

# Hybrid Evolutionary Optimization/Heuristic Technique for Water System Expansion & Operation

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## Abstract

This paper presents a methodological solution to The Battle of Background Leakage Assessment for Water Networks (BBLAWN) competition. The methodology employs two constrained multiple-objective optimization problems and is implemented in the context of a software application for the generic hydraulic optimization and benchmarking of Water Distribution System (WDS) problems. The objectives are the combined infrastructure and operational costs and system-wide leakage, both to be minimized. In order to accelerate the evaluation of potential solutions, a distributed computing approach permits multiple EPANET solutions to be evaluated in parallel. A pressure-driven demand extension to EPANET assists the optimization in accurately ranking near-feasible solutions and to dynamically allocate leakage demand to nodes. Pressure Reducing Valves (PRVs) have been located in two ways: *a priori*, with respect to the optimization analysis and *a posteriori* after the infrastructure optimization to reduce excess pressure and pipe leakage. The latter demonstrates better overall fitness, leading to optimal configurations dominating those obtained with the former. Several temporal resolutions for PRV settings have been evaluated to contrast the optimal solutions with the computational effort required.

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25 *Author Keywords:* Multiple Objective Optimization; Evolution Algorithms; BBLAWN; Water  
26 Distribution Systems; Leakage

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## 27 **Introduction**

28 The Battle of Background Leakage Assessment for Water Networks competition (BBLAWN)  
29 (Giustolisi *et al.*, 2015) presents a challenge in optimizing a water network both in terms of  
30 design/expansion and also operation. The analysis requires the reinforcement through the  
31 replacement or augmentation of its components (tanks, pipes, etc.) and better management of the  
32 system operation by regulating the use of pumps and the installation of PRVs in order to minimize  
33 leakage and operational costs in terms of energy consumption.

34 Population-based optimization techniques have emerged over the last few decades as a popular  
35 technique for application to water distribution system design, operation and rehabilitation  
36 problems. Within this class of technique, a number of approaches have been proposed including  
37 genetic algorithms (Savić and Walters, 1997) and memetic algorithms such as the Shuffled Frog  
38 Leaping Algorithm (Eusuff and Lansey, 2003) and Ant Colony Optimization (Maier *et al.*, 2003).  
39 A number of these have been applied to the BBLAWN problem in order to determine which might  
40 be the most effective approach (Morley and Tricarico, 2014). The high dimensionality of the  
41 problem appeared to cause many of the techniques to struggle with the optimization – leading to  
42 the selection of the Omni-Optimizer algorithm (Deb and Tiwari, 2008) coupled with both in-  
43 process and post-processing heuristics – which produced the runner-up solution in the competition.  
44 In this paper, the analysis has been refined by analysing in greater detail the optimal component

45 of the system and by allocating the location of PRVs in the system by means of two different  
46 approaches, *a priori* with respect to the optimization runs or *a posteriori* in which an initial  
47 optimization of the infrastructure with no PRVs installed is undertaken. This latter method has  
48 demonstrated solutions characterized by a better fitness with respect to those obtained by the  
49 original methodology applied by Morley and Tricarico (2014).

## 50 **Methodology**

51 The software methodology employed combines a revised, pressure-driven version of EPANET  
52 (Morley and Tricarico, 2008; Rossman, 2000) with a C++ implementation of optimization  
53 algorithm to model the effect on the hydraulic network performance under the varying system  
54 parameters derived through the optimization process. This combination is embodied in a unified,  
55 generic WDS optimization application, also developed in C++.

## 56 **Objectives**

57 The BBLAWN optimization has been formulated as twin-objective optimization problem to  
58 minimize:

- 59 1. Total Cost – the sum of annualized infrastructure upgrade costs (pipe replacement and  
60 duplications, tank, pump and valve installation) and annual operational (pumping) costs.
- 61 2. Leakage – the absolute annual volume of water lost as leakage.

## 62 **Hydraulic Solver**

63 The BBLAWN problem introduces a leakage model whereby leaks are calculated on a per-pipe  
64 basis and then aggregated into the demand nodes as per Giustolisi, *et al.* (2015).

65 Since the leakage ascribed to a particular node is a function of the pressure both at itself and at the

66 nodes at the end of each attached link, it is not possible to use the standard EPANET emitter  
67 component to model the leakage which operates on the basis of the available pressure at a single  
68 node. One approach would be to run the EPANET model normally and then adjust the demands  
69 to account for the leakage and to rerun the model repeatedly until convergence was reached.  
70 Whilst this has the advantage of not requiring any modifications to EPANET directly, it was  
71 discounted because of the extended run-times that such a strategy would necessarily entail.  
72 Having successfully retrofitted a pressure-driven extension to EPANET previously (Morley and  
73 Tricarico, 2008) the authors have experience in adapting and extending the hydraulic solver and,  
74 accordingly, the leakage model described above has been incorporated directly into the C language  
75 source code of the EPANET toolkit. A number of functions have been modified (detailed in Table  
76 1) to accommodate the leakage model as part of the normal iterative cycle employed by EPANET  
77 to produce the hydraulic solution. In addition, further variables were added to EPANET in order  
78 to store the leakage parameters alpha and beta for each link as well as the calculated leakage on a  
79 per-link and per-node basis. This approach has the advantage that by directly manipulating the  
80 solution matrices employed by EPANET, it is relatively straightforward to allocate leakage to  
81 tanks (as is required according to the rules). Ordinarily, EPANET does not allow the direct  
82 assignation of demands to tanks as would be necessary in this instance – requiring the introduction  
83 of additional dummy nodes and pipes in order to model this leakage correctly.

84

85 TABLE 1 TO BE INSERTED HERE

86

87 The use of EPANET with a stochastic optimization process commonly results in a large number

88 of hydraulically-infeasible solutions being generated and subsequently evaluated by the hydraulic  
89 solver. The evaluation of these infeasible solutions takes additional time as, typically, the  
90 maximum number of solver iterations is expended attempting to converge the model and,  
91 additionally, large numbers of intermediate timesteps may be introduced into the evaluation. The  
92 algorithm used seeks to avoid the worst impacts of infeasible solutions by terminating their  
93 execution after the first timestep in which they demonstrate hydraulic infeasibility. Instead of  
94 penalizing the solution heavily in order to hasten its departure from the population, the solution is  
95 marked as infeasible and estimates of its constraint violations are extrapolated, weighted by the  
96 proportion of the extended period simulation that had been successfully completed prior to the  
97 infeasibility. This results in a commensurate reduction in the runtime “wasted” in evaluating  
98 infeasible solutions as well as preserving the genetic diversity of the population to the maximum  
99 extent possible.

## 100 **Optimization Environment**

101 The software presented in this paper also includes a distributed-processing system in order to  
102 militate against the extended runtimes that are a common issue when optimizing with evolution  
103 algorithms. The BBLAWN optimization is characterized by a particularly high number of decision  
104 variables as seen in Table 2. As a consequence of this, the deEPANET system (Morley et al.,  
105 2006), which employs the industry standard Message Passing Interface (MPI) protocol to  
106 parallelize the hydraulic simulation computation, was incorporated into the methodology. This  
107 system permits the concurrent evaluation of a large number of potential solutions either on local  
108 processors or to other computers on a LAN. Owing to the relatively long runtimes of the hydraulic  
109 simulations compared to the data transfer speeds across a modern Gigabit LAN, near linear

110 improvements in GA runtime are achievable as processing cores are added to the cluster. For the  
111 purposes of this optimization the software was deployed across a cluster of three workstations,  
112 each equipped with two Intel Xeon E5645 CPUs packages which comprise six cores running at  
113 2.4 GHz.

## 114 **Decision Variables**

115 For the purposes of optimizing the BBLAWN problem, no attempt was made to simplify the  
116 problem. Legitimate approaches to doing this might have included grouping adjacent pipes with  
117 similar characteristics or restricting the application of the optimization to pipes over a given length.  
118 Table 2 enumerates the configuration of the decision variables used in the optimization. In the  
119 first instance, as in Morley and Tricarico (2014), the potential sites for the 39 possible PRV  
120 installations were determined through engineering judgment prior to starting the optimization and,  
121 naturally, this will have biased the range of potential solutions, accordingly. The resolution  
122 afforded the settings of the PRVs has been considered (Table 3) and separate optimizations have  
123 been undertaken for four different schemes: one single fixed setting for each PRV for the entire  
124 simulation; a daily variation in which it has been assumed a different setting for each day of the  
125 simulation– i.e. 7 values for each PRV in total; one value for every 6 hours of the simulation– i.e.  
126 28 values for each PRV; and one value for each hour of the simulation the maximum resolution  
127 permissible under the rules of the competition giving 168 settings for each PRV.

128 As a second stage, as reported below, the problem has been reformulated as a two-stage  
129 optimization in which PRV locations have again been placed according to engineering judgment  
130 but following an initial optimization without considering PRVs which determines the optimal  
131 infrastructure arrangement *a priori*. With the network so optimized, the valves have been located

132 using two criteria: (1) available head for pressure reduction (i.e. making zones out of nodes that  
133 have significant excess pressure for the majority of the simulation); (2) Analysis of the maximum  
134 quantity of downstream leakage that can be reduced in order to save money. This is achieved by  
135 assuming that the each node in the network can be reduced to a theoretical minimum pressure of  
136 20m – the minimum permissible. This allows the quantification of a maximum amount of leakage  
137 that can be saved for each pipe. With this further analysis the number of PRVs to be located in  
138 the network have been reduced to 33. The pressure-setting for each PRV has been considered as  
139 detailed previously (i.e. fixed, daily, every 6h and every 1h).

140  
141 TABLE 2 TO BE INSERTED HERE

142 TABLE 3 TO BE INSERTED HERE

### 143 **Constraints**

144 During the evaluation of potential solutions a number of “hard” constraints are employed to  
145 ensure that the solution under consideration meets the minimum criteria to be considered as a  
146 solution. The constraints are divided into those general constraints which are applicable to all such  
147 optimizations such as hydraulic feasibility and avoiding negative pressures, disconnected nodes  
148 and pumps operating outside their normal flow regime. In addition there are a number of problem-  
149 specific constraints for the BBLAWN optimization, comprising: all demand nodes with a demand  
150 meeting a minimum pressure requirement of 20m; tanks not being permitted to empty at any time  
151 through the simulated time horizon and the final levels of tanks being at least as high as their initial  
152 levels to ensure that a solution is repeatable over successive weeks. Differential constraint  
153 weightings are used to signify the relative importance of meeting the optimization constraints. The

154 EPANET Error and EPANET Warning constraints are given the highest priority in order to  
155 prioritise the generation of feasible solutions by the optimization. Solutions which violate hard  
156 constraints are considered unfeasible by the optimization algorithms and as such are unlikely to  
157 play a significant role in the evolution of the population once more favourable, feasible solutions  
158 have been identified.

### 159 **Inline heuristics**

160 The formulation of the BBLAWN problem includes a pricing differential between the cost of  
161 replacing the pipe and the installation of a duplicate, parallel pipe. This is realised as a premium  
162 of 20% on the parallel pipe, ostensibly to cover the additional costs of installing an entirely new  
163 pipe. As detailed above, the optimization has complete freedom to select either replacement  
164 (including closure) and/or duplication options for each existing pipe. Accordingly, a number of  
165 heuristics were added to the objective function to ensure that the most cost-effective option is  
166 selected in each instance. These heuristics include:

- 167 • If a pipe is to be closed and also duplicated, then the selected duplicate pipe diameter is  
168 chosen as a replacement pipe – given that this will necessarily be 20% cheaper to install.
- 169 • If a pipe is to be duplicated as well as replaced, and the selected duplicate pipe diameter is  
170 larger than the replacement (and is therefore more expensive), the pipe diameters are reversed so  
171 that it is the cheaper pipe that attracts the 20% premium.
- 172 • If a pipe is to be duplicated and the existing pipe is not to be closed, a test is made to see if  
173 it is more cost-effective to install a single pipe with the same or greater cross-sectional area to the  
174 two pipes combined.

## 175 **Post-processing heuristics**

176 Owing to the very high dimensionality of the problem as formulated, it was considered likely  
177 that there would be scope to further improve the quality of the solutions obtained during the  
178 evolutionary algorithm phase of the optimization. To that end two heuristics are applied to the  
179 resulting solutions in order to identify feasible, incremental improvements can be applied to a  
180 given solution. The two heuristics operate in a mutually exclusive fashion and can be repeated a  
181 number of times.

182 In order to reduce the installation cost of the pipe infrastructure, at the expense of available  
183 pressure, the first heuristic attempts to reduce, sequentially, the pipe diameters in the network. The  
184 heuristic operates recursively from the extremities of the network, inward, and can be seen to work  
185 well for purely dendritic networks. In the event of the recursion encountering a loop, each branch  
186 of the loop is evaluated separately in turn and the most cost-effective combination implemented.

187 The second heuristic varies (downward) the pressure settings of each of the PRVs in the network  
188 for each timestep in the simulation in an attempt to further reduce available pressure and thus  
189 reduce the pressure-dependent leakage accordingly. PRVs are considered for this reduction in the  
190 order of the highest differential from upstream to downstream, for each timestep.

## 191 **Discussion of Results**

### 192 **Issues**

193 In contrast to the previous Battle problem (Marchi et al., 2014; Morley et al., 2012), the outputs  
194 of the hydraulic solver do not need to be directly compared with a reference version of the  
195 EPANET solver. As a consequence, minor variations in the computation are no longer as critical

196 for assessing the suitability of the proposed solution. However, the scale of the unconstrained  
197 problem as described above has introduced further challenges related to memory capacity. 32-bit  
198 computers are limited to accessing 4GB of memory whilst 32-bit operating systems may introduce  
199 further constraints – in the case of Microsoft Windows, each process may access a maximum of  
200 around 1.6GB. The unconstrained problem, as outlined above, requires a greater amount of  
201 memory, particularly when considering the large population sizes that the algorithms under  
202 consideration require when contemplating such large decision spaces. In order to consider a full  
203 evaluation of the unconstrained problem, using all of the decision variables, it proved necessary to  
204 move to a 64-bit implementation of the software to avoid this process limit. As has been seen  
205 previously with variations between single and double-precision versions of EPANET, the move to  
206 a 64-bit version revealed appreciable differences between the numeric solutions achieved with the  
207 32-bit version. It is thought that these numerically minor variations are present as a consequence  
208 of differing standard libraries being employed by the 32-bit and 64-bit varieties of C++ being  
209 employed. For the purposes of the analysis herein, all results were evaluated using a 64-bit, double  
210 precision version of the EPANET solver.

211 Accurately establishing pump energy consumption is somewhat problematic using the EPANET  
212 toolkit API. Instead of returning an average consumption (or total consumption) over the reporting  
213 timestep, the EN\_ENERGY result returns an instantaneous value for energy consumption. As a  
214 consequence of this, retrieving the total energy consumption for a network which has many state  
215 changes (introducing intermediate timesteps) is a somewhat contrived process. Therefore, it is  
216 necessary to recalculate both energy consumption and leakage at all intermediate timesteps in a  
217 simulation in order to obtain accurate values for both.

## 218 **Result overview and comparison**

219 For the most part, it can be seen that the optimal solutions preferred for the BBLAWN problem  
220 using this methodology are characterized by the replacement of most or all of the pipes in the  
221 network and a very small number of, or no, pipe duplications. This is a sensible outcome given  
222 the 20% cost penalty associated with pipe duplications. A more surprising and common feature  
223 in the results is the absence of any supplementary tank storage. All of the optimization techniques  
224 tested in this study employed enlarged tanks in the early stages of their evolution but later were  
225 seen to remove these from later solutions as the optimizations progressed.

### 226 227 ***A priori* PRV optimization**

228 By considering the *a priori* allocation of PRVs in the system for 39 PRVs, the Pareto Fronts  
229 obtained by varying the PRVs setting are illustrated in Figure 1. Table 4 reports the summary of  
230 the results obtained for each optimization, considering the solution with minimum total cost and  
231 minimum leakages (the solutions circled on each pareto front in Figure 1). Given the absence of  
232 any reliability criterion, it is not surprising that the GA opted for dendritic network forms,  
233 removing all but one loop from each of the candidate solutions in the resultant populations. It is  
234 interesting also to note that, in contrast to the results obtained in Morley and Tricarico (2014) these  
235 optimizations have not preferred not to isolate the tank T6.

236 FIGURE 1 TO BE INSERTED HERE

237 TABLE 4 TO BE INSERTED HERE

238  
239 From the results reported the lowest total cost solution is that of the single fixed value for each  
240 PRV. The results for the optimizations with greater degrees of freedom for the PRV settings

241 compare unfavorably. In particular, the optimization with hourly settings for the PRVs struggled  
242 to match the objective values of the other runs – likely owing to the significantly larger search  
243 space associated with this configuration.

#### 244 ***A posteriori* PRV optimization**

245 Along with the single stage optimizations, outlined above, a two stage optimization was also  
246 formulated in which location of the PRVs is determined through expert judgment following an  
247 initial optimization to determine the optimal infrastructure. A solution has been selected from the  
248 resulting Pareto front, representing the lowest overall cost solution (€1,314,874). A number of  
249 thematic maps (for example, Figure 2) were generated to assist in the placement of the PRVs,  
250 including quantification of total leakage from each pipe and the mean surplus pressure experienced  
251 at each node. Prior to PRV installation, this network configuration experiences annual leakage,  
252 by value, of €609,520. By considering a theoretical scenario in which the pressure can be reduced  
253 to the minimum required of 20m at each node, it is possible to place a lower bound on the leakage  
254 for this particular network configuration, amounting to €240,823/year. Expert analysis yielded 33  
255 pressure zones for installation – reduced from the 39 potential pressure zones identified at the  
256 outset and employed in the other optimization strategy. This reduction can be attributed to the  
257 simplification of the network into a dendritic form and a better understanding of the distribution  
258 of surplus pressure within the network. The dendritic form, whilst cheaper to implement, may be  
259 considered a less reliable topology – an objective not considered by the BBLAWN optimization.

260  
261 FIG. 2 TO BE INSERTED HERE

262 The Pareto Fronts resulting from the *a posteriori* analysis undertaken (Figure 1) demonstrate  
263 lower total costs than those obtained previously with the *a priori* PRV allocation, employing at  
264 the same time a reduced number of decision variables, reducing the computational effort required.  
265 The Summary of the optimal results obtained have been reported in Table 5 in which at the “no  
266 PRVs” solution have been compared the optimal solutions obtained by letting fixed or vary the  
267 pressure settings of the PRV as before.

268  
269 TABLE 5 TO BE INSERTED HERE

## 270 **Conclusions**

271 A novel methodology for the expansion and operation optimization problem for the BBLAWN  
272 case study has been applied and solved by means of a population-based algorithm incorporating  
273 heuristics both within the optimization process and in post processing. The problem has been  
274 considered has a two objectives one in which it was necessary to minimize both operational/design  
275 costs and leakages. The BBLAWN leakage model has been directly incorporated into a pressure  
276 driven extension of EPANET hydraulic solver to maximize the efficiency of the leakage  
277 evaluation. Evaluation of the problem has been distributed on a local cluster computing resource  
278 using the deEPANET software for parallelizing the hydraulic simulations associated with each  
279 individual solution generated by the optimization. The analysis of the problem in order to respect  
280 the “battle” criteria has been solved by means of a methodology based on engineering judgment  
281 supported by the optimization algorithm. The problem has been solved by means of two different  
282 approaches in which the PRVs location have been located *a priori* respect to the optimization or *a*  
283 *posteriori* following an initial optimization of the infrastructure alone. The latter analysis has  
284 demonstrated better solutions for both of the objectives under consideration. In addition, different

285 pressure-setting schemes for the PRVs have been considered although the results demonstrate that  
286 there is an insignificant difference in terms of objective values achieved if a fixed pressure setting  
287 is assumed compared to those schemes which required greater computational effort.

288 As a recommendation for future work, a better cost model would consider the cost and reliability  
289 implications of pump status/valve setting switches and would allow the optimization to attempt to  
290 minimize this type of cost.

## 291 **References**

292 Deb, K. and Tiwari, S. (2008). “Omni-Optimizer: A Generic Evolutionary Algorithm for Single  
293 and Multi-Objective Optimization”, *European Journal of Operational Research*, **185**(3),  
294 pp.1062-1087.

295 Eusuff, M.M. and Lansey, K.E. (2003). “Optimization of Water Distribution Network Design  
296 Using the Shuffled Frog Leaping Algorithm”, *Journal of Water Resources Planning and  
297 Management - ASCE*, **129**(3), pp.210-225.

298 Giustolisi, O., Berardi, L., Laucelli, D., Savić, D., Kapelan, Z. (2015). “Operational and Tactical  
299 Management of Water and Energy Resources in Pressurized Systems: the competition at  
300 WDSA2014” *Journal of Water Resources Planning and Management*, Special Issue on  
301 The Battle of Background Leakage Assessment for Water Networks [doi:  
302 10.1061/(ASCE)WR.1943-5452.0000583]

303 Maier, H.R., Simpson, A.R., Zecchin, A.C., Foong, W.K., Phang, K.Y., Seah, H.S., Chan Lim  
304 Tan, H.S. (2003). “Ant Colony Optimization for Design of Water Distribution Systems”,  
305 *Journal of Water Resources Planning and Management - ASCE*, **129**(3), pp.200-209.

306 Marchi A., Salomons E., Ostfeld A., Kapelan Z., Simpson A., Zecchin A., Maier H., Wu Z.,

307 Elsayed S., Song Y., Walski T., Stokes C., Wu W., Dandy G., Alvisi S., Creaco E.,  
308 Franchini M., Saldarriaga J., Páez D., Hernández D., Bohórquez J., Bent R., Coffrin C.,  
309 Judi D., McPherson T., van Hentenryck P., Matos J., Monteiro A., Matias N., Yoo D.,  
310 Lee H., Kim J., IglesiasRey P., MartínezSolano F., MoraMeliá D., RibellesAguilar J.,  
311 Guidolin M., Fu G., Reed P., Wang Q., Liu H., McClymont K., Johns M., Keedwell E.,  
312 Kandiah V., Jasper M., Drake K., Shafiee E., Barandouzi M., Berglund A., Brill D.,  
313 Mahinthakumar G., Ranjithan R., Zechman E., Morley M., Tricarico C., de Marinis G.,  
314 Tolson B., Khedr A., Asadzadeh M. (2014). “Battle of the Water Networks II”, *Journal*  
315 *of Water Resources Planning and Management - ASCE*, **140(7)**, pp. 1-14.

316 Morley, M.S., Tricarico, C., Kapelan, Z., Savić, D.A. and de Marinis, G. (2006). “deEPANET:  
317 A Distributed Hydraulic Solver Architecture for Accelerating Optimization Applications  
318 Working with Conditions of Uncertainty”. In: GOURBESVILLE, P., CUNGE, J., GUINOT, V.  
319 & LIONG, S-Y. (eds.) *Proceedings 7th International Conference on Hydroinformatics*,  
320 Nice, France, pp.2465-2472.

321 Morley, M.S. and Tricarico, C. (2008). *Pressure Driven Demand Extension for EPANET*  
322 *(EPANETpdd)*, Technical Report 2008-02, Centre for Water Systems, University of  
323 Exeter, UK, 11pp. DOI:10.13140/2.1.2094.9760

324 Morley, M.S., Tricarico, C. and de Marinis, G. (2012). “Multiple Objective Evolutionary  
325 Algorithm Approach to Water Distribution System Model Design”, *Proceedings WDSA*  
326 *2012 – 14th Water Distribution Systems Analysis Conference*, Adelaide, Australia,  
327 pp.551-559.

328 Morley, M.S. and Tricarico, C. (2014). “A comparison of population-based optimization

329 techniques for water distribution system expansion and operation”, *Procedia*

330 *Engineering*, **89**, pp13-20. DOI:10.1016/j.proeng.2014.11.154.

331 Rossman, L.A., *EPANET 2 User’s Manual*, United States Environmental Protection Agency,

332 Washington, USA, 2000, 200pp.

333 Savić, D.A. and Walters, G.A. (1997). “Genetic algorithms for least-cost design of water

334 distribution networks”, *Journal of Water Resources Planning and Management – ASCE*,

335 **123**(2), pp.67-77.

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340 model

341 **Table 2** – Common Decision Variable configuration

342 **Table 3** – Decision Variable configuration for PRV optimization

343 **Table 4** – Summary of Results (PRV locations determined *a priori*)

344 **Table 5** – Summary of Results (PRV locations determined *a posteriori*)

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**Fig. 1.** - Consolidated results showing *a priori* pareto fronts and selected solutions for *a posteriori* optimizations

**Fig. 2.** - Thematic map showing leakage (pipe thickness), surplus pressure (node size) and the 33 pressure zones identified for the *a posteriori* optimizations