Title:

Universal response spectrum procedure for predicting walking-induced floor vibration

Name and address of the authors:

James Brownjohn	PhD, Professor, *corresponding author*
	College of Engineering Mathematics and Physical Sciences, University of Exeter, UK
	Harrison Building, North Park road, Exeter EX4 4QF
	Email: j.brownjohn@exeter.ac.uk
	+44 1392 723698
	Director
	Full Scale Dynamics Ltd,
	The Sheffield Incubator,
	40 Leavygreave Road,
	Sheffield S3 7RD
	Email: j.brownjohn@fullscaledynamics.com
Vitomir Racic	PhD, Lecturer
	Department of Civil and Structural Engineering, University of Sheffield, UK
	Email: v.racic@sheffield.ac.uk

Jun Chen: PhD, Professor
State Key Laboratory for Disaster Reduction in Civil Engineering
Tongji University, Shanghai, P.R. China
Email: cejchen@mail.tongji.edu.cn

Universal response spectrum procedure for predicting walking-induced floor vibration

JMW Brownjohn, V Racic, J Chen

ABSTRACT: Floor vibrations caused by people walking are an important serviceability problem both for human occupants and vibration-sensitive equipment. Present design methodologies available for prediction of vibration response due to footfall loading are complex and suffer from division between low and high frequency floors. In order to simplify the design process and to avoid the problem of floor classification, this paper presents a methodology for predicting vibration response metrics due to pedestrian footfalls for any floor type having natural frequency in the range 1 Hz to 20 Hz.

Using a response spectrum approach, a database of 852 weight-normalised vertical ground reaction force (GRF) time histories recorded for more than 60 individuals walking on an instrumented treadmill was used to calculate response metrics. Chosen metrics were peak values of 1 second peak root-mean-square (RMS) acceleration and peak envelope one-third octave velocities. These were evaluated by weight-normalising the GRFs and applying to unit-mass single degree of freedom oscillators having natural frequencies in the range 1-20 Hz and damping ratios in the range 0.5-5%. Moreover, to account for effect of mode shape and duration of crossing (i.e. duration of dynamic loading), the recorded GRFs were applied for three most typical mode shapes and floor spans from 5 m to 40 m.

The resulting peak values as functions of frequency i.e. spectra are condensed to statistical representations for chosen probability of being exceeded over a wide range of applications. RMS (acceleration) spectra show strong peaks corresponding to the first harmonic of pacing rate followed by clear minima at approximately 3.5 Hz, a second much smaller peak corresponding to the second harmonic and a steady decline with increasing frequency beginning around 5 Hz. One-third octave spectra show asymptotic trends with frequency, span and damping.

A comprehensive validation exercise focusing on the acceleration RMS spectra was based on a representative range of floor samples for which modal properties had been identified and walking response studied during experimental campaigns of vibration serviceability evaluation. Due to the statistical approach an exact validation would not be possible, hence measured peak RMS values were matched to distributions for the equivalent idealized structure. In the vast majority of cases the measured values, intended to represent worst-case conditions fitted the upper decile of the corresponding simulated spectra indicating consistency with the proposed approach.

Key words: vibration serviceability; human walking; response spectrum; low frequency floor; high frequency floor

Highlights:

- Simulations used database of 852 ground reaction forces (GRFs) recorded by treadmill
- Response spectra of 1 second RMS weighted accelerations generated from GRFs
- Simulations applied for combinations of floor span, mode frequency and damping ratio
- Characteristics and statistical distributions of spectra presented
- Comparison made against extensive database of full-scale performance data

1 INTRODUCTION

With primary concern for floor design for ultimate limit state i.e. strength and safety, vibration serviceability often gets overlooked. While the problem of vibration serviceability is well known in footbridges due to high profile public 'failures' such as the London Millennium Bridge and Passarelle Solferino in Paris [1], [2], for floors the failures (in design) rarely surface in the public domain and are usually hidden due to legal and public relations concerns. Experiences of dealing with these problems are documented by industry specialists [3] and research findings are incorporated into design guidance available from many trade organisations such as American Institute of Steel construction (AISC) and in the UK the Steel Construction Institute (SCI), Concrete Society (CS) and Concrete Centre (CC).

However, first author's own experience through numerous consulting projects is that despite such guidance, problems with excessive floor vibrations due to human footfall loading still occur, thus indicating the lack of reliable tools and procedures for vibration serviceability design. That is, even when such guidance is followed the outcome can be satisfactory and may even lead to litigation. The problems are sometimes due to unexpected or unpredictable factors, such as change of floor use or unreliable prediction of modal properties at the design stage. However, it appears that two recurring factors are inappropriate assessment criteria and unrepresentative (footfall) loading models.

Vibration serviceability of floors is commonly addressed at the design stage in two ways: (1) setting a lower bound value for the floor's fundamental frequency [4] with the intention to avoid the possibility of resonant response to footfall, or (2) setting an upper bound value for the floor vibration response according to an appropriate design measure [5]. The latter is more common in design practice and is characterized by performance-based design approach in which walking loading is defined and applied to a numerical representation of the floor. Evaluation of the resulting response depends on the floor usage and the vibration receivers. In cases when the receivers are humans, evaluations of the vibration response is most often compared to the maximum permitted value of root mean square (RMS) acceleration, with filtering or frequency weighting to limit the calculation to frequency ranges to which humans are most sensitive to vibrations [6]. For vibration-sensitive machinery, aside from occasional machine-specific requirements based on some measure of velocity or displacement, an accepted metric is the maximum value of RMS velocity in any single one-third octave band [7].

The UK guidelines for floor vibration serviceability design [8–10] determine response in a floor vibration mode either based on resonant forcing by a harmonic component of quasi-periodic loading, or on transient response to an impulse whose magnitude depends on both pacing rate and floor frequency. Consistent with the 'frequency control' approach, the resonant or transient approach is adopted according to whether or not the first mode natural frequency of the floor exceeds a threshold accepted as 10 Hz [10] and results in floor classification as 'low frequency' or 'high frequency' regardless of usage. Low frequency floors are supposed to develop resonance due to the periodicity inherent in walking. On the other hand, high frequency floors are supposed not to sustain resonance since their natural frequencies are high enough for response to a footfall to decay heavily between successive steps.

In both cases modal responses are superposed, by square-root-sum-of-squares for harmonic forcing, and directly for transient response. At the design stage modal parameters can be derived by finite element modeling or by empirical formulae offered in the guideline, while modal testing is preferable for the existing floors. On the other side of Atlantic, the American Institute of Steel Construction guidance [11] is more rational and adopts different evaluation approaches depending whether design is for human comfort or sensitive equipment.

Hence, despite a decade of progress in addressing vibration serviceability design of floors there are still deficiencies in and differences among design approaches to the exact same problem. While simple and logical, the UK approaches do not work in the many cases observed in (consulting) practice where 'high frequency' floors show clear evidence of resonant response or where 'low frequency' floors have localized high frequency modes with low modal mass that are readily excited by footfall transients. On the other hand, the US approach suffers from opaque methodology and often apparently impossible physics [12].

The approach proposed in this study advocates using response spectra to avoid the need for distinction by floor frequency or by application. Although response spectra have commonly been used as an efficient way to estimate peak dynamic response due to other key dynamic loads of structures, such as earthquakes and winds, they do not feature in the current design guidelines pertinent to human-induced vibrations. However, some researchers have considered their application in vibration serviceability design of footbridges [13] and long span floors [14]. While the footbridge study used Fourier-based numerical walking load models which are now regarded as a too conservative and unreliable representation of real walking [15], the long span floor study [14] used artificial force time histories synthesized by replicating a single footfall data measured on force plates with footfall timing data for successive steps from optical motion tracking technology.

This paper uses directly measured footfall ground reaction forces (GRFs) from continuous walking on a force measuring treadmill, thus representing variation in both timing and amplitude between successive footfalls. It extends to the full range of floor frequencies experienced by the authors and includes frequency weighting and a range of performance metrics. Moreover, the GRF records were used to establish an elaborate database of force time histories that exceeds the size and standard of similar data sets reported previously [16].

First the GRF database and its creation are described, then the straightforward methodology used to generate response spectra for a comprehensive set of representative parameters is explained. A sample of results is presented graphically, principally for moving RMS of weighted acceleration but covering one-third octave RMS velocity and peak acceleration. Characteristic features of the spectra and their statistical distribution are presented, useful for identifying the likelihood of acceptable performance according to floor characteristics. Finally a validation exercise is presented, selecting a representative range of floors among the dozens examined experimentally by the authors over the previous 20 years of research and consulting projects. The statistical nature of the process precludes an absolute proof of reliability, but the validation shows consistency with observations that can be judged by the reader.

2 WALKING LOADS FOR DEVELOPING RESPONSE SPECTRA

An essential element for developing the response spectra presented is a comprehensive database of force-time histories generated by many individuals walking at a wide range of pacing rates. In this study, such a database was established using a state-of-the-art force measuring treadmill, which design is described in Section 2.1. The choice of the equipment and the test protocol (Section 2.2) were motivated by recent studies [17,18] that proved the essential statistical equivalence between treadmill and overground locomotion in biomechanical domain, such as measuring performance of healthy athletes [19] and design of "blade runners" for disabled athletes [20]. Therefore, there is no doubt that treadmill force records are suitable for design of less delicate floor structures.

2.1 Experimental setup

The walking tests were carried out in the Light Structures Laboratory in the University of Sheffield. Continuously measured vertical force (GRF) time histories were recorded by an instrumented treadmill ADAL3D-F (Figure 1).

Figure 1: Experimental setup.

All components of the ADAL treadmill, including brushless servo motors equipped with internal velocity controllers, belts and secondary elements, are mounted on a rigid metal frame and mechanically connected to the supporting ground only through four Kistler 9077B tri-axial piezoelectric force sensors. The sensors have high stiffness to avoid the treadmill dynamic characteristics affecting the measurements. The whole system is mechanically isolated, i.e. the sensors measure only external walking forces, while the internal forces due to belt friction and belt rotation are not detected by the sensors [21].

Speed of the belt rotation (here also called “treadmill speed”) can be controlled and monitored remotely in the range 0-10 km/h either with a control panel or with bespoke software, run from the data acquisition PC. Similar to fitness treadmills, the remote control panel and the treadmill itself are equipped with a safety stop switch.

2.2 Test sequence

Prior to the force measurements, the Research Ethics Committee of the University of Sheffield required each prospective test subject to complete a Physical Activity Readiness Questionnaire and pass a preliminary fitness test (by satisfying predefined criteria for blood pressure and resting heart rate) to check whether they were suited for the moderate physical activity required during the experiment. Measurements of the body mass, age and height were taken for test subject who passed the preliminary test.

All participants wore comfortable footwear. Those who had no experience with treadmill walking were given a brief training prior to the force collection supervised by a qualified instructor. Each participant had at least ten minutes of warming up on the treadmill, which included walking while the speed was varied randomly and controlled by the speed of rotation of the treadmill belts.

During each test participants were asked to walk on the treadmill at a fixed treadmill speed. The actual walking speed could vary on a step-by-step basis around the given treadmill speed as the 1 m long belts allowed test subjects to move forwards and backwards on the treadmill, thus to slow down and speed up while walking. This made treadmill walking natural and allowed variability of successive footfalls naturally existing in overground walking [17]. The acquisition of walking forces started at a speed of 2 km/h and continued in increments of 0.5 km/h up to the maximum walking speed, i.e. an ultimate self-selected walking speed at which jogging, rather than walking, was more comfortable for an individual. In very few examples of young daring individuals this speed reached the maximum treadmill speed of 10 km/h but in most of the cases the maximum speed attainably safely was 7 km/h. Pacing rate was not prompted by any stimuli such as a metronome, and it was determined only from subsequent analysis of the generated force signals. Each test was completed when at least 64 successive footfalls were recorded and rests were allowed between successive tests.

In total, 85 volunteers (57 males and 28 females, body mass 75.8±15.2 kg, height 174.4± 8.2 cm, age 29.8±9.1 years) were drawn from students, academics and technical staff of the University of Sheffield and occasional research visitors. On average, forces corresponding to ten different walking speeds were collected for each test subject depending on their maximum comfortable walking speed. All together they generated 852 vertical walking force time histories of the kind illustrated in Figure 2. All recorded force signals were sampled at 200 Hz. Average pacing rate (and corresponding stride) was determined from analysis of the Fourier spectrum.

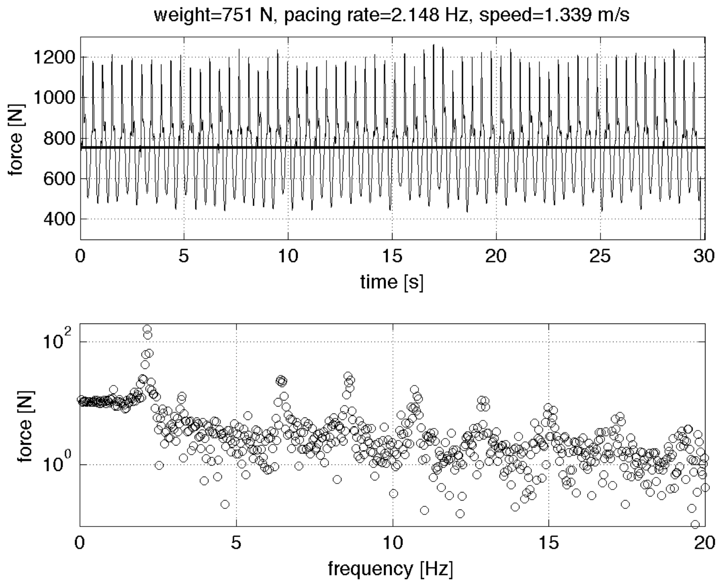

Figure 2: W0819 time history and Fourier amplitudes.

3. RESPONSE SPECTRA FROM RECORDED FOOTFALL TRACES

The vertical vibration response of a floor with span *S* to the kth walking force time history is given in terms of generalized coordinates *Y* for mode *j* as:

$$\ddot{Y}_j(t)+2\omega_j\zeta_j\dot{Y}_j(t)+\omega_j^2Y_j(t)=\frac{G_k}{M_j}p_k(t)\phi_j\left(f_kL_kt/S\right) \qquad (1)$$

where G_k is pedestrian weight, p_k(t) is ground reaction force time history normalized to unit pedestrian weight, f_k is pacing rate and L_k is the average step length with f_kL_k being the average walking speed, i.e. equal to the given treadmill speed controlled by the belt rotation. For (floor) vibration mode *j* with circular frequency ω_j and damping ratio ζ_j , modal mass M_j is normalized using a mode shape $\phi_j(x)$, $0<x<S$, having unit maximum (absolute) value.

The database of 852 treadmill GRF recordings were used to compute response time histories for spans varying from 5 m to 40 m (in 5 m increments), for damping ratios of 0.5 %, 1 %, 2 %, 3 % and 5 % and for floor frequencies from 1 Hz to 20 Hz (in 0.1 Hz increments). The frequency spacing is linear and chosen to provide a god balance of resolution vs. computational time and of course is not related to duration of the GRF time series.

While actual floor spans as high as 40 m are rare [14], experimentally observed half-sine mode shapes can span this distance and as shown in the validation exercise are in fact more relevant than the structural dimensions. Of course longer spans have frequencies in the lower range, while the shortest spans typically have frequencies in the higher range. Also, damping ratios of in-service floors are unlikely to be as low as 0.5%. Nevertheless, all these extremes were included for completeness and to demonstrate trends.

For a given span, treadmill force time histories were truncated to the span crossing time at the average walking speed, then modulated by one of three functions representing typical mode shapes:

- Half-sine representing first mode of a simply supported panel
- Full sine representing second mode of a simply supported panel
- Offset full cosine representing first mode of a fully fixed panel

Acceleration and velocity responses were calculated for the range of oscillator frequencies and the following metrics evaluated in each case:

- Peak acceleration, which is applicable if the floor is used as a footbridge or walkway.
- Maximum RMS of frequency-weighted acceleration over 1 second windows starting with 0.1 second increments. The result is 'maximum transient vibration value' (MTVV), with so called 'b-weighting' used to attenuate response outside the frequency range in which humans are most sensitive to vertical vibrations. This frequency weighting is commonly used when assessing floors in hospitals, workplaces and dwellings [9]. 1 second averaging was chosen as it is conventional in the UK practice for floor assessment and it is referenced in international standards [21]. Moreover, it is conservative since crossing durations for short spans at high pacing walking speeds could be as low as two seconds.

- Maximum RMS of unweighted velocity (also for moving 1 second windows) and evaluated in one-third octave bands with centre frequencies at least 8 Hz. This is the common metric for vibration sensitive equipment such as micro-electronics manufacturing facilities [7].

Peak factors were also available from the ratio of maximum to MTVV acceleration.

The process of deriving a response spectrum for a single time history and selected floor span, damping ratio and natural frequency corresponding to the pacing rate is summaried in Figure 3 in a sequence that runs from left to right across the first then second rows.

The crossing time T for the given span S is evaluated from the pacing rate f_k and stride length L_k then a T-second segment is chopped from the de-trended and weight-normalised time history. This is then modulated by the relevant mode shape (tapering the GRF segment ends to zero) and the T-second response for a unit mass SDOF oscillator with specified frequency (in this case the exact pacing frequency) and damping ratio and zero initial conditions is calculated. The second row shows the b-weighting filter applied to the response leading to reduced levels since in this example the oscillator frequency is away from the range of maximum human sensitivity to acceleration. The moving RMS trend is shown and the MTVV indicated. The final plot is the response spectrum which is evaluated for frequencies 1 to 20 Hz in 0.1 Hz increments.

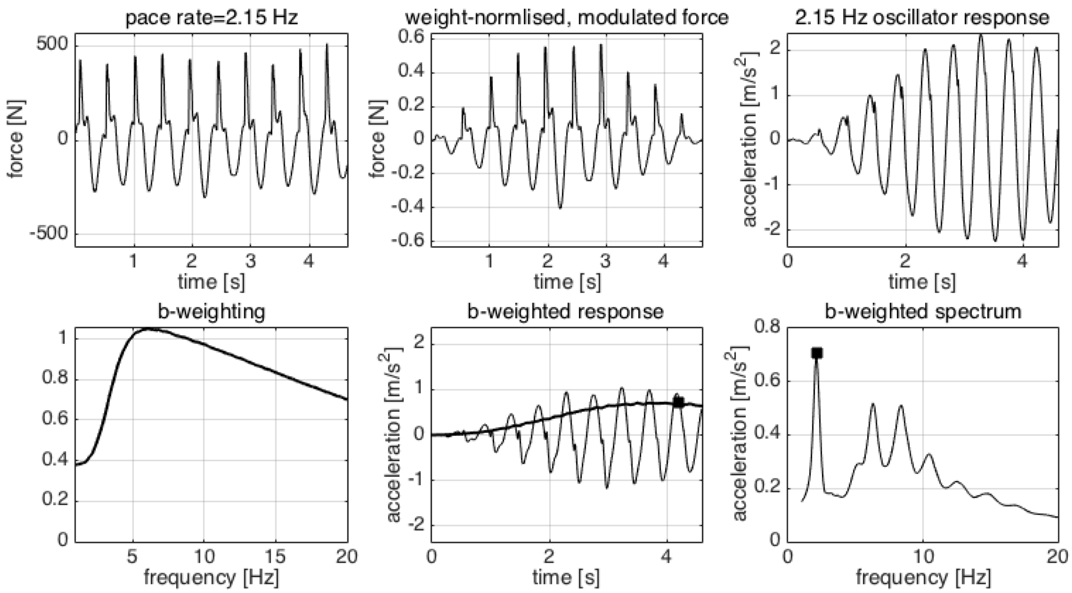

Figure 3: Response spectrum evaluation procedure for GRF of Figure 2 and 10 m span floor with 5 % damping, half-sine mode shape and frequency matching pacing rate. Lower right plot shows MTVV for oscillators in the 1-20 Hz range, with the marker mapping the MTVV from time domain.

4 A SELECTION OF RESULTS FOR GROUND REACTION FORCE RESPONSE SPECTRUM (GRFRSP)

Out of the large set of simulations, only a few examples are presented here to illustrate specific features and differences between spectra.

Figure 4 shows ensemble response spectra of MTVVs with b-weighting for a) short (5 m), b) ‘medium’ (15 m) and c) long (40 m) span floors with different (and appropriate) damping ratios and for a half-sine first vibration mode consistent with simple supports.

One obvious common feature is the strong band centred close to 2.5 Hz and corresponding to the first harmonic of pacing rates. Likewise, there is a much broader band corresponding to the second harmonic range. It is separated from the first band by a distinct trough with minimum close to 3.5 Hz which appears in every single one of the 8 (spans) x 5 (damping ratios x 3 (mode shapes) spectra. There is a less distinct trough following the second harmonic band and from approximately 8 Hz on there is a monotonic declining trend of spectra amplitudes. The major differences between the spectra are the absolute and relative amplitudes of the two peaks. The higher levels and proportionally stronger harmonic bands for longer spans reflect the opportunities to establish resonance and observe stronger transient response due to longer crossing times. The enhanced response for longer floors is recognised in guidance e.g. [10], where perfect resonance is assumed.

The wide range and distribution of the 852 individual response spectra are reflected by the mean, 95th percentile and 99th percentile values, as well as a few outlying spectra values that are proportionally greater for longer spans. This leads to a question as to what is a representative percentile value if this approach is to be used for design. 75th percentile is applied to values of impulse used in the UK guidance for high frequency floors[5,8], but it is clear from Figure 4 that a much higher percentile would need to applied here due to the very long tails of the distributions.

Figure 5 shows ensemble MTVV spectra for the three different mode types for 10 m span and 2% damping. There are no obvious differences other than small changes in overall scale suggesting that there is little to be gained by attempting exact representation of a mode shape that does not match one of the three variants.

Figure 6 shows different forms of response evaluation for 10 m span with 2 % damping and half-sine mode. Compared to Figure 5, Figure 6a illustrates the attenuating effect of the weighting on the first-harmonic response in the frequency range where humans have reduced sensitivity (Figure 3). If the unweighted spectra with dominant 1st harmonic peak were to be used, a very simple representation could be to fit a bell-shaped function around the first harmonic hump merging with a single overlaying line that decays with frequency and conservatively overestimates at the two troughs.

For floor vibration serviceability evaluation peak accelerations are seldom used as a response metric but there are some situations where floors can serve as walkways, and the response spectra models could also be applicable to footbridges where peak accelerations are relevant for vibration serviceability design [23] and Figure 6b provides one example. Around the first harmonic peak, peak accelerations appear to be about 40% higher than the unweighted MTVV, which is consistent with the resonant response that the hump represents. For higher frequencies the downtrend is gentler and range of peak accelerations lower than for MTVVs, a point discussed later.

Since one-third octave spectra are in practice used for ‘high frequency floors’ and since the vibration criteria (VC) levels are constant above 8 Hz [24], Figure 6c shows the third-octave maximum velocities for floors (oscillators) with frequencies upwards of 8 Hz. The trend in Figure 6c potentially offers a very simple spectrum for design of high-frequency floors model via an exponential or hyperbolic fit to the data. The 1-second averaging time is used here allows assessment of the shortest spans but is conservative compared to the 10 seconds often used for assessment of low-vibration manufacturing facilities (e.g. for hard disk drives and micro-electronics). For such applications there appears to be no specific guidance on averaging time other than the need for adequate frequency resolution, in this case for minimum 8 Hz band centre frequency.

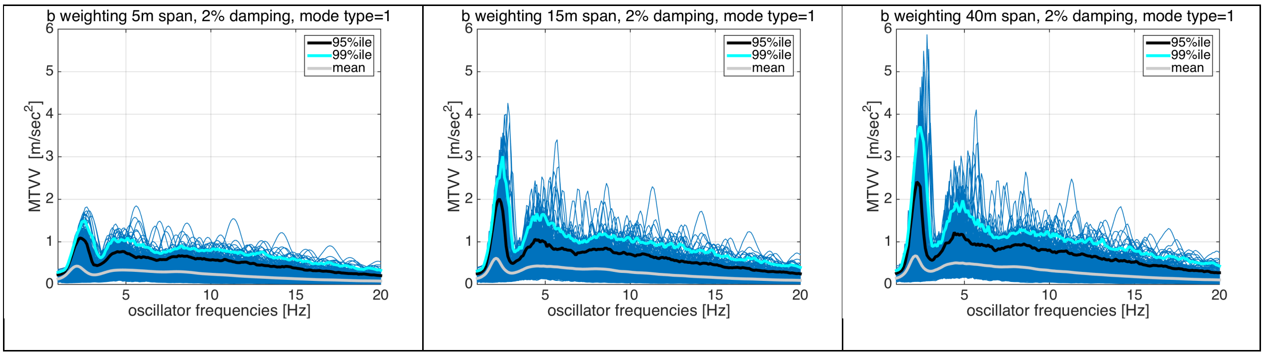

Figure 4: MTVV for b-weighting and a range of simply-supported spans in first mode. 95%ile is 95th percentile etc.

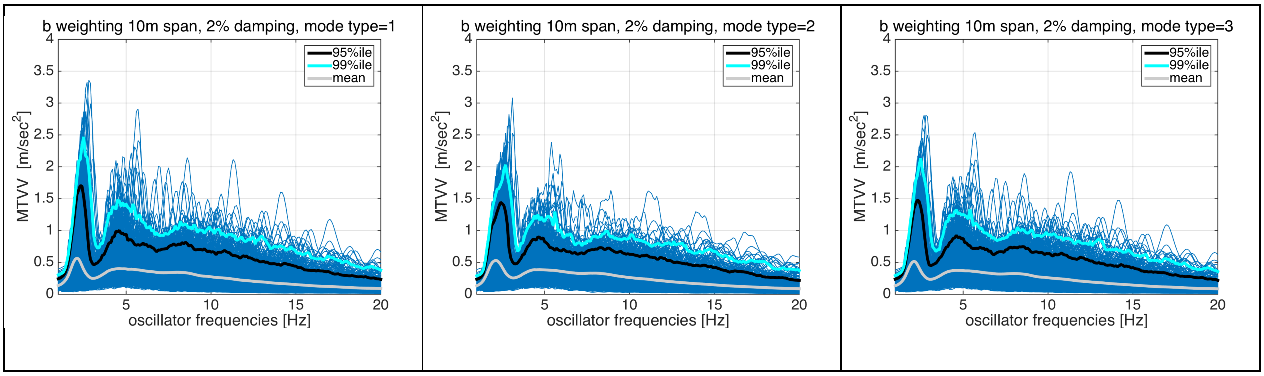

Figure 5: MTVV for b-weighting and three span types: a) simply supported first mode, b) simply-supported second mode and c) fixed end first mode. 95%ile is 95th percentile etc.

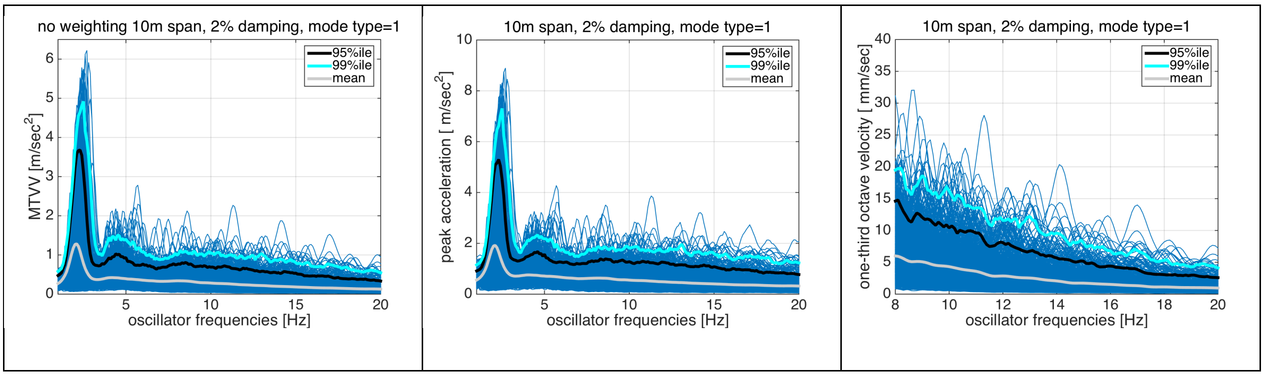

Figure 6: a) MTVV for no weighting, b) unweighted peak acceleration and c) one-third octave velocities, all for 10 m span and 2% damping. 95%ile is 95th percentile etc.

4.1. Surface plots

Because it is conservative and minimised outliers, 99th percentile is chosen as the best representative value for combining examples such as shown in Figure 4 to Figure 6 to reveal trends via surface plots with combinations of two of the three modal parameters as independent variables: floor frequency, damping and span. Other variants are mode type, weighing (none and b) and metric (MTVV and one-third octave velocity) so that only a sample projection of the parameter space can be illustrated in a single figure.

Figure 7 shows 99th percentile MTVVs vs. a) frequency and damping for 5 m span and b) against frequency and span for 1 % damping. Logarithmic scales are used for the two common axes, i.e.

frequency and MTVV, and axes are rotated (with frequency axes reversed) for best view of the important features. MTVV trends for the first harmonic are consistent with the behaviour of damped harmonic oscillator with resonant amplitude depending on inverse of damping ratio (Figure 7a) and asymptotic build-up to steady state response (Figure 7b).

For higher frequencies the strong dependence on damping is at first glance surprising given that vibration response of high frequency floors is assumed to be governed by impulsive nature of heel strikes where the level of the resulting transient response depends primarily on oscillator (floor) mass and frequency.

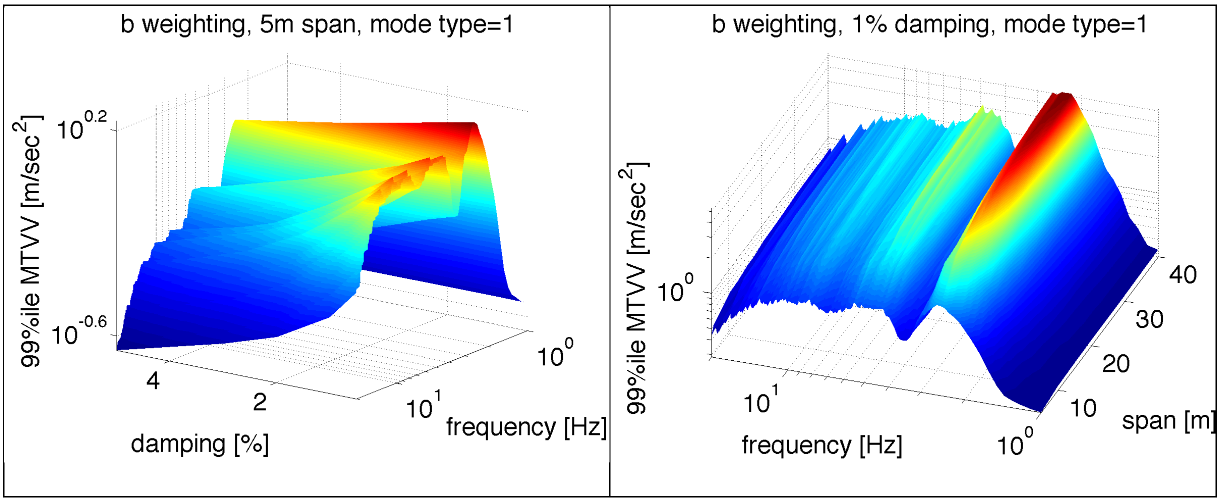

Figure 7: MTVV vs a) frequency and damping and b) vs frequency and span for simply supported first mode.

For one-third octave velocities, Figure 8, the surfaces use only linear axes and the most remarkable feature is (for the lower frequencies) an asymptotic buildup resembling the result of resonant forcing.

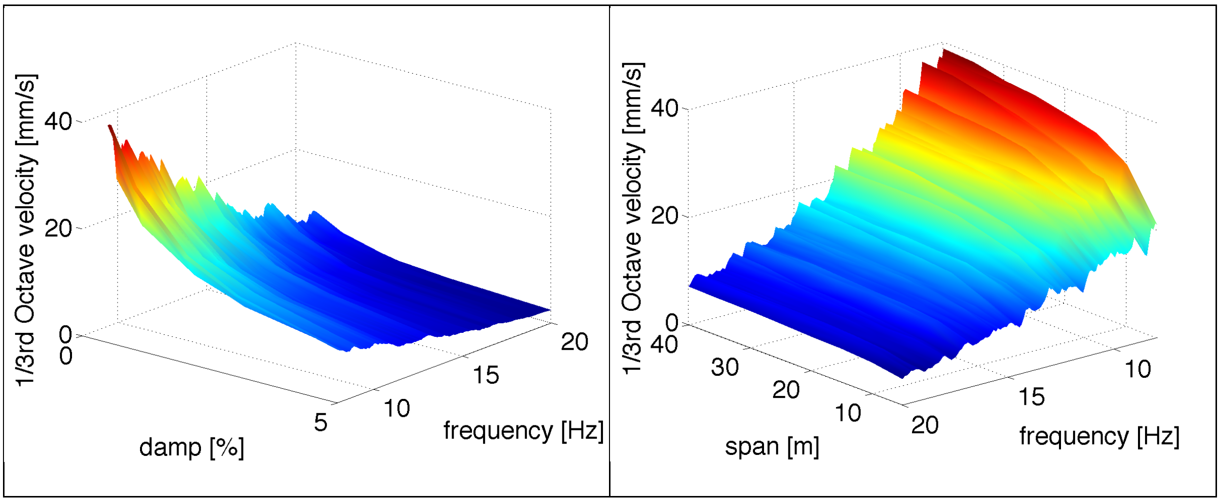

Figure 8: One-third octave RMS velocities vs a) frequency and damping for 10m span and b) vs frequency and span for 1 % damping, both for simply supported first mode.

4.2 Statistical Analysis of Spectrum Parameters

The spectra overlays of Figure 4 to Figure 6 do not reveal the full statistical properties of the various metrics, but it is at least clear that conservatively high percentiles values need to be used for design purposes and that any fitted distribution function would need to be asymmetric and have long tails for extreme values. For illustration Figure 9a shows as a contour plot density probability function of MTVV values corresponding to Figure 5a. Two bands are visible, the first for low frequencies and corresponding to the first harmonic plateau clearly showing the trend of riding MTVV but diminishing probability as the most likely range of MTVVs switches to the second harmonic plateau.

This is responsible for the trough between first and second harmonics in the spectra overlays of Figure 4 to Figure 6 and leads to a bi-modal distribution for low frequency floors (e.g. 2 Hz) as opposed to a single mode for higher frequency floors (e.g. 18 Hz).

Horizontal sections of Figure 9a at 2 Hz and 18 provide probability density functions from which cumulative density functions (CDFs) are derived and shown in Figure 9b in which the bimodal distribution for 2 Hz is clearly visible. Also note that the MTVV axes are logarithmic in both plots showing that values might need to be represented by a log-normal distribution.

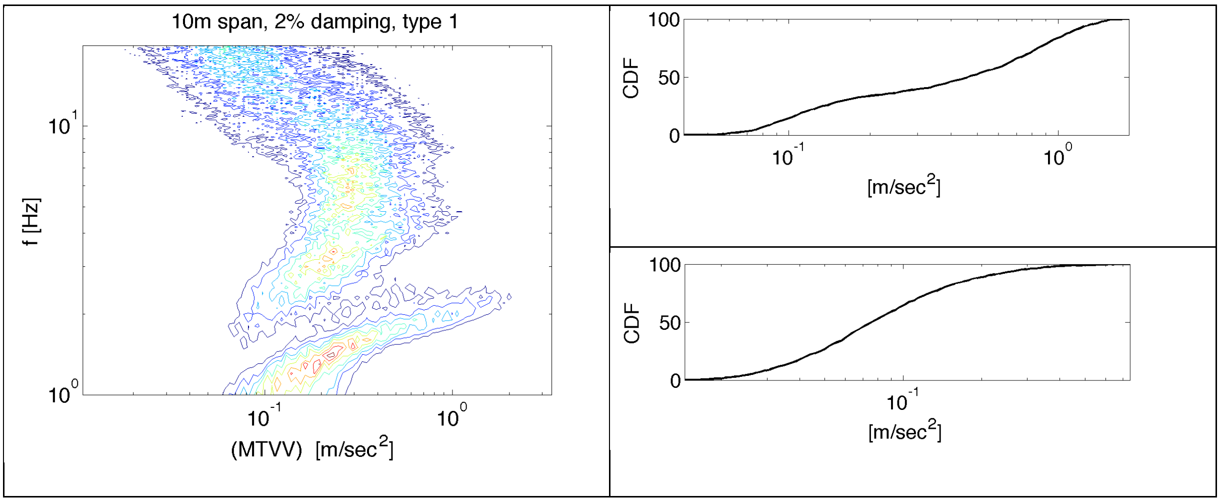

Figure 9: a) Typical MTVV probability density function (left) and b) cumulative density functions at 2 Hz and 18 Hz (right).

4.4 Peak factors

Figure 6a,b showed the relationship between unweighted MTVV and peak acceleration. Figures 11 and 12 explore the relationship more systematically in the form of peak factors which are here defined as ratios of peak acceleration to MTVV rather then to overall RMS.

For 2 Hz oscillators peak factors converge to a minimum value larger than for a pure sinusoid ($\sqrt{2}$) that is consistent with pure harmonic response, and beyond 5 Hz values diverge with damping ratio, whereas there is little variation with span. Bear in mind that MTVV for walking across a large span should already capture more of the variability as more RMS values are generated, so the classical peak factor relationship with averaging time is not expected.

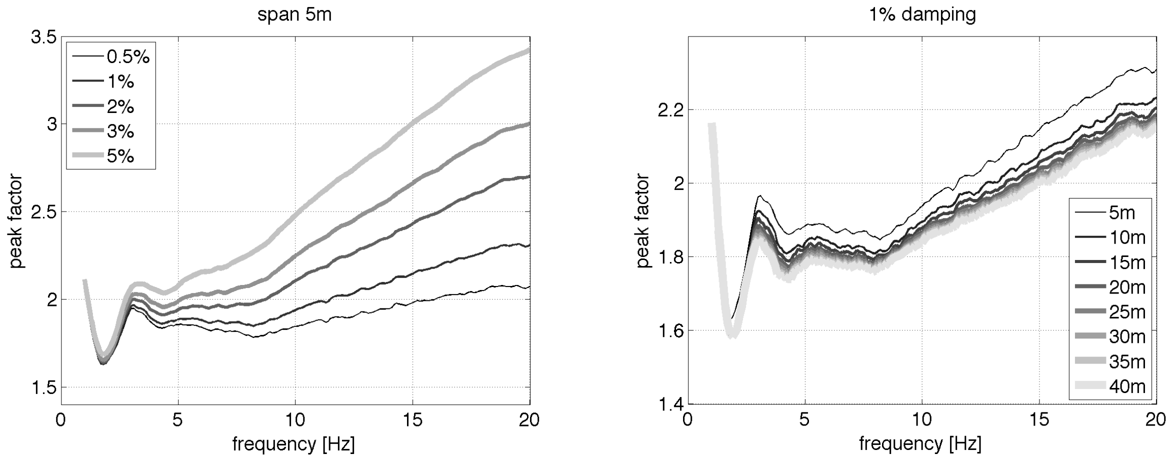

Figure 10: Peak factor dependence on damping ratio (left) and span (right).

As with the MTVVs, peak factors are not exact and have their own distributions, as shown in Figure 11. Distributions are tight around 2 Hz but have greater range for higher frequencies appearing to follow a log normal distribution. If peak responses are actually needed then RMS spectra (e.g. Figure 6b) would not be appropriate since their variability is compounded by variability of peak factors.

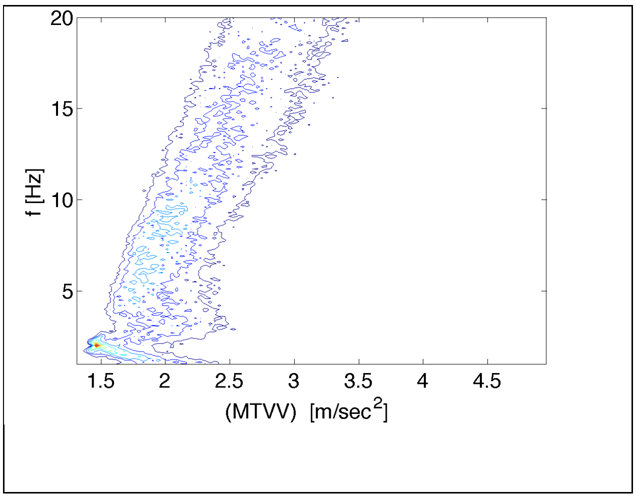

Figure 11: Distribution of peak factors.

4.5 Distillation of key metrics for GRFRSP

For design purposes, the trends shown in Figure 4 to Figure 11 require empirical representation such as used [14] for the case of low frequency floors, representing the effect of span, frequency, damping and percentile value. No one single empirical representation works well enough for MTVVs so for typical cases i.e. floors with span 5 m, 10 m and 15 m and for 1%, 3% and 5% damping, curves are provided in Figure 12. These are divided into two regions above 10 Hz where simple quadratic approximations fit reasonably well and a linear axis is used, and below 10 Hz where the shapes are complex and a logarithmic axis is used to enhance the low frequency zone. To apply these results the values must be multiplied by pedestrian weight and divided by floor modal mass.

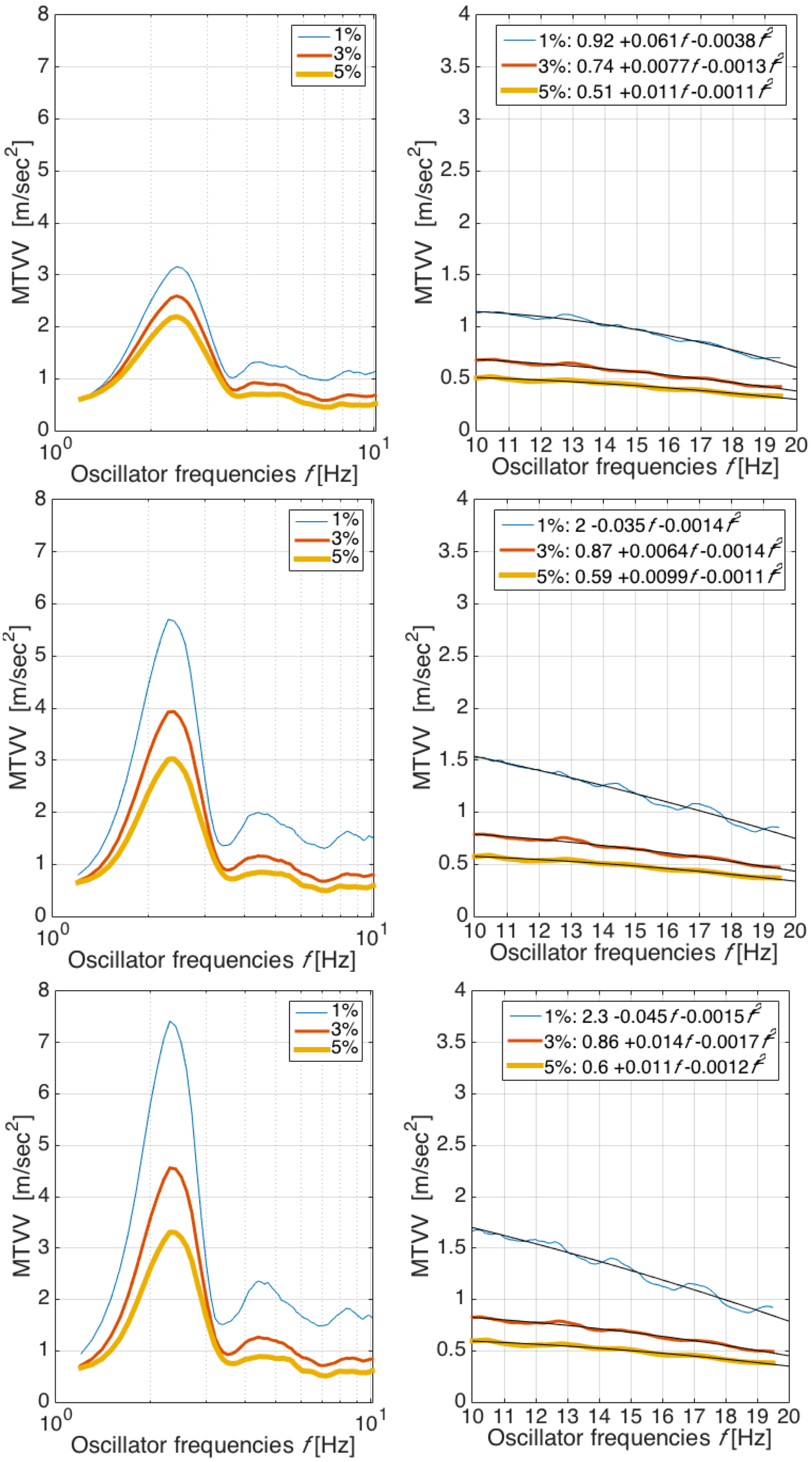

Figure 12: 99%ile b-weighted MTVV curves for 5 m (upper), 10 m (middle) and 15 m (lower) spans.

The first harmonic peak and subsequent trough vary in both scale (MTVV) and location (frequency); Figure 13 illustrates these parameters vs. damping and span for the first harmonic peak. The dependence of MTVV level on damping and span (duration of forcing) is consistent with known behaviour of oscillators driven at resonance. For the trough the values follow the same trend are visible in the minima, and the minimum frequency ranges from 3.1 Hz to 3.5 Hz.

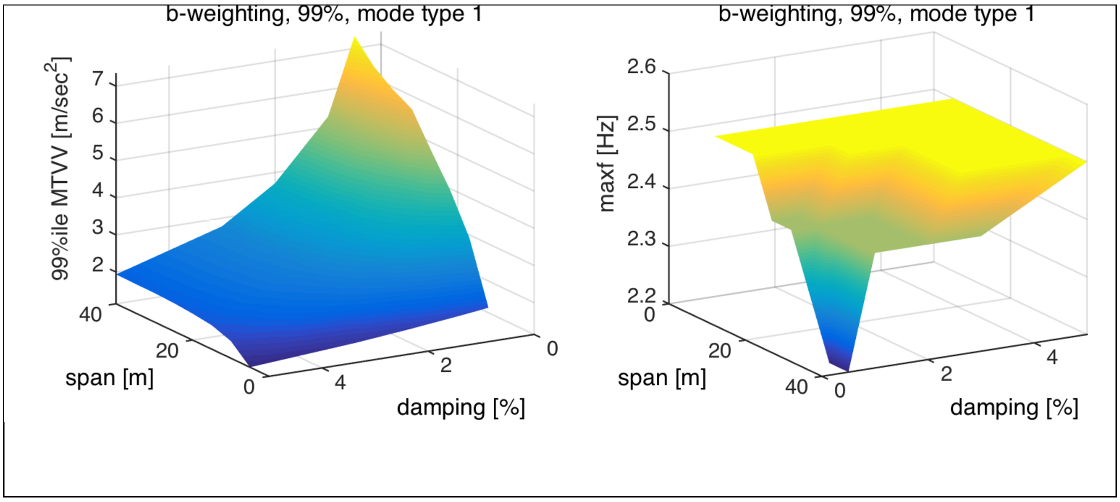

Figure 13: First harmonic peak value, and first trough minimum value.

5 VALIDATION OF GRFRSP PROCEDURE

Because the spectra are presented in a statistical form, validation cannot be achieved through a single example, rather confidence in its reliability might be established by comparing recorded MTVVs for sample structures and single pedestrians with those for a given percentile (e.g. 99%) for the closest matching combination of span, mode type and damping ratio. This means that reliable estimates of mode frequency, damping and mass must be available, and the mass or weight of the pedestrian known. Interpolation in the results database (or fitted empirical formulae) could be used, but one major problem is that few in-operation floors can be represented as perfect simple-supported spans. In reality the span needs to be judged as the effective length of the dominant mode, which as the examples given will show is rarely the same as either the full length of the structure or the bay size.

Because such a comparison is difficult to quantify, an alternative process is to take the cumulative density function (for the mode frequency, damping ratio mass, along with effective mode length as span) such as Figure 9b and read off the percentile value corresponding to the MTVV measured on the full-scale floor after normalising it to unit floor mass (multiply by this value) and pedestrian weight (divide by this value). This means that the only examples that can be used are where modal both modal mass (estimates) and pedestrian weights are known.

For an effective comparison a representative range of floor types is required with (for each floor), a full set of modal data and walking time histories. Such data are available thanks to over two decades of research and engagement with industry on problems in vibration serviceability of floors [24,25] involving a range of floors of different construction and dynamic characteristics. Since the purposes of the research and consulting do not always require both full modal data (including modal mass) and walking response time series, the set of candidates is narrowed, but there are enough examples to provide a useful comparison. Walking tests for serviceability evaluation are normally done with an experienced engineer pacing along the line of strongest response (maximum modal ordinates) using a metronome to keep time, and repeating the exercise for pacing rates ranging (for example) as 1.5:0.1:2.4 Hz, and often with a 'lap' of walking in both directions, so that any possible resonance is given the maximum opportunity to develop, in other words such tests should be at the 'worst case' (high percentile) end of a statistical range.

Table 1 summaries the most relevant properties of the floors used in the validation exercise. Examples were chosen where a single mode dominates measured response with bandpass filtering as appropriate. The set includes floors normally classified as ‘low frequency’ and ‘high frequency’, different structural types, materials and panel spans. Effective mode length is estimated based on mode shape plots and rounded to the nearest 5 m and classification match (type 1, 2 or 3). Identities of the floors are mostly disguised, but research involving some of the examples has been published, as indicated.

Table 1: Example floor structural details and lowest dominant mode parameters

ID	use	construction	panel sizes /m	mode length /m (type)	$M_1/10^3$ kg	f_1 /Hz	ζ_1/%
S-S1 [26]	light industrial	RC PC plank	12 ×12	20 (1)	30	14.05	5
S-S2 [26]	light industrial	ditto	12 ×12	20 (1)	47	12.4	3
S-S3 [26]	warehouse	ditto	7.5 ×18	25 (1)	120	10.3	2.9
Poly-S [12]	entertainment	RC in situ	21 × 9	20 (1)	37	10.64	2.65
L-T1	office	composite	10.5 × 9	20 (2)	10.5	6	3.65
L-T2	“	“		10 (1)	39	4.9	2.37
L-T3	“	“		15 (2)	17.7	5.98	2.25
L-G	office	composite	3.75 × 2.7	15 (1)	19.7	7.02	2.8
D-H1	entertainment	composite	15 × 3	30 (1)	102	4.92	1.03
D-H2	“	“		10 (1)	23.7	5.15	1
SBS-S	biology lab	PT flat slab	9.6 × 11.2	20 (1)	100	10.34	2.5
R-P	car park	waffle slab	9 × 7.2	20 (1)	35 -FEM	7.67	1.5
WSP-L [27]	office	composite	6 × 3	20 (1)	20	6.37	3

J-C [14]	test structure	slab	10×6.3	10 (1)	8	3.49	1.5

Figure 14 to Figure 17 provide details of four representative examples along with mode shapes corresponding to the modes indicated in Table 1, illustrating the difficulty in identifying a ‘span’ length.

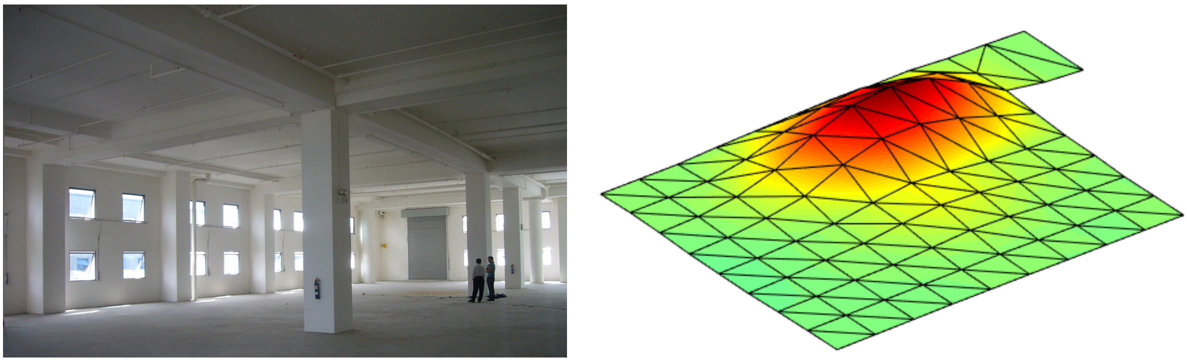

Figure 14: S-S1 unoccupied industrial unit, one-way 12 m span hollow core planks. The structural arrangement is similar for the upper level (roof visible) and the floor tested (engineer visible for scale).

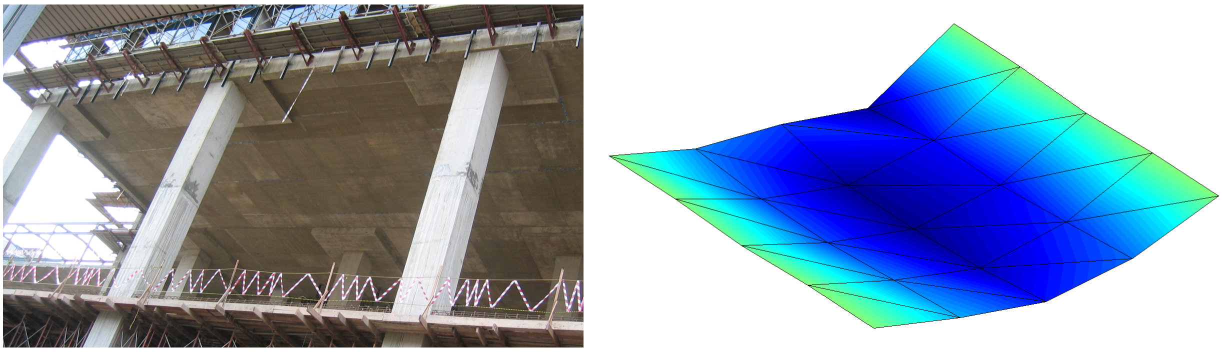

Figure 15: SBS-S bare laboratory floor shown from below post-tensioned flat slab with drop panels.

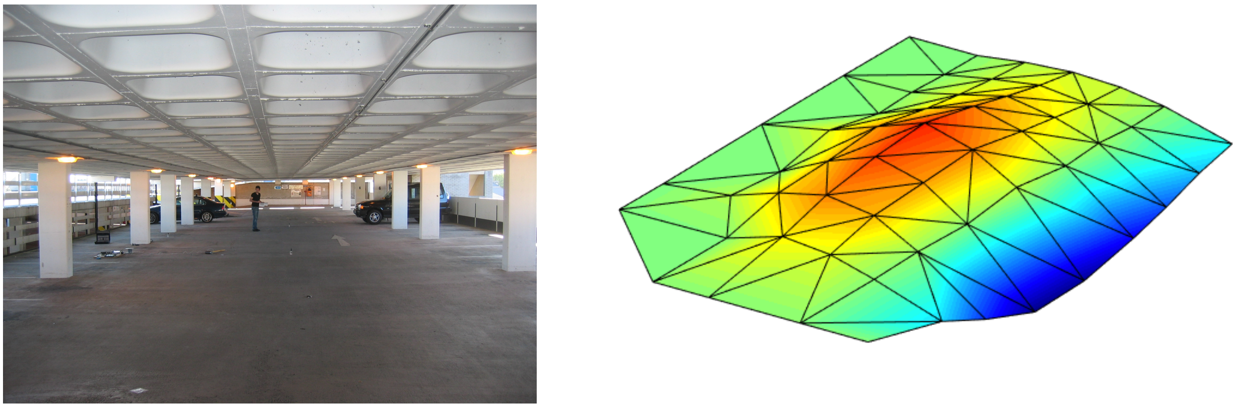

Figure 16: R-P car park, waffle-slab; the mode shape engages the majority of the car park level, whose structural form is the same as the upper level.

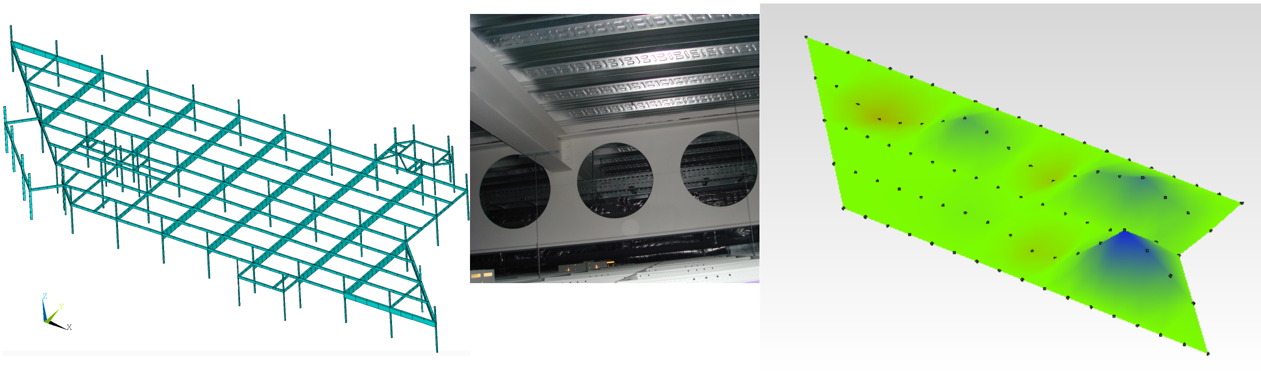

Figure 17: WSP-L engineering consultant office, composite with cellular primary beams at 6 m centres.

The modal parameters indicated in Table 1 (which include rarely reported modal mass) are all estimates obtained using the global rational fraction polynomial (GRFP) method implemented in commercial modal analysis software (ME'scope by Vibrant Techology Inc.). where possible mode frequency, damping and mass estimates were cross-checked with circle-fit or free decay parameter estimation methods.

In principle according to the methodology of deriving the GRFRSPs the worst case pacing rate should be covered by the data set of walking time histories, while in the testing the worst case was desirable if not actually achieved. The end result is that the walking tests should produce percentile values in the upper 90s.

Figure 18 evaluates the hypothesis for 90 comparisons of measured and simulated MTVVs. 63% of examples indicate a match in the 90-100th percentile range of simulations, with 85% above 70th percentile. There are several cases of low percentiles that are worth examining, many of them occurring for L-G. This is a composite structure that exhibited annoying vibrations at one end of the office floor. An incomplete mode shape was provided by modal testing, but it was sufficient to indicate that the edges of the floor were not behaving as full supports resulting in an element of cantilever behavior. As such the measured response would be larger than that recorded using an assumed half-sine mode shape.

Other examples include J-C which has a low recorded damping, and a value of 1% used in simulations assuming a positive bias on the estimation procedure (which is quite common with modal testing). Low values for D-H1 and D-H2 are not so simply explained away, however overall the comparison appears reasonable.

The values in Figure 18 are plotted against pacing rate simply to distribute values for presentation, although there is a pattern for L-G only. Likewise the marker size is made proportional to floor mode frequency in case there is any correlation with high or low frequency mode type; which appears not to be the case.

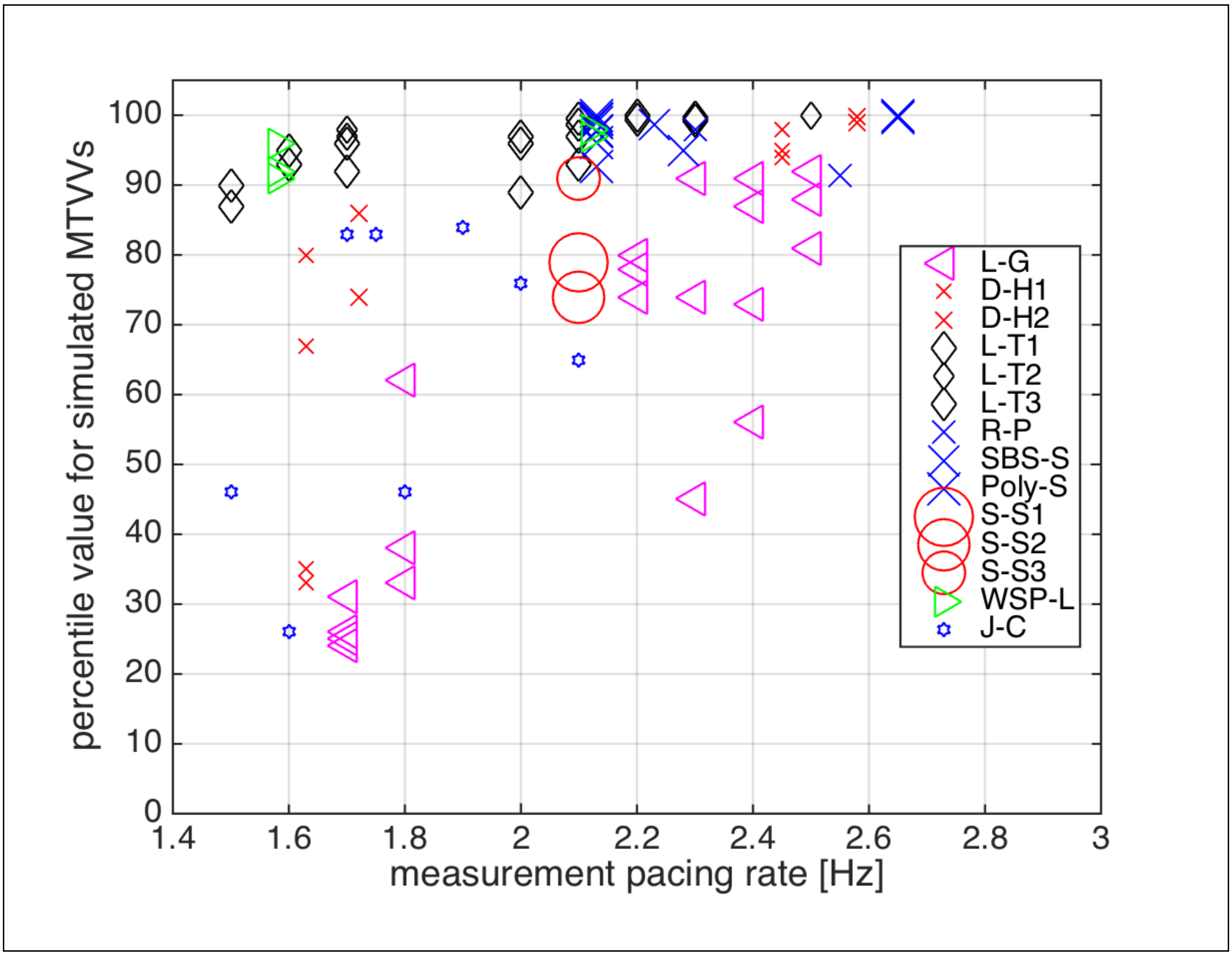

Figure 18: Percentile values in simulation corresponding to measured MTVVs. Marker size is proportional to mode frequency.

Finally for one example, WSP-L a comprehensive monitoring exercise [27] was carried out over one week of normal operation (the busy Leeds, UK office of consulting firm WSP). Acceleration data were obtained for the antinode in Figure 17 (lower right in mode shape plot). The monitored MTVVs are not restricted to 'events' where a single pedestrian crosses over the exact 'ridge' line of maximum mode shape (i.e. in the middle of a bay) and include periods of zero activity. Hence the monitoring would be bound to produce a lower proportion of strong responses compared to the simulated sequence of 'perfect' crossings.

The density function for a day of monitoring (using 1 second MTVVs) is compared with the closest equivalent for simulated MTVVs in Figure 19. While a comparison is not being made for the same situation, there should be some relationship and it is not a surprise that the monitored MTVV distribution is shifted down by a factor of approximately two with respect to the simulations. This provides a degree of validation, although there can be no direct proof that the approach is valid.

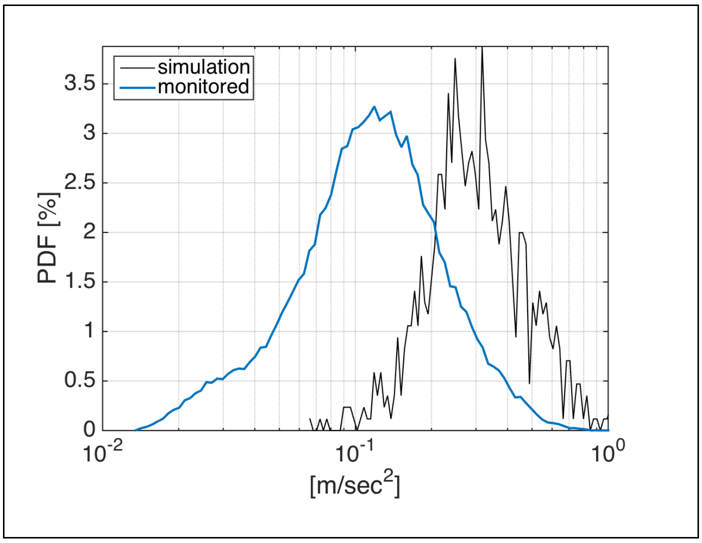

Figure 19: Comparison of density function of MTVVs from one whole working day of non-stop monitoring with density function of MTVVs for pedestrian data set and modal parameters corresponding to the monitored floor.

6 CONCLUSIONS

A comprehensive database of 852 walking time histories has been used to generate response spectra of typical vibration response metrics, principally the 'maximum transient vibration value', which is a moving average of root mean square acceleration, accounting for weighting of signals for application to occupant comfort.

The resulting spectra show a number of significant features. First, there is a broad ‘hump’ that represents the first harmonic of walking. This is followed by a distinctive dip (notch) and a small but diffuse secondary hump. Practically it appears that response spectra value decrease monotonically from about 5 Hz showing that the arbitrary distinction between high and low frequency floors lacks scientific basis.

Distributions of values for each oscillator frequency (and same conditions of span, damping etc.) appear to be lognormal, leading to an issue in defining an appropriate percentile level, which in our case has been set at 99%.

The method has been checked against a database of measured modal properties and matching walking response data for representative structures showing that there are some complications, such as defining span through the observed shape rather than the structural information. However the comparison with measured data shows consistency.

It was not possible to evaluate the technique for multi-mode response due to the much diminished set of full-scale test data, but in principle the square root sum of square approach could be applied.

7 ACKNOWLEDGEMENTS

The authors are grateful to Prof. Paul Reynolds for the WSP data, to all participants in the treadmill database data capture exercise and to the researchers who helped collect the full scale walking and modal test data. Thanks also to Full Scale Dynamics Ltd for providing anonymised floor performance data for the validation exercise.

The database of walking forces was created courtesy of funding by the UK Engineering and Physical Sciences Research Council, grant EP/E018734/1: Human walking and running forces: novel experimental characterisation and application in civil engineering dynamics.

REFERENCES

[1] P. Dallard, A. J. Fitzpatrick, A. Flint, S. le Bourva, A. Low, R. Ridsdill Smith, and M. R. Willford, “The London Millennium Footbridge,” Struct. Eng., vol. 79, no. 22, pp. 17–33, 2001.

[2] P. Dziuba, G. Grillaud, O. Flamand, S. Sanquier, and Y. Tetard, “La passerelle Solferino: Comportement dynamique (Solferino bridge: Dynamic behaviour),” Bull. ouvrages métalliques, no. 1, pp. 34–57, 2001.

[3] H. Bachmann and W. Ammann, Vibrations in Structures Induced by Man and Machines. Zürich, Switzerland: International Association of Bridge and Structural Engineering CH-8093 Zurich, switzerland, 1987.

[4] BSI “BS 6399: 1996 Loading for buildings.” British Standard Institution, London, 1996.

[5] M. R. Willford and P. Young, “A design Guide for Footfall Induced Vibration of Structures (CCIP-016).” The Concrete Centre, London. pp. 1–84, 2006.

[6] BSI, “Guide to evaluation of human exposure to vibration in buildings Part 1 : Vibration sources other than blasting”. Bristish Standards Insitution, London, 2008.

[7] C. G. Gordon, “Generic criteria for vibration-sensitive equipment,” in Proceedings of International Society for Optical Engineering (SPIE), vol 1619, pp. 71–85, 1992.

[8] A. Pavic and M. R. Willford, “Appendix G: Vibration serviceability of post-tensioned concrete floors,” The Concrete Society, Slough, UK, pp. 99–107, 2005.

[9] A. L. Smith, S. J. Hicks, and P. J. Devine, “Design of floors for vibration: a new approach (P354),” SCI P354. The Steel Construction Institute, Ascot, pp. 1–128, 2009.

[10] C. J. Middleton and J. M. W. Brownjohn, “Response of high frequency floors: A literature review,” Eng. Struct., vol. 32, no. 2, p337-352, 2010.

[11] T. M. Murray, D. E. Allen, and E. E. Ungar, “Floor vibrations due to human activity”, DG-11. American Institute of Steel Construction, Chicago, 1997.

[12] J. M. W. Brownjohn and C. J. Middleton, "Procedures for vibration serviceability assessment of high-frequency floors," Eng. Struct., vol. 30, no. 6, pp. 1548–1559, 2008.

[13] C. T. Georgakis and E. T. Ingolfsson, "Vertical footbridge vibrations: the response spectrum methodology," in Footbridge 2008, Porto, 2008.

[14] J. Chen, R. Xu, and M. Zhang, "Acceleration response spectrum for predicting floor vibration due to occupant walking," J. Sound Vib., vol. 333, no. 15, 3564-3579, 2014.

[15] V. Racic and J. M. W. Brownjohn, "Stochastic model of near-periodic vertical loads due to humans walking," Adv. Eng. Informatics, vol. 25, pp. 259–275, 2011.

[16] V. Racic, A. Pavic and J.M.W Brownjohn, "Experimental identification and analytical modelling of human walking forces: literature review," J. Sound Vib., vol. 326, 1-49, 2009.

[17] M. van de Putte, N. Hagemeister, N. St-Onge, G. Parent, and J. A. de Guise, "Habituation to treadmill walking," Bio-Medical Materials and Engineering, vol. 16. pp. 43–52, 2006.

[18] P. O. Riley, G. Paolini, U. Della Croce, K. W. Paylo, and D. C. Kerrigan, "A kinematic and kinetic comparison of overground and treadmill walking in healthy subjects," Gait and Posture, vol. 26. pp. 17–24, 2007.

[19] R. Bartlett, J. Wheat, and M. Robins, "Is movement variability important for sports biomechanists?," Sports Biomech., vol. 6, no. 2, pp. 224–43, May 2007.

[20] S. Bailey, "Ahtlete First: A history of the paralympic movement," John Wiley & Sons, Chichester, 2008.

[21] A. Belli, P. Bui, A. Berger, and J. R. Lacour, "A treadmill for measurements of ground reaction forces during walking," Proceedings of the XVth Congress of the International Society of Biomechanics. Jyvaskyla, Finland, pp. 100–101, 1995.

[22] ISO, "Mechanical vibration and shock - Evaluation of human exposure to whole-body vibration - Part 2: Vibration in buildings (1Hz to 80Hz)", ISO 2731-2, International Standards Organisation, Switzerland, 2003.

[23] BSI, "UK National Annex to Eurocode 1: Actions on structures – Part 2: Traffic loads on bridges.", NA to BS EN 1991-2:2003, British Standards Institution, London, 2003.

[24] C. H. Amick, M. Gendreau, T. Busch, and C. G. Gordon, "Evolving criteria for research facilities: I - Vibration," pp. 1-13. Proceedings of International Society for Optical Engineering (SPIE). San Diego, 2005.

[25] A. Pavic, Z. Miskovic, and P. Reynolds, “Modal testing and finite-element model updating of a lively open-plan composite building floor,” ASCE Journal of Structural Engineering, vol. 133, no. 4. pp. 550–558, 2007.

[26] J. M. W. Brownjohn, C. J. Middleton, T.-C. Pan, S. C. Tan and G. C. Yang, , “Floor vibration serviceability in a multistory factory building”. ASCE Journal of Performance of Constructed Facilities (accepted)

[27] M. J. Hudson and P. Reynolds, “Analytical and Experimental Evaluation of Active Vibration Control of an Office Floor Structure,” in Proceedings of 5th World Conference on Structural Control and Monitoring, Tokyo, 2010.