Combining scaffolding for content and scaffolding for dialogue to support conceptual breakthroughs in understanding probability

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Abstract

In this paper, we explore the relationship between scaffolding, dialogue, and conceptual breakthroughs, using data from a design-based research study that focuses on the development of understanding of probability in 10–12 year old students. The aim of the study is to gain insight into how the combination of scaffolding for content using technology and scaffolding for dialogue can facilitate conceptual breakthroughs. We analyse video-recordings and transcripts of pairs and triads of students solving problems using the *TinkerPlots* software with teacher interventions, focusing on moments of conceptual breakthrough. Data show that dialogue scaffolding promotes both dialogue moves specific to the context of probability and dialogue in itself. This paper focuses on episodes of learning that occur within dialogues framed and supported by dialogue scaffolding. We present this as support for our claim that combining scaffolding for content and scaffolding for dialogue can be effective in students' conceptual development. This finding contributes to our understanding of both scaffolding and dialogic teaching in mathematics education by suggesting that scaffolding can be used effectively to prepare for conceptual development through dialogue.

Keywords: scaffolding, dialogue, probability, technology

1 INTRODUCTION

The notion of scaffolding of learning traditionally refers to a way of structuring the learning task where an adult helps a learner "to internalize knowledge and convert it into a tool for conscious control" (Bruner, 1985, p. 25). In this context a teacher or a more experienced adult usually offers explicit support and guidance to help a learner to perform a difficult task beyond his/her own capacity. As such, creativity is important to moments of conceptual breakthrough. However, from a scaffolding of learning perspective, creativity does not appear to easily fit the model of a cognitive tool that one can consciously control. Bakhtin's (1981, 1986) theory of dialogism provides an inspiration to promote conceptual development indirectly by opening up spaces that can stimulate creativity. For example, a student asking another student the simple open question 'Why?' can, under certain circumstances, trigger a pause in the dialogue in which more possibilities are considered and as a result, a better answer to a problem may be found (Kazak, Wegerif & Fujita, 2013; Kazak, Wegerif & Fujita, 2015). We refer to this kind of move as 'opening a dialogic space' (Wegerif, 2013). Introducing a new perspective is a way of widening a dialogic space and questioning assumptions is a way of deepening a dialogic space. The notion of a 'dialogic space' is not the same as the notion of a cognitive tool, because, as we explain below, dialogic space belongs to a different model of cognitive development. Nonetheless, there are ways of scaffolding for the opening, widening, and deepening of dialogic space in a classroom in the same way that a teacher can scaffold for the conscious use of cognitive tools (Wegerif, 2007).

In this paper we focus on extending the concept of scaffolding for dialogue from a perspective of dialogic learning inspired by Bakhtin (1981, 1986). We use data from a design-based research (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) study using *TinkerPlots* software (Konold & Miller, 2011) to support 10–12 year old students' understanding of probability to argue that combining scaffolding for content knowledge using technology and scaffolding for dialogue is useful in promoting students' conceptual development in mathematics. The main research question that we explore is: In what ways can the combination of scaffolding for content using technology and scaffolding for dialogue support students' understanding of probability concepts?

2 LITERATURE AND BACKGROUND

2.1 Scaffolding

In dyadic problem solving settings, Wood, Bruner, and Ross (1976) describe a scaffolding process as the interaction between an adult and a child that "enables a child or a novice to solve a problem, carry out a task, or achieve a goal which would be beyond his unassisted efforts" (p. 90). To facilitate learning, an adult provides a temporary support for the parts of the task that are beyond the learner's ability and engages him/her in focusing on and finishing the components that he/she can accomplish alone without assistance. Presumably this process eventually leads to the development of cognitive and social skills required by the task that can be used independently in other contexts (Wood et al., 1976).

This idea of scaffolding has been used in different educational settings. For example, Fernandez, Wegerif, Mercer, and Rojas-Drummond (2001) report on the application of scaffolding of learning among peers in peer groups where children of the same age and social status interact with each other to solve a problem together. In two studies that are conducted in Mexico and the UK, children worked together in groups to solve non-verbal reasoning problems. Both studies used interventions that focused on developing effective ways of talking together in small groups to encourage students to "engage critically but constructively with each other's ideas, offering justifications and alternative hypotheses" (p. 42). The way of talking together promoted by the intervention is called Exploratory Talk. Researchers find that Exploratory Talk provides a scaffolding of shared learning in symmetrical peer-to-peer interactions. For example, sharing their ideas and their understanding of the problem with others and explaining solutions allows children to support each other through difficult parts of the common task without the assistance of an adult. Not only could scaffolding be seen in symmetrical peer-to-peer interactions, it could also be found in the phenomenon of a group version of Vygotsky's 'Zone of Proximal Development' (ZPD) (1978). When problems are too easy or too difficult to handle, Exploratory Talk is not used by the group. Exploratory talk is used for problems that are only solvable with the support of the scaffolding offered by Exploratory Talk. Through Exploratory Talk, the group is able to spontaneously reproduce the role of the teacher who, from a Vygotskian perspective, offers scaffolds for problems that are within the ZPD of the learner.

Smit and colleagues apply the notion of scaffolding to whole-class interaction (Smit, van Eerde, & Bakker, 2013) by focusing on the teacher's role (Smit & van Eerde, 2011) in scaffolding students' development of mathematical language in a multilingual classroom. The key aspects of the scaffolding in one-on-one interactions (i.e. teacher-student), including diagnosis, responsiveness, and handover to independence, are also observed in classroom settings (Smit et al., 2013). Moreover, the researchers identify three features of whole-class scaffolding relevant to these processes. First, whole-class scaffolding is 'layered' in the sense that its diagnosis, responsiveness, and handover to independence aspects function both during and outside of whole-class interaction. Its 'distributed' nature implies that the occurrences of these three characteristics are scattered over several episodes. Finally, the 'cumulative' nature of whole-class scaffolding refers to the fact that the collective outcome of distributed diagnoses and responsiveness during and outside of classroom interaction fosters students' learning processes.

Others adapt the notion of scaffolding to computer-based learning environments and refer to this as scaffolding mediated by technology tools (Guzdial, 1994; Reiser, 2004; Quintana, et al., 2004). Reiser (2004) suggests two mechanisms of scaffolding with software tools to influence learning process: (1) providing structure for learners' engagement in a task, and (2) problematizing content knowledge. A detailed design framework for scaffolding with software tools to support science inquiry proposed by Quintana et al. (2004) also suggests the use of tools to structure tasks in order to bridge learners' intuitive ways of thinking and prior understanding with the formal concepts and procedures used in the subject area. Such tools enable learners to inspect multiple representations to build meaning relevant to content, to test their ideas, and to get feedback. Some software designs also include providing prompts for explicit arguments, reflection and alternative explanations to engage students in various cognitive activities during scientific inquiry tasks. Both Reiser (2004) and Quintana et al. (2004) note that software tools do not function alone. Rather, teacher, learners/peers, instructional materials, and software tools work together as a system to support learning.

These studies have implications for re-conceptualisation of Wood et al.'s (2006) notion of scaffolding within mathematics classrooms for supporting the effective use of dialogic approaches to teaching and learning. Rather than focussing on one aspect of scaffolding, such as student talk, teacher, or technology, we are interested in the role of scaffolding in the interplay between students' dialogue in small groups and their use of computer tools in this study. Hence, our use of the notion of scaffolding in the context of dialogic learning is twofold: (1) the teacher 'scaffolds' for dialogue by providing prompts for questions that reduce the learners' degrees of freedom and encourage them to engage dialogically even when the prompts are removed, and (2) the *TinkerPlots* software scaffolds for content understanding in that it simplifies, speeds up, and represents the stochastic processes thereby making it easier for students to grasp the concepts of probability. Next, we elaborate key concepts relevant to the dialogic approach we use in our study.

2.2 Talk and dialogic space

The research of Mercer, Wegerif and colleagues on Exploratory Talk, referred to above (Fernandez et al., 2001), led to an understanding of scaffolding as something that can be found within interaction between peers and also to the idea of a group ZPD which Mercer referred to as an 'intermental development zone' (Mercer, 2000). While agreeing with this, Wegerif (2011) went further to argue that scaffolding in talk did not only support the internalisation of specific cognitive tools but could also support the learning of creative thinking through the opening of 'dialogic space', a notion inspired by Bakhtin's dialogism. Dialogic space is not in contradiction to the idea of a 'development zone' but takes the zone itself seriously as a real space of possibility and potentiality that opens up between incommensurate perspectives. Vygotsky and neo-Vygotskians like Mercer tend to present learning to think in terms of internalization, personal appropriation or 'co-construction' of cognitive tools, such as signs, through the medium of dialogue. For example, Vygotsky claims that children learn to think logically through formal schooling, which allows them to personally internalize the categorizing practices of school mathematics and science using cultural mediating means such as tables in text books, long-division procedures, etc. (Vygotsky, 1986, p. 156). Wegerif (2011) argues that while this model can account for the way children learn to use specific concept words and tools appropriately, it does not account for how children might learn to think in an open, critical, and creative way. Wegerif (2007; 2014) argues that in order to account for children's ways of thinking, we can extend the Vygotskian concept of learning with an understanding that children do not only internalize or personalize appropriate cultural tools, they also internalize the dialogic space that precedes and makes possible the use of signs. Learning to think for oneself is learning to open a dialogic space through which anything and everyone can be questioned and seen in a new way. This space is not a cognitive tool to be used consciously as described by Bruner (1985), but what is discovered when the learner 'steps back' from self and is 'open to the other'. However, the learning of dialogic space or 'becoming dialogue', as Wegerif (2013) puts it, can be scaffolded through actions and interactions that open, widen, and deepen dialogic space. The idea behind the phrase 'becoming dialogue' is that learning involves shifts in the identifications of learners. In order for creative learning to occur, it is necessary for the learner to take a step back from identification with a bounded image of self and identify, however temporally, with the open-ended flow of dialogue in which self and other are united within dialogic space. For example, as we mentioned earlier, asking 'why?' is a key move in Exploratory Talk. It is more than just a way to teach the cognitive tool of explicit reasoning. It also serves the purpose to open a space of reflection and creative emergence. This is sometimes observed in pregnant pauses in dialogues that can occur after asking difficult questions and preceding the emergence of a new idea. In a whole-class context, a teacher pausing for a longer period of time after asking questions has been found to be effective in promoting children's creative thinking (Dillon, 1988). This move should not be mistaken as scaffolding for the appropriation of a cognitive tool, something which is personal and would normally take more time than a pause in the conversation. This move can be understood as scaffolding for the opening of dialogic space, something which is collective and almost instantaneous.

2.3 TinkerPlots software understood as a scaffold

As previously discussed, the concept of scaffolding has been adapted to apply to the case of educational software. *TinkerPlots* (TP), the software that is used in this study, offers visual and dynamic tools to support the exploration of concepts in probability. Makar, Bakker and Ben-Zvi (2011) particularly highlighted the role of TP as scaffolding in promoting students' informal inferential reasoning skills. Using TP, students can test and revise their intuitive ideas and theories as well as articulate their ideas to their peers. For example, in their study with 10–12 year-olds, Fitzallen and Watson (2010) suggests that TP allows students to generate different sorts of plots that make sense to them and allow them to utilize these effectively in drawing their conclusions from their data. Fitzallen and Watson (2010) also point out that TP facilitates students' thinking processes, and allow them to move back-and-forth between formulating hypotheses and constructing graphs in order to make sense of the data. Moreover, Ben-Zvi (2006) report that students use TP not only as a visual tool, but also as an argumentation tool in articulating ideas to others. These uses can be understood as scaffolding because the software reduces the students' degrees of freedom by focusing on the most salient aspects of problems, making it easier for students to explore the domain of probability and communicate their reasoning with others. A detailed description of TP as tools to support students' reasoning and argumentation can be found in Section 3.1.

2.4 Children's Conception of Probability

Various research studies (e.g., Lee, Angotti & Tarr, 2010; Pratt, 2000) have shown that the context of fairness can lead to a motivating and fruitful area of inquiry to engage students in probabilistic ideas. These findings suggest that when analysing the fairness in games of chance involving dice, students seem to rely on their intuitions about fairness in judging whether each player has an equal chance of winning or whether the chance of each possible outcome is equal. In order to determine fairness, students are expected to enumerate all possible equal outcomes (theoretical model, e.g., a uniform distribution for fair outcomes) and compare them with the relative frequency of each outcome in the long run (empirical distribution of outcomes). However, relevant concepts and ideas, such as sample space and the law of large numbers, can pose challenges for students in probability problems.

Regarding the concept of sample space, Francisco and Maher (2005) finds that students have difficulties in determining all possible outcomes in a probability context. Even when students know how to generate a complete set of possible outcomes, they tend to fail to make proper probability predictions based on an analysis of sample space for compound events (Polaki, 2002). Students also often fail to consider the order of outcomes in two or more stage events. For example, in the context of the sum of two dice, some students ignore the order of individual outcomes in rolling the dice because they believe that due to the commutative property of addition, the individual outcome (the rolling of each die) does not affect the sum of two dice (Horvath & Lehrer, 1998).

An application of the law of large numbers in comparing the empirical outcomes with a theoretical model is to acknowledge that as the number of trials increases, the aggregate characteristics of the empirical distribution tend to closely resemble the expected distribution based on the sample space (theoretical model). Lee et al. (2010) suggest that using technology can promote 11 to 12 years old students' development of this important idea. A study by Pratt, Johnston-Wilder, Ainley and Mason (2008), however, reveals some challenges students (age 10–11) encounter in grasping the idea of the law of large numbers while using computer simulation tools. In their study, students focused on local properties, such as the changes from sample to sample, and failed to see the stability in frequencies of simulated outcomes as the number of trials increases.

In an interview-based study, Watson and Moritz (2003) examine hierarchical levels of students' beliefs about fairness of dice and the strategies they use to decide whether the dice used are fair. At the lowest level of student beliefs, Watson and Moritz (2003) find inconsistencies in students' responses where students believed the dice to be both fair and unfair. For instance, a student believed that all dice are fair because all numbers have a chance also believed that certain numbers occur more often than others based on personal experience or other beliefs. In such cases, Watson and Moritz argue that these students showed "some indication of a subtle distinction between frequencies and chances" (p. 296). They suggest that for these students the idea of chance could be relevant to a future event or taken from a theoretical perspective while frequency of outcome could be interpreted as what really happened. At the higher levels of student strategies, majority of the students used observational strategies (i.e., the symmetry of the die or a few unsystematic trials) while a few students performed systematic trials to decide the fairness of the dice.

Based on previous research, it is apparent that students experience conceptual challenges when dealing with problems that involve probability. In addition to the students' contradictory ideas about fairness, inconsistencies in students' probabilistic reasoning appear when they use a heuristic or outcome approach (e.g., Konold et al. 1993) to solve problems that involve probability. In this study, we are interested in the moments where students have a new insight as they switch from an intuitive reasoning to a normative probabilistic reasoning through scaffolding. We refer to these moments of insights as conceptual breakthroughs.

3 METHODS

3.1 Design-Based Research Study

A design-based research study is utilized with the overall aim to investigate the development of young students' conceptual understanding of key ideas in statistics and probability through the mediating roles of technological tools and students' dialogues. This paper focuses on exploring the role and relationship of scaffolding and dialogue in conceptual growth in probability.

A design study involves an iterative process to develop theories of students' learning and ways of supporting their learning in domain specific content (Cobb et al., 2003). The initial design, including the instructional materials, is improved through testing and revising conjectures based on an on-going analysis of students' reasoning and the learning environment while the investigation is in progress. In our study, the iterative research cycle involves (1) designing instructional tasks and a learning environment that support students' conceptual development in statistics and probability based on our literature review, (2) conducting a teaching experiment, and (3) retrospective analysis. The new insights gained from the retrospective analysis of the first cycle then form the basis for the subsequent teaching experiment. During the study, we conducted three teaching experiments. Within this iterative process, some tasks required specific changes to be used in the subsequent experiment as we developed understanding about how the combination of scaffolding for content using *TinkerPlots* software and scaffolding for dialogue could support students' conceptual development in statistics and probability. The empirical data we discuss in this paper come from a selection of tasks from these teaching experiments.

Scaffolding for dialogue. Drawing on the 'Thinking Together' approach (Dawes, Mercer & Wegerif, 2000), we aim to foster a dialogic way of talking about effective collaboration in mathematical activities that involve group work by setting up a learning environment that promotes effective small group dialogues. The first two lessons in the study focused explicitly on discussing with students specific expectations or 'ground rules' that they need to follow for effective dialogues to occur. Two examples of these expectations are that claims should be challenged and that students should provide reasons when challenging others' claims. Students then practised these 'ground rules' in class in order to set up a learning environment that promotes effective talk in small group work.

In the first lesson, students were put into groups of two or three where each student could make a positive contribution. The aim of this lesson was to discuss ways of effectively working together using technology tools in small groups. We began the lesson with a whole-class discussion about group work around computers, which was the essential component of our study. Students were made aware of how they are expected to work and talk with others when working around a computer through some discussion questions. Examples of these expectations include, 'Who thinks they are good in groups? Why?', 'What sort of things can we do by talking together around a computer?', and so on. We then showed two short video clips of a group of primary school students working together to solve a task before and after an intervention using the Thinking Together programme (Dawes et al., 2000). We asked students to discuss in their groups to decide how the group in the first video clip was working together and explain what they liked or didn't like. This was followed by a whole-class discussion to identify the different ways of working together they observed in the video. After showing students the second video clip, the discussion focused on the similarities and differences students noticed between the first and the second video.

The second lesson focused on raising students' awareness of implicit expectations for group work, reflecting on these expectations and deciding on a set of expectations for group work. In this session students worked in their groups and were given a set of cards with a potential ground rule printed on each card (Figure 1). Students were asked to identify the rules they thought were conducive to effective group work with an explanation to stimulate discussion in their groups. They were also encouraged to add any expectations that they thought were missing.

Listen to others' ideas	Be brave, challenge conventions and say what you really think	Question other's ideas	Change your mind if you hear a good reason
Let the oldest start	There should be no jokes	Alternatives should be discussed before taking a decision	Disagree to other's ideas with reasons
Give reasons for one's ideas	Criticise ideas, not people	Stick to your own opinion whatever the others say	Elect a leader and let them take decisions

Figure 1. An example of the ground rules (Dawes et al., 2000) given to students to discuss in their groups.

After a whole-class discussion of the groups' reports, we listed a set of shared expectations for group work the students generated (Figure 2). Next, students practised these expectations in a group activity. We used the list shown in Figure 2 throughout the study to help students become aware of these expectations while working together on the following tasks.

The 10+1 Club's Expectations for Group Work

- Pay attention to other people.
- Don't just say no to an idea–always ask 'why do you think that?'
- It's good to disagree but if you disagree, always give a reason.
- Ask everyone 'what do you think?' to bring them in.
- Be creative–try to see things in a new way.
- Work together as a group and take joint responsibility for what we agree.

Figure 2. The student-generated list of shared expectations for group work.

In order to promote scaffolding of effective talk, we encouraged students to adhere to these expectations by giving them sticky notes to record the number of "why" questions they heard and the number of reasons given in their groups. Students used tally marks to record the number of times a group member asked another student to explain his/her idea and the number of times a group member challenged someone else's idea and provided a reason for the challenge (Figure 3).

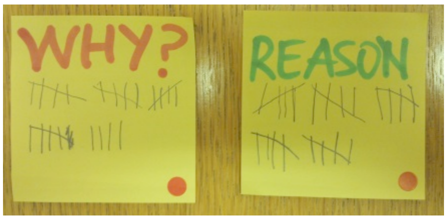


Figure 3. Record of the number of “why” questions and reasons given in one group session.

TinkerPlots for scaffolding content. *TinkerPlots 2.0* is a data exploration and modelling tool that builds on children’s intuitive knowledge about data representations and analysis. The software offers a variety of tools to visualize data and to conduct probability simulations. With the Sampler tool, users can build their own chance models using various devices, such as mixers and spinners, and fill them with various elements. Then the software enables users to collect outcomes from the Sampler and carry out a large number of trials very quickly. Users can also view simulation data in a graph as they occur. These tools help students test and revise their conjectures about probability of events. Fitzallen and Watson (2010) show similar affordances of TP in the context of data analysis. Moreover, the studies by Konold and colleagues (Konold, Harradine & Kazak, 2007; Konold & Kazak, 2008) suggest that the new modelling and simulation features of TP could support students’ development of integrated sets of statistical and probabilistic ideas.

As an example, Figure 4 shows a *TinkerPlots* model of the Wink, Blink, Stare Game discussed in section 4.2. The game involves drawing twice with replacement from a bag containing two counters, one labelled with a dot (•), the other with a dash (-). If the draws result in two dots, Player 1 wins. If the draws result in two dashes, Player 2 wins. If the results of the two draws are different, Player 3 wins.

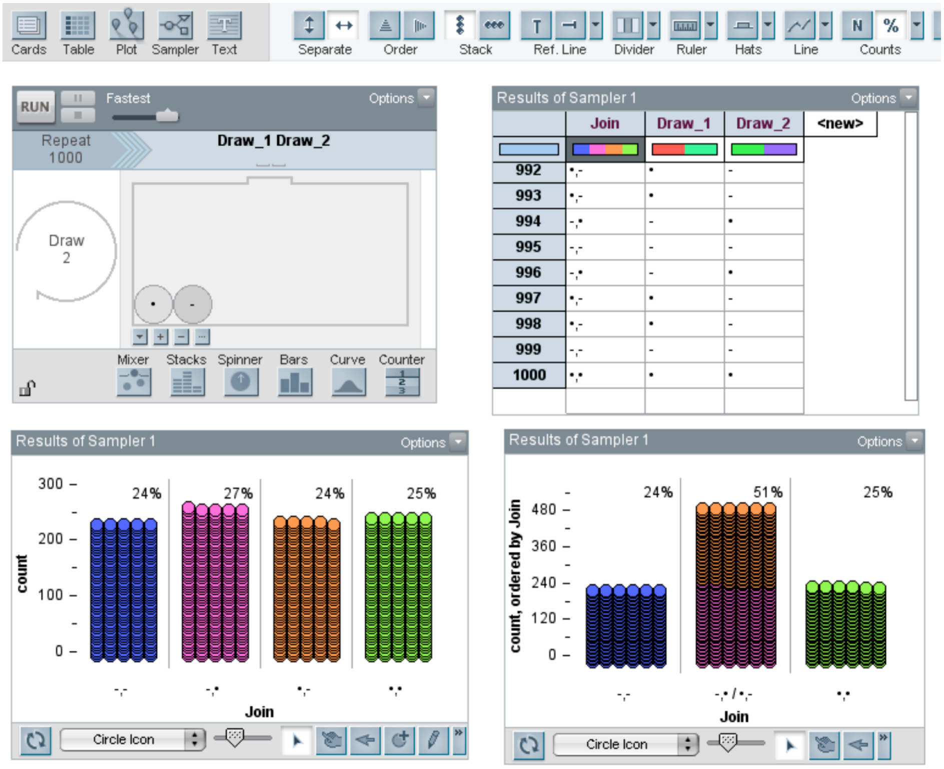


Figure 4. *TinkerPlots* model of the Wink, Blink, Stare game

In Figure 4, a single mixer device in the Sampler is created to draw twice from the bag with 1000 repetitions. Students can set the repetition number that they want and run the simulation several times to collect more data. The table on the right shows the results of each repetition as they are drawn. The graph on the left displays the percentage of outcomes for each event (-,- (blink); •,- (wink); -,• (wink); •,• (stare)). In the graph on the right, the two outcomes (•,- and -,•) are combined into a single bin by dragging one into the other to show the total percentage of wink outcome. We anticipate that this feature of combining two outcomes in the process of creating and interpreting a visual display of the simulation outcomes can lead students to realize that Wink is about twice as likely as Blink or Stare because it is made up of two different outcomes. This insight supported by TP does not naturally come to students since many of them tend to consider the game fair because they believe there are only three possible outcomes rather than four (Konold & Kazak, 2008). However, creating and interpreting graphs seen in Figure 4 can provide students with an opportunity to make counter-arguments for the fairness of the game. As suggested by Ben-Zvi's (2006) study, TP environment can provide a space for dialogue as well as a tool for mathematical thinking.

3.2 Setting and Participants

The first teaching experiment was involved five Year 7 (age 11–12) students, three boys and two girls, who voluntarily participated in an afterschool club at a private secondary school in Exeter, UK. In the second teaching experiment, six Year 6 (age 10–11) students, two boys and four girls, from a local primary school participated in a mathematics enrichment class. The third teaching experiment was conducted the following year at the same primary school during a mathematics enrichment class in which eleven Year 6 (age 10–11) students, five boys and six girls, participated. These students had some prior knowledge about probability that involves simple events. The lessons were taught by the researchers.

3.3 Data collection and analysis

During the lessons, students were seated to work in pairs or groups of three at tables with a computer. Each group's work was video-recorded to capture the students' interactions. The data for analysis include video footage of group work and lessons, computer screen captures, and student worksheets.

Drawing upon Mercer's (2004) work, a sociocultural and dialogic discourse analyses were used to analyse and evaluate students' dialogues as they worked in pairs or groups on computer-based activities. Based on the field notes of each teaching session we initially identified possible episodes of students' group work for further collaborative analyses to answer our research question. We then watched the video-recordings together to discuss and classify critical events of dialogues (student-student and student-teacher) that led to a new insight in students' understanding in these episodes. Using the selected video segments and other sources of data (computer screen captures and worksheets), we formulated and agreed upon conjectures about how students' emerging understanding and reasoning were supported by the combination of the scaffolding for content using *TinkerPlots* and scaffolding for dialogue. For more detailed analyses, selected key episodes were transcribed. These excerpts were coded for the critical instances where (1) the software tools provided students with a mathematical insight, (2) the teacher asked students to make their ideas and explanations explicit to others, (3) the teacher asked group members to listen to each other's ideas, or (4) the students added a new idea or challenged an idea that led to a progress in their collective thinking. Afterwards, we interpreted these segments and paid particular attention to the emergence of a new insight, the discourse in students' conversations, such as explaining, challenging, arguing, disagreeing, asking and justifying, as well as actions afforded by the tools.

4 FINDINGS

In the next section, we present our analyses of three episodes of students working together in pairs or groups to explore the role of scaffolding for content, the role of scaffolding for dialogue, and their combined role on developing students' understanding and reasoning three different probability tasks.

4.1 Episode 1: Chips Activity

The episode from the first teaching experiment focused on two 12-year-old boys, Chris and Jacob (pseudonyms), as they investigated their initial predictions about the fairness of a chance game:

> There are two bags containing game chips of two colours–red and blue. To play the game, you will randomly select a chip from each bag. If the two chips are the same colour, you will win. If they are different colour, the teacher will win. Bag one has 4 red chips; bag two has 2 red chips, 2 blue chips. Is this a fair game?

Both students were initially quite confident that the game was not fair. For instance, Chris explained his reasoning and said, "*Bag two is fair. Bag one is all red chips. It is impossible to pick out a blue chip from there. It is all made up of red chips*." It appeared that students intuitively focused on the number of red and blue chips in each bag rather than the combined outcomes of a two-stage experiment, i.e., the same and mixed colour pair of chips. Next, Chris and Jacob built a model for this game in *TinkerPlots* (see Figure 5) to test their initial conjecture as part of the task. The model that they built involved two spinners (on the left in Figure 5), one with 100% red and the other with 50% red and 50% blue sections showing the proportions of red and blue chips in bag 1 and bag 2 respectively. They set the number of trials of randomly drawing a chip from each bag at 1000. The plot on the right displays the percentage of each combined outcome (the same colour and the mixed colour) in 1000 repetitions.

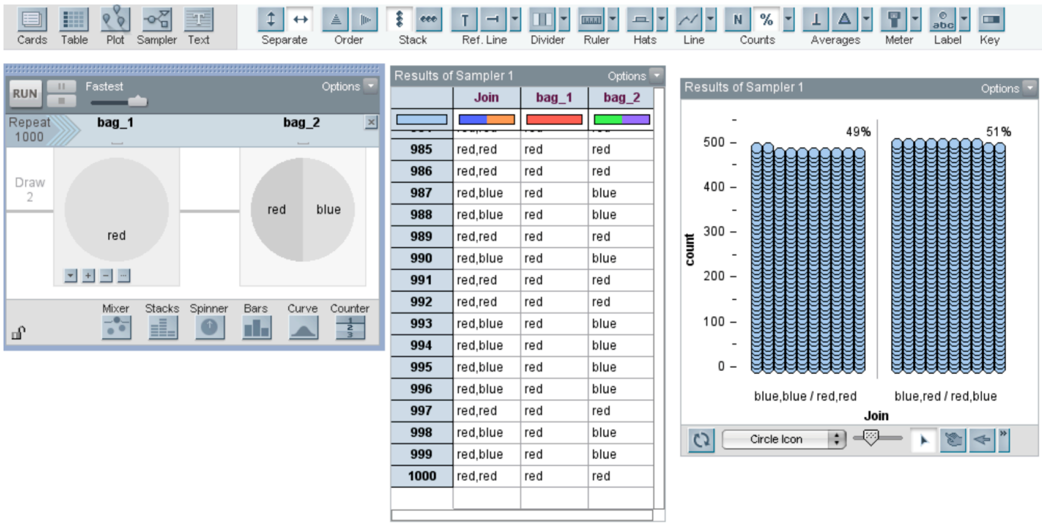


Figure 5. The two-spinner model of the chips game built by Chris and Jacob in *TinkerPlots*.

Excerpt 1 shows the exchange primarily between the two students once they completed building their model in TP. Students initially believed that the game was unfair but after running the TP simulation to collect more data and seeing the even results on the plot led them to develop a new insight about the chance events involved in the game. We focus on this specific episode which illustrates how TP as an external tool scaffolded students' reasoning about the fairness of the game as students discussed results generated by TP.

Excerpt 1 (lines 1-19)

1 Teacher: ((*After students built and explained their model in TinkerPlots*))[1]
2 Okay. And you think that you guys will win most of the time, huh?
3 Chris: I think we will actually win most of the time.
4 Jacob: Actually, I am actually debating now ((*while he presses the run*
5 *button to collect 1000 data*))
6 Jacob: Oh yes, it is fifty fifty because oh yeah!
7 Chris: Jeez, we got an entire (.) army on our side!
8 Jacob: No, no, Chris you don't get it. The first one ((*pointing to the first*
9 *spinner on the screen*)) you always get hundred percent red
10 Chris: Exactly
11 Jacob: [Then the next
12 Chris: [So, then] the next one you could get ((*pointing to the second*
13 *spinner on the screen*))
14 Jacob: It's a fifty fifty chance of getting either the same (0.5) ((*he is*
15 *laughing and almost speechless*))
16 Chris: I don't get it!

[1] Transcription notations used in this paper include: (.)=Micro pause; (1.2)=Timed pause; [word]=Overlapping talk; (( ))=A note (in italics) describing action or giving commentary.

17 Jacob: So basically the first time you will get a red, next time you got a fifty
18 fifty chance of either getting the same or something different ((*covering*
19 *his face with his hands, leaning down the desk and laughing*))

As seen in the segment above, scaffolding of students' reasoning about the fairness of the game occurred through the combination of the use of *TinkerPlots* (technical tool) and talk (dialogue). Jacob noticed immediately why the game was fair after seeing the results of the simulation on the plot (Figure 5). By paying attention to how the two-spinner model in TP worked, Jacob was able to articulate the significance of the two stages of the game and explain the two actions (first drawing from one bag and then from the other one) in relation to the expected outcomes (Excerpt 1, line 17). This observation enabled Jacob to see the problem from a different perspective, which eventually led to a change his understanding of the chance situation.

Excerpt 1 (lines 20-48)

20 Chris: Jacob, I don't get this at all. Why are you laughing?
21 Teacher: Yeah yeah, I agree, explain
22 Chris: Jacob, why are you laughing? Just calm down, calm down ((*taking*
23 *a deep breath*))
24 Jacob: ((*now talking to Chris*)) First one you will definitely [get a red]
25 Chris: [get a red],
26 Jacob: so the next one you would get either a red or a blue. So basically
27 you can either get fifty percent
28 Chris: [Yeah]
29 Jacob: you will get [red]
30 Chris: [Red] Yeah. So it is
31 Jacob: Fifty percent you will [get blue] ((*laughing*))
32 Chris: [It is, it is fifty fifty]
33 Jacob: So fifty fifty
34 Teacher: I think I think, so this is the bag one, hundred percent red. Yeah. And
35 Chris: Yeah I think it is fifty (.) I think it is fifty fifty. What Jacob said, it
36 is fifty fifty. That is fair.
37 Jacob: Because in the first one [you will get a red]
38 Chris: [you will get a red]
39 The next one you will, you will probably get a fifty fifty chance of
40 getting a red [or a blue]. So it is either you get [red or blue]
41 Chris: [or a blue] [red or blue]
42 Jacob: So basically if you get red, you get the same. Blue is different. So
43 basically they are fifty fifty. The first bag [(inaudible)
44 Chris: [the first bag is just red.
45 What (.) what (.) the whole competition is in the second bag.
46 Jacob: We believe that this game four [is fair], like [game three]
47 Chris: [is fair] [game three]

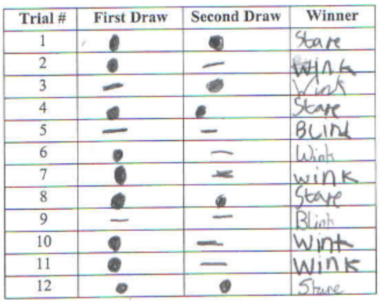
48 Jacob: although it does not look fair when you look at it.

As seen in the segment above, the dialogic talk also helped Chris change his mind when he and Jacob engaged in a dialogue around the TP model on the computer screen. Initially Chris did not seem to understand how Jacob came to understand why the results came out even when he said "*Jacob, I don't get this at all. Why are you laughing?*" (Excerpt 1, line 20). After that Jacob began to explain his idea of even results in the game as a result of scaffolding by the TP model. Then we see a shift in Chris's understanding during this exchange (Excerpt 1, line 45) while he was appropriating Jacob's reasoning. Therefore, his insight did not emerge from the scaffolding provided by the software alone, but within the dialogue scaffolding that occurred between the two students using the TP model as a reference (Excerpt 1, lines 20-47).

4.2 Episode 2: Wink, Blink, Stare Game

The next episode from the third teaching experiment focused on three 10-11-year-old boys, Justin, Owen and Matt (pseudonyms), while they discussed the results of another chance game called Wink, Blink, Stare Game (Konold & Kazak, 2008). Students first played the game 12 times by (1) drawing a chip randomly from a bag that contains two chips, one with a dot (•) and one with a dash (–), (2) returning the chip into the bag and shaking the bag, and (3) drawing from the bag again. If the two chips drawn were different (-,• or •,-), wink wins. If both chips were dashes (-,-), blink wins. If both chips were dots (•,•), stare wins. Students were initially quite confident that this game was fair because there was an equal number of each chip (a dot and a dash) in the bag. So they each picked an event (wink, blink or stare) to see who would win. Owen called 'wink' first, then Justin picked 'stare', and Matt got 'blink'. They recorded each game's result in the table on their worksheet (Figure 6). Accordingly, wink (Justin) won six times, blink (Matt) won twice and stare (Owen) won four times. Based on these results they made predictions for each event if they played the game 100 times. Their predictions were proportional to the results they got from playing the game 12 times.

After playing the game 12 times, they still believed that it was a fair game even though their predictions did not match the results of the game. Since we did not follow up this response given by the students on their worksheet during the group work, we could only offer our interpretation of this contradiction based on our knowledge about the background of these students and prior research. That is, while students had the notion of fairness as ‘each player is equally likely to win’, they based their predictions on the empirical results from a small number of trials and regarded the unequal frequencies as something that happened by chance or simply believed that ‘anything could happen’, by using the outcome approach (Konold, 1989). Students’ contradictory ideas about fairness and inconsistencies in probabilistic reasoning have also been shown by others (Konold et al. 1993; Watson & Moritz, 2003). Since this was the first probability task explored in the teaching experiment (before the revised chips game), we did not think that students’ response was based on a sophisticated understanding of the law of large numbers (i.e. an expectation of more divergence in small number of trials). Literature suggests that the law of large numbers could be challenging for some students to understand (e.g., Pratt et al., 2008).



Trial #	First Draw	Second Draw	Winner
1	•	•	Stare
2	•	-	Wink
3	-	•	Wink
4	•	•	Stare
5	-	-	Blind
6	•	-	Wink
7	•	-	wink
8	•	•	Stare
9	-	-	Blink
10	•	-	Wink
11	•	-	Wink
12	•	•	Stare

Figure 6. Wink, Blink, Stare Game results recorded by Justin, Owen and Matt.

Next, students built a model of the game in TP and played the game 100 times. The results of their first run is shown in Figure 7, i.e., 23 (-,-), 26 (-,•), 24 (•,-) and 27 (•,•). When deciding who won the game based on these results, Justin and Owen had a disagreement about what counted as ‘wink’. It seemed that the way data were organised in four bins with the counts shown on the top in TP led to Justin and Owen’s dispute over who won. The results also came out in a way that if students did not count both -,• and •,- as outcomes for wink, stare would win by one point, and the game remained fair. As we see in students’ predictions both before and after playing the game, they did not seem to understand that wink was comprised of two outcomes.

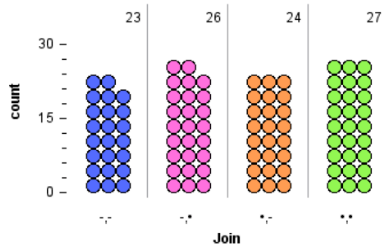


Figure 7. First simulation results for 100 games in *TinkerPlots*.

In Excerpt 2, after observing the conflict between Owen and Justin, the teacher used questioning as a means of scaffolding the development of students' understanding and reasoning. So far neither of the students provided a convincing argument for which outcomes to count. The teacher's scaffolding for talk involved encouraging students to make their thoughts and reasons explicit to each other when discussing whether both (-,•) and (•,-) should count as wink (Justin's idea) and why the game was not fair (Owen's idea).

Excerpt 2 (line 1-12)

Justin:	So, who won?
Owen:	Me!
Justin:	Me.
Owen:	Looks like
Justin:	((*inaudible*)) because both of them technically
Owen:	No. Yeah whoever is, I've won.
Justin:	Looks like I am both
Owen:	You can't put them both together.
Justin:	Fine! I'll have my 26.
Teacher:	Wait a minute. Let's. Can you just discuss that? Should we count both or not?
Justin:	Yea, because technically it is the same.

After seeing the results on the plot as presented in Figure 6, Justin believed that the two outcomes, (-,•) and (•,-), were basically the same and both should count as wink (Excerpt 2, lines 5 and 7). Owen, however, protested and said, "You can't put them both together". While Justin was not happy, he did not challenge Owen. At this point the teacher posed a question to encourage the students to discuss with each other about why they should or should not consider both outcomes as wink.

Since Justin's explanation did not go beyond "because technically they are the same" and Owen thought that combining them was "not fair", the teacher then used a feature of TP to show the students how to combine two outcomes in the plot and let them decide whether that would make sense. Then they decided who won in four trials of the simulated version of the game where both (-,•) and (•,-)

were counted as winks in the plot. Wink won about twice as many times as the other two outcomes.

Excerpt 2 (lines 44-53)

Owen: You always win J!
Teacher: OK, so maybe, so you want to, you said, Owen said, did you listen to him?
Justin: Yes, it's not fair 'cause I got two ((*pointing to the wink outcome on the plot*)).
Teacher: OK. So try to come up with an answer, why it is not fair because we will have a discussion together now.
Justin: It's not fair because whoever's got wink has got two so they are bound to win.
Matt: They've got two chances.

As Justin won every time during the simulations, Owen kept saying that it was not fair. His notion of fairness seemed to be an emotional one because he did not articulate why it was not fair in terms of chances of winning the game for each player. He simply did not want to lose. When he exclaimed: "You always win J!" (Excerpt 2, line 44) after four trials, the teacher tried to draw Justin's attention to Owen's observation and to scaffold for explanations and justifications for the simulation results (Excerpt 2, lines 45 and 49). Finally, Justin and Matt explicitly justified the observed outcomes by articulating clearly that there were two possibilities to get wink in the game and only one possibility for the other two outcomes.

4.3 Episode 3: Designing a fair game

After the first teaching experiment, the chips game (described in section 4.1) was revised to include an extension to gain deeper insights into students' conceptions of fairness and probability. In the second teaching experiment, after students completed the first part of the revised chips game, students were asked to design a fair game using 5 chips in each bag (blue and red) following the same rules. This episode focused on the group where Gabby, Blair and Flora (pseudonyms) came up with three red and two blue chips in bag one and two red and three blue chips in bag two. To test their conjecture, they built a model of their game in *TinkerPlots* and ran it several times with 1000 repeats each time. The results were mostly 52% mixed colour and 48% same colour. They got a few 51%-49% (mixed colour-same colour) and occasionally a 50%-50%. Based on these results, Gabby seemed to believe that their game was fair. In fact, it would be difficult to interpret the results without collecting systematic data on the percent of the outcomes in a large number of trials and with no understanding of the distinction between sample-to-sample variability and variability due to the chance setup. Alternatively, a theoretical analysis of the sample space was needed to count the number of ways each combined event could happen. Blair, who had an intuitive idea, was not totally sure about the fairness of the game: “It’s roughly…It feels like it’s as close as you can because there is an odd like number in each bag.” This uncertainty due to the results being close to 50-50, but not quite even, motivated a further joint exploration, which led students to analyse the number of possible outcomes for each combined event (mixed colour and same colour).

In Excerpt 3, the teacher intervened when the group laid the chips from each bag on the table to make sense of the simulation results. Through questioning, she tried to scaffold students’ understanding of the problem. While Gabby thought that their game was fair, Blair disagreed with her. Blair began to focus on the number of red and blue chips in each bag and the amount of mixed colour chips that one could get in the game to explain her idea. By manipulating the chips on the table, she suggested a new idea for why the game could not be fair. Then the teacher’s scaffolding helped Gabby to take on Blair’s idea and to develop a new insight about the outcomes of the game.

Excerpt 3 (lines 1-22)

1	Teacher:	So when you put two like this, and then like this ((*a group of two*
2		*red chips and three blue chips and a group of two blue chips and*
		three red chips)) huh?
3	Gabby:	Yeah. That’s fair.
4	Teacher:	That is fair?
5	Blair:	I don’t think it is completely fair, because, there is, like a different
6		amount of reds in each bag, a different amount of blues, but then
7		there is the same amount of the opposite colour, like, so, in each bag.
8	Teacher:	Oh what does that mean? Interesting, did you follow her? Gabby?
9	Gabby:	Yeah
10	Teacher:	Does it matter? She says that they have the same number of,
		different colour?
11	Blair:	The same amount, there is the same number of different colours,
12		like three blues and three reds, like that, but then, there is not the
13		same amount of the same colours, because there is two and three
14	Teacher:	Huh, in each bag?
15	Blair:	So it is almost impossible to get them the same, because if you like
16		move this one for this one and you’d have... ((*showing two groups:*
17		*one with one blue chip and four red chips and another with one red*

chip and four blue chips))

It is evident that the teacher's scaffolding for dialogue by questioning (i.e., "*Oh what does that mean? Interesting, did you follow her? Gabby?*" and "*Does it matter? She says that they have the same number of, different colour?*"), combined with the scaffolding for content afforded by the manipulatives, encouraged Blair to explain her ideas more clearly (Excerpt 3, lines 5-7 and 11-13) and enabled Gabby to take on Blair's idea by listening to her. By manipulating the blue and red chips on the table, Gabby began to count all possible outcomes to get two same coloured chips in the game (see Figure 8). It can be argued that a dialogic switch (Wegerif, 2013) occurred when Gabby had a new way of seeing the problem as she reflected on the problem through Blair's perspective. The idea of a dialogic switch is a change in perspective that arises in a dialogue when one person sees the perspective of another or sees the perspective of a new outside or witness position. Using this new insight Gabby figured that there were 12 different ways to get two same coloured chips whereas there were 13 different ways to get two different coloured chips. Gabby then concluded that "*there is one more chance that you will get* [two different coloured chips]" to explain why the game was not fair.

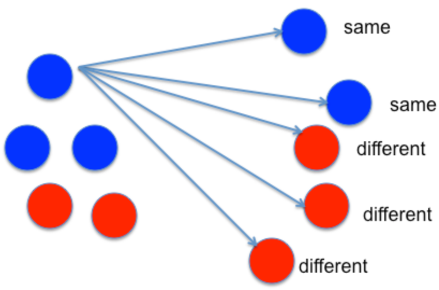


Figure 8. An illustration of how Gabby used the real objects to count the number of possible outcomes.

5 Discussion and Conclusion

Our research question was: In what ways can the combination of scaffolding for content using technology and scaffolding for dialogue support students' understanding of probability concepts? In the study, we focused on both scaffolding moves and dialogues in small groups that helped students develop their mathematical understanding during group activities in the context of probability. In order to scaffold for content knowledge, *TinkerPlots* software was utilized as it provided simplification, visualization and abstraction of mathematical ideas inherent in various probability experiments. We also used non-computer based pedagogical tasks derived from the 'Thinking Together' approach (Dawes et al., 2004) to promote effective talk as scaffolding for dialogue. These tasks include developing ground rules, using prompt cards to ask 'why' questions, and giving reasons when challenging others' ideas.

Our analyses of the three episodes showed that various moments of insight leading to a new understanding were linked to content scaffolding (*TinkerPlots*) and dialogue scaffolding (Thinking Together). For instance, in the first episode the two-spinner model built in TP allowed Jacob to see the two-stage nature of the probability experiment. His insight became apparent through the combination of the TP model (an external scaffold) and his dialogue between himself and an outside other, or a 'superaddressee' in Bakhtin's (1986) terms. Moreover, the dialogue between Jacob and Chris used the TP plots as visualization supports and enabled a shift in Chris' understanding of the problem. This peer-to-peer scaffolding through dialogue was enabled by a dialogic relationship in which Chris was able to ask for a reason when Jacob had a new insight.

In the second and third episodes, the dialogues that led to conceptual breakthroughs in other similar probability contexts were mostly scaffolded by the teacher through questioning. These questions allowed group members to reflect on what they were doing or thinking. In these examples, the teacher's scaffolding moves were also linked to the external scaffolding by *TinkerPlots*, which initiated students' discussion (in episode 2) and students' theoretical analysis using manipulatives (in episode 3). Similarly, Rojas-Drummond and Mercer (2003) report that effective teachers use 'why' questions to promote students' reasoning skills and to encourage them "to give reasons for their views, [to organize] interchanges of ideas and mutual support amongst pupils and [to encourage] pupils to take a more active, vocal role in classroom events." (p. 107). This method is effective because it allows group members to share alternative ideas and to be open to each other. According to a dialogic theory inspired by Bakhtin (1981, 1986), a switch in perspective entails being able to take the point of view of the other and thus openness to the other is needed. This process was facilitated by our dialogic approach described in Section 3.1 where students were encouraged to listen to each other, to understand each other's ideas, and ask for explanations. However, sometimes students needed additional support as they engaged in group work.

Both peer-to-peer scaffolding and teacher scaffolding for dialogue in our examples show the significance of moving tool-based scaffolding into dialogue. Hence we argue that learning does not happen solely in scaffolding for content knowledge but also in dialogues that are prompted and framed by scaffolding for dialogue. As such, scaffolding for dialogue becomes a part of the overall pedagogy.

Our analysis of these three incidents of conceptual breakthrough suggests that they result from, or at least are facilitated by, two different kinds of scaffolding. Without dialogues and dialogic reflections that are supported by scaffolding for dialogue, it is unlikely, in each of the three examples, that scaffolding for conceptual understanding alone is sufficient for conceptual breakthroughs to occur. Even when students used the physical bags and chips to play games in the revised version of the chips task and the wink, blink and stare task, none of the students in the second and third teaching experiments used the manipulatives to make their probability predictions. Only students in Episode 3 were able to show whether the game was fair with scaffolding for dialogue. This could be partly due to the lack of students' attention to consider the process of what each combined outcome is comprised of without any prompt, which was similar to the findings of Polaki (2002) on the analysis of sample space in compound events. In each episode we analysed, conceptual scaffolding was present. However, a breakthrough in understanding did not occur until the learners were able to see the problem from a new perspective through the medium of a dialogic switch. This is particularly apparent if we look at the role of the *TinkerPlots* software. In episode 1 and episode 2 the model on the screen clearly showed the solution of the problem but the learners did not use it as a scaffold for their understanding until it appeared within their dialogues (both audible and inaudible or 'internal') as an answer to a question that they had asked. At first, Jacob did not see the significance of the two-part model of the probability problem that he created on the computer screen. He was not sure, and said, "I am still debating". When he ran the simulation, the significance of the model became so obvious that he could not help laughing. Chris, who had not been debating, did not see this until Jacob pointed it out to him. The TP model only became functional as a scaffold when it appeared to him as an answer to his question. The model on the *TinkerPlots* screen in a way became a tool for argumentation, as Ben-Zvi (2006) argues, through the dialogue between Chris and Jacob.

It follows from this analysis that even if we just want to teach content knowledge we need to scaffold not only for the personal appropriation of conceptual tools but also for dialogues and dialogic reflection. Scaffolds, of the neo-Vygotskian kind, will only work as scaffolds within dialogues in which questions are asked, which the scaffolds by a teacher can then help to answer. Our scaffolds for dialogue are different. They do not only help to teach concept knowledge within a domain but also that kind of dialogic thinking that facilitates having new insights as a result of a dialogic switch in perspective. The ability to switch perspective is essential to learning in every domain.

Acknowledgements

This research was supported by a Marie Curie Intra European Fellowship within the 7th European Community Framework Programme to the first and second authors. We would also like to acknowledge the generous support of the schools, teachers and students who worked with us on this project.

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