about the teacher's role in the classroom, their personal competence and the relevance to their lives and mathematics as an inaccessible subject. A more negative outcome for the fourth factor of the MRBQ scale, 'mathematics as an inaccessible subject', resulted for all participants (experimental and control combined) with a moderate effect of $\eta^2=0.09$. Findings from the qualitative data indicated the experimental participants found mathematics to be a difficult but useful subject.

Findings, overall, revealed no significant differences between the experimental and control classes, indicating the new learning environment had not had a positive impact on the beliefs examined. Possible factors identified were the length of the intervention, the ages of participants and the socio-economic status of the majority taking part in this study. Qualitative data also indicated participants in the treatment class had found some of the activities used in the intervention to be interesting and enjoyable. Responses to the use of group work indicated participants were both willing and able to enter into communities of learners.

Other results showed that participants with the highest achievement scores appeared to be the most confident learners of mathematics. Participants appeared to accept the need to have patience and perseverance when solving difficult problems but this was not translated into action in the classroom. The importance of understanding mathematics appeared to be accepted by participants.

Implications for methodology, research and practice are discussed in light of these findings.

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Chapter 1) Introduction

This research examines aspects of beliefs held by students about mathematics and its teaching and learning in an Irish mathematics classroom. The purpose of this study is to determine whether, and to what extent, changing the learning environment in the classroom can have a positive and enhancing impact on students' beliefs about mathematics and its teaching and learning. The intervention in the mathematics classroom included the use of active learning methodologies and small group work with a focus on student effort to learn. The role of the teacher changed from one of a traditional approach with teacher as expert to one of teacher as a facilitator of students' learning emphasizing conceptual thinking and problem solving.

1.1 Motivation

What motivates a practising teacher to examine students' beliefs and attempt to create a different classroom environment that might or might not positively impact on those beliefs? The author's interest in students' beliefs arises from their apparent influence on students' interest in mathematics, on their ability to problem solve and on their achievement in mathematics. 'Students' beliefs are said to direct their actions and subsequent learning' (Lester, 2002:351).

Students often voiced these beliefs in the classroom even when not asked for them, in this researcher's experience as a practitioner. It is difficult to measure how common are comments such as 'I hate mathematics' and 'I'm bored'. Such comments about mathematics give rise to questions for researchers and practitioners in the classroom about what students actually think learning mathematics is all about and about the relevance of this subject to their lives.

Teaching for many years within a school that is designated disadvantaged in the 'DEIS'¹ (Delivering Equality of Oppportunity in Schools) scheme in Ireland, resulted in the author undertaking many changes that sought to improve the learning outcomes for students. New ways of teaching and learning mathematics were attempted both with and without information technology. Schoenfeld (2014) asks 'How can fundamental research and real-world practice thrive in happy synergy?' A possible answer to Schoenfeld's question, this

¹ 'Deis' (pronounced "desh") is the Irish word for "opportunity"

author believes, is that the research-practice divide can be bridged more successfully if practising teachers undertake more research in their own classrooms. This could help to inform researchers and practising teachers and might create a closer community of inquiry where context, both in this researcher/teacher's view and within the framework of socio-cultural theory, is a key element. It could also provide a fuller perspective on the teaching and learning that takes place in mathematics' classrooms and perhaps also inspire other practitioners to become involved in research. 'It is relatively easy for us to see teaching from a more abstract level if we are not part of the act' (Cooney, 2001:256).

As a classroom practitioner the author had been told on numerous occasions, without invitation, by students, parents and also fellow teachers of other subjects that they 'never liked mathematics'. For students to believe that mathematics is a both a subject of beauty and power, efforts must be made to discover what helps students' enjoyment and understanding, as well as what will foster a love and appreciation of mathematics. As practitioners and researchers, the time has come to listen to students more as this may, hopefully, create a richer learning environment that will satisfy not only their curricular needs but may also allow them to enjoy learning mathematics more in the future.

1.2 Background and Context

'Beliefs might be thought of as lenses through which one looks when interpreting the world' (Philipp, 2007:257). The world of the mathematics classroom is a very complex one that has the responsibility for the development of the mathematicians and the problem solvers of tomorrow. Teachers who endeavour to understand this world, with the goal of achieving positive outcomes for students, can find it to be extremely challenging. Each student and each teacher brings an outlook based on a set or sets of prior experiences to the mathematics classroom at the start of a new academic year. To some extent, these underpin their beliefs about mathematics and the learning and teaching of mathematics.

There has been substantial research in the literature on student affect, which has been described as comprising of emotions, attitudes and beliefs (Philipp, 2007: 259). Philipp (2007:259) maintains emotions are feelings or states of consciousness, attitudes are manners of acting, feeling or thinking and beliefs are psychologically held understandings, premises, or propositions about the world that are thought to be true. However, much of the literature on affect does not provide a clear definition of these constructs or how they are structured and relate to each other. What both teachers and students believe, in relation to mathematics and learning mathematics, has been and still is a focus of the literature. Mathematics teachers would appear to play a significant role in the formation of the mathematical beliefs that are held by their students (Wilson and Cooney, 2002:127).

The context surrounding this study was unusual in that it was carried out at a time when a major mathematics curricular initiative had just been introduced into Irish secondary mathematics classrooms. A review of second-level mathematics education in Ireland took place in 2005, in preparation for root and branch reform, and was the first such opportunity in Ireland for over forty years (Conway and Sloane, 2006:202). Mathematics has been a compulsory subject in Ireland throughout post-primary schooling up to the Leaving Certificate (Age 18-19). Changes made to syllabuses over the years had focused on mathematical content with minor changes to structure and format of state examinations and less consideration to classroom culture and to teaching and learning styles (Oldham, 2006:47). Syllabuses had also perhaps an undue emphasis on formal notation and abstraction with insufficient emphasis on application and problem solving in real-life contexts (Oldham, 2006:30). This presented challenges in redefining and reforming post-primary mathematics education in Ireland in order to meet the needs of students with diverse needs and capabilities (Conway and Sloane, 2006:201). At the time of this study the new curriculum initiative called 'Project Maths' had been partially introduced to both first year and fifth year in the schools but had not yet been introduced to this study's participants. Project Maths placed greater emphasis on students' understanding of mathematical concepts with increased use of contexts and applications. Full implementation to all years at second-level was due to be completed by 2015. There was considerable controversy amongst the

stakeholders about the changes that were being introduced into Irish mathematics classrooms at that time. Further details on the historical and policy context of the study are in Section 3.5.2.1.

1.3 The Study

As a starting point, this study accepted that the teacher's role in developing students' beliefs would appear to be significant. The literature on teachers' beliefs and their impact on students' beliefs is immense and hence cannot be explored in this study in a significant way. Teachers' beliefs are discussed in the context of belief definitions, the teacher's role and the influences of beliefs in the classroom context. The author recognises that teachers should be concerned with the kind of beliefs they might be promoting in their classrooms through their practices (Francisco, 2002:15). This study was concerned with seeking better learning outcomes for students in the classroom with the author's primary interest and focus being with students' beliefs.

This study attempted to change students' beliefs on certain aspects of mathematics and its teaching and learning. Lester (2002:8) considered there are intrinsic difficulties in researching others' beliefs. Lester (2002:346) writes that research on beliefs is relatively new and extremely problematic and complex. However, Yackel et al. (2002:313) claim that changes in students' mathematical beliefs are possible through coordinating sociological and psychological perspectives. The aim of this current study was to undertake an intervention in the classroom, that attempted to change some aspects of students' beliefs so that they become more positive in nature. Through this process it was hoped to further highlight what teachers in their classrooms needed to do to develop more positive beliefs about mathematics in their students. Students, it is thought, develop beliefs from their experiences and interactions during the classroom activities in which they engage (Greer et al., 2002:274). So students' beliefs were measured, before and after a teaching intervention in the mathematics classroom was undertaken, using a questionnaire and focus group and individual interviews.

The learning environment created in the classroom for this study was significantly more challenging than the author had attempted before. It was

guided by current research on what might be appropriate to change to enable students' beliefs about mathematics and its teaching and learning to be enhanced. The framework of the new learning environment was based on Verschaffel et al.'s model (2000:97). The new environment created aimed to provide an inquiry/discovery atmosphere in the classroom with students actively involved in their learning of mathematics with a focus on student effort to learn. It also attempted to give students more discussion time when solving problems, with group work being introduced to the class for the first time. 'The use of small groups requires fundamental changes not only in the organization but also in ways of learning' (Kramarski and Mevarech, 2003:282). The research outcomes link learning environments in mathematics' classrooms to more positive students' beliefs about mathematics and learning the subject. '... as classroom norms are renegotiated, there is a concomitant evolution of individual beliefs' (Yackel and Rasmussen, 2002:328).

An examination in the literature on the available models of students' mathematical beliefs was undertaken (Section 2.3). This showed 'a considerable diversity in the ways in which beliefs' were explored (Leder et al., 2002b:7). This current study was conducted with its primary focus on four particular sets of beliefs that held by students. They are based on a framework integrating the major components of common models of students' beliefs (Op't Eynde et al., 2002C:13).

Using this framework, the beliefs examined were therefore students' mathematics-related beliefs on:

1. The role of the teacher in the mathematics classroom
2. Students' view of their own competence in mathematics
3. Relevance of the mathematics they are learning to their lives
4. Inaccessibility of mathematics as a subject

The study was carried out through a quasi-experimental mixed method design. The rationale for the design choice is given in Chapter 3, Section 3.4.1. A well-tested questionnaire was used to obtain a value for the strengths of beliefs (positive and negative) in the four areas listed above. The study used

the MRBQ (Mathematics Related Beliefs Questionnaire (Diego-Mantecon et al., 2007). Data collected from the questionnaire were supported by a number of small groups focus interviews before and after the intervention. A number of individual interviews were also conducted as soon as possible at the end of the classroom intervention. The questions used in the interviews were sourced using the MRBQ scale results collected prior to the intervention and from the Student Interview Instrument (Kloosterman, 2002) used in the pilot study. The interviews were intended to bolster the outcomes from the MRBQ scale.

The study was conducted in the academic year 2010-2011 with second-year students in a secondary post-primary community school². The students were aged between 13-14 years and they had already completed one year of secondary education. The classes of students taking part in the study were chosen as they were considered to be age appropriate and, also at the beginning of the year, all of the students were studying curricular material from the Ordinary level Junior Certificate mathematics course (NCCA, 2009), albeit at different paces. The classes were not involved in the new 'Project Maths' (NCCA, 2010) that was introduced in 2010 as this would have been an additional factor to consider when assessing any changes in their beliefs. However, a number of appropriate activities developed by Project Maths were used with students during the classroom intervention.

The work described in this research is organized as follows:

Chapter 2 discusses the relevant educational research literature

Chapter 3 discusses the research design, methodology and methods

Chapter 4 describes the implementation of the plan and the results

Chapter 5 discusses the interpretation of the results and their significance

² Community schools represent a coming together of two traditions in post-primary education in Ireland— the secondary and the vocational. Community schools are owned by the Minister for Education who vests the ownership in Religious and Vocational Education Committee (VEC) Trustees.

Chapter 2) Review of the Literature

Different approaches to learning have influenced mathematics education over the last one hundred years (Conway and Sloane, 2006:89). Considerable progress has also been made in explaining the processes involved in learning and teaching mathematics and in linking beliefs to both affective and cognitive processes (McLeod and McLeod, 2002:115). But there are a variety of different perspectives on beliefs and their significance in the teaching and learning of mathematics in the literature (Leder et al., 2002:3). The areas below chosen for examination in this chapter are those that are central to beliefs research, the teaching and learning of mathematics and the possibilities of effecting change in naïve beliefs.

The structure of this chapter is:

2.1 Learning theories: A brief overview

2.1.1 Learning theories and the Irish Educational System

2.1.2 Researcher's background and views in relation to learning theories

2.2 The Affective Domain: Affect

2.2.1 Beliefs and Belief Systems

2.3 Students' beliefs about mathematics and its teaching and learning in the classroom

2.3.1 Literature on changing students' beliefs

2.4 Literature on interventions

2.4.1 Research about classroom interventions

2.4.2 Interventions carried out that aimed to change students' beliefs about mathematics and its teaching and learning

2.4.3 Summary of main characteristics of learning environments above

2.5 Analysis of the literature review

2.1 Learning Theories: A brief overview

It is important to consider what is already known about how students learn and the context within which effective learning would seem to take place. Current information on the individual classroom for this study, as it pertains to learning

theories, is also described in order to provide a background and context for this study.

There are various perspectives on learning that have had a significant impact on mathematics education in recent decades. Two of them are referred to as the behaviourist approach and the cognitive approach. A third perspective includes a constructivist approach and yet another is the socio-cultural approach. The behaviourist and cognitive approaches have been the most researched during the last one hundred years with the sociocultural perspective now becoming a widely researched and influential approach (Conway and Sloane, 2006:87).

Defining behaviourism is not easy as it would seem to have changed over time. However 'a useful definition is that it is the belief that learning takes place through stimulus-response connections, that all human behaviour can be analysed into stimulus and response' (Orton, 1992:39). Key theorists include Skinner, Thorndike and Watson. Elwood (1930) describes Behaviourism as lacking the ability to show the true nature of human institutions based on values and valuing processes. In the behaviourist tradition, teachers are seen as experts in the knowledge of their own subject. Learning proceeds through a sequence of bite-size pieces of information that depends for its success on the quality of sequencing (Orton, 1992:39). In this theory, teaching was perceived as the reinforcement of desirable behavior, which was achieved through providing rewards or punishment but which ignored factors such as beliefs, (Orton, 1992; Conway and Sloane, 2006). The mind of the learner is seen as a black box where it is not necessary to know about what goes on inside. The limits of behaviourism for students became apparent with problems identified in the literature such as the lack of task authenticity (Conway and Sloane, 2006:95).

The cognitivist tradition emerged in the 1960s. Conway and Sloane (2006:98) argue that in the context of mathematics education the cognitivist perspectives on learning inform us in four ways. These are:

- (i) the concept of active learning
- (ii) the notion of cognitive challenge

- (iii) the concept of expert problem-solving and
- (iv) the demonstration of literature into teaching self-regulated learning using various strategies. Improvement of self-regulation in individuals giving consideration to memory issues, attention, problem-solving strategies, understanding of one's own learning and thinking processes and monitoring one's own moods, feelings and emotions (Duckworth et al., 2009:6).

Conway and Sloane write that much of the appeal of cognitive theories grew out of a desire to move away from didactic and transmission-oriented teaching. In the context of this study, recent literature on mathematics teachers in post-primary schools in Ireland indicates that much instruction was directed at preparing students to do well in public examinations and that teaching tended to be mainly didactic (Cosgrove et al., 2004:21). Cognitive theorists, on the other hand, view learning as involving the acquisition or reorganization of the cognitive structures through information processing (Good and Brophy, 1990:187). Greer (1981:21) writes of the difference between the beliefs held by cognitive theorists, in contrast to the behaviourists, in the existence of 'mental processes' that 'intervene between stimuli and responses'. The behaviourist and cognitive theories, despite being based on very different assumptions about learning, do share one defining feature in that they focus on the individual learner with little emphasis on the cultural and historical context of learning mathematics (Conway and Sloane, 2006:103).

Constructivism emerged from the work of cognitive psychologists such as Piaget, Bruner and Vygotsky. Two perspectives emerged: (i) Piagetian Constructivism, that conceptualized learning as individuals constructing meaning based on their own experience and prior knowledge and (ii) Social Constructivism espoused by Vygotsky and others who took a social and cultural perspective of knowledge creation (Lowenthal et al., 2008:3).

Constructivist perspectives on learning adopt a view that sees learners as being active in their own learning, constructing new knowledge from their prior knowledge through interaction with the environment, and with their peers and teacher (Cobb, 1990:127). Real world application of mathematics would also

seem to be important to the constructivist classroom learning environment as students build an understanding of mathematics from their day to day experiences in the classroom. A study carried out in Ireland on a sample of mathematics teachers showed that they spent only a small percentage (4-5%) of total time in mathematics classes devoted to the transfer of knowledge to real-life situations (Cosgrove et al., 2004:40).

Social constructivism grew out of Piaget's constructivism but there is a much stronger emphasis on the role of language, communication and instruction. The assumptions/premises behind social constructivism are that:

- (i) Reality is constructed in social activity
- (ii) Knowledge is a human product and is socially and culturally constructed
- (iii) Learning is not passive, as meaningful learning is constructed by humans when they are engaged in social situations (Kim, n.d.)

Social constructivists see as crucial the context in which students learn and the social context they bring to their learning environment and that they learn from being part of it (Kim, 2006:4, Jaworski, 2007:73). Students' personal learning goals create a framework for achievement and effort expenditure on mathematical tasks (Seegers et al., 2002:366).

Vygotsky's work on social development theory is considered to be hugely relevant to social constructivism. His zone of proximal development is described as the difference between what a child is able to do when solving a problem alone and what can be achieved with the help of peers or teacher or both (Nickson in Haggarty, 2002:238). Goos (2008:236) recommends that Valsiner's theories of child development be applied to classrooms and includes two additional zones. They are the zone of promoted action (ZPA) and the zone of free movement (ZFM). 'ZFM characterizes the child-environment relationship, at a particular time and in a certain environment' and the ZPA refers to activities, objects or areas in the environment in which child's actions are promoted (Hussain et al., 2011:2). Goos writes that these zones structure individuals' access to the environment and objects or areas of the environment in which

person's actions are promoted. Goos (2008:236) argues that all zones are interrelated and that the teacher establishes a ZFM/ZPA complex that 'characterizes the learning opportunities experienced by the students'. Blanton et al. (2005:5) describes the ZPA zone as what the teacher has promoted and the ZFA zone as what the teacher has allowed. Exploring teacher conceptual development, Blanton characterises as an illusionary zone (IZ) one where the teacher does not allow students to experience actions or events that he/she has promoted.

Metacognition is described as having many meanings, including 'knowledge about one's thought processes and self-regulation during problem solving' (Schoenfeld, 1992:2). Holton et al. (2004:127) situates metacognition 'within a framework derived from the social activity of scaffolding'. This, they write, forms a bridge between the support of the teacher and the learner's self-control. Orton (1992:165) states that the teacher should provide appropriate scaffolding to enable the student to progress and this may mean non-intervention for some students but for other students some help may be required to help them construct meaning. Providing the correct scaffolding for every student in a class, in order to promote independent responsibility for learning, is a considerable challenge for teachers. Knowledge is actively constructed by the learner, who searches for meaning and who is responsible for his/her own learning. Part of the teacher's role, it is claimed, is to provide students with high levels of both cognitive and affective support in a balanced way (Turner et al., 2002:103). Feuerstein's mediated learning experience (MLE) focuses on students' cognitive processes (Kozulin et al., 1995:70). The authors write that the real objective of Feuerstein's mediated interaction is students thinking infused with meaning by the teacher with the underlying principal being identified.

A focus on learning as a social act emerged in the literature in the mid 1980s in western literature, leading to a potential unification of aspects of cognitive and social perspectives in enculturation (Schoenfeld, 1992:39). More recent sociocultural models include Activity Theory. Activity theory was developed by Russian psychologists Vygotsky and Leont'ev from the 1920's onwards. Engestrom further developed the theory extending the three interacting entities of the individual, object and tools to include other components such as division

of labour, community and rules (Karasavvidis, 2009:438). Karasavvidis provides an example of a representation of an activity system, in terms of education, where the teacher is the subject, the object of the teacher's activity is the student and his/her learning, the mediating means (tools) include textbooks, technology, materials, instructional strategies and the rules include national curricula and school rules. Activity is mediated by community with the roles of participants, responsibilities and power being continuously negotiated (Hardman, 2008:74). Sociocultural theory and activity theory attempt to provide an account of learning and development as mediated processes (Daniels, 2004:121). Daniels goes on to say that with activity theory it is the activity itself which takes centre stage in the analysis and that both sociocultural and activity theory provide methodological tools for investigating the processes which shape human functioning.

2.1.1 Learning theories and the Irish Educational System

This section gives an overview of the Irish mathematics classroom in the last four to six decades from a learning theories perspective. It provides the background prior to a deep review in 2005 of Irish second-level mathematics education and the subsequent introduction of the new curriculum initiative 'project Maths'. This is described in detail in Section 3.5.2.1.

The philosophy behind the 'new mathematics' movement of the 1950s and 1960s, adopted by Ireland and other countries, considers mathematics to be the study of abstract structures whose hallmark is one of 'vertical mathematising'. This involves working within the mathematical system itself and that means moving within the world of symbols to achieve greater sophistication in understanding that world. In contrast, 'horizontal mathematising' involves moving between the real world and the world of symbols and back.

Ireland's post-primary mathematics education culture has been described as traditional in manner with teacher initiated interaction comprising of 96% of interactions in the classroom, using a procedural approach, for the most part, to teaching (Lyons et al., 2003:363). Lyons et al. had used a small sample. The teaching approach used would seem to be somewhat consistent with the vertical mathematising agenda. In contrast, the cognitive tradition believes

students should be involved actively in their own learning with an emphasis on problem solving as a teaching approach. Connolly (2007:268) describes traditional teaching as one where very little initiative or independent thought is required of the student. In the Irish context, a 2009 study was carried out on 210 student teachers, some of which took mathematics as a major teaching subject and the others as a minor or second teaching subject, described their views on learning mathematics at second level. The study revealed the predominant view was one that described mathematics as a subject that did not require creativity, opinions or debate and that it was difficult to make the subject relevant or interesting to students (Meehan and Paolucci, 2009:260).

Irish mathematics education would seem to have been influenced, to some degree, by behaviourism, which has contributed to shaping the teaching and learning of mathematics in secondary schools over forty years since 1960. This influence has not to this researcher's knowledge been systematically examined in the literature (at least until the time of writing in September 2014). However, it is possible to find evidence of some aspects of behaviourism in the Irish mathematics classroom from official education documents. Two particular aspects of behaviourism in the mathematics classroom are discussed here, in relation to the Irish mathematics classroom. These are rote learning and a lack of learning in the context of the real world. Rote learning can be described as learning or memorization by repetition often without an understanding of the reasoning or ideas behind it. These are chosen for discussion, as they are important issues to examine in mathematics education and there is some evidence in the education documents to support their inclusion. A 2003 video study 'Inside Classrooms' explored the culture in a small number of mathematics classrooms in Ireland and focused on the approaches used to teaching mathematics. Results from the study indicated that the teaching of mathematics in Ireland presented the subject as static, formal and remote rather than relevant and accessible (Lyons et al., 2003:363). The behaviourist tradition has been commonly associated with the use of rote learning (Jaworski in Johnston-Wilder et al., 2003:43). Recent studies have claimed that there has been an emphasis on rote learning and that cramming is paramount in Irish mathematics classrooms (Carroll and O'Donoghue, 2009:245). There is also some evidence from reports written by chief examiners, on behalf of the Irish

Department of Education and Skills, on students' answers in the state examinations to support this claim. They would seem to indicate that a significant number of teachers use 'rote learning' to some degree in their classrooms. This, however, has not been quantified. The Chief Examiner's report (S.E.C., 2006:16) on mathematics in the Junior Certificate examination (age 15-16) in Ireland, recommended to teachers that they 'should encourage the understanding of concepts, rather than relying on "rote" learning' and again when commenting on students' answers to higher level geometry the examiners indicated that answers to questions were weak and that 'rote learning could be a contributory factor there'. In another Chief Examiner's report for the terminal examination at second level in Ireland (Leaving Certificate 2005, ages 17-19), commenting on candidates answers to higher level questions, the examiner wrote that 'weaknesses continue to stem from inadequate understanding of mathematical concepts and underdeveloped problem-solving and decision-making skills'. Summarising, the Chief Examiner wrote 'strong performance was most evident in procedural questions where a definite sequence of steps was required'. There are also indications from the literature that students' achievement of higher-order objectives in state examinations is poor (Oldham and Close, 2009:295). Is rote learning associated, in its entirety, with a behaviourist approach to the teaching and learning of mathematics? Orton (1992:3) warns that it would be wrong to tie rote learning too closely to the behaviourist tradition and to suggest that it has no place within a cognitive or constructivist approach. There are arguments, however, in the literature that maintain certain tasks such as rote memorisation would appear to be facilitated by a behavioural approach in that they require a low degree of processing (Mergel, 1998:20).

The second aspect of the Behaviourist tradition that is examined here is that of teachers failing to provide sufficient real-life context for the teaching and learning of mathematics in Irish classrooms. Learning is considered to be context independent under the behaviourist paradigm (Laliberte, 2005:4). There is evidence in the Irish Chief Examiner's report (2006:22) for Junior Certificate (age 15-16) that students have a problem with context: 'Some added tax to get the total take home pay, displaying an inability to relate their learning to a practical real life situation'. Evidence from Chief Examiners' reports include

statements indicating there are 'significant weaknesses regarding sound conceptual understanding with corresponding weaknesses in its application in contexts (DES, 2001:21).

During the 20th century theories of learning have been central to influencing the changes taking place in mathematics education. There has been a consensus growing over the last twenty years that social and cultural influences effect cognition and learning (Conway and Sloane, 2006:88). Despite the lack of significant change in secondary education in Ireland over the forty years from 1969 to 2009, there have been studies that attempted to examine whether constructivist approaches to teaching were being implemented. Gash and McCloughlin (2010), in a cross-national study that included Ireland, attempted to assess dimensions of students' thinking about learning science, that reflected aspects of the constructivist approach to teaching and learning. Outcomes from that study, using teacher and student questionnaires, indicated primary teachers were more open to constructivist approaches than secondary teachers who favoured instructional approaches, reflecting more traditional views where memory is most important (Gash and McLoughlin, 2010:2).

2.1.2 Researcher's Background and views in relation to learning theories:

The author was educated in a mathematics classroom as a student in a learning environment dominated by the 'new mathematics' movement at secondary school in the late 1960s. This meant learning set theory and number bases, including binary and hexadecimal, transformation geometry and abstract algebra in the final years of secondary education. Training as a teacher of mathematics followed a degree course in mathematics at university.

Behaviourism was the learning theory espoused by the university lecturers, in the author's view, and taught and assimilated by student teachers. A career of over thirty years in a secondary school teaching mathematics followed. The first introduction to constructivism as a learning theory came twenty years after the author had first started teaching mathematics. Over the years there had been some minor changes to syllabuses and recommended pedagogy. However, teachers were, in the author's view, largely motivated to make changes in their pedagogy by annual examination papers prepared by the Department of Education and Skills. Opportunities for professional development were minimal

during the author's first sixteen years in the classroom (1972-1988). In 1982, for example, the Department of Education syllabus committee had as their main focus dealing with unfinished business from the radical reforms of the 1960s rather than re-conceiving mathematics education in Ireland for the 1980s or 1990s and there was no plan due to lack of funds to provide significant teacher in-service (Oldham, 2006a:162). Recommendations from a recent study indicate that a lack of purposeful continuing professional development in Ireland is a contributory factor to poor teaching (Prendergast and O'Donoghue, 2009:328). This has changed significantly since 2008 with the introduction of Project Maths and on-going universal professional development being provided for teachers of mathematics (NCCA, 2005).

On a personal level, the author made efforts to improve the teaching and learning that was taking place in the mathematics classroom resulting with some degree of success. These efforts did include the use of rote learning in the classroom on a reasonably regular basis but the author also attempted to teach the understanding of concepts. Also included were attempts to teach a cohesive course that connected mathematical ideas together where possible. The use of ICT was included to aid the teaching and learning of mathematics in the classroom.

The author does accept that learning is a social act and that promoting efficient learning is both a complex and difficult task for the teacher, and equally so for the learner. Social constructivism as a theory of learning has opened up and taught us more about the process of learning in a positive way. If the research community can bridge the divide between researchers and practitioners the author believes that further progress in learning how to teach and learn mathematics well will result from this in the future. One way forward is for teachers to research the teaching and learning that is taking place in their own classrooms.

2.2 The Affective Domain: Affect

Do teachers of mathematics need learning theories to enable them to perform successfully in the classroom? Orton (1992:1) debates this issue and argues that education is too important to dismiss them and that these theories are

needed as a basis for decision-making in the classroom. If we accept that we do need to know learning theories to teach mathematics in the classroom what other scholarly theories about students learning from the research can provide teachers of mathematics the opportunity to engage in best practice in their classrooms? Little attention was given to the interaction between the emotional and cognitive aspects of learning up to the late 1980's (Di Martino et al., 2010:28). This led to much interest in the field of affect and mathematics education.

Teachers typically set learning or educational outcomes for students in their classes within required mathematics syllabuses, school and state demands. Bloom's taxonomy divides educational objectives into three domains. They are the cognitive, affective and psychomotor domains of learning. It has been claimed that the educational system has focused significantly on research into the cognitive domain until recent decades (Reeves, 1990; Schoenfeld, 1992; Tapia, 2004). Throughout the 1980s, psychological research tended to focus on cognitive architecture (Schoenfeld, 1992:40). Lester et al. (1989:75) write that the overwhelming majority of problem-solving researchers have restricted their investigations to cognitive aspects of performance. Chamberlin (2010:167) writes that increased attention to standardized assessments has led to a neglect of attention to components in the research such as dispositions and motivation. Rovai et al. (2009: 7) argue that several taxonomies have been developed to measure the perceived products of learning and that these all address Bloom's three overlapping domains. Rovai reports their attempts to develop a self-report instrument that could be used to measure learning across all three domains as they considered it essential to assess students across all these aspects. Their instrument (CAP Perceived Learning Scale) was developed for use with third level students and it claims to generate an overall Perceived Learning Score across all three of Bloom's Domains.

Curriculum documents over this period, including the Cockcroft report (1982) and the Australian Education Council (Leder and Forgasz, 2006:407), do refer to the importance of engaging students affectively as well as cognitively in the study of mathematics. At the same period, Irish curriculum documents for the Junior Certificate State examination (age 15-16) advised teachers to develop in

their students' appreciative attitudes to mathematics and how it is used (NCCA, 2000).

Research on affect over the three decades from 1980s onwards has explored students' responses to mathematics and to its learning in schools. Early research on affect focused on mathematics anxiety and attitude towards mathematics (Zan et al., 2006, Malmivouri, 2001). There would seem to be some agreement in the literature that affect is a significant and critical dimension of learning as recognized in this study (Zembylas, 2004, McLeod, 1992). However, not all studies accept that learning outcomes are influenced to a significant extent by students' attitudes and beliefs and it is argued they may not be used to predict achievement (Papanastasiou, 2000:39).

What, then, is affect and how is it defined? Affect is described as comprising of 'emotions, attitudes and beliefs' (McLeod, 1992; Philipp, 2007). Philipp (2007:257) writes that '...affect might be thought of as a disposition ...one takes toward some aspect of his or her worldthe beliefs and affect one holds surely affect the way one interacts with his or her world'. McLeod (1992) viewed them as ranging along a dimension with increasing stability and decreasing intensity with beliefs being the most stable and least intense. The use of a linear continuum to describe the affective domain, with an increasing level of cognitive component and stability, is also used by others in the literature (Phillippou, 2002:213). Muis (2004:323) distinguishes beliefs according to how they advantage or avail learning and describes non-available beliefs as having no influence or a negative influence on learning outcomes.

The literature has identified some different perspectives on affect. Lester et al. (1989) supported the notion that emotions and cognitive actions interact in important ways. Orton (1992:10) claims that the cognitive cannot be separated from the affective domain. Op't Eynde et al. (2006a:194) sees affect grounded in and defined by social context. Malmivouri (2006:161) views the powerful connection between affect and cognition as one of linking affective experience to a combination of the constructs of self and self-regulation. DeBellis and Goldin (2006:131) construct a theoretical framework based on affect as an internal representation parallel to cognitive systems in individuals examining a

model on how they process information. They argue that the affective system is very complex and that 'it involves emotions, attitudes, beliefs, morals, values and ethics, the last three of these belonging to a fourth subdomain of affect (DeBellis et al. 2006: 132). Zan et al. (2006:117) includes constructs such as motivation, mood and interest in the field of affect. Hence, research on understanding the interrelationship between affect and cognition would seem to be an important problem for mathematics educators to investigate (Zan et al., 2006:117). Questions that immediately arise are: What is the nature of that relationship and how do you explain it? What might be the mediating factors? One suggestion has been to take the constructs and processes of self and self-regulation as a powerful feature of affect and cognition stressing the importance for students' self-appraisals when problem-solving (Malmivuori, 2006:149). In more recent years affect as an integral component of cognition is promoted (Chamberlin, 2010:175). It would seem that there is not yet agreement in the literature on the relationship between affect and cognition.

Goldin (2002:62) developed the notion of meta-affect that he describes as 'affect about affect, affect about and within cognition that may again be about affect, the monitoring of affect, and affect itself as monitoring'. Beliefs, Goldin (2002:69) argues, form meta affective conditions for the experience of emotions connected to beliefs which can be strong enough for students to maintain beliefs even in the face of conflicting evidence. Pathways are established and interwoven with cognition and a meta-affective context for learning mathematics that is reasonably comfortable for individuals is developed. Meta-affect is a tool that should be used more frequently by mathematics teachers, according to Moscucci (2009:1814) where the teachers become aware of their own belief systems and emotions towards mathematics as a learner in the past and as a teacher currently. As students engage in problem solving, the emotions they experience have a direct impact on their achievement due to the beliefs that they hold about their own general mathematics competence (Op't Eynde et al., 2006a:202). Hannula (2007:197) based on a virtual panel of belief researchers arranged by Pehkonen concludes that there are different views about how much emotions are part of beliefs.

Attitudes are considered to belong to the Affective domain and studies of the domain were initially limited to their investigation (Ignacio et al., 2006:16). Efforts were made to unravel the structure and function of systems of attitudes in the 1980s and 1990s and that was followed by research on the role of cognition in the structure and shaping of attitudes (Leder and Forgasz, 2006:405). Attitude has been defined as having moderately stable tendencies towards how one feels in groups of situations (Goldin, 2002:61). In particular, negative attitudes towards mathematics have been researched. Ma and Kishor (1997), in a study synthesizing over one hundred studies of the relationship, found a stronger correlation for Grades 7 to 12 between attitude and achievement compared to other grades (Hannula, 2002:25). Attitudes can be influenced by other factors operating in the classroom. A negative attitude towards mathematics, for example, may be viewed by students as part of a functional coping strategy in the light of social goals (Hannula, 2002:43). Schorr et al. (2008:132) argue for a deeper study of affect as studies of attitude and achievement alone, they claim, show a weak relationship between them. Di Martino and Zan (2010:27) attempt to clarify the construct attitude theoretically while at the same time keeping in touch with the practice that motivates its use. Outcomes from their study suggest that it is never too late to change students' attitudes.

The impact of the literature discussed in the section above on this present study was two-fold:

- (i) It prompted the author, as the practitioner in the classroom, to examine her own beliefs about mathematics and its teaching and learning and the possible consequences for the students learning mathematics in her care. This resulted in the author being made aware of the need to observe more carefully what was actually happening in the classroom during the classroom intervention.
- (ii) It also enabled the author to be aware of the factors pertaining to 'affect' that would appear to influence students' view of mathematics and their learning of it (Section 2.2). This was used in this study to help shape the data collection instruments.

2.2.1 Beliefs and Belief Systems:

Beliefs are also considered to belong to the Affective domain. Studies in the literature on beliefs and belief systems indicate different classifications and perspectives of beliefs and belief systems (Pajares, 1992:313). This is possibly due, for example, to studies that have taken place in the different disciplines of sociology, psychology and anthropology. It would also seem to be due to the complex nature of beliefs and belief systems (Liljedahl et al., 2007:278). Some researchers argue that beliefs belong in the 'zone' between the cognitive and the affective domain of psychological functioning, and therefore have a component in both domains (Pehkonen et al., 2004:1).

Reaching agreement by the research community on a clear and unambiguous definition of 'belief' is ongoing. However, Torner (2002:91) writes that 'only in rare cases can a final precise definition of all components of a belief definition be achieved in a specific context'. Beliefs have been loosely defined as mental constructs (Malmivouri, 2001:42). McLeod et al. (2002:118) sought a shared understanding of the term 'belief' and discovered certain identifiable commonalities, agreed by a panel of experts, in the construct. McLeod et al. recommend several types of definitions of 'belief' be tailored to the audience ranging from an informal to a formal definition. Leder and Forgasz (2002:96) argue that an agreed definition is not possible because of the overlapping nature of terms such as attitude, disposition and belief. Goldin et al. (2007:13) analyzed mathematical beliefs and values and declared them not only to be organized and belong to belief systems but that they are embedded in complex structures. Torner (2002:73) states 'that anything that shares a direct or indirect connection to mathematics can function as a belief object'. Some objects are abstract, Torner writes, and others are more concrete e.g. school mathematics. Belief systems are loosely bounded networks with highly variable and uncertain linkages to events, situations and knowledge systems (Calderhead, 1996:719). Indications from the research are that studying beliefs as single objects is not sufficient with analysis of belief systems being a priority (Torner, 2002:84). As such, he maintains that these belief systems are important in that they help the understanding of students' motivational and behaviour patterns.

Osterholm (2010:2) sees the lack of an agreed definition for beliefs as a

problem highlighting on the one hand conceptual/theoretical problems in belief research and on the other hand missing connections between aspects of theory and empirical research. Skott (2009:45) suggests a shift to an alternative approach in belief research that he calls patterns-of-participation in order to discover how teachers' practices are formed. Skott is concerned with teacher identity and hence his research seeks a coherent understanding of the teacher's role in the mathematics classroom.

Osterholm (2010:14) too argues for a participatory perspective to be used in beliefs research where the focus shifts from viewing beliefs as mental entities to a focus on patterns of participation in different practices. The different practices explored in Osterholm's study on teachers' beliefs are the teacher education programme and the teacher's instructional approaches with data being collected from interviews and observations. Skott (2009:27) claims that focus on teachers' beliefs about mathematics and the teaching and the learning of mathematics to explain belief-practice relationship should be extended to include the notion of context and practice.

Philipp (2007:259) defines belief systems as an image where beliefs are organized in a cluster around a particular idea or object. He maintains they are associated with three aspects:

- I. Beliefs may be primary or derivative
- II. Beliefs may be central or peripheral
- III. Beliefs exist in clusters

Schoenfeld (1985:45) describes belief systems as a person's view of the mathematical world, in addition to a viewpoint as to how one approaches mathematics and mathematical tasks. Lester et al. (1989) argue that 'beliefs constitute the individual's subjective knowledge about self, mathematics, problem solving...' that can influence decisions made during problem solving. And more recently, Di Martino (2004:272) considers it fundamental to consider the structure of belief systems, which he describes as including the content of the belief and the way that people hold it.

The review above has identified beliefs as significant for the effective teaching of mathematics. It identified important perspectives of beliefs and belief system. Through this review it improved the understanding of the researcher on how best to shape this study on belief enhancement.

2.3 Students' beliefs about mathematics and its teaching and learning in the classroom:

There have been diverse approaches in the conceptualization and study of beliefs (Leder et al., 2002:2). Hence, the study of beliefs can be classified and organized using different themes. Op 'T Eynde et al. (2002:28), in their review of publications studying students' beliefs, attempted to categorize topics on mathematics-related beliefs that aimed to integrate the major components of models in the literature at that time. The authors present it as a review of available models of students' beliefs related to mathematics learning and problem-solving. Their chosen categories were:

- i. Beliefs about mathematics education. Subcategories include beliefs about mathematics as a subject and its teaching and learning;
- ii. Beliefs about self in the context of learning mathematics. These are differentiated between goal orientation beliefs, task value beliefs and control beliefs;
- iii. Beliefs about the social context including the role and functioning of student and teacher.

Mathematics, as a subject, has been viewed as being somewhat less popular than other subjects by students at second level both in Ireland and elsewhere (Brumbaugh et al., 2006, Shernoff, 2003). Philipp (207:257) contends that the beliefs that students carry about a subject are just as important as the knowledge they learn of the subject. Beliefs about the nature of mathematics and its connection to real life are often fuzzy and difficult to recognize and these can sometimes operate as a central compulsion in the construction of specific beliefs, for example the belief that mathematics is a discipline that is appropriate only for scientists to learn due to its abstract nature (Malmivouri, 2001: 52). In Ireland it would seem that students are unable to appreciate the role of mathematics in everyday life and its influence on their future life and

work (Carroll and O'Donoghue, 2009:224). This, the authors claim, is perhaps due in part to the instructional practices used by their mathematics' teachers in the classroom.

Students, when studying word problems, pay little attention to even the simplest of reality constraints (Verschaffel et al. 2000:29, Mason 2003:74). Mason (2003:79), in her study on students' beliefs, found students with better mathematics grades had a belief in their ability to solve difficult problems, did not follow memorised rules, thought mathematics was a useful subject and realised that it was important to understand concepts. Towards the end of the twentieth century there has been a stronger emphasis on teaching mathematics through problem solving and this shows promise (Cai, 2003; Nickson, 2004).

Previous experiences in the mathematics classroom would also appear to influence the learning that individuals achieve. Spangler (1992:19) maintains students' prior learning experiences of mathematics are likely to influence their beliefs and these in turn influence their approach to new learning experiences. Moscucci (2009:1812) describes every belief as a sort of axiom garnered as a result of personal experience. Muse (2004:320) and Schoenfeld (1989) argued that beliefs develop and change over time and that they are influenced by our experiences. Depaepe (2009) found from interviewing students that students' beliefs were mostly in line with the classroom practices they experienced. The author suggests this is useful information for this current study, because it suggests that new practice in the classroom could make a fundamental difference to students' beliefs. Ignacio et al. (2006:16) carried out a study aiming to analyse students' beliefs, attitudes and emotional reactions in the process of learning mathematics. Outcomes from the study indicated that boys had a better-adjusted mathematics self-concept than girls. According to Ignacio et al. (2006:28) the girls attributed some success/failure to the teacher's behaviour. Ignacio et al. recommend programmes of prevention and intervention in the difficulties of learning mathematics, and of emotional education aiming to stimulate a taste for mathematics and to improve attitudes and beliefs. Presmeg (2002:293) claims students' beliefs about the nature of mathematics and its teaching are fundamental in understanding the learning of mathematics and hence are fundamental in attempting to change practices.

Presmeg goes on to say that these beliefs both enable and constrain students' construction of conceptual bridges between school and life mathematics. Lester (2002:348), in a critique of Presmeg's study, acknowledges the use of a new tool for researchers studying formation of beliefs but says that her study provided no direct implications for classroom practice.

The roles of both teacher and student in the classroom have also been studied in connection with beliefs research. Students' beliefs about the role of the teacher in the classroom would appear to be closely linked to what they themselves believe to be their role in the classroom (Ponte et al. 1992; Verschaffel et al., 2000). Students participate in the traditional classroom by cooperating in the learning of procedures and practising similar problems to the one demonstrated by the teacher. Expectations of students in classroom environments that are described as holding a strong procedural and rule-oriented view of mathematics expect their role to be one of receiving mathematical knowledge and being able to demonstrate it (Ponte et al. 1992; Quinlan, 2009). Verschaffel et al. (2000:73) discuss the effects of a teacher's own disposition toward realistic mathematical modeling influencing their teaching behaviour and consequently the impact on their students' learning processes and outcomes. This, in turn, would seem to influence how students perceive their role in the mathematics classroom. According to the results of an OECD study (2009:241), teachers' beliefs are connected to their teaching practice in the classroom and are considered to be influential in the development of an effective learning environment. Turner et al. (2009:361) claim that teachers' beliefs shape their instructional behaviours and hence what students learn. Raymond (1997:550) claims that teachers' beliefs and practice are not wholly consistent. Practice, she claimed, was more closely related to beliefs about content than to beliefs about mathematics pedagogy. Teachers' beliefs transformed into practice, Ernest (1989:4) argues, are affected by two factors: the social context constraining freedom of choice and action and the level of teachers' thought. There is evidence from the Irish perspective that students, in their first year of university education, are perceived to have developed study habits that are almost entirely procedure driven, with little evidence of a desire in most students' mathematical activity to remedy weaknesses in their conceptual understanding (Quinlan, 2009:423). Thompson

(in Philipp, 2007:258) noted the importance of researchers making explicit their own perspectives on mathematics as a subject and its teaching and learning as these greatly influence their interpretations of their work. Goos et al. (1999:58) write that a barrier to change in classrooms is the teacher's beliefs formed as a consequence of their own schooling.

The influence of teachers' beliefs on their students' beliefs and attitudes about mathematics and its learning has resulted in attempts being made to train teachers themselves to adopt appropriate beliefs. According to Tripathi (2009:171), research has been less focused on conceptions about mathematical thinking, concentrating instead on metacognition, critical thinking and mathematical practices as more important aspects of mathematical thinking. In order to translate this perspective into the classroom, Trepahi claims that it requires a revolutionary change in our attitude and belief structures about mathematics. Her study involving trainee elementary school teachers, providing them with instruction on problem solving, brought about a positive change in attitudes and beliefs. These teachers Trepahi maintains, in turn would be agents of change for their own students' beliefs and attitudes.

The above literature would seem to indicate that students' beliefs are influenced by what happens in the classroom and hence with the teacher providing a suitable learning environment that may, in turn, enhance beliefs is worthwhile.

2.3.1 Literature on Changing Students' Beliefs

There would seem to be agreement in the literature that to change students' beliefs is not an easy task but that it is possible (Philippou et al. 2002:213). McLeod (1992:575) concluded that beliefs tend to develop gradually and that cultural factors play a key role in their development. Working on beliefs would seem to be important as they may otherwise form an insurmountable wall to learning mathematics (Moscucci, 2007:300).

Some of the literature, mentioned above, on changing students' beliefs starts from the premise that teacher beliefs need to be a starting point. Moscucci (2007:298) regards teacher beliefs to be of great importance as students' personal approach to mathematical objects result from received stimuli from their teachers' beliefs about mathematics. Students, by the age of 12 years, will

have experienced the learning of mathematics from a number of different teachers. Skott (2009:45), on the other hand, views practice and context socially and as a consequence discards the idea that teachers' beliefs are the main determinant of how mathematics is taught and learnt. Prawat (1992:358) questions the amount of time in a class spent on delivery in comparison to the time spent on what he calls meaning making. Richardson (1990:11) maintains that individual behaviour, and the decision to change, are influenced by individual's beliefs, attitudes, goals and the environment.

Mason and Scrivani (2004:173) claim that the teacher's beliefs and behaviour should be the starting point for classroom interventions aiming to enhance students' mathematical beliefs as they play an essential role in helping or constraining more advanced convictions. The authors worked with the usual teachers to implement the new learning environment. The environment, used in their study, is described by Verschaffel et al. (2000) and it attempted to promote enhanced students' mathematics beliefs. Details of the environment are discussed in Section 2.4.1. In a more recent study, Hassi et al., (2009:119) argue that promoting cognitive, affective and social skills in an inquiry based learning environment points to positive impacts on students' beliefs.

In summary, beliefs are thought to be important to successful learning of mathematics (Moscucci, 2007; McLeod, 1992). Cultural and context factors are thought to be some of the main determinants to consider when attempting to change beliefs (McLeod, 1992; Skott, 2009; Hassi, 2009). Individual students and teachers must be prepared to change in order to impact positively on their beliefs.

2.3.1.1 Assessing beliefs

A practical starting point for this study would seem to be to assess the beliefs held by the participants. If an appropriate instrument were available to teachers of mathematics to enable them to measure their students' beliefs about mathematics and the teaching and learning of mathematics, it would allow a modification to be made in instruction to aid improvement in beliefs (Kloosterman et al., 1992:109). Scales and interview instruments to measure attitude and beliefs have been developed in the literature. They include the

Indiana Mathematics Beliefs Scales (Kloosterman, 1998) and Fennema-Sherman Mathematics Attitudes Scales (1976) and these are discussed in detail later in this study.

The literature recommends that students be made aware of their own beliefs in a conscious way as a process of changing negative beliefs (Spangler, 1992; Moscucci, 2007; Osterholm, 2010). Suggestions from the literature on how this can be done includes group discussions allowing beliefs to be challenged and possibly modified (Spangler, 1992) or through the positive role of making students aware of their own beliefs and monitoring difficulties (Moscucci, 2007:300). Students' and teachers' awareness of their own beliefs is considered to be fundamental to change taking place (Osterholm, 2010:12). Moscucci (2007:300) developed an activity attempting to restructure a relationship with mathematics by making students and teachers more aware of their beliefs. This included the creation of an environment that allowed learners to acquire awareness of their own beliefs independently and helped them to build a productive relationship with mathematics. Lester (2002:352) is skeptical of the claims that it is possible for teachers to access core beliefs of students using interviews or self-written reports. Most students, he argues, are not very aware of their beliefs and there is a considerable amount of work to be done in this area. He does accept the influence of socio-cultural contexts on the formation of beliefs and encourages the study of the role of beliefs in mathematics' learning. Gaps in the literature would appear to include questions as to whether students are aware of their mathematical beliefs and, if they are, is this age related?

Phillipou et al. (2002:213) maintain changes in beliefs occur as a consequence of new information and experiences that come into conflict with established beliefs. Examining the learning experiences that are provided by an individual teacher may inform that teacher as to what changes could be made that might better the learning environment he/she provides for the students. Verschaffel et al. ((2000:55) changed the instructional learning environment in the classroom with a view to changing students' beliefs and conceptions. Yackel and Rasmussen (2002:313) claim changes in beliefs might be initiated and fostered in mathematics classrooms by coordinating sociological (classroom social and

socio-mathematical norms) and psychological (students' beliefs) perspectives. They claimed that it is possible to explain how changes in beliefs might be initiated and fostered. The purpose of their study was to show that changes in beliefs and the negotiation of classroom norms are closely linked (Yackel and Rasmussen, 2002:328). They argue that social and socio-mathematical norms and individual beliefs evolve together. Boaler (2002:42) says students do not accept the norms of the classroom without question but play a part in forming, accepting or resisting such norms. She warns that it may take time for some students to accept changes. Students should be encouraged to initiate and negotiate socio-mathematical norms as these will lead to better agreement on expectations amongst students (Gerson et al., 2011:1). Lester (2002:349) asserts that Yackel and Rasmussen's framework assumes that students' beliefs are essentially cognitive in nature but that they provide a compelling example of how classroom norms influence the development of beliefs.

2.4 Literature on Interventions

The purpose of this study was to implement an intervention in the classroom with the aim of enhancing students' mathematics related beliefs.

What is meant by an intervention? In this case, the word intervention would seem to include a modification of the teaching and learning environment in some way provided by a teacher for a group of students with specific desired outcomes.

2.4.1 Research about classroom interventions:

Traditional learning environments were found to be failing students in that a significant number of the students appeared to dislike mathematics and/or found it to be a difficult subject to learn. School mathematics is widely 'hated' but the mathematics of life is much more enjoyable (Boaler, 2009:3). Redefining and reforming mathematics education is a powerful educational movement sweeping around the world (Conway and Sloane, 2006:201). Vauras et al. (1999:528) claim that there is a long way to go in instructional intervention research but that a successful first step seems to require innovative learning environments.

Boekaerts et al., (2005:212) assert that models of intervention have evolved over time and that they in turn have influenced later interventions. They claim that the quality of the intervention would seem to be important and also the length of time that it is carried out. Boeraerts and Corno (2005:218) write of the development of second-generation classroom interventions during the 1990s. This, they argue, was due to an infusion of sociocultural objectives and features to interventions, through the use of new technological tools for mediated learning. They include learner-to-learner collaboration within communities. Hoek et al.(1997: 361) used instructional approaches in their intervention that included a mixture of whole class teaching, group and individual work. Outcomes from the study showed the expected positive effects with the low achieving students in the experimental group outperforming similar students from the control groups.

What elements of a classroom environment are considered to be helpful in creating an atmosphere conducive to effective learning? Schorr et al., (2008:131) places emphasis on the roles of dignity and respect in creating an emotionally safe environment and suggest that attention to affective development may be particularly valuable in schools, as in this study, where social conditions seem discouraging and instruction is primarily procedural and test oriented. Bjarnadóttir (2011:80) writes that students value teachers who are cheerful, patient and care for students but they appreciate most teachers who explain the syllabus in a manner in which they can understand it. Turner et al. (2002:103) found classrooms with an emphasis on mastery correlated with students' perceptions of caring, respectful teachers. Schorr et al. (2008:134) define a safe environment as one where students are not embarrassed and/or humiliated and do not lose dignity or respect when dealing with mathematical impasse and/or frustration in solving mathematical problems. In her short exploratory intervention using the software 'simcalc' she argues that affective pathways culminating in pride and elation had been identified (Schorr et al., 2008:145). Self-efficacy is increased when students are provided frequent feedback and encouraged to attribute the feedback to their own efforts and that in turn encourages them to work harder (Pajares and Schunk, 2001:8).

2.4.2 Interventions carried out that aimed to change students' beliefs about mathematics and its teaching and learning:

There are several thousand intervention studies in the literature. Many of these studies focus on changing teachers' beliefs about mathematics and mathematics pedagogy (Ernest, 1989; Wilson et al., 2002; Hart, 2002; Lerman, 2002). Evidence from interventions previously carried out that seem to focus on enhancing students' beliefs were important to consider for this study. Garofalo (1989: 504) writes that teachers should devise classroom environments that help students to develop more realistic beliefs about mathematics. Only a small number of classroom interventions that focus directly on changing student's mathematics' beliefs, to this researcher's knowledge, are available in the literature. This researcher has provided a list of these studies in Table 1 below. Each study provides some insight into the design of an appropriate learning environment that may result in positive change to students' beliefs.

Author/Authors and Date	Sample Size	Duration	Aims of Study	Instruments used in study to collect data	Outcomes from study
Higgins, Karen, (1997)	n=137 Heuristic group=74 Traditional Group=63	1 Year	Explores the relationship between students' beliefs, attitudes and the problem-solving instruction given to students Learning environment to develop pupils' acquisition of overall metacognitive strategy solve problems. Replace pupils' erroneous beliefs by positive beliefs.	Quasi-experimental Design. Questionnaire using Likert Scale given before and after the Intervention for all students. Semi-structured individual interviews with a small number of students groups Written tests pretest, posttest and retention test. A questionnaire on beliefs and attitudes on teaching and learning of mathematics. Video tape of three pairs of students problem-solving	Positive on problem-solving tasks and on attitudes re: usefulness of maths. Learning environment had a positive impact on pupils' enjoyment and persistence in solving application problems and on their beliefs and attitudes towards problem solving. CLIA-based learning environments model developed later (De Corte, 2004:378)
Verschaffel, Lieven and De Corte, Erik et al., (1999)	N=11 classes 4 Exp. classes and 7 Control classes	4 Months	Explored possibility of nurturing positive beliefs about mathematics and learning of same in classroom using powerful learning environment	Pre and Post tests using Questionnaire with a 5-point Likert scale	Innovative learning environment showed students having more adaptive and stable beliefs about themselves (Mason & Scriver, 2004:172)
Mason, Lucia and Scriver, Luisa, (2004)	n=86 Exp. Group =40 Control =46	3 Months	Learning environment aims: students' acquisition of self-regulatory metacognitive strategy for solving problems and appropriate beliefs and attitudes of mathematics and problem-solving.	Beliefs and Attitudes Questionnaire and word problem solving test. Standardized achievement Tests. Video providing classroom observations.	Students from reform classroom applied more heuristics when problem- solving. Traditional classroom student beliefs slight improvement. Slight decline reform classroom beliefs.
Depaepe, F., De Corte, E., Verschaffel, Lieven, (2007)	n=10 classes	7 Months	Intervention aimed to monitor beliefs and problem solving behaviours of students who are learned-helpless	Pre and Post study surveys. Journal, Achievement tests. Student and Teacher Interviews. Video in class sometimes.	Inconclusive outcomes determining change after intervention

Table 1: Interventions

Higgins (1997) conducted a year-long intervention focusing on mathematical problem-solving instruction. Her study showed very much better outcomes for students compared to the students who had been taught in a traditional manner. Higgins examined students' beliefs and attitudes about mathematics and their own problem solving abilities including the solving of a number of non-routine problems. The student's role, Higgins maintains, is one of the discoverer not that of passive recipient of knowledge (1997:6). Problem-solving skills were taught to the students using direct method of instruction over a period of weeks. The tasks chosen to give to students to solve were open-ended problems.

Verschaffel et al. (1999) reported on two interventions that aimed to create a classroom climate conducive to the development of students' appropriate beliefs about mathematics and mathematics modeling and problem solving. Teaching methods that they recommended for use included guided practice included small-group work, whole class discussions on evaluation and reflection and different solution strategies (Verschaffel et al., 2000:117). The authors acknowledge the wider political and institutional factors at work in schools that reinforce traditional instructional practices and they hope that more enlightened decision-makers, parents and the public in general would support the implementation of the approach they advocated (Verschaffel et al., 2000:181). De Corte et al. (2008:34), in a continuation from Verschaffel's study examined the relationship between students' mathematics-related beliefs and the classroom culture and they made changes to four areas of the learning environment. These included the content of the learning and teaching, the nature of the problems, the instructional techniques and the classroom culture. New classroom norms about teaching and learning problem solving aimed at fostering positive mathematical beliefs. The effects of the environment were tested using a pre- and post-test retention test with experimental and control groups. The intervention significantly boosted students' cognitive and metacognitive competencies with word problems with an effect of .31. It also fostered experimental classes mathematics-related beliefs although the effect was quite small (De Corte et al., 2008:30). De Corte et al. (2004:370) characterize a powerful learning environment as one that has as having a good balance between discovery and personal exploration with systematic instruction

and guidance taking into account individual differences (De Corte et al., 2004:372).

To enhance student's mathematics' beliefs, Mason and Scrivani (2004:156) sought to establish a new classroom culture through the negotiation of new socio-mathematical norms. The student's role, established by their teacher, was one where they were encouraged 'to do' maths, gradually being asked to take responsibility for their own learning (Mason et al., 2004:159). The teacher's role in Mason's study was one of stimulation, encouragement and scaffolding, underpinned by a model of competent problem-solving. The model underlying the learning environment was that as described by Verschaffel et al. (1999:97). Mason's study was carried out with students aged 10 and involved the students' usual teacher carrying out the intervention assisted by the researchers.

Depaepe et al. (2007) sought to investigate the impact of changing perspectives on current mathematics teaching underpinning a new generation of textbooks. Results indicated that some reform-based aspects seemed easier to implement than others e.g. embedding tasks in realistic context was easier to implement than, say, use of group work. Depaepe et al. created a learning environment that de-emphasised the teaching and practising of procedures and algorithms and stressed the importance of reasoning and problem-solving skills.

In Martinez's (2011) intervention study above, students who were learned-helpless were identified from a number of questions in the questionnaire given to all students prior to the start of the study. The jigsaw instructional method was used to implement the study to encourage cooperative learning within the classroom. Students studied word problems and chose the concept they wished to investigate and they kept a goal log for each class period. The jigsaw instructional approach allowed students to work jointly on an activity. The students worked on small problems and they, in turn, became part of the solution to a whole problem. This supported differentiated work in the classroom. Students viewed an instructional video on a laptop about the problem they attempted to solve. The study aimed to foster the belief that success is achievable in mathematics through effort.

2.4.3 Summary of main characteristics of learning environments above

The learning environments created in classrooms for the above studies in section 2.4.2 included in their design some or all of the following characteristics:

INTERVENTION	STUDIES					
	Higgins 1997	Verschaffel 2000	De Corte 2004	Mason 2004	Depaepe 2007	Martinez 2011
Active learning methodologies	✓	✓	✓	✓	✓	✓
Learner reflection with peers through small group work	✓	✓	✓	✗	✓	✓
Problem-solving Instruction	✓	✓	✓	✓	✓	✗
Awareness training of different phases of competent problem solving	✗	✓	✓	✗	✗	✗
Use of routine and non-routine problems	✗	✓	✓	✓	✗	✓
Application to real-life situations	✗	✓	✓	✗	✓	✓
Teacher promoted success through effort	✗	✓	✓	✗	✗	✓

Table 1A: Interventions and Studies.

These characteristics support effective learning environments, as evidenced in a number of other studies from mathematics education research. These characteristics are now explored:

(i) Active learning methodologies: For students to learn mathematics well it would seem to be necessary for them to be active learners (Boaler, 2009; Leikin et al., 1997; Michael, 2006). It would appear, too, from the literature, that some of the mathematics tasks that teachers use in class with students do not promote students' active task-related interaction (Leikin et al., 1997; King, 1993). Boaler (2009:58) examined the learning environments in different mathematics' classrooms in her study. In one classroom using a traditional approach to teaching some students were convinced of the need to memorise the methods they were shown by their teacher and they did not see any place for thinking in the classroom. Lester (2002:352) warns of the mismatch between a student's beliefs and the external knowledge of the mathematics community, alerting

teachers to the importance of providing students with a wide range of ways to think about and learn mathematics. By being active in their own learning students would seem to be more able to acquire skills that enable them to take charge of their own learning (Anthony, 1996; Michael, 2006). McClintock et al. (2005:140) describe this as students actively building new knowledge of mathematics from their experience in the classroom and prior knowledge. Research in Ireland indicates a culture of passive learning in the mathematics classrooms where teachers would seem to be focused on well-rehearsed procedures and examinations with readymade mechanisms used to aid memorization of procedures (Breen and O'Shea, 2011:46). There is a lack of research on the nature of post-primary mathematics textbooks in Ireland. However, they would seem to 'rely on a similar pattern of exposition as evidenced in examination papers' described as focusing on 'precise terminology, symbol manipulation and abstraction with little attention devoted to rich contexts' (Conway and Sloane, 2006:217).

(ii) Small group work: The literature on using small group work in the classroom has been heralded as providing students with more, improved achievement outcomes (Good et al., 1992:166, Cohen, 1994; Dekker, 2004; Swan, 2005; Brodie, 2000). Outcomes from studies that have used small group work in the classroom may help to inform the implementation of the learning environment in this study.

Some aspects of the implementation of group work in the classroom are therefore discussed below.

Boekaerts and Corno (2005:220) claim that collaborative learning supports self-regulation as more intellectually able peers deepen their own learning by explaining concepts to peers in need of support during small group activities. Good et al., (1992:166) argue that they believe that small-group instruction used in the classroom can facilitate students' affective and cognitive growth with the proviso that most important of all is the quality of the instruction provided by teachers. Cohen et al., (1994) and Barnes (2005) raise the issue of equity in the classroom and the status of low ability students within groups. The question they raised in their study was how a teacher might ensure that all members of a

small group could learn within the group. The advice is that the teacher must specifically boost the participation of low status students so that other group members recognize and value their contribution to solving the problem (Cohen et al., 2009; Barnes, 2005; Boaler, 2009). More evidence from the research is needed on how teachers can successfully increase the participation of low status students.

Wenglinisky (2000:33) claims there are some studies that claim to show no benefits to students learning from using group work in mathematics or science classes. Mercer and Hodgkinson (2008:7) argue that group work 'is a valuable resource in a teacher's repertoire but it is not a universal remedy'. Vidakovic & Martin (2004:467) believe that small-group problem solving takes place when the shared knowledge stays within the ZPD of the group members. Zack & Graves (in Armstrong, 2008:115) claim that the development of a collaborative ZPD does not always occur through interaction even under optimal conditions. Depaepe et al. (2007:266) argues that group work is more difficult to implement in classrooms than for example placing a strong focus on heuristic skills.

Effective communication within the classroom would seem to be challenging for all. Brodie (2000:2) talks about the difficulties teachers have in supporting students' group discussions as they struggle to communicate and learn from each other. Mercer (2003) says teachers can help students to learn to think by encouraging 'dialogic talk' using group work and also in whole class discussions., He describes Dialogic talk as an effective type of classroom interaction, where both teacher and student make substantial and significant contributions. Interactions between students and teacher are characterised by the quality of the arguments given and considered together for solving problems. Mercer claims that role of dialogue is a key component of conceptual change (Mercer in Dillon, 2000). Liljedahl et al. (2007:285) contend that the theory of conceptual change is an ideal framework for the examination and explanation of belief rejection.

Dekker et al. (2004:62) discuss two types of help that are given by teachers to groups of students. They call them 'process help' which aims to help the interaction process and 'product help' in which the teacher generally acts as a

part-time assistant. Teachers, Rojas-Drummond et al. (2003:99) suggest, should focus on certain interactional strategies hence enabling students to manage learning activities through individual and joint reasoning. The strategies recommended included:

1. Asking students 'why' questions to get them to reason and reflect on what they were doing.
2. Teaching problem-solving strategies and encouraged children to make explicit their own thought processes.
3. Learning to be treated as a social and communicative process.

Mercer and Littleton (2007:2) believe that insufficient attention has been given to the relationship between the quality of talk and learning outcomes and that 'learning to use language for reasoning is a valuable goal in its own right'. They argue that both teacher-student and student-student discussions have special functions. Dialogic teaching is a term used by Alexander (2004:32) to indicate certain characteristics of classroom interaction that include: carefully structured questions to elicit thoughtful answers. Answers to questions are seen as the building blocks to dialogue rather than end points and teacher-student and student-student dialogues are a sequence of lines of reasoning. Mercer and Hodgkinson (2008:162) argue that 'exploratory talk' will be most productive when the following conditions apply:

- i) Individuals have opportunities to contribute opinions, suggestions etc.
- ii) Others are willing to listen attentively and critically
- iii) There are opportunities for all participants to discuss whether and in what ways different contributions are relevant
- iv) Teacher shares control and right to evaluate with students
- v) Topic under discussion is, or becomes of interest to participants

Imm et al., (2012:144) classified the discourse in classrooms as:

- (i) High discourse where teachers valued inclusive, purposeful mathematical conversations;
- (ii) Low discourse where one-directional telling is the norm and

- (iii) Hybrid discourse reflecting a mix of the features of the other two which they describe as teachers promoting exploration and justification of ideas but funneled the conversation towards a particular answer (Imm et al., 2012:141).

The relationship between the activities given to students and talk, they claim, is not clear. The selection of an appropriate activity alone does not guarantee students' rich discussions and engagement with a task. A coordination of two frameworks, the role of cognitively demanding tasks and the role of discourse, needs to be undertaken to illuminate this particular relationship (Imm et al., 2012:145). It would seem that the research on effective communication within small group work requires further studies.

Group flow is what the teacher hopes to develop within groups of students in the classroom. Group flow has been defined as having eight dimensions including clear goals, immediate and unambiguous feedback, a balance between the challenges of an activity and the skills required to meet those challenges and concentration on the task in hand (Zheng et al., 2011:192). A theory of optimal experience, based on the concept of flow, is described by Csikszentmichalyi (1990:4) as being in a state of deep absorption in an activity where nothing else matters and Armstrong (2008:102) describes group flow as 'a state of experience in which an individual is intensely focused on and absorbed by an activity, so that the experience is inherently self-motivating'. Concentration, interest and enjoyment in an activity must be experienced simultaneously for flow to occur (Shernoff et al., 2003:161). Further research is required to advise teachers on how to achieve this.

(iii) Problem-solving Instruction

The literature on the design of a good problem-solving lesson is still developing. Problem solving abilities, beliefs and attitudes develop in contexts and are influenced by the activities used by the teacher (Wilson et al., 2002). Wilson et al. advise teachers not to use Polya's steps for problem solving as a linear process as this promotes the solving of problems as a procedure to be memorized and practiced and leads to an emphasis on answer-getting. Students should engage in thinking about the various tactics, patterns and the strategies available to them (Polya, 1973). Fan and Zhu (2000) examined two

secondary mathematics textbooks used in Singapore concerning their representation of problem solving tasks. The textbooks were found to give a strong foundation in problem solving to students with a recommendation for further exposure to the general strategies of Polya's model. Recent literature has focused on using particular activities and teacher guidance to maximize opportunities for students to make visible their current understanding and reasoning to teachers, thus enabling a better response to individuals learning needs (Evans and Swan, 2013). This contrasts sharply with the didactic teaching style which may be successful in delivering good examination grades but is less effective in developing understanding and the thought and persistence needed for successful problem-solving (Haggarty, 2002:39). In Irish classrooms, recent research, as mentioned earlier, shows the didactic teaching style is prevalent in mathematics classrooms.

(iv) Awareness training

Making students aware of their beliefs and monitoring their difficulties, as mentioned above, has been recommended by Moscucci (2007). De Corte et al. (2004) also mentioned above, recommended awareness training for students of the different phases of competent problem solving. Damon et al. (2006:28) writes of the necessity for students to become aware of the problem-solving strategies they are using in the different phases of the solution process with a view to fostering self-regulatory skills. These are the same models, they argue, proposed by Schoenfeld (1985) and Lester, Garofalo (1989). Kwang (2000) advises in order to help students become better monitors of their own mathematical problem solving actions teaching should take place in the context of learning concepts and techniques.

(v) Routine, non-routine tasks and real-life contexts

Yeo (2009:23) examined students' difficulties in solving non-routine problems in secondary mathematics classrooms and found students were observed using only one strategy in attempting to solve problems. They showed no flexibility in trying to apply another strategy or to check their solutions in their attempts to solve the problems. Teachers are advised by Yeo to make students aware of their difficulties, arguing that students must have relevant knowledge skills and be able to coordinate those skills to solve problems. Kloosterman (2002:262)

claims more needs to be done to examine students' reaction to challenge. Contexts that are meaningful to students may hold their interest in the subject area (Nickson, 2000; Mitchell, 1993). Chu et al. (2013) include making explicit connections between school mathematics and out of school phenomena when teaching for understanding. Real world contexts are considered to be potentially significant for developing understanding in mathematics (Koedinger & Nathan, 2004; Moses & Cobb, 2001; Van den Heuvel-Panhuizen, 2005).

(vi) Teacher promotes success through effort

Work avoidance goals can be attributed to teachers as well as to students of mathematics. This study acknowledges the vast area in the literature on motivation including achievement goals applied to all aspects and areas of the mathematics classroom. It is only possible to discuss in this section the literature on the teacher promoting the achievement of success in learning mathematics through an individual's efforts.

Butler (1988:13) in a study on different types of feedback to students found that praise that was frequent, credible, specific and genuine and related to factors within students' control were most effective in helping to raise achievement. Comments, by way of feedback to one section of the students only showed, in the second lesson examined in her study, a substantially improved quality of work. Boston (2002) argues that "comment only marking" encourages students to focus on thinking rather than getting simply getting the correct answer. This, Boston maintains, may be helpful to lower achieving students because of its emphasis on improvement through effort rather than by innate ability. Feedback, according to Black and Wiliam (2006), has three elements, 'recognition of the desired goal, evidence of the present position and some understanding of the way to close the gap between the two'. Wiliam (1999) examined the effect of previous feedback to students on their present performance and on their attitude to their learning. Wiliam's study found the research on feedback to be remarkably clear with recommendations that teachers focus on what learners need to do to improve rather than on how well they have done and that they should avoid comparison with others. Shepard (2005:11), remarking on formative assessment, says the case for its use in the classroom is compelling. It helps, Shepard maintains, to counteract students'

obsession with grades and helps to redirect interest and effort towards learning. However, she recommends sustained professional development for teachers implementing it in the classroom and further argues the difficulties interim assessments measuring progress may impede its successful implementation in the classroom.

2.5 Analysis of the literature review

From the rich picture provided in the literature relevant to this study the aspects focused on are the following:

(i) Beliefs:

The review showed that there is not yet agreement in the literature on a definition for belief and belief systems (Leder et al., 2002).

More recent beliefs research suggests a change away from looking at beliefs as mental entities to an alternative approach that is described as a participatory perspective. For the purposes of this study, 'Belief' and 'Belief Systems' are defined as cognitive/affective configurations to which the holder attributes some kind of 'truth' value (Goldin, 2002:64). The use of a continuum to describe the Affective domain where beliefs are considered to be more cognitive and stable and least intense is also to be found in the literature (Phillippou, 2002). Beliefs develop slowly, it is claimed, and are the weak element of the Affect structure and hence the best to act on with a view to enacting change (Moscucci, 2007:299). This is the view adopted by this study.

(ii) Students' Beliefs:

This study has chosen to focus on students' beliefs about mathematics, as listening to students and attempting to support their learning needs is vital, in this researcher's view, if teachers are to be successful in the classroom. Much of the literature puts forward an exploration of a set of beliefs about the subject mathematics, about its learning and teaching and the social context (McLeod, 1992; Kloosterman, 1996, Pehkonen, 1996). This, it is claimed, placed students' beliefs on the research agenda (Op't Eynde et al., 2002:20). The authors argue for the development of an overall model and categorization of mathematical-related beliefs. The review carried out by Op't Eynde et al. includes students' beliefs commonly found in the literature, and would seem to be comprehensive

in nature. Hence their categorization of beliefs is accepted for use with this study. The categories include: Beliefs about mathematics, self and beliefs about the social context of learning. This would therefore appear to be a reasonable starting point for this study.

Links have been made in the literature from students' previous learning experiences to the beliefs that they hold about mathematics (Schoenfeld, 1992). Students hold beliefs about mathematics and the context in which it is learned. Classrooms that stress understanding of concepts over memorization of rules can sometimes result in students quitting trying to learn (Kloosterman, 2002:247). Kloosterman goes on to say that there is evidence that with time and discussion of issues such as what the subject of mathematics is should enable students to thrive in a reform-learning environment. With a view to optimizing the possibility for a successful outcome for this study, the intervention in the classroom will be carried out for the maximum possible time over the course of one academic year.

Teaching through problem solving is one approach recommended to improve students' experiences and successes in the mathematics' classroom (Nickson, 2000; Cai, 2003; Vertschaffel, 2000; Boaler, 2009). This approach is valued and adopted for this study as it recommends a supportive environment, uses worthwhile activities, encourages mathematical discourse and promotes sense making (Anderson, 2005: 89). Moving from a didactic, traditional approach to a problem-solving environment successfully over one academic year is a considerable challenge that will no doubt include some setbacks (Turner, 2009:368). These challenges are expected by this study.

(iii) Measuring Students' Beliefs:

Attempts to measure students' or teachers' beliefs about mathematics and its teaching and learning are ongoing (Kloosterman, 1992; Hart, 2002; Lerman, 2002; Lester 2002).

Appropriate instruments and scales to measure students' beliefs and attitudes are available in literature. These are examined in Section 3.5.3.

(iv) Interventions:

Interventions reported in the literature provided some indication of what type of learning environment might support the development of students' positive beliefs, to this researcher's knowledge, are not many to date. Information is available on what the characteristics of a good environment might be but most of those are examined for improved achievement outcomes alone. There is evidence that prevailing teachers' practices and the classroom culture are hugely responsible for students' naïve/ negative beliefs (De Corte et al., 2008:34). The view of this study is that it is not only necessary, but also essential, for studies to focus on students' beliefs. Teachers desperately need to be provided with direct help in how they might change students' naïve beliefs about mathematics.

Deciding on the characteristics of the learning environment to implement in the classroom for this study is predicated on the above literature review and what aspects of the environments they indicate might be necessary to include.

The aspects chosen for implementation in the learning environment in this study are indicated in Table 2 below:

Characteristics of the learning environment adopted for use in this study	
Characteristic of Learning Environment	Intervention: Author/Year
Active Learning Methodologies	<u>Verschaffel et al. 2000</u> ; Lester, 2000; <u>McClintock et al., 2005</u> ; <u>Boaler, 2009</u>
Explicit Instruction in Problem-Solving Skills	<u>Higgins, 1997</u> ; <u>Verschaffel et al., 2000</u> ; De Corte et al., 2000; Mason et al., 2004; <u>Depaepe et al., 2007</u> ; <u>Rojas-Drummond et al., 2003</u>
Students' asked to gradually take responsibility for their own learning in the classroom	<u>Shoenfeld, 1991</u> ; De Corte et al., 2000; <u>Boekaerts and Corno, 2005</u>
Use of routine and non-routine problems	<u>Higgins, 1997</u> ; Mason et al., 2004; <u>Verschaffel et al., 2002</u>
Collaborative Learning-Small Groups	<u>Good et al., 1992</u> ; Mason et al., 2004; De Corte et al., 2000; <u>Mercer et al., 2008</u>
Students made aware of their beliefs	<u>Moscucci, 2007</u> ; De Corte et al. 2008;
A Focus on effort in the classroom	<u>Martinez, 2011</u> ; <u>Butler, 1998</u> ; <u>Black & William, 2006</u>

The next chapter, Methodology, specifies how this literature review informed the research questions generated for this study.

Chapter 3) METHODOLOGY

The review of the literature (Chapter 2) identified the current research in relation to students' beliefs about mathematics and its teaching and learning. It showed that beliefs research is relatively new in comparison to many other areas of concern to mathematics education. The review highlighted shortcomings in the current research and it also indicated a very complex field requiring much more investigation.

The research goal and research questions for this study, arising from the research area of interest informed by the literature review, are first stated. This chapter then describes the research paradigm, methodology and methods used and the reasons they were considered to be the most appropriate for this study. The planned methods section discusses the participants involved in the study, the instruments used to collect the data, the recommendations arising from a pilot study undertaken in preparation for this study and the data analysis carried out. The chapter also discusses the procedures used in implementing the study in the classroom. These include ethical considerations in the collecting of the data, the process chosen in the selection of individuals and focus groups for interviews and the implementation of both.

This chapter structure comprises 3.1 which states the research goal and research questions, 3.2 the research paradigm and theoretical background used in the current study, 3.3 the chosen research design, 3.4 the rationale behind that choice, 3.5 Data collection including planned methods, sample used and chosen instruments, 3.6 the Pilot study, 3.7 Procedure, ethical considerations and administration of data collections 3.8 the Intervention and 3.9 data analysis

3.1 Research Goal:

The aim of this study was to develop, in a positive way, students' beliefs about mathematics and its teaching and learning through the implementation of a classroom intervention. Students' beliefs are measured in order to examine changes after the intervention. Newman et al. in Onwuegbuzie et al.

(2006b:478) identified nine types of research goals in education. Measuring change is identified as an appropriate type of goal.

3.1.1 Research Questions:

The research questions in this study are directly linked to the research objectives for this study. Beliefs about mathematics and its teaching and learning are measured to assess the impact of the classroom intervention. The review of the literature (Chapter 2) identified students' beliefs that are commonly found in the literature. They included students' beliefs about the role of the mathematics teacher, beliefs about their competence in mathematics, the relevance of mathematics, and mathematics as an inaccessible subject (Op't Eynde et al., 2002:20). These beliefs were chosen for exploration in this study as they provided a variety of perspectives on different aspects of beliefs about mathematics and its teaching and learning at second level. The pedagogical changes to the learning environment made in this current study were informed by the literature on interventions in the last chapter. The research questions below attempt to understand these aspects of the effects of the changed learning environment.

The research questions for this study are:

Research Question 1: Are students' beliefs about their teacher's role in the mathematics classroom changed in a positive direction by the altered learning environment in the mathematics classroom?

This question was asked as the literature review highlighted the influence of the teacher's role in the classroom on students' beliefs (Ch. 2, Section 2.3).

Research Question 2: Are students' beliefs about their perception of their own competence in mathematics changed in a positive direction by the altered learning environment in the mathematics classroom?

This question was asked, as beliefs about self are important in the context of learning mathematics (Op't Eynde et al., 2002:17).

Research Question 3: Are students' beliefs about the relevance of mathematics to their lives changed in a positive direction by the altered learning environment in the mathematics classroom?

This question was asked as it is thought to be a component of mathematical disposition (Op't Eynde et al., 2002:15).

Research Question 4: Are students' beliefs about mathematics as an inaccessible subject changed in a positive direction by the altered learning environment in the mathematics classroom?

This question was asked, as the subject of mathematics might be thought of as difficult and boring with little connection to real-life.

3.2 Research Paradigm/Theoretical Background:

3.2.1 Background

In choosing a theoretical perspective for a study, a researcher creates the basis for their interpretation of the world, in this case the world of the secondary mathematics' classroom. Leona Burton (2005:3) challenges the lack of acknowledgment by researchers of the assumptions behind their choices of theoretical stances in their studies. She states that she 'does not believe there is ever a case where the researcher's beliefs, attitudes, and values have not influenced a study'. Crotty (1998:2) states that in order for the research community to take seriously the outcomes from a study, it is necessary to state the philosophical stance that informs the theoretical perspective of a study and it, in turn, informs the methodology and methods chosen.

3.2.2 Paradigms

There are a number of research paradigms recognized in educational research, each with its own assumptions about knowledge and learning, about the world and how knowledge is obtained (Ernest, 2005:29). They include the scientific research paradigm, interpretative, critical and pragmatic paradigms. A research paradigm is described essentially as the worldview held by the researcher and this includes a whole framework of beliefs, values and methods within which the

research takes place (Ernest, 2005; Morgan, 2007; Greene, 2007; Creswell and Plano Clarke, 2007, Mertens, 2010). Categorizing educational research into a few paradigms is considered to be complex, if not impossible, with some paradigms overlapping (Mertens, 2010:8). In the author's view choosing a paradigm for a particular study, the main consideration must be based on what is deemed to be most appropriate to use with a particular study.

The scientific research paradigm is one that views knowledge as objective, and demands an observer role for researchers and searches for general laws that predict future educational outcomes (Ernest, 2005; Cohen, 2007). This paradigm is inappropriate for use with this study as it is not possible to attempt to produce time and context-free generalizations from the outcomes (Johnson et al., 2004:14).

Traditional interpretive researchers, on the other hand, begin with individuals and try to understand their interpretations of their worlds (Cohen, 2007:22). This paradigm is possibly suitable for use with this study. However, other paradigms were deemed to be, by the author, to have been more appropriate for this study as explained below. It is acknowledged, however, that the nature of knowledge relation within the interpretative paradigm described as having an interactive link between the researcher and participants did resonate with this study (Mertens, 2010:11).

The Critical Paradigm, sometimes described as transformative, places knowledge as socially and historically situated (Cohen et al., 2007:33). It also focuses on the need to address issues of power and trust within relationships between the stakeholders. In this instance these include management, teachers and special needs staff in the school; These are not issues that were addressed in this study, and hence it was not the most appropriate paradigm to use with this study. An additional reason for this was that the framework is marked by an intention to advocate for improvements in social justice and human condition, especially with marginalized groups in society (Sweetman et al., 2010, Feldman et al., 2004). This study was undertaken in a school where a large proportion of its students are marginalized in society because of their socio-economic status. Sweetman et al., (2010:445) cite ten characteristics of transformative studies.

These characteristics include research questions written with an advocacy stance, a literature review that includes discussions of diversity and oppression and a problem in a community of concern. None of these characteristics of a transformative lens used in the research applied to this study. Hall (2012:4) argues that this focus of a transformative lens limits its application to a small range of social science research although it might also be argued that a new learning environment might prove transformative within the confines of an individual school. Heidegger, in Mertens (2011:26), argues that all meanings including the meanings in research findings are interpretative. There is recognition too in the literature that there is no rule that forbids the eclectic mixing of the pragmatic and emancipatory orientations (Johansson et al., 2008:102).

The fourth paradigm is the pragmatic paradigm. Mertens (2010:11) describes the nature of knowledge relation between the knower and the would-be known for the pragmatic paradigm as one that is determined by what the researcher considers to be appropriate for the particular study. Morgan (2007:50) commits to what he describes as Kuhn's concept of paradigms shifts as a tool for examining changes in social science research approaches. The pragmatic approach, Morgan claims, will build upon how our worldviews influence the research that we do. Johnson et al. (2007:125) propose that one or more of the pragmatisms can provide a philosophy that helps mixed research to coexist with the philosophies that support scientific and interpretative research. Johnson et al. (2007:113) write that 'the primary philosophy of mixed research is that of pragmatism'. They have created a version of pragmatism around the ideas of classical pragmatist philosophers such as Charles Sanders Peirce, William James and John Dewey. Mixed methods is defined as a type of research where both qualitative and quantitative approaches are used in research methods, data collection and analysis (Teddle and Tashakkori, 2009:7). The Pragmatic Approach using mixed methods was the chosen theoretical underpinning for this study as the author considered it was the most suitable paradigm to use with the research questions that were asked.

There are those in the literature who challenge attempts to place mixed methods within the notion of pragmatism, claiming that adequate criteria have

not been provided and that serious debates are still to take place in relation to mixed research (Miller et al., 2006: 596). Despite the challenges of mixing two methodological orientations consensus would seem to have been accepted that quantitative and qualitative methods should be mixed in order to understand the political reality (Mastenbroek et al., 2007:18). Johnson et al. (2004:16) argue that research approaches should be mixed in ways that offer the best opportunity for answering important research questions.

3.2.3 Methodology

Pragmatism was the chosen paradigm for this study as explained above. In order to operate within this paradigm an appropriate methodology was required. Teddlie and Tashakkori (2009:103) argue that the pragmatic perspective may be employed as an underlying paradigm for the use of a mixed methods methodology. Denscombe (2008:273) claims one reason for the use of mixed methods is to produce a more complete picture through the combination of complementary sources. This highlights the most significant reason for the choice of mixed methods used in this study.

Creswell (2008:326) provides multiple perspectives on the nature of mixed methods:

1. One group of researchers focuses on the research methods without being encumbered by a philosophy (Creswell, 2003)
2. A second group views mixed methods as a process with a mix of all stages of the research approach and calls on researchers to mix worldviews in a study (Burke Johnston and Onwuegbuzie, 2004:19)
3. A third group focuses on the philosophical assumptions as the focus of the inquiry (Morgan, 2007; Tashakkori and Teddlie, 2003).

The author places this study within the second group above where the stages of the research approaches and worldviews are mixed. Within the pragmatic paradigm, the choice of mixed methods is considered to be an appropriate methodology. This allows methods to be matched to specific questions and to the purposes of the research (Mertens, 2010:11). Mixed methods research is described as research where quantitative and qualitative techniques, methods and approaches are used in a single study (Burke Johnson et al., 2007:17).

Reasons for this choice in this study include the belief that multiple methods are considered useful for addressing the research questions proposed in this study. This leads, it can be assumed, to a better understanding of research problems than either the scientific or interpretative paradigms alone (Creswell, 2008:322). There are, however, significant controversies about mixed methods studies in the literature questioning its uncritical acceptance as an emerging dominant discourse (Creswell, 2011:277). This study acknowledges the messiness of mixed methods research and that it is early days in the development of the field.

3.2.3.1 Teacher as Researcher

Insider research is defined in the literature as Action Research, in comparison to what is described as outsider research (Zeni, 2001:3). Zeni writes that the goals of both methods of research may be to change or improve the teaching and learning in the classroom but that Action Research documents one's own practice and not someone else's.

Insider researcher includes practitioners in the educational system carrying out individual or collaborative inquiries (Clarke and Erickson, 2003:3). There are assertions in the literature that claim that only an impartial outsider can provide an objective account of human interaction (Mercer, 2007:5). However, there is no doubt that the researcher's position (whether insider or outsider) will impact on the research process and the validity of the process. (Even a researcher working in the scientific paradigm and attempting to be as objective as possible in their collection and interpretation of data will bring their own position to bear on, for example, their choice of topic or the focus of their data collection instruments.) Rather than attempt to avoid such difficulties by recourse to the 'impartial' external researcher it is, perhaps more appropriate to consider the benefits and challenges to the validity of outcomes from the insider researcher. Westberry (2011:13) includes credibility and the use of implicit knowledge as benefits and the lack of objectivity as a challenge. Teacher-researchers bring their expert knowledge to bear in undertaking the research of their practice (Loughran et al., 2003:181). Morse et al. (2002:17) includes a number of strategies that researchers should use to attain validity and reliability in their work. These include methodological coherence, sampling adequacy and an active analytical stance. Cochran-Smith (2005:224) claims that teachers need

to be smart consumers of research and need to function as both researcher and practitioner. Practitioner researchers have the ability to produce rich, detailed insightful analyses about teaching and learning from the inside (Cochran-Smith et al., 2009:5). This study identifies with the concept of insider/outsider abstractions occurring on a continuum and that the boundaries between the two are fluid (Mercer, 2007:4). This study provides multiple examples of direct quotations from participants that are used to support inferences drawn from the data.

The literature has also examined what has been described as 'the tension between the teaching role and the research role' for teachers researching their own classrooms (Mitchell, 2004:1393). The dual role is considered by some researchers to create conflict between the roles and this is considered to be unavoidable (Wong, 1995; Hammack, 1997). Mitchell (2004:1403) drawing on the work of Zeni (2001) argues the existence of a 'zone of accepted practice' where students collaborate in research undertaken on the teacher's practice. Classroom interventions, Mitchell (2004:1407) claims are constantly undertaken where the teacher researches with the students rather than on them. In this instance, the roles of the teacher and the researcher are considered to work together. Students taking part in this study were encouraged and invited to share their views on the changes made to the teaching and learning environment. There are ethical issues associated with all aspects of the research process. These are discussed later in the chapter.

Action research developed from dissatisfaction amongst educational practitioners to positivist approaches and a desire to understand and effect change in professional practice (Scott and Morrison, 2005:4). Action Research is described in the literature for example as research conducted by practitioners in educational settings (Glanz, 1998:20). Teachers can work alone or collaboratively seeking answers to research questions of interest to them. Ball (2000:373) claims 'first person research enters a teacher's voice and perspective into the discourse of scholarship', Heaton (1994) developed a strategy 'for straddling the inside and the outside'. Simon (1995) posited a 'provisional theoretical model' of design work using his own teaching. The challenge of using the self while standing back is an endemic challenge that

might be illuminated by looking at the development of method and scholarship in other fields (Ball, 2000:402). It may be, for example, the structure of a professional EdD. study, together with its supervisory framework, provides support to the development of scholarly approaches to first-person research.

These questions may be about students' beliefs about teaching and learning in the classroom, classroom behaviours or indeed pedagogical issues. This study was driven by a desire to improve practice through the evaluation of specified actions taken in the mathematics' classroom. An attempt was made in this study to evaluate the impact of actions carried out in the classroom intervention on students' beliefs. The research questions in this study sought to enhance the beliefs of students about mathematics as a subject and its teaching and learning in the classroom (Section 3.1.2). The researcher implemented a classroom intervention designed to measure students' beliefs afterwards and through this it sought to improve practice. Consequently, Action Research was considered to be an appropriate methodology for use with this study.

How is Action Research framed within acceptable worldviews and what theoretical lens is considered appropriate to use with it? Some of literature claims that there is a high congruence between the philosophical assumptions behind Action Research and the pragmatic paradigm instrumented through the use of mixed methods (Philips et al., 2009:215). Indeed the roots of Action Research are commonly associated with pragmatism as a tradition within philosophy (Johansson et al., 2008:98). The argument in the literature is that mixed methods are easily reconciled within the dynamic contents of action research (Philips et al., 2009:213). This view coincides with the choices that have been made in this study by the author.

3.3 Research Design

The research design utilized in this study is described in the literature as action research using quasi-experimental sequential explanatory mixed methods. The justification for this choice is discussed below.

3.3.1 Research Objectives:

The first objective for this study was to attempt to develop students' positive beliefs about mathematics and its teaching and learning. A secondary objective for this study was to assess the impact of a classroom intervention as a means of fostering students' mathematics related beliefs.

Johnson and Christensen (2004:26) define five major standard research objectives that are pertinent for the quantitative and qualitative phases of a mixed methods research study. They are defined as follows:

1. *Exploration* attempts to generate ideas or theories about phenomenon.
2. *Description* attempts to describe the characteristics of a phenomenon.
3. *Explanation* attempts to show how and why a phenomenon operates as it does.
4. *Prediction* attempts to predict and forecast a phenomenon.
5. *Influence* described as attempts to apply research to make certain outcomes occur.

Two of the above standard research objectives were applied to this study. The objective described above as *Influence* was an appropriate objective. In this study a new learning environment, informed by current literature, was implemented in the mathematics classroom that sought to aid the development of more positive student' beliefs. The second objective above that was considered to be appropriate to apply to this study is *description*. The reason for this was that this study attempted to report on students' beliefs about mathematics and its teaching and learning.

3.4 Rationale for Design Choice

Teddlie and Tashakkori (2009:140) assert the use of mixed methods is a choice made by researchers and driven in the main by the situation and the research questions. Nastasi et al. (2005:177) writes about the added value of using mixed methods in intervention research. Complex social phenomena are thought to be best comprehended through quantitative and qualitative lenses (Waysman et al.,1997; Natasi et al., 2005). This is the underlying rationale, as mentioned earlier in this chapter, for the choice of mixed methods as a methodology for this study.

Collins et al., (2006:76) identifies four themes from the literature representing different rationales for the employment of mixed methods research designs.

They are:

1. Participant enrichment-with a view to optimizing the sample
2. Instrument fidelity- with a view to assessing the appropriateness of the instrument used
3. Treatment Integrity- assesses the fidelity of interventions
4. Significance of Enhancement- facilitates the collection of rich data and hence augments the interpretation of findings.

Creswell et al. (2006:6) write that reasons for the use of mixed methods include:

1. Validating the quantitative results with the voices of participants
2. Improve an intervention design
3. Develop a model to explain a process
4. Develop an instrument
5. Provide a means to examine a trend in a national study

Assessing the fidelity of interventions (Collins et al., 2006) and the desire to improve an intervention design (Cresswell et al., 2006) are appropriate rationales for this study. Similarly, augmenting the interpretation of the findings (Collins et al.) and validating the quantitative results with the voices of participants match the goals of this study (Creswell et al.).

Within mixed methods designs, there are several purposes mentioned in the literature to justify the use of mixed methods (Waysman et al., 1997:228).

Waysman et al.'s purposes for the use of mixed methods were explored for their suitability for application to this study. They are:

1. Expansion: described as seeking to extend range of the inquiry by using different methods for different inquiry components (Greene et al., 1989:29). There was no attempt to extend the range of the inquiry in this study.
2. Development: The development characteristic of mixed methodologies seeks to use the findings from the first method to help develop or inform

the second method (Onwuegbuzie et al., 2007:291). This purpose seemed to match the design of this study. Outcomes from participants' answers to the MRBQ questionnaire prior to the classroom intervention informed the development of the questions for the focus groups and individual interviews.

3. Initiation: described as seeking to discover contradiction and new perspectives of frameworks (Greene, 1989; Onwuegbuzie and Collins, 2007). This study did not aim to explore perspectives on frameworks.
4. Elaboration (complementarity): Complementary approaches seek elaboration and enhancement of outcomes with the rationale of increasing interpretability and the validity of results (Greene et al., 1989:258). The quantitative data collected from the survey questionnaire in this study was complemented by the qualitative data collected from the focus groups and individual interviews. Data from the focus groups and individual interviews was expected to help clarify and interpret data from the survey instrument (Waysman, 1997:234).
5. Corroboration (triangulation): Corroboration was not an appropriate purpose for this study as it seeks to confirm findings from one method by the use of another method. The reason for this was that the data was collected sequentially and not concurrently in this study. Findings from the first approach might have influenced those collected from the second approach, thereby positively biasing any comparisons (Onwuegbuzie and Collins, 2007:291).

3.4.1 Mixed Methodology Design

There are families of mixed methods research designs with several points of view described in the literature (Teddlie et al., 2009, Leech, 2009; Johnson et al., 2004; Hanson et al., 2005; De Lisle, 2011). Teddlie et al. argue that it is not possible to expect a complete menu of designs from which to choose the right one. This, they claim, is due to the designs' capacity to mutate into other forms (Teddlie et al., 2009:141). The choice of design for this study was made in keeping with literature on mixed methods designs at the time.

3.4.1.1 Typology of Mixed Methods Research

The challenge is always to select the most suitable mixed methods research design for a study (Leech and Onwuegbuzie, 2009:266). Studies describing mixed methods designs use different models. An example from the literature describes a nested relationship as one that infers that sample members selected for one phase of a study represent a subset of the participants chosen for another phase of the investigation (Hanson et al., 2005; Onwuegbuzie et al., 2007).

Leech et al. (2009:268), in an attempt to create an integrated typology of mixed methods designs wrote that these are represented as a function of three dimensions:

- (i) Level of mixing of quantitative and qualitative approaches (partially or fully).
- (ii) Time orientation (concurrent or sequential)
- (iii) Emphasis of approach (Equal status or dominant).

The three dimensions above provided a total of eight different types of mixed research designs (Leech and Onwuegbuzie, 2009:268). This typology seemed to be acceptable as a reasonable list of the current mixed research designs available in the literature. It is acknowledged by the author that the mixed methods paradigm is still in its adolescence and that it is evident that there is still a way to go (Leech, 2009:265).

Of the eight different types of research designs defined above by Leech et al., (2009:269) the fully mixed sequential dominant status design was the chosen design for this study. It was chosen for use in this study for the following reasons:

1. This study connected the quantitative and qualitative phases of the research process (Figure 1, Section 3.7.2).
2. The quantitative and qualitative phases occurred sequentially across the stages of this study.
3. The quantitative approach was the dominant approach used in this study. The data collected from the interviews used in this study were designed

to strengthen the outcomes from the quantitative data.

Four of the eight mixed method designs use partially mixed methods in the mixing dimension (Leech et al., 2009:268) and hence were inappropriate for use with this study.

The remaining three mixed method designs use fully mixed methods. Two of these employ phases that are carried out concurrently also making them unsuitable for this study. Sequential designs are described as those whose quantitative and qualitative phases occur one after the other which was the case for this study (Leech et al., 2009:268). Sequential designs are also considered to be appropriate for use in a study whose purpose is development and complementarity (Onwuegbuzie et al., 2007:292).

The third dimension defines the equal status or dominant status of the data (qualitative v quantitative). The dominant status lay with the quantitative phase.

The literature above and in Chapter 2 had sought to establish an appropriate design for this study.

This study was described as having a quasi-experimental mixed methods design. Cohen (2007:283) describes a quasi-experimental study as one that is similar to a classical 'controlled experiment' but where it is not possible to assign participants to random groups. Students participating in this study were assigned to particular groups by school management. The study consists of experimental and control groups. The experimental group is subject to the classroom intervention that is carried out in the mathematics classroom. The control group is taught in the usual manner. Battista et al. (2009:241) challenge the view that experimental/quasi experimental studies on their own provide the gold standard for addressing all the important questions in educational research. Fraenkel et al. (1993:16) argue researchers need tight controls for experiments to be successful. Researchers, they recommend, should try to control extraneous factors such as history and maturation due to the passage of time in studies through for example investigating interactions between independent variables. This advice was applied to this study through an investigation of any

possible interaction effect on gender by class (Table 21, Chapter 4). By including the collection of the qualitative data in this study it was thought possible for participants to explain their thinking on the changes that had been made to their learning environment and their reasons behind their beliefs.

3.5 Data Collection Strategies

There are many strategies used in the collection of data discussed in the literature that have been associated with different methodological approaches. Well known strategies include the use of questionnaires, interviews, focus groups, tests, observations and secondary data with quantitative, qualitative or mixed methods approaches (Teddlie and Tashakkori, 2009:206). Teddlie and Tashakkori (2009:209) write that 'methods should be mixed in a way that has complementary strengths and non-overlapping weaknesses'. One of the challenges, they claim, for mixed methods researchers is that researchers use two different sets of standards for assessing the data quality of the quantitative and qualitative strands. They write, for example, that data quality issues include, ethical considerations, determining the measurement validity of the questionnaires used and the use of triangulation techniques. There are two questions, Teddlie and Tashakkori claim, that researchers must be able to answer with respect to their chosen data collection strategies. These comprise, in the context of this study, firstly whether the scores obtained from the questionnaire and the data from the focus groups and individual interviews are true indicators of the beliefs students' hold about these constructs and secondly whether the methods used are capturing what was intended, in this instance the measurement of the constructs with respect to mathematics and its teaching and learning.

Goodchild and English (2002) in their series foreword wrote that mathematics education was a very eclectic discipline in the research methods used and promised a new focus on methods used in studies and the rationales behind them. Heyvaert et al. (2013:323) stated that a consensus on the critical appraisal of mixed methods studies is still lacking in the literature. They sought to develop a critical appraisal checklist for the purpose of evaluating methodological quality of mixed methods research articles.

3.5.1 Planned Methods

Choosing appropriate methods to use with this study was informed by examination of the data collection techniques used in the literature with mixed methods designs and with what was possible in the context of this study.

Three separate, specific approaches were selected for use with this study in keeping with the research questions asked. They included:

1. Questionnaire
2. Focus groups and
3. Individual interviews.

Other possible approaches that could have been used included observation, unobtrusive (secondary) measures and tests (Teddlie and Tashakkori, 2009:239). Observation was not possible as the practitioner in the classroom was also the researcher in this study. Unobtrusive measures were not considered appropriate to use with this study, as archived research data from similar classrooms was not available at the time. Tests were included to examine achievement over the period of the intervention but they were considered to be unsuitable as a measure to provide insight into students' beliefs about mathematics and its teaching and learning.

1. The Questionnaire

There are a number of reasons why researchers use surveys to collect data in the literature. These reasons include the ability to make comparisons between groups and the ability to quantify the attitudes and opinions of a particular population (Sukamolson, n.d). Questionnaires are thought to be good for measuring attitudes and beliefs (Teddlie and Tashakkori, 2009; Strange et al., 2003).

2. The decision to use a survey instrument in this study was for the following reasons:

- a. The benefits of using a survey instrument to collect data was that it provided a quick and easy method of collecting answers from a large number of questions with the full group of participants
- b. It provided the ability to collect and measure uniform data on the constructs held by participants that were of interest to this study.

3. Focus Groups:

Teddlie and Tashakkori (2009:227) describe focus groups as both an interview and an observational technique allowing access to the attitudes and experiences of participants. Complementary interviews were chosen as they offered the possibility of explanations from participants about the beliefs that they held about the constructs of interest to this study. Focus groups are used, in general, to find out participants views on the topic that is being researched. It was hoped that they might produce more information than might otherwise have been obtained and it allowed a large number of participants in the intervention class to be interviewed. It was also hoped that using a focus group might provide a safe environment for participants to speak more freely. The questions used with the focus groups in this study consisted of a number of questions developed from the questions in the survey questionnaire and the pilot study. The questions were open-ended allowing students to reveal what was on their minds (Krueger et al., 2000:57). Morgan (1996:130) asserts interaction in a group discussion is where the source of the data is located. Stewart et al., (2007:9) defines the variety of common approaches to focus group interviews. They are:

- i) Purpose of a focus group is to explore a shared experience among participants
- ii) Interviewer should be aware of how group dynamics affects contributions to discussions
- iii) Interviewer should be active in their listening, empathetic and open to participants.

The status of the focus groups used in this study is that of being an auxiliary method that generates data to aid interpretation of the quantitative results (Freeman, 2009:232). Hence, the focus groups provide the possibility of adding

valuable insight into the results collected from the survey questionnaire (Frey et al., 1991:175). Kitzinger (1994:107) argues that this data provides both the similarities and the differences between group participants. Some of the disadvantages of using focus groups include the possibility of one participant dominating discussions resulting in possible bias in the results by that person's views (Stewart et al., 2007:11). Open communication with students about the value of their input to the study is important to establish from the beginning of the study and during the interview stages (Green, 2011:61).

3. Individual Interviews:

The advantages of face-to-face interviews include:

- i) No significant time delay between question and answer and hence the answer given is more spontaneous.
- ii) Researcher can create a good ambience.

Standardization of the interviews is possible (Opdenakker, 2006:4).

Individual interviews are the third approach used in this study. Interviews were included in this study to encourage participants to provide a rich depth to discussions (Cohen, 2007:150). The use of individual interviews was intended to help offset the disadvantages of the focus groups and provided the possibility of confirming or otherwise the data collected from the other methods. The individual interviews took place at the end of the study. The questions asked in the individual interviews were developed from information shown in a preliminary analysis of the survey instrument, the transcripts of the focus groups and trends/ issues shown in the diary.

In this researcher's view, students were more likely to give a true answer to a question when no time was available for reflection. The participants were reassured that the answers that they provided to the questions would be kept confidential and not shared with any individuals in the school setting in which the study was undertaken. This reassurance included a reminder to say what they really believed when answering the questions asked. Disadvantages to face-to-face interviews included asking sensitive questions, particularly on the teacher's role in the classroom. Students may have been reluctant to tell the full truth to the author's face.

Face-to-face interviews are further defined in the literature by their particular characteristics. These include the approach used in the interviews (structured or unstructured), the questions asked of participants (open or closed).

Standardized open-ended interviews use the same basic questions in the same order and are worded in an open-ended format (Patton, 1987:116). This is the type of individual interview chosen for use with this study. The reason for this choice is because it enables the responses given by students from the survey questionnaire and the focus groups to be compared to responses from some of the same questions by individual students. Unstructured or semi-structured interviews would be unsuitable for use with this study for two reasons. The individual interviews are designed to aid the interpretation of the outcomes from the survey instrument used. A standardized open-ended interview is the most useful method for grouping answers from different students in order to analyse different perspectives on critical issues (Green, 2011:69).

3.5.2 Participants

Action Research was the chosen methodology for this study as discussed earlier in this chapter. The author's own practice was investigated. Hence, the sample used is one of convenience making statistical generalization impossible as the sample was not representative of any population. Convenience samples are described as either captive or volunteer in the literature (Teddlie and Tashakkori, 2009; Cohen 2007).

The participants invited to partake in this study attended a secondary community school with six different year groups comprising of a total of approximately 500 students. The school was chosen for use because of it being the workplace of the author. The sample can be described as captive, as the class who experienced the classroom intervention was taught mathematics by the author and had not volunteered to be part of the intervention class.

Individual students who may not have wished to participate in this study would have found it difficult not to do so. A number of individual students did volunteer to take part in the focus and individual interviews and appeared to be happy to do so.

The sample consisted of the second year secondary students, numbering 81 in total. They were divided into four classes. Class 1 experienced the classroom intervention and experienced the changed learning environment. All classes, experimental and control, in the year group completed the survey questionnaire both before and after the intervention was completed in the classroom.

There are various recommendations in the literature recommending the optimal number taking part in focus groups. Some studies consider four to be the minimum number that should be considered (Morgan, 1988:43). Hatch (2002:135) advises that most texts recommend 6-12 as a group size and that small groups do provide space for individuals to go more deeply into a topic. The students in this study were working in groups of 3-4 as they discussed mathematical topics. It would seem to be reasonable to have used a number of these same groups of students to take part in the focus groups interviews together. The implications for the generalizability of outcomes, for this study using this sample, are discussed in Chapter 5.

3.5.2.1 Historical and School Context

As discussed in Section 1.2, the need to urgently reform second-level mathematics education in Ireland in 2005 arose from issues such as the disenchantment with the overly abstract focus of the 'new mathematics' curricular culture, business communities' anxiety about students' limited capacity to apply knowledge to new contexts and recent PISA results (Conway and Sloane, 2006:202). At the time this study was carried out (September 2010-June 2011) the Irish educational landscape in relation to mathematics education at second-level this study was carried out was one of significant change and was characterized by divisions amongst the stakeholders. Media sources at the time of this study including newspapers, radio and television programmes focused on the new curriculum. They discussed the merits and challenges of new Project Maths with parents, students and education experts. At a meeting attended by the author there was conflict between some Irish mathematics teachers associations and the National Council for Curriculum and Assessment (NCCA) who devised the new syllabuses on behalf of the government's Department of Education and Skills.

The Project Maths initiative was implemented in the first instance in 24 pilot schools followed by a phased implementation to all other schools in Ireland. This study coincided with the introduction into all schools of the first two strands (of five) into all schools. The implementation of the Project Maths new curriculum caused controversy with teachers, students and parents complaining in particular that its introduction to the 5th year Leaving Certificate students (first year of senior cycle, age 17 approximately) caused significant stress. A study on the impact of the introduction of new syllabus to senior cycle in the pilot schools refers to the leaving certificate students being stressed, afraid and lacking in confidence in their study of mathematics (NCCA, 2012:17). Although there was widespread acceptance that reform was needed asking senior students to absorb a new way of learning mathematics over their final two years at second-level that ended in a high stakes examination was considered unacceptable and highly stressful for all. Leaving Certificate students did find the change in learning approach to be challenging (Jeffes et al., 2013:32). Teachers also expressed views at association meetings arguing that the new course was dumbing down content and that it would not improve achievement scores. Some teachers expressed a reluctance to change what was happening in their mathematics classrooms and a reluctance to engage with new materials citing the time challenges involved in implementing change at a time of significant personnel and resource cuts to schools and to teachers' salaries. Discussions amongst teachers in the staffroom of the school involved in this study were at times very negative towards the changes being made to the mathematics curriculum. A more recently published report indicated frequent uses of activities associated with the revised syllabuses but more traditional approaches also continue to be widespread (Jeffes et al., 2013:21).

3.5.2.2 Sample Composition

The school in which the study was undertaken is designated Disadvantaged within the DEIS scheme (Delivering Equality of Opportunity in Schools) in Ireland. The school had six separate years ranging from first year to sixth year. The students' age ranges are from 12-18 years. In the first three years the students study for the Junior Certificate in Education at the end of which they take a state examination. These students were aged between 12-15 years. The

second year students were chosen, as participants for this study, by the author for the following reasons:

1. The students in second year were aged 13 -14 years old. Younger students were considered to be more suitable participants for the study for two reasons. The beliefs held by the younger students about mathematics and its teaching and learning may be less entrenched and hence may be more open to change. Students at age 12, irrespective of gender or nationality were a half a point more positive than students at age 15 in their beliefs about learning mathematics (Andrews et al., 2007:214).
2. Changes were being introduced countrywide in Ireland to the teaching of mathematics at the time. Project Maths, as the new curriculum has been called, is a substantial curriculum innovation in second-level mathematics (N.C.C.A., 2008). The second year students were not to be included in the implementation of these changes until 2013, after the fieldwork had been completed. In attempting to measure changes in students' beliefs over a period of time for this study it was considered that introducing additional variables such as involvement in Project Maths might make it more difficult to assess outcomes from this study, so this ruled out the use of other year groups.

The second year students were divided into classes as follows:

Classes	Number of Participants
Class 1	27
Class 2	27
Class 3	17
Class 4	10
Total	81

Of the 81 second-year students in the school 78 took part in this study. The breakdown of the classes by gender was as follows:

Table 4		
Classes	Number of Boys	Number of Girls
Experimental (Class 1)	11	14
Control (Class 2)	15	11
Control (Class 3)	10	7
Control (Class 4)	7	3
Total	43	35

The students attending the school, in the main, had transferred from five or six local primary schools. The year group was divided into classes on the basis of the standardized Drumcondra mathematics and literacy tests of achievement (Drumcondra, 2010). This test gave an indication of the standards achieved by students in mathematics in the primary school. However, in a small number of these schools, the primary syllabus was not completely covered making these scores somewhat unreliable. Class lists of students were routinely adjusted in the school after achievement in the December school tests at the end of term one in first year secondary were examined. The December scores arose from school-based tests designed by teachers and completed prior to Christmas. The scores achieved in the Drumcondra tests did still provide an overall picture of the sample group taking part in this study in comparison to the national scores at the time. The tests were given to students in March 2009 prior to their entry to the school. Student scores (some students were absent) from the mathematics test are indicated in the table below:

Drumcondra Standardized Mathematics Achievement Test		Frequency	Percentage
Lowest 20%		31	39.7
Between 20 and 40%		25	32.1
Between 40 and 60%		8	10.3
Between 60 and 80%		2	2.6
Top 20%		1	1.2
Total		67	85.9
Missing		11	14.1
Total		78	100.0

All students were expected to study the ordinary level mathematics course for the school year in which the study was undertaken, the second year of the Junior Certificate cycle. Within this framework school management expected a minimum of one class to undertake the higher-level mathematics course, two groups were expected to undertake the ordinary level course and the small class four would follow the foundation level course Junior Certificate course. Outcomes from the Drumcondra mathematics test for the participants in this study indicated that 82.1% of the participants in this study lay within the 0th - 60th percentile nationwide and 3.9% in the 60-100th percentile. Comparing these figures from the table above with national statistics from Table 6 below on percentages of candidates studying each of the available levels for examination indicate differences.

Comparison with National Statistics		
Percentages (aiming at different levels of mathematics)	Drumcondra Scores (All Participants at the time of this study)	National Statistics who took these levels in state examination (SEC, 2012)
Higher Level	33.333%	48.07%
Ordinary Level	54.333%	44.68%
Foundation Level	12.333%	7.25%

The percentages given for the participants studying each level in the table above indicates those placed in these second year classes by management policies. The number of participants given in Table 6, who actually did sit the higher level papers in June 2012, was lower than those given. The sample was

of lower ability than the national average, as shown above. The lower proportion of those actually taking the higher level suggests that the work was challenging for those participants.

The school would seem to be generally well-respected by local communities with full enrollment for the past number of years and a waiting list in operation. Parents had a number of schools to choose from in the area including 3 mixed community schools, 2 single sex girls schools, 2 single sex boys schools. The school operated a strict first come first served policy in enrolling students for the academic year.

All of the classes were studying the Junior Certificate Ordinary Level syllabus, although at different paces for the academic year. Class 1 was expected to complete the ordinary level syllabus over the course of the year, Class 2 a large part of the syllabus, Class 3 was expected to make adequate progress with the content and Class 4 was expected to make slow progress. Entry expectations, on entering secondary on the level of mathematics that would be studied, by students and their parents were most often ordinary level standard. This was based on the author's long experience of teaching these students. A number of topics from the syllabus were chosen for study at the beginning of an academic year at the teacher planning meetings. Within these general guidelines individual teachers were free to plan their own individual teaching periods. Class 1 was chosen to be the experimental class for this study as it was reasonably sized and it had been allocated to the researcher's teaching timetable for that academic year. The remaining three classes formed the control classes for this study.

3.5.3 The Instruments

Considerable thought was given to choosing the survey instrument used with this study. In the first instance an investigation from the literature was conducted to ascertain what scales to measure affect were available for use and what constructs and/or sub-constructs of affect did they aim to measure. It was important to choose an instrument whose validity could be defended and whose reliability was sound (Leder and Forgasz, 2002:98). A valid measure is one that measures the construct it is expected to measure. Scales are

measured for validity in the literature. Factor analysis can be used to construct a questionnaire that measures an underlying variable and hence it would appear to support the validity of the scale (Field, 2005:619).

What is common amongst all of these scales is that the measurements that were developed by the research community were to a large extent developed for use with mathematics students and all were examined for their reliability and validity. Nunnally in Chamberlin (2010:171) regards a 0.80+ internal consistency level as one that provides a sound instrument for the mathematics education community. More recently a maximum level of 0.90 is recommended as higher may indicate that some items in the scale are redundant (Tavakol et al., 2011:54). Many of the self-report questionnaires would be constructed to use a Likert-scale format.

Instruments used to measure affect include:

1. Observations in mathematics' classrooms usually carried out by researchers
2. Checklists
3. Questionnaires using Likert scales
4. Interviews of individuals
5. Focus groups
6. Collected dialogue, reflections, journal entries (Leder and Forgasz, 2002:101).

The most common instruments used in studies on measuring beliefs and attitudes showed that questionnaires and interviews and journal entries predominated (Leder and Forgasz , 2006:411). One criticism of the instruments developed is that they are rarely created for the individual teacher to use in the classroom and a view is expressed that affective instruments should also be easy to implement (Chamberlin, 2010:177).

Table 7

Instruments developed in the literature to measure beliefs/attitudes		
Scale	Instruments	Subscales
Fennema-Sherman Mathematics Attitude Scale (1976)	Measured Attitudes	
Attitudes Towards Mathematics Inventory (ATMI) (Tapia, 2004)	Measured Attitudes -5-point Likert Scale	Self-confidence, Value (Beliefs), Enjoyment, Motivation
Academic Emotions Questionnaire (AEQ-M) (Frenzel, 2007)	Measured Emotions -5-point Likert Scale	Measured enjoyment, anxiety, anger and boredom in relation to subject mathematics
Experience Sampling Method (ESM) (Leder and Forgasz, 2002:105)	Charts snapshots of individuals' Daily Behaviours and concomitant affects using Diary	
Indiana Mathematics Beliefs Scales and Student Interview Instrument (Kloosterman, 2002:247)	5-point Likert Scale (Beliefs Scale) 51 questions in Student Interview Instrument	Motivation, Self-confidence, perceptions of ability, goal orientation, mathematical content, assessment practices and expectations of teachers, nature of mathematics
Mathematics Related Beliefs Scale (MRBQ) (Andrews et al., 2007:211)	6-Point Likert Scale	Students beliefs about teacher's role in classroom Students beliefs about their own competence in mathematics Beliefs about relevance of mathematics. Students' beliefs about mathematics as an inaccessible subject.
Indiana Mathematics Beliefs Scales and Usefulness of Mathematics Scale (Fennema and Sherman Scale) Mason (2003:73)	5-Point Likert Scale followed by Individual Interviews	

Later research has questioned the validity, reliability and integrity of the Fennema-Sherman Mathematics Attitude Scale scores (Tapia, 2004:1). Chamberlin (2010:173) cautions that estimates of reliability and validity of scales may become less stable over several decades due to for example changes in word meanings. The Fennema-Sherman scale was used

extensively for many years to measure students' attitudes to mathematics. This makes the scale unsuitable for use with this study due to the passage of time as validity and reliability of an instrument should be established for the particular group on which the instrument is used. More recently the scale 'Attitudes Towards Mathematics Inventory (ATMI)' was designed to measure students' attitudes to mathematics that the researcher believed must be a shorter instrument than the 108 items in the Fennema-Sherman scale (Tapia, 2004:19). Four sub-scales were included in the ATMI scale and they included self-confidence, value, enjoyment and motivation where the value category was designed to measure students' beliefs on the relevance of mathematics to their lives now and in the future (Tapia, 2004:17). The ATMI scale was designed to be used with adolescents (Tapia, 2004:21). Only a small section of this scale was suitable for the measurement of students' beliefs and hence was not considered suitable for use with this study.

The Academic Emotions (AEQ) was used alongside four self-developed scales measuring characteristics of the classroom environment. The authors (Frenzel et al., 2007:483) argued that students' perceived learning environments are significantly related to emotional and social outcomes. The study conjectured that teachers were able to influence and mould students' value beliefs in the subjects that they taught describing the teacher's enthusiasm for their subject as emotional contagion (Frenzel et al., 2007:493).

The Experience Sampling Method (ESM), the authors claims, allows insight into motivations, attitudes and beliefs associated with an individual's behaviours (Leder and Forgasz, 2002:105).

Kloosterman (2002:262) combined the collection of data through the use of a questionnaire and student interviews. In analyzing the outcomes from his study Kloosterman maintains that the interview instrument proved to be more effective than Likert scales in assessing students' beliefs, attitudes and overall motivation.

The growing body of research indicating the importance of students' beliefs about mathematics and its learning prompted the development of the Mathematics-Related Beliefs Questionnaire (MRBQ) (Andrews et al., 2007:211).

The development of the scale was also prompted by the lack of integration of different categories of beliefs in previous studies (Andrews et al., 2007:211). An instrument was developed consisting of a number of factors and sub-factors to measure students' mathematics-related beliefs. The focus in the MRBQ scale was on belief systems, relevant categories of beliefs and the way they relate to each other (Op't Eynde and De Corte, 2003). This instrument was further developed and refined and yielded four conceptually different and reliable scales (Diego-Mantecon et al., 2007: 229). The questionnaire uses a 6-point likert scale to assess students' beliefs.

The MRBQ scale was developed at the University of Leuven, Belgium by Op 't Eynde and de Corte in 2003 (Diego-Mantecon et al., 2007:229). The questionnaire was developed for use with fourteen year old Flemish students which suggests it could be appropriate for use with this current study. The instrument was refined subsequently and was shown to yield four reliable factors, each with at least two reliable sub-factors. The study to test the instrument for transferability between cultures was carried out in England, Spain, Northern Ireland and Slovakia which resulted in the conclusion that the scale was sensitive to context (Andrews et al., 2007:209). The MRBQ has four factors as identified by the results of a principal components analysis which refines and reduces items in a scale to form a smaller number of coherent factors (Pallant, 2007:179). They are:

Beliefs about the role of their own teacher.

Beliefs about their own competence in mathematics.

Beliefs in the relevance of mathematics.

Beliefs in mathematics as an inaccessible subject.

Cronbach's alphas for these factors were 0.92, 0.89, 0.65 and 0.69 respectively (Op 't Eynde and Hannula, 2006:123). Cronbach's alpha values, usually between 0 and 1 (though occasionally negative), give the average correlation among all the items that make up the scale (Pallant, 2007:6). The higher values indicate greater reliability.

A 6-point scale was used as the scale developers believed that forcing a positive or a negative answer from respondents would lead to better quality data (Diego-Manetcon et al., 2007:3). A 6-point Likert scale is used in the questionnaire with responses to questions scoring from strongly agree (1) to strongly disagree (6).

The 6 points are equidistant from one another. The total for each of the factors is the sum of each of the items scores. Questions are generally positively worded except for the last category (Beliefs in mathematics as an inaccessible subject) being negatively worded. An example of a positively worded item is 'My teacher tries to make the mathematics lessons interesting' (Beliefs about the role of their own teacher). In negatively worded items the scoring is reverse scoring from strongly agree (1) to strongly disagree (6). An example of a negatively worded item is 'If I cannot solve a mathematics problem I quit trying' (Beliefs in mathematics as an inaccessible subject).

3.6 The Pilot Study

A pilot study was carried out in the autumn of 2009. The reasons for carrying out the pilot study were four-fold:

1. As a practice session prior to carrying out the main study
2. To establish the suitability of the instruments for use with the main study.
3. To test the characteristics of the new learning environment as a support for the development of students' more positive beliefs about mathematics and its teaching and learning
4. To assess the proposed data analysis techniques.

The pilot study was carried out with a sample of 37 students. This included 22 students in the intervention group and 15 students in the control group. A new learning environment was introduced to the intervention group in the mathematics classroom. A short intervention was carried out involving a sequence of 6 separate mathematics' class periods. Each class period consisted of a single class period of duration of 40 minutes carried out with the intervention group. The short time used in carrying out the intervention was all that was possible at that time due to local circumstances. The sample was

chosen from a population of convenience in the school available to the researcher.

The framework of the new learning environment was based on Verschaffel et al.'s (2000:97) model. The new learning environment gave prominence to:

Addressing beliefs, especially maladapted or hindering ones.

Rich discussion in the classroom.

An inquiry/discovery atmosphere in the classroom with actively participating students.

Encouragement of reflective practice.

Instruments (Pilot Study)

1. The MRBQ scale (Diego-Mantecon et al. (2007:233)
2. Student Interview Instrument (Kloosterman, 2002:266).
3. Classroom conversations audio recorded.

Data Analysis Techniques (Pilot Study)

Statistical software SPSS was used to conduct the analyses of the data collected from the questionnaire. Cronbach's alpha tests carried out on the data collected from the MRBQ scale before the changes were made to the learning environment showed:

Factor	Cronbach's Alpha Score
Teacher	0.684
Competence	0.821
Relevance	0.856
Inaccessible Subject	0.826

The alpha scores indicated that the scales were exceptionally reliable. Tests conducted on the data included mean and standard deviations for each of the factors of the MRBQ scale both before and after the learning environment was changed.

The 50 questions asked in the Student Interview Instrument (Kloosterman, 2002) and the data collected from 3 students were matched to the questions asked in the MRBQ scale. Relevant data that enhanced the data collected from the MRBQ scale was examined.

Data collected from the recorded conversations was transcribed and analysed for their ability to provide additional information on data about students' beliefs in the MRBQ scale.

Outcomes from the pilot study indicated no significant change in either the competence or relevance component of the scale for either the intervention or control groups. There were significant differences in the teacher's role component after the intervention but this applied to both the intervention and control groups. The fourth component mathematics as an inaccessible subject showed significantly more negative scores after the intervention. At the same time there was no significant change in the control group.

Lesson learnt from Pilot Study

The MRBQ scale was found to a suitable instrument to use with the main study. The Student Interview Instrument was found to be unsuitable for use due to the small number of questions in the instrument that matched those in the MRBQ scale. The data collected from the recorded conversations had also been poor. Hence, the ability of the Interview Instrument and the Conversations could not fulfill their expected purpose, which had been expected to enhance the data collected from the questionnaire.

Expert feedback from the pilot study recommended the Student Interview Instrument and the classroom conversations should be replaced by a number of focus groups and individual interviews (Ernest, 2010). This would allow groups of students to discuss a number of pertinent questions from the scale on the four belief components.

A longer period of time was recommended for the classroom experiment (Oldham, 2010). Recommendations also included further work was required on the reporting of the results (Ernest, 2010; Oldham, 2010). A particular issue with

the inaccessibility belief component of the scale was highlighted indicating the questions were negatively worded when compared to the other three belief components. These questions required reverse coding for the full study to enable easy comparison of outcomes with the other three components.

3.7 Procedure:

This section of the chapter covers the proposed implementation of the classroom intervention based on the discussions above. It also covers the actual administration of the survey, the selection of the focus groups and individuals to be interviewed, the timeline of the phases of the intervention etc.. The implementation of the focus groups and interviews are described and the choice of data analysis methods used in this study.

3.7.1 Ethical Considerations

There are serious ethical considerations in the carrying out of research and particularly so with young children. According to Mitchell (2004:1419), there are three components in practitioner research that must be considered on ethical grounds. They are:

1. The Intervention
2. Data Collection
3. Data Analysis and Reporting

For this study consent from students and parents for the intervention in the classroom to take place was considered to be inappropriate as it belonged to what Mitchell (2004:1420) calls the zone of accepted practice. This he describes as the teacher constantly responding to learning outcomes by refining some facets of teaching on an ongoing basis. Consent issues, in this instance, had already been ceded to schools and teachers. Mitchell (2004:1420) does identify one risk that can be real in any teaching innovation and that is curriculum coverage may be lost. This challenge is acknowledged by this study. The practitioner did ensure that a reasonable quantity of the required syllabus was covered to a reasonable depth during the academic year.

It was necessary to seek permission from the Principal and Board of Management of the school to undertake the research. Preparations for the administration of the survey instrument, focus groups and interview instruments began with a letter to the Board of Management of the Community School seeking permission to undertake the study with the students (Appendix C). The Board with the Principal (Head) acting as secretary to the Board, meets once a month during the academic year. Permission to conduct the study with the second year group was sought and granted in September 2010.

The second component above was that of ethical considerations in relation to data collection procedures. The data collection from the intervention involved the researcher keeping a diary of classroom events. Reporting of these events involved making sure that no student would be identifiable. To facilitate this, students were assigned an individual number that was used in the reporting of the study. This preserved student anonymity.

Following a discussion with management, permission from parents to engage their children in the research process was sought. Letters advising of the commencement of the study were sent to the parents of all students in the intervention and control groups. The use of the letter was in keeping with normal school communication policies. The letter to the intervention group and the control group advised parents that students would be invited to complete questionnaires about their learning of mathematics. The letter emphasized that participation in the study was completely voluntary. Parents were invited to contact the author for further detail on the study should they wish to do so. A letter sent to parents of the students in the intervention group contained an additional paragraph, advising parents of the focus group discussions that would take place later in the academic year. No parent contacted the school seeking further information. A parent-teacher meeting took place in January 2011 and parents, on an individual basis, were invited to discuss the research. Only one student's parents were interested in the research. They were positive about the changes that had been made to the learning environment and were happy for their son to be involved in the interviews.

Permission for students to be interviewed individually and in the focus groups was obtained from parents by phone. The students had been invited to volunteer to take part in the interviews and had agreed to do so prior to the phone calls to their parents. Students were asked to explain to their parents what the involvement entailed and to expect a phone-call seeking permission. Students who took part in the focus group and individual interviews were reminded verbally that it was voluntary. All were happy to be involved. Direct contact with parents by phone was normal practice within the school. Parents were reminded of the voluntary nature of student involvement in the interviews. All were happy for their children to be involved. It is difficult to know if parents gave permission because they felt that they did not have a choice. However, other situations arising in the school would seem to indicate that if the students were happy to volunteer their parents were happy to give permission. In relation to confidentiality, students were advised that information given in the interviews would not be identified with an individual student.

The teachers of the three other groups in the year, who participated by completing the survey questionnaire before the intervention was carried out and afterwards, were happy to be involved in this manner. A more ideal situation would have been for a teaching colleague rather than myself to undertake the intervention in the classroom. Individual research can lack benefits from working with colleagues such as the opportunity it provides for dialogue and sharing are less likely to occur (Hewitt et al., 2005:3). Colleagues were happy to be involved in distributing the surveys to their students. My colleagues were not prepared to have any further involvement in the intervention in the classroom. This was possibly due to the work involved and/or a lack of information on current methodologies used in mathematics' classrooms.

3.7.2 Administration of the Data collection:

A visual model of the process carried out in this study in collecting the data for this study is shown in figure 1 below. The value of providing a visual model of procedures in mixed methods research is recognized in the literature (Creswell et al., 2007; Ivankova et al., 2006; Morse, 1991).

(The Drumcondra standardised testing was a routine procedure carried out yearly. Data was available to this study)

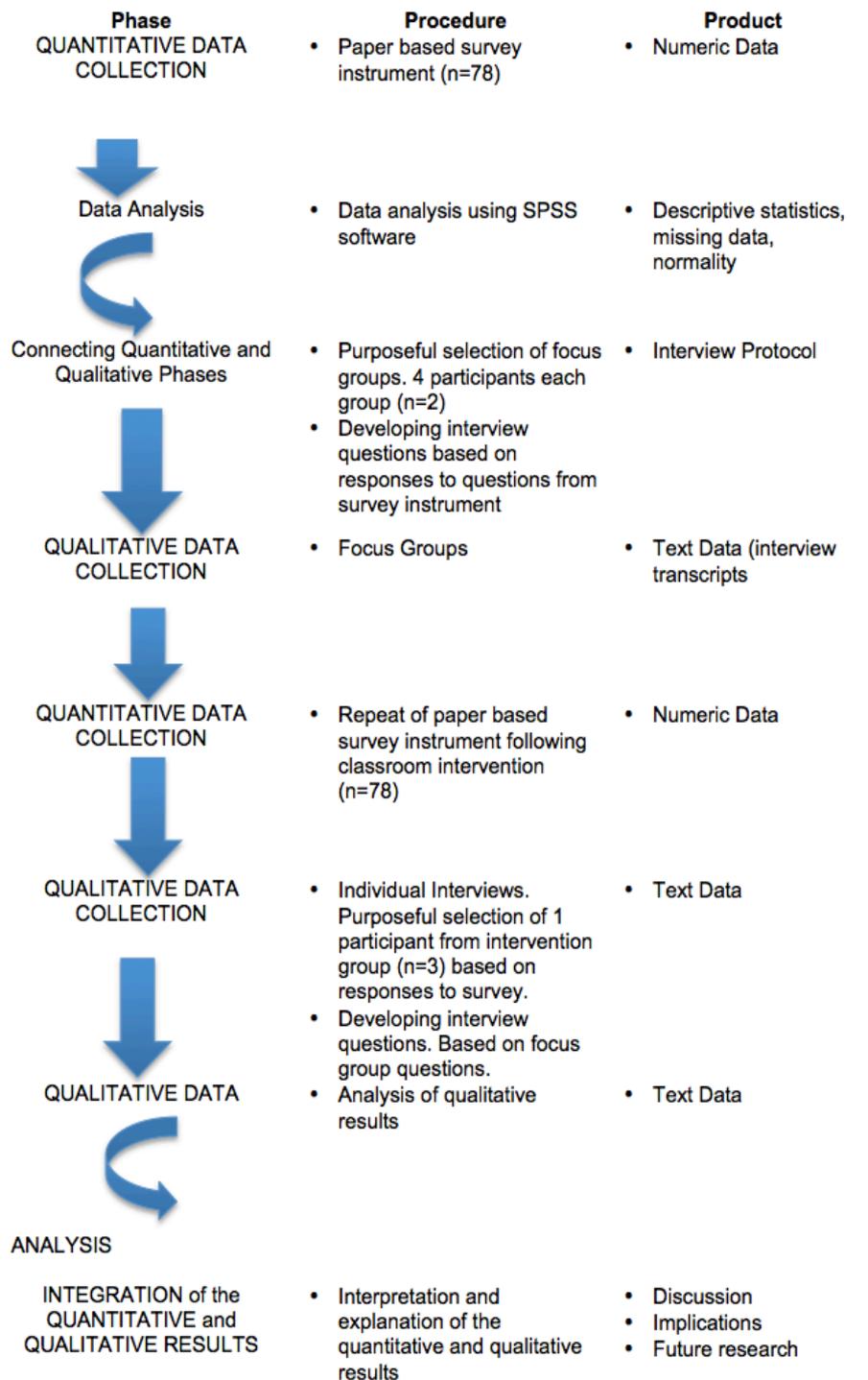


Figure 1: Visual Model for Mixed Methods Sequential Explanatory Design Procedures (Adapted from Ivankova et al., 2006:16)

The data was collected sequentially as per the following timeline:

Phase	Instrument	Timeline
1	MRBQ Survey Instrument completed	November 2 nd 2010
	Classroom Intervention	Late November- early December, 2010
2	Focus Groups	January 2011
	Classroom Intervention cont.	January-May 1 st , 2011
3	MRBQ Survey Instrument repeated	May 2 nd , 2011
4	Focus Groups Repeated	May, 2011
	Individual Interviews	May, 2011

The academic year spanned late August 2010 to June 1st 2011. There were challenges to completing the study in the short school year available. As a result there were time constraints in the implementation of the process. The learning environment for the months of September and October used with students in all of the mathematics classrooms, including the experimental class, was traditional, focusing on procedural learning with direct teaching of content by the teacher using whiteboard and standard textbook. This was to allow fair comparison to be made between the traditional learning environment and the Intervention learning environment. The classroom intervention began November 20th, 2010 and continued until May 1st 2011. This necessitated the first focus group interviews taking place a few weeks after the intervention had been started in the classroom. The time allocated to the intervention was influenced by similar intervention studies in the literature (Mason, 2004; De Corte et al., 2004; Higgins, 1997). The preparation for the focus groups included the analysis of the survey instrument followed by the preparation of appropriate questions. This preparation took some time.

Phase 1: The first phase of the study used the Mathematics Related Beliefs Questionnaire (Appendix A) survey. The details of the constructs measured are listed in Section 3.5.3 above. The scale consists of a total of 57 questions over four categories.

The survey was distributed to the four classes of students through their mathematics teachers. The teachers had agreed to begin the distribution on the same day at the same time on November 2nd, 2010. The questionnaires were completed by each of the groups on that day and stored securely. The

information from the questionnaires was not shared with colleagues thus maintaining confidentiality. All students were invited to complete the questionnaire voluntarily. Absent students were excluded from the study.

Data from the survey questionnaire was then entered into SPSS in preparation for analysis. Data provided by each of the participants were entered into the database (Appendix) using the following coding of variables:

Class	Student ID Code	Gender	Drumcondra Standardized Numeracy Score
1=Experimental	101- 125	Male=1	0=Missing
2=Control	201-225	Female=2	1= Lowest 20%
3=Control	301-317		2= Between 20 to 40%
4=Control	401-417		3= >40% to 60%
			4=>60% to 80%
			5= Top 20%

The Drumcondra scores are produced as raw scores and percentile scores. The percentile scores were chosen to include as they are easily recognizable to readers.

Each of the factors in the questionnaire had a number of individual questions that needed to be answered by participants e.g. Teacher’s role. The questions in the survey were given to students to complete on two occasions: before and after the classroom intervention. The variables were coded in SPSS as follows:

Survey Questions Data Prior to classroom Intervention	
Survey Questions Data Prior to classroom Intervention	Variable Name used SPSS
Teacher’s Role Factor	TeacherQ1 to TeacherQ14
Competence Factor	CompetenceQ15 to CompetenceQ28
Relevance Factor	RelevanceQ29 to RelevanceQ44
Inaccessible Factor	InaccessibleQ45 to InaccessibleQ57
Inaccessible Factor Negatively Worded Questions Re-Coded	InaccessibleQ45R to InaccessibleQ57R
Classroom Intervention	
Survey Questions Data After classroom Intervention	Variable Name used SPSS
Variables for Questions are entered into database with ‘after’ added to name used above.	e.g. TeacherQ1 Becomes TeacherQ1after etc.

Initial tests were made to check the data for accuracy using descriptive statistics. The data from the MRBQ questionnaire was then analysed using tests described below.

Phase 2: Focus Groups

Preparation of the questions to be used with the focus groups considered the content, quantity and language used in them. The questions were based on the outcomes from the 57 questions in the MRBQ survey questionnaire used in the first phase of the study. A total of 5 questions from each of the factors were included making a total of 20 questions in all. The questions were designed to probe beyond the questions asked in the MRBQ scale. The focus group interviews took place within a single class period of duration forty-minutes during the school day. This was considered to be a reasonable time period for students aged 13-14 years of age to engage in discussions.

A number of the questions from the MRBQ questionnaire were combined to make a single question in the interviews. An example of this is:

MRBQ Questionnaire (Appendix)	Focus Groups Questions (Appendix)
Q.4 My teacher appreciates it when we try hard, even if our results are not so good. Q.12 My teacher thinks mistakes are okay as long as we are learning from them.	Does your teacher appreciate it when you try hard, even if your results are not so good. Are mistakes ok when you are trying to solve a problem in your own way? (Why/ why not? Give an example)

The language used in some of the questions was altered to aid students' understanding of the question. An example of this is Q. 42 of the survey instrument that states 'I think it is important to learn different strategies for solving the same problem?' The language in the question was changed to 'Do you think that it is important to learn different approaches for solving the same problem?'

The format adopted for use in the groups and individual interviews was designed to make students consider each individual question one at a time. Stimulating discussion was addressed by prompts 'Why/Why not?' This did not include the questions on teacher's perception, as the participants would not

know why the teacher had done or not done something. Also, included were requests to students to 'give an example' in order to illustrate and allow them to expand upon their answers. An open question at the end of the interviews was also included to encourage students to give any other views they might wish to include in the discussion. In the beginning of each of the focus group interviews the participants were reminded about the purpose of the study and the confidentiality of the group's discussions. The group was advised that their discussion would be recorded in audio. The participants did not appear to have a problem with this. The participants were thanked and advised that each of their contributions was welcomed. The students were then asked to say exactly what they thought in answer to the questions on the role of the teacher in the classroom.

The focus groups had not been a part of the original pilot study (discussed earlier in this chapter) that had taken place in the previous academic year. A focus group pilot study was carried out to test the suitability of the questions and the language used in them. Feasibility is a concern in many studies (Hatch, 2002:51). The interview was audio recorded in the mathematics classroom. This provided a familiar and quiet environment and enabled the discussions to be audio recorded satisfactorily.

The following instructions were given to the group who had volunteered to take part in the pilot. 'Read the questions one at a time and say what you believe is true. If you are not sure what the question is about you can ask me to explain it to you'. All of the focus group interviews, including the pilot, were transcribed on the same evening that they had taken place (Appendix B). The majority of the questions appeared to have been understood by the participants. A small number of changes were made to the wording of the original questions. An example of this is:

Wording used in the Focus Group Pilot Study	Wording used in the Focus Groups
Q. Do you think mathematics has any relevance to your life?	Q. Do you think mathematics has any use in your life?

The role of the moderator is a crucial one in determining the success or otherwise of the focus groups. Too much control of discussions or too little may

stifle discussions or individuals may led to biased findings (Hatch, 2002:132). The goal was to make the students feel responsible for managing the discussions themselves and hence produce data for this study (Morgan, 1996:49). Reliance was placed on the interaction within the groups based on the questions provided (Morgan, 1996:2). The questions were given to the groups and a request was made for one student to volunteer to read out loud the first set of questions to be discussed. As moderator, the aim was to ensure that every individual was given the same chance to give their personal view on the questions and each participant was also encouraged to generate discussion concentrated on the questions asked (Hatch, 2002:132).

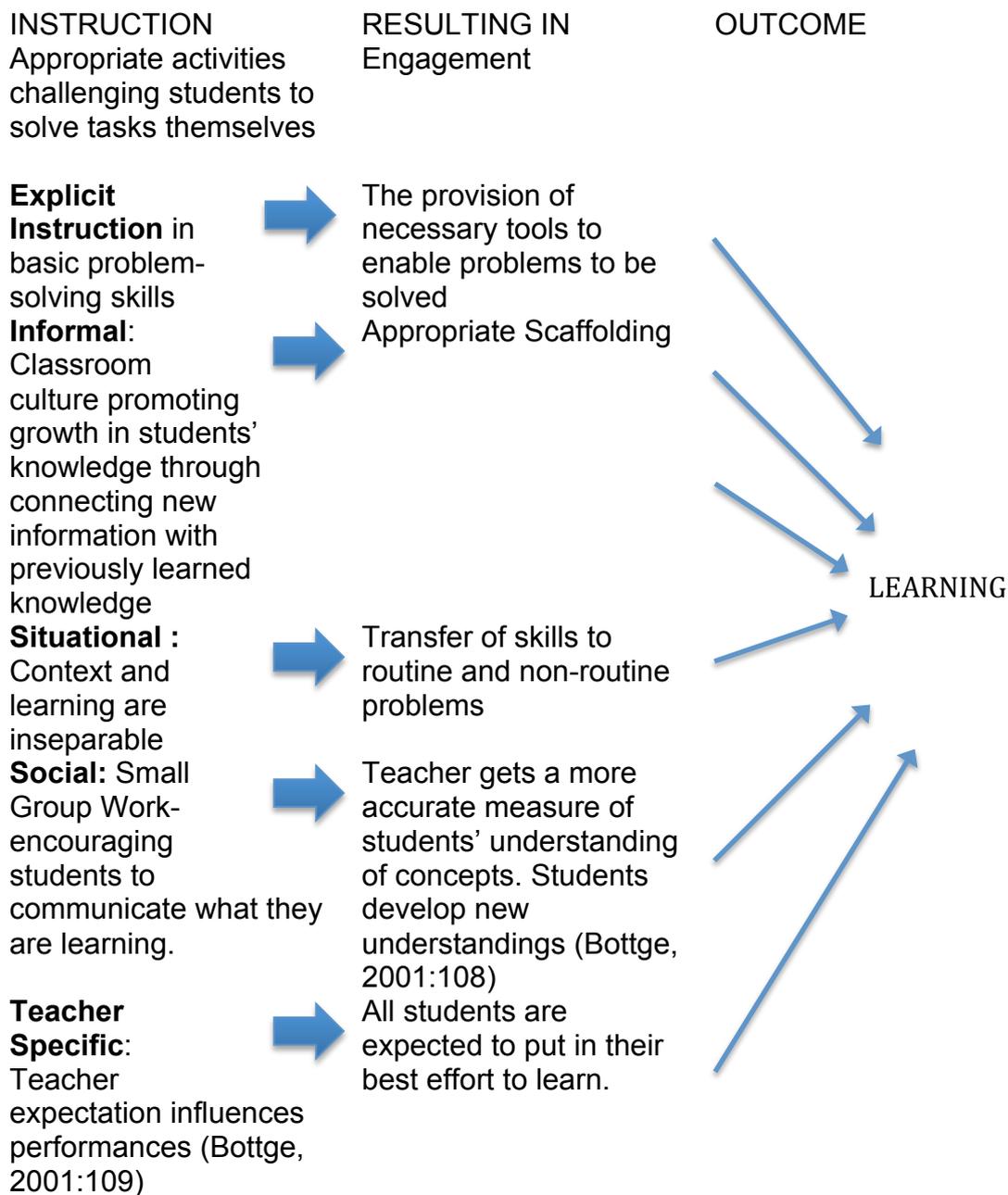
Following the completion of the pilot focus group and subsequent changes to the questions two groups were invited to take part in discussions. They consisted of a group of four boys and a group of four girls (Appendix B). The classes had been divided into small groups for the purposes of collaborative work solving problems in the classroom. These two groups were invited to take part, as they appeared to be a reasonable representation of the intervention class. Ethical considerations were as described earlier in this chapter.

3.8 The Intervention (Phase 2)

Andrews et al. (2000:259) recommended teachers should use classroom interventions with a view to enhancing performance. Much research has been carried out into mathematical instruction on what might help students' understanding of concepts. Methods such as direct teacher explanations, strategy instruction are but two found to be successful in improving mathematics skills in students (Bottge, 2001:102). Bottge's (2001:104) key model of problem solving was adapted for use with the classroom intervention in this study (Figure 2). The literature review (Chapter 2) of this study defined the characteristics of the learning environment that should be employed in the classroom. The characteristics were:

1. Active Learning Methodologies
2. Explicit instruction in problem-solving skills
3. Students gradually take responsibility for their own learning
4. Use of routine and non-routine problems
5. Small group work introduced
6. Students made aware of their beliefs

7. A focus on effort in the classroom



Adapted from Key Model of Problem Solving (Bottge, 2001:104)

Figure 2

1. In this study, Active Learning Methodologies were promoted by the use of appropriate activities, matched to syllabus content, that required students to understand and generate meaning about concepts (Zweck, 2006:5). The aim was to develop a community of learners whose members learned together and from each other. The role of teacher helped students make sense of the mathematics by asking questions that prompted them to clarify, elaborate, justify and critique their conjectures and solutions (Goos et al., 2004:105).

Success was measured by students' understanding, engagement and achievement.

2. The focus is on teaching mathematics via problem-solving by engaging students in doing mathematics, creating, conjecturing, exploring, testing and verifying (Project Maths, 2011). Effective questioning supported students in the monitoring of their understanding. Questions used with students in the classroom when solving problems included:

'What do I know?

What do I want?

What do the words mean?

Can I draw a diagram?

Can I find an example?

Will it always work?

How is it similar or different to what I have done before?'

(Johnston- Wilder et al., 2003:252)

3. Making students responsible for their own learning was a challenge for a number of reasons. The learning environment as described above had been one where transmission learning has been paramount geared to the external examination system. Assessment for Learning (AFL) was used as a tool to improve students' learning and involved them directly in the learning process. The characteristics of AFL that the practitioner attempted to include in the intervention were:

- a. Sharing learning goals with students
- b. Helping students to recognise the standards they are aiming for
- c. Involving students in assessing their own learning
- d. Providing feedback, which helps students to recognise what they must do to close any gaps in their knowledge or understanding
- e. Communicating confidence that every student can improve
- f. Adjusting teaching to take account of the results of assessment (NCCA, 2003).

4. The aim was to provide the students with rich tasks. Rich tasks are tasks that:
- a. Offer different levels of challenges making them accessible to a wide range of learners
 - b. Encourage collaboration and discussion
 - c. Have the potential to reveal underlying principles and connections with other areas of mathematics
 - d. Encourage learners to be confident, independent and critical thinkers (NRich, 2008)

An example of an activity used with students is in Appendix B.

5. Small group work: there is general agreement in the literature that small group work has positive effects on learning but that it can be challenging to implement in the classroom (Swan, 2005:8). The aim in this study was to develop genuine collaboration amongst the groups working together. Mercer et al., (2008:31) emphasize the task design. Tasks that were considered to be too simple or indeed too complex were rejected in favour of open-ended, challenging tasks with clear task structure. The quality of relationships in the groups would seem to be important too (Mercer et al., 2008:31). Participants within groups in this study were encouraged to relate in positive ways by being given roles to play in their group. They were given four particular roles with the intention of making individual students responsible for active participation in solving the task (Cohen, 1994:17). These roles were:

- a. Facilitator: who makes sure everybody is listened to, contributes and that everybody agrees and understands the solution.
- b. Captain: Keeps the group on task with the work to be completed
- c. Reporter: writes down the group answers agreed by all and checks it is written down correctly
- d. Resource: Alerts the group to the possible strategies available for solving the problems (CPM, 2009:11).

Participants were given ground rules for working in groups. These rules were reinforced regularly and also at times when they appeared to be necessary. Using group work with the students in the classroom was new to the practitioner.

This provided an additional challenge when implementing the new learning environment.

6. Students to be made aware of their beliefs about mathematics and its teaching and learning: Moscucci's (2005:301) 5-step meta-beliefs systems activity (mBSA) was not carried out formally in the classroom due to the time constraints but a number of the steps were used and helped to inform the creation of the learning environment.
 - a. Step 1 of the mBSA activity advised a logbook be kept on the quality of every learner's beliefs systems. With 28 students in the classroom this was not considered to be feasible.
 - b. Step 2 involved the carrying out of class conversations to discover learners' interests and their expectations without referring to mathematical interests. This step was adapted for use in this study by students being encouraged to express their opinions on all aspects of the teaching and learning in the classroom. During whole class discussions, students' advice and opinions on the value of the activity being used, the teaching approach, the type of scaffolding that would help the understanding of the concept being taught was sought. A diary was kept recording information on students' contributions (Appendix B).
 - c. In Step 3 of the mBSA (Moscucci, 2005:302) a learning environment is created wherein questions about belief categories arise spontaneously. To help students' gain awareness of their beliefs about their relationship with mathematics whole class discussions were encouraged at specific appropriate times. Questions of particular interest included Is there an inborn aptitude to mathematics? and Do you have to be intelligent to be good at mathematics?. This was linked to the belief that persistent effort to learn mathematics is what matters when seeking success.
 - d. Steps 4 and 5 were not considered for implementation in this study. An analysis of the role of problem solving in constructing mathematical thinking was inappropriate for this age group.

7. As the practitioner, the author placed an emphasis on the effort to learn, to enjoy and to understand mathematics. Participants were encouraged to persist in their efforts to learn mathematics and were reminded that this would bring success over time. Each month participants were rewarded for their effort with a small nominal prize.

Phase 3 - On Monday May 2nd 2011 the survey questionnaire was distributed and completed for the second time following the completion of the classroom intervention. The distribution process remained the same as from phase 1 above.

Phase 4 - The final phase consisted of the two focus groups discussing the same set of questions as above. The individual interview questions consisted of three questions from the Student Interview Instrument (Kloosterman, 2002) that had matched questions from the MRBQ scale and seven additional questions arising from the data collected from Phase 1 (MRBQ questionnaire) and Phase 2 of the focus groups. A pilot individual interview was undertaken and three further individual interviews followed.

3.9 Data Analysis

The literature provided a typology of techniques available for analysing data for mixed methods studies (Onwuegbuzie et al., 2007b; Johnson and Turner, 2003; Teddlie and Tashakkori, 2009). Waysman et al. (1997:236) warns that researchers still lack 'clear operative guidelines' for blending mixed methods evaluations and that great care should be taken not to misinterpret the findings. Ivankova et al. (2009:18) claims the priority of the quantitative or qualitative data is determined by the study purpose and the research questions asked in a study. Recently Heyvaert (2013:323) argues that a critical appraisal framework for the evaluation of the methodological quality of mixed methods studies is overdue.

A mixed analysis matrix involving mixed methods consists of a number of analysis types generating a general typology (Onwuegbuzie et al., 2007b:8). The authors discuss what they describe as the fundamental principle of mixed analysis. They describe this principle as one that involves the use of

quantitative and qualitative techniques that are used either concurrently or sequentially (Onwuegbuzie, 2007b:5). Johnson and Turner (2003:298) provide a matrix of data collection strategies for mixed methods research. This matrix was adapted by Teddlie and Tashakkori(2009:207) who emphasised its use with mixed methods research. They describe the two basic mixed method data collection strategies for use with MM studies. The authors describe these as Within-strategy and Between-strategies data collection strategies. The within-strategy involves gathering quantitative and qualitative data using the same data collection strategy (Teddlie and Tashakkori, 2009:18). The between-strategies mixed methods data collection is referred to quantitative and qualitative data that use more than one data collection strategy. Between strategies mixed methods may be associated with sequential designs and as this design had already been chosen for this study (Section 3.4.3.1 above) it was considered appropriate for use as a data collection strategy for this study.

Quantitative Data Analysis:

SPSS software was used to analyse the data collected on all of the research questions from the MRBQ survey instrument. The results of the statistical tests are described in detail and illustrated in Chapter 4.

In this study the internal consistency of the four factors of Mathematics Related Beliefs Scale (MRBQ) were tested using Cronbach's Alpha. This measured the internal consistency among the items of the scale and was useful for establishing reliability in multi-item scales (Cohen, 2007, Tavakol, 2011).

Tavakol goes on to say that alpha is a property of the scores on a test from a sample group of individuals and hence alpha should be measured each time the test is administered.

Related t-tests were chosen as an appropriate measurement of analysis of the data. The tests were carried out on the total scores for each of the factors.

These tests were considered to be appropriate as each student produced a pair of scores from the survey MRBQ Scale, one score from before the intervention and one after the intervention was completed. The rationale for calculating the t-tests was to see if there was any change in the scores on any of the four factors

from before to after the intervention. Means are calculated to see if they differ a little or a lot (Field, 2005:286).

Independent t-tests were carried out to see if there was a difference between the intervention group and the control classes on students' beliefs about the teacher's role, their competence in mathematics, relevance of mathematics to their lives and mathematics as an inaccessible subject.

Analysis of Variance (ANOVA) was then chosen as the appropriate method of analysis. Use of the Anova test was to determine how the students' beliefs fared from before to after the intervention in the classroom. A two-way Analysis of Variance (ANOVA) was used to test for differences between the four classes of students and/or differences in gender.

Focus Groups and individual interviews Data Analyses:

The data collected from the focus groups and individual interviews were analysed separately. The data from the focus groups was analysed using two types of analysis as this is thought to strengthen the trustworthiness of the findings. Onwuegbuzie et al. (2009b:25) used a 2-dimensional matrix indicating analytical techniques as a function of approach (Quan. v Qual.) and analysis emphasis (case v variable). Case oriented techniques include Constant comparison analysis, Keywords-in-Context, Classical Content Analysis, Text mining, Member Checking, Micro-interlocutor analysis (Onwuegbuzie, 2009:25). Constant comparison analysis is commonly used to analyse qualitative data (Onwuegbuzie, 2009; Teddlie and Tashakkori, 2009; Angell and Townsend, 2011). Constant comparison analysis and micro-interlocutor analysis are both considered to be suitable for qualitative phases of a mixed methods study (Onwuegbuzie et al., 2009b:25). Member checking was considered for use with this study to confirm the information interpreted from the focus group data. However, it was not possible to use that method as some of the individuals in the focus groups were no longer present in the school.

Hence, the analysis types used in this study were:

- a. Constant comparison analysis (Leech et al., 2007:565) which has been termed coding and
- b. Micro-interlocutor analysis that attempts to assess the level of consensus in answers given in the focus groups (Owuegbuzie et al., 2009:7).

Data collected from the individual interviews was analysed using the same methods as used with the focus groups above. In both the focus groups and individual interviews constant comparison analysis was undertaken inductively with the codes emerging from the data (Leech et al., 2007:565). The data collected from focus groups acted as a follow-up that might assist in interpreting the survey results (Morgan, 1996:135). Explicit comparisons of survey and focus group results shows the biggest difference found between the methods was the ability of the focus groups to produce more in-depth information (Morgan, 1996:137).

3.9.1 Conclusion

This study used a quasi-experimental sequential explanatory mixed methods design. It attempted to measure the change in students' beliefs about mathematics and its teaching and learning after a classroom intervention was carried out in the classroom. Decisions made on the design of this study were influenced by the current literature at the time the study was implemented. The study was predominately quantitative with a qualitative approach included to extend and enhance the findings. The quantitative data collected used a psychometrically tested survey instrument and was analyzed using statistical methods. The qualitative data consisted of focus groups and individual interviews.

Chapter 4) Results

The purpose of this study was to measure changes in students' beliefs about mathematics and its teaching and learning following a classroom intervention. This chapter presents the results of this study. The quantitative and qualitative data collected in this study were separately analysed and the results are reported below. The results from the quantitative and qualitative data are then combined. A discussion of the results is in the conclusions chapter (Chapter 5) that follows.

Research on the analysis stage of the mixed methods research process is a very undeveloped area in the literature according to Onwuegbuzie et al., (2009b:15) who advise that extra care is needed when combining interpretations stemming from quantitative and qualitative data findings. However, an agreed comprehensive framework for mixed data analysis does seem to be developing in the literature currently. The framework used with the data from this present study was chosen as being in keeping with the current literature.

This chapter is therefore structured as follows:

1) Analysis of the quantitative results:

Descriptive statistics were used to summarise the data. As previously stated in Chapter 3, the fully mixed sequential dominant status design was the chosen design for this study, with the quantitative approach being dominant. The study's focus was on changes in students' beliefs about their teacher's role, about their personal competence, about the relevance of mathematics and about mathematics as an inaccessible subject after the intervention had been completed, all factors of the MRBQ scale. The analysis first provides an overview with calculations on these factors on all participants (experimental and control) in the study combined together. The statistical tests in this study were used to analyse separately the two sets of scores (i.e. before and after the classroom intervention) from the MRBQ questionnaire. Statistical tests carried out included related (paired) t-tests and independent t-tests and ANOVA. The aim was to discover whether there was a quantitative relationship between the

classroom intervention and changes, if any, to students' belief scores for these factors of the scale.

(2) Analysis of the qualitative results:

The data collected from the focus groups and individual interviews were analysed separately and the results are recorded below. The constant comparison analysis tool was used to analyse the focus groups and the individual interviews (Onwuegbuzie, 2009:27). As mentioned previously in Chapter 3 a second line of analysis, that of the focus group interviews, was informed by the micro-interlocuter (MIC) approach: this was intended to reveal the level of consensus and dissention in the data from the focus groups (Onwuegbuzie et al., 2009:10). The MIC approach was valuable because consensus can otherwise go unnoticed. This approach was chosen to provide confidence that the interpretations of the data were properly made. The opportunity to return to the students for member checking was not available. If this had been possible, it would have informed further discussion about the extent to which the interpretations made from the data collected were valid. Appendix B contains data from the interviews and results of analyses carried out on them.

The reports below, of all of the outcomes, have each been divided into three sections that matched the four research questions on the factors belonging to the MRBQ scale. The different sections are:

- a) Constant comparison analysis of the focus group data
- b) Micro-interlocuter analysis of the focus group data
- c) Constant comparison analysis of the individual interviews data

3. Comparison and Contrast: The final section combines the outcomes from the quantitative and qualitative data.

4.1. Quantitative Data Results

This section of the study is structured as follows:

A description of the MRBQ questionnaire used is presented in 4.1.1. This checks for normality of the data, internal consistency of the scale, the factors and sub-factors of the scale tested. It then discusses the chosen statistical tests arising from these tests that were used in this present study. (Quantitative results from these tests are given in 4.1.2).

4.1.1 MRBQ Scale

The questionnaire used to collect the quantitative data in this study was the Mathematics Related Beliefs Scale (MRBQ). Details on why this questionnaire was chosen are in Chapter 3, Section 3.5.3.

The factors and sub-factors of the MRBQ (De Corte et al., 2004) are:

Table 11	
MRBQ Scale	
Factors	Sub-factors
Teacher's role	Teacher attends to students' meaningful knowledge
	Affective dimensions and perceptions of teacher interest
Personal Competence	Student perception of enjoyment in intellectual demands of mathematics
	Absolute or intrinsic mathematical competence
	Relative or extrinsic mathematical competence
Relevance	Sense of mathematics as personally relevant
	Issues of global relevance
	Student perception of different strategies in learning mathematics and problem-solving
Inaccessible subject	Mathematics unattainable to all but able child
	Mathematics as a fixed body of knowledge requiring mainly good memory

Normality of the MRBQ scale:

To reduce the number of separate analyses that are reported, total scores were calculated for each of the factors and subfactors in the MRBQ scale. For example TotalTeacherScoresBefore is the sum of the scores on the teacher role before the intervention was implemented in the classroom.

A requirement for the use of parametric statistical tests is that the scores be normally distributed. A statistical procedure indicating the normality of a set of data is given in the results of the Kolmogorov-Smirnov (KS) test which for the variables in this study, yielded the following results:

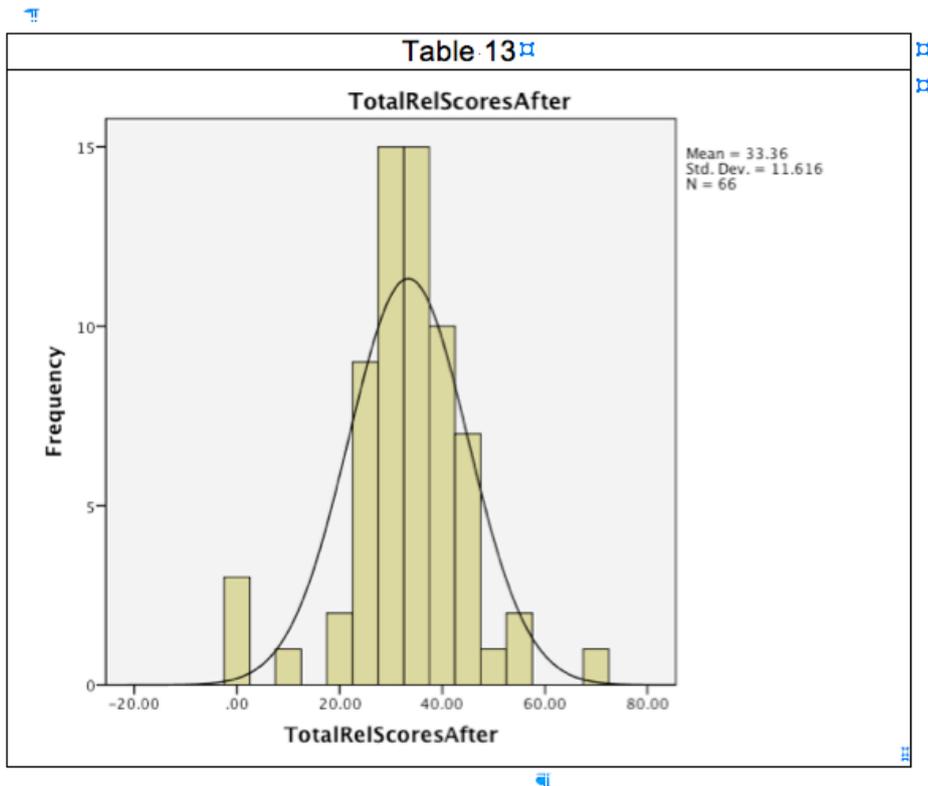
Table 12	
Kolmogorov-Smirnov Scores MRBQ Factors	
Factor	Statistic
TotalTeacherScoresBefore	.200
TotalTeacherScoresAfter	.200
TotalCompetenceScoresBefore	.050
TotalCompetenceScoresAfter	.200
TotalRelevanceScoresBefore	.200
TotalRelevanceScoresAfter	.010
TotalInaccessibleScoresBefore	.200
TotalInaccessibleScoresAfter	.008

A non-significant result ($>.05$) indicates normality of the distribution of scores.

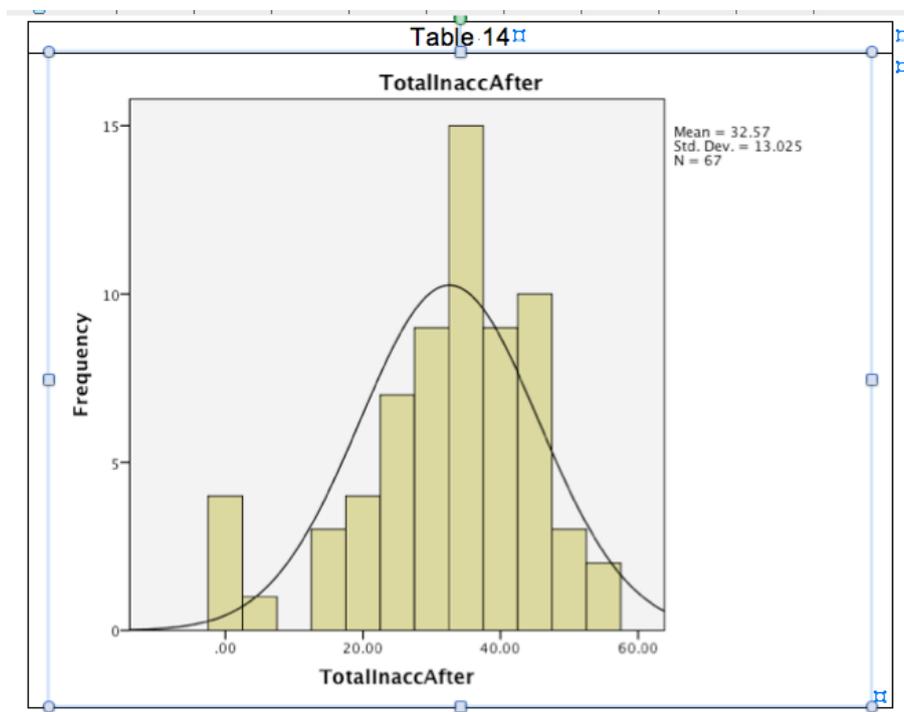
The literature on testing the normality of a distribution recommends the data should be also be plotted to aid an informed decision as deviations from normality may be enough to show up in statistical procedures and with large samples, quite small deviations from normal may show up as statistically significant and yet have little impact should parametric tests be used for subsequent analysis (Field, 2005; Analyse-it, n.d).

Histograms were plotted for the total scores of each of the four factors from before and after the classroom intervention was undertaken e.g. Total teacher scores before and Total teacher scores after (Appendix B).

Because the KS scores for TotalRelevanceScoresAfter (= 0.010) and TotalInaccessibleScoresAfter (= 0.008) gave significant results in the table above, the histograms for these two variables are also shown in Tables 13 and 14 below



and



The histograms for these variables indicated that the scores for TotalRelScoresAfter and TotalInaccAfter were reasonably normally distributed. Hence, taken together, the plotted histograms and Kolmogorov-Smirnov statistic for the TotalRelevanceScoresAfter and TotalInaccessibleScoresAfter

scales suggest that the data match normal distributions sufficiently well to treat them as normal distributions. Consequently, parametric statistical tests were considered to be appropriate for analyzing the data in the present study.

Internal Consistency of MRBQ scale:

The whole set of data collected, using the MRBQ questionnaire, collected for this study, was analysed for internal consistency by its authors and yielded a Cronbach's alpha score of $\alpha = 0.939$ (Diego-Mantecon et al., 2007). Values of alpha indicating the overall reliability of a scale that provide magnitudes of 0.7 or 0.8 is thought to indicate good reliability in a scale (Field, 2005:673). The four factors of the MRBQ scale were also tested for good internal consistency. Gleim and Gleim (2003:88) write of the importance of calculating and reporting Cronbach's alpha scores for factors and sub-factors as it is not appropriate for single items. They further argue that subsequent analyses of the data must use the factors and not individual items.

Although this is encouraging, the psychometric properties of a test are dependent on the sample with which it is used and are not a transferable property of the test itself. General measures of validity or consistency do not therefore guarantee that a test will work adequately in a particular study. Therefore, the data collected on the MRBQ scale was examined for its reliability for the actual sample used in this study in order to determine its internal consistency. It yielded a Cronbach's Alpha score of $\alpha = 0.919$ for the set of responses given by students to the MRBQ scale prior to the intervention (75 responses) and a Cronbach's Alpha score of $\alpha = 0.903$ for the responses given by students to the MRBQ scale after the intervention (a total of 67 responses). These results indicated that the scale had high levels of internal consistency when used with the sample involved in the current study.

Cronbach alpha scores for each of the factors and sub-factors are shown in the table below indicating satisfactory internal consistency for all of the factors in this study.

Table 15		
Cronbach alpha scores for the sample		
	Factor	Sub-factors
Teacher's Role	Before= .809	Sub-factor1Before= <u>.748</u> Sub-factor2Before= <u>.234</u> (Inter)
	After= .829	Sub-factor1After = <u>.811</u> Sub-factor2After = <u>.448</u> (Inter)
Personal competence	Before= .871	Sub-factor1Before= <u>.870</u> Sub-factor2Before= <u>.483</u> (Inter) Sub-factor3Before= <u>.467</u> (Inter)
	After= .925	Sub-factor1After = <u>.912</u> Sub-factor2After = <u>.594</u> (Inter) Sub-factor3After = <u>.785</u> (Inter)
Perception mathematical relevance	Before= <u>.794</u>	Sub-factor1Before= <u>.113</u> (Inter) Sub-factor2Before= <u>.334</u> (Inter) Sub-factor3Before= <u>.481</u> (Inter)
	After= <u>.849</u>	Sub-factor1After = <u>.337</u> (Inter) Sub-factor2After = <u>.477</u> (Inter) Sub-factor3After = <u>.506</u> (Inter)
Inaccessible subject	Before= <u>.819</u>	Sub-factor1Before= <u>.299</u> (Inter) Sub-factor2Before= <u>.207</u> (Inter)
	After = <u>.886</u>	Sub-factor1After = <u>.457</u> (Inter) Sub-factor2After = <u>.360</u> (Inter)

Initially, the Cronbach alpha score was calculated as $\alpha = 0.546$ for the sum of the 14 items in the teacher's role factor for the present study. Questions 13 and 14 shown below, that would appear to be negatively oriented, were reverse coded and reliability was re-calculated producing the alpha score of 0.809 above.

Factor Teacher's role- items	DiegoMantecon et al. (2007:233-236)
Question 13	My teacher is too absorbed in the mathematics to notice us
Question 14	My teacher does not really care how we feel in class

The removal of questions 13 and 14 either together or separately would not have improved the reliability of the teacher factor. Hence, the reverse coded questions 13 and 14 were included in the quantitative calculations below.

Sub-factors with less than 10 items from the table above have the mean inter-item correlation reported as recommended in the literature (Pallant, 2007:95).

The mean inter-item correlation is not thought to be influenced by the length of the scale (Briggs and Cheek, 1986:115). Sub-factors in the table above where the mean inter-item has been reported are identified with Inter after score. An item correlation between .2 and .4 is considered optimal offering an acceptable balance between bandwidth and fidelity (Briggs and Cheek, 1986:115). These were calculated using the data collected before and after the classroom intervention.

Parametric tests used with this study:

This study tested the difference between participants who were subject to the classroom intervention with participants who were taught in the normal way. The MRBQ scale used in this study showed the factors were normally distributed, as shown above. Hence, the parametric tests chosen were considered appropriate for use.

(i) t-tests: related

Related or paired t-tests calculated the differences between the MRBQ scores from before to after the classroom intervention for the participants in this study. The scores were calculated for each of the four factors and sub-factors of the MRBQ scale. Differences in the scores would indicate that the students' beliefs about mathematics and the learning of mathematics had been enhanced or impaired by the changes made to the learning environment. t-tests were also carried out on the classes of participants who were part of the control classes to assess belief differences.

(ii) t-tests: Independent

Independent t-tests make a direct comparison between the experimental class and the control classes in the study. Gain scores were used to make the comparisons from the beginning of the study to the end of this study. On the one hand, the class of participants who experienced a new learning environment were the experimental class and on the other hand classes of participants who were taught in the usual manner made up the control group. t-tests were again calculated for each of the factors of the MRBQ scale.

(iii) Two-way ANOVA:

A two-way ANOVA sought to explore the impact of class grouping and gender on students' belief scores. The ANOVA calculated the beliefs, held by participants, to see if they were impacted by gender or by class. The scores used in the test were the gain scores for each of the factors of the MRBQ scale e.g. Total teacher score after – total teacher score before for each participant. The procedure aimed to quantify influences of gender or class differences on the gain scores for the four classes of participants.

4.1.2 Quantitative data collected in this study:

Results were organized to provide information on the outcomes for this study in keeping with the research questions. The classes consisted of the experimental class (n=25), who were the subject of the intervention in the classroom, and the three control classes who were taught as normal (n=53). 4.1.2.1 provides an overview with calculations on all participants together, experimental and control classes, who took part in this study.

4.1.2.2 provides the data that was analysed to provide outcomes to the research questions in this study.

4.1.2.1 Overview calculations for all participants (Experimental and Control classes):

Related t-tests:

A related (paired) t-test calculates the differences between the scores at two different times for the same group of students (Greene and D'Oliveira, 2009:49). The calculations for the related t-test on the total scores for all of the participants in this study from before the classroom intervention are compared to the same scores collected after the classroom intervention was carried out. The tests were carried out on the four factors of the MRBQ scale completed by the participants.

Table 16 below shows the results of the related t-tests for the all participants (Experimental and Control) comparing scores before with those after the classroom intervention. The 4 factors tested teacher's role, perception of competence, relevance about mathematics to their lives and mathematics as an inaccessible subject. Results are shown in the tables below:

Table 16				
Related (Paired) t-tests Factors (All participants)				
Subscale	Teacher	Competence	Relevance	Inaccessible
Mean Score Before	32.76 (n=67)	45.34 (n=67)	40.17 (n=66)	36.49 (n=49)
S.D.	8.20	12.34	9.31	10.31
Mean Score After	32.72 (n=67)	44.90 (n=67)	43.55 (n=66)	40.76 (n=49)
S.D.	9.13	15.60	15.02	12.21
t value	0.48	0.233	-1.65	-2.207
df	66	66	65	48
Sig. (p-value)	0.962	0.816	0.103	0.032*

i) Students' beliefs about the role of their teacher:

Results from the t-tests in the table above show the mean decrease of 0.04 (32.76 - 32.72) in the teacher's role belief scores, for all participants together, was not statistically significant ($p > .05$) and therefore could not be attributed simply to chance. This would seem to indicate that there was no real change in the participants' beliefs scores about their teacher's role over the period of the classroom intervention. Hence, students' beliefs about the role of their teacher had remained constant across the intervention whether they had experienced the new learning environment or not.

ii) Students' beliefs about their personal competence and students' beliefs about the relevance of mathematics to their personal lives:

The t-tests, in the table above, showed that there were no significant differences ($p > .05$) in the scores of all participants (experimental and control) over the period of the intervention.

iii) Students' beliefs about mathematics as an inaccessible subject:

Students' beliefs about mathematics as an inaccessible subject was found to be statistically significant ($p < .05$) at the 0.05 level as shown in Table 16 above. The mean scores increased on this occasion over the period of the classroom intervention. This indicated students' beliefs about mathematics as an

inaccessible subject had become more negative over the period of the intervention whether they had experienced the new learning environment or not. The eta squared statistic of 0.09 indicated a moderate effect. An appropriate interpretation of these results could be to acknowledge the increased academic challenges arising from the curricular content in that particular school year for all participants. The content would have been considerable greater in quantity and more challenging than in the previous academic year, particularly for the experimental Class 1 and for Class 4, the weakest students in the year. Further related t-tests were then carried out on all participants' beliefs for the sub-factors of each of the factors above in the scale. Results for each of the sub-factors are shown in the table below.

Table 17					
Related t-tests: sub-factors for each factor MRBQ scale (All participants)					
Teacher Sub-Factors					
Sub-factor	t-value	df	p value	Mean Difference	S.D.
1. Teacher attends students' meaningful learning	0.804	66	0.424	0.672	6.841
2. Perceptions of teacher interest	-2.049	66	0.044*	-0.6866	2.742
Competence Sub-factors					
1. Perception of enjoyment	-0.421	66	0.675	-0.582	11.309
2. Intrinsic mathematical competence	0.593	66	0.555	0.403	5.562
3. Extrinsic mathematical competence	2.211	66	.031*	0.940	3.481
Relevance Sub-factors					
1. Mathematics personally relevant	-1.184	65	0.241	-1.242	8.523
2. Global Relevance	-2.041	66	0.045*	-1.896	7.600
3. Perception of different strategies in learning maths	-1.648	66	0.104	-0.955	4.743
Inaccessible Subject Sub-factors					
1. Mathematics only attainable able child	-2.910	54	.005*	-3.218	8.200
2. Mathematics fixed body knowledge	-2.238	50	.030*	-2.431	7.760

The first sub-factor from the teacher's role factor has been called 'teacher attends to students' meaningful learning' (Diego-Mantecon et al., 2007:232). From the table above this gives a mean difference over the period of the intervention of 0.672. This sub-factor includes 12 of the 14 questions from the teacher factor questions. The results showed no change for the outcomes of this factor. The finding for the second sub-factor 'Perceptions of teacher interest' was significant ($p < 0.05$), as shown in the Table 17 above, for the whole set of participants in this study over the period of the intervention in the classroom. The second sub-factor had used 3 of the 14 questions from the teacher factor.

Two of the sub-factors from the Competence factor 'Perception of enjoyment' and 'Intrinsic mathematical competence' showed no significant change from before to after the intervention. There is a statistically significant result, as shown in the table above, The finding for the third sub-factor 'Extrinsic mathematical competence' was significant ($p < 0.05$) as shown in Table 17 above. The mean increase in scores was 0.940 at the 0.05 level. This sub-factor used only 2 questions from a total of 14 questions in the factor. A reasonable interpretation of this would be to acknowledge that only two questions were included in the sub-factor and that this may indicate that the outcome requires further investigation to verify that change in these beliefs had taken place.

The belief scores for related t-tests on each of the sub-factors from the relevance factor are also shown in the Table 17 above. There were no significant statistical differences, at the level of 0.05, between the scores from before to after the intervention for two of the sub-factor scores. The third sub-factor 'Global relevance of Mathematics' was found to be significant ($p < 0.05$) as shown in Table 17 above. Diego-Mantecon et al. (2007:235) argued that this second factor focused on the collective relevance of mathematics. A later study by Andrews et al. (2011:12) claims the sub-factors for the factors of this scale require further analyses. It may also be that participants perception of mathematics as globally relevant had become more negative over the period of the intervention.

The fourth factor, mathematics as an inaccessible subject, has two sub-factors. Both of these sub-factors showed significant statistical differences ($p < 0.05$) in the scores from before to after the intervention (Table 17). The finding from first sub-factor 'Mathematics is only attainable to able child' showed a mean increase of 3.218 over the period of the intervention at the level of 0.05. The eta squared statistic (0.18) indicated a large effect size. The second sub-factor, indicating 'Mathematics as a fixed body of knowledge' only requiring a good memory to succeed, showed a mean increase of 2.431 at the level of 0.05. The eta-squared statistic (0.091) indicated a moderate effect size.

The scores in Table 17 above for the factor 'mathematics as an inaccessible subject' had shown a statistically significant difference in a negative way from before to after the intervention in the classroom. The related tests on the sub-factors have shown a more negative outcome for 'mathematics only being attainable for the able child'. An appropriate interpretation, in this researcher's view, of the results would be that the participants had found mathematics to be more difficult for 2nd year of secondary school. The participants, who might have possibly shown more enhanced scores, had been seriously challenged with the pace of work covered with the requirement for the experimental class to complete the ordinary level course during that academic year.

4.1.2.2 Quantitative outcomes on the Research Questions:

This section presents the results of the analyses of the quantitative data seeking to answer the research questions (i.e. examining the MRBQ scores for the experimental class across the intervention and also comparing those with the MRBQ scores collected from the control classes).

Related t-tests Scale Factors: Experimental and Control Classes:

As mentioned earlier, Class 1 was the experimental class with classes 2,3 and 4 the control classes. The outcomes from Related t-tests for the four factors of the MRBQ scale are shown in the table below. The related scores used were summed scores for each factor before and after the intervention for the experimental and control classes. Two sets of related scores were compared e.g. ExpTotalCompBef (the summed set of scores of each participant from the experimental class for the Competence factor at start of the study) and

ExpCompTotAft (the summed set of scores of each participant for the experimental class for the Competence factor at the end of the study). Table 18 below shows the resultant scores for these Related t-tests.

Table 18						
Related t-tests MRBQ factors Experimental and Control						
	t-value	df	p -value	Mean	SD	
Teacher Factor						
Experimental	0.878	21	0.39	Before	39.86	Before 5.60
				After	38.41	After 7.707
Control	-0.567	44	0.573	Before	29.29	Before 6.95
				After	29.93	After 8.52
Competence Factor						
Experimental	-0.395	21	0.697		47.73	Before 14.30
					48.68	After 15.54
Control	0.433	44	0.667		44.18	Before 11.254
					43.04	After 15.47
Relevance Factor						
Experimental	-2.074	20	.051		41.905	Before 9.914
					46.952	After 11.771
Control	-.935	44	.355		39.356	Before 9.010
					41.956	After 16.191
Inaccessible Subject Factor						
Experimental	-0.877	14	0.395		40.933	Before 9.362
					43.333	After 12.793
Control	-2.018	33	0.52		34.529	Before 10.228
					39.618	After 11.957

The teacher factor in Table 18 above indicated no statistically significant change at the level of 0.05, either positive or negative, in **students' beliefs** scores ($t(21)=0.878$, $p>0.1$ (two-tailed)) about the teacher's role had taken place for the participants in the experimental class over the period of the intervention. The scores also indicated that the beliefs of the control classes about the **teacher's role**, who were taught as normal, had also not changed significantly for the teacher's role ($t(44)=0.573$, $p>0.5$ (two-tailed)).

The results of the related t-test for the **competence factor** also showed no statistically significant change for the experimental class $t(21)= -0.395$, $p>0.5$ at the level of 0.05. Similar non-significant results for the control classes for the competence factor are shown in the table above ($t(44)=0.433$, $p>0.5$ at the level of 0.05).

The related t-tests for the third factor, **relevance of mathematics to students' lives**, showed no significant changes in participants' belief scores for the intervention period for both the experimental and also for the control classes calculated separately. The intervention class scores showed $t(20)=-2.074$, $p>0.5$ at the level of 0.05. The control classes similarly showed no statistically significant change over the period of the study, $t(44)=-0.935$, $p>0.1$ at the level of 0.05. The finding for the experimental class for the Relevance factor was shown to be close to significance, at the level of 0.05, giving a p-value of 0.051 (Table 18). This contrasts sharply with the far from significant finding ($p=0.355$) for the same Relevance factor for the control classes.

Similar non-significant results for the related t-tests for the fourth factor, **beliefs about mathematics as an inaccessible subject**, are shown in the table above for both the experimental classes and control classes. The experimental class scores showed $t(14)=-0.877$, $p>0.1$. The control classes scores showed $t(33)=.52$, $p>0.05$ at the level of 0.05. This contrasts with the results for the whole sample taken together where, perhaps, the larger sample size makes the non-significant changes for the experimental and control classes separately, show as significant when they are combined.

Independent t-tests

Independent t-tests were carried out to directly compare the scores for the experimental and control classes over the period of the study. Independent samples t-tests (unrelated) were used to compare the means of the different classes under two experimental conditions (Field, 2005:296). In this study the experimental class ($n=22$) was subject to the classroom intervention and the other three classes ($n=45$) in the control classes were taught as normal. The differences between participants' scores on each of the factors in the MRBQ scale were calculated. Using each question for each of the factors the differences in scores were calculated from before to after e.g. $\text{GainTeacher1}=\text{Teacher1After}-\text{Teacher1}$, where Teacher1 is question 1 before the classroom intervention on the teacher's role from the questionnaire etc. These gain scores were then summed for each of the four factors of the scale for each participant. The calculations of these scores provided a direct

comparison between participants' gain scores in the intervention class to those in the control classes in the study.

Results from statistical tests on the scores from the factors of the MRBQ scale indicated no statistically significant difference in the mean gain scores for each belief factor as shown in the table below.

The scores showed virtually no significant differences, at the level of 0.05, in students' beliefs about their **teacher's role** for both the experimental and control classes as shown in Table 19 below.

The second factor, **personal competence**, also showed scores indicating virtually no change to the beliefs of either of the groups. The researcher's interpretation of this was that students' beliefs about their competence remained constant whether they belonged to the class who experienced the new learning environment or those that did not. The remaining two factors in the scale, **relevance of mathematics to participants' lives** and **mathematics as an inaccessible subject** showed gain scores indicated somewhat constant beliefs from before to after the intervention period.

Factor		N	Mean Gain	P value	S.D.
Teacher	Experimental	22	-1.45	0.297	7.769
	Control	45	0.64		
Competence	Experimental	22	0.500	0.98	11.7666
	Control	45	-0.711		
Relevance	Experimental	21	5.050	0.58	11.151
	Control	45	2.60		
Inaccessible Subject	Experimental	15	1.40	0.862	12.620
	Control	34	2.18		

Further independent t-tests were carried out on the above gain scores examining the sub-factors for each of the four factors of the MRBQ scale. All the gain scores for participants in the study were binned into 1 (experimental) and 2 (Control) for each of the sub-factors. The aim was to compare the gain scores for both experimental and control to discover if any of the sub-factors showed differences to students' beliefs over the period of the study.

Table 20					
Independent t-tests: sub-factors for each factor MRBQ scale					
Teacher Sub-Factors					
Sub-factor	t-value	df	p value	Mean	S.D.
1. Teacher attends students' meaningful learning	-1.387	65	0.170	Exp. = -2.318 Control =0.133	Exp. = 6.778 Control=6.801
2. Perceptions of teacher interest	1.359	65	0.179	Exp. = 0.9091 Control =-1.045	Exp. = 4.219 Control=6.053
Competence Sub-factors					
1.Perception of enjoyment	1.199	58.5	.913	Exp. = 0.773 Control =0.489	Exp. = 8.4229 Control=12.567
2. Intrinsic mathematical competence	1.359	65	.179	Exp. = 0.909 Control =-1.04	Exp. = 4.2191 Control=6.0526
3. Extrinsic mathematical competence	.800	64.9	0.427	Exp. = -0.546 Control =-1.13	Exp. = 1.9935 Control=4.2040
Relevance Sub-factors					
1. Mathematics personally relevant	1.052	64	0.297	Exp. = 2.86 Control =0.489	Exp. = 7.1084 Control=9.0845
2. Global Relevance	0.012	64.7	0.990	Exp. = 1.909 Control =1.889	Exp. = 4.5246 Control=8.765
3. Perception of different strategies in learning maths	-0.601	65	0.550	Exp. = 0.455 Control=1.200	Exp. = 3.0036 Control=5.4088
Inaccessible Subject Sub-factors					
1. Mathematics only attainable able child	-.793	51.7	.432	Exp. = 2.167 Control =3.730	Exp. = 5.2608 Control=9.326
2. Mathematics fixed body knowledge	-1.090	49	.281	Exp. = 6.822 Control =8.082	Exp. = 1.7615 Control=1.3469

The results (Table 20) showed reasonably constant beliefs held by the participants whether they belong to the experimental or the control classes at the end of this study.

Analysis of Variance

Analysis of Variance (ANOVA) is a statistical method designed to test equality among the means of two or more groups. A sufficiently large F-statistic, a comparison of the variability between groups to the variability within groups, indicates a significant difference among the means of the groups. A two-way ANOVA tested the possibility that there might be a differential effect called an interaction between class and gender in this study. This enabled an investigation of the interaction effects of class (experimental or control) and gender to discover whether they had influenced each other or not.

A two-way between-groups ANOVA was conducted on the gain scores in order to explore simultaneously the impact, if any, of class and gender on outcomes of the variables tested. Calculations were made on teacher gains, competence, relevance and inaccessible gains from before to after the classroom intervention. The four class groups included the experimental class1 and the control classes numbered 2,3 and 4 of participants. The following tables show the scores for the two-way ANOVA for each of the four factors of the MRBQ scale.

The objective was to measure the interaction effects of class and gender for outcomes on the teacher's role.

Table 21				
Two-Way ANOVA				
Factor: Gain (Teacher)	f-value	df	Error	Significance (p-value)
Interaction effects	0.259	3	59	0.85
Class Group	0.655	3	59	0.221
Gender	0.221	1	59	0.583

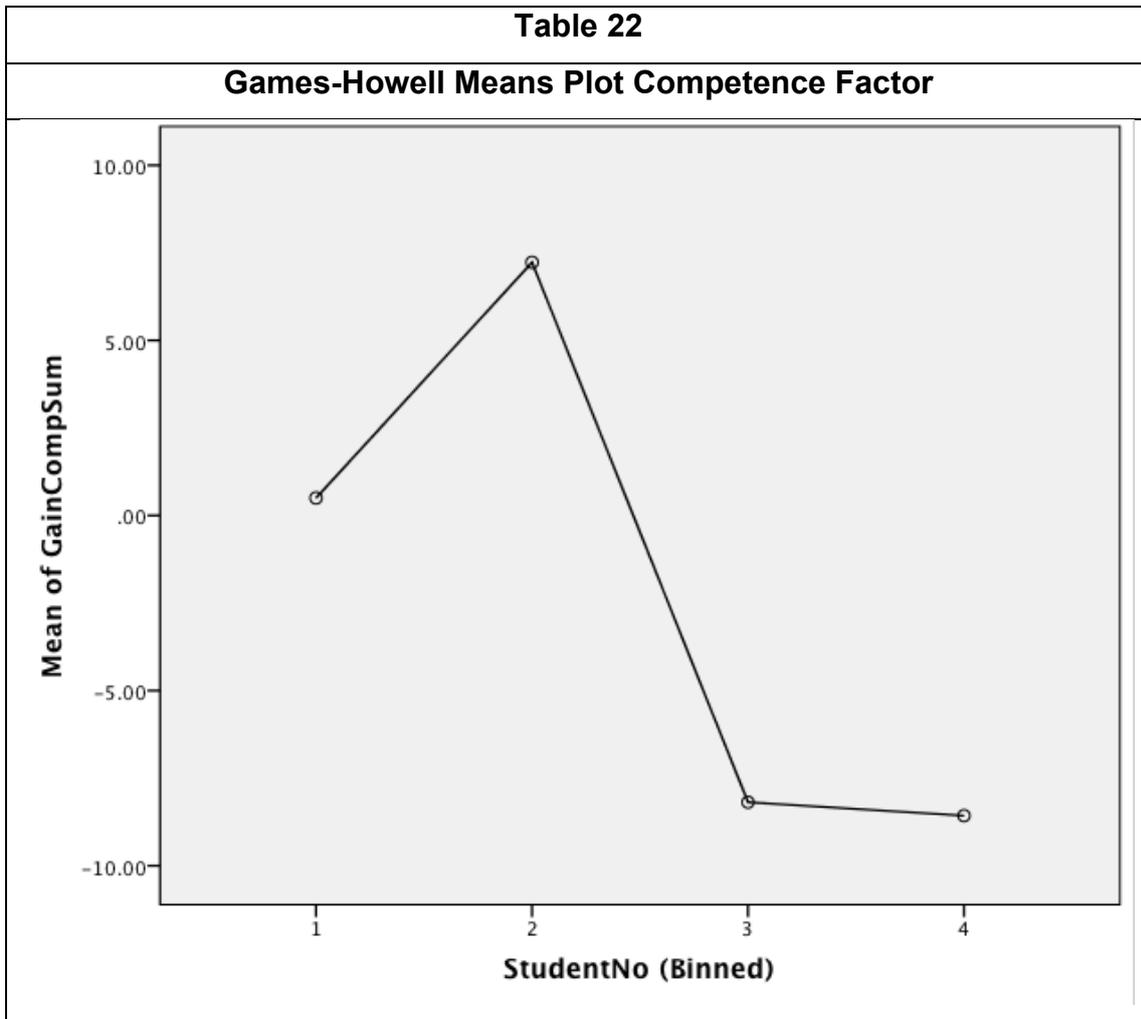
Levene's test of equality of error variance was non-significant ($p > .05$) for Gain (teacher) variable. As indicated in the table above there was no interaction effect. That suggests that males and females responded in the same way to being in the experimental and control classes: the intervention did not make a bigger difference for either gender category.

The main effects scores for classes were also not significant and hence classes do not differ in terms of their gain scores for the teacher factor. Similarly, the main effects scores for gender were not significant and hence there was no evidence suggesting gender differences in the ways in which students' beliefs about their teacher's role changed for the sample studied.

The second research question was to assess belief change in students' **personal competence in mathematics** over the period of the classroom intervention. The ANOVA statistical test on the competence gain scores were not reliable as they gave a significant result for Levene's test of equality of error

variances with a significant result ($p=0.018$) (Appendix B). Hence, the variance, of the competence gain scores, across the classes were not equal. A variant of one-way Anova, Welch F was then used to check for difference amongst the classes. The Welch F test was chosen for use (rather than the Brown Forsythe) because the Welch test is more powerful and better at detecting an effect where it exists (Field, 2005:348). Results indicated Welch's $F(3, 26.32)=4.517$, $p<.05$. Hence, there was a significant difference between the classes on gain scores for the **competence factor**. Post-Hoc tests were also carried out to detect where the changes in beliefs existed within the classes. The Games-Howell post hoc procedure was intended to show the pattern of changes within the classes. It tested every class against every other class e.g. Class1 (Experimental) with Class4 (Control). The Games-Howell procedure is recommended for use when there is doubt about the homogeneity of variance using Levene's test and because it generally seems to offer the best performance (Field, 2005:341). The scores indicate the only classes that differed significantly were classes 2 and 4 (0.014). This can be seen from the means plot in Table 22 below.

Both classes 2, 3 and 4 belonged to the control in this study. Class 2 scores had 22 students and class 4 had 7 students. Interpreting the results, it was clear that this difference was not related to the intervention as these were control classes. An explanation of this difference may be that Class 2 had been considerably less challenged by the curricular content and pace of delivery than Class 4 who could be described as seriously challenged academically. Class 4 participants had been placed in the small class on their entry to the school due to the requirement to provide for their special educational needs. The effect size was calculated giving $\omega = .01$ indicating a small effect size.



The assessment of the interaction with respect to the **relevance of mathematics** to students' lives was also calculated. The 2-way ANOVA test scores were not reliable as they gave a significant result for Levene's test of equality of error variances with a significant result (0.018). As above the Welch test was calculated giving results of $F(3,20.55)=0.527$, $p>0.05$. Hence, there was no evidence that class or gender had influenced the outcomes over the period of the intervention.

The fourth research question was to assess belief change in participants with respect to **mathematics as an inaccessible subject**. Levene's test of equality of error variance was non-significant ($p>.05$) for Gain (inaccessible) variable so ANOVA was an appropriate tool. Results from the ANOVA statistical test indicated the following:

Table 23				
Two-Way ANOVA				
Factor:	f-value	df	Error	p values
Gain (Inaccessible Subject)				
Interaction effects	3.134	3	41	0.036
Class Group	0.205	3	41	0.893
Gender	1.159	1	41	0.288

There were no main effects i.e. no gender differences or class differences in gain scores for mathematics as an inaccessible subject (Table23 above). There was a significant interaction effect ($p=0.036$) indicating an influence on Gain scores by gender, which was different in the different classes, in the study. The nature of this interaction can be understood by inspecting the mean scores for males and females. The means show that the size of the difference between male and female results was different in different classes, but there was nothing to mark out the experimental class from the control classes. The effect size for this interaction effect is modest (partial $\eta^2= 0.187$) with the interaction being unlikely to be simply the result of chance but this is not likely to have important practical implications (Appendix B).

Achievement Tests:

Almost all of the students completed the “Drumcondra” standardized mathematics test, administered by the secondary school, prior to their entrance in September 2009 (Drumcondra, 2010). It was not possible, however, to assess students’ progress in their learning of mathematics since their entry to secondary school through relying entirely on these standardized test scores as a base. The reasons are outlined below.

The majority of the students, that were admitted each year to first year in the secondary school, came from the same five or six primary schools in the area. These standardised tests were and are used for assessment purposes in all years of primary school. There was no formal mechanism in place that allowed secondary schools to compare outcomes from different primary schools and no publication of the results of these tests were available. The author’s experience of teaching students from these schools over many years would seem to show that a small number of these schools regularly did not complete the required primary syllabus to a significant extent for all 6th class students. In some instances the students had not completed any of the mathematics’ syllabus content for the final primary school year (6th Class/age 12). The achievement levels of participants were included in the results to help identify progress in participants’ learning of mathematics. The scores covered the two-year period in which the cohort of participants completed 1st and 2nd year in secondary school.

Students' scores achieved in the Summer school examination at the end of their first year in secondary school (June 2010) and at the end of the 2nd academic year (June 2011) were compared using a paired t-test. All of the participants studied the same syllabus content with differing amounts to be covered resulting in different paces of delivery, particularly during the year the classroom intervention was carried out. To reflect these differences, three separate examination papers were set at the end of that year. A sample end of year examination question is shown in Appendix D. The experimental class was examined on the three sections of the question. The control classes answered sections (a) and (b) with class 4 completing the graph of a linear instead of the quadratic function. The results shown in the table below indicated that there were no statistical differences in the scores from June 2010 to June 2011 for the experimental and control participants. The figures presented take into consideration neither the larger volume of academic content studied by the experimental class, nor the consequent faster delivery they experienced during this study. The impact of different teachers on outcomes has also not been measured.

Table 24		
Related t-test		
School End of Year Achievement Scores		
	Experimental Class	Control Classes
Mean June 2010	54.0455	41.3469
S.D.	17.6189	18.70556
Mean June 2011	56.7273	41.2857
S.D.	16.79904	18.47634
t value	0.820	-0.024
df	21	48
Sig. (p-value)	0.422	0.981

4.1.2.3 Summary of Quantitative Results:

Results giving an overview of outcomes (Table 16) from the quantitative data for all participants (combining experimental and control classes) indicated no changes in beliefs following the classroom intervention for the factors '**teacher's role**', '**personal competence**' and '**relevance to students' lives**' of the scale had taken place over the duration of the study ($p > 0.05$). The fourth factor of the MRBQ scale '**mathematics as an inaccessible subject**' gave a finding in Table 16 above that was significant ($p < 0.05$). It showed more

negative belief scores for this factor. Results of related t-tests for the sub-factors of each of the chosen MRBQ factors showed a non-significant outcome for some of the sub-factors of the **Teacher**, **Competence** and **Relevance** factors ($p > 0.05$). The teacher sub-factor 'Perceptions of Teacher Interest' was significant ($p < 0.05$). As mentioned earlier only three questions from 14 were used in this sub-factor. The **Competence** factor gave a non-significant finding at the level of 0.05 for two sub-factors but was significant ($p < 0.05$) for the 3rd sub-factor '**extrinsic mathematical competence**' but because of the small number of questions included, further scholarship on the sub-factors for all factors was recommended, in line with more recent literature (Andrews et al., 2011:22). One of three sub-factors of the **Relevance** factor 'Global relevance' gave a significant finding ($p < 0.05$) in Table 17 above. Both sub-factors of the **Inaccessible factor** gave significant results ($p < 0.05$). These findings may have been due to the larger sample size of the combination experimental and control classes. This is discussed in Chapter 5.

Focusing on the research questions for this study Related t-tests for the experimental and control classes were calculated and analysed separately. Outcomes were shown to be not significant for the '**teacher's role**', '**personal competence**' and '**relevance of mathematics to their lives**' and '**mathematics as an inaccessible subject**'. This indicated answers to the research questions seeking more positive beliefs about the four factors following the classroom intervention had not been realized for the experimental class. This contrasted with significant outcomes for the '**inaccessible subject**' factor, discussed above (for the combined scores for experimental and control classes).

Independent t-tests for the factors and sub-factors of the scale were carried out on gain scores calculated over the duration of the study. Scores showed non-significant results for both factors and sub-factors of the scale indicating no differences between the experimental and control classes were evident following the completion of the classroom intervention.

An ANOVA test looked for the impact of class or gender on the gain scores across the intervention. There was no evidence of influence of class or gender

on the outcomes from experimental Class 1 and Classes 2, 3 and 4 of the control. A significant interaction effect was shown for classes 2 and 4 of the control classes for the Competence factor. This showed an impact of class on students' beliefs about their competence. Possible reasons for this, the author suggests, are the lower level of academic challenge for class 2 (strongest ordinary-level class and the high challenge for class 4 (special educational needs) for that academic year. It is also possible that the teacher, that had been allocated to teach the classes, had had an influence on these outcomes.

Qualitative data were also collected and are analysed below.

4.2 Qualitative Data Results

A variety of tools has been developed in the literature with which to analyse qualitative data. As mentioned previously, these techniques are governed by an attempt to yield at least one type of generalization and recommend that one should aim to be careful to make meta-inferences that have interpretative consistency (Onwuegbuzie et al., 2009b:24). This study focused entirely on the selected cases, hence a case-oriented analysis was appropriate to apply to the qualitative data. The techniques chosen for this study (Chapter 3) are the method of constant comparison analysis and the micro-interlocutor analysis (MIC). The constant comparison method codes and analyses the data by comparing specific incidents, refines concepts and explores their relationships to one another (Taylor and Bogdan, 1998:137). This constant comparison method is one of the most frequently used strategies (Teddlie et al., 2009:254). The MIC was used to supplement and complement the voices of participants and serves both as a validation and representation tool to increase representation (Onwuegbuzie et al., 2010:715). It aimed to measure agreement in the focus groups that would not be evident from the constant comparison method. Kolb (2012:83) states that the benefits of using constant comparison analysis are that the research begins with raw data and through constant comparisons a theory will emerge. Kolb goes on to say that there are limitations that the researcher must be aware of in the process such as managing the data and credibility. Onwuegbuzie et al. (2009:1) claims MIC offers great potential for analyzing focus group data. The micro-interlocutor

analysis was used with the individual interviews seeking consensus or dissent across participants' views.

The constant comparison analysis, as discussed earlier, involves three steps. They are:

1. Researcher reads through the data and underlines chunks or phrases of the data
2. Each chunk is assigned a code. Other codes to be checked to see if there is an existing similar code.
3. Codes are combined and the themes developed (Leech et al., 2007:566).

An example of this process from the analysis of the focus groups is shown in Table 25 below. Each of the lines represents an individual student's comment from the one of the three focus group interviews. Answers of Yes or yea indicated agreement with the question.

Table 25			
Example of Constant Comparison Method			
Question 2			
Does your teacher appreciate it when you try hard, even if your results are not so good? Are your mistakes ok when you are trying to solve a problem in your own way?			
Before Intervention		After Intervention	
Units	Code for each unit	Units	Code for each unit
<i>Ah don't know [appreciate hard work] (M1)</i>	Not sure		
<i>She does appreciate it when we try hard (M2)</i>	Appreciate hard work		
<i>She does appreciate it when you try hard because she gives out prizes for those who try hardest in class (M2, M4, M5, M6, M7)</i>	Effort prizes given		
<i>I think it is ok (M3)</i>	Mistakes ok		
<i>Yea (M4, 5, 6,7)</i>	Mistakes ok	<i>The mistakes are good as long as you are learning from them (M5)</i>	Mistakes ok
<i>She gives effort prizes (M4)</i>	Effort prizes given	<i>Yea as long as you try (M6)</i>	Mistakes ok
<i>Effort prizes for effort (M7)</i>	Effort prizes given	<i>Most of the time (M6, M7)</i>	Mistakes ok

The micro-interlocutor analysis assessed the level of consensus or dissent with themes from the data collected from both the focus groups and individual interviews in this study. Leech et al., (2009:8) provided five notations to assess consensus. The last notation S (this indicated the participant answered sometimes true and sometimes it was not true e.g. in question 4, Table 'Are you

sometimes given new problems that you have to solve in your own way?') was added for this study.

The notations used were:

A=Indicated agreement with question asked (i.e. verbal or nonverbal)

D=Indicated dissent with question asked (i.e. verbal or nonverbal)

SE=Provided significant statement or example suggesting agreement

SD=Provided significant statement or example suggesting dissent

NR=Did not indicate agreement or dissent including don't know (i.e. nonresponse.

S=Indicated answered sometimes

An example of this process from the focus groups is shown in the table below.

Agreement (A in the table) is a yes answer to the teacher question.

Focus Group A (all female) Teacher Component Questions	Before Intervention				After Intervention		
	Member 4	Member 5	Member 6	Member 7	Member 4	Member 5	Member 6
Does your teacher manage to make the mathematics lessons interesting?	SE	A	A	A	A	A	A
Does your teacher appreciate it when you try hard, even if your results are not so good? Are mistakes ok when you are trying to solve a problem in your own way?	SE	A	SE	NR	NR	SE	SE
Does your teacher always show you, step by step, how to solve a mathematical problem?	SE	A	SE	NR	NR	A	NR

A summary table of changes in consensus for all factors over the period of the study is in Table 35 later in this chapter.

4.2.1 Focus groups Results

As previously mentioned (Chapter 3) the classroom intervention began on November 20th 2010 and the first set of focus groups took place in January 2011. As a result of this overlap some of the data in the first interviews refers to answers indicating changes that belonged to the classroom intervention e.g. group work. These are flagged by an * in the tables below.

a. MBRQ **Teacher factor** research question:

Table 27	
Emergent Themes from data on Teacher Factor MRBQ Scale Focus Groups	
Subject of Research Question: Students' beliefs about teacher's role in the mathematics' classroom Focus group interview Questions (Appendix A) Constant comparison analysis Focus groups (Appendix B)	
Question 1	
Does your teacher manage to make mathematics lessons interesting?	
Codes	Themes Developed
Sometimes interesting (M2) Group work (M3)* Videos (M4)* Oranges interesting (M10)* Diagrams and all (M8)	Before Intervention Interesting Activities
Interesting lessons (M4, M5, M7) Play games (M4, 5, 6, 7) Games interesting (M8) Games helped (M9) Not really (M10)	After Intervention Interesting Activities
Question 2	
Does your teacher appreciate it when you try hard, even if your results are not so good. Are mistakes ok when you are trying to solve a problem in your own way?	
Codes	Themes Developed
Not sure (M1) Appreciate hard work (M2) Effort prizes given (M2)* Think mistakes ok (M3) Effort prizes given (M4)* Effort prizes given (M11)*	Before Intervention Teacher appreciates hard work
Mistakes ok (M5) [as long as you are learning and trying] As long as you try (M6) Most of the time (M6, M7)	After Intervention Teacher appreciates hard work
Question 3	
Does your teacher always show you, step by step, how to solve a mathematical problem before giving you exercises?	
Codes	Themes Developed
Shows us on computer (M2)* She writes it on whiteboard (M4) Yea (M5) Shows video (M6)* Yea on whiteboard (M8)	Before Intervention Given step by step instruction
Yea (M5) Most of the time (M7)	After Intervention No theme (Insufficient data)

TABLE 27 (Continued)	
Question 4	
Are you sometimes given new problems that you have to solve in your own way?	
Codes	Themes Developed
Not given solve new problems (M3)	Before Intervention
Explains on the board (M1) (M2)	Teacher methodology sometimes problem-solving
Given solve new problems (M4, M5, M6, M7)	
Not given solve new problems (M8, M11)	
Sometimes given solve new problems (M9, M10)	After Intervention
Sometimes given solve new problems (M5, M7)	Teacher methodology sometimes problem-solving
Some problems more complicated (M5)	
Some problems more complicated tests (M7)	
Question 5	
Does your teacher give you time to explore new problems and try out different ways of solving problems? Does your teacher discuss your own ways of solving problems with you so that you can see their strengths and weaknesses?	
Codes	Themes Developed
Teacher put squares in a box (M1) *	Before Intervention
Explore solutions in groups (M4, M5)*	Teacher sometimes gives time explore new problems
Group work new stuff (M9)*	
Depends on work and time (M5)	After Intervention
Shows us different solutions (M5, M6)	Teacher sometimes gives time explore new problems

A number of themes emerged from the data on the **teacher factor** from the table above. These are:

1. Interesting lessons (Before and after intervention)
2. Teacher appreciates hard work (Before and after intervention)
3. Teacher gives step by step instruction (Before intervention)
4. Teacher methodology sometimes problem-solving (Before and after the intervention methodology used in the mathematics' classroom)
5. Teacher sometimes gives time to explore new problems (Before and after intervention)

The data indicated participants seem to have found a number of activities used during the classroom intervention to be interesting. All of these activities, with the exception of one ('Diagrams and all', Table 27) belonged only to the intervention period in the classroom. The participants mentioned videos, group work and a practical exercise that used oranges to help students derive the formula for the surface area of a sphere (Table 27). Following the intervention,

data indicated the students had enjoyed playing computer-based mathematics games with one student dissenting. M10 indicated the lessons were not really interesting (Table 27).

b: MIC Teacher Factor:

The second method of analyzing the data sought to examine the consensus or not arising from the views of participants as expressed in the interviews. The table below shows two groups of four participants who took part in the focus groups before and after the intervention.

Micro-interlocutor Analysis Teacher Factor: Focus Groups								
Focus Group A (all female) Teacher Factor Questions	Before Intervention				After Intervention			
	Member 4	Member 5	Member 6	Member 7	Member 4	Member 5	Member 6	Member 7
1. Does your teacher manage to make the mathematics lessons interesting?	SE	A	A	A	A	A	A	SE
2. Does your teacher appreciate it when you try hard, even if your results are not so good? Are mistakes ok when you are trying to solve a problem in your own way?	SE	A	SE	NR	NR	SE	SE	NR
3. Does your teacher always show you, step by step, how to solve a mathematical problem?	SE	A	SE	NR	NR	A	NR	S
4. Are you sometimes given new problems you have to solve in your own way?	SE	A	A	A	NR	S	NR	S
5. Does your teacher give time to explore new problems and try out different ways of solving problems? Does your teacher discuss your own ways of solving problems with you so that you can see their strengths and weaknesses?	SE	SE	A	A	A	S	SE	NS

Focus Group B (all male) Teacher Factor Questions	Member 8	Member 9	Member 10	Member 11	Member 8	Member 9	Member 10	Member 11
	1. Does your teacher manage to make the mathematics lessons interesting?	SE	SE	SE	NR	SE	SE	D
2. Does your teacher appreciate it when you try hard, even if your results are not so good? Are mistakes ok when you are trying to solve a problem in your own way?	A	A	A	SE	A	A	A	A
3. Does your teacher always show you, step by step, how to solve a mathematical problem?	SE	NR	NR	NR	A	A	SE	SE
4. Are you sometimes given new problems you have to solve in your own way?	S	S	S	S	NR	D	SD	NR
5. Does your teacher give time to explore new problems and try out different ways of solving problems? Does your teacher discuss your own	SD	SD	SD	SD	NR	NR	NR	NR

The first focus group contained all female members and the second group all male members. The girls would appear to have agreed with each other when answering the questions about their teacher and would appear to be reasonably consistent in the answers that they gave after the classroom intervention. The second focus group, all boys, showed a similar level of consensus in their answers. Hence, the data indicated no major change had taken place in **participants' beliefs about their teacher's role**. In summary, results from the constant comparison analysis and micro-interlocutor above showed reasonable consensus in participants' answers arising from the focus group data. These

outcomes support the quantitative results earlier in this chapter indicating non-significant outcomes for this factor.

c. **Competence** factor questions

Table 29 below shows the themes that emerged from the data collected on this factor from the focus groups:

TABLE 29	
Emergent Themes from data on Competence Factor (Focus Groups Constant Comparison)	
Subject of Research Question: Students' beliefs about their competence in mathematics	
Focus group interview Questions (Appendix A)	
Constant comparison analysis Focus groups (Appendix B)	
Question 1	
Are you interested in mathematics? Do you like what you are learning this year?	
Codes	Themes Developed
Getting used geometry (M1); Geometry easy (M2); Kind of like algebra (M9)	Before Intervention Like geometry, algebra
Don't like maths (M4); Don't like geometry (M5); Geometry hard (M8)	Don't like geometry
After Intervention Interested maths (M5); Like maths brutal at first (M10); Sort of like maths(M9)	After Intervention Like syllabus content this year
Not interested maths (M4, M6); Don't like content this year (M6); Maths boring and hard (M7); Don't understand multiplying (algebra) (M9); Didn't understand algebra last year (M10); Understand when told by (M10, M8)	Don't like syllabus content this year
Question 2	
Do you feel you are behind, where you need to be or ahead of what your class is doing in mathematics this year? What do you feel about this and what do you do about it? Do you think that you will do well in mathematics this year?	
Codes	Themes Developed
Not behind (M2, M3); My marks starting come up (M1); Do ok (M3); Not sure (M6); Definitely pass (M11)	Before Intervention Do well this year
Too hard (M4) (M5); Definitely behind (M8)	Will not do well this year
Kind of ahead in some (M5); Will do alright (M6)	After Intervention Do well this year
Too hard (M4); Not really (M10) No better in Maths than French (M8)	Will not do well this year

TABLE 29 (Continued)	
Question 3	
Do you find that you can do hard mathematical problems if you approach them with patience?	
Codes	Themes Developed
If you have patience (M3, M5, M8, M9, M10) Yes, slowly (M3); Yea (M5)	Before Intervention Can do hard problems with patience
Mixing myself up (M1)	Cannot do hard problems
If you have patience (M5); Concentrate and have patience (M4) Depends on problem (M7) Have patience (when in class) (M10, M11)	After Intervention Can do hard problems with patience
No I've no patience (M8) Can't understand give up (M8)	Cannot do hard problems with patience
Question 4	
Do you have your own way of solving mathematical problems? If yes, do they work? Where did you learn them?	
Codes	Themes Developed
Sometimes own ways use common sense (M2) (M3) People in group give opinion (M1) (M3) Own ways don't work (M4)	Before Intervention Own ways solve problems
Yea sometimes (M5) Mine never work (M4) Own ways head (M7) (M8) (M9) Own ways primary (M10)	Own ways work
After Intervention	After Intervention
Sometimes (M5) No (M4)	Own ways solve problems
Work most of the time (M5) Don't know learn (M5) In my house and in class (M10)	Own ways work
Question 5	
Do you like challenging classwork that helps you learn new things?	
Codes	Themes Developed
Like challenging group work (M3) Someone gives their opinion (M1) (M3)	Before Intervention Likes challenging group work
Sometimes (M4) (M9) Sometimes interesting, sometimes hard (M5)	Sometimes likes challenging work
No (M8) No can't do (M11)	Don't like challenging work
Yea (M5)	After Intervention
Yea as long as I get it right (M10)	Like challenging work
No (M4)	Don't like challenging work

Question 1, above, asked participants if they were interested in mathematics and whether they liked the syllabus content they were learning in the academic year. The females, M4, M6 and M7 continued to find the study of mathematics 'boring and hard' (M7) or were 'not interested maths' (M4, M6). M5 is the exception, indicating geometry was difficult before the intervention and changed

to being interested in mathematics after the intervention. As their teacher, these participants had presented themselves at the beginning of the academic year with similar beliefs about the subject of mathematics.

There was no evidence from the data in the table above arising from question 2 in focus groups that students believed themselves to be more **competent** as a result of the new learning environment. Some of the individual comments indicated signs of enhanced beliefs. M6 was 'not sure' she would do well in mathematics this year compared to after the intervention 'will do alright'. M4 on the other hand continued to find mathematics 'too hard'.

Question 3 asked focus group participants about solving challenging problems with patience. From the evidence in the data above, it would appear to show students had unchanged beliefs about their ability to persist in trying to solve difficult problems. Question 4 on finding and using your own ways to solve problems also showed no change in students' beliefs of their own competence after the learning environment was changed in the study.

The data collected on the final question in the competence factor also indicated no shift in students' beliefs about enjoying challenging work in mathematics. Only one student M5 indicated her view had changed from before to after the intervention. This may be due to M5's attendance pattern during the academic year. She had been absent frequently early in the school year due to family difficulties but this had changed significantly following contact between the school authorities and her parents.

MIC Consensus on the **Competence** factor:

Table 30									
Consensus Tables Micro-interlocutor Analysis Competence Factor (Focus Groups)									
Focus Group A (all female) Competent Factor Questions	Before Intervention				After Intervention				
	Member 4	Member 5	Member 6	Member 7	Member 4	Member 5	Member 6	Member 7	
	1. Are you interested in mathematics? Do you like what you are learning this year?	SD	SD	SD	D	D	A	D	SD
	2. Do you feel you are behind, where you need to be, or ahead of what your class is doing in mathematics this year? How do you feel about this and what do you do about it? Do you think you will do well in mathematics this year?	SD (Behind)	SD (Behind)	NR	S (Not sure)	SD (Behind)	SE (Ahead Some parts)	SA	S
	3. Do you find that you can do hard mathematical problems if you approach them with patience?	SE	SE	NR	NR	SE	A	NR	S
	4. Do you have your own way of solving mathematical problems? If yes, do they work? Where did you learn them?	SD	S	NR	A	D	S	NR	NR
5. Do you like challenging class work that helps you learn new things?	S	S	NR	NR	D	A	NR	D	
Focus Group B (all male) Competent Factor Questions	Before Intervention				After Intervention				
	Member 8	Member 9	Member 10	Member 11	Member 8	Member 9	Member 10	Member 11	
	1. Are you interested in mathematics? Do you like what you are learning this year?	D	S	S	NR	SD	S	SD	A
	2. Do you feel you are behind, where you need to be, or ahead of what your class is doing in mathematics this year? How do you feel about this and what do you do about it? Do you think you will do well in mathematics this year?	SE (Behind)	SE (ok)	SE (ok)	SE (ok)	SE (Behind)	NR	SE (Behind)	SE (Ahead)
	3. Do you find that you can do hard mathematical problems if you approach them with patience?	A	A	A	SE	SD	A	A	SE
	4. Do you have your own way of solving mathematical problems? If yes, do they work? Where did you learn them?	A	A	SE	NR	NR	NR	SE	NR
5. Do you like challenging class work that helps you learn new things?	D	S	NR	SD	NR	NR	SE	NR	

The consensus tables above indicated that the focus group consisting of girls would seem to agree to a reasonable level on the answers that they provided to the questions asked prior to the classroom intervention. This level of consistency would not seem to have changed significantly afterwards. There were participants e.g. M5 who indicated some minor changes had taken place in individuals' beliefs about their own competence in learning mathematics. The table above shows the consensus in the boys' focus group, for the most part, is in agreement with the answers that they provided to the questions. Again, there were some minor differences e.g. M10 would appear to believe that he was doing 'ok' academically before the classroom intervention but was 'behind' after it.

There are indications from the data that the girls' focus group compared to the boys' focus group were less interested in the subject of mathematics.

In summary, the intervention would not appear to have changed the level of consensus shown by students about their beliefs with respect to their competence in mathematics. This result supports the quantitative results discussed earlier in this chapter.

c. **Relevance** Factor themes

Table 31	
Emergent Themes from data on Relevance Factor Constant Comparison (Focus Groups)	
Subject of Research Question: Students' beliefs about the relevance of Mathematics	
Focus group interview Questions (Appendix A)	
Constant comparison analysis Focus groups (Appendix B)	
Question 1	
Do you think that mathematics is an important subject in itself?	
Codes	Themes Developed
Need maths job (M3, M2, M1, M8)	Before Intervention
Need maths certain jobs (M4)	Mathematics is an important subject
Need maths all jobs (M5, M8, M9, M10, M11)	
Need maths later (M11)	
Yea (M7)	After Intervention
Need maths most jobs (M5)	Mathematics is an important subject
Wish it wasn't (M8)	
Why (M4)	Why do we need maths?
Question 2	
Do you think that mathematics has any use in your life? Will mathematics help you earn a living?	
Codes	Themes Developed
Need maths life (M3, M1, M2, M4, M5, M6, M7, M8)	Before Intervention
Need maths architect (M1)	Mathematics useful subject
Need maths engineer (M3)	
Need maths earn living (M4, M5, M6, M7)	
Definitely (M8)	
Its ok (M9)	Maths ok subject
Not my favourite, depends sections (M10)	
Yea (M4, M5, M6, M7)	After Intervention
Needs maths earn living (M5, M7, M9, M11)	Mathematics useful subject
Everywhere (M8, M9, M10, M11)	
Yea banker (M11)	

Table 31 (continued)	
Question 3	
Do your friends like mathematics? Do they find it useful? Do you think that your friends are right about this?	
Codes	Themes Developed
Some of them do (M2) (M6)	Before Intervention
Yea (M3)	Some friends like maths
Friends say don't need maths (M1)	Friends don't need maths
Not a chance (M1)	Don't agree need maths
Don't agree (M3, M4, M5, M6, M7)	
Friends don't like (M4, M5, M6, M7, M9, M10)	After Intervention
	Friends don't like Maths
Need Maths life (M5, M6)	Maths useful subject
Question 4	
Do you think that anyone can learn mathematics?	
Codes	Themes Developed
If they try (M1, M7, M9)	Before Intervention
Am making progress (M1, M11)	Effort not talent learn maths
Twin brother not good maths (M3)	Need talent maths
Yea (M2)	
Can't learn maths (M8)	
Have right approach (M5)	After Intervention
Have patience (M6)	Effort not talent learn maths
Question 5	
Do you think that seeing and discussing different solutions to a mathematics problem is a good way of learning mathematics? Do you think that it is important to learn different approaches for solving the same problem?	
Codes	Themes Developed
You would have more understanding (M2)	Before Intervention
People remember different things (M5)	Learn better different approaches
Some easier (M4)	
One way easier someone else other way (M7)	
Sometimes (M9)	
Too much in your head (M1)	Best learn one solution
For some people maybe (M2)	
Might get mixed up (M3)	
Better learn one way (M8)	
Confusing (M11)	
Understand one and not other (M7)	After Intervention
Try one way than other (M6)	Learn better different approaches
Sometimes yea (M5)	
Group work gets everyone's opinion (M10)	
No (M11)	Best learn one solution
Confusing (M8)	

All of the participants in the focus groups consistently agreed that mathematics was an important and useful subject to learn, both prior to and after the study was undertaken. One participant, M4, conceded the importance of mathematics

as a subject but argued that it was only needed for 'certain jobs' and after the intervention questioned others in her group asking why. The author interprets this as M4 questioning the need for all to learn some of the content on the syllabus that year. Other male participants mentioned careers that needed mathematics such as Engineering (M3), Architecture (M1) and Banking (M11) as particularly requiring knowledge of mathematics. All of the participants in the focus groups agreed that mathematics was required to earn a living both before and after the classroom intervention. There was no evidence from the data that participants in the focus groups had found the mathematics syllabus content studied that year to be more relevant to their lives than it had been prior to this.

All participants in the focus groups acknowledged that some or all of their friends didn't like mathematics and that they thought it not to be a useful subject. The classroom intervention would appear to have had no impact on participants' beliefs with respect to their friends. It would seem that the participants in the focus groups did not agree with their friends that you did not need to know mathematics. Some of these friends belonged to either the experimental class or the control classes in this study. The author's interpretation on the answers provided by participants was that they were the expected views for this cohort of participants. Some of the participants in the experimental class were children of families that were known to the author as a teacher for many years. A small number of these families particularly valued achievement in mathematics as an essential part of a good education for their children. This type of parent would generally have encouraged their child to stay with the study of the higher-level mathematics course particularly at the Junior Certificate stage. This type of parental attitude or its influence on their children was not measured in this study.

One specific characteristic that the new learning environment attempted to instill in participants was that effort to learn would, over time, increase the understanding of and achievement in mathematics. From the small amount of data collected on this question, there was no evidence that any changes had taken place in these participants' beliefs on the value of effort by students in the mathematics' classroom.

Participants' beliefs about learning different approaches to solving a mathematical problem, as a means to learning mathematics better, would seem to be unchanged following the classroom intervention.

Consensus matrix for relevance factor:

Table 32								
Micro-interlocutor Analysis Relevance factor (Focus Groups)								
Focus Group A (all girls) Relevance Factor Questions	Before Intervention				After Intervention			
	Member 4	Member 5	Member 6	Member 7	Member 4	Member 5	Member 6	Member 7
1. Do you think mathematics is an important subject in itself?	SE	SE	A	A	NR	SE	A	NR
2. Do you think that mathematics has any use in your life? Will mathematics help you earn a living?	A	A	A	A	A	A	A	A
3. Do your friends like mathematics? Do they find it useful? Do you think your friends are right about this?	D	S	S	D	D	SD	D	D
4. Do you think that anyone can learn mathematics?	NR	NR	NR	SE	NR	SE	A (patience)	S
5. Do you think that seeing and discussing different solutions to a mathematics problem is a good way of learning mathematics? Do you think that it is important to learn different approaches for solving the same problem?	SE	A	A	SE	NR	S	A	SE

Focus Group B (all male) Relevance Factor Questions	Before Intervention				After Intervention			
	Member 8	Member 9	Member 10	Member 11	Member 8	Member 9	Member 10	Member 11
1. Do you think mathematics is an important subject in itself?	SE	A	A	SE	SE	A	A	A
2. Do you think that mathematics has any use in your life? Will mathematics help you earn a living?	S	A	S	NR	A	A	A	S
3. Do your friends like mathematics? Do they find it useful? Do you think your friends are right about this?	NR	NR	NR	NR	NR	D	SD	NR
4. Do you think that anyone can learn mathematics?	SD	SE	NR	A	NR	NR	NR	NR
5. Do you think that seeing and discussing different solutions to a mathematics problem is a good way of learning mathematics? Do you think that it is important to learn different approaches for solving the same problem?	SD	S	NR	SD	SD	NR	SE	D

There was clear evidence in Table 32 (above) for the girls' focus group that consensus was strong, indicating similar beliefs about mathematics held by the group. The boys' focus group shows very clear consensus amongst those participants too with some incomplete data.

Overall, the qualitative data collected on students' beliefs about the relevance of mathematics to their lives showed no significant changes from before to after the participants had experienced the new learning environment and hence supports the quantitative data.

d. Emerging themes: **Mathematics as an inaccessible subject** factor.

Table 33	
Emergent Themes from data on Inaccessible Factor Constant Comparison (Focus Groups)	
Subject of Research Question: Students' beliefs about mathematics as an inaccessible subject	
Focus group interview Questions (Appendix A)	
Constant comparison analysis Focus groups (Appendix B)	
Question 1	
Do you think that if you cannot solve a mathematics problem quickly you quit trying?	
Codes	Themes Developed
Keep on doing them (M2)	Before Intervention
Try find answer (M3)	Persevere solve problems
Just keep trying (M1)	
I do (M5)	
I'm not bothered anymore (M4)	Cannot solve quit
Gets too complicated (M6)	
Gets messed up (M7)	
Harder I try worse gets (M11)	
Mind goes blank (M10)	
Yea (M4, M7)	After Intervention
Yea I do (M6)	Cannot solve quit
Question 2	
Do you think that only very intelligent students can succeed at mathematics?	
Codes	Themes Developed
Good at maths bad at other subjects (M2)	Before Intervention
Teacher said try hard succeed maths (M1)	Don't need to be intelligent succeed maths
Am too good not study (M3)	
No (M4, M5, M6, M7)	
No (M5, M6, M7, M9, M10)	After Intervention
It depends (M11)	Don't need to be intelligent succeed maths

Table 33 (Continued)	
Question 3	
Do you think that learning mathematics is mainly about having a good memory? Can people do ok in mathematics who are not very good at memorization?	
Codes	Themes Developed
You have to remember how to do the method (M1) There's formulas you have to remember (M4) Have to learn theorems (M8) Yes (M4, M5, M6, M7)	Before Intervention Need good memory succeed at maths
Yea (M6, M7) Have to learn theorems (M5) Won't do best (M5) Have to remember equations (M9) Have to remember angles and stuff (M8)	After Intervention Need good memory succeed at maths
Question 4	
Do you think that getting the right answer in mathematics is more important than understanding why the answer works?	
Codes	Themes Developed
Figure it out yourself (M1) What's point don't know how it works (M4) Won't remember it again (M2) No (M5, M6, M7) Only answer test less marks (M11)	Before Intervention Understanding is more important than right answer
	After Intervention
No once you get it (M4) No if you don't understand it (M5) You need to know how (M9) Learn from mistakes (M10) Understand better chance solve (M9)	Understanding is more important than right answer
Question 5	
Do you think that there is only one way to find the right answer to a mathematical problem?	
Codes	Themes Developed
There's lots (M3) (M2) There are different solutions (M4) There's a few (M1) No more than one way (M4, M5, M6, M7) Always another way (M10)	Before Intervention Many solutions mathematics problems
	After Intervention No theme insufficient data

This factor (**mathematics as an inaccessible subject**) consisted of the last set of 13 questions from a total of 57 questions that participants were required to answer. It resulted in the collection of the smallest amount of data from all of the factors explored in the focus group data.

There are clear indications from the data above that, prior to the intervention, participants believed that they were not able to persist in trying to solve mathematical problems. Evidence from females M4, M6, M7 and males M10 and M11 indicate that they give up if it became difficult. Evidence after the intervention shows M4, M6 and M7 still believed they quit if they cannot solve a problem quickly.

Asked (in question 2) whether a student needed to be intelligent or not to succeed when learning mathematics, all the female participants in their focus group gave a negative answer. After the intervention there was no change in this expressed belief found in the data above (Table 33).

There was also general agreement amongst the participants that a good memory was necessary to succeed at learning mathematics from the start to the end of this study (M4, M5, M6, M7 and M8). Reasons given, both prior to and after the intervention, for the necessity of a good memory included having to learn the proofs of the theorems and formulae, and remembering equations and angles. The change of learning environment would seem to have had no impact on these beliefs.

Participants distinguished between getting the right answer to a problem from understanding the solution very clearly when answering question 4. All of the students who contributed to the data were adamant that you needed to understand a solution to a problem. Getting the right answer and not understanding the solution was not satisfactory. 'What's the point?: don't know how it works' (M4). These beliefs on the need to understand would not have appeared to change as evidenced from the data collected above.

The final question asked in the focus groups for this factor was whether they thought that there was only one right way to solve a problem. Data was only available from before the intervention, as participants had not supplied answers afterwards. The evidence would seem to show that most of the participants believed that there was more than one method of solving problems (M1, M2, M3, M4, M5, M6, M7, M10).

e) Consensus matrix for **mathematics as an inaccessible subject**

Table 34								
Micro-interlocutor Analysis Inaccessible Factor (Focus Groups)								
Focus Group A (all female) Inaccessible Factor Questions	Before Intervention				After Intervention			
	Member 4	Member 5	Member 6	Member 7	Member 4	Member 5	Member 6	Member 7
1. Do you think that if you cannot solve a mathematics problem quickly you quit trying?	SE	SD	SE	SE	A	NR	SE	A
2. Do you think that only very intelligent students can succeed at mathematics?	D	D	D	D	NR	D	S	S
3. Do you think learning mathematics is mainly about having a good memory? Can people do ok in mathematics who are not very good at memorization?	SE	A	A	A	NR	A	A	A
4. Do you think that getting the right answer in mathematics is more important than understanding why the answer works?	SD	SD	D	D	SD	SD	SE	SE
5. Do you think there is only one way to find the right answer to a mathematics problem?	SD	D	D	D	NR	NR	SE	SE

Focus Group B (all male) Inaccessible Factor Questions	Before Intervention				After Intervention			
	Member 8	Member 9	Member 10	Member 11	Member 8	Member 9	Member 10	Member 11
1. Do you think that if you cannot solve a mathematics problem quickly you quit trying?	SE	SD	SE	SE	NR	NR	NR	NR
2. Do you think that only very intelligent students can succeed at mathematics?	NR	NR	NR	NR	NR	D	D	D
3. Do you think learning mathematics is mainly about having a good memory? Can people do ok in mathematics who are not very good at memorization?	S	A	A	SE	S	SE	S	NR
4. Do you think that getting the right answer in mathematics is more important than understanding why the answer works?	A	D	D	SD	A	SD	SD	NR
5. Do you think there is only one way to find the right answer to a mathematics problem?	D	D	SD	D	NR	D	D	SD

The consensus tables (Table 34, above), for the fourth factor ‘mathematics as an inaccessible subject’, indicated consistency amongst students’ beliefs over the duration of the study. There were some changes indicated in the answers given by two girls in their focus group. These changes show a somewhat more negative belief on the part of participants M6 and M7 on the need to be intelligent to succeed at mathematics and that getting the right answer to a problem is more important than understanding a solution. The author’s interpretation of this would be that M6 and M7 had struggled with the pace of work required in the experimental class during the academic year. However, both M6 and M7 had parents who valued their children studying the higher-level mathematics course and who may have insisted that they stay studying the higher-level course.

The tables above show that participants did, for the most part, display a consistent level of consensus in answers given in the data above. The table below highlights the minor changes in answers from before to after the

classroom intervention was completed in the data from the consensus tables above for all of the factors examined.

Table 35				
Changes in Consensus (Micro-interlocutor Analysis-Focus Groups)				
Factor	Participant	Before	After	Information Sought
Teacher	M10	SE	D	Are lessons interesting?
Competence	M5	SE 'Behind'	SE 'ahead some parts'	Are you behind or ahead of your class this year?
	M10	SE 'OK'	SE 'Behind'	
	M11	SE 'ok'	SE 'Ahead'	
Relevance				
Inaccessible	M6	D	S	Only very intelligent students can succeed at mathematics?
	M7	D	S	

Table 35 shows the small number of changes that took place in answers over the study indicating the level of consensus amongst participants had remained constant.

In summary, from the small quantity of data collected, there is no evidence that participants' beliefs about mathematics as an inaccessible subject had been altered over the duration of this study.

4.2.2 Results of Individual Interviews

Following the completion of the focus groups the individual interviews were carried out. Unlike the focus groups, there had been no individual interviews prior to undertaking the classroom intervention. Hence, the data from the individual interviews can only serve to corroborate, enhance or contradict the beliefs professed by the students from the other data collected. The reasons for the inclusion of the interviews were to identify individual beliefs and to provide an opportunity for participants to say anything that they may have felt unable to say in a focus group situation. The interviews took place between May 13th and May 21st, 2011. A pilot interview, as previously mentioned, was conducted with one student (M2) and this was followed, after a few minor changes to the questions, by three other individual interviews with M5, M12 and M13 from the

experimental class: these are noted below in the analysis. M2 and M5 were members of the focus groups in the study unlike M12 and M13. Two participants were male and two female. M12 had volunteered for the interview and M13 had been invited to take part by the author and did appear to be delighted to accept the invitation. M13 had appeared to be sufficiently confident to express her true beliefs to the author. A brief introduction to these students can be found in Appendix C.

The ten questions used with participants in the interviews were chosen, as described in Chapter 3.

The data discussed in 4.2.2.1 below indicate the themes that emerged from the individual interview data collected about the teacher's role, personal competence, relevance of mathematics to their lives and mathematics as an inaccessible subject.

4.2.2.1 Students' beliefs about teacher's role in the mathematics classroom

Questions 1 and 2 in the individual interviews provided data on the students' beliefs about their teacher's role in the classroom:

Question 1:

Do you do a lot of group work in maths class this year? Do you feel that working in a group helps you when you are solving problems in maths? In what way does it help/not help? What do the members of your group think? Do you agree? Why/Why not?

Question 2:

What else could your teacher do to make teaching work well?

The answers provided by participants to these questions is discussion next:

(i) Group work

Participants who made comments about group work, which was new to them, indicated that they appeared to be positive towards its use in the mathematics' classroom. The literature on small group work in mathematics' classrooms

shows that it provides opportunities for higher-order thinking and problem solving when compared to traditional mathematics lessons (Mulryan, 1994:289). One participant (M13) was adamant that she did not like group work. This was also consistent with the research that most students, but not all, like working with other students (Whicker et al., 1997:47).

M13 did expand her dislike of group work at the end of the interview when she said:

‘Groups are a joke. If everyone was doing their work [individually] and not allowed to talk [with other students]. I would prefer [to work] by myself. I just switch off [during group work]’. (Appendix B).

M13’s reaction to the use of group work was largely ignored by other participants in the class. Prior to the classroom intervention the participants were expected to work in a quiet classroom where some whispering did take place as participants requested help from those sitting beside them. In the author’s view, this atmosphere in the mathematics classroom could be described as ‘usual’ in Ireland prior to 2010.

Other answers on group work included:

‘Because if you don’t know something the other person might know it’ (M12, Appendix B).

‘If I don’t understand a part another person might and if they don’t know it I might know it or she may know what I don’t know’ (M5, Appendix B).

Participants were expected to work together to solve various problems in formal cooperative groups with the goal of teaching specific mathematical content. Shared goals were set for groups to achieve, discussion of strategies for proposed solutions were expected, also helping each other to understand the material making the group more than the sum of its parts (Johnson and Johnson, 1999:68). Participants had been introduced to the use of group work through the use of appropriate activities to help support their understanding of the process.

M13 made several attempts to get the author to desist from using group work. On two occasions she refused to join her group and insisted on being given a

copy of the activity to work on by herself (Appendix B). School procedures were used over a number of periods to eventually achieve compliance. The author had opted for a gentle approach initially to solve this difficulty with a view to encouraging a positive view in M13 of the use of group work. Information later showed M13 had exhibited significant behavioural difficulties at that time with many of her teachers.

(ii) How the teacher can make teaching work well

Participants' beliefs on how the teacher could make teaching work well, possibly due to their age, yielded little clear information. Comments included the following:

- 'Can't think of anything' (M2)
- 'Don't know' (M5, 12)
- 'Play more games' (M5)
- 'Come to me first' [when teacher is helping in class] (M13)

4.2.2.2 Students' beliefs about their own competence in mathematics

Questions asked in the individual interviews that related to participants' perceptions of their own competence in mathematics were:

Question 3: Do you like what you are learning in maths class this year? Are there some topics that you like better than others? Can you tell me which and why? What do your friends think about maths? Do they like it? Do you agree?

Question 5: Do you do the best that you can in maths class this year? Why is that? Do you want to get a higher score than other students in your class in mathematics? Why/Why Not? Is it important for you to score a higher mark in maths tests than other students? Why/ Why not?

Question 8: Do you think that everybody has to think hard to solve a maths problem? Is it the same for all the topics covered in maths class this year? Can you tell me why/why not?

Five separate themes arose from the individual data collected from this factor of the MRBQ. They are:

- (i) I like what I am learning this year [in mathematics]

- (ii) Effort made to learn and the reason for making that effort
- (iii) I want to be top student in mathematics
- (iv) I had to think hard for some problems
- (v) Some topics problems are hard

i) I Like what I am learning in Mathematics this year

All four interviewed participants indicated that their friends did not like mathematics as a subject although not all agreed with them. One female participant, when working on co-ordinate geometry 'make your own code' activity, created a message saying 'I hate maths, It is stupid' (M4). M4 had completed the exercise perfectly. This indicated M4's beliefs about mathematics had not been influenced over the duration of the study.

Comments from interviews (Appendix B) on mathematics as a subject included:

'Something [Mathematics] that is really hard and is really boring' (M13)

'They [friends] don't like it. They think it is too hard' (M2)

All four participants interviewed individually indicated that they liked/did not like some of syllabus content taught during the year. Comments included:

'Algebra is better and geometry than multiplying and dividing' (M2) and
'I like the graphing. I don't like percentages' (M5)

'I like graphs and Pythagoras' (M12)

'I like algebra because I can do it. I hate the rest 'cause I can't do them.
'Everything else takes me ages to cop on. I hate money problems
because I can never work them out' (M13)

These comments suggest that no particular topics from the syllabus would seem to have been liked by all participants with some emphasis on the difficulty in learning the topic being a reason for making choices. Ferla et al. (2009) suggest that participants' academic self-concept strongly influences their academic self-efficacy beliefs.

ii) Effort made to learn and the reason for making the effort

Two of the students (M2, M5) indicated that they did their best to learn in class. M12 indicated that he had not worked well prior to Christmas but did work hard afterwards. M13 indicated that she tried to work hard some of the time. Kloosterman (2002:248) writes of students' beliefs affecting the effort they make when learning mathematics. Knowledge and beliefs, he maintains influences action. Comments from the data included the following:

'[I] tried really hard doing problems' (M2)

'Yea. In class-time I didn't really try before Christmas. When I seen my 'Christmas result I've tried really hard [since then]' (M12)

'Well I try' [All the time?] 'No' (M13)

Participants indicated a number of answers as to why they made an effort to learn mathematics. These included more success in life, Liking mathematics as a subject and it helps you to get a good job later in life.

Comments made by participants included:

'You will do better in life with a high score' (M2)

'cause I like maths' (M5)

iii) I want to be the top student at mathematics

Participants ambitions to be the best at mathematics varied in those interviewed.

Comments made included:

- 'I want to be the best' (M13)
- 'Not important [to get highest score] but it is good to have a good score' (M2)
- 'You can't compare yourself to the best person in maths- you just have to try your best' (M12).

M13's aspiration to be the best student of mathematics was consistent with one of the out-of school aspects of her life. She had had some experience in competing at world level in hip-hop dancing competitions and had achieved a 2nd place result in one area of dancing at that competition.

iv) I had to think hard to solve some problems

Participants' beliefs about the need to think hard to be able to solve some mathematical problems was unanimous. All agreed that it depended on the

topic being learned. Comments from the data included:

‘There are different parts that you have to work harder for’ (M5)

‘On some topics you have to think hard. On others you just know the answer straight away’ (M12)

‘It depends on the topic’ (M13).

f. For some topics problems are hard to solve

Topics mentioned in the data varied about the type of problems that were difficult and hard. They included algebra, percentages, equations and perimeter and area, geometry. M2 gave data on difficult problems within sections of the course.

‘Algebra with brackets.... you have to count every x and y’s. Pretty hard’ (M2)

‘I don’t have to think hard for algebra’ (M13)

‘For me its percentages, it’s so kind of complicated (M5).

‘Geometry [is hard]. Constructions in geometry, I don’t have to think hard about them’ (M12).

Individual participants perception of their difficulties with some of the syllabus content does not match achievement scores in some instances. This is discussed in Chapter 5.

4.2.2.3 Students’ beliefs about relevance of mathematics to their lives

The question asked in the individual interviews with respect to the third research question and factor of the MRBQ scale ‘relevance of mathematics to students lives’ was:

Question 4: Is maths a useful subject? Why do you think that? Do your friends see it as useful? Do they work hard?

The data collected in answer to this question are discussed next:

i) Maths is a useful/useless subject.

All four participants interviewed expressed their belief that maths is a useful subject. In a review of post-primary mathematics in Ireland in 2005 (NCCA, 2005:19), results indicated that almost all of the students believed that mathematics is a useful subject. The belief that you need to know mathematics

for your job was mentioned by three of the participants and the fourth participant believed mathematics was everywhere. Three of the four expressed their belief that some friends thought that mathematics was a useless subject. One participant thought that some of her friends didn't think they would need any job and hence they did not think they would need to learn mathematics. Another participant mentioned that some students believed that they did not need maths for their particular career choice. There was some confusion with M12 as to how mathematics was linked to certain careers with a view being expressed that mechanics and carpenters do not need mathematics. The literature identifies students only putting effort into learning when they perceive outcomes will help them fulfill their personal goals (Kloosterman, 2002:248).

Comments from the data about the usefulness of mathematics as a subject included:

- 'Maths is very [important]. Every day in every job' (M2).
- 'They don't think they'll need a job so they don't think they'll need it' (M5)
- 'Others think they don't need it for what they want to be' (M12)
- '[Friends] They know its useful' (M13)
- 'yea say your mam and dad go to the bookies and put money on a horse- say 11/1 and you put a fiver on it you have to add up what you will win' (M12)

4.2.2.4 Students' beliefs mathematics is an inaccessible subject

Questions asked in the individual interviews with respect to the fourth research question and factor of the MRBQ scale 'relevance of mathematics to students lives' were:

Q. 6: Do you think it takes a special talent to do well in maths? Do you have such a talent? Can people do ok even without special talent? Why?
Q. 7: Is memory important in maths? Are you good at memorizing? Can someone who is not good at memorization be good in maths (or even ok in maths)?
Q. 9: Do you think that it is important to get a good grade in maths? Why? Is the grade achieved the only thing that is important when studying maths? What do your friends think? Do they agree?
Q. 10: Does your teacher think that she knows everything best? What do you think?

10 themes have emerged from the data provided by the participants in the individual interviews. They are:

- i) I have talent [mathematics]
- ii) You need a special talent to succeed in mathematics
- iii) Memory is important when learning mathematics
- iv) Having the right attitude to learning mathematics
- v) Grade achieved in mathematics is important
- vi) Different approaches to solving problems
- vii) Understanding [mathematics] is important
- viii) Maths is harder in second year (compared to first year)
- ix) Effort to learn
- x) Teacher thinks she knows best

i) I have talent [mathematics]

Participants provided a variety of answers as to whether they believed that they have talent or not in mathematics. One participant hoped that he had talent in mathematics, another was unsure if she did have talent, a third thought he had talent for certain topics. A fourth believed that she did not have any talent for mathematics: she was the one who wanted to be the top student in the class at mathematics.

Leonard (2012:14) argues that self-concept has a significant impact on achievement when learning mathematics. The achievement levels (Appendix B) of these participants in mathematics at the end of this study indicated that M2, who hoped he had talent in mathematics, increased his score by 12%, M5 was a little unsure if she had talent and M12 believed he had talent for some topics in mathematics. Both M5 and M12 maintained their achievement levels from 1st year to 2nd year. M13, who believed that she had no talent in mathematics, decreased her score by 8%.

ii) You need a special talent to succeed in mathematics:

Comments from the data collected on this question included:

- 'No, everybody can learn how to do maths' (M2)
- 'I'm not too sure. I'm quite good' (M5)
- 'Not really. I'm alright at algebra and the graphs' (M12)
- 'I don't think so' (M13).

Participants showed subject to qualification, beliefs held showed mathematics can be learned without a special talent subject to qualifications. These qualifications included a good memory was needed to learn theorems (M5), depends on the topics being studied (M12). The literature shows that as students get older they often attribute their lack of success to a lack of talent rather than other reasons (Lafortune et al., 1996:84). Other studies concentrate on parental influence on the need to have a special talent. Lyons et al. (2003:19) found that the parents interviewed in the Irish study believed that success in mathematics was dependent on having a natural ability in the subject. In the same study one particular group of parents, not quantified, who were concerned about their children's performance and attitudes towards mathematics believed they were linked in some way to parents' personal negative experience of learning mathematics at school. Comments from the data collected included:

'There are different parts [of syllabus] that people won't do the best in, some people might like that part and not like other parts' (M13)
'No. Not really. Some of the best people [at maths] could be bad at one thing [topic] and you could be good at that' (M12)
'If they really want to yea but if they don't bother they're not going to [do well]' (M13). Although M13 made this comment she had generally not acted on it herself when in the classroom.

iii) Memory is important when learning mathematics

All of the students believed that memory was important, to some extent, in the learning of mathematics. Two of the participants indicated that a good memory was necessary for examinations. One participant thought it was very important and the final participant believed that a good memory was not the only requirement to succeed at mathematics. Despite this, participants appeared to believe that you could do reasonably well even if you did not have a good memory. Students' belief that people who are poor at memorizing can still do well in mathematics, despite the inherent contradiction, is typical of many students (Kloosterman, 2002:263). In the Irish context research on gender showed that the majority of questions in the state examination (2001/2002 Junior Certificate (Age 15/16)) across all levels focused on memory through the recall of formulae and procedures rather than an application to problems

(Elwood et al., 2003:74). The research would seem to support participants' views of the importance of memory to some extent at least when undertaking state examinations at that time.

Kloosterman (2002:260) says mathematics taught traditionally emphasizes memorization of procedures and that this can affect students' motivation to learn if they have doubts about their ability to memorise formulas and procedures. All four participants interviewed appeared to believe that they had a reasonable memory but that sometimes it was linked to particular topics in the syllabus. M13 linked the quality of her memory to whether she had enjoyed learning the topic or not. Has the intervention had an impact on these participants' beliefs? The small amount of data collected can only inform about these students' beliefs. With this proviso the evidence would seem to indicate no change in these beliefs had taken place in this study.

Comments from the data include:

'I think I am ordinary [memory] '(M2)
'cause as I said memory isn't everything. So you don't need to memorise everything' (M5)
If I like it yea but if I didn't I'd probably not remember [it] if I didn't enjoy it' (M13)

iv) Having the right attitude to learning mathematics

Negative attitudes enable students to experience learning mathematics as a burden (Lafortune et al., 1996:83). M5 believed that having the right attitude was required, even though some of her friends did not have it all of the time.

Comments made:

'You have to enjoy it. You can't just sit there and go like I hate this then like try to get really good you have to like not enjoy it but have the right attitude to it' (M5).

v). Grade achieved in mathematics is important

According to the results of the PISA 2003 mathematical literacy scores for Irish 15 year olds showed, compared most other countries showed a relatively low number of students scoring at the high end (Conway and Sloane, 2009:241).

All four participants interviewed seem to think that achieving a good grade in mathematics was important. Further questioning on exactly what they meant by a good grade showed that M2 believed a grade B or better in the higher or ordinary paper, M5 believed full marks were not necessary but a bare pass (40%) would need to be worked on, M12 believed 50-60% in higher paper or 70% in ordinary paper or 80% in the foundation level was good and M13 believed a grade A or B or C was good on any paper. None of the participants commented on the fact that taking the higher paper was significantly more valuable for their futures than taking the ordinary paper.

Comments from the data include:

'A, B or higher for me and [same for] ordinary [level] (M2)
'You don't have to get like 100 but if you don't get 40 you have to work really hard but if you get in between it will be alright' (M5)
'In Higher 50-60%, in Ordinary 70%, Foundation is that lower than Ordinary..then 80%' (M12).

vi) Different approaches to solving problems

M2 was the only one of the four participants interviewed who, when asked what else other than a good grade was important, discussed learning different approaches to solve problems. Comments made:

'You learn different ways to solve that you can solve any problems you might associate it ...you might think of a few ways to solve it' (M2)
'Associating [it] with what you know- a cylinder with a tree trunk for example' (M2)

vii) Understanding [mathematics] is important

Three of the participants believed that understanding the mathematics to be learned was important. Comments made included:

'I think being good at understanding' (M2)
'Because like anyone can just type numbers into a calculator and get an answer like but if you understand how you do it how you got how it works that is the main thing' (M5)
'Knowing that you can do the sum after the teacher has explained it' (M12)
'No you have to know it not just get the grade like' (M13)

viii) Mathematics is harder in second year [compared to first year]

In a 2004 study carried out in Ireland students indicated that they found mathematics harder or the same in secondary school when compared to mathematics in the primary school. This was particularly true, Smyth et al, claim, of students studying the higher level and this compares unfavourably with the study of the subjects English or Irish (Smyth et al., in NCCA, 2005:19). M5 raised the increased level of difficult mathematical content that the students had encountered in second year when comparison to the previous year. She said:

‘Some of us loved maths in first year and in second year it was very hard at higher level’ (M5)

..because it’s not like we didn’t do much last year. We kind of revised all of the stuff we were meant to know in the primary school so now we have to do loads this year’ (M5).

The author had heard similar comments made over many years.

ix) Effort to learn

M12 raised as important the need to work hard by commenting:

‘If you didn’t put in all the work you wouldn’t get the high grade’ (M12)

x) Teacher thinks knows best

A final question asked participants whether they thought that the author knew everything best. All interviewees were reminded to answer the question truthfully. Comments made were:

‘She does think she knows best but for some people they might know their own way which is best for them’ (M2)

‘No because we have different ways. She shows us one way and we’d have another way that she didn’t show us...whichever way works best for you that’s grand (M5)

‘Well she knows everything about maths! She helps her students an all’ (M12).

There is evidence from the data collected from the individual interviews students’ beliefs about ‘mathematics as a subject’ had remained somewhat constant across the intervention.

The data collected from the focus groups and the individual interviews was similar in outcomes and would appear to add no new insights to this study.

4.3 Combination of Quantitative and Qualitative Results

The fully mixed sequential dominant status design was used in this study, described earlier in Chapter 3, wherein the quantitative approach had dominant status. The qualitative approaches were expected to provide data that might support, or not, and also help to expand the outcomes from the quantitative findings.

The research questions in this study had sought to find whether students' beliefs about the 'role of their teacher', 'perception of their competence', 'the relevance of mathematics to their lives' and 'mathematics as an inaccessible subject' had become more positive after the implementation of a classroom intervention.

Outcomes from the analysis of the quantitative data produced non-significant results indicating no positive changes had taken place in students' beliefs of the factors examined following the implementation of the new learning environment in the classroom. The qualitative data from the focus groups and individual interviews supported the quantitative outcomes indicating a small number of differences in the beliefs explored. Had there been a difference between the outcomes from the quantitative and qualitative data the author would have accepted the outcomes from the quantitative data as it was the dominant approach used in this study (Chapter 3).

Chapter 5) Conclusions

The main part of this chapter reviews and interprets the results from this study, links them to earlier research and examines their significance. Insights are drawn together from these results providing answers to the research questions set down at the beginning of this study. This is followed by the implications arising from these outcomes for the author's practice, for other teachers, policy makers, the system and for theory. The following section reflects on some of the strengths and weaknesses of the study and their implications for further research and practice in mathematics education. Future relevant research is then identified in terms of future scholarship. The chapter concludes with some general comments on the study.

5.1) Discussion of Results

This study sought to enhance in a positive way students' mathematical beliefs by changing the learning environment in the classroom. The results from this study are examined and assessed below in answering each of the research questions explored. The findings from each of the research questions are described below. They show, for the main part, non-significant positive changes to students' beliefs about mathematics and its teaching and learning. What may be viewed positively is the singular lack of negative outcomes in the findings. The outcomes from this study also highlight the additional challenges involved in changing the beliefs of learners in DEIS schools about mathematics and its teaching and learning.

The changes made to the learning environment identified in this study are shown to have been a positive change for learners who for the most part welcomed them. The point in time that the study was carried out, discussed earlier in Sections 1.2 and 3.5.2.1, was a time of flux in second-level mathematics education over many levels for teachers and schools. Black and William (1998:2) write that there is widespread evidence that fundamental change with new modes of pedagogy can only be achieved slowly. Difficulties and obstacles to change taking place inside classrooms do present challenges to teachers and researchers particularly where classrooms as in this case had experienced few changes prior to this study over previous decades. The new

learning environment was welcomed and with that the author can be somewhat confident that over time students' beliefs will become more positive in the future.

5.1.1) Research Question 1

Issue

Following the classroom intervention with the experimental class this study sought to discover whether **students' beliefs about their teacher's role** had been changed by their experience of the new learning environment.

Findings

Results were clear and unambiguous from both the quantitative and the qualitative data collected and analysed. The results showed that no significant changes, either positive or negative, had taken place in the beliefs of participants in the experimental class about their teacher's role in the mathematics classroom.

Quantitative results (Table 17) for all participants (experimental and control) carried out on the data for the sub-factor 'Perception of teacher interest' using related t-tests was shown to be significant ($p=0.045$). This may have been due to the larger sample size.

Sub-factors of beliefs about the teacher's Role

Independent t-tests were also carried out on the sub-factors of the teacher's role factor for both the experimental and control classes. The figures would seem to confirm the somewhat constant beliefs held by participants throughout the duration of this study on the teacher attending to students' meaningful learning and students' perception of teacher interest.

Two independent variables in this study, class and gender, were also tested for possible influences on the outcomes of the scores on the teacher factor and sub-factors. Table 21 shows that there were no main effects of gender or grouping and no interaction effect. There is therefore no evidence that gender interacts with class with respect to the teacher's role in the classroom in this study. Hence, any initial gender differences and class differences remained

constant in the sample studied after the intervention with respect to the teacher's role in the classroom.

The qualitative data collected on the teacher's role in the classroom from the focus groups and individual interviews were analysed using constant comparison analysis and micro-interlocuter analysis. Despite the qualitative outcomes showing no major change in students' beliefs about the role of their teacher in the mathematics classroom, there was some indication that participants in the experimental class welcomed some of the characteristics of the new learning environment.

The main themes arising from the qualitative data are now discussed:

i) Teacher used interesting activities that made teaching work well:

As indicated by the focus group transcripts (Table 27), the participants found some of the new activities used by the teacher to be interesting (M4, M5, M7). There was evidence from the diary (Appendix B) that participants had enjoyed, for example, a card-matching exercise in algebra, matching words with corresponding expressions and an exercise using oranges to enable students to discover the formula for the surface area of a sphere. The benefits of using rich meaningful activities and tasks with students that focused on discussion in the classroom is well supported in the literature (Boaler, 2009; Swan, 2004; Peressini et al., 2004).

Stylianides et al. (2014:27) reported on a key idea underpinning their instructional intervention that attempted to engineer a powerful and dramatic episodic memory with a view to overshadowing students' earlier memories that had shaped their problem solving beliefs. Stylianides et al. used a problem they described as having 'memorable characteristics'. This, they claimed, impacted on students' problem-solving beliefs in a positive way and helped to encourage perseverance in solving problems with lasting learning gains. In the current study, the success of the orange activity in this study to discover the surface area of a sphere might, perhaps, be understood in similar terms i.e. as providing a memorable learning experience for some participants (Table 27). However, there was no evidence from this study that the activity had had a

major impact on participants' beliefs or a significant impact on their achievement scores. One possible reason may be that the participants in the Stylianides et al. study were undergraduate students and hence cannot be compared to the younger participants in this study whose profiles included a very different socio-economic status. The author, as a result of her extensive experience in the classroom, suggests that providing this type of memorable experiences for second-level learners in a DEIS school are insufficient, of themselves, to impact in a major way on their mathematical beliefs.

On a methodological - rather than substantive - point, M10 showed in his response that his view on what is an interesting lesson had changed from before to after the intervention. In considering lessons beforehand on what he had found interesting he declared 'Oranges [activity] interesting' though after the intervention he regarded this same activity as 'not really' interesting. The 'oranges' activity had, in fact, belonged to the intervention period but the first focus group took place a number of weeks after it had begun due to the condensed academic year. These comments suggest that the qualitative data may under-report the impact of the intervention as the initial impact may have affected the view expressed at the start of the data collection. The micro interlocutor analysis did help to highlight this individual's changing views as represented in the focus group data. Hence, it supports the view in the literature that the micro interlocutor analysis can help to strengthen the analysis of focus group data (Onwuegbuzie et al., 2009c:7).

Individual interviews (Appendix B) carried out at the end of this study indicated that participants were too young and inexperienced, when asked 'how the teacher could make teaching work well in the mathematics classroom, to discuss this type of question. M5 (Appendix B) in her individual interview did, however, suggest that the teacher could

'Play more games, far more. Do it in a memory way, not just look at it and memorise it, but not in a game but kind of if you do it in a playful way'.

M5 would seem to be suggesting ways the teacher might establish a productive frame of mind in students. Harasymowycz (2008:292) argues that it is the frame of mind, rather than the activity itself, that creates the difficulty. Students,

she argues, spend most of their time doing repetitive and mundane activities in mathematics classes in comparison to art classes where they are engaged most of the time showing what they can do with what they have learned and/or where they can show off a personal creation.

ii) Teacher showed her appreciation of hard work

Other themes in relation to the teacher's role derived from the qualitative data included the participants' belief that the teacher appreciated the hard work of students and that making mistakes was expected, when learning mathematics, as they are part of becoming a successful learner of mathematics. M2, M4 and M11, when asked in their focus-group interview if the teacher valued hard work, mentioned that their teacher had given small prizes for sustained effort to learn mathematics at the end of each month. Kloosterman (2002:248) argues that students only put in the effort to learn when this results in fulfilling a person goal. Stipek (2001:214) links extrinsic rewards to a performance examination-oriented model generally used by teachers using traditional textbook-teaching methods. Despite this evidence from the literature the author argues, from her long experience of teaching in this DEIS school, that the use of the small effort rewards did create a sense of competition but, more importantly, they were seen by participants as recognition of their hard work. This would seem to be supported by the focus group data. This may, the author suggests, have been due to the age of the student cohort and/or their socio-economic status.

iii) Teacher methodology is sometimes problem solving i.e. when using group work:

Evidence from the focus groups in relation to the methodologies used during the classroom intervention indicated the participants had recognised the introduction of some of the changes made to the environment. Most of these changes were not hugely evident in the data collected. However, exceptions were the use of group-work and the use of some interesting activities, as mentioned above. The fact that participants noticed the introduction of small group work in the classroom was evident in both the focus group data and also in the individual interview data. There was general approval for the use of group-work in the classroom by the teacher. M2, M4, M5 and M8 indicated that the teacher had given them step-by-step instructions when solving problems

(Table 27). Data are limited and somewhat confused, in that some contradictions were evident, but the study does indicate that participants seemed to link solving new problems and exploring different solutions to when group-work was used in the classroom. This may help to support De Corte et al.'s (2008:378) point of view that old learning environments, with embedded associated beliefs and habits, might need to be deconstructed to enable a more productive outcome.

Possible Explanations for outcomes:

The students' beliefs about their teacher's role in this study were not more positive - but neither had they become more negative - across the intervention. Some other studies had shown small positive changes to students' beliefs following the changing of the learning environment (Verschaffel et al., 2000; Mason, 2004). On the other hand, Martinez's (2011) study had not shown significant changes in students' beliefs, in general, about mathematics and its teaching and learning.

A fundamental change to the culture of the classroom took place with the participants in the experimental class in this study. Changing to a predominately student-centred perspective is challenging for students and it is also not without its challenges for the teacher. Hoyle (1982:368) found some students liked the challenge presented in tasks while others preferred the teacher to make it easy for them, possibly because of anxiety or feelings of inadequacy and shame developed from bad experiences when learning mathematics. Hoyle maintains this highlights certain problems for the teacher identifying conflict between students and their expectations of the teacher that may be at variance with good educational practice. The teacher focused on developing students' procedural knowledge providing easy steps for learners to aid them when solving problems. Prior to the intervention, this was the approach that the participants in this study were familiar with. The focus group data supports Hoyle's view, for example 'No' (M4, M8) 'No Can't do it' (M11), when answering whether they liked challenging classwork (Table 29). Developing new socio-cultural norms within the classroom takes time and is challenging for both students and teachers. Issues identified in this study as being new demands for participants included (i) the requirement to think for themselves in finding

solutions to problems and (ii) the reporting of these solutions to the class during plenary sessions. Information from the teachers' diary a few weeks after the intervention had begun showed the participants to be uncomfortable in reporting their solutions to the class during whole class discussions (Appendix B). This provides evidence of low self-esteem held by some of the participants in the experimental class. Oldham et al. (2009:304), in a paper on the challenges to implementing the new Project Maths reform of mathematics education at second level in Ireland, write of the challenges to teachers to implementing more problem-solving philosophies and approaches in the classroom. This, Oldham et al. (2009:302) claim, raises questions about Irish students' reactions and their developmental readiness to engage with the new approaches. Bourdieu's notion of 'cultural habitus' from the literature resonates with this study too. Habitus has been described as being internalized in the form of 'dispositions to act, think and feel in certain ways' (Fleming, 2005). Fleming claims habitus is acquired through our 'aculturation into social class, gender, family, peer and even our nationality'. In a school community where the teachers were largely from middle class socio-economic backgrounds and students belonged, to a great degree, to working class/unemployed backgrounds, it may be that further consideration should be given to the influence of teachers' beliefs on their own practices in the classroom (Pajares, 1992:310). Bol and Berry (2005:40) found teachers associated achievement in mathematics more strongly with student factors, including family support, than with curriculum and instruction factors. Social processes are thought to play a critical role in shaping everyday cultural activities with family and classroom culture effects on learning and schooling outcomes (Nasir, 2006:468).

Longer Time Needed with Young Cohort

This author would speculate that, as the research cohort was aged 13-14 years old, more time might be required to achieve a significant change in these participants' beliefs. De Corte et al., (2004:374) had reported with participants age 10-11, in an earlier study (Verschaffel et al., 2000), a significant positive change (small effect size 0.04) on students' mathematics-related beliefs on the teacher's role from the impact of a similar learning environment to this study. Past history as a learner of mathematics will have had an influence on students'

beliefs, as the participants in this study had had 9 years of learning mathematics prior to this study being carried out. This age cohort, in general, have been seen to show a reduction in positive beliefs from age 12 to age 15 with respect to three of the four factors explored in this study including that of the teacher's role in the classroom (Andrews et al., 2007:214).

In considering outcomes from intervention studies, it is also important to consider how well changes to classroom environments have been implemented. De Corte et al. (2004:374) claim four teachers had implemented their intervention in an appropriate way in their study. Depaepe et al.'s study (2007:374) in contrast showed substantial differences between 10 teachers in implementing Verschaffel's(2000) learning environment.

School's Passive Learning Environment

The author, in her view of the school environment used in this study, had over many years, adopted significant responsibility for students' learning and, as a result, students had come to expect this of their teachers. Furthermore, senior school management had prioritised state examination results for students and had little interest or input into the teaching methodologies used by teachers in the classroom. This had not, in the author's view, improved over time. Subject Head of Department structures were non-existent, with responsibility for yearly planning being made collectively during two planning days at the start of the academic year; a volunteer completed the paperwork. With little professional development provided by the Department of Education and Skills, the situation had remained static for many years. Formal structures, including leadership responsibility for individual subjects, had not been set up by the Department of Education and Skills for second-level schools in general. For these reasons and others, in the author's view a passive learning culture was very evident amongst the students in the school. This, as mentioned earlier, had changed recently to some degree with the first introduction of the Project Maths³. Some of the new methodologies used in Project Maths were used with other mathematics

³ Project Maths- The web-site for Project Maths describes Project Maths to be "an exciting, dynamic development in Irish education. It involves empowering students to develop essential problem-solving skills for higher education and the workplace by engaging teenagers with mathematics set in interesting and real-world contexts".

classes by the author during the intervention period. One student in a senior class, aged 18 years, demanded that she be given the steps that would solve a routine mathematical problem on perimeter/area of a rectangle. She had added that her last teacher had given her all the required steps to solve mathematical problems. This example highlights some of the obstacles that teachers may meet as they seek to make changes to the teaching and learning environment in their classrooms.

Timing of Focus Groups

The qualitative data from the focus groups, which ideally should have been collected prior to the implementation of the intervention, actually took place in mid/late January 2011. The intervention had been started in the classroom on November 20th 2010. It is evident from the data collected that participants had already reacted positively to some of the changes that had been made to the learning environment e.g. the change to 'Explore solutions in groups' (Table 27). Positive data on the use of group work found in the first focus groups could not have been found there prior to the intervention, as participants had not experienced its use up to that point. The focus groups, after the intervention, showed the playing of games had helped to make the classes more interesting. This showed participants had recognised this change that had been made in the teaching approach used as they had not experienced playing games in the mathematics classroom until after the first focus groups had been completed. The timing of the first focus groups may have made it more difficult to discover changes to students' beliefs about their teacher's role. However, the quantitative data (Dominant approach used) had also shown no significant positive changes in students' beliefs about their teacher's role in the classroom across the intervention.

The sample used in the current study was different with respect to the sample used in most of the studies with which it has been compared above. Samples used in the other studies included caucasian, middle class social backgrounds (Mason et al., 2004) and participants of comparable ability and socio-economic status (Verschaffel et al., 2000). These profiles do not match the sample used in this current study with, perhaps, the exception of Martinez (2011) whose

participants were of similar age and, to some extent, socially alike and finding somewhat similar outcomes.

Timing of Intervention

The author would suggest that different time scales for interventions are required for different ages and groups of students, different beliefs and needs in order to significantly impact in a positive way on their mathematical beliefs. The timing of the intervention may also be important. It may be that this intervention would have resulted in more enhanced beliefs if it had been carried out one year earlier, at the time of the transfer from primary to secondary school: this is when students' expectations about what classrooms are likely to be more malleable because the students are unsure about their expectations of their new environment. It may be also that the time span of a classroom intervention seeking positive change in students' beliefs is influenced by particular variables. The author's experience as a teacher suggests that two such variables must be the age cohort of the participants and the majority socio-economic status in the classroom/school.

5.1.2) Research Question 2

Issue

Is there evidence from the data that **students believed themselves to be more competent mathematically** following the classroom intervention?

Findings

The results from this study with respect to the second research question were clear, arising from both the quantitative and qualitative data collected. No significant changes were evident from the data, with respect to the experimental participants' beliefs for the competence factor. Hence, the participants did not believe they were more competent in their learning of mathematics across the intervention.

Sub-Factors

Two of the sub-factors of the Competence factor 'perception of enjoyment' and 'intrinsic mathematical competence' also showed no significant change across the period of the intervention. There **was** a statistically significant result for the

third factor 'extrinsic mathematical competence'. This sub-factor tested participants' beliefs about their competence compared to others in Class 1 and whether they had tried to show their teacher that they believed they were better than others in Class 1. However, only 3 questions out of a total of 14 questions from the competence factor were relevant to this sub-factor. DeCoster (1998:4) questioned the trustworthiness of a factor with a very small number of items. This third sub-factor of the Competence factor had been shown to have moderate reliability for the MRBQ scale (Diego-Mantecon, 2007:234) and also for the sample in this study (Table 15). A further study by Andrew et al. (2011:22) described their secondary analysis of the second factor of the MRBQ scale as provisional and questioned the dichotomous nature of items in the factor. Outcomes from this study support further investigation to identify more clearly the sub-factors for this Competence factor of the MRBQ scale. Indeed, the author with a long history of teaching in this DEIS school, would not have experienced many students who were sufficiently confident about their mathematical competence to actively present themselves to her as better at mathematics than other classmates in the classroom.

Independent t-tests were carried out on the gain scores seeking differences between the experimental and control classes on the Competence factor after the intervention period (Table 19). The scores did not show a statistically significant difference between them. Independent t-tests were also carried out on the sub-factors of the competence factor. They also indicated that no major differences in the scores had taken place for the competence sub-factors by participants, both experimental and control classes, over the course of the classroom intervention (Table 20).

As mentioned earlier, Section 4.1.2, the Welch F test (a replacement for ANOVA because Levene's test was significant), was carried out on the gain scores to check for influences on outcomes for gender or class. Results had indicated that there was a significant difference between some of the gain scores calculated on the classes of participants who took part in the study. A Games-Howell post-hoc procedure tested each of the 4 classes of participants with each other in turn. There were no significant differences between the experimental Class 1 with any of the three control Classes 2, 3 and 4. This was

not surprising to the author, as the academic challenges presented to Class 1 had been much larger than e.g. Class 2 had faced that year. Classes 2 and 4, however, differed significantly indicating participants in those classes had shown major differences in their competence levels after the intervention period with a small effect size $\omega=0.01$ (Table 22). The author's experience as a teacher in this school leads her to believe this result may be related to the fact that Class 2 could be considered to be the class most likely to be least challenged by the quantity and difficulty of academic content covered for that academic year and the consequent slower pace of delivery compared to Class 1. Class 2 would also have had a number of participants who had similar Drumcondra scores to some of the participants in Class 1 as discussed earlier in Chapter 3. In contrast, the participants in Class 4 were the weakest academically in the year and struggled academically. Amongst other possible reasons for this difference may be the teachers of the classes who may have also influenced participants. Stipek (2008:216) identifies differences in teachers citing, for example, teacher self-confidence being linked to more enjoyment learning mathematics with teachers who are not confident having difficulty in fostering these beliefs in their own students.

Results from the qualitative data collected on this factor do support the quantitative outcomes on the beliefs held by students about their own competence in mathematics. The micro interlocutor table constructed for this factor (Table 30) showed reasonably similarly-held beliefs from before to after the classroom intervention. Two individuals, a male M10 and a female M5, indicated a minor change had taken place in their beliefs about their own competence. Before the intervention M5 thought she was behind in her learning but after the intervention she felt she was ahead in some parts. M10, on the other hand, believed he was 'ok' in his learning beforehand but 'behind' after the intervention. The quantitative results had yielded no significant change in the group of participants' beliefs about their competence as a result of the classroom intervention: the qualitative data suggests that this may have been due to different participants being affected differently by the intervention.

A number of themes arose from the data collected. These are now discussed:

i) Participants like/don't like curricular content this academic year

The focus groups (Table 29) indicated that some of the students liked geometry and/or algebra while other students declared they didn't like geometry and found algebra difficult. Data indicated M1, M2 and M9 had a liking for geometry/algebra with M4, M5 and M8 not liking geometry and finding it hard. 'Like maths, brutal at first' and 'Didn't understand Algebra last year' (M10). M6 did not like the content this year and M9 didn't like the multiplying in algebra. M13 declared 'I like algebra because I can do it. I hate the rest 'cause I can't do them'. It occurs to the author that a common thread through all these comments appears to show that they believed they were making some progress in their learning of these mathematical topics and that perhaps a longer period of time may have been needed to make a major impact on their competence beliefs. Participants appear to find learning mathematics hard and this is a common reason given for disliking mathematics as a subject (Simmers, 2011).

ii) Participants can do hard problems with patience/They have their own ways of solving problems/They find some curricular topics hard to learn

The participants in the focus groups were very clear before the intervention that they could do hard problems with patience. M3, M5, M8, M9, and M10 agreed they could solve difficult problems if they had patience. After the intervention M7 thought it depended on the problem and M10 and M11 thought you had to have patience to solve the problem but it was possible when in the mathematics classroom rather than at home on their own. M8 decided afterwards that he did not have the patience to solve difficult problems. M13 indicated she had no difficulties with algebra but this was not reflected in her achievement scores. M2's perception of his competence in mathematics was surprisingly low as he was a talented student of mathematics. There are contradictions in the comments provided by these participants making it difficult to assess the responses. Despite all of the above responses, the author as teacher would have viewed a lack of persistence in solving problems in participants regularly in the mathematics' classroom.

iii) Participants like/don't like challenging group work

Participants in the focus groups appeared to link challenging work to small group work in the classroom. M4, M5 and M9 expressed the belief that

sometimes they liked it. M10 liked the challenging work when he was successful. This is an encouraging response to the introduction of group work approaches by the teacher in the classroom. Overall the results were consistent with Middleton and Spanias (199:82) who indicate students perceive the input of group members in small group work in both positive and negative ways.

iv) Participants think they will do well/not do well this year

The focus groups showed participants were not very confident of the grades they expected to achieve in the academic year. The author did not find this unexpected as the confidence levels, in general, held by students in this DEIS school, were not high. M4, M5 and M8 believed they would not do well before the intervention saying '[mathematics is] too hard' and '[I am] definitely behind' and afterwards believed mathematics remained 'too hard' [M4]. M6 appeared to believe she 'will do alright'. M10 appeared to be less confident after the intervention believing he would 'not really [do well]' (Table 29). End of second year achievement scores (Table 36 below) shows some of these participants were more competent than they had expressed in the focus groups.

Overall the qualitative data collected did not indicate any improved belief, by participants, of their competence in mathematics after the intervention had taken place. Lester (2002:348) claims that beliefs are 'notoriously resistant to change' linking this resistance to the lack of sense-making in school mathematics. Lester challenges the notion that interview data can enable a researcher access to students' beliefs. He claims the students may provide answers that they thought were those expected by a researcher. In this study, the answers expressed by the participants were perceived by the author as genuine beliefs held about their competence due to her extensive knowledge of the participants in the experimental class. Students in this school were known to express their opinions frankly, informing their teachers about themselves personally and/or aspects of the teaching they were receiving from their perspectives. Ball (2000:366) claims first-person research, unlike outsider research, can see and hear what lies at the heart of the puzzles of practice.

Possible Explanations for Outcomes

The Independent t-tests did not show a statistically significant change either positively or negatively but they did indicate a small, although not significant, increase in scores for the experimental class and a corresponding small decrease in scores for the control classes. An increase in scores indicates a more negative belief score about participants' own competence. This would seem to indicate that participants in the experimental class believed themselves to be somewhat less competent than before. The author's interpretation of this is that, as expressed above, it was due to the quantity and additional complexity of the new academic content taught to the experimental class during the intervention. This was particularly true for participants who were the least academic in this class. This view is supported by examination of the gain scores for participants in the experimental class. Table 36 below shows the differences in the sum of gain scores for the competence factor across the study for each of the participants in the experimental class. It also identifies the gender of each participant and their identifying code when interviewed, if applicable. The scores were categorised in the table below into two groups: one group consisted of those who got marks of 55% and upwards corresponding to grades C, B and A; the second group consisted of those who got grades lower than 55%. Historically, these grades were accepted as 'honours' grades with Grade D (40%-54%) thought to be a 'pass' grade and below 40% a fail grade.

Student ID	Students Achieve Grade C (55%) or higher June 2011 (Gender f or m)	Gain Score Sum Competence Factor	Student ID	Students Achieve Grade D or lower June 2011 (Gender f or m)	Gain Score Sum Competence Factor
101	83(f)	-20	103	51(m)	-6
102	77(f)	11	104	50(f)	-3
106	81(f)	3	105(M4)	53(f)	25
107(M11)	74(m)	-2	108(M8)	41(m)	15
111	56(f)	-4	109	50(m)	4
114 (M3)	76(m)	-13	110	45(f)	-21
115 (M2)	90(m)	-4	112(m7)	45(f)	1
117 (M6)	55(f)	0	113(M9)	43(m)	-1
118	55(m)	0	116	41(f)	27
120 (M5)	55(f)	-3	119	32(m)	7
125 (M10)	63(m)	-6	121(M13)	31(f)	1
Total	N=11	-38		N=11	49

The table indicates that the higher-achieving experimental participants (June 2011 end of year examination) summed together gave a total score of (-38) for the gain in the competence factor of the scale and the participants who could be described as more challenged academically by the content and were struggling provided a total score of (49) for the same factor. A positive score indicated students' beliefs about their own competence had become more negative over the duration of the study. It occurs to the author that these participants may have been somewhat overwhelmed and a new learning environment, however appropriate, was insufficient of itself to impact on their beliefs on their personal competence. It may also be that the participants' expectations in relation to both the quantity and demands of the second year higher-level syllabus covered in the classroom during that time did not match. Roesken et al. (2011: 503) also found that students studying higher tracks or advanced courses expressed greater self-confidence and felt more able and more competent to do mathematics. The data above would appear to support that view. Not all of the participants in the experimental class were expected to continue studying the higher-level course through to the state examination one year later.

There was evidence from the focus groups, too, of some gender differences on whether mathematics was interesting or not as a subject. These differences were evident in the data prior to the intervention. In the all-female focus group, three out of four were not interested in mathematics as a subject both before and after the intervention (Table 29). The male focus group was somewhat more positive about mathematics, with two declaring they liked mathematics. The data provided no evidence that participants had found the subject of mathematics to be more interesting at the end of the study. Hannula (2009) found that mathematical confidence was influenced by gender and students' perception of their competence in mathematics and was linked mainly to students' achievements. The scores above would seem to support the view that perception of competence was linked to achievement in the subject. Andrews et al. (2007:215) found girls less positive in their beliefs about their personal competence in mathematics irrespective of whether their nationality was Spanish or English. Spencer et al. (1999:25) argue that females risk being judged as having weaker mathematical ability than male learners, which they claim is probably most disruptive when they encounter new mathematics material at the limit of their skills. The author, from experience in the classroom, had found that females were likely to be less confident in themselves when learning mathematics and also less likely, compared to their male counterparts, to take risks when problem-solving. This experience leads her to believe that the girls in this DEIS school were more likely to have low self-esteem than their male classmates. Low self-confidence and a belief that mathematics is not important make it very difficult to motivate students for mathematics (De Corte, 2002:100). Perhaps these are areas that need to be addressed explicitly in any intervention.

Evidence from the data in this study showed that M4 and M6 expressed their dislike of mathematics through the co-ordinate geometry encrypted exercise, indicating strong emotions about learning mathematics. Recent studies have examined emotions and their impact on students learning of mathematics. Di Martino and Zan (2010:46) claim teachers need to learn to deal with students' emotions, their vision of mathematics and their perceived competence. Hanula (2010:55) suggests there is a need to overcome division between positive and negative attitudes and set about identifying different profiles of learners with

negative attitudes towards learning mathematics. No serious attempt was made in this study to conquer negative emotions of participants. This study recommends future interventions should include consideration of this particular issue.

Elwood and Carlisle (2003) indicated superior performance of girls in mathematics in both state examinations in Ireland at age 15/16 and ages 17/18 due to better organization and knowing the rules and formulae. Jeffes et al. (2013) found, regarding the impact of the new Project Maths curriculum initiative on Irish students, that gender was a strong predictor of confidence in mathematics for outcomes from state examinations, with girls being less confident and with lower achievement at ages 15-16 (i.e. they did worse in Project Maths). The conclusion from the current study seemed to show changing to problem-solving approaches were more challenging for girls than for the boys in the experimental class.

5.1.3) Research Question 3

Issue Is there evidence that the **subject of mathematics has become more relevant** to students' lives?

Findings

The evidence from the data collected shows participants' beliefs about the relevance of mathematics to their lives had not changed as a result of the classroom intervention.

Quantitative scores calculating related t-tests on all participants showed no major change had taken place in participants' beliefs about the relevance of mathematics to their lives. Mathematics had not become more personally or more globally relevant for participants and they had not perceived different strategies in learning mathematics in a major way across the intervention (Tables 16,17).

Quantitative results (Table 17) carried out on the data for the sub-factor 'Global Relevance' for all participants for related t-tests was significant ($p=0.045$). This

may have been due to the larger sample size and/or it may also have been due to the increase in abstract content in the curriculum content.

Independent t-tests on gain scores, seeking differences between the experimental and control classes, showed no differences to Relevance beliefs for all the participants after the period of the intervention (Table 19). A Welch F test, calculated on the experimental and control classes, indicated there was no influence from gender or class on the outcomes. Hence, the quantitative findings from the data are quite clear, indicating no major change had taken place in participants' beliefs about the relevance of mathematics to their lives despite Class 1 participants having experienced the new learning environment. The author suggests that this shows the learning environment implemented in Class 1 had been insufficient of itself to make the mathematics more relevant for participants. This was despite significant effort being made by the author to link mathematics to the real world.

The findings from the qualitative data support the quantitative outcomes on students' beliefs about the relevance of mathematics. The micro interlocutor matrix constructed to examine consensus shows strong consistency in responses for the girls' focus group from before to after the intervention. Mathematics was seen as a necessary and important subject due to its requirement for most third level courses but there was no indication that the subject of mathematics and the skills it taught were seen by participants any differently or more valuable for life outside the secondary school than they had prior to the intervention.

Participants' beliefs about the relevance of mathematics were clarified when answering questions in the interviews. These are now discussed:

i) Participants found mathematics to be an important/useful subject

All of the focus group participants would seem to believe that knowledge of mathematics is important and is necessary to earn a living both before and after the intervention. All the boys M8, M9, M10 and M11 and one girl, M5, spoke of the need to know mathematics for all careers before the intervention (Table 31). After the intervention M5 believed 'you need maths most jobs'. Whether M5 was

influenced or not by the more abstract curricular content during the intervention is difficult to say. The focus group participants did link a requirement to know mathematics to particular careers such as architecture and engineering before the intervention and to banking after the intervention. M4 did challenge the other students as to why knowing mathematics was important to getting a job at the end of the study but did not receive any answer to her question from the other participants. All of these comments support the reasonably constant beliefs on the relevance of mathematics to their lives held over the period of the study. Higgins (1997:25) found in her study that only high ability students and the experimental group perceived mathematics as useful. Every experimental participant in the current study believed mathematics was a useful subject. Perhaps this was partly because it was needed for the majority of third level courses in Ireland. The teachers of mathematics in this school would also have emphasised the need to achieve a grade D in ordinary level Leaving Certificate (age 18-19) mathematics as a necessary requirement for the next stage in their futures. The careers that were open at third-level to students without mathematics at ordinary level were not those that had been mentioned by participants such as architecture, engineering and banking.

ii) Friends of Participants like/don't like subject of mathematics

The liking or not liking mathematics as a subject provided very clear-cut answers by participants who declared 'Friends don't like [mathematics]' (M4, M5, M6, M7, M9 and M10, Table 31). This is in keeping with the literature indicating the majority of students don't like mathematics: included in these are some of the more able students (Alenezi, 2008:97). Focus group participants were asked if they agreed with their friends. M5 and M6 provided answers that focused on the usefulness of knowing mathematics rather than placing an emphasis on liking the subject of mathematics itself (Table 31). One insight from the individual interviews introduced a different perspective on the need to know mathematics with M5 (Transcript, Appendix B) saying 'they don't think they'll need a job so they don't think they'll need it' and M12 declared at the same time '[friends] they know it's useful'. The author concludes participants were clear that mathematics was not a subject that was appreciated, with some of the participants showing a poor disposition towards mathematics. These comments would seem, the author suggests, to identify some of the challenges

for teachers of mathematics in DEIS schools and/or schools with similar school communities.

iii) Participants believe you learn better when you use different approaches/solutions to solving problems

The third sub-factor examined 'students' perception of using different strategies' when learning mathematics. Changes explored in participants' beliefs about the use of different strategies to solving problems indicated somewhat constant beliefs across the intervention. M8 would appear to prefer a single solution to a problem to be worked on, with M11 finding the use of more than one strategy 'confusing' (Table 31). The author would not have expected M11 to express this belief, as he was a very high achieving student. M8, on the other hand, struggled with understanding the concepts taught and preferred learning the procedures by rote. These comments, on using different strategies, would seem to be referring to the old approach to teaching mathematics that had a focus on procedural learning. It would also seem to indicate that M8 and M11 were more reluctant than the others in the focus groups to adopt the new pedagogical approaches used in the intervention. Interestingly, M4, M5 and M7 appeared to welcome different approaches to problem-solving both before and after the intervention. Hence, the intervention had not changed these beliefs held by the participants. Swan (2005:27) found learners increased in confidence, flexibility and became more powerful problem-solvers when they compared and discussed alternative solution strategies to problems. Possible reasons for M8 and M11 continuing to cling to a single right answer and one approach may have been the individuals themselves, or that there had been insufficient time to become comfortable with the new pedagogical approaches.

iv) You must have talent to learn mathematics/Individual effort to learn mathematics works

Data on participants' beliefs from the focus groups (Table 31) showed that putting in the effort to learn mathematics works was minimal and cannot be said to provide information on any changes that might have taken place as a result of the intervention.

In the individual interviews M2 indicated that he had 'tried really hard doing problems (M2, Appendix B) and 'Well I try' and added 'some of the time' (M13,

Appendix B). M12 and M13 expressed a belief that they had talent for some parts of the syllabus and not other parts was possible. The small amount of data did not provide any evidence that these participants' beliefs had changed after experiencing the classroom intervention. Reasons for the lack of change or enhancement of relevance beliefs may include lack of confidence and perhaps mathematics anxiety.

Participants were not asked to define what they meant by achieving success in learning mathematics. The author speculates that success, in some participants' view, may have been a 'pass' grade D at ordinary-level standard (middle-level course) could be compared favourably to a similar grade on the higher-level course (most advanced course). This raises the question of both student and parental expectations in this DEIS school being a factor in determining the beliefs held about the relevance of mathematics to their lives.

Possible Explanations for Outcomes

All of the participants in the focus groups in this study perceived mathematics to be useful, although this was qualified by linking certain careers with knowledge of the mathematics expected. This perception by participants may have been influenced by the utilitarian necessity to achieve a pass grade in ordinary level mathematics for entry to the majority of post-Leaving Certificate (ages 18-19) courses (Careersportal, 2014). The author's experience in this school led her to believe that the student body and their parents were aware of the requirement for an ordinary-level grade C or above in Junior Certificate mathematics in, for example, apprenticeships with semi-state bodies such as the Electricity Supply Board requiring applicants to have (ESB, 2014). This utilitarian view of mathematics, in this author's opinion, is a clear indication, that participants had very strongly-held beliefs about the subject of mathematics. Boaler et al. (2000:1) write that students do not see success in mathematics relevant to their developing identities except to allow access to future education and careers. The data from this study would seem to support this view. Before and after the intervention, participants had spoken about the need to know mathematics for specific careers such as engineering. Presmeg (2002:310) found evidence of the need to broaden students' beliefs about the nature of mathematics and how it relates to everyday activities. The literature identifies the need for teachers to connect real life use of mathematics to what is taking place in the classrooms

thereby making the subject more relevant to students' lives (Presmeg, 2002; Bonotto, 2005; Conway and Sloane, 2006). Efforts had been made in this current study to use suitable activities, strongly linked to real-life with students in the intervention classroom e.g. Shape, Space, Measure activity. There is no evidence from the data collected that relevance beliefs had been enhanced as a result. Possible reasons include that they had not had time to adjust to the significantly changed classroom environment that had been implemented. The author suggests further time may have been required for the participants to absorb these huge changes and perhaps benefit from them. Conway and Sloane (2006:216) help to highlight the problems inherent in making major changes to the learning environment such as those made in this study. At that time, the participants had not yet experienced any of the new methodologies, activities or approaches associated with the new Project Maths described earlier in this study. Hence, the very different learning environment may have been more challenging and more significant for participants than expected and this may have placed overly large demands on participants in the experimental class. The design of this study did not include assessing the measure or significance of these additional challenges as experienced by the participants.

5.1.4) Research Question 4

Issue

Did the students' beliefs about **'mathematics as an inaccessible subject'** become more positive following their experience of the new learning environment?

Findings

This factor produced the smallest number of answered questions by participants both before and after the study. A total of 49 participants answered these questions, compared to 66 or 67 participants for each of the other factors in the MRBQ scale used in this study. There may have been a number of reasons why participants, despite the encouragement from their teachers, did not complete as many of this factor's questions. One obvious reason would be that the questions belonged to the final section of the questionnaire and the participants were tired or had lost interest and were fed-up. The author's own interpretation would be that the participants, as part of the wider school

community, would have shown a lack of persistence in fully completing documents, including examination papers, despite teachers' regular exhortations of them to do so. The reduced data collected may have impacted on the outcomes from the classroom intervention for this factor. It may also be that the length of the MRBQ questionnaire was too long for these participants to complete in one setting.

Quantitative results (Table 16) carried out on the data for this factor indicated a statistically significant result for related t-tests for all participants together in the study i.e. a more negative belief in 'mathematics as an inaccessible subject' was held by the combined participants, experimental and control classes, after the classroom intervention had taken place. The effect was calculated as moderate (eta squared 0.09). As mentioned in Chapter 4 the reason for this may have been the larger sample size. The author's experience in this DEIS school, at both higher-level and ordinary-level, had found learners were more challenged by the second year course than they had been by the first year secondary mathematics' course. It may also be that students' expectations of the secondary mathematics classroom had been absorbed during the previous academic year. First year secondary mathematics had included a large proportion of revision of the Primary school mathematics syllabus and hence was, in the author's view, much less challenging for learners.

Results from related t-tests, for all participants combined, on the sub-factors (Table 17) indicated 'mathematics only attainable by able child' gave a significant result (.005). The second sub-factor 'mathematics as a fixed body of knowledge' also gave a significant result (.030) (Table 17). Hence, the results indicated a negative result for all participants together. One other possible reason for this result may be the age of the students. Andrews et al. (2011:20) state that it is not surprising that students' beliefs about all aspects of mathematics become less positive as students get older.

To answer the research question above, related t-tests on this factor were calculated for both the experimental and control classes separately. Results for the set of participants in the experimental class indicated no significant change had taken place across the intervention for this factor and its sub-factors. Hence,

the intervention had not impacted on those beliefs in a positive way. Similar t-tests on the control classes also were shown to be not significant. Hence, mathematics had not become more accessible whether they had experienced the new learning environment or not.

Further comparisons between the experimental and control classes were made by carrying out Independent t-tests on the gain scores. Results show that there were no differences found between the experimental and control classes (Table 19). It may also be that the final sample (n=22) from the experimental class, from which the quantitative scores were calculated, does not represent adequately the beliefs of the entire experimental class (n=28). It may also be the significantly increased volume of new mathematical content in the second year of the Junior Certificate to the students who attended this DEIS school proved very challenging. The first year of the post-primary syllabus was intended to revisit significant primary curricular material on number, measure, space and shape etc. much of it familiar to all participants but not to all participants as mentioned in Chapter 3. In the second year of the course there was a major increase in the amount and difficulty level of new mathematical concepts and content introduced, including a significant increase in the level of abstraction. The author's experience had taught her that, generally, students took until the third year of the course to get comfortable with the faster pace of delivery of more challenging content than they had experienced up to that year and sometimes this resulted in students of reasonable ability deciding to study the ordinary level course (i.e. they chose a lower level) for the third and final year of the Junior Certificate course. One of the individuals interviewed did speak about the challenging amount of academic content covered during the study's intervention. M5 said 'We kind of revised all ..the stuff...in the primary school [1st year] so now we have loads to do this year [2nd year]' (Appendix B).

An ANOVA test indicated no differences for all participants indicating intervention had not made a bigger difference for either gender or class.

The qualitative data collected on the fourth factor (Table 33) cannot be said to either support or not support quantitative outcomes above showing no significant change in students' beliefs about mathematics as an inaccessible

subject because of its limited quantity. The micro-interlocutor analysis carried out on the limited qualitative data for this factor looked at the consistency of students' answers in the focus groups from before to after the classroom intervention. The data collected for this factor indicated that very little change had taken place in students' answers for this fourth factor of the MRBQ scale as a result of the classroom intervention (Tables 33, 34). Participants appeared to express beliefs similar to what they had expressed earlier in this study.

A number of themes emerged from the qualitative data. A number of these are now discussed below:

(i) Participants believed they had talent in mathematics/You need to have a special talent needed to succeed in learning mathematics well

Four students in the individual interviews (Appendix B) answered this question. The data indicated a less-than-positive self-concept by the students with none of the four expressing a view that they had talent in mathematics although M2 hoped that he had talent (Appendix B). M13 (Appendix B) believed that she had no talent in mathematics. The remaining students believed that it depended on the curricular topic, as one part of the syllabus would be easier to learn than another. This seems to indicate that some of the participants believed they were making some progress in their learning of mathematics. This would suggest the possibility of making an improvement in participants' beliefs over a longer time period than had been used with participants in this study.

(ii) Memory is important when you are learning mathematics

There was strong agreement amongst the individual and focus group interviewees that memory was important for learning mathematics (M2, M4, M5, M6, M7, M8, M9, M12, M13). In contrast to this, the interviewees also thought you could succeed reasonably well at mathematics without it, although it was unclear exactly what this meant for participants. M5 thought it meant a pass grade is achievable (Appendix B). The participants also indicated some belief in the need to have a good memory to succeed at mathematics, stating a learner would 'have to learn theorems' (M5) and would 'have to remember angles and stuff' (M8). M5 stated the importance of having the right attitude (Transcript, Appendix B) and the need to enjoy learning the content (M5, M13).

Kloosterman (2002:263) found the same contradiction in students' statements about memory. Kloosterman (2002:256) went on to say that he found a majority of the students believed that memorization skills were important to learning mathematics and he assumed that this must have a significant impact on the students' efforts to learn. Other research recommends teachers reduce the load on students' working memory, which they argue is limited in capacity, by encouraging fluency in calculation (Sullivan, 2011:7). Raghaur (2010:119) argues there is more research required, citing the lack of a sufficiently comprehensive model of mathematical processing in relation to skill acquisition that is considered to be in line with current findings on working memory. The results from the current study would seem to indicate that there is a need for further research to help enlighten teachers about working memory to enable them in turn to help their students learn mathematics better. There was no evidence of a change in the beliefs expressed in the data by participants about the importance of memory to learning mathematics.

(iii) Participants believed the grade you achieved for mathematics was important

Individual interviews indicated that all four participants believed getting a good grade in mathematics was important. Their comments indicated a lack of discernment in relation to the different levels offered in the syllabus. As mentioned earlier three levels of mathematics Higher, Ordinary and Foundation were available for students to study. The data would appear to show that a good grade is acceptable irrespective of the level at which that grade is achieved (M2, M13, M12, M5). High expectations were demanded by teachers in this school but had not, in the author's view, been realised with respect to the subject of mathematics by many students.

(iv) Effort is what is important in order to learn mathematics/You do not need to be intelligent to succeed at mathematics

Focus group interviews indicated before and after the classroom intervention that participants thought that you didn't need to be intelligent to succeed at mathematics with M11 being unsure (Table 33). Zimmerman (2002:66) writes that students are proactive in their effort to learn as a result of being made aware of their strengths and limitations. The issue here would seem to be a lack

of persistence on the part of these participants. Martinez (2011:38) found the focal students interviewed in his study had expressed sentiments towards increased effort to learn but did not follow through in the classroom. There is evidence to support this finding in this study. M13, for example, had claimed she had tried to work hard but had not done so all of the time (Appendix B).

(v) Understanding is more important than getting the right answer when solving a mathematical problem:

Almost all of the participants in the focus groups, boys and girls, shared the belief that understanding is more important than the right answer to a problem both before and after the classroom intervention was undertaken (M4, M5, M6, M7, M9, M10, M11). M8 alone believed getting the right answer was more important than understanding the problem (Appendix B). Participants wanted to understand the concepts being taught as expressed clearly in the focus groups. The author speculates that this indicated the participants had understood that you were more likely to learn mathematics well if you understood the concept.

Possible Explanations for Outcomes

Mathematics had not become more accessible to participants in the experimental class at the end of the classroom intervention. The evidence does show the participants had believed across the intervention that you needed to have a special talent in mathematics to succeed. They also thought you could do ok if you put in the effort and did the necessary work. The evidence of a poor self-concept in mathematics held by these participants could, the author suggests, be linked to this type of behaviour. Reasons for this may be their socio-economic status and with that, their own and their parents' lower expectations for their futures.

Participants appeared to believe in the importance of succeeding at mathematics and also in the effort to learn being important to succeed but there was insufficient follow-through and some poor persistence levels in the day-to-day classroom. Martinez (2011) has shown, that despite having confidence in their ability to solve challenging problems, this did not facilitate a change in students' behaviour. The habit of challenge avoidance had persisted throughout

his study. The participants in this study were the same age as those who took part in Martinez's (2011) study and they were of similar socio-economic status. Much of what he has described in his study resonates with this author. This may reflect, the author suggests, the dominant socio-economic demographic of this school. This community school is located in an area that provides a choice for parents of a number of single-sex faith based private and non-private schools, a Gael-Scoil where students are taught through the medium of the Irish language, a comprehensive school and community schools.

Few participants indicated they liked or appreciated the nature of the subject mathematics. M5 (Appendix B) said in her interview 'cause I like maths'. This was one of the few comments made about liking the subject. Many participants spoke about how it hard they found mathematics to learn. This appeared to be one of the main reasons for not liking the subject. M2 declared 'Algebra with brackets....you have to count every x and y's.. pretty hard' (Transcript, Appendix B). Roesken et al. found that students' views of themselves on the core dimensions of ability, difficulty of mathematics and success are critical as they engage in the learning of mathematics as it simultaneously becomes more difficult. These views may be relevant to this study's outcomes due to the increased challenges demanded of participants in this study.

5.2) Summary of the main findings

The purpose of this study was to measure students' beliefs about mathematics and its teaching and learning following changes to the learning environment in the classroom.

Overall impact:

The impact of the new learning environment on students' beliefs about their teacher's role in the classroom, their competence, the relevance of mathematics to their lives and mathematics as an inaccessible subject were shown not to be statistically significant. The changes made, some of which were welcomed by the participants, proved to be insufficient to alter in a positive way the beliefs of the participants tested.

Teacher's Role:

The teacher's role, post-intervention, appeared to be viewed by participants in a similar manner to what it had been prior to the implementation of the classroom intervention. The author had found this result somewhat surprising due to the changes that had been implemented in her role in the classroom. Perhaps the process of changing from being a passive learner to a more active connected learner had not been easily evident to learners due to their age.

Personal Competence

Beliefs of the experimental participants about personal competence in mathematics had remained unchanged too. This was not surprising to the author due to her long experience in this DEIS school. Participants experienced the additional challenge of an increased volume of new curricular content (for those studying higher level) leading to a much faster delivery of content. Low self-esteem held by students was also evident in the school community, with teachers struggling to raise academic expectations. It may have been difficult for participants to recognize their increasing levels of competence in mathematics in themselves. Evidence of increased competence was shown in the scores from the achievement tests at the end of the year. Participants had generally maintained their grade, from the end of the previous year, which was commendable, given the increased challenges for them.

Influence of Class and Gender:

One difference found in participants in the competence factor was highlighted by an ANOVA test seeking to look at possible influences of class or gender on outcomes. A significant difference was shown between Class 2 and Class 4 from the control participants, showing this difference was not related to the intervention as neither class had experienced the intervention. The author suspects this was due to the relative ability of these participants to cope with the quantity of new curricular content and the pace of delivery. Class 2's participants were more able and had to absorb less curricular content delivered at a more reasonable pace than the experimental Class 1. Class 4's participants were small in number and had special educational needs. This links, the author suggests, to the level of challenge learners are faced with arising from the volume of work to be covered that academic year and to its complexity.

Relevance of Mathematics:

There was no significant evidence that participants viewed mathematics as more relevant to their lives at the end of the study. There was a strong belief, before and after the intervention, that mathematics was an important and useful subject to know. Participants, when speaking about their beliefs about mathematics, spoke about its usefulness with little or no appreciation or love of the subject of mathematics itself. This was in spite of a serious effort on behalf of the author to introduce context to the mathematics' classroom through the use of appropriate rich activities and approaches. The result does highlight some of the challenges for teachers of mathematics in raising students' beliefs about their own competence.

Accessibility of Mathematics as a subject:

Mathematics had not become a more accessible subject to participants at the end of the study. Outcomes had shown non-significant results for experimental and control participants. No significant differences were evident between those who had experienced the new learning and environment and those who had not. A significant result had been found for this factor and its sub-factors when scores from all participants together had been combined. Significant outcomes from the combined scores may have been due to the larger sample size.

Group Work:

Almost all of the participants were very positive about the use of group work in the classroom by their teacher. This was their first experience of learning mathematics using this teaching approach in this secondary school. Outcomes indicated participants saw that it provided the possibility of increasing their understanding of the mathematical concepts taught in the classroom.

Change of Beliefs:

The author suggests that this outcome from the scores collected from the experimental Class 1 indicating no change in beliefs could be viewed as not unexpected. The challenges that Class 1 had experienced during the intervention included coping with the expectations of more independent active learning in the classroom coupled with a larger volume of new more abstract content delivered at a faster pace than the previous year's content had been.

5.3) Limitations

Sample of Convenience:

One of the limitations to this study was the sample used. The study acknowledged in Chapter 3 that random sampling is the optimal choice when choosing participants to be part of an experimental class. The sample in this instance was a sample of convenience. This would seem to be almost inevitable in an action research study, as a researcher has to work with classes in which he/she can intervene. This means choosing participants from pre-arranged classes in the school setting. The impact of this choice of sample is that no generalization of outcomes is possible. Transferability to similar classes in similar schools has to consider that the sample used in this study is unique. However, it is hoped that it will provide information and some insights for other researchers examining similar type samples in similar school communities.

Number of experimental and control classes:

It may also have been better to use two experimental classes and two control classes giving equal numbers of participants in both. Some similar quasi-experimental studies have chosen to do this (Higgins, 1997; Mason, 2004; Verschaffel et al., 1999).

Choice of Interviewer:

It may be that a better decision, for this study, would have been for a colleague to carry out the interviews on the author's behalf. This decision was not taken, as colleagues were exceptionally busy during the final month of the academic year. They were happy to engage in the study to the extent that the teachers of the control classes completed the MRBQ questionnaire before and after the classroom intervention.

Implementation of Intervention:

Was the intended intervention implemented as planned? Fidelity of implementation of the intervention is explored in the literature (Stylianides et al., 2014:21). No consideration was included in this study to build in any checks to ensure the intervention was implemented as planned. This may have been

more important, as the author was both researcher and teacher for this study. In addition to this, colleagues, who might have been able to support the intervention, had been approached early in the study with a view to discussing and advising on the new learning environment but none appeared sufficiently confident or interested to provide an opinion. Nelson et al. (2010) suggest keeping a log to support a reliable and valid implementation of an intervention.

The outcomes from this study, as interpreted by the researcher, should be considered in the light of these limitations and the implications arising from them.

5.4) Implications for practice

The outcomes show a non-significant result for the research questions on the changed learning environment. The qualitative data collected provided information on some aspects of the learning environment that participants welcomed and may be worthwhile developing. Other research identifies challenges that may arise when changing the learning environment from a traditional approach with a lesser focus on higher order skills and interactive learning. De Corte et al. (2004:378) argue that it may be necessary for old habits and beliefs to be 'deconstructed'. Garcia et al. (2004:164) writes, in her study on deconstructing deficit thinking in teachers, that middle-class teachers 'did not question the effectiveness of the educational system in providing equitable, culturally responsive, learning environments in which all students can be successful'. This perspective may help to highlight additional challenges that might be present in a DEIS school when new learning environments are being implemented in the classroom.

5.4.1) Implications for the author's own practice

The author, despite being an experienced practitioner in the classroom, learned a great deal about the teaching of mathematics. Specifically, the author learned more about individual students' mathematical thinking than prior to this point and this enabled help to be given to participants to make further progress in their understanding of mathematical concepts. It would seem, too, that consideration of students' expectations of what the mathematics class will be like are important for the teacher to engage with. The author's practice has

changed significantly, with the introduction of group-work and a more student-centred learning environment, following the completion of this study. These changes make her a more competent teacher of mathematics who aims to facilitate students' learning more in the classroom.

5.4.2) Implications for colleagues

The participants in the experimental class enjoyed some activities and this helped to make them more active in their own learning. This outcome has been shared with colleagues.

Group work was well received by the students as a whole and it should be included as a teaching methodology in every teacher's repertoire. Carrying out group-work successfully in the classroom is also challenging and it should be implemented using current and future recommendations from the literature.

The outcomes from this study are useful for the author's own school and, perhaps in particular, for other DEIS schools in Ireland. It may be of interest to the Principal of the school and the parents of the school as the school seeks to raise standards of numeracy in the school. It may, for example, encourage senior management to take a hands-on approach to the introduction, use of and assessment of new teaching methodologies in the mathematics' classroom. It may also be of interest to colleagues teaching mathematics in other schools in Ireland.

5.4.3) Implications for Future Research

Further research should be carried out on how to enhance students' mathematical beliefs at secondary level. Age does appear to be a factor possibly because a longer history of learning mathematics requires additional time to achieve a positive change in beliefs. Whilst acknowledging that beliefs are formed in particular education experiences, there is also a need to identify any additional factors arising in different types of schools that serve the needs of different types of students.

This is the first such intervention of this scale, to the author's knowledge, in a Republic of Ireland secondary classroom attempting to enhance students'

mathematical beliefs. DEIS schools provide unique challenges to teachers of mathematics. This type of school cannot be said to take into first year secondary (age 12-13) each year, the national average cohort of students as evidenced by the results of the Drumcondra testing (Appendix B) that showed 3 students from the entire year were placed in the top 20th percentile of students of same age with respect to numeracy levels. Low attainment in mathematics is an issue for the mathematics teachers and the management of the school. The author speculates that the aspects of the learning environment that participants enjoyed may serve as a starting point for the development of a learning environment that may help over time to enhance students' mathematical beliefs.

Francisco (2013:481) has called for more research on the mathematical beliefs of students at second level and how these impact on their mathematical behaviour as they engage in challenging mathematical tasks. Much of the research in mathematics education since 2004 has called for greater attention to be paid to teachers' beliefs. Bringing about dramatic changes in mathematics instruction has been linked to teachers' beliefs about mathematics learning and teaching (Lloyd, 2002:149). The experimental class in this study appeared to believe that understanding mathematics was important. An emphasis on sense-making suggests a commitment to understanding is argued by Francisco (2013:491). This may be a fruitful starting point for a new study.

This study has made the author aware of some of the challenges associated with changing students' beliefs about mathematics. It has encouraged the author to believe that students' beliefs will change given sufficient time and taking into consideration the age of the students and their socio-economic backgrounds. The author suggests that parents need to become more involved with the learning of mathematics by their children, particularly when they first enter secondary school.

5.4.4) Implications for theory

This study is a mixed methods study. The findings from this study confirm the use of mixed methods was appropriate for this type of educational study. The quantitative results on their own would have shown the participants mathematical beliefs had not changed significantly, positively or negatively, as a

result of experiencing the new learning environment. The qualitative data provided additional and confirmatory insights into these beliefs and information on what aspects of the new learning environment participants could be considered to be worthwhile.

Hence, this study would seem to support the view that advocates the use of mixed methods that might best support answers to the research questions and make the findings stronger (Teddlie and Tashakkori, 2009:7).

5.4.5) Areas of Improvement

Start at beginning of Year

The decision to teach in the traditional manner for the first two months of the school year may have had an impact on outcomes. This impact was not evaluated at any stage in the carrying out of this study. It may be that participants' expectations of the mathematics classroom - and their role in it - had been absorbed during that time period. There is some evidence in the literature to support changes made by a teacher new to a class, provided this was started at the beginning of the school year (Haggarty, 2002:283).

Use First-Year Students

It is possible too, as mentioned earlier, that the new learning environment should have been introduced into first year secondary, as students made the transition from primary to secondary school.

Extended Time for Intervention

One improvement that could have been made with this study relates to time management in implementing the new learning environment. Changing the learning environment began in November 2010 as described in Chapter 3 following the completion of the MRBQ questionnaires by participants for the first time. An analysis was made of these answers and the focus group questions arose from this analysis. As a practitioner in the classroom with the usual duties of the practising teacher the first focus group did not take place until January 2011 giving an overlap between the implementation of the new environment. This decision was made to extend the time that participants could experience the new learning environment. The additional time had the merit of a longer

implementation for the new learning environment. In hindsight, it would have been much better to have had carried out the study over two academic years, had this been possible.

Further follow-up focus Groups (in year following intervention)

It may also have been advisable to undertake follow-up focus groups with the same participants in the academic year following the completion of the study. Anecdotal evidence from other teachers of the group indicated that four of the students from the experimental class had suggested to them that they missed this teacher's presence in their mathematics' classroom. At a practical level this was not possible.

Awareness of Non-Verbal data in focus groups and interviews

With respect to the conditions under which the focus groups and individual interviews had been carried out, some issues are noted. No detailed data had been collected on the non-verbal communication that had taken place between participants during the interviews. Data on participants' non-verbal communication etc. might have enriched the outcomes from the micro-interlocutor analyses included in this study.

5.4.6) Implications for researching practice

The use of a mixed methods design has been proven to be more productive in this study than if a single quantitative method had been used as mentioned above.

This study encompassing insider research is encouraging for two reasons: It may help to add to information in the future about first-person research enabling distance and insight for this genre of research to be realized effectively (Ball, 2000:401). It may also highlight the value of insider research to practitioners, with the possible benefits to their students learning of mathematics and their own success as teachers and through that help to close the research-practice divide.

5.5) Personal Reflection

At the completion of this study the author reflected on many years as a practitioner in the mathematics classroom and the newer influences of her

experiences as a researcher. The author's understanding of educational research and its links to practice in the classroom had changed considerably since engaging in the research process. A perceived lack of acknowledgement/understanding of day-to-day classroom realities by researchers and the disconnection to the challenges that existed for teacher and students can perhaps describe the author's beliefs prior to engaging in the research process. Mathematics' pedagogy and improvements in learning outcomes was of serious interest to the author as expressed previously in section 2.1.2. At the end of the research process, the author's beliefs with respect to research in mathematics education are vastly different. Research is now viewed as essential to advancing more successfully the learning and teaching in classrooms for all students. It would appear to offer the promise of improvements in mathematics pedagogy with better learning outcomes. The challenge in Ireland as elsewhere would appear to be to engage practitioners in the researching of mathematics classrooms and to work towards closing the division between research and practice.

The research process did, the author believes, make her a more effective, more confident teacher of mathematics. Knowledge gained from the literature was enlightening and provided exciting new information to be acted on in the classroom. The research had put a framework around all the author's knowledge and experience that had been gained in the classroom over thirty years and had expanded and explained more fully the processes involved in teaching and learning. Students had appeared to benefit from the new methodologies and rich activities used with them in the classroom during the intervention. The qualitative data collected in this study shows students were very positive about the changes made in the classroom (Section 4.2). In the academic year that followed a number of students had (anecdotally) voiced a view that they had missed the presence of the author-and her teaching practices- in their mathematics classroom.

Instruments

STUDENTS' MATHEMATICS RELATED BELIEFS QUESTIONNAIRE

(MRBQ) Scale (DiegoMantecon et al., 2007:233-236)

A 6-point Likert scale was used with the questionnaire, with responses to questions ranging from strongly agree, agree, somewhat agree, somewhat disagree, disagree, strongly disagree.

Factor 1: Beliefs about Teacher's Role in the classroom questions

My teacher wants us to enjoy learning new things.

My teacher understands our problems and difficulties with mathematics.

My teacher tries to make the mathematics lessons interesting.

My teacher appreciates it when we try hard, even if our results are not so good.

My teacher always shows us, step by step, how to solve a mathematical problem, before giving us exercises.

My teacher listens carefully to what we say.

My teacher is friendly to us.

My teacher always gives us time to really explore new problems and try out different solution strategies.

My teacher wants us to understand the content of this mathematics course.

My teacher explains why mathematics is important.

We do a lot of group work in this mathematics class.

My teacher thinks mistakes are okay as long as we are learning from them.

My teacher is too absorbed in the mathematics to notice us.

My teacher does not really care how we feel in class

Factor 2: Beliefs in personal Competence questio

I think that what I am learning in this class is interesting.

I like what I am learning in this class.
I'm very interested in mathematics.
I prefer class work that is challenging so I can learn new things.
I expect to do well on the mathematics tests and assessments we do.
I prefer mathematics when I have to work hard to find a solution.
I find that I can do hard mathematics problems with patience.
I am certain I can learn how to solve the most difficult mathematics problem.
I don't have to try too hard to understand mathematics.
I think I will do well in mathematics this year.
I can usually do mathematics problems that take a long time to complete.
I can understand even the most difficult topics taught me in mathematics.
By doing the best I can in mathematics I try to show my teacher that I'm better than other students.
I try hard in mathematics to show the teacher and my fellow students how good I am.

Factor 3: Beliefs about the Relevance of mathematics to their lives

Mathematics has no relevance to my life.
Studying mathematics is a waste of time.
Mathematics is a worthwhile and necessary subject.
I study mathematics because I know how useful it is.
Knowing mathematics will help me earn a living.
I think mathematics is an important subject.
I think that what I am learning in this class is useful for me to know.
Mathematics enables us to better understand the world we live in.
Everyone can learn mathematics.
Mathematics is used all the time in people's daily life.
If I try hard enough I understand the mathematics we are taught.
I can use what I learn in mathematics in other subjects.
Discussing different solutions to a mathematics problem is a good way of learning mathematics.
I think it is important to learn different strategies for solving the same problem.
Time used to understand why a solution works is time well spent.
Routine exercises are very important in the learning of mathematics.

Factor 4: Beliefs about mathematics as an Inaccessible Subject questions

If I can not solve a mathematics problem quickly, I quit trying.

Only very intelligent students can understand mathematics.

Only the mathematics to be tested is worth learning.

Ordinary students cannot understand mathematics, but only memorise the rules they learn.

I I can not do a mathematics problem in a few minutes, I probably can not do it at all,

It's a waste of time when our teacher makes us think on our own.

My teacher wants us just to memorise the content of this mathematics course.

Mathematics learning is mainly about having a good memory.

There is only one way to find the correct solution to a mathematics problem.

Everybody has to think hard to solve a mathematics problem.

My teacher thinks she/he knows everything best.

Getting the right answer in mathematics is more important than understanding why the answer works.

My only interest in mathematics is getting a good grade.

Focus Group Interview Questions

Teacher Factor Questions

1. Does your teacher manage to make the mathematics lessons interesting?

(Give an example)

2. Does your teacher appreciate it when you try hard, even if your results are not so good.

Are mistakes ok when you are trying to solve a problem in your own way?

(Why/ why not? Give an example)

3. Does your teacher always show you, step by step, how to solve a mathematical problem before giving you exercises?

(Give an example)

4. Are you sometimes given new problems you have to solve in your own way?

(Give an example)

5. Does your teacher give time to explore new problems and try out different ways of solving problems? (Give an example)

Does your teacher discuss your own ways of solving problems with you so that you can see their strengths and weaknesses?

Competence Factor Questions

1. Are you interested in mathematics? Do you like what you are learning in mathematics this year? (Why/ why not? Give an example)

2. Do you feel you are behind, where you need to be, or ahead of what your class is doing in mathematics this year? How do you feel about this and what do you do about it? Do you think you will do well in mathematics this year?

(Why/ why not? Give an example)

3. Do you find that you can do hard mathematical problems if you approach them with patience (Give an example)

4. Do you have your own way of solving mathematics problems? If yes, do they work? Where did you learn them?

5. Do you like challenging class work that helps you learn new things?

(Why/ why not? Give an example)

Relevance Factor Questions

1. Do you think mathematics is an important subject in itself?

(Why/ why not? Give an example)

2. Do you think that mathematics has any use in your life? Will mathematics help you earn a living?

3. Do your friends like mathematics? Do they find it useful? Do you think your friends are right about this? (Why/ why not? Give an example)

4. Do you think that anyone can learn mathematics? (Why/ why not? Give an example)

5. Do you think that seeing and discussing different solutions to a mathematics problem is a good way of learning mathematics?

Do you think that it is important to learn different approaches for solving the same problem? (Why/ why not? Give an example)

Inaccessible Subject Factor

1. Do you think that if you cannot solve a mathematics problem quickly you quit trying? (Why/ why not? Give an example)

2. Do you think that only very intelligent students can succeed at mathematics? (Why/ why not?)

3. Do you think learning mathematics is mainly about having a good memory? Can people do ok in mathematics, who are not very good at memorisation? (Why/ why not? Give an example)

4. Do you think that getting the right answer in mathematics is more important than understanding why the answer works? (Why/ why not? Give an example)

5. Do you think there is only one way to find the right answer to a mathematics problem? (Why/ why not? Give an example)

Final General Question

Do you have anything else you would like to say about mathematics and learning mathematics in school?

Individual Interview Questions

1. Do you do a lot of group work in maths classes this year? Do you feel that working in a group helps you when you are solving problems in maths? In what way does it help/not help? What do the members of your group think? Do you agree? Why/Why not? (Teacher's Role Factor)
2. What else could your teacher do to make teaching work well? (Teacher's Role Factor)
3. Do you like what you are learning in maths class this year? Are there some topics that you like better than others? Can you tell me which and why? What do your friends think about maths? Do they like it? Do you agree? (Competence Factor)
4. Is maths a useful subject? Why do you think that? Do your friends see it as useful? Do they work hard? (Relevance Factor)
5. Do you do the best that you can in maths class this year? Why is that? Do you want to get a higher score than other students in your class in mathematics? Why/Why Not? Is it important for you to score a higher mark in maths tests than other students? Why/ Why not? (Competence Factor)
6. Do you think it takes a special talent to do well in maths? Do you have such a talent? Can people do ok even without special talent? Why? (Inaccessible Factor)
7. Is memory important in maths? Are you good at memorizing? Can someone who is not good at memorization be good in maths (or even ok in maths)? (Inaccessible Factor)

8. Do you think that everybody has to think hard to solve a maths problem? Is it the same for all the topics covered in maths class this year? Can you tell me why/why not? (Competence Factor)

9. Do you think that it is important to get a good grade in maths? Why? What do you think is a good grade in maths? Is the grade achieved the only thing that is important when studying maths? Can you tell me what other things are important when studying maths? What do your friends think? Do you agree? (Relevance Factor)

10. Does your teacher think that she knows everything best? What do you think? (Inaccessible Factor)

(All questions may require prompts)

Appendix B

Quantitative and Qualitative Data Results

(Appendix includes some exemplars of various tests
- All data and tests available on request)

Quantitative Data Results

Related t-tests TEACHER FACTOR MRBQ scale (Experimental and Control)

T-Test

		Notes
Output Created		30-Jun-2013 11:24:39
Comments		
Input	Data	/Users/alice/Documents/ThesisData Feb42013.sav
	Active Dataset	DataSet1
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	78
Missing Value Handling	Definition of Missing Cases Used	User defined missing values are treated as missing. Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
Syntax		T-TEST PAIRS=TotalTeacherIntervBefore TotalTeacherControlBefore WITH TotalTeacherIntervAfter TotalTeacherControlAfter (PAIRED) /CRITERIA=CI(.9500) /MISSING=ANALYSIS.
Resources	Processor Time	00 00:00:00.006
	Elapsed Time	00 00:00:00.000

[DataSet1] /Users/alice/Documents/ThesisDataFeb42013.sav

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	TotalTeacherIntervBefore	41.77	22	3.741	.798
	TotalTeacherIntervAfter	38.59	22	6.674	1.423
Pair 2	TotalTeacherControlBefore	35.47	45	5.430	.809
	TotalTeacherControlAfter	35.09	45	7.813	1.165

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	TotalTeacherIntervBefore & TotalTeacherIntervAfter	22	.376	.085
Pair 2	TotalTeacherControlBefore & TotalTeacherControlAfter	45	.255	.092

Paired Samples Test

		Paired Differences				
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference	
					Lower	Upper
Pair 1	TotalTeacherIntervBefore - TotalTeacherIntervAfter	3.182	6.307	1.345	.386	5.978
Pair 2	TotalTeacherControlBefore - TotalTeacherControlAfter	.378	8.302	1.238	-2.116	2.872

Paired Samples Test

		t	df	Sig. (2-tailed)
Pair 1	TotalTeacherIntervBefore - TotalTeacherIntervAfter	2.366	21	.028
Pair 2	TotalTeacherControlBefore - TotalTeacherControlAfter	.305	44	.762

IF (ID<200) TotalCompIntervBefore=TotalCompetenceScoresBefore. EXECUTE.
 IF (ID>199) TotalCompControlBefore=TotalCompetenceScoresBefore. EXECUTE.
 IF (ID<200) TotalCompIntervAfter=TotalCompScoresAfter. EXECUTE.
 IF (ID>199) TotalCompControlAfter=TotalCompScoresAfter. EXECUTE.
 T-TEST PAIRS=TotalCompIntervBefore TotalCompControlBefore WITH
 TotalCompIntervAfter TotalCompControlAfter (PAIRED)
 /CRITERIA=CI(.9500)
 /MISSING=ANALYSIS.

T-Test COMPETENCE FACTOR MRBQ scale (Experimental and Control)

Notes

Output Created		30-Jun-2013 11:32:35
Comments		
Input	Data	/Users/alice/Documents/ThesisDataFeb42013.sav
	Active Dataset	DataSet1
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Missing Value Handling	Definition of Missing Cases Used	User defined missing values are treated as missing. Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
Syntax		T-TEST PAIRS=TotalCompIntervBefore TotalCompControlBefore WITH TotalCompIntervAfter TotalCompControlAfter (PAIRED) /CRITERIA=CI(.9500) /MISSING=ANALYSIS.
Resources	Processor Time	00 00:00:00.007
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[DataSet1] /Users/alice/Documents/ThesisDataFeb42013.sav

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	TotalCompIntervBefore	47.73	22	14.300	3.049
	TotalCompIntervAfter	37.64	22	12.561	2.678
Pair 2	TotalCompControlBefore	44.18	45	11.254	1.678
	TotalCompControlAfter	32.98	45	12.844	1.915

Paired Samples Correlations

	N	Correlation	Sig.
--	---	-------------	------

Pair 1	TotalCompIntervBefore & TotalCompIntervAfter	22	.714	.000
Pair 2	TotalCompControlBefore & TotalCompControlAfter	45	.204	.179

Paired Samples Test

		Paired Differences				
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference	
					Lower	Upper
Pair 1	TotalCompIntervBefore - TotalCompIntervAfter	10.091	10.286	2.193	5.530	14.651
Pair 2	TotalCompControlBefore - TotalCompControlAfter	11.200	15.253	2.274	6.617	15.783

Paired Samples Test

		t	df	Sig. (2-tailed)
Pair 1	TotalCompIntervBefore - TotalCompIntervAfter	4.601	21	.000
Pair 2	TotalCompControlBefore - TotalCompControlAfter	4.926	44	.000

```

IF (ID<200) TotalRelevIntervBefore=TotalRelScoresBefore. EXECUTE.
IF (ID>199) TotalRelevControlBefore=TotalRelScoresBefore. EXECUTE.
IF (ID<200) TotalRelevIntervAfter=TotalRelScoresAfter. EXECUTE.
IF (ID>199) TotalRelevControlAfter=TotalRelScoresAfter. EXECUTE.
T-TEST PAIRS=TotalRelevIntervBefore TotalRelevControlBefore WITH
TotalRelevIntervAfter TotalRelevControlAfter (PAIRED)
/CRITERIA=CI(.9500)
/MISSING=ANALYSIS.

```

T-Test RELEVANCE FACTOR MRBQ scale

Notes

Output Created	30-Jun-2013 11:39:49		
Comments			
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	Weight	<none>	
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	N of Rows in Working Data File	78	
Missing Value Handling	Definition of Missing	User defined missing values are treated as missing.	
	Cases Used	Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.	
Syntax	T-TEST PAIRS=TotalRelevIntervBefore TotalRelevControlBefore WITH TotalRelevIntervAfter TotalRelevControlAfter (PAIRED) /CRITERIA=CI(.9500) /MISSING=ANALYSIS.		
Resources	Processor Time	00 00:00:00.008	
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[DataSet1] /Users/alice/Documents/ThesisDataFeb42013.sav

Paired Samples Statistics

	Mean	N	Std. Deviation	Std. Error Mean
Pair 1 TotalRelevIntervBefore	41.90	21	9.914	2.163
TotalRelevIntervAfter	36.5714	21	8.81800	1.92425
Pair 2 TotalRelevControlBefore	39.3556	45	9.01049	1.34320
TotalRelevControlAfter	31.8667	45	12.52198	1.86667

Paired Samples Correlations

	N	Correlation	Sig.
Pair 1 TotalRelevIntervBefore & TotalRelevIntervAfter	21	.372	.096
Pair 2 TotalRelevControlBefore & TotalRelevControlAfter	45	-.001	.997

Paired Samples Test

	Paired Differences
--	--------------------

	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference	
				Lower	Upper
Pair 1 TotalRelevIntervBefore - TotalRelevIntervAfter	5.33333	10.53249	2.29838	.53900	10.12767
Pair 2 TotalRelevControlBefore - TotalRelevControlAfter	7.48889	15.43111	2.30033	2.85287	12.12491

Paired Samples Test

	t	df	Sig. (2-tailed)
Pair 1 TotalRelevIntervBefore - TotalRelevIntervAfter	2.320	20	.031
Pair 2 TotalRelevControlBefore - TotalRelevControlAfter	3.256	44	.002

IF (ID<200) TotalInaccIntervBefore=TotalInaccBefore. EXECUTE.
IF (ID>199) TotalInaccControlBefore=TotalInaccBefore. EXECUTE.
IF (ID<200) TotalInaccessInterAfter=TotalInaccAfter. EXECUTE.
IF (ID>199) TotalInaccessControlAfter=TotalInaccAfter. EXECUTE.
T-TEST PAIRS=TotalInaccIntervBefore TotalInaccControlBefore WITH
TotalInaccessInterAfter TotalInaccessControlAfter (PAIRED)
/CRITERIA=CI(.9500)
/MISSING=ANALYSIS.

WELCH-F TEST RELEVANCE FACTOR ACROSS INTERVENTION

Oneway

Notes

Output Created	15-Jul-2013 12:44:27	
Comments		
Input	Data	/Users/alice/Dropbox/Thesis/ThesisDataJuly13.sav
	Active Dataset	DataSet2
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	78
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics for each analysis are based on cases with no missing data for any variable in the analysis.
Syntax	ONEWAY GainRevSum BY Class /STATISTICS DESCRIPTIVES HOMOGENEITY BROWNFORSYTHE WELCH /PLOT MEANS /MISSING ANALYSIS /POSTHOC=TUKEY GH ALPHA(0.05).	
Resources	Processor Time	00 00:00:00.562
	Elapsed Time	00 00:00:01.000

[DataSet2] /Users/alice/Dropbox/Thesis/ThesisDataJuly13.sav

Descriptives

GainRevSum

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Min	Max
					Lower Bound	Upper Bound		
1	21	5.05	11.151	2.433	-.03	10.12	-12	35
2	22	1.82	9.845	2.099	-2.55	6.18	-16	25
3	16	5.81	26.453	6.613	-8.28	19.91	-39	75
4	7	-2.29	20.089	7.593	-20.87	16.29	-45	14
Total	66	3.38	16.582	2.041	-.70	7.46	-45	75

Test of Homogeneity of Variances

GainRevSum

Levene Statistic	df1	df2	Sig.
2.835	3	62	.045

ANOVA

GainRevSum

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	431.439	3	143.813	.511	.676
Within Groups	17440.091	62	281.292		
Total	17871.530	65			

Robust Tests of Equality of Means

GainRevSum

	Statistic ^a	df1	df2	Sig.
Welch	.527	3	20.555	.668
Brown-Forsythe	.415	3	26.404	.744

a. Asymptotically F distributed.

RELEVANCE Post Hoc Tests

Multiple Comparisons

Dependent Variable: GainRevSum

	(I) Student No (Binned)	(J) StudentNo (Binned)	Mean Difference (I-J)	Std. Error	Sig.
Tukey HSD	1	2	3.229	5.117	.922
		3	-.765	5.566	.999
		4	7.333	7.320	.749
	2	1	-3.229	5.117	.922
		3	-3.994	5.511	.887
		4	4.104	7.278	.942
	3	1	.765	5.566	.999
		2	3.994	5.511	.887
		4	8.098	7.600	.712
	4	1	-7.333	7.320	.749
		2	-4.104	7.278	.942
		3	-8.098	7.600	.712
Games-Howell	1	2	3.229	3.214	.747
		3	-.765	7.047	1.000
		4	7.333	7.973	.796
	2	1	-3.229	3.214	.747
		3	-3.994	6.938	.938

	4	4.104	7.878	.951
3	1	.765	7.047	1.000
	2	3.994	6.938	.938
	4	8.098	10.069	.851
4	1	-7.333	7.973	.796
	2	-4.104	7.878	.951
	3	-8.098	10.069	.851

Multiple Comparisons

Dependent Variable: GainRevSum

	(I) StudentNo (Binned)	(J) StudentNo (Binned)	95% Confidence Interval	
			Lower Bound	Upper Bound
Tukey HSD	1	2	-10.28	16.74
		3	-15.46	13.93
		4	-11.99	26.66
	2	1	-16.74	10.28
		3	-18.54	10.55
		4	-15.11	23.32
	3	1	-13.93	15.46
		2	-10.55	18.54
		4	-11.97	28.16
	4	1	-26.66	11.99
		2	-23.32	15.11
		3	-28.16	11.97
Games-Howell	1	2	-5.39	11.84
		3	-20.57	19.04
		4	-18.80	33.46
	2	1	-11.84	5.39
		3	-23.60	15.61
		4	-22.03	30.24
	3	1	-19.04	20.57
		2	-15.61	23.60
		4	-20.90	37.10
	4	1	-33.46	18.80
		2	-30.24	22.03
		3	-37.10	20.90

Homogeneous Subsets

GainRevSum

StudentNo (Binned)	N	Subset for alpha = 0.05
		1

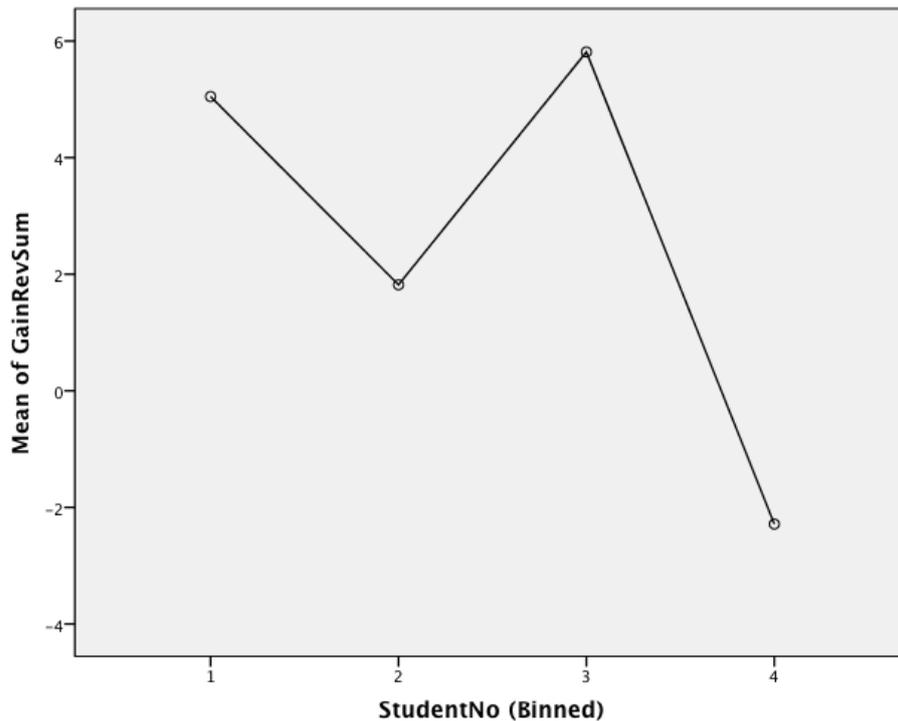
Tukey HSD ^{a,b}	4	7	-2.29
	2	22	1.82
	1	21	5.05
	3	16	5.81
	Sig.		.598

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 13.403.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Means Plots



```

ONEWAY GainRevSum BY IDTwoGroups
/STATISTICS HOMOGENEITY BROWNFORSYTHE WELCH
/PLOT MEANS
/MISSING ANALYSIS
/POSTHOC=TUKEY GH ALPHA(0.05).

```

Qualitative Data Results

Sample of the data obtained from the FOCUS GROUPS (transcripts).

Focus Group B (Male) Members: M8, M9, M10, M11

TEACHER FACTOR

M8: reads the questions to the others

Q. Does your teacher try to make the mathematics lessons interesting?

(Give an example)

M8: *Answer the question*

M9: *Yea. The videos*

M10: *Sometimes. The oranges*

M8: *Diagrams and all*

Q. Does your teacher appreciate it when you try hard, even if your results are not so good.

Are mistakes ok when you are trying to solve a problem in your own way?

(Why/ why not? Give an example)

All: *yea*

M11: *Effort prizes for effort*

Q. your teacher always show you, step by step, how to solve a mathematical problem

before giving you exercises?

(Give an example)

M8: *Yea on the board*

Q. Are you sometimes given new problems you have to solve in your own way?

(Give an example)

M8: *No*

M9: *Sometimes*

M10: *Yea Sometimes Yea*

M11: *Rarely ever*

Q. Does your teacher give time to explore new problems and try out different ways of solving problems? (Give an example)

M9: *No not really- in group work we do work new stuff*

M8: *Except groupwork*

M11: *Yea in groupwork*

M10: *Yea we discuss it an all*

COMPETENCE FACTOR

M11 reads this section

Q. Are you interested in mathematics? Do you like what you are learning in mathematics this year? (Why/ why not? Give an example)

M9 *I kinda like algebra-Geometry I find hard*

M10: *I like geometry*

M8: *No*

Q. Do you think you will do well in mathematics this year?

(Why/ why not? Give an example)

M8: *I'm definitely behind*

M9:, C:, D: *I think that I'm on track*

M9: *Allright. I think I will do ok*

M10: *I'll definitely pass*

Q. Do you find that you can do hard mathematical problems with patience?

(Why/ why not? Give an example)

All: *Yea*

M11: *A lot of patience*

Q. Do you have your own way of solving mathematics problems? If yes, do they work? Where did you learn them?

M9: *yea*

C:*yea. From primary school*

M8 *Yea*

Q. Do you like challenging class work that helps you learn new things?

(Why/ why not? Give an example)

M11: *No Because I can't do it*

M9: *Sometimes*

M8: *No*

EXEMPLAR INDIVIDUAL INTERVIEW DATA

Student advised to say what he thinks.

INDIVIDUAL INTERVIEW QUESTIONS

11. Do you do a lot of group work in maths classes this year?

M2: Yes We started doing a lot of them. Before we didn't do that much but now we do

Do you feel that working in a group helps you when you are solving problems in maths?

M2: Yes because then I asked other people how to solve problems that I don't know

In what way does it help/not help?

M2: Yes

Is there any way that it does not help?

M2: Well sometimes if a person does not understand and you have to explain it to them it can slow you down.

What do the members of your group think?

M2: I hope so. I didn't ask

Do you think that they agree with you? Why/Why not?

M2: Yea they seem happy in the group

12. What else could your teacher do to make teaching work well?

M2: I can't think of anything else.

13. Do you like what you are learning in maths class this year?

M2: Yea its pretty good. I never learned stuff like that.

Are there some topics that you like better than others?

M2: algebra is better and geometry than multiplying and dividing.

Can you tell me which and why?

M2: I like [algebra and geometry] equally.

M2:

What do your friends think about maths?

M2: Something that is really hard and is really boring.

Do they like it?

M2: some do – the smart people who get the most marks. The other people they give up!

Do you agree?

M2: Not really because I think maths is really important and is not to be given up on.

Why is that?

M2: Because you use it every day in your life.

14. Is maths a useful subject? Why do you think that?

M2: Yes very every day in every job

Do your friends see it as useful?

M2: Yes some of them do some think it is useless.

Do they work hard?

M2: Yes lots of them do

15. Do you do the best that you can in maths class this year?

M2: Yes

Why is that?

M2: Because I tried really hard doing problems.

Do you want to get a higher score than other students in your class in mathematics?

M2: Yes

Why/Why Not?

M2: You will do better in life with a high score.

Is it important for you to score a higher mark in maths tests than other students? Why/ Why not?

M2: Not important but it is good to have a good score.

16. Do you think it takes a special talent to do well in maths?

M2: No everybody can learn how to do maths

Do you have such a talent?

M2: I hope I do

Can people do ok even without special talent?

M2: They can- not really ok

Why?

M2: [They can] normal

Can you explain what you mean by normal? Student reminded that there are 3 levels of mathematics courses Higher, Ordinary and Foundation

M2: Ordinary

Is that a person without a special talent in mathematics?

M2: Yes

What about higher level?

M2: Yes if he works

In higher level what would you think is an ok score?

M2: a B Plus

17. Is memory important in maths?

M2: Yes if a question comes up in the exam and you don't remember how to do it

Are you good at memorising?

M2: I think I am ordinary

Can someone who is not good at memorization be good in maths (or even ok in maths)?

M2: He'll be ok but he will have problems remembering the rules.

What could you do to get around that?

M2: You could associate it with something that is part of your life

18. Do you think that everybody has to think hard to solve a maths problem?

M2: For some maths problems you have to think hard but for others ordinary

Is it the same for all the topics covered in maths class this year?

M2: It varies

Can you give me an example?

M2: Let me think. The algebra with the brackets

Can you tell me why that was hard?

M2: you have to count every x and y 's. Pretty hard

19. Do you think that it is important to get a good grade in maths? Why?

What do you think is a good grade in maths?

M2: a B plus higher for me and ordinary [in general].

Is the grade achieved the only thing that is important when studying maths?

M2: No the thing that is important is what you learn

What are you learning when you are solving problems:

M2: You learn different ways to solve that you can solve any problems you might associate it with another problem that doesn't include maths and you might think of a few ways to solve it.

How can you find a few ways to solve a problem?

M2: If you can't solve a problem efficiently that's ok

Can you tell me what other things are important when studying maths?

M2: Can't think of anything

Is that a difficult question to answer?

M2: Yes because I don't know the answer

We talked about memory and learning maths. Is there anything else

M2: I think being good at understanding

How can you get good at understanding?

M2: By practising and trying to understand

What would help in class or at home with understanding?

M2: associating with what you know- a cylinder with a tree trunk for example

Do you mean everyday objects?

M2: Yes

What do your friends think?

M2: It is pretty hard to answer questions about your friends –you don't know what they think

Do boys share their thoughts in the same way as girls about these things?

M2: Girls talk about things in their lives. Boys talk about sports

Do you agree?

The last question is about your teacher (Author) I want you to feel free to answer what you think. It is a hard question to answer to somebody's face but it is an important question.

20. Does your teacher think that she knows everything best? What do you think?

M2: she does think she knows best but for some people they might know their own way which is best for them

A discussion follows on what help improve the learning in the class? M2 raises the short time available to get help from the teacher due to large class. He suggests a classroom assistant that he says he has in his art class.

Table Codes From Constant Comparison Analysis Relevance Component Questions

Question 1			
Do you think that mathematics is an important subject in itself?			
Before Intervention		After Intervention	
Units	Code for each unit	Units	Code for each unit
<i>Yea people use it if you get a job</i> (M3)	Need <u>maths</u> job		
<i>It is definitely</i> (M2)	Definitely		
<i>Yea</i> (M1)	Yea		
<i>Yes</i> (M4, M5, M6, M7)	Yes	<i>Yea</i> (M7)	Yea
<i>You need it for certain jobs</i> (M4)	Need <u>maths</u> certain jobs	<i>Why</i> (M4)	Why
<i>You need it for all jobs</i> (M5)	Need <u>maths</u> all jobs	<i>Because most jobs you need <u>maths</u> for</i> (M5)	Need <u>maths</u> most jobs
<i>Yea</i> (M8, M9, M10, M11)	Yea	<i>Yea</i> (M6)	Yea
<i>You need it for later on</i> (M11)	Need <u>maths</u> later	<i>I wish it wasn't [important]</i> (M8)	Wish it wasn't
<i>You need it for a job</i> (M8)	Need <u>maths</u> job		

Question 2			
Do you think that mathematics has any use in your life? Will mathematics help you earn a living?			
Before Intervention		After Intervention	
Units	Code for each unit	Units	Code for each unit
<i>Oh no you need <u>maths</u> in your life</i> (M3)	Need <u>maths</u> life		
<i>It depends say if you wanted to be an architect you need <u>maths</u></i> (M1)	Need <u>maths</u> architect		
<i>Or engineering</i> (M3)	Need <u>maths</u> engineer		
<i>Its impossible [to do without <u>maths</u>]</i> (M2)	Need <u>maths</u> life		
<i>Yes</i> (M4, M5, M6, M7)	Yes	<i>Yea</i> (M4, M5, M6, M7)	Yea
<i>Yea [earn living]</i> (M4, M5, M6, M7)	Need <u>maths</u> life	<i>Yea [Earn living]</i> (M5, M7)	Need <u>maths</u> life
<i>Yes. Definitely</i> (M8)	Definitely	<i>Yea</i> (M11)	Yea
<i>Its ok</i> (M9)	Its ok	<i>Yea</i> (M9)	Yea
<i>Its ok. It's not my favourite but its not my least favourite either. Depends on sections</i> (M10)	Like <u>maths</u> depends sections	<i>Everywhere. When you're doing your household bills</i> (M10)	Everywhere
		<i>Yea</i> (M8, M9, M10, M11)	Yea
		<i>Yea if you're a banker</i> (M11)	Yea banker

Question 3			
Do your friends like mathematics? Do they find it useful? Do you think your friends are right about this?			
Before Intervention		After Intervention	
Units	Code for each unit	Units	Code for each unit
<i>Some [friends] of them do</i> (M2)	Some friends like <u>Maths</u>		
<i>Yea</i> (M3)	Yea		
<i>Some [friends] of them say you don't need <u>maths</u></i> (M1)	Friends say don't need <u>Maths</u>		
<i>No [friends not right]</i> (M3)	Don't agree		
<i>Not a chance</i> (M1)	Not a chance		
<i>No. It's not boring. Its just hard for them to learn</i> (M2)	Hard for friends learn <u>Maths</u>		
<i>Some [friends] of them</i> (M6)	Some friends like <u>Maths</u>	<i>No [friends]</i> (M4, M5, M6, M7)	No Friends don't like
<i>No [friends right]</i> (M5)	Don't agree	<i>Not really [agree friends]</i> (M5)	Not really
<i>Not really [friends]</i> (M7)	Friends not right	<i>No [friends]</i> (M5, M6, M7)	Friends say don't need <u>maths</u>
<i>No [friends right]</i> (M4, 5, 6, 7)	Friends not right	<i>Yes you do need it [<u>maths</u>] in life</i> (M5)	Need <u>maths</u> life
		<i>You do need It [<u>maths</u>]</i> (M6)	Need <u>maths</u> life
		<i>No just us</i> (M10)	No friends don't like
		<i>No [Friends]</i> (M9)	No friends don't like

Question 4			
Do you think that anyone can learn mathematics?			
Before Intervention		After Intervention	
Units	Code for each unit	Units	Code for each unit
<i>Yea If they try they can</i> (M1)	If they try		
<i>Some people (my twin brother) are just not good at <u>maths</u></i> (M3)	Twin brother not good at <u>maths</u>		
<i>Yea [agrees with M3]</i> (M2)	Yea		
<i>I am making progress</i> (M1)	Am making progress		
<i>Yea if you put your head to it</i> (M7)	If they try	<i>No yea</i> (M7)	No Yea
<i>Yea if they try it</i> (M9)	If they try	<i>Yea if they have the right approach to it</i> (M5)	Have right approach
<i>No</i> (M8)	No	<i>You have to have patience</i> (M6)	Have patience
<i>Yea</i> (M11)	Yea		
<i>I can't learn <u>maths</u></i> (M8)	Can't learn <u>maths</u>		

Question 5			
Do you think that seeing and discussing different solutions to a mathematics problem is a good way of learning mathematics? Do you think that it is important to learn different approaches for solving the same problem?			
Units	Code for each unit	Units	Code for each unit
<i>Yea you would have more understanding</i> (M2)	You would have more understanding		
<i>I don't think so- too much in your head</i> (M1)	Too much in your head		
<i>For some people maybe</i> (M2)	For some people		
<i>Not really good- you might get mixed up. If a method is right you don't need another</i> (M3)	Might get mixed up		
<i>Some people can remember different things</i> (M5)	People remember different things	<i>Yea</i> (M5)	Yea
<i>Some are easier</i> (M4)	Some easier	<i>Yea because you find the one that you are best at doing</i> (M7)	Find best one
<i>You can find one way easier and someone else the other way</i> (M7)	Some easier	<i>Kind of confusing</i> (M6)	Confusing
<i>No better to learn one way</i> (M8)	Better learn one way	<i>Sometimes you don't get either of them</i> (M5)	Don't get either
<i>Sometimes</i> (M9)	Sometimes	<i>Sometimes yea</i> (M5)	Sometimes
<i>No way you don't want to confuse yourself</i> (M11)	Confusing	<i>You are able to understand one and not the other. You are able to do one and answer the question</i> (M7)	Understand one not other
		<i>If it doesn't work one way you can try the other way</i> (M6)	Try one way than other
		<i>Yea it's the group work. You get everyone's opinion</i> (M10)	<u>Groupwork</u> gets everyone's opinion
		<i>No</i> (M11)	No
		<i>It's just too confusing</i> (M8)	Confusing

Emergent themes Individual Interviews

Exemplar themes

Beliefs about mathematics as an inaccessible subject: Question 6, 7, 9, 10	If I get higher in maths then it means that I work harder. I'm not going to like try and beat them (M5)	Need special talent
	I want to be the best (M13)	
	No not really (M12)	
	If they really want to yea but if they don't bother they're not going to [do well] (M13)	
	I don't think so (M13)	
	Not important [to get highest score] but it is good to have a good score (M2)	
	It's important to do good for myself and not to be better than everyone (M13)	
	Memory quite important	
	Memory isn't everything	
	Remember	
	Not good at memorizing some things	
	If I like it	
	Revise a lot	
	Like not more important than getting the right answer	
	Don't remember	
	Ok memory	
	Problem remembering rules	
	Associate with something	
	Very important	
	Not good at memorizing some things	
		Important things when studying maths
	Understanding	
	Have right attitude	
	Kind of attitude	
	Enjoy it	Teacher knows best
	She does think she knows best (M2)	
	No because we have different ways (M5)	
	She knows everything about maths (M12)	Teacher knows best
	Sometimes she knows best (M13)	

DIARY

Exemplar extract.

Thursday December 9th 2011 (130-245 pm)

More challenging questions on combined shapes calculating perimeter and area are given to the students from their textbook. Shapes include combined circles, semicircles, rectangles and squares and triangles.

Students were asked to work in their groups. Roles within the groups were emphasized and the requirement that all the members of the groups had to understand the solutions and were able to explain to the whole group.

I did notice a number of the groups worked extremely well together. It was also evident that two groups were not working well together. **** left her group to join two girls in another group without asking. When I asked why I was told the girls were helping her to understand the problem. **** also in the same group decided to work on his own. When I looked at his work there were errors despite a good knowledge of the mathematical content involved.

This was a much more successful class. I acted as facilitator providing scaffolding to some groups (of girls) by asking questions that helped the students to provide their own solutions.

APPENDIX C:

Exemplars of profiles of Interviewees

M2 (Individual Pilot, Appendix B)

M2 was a foreign national who had come to Ireland a number of years earlier. He was a very conscientious student who was anxious to do well in his studies. He was co-operative and respectful to all his teachers and was well regarded by the staff in the school. He was in the highest class academically for his year but had not completed the Drumcondra standardized test. At times he did not appear to be very confident in his work. He appeared to like the subject of mathematics and saw it as an important subject.

M5 (Individual Interview, Appendix B and focus group)

M5 would seem to have a reasonably positive view of mathematics as a subject. She viewed mathematics as an important subject unlike some of her friends and she seemed to be interested in learning it. Her home background proved to be very challenging during the academic year when the study was carried out and she had much responsibility at home. She did not mix well with her peers generally and was not very popular.

M12 (Individual Interviews, Appendix B)

M12 came from a very supportive working class background. Compared to his brothers who had attended the same school he was more successful academically. He was interested in doing well in school but lacked persistence and was an intermittent worker.

M13 (Appendix B, Individual Interviews)

M13 was a student with some challenging behaviour in the classroom at times. She was successful at hip-hop dancing competitions winning prizes at home and internationally. Her general ability placed her at the bottom 30% of the class. Her parents had had to visit the school on a number of occasions to sort out problems that had arisen with her teachers in the classroom.

Letter to Parents seeking permission for their children to take part in this study

Dear Parent/Guardian,

I am conducting research into the teaching and learning of mathematics in the school this year. The purpose of the research is to explore how teachers can help their students to learn mathematics more easily and more effectively.

I will be asking all second year students to complete a questionnaire about their learning of mathematics in the coming week and at the end of the year. Completing the questionnaire is voluntary. The completed questionnaires will provide helpful information on students' views on how they learn mathematics.

Later in the year I will ask, at a suitable time, small groups of students to discuss and give their views on a question on the learning of mathematics.

If you have any queries about the above please contact me at the school.

Thank you

Alice McDonnell
Mathematics Teacher

Letter to Board of Management of the Community School seeking permission to carry out the research.

September 5th 2010

Dear Chairperson

I wish to carry out research with the second year group of students during this academic year. I am currently studying for an EDD. in Mathematics Education at the University of Exeter.

The purpose of the research is to study students' beliefs about mathematics and its teaching and learning.

Permission will be sought from parents and participants to take part in this study. Participation is voluntary.

I look forward to hearing from you.

Yours truly,

Alice McDonnell



Graduate School of Education

Certificate of ethical research approval

MSc, PhD, EdD & DEdPsych theses

To activate this certificate you need to first sign it yourself, and then have it signed by your supervisor and finally by the Chair of the School's Ethics Committee.

For further information on ethical educational research access the guidelines on the BERA web site: <http://www.bera.ac.uk/publications> and view the School's Policy online.

READ THIS FORM CAREFULLY AND THEN COMPLETE IT ON YOUR COMPUTER (the form will expand to contain the text you enter). **DO NOT COMPLETE BY HAND**

Your name: Alice McDonnell

Your student no: 560025862

Return address for this certificate: 6 Barnhill Park, Dalkey, Co. Dublin, Ireland

Degree/Programme of Study: Ed.D (Mathematics Education)

Project Supervisor(s): Dr. Keith Postlethwaite

Your email address: ammcdonnell@eircom.net

Tel: +353 1 2852400 or +353 86 0816468

I hereby certify that I will abide by the details given overleaf and that I undertake in my thesis to respect the dignity and privacy of those participating in this research.

I confirm that if my research should change radically, I will complete a further form.

Signed: Alice McDonnell. **Date:** March 5th2015

Certificate of ethical research approval

TITLE OF YOUR PROJECT:

Developing the mathematical beliefs of second-level students: An Intervention Project

1. Brief description of your research project:

The classroom intervention consisted of changing the learning environment in a second-level classroom mathematics classroom in Ireland with a view to developing more positive beliefs in students about mathematics as a subject and its teaching and learning.

2. Give details of the participants in this research (giving ages of any children and/or young people involved):

81 participants aged 13-14 boys and girls in year 2 in a second level school in Ireland

Give details (with special reference to any children or those with special needs) regarding the ethical issues of:

3. informed consent: Where children in schools are involved this includes both headteachers and parents).

An overview of the main ethics decisions was provided in the thesis and is attached here as the file Ethics.docx. Normal school practice was followed. Permission was sought from the school to carry out the research (Appendix C attached). I was informed face-to-face by the Principal, in his role as Secretary of the Board of Management, that permission had been given for the research. There was no written confirmation.

Permission was sought from parents, to whom the voluntary nature of getting involved in the study was explained (Appendix C attached).

4. Anonymity and Confidentiality

- *Pseudonyms were used in the write-up.*
- *Data were stored securely in a locked cupboard in the staff room to which I held the only key.*

5. Give details of the methods to be used for data collection and analysis and how you would ensure they do not cause any harm, detriment or unreasonable stress:

*Students were invited to take part in the interviews
Focus groups invitations followed school practice (see Ethics.docx)
Individual interviews - see Ethics.doc. (I asked for volunteers; phoned parents to seek permission and stressed that involvement was voluntary).*

Chair of the School's Ethics Committee
updated: March 2013

6. Give details of any other ethical issues which may arise from this project - e.g. secure storage of videos/recorded interviews/photos/completed questionnaires, or

Audio recordings were stored securely in the locked cupboard. Pseudonyms were used in the transcripts.

7. Special arrangements made for participants with special needs etc.

N/A

8. Give details of any exceptional factors, which may raise ethical issues (e.g. potential political or ideological conflicts which may pose danger or harm to participants):

None

*This form should now be printed out, signed by you on the first page and sent to your supervisor to sign. Your supervisor will forward this document to the School's **Research Support Office** for the Chair of the School's Ethics Committee to countersign. A unique approval reference will be added and this certificate will be returned to you to be included at the back of your dissertation/thesis.*

N.B. You should not start the fieldwork part of the project until you have the signature of your supervisor

This project has been approved for the period: 06.09.2010 **until:** 30.06.2011

By (above mentioned supervisor's signature):

Keith Pople

...date:.....06.09.2010...

N.B. To Supervisor: Please ensure that ethical issues are addressed annually in your report and if any changes in the research occur a further form is completed.

GSE unique approval reference:.....*D/14/15/32*.....

Signed:.....*[Signature]*.....**date:** *19/3/15*.....
Chair of the School's Ethics Committee

APPENDIX D

Sample Question Ordinary Level paper, Junior Certificate 2003 (SEC, 2003)

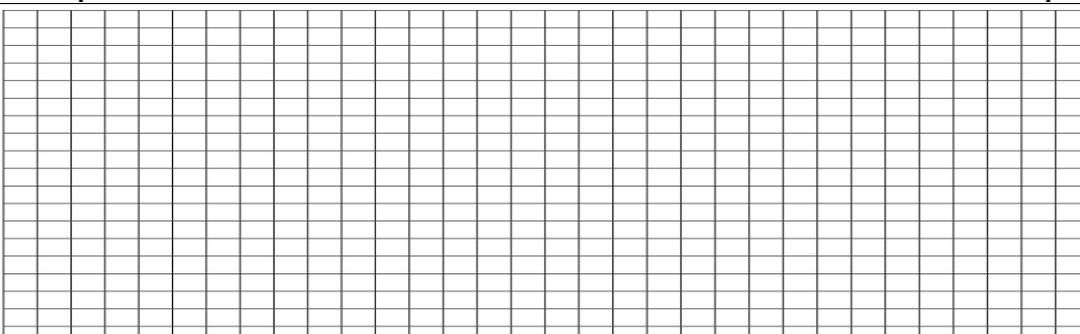
(a) $f(x) = 3x - 5$. Find:

 (i) $f(2)$

 (ii) $f(-1)$

(b) Draw the graph of the function

$g : x \rightarrow x^2 - 2x - 2$
in the domain $-1 \leq x \leq 3$, where $x \in \mathbf{R}$.



(c) (i) Draw the axis of symmetry of the graph drawn in (b) above.

(ii) Use the graph to estimate the value of $x^2 - 2x - 2$ when $x = 1.5$.



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