Anyon exciton revisited

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Abstract. We review the main results of the anyon exciton model in light of recent criticism by Wójs and Quinn. We show that the appearance of fractionally charged anyon ions at the bottom of their numerically calculated excitation spectra is an artefact caused by finite-size effects in a spherical geometry.

1. Review of anyon exciton model

The anyon exciton model [1, 2] provides a full classification of the multiple-branch spectra of a four-particle anyon exciton, a neutral composite particle consisting of a valence hole and three anyons with charge $-e/3$. This model is only applicable for large enough separation between a photoexcited hole and a two-dimensional electron gas at exact fractional filling factor, when the Coulomb field from the hole cannot destroy the incompressible quantum liquid.

As a neutral particle, the anyon exciton possesses an in-plane momentum $\mathbf{k}$. At $k = 0$ it is also described by the angular momentum $L_z = -L$, where L is the degree of the polynomial symmetric with respect to interchange of anyon coordinates. There is an additional quantum number which enumerates different polynomials of the same degree. An important parameter of the problem is the separation h between electron and hole confinement planes, measured in units of magnetic length l. Two examples of numerically calculated energy spectra for $h = 2$ and 3 are shown in Fig. 1. For $h = 2$, the minimum of the spectrum occurs for non-zero k, which means that the exciton in the lowest energy state possesses a non-zero dipole moment. The negative dispersion in Fig. 1a arises because of the mutual repulsion of $L = 2$ and $L = 3$ branches. For higher values of h, the ground state is always at $k = 0$, as in Fig. 1b, and the lowest-branch dispersion is always positive. The value of L at $k = 0$ for the lowest branch increases with increasing h, and thus direct optical transitions from the ground state are forbidden. However, for $k \neq 0$ the wavefunction is a mixture of states with different L values, and magnetoroton-assisted transitions become possible. Fig. 2, which shows the negative charge distribution around a valence hole for two different values of k, implies a simple qualitative picture of such a transition. A quasihole appears as a result of recombination of the hole with two negatively-charged anyons which are close to it. The quasihole and a split-off anyon form a magnetoroton. Notably, for the particular value of $h = 2$ used in Figs. 1a and 2, the asymmetric $k \neq 0$ distribution has a lower energy than the symmetric one. Nevertheless, we wish to emphasise that the exciton remains neutral, contrary to the statements in Ref. [3].

Such features of the anyon exciton model as multiple-branch spectra, dark ground states, and absence of $L = 1$ states coincide with the results of exact diagonalisation for a few-electron system in the spherical geometry [3, 4]. Similarities between the electron density distribution and pair correlation functions for low-lying states in planar and spherical geometries are discussed in Ref. [2]. We wish to emphasise that such a comparison is only

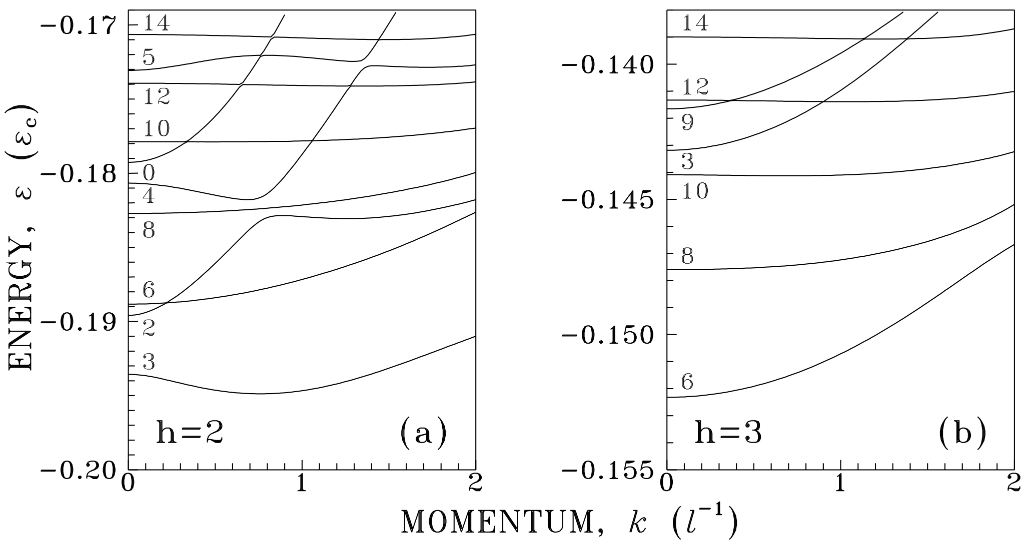


Figure 1. Anyon exciton dispersion $\epsilon(k)$ for two values of electron-hole separation h. Numbers show L values; h is in units of magnetic length l.

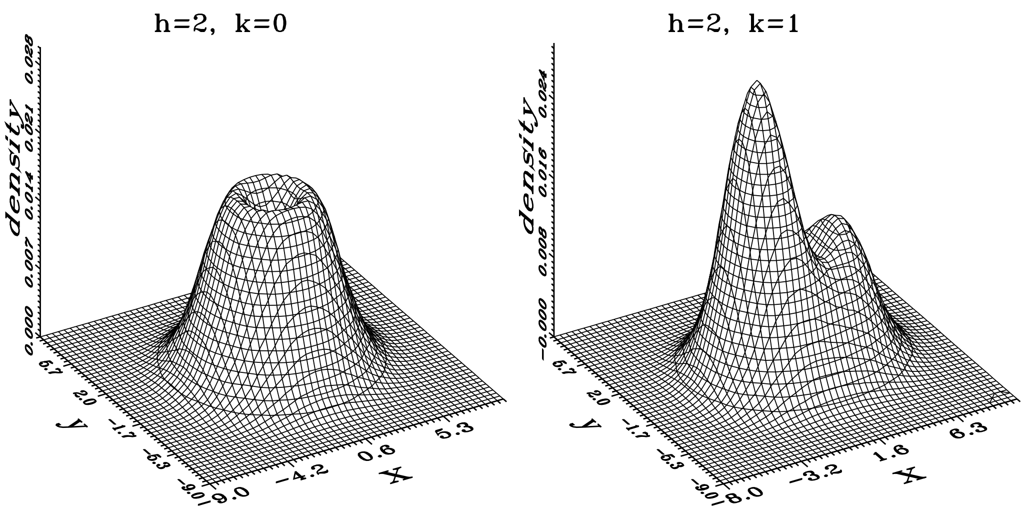


Figure 2. Electron density distribution in an anyon exciton for different values of the exciton in-plane momentum k. The distance h between the hole and the incompressible electron liquid is equal to two magnetic lengths. The hole is at the origin; the x-axis is chosen in the direction of the exciton dipole moment.

valid in the region of intermediate electron-hole separation, $h \approx 2$, where both approaches are applicable.

The presence of a 'tight' $(L = 0)$ state in the upper part of the exciton energy spectrum, together with a magnetoroton-assisted transition from the lower part of the spectrum, may explain the double-line feature observed in the intrinsic spectroscopy of incompressible quantum liquids [5].

2. Classical limit in planar and spherical geometry

When the separation h exceeds two magnetic lengths (which is a requirement for the anyon exciton model to be valid), the ground state at $k = 0$ is formed by the states with angular momentum obeying the superselection rule $L = 3N$, where $N \geq 2$. All these states are hard-core states, i.e. their wavefunctions are zero if any two of the anyon coordinates coincide. This means that the three anyons in the exciton form an equilateral triangle; this is in complete agreement with the classical picture. In fact, for large values of separation, the result of a quantum mechanical calculation for the ground state energy is the same as that obtained by a simple classical approach. This is true for quasielectrons obeying both anyonic and bosonic statistics [2].

It is natural to compare the results of classical calculations for anyon excitons and ions in both planar (Fig. 3) and spherical (Fig. 4) geometries. These calculations are based on minimising the classical potential energy of the system, and the hole and anyons are considered as point charges. Fig. 3a shows the 'classical' $k = 0$ anyon exciton, the energy of which is given by

$$E = \frac{1}{3\sqrt{3}r} - \frac{1}{(h^2 + r^2)^{1/2}} \, , \tag{1}$$

where r is the distance from each anyon to the centre of negative charge. Here, and in what follows, charge is measured in units of electron charge e. Minimising this energy with respect to r gives

$$E_{min} = -\left(\frac{2}{3}\right)^{3/2} \frac{1}{h} \approx -\frac{0.544}{h} \, . \tag{2}$$

This result coincides exactly with quantum mechanical calculations in the limit of large h [2]. A similar calculation for an ion (Fig. 3b) gives a potential energy

$$E = \frac{1}{18r} - \frac{2}{3(h^2 + r^2)^{1/2}} \, , \tag{3}$$

which, when minimised, leads to

$$E_{min} = -\frac{(12^{2/3} - 1)^{3/2}}{18h} \approx -\frac{0.485}{h} \, . \tag{4}$$

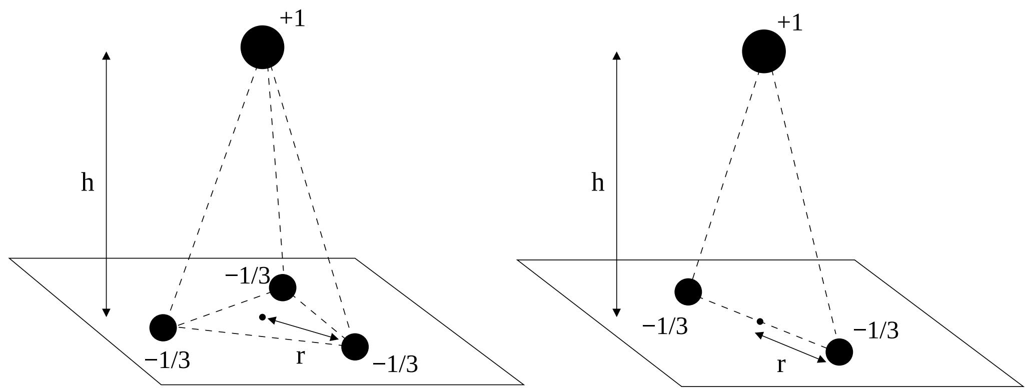
Therefore, as expected, the neutral exciton is more energetically favourable than a positively-charged ion.

For a spherical geometry, with the valence hole at the north pole of the sphere, the anyon exciton corresponds to the configuration shown in Fig. 4a, whereas for the ion, one of the anyons goes to the south pole of the sphere (Fig. 4b), which corresponds to infinity on the plane. Note that the electron-hole separation is taken into account by introducing an effective interaction $V = -(r_c^2 + h^2)^{-1/2}/3$, where r_c is taken as the anyon-hole chord length. The expression for the energy (with the radius of the sphere taken as unity) is

$$E = \frac{\sqrt{3}}{9(1 - d^2)^{1/2}} - \frac{1}{(h^2 - 2d + 2)^{1/2}} \, , \tag{5}$$

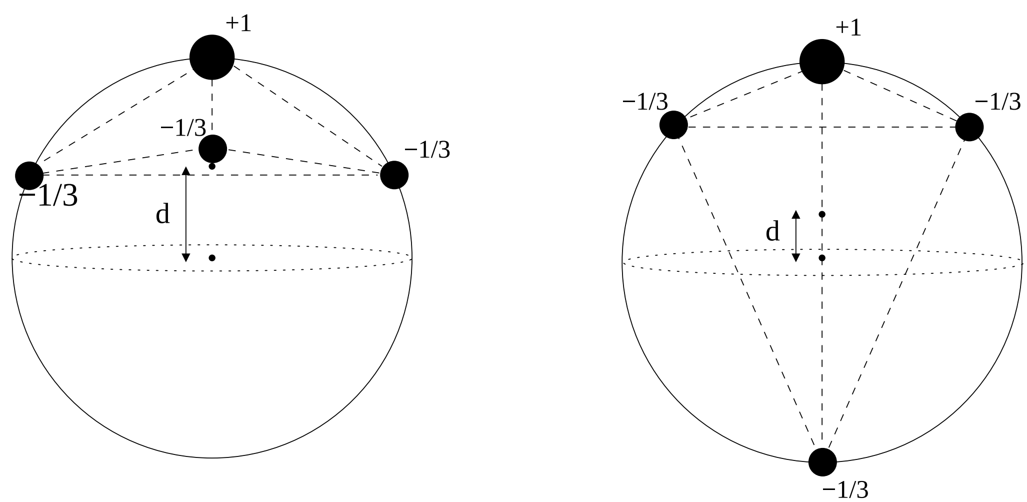
where d is the distance between the centre of negative charge and the centre of the sphere. Minimising this expression in the limit of large electron-hole separation h, we obtain

$$E_{min} = \frac{1}{3\sqrt{3}} - \frac{1}{h} + \frac{1}{h^3} - \frac{3}{2h^5} + O\left(\frac{1}{h^6}\right) \, . \tag{6}$$



(a) Anyon exciton (b) Anyon ion

Figure 3. 'Classical' anyon exciton (a) and ion (b) in the planar geometry, where h is the separation between the electron and hole confinement planes and r is the distance between each anyon and the centre of negative charge. Note that charges are scaled by the electron charge e.



(a) Anyon exciton (b) Anyon ion

Figure 4. 'Classical' anyon exciton (a) and ion (b) in the spherical geometry, where d is the distance between the centre of negative charge and the centre of the sphere.

A similar calculation for the configuration in Fig. 4b gives a potential energy

$$E = \frac{\sqrt{3}}{27(1+d)^{1/2}(1-3d)^{1/2}} + \frac{2\sqrt{3}}{27(1+d)^{1/2}} - \frac{2}{3(h^2-3d+1)^{1/2}} - \frac{1}{3(h^2+4)^{1/2}}\,, \tag{7}$$

which, when minimised, yields

$$E_{min} = \frac{1}{3\sqrt{3}} - \frac{1}{h} + \frac{1}{h^3} - \frac{9}{4h^5} + O\left(\frac{1}{h^6}\right)\,. \tag{8}$$

Comparing Eqs. 6 and 8, one can see that for large electron-hole separation in a spherical geometry, the positively-charged ion is energetically more favourable than the neutral exciton, which is not the case in the more realistic planar geometry.

Introducing realistic form factors [3], which reduce anyon-anyon repulsion at large distances, is unlikely to push one of the anyons to infinity and make the ion energetically favourable in the planar geometry.

In conclusion, we believe that the appearance of fractionally-charged anyon ions at the bottom of the numerically calculated excitation spectra [3] is an artefact caused by finite-size effects in the spherical geometry.

References

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