Multi-criterion Water Quality Analysis of the Danube

River in Serbia: A Visualisation Approach

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Abstract

River quality analysis is an important activity which, in Serbia, has been performed using the Serbian Water Quality Index (SWQI). This is a measure based on a weighted aggregation of 10 water quality parameters. In this work, alternative methods drawing on visualisation approaches used in multi-criterion decision analysis are applied to the problem of evaluating river quality in the Danube. Two methods are considered: one which constructs a graph using the dominance relation combined with a further multi-criterion ranking method, average rank, and the other in which the dimensionality of the data is reduced using PCA for visualisation. Results for data collected in 2010 are analysed and compared with the corresponding SWQI values for the river in that year, and we find that by employing these methods it is possible to reveal more information within the data than is possible by using SWQI alone.

1. Introduction

Water quality plays a vital role in all aspects of human and ecosystem survival. All living and industrial activities are controlled by physical, chemical, biological and microbiological activities (Mahapatra et al., 2011; Vasiliev et al., 2014). Anthropogenic influences and natural processes degrade surface waters and impair their use for drinking, industry, agriculture, recreation and other purposes (Sánchez et al., 2007). In aquatic environments organisms are exposed to mixtures of pollutants whose effects are hardly interpreted and predicted exclusively from chemical analyses. Moreover, the analyses of pooled chemicals, present at different compounds, increase uncertainty when evaluating water quality (Monferrán et al., 2011). Hence, analysing the quality of river water requires a range of quality indicators, from which an overall measure of quality can be produced. Evaluation of water quality parameters is necessary to plan and develop better water resource management (Mahapatra et al., 2011). To establish water quality criteria, measures of chemical and
physical constituents must be specified, as well as methods for reporting and comparing
results of water analysis (Saxena and Gangal, 2010). In order to evaluate water quality
synthetically, many techniques have been introduced to monitor and evaluate the effects of
pollution: traditional methods, modelling approaches, water quality indices (WQIs, e.g.
Armitage et al. (1983) and Prati et al. (1971)), multivariate statistical techniques, such as
principal component analyses (PCA), artificial neural networks, artificial intelligence, fuzzy
logic, as well as combinations of some of them (Ma et al., 2013; Othman et al. 2012;
Monferrán et al. 2011; Taner et al., 2011; Saxena and Gangal, 2010; Simões et al. 2008;
Nasirian, 2007). The work presented in this paper is distinct from these examples in that none
of them are based on multi-criterion visualisation techniques constructed in terms of the
water quality parameters on which WQIs are based.

Multi-criterion decision making (MCDM) is a process by which a set of options can be
assessed according to a set of criteria. MCDM is often applied in cases where the selection of
a single choice from a set of alternatives is required, and provides techniques for drawing
together information from all of the criteria so that a decision maker can make an appropriate
selection. Often the criteria are in conflict with each other, and selecting an option which
performs well against one criterion means accepting poor performance on another criterion.
A range of MCDM techniques exist, and in this work we are concerned with those in which
alternatives are ranked, providing an ordering of the alternatives so that the best one can be
readily identified through visualisation. Visualisation of alternatives allows the decision
maker to observe how they relate to each other, for example, making use of spatial
information gleaned from the placement of alternatives in the visualisation, to better
understand the nature of the alternatives from which they must choose (e.g., Walker et al,
2013).
In addition to selecting the best alternative, it can also be useful to identify the worst alternatives. Engineers often undertake activities such as constructing maintenance schedules. It is not usually feasible to maintain all of the components of an infrastructure, so those components most urgently in need of maintenance must be identified so that they can be prioritised in the schedule. This too is often formulated as a MCDM task, and one of the visualisation methods used in this study has been applied in this area (for ranking district metered areas in a water distribution network (McClymont et al., 2011) and for analysing the performance of the wireless access points in a mobile telephone network (Walker, 2013) for the purpose of constructing maintenance timetables). MCDM techniques of this variety are particularly well suited to river quality analysis as they allow engineers and scientists to identify regions of the river that have particularly poor quality so that remedial action can be taken.

In this work, we examine the quality of water in the Danube in Serbia. Due to its great international importance, the Danube River has been the subject of numerous water quality studies (Živadinović et al., 2012; Micić et al., 2011; Bird et al., 2010; Kirschner et al., 2009; Micić and Hofmann, 2009; Maljević and Balač, 2007; Relić et al., 2005).

Data was collected at a range of stations; in the MCDM context, these stations are the alternatives, or individuals. A range of water quality parameters have been collected for each of the stations at approximately monthly intervals for a year. These parameters are used as criteria in order to compute the Serbian Water Quality Index (SWQI) which ranks stations according to 10 criteria.

This paper presents an analysis of the water quality data using the criteria constructed under the scheme. We use multi-criterion visualisation techniques, one based on methods for ranking multi-criterion sets (Walker et al., 2010) and the other employing principal
component analysis (Jolliffe, 2002). The visualisations are used to provide new insight into the data, before which we discuss avenues of future work arising from this study.

2. Multi-criterion Visualisation

Visualisation is an active area of research within the MCDM community because of the potential for revealing more about the characteristics of a multi-criterion dataset, or population of individuals. Visualising a population of multi-criterion individuals is usually a nontrivial task, since it is often the case that a large number of criteria must be incorporated into the visualisation. In the case of a two or three dimensional population the task is relatively straightforward: we must simply construct a scatter plot in two or three dimensions, a visualisation approach with which most people are familiar and able to use. Unfortunately, since most people are not able to comprehend four or more spatial dimensions visually, this is not possible for populations comprising four or more criteria. It is therefore necessary to develop techniques for visualising such populations, and one of two approaches can be taken: either reduce the dimensionality of the population so that it can be visualised with a scatter plot, or find a way of visualising all of the data in an intuitive fashion.

A range of methods have been developed for visualising a population in terms of the full set of criteria. A considerable amount of work in visualising populations has been done in the optimisation community; optimisation problems are often formulated in terms of a set of problem objectives.

One of the early approaches developed was the Pareto race (Korhonen and Wallenius, 1988), which enabled a decision maker to “drive” through the space of possible solutions to an optimisation problem in order to steer an interactive optimisation procedure. More recently,
another method developed as part of an interactive optimisation framework was the Pareto
navigator (Eskelinen et al., 2010).

Vilfredo Pareto defined the Pareto dominance relation; dominance is one of the most
frequently used methods for comparing multi-criterion individuals, and is used in MCDM so
that the relative quality of individuals can be considered without requiring a weighted
summation of their respective criterion values. The dominance relation will be formally
introduced shortly. Other techniques for visualising the complete criterion set are heatmaps
(Pryke et al., 2006; Walker et al., 2013), which represent individuals as the rows in a matrix,
criteria as the columns, and criterion values with colour (where “cool” colours indicate low
values and “warm” colours indicate high values). The alternative is to use a dimension
reduction method to identify a two or three dimensional representation of the individuals that
can be visualised with a scatter plot. Two varieties of dimension reduction can be used:
feature selection and feature extraction. In feature selection, the most representative features
are retained, and the remainder are discarded. In MCDM, this means finding the two or three
most important criteria; methods for doing this have been demonstrated (e.g. Saxena et al.,
2013). Under feature extraction, a completely new set of coordinates is identified which
represent the individuals. Common examples are PCA, which seeks to preserve as much of
the variance within the data as possible, and multidimensional scaling (MDS) which
preserves pairwise distances between individuals. MDS was recently used in combination
with a metric defined in terms of dominance to visualise multi-criterion populations (Walker
et al., 2013). The application of other feature extraction techniques, neuroscale and
generalised topographic mappings, were examined by Fieldsend and Everson (2005), and
Obayashi (2002) demonstrated the use of self-organising maps for visualising criterion data.

Of particular relevance to this work are interactive decision maps, which have been used for
analysing river quality (Lotov et al., 2004). Interactive decision maps are used to identify
goal points, regions of criterion space that are perceived to be of high quality. This is done by allowing the decision maker to experiment with different combinations of criterion values to find the most satisfactory trade-off possible. In their work, Lotov et al. (2004) present two relevant analyses using interactive decision maps. The first tackles the problem of designing wastewater treatment strategies that would enhance water quality, while the second optimises the parameters of a water quality decision support system. Another relevant application of multi-criterion visualisation was reported by Udias et al., (2012) in which a watershed management system was presented. Their visualisation was based on a combination of interactive decision maps and 2-dimensional scatter plots. The scatter plots displayed the overall quality in the ordinate axis and the monthly cost for each of four criteria in the abscissa.

Each visualisation method has features that make them attractive for specific uses. For example, heatmaps are useful because they incur no information loss and the original criterion values can be recovered from the visualisation. Projection techniques, on the other hand, incur information loss in that the original criterion values are discarded, however are very useful for identifying spatial relationships in the data. In this work, in order to overcome the shortcomings of existing approaches, we employ two methods. The first method uses dominance and rank information to visualise the water quality stations in such a way that no information is discarded. The second uses PCA to project the stations into two dimensions so that they can be visualised using scatter plots, in order to compare the resulting spatial arrangement with the dominance-based approach.

2.1 Pareto Sorting Visualisation

Ranking is an important component of MCDM framework, as well as in evolutionary algorithms (EAs) used to solve multi-objective optimisation problems. The first method we
use draws on ranking methods from multi-objective EAs. One of the most common
approaches to ranking a population of multi-criterion individuals is Pareto or non-dominated
sorting, particularly well known for its use in the popular non-dominated sorting genetic
algorithm (NSGA) (Srinivas and Deb, 1994), and its successor NSGA-II (Deb et al., 2002).
Non-dominated, or Pareto sorting is a technique that is based on the dominance relation.
Under dominance, an individual \( u \) dominates individual \( v \) if its criterion values \( u_m \) are no
worse than those of \( v \), and are better than \( v \) on at least one criterion. More formally:

\[
\forall m (u_m \leq v_m) \land \exists m (u_m < v_m)
\]

(1)

If an individual exists such that it is dominated by no other individuals within the population,
it is called non-dominating. A pair of individuals where neither dominates the other are called
mutually non-dominating. Pareto sorting follows a simple procedure by which a partial
ordering of solutions is constructed. This begins by identifying all of the individuals in the
population that are non-dominated. Those individuals are the strongest, and are assigned to
the first Pareto shell before being temporarily discarded. The removal of shell 1 individuals
means that a new subset of the individuals in the population are non-dominated (those
previously dominated only by shell 1 individuals). These become the second Pareto shell, and
are themselves discarded from the population, leaving a third subset of non-dominated
individuals. This procedure continues until the entire population has been assigned to a Pareto
shell.

A visualisation method was presented in Walker et al. (2010) that uses dominance
information and the partial ordering resulting from Pareto sorting to construct a visualisation
of a population. The population is cast as a directed graph, such that individuals are nodes
arranged into columns according to their Pareto shell (each column represents a shell) and
edges are used to show dominance relations between individuals in adjacent shells.
2.2 Average Rank

The Pareto sorting visualisation furnishes us with a way of discriminating between individuals in different Pareto shells, however it provides limited information about the difference between individuals in the same shell. We therefore enhance the visualisation with a complementary ranking method. Additionally, while dominance is capable of distinguishing between individuals comprising a small number of criteria, it is known that individuals comprising a large number of criteria (often called “many-objective” or “many-criterion” individuals) are likely to be mutually non-dominating (Farina and Amato, 2003). As such, various alternative schemes for ranking multi-criterion populations have been developed; one of these methods is average rank (Bentley and Wakefield, 1998). In order to calculate the average rank \( \bar{r}_i \) of an individual \( \mathbf{y}_i \) the population is ranked \( M \) times, once for each criterion. This converts the population to rank-coordinates, such that each criterion is on the scale 1 to \( N \); the best individual has a score of 1, and the worst has a score of \( N \). Then, an average of the rank-coordinate values for the individual is taken:

\[
\bar{r}_i = \frac{1}{M} \sum_{m=1}^{M} r_{im} \tag{2}
\]

Other multi-criterion ranking methods can be employed instead of average rank; several were demonstrated in Walker et al., (2010), and were found to provide complementary rankings to the partial ordering resulting from Pareto sorting. In fact, any colour scheme can be used. In the next section, we also colour the nodes according to the classification of the stations under SWQI.

2.3 Principal Component Analysis

A widely used visualisation method is Principal Component Analysis (PCA) (Jolliffe, 2002), and we use it here to perform multi-criterion analyses. PCA is well suited to reducing the
dimensionality of multi-criterion individuals. It has, for example, been used to reduce the
dimensionality of populations of solutions in evolutionary algorithms (e.g., Deb and Saxena,
205). In terms of water quality analysis, Astel et al. (2007), Kowlakowski et al. (1971) and
Vega et al. (1998) presented visualisations of water quality using PCA. Other studies have
used PCA for reasons other than visualisation, such as data clustering. Examples include
Koklu et al. (2010), Simeonov et al. (2003), Singh et al. (2004), Alberto et al. (2001) and
Zhang et al. (2010).

PCA projects data points into a low-dimensional space such that their new low-dimensional
representation retains as much of the variance contained within the original, high-dimensional,
data as possible. The low-dimensional space does not comprise any of the original criteria. In
the context of MCDM, each individual is a data point, and the original dimensions are the
criteria. Projecting the individuals into a low-dimensional space should therefore preserve as
much of the original variance within in the criteria, so that the most important information is
preserved. The principal components are identified by first computing the covariance matrix
of the data. Given the covariance matrix, the principal components are the eigenvectors
corresponding to the eigenvalues, which correspond to the original criteria, with largest
magnitude. In this study, as the goal is to produce a visualisation, the first two principal
components are retained. Having projected the individuals onto the first two principal
components, they can be visualised with a two-dimensional scatter plot.

3. Case Study

We apply the methods described above to the analysis of water quality in the Serbian
stretches of the Danube; this region of the river is 588km long and constitutes 20.6% of the
total 2857km river (Fig 2a). Data is collected for eleven monitoring stations along the river,
shown in Fig. 2b. The available data is for 2010, and was collected at monthly intervals with
some exceptions. Number of measurements for each constituent was following: 1) Bezdan station: 11 measurements for 8 constituents, 10 for Oxygen Saturation and BOD; 2) Bogojevo station: 9 measurements for all constituents; 3) Bačka Palanka station: 8 measurements for all constituents except 7 for BOD; 4) Novi Sad station: 12 measurements for all constituents; 5) Slankamen station: 11 measurements for all constituents; 6) Čenta station: 11 measurements for all constituents; 7) Smederevo station: 12 measurements for all constituents; 8) Banatska Palanka station: 10 measurements for all constituents; 9) Veliko Gradište station: 12 measurements for all constituents; 10) Dobra station: 12 measurements for all constituents except 11 for BOD; 11) Radujevac station: 12 measurements for all constituents. E. Coli criterion is omitted from this study, because of small number of measurements in all stations. Where a station was omitted from data collection, it is omitted from the results presented herein. No attempt was made to impute missing values.

The parameters collected in the study (Jakovljević, 2012) are used to calculate the Serbian Water Quality Index (SWQI). SWQI is an environmental indicator, developed by Serbian Environmental Protection Agency and based on the Water Quality Index method (Scottish Development Dept., 1976). SWQI uses ten quality parameters: oxygen saturation, biochemical oxygen demand (BOD₅), ammonium, pH, total nitrogen oxides, orthophosphate, suspended solids, temperature, conductivity and most probable number of coliform bacteria (E. Coli/MPN). Each of these parameters has value $q_i$ (the water quality of the $i$-th parameter) and weight unit $w_i$ (the weight attributed to the $i$-th parameter). Parameters have varying degrees of importance on the overall water quality, specified by an appropriate weight ($w_i$) where the sum of all weights is 1. By summarizing the products of all quality parameters ($q_i$) and all weights ($w_i$) an index is created representing a weight sum of all parameters ($q_i$). (Veljković, 2013; Veljković et al., 2010; Veljković et al., 2008). SWQI is then calculated as the sum of $q_i \times w_i$. The maximum value of each parameter is shown in Table 1.
Table 1: SWQI parameters and their corresponding maximum $q_i \times w_i$ value (Veljkoirić et al., 2010).

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Maximum $q_i \times w_i$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxygen Saturation (%)</td>
<td>18</td>
</tr>
<tr>
<td>BOD$_5$ (mg/l)</td>
<td>15</td>
</tr>
<tr>
<td>Ammonium (mg/l)</td>
<td>12</td>
</tr>
<tr>
<td>pH</td>
<td>9</td>
</tr>
<tr>
<td>Total Nitrogen oxides (mg/l)</td>
<td>8</td>
</tr>
<tr>
<td>Orthophosphates (mg/l)</td>
<td>8</td>
</tr>
<tr>
<td>Suspended solids (mg/l)</td>
<td>7</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>5</td>
</tr>
<tr>
<td>Conductivity ($\mu$S/cm)</td>
<td>6</td>
</tr>
<tr>
<td>E. Coli (MNP/100 ml)</td>
<td>12</td>
</tr>
</tbody>
</table>

$SWQI = \sum(q_i \times w_i)$

100
For each SWQI range a descriptive quality indicator has been defined ranging from very poor (0–38), poor (39–71), good (72–83), very good (84–89), and excellent (90–100) (Veljković et al., 2008). Parameter values are shown in Table 2.
### Table 2: Parameters concentration corresponding to $Q_i \times W_i$ (Scottish Development Department, 1976)

<table>
<thead>
<tr>
<th>Water quality (q x w)</th>
<th>Oxygen saturation (%)</th>
<th>BOD (mg/l)</th>
<th>Ammonium (mg/l)</th>
<th>E.coli (coli/100ml)</th>
<th>Suspended solids (mg/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>93-109</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>17</td>
<td>88-92</td>
<td>110-119</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>16</td>
<td>85-87</td>
<td>120-129</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>81-84</td>
<td>130-134</td>
<td>0</td>
<td>0.9</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>78-80</td>
<td>135-139</td>
<td>1.0</td>
<td>1.9</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>75-77</td>
<td>140-144</td>
<td>2.0</td>
<td>2.4</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>72-74</td>
<td>145-154</td>
<td>2.5</td>
<td>2.9</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>69-71</td>
<td>155-164</td>
<td>3.0</td>
<td>3.4</td>
<td>0.10</td>
</tr>
<tr>
<td>10</td>
<td>66-68</td>
<td>165-179</td>
<td>3.5</td>
<td>3.9</td>
<td>0.15</td>
</tr>
<tr>
<td>9</td>
<td>63-65</td>
<td>180-191</td>
<td>4.0</td>
<td>4.4</td>
<td>0.20</td>
</tr>
<tr>
<td>8</td>
<td>59-62</td>
<td>-</td>
<td>4.5</td>
<td>4.9</td>
<td>0.25</td>
</tr>
<tr>
<td>7</td>
<td>55-58</td>
<td>-</td>
<td>5.0</td>
<td>5.4</td>
<td>0.30</td>
</tr>
<tr>
<td>6</td>
<td>50-54</td>
<td>-</td>
<td>5.5</td>
<td>6.1</td>
<td>0.40</td>
</tr>
<tr>
<td>5</td>
<td>45-49</td>
<td>-</td>
<td>6.2</td>
<td>6.9</td>
<td>0.50</td>
</tr>
<tr>
<td>4</td>
<td>40-44</td>
<td>-</td>
<td>7.0</td>
<td>7.9</td>
<td>0.60</td>
</tr>
<tr>
<td>3</td>
<td>35-39</td>
<td>-</td>
<td>8.0</td>
<td>8.9</td>
<td>0.70</td>
</tr>
<tr>
<td>2</td>
<td>25-34</td>
<td>-</td>
<td>9.0</td>
<td>9.9</td>
<td>0.80</td>
</tr>
<tr>
<td>1</td>
<td>10-24</td>
<td>-</td>
<td>10.0</td>
<td>14.9</td>
<td>1.00</td>
</tr>
<tr>
<td>0</td>
<td>-</td>
<td>0.9</td>
<td>15.0+</td>
<td>10.0+</td>
<td>750000+</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Water quality (q x w)</th>
<th>pH</th>
<th>Total nitrogen oxides (mg/l)</th>
<th>Orthophosphate (mg/l)</th>
<th>Conductivity (µS/cm)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>17</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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<tr>
<td>16</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
According to the Regulation – Official Gazette 1978, all surface waters in Serbia are categorized in four classes (class I – best water quality). Parameters from Regulation were used as input parameters for SWQI calculation. Maximum Concentration Level (MCL) is defined for each of these classes; this is shown in Table 3. MCL values have been established by the Regulation 1978 and they have been constant. There were no their changes, as well as maximum $q_i \times w_i$ values have not changed during the time. This is an important for calculation of long term trends by SWQI method.
Table 3: Correlation between SWQI and Maximum Concentration Level (MCL), (Veljković et al., 2010).

Temperature and conductivity are omitted as they are not used in water quality characterisation using MCL.

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Max value</th>
<th>MCL Class I</th>
<th>MCL Class II</th>
<th>MCL Class III</th>
<th>MCL Class IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxygen saturation (%)</td>
<td>18</td>
<td>90-105</td>
<td>70-90</td>
<td>50-75</td>
<td>30-50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>105-115</td>
<td>115-125</td>
</tr>
<tr>
<td>BOD (mg/l)</td>
<td>15</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>Ammonium (mg/l)</td>
<td>12</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>pH</td>
<td>9</td>
<td>6.8–8.5</td>
<td>6.8–8.5</td>
<td>6–9</td>
<td>6–9</td>
</tr>
<tr>
<td>Total Nitrogen Oxides (mg/l)</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Orthophosphate (mg/l)</td>
<td>8</td>
<td>0.005</td>
<td>0.005</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Suspended solids (mg/l)</td>
<td>7</td>
<td>10</td>
<td>30</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Conductivity (µS/cm)</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>E.coli (coli/100ml)</td>
<td>12</td>
<td>200</td>
<td>10000</td>
<td>20000</td>
<td>20000</td>
</tr>
<tr>
<td>$\sum (q_i \times w_i) = WQI$</td>
<td>100</td>
<td>85–84</td>
<td>69–71</td>
<td>44–48</td>
<td>35–36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>74–71</td>
<td>56–52</td>
<td>51–46</td>
</tr>
</tbody>
</table>

4. Results

The two visualisation methods described above are now applied to the case study data. Due to its absence in many cases, the E. Coli criterion is omitted from these results. The results
presented herein are based on the remaining 9 SWQI parameters. In all cases, the criteria were arranged for minimisation, such that small criterion values are preferred to large ones when stations are ranked. This is in some ways an arbitrary choice, but was made because in a ranking of $N$ items 1 is generally the best rank and $N$ is the worst. The methods were applied to all of the populations (one for each month that data was collected), and we begin by illustrating the features of each visualisation.

4.1 Rank-based Visualisation

Figure 3 illustrates the population describing the water quality stations in February 2010. The left-hand plot shows the data visualised as a Pareto shell graph. The stations sort into four Pareto shells; the most powerful (best) station is Smederevo, which is in shell 1 and has the best average rank (light colours indicates a good average rank; dark colours indicates a poor average rank). Generally, the ordering of individuals according to Pareto sorting corresponds to that average ranking; stronger individuals are on the left-hand side of the visualisation and weaker individuals are on the right-hand side. This corresponds to the findings presented in Walker et al., (2010). That said, according to the Pareto sorting Veliko Gradište is the weakest station, Bezdan (shell 2) has the worst average rank of any in the population. This highlights a useful feature of the method first observed in Walker et al., (2010), whereby it is possible for an individual with an extremely poor average rank to be placed in a high Pareto shell. In order for this to happen, the individual must have a very strong score on one of the criteria, making it very hard for other individuals to dominate it; this means that the individual is likely to be placed into a strong Pareto shell. If the remainder (and majority) of the criterion values are extremely poor, the overall average rank for the individual will be very poor. In fact, Bezdan has a very strong score on the BOD criterion, and poor scores on the other eight criteria. By combining the two ranking methods we prevent stations from
being unduly rewarded for extreme criterion values, as was demonstrated by Walker et al. (2010).

Figure 4 presents the visualisations shown in Figure 3, this time coloured according to the quality rating assigned to each station using SWQI. With the exception of three stations (Smederevo, Čenta and Radujevac) all are “good”. Of the three that are not, two are “very good” and one (Smederevo) is “excellent”. This corresponds with the ranking induced by average rank, under which the strongest station was Smederevo, followed by Radujevac and Čenta. This is a useful result, as it shows that the average rank procedure allows for comparison between individuals that were incomparable under the SWQI scheme without conflicting with the partial ordering produced under SWQI. Average rank scores for the stations throughout the year are shown in Table 4. Note, that these scores do not take account of the absence of some stations from the data in some months, which causes artificially low average rank scores (this is particularly prevalent in December, where the measurement of just five stations results in a maximum average rank of 5). Figure 6 presents the distribution of average ranks graphically; in order to facilitate comparison of stations, the average rank values shown in Table 4 have been ranked, placing them on the scale 1, ..., N (for N stations in a given month) and then normalised to the range (0, 1).

Table 4: Monthly average rank scores for water quality stations.
<table>
<thead>
<tr>
<th>Location</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bačka Palanka</td>
<td>-</td>
<td>-</td>
<td>11</td>
<td>9</td>
<td>8</td>
<td>11</td>
<td>3</td>
<td>4</td>
<td>6.5</td>
<td>10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Banatska Palanka</td>
<td>7.5</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>-</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Bezdan</td>
<td>4.5</td>
<td>9</td>
<td>3</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>6.5</td>
<td>11</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Bogojevo</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>Čenta</td>
<td>7.5</td>
<td>3.5</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>5.5</td>
<td>-</td>
</tr>
<tr>
<td>Dobra</td>
<td>4.5</td>
<td>7</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>1</td>
<td>3.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Novi Sad</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td>11</td>
<td>11</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>5</td>
<td>3</td>
<td>3.5</td>
<td>5</td>
</tr>
<tr>
<td>Radujevac</td>
<td>2</td>
<td>2</td>
<td>9</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>8</td>
<td>1</td>
<td>2</td>
<td>9</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Slankamen</td>
<td>3</td>
<td>3.5</td>
<td>8</td>
<td>6.5</td>
<td>5</td>
<td>8.5</td>
<td>10</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>5.5</td>
<td>-</td>
</tr>
<tr>
<td>Smederevo</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>8.5</td>
<td>1</td>
<td>5</td>
<td>11</td>
<td>5</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>Veliko Gradište</td>
<td>6</td>
<td>8</td>
<td>2</td>
<td>6.5</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>11</td>
<td>9</td>
<td>2</td>
<td>9</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Radujevac had the highest quality according to Pareto sorting. It achieved rank 1 on nine occasions and rank 2 twice. One of these two occasions was February, in which it was dominated by Smederevo. Radujevac in turn dominated Dobra and Veliko Gradište. The SWQI values for these four stations agree with the ordering according to dominance:
Smederevo is rated excellent, Radujevac is very good, while Dobra and Veliko Gradište are rated good. The parameter that causes this relationship is BOD. In the case of Smederevo, the BOD result was 2mg/l (14 according to SWQI and class I according to the maximum concentration level (MCL)). Radujevac had BOD of 2.8mg/l (12 according to SWQI, class II MCL), while Dobra and Veliko Gradište measured 4.8mg/l (8 under SWQI and class III MCL) and 5.3 mg/l (7 under SWQI, class III MCL). The average rank results support this ordering too: Smederevo and Radujevac are the best two individuals in the population, while Dobra and Veliko Gradište are two of the worst. This agreement between the Pareto sorting method and well understood measures such as SWQI and MCL is reassuring, as it provides a simple approach to visualising the relationship between stations in a context with which engineers and scientists are familiar. This, when combined with its ability to compare stations without requiring WQIs to be weighted, illustrates the potential of the visualisation method for analysing multi-criterion water quality data. That said, if weights are available, as is the case here, then they can be incorporated via the colouring approach taken (e.g., there is an optional provision for incorporating weights into average rank).

The other occasion in which Radujevac was dominated was October, when it was dominated by Veliko Gradište. Both stations were rated very good under SWQI, however Veliko Gradište’s Ammonium value was better than that of Radujevac (0.05mg/l, or 12 according to SWQI and MCL class I-II, in the case of Veliko Gradište; 0.19mg/l, or 10 under SWQI and MCL class III-IV in the case of Radujevac). Interestingly, however, Radujevac has a poor average rank. This, in concert with the fact that Radujevac does not dominate anything in the next shell, indicates overall poor quality. By observing the average rank results for the rest of the year we can see that its performance according to average rank was poor in five months. In three of these cases, March, July and December, its SWQI rating is good, the worst classification assigned to a station in those months. That said, when considering the
distribution of normalised ranks in Figure 6, Raduèevac has the best overall quality according to average rank. While it might be tempting to interpret Smederevo as the best station (it achieves rank 1 in five months, more than any other station) its ranking in some months is particularly poor. Raduèevac, on the other hand, has more consistent performance. It appears in the top three positions in the ranking in all but four months of the year.

One of the important utilities of this type of analysis is that hydrologists can use them to observe stations with low quality, so that efforts can be made to improve river quality at those locations. Two stations with poor performance under Pareto sorting were Veliko Gradište and Bogojevo. Veliko Gradište was in rank 1 on 8 occasions, rank 2 twice, and on one occasion was the sole member of rank 4. Though it achieved a good rank in some of the months, its lower rank on other occasions reduced its overall quality. On the occasion it appeared in rank 4, it had one of the lowest average ranks for that month (February). Bogojevo was in rank 1 on 7 occasions, rank 2 twice and rank 4 once. Bačka Palanka also achieved poor results. Though it was in rank 1 on 5 occasions, there was also a month in which it appeared in ranks 2, 3 and 4, respectively. This corresponds to relatively low quality SWQI results. It was predominantly classified "good" under SWQI, and was classified "very good" just once; most of the other stations achieved a "very good" classification multiple times. In June it was classified "poor", the worst possible classification under SWQI. Novi Sad was the worst station according to Pareto sorting. It appeared in rank 1 only once, and was mainly placed into rank 2. It was also placed into ranks 3 and 4 on one occasion each.

Beyond analysing the relative performance of individual months, considering the visualisations for the year as a whole also offers useful insight. Under SWQI, the months with lowest water quality are the summer months, June to September, inclusive. June is the only month in which stations were classified "poor", and both July and August are entirely comprised of "good" classifications, with no station achieving "very good" or "excellent".
Examining the Pareto shell visualisations for these months shows that they are also the months in which the overall structure of the dominance graph is flattest. Figure 7 shows the distribution of values throughout the year for the temperature and oxygen saturation parameters. As can be seen, there is a significant peak in the values for the summer months, which implies that these parameters have significant influence on the overall structure of the population; we can infer a large degree of conflict between these parameters and one of the other parameters, as the increased temperature appears to cause the stations in the data to become mutually non-dominating. In order to properly evaluate this result it would be necessary to collect data over a number of years, and currently this data is unavailable.

4.2 PCA Visualisation

We also applied PCA to the data, in order to produce two dimensional scatterplots of the data. In this work we did not consider the data a time series; rather, each month was treated as its own case, unrelated to the other data. One of the potential difficulties with using PCA is that a loss of information is incurred. The data was projected onto the first two principal components, which, as explained earlier, represent the stations in a new coordinate space that may be, but is not necessarily, correlated with the original criterion values. Inspection of the eigenvalues indicates in all cases between 89 and 99% of the variance in the data was contained in the first two eigenvectors. This means that the majority of the information in the data has been retained for all 12 months.

As with the Pareto sorting visualisations, the data is presented in terms of both average rank and SWQI classification. The lower panel of Figure 3 illustrates the PCA projection of the data for February, and examining the clustering reveals an interesting result. The data can be broadly grouped into four clusters. The first cluster contains four stations: Čenta, Slankamen, Bezdan and Novi Sad. Examining the Pareto shell visualisation of February (Figure 2) shows
that these four stations form a cluster in terms of their dominance relations too. Slankamen
and Čenta are Pareto optimal, and both Bezdan and Novi Sad are dominated by Čenta.
Bezdan is also dominated by Čenta. The second cluster includes Radujevac, Banatska
Palanka and Smederevo. Smederevo is Pareto optimal, and dominates both Radujevac and
Banatska Palanka. The final two clusters contains the final two stations, Dobra (in a shell of
its own and dominated only by Radujevac) and Veliko Gradište (dominated only by Dobra,
again the sole member of its shell). This is an interesting result, as it shows potential for
revealing spatial coherence in the data using PCA even with such small datasets. Figure 8
shows examples for other months, and this effect is again seen in October. In that case,
Bezdan is the station with the worst quality and is placed far away from the main grouping of
stations.

Further insight into the data for August is possible using this technique. As can be seen in the
average rank case the stations have been arranged such that those with a poor average rank
are together and those with a strong average rank are further away. In the extreme, Radujevac
is placed away from the main cluster of stations; it has the best average rank for that month
and in the Pareto sorting example is Pareto optimal but dominates nothing in the subsequent
shell.

While it is possible to observe relative quality between individuals using the Pareto shell and
PCA visualisations, the actual WQI values of the stations are either not conveyed, in the case
of the Pareto shell visualisation, or discarded, in the case of PCA. Having provided this
information, we enhance their decision making potential by producing corresponding parallel
coordinate plots (Inselberg, 2009). The examples shown in Figure 9 are for February (which
corresponds to the results shown in Figure 4) and May. Each line represents a station, and is
coloured according to its SWQI score for that month (NB: stations achieving a SWQI score
of "excellent" are in this case represented with a dashed line, rather than colouring with
white). It can be seen that the station with excellent SWQI is generally at the lowest point of
the graph, indicating its superiority. It is, however, difficult to infer the overall quality of a
station from these visualisations alone, and thus we recommend that they are used in
combination with the Pareto shell or PCA visualisations.

As previously mentioned, the station with the best water quality was Raduževac. This was
caused by better quality of specific parameters comparing with other stations. The other
station with the high water quality was Smederevo, which was caused by high average water
quality. At the other hand the station with the worst water quality was Novi Sad, because of
low average water quality and Bačka Palanka due to bad quality of specific parameters.
Considering the location of these stations, this suggests that downstream stations had a better
water quality than upstream ones. This can be explained by high selfpurification of the
Danube River, which enabled reduction of organic loading. The other unexpected outcome
was the best water quality in the June according to the Pareto sorting visualisation, which was
the month with the worst water quality according to the SWQI. The explanation is in the fact
that due to the impairment of water quality in all stations and in most parameters, there was
no possibility that the stations dominated each other, except in one case. This produced the
result that the stations with the worst water quality were in the Rank 1. The parameter which
caused the water quality decline was Oxygen saturation with the following values: 34%
(Bačka Palanka), 52% (Banatska Palanka), 59% (Čenta) and 64% (Slankamen). Due to this
parameter all these stations had poor water quality according to the SWQI as well as III class
(Banatska Palanka, Čenta and Slankamen) and IV class (Bačka Palanka). This case is very
important in terms of environmental conditions, because low Oxygen saturation values can
threaten life of aquatic organisms.
5. Conclusions

Analysing river quality is an important task for environmental scientists and engineers. Given the range of available criteria according to which river quality can be measured, multi-criterion visualisation is a natural candidate for presenting the information. This work has illustrated the application of such methods to the analysis of water quality in the Serbian stretches of the Danube in 2010.

One of the methods employed multi-criterion ranking methods; the first used Pareto sorting, based on the dominance relation, to produce a partial ordering of water quality stations on which a directed graph can be constructed. This graph was shown to produce comparable results to those achieved using SWQI, which is a well-known measure of river quality used in Serbia. Additional information can be included in the visualisation by illustrating the average rank of each station with the colour of its corresponding node in the graph. This also revealed additional information, such as identifying stations with poor quality that had been unduly promoted to a high rank by the Pareto sorting procedure. Examining the average rank of stations provides a useful insight into the overall quality of a station; for example, by inspection it was clear that Radujevac was the station with highest quality in the data used for this study. Principal component analysis provided some additional insight into the data, however it is likely to be more useful in cases where larger numbers of stations are employed. We note that while in some applications using PCA can cause unacceptable levels of information loss, in this case the vast majority of variance in the data was retained in the two principal eigenvalues (89% in the worst case). Using PCA in combination with the Pareto shell visualisations facilitated further insight into the data, as well as the identification of facets of the data that appeared in both visualisations. We plan to extend the use of PCA in this area by considering the criteria as a time series, which we feel will provide further information to the decision maker given the temporal nature of the data.
The analyses described in this work have considered each visualisation a description of the
river at a specific point in time. Relationships between stations were described in that context,
as well as in the context of their results throughout the year. The criteria on which the
visualisations are based do not take account of any temporal variation in the parameter values,
however the results discussed clearly show that there is a seasonal aspect to them. The rise in
temperature and oxygen saturation levels in the summer months demonstrate this. Therefore,
beyond the demonstration that the techniques illustrated in this study can provide useful
information about river quality, a useful direction of future work would involve incorporating
this temporal or seasonal variation into the visualisation. One possibility that is currently
under investigation would be to construct additional criteria so that the historical quality of
river water can be understood, and issues regarding the sensitivity of the data can be
addressed.

Water quality has traditionally been assessed in terms of complex variable–by–variable and
water body–by–water body summaries. This type of information is of value to water quality
experts, but needs to be improved for users who want to know about the state of their local
water bodies and for managers and decision makers who require concise information about
those water bodies. Water quality index methodologies partially overcome the shortcomings
of these methods, and provide the ability to describe water quality with a single value based
on arrange of indicators and measurements. This facilitates simple communication of water
quality results to interested parties. Disadvantages of such methods include the sensitivity of
the results to the formulation of the index and the loss of potentially important interactions
between variables. By using the multi-criterion visualisation approaches proposed in this
paper, some of these disadvantages can be ameliorated.

The Pareto shell visualisation is advantageous in the visualisation of data of this type. It does
not require any dimension reduction, presenting the decision maker with a visualisation based
on all available data, and does not require the *a priori* selection of importance weights for the
criteria (although they can be incorporated into the secondary ranking method used to colour
the nodes in the graph if they are available). The method is generally very scalable, and can
be combined with well known domain-specific techniques, as was done with SWQI in this
paper. That it does not visually represent specific criterion values might be seen as a
limitation, however we feel that this is easily addressed by combining it with a separate
visualisation, such as the parallel coordinate plots demonstrated in this work. PCA is a
suitable choice as it relies on spatial proximity to convey similarity, which decision makers
can generally comprehend easily. The obvious limitation with this method is that it discards
potentially important information, however it was demonstrated here that the amount of
information lost was minimal.

Many studies have applied multi criterion decision making to conduct sustainability
assessment (Cinelli et al, 2014) in environmental and human health risk assessment (Topuz et
al., 2011), to assess habitats and wildlife (Cortina and Boggia, 2014), in the case of urban
water strategies (Moglia et al, 2012) and water supply infrastructure planning and
rehabilitation (Scholten et al., 2015; Schoelten et al., 2014) in order to support decision
makers, in agricultural systems according to the decision-makers' expectations (Carof et al.
2012), in the risk assessment of contaminated ground water (Khadam et al., 2003).
Techniques presented in this paper could also help decision makers to determine the best and
worst quality sites, as well as to determine how these water quality estimation tools relate to
land characteristics management choices, urban versus rural designations etc. Ravier et al.,
(2015) have used PCA for water quality preservation programme. Pareto optimal set was
used in different land uses, crops, pastures, forestry and soil water conservation practices at
the basin scale in the Pampas in Argentina (Cisneros et al., 2011), minimising probable flood
damages and maximizing water demand supply (Malekmohammadi et al., 2011) as well as
Pareto frontier visualisation in support of decision makers in rehabilitation of water quality in Googong Reservoir in Australia (Castelletti et al., 2010). It can be inferred that these water quality estimation techniques have already played important role as support in the process of decision making and their importance will be increase in a future.

6. Acknowledgement

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Multi-criterion Water Quality Analysis of the Danube River in Serbia: A Visualisation Approach

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Keywords: river quality, visualisation, multi-criteria analysis, ranking, water quality indicators
Abstract

River quality analysis is an important activity which, in Serbia, has been performed using the Serbian Water Quality Index (SWQI). This is a measure based on a weighted aggregation of 10 water quality parameters. In this work, alternative methods drawing on visualisation approaches used in multi-criterion decision analysis are applied to the problem of evaluating river quality in the Danube. Two methods are considered: one which constructs a graph using the dominance relation combined with a further multi-criterion ranking method, average rank, and the other in which the dimensionality of the data is reduced using PCA for visualisation. Results for data collected in 2010 are analysed and compared with the corresponding SWQI values for the river in that year, and we find that by employing these methods it is possible to reveal more information within the data than is possible by using SWQI alone.

1. Introduction

Water quality plays a vital role in all aspects of human and ecosystem survival. All living and industrial activities are controlled by physical, chemical, biological and microbiological activities (Mahapatra et al., 2011; Vasiliev et al., 2014). Anthropogenic influences and natural processes degrade surface waters and impair their use for drinking, industry, agriculture, recreation and other purposes (Sánchez et al., 2007). In aquatic environments organisms are exposed to mixtures of pollutants whose effects are hardly interpreted and predicted exclusively from chemical analyses. Moreover, the analyses of pooled chemicals, present at different compounds, increase uncertainty when evaluating water quality (Monferrán et al., 2011). Hence, analysing the quality of river water requires a range of quality indicators, from which an overall measure of quality can be produced. Evaluation of water quality parameters is necessary to plan and develop better water resource management (Mahapatra et al., 2011). To establish water quality criteria, measures of chemical and
physical constituents must be specified, as well as methods for reporting and comparing
results of water analysis (Saxena and Gangal, 2010). In order to evaluate water quality
synthetically, many techniques have been introduced to monitor and evaluate the effects of
pollution: traditional methods, modelling approaches, water quality indices (WQIs, e.g.
Armitage et al. (1983) and Prati et al. (1971)), multivariate statistical techniques, such as
principal component analyses (PCA), artificial neural networks, artificial intelligence, fuzzy
logic, as well as combinations of some of them (Ma et al., 2013; Othman et al. 2012;
Monferrán et al. 2011; Taner et al., 2011; Saxena and Gangal, 2010; Simões et al. 2008;
Nasirian, 2007). The work presented in this paper is distinct from these examples in that none
of them are based on multi-criterion visualisation techniques constructed in terms of the
water quality parameters on which WQIs are based.

Multi-criterion decision making (MCDM) is a process by which a set of options can be
assessed according to a set of criteria. MCDM is often applied in cases where the selection of
a single choice from a set of alternatives is required, and provides techniques for drawing
together information from all of the criteria so that a decision maker can make an appropriate
selection. Often the criteria are in conflict with each other, and selecting an option which
performs well against one criterion means accepting poor performance on another criterion.
A range of MCDM techniques exist, and in this work we are concerned with those in which
alternatives are ranked, providing an ordering of the alternatives so that the best one can be
readily identified through visualisation. Visualisation of alternatives allows the decision
maker to observe how they relate to each other, for example, making use of spatial
information gleaned from the placement of alternatives in the visualisation, to better
understand the nature of the alternatives from which they must choose (e.g., Walker et al,
2013).
In addition to selecting the best alternative, it can also be useful to identify the worst alternatives. Engineers often undertake activities such as constructing maintenance schedules. It is not usually feasible to maintain all of the components of an infrastructure, so those components most urgently in need of maintenance must be identified so that they can be prioritised in the schedule. This too is often formulated as a MCDM task, and one of the visualisation methods used in this study has been applied in this area (for ranking district metered areas in a water distribution network (McClymont et al., 2011) and for analysing the performance of the wireless access points in a mobile telephone network (Walker, 2013) for the purpose of constructing maintenance timetables). MCDM techniques of this variety are particularly well suited to river quality analysis as they allow engineers and scientists to identify regions of the river that have particularly poor quality so that remedial action can be taken.

In this work, we examine the quality of water in the Danube in Serbia. Due to its great international importance, the Danube River has been the subject of numerous water quality studies (Živadinović et al., 2012; Micić et al., 2011; Bird et al., 2010; Kirschner et al., 2009; Micić and Hofmann, 2009; Maljević and Balač, 2007; Relić et al., 2005).

Data was collected at a range of stations; in the MCDM context, these stations are the alternatives, or individuals. A range of water quality parameters have been collected for each of the stations at approximately monthly intervals for a year. These parameters are used as criteria in order to compute the Serbian Water Quality Index (SWQI) which ranks stations according to 10 criteria.

This paper presents an analysis of the water quality data using the criteria constructed under the scheme. We use multi-criterion visualisation techniques, one based on methods for ranking multi-criterion sets (Walker et al., 2010) and the other employing principal
component analysis (Jolliffe, 2002). The visualisations are used to provide new insight into
the data, before which we discuss avenues of future work arising from this study.

2. Multi-criterion Visualisation

Visualisation is an active area of research within the MCDM community because of the
potential for revealing more about the characteristics of a multi-criterion dataset, or
population of individuals. Visualising a population of multi-criterion individuals is usually a
nontrivial task, since it is often the case that a large number of criteria must be incorporated
into the visualisation. In the case of a two or three dimensional population the task is
relatively straightforward: we must simply construct a scatter plot in two or three dimensions,
a visualisation approach with which most people are familiar and able to use. Unfortunately,
since most people are not able to comprehend four or more spatial dimensions visually, this is
not possible for populations comprising four or more criteria. It is therefore necessary to
develop techniques for visualising such populations, and one of two approaches can be taken:
either reduce the dimensionality of the population so that it can be visualised with a scatter
plot, or find a way of visualising all of the data in an intuitive fashion.

A range of methods have been developed for visualising a population in terms of the full set
of criteria. A considerable amount of work in visualising populations has been done in the
optimisation community; optimisation problems are often formulated in terms of a set of
problem objectives.

One of the early approaches developed was the Pareto race (Korhonen and Wallenius, 1988),
which enabled a decision maker to “drive” through the space of possible solutions to an
optimisation problem in order to steer an interactive optimisation procedure. More recently,
another method developed as part of an interactive optimisation framework was the Pareto
navigator (Eskelinen et al., 2010).

Vilfredo Pareto defined the Pareto dominance relation; dominance is one of the most
frequently used methods for comparing multi-criterion individuals, and is used in MCDM so
that the relative quality of individuals can be considered without requiring a weighted
summation of their respective criterion values. The dominance relation will be formally
introduced shortly. Other techniques for visualising the complete criterion set are heatmaps
(Pryke et al., 2006; Walker et al., 2013), which represent individuals as the rows in a matrix,
criteria as the columns, and criterion values with colour (where “cool” colours indicate low
values and “warm” colours indicate high values). The alternative is to use a dimension
reduction method to identify a two or three dimensional representation of the individuals that
can be visualised with a scatter plot. Two varieties of dimension reduction can be used:
feature selection and feature extraction. In feature selection, the most representative features
are retained, and the remainder are discarded. In MCDM, this means finding the two or three
most important criteria; methods for doing this have been demonstrated (e.g. Saxena et al.,
2013). Under feature extraction, a completely new set of coordinates is identified which
represent the individuals. Common examples are PCA, which seeks to preserve as much of
the variance within the data as possible, and multidimensional scaling (MDS) which
preserves pairwise distances between individuals. MDS was recently used in combination
with a metric defined in terms of dominance to visualise multi-criterion populations (Walker
et al., 2013). The application of other feature extraction techniques, neuroscale and
generalised topographic mappings, were examined by Fieldsend and Everson (2005), and
Obayashi (2002) demonstrated the use of self-organising maps for visualising criterion data.

Of particular relevance to this work are interactive decision maps, which have been used for
analysing river quality (Lotov et al., 2004). Interactive decision maps are used to identify
goal points, regions of criterion space that are perceived to be of high quality. This is done by allowing the decision maker to experiment with different combinations of criterion values to find the most satisfactory trade-off possible. In their work, Lotov et al. (2004) present two relevant analyses using interactive decision maps. The first tackles the problem of designing wastewater treatment strategies that would enhance water quality, while the second optimises the parameters of a water quality decision support system. Another relevant application of multi-criterion visualisation was reported by Udias et al., (2012) in which a watershed management system was presented. Their visualisation was based on a combination of interactive decision maps and 2-dimensional scatter plots. The scatter plots displayed the overall quality in the ordinate axis and the monthly cost for each of four criteria in the abscissa.

Each visualisation method has features that make them attractive for specific uses. For example, heatmaps are useful because they incur no information loss and the original criterion values can be recovered from the visualisation. Projection techniques, on the other hand, incur information loss in that the original criterion values are discarded, however are very useful for identifying spatial relationships in the data. In this work, in order to overcome the shortcomings of existing approaches, we employ two methods. The first method uses dominance and rank information to visualise the water quality stations in such a way that no information is discarded. The second uses PCA to project the stations into two dimensions so that they can be visualised using scatter plots, in order to compare the resulting spatial arrangement with the dominance-based approach.

2.1 Pareto Sorting Visualisation

Ranking is an important component of MCDM framework, as well as in evolutionary algorithms (EAs) used to solve multi-objective optimisation problems. The first method we
use draws on ranking methods from multi-objective EAs. One of the most common approaches to ranking a population of multi-criterion individuals is _Pareto_ or _non-dominated sorting_, particularly well known for its use in the popular non-dominated sorting genetic algorithm (NSGA) (Srinivas and Deb, 1994), and its successor _NSGA-II_ (Deb et al., 2002).

Non-dominated, or _Pareto_ sorting is a technique that is based on the _dominance_ relation. Under dominance, an individual _u_ dominates individual _v_ if its criterion values _u_m_ are no worse than those of _v_, and are better than _v_ on at least one criterion. More formally:

\[ u < v \iff \forall m(u_m \leq v_m) \land \exists m(u_m < v_m) \]  

(1)

If an individual exists such that it is dominated by no other individuals within the population, it is called _non-dominating_. A pair of individuals where neither dominates the other are called _mutually non-dominating_. Pareto sorting follows a simple procedure by which a partial ordering of solutions is constructed. This begins by identifying all of the individuals in the population that are non-dominated. Those individuals are the strongest, and are assigned to the first Pareto shell before being temporarily discarded. The removal of shell 1 individuals means that a new subset of the individuals in the population are non-dominated (those previously dominated only by shell 1 individuals). These become the second Pareto shell, and are themselves discarded from the population, leaving a third subset of non-dominated individuals. This procedure continues until the entire population has been assigned to a Pareto shell.

A visualisation method was presented in Walker _et al_. (2010) that uses dominance information and the partial ordering resulting from Pareto sorting to construct a visualisation of a population. The population is cast as a directed graph, such that individuals are nodes arranged into columns according to their Pareto shell (each column represents a shell) and edges are used to show dominance relations between individuals in adjacent shells.
2.2 Average Rank

The Pareto sorting visualisation furnishes us with a way of discriminating between individuals in different Pareto shells, however it provides limited information about the difference between individuals in the same shell. We therefore enhance the visualisation with a complementary ranking method. Additionally, while dominance is capable of distinguishing between individuals comprising a small number of criteria, it is known that individuals comprising a large number of criteria (often called “many-objective” or “many-criterion” individuals) are likely to be mutually non-dominating (Farina and Amato, 2003). As such, various alternative schemes for ranking multi-criterion populations have been developed; one of these methods is average rank (Bentley and Wakefield, 1998). In order to calculate the average rank \( \bar{r}_i \) of an individual \( y_i \) the population is ranked \( M \) times, once for each criterion. This converts the population to rank-coordinates, such that each criterion is on the scale 1 to \( N \); the best individual has a score of 1, and the worst has a score of \( N \). Then, an average of the rank-coordinate values for the individual is taken:

\[
\bar{r}_i = \frac{1}{M} \sum_{m=1}^{M} r_{im}
\]

(2)

Other multi-criterion ranking methods can be employed instead of average rank; several were demonstrated in Walker et al., (2010), and were found to provide complementary rankings to the partial ordering resulting from Pareto sorting. In fact, any colour scheme can be used. In the next section, we also colour the nodes according to the classification of the stations under SWQI.

2.3 Principal Component Analysis

A widely used visualisation method is Principal Component Analysis (PCA) (Jolliffe, 2002), and we use it here to perform multi-criterion analyses. PCA is well suited to reducing the
dimensionality of multi-criterion individuals. It has, for example, been used to reduce the
dimensionality of populations of solutions in evolutionary algorithms (e.g., Deb and Saxena,
2005). In terms of water quality analysis, Astel et al. (2007), Kowalkowski et al. (1971) and
Vega et al. (1998) presented visualisations of water quality using PCA. Other studies have
used PCA for reasons other than visualisation, such as data clustering. Examples include
Koklu et al. (2010), Simeonov et al. (2003), Singh et al. (2004), Alberto et al. (2001) and
Zhang et al. (2010).

PCA projects data points into a low-dimensional space such that their new low-dimensional
representation retains as much of the variance contained within the original, high-dimensional,
data as possible. The low-dimensional space does not comprise any of the original criteria. In
the context of MCDM, each individual is a data point, and the original dimensions are the
criteria. Projecting the individuals into a low-dimensional space should therefore preserve as
much of the original variance within in the criteria, so that the most important information is
preserved. The principal components are identified by first computing the covariance matrix
of the data. Given the covariance matrix, the principal components are the eigenvectors
corresponding to the eigenvalues, which correspond to the original criteria, with largest
magnitude. In this study, as the goal is to produce a visualisation, the first two principal
components are retained. Having projected the individuals onto the first two principal
components, they can be visualised with a two-dimensional scatter plot.

3. Case Study

We apply the methods described above to the analysis of water quality in the Serbian
stretches of the Danube; this region of the river is 588km long and constitutes 20.6% of the
total 2857km river (Fig 2a). Data is collected for eleven monitoring stations along the river,
shown in Fig. 2b. The available data is for 2010, and was collected at monthly intervals with
some exceptions. Number of measurements for each constituent was following: 1) Bezdan station: 11 measurements for 8 constituents, 10 for Oxygen Saturation and BOD; 2) Bogojevo station: 9 measurements for all constituents; 3) Bačka Palanka station: 8 measurements for all constituents except 7 for BOD; 4) Novi Sad station: 12 measurements for all constituents; 5) Slankamen station: 11 measurements for all constituents; 6) Čenta station: 11 measurements for all constituents; 7) Smederevo station: 12 measurements for all constituents; 8) Banatska Palanka station: 10 measurements for all constituents; 9) Veliko Gradište station: 12 measurements for all constituents; 10) Dobra station: 12 measurements for all constituents except 11 for BOD; 11) Raduževac station: 12 measurements for all constituents. E. Coli criterion is omitted from this study, because of small number of measurements in all stations. Where a station was omitted from data collection, it is omitted from the results presented herein. No attempt was made to impute missing values.

The parameters collected in the study (Jakovljević, 2012) are used to calculate the Serbian Water Quality Index (SWQI). SWQI is an environmental indicator, developed by Serbian Environmental Protection Agency and based on the Water Quality Index method (Scottish Development Dept., 1976). SWQI uses ten quality parameters: oxygen saturation, biochemical oxygen demand (BOD$_5$), ammonium, pH, total nitrogen oxides, orthophosphate, suspended solids, temperature, conductivity and most probable number of coliform bacteria (E. Coli/MPN). Each of these parameters has value $q_i$ (the water quality of the $i$-th parameter) and weight unit $w_i$ (the weight attributed to the $i$-th parameter). Parameters have varying degrees of importance on the overall water quality, specified by an appropriate weight ($w_i$) where the sum of all weights is 1. By summarizing the products of all quality parameters ($q_i$) and all weights ($w_i$) an index is created representing a weight sum of all parameters ($q_i w_i$). (Veljković, 2013; Veljković et al., 2010; Veljković et al., 2008). SWQI is then calculated as the sum of $q_i w_i$. The maximum value of each parameter is shown in Table 1.
Table 1: SWQI parameters and their corresponding maximum $q_i \times w_i$ value (Veljković et al., 2010).

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Maximum $q_i \times w_i$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxygen Saturation (%)</td>
<td>18</td>
</tr>
<tr>
<td>BOD$_5$ (mg/l)</td>
<td>15</td>
</tr>
<tr>
<td>Ammonium (mg/l)</td>
<td>12</td>
</tr>
<tr>
<td>pH</td>
<td>9</td>
</tr>
<tr>
<td>Total Nitrogen oxides (mg/l)</td>
<td>8</td>
</tr>
<tr>
<td>Orthophosphates (mg/l)</td>
<td>8</td>
</tr>
<tr>
<td>Suspended solids (mg/l)</td>
<td>7</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>5</td>
</tr>
<tr>
<td>Conductivity (µS/cm)</td>
<td>6</td>
</tr>
<tr>
<td>E. Coli (MNP/100 ml)</td>
<td>12</td>
</tr>
<tr>
<td>$SWQI = \Sigma (q_i \times w_i)$</td>
<td>100</td>
</tr>
</tbody>
</table>
For each SWQI range a descriptive quality indicator has been defined ranging from very poor (0–38), poor (39–71), good (72–83), very good (84–89), and excellent (90–100) (Veljković et al., 2008). Parameter values are shown in Table 2.
### Table 2: Parameters concentration corresponding to $Q_i \times W_i$ (Scottish Development Department, 1976)

<table>
<thead>
<tr>
<th>Water quality ($q_i \times w_i$)</th>
<th>Oxygen saturation (%)</th>
<th>BOD (mg/l)</th>
<th>Ammonium (mg/l)</th>
<th>E.coli (coli/100ml)</th>
<th>Suspended solids (mg/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>93-109</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>17</td>
<td>88-92</td>
<td>110-119</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>16</td>
<td>85-87</td>
<td>120-129</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>81-84</td>
<td>130-134</td>
<td>0.9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>78-80</td>
<td>135-139</td>
<td>1.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>75-77</td>
<td>140-144</td>
<td>2.0</td>
<td>2.4</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>72-74</td>
<td>145-154</td>
<td>2.5</td>
<td>2.9</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>69-71</td>
<td>155-164</td>
<td>3.0</td>
<td>3.4</td>
<td>0.10</td>
</tr>
<tr>
<td>10</td>
<td>66-68</td>
<td>165-179</td>
<td>3.5</td>
<td>3.9</td>
<td>0.15</td>
</tr>
<tr>
<td>9</td>
<td>63-65</td>
<td>180-185</td>
<td>4.0</td>
<td>4.4</td>
<td>0.20</td>
</tr>
<tr>
<td>8</td>
<td>59-62</td>
<td>-</td>
<td>4.5</td>
<td>4.9</td>
<td>0.25</td>
</tr>
<tr>
<td>7</td>
<td>55-58</td>
<td>-</td>
<td>5.0</td>
<td>5.4</td>
<td>0.30</td>
</tr>
<tr>
<td>6</td>
<td>50-54</td>
<td>-</td>
<td>5.5</td>
<td>6.1</td>
<td>0.40</td>
</tr>
<tr>
<td>5</td>
<td>45-49</td>
<td>-</td>
<td>6.2</td>
<td>6.9</td>
<td>0.50</td>
</tr>
<tr>
<td>4</td>
<td>40-44</td>
<td>-</td>
<td>7.0</td>
<td>7.9</td>
<td>0.60</td>
</tr>
<tr>
<td>3</td>
<td>35-39</td>
<td>-</td>
<td>8.0</td>
<td>8.9</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>25-34</td>
<td>-</td>
<td>9.0</td>
<td>9.9</td>
<td>2.00</td>
</tr>
<tr>
<td>1</td>
<td>10-24</td>
<td>-</td>
<td>10.0</td>
<td>14.9</td>
<td>4.00</td>
</tr>
<tr>
<td>0</td>
<td>0.9</td>
<td>15.0+</td>
<td>10.0+</td>
<td>75000+</td>
<td>120+</td>
</tr>
</tbody>
</table>

### Table: Water quality, pH, Total nitrogen oxides, Orthophosphate, Conductivity, Temperature

<table>
<thead>
<tr>
<th>Water quality ($q_i \times w_i$)</th>
<th>pH</th>
<th>Total nitrogen oxides (mg/l)</th>
<th>Orthophosphate (mg/l)</th>
<th>Conductivity ($\mu$S/cm)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>17</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>16</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>6.5-7.9</td>
<td>6.0-6.4</td>
<td>8.0-8.4</td>
<td>0</td>
<td>0.49</td>
</tr>
<tr>
<td>----</td>
<td>----------</td>
<td>----------</td>
<td>---------</td>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>9</td>
<td>5.8-5.9</td>
<td>8.5-8.7</td>
<td>0.50</td>
<td>1.49</td>
<td>0.030</td>
</tr>
<tr>
<td>8</td>
<td>5.6-5.7</td>
<td>8.8-8.9</td>
<td>1.50</td>
<td>2.49</td>
<td>1.060</td>
</tr>
<tr>
<td>7</td>
<td>5.4-5.5</td>
<td>9.0-9.1</td>
<td>2.50</td>
<td>3.49</td>
<td>0.100</td>
</tr>
<tr>
<td>6</td>
<td>5.2-5.3</td>
<td>9.2-9.4</td>
<td>3.50</td>
<td>4.49</td>
<td>0.130</td>
</tr>
<tr>
<td>5</td>
<td>5.0-5.1</td>
<td>9.5-9.9</td>
<td>4.50</td>
<td>5.49</td>
<td>0.180</td>
</tr>
<tr>
<td>4</td>
<td>4.5-4.9</td>
<td>10.0-10.4</td>
<td>5.50</td>
<td>6.99</td>
<td>0.220</td>
</tr>
<tr>
<td>3</td>
<td>3.5-4.4</td>
<td>10.5-11.4</td>
<td>7.00</td>
<td>9.99</td>
<td>0.280</td>
</tr>
<tr>
<td>2</td>
<td>0-34</td>
<td>11.5-14</td>
<td>10.00+</td>
<td>0.370+</td>
<td>810+</td>
</tr>
</tbody>
</table>

According to the Regulation – Official Gazette 1978, all surface waters in Serbia are categorized in four classes (class I – best water quality). Parameters from Regulation were used as input parameters for SWQI calculation. Maximum Concentration Level (MCL) is defined for each of these classes; this is shown in Table 3. MCL values have been established by the Regulation 1978 and they have been constant. There were no their changes, as well as maximum \( q_i \times w_i \) values have not changed during the time. This is an important for calculation of long term trends by SWQI method.
Table 3: Correlation between SWQI and Maximum Concentration Level (MCL), *(Veljković et al., 2010).*

Temperature and conductivity are omitted as they are not used in water quality characterisation using MCL.

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Max ( q_i \times w_i ) value</th>
<th>MCL Class I</th>
<th>MCL Class II</th>
<th>MCL Class III</th>
<th>MCL Class IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxygen saturation (%)</td>
<td>18</td>
<td>70-90</td>
<td>50-75</td>
<td>30-50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>90-105</td>
<td>105-115</td>
<td>115-125</td>
<td>125-130</td>
<td></td>
</tr>
<tr>
<td>BOD (mg/l)</td>
<td>15</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>Ammonium (mg/l)</td>
<td>12</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>pH</td>
<td>9</td>
<td>6.8–8.5</td>
<td>6.8–8.5</td>
<td>6–9</td>
<td>6–9</td>
</tr>
<tr>
<td>Total Nitrogen Oxides (mg/l)</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Orthophosphate (mg/l)</td>
<td>8</td>
<td>0.005</td>
<td>0.005</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Suspended solids (mg/l)</td>
<td>7</td>
<td>10</td>
<td>30</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Conductivity (µS/cm)</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>E.coli (coli/100ml)</td>
<td>12</td>
<td>200</td>
<td>10000</td>
<td>20000</td>
<td>20000</td>
</tr>
<tr>
<td>( \sum (q_i \times w_i) = WQI )</td>
<td>100</td>
<td>85–84</td>
<td>69–71</td>
<td>44–48</td>
<td>35–36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>74–71</td>
<td>56–52</td>
<td></td>
<td>51–46</td>
</tr>
</tbody>
</table>

4. Results

The two visualisation methods described above are now applied to the case study data. Due to its absence in many cases, the *E. Coli* criterion is omitted from these results. The results
presented herein are based on the remaining 9 SWQI parameters. In all cases, the criteria were arranged for minimisation, such that small criterion values are preferred to large ones when stations are ranked. This is in some ways an arbitrary choice, but was made because in a ranking of *N* items 1 is generally the best rank and *N* is the worst. The methods were applied to all of the populations (one for each month that data was collected), and we begin by illustrating the features of each visualisation.

### 4.1 Rank-based Visualisation

Figure 3 illustrates the population describing the water quality stations in February 2010. The left-hand plot shows the data visualised as a Pareto shell graph. The stations sort into four Pareto shells; the most powerful (best) station is Smederevo, which is in shell 1 and has the best average rank (light colours indicates a good average rank; dark colours indicates a poor average rank). Generally, the ordering of individuals according to Pareto sorting corresponds to that average ranking; stronger individuals are on the left-hand side of the visualisation and weaker individuals are on the right-hand side. This corresponds to the findings presented in Walker *et al.*, (2010). That said, according to the Pareto sorting Veliko Gradište is the weakest station, Bezdan (shell 2) has the worst average rank of any in the population. This highlights a useful feature of the method first observed in Walker *et al.*, (2010), whereby it is possible for an individual with an extremely poor average rank to be placed in a high Pareto shell. In order for this to happen, the individual must have a very strong score on one of the criteria, making it very hard for other individuals to dominate it; this means that the individual is likely to be placed into a strong Pareto shell. If the remainder (and majority) of the criterion values are extremely poor, the overall average rank for the individual will be very poor. In fact, Bezdan has a very strong score on the BOD criterion, and poor scores on the other eight criteria. By combining the two ranking methods we prevent stations from
being unduly rewarded for extreme criterion values, as was demonstrated by Walker et al. (2010).

Figure 4 presents the visualisations shown in Figure 3, this time coloured according to the quality rating assigned to each station using SWQI. With the exception of three stations (Smederevo, Čenta and Radujevac) all are “good”. Of the three that are not, two are “very good” and one (Smederevo) is “excellent”. This corresponds with the ranking induced by average rank, under which the strongest station was Smederevo, followed by Radujevac and Čenta. This is a useful result, as it shows that the average rank procedure allows for comparison between individuals that were incomparable under the SWQI scheme without conflicting with the partial ordering produced under SWQI. Average rank scores for the stations throughout the year are shown in Table 4. Note, that these scores do not take account of the absence of some stations from the data in some months, which causes artificially low average rank scores (this is particularly prevalent in December, where the measurement of just five stations results in a maximum average rank of 5). Figure 6 presents the distribution of average ranks graphically; in order to facilitate comparison of stations, the average rank values shown in Table 4 have been ranked, placing them on the scale 1, ..., N (for N stations in a given month) and then normalised to the range (0, 1).

Table 4: Monthly average rank scores for water quality stations.
<table>
<thead>
<tr>
<th>Location</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bačka Palanka</td>
<td>-</td>
<td>-</td>
<td>11</td>
<td>9</td>
<td>8</td>
<td>11</td>
<td>3</td>
<td>4</td>
<td>6.5</td>
<td>10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Banatska Palanka</td>
<td>7.5</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>-</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Bezdan</td>
<td>4.5</td>
<td>9</td>
<td>3</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>6.5</td>
<td>11</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Bogojevo</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>Čenta</td>
<td>7.5</td>
<td>3.5</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>5.5</td>
<td>-</td>
</tr>
<tr>
<td>Dobra</td>
<td>4.5</td>
<td>7</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>1</td>
<td>3.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Novi Sad</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td>11</td>
<td>11</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>5</td>
<td>3</td>
<td>3.5</td>
<td>5</td>
</tr>
<tr>
<td>Radujevac</td>
<td>2</td>
<td>2</td>
<td>9</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>8</td>
<td>1</td>
<td>2</td>
<td>9</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Slankamen</td>
<td>3</td>
<td>3.5</td>
<td>8</td>
<td>6.5</td>
<td>5</td>
<td>8.5</td>
<td>10</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>5.5</td>
<td>-</td>
</tr>
<tr>
<td>Smederevo</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>8.5</td>
<td>1</td>
<td>5</td>
<td>11</td>
<td>5</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>Veliko Gradište</td>
<td>6</td>
<td>8</td>
<td>2</td>
<td>6.5</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>11</td>
<td>9</td>
<td>2</td>
<td>9</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Radujevac had the highest quality according to Pareto sorting. It achieved rank 1 on nine occasions and rank 2 twice. One of these two occasions was February, in which it was dominated by Smederevo. Radujevac in turn dominated Dobra and Veliko Gradište. The SWQI values for these four stations agree with the ordering according to dominance:
Smederevo is rated excellent, Radujevac is very good, while Dobra and Veliko Gradište are rated good. The parameter that causes this relationship is BOD. In the case of Smederevo, the BOD result was 2mg/l (14 according to SWQI and class I according to the maximum concentration level (MCL)). Radujevac had BOD of 2.8mg/l (12 according to SWQI, class II MCL), while Dobra and Veliko Gradište measured 4.8mg/l (8 under SWQI and class III MCL) and 5.3 mg/l (7 under SWQI, class III MCL). The average rank results support this ordering too: Smederevo and Radujevac are the best two individuals in the population, while Dobra and Veliko Gradište are two of the worst. This agreement between the Pareto sorting method and well understood measures such as SWQI and MCL is reassuring, as it provides a simple approach to visualising the relationship between stations in a context with which engineers and scientists are familiar. This, when combined with its ability to compare stations without requiring WQIs to be weighted, illustrates the potential of the visualisation method for analysing multi-criterion water quality data. That said, if weights are available, as is the case here, then they can be incorporated via the colouring approach taken (e.g., there is an optional provision for incorporating weights into average rank).

The other occasion in which Radujevac was dominated was October, when it was dominated by Veliko Gradište. Both stations were rated very good under SWQI, however Veliko Gradište’s Ammonium value was better than that of Radujevac (0.05mg/l, or 12 according to SWQI and MCL class I-II, in the case of Veliko Gradište; 0.19mg/l, or 10 under SWQI and MCL class III-IV in the case of Radujevac). Interestingly, however, Radujevac has a poor average rank. This, in concert with the fact that Radujevac does not dominate anything in the next shell, indicates overall poor quality. By observing the average rank results for the rest of the year we can see that its performance according to average rank was poor in five months. In three of these cases, March, July and December, its SWQI rating is good, the worst classification assigned to a station in those months. That said, when considering the
distribution of normalised ranks in Figure 6, Radujevac has the best overall quality according to average rank. While it might be tempting to interpret Smederevo as the best station (it achieves rank 1 in five months, more than any other station) its ranking in some months is particularly poor. Radujevac, on the other hand, has more consistent performance. It appears in the top three positions in the ranking in all but four months of the year.

One of the important utilities of this type of analysis is that hydrologists can use them to observe stations with low quality, so that efforts can be made to improve river quality at those locations. Two stations with poor performance under Pareto sorting were Veliko Gradište and Bogojevo. Veliko Gradište was in rank 1 on 8 occasions, rank 2 twice, and on one occasion was the sole member of rank 4. Though it achieved a good rank in some of the months, its lower rank on other occasions reduced its overall quality. On the occasion it appeared in rank 4, it had one of the lowest average ranks for that month (February). Bogojevo was in rank 1 on 7 occasions, rank 2 twice and rank 4 once. Bačka Palanka also achieved poor results.

Though it was in rank 1 on 5 occasions, there was also a month in which it appeared in ranks 2, 3 and 4, respectively. This corresponds to relatively low quality SWQI results. It was predominantly classified "good" under SWQI, and was classified "very good" just once; most of the other stations achieved a "very good" classification multiple times. In June it was classified "poor", the worst possible classification under SWQI. Novi Sad was the worst station according to Pareto sorting. It appeared in rank 1 only once, and was mainly placed into rank 2. It was also placed into ranks 3 and 4 on one occasion each.

Beyond analysing the relative performance of individual months, considering the visualisations for the year as a whole also offers useful insight. Under SWQI, the months with lowest water quality are the summer months, June to September, inclusive. June is the only month in which stations were classified "poor", and both July and August are entirely comprised of "good" classifications, with no station achieving "very good" or "excellent".
Examining the Pareto shell visualisations for these months shows that they are also the months in which the overall structure of the dominance graph is flattest. Figure 7 shows the distribution of values throughout the year for the temperature and oxygen saturation parameters. As can be seen, there is a significant peak in the values for the summer months, which implies that these parameters have significant influence on the overall structure of the population; we can infer a large degree of conflict between these parameters and one of the other parameters, as the increased temperature appears to cause the stations in the data to become mutually non-dominating. In order to properly evaluate this result it would be necessary to collect data over a number of years, and currently this data is unavailable.

**4.2 PCA Visualisation**

We also applied PCA to the data, in order to produce two dimensional scatterplots of the data. In this work we did not consider the data a time series; rather, each month was treated as its own case, unrelated to the other data. One of the potential difficulties with using PCA is that a loss of information is incurred. The data was projected onto the first two principal components, which, as explained earlier, represent the stations in a new coordinate space that may be, but is not necessarily, correlated with the original criterion values. Inspection of the eigenvalues indicates in all cases between 89 and 99% of the variance in the data was contained in the first two eigenvectors. This means that the majority of the information in the data has been retained for all 12 months.

As with the Pareto sorting visualisations, the data is presented in terms of both average rank and SWQI classification. The lower panel of Figure 3 illustrates the PCA projection of the data for February, and examining the clustering reveals an interesting result. The data can be broadly grouped into four clusters. The first cluster contains four stations: Čenta, Slankamen, Bezdan and Novi Sad. Examining the Pareto shell visualisation of February (Figure 2) shows
that these four stations form a cluster in terms of their dominance relations too. Slankamen
and Čenta are Pareto optimal, and both Bezdan and Novi Sad are dominated by Čenta.
Bezdan is also dominated by Čenta. The second cluster includes Raduževac, Banatska
Palanka and Smederevo. Smederevo is Pareto optimal, and dominates both Raduževac and
Banatska Palanka. The final two clusters contains the final two stations, Dobra (in a shell of
its own and dominated only by Raduževac) and Veliko Gradište (dominated only by Dobra,
again the sole member of its shell). This is an interesting result, as it shows potential for
revealing spatial coherence in the data using PCA even with such small datasets. Figure 8
shows examples for other months, and this effect is again seen in October. In that case,
Bezdan is the station with the worst quality and is placed far away from the main grouping of
stations.
Further insight into the data for August is possible using this technique. As can be seen in the
average rank case the stations have been arranged such that those with a poor average rank
are together and those with a strong average rank are further away. In the extreme, Raduževac
is placed away from the main cluster of stations; it has the best average rank for that month
and in the Pareto sorting example is Pareto optimal but dominates nothing in the subsequent
shell.
While it is possible to observe relative quality between individuals using the Pareto shell and
PCA visualisations, the actual WQI values of the stations are either not conveyed, in the case
of the Pareto shell visualisation, or discarded, in the case of PCA. Having provided this
information, we enhance their decision making potential by producing corresponding parallel
coordinate plots (Inselberg, 2009). The examples shown in Figure 9 are for February (which
corresponds to the results shown in Figure 4) and May. Each line represents a station, and is
coloured according to its SWQI score for that month (NB: stations achieving a SWQI score
of "excellent" are in this case represented with a dashed line, rather than colouring with
white). It can be seen that the station with excellent SWQI is generally at the lowest point of
the graph, indicating its superiority. It is, however, difficult to infer the overall quality of a
station from these visualisations alone, and thus we recommend that they are used in
combination with the Pareto shell or PCA visualisations.

As previously mentioned, the station with the best water quality was Radujevac. This was
caused by better quality of specific parameters comparing with other stations. The other
station with the high water quality was Smederevo, which was caused by high average water
quality. At the other hand the station with the worst water quality was Novi Sad, because of
low average water quality and Bačka Palanka due to bad quality of specific parameters.
Considering the location of these stations, this suggests that downstream stations had a better
water quality than upstream ones. This can be explained by high selfpurification of the
Danube River, which enabled reduction of organic loading. The other unexpected outcome
was the best water quality in the June according to the Pareto sorting visualisation, which was
the month with the worst water quality according to the SWQI. The explanation is in the fact
that due to the impairment of water quality in all stations and in most parameters, there was
no possibility that the stations dominated each other, except in one case. This produced the
result that the stations with the worst water quality were in the Rank 1. The parameter which
caued the water quality decline was Oxygen saturation with the following values: 34%
(Bačka Palanka), 52% (Banatska Palanka), 59% (Čenta) and 64% (Slankamen). Due to this
parameter all these stations had poor water quality according to the SWQI as well as III class
(Banatska Palanka, Čenta and Slankamen) and IV class (Bačka Palanka). This case is very
important in terms of environmental conditions, because low Oxygen saturation values can
threaten life of aquatic organisms.
5. Conclusions

Analysing river quality is an important task for environmental scientists and engineers. Given the range of available criteria according to which river quality can be measured, multi-criterion visualisation is a natural candidate for presenting the information. This work has illustrated the application of such methods to the analysis of water quality in the Serbian stretches of the Danube in 2010.

One of the methods employed multi-criterion ranking methods; the first used Pareto sorting, based on the dominance relation, to produce a partial ordering of water quality stations on which a directed graph can be constructed. This graph was shown to produce comparable results to those achieved using SWQI, which is a well-known measure of river quality used in Serbia. Additional information can be included in the visualisation by illustrating the average rank of each station with the colour of its corresponding node in the graph. This also revealed additional information, such as identifying stations with poor quality that had been unduly promoted to a high rank by the Pareto sorting procedure. Examining the average rank of stations provides a useful insight into the overall quality of a station; for example, by inspection it was clear that Radujevac was the station with highest quality in the data used for this study. Principal component analysis provided some additional insight into the data, however it is likely to be more useful in cases where larger numbers of stations are employed.

We note that while in some applications using PCA can cause unacceptable levels of information loss, in this case the vast majority of variance in the data was retained in the two principal eigenvalues (89% in the worst case). Using PCA in combination with the Pareto shell visualisations facilitated further insight into the data, as well as the identification of facets of the data that appeared in both visualisations. We plan to extend the use of PCA in this area by considering the criteria as a time series, which we feel will provide further information to the decision maker given the temporal nature of the data.
The analyses described in this work have considered each visualisation a description of the river at a specific point in time. Relationships between stations were described in that context, as well as in the context of their results throughout the year. The criteria on which the visualisations are based do not take account of any temporal variation in the parameter values, however the results discussed clearly show that there is a seasonal aspect to them. The rise in temperature and oxygen saturation levels in the summer months demonstrate this. Therefore, beyond the demonstration that the techniques illustrated in this study can provide useful information about river quality, a useful direction of future work would involve incorporating this temporal or seasonal variation into the visualisation. One possibility that is currently under investigation would be to construct additional criteria so that the historical quality of river water can be understood, and issues regarding the sensitivity of the data can be addressed.

Water quality has traditionally been assessed in terms of complex variable–by–variable and water body–by–water body summaries. This type of information is of value to water quality experts, but needs to be improved for users who want to know about the state of their local water bodies and for managers and decision makers who require concise information about those water bodies. Water quality index methodologies partially overcome the shortcomings of these methods, and provide the ability to describe water quality with a single value based on arrange of indicators and measurements. This facilitates simple communication of water quality results to interested parties. Disadvantages of such methods include the sensitivity of the results to the formulation of the index and the loss of potentially important interactions between variables. By using the multi-criterion visualisation approaches proposed in this paper, some of these disadvantages can be ameliorated.

The Pareto shell visualisation is advantageous in the visualisation of data of this type. It does not require any dimension reduction, presenting the decision maker with a visualisation based
on all available data, and does not require the *a priori* selection of importance weights for the
criteria (although they can be incorporated into the secondary ranking method used to colour
the nodes in the graph if they are available). The method is generally very scalable, and can
be combined with well known domain-specific techniques, as was done with SWQI in this
paper. That it does not visually represent specific criterion values might be seen as a
limitation, however we feel that this is easily addressed by combining it with a separate
visualisation, such as the parallel coordinate plots demonstrated in this work. PCA is a
suitable choice as it relies on spatial proximity to convey similarity, which decision makers
can generally comprehend easily. The obvious limitation with this method is that it discards
potentially important information, however it was demonstrated here that the amount of
information lost was minimal.

Many studies have applied multi criterion decision making to conduct sustainability
assessment (Cinelli et al, 2014) in environmental and human health risk assessment (Topuz *et
al.,* 2011), to assess habitats and wildlife (Cortina and Boggia, 2014), in the case of urban
water strategies (Moglia *et al.,* 2012) and water supply infrastructure planning and
rehabilitation (Scholten *et al.,* 2015; Schoelten *et al.,* 2014) in order to support decision
makers, in agricultural systems according to the decision-makers' expectations (Carof *et al.
(2012), in the risk assessment of contaminated ground water (Khadam *et al.,* 2003).

Techniques presented in this paper could also help decision makers to determine the best and
worst quality sites, as well as to determine how these water quality estimation tools relate to
land characteristics management choices, urban versus rural designations etc. Ravier *et al.,
(2015) have used PCA for water quality preservation programme. Pareto optimal set was
used in different land uses, crops, pastures, forestry and soil water conservation practices at
the basin scale in the Pampas in Argentina (Cisneros *et al.,* 2011), minimising probable flood
damages and maximizing water demand supply (Malekmohammadi *et al.,* 2011) as well as
Pareto frontier visualisation in support of decision makers in rehabilitation of water quality in Googong Reservoir in Australia (Castelletti et al., 2010). It can be inferred that these water quality estimation techniques have already played important role as support in the process of decision making and their importance will be increase in a future.

6. Acknowledgement

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7. References


Figure 1: A simple Pareto shell visualisation of a population comprising eight individuals. Edges represent dominance relations, e.g., individual B (shell 1) dominates individuals D and E (shell 2). Although B may also dominate individuals in later shells (e.g., individual F in shell 3) this relationship is omitted to preserve the clarity of the visualisation.
Figure 2: (top) Danube River Basin (Relić et al, 2011); (bottom) A map showing the locations of the 11 monitoring stations used in this study. The stations are as follows: 1 - Bezdin, 2 - Bogojevo, 3 - Baka Palanka, 4 - Novi Sad, 5 - Slankamen, 6 - Čenta, 7 - Šmederevo, 8 - Banatska Palanka, 9 - Veliko Gradište, 10 - Dobra, 11 - Raduževac.
Figure 3: Visualisations of the water quality stations for February 2010. The upper plot shows a Pareto shell visualisation of the population in which the nodes are coloured according to average rank. The lower plot shows the corresponding PCA embedding of the individuals, again, coloured according to average rank.
Figure 4: The visualisations of the February 2010 population shown in Figure 3. Rather than the colour scale used in the earlier average rank visualisations (Figure 3) these stations are coloured according to the class indicated by their SWQI scores.
Figure 5: The complete set of Pareto shell visualisations for 2010 (for February, see Figure 3), coloured according to SWQI scores.
Figure 6: The distribution of ranks for the stations throughout the year. This type of visualisation makes it possible to begin identifying the best and worst stations throughout the year according to average rank; Radujevac is the best station, with the joint lowest normalised rank and the lowest median rank of any of the stations.
Figure 7: The distribution of temperature (top) and oxygen saturation (bottom) values throughout the year; each line represents a station. The peak corresponds to the months in which the Pareto shell visualisations are flattest, indicating that this parameter has a strong structural influence on the data.
Figure 8: PCA projections for August (left-hand column) and October (right-hand column). The visualisations in the top row are coloured according to average rank while those in the bottom row are coloured according to the stations' SWQI classification.
Figure 9: Parallel coordinate plots of the populations for February (top) and May (bottom). The colours correspond with that used for the SWQI scheme in earlier figures; white stations are now shown with a dashed line. With these plots it is possible to observe the individual criterion values of an individual, however it is more difficult to compare the overall quality of two stations.