Extreme Downside Risk: Implications for Asset Pricing and Portfolio Management

Submitted by Linh Hoang Nguyen, to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Finance, May 2015

I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

(Signature)..

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ABSTRACT

This thesis investigates different aspects of the impact of extreme downside risk on stock returns. We first investigate the impact at market level, where the return of the stock market index is expected to be positively correlated to its tail risk. More specifically, we incorporate Markov switching mechanism into the framework of Bali et al. (2009) to analyse the relationship between risk and returns under different market regimes. Interestingly, although highly significant in calm periods, the tail risk-return relationship cannot be captured during turbulent times. This is puzzling since this is the time when the distress risk is most prominent. We show that this pattern persists under different modifications of the framework, including expanding the set of state variables and accounting for the non-*iid* feature of return process. We suggest that this result is due to the leverage and volatility feedback effects. To better filter out these effects, we propose a simple but effective modification to the risk measures which reinstates the positive extreme risk-return relationship under any state of market volatility. The success of our method provides insights into how extreme downside risk is factored into expected returns.

In the second investigation, this thesis explores the impact of extreme downside risk on returns in a security level analysis. We demonstrate that a stock with higher tail risk exposure tends to experience higher average returns. Motivated by the limitations of systematic extreme downside risk measures in the literature, we propose two groups of new 'co-tail-risk' measures constructed from two different approaches. The first group is the natural development of canonical downside beta and comoment measures, while the second group is based on the sensitivity of stock returns on innovations in market systematic crash risk. We utilise our new measures to investigate the asset pricing implication of extreme downside risk and show that they can capture a significant positive relationship between this risk and expected stock return. Moreover, our second group of 'co-tail-risk' measures show a highly consistent performance even in extreme settings such as low tail threshold and monthly sample estimation. The ability of this measure to generate a number of observations given limited return data solves one of the most challenging problems in tail risk literature.

In the last investigation, this thesis examines the influence of extreme downside risk on portfolio optimisation. It is motivated by the evidence in Chapter 4 regarding the size pattern of the extreme downside risk impact on stock returns where the impact is larger for small stocks. Accordingly, portfolio optimisation practice that focuses on tail risk should be more effective when applied to small stocks. In comparing the performance of mean-Expected Tail Loss against that of mean-variance across size groups of Fama and French's (1993) sorted portfolios, we confirm this conjecture. Moreover, we further investigate the performance of different switching approaches between mean-variance and mean-Expected Tail Loss to utilise the suitability of these optimisation methods for specific market conditions. However, our results reject the use of any switching method. We demonstrate the reason switching could not enhance performance is due to the invalidity of the argument regarding the suitability of any optimisation method for a specific market regime.

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LIST OF ABBREVIATIONS

ACY	Ang, Chen, and Yuhang (2006)
AGARCH	Asymmetric Generalized Autoregressive Conditional Heteroskedasticity
AIC	Akaike Information Criterion
AMEX	American Stock Exchange
AR	Autoregressive
ARCA	Archipelago Exchange
ARCH	Autoregressive Conditonal Heteroskedasticity
ARIMAX	Autoregressive Moving Average with Explanatory Variables
ARMA	Autoregressive Moving Average
BDL	Bali, Demirtas and Levy (2009)
BL	Bawa and Lindenberg (1977)
BM	Book-over-Market
CAPM	Capital Asset Pricing Model
CDF	Cumulative Density Function
CoVaR	Systematic Value-at-Risk in Adrian and Brunnermeier (2011)
CRSP	Centre for Research in Security Prices
CVB	Conditional Value-at-Risk Beta
DCRP	Change in Credit Risk Premium
DIF	Change in Inflation
DO	Change in Oil Price
DTRP	Change in Term Structure Risk Premium
DY	Dividend Yield
EDB	Extreme Downside Beta
EDC	Extreme Downside Comoment
EDH	Extreme Downside Hedge
EDH_O	Orthogonalised Extreme Downside Hedge
EGARCH	Exponential Generalized Autoregressive Conditional Heteroskedasticity
ES	Estrada (2007)
ETL	Expected Tail Loss
FF	Fama and French
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GJR GARCH	Glosten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroskedasticity
iid	independent and identically distributed
IID	Tail risk measures based on iid return distribution
IPG	Industrial Production Growth
LTD	Left Tail Dependence
MBG	Monetary Base Growth
MS	Markov Switching
MS-BDL	Bali et al. (2009) model with Markov switching integrated
NASDAQ	National Association of Securities Dealers Automated Quotations
NIID	Tail risk measures based on non-iid return distribution

NYSE	New York Stock Exchange
PDF	Probability Density Function
RFD	De-trended Risk Free Rate
US	United States
VaR	Value-at-Risk
VIX	Chicago Board Options Exchange Market Volatility Index

AUTHOR'S DECLARATION

I hereby declare this thesis incorporates materials that are the result of joint research, as follows:

The studies reported in Chapter 3, Chapter 4, and Chapter 5 are based on three working papers co-authored with Professor Richard Harris and Dr. Evarist Stoja. The paper in Chapter 3 was presented at the '8th Financial Risk International Forum' (Paris, March 2015) under the name 'Extreme downside risk and financial crisis'. The paper in Chapter 4 was presented at the 'Young Finance Scholars' Conference' (Brighton, May 2014) under the name 'Systematic extreme downside risk'. Professor Richard Harris and Dr. Evarist Stoja provided editorial advice and guidance throughout the development of the models and the papers. I developed the models, carried out the analysis and wrote most of the papers.

I am aware of the University of Exeter's regulation and I certify that I have properly acknowledged the contribution of other researchers to my thesis, and I have obtained permission from them to include the above materials in my thesis.

I certify that, with the above qualification, this thesis, and the research to which it refers, is the product of my own work.

CHAPTER 1: INTRODUCTION

1.1. Background and rationale

Over the last decade, the financial market has experienced many extreme movements of different natures, including economic crisis, natural disaster, terrorist attacks, among others. Any of these extreme events can trigger severe collapses of asset price both individually and systematically. In this context, the knowledge about the predictability of extreme events, the behaviours of asset prices in these events, as well as the risk premium corresponding to this type of risk are vital for investors in managing their risk-return trade-off. Accordingly, the extreme downside risk study has now become one of the fastest-growing frontiers of the literature.

Recent developments in this literature have shifted from simply reporting the behaviours of asset price in extreme events to the potential price impact of investors' fear of extreme events. The fact that assets are fire sold when disastrous events happen embeds a constant fear in the mind of investors. This fear, to a certain extent, has a persistent impact on asset price, which exists even when no actual disaster is realised. This impact is actually captured in many studies, namely those by Rietz (1988), Barro (2006), Gabaix (2012) and Wachter (2013), among others. They all demonstrate that the impact of the fear about a disaster taking place is significant and can solve numerous financial market puzzles. The approach of these studies in investigating extreme downside risk based on the probability of economic disaster happening is desirable since it explains the risk from its most fundamental cause. Ultimately, it is only a disaster with a substantial economic consequence that matters to economic agents. However, the room for the development and applicability of this strand of the literature is very limited due to the unavailability of rare event observations.

An alternative approach for disaster investigation is to work directly on the crash probability of asset prices, since there are much more data available. In other words, disaster risk could be investigated directly from the tail part of the return distribution of asset prices. From this perspective, disaster risk could also be referred to as tail risk or extreme downside risk. We, therefore, will use these terms interchangeably in this thesis. Given the availability of return data, many studies have examined different aspects of tail risk and their influence on asset returns. Specifically, components of tail risk including skewness, kurtosis, and higher moments are demonstrated to have significant impacts on returns, as shown in Kraus and Litzenberger (1976), Harvey and Siddique (2000), Dittmar (2002), Chung et al. (2006), among others. They consistently imply a positive risk premium attributable to fat tail.

However, research studies on the direct impact of tail fatness rather than its components are still sparse due to data scarcity. This is because direct tail risk measures require a large number of observations and the trading data is still not sufficient to generate the variability of these measures over time. Only a few papers have managed to work around this limitation and capture a significant tail risk premium, such as Bali et al. (2009), Bollerslev and Todorov (2011), Ruenzi and Weigert (2013). Ultimately, investors care about skewness risk, kurtosis risk, or higher moment risk because they are concerned about downside risk and particularly tail risk. Therefore, the imbalance between evidence regarding tail risk premium and its components' risk premium is unjustifiable. Motivated by this gap, the main theme of this thesis is the direct impact of tail risk on stock returns, where we propose new tail risk measures and investigate the tail risk impact on returns from different perspectives, including the cross-section of individual stocks, the portfolio of stocks, as well as the aggregate stock market level.

1.2. Research questions and scopes

In this thesis, we follow the canonical approach of asset pricing literature to work on systematic risk rather than idiosyncratic risk, assuming investors' homogeneity and full diversification. We examine this systematic risk from two perspectives. First, we investigate the tail risk at general market level. Under this perspective, the tail risk of the market is essentially its systematic risk and therefore we could examine its relationship with returns directly without filtering any idiosyncratic part. However, this seemingly straightforward exercise turns out to be challenging due to the work of leverage and volatility feedback effects, which cause a significantly negative relationship between any dispersion-related risk, including tail risk, and concurrent returns (see for example, Black, 1976; Campbell and Hentschel, 1992). In other words, tail risk premium turns out to be counter-intuitively negative. Given this context, Bali et al. (2009) propose a simple effective framework that captures a significant positive risk premium of the market tail risk by relating its expected value with expected returns. The simplicity of this framework provides a flexible platform for market tail risk studies to examine the risk behaviour from different perspectives. Given the time-variant nature of the tail risk, we are interested in the state-dependent behaviour of this risk-return relationship, particularly in distress state. By incorporating Markov switching mechanism into Bali et al.'s (2009) framework, we will examine how market tail risk influences its returns in calm (low volatility) state and turbulent (high volatility) state. In fact, the standard norm in the literature is to consider the turbulent state as distress state, as the timing of this state tends to cover every distress time in the market.

From the second perspective, we investigate the systematic tail risk of an individual asset. This is the risk of an asset that is attributable to the tail risk of the system (market). In other words, it shows how the performance of an asset is influenced by the tail risk of the system. An asset whose return is more sensitive to the market tail risk will have a higher systematic tail risk. Since a higher tail risk is undesirable, we are interested in whether this systematic tail risk is related to a significant positive risk premium, as postulated by asset pricing theory.

Furthermore, given the positive relationship between returns and any type of risk, it is essential for any investor to understand how to maximise returns while minimising one or several risks. As downside risk and extreme downside risk are the main concerns of practical investors rather than dispersion risk, the mean-downside risk problem is receiving significant research interest. However, in contrast to the canonical mean-variance optimisation problem, researchers have not reached any consensus regarding the optimal solution for the mean-downside risk problem. This is because the complication of downside risk measures causes difficulty in handling the optimisation problem as well as prevents it from having a close form solution. Particularly, nonlinearity and non-convexity are the most troublesome issues of the problem. Accordingly, different optimisation models propose different ways to transform the problem and make it numerically optimisable by programming algorithms. Therefore, a number of research questions in this area are still open for research studies.

Having realised the size pattern of the impact of tail risk on stock returns from the literature as well as from our study, we investigate how this size pattern affects the relative performance between portfolio optimisation practices that weight tail risk differently. As tail risk has a more prominent impact on small size stocks, we conjecture that the tail-risk-focused optimisation practice is more effective, in comparison to the mean-variance optimisation, when being applied to small stocks. This constitutes the third research question of the thesis. Furthermore, in addition to comparing the two optimisation methods, it is also desirable to understand whether a switching mechanism between the two methods based on some statistically determined or economically determined indicators can outperform both the constituent methods. One might argue that it is optimal to use the tail-risk-focused method in a distress period and the standard mean-variance method in normal periods. Therefore, we also aim to examine the feasibility of this suggestion in the last research question of the thesis.

Thus, in this thesis, we will address four main research questions as follows:

1- How does market tail risk influence its returns in calm and turbulent states?
2- Is the systematic tail risk of a stock associated with a significant positive risk premium?
3- Is the tail-risk-focused portfolio optimisation method more effective when applied to small stocks?
4- Does the switching method between tail-risk-focused portfolio optimisation and standard mean-variance optimisation methods enhance performance?

In answering these questions, we restrict our investigations to the US stock market. More specifically, our investigated assets are the US stock index and stocks in its main exchanges, including NYSE, AMEX, NASDAQ, and ARCA. Furthermore, we aim to examine tail risk over a long period of time. Therefore, we base our work on daily and monthly returns in order to expand our sample to the far past. With this intention, we could not work on high frequency intraday data because this type of data, although very promising, is only available for several recent years. Its application will be examined in future research studies.

1.3. Main findings

Regarding the first research question, we discover an interesting unexpected inconsistency. Specifically, the positive risk-return relationship documented by Bali et al. (2009) only holds in calm state of the market, while it disappears and even tends to be negative in turbulent state, which is when it is expected to be more prominent. This conflicting evidence is still robust in different modifications of Bali et al.'s (2009) framework, including accounting for a broader set of state variables and non-*iid* features of return distribution. This robust inconsistency suggests a fundamental issue in capturing this type of risk-return relationship. We argue that this is the leverage effect and volatility feedback effect. We demonstrate that these effects spread out over multiple periods during turbulent times, causing Bali et al.'s (2009) autocorrelation-based expected tail risk measure to be negatively related to returns just as a concurrent risk measure. Accordingly, we propose a new measure which better filters out the leverage and volatility feedback effects and produces a consistent performance in both market states. It reveals insights into the asset pricing mechanism of how investors' distress risk aversion is factored into expected returns.

In our investigation of the second research question, we propose two groups of systematic extreme downside risk measures. The first group of measures are constructed by pushing the standard CAPM beta and comoment measures to the tail of the distribution. The second group is constructed based directly on the argument of the hedging need against the extreme downside risk of investors. Using these new measures, we capture a significantly positive tail risk premium in both portfolio sorting investigation and Fama and Macbeth's (1973) cross-sectional regression. However, due to the tail observation scarcity problem, our first measure group is only valid in a moderate tail threshold and is not robust in a low tail threshold. This is because they are only based on a few observations in the low threshold setting. On the other hand, our second group of measures deliver a consistent performance in any setting, even at a low tail threshold in the monthly sample estimation.

The thesis also confirms our conjecture in the third research question. Specifically, using the Fama and French (1993) Size and Book-over-Market sorted portfolios; we show that due to the size-pattern of the tail risk impact on returns, the tail-risk-focused portfolio optimisation significantly outperforms standard mean-variance optimisation in small size stocks, while their performances are roughly similar in large size stocks. This result is robust even after controlling for skewness and different asset universes. Furthermore, in addition to confirming standard understanding regarding the better performance of small stock investments and high risk tolerance strategies, we also capture an interesting pattern that the tail-risk-focused optimisation method consistently outperforms the standard mean-variance method. This implies a strong recommendation for practitioners to shift to tail-risk-focused optimisation strategies, especially given the highly crash sensitive financial market in recent times.

Finally, we reject the hypothesis in the fourth research question. We propose different switching strategies between mean-variance and mean-tail risk, based on a Markov switching indicator or binary response indicator. However, even under the assumption of perfect regime awareness and perfect foresight binary response regression, the switching method tends to average the performances of the two optimisation methods rather than enhance them. We demonstrate that the reason for this failure is the unpredictability of the better-performance optimisation method for the next period. Thus, evidence in this analysis suggests that the selection of a portfolio optimisation method should be based on the specific risk perspective of the manager rather than being market-context dependent. This recommendation should be valid given that general market information and investment strategy are applied consistently over a long horizon.

1.4. Main contributions

Our first contribution to the literature is to provide insights regarding the way the tail risk-return relationship at general market level could be captured. We unfold the assumption underlying the framework of Bali et al. (2009) to capture a positive tail risk premium, which is the one-month horizon of leverage and volatility feedback effects. This helps to explain the inconsistency generated by their framework in distress periods and serves as the basis for our successful measure that works consistently in any market state. In fact, our analysis of the leverage and volatility feedback effect, Bali et al.'s (2009) measures, and our new measures reassure the mechanism of how tail risk and dispersion-related risks are factored into asset returns. Specifically, asset fire sale is to assure the correct high expected risk-high expected return relationship. Furthermore, our analysis indicates a general timeframe for this fire sale in different market states, which is one month for calm state and two months for turbulent states. This in turn is suggestive for practitioners in timing their market actions.

Another main contribution of this thesis is the proposal of several new systematic tail risk measures, which all confirm the existence of a positive tail risk premium. In particular, the ability of our second measure group to work under extreme settings such as a low tail threshold and monthly sample is valuable for the literature. This solves the dilemma in testing systematic extreme downside risk. Specifically, researchers do not need to trade-off between the precision in estimating measures by using a long estimation period and the quality of their over time statistics based on a limited number of observations. This, in turns, clears the most problematic obstacle in the extreme downside risk literature, paving the way for other research ideas.

In this thesis, we also demonstrate the significant impact of tail risk on portfolio optimisation choices. We show that the choice between standard mean-variance optimisation and tail-risk-focused optimisation does matter. We provide varying evidence in support of the consistent choice of the tail-risk-focused optimisation method over the standard mean-variance method. This is consistent with the practical concern of investors regarding downside risk and extreme downside risk. Furthermore, our confirmation about the effectiveness pattern of tail-risk-focused optimisation methods when applying for large and small stocks is useful for the size-focused investment strategy, one of the most popular strategies in practice.

1.5. Structure of the thesis

Before reporting our main research, this thesis introduces the literature review chapter where we summarise the related studies in the extreme downside risk literature. This review contains common research studies which are referred to by the other three main chapters of the thesis (Chapters 3 to 5). This is to avoid overlapping between the literature reviews of the three main chapters, as they deal with the common topic of extreme downside risk. Other research studies that are specific to each chapter will be mentioned within the corresponding chapter. We believe this is the most effective structure for reporting our main studies which are in the same area of literature but deal with distinguished research questions.

We then introduce our three main studies in Chapters 3 to 5. Chapter 3 is our market level analysis of the tail risk, which answers the first research question. Chapter 4 deals with tail risk in the individual stock level and it addresses the second research question. Chapter 5 is then introduced to report our portfolio optimisation analysis with respect to tail risk impact. It addresses the last two research questions about the size-pattern in tail-risk focused portfolio optimisation and the ability of the switching method to enhance portfolio performance. We conclude the thesis in Chapter 6, where we summarise the thesis, acknowledge possible limitations, and provide suggestions for future research.

Within each chapter, excluding the Introduction (Chapter 1) and Conclusion (Chapter 6), we include an introduction at the beginning to introduce the study we are going to report and summarise the structure of the chapter. In the three main chapters (Chapters 3 to 5), the introduction also includes additional literature which is relevant to the corresponding studies in the chapters but is not mentioned in the common Literature Reviews Chapter (Chapter 2). At the end of each chapter, when necessary, we include an Appendix to report additional results of the investigations in the main body of the chapter. Given the empirical nature of our studies, where the examinations are altered in a number of ways, this will help to keep the main body of the thesis clear and concise, which is useful for conveying our messages.

CHAPTER 2: LITERATURE REVIEW

2.1. Introduction

In this chapter, we collectively summarise and analyse common literatures for the three main studies in the following chapters. Since our studies investigate different aspects of the same topic of extreme downside risk, overlapping between the literatures consulted is unavoidable. Therefore, collecting their common literatures in a separate chapter, while referring to topic-specific literature within each subsequent chapter, is an effective way to report our research. Additionally, since some of the methodologies will be used in more than one chapter, such as the Markov switching model and the conditional volatility model, to avoid repetition, we also introduce the standard formats of these methodologies as well as their related literatures in this chapter. The specific configurations will be provided when they are applied in each chapter.

We first review the literature regarding the asset pricing relationship between extreme downside risk and returns. We start from canonical papers using economic disaster risk to explain a significant part of equity risk premium, and then refer to studies on tail risk and components of tail risk using observable stock return distributions. We then summarise the most commonly used distributions of stock return modelling in the literature, including both Normal distribution and a range of fat-tail distributions such as Student-t, Mixture Gaussian and Generalised Pareto, among others. A section about Markov switching analysis is then introduced, which summarises both its standard analytical framework as well as the related literature, as explained above. We spend the final section reviewing the Value-at-Risk literature as well as related estimation models which will be used throughout the thesis. This section also includes a brief introduction to Expected Tail Loss as it is a close variation of Value-at-Risk that is becoming of more and more interest to practitioners and regulators.

2.2. Asset pricing relationship between extreme downside risk and returns

Asset pricing literature has long documented the nontrivial impact of the risk of extreme movements on asset prices, particularly extreme crashes. For example, Nikkinen et al. (2008) report evidence about the return and volatility impacts of the September 11 attack on a number of equity markets all over the world. Rose (2011) discusses the 'Flash Crash' of May 6th 2010 where the S&P 500 Index dropped sharply by 6.2 percent within 20 minutes. Ferstl et al. (2012) provide evidence of the significant impact of Japan's Fukushima earthquake nuclear disaster in 2011 on the stock returns of related industries worldwide.

Meanwhile, many studies have gone further than the simple direct impact of extreme events on prices to demonstrate that it is the probability of those crashes happening that consistently imposes a nontrivial premium on asset returns. One of the very first building blocks for this idea dates back to the eighties of the last century in Rietz's (1988) framemwork, where he solves the equity risk premium puzzle of Mehra and Prescott (1985) by introducing an additional rare disaster state into the two-state economy. He demonstrates that the extended model can generate any high level of equity risk premium given a severe enough distress risk. Rather than a restricted model with specific assumptions about risk aversion, time preference, or probability of distress, his paper provides a flexible framework to obtain the equity risk premium generated by any combination of these parameters. As a result, he demonstrates that reasonable ranges of these parameters can result in the observed equity risk premium and risk free rate, and thus resolve the puzzle.

Rietz's (1988) framework is developed by Barro (2006) with a specific disaster definition and a practical approach to measure disaster probability. He demonstrates the ability to derive the observed high level of equity risk premium using available data about disaster events, including economic crisis, wars, natural disasters, and epidemics of 35 countries around the world in the 20th Century. He also suggests an expanded capital-investment-included model to replace the simple Lucas-economy-setting model, but without a calibration for its parameters.

More recently, this trend of studies has been further accomplished by Gabaix (2012) and Wachter (2013) by accounting for the variation in the disaster risk over time as well as the relative performance of assets in these distress times. Gabaix (2012) develops the models of Rietz (1988) and Barro (2006) to incorporate the severity of stochastic disaster and shows that his disaster risk model solves a number of puzzles in economics and finance. He also examines the impact of time-varying disaster probability through the common movement between the returns of stocks and bonds, and concludes that its contribution is only marginal. On the other hand, assuming consumption follows a diffusion process with jumps that follow the time-varying intensity Poisson process, Wachter (2013) proposes the first complete model that accounts for disaster risk with time-varying severity and probability. This model reveals the nontrivial roles of both time-varying severity and time-varying probability in determining the equity risk premium.

However, the rare economic disaster observation limits the room for development in this strand of literature. In fact, severe economic crises occur too infrequently, causing great difficulty in constructing justifiable disaster risk measures. For example, in Rietz (1988), in order to result in a 5 to 7 percent equity risk premium given that risk free rate is less than 3 percent, under reasonable ranges of risk aversion and time preference coefficients, the output may, about once every 200 years, need to crash to half of the expected level within only one year, which is the entire destruction level of the Great Depression (1929-1932). Similarly, Barro (2006) uses international data to measure the US rare disaster risk and comes up with a probability of 1.5 to 2 percent for the US real GDP to experience a fall of over 15 percent every year.

An alternative approach for examining the implications of extreme downside risk, which receives much more research interest, is to directly investigate the impact of the tail of return distribution on returns. In contrast to the unavailability problem of economic disaster data, asset return is readily observable at any frequency. Therefore, analysts could construct different measures corresponding to any aspect of its distribution. A review about asset return distribution will be provided in the next section. In fact, the tail is a non-linear function of variance, skewness, kurtosis, and higher moments of the return distribution. Therefore, each of these components could be examined separately or collectively to reveal different aspects of the asset pricing implications of tail risk. In the standard approach of asset pricing literature, given investors' homogeneity and portfolio investment perspective, studies of these 'moment risks' tend to focus on their systematic part (the 'co-' part). Meanwhile, idiosyncratic risk is assumed to be diversified away.

Coskewness is the first, and probably the most popular, higher comoment whose relationship with asset returns is investigated. In addition to the canonical systematic second-moment risk of CAPM beta, many papers have proved that coskewness has a significant impact on expected returns. For example, Kraus and Litzenberger (1976) develop a three-moment CAPM, which gives rise to the role of coskewness. Furthermore, given that investors exhibit non-increasing absolute risk aversion, three-moment CAPM postulates that the price of coskewness risk (gamma) has the opposite sign to the skewness of the market. In other words, if the market exhibits positive skewness, the price of coskewness risk should be negative: an asset with higher coskewness is desirable and traded with a price premium (high price and low expected returns). On the other hand, if the market exhibits negative skewness, the price of gamma is expected to be positive: an asset with higher coskewness is risky and requires higher expected returns. Using all the stocks listed in the NYSE from 1926 to 1970, the authors successfully demonstrate that coskewness is priced and the sign of the premium is exactly what is predicted by the three-moment CAPM. Harvey and Siddique (2000) further develop a conditional version of three-moment CAPM using a quadratic version of the stochastic discount factor and demonstrate the significance of 'systematic skewness' or coskewness, given the presence of size and Book-to-Market factors. They further find evidence that coskewness is related to momentum effect. Finally, they also suggest that the ignorance of coskewness risk partially explains the equity premium puzzle.

Findings in Kraus and Litzenberger (1976) and Harvey and Siddique (2000) are supported by numerous empirical evidence, namely Ang et al. (2006a), Moreno and Rodríguez (2009), among others. In more detailed investigations, coskewness risk is found to be more prevalent in bull markets as compared to bear markets (see, for example, Friend and Westerfield, 1980; Guidolin and Timmermann, 2008). As coskewness can be interpreted as a measure of the ability of an asset to hedge against the volatility risk of the market, these results imply that investors' need for volatility hedging would be higher in good times compared to in bad times, which is consistent with the evidence that shows investors tend to be risk-seekers when faced with loss in behavioural finance literature.

In addition to coskewness, higher comoments are also shown to have an impact on returns. Indeed, the three-moment CAPM used in demonstrating the role of coskewness is consistent with the well-recognised investor's preferences in financial economics, namely positive marginal utility, risk aversion, and non-increasing absolute risk aversion. Dittmar (2002) further incorporates Kimball's (1993) decreasing absolute prudence to argue for the use of a conditional cubic form of the pricing kernel, which corresponds to a fourth-moment CAPM, implying the relevance of conditional forth-order comoments, or conditional cokurtosis. Using total wealth which includes both incomes from equity investment and human capital that is argued to account for the majority of the total wealth, the author captures a significant coefficient for the cubic component of the kernel. He even demonstrates that the Fama-French factors are redundant when incorporated in the cubic pricing kernel. The risk premium corresponding to cokurtosis risk is expected to have a positive sign since an asset with higher cokurtosis offers poor returns when the market is left-skewed, which is undesirable. Many other studies have also captured this positive risk premium in stock returns, such as Ang et al*.* (2006a), Guidolin and Timmermann (2008), Yang and Chen (2009), Kostakis et al*.* (2012), among others.

Regarding higher levels of moments, there is evidence that comoments higher than cokurtosis also explain returns. Chung et al*.* (2006) demonstrate that higher comoments of up to order ten collectively explain more than 80 percent of the Fama and French (1993) Size and Book-to-Market factor loadings. Therefore, these loadings become redundant in explaining the cross-sectional asset returns if comoments are included. Furthermore, they only observe these results using comoments rather than 'total' moment and therefore validating the argument for the use of pure systematic risk in the standard asset pricing approach. In the context of our analysis, we call a measure calculated directly from returns a 'total' measure. This is to distinguish it from systematic ('co-') measures which account for the contribution of the stock to the corresponding risk of the portfolio, or idiosyncratic measures which account for the residual risk of the stock in excess of its systematic risk.

On the other hand, supporting evidence for the roles of these comoments are weak in some investigations, as in the studies of Chi-Hsiou Hung et al. (2004), Galagedera and Maharaj (2008), Carvalhal da Silva (2006), Post et al*.* (2008), Mishra et al*.* (2008). One possible explanation could be that of Brockett and Kahane (1992) where they show cases when the commonly used practice of relating derivatives of the utility function to investors' moment preferences is not solid. For example, they show the existence of cases when an agent with any commonly used utility function will prefer an opportunity with lower expected returns, higher variance, and lower skewness. In other words, it is not sound enough to assume about the aversion and preference of investors regarding moments of return from the signs of the derivatives of the utility function.

Moreover, as the validity of comoments is only as good as the assumption of fully diversified homogeneous investors in the general equilibrium asset pricing models, another explanation could be the failure of this assumption in practice. This in turn gives rise to arguments supporting the roles of idiosyncratic tail risks. For example, just like coskewness, idiosyncratic skewness attracts researchers in downside risk investigations. A number of papers have reported about the nontrivial influence of idiosyncratic skewness on asset returns, which could be even more robust than that of coskewness, for example, Mitton and Vorkink (2007), Boyer et al*.* (2010), Conrad et al*.* (2013). In fact, Mitton and Vorkink (2007) demonstrate that the relevance of idiosyncratic skewness is not simply a consequence of investors' underdiversification. They show that it is the desire of investors to capture a higher level of skewness that causes them to remain underdiversified. In their model, two distinguished investors are modelled: a 'Traditional' investor having a standard quadratic utility function and a 'Lotto' investor having a utility function with skewness preference. Using this simple setting, given that asset returns are skewed, the authors show that both investors remain undiversified, especially the 'Lotto' investor. Their argument is supported by the dataset of 60,000 actual accounts of individual investors, where the stocks chosen by 'less diversified' investors have much higher skewness and idiosyncratic skewness compared to those included in the portfolios of 'more diversified' investors.

In contrast to a large number of studies exploring extreme downside risk through moments of return distribution, there are fewer research studies that consider the asset pricing implication of this risk via some direct measure of the fat tail. A seemingly convenient approach is to examine this risk-return relationship at general market level if one could show the tendency of market return to be higher when its tail risk is higher. However, in fact, this turns out to be a challenge due to the fact that leverage and volatility feedback effects cause a significantly negative relationship between returns and any commonly used turbulent-related measures, including tail risk measures. This phenomenon will be discussed in detail in Chapter 3. Thus, only a few studies have successfully captured the significantly positive tail risk-return relationship at general market level. For example, Bali et al*.* (2009) use the expected Value-at-Risk of the market to capture the positive expected tail risk-expected return relationship, controlling for some commonly used state variables. Meanwhile, Bollerslev and Todorov (2011) use high-frequency data of S&P500 futures and options to identify and estimate an “Investor Fear Index” and capture a significant return premium corresponding to this fear of disaster.

Similar to the studies of comoments and idiosyncratic moments above, the more standard approach for the investigation of the asset pricing impact of direct tail risk measures is cross sectional analysis that uses the whole stock universe. One of the first papers of this kind is Bali and Cakici (2004). In this study, using individual stocks in the main US markets (NYSE, AMEX, and NASDAQ); the authors show that Value-at-Risk significantly affects returns. Specifically, under portfolio sorting, stocks with higher Value-at-Risk tend to have higher returns. This result is further confirmed by Fama-Macbeth’s (1973) two-stage analysis and the analysis of the High-minus-Low Value-at-Risk factor. They further show that the impact of Value-at-Risk is still robust after taking into account volatility and liquidity. However, though empirically interesting, the tail risk measure in this study is simply the raw Value-at-Risk of a security, which is essentially only a total risk measure. Although studies on the impact of total measures of risk including variance, skewness and kurtosis, among others, are abundant in the literature, such total measures lack a theoretical base since they essentially imply that investors view each asset in isolation. From the classical point of view, when the portfolio perspective is taken into account, idiosyncratic risk is assumed to be diversified away and only systematic risk matters. Therefore, similar to the case of moment risk, a ‘co-tail-risk’ measure would be more appealing theoretically.

Unfortunately, the number of such ‘co-tail-risk’ measures in the literature is still limited. Some candidates like CVaR Beta of Kaplanski (2004) or CoVaR of Adrian and Brunnermeier (2011) are exposed to certain disadvantages. Specifically, the Conditional Value-at-Risk Beta (CVB) in Kaplanski (2004) measures the marginal contribution of a stock to the Conditional Value-at-Risk of the market, which is in line with the idea of a systematic measure. This measure only has a closed form when the stock return is assumed to follow a specific parametric distribution. However, when investigating several distributions, Kaplanski (2004) shows this measure is highly correlated to CAPM beta and the correlation coefficient is around 98 percent. Even in the case when the return is not assumed to follow any specific distribution by using pure empirical distribution, and CVB is calculated using some numerical algorithms, its correlation to beta is still about 85 percent. Therefore, the usefulness of this measure is limited.

Similarly, the CoVaR measure in Adrian and Brunnermeier (2011) is also of limited security-wise applicability. This measure is proposed by researchers at The Federal Reserve Bank of New York in order to capture the contribution of a financial institution to the systematic risk of the financial system alone. Using quantile regression, the contribution is calculated as the change in Value-at-Risk of the financial system predicted by the returns of a member when the member moves from normal state to distress state. However, this measure is subjected to serious endogeneity as it implicitly assumes the tail risk of the individual asset drives the systematic tail risk. This problem might be less serious when the investigated assets are closely related and each can significantly affect the portfolio, such as in the case of assets in the banking and financial sectors. However, applying this method for a general universe of stocks will be of much more concern.

Another tail risk measure that has been tested in the literature is the tail risk measure of Huang et al*.* (2012). However, even though it has been proved to significantly affect returns given other risk factors; this is only an idiosyncratic measure rather than a systematic measure. Moreover, this idiosyncratic measure still essentially contains the ‘co-tail risk’ effect as it is calculated from the residuals of the Carhart model, which has not taken the systematic tail risk into account. In other words, it would be questionable whether the significant role of Huang et al*.'s* (2012) risk measure is essentially due to the systematic part or the idiosyncratic part of the extreme downside risk.

Bali et al. (2014) demonstrate how both systematic and idiosyncratic extreme downside risk measures fail to explain asset returns in expected direction. They, therefore, propose a hybrid tail risk measure which is the mixture of both systematic and idiosyncratic measures in order to capture a significant extreme downside risk premium. Their measure is essentially a co-downside measure between a stock return and the market return, conditional on the tail event of the stock rather than that of the market. The rationale for this hybrid measure is that underdiversified portfolio of investors are influenced by both general market returns as well as individual stock returns.

To my knowledge, the newly proposed Left Tail Dependence (hereafter LTD) measure of Ruenzi and Weigert (2013) is the only successful 'co-tail-risk' measure so far in the literature. Their LTD represents the crash sensitivity of an individual stock relative to the market, constructed using the Copular of the joint distribution between the individual stock returns and market returns. Essentially, it provides information about the probability of the stock return to be in its extreme lower tail conditional on the tail event of market returns. The risk premium corresponding to this tail dependence is expected to have a positive sign as stocks with high LTD tend to offer low returns when investors' wealth is low. Investors require price discounts in order to hold these assets. In their empirical analysis, Ruenzi and Weigert (2013) successfully capture this significant positive risk premium. However, the study of Ruenzi and Weigert (2013) also suffers from some shortcomings. In particular, the cumbersome estimation of their LTD measure requires too many return observations that the cross-sectional regression framework must be set at a yearly frequency. Furthermore, it also ignores the crash severity. More importantly, the level of LTD does not have the same implication with the ability to hedge against tail risk. Strictly speaking, an asset with low LTD is not necessarily a hedge because it does not guarantee to offer good returns when systematic crashes happen. Instead, it just has a lower probability to crash in these times. Although, given the numerous difficulties in working with rare disaster risk, this is still an important measure in systematic tail risk literature.

2.3. The distribution of stock return

Finance literature on asset return distribution has been thoroughly scrutinised over many decades with innumerable developments and applications. The most widely used distribution is the Gaussian (Normal) distribution, which is symmetrical, bell-shaped, and fully specified by its mean and variance. The Probability Density Function (PDF) of a Normally distributed random variable $x \sim N(\mu, \sigma^2)$ is given as:

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{2.1}$$

The popularity of Normal distribution is partially due to the Central Limit Theorem, which governs the distribution of the mean of a sample taken from any arbitrary distribution. Moreover, its simple parameterisation and the characteristic of a stable distribution, where the sum of Normally distributed variables also has Normal distribution, make Normal distribution highly manoeuvrable in any analytical framework. As a consequence, it is the building block for almost all canonical models in finance, including Markowitz's (1952) portfolio theory or Sharpe's (1964) and Lintner's (1965) CAPM. However, in reality, asset returns tend to exhibit 'non-normality' features such as skewness and tail fatness. In fact, a number of studies have tested and rejected normality in asset returns at any frequency. For example, Chung et al*.* (2006) reject normality for stock returns at 5 different frequencies ranging from daily to semi-annual, while Campbell et al*.* (1997) claim that non-normality is more prominent in returns of shorter intervals. Moreover, researchers also reject normality after filtering out autocorrelation and heteroskedasticity from the return process. Rachev et al*.* (2005) show that the residuals of the ARMA-GARCH model for the stock returns of a large number of companies in S&P 500 still exhibit significant non-normality, which is consistent with evidence in Bollerslev (1987) and Nelson (1991a).

In order to account for the non-normality features in asset returns, different families of distribution have been developed. The closest version to Gaussian is the Student-t distribution. However, although able to account for fatter tail, Student-t is still symmetrical. Although Campbell et al*.* (1997) show that evidence for skewness is weaker than excess kurtosis in asset returns, a desirable distribution should be able to reflect both of these features. As a result, Hansen (1994) develops a version of Student-t distribution which is able to exhibit asymmetry. The density function of this Skewed Student-t distribution is specified as:

$$f(z_t|\eta,\lambda) = \begin{cases} bc\left(1+\frac{1}{\eta-2}\left(\frac{bz_t+a}{1-\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & if\ z_t < -\frac{a}{b} \\ bc\left(1+\frac{1}{\eta-2}\left(\frac{bz_t+a}{1+\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & if\ z_t \geq -\frac{a}{b} \end{cases} \qquad (2.2)$$

where $z_t = (r_t - \mu)/\sigma$ is the standardised version of return r_t with mean μ and standard variance σ; η is the degree of freedom (2< η <∞); λ is the skewness parameter (-1< λ <1); and the constants a, b, c are determined as:

$$a = 4\lambda c\frac{\eta-2}{\eta-1} \qquad (2.3)$$

$$b^2 = 1 + 3\lambda^2 - a^2 \qquad (2.4)$$

$$c = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\Gamma\left(\frac{\eta}{2}\right)}, \Gamma \text{ is the gamma function} \qquad (2.5)$$

Thus, this Skewed Student-t distribution is able to account for both fat tail, as it is based on the Student-t density function, and skewness. The distribution will be right-skewed if $\lambda > 0$ and left-skewed if $\lambda < 0$. Following the proposal of Hansen (1994), many studies have developed and applied different versions of Skewed Student-t to better fit the data, including Fernandez and Steel (1998), Jones and Faddy (2003), Patton (2004), among others.

Another type of widely-used distribution that accounts for non-normality is the Gaussian Mixture distribution. Kon (1984) claims that the use of a mixture of Normal distribution fits stock returns by arguing that the behaviours of stock returns might be significantly different upon the arrival of important information or at different timings of the market. Thus, even though stock returns truly follow Normal distribution in a specific regime, its historical distribution would exhibit substantial non-normality due to the combination of different Normal distributions belonging to different regimes. He then demonstrates the ability of Gaussian Mixture to nicely fit return data, capturing both skewness and excess kurtosis. The density function of a variable x following a mixture of M Normal distribution is given as:

$$f(x|\lambda) = \sum_{i=1}^{M} w_i g(x|\mu_i, \sigma_i) \quad (2.6)$$

where $g(\cdot)$ is the Normal density function; $\lambda = \{w_i, \mu_i, \sigma_i\}$ for $i = 1,2, \dots, M$ is the set of parameters, including the weights, mean, and standard deviation of each component Normal distribution. Other studies that apply this type of distribution to model stock returns include Kim and Kon (1994), Aparicio and Estrada (2001), Kuester et al*.* (2006), among others. In fact, the idea that asset returns follow a combination of regimes is strongly supported by extensive literature that uses the Markov switching mechanism, which will be introduced in the next section.

Until recently, two families of distribution that specifically deal with the tail of return distribution have been attracting much research interest. Under Extreme Value Theory (Fisher and Tippett, 1928; Gnedenko, 1943; Embrechts et al., 2011), the tail of any empirical distribution could be fitted using Generalised Extreme Value Distribution and Generalised Pareto Distribution, dependent on the estimation method being used, which are block maxima and 'Peak over Threshold' respectively. Thus, the Extreme Value Theory allows researchers to focus on modelling the tail of asset return distribution without being concerned about an optimal model to fit the whole distribution. This 'zooming' function would result in a more precisely fitted tail, improving the quality of tail-based applications. In asset pricing literature, the tail indexes of these distributions, which represent the shape of the tail, are excellent candidates to serve as tail risk measures or building blocks of a tail risk measure. Huang et al. (2012) is an example of studies utilising this measure. A complete introduction and analysis of the Extreme Value Theory and these extreme value distributions can be found in Embrechts et al. (2011).

2.4. Markov switching analysis

Given the dynamic of asset returns, there is no doubt that most researchers believe in the existence of different regimes in the return process. As information drives returns, each distinguished informational context should govern a regime. These could be period-specific regimes such as ones corresponding to a new government policy, a war, or an economic crisis. They could also be recurring regimes such as those suggested in Kon (1984), including profit announcement dates of companies, trading days of a week, trading months of a year, among others. The case of a 'one-off' regime could be analysed simply using a dummy variable while the regime timing, if unknown, could be approximated by some form of structure break analysis. A dummy variable could also be helpful in case of recurring regimes if one could explicitly identify the regimes.

However, in many cases, it is impossible to precisely identify when a recurring regime takes place. For example, it might be the case when the regime is driven by some latent (hidden) variables. In these situations, Markov switching analysis provides an effective way to estimate both the parameters of the model and the probability-based timing of each regime. Specifically, assuming K regimes evolve among one another under a first-order Markov chain summarised in the transition probability matrix P:

$$P \equiv \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{22} & \cdots & p_{2K} \\ \vdots & & \ddots & \vdots \\ p_{K1} & p_{K2} & \cdots & p_{KK} \end{bmatrix}$$

where $p_{ij} = Pr\{S_t = j | S_{t-1} = i\}$ with $i, j = 1,2, \dots, K$ are K regimes of the market; S_t is the regime (state) at time t; then the Markov switching model for returns r_t could be specified as:

$$r_t = \mu_{S_t} + \sum_{i=1}^{n} \beta_{i,S_t} X_{it} + \sigma_{S_t} \varepsilon_t \qquad \varepsilon_t \sim iid\ N(0,1) \qquad (2.7)$$

where μ_{S_t} is state-dependent intercept; β_{i,S_t} for $i = 1,2, \dots, n$ are state-dependent coefficients of n explanatory variable X_i; σ_{S_t} is the state-dependent standard deviation of the error terms.

Thus, the ability of parameters to change between regimes in the Markov switching model enables a great deal of dynamics in the relationship between variables. Moreover, given a known state at the starting point and the estimated transition probability matrix, one could obtain the probability of the market to be at any regime at any point of time. This is an objective complete picture of the states of the market during the investigated sample. Additionally, researchers could further set up a cut-off threshold to determine, approximately, what specific regime is prevalent at any point in time ex-post. He might then incorporate further information that is available at the time of a regime to gain more insights into its specific economic context.

This standard framework of Markov switching analysis, as well as its estimation procedures, are developed by Hamilton (1989,1990). Following his innovation, researchers have used Markov switching mechanism in different contexts to explain patterns in all asset classes, including stock returns (Guidolin and Timmermann, 2006; Chang-Jin et al., 2004), interest rates (Ang and Bekaert, 1998; Bansal and Hao, 2002), foreign exchange rates (Spagnolo et al., 2005; Nikolsko-Rzhevskyy and Prodan, 2012), commodity price (Alizaded et al., 2008; Tien-Yu and Shwu-Jane, 2009), among others. In addition to applying Markov switching for the return equation directly as described above, which is the most common application, some studies also apply this technique for the conditional volatility model to introduce MS-ARCH or MS-GARCH. For example, see Hamilton and Susmel (1994), Elliott et al*.* (2012), among others. Another development of Markov switching analysis is the use of time varying transition probability instead of static probability to incorporate the fact that the probability of any state to happen could be influenced by some variables. For example, the components of the transition probability matrix could be modelled as

$$Pr\{S_t = k | \mathcal{F}_{t-1}\} = \Phi\left(\sum_{i=1}^{n} \gamma_{i,k} X_{i,t-1}\right) \quad (2.8)$$

where $\Phi(\cdot)$ is the Normal CDF to ensure the probability is within $(0,1)$ range. Gray (1996) uses this version of Markov switching analysis to model the US short-term interest rate and shows that it is better than existing short-term rate models.

Regarding out-of-sample analysis, evidence about the performance of Markov switching models is mixed. For example, Engel and Hamilton (1990) demonstrate the ability of Markov switching model to beat the performance of the random walk process in exchange rate forecasting. On the other hand, Clements and Krolzig (1998) show that a simple linear model could perform well in comparison with a Markov switching model, even when the data is non-linearly generated.

2.5. Value-at-Risk

Value-at-Risk (hereafter VaR) is, arguably, one of the most common risk measures used by practitioners in the modern financial market. If the classical volatility measure of standard deviation has a neutral view of risk by equally treating upward and downward deviation, VaR accounts for risk from the perspective of a risk-averse agent. By definition, VaR is the maximum loss that one could experience given a specific level of confidence. Accordingly, in addition to the risk-averse perspective, VaR inherently provides information regarding stressed scenarios. As a result, it is the most important risk measure in the banking system. It has been chosen by Basel Accords to be the standard method for measuring market risk and calculating Market Risk Charge to be used in determining the regulatory capital of a bank. Moreover, as VaR is commonly measured in terms of the value of loss, it could be used universally across all types of assets (activities) within a portfolio (business). Under some standard assumptions, these individual risks could be aggregated to form the total risk measure for the portfolio (business). Thus, VaR provides an effective tool for the risk manager to monitor, assign, and control risk behaviour across all departments under his supervision.

Mathematically, VaR is the quantile of distribution of the Profit-and-Loss (P&L) amount of an investment (business). For example, by definition, a $100\alpha\%$ VaR means at $100(1-\alpha)\%$ level of confidence, one could assure that his P&L level will not be worse than the VaR level. Thus,

$$Pr\{P\&L \geq VaR\} = 100(1-\alpha) \tag{2.9}$$

or

$$Pr\{P\&L < VaR\} = 100\alpha \tag{2.10}$$

In other words, $100\alpha\%$ VaR is exactly the $100\alpha\%$ quantile of the P&L distribution. This straightforward mathematical meaning creates a very convenient set-up for VaR estimation models. Accordingly, different methods to estimate VaR are mainly distinguished in the way P&L distribution is modelled. We summarise the most commonly used VaR models in the literature as follows. A point that should be made clear before we introduce these models is that they are VaR models for returns rather than for the dollar value of P&L. In fact, defining VaR using returns rather than the dollar value is a common practice in the literature, which is different from industrial practice. However, as return is just P&L scaled by the initial value of the investment, the two practices are essentially the same.

The most simple and straightforward VaR estimation model is Historical Simulation, where one utilises the empirical distribution of returns collected within a sample period. If we denote $Q = \{r_t\}$, $t = 1,2, \ldots, N,$ the collection of all (N) returns of the subject investment within a period, then $100\alpha\%$ VaR is the $(100\alpha/N)^{th}$ (rounded up to the nearest integer) worst return of Q. The underlying assumption of this model is that r_t is independent and identically distributed (*iid*), so that all observations are equally treated and the future distribution is similar to the past. This naïve assumption is clearly not satisfied in reality. Thus, although simple, Historical Simulation is not robust in practice. However, a modification of this method could significantly enhance its performance. Some researchers have demonstrated that if the return process is filtered using a location-scale model such as ARMA-GARCH to obtain *iid* observations, then the Historical Simulation will perform reasonably well. For example, see Kuester et al*.* (2006), Barone-Adesi et al*.* (2002), among others. In backtesting alternative VaR models, Kuester et al*.* (2006) even show evidence that this modified Historical Simulation is the most optimal one. Later in this section, we will further discuss the location-scale filtering models commonly used in VaR estimation methods.

Another popular estimation method is the parametric VaR. As its name suggests, the distribution from which VaR is calculated is estimated by fitting some parametric distribution on the observed return data. Different assumptions about the fitted distribution result in different parametric VaR models, including Gaussian VaR, Student-t VaR, Skewed student-t VaR, Extreme Value Theory VaR, among others. Similar to the case of pure Historical Simulation, one could estimate the distribution parameters directly from observed returns, assuming they are independently and identically distributed. Otherwise, it is possible to location-scale filter the returns and estimate the parameters of the distribution using the standardised residuals. The latter should always outperform the former since the actual return process is non-*iid*. Kuester et al*.* (2006) provide an intensive comparison between these parametric models, with or without filtering. The results reported in this paper clearly reject the use of un-filtered models in favour of the filtered ones. Among popular distributions, Extreme Value Theory VaR and Skewed Student-t VaR tend to produce the best performance, which is not surprising since they account for both skewness and tail fatness features of return distribution. On the other hand, McNeil and Frey (2000) show some evidence that justifies the use of Normal distribution when the level of significance is larger than 5 percent.

In a completely different approach, VaR could also be estimated directly using quantile regression rather than indirectly through fitting distribution. In quantile estimation, a specific quantile of a distribution is estimated by explanatory variables. This is somewhat similar to the way the standard regression model is used to estimate the expected value of dependent variables using exogenous variables. Intuitively, considering the case of linear regression, OLS estimates a line passing through the means of the dependent variables. On the other hand, α quantile regression estimates a line passing through the α quantiles of the dependent variable when the explanatory variables move. The way to estimate coefficients in quantile regression is, therefore, different from OLS. Instead of minimising the sum of squared error, quantile regression minimises a function that assigns different weights for positive and negative errors. Positive errors are weighted by the quantile level α, while negative errors are weighted by 1 – α. Specifically, considering a quantile regression model:

$$\widehat{Q_\alpha}(y|X) = X\widehat{\beta_\alpha} \qquad (2.11)$$

where $\widehat{Q_\alpha}$ is the estimated quantile of y corresponding to the level of significance α; y is $n \times 1$ vector of the dependent variables; X are $n \times k$ matrix of k independent variables (which may include an intercept); $\widehat{\beta_\alpha}$ is $k \times 1$ vector of estimated quantile regression. Then, estimated coefficient $\widehat{\beta_\alpha}$ will be the one that minimises the loss function:

$$\left(\alpha \times \iota - 1_{y \le X\widehat{\beta_\alpha}}\right) \times \left(y - X\widehat{\beta_\alpha}\right) \qquad (2.12)$$

where ι is $1 \times n$ vector of 1; $1_{y \le X\widehat{\beta_\alpha}}$ is $1 \times n$ vector with i^{th} component takes the value of 1 if $y_i \le X_i\widehat{\beta_\alpha}$ and 0 otherwise (X_i is i^{th} row of X). A detailed review about quantile regression can be found in Koenker (2005).

The focus of quantile regression perfectly matches with VaR, which is essentially the quantile of a distribution. Accordingly, one could use quantile regression to directly estimate VaR at any point of time using some explanatory variables, as in Bao et al. (2006), Taylor (2008), Adrian and Brunnermeier (2011), among others. Engle and Manganelli (2004) further incorporate other components into the regression model such as autocorrelation or asymmetric response to lagged returns to improve VaR prediction and demonstrate the success of these advanced models.

Regarding the location-scale models to filter the non-*iid* return process, a combination of an ARMA model to filter autocorrelation and a conditional volatility model to filter heteroskedasticity is the most commonly used approach in the literature. For example, considering the simple symmetric Bollerslev (1986) GARCH model for volatility function, an ARMA(p,q)-GARCH(r,s) model for returns R_t could be specified as

$$R_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t z_t \qquad (2.13)$$

$$\mu_t = a_0 + \sum_{i=1}^{p} a_i R_{t-i} + \sum_{j=1}^{q} b_j \varepsilon_{t-j} \qquad (2.14)$$

$$\sigma_t^2 = c_0 + \sum_{i=1}^{r} c_i \sigma_{t-i}^2 + \sum_{j=1}^{s} d_j \varepsilon_{t-j}^2 \qquad (2.15)$$

where $z_t = \varepsilon_t / \sigma_t$ is the standardised residual which is expected to be *iid* since both autocorrelation and heteroskedasticity are eliminated. As a result, one could fit any of the mentioned statistical distributions on z_t and calculate the α-quantile correspondingly. The VaR of returns at any point in time is then estimated by location-scale updating this quantile of the standardised residual. Specifically,

$$\widehat{VaR_t} = \widehat{\mu_t} + \widehat{\sigma_t}\hat{q} \qquad (2.16)$$

where $\hat{q}$ is the estimated quantile of the standardised residual, which is assumed to be constant over time as $z_t$is *iid*; $\widehat{\mu_t}$ and $\widehat{\sigma_t}$ are the estimated mean and standard deviation of the return process at time t, obtained from equations 2.14 and 2.15 above. Since the distribution from which the quantile is obtained is estimated from an *iid* process and then is location-scale updated using the most current information, VaR predicted by this approach delivers quite a robust performance. In fact, this approach is used extensively in the literature to estimate VaR, such as Berkowitz and O'Brien (2002), Brooks et al. (2005), Kuester et al. (2006), among others.

Additionally, researchers could also use a more advanced location-scale filter in the VaR model. For example, one could use an ARMAX model to incorporate the predictability of other exogeneous variables in determining the mean, or an asymmetric GARCH model in determining the conditional volatility. In fact, asymmetric GARCH takes into account the phenomena that a negative shock tends to have a different impact on volatility than a positive shock of the same magnitude. Alexander (2008) argues that “an asymmetric GARCH model is almost always the better fit to daily data, and for equities, equity indices and commodities at any frequency”. Accordingly, versions of asymmetric GARCH such as AGARCH (Engle, 1990; Engle and Ng, 1993), GJR GARCH (Glosten et al., 1993), and EGARCH (Nelson, 1991), among others, could be used to model the ‘scale’ component of the filter. For example, the GJR GARCH proposed by Glosten et al. (1993) is specified as:

$$\sigma_t^2 = c_0 + \sum_{i=1}^{r} c_i \sigma_{t-i}^2 + \sum_{j=1}^{s} d_j \varepsilon_{t-j}^2 + \sum_{j=1}^{s} \gamma_j I\left[\varepsilon_{t-j} < 0\right] \varepsilon_{t-j}^2 \quad (2.17)$$

where $I(\cdot)$ is the indicator function that takes the value of 1 when $\varepsilon_{t-j} < 0$ and 0 otherwise. Thus, the γ coefficients are to account for the asymmetry of the volatility response to shocks. For stock returns, it is expected to be greater than 0 since negative shocks should increase volatility more compared to positive shocks. A detailed introduction about other asymmetric GARCH models can be found in Alexander (2008).

The quality of any VaR estimation model is normally tested by examining the series of violations of returns R_t with respect to the estimated VaR. In other words, we would like to test whether the realised probability of violation $(R_t \leq VaR_t)$ is 100α percent or not. Denoting a hit $H_t = I(R_t \leq VaR_t)$ which takes the value of 1 when the condition in brackets is satisfied and 0 otherwise, a test of a VaR model will examine whether

$$E(H_t|\mathcal{F}_{t-1}) = \alpha \tag{2.18}$$

where $\mathcal{F}_{t-1}$ is the set of information available up to time $t-1$. Moreover, with the rule of iterated expectation, $E(H_t) = E[E(H_t|\mathcal{F}_{t-1})] = E(\alpha) = \alpha$, this implies that a good model should also produce an *iid* process of violation with expected value equals to α. These are summarised in the 'correct conditional coverage' condition (see Kuester et al., 2006):

$$H_t|\mathcal{F}_{t-1} \sim iid\ Bernoulli(\alpha) \tag{2.19}$$

Accordingly, different aspects of this condition could be tested, namely using the 'unconditional coverage test' to examine whether $E(H_t) = \alpha$, the 'independence test' to examine whether H_t is independent over time, or the 'conditional coverage test' to examine the whole 'correct conditional coverage' condition. Further details about these tests can be found in Christoffersen (1998). Additionally, regression-based tests as in Engle and Manganelli (2004) are also popular. In these tests, H_t is regressed against some explanatory variables including, among others, its lags and the corresponding estimated VaRs. The values and significance of estimated coefficients provide information about the expected value of H_t as well as whether it is independent over time (explanatory variables have no predictability).

Expected Tail Loss (Conditional Value-at-Risk)

Despite a number of advantages, VaR is still subjected to some limitations, leading to the use of Expected Tail Loss (ELT) which is becoming popular. For example, VaR only gives information about the 'threshold loss', which is the loss that we expect an investment P&L will not exceed given a level of confidence. Thus, VaR is quiet about the severity of the loss when it exceeds VaR. In fact, information about the severity of the distress scenario is arguably even more important than the threshold information since in many cases jumps happen rarely but are extreme when they actually occur. Moreover, distress risk management is more about how to deal with distress situations when they occur rather than simply specifying how frequently they may happen. Given this problem, Expected Tail Loss is introduced to provide information about the severity of large losses when they exceed the VaR level, which explains why ETL is also referred to as Conditional VaR. Specifically, it is the expected value of the loss when it exceeds the VaR level. Mathematically, ETL is expressed as:

$$ETL_{\alpha} = E[R|R \leq VaR_{\alpha}] = \frac{1}{\alpha}\int_{0}^{\alpha} VaR_{\theta}d\theta = \frac{1}{\alpha}\int_{R\leq VaR_{\alpha}} p(R)RdR \qquad (2.20)$$

where R is the returns being examined; VaR_{θ} is VaR at θ level of significance; α is the significance level corresponding to ETL; and $p(\cdot)$ is the probability of a level R to happen.

Furthermore, from the perspective of portfolio management, ETL is even better than VaR given the fact that ETL is sub-additive while VaR is not. In other words, the ETL of a portfolio is smaller than the sum of the ETL of its components, while this is not always true for VaR. More generally, Artzner et al*.* (1999) show that ETL is a ‘coherent’ risk measure, satisfying all four desired criteria of a risk metric, namely monotonic, homogeneous, risk-free condition, and sub-additive. As a result, Rockafellar and Uryasev (2000) develop an effective mean-ETL portfolio optimisation procedure while VaR ‘ill-behaves’ in the portfolio optimisation context. Thus, not surprisingly, in the newest version of the Basel Accord (Basel III Accord), banking regulators started to recommend the use of ETL to replace VaR in calculating losses (see Basel Committee, 2012).

CHAPTER 3: THE IMPACT OF EXTREME DOWNSIDE RISK ON STOCK RETURNS: MARKET LEVEL ANALYSIS

3.1. Introduction

The notion of tail risk, or extreme downside risk, has become increasingly prominent in the asset pricing literature. In particular, in contrast with the assumptions of the standard CAPM of Sharpe (1964) and Lintner (1965), in which portfolio risk is fully captured by the variance of the portfolio return distribution, asset returns display significant negative skewness and excess kurtosis, both of which increase the likelihood of extreme negative returns. A number of studies have examined the importance of these higher moments for asset pricing. Kraus and Litzenberger (1976) develop a three-moment CAPM, in which returns are determined, in part, by co-skewness with the market portfolio. This finding is supported by Harvey and Siddique (2000), who consider the role of co-skewness in a conditional asset pricing framework. Dittmar (2002) shows that co-kurtosis is also priced. Using moments of the return distribution implied by option prices, Conrad et al. (2013) show that individual securities' risk-neutral skewness and kurtosis are strongly related to future returns. Ang et al. (2006a) find that comoment risks are still significant even after general downside risk is taken into account through downside beta.

Other studies focus directly on the likelihood of extreme returns, rather than indirectly on the moments of the return distribution. For example, using a copula-based approach, Ruenzi and Weigert (2013) show that stocks with high crash sensitivity, measured by lower tail dependence with the market, are associated with higher returns that cannot be explained by traditional risk factors, downside beta, coskewness or co-kurtosis. Relatedly, Huang et al. (2012) propose a measure of extreme downside risk based on the tail index of the generalised extreme value distribution, and show that it is associated with a premium in cross-section stock returns, even after controlling for market, size, value, momentum, and liquidity effects.

The studies described above examine the variation in expected returns across individual stocks. An alternative strand of the literature is concerned with the variation in tail risk over time, and its impact on aggregate equity returns. This is a more challenging objective because commonly used measures that are related to tail risk are unable to explain contemporaneous market returns in the standard, positive risk-return relation postulated by asset pricing theory. For example, since investors prefer skewness, an investment with higher skewness should correspond to lower expected returns. However, skewness is, by construction, associated with large positive returns, and so there will be a tendency for skewness to be positively related to returns. Additionally, owing to leverage and volatility feedback effects, high volatility tends to be associated with lower contemporaneous returns (see, for example, Black, 1976; Campbell and Hentschel, 1992). As a result, market tail risk measures such as VaR and ETL, which are positive functions of return volatility, will tend to have a negative relation to returns. Thus, while there are a number of studies that consider the cross-sectional relation between tail risk and returns for individual stocks, there is little evidence concerning tail risk at the aggregate level. Recognising this difficulty, Kelly and Jiang (2014) develop a measure of aggregate market tail risk that is based on the common component of the tail risk of individual stocks. They show that their tail risk measure is highly correlated with the tail risk implied by equity options, and has significant predictive power for aggregate market returns. A similar approach is proposed in Allen et al. (2012) where market tail risk measure is obtained as the VaR of the crossectional distribution of pooled stock returns, which is then demonstrated to significantly predict economic downturns.

Bali et al. (2009) (hereafter BDL) directly investigate the intertemporal relation between market returns and tail risk. In order to circumvent the inherent endogeneity of tail risk described above, they measure tail risk by the previous month’s one-month ahead expectation of the VaR of the market return. Using monthly data over the period July 1962 to December 2005, they show that there is a statistically and economically significant positive relation between market returns and tail risk. Moreover, the relationship between returns and tail risk is stronger than between returns and conditional volatility, and it is robust to different VaR measurement methods, different VaR confidence levels, alternative measures of tail risk, different measures of the market return and the inclusion of macroeconomic variables to control for business cycle effects.

In this chapter, we investigate the nature of the relation between returns and tail risk under different market conditions. This is motivated by empirical evidence that other, closely related risks, such as coskewness risk, affect returns differently in different states of the world (see, for example, Friend and Westerfield, 1980; Guidolin and Timmermann, 2008). In order to model the state-dependent relation between tail risk and return, we incorporate the BDL model into a two-state Markov switching framework. We estimate the Markov switching model using an extended sample that covers the period July 1962 to June 2013, which includes the recent financial crisis. The two states in the estimated Markov switching model are characterised by a relatively infrequent high volatility state and a relatively frequent low volatility state.

Surprisingly, we find that the positive risk-return relation documented by BDL holds in the low volatility state, but disappears in the high volatility state. To shed light on the reason for this finding, we estimate the BDL model using two sub-samples and show that, while the risk-return relation is significantly positive during the 1962-2005 period considered by BDL, it tends to be negative during the 2006-2013 period that includes the recent financial crisis. The failure of the BDL model to capture the risk-return relationship during crisis periods is counter-intuitive since tail risk is expected to be more relevant during these times. In order to rule out omitted variable bias, we expand the set of state variables that are included in the original BDL model to control for business cycle effects. This yields a stronger and more significant positive risk-return relation in the original BDL sample, but also a more negative risk-return relation in the 2006-2013 sample. We also consider the possibility that the results are driven by the non-*iid* nature of the return generating process, and compute tail risk measures using returns that are standardised by time-varying conditional volatility. This yields a positive risk-return relation in both sub-samples, but it is not statistically significant in the 2006-2013 period.

The BDL model critically depends on the assumption that leverage and volatility feedback effects dissipate within one month, so that the one-month ahead expectation of VaR, lagged by one month, can be considered pre-determined. We show, however, that leverage and volatility feedback effects take longer to dissipate during periods of high volatility, and so the one-month ahead expectation of VaR is endogenous, even when lagged by one month. In order to circumvent the endogeneity of the tail risk measures of the BDL model in the high volatility state, we consider longer horizon expectations of market VaR at correspondingly longer lags. We show that by using the two-month ahead expectation of VaR, lagged by two months, there is a statistically significant and positive relation between market returns and tail risk in both states.

We carry out different robustness checks for the validity of our analysis and inferences above, using alternative non-iid return models, different significance levels in VaR measures, and ETL measures. The uses of these alternative choices do not induce any changes in any of our conclusions. We further examine how the inferences could change if the left tail risk measures are replaced by the corresponding right tail ones and find that right tail measures perform exactly the same as their left tail counterparts. This finding suggests that the risk analysed in the BDL framework (and our modified versions) is essentially extreme movement risk rather than simply extreme downside risk.

We make the following contributions to the literature. First, we explore different alternatives to improve the performance of the BDL framework in explaining the positive tail risk-return relationship. Secondly, we show that the cause of the inconsistent performance of the BDL framework during turbulent times is due to the leverage and volatility feedback effects. Thirdly, we propose a new risk measure to address this inconsistency and examine its effectiveness with an extensive array of robustness studies. Finally, we illustrate the underlying mechanism of how the tail risk and return relationship materialises in the market. Specifically, we show that during calm periods, the high risk-high expected return relationship at the market level materialises after just one month. However, in turbulent periods, the same relationship needs two or more months to materialise.

The remainder of the chapter is organised as follows. Section 3.2 describes the methodology and the data used in the original BDL framework as well as in our expanded framework. Section 3.3 provides our empirical results, including the conflicting evidence of the BDL framework during turbulent times and in several modifications of their framework that aim to address this problem. Section 3.4

examines the robustness of our findings. Section 3.5 summarises the chapter and offers some concluding remarks.

3.2. Methodology and data

3.2.1. Methodology

3.2.1.1. Extreme downside risk-return relationship in the BDL framework

In order to examine the dynamics of the extreme downside risk-return relationship over different states of market volatility, we utilise the BDL framework summarised briefly in this section (see Bali et al., 2009 for further details). Bali et al. (2009) capture the impact of extreme downside risk on returns by regressing the next month's market excess return R_{t+1} on the next month's expected extreme downside risk measure $E_t(VaR_{t+1})$ and other control variables X_t:

$$R_{t+1} = \alpha + \beta E_t(VaR_{t+1}) + \gamma X_t + \varepsilon_{t+1} \qquad (3.1)$$

The control variables, X_t, include a range of macroeconomic variables to proxy for business cycle fluctuations, the lagged excess market return, and a dummy variable for the October 1987 crash. The risk-return relationship is reflected in the sign and the significance of the coefficient β. BDL measure VaR both parametrically and non-parametrically, using the most recent one to six months of daily market returns. Parametric VaR is obtained by fitting the Skewed Student-t distribution of Hansen (1994) to market returns over the last one month, the last two months, and so on, and calculating the corresponding quantile in each case. Non-parametric VaR is measured as the quantile of the empirical distribution of the daily market return over the past one to six months. In particular, BDL use the lowest return over the last one month (which corresponds to a VaR significance level of 4.76%, assuming that there are 21 trading days each month), over the last two months (which corresponds to a VaR significance level of 2.38%), and so on up to six months.

BDL estimate the conditional expectation of VaR in two approaches. In the first approach, they assume that $E_t(VaR_{t+1}) = VaR_t$, which would be equal to the true conditional expectation only if VaR follows a random walk. In the second approach they assume that VaR is mean reverting and estimate an AR(4) model:

$$VaR_t = \theta_0 + \sum_{i=1}^{4} \theta_i VaR_{t-i} + v_t \tag{3.2}$$

The conditional expectation of VaR is then given by $E_t(VaR_{t+1}) = \hat{\theta}_0 + \sum_{i=0}^{4} \hat{\theta}_i VaR_{t-i}$. We refer to these two measures as raw VaR and AR4 VaR, respectively. BDL estimate the regression given by (3.2) using monthly data over the period July 1962 to December 2005, and show that there is a statistically and economically significant positive relation between market returns and tail risk. Moreover, the relationship between returns and tail risk is stronger than between returns and conditional volatility, and is robust to the different VaR measurement frameworks, different VaR confidence levels, alternative measures of tail risk, different measures of the market return, and the inclusion of macroeconomic control variables.

An important aspect of the BDL approach is that they use the conditional expectation of the risk measure rather than its realisation in order to offset the leverage and volatility feedback effects in returns. The use of the one-month ahead expectation, lagged by one month, implicitly assumes that these leverage and volatility feedback effects are short lived, lasting only a single month. This subtle but important observation is the basis of our modification of the BDL framework, as detailed in Section 3.3.2.

3.2.1.2. Extreme downside risk in different market states: Markov switching mechanism

In order to examine the state-dependent dynamics of the tail risk-return relationship, we introduce a Markov switching mechanism into the BDL framework. The Markov switching mechanism has been applied in many contexts to capture significant changes in variables' behaviour under different states of some underlying (economic) variables (see Hamilton, 1989; Hamilton, 1990; Gray, 1996; Nikolsko-Rzhevskyy and Prodan, 2012; among others). Indeed, many studies employ a Markov switching framework to examine the time-varying impact of volatility risk, which is closely related to the extreme downside risk, on stock returns. For example, Turner et al. (1989) employ

Markov switching models to examine how the expectation of market volatility affects market excess returns in different market contexts and at varying levels of investors' awareness. Similarly, Chang-Jin et al. (2004) use Markov switching to directly model volatility feedback effects on returns. Thus, it is natural to extend the BDL framework with a Markov switching mechanism to examine the dynamics of the extreme downside risk-return relationship in different states of market volatility. Additionally, since the Markov switching mechanism takes into account the conditional distribution of the latent state variable, our analysis could be carried out without the need to arbitrarily specify the timing of turbulent periods.

Given a large number of control variables, to incorporate Markov switching into the BDL framework, we choose the simplest setting with a first-order Markov process and two regimes. This is one of the most widely used settings in studies employing Markov switching models (see, for example, Bansal and Hao, 2002; Guidolin and Timmermann, 2006). Thus, our Markov switching-BDL regression model (hereafter MS-BDL) is given by:

$$R_{t+1} = \alpha_{S_{t+1}} + \beta_{S_{t+1}} E_t(VaR_{t+1}) + \gamma_{S_{t+1}} X_t + \varepsilon_{S_{t+1}t+1} \qquad (3.3)$$

where $\varepsilon_{S_t t} \sim N(0, \sigma^2_{S_t})$ and $S_t = \begin{cases} 1, if\ state\ 1\ occurs\ at\ time\ t \\ 2, if\ state\ 2\ occurs\ at\ time\ t \end{cases}$.

The statistical properties of the expected tail risk measure coefficient β in the two states provide insights into the extreme downside risk-return relationship during different states of the market volatility. The low volatility $\sigma_{S_t} = \sigma_1$ and the high volatility $\sigma_{S_t} = \sigma_2$ represent calm and turbulent states, respectively. Further, since the Markov switching mechanism takes into account the different regimes of the market, the October 1987 dummy variable can be eliminated from X_t.

3.2.2. Data

Following BDL, we use the value weighted Center for Research in Security Price (CRSP) index which includes all stocks in the major stock exchanges in the US (specifically, NYSE, AMEX, NASDAQ and ARCA markets) to represent the return of the market. The excess market return is computed as the difference between the market return and the one-month T-bill rate obtained from Kenneth R. French's website. Our sample period is July 1962 to June 2013, which covers the original period of July 1962 to December 2005 studied by BDL, as well as the more recent period that includes the financial crisis of 2007-08. In Table 3.1, we provide summary statistics (Panel A) and correlations (Panel B) for monthly excess returns and a range of realised risk measures, computed using daily returns, over the full sample. The risk measures are standard deviation, mean absolute deviation, skewness, kurtosis and maximum loss (which is the non-parametric estimate of VaR used by BDL). In Panel C, we report the estimated coefficients and corresponding t-statistics for the AR(4) models of these risk measures.

[Table 3.1]

From Panel B of Table 3.1, it is clear that none of the commonly used realised tail risk measures can explain returns in a way that could be considered consistent with asset pricing theory. In particular, skewness is positively related to returns, while the other measures are negatively related to returns. The incorrect signs of the coefficients are not surprising. Skewness is, by construction, associated with large positive returns, while the other risk measures are closely related to volatility, which is significantly negatively correlated with concurrent returns due to leverage and volatility feedback effects. It is these observations that motivate the use of expected risk measures, rather than realised risk measures, in the BDL framework.

Regarding the macroeconomic variables in the regression framework, we employ an extended set of explanatory variables relative to the one used in the original BDL framework. Our set includes the BDL variables, which are the de-trended risk free rate, change in the term structure risk premium, changes in the credit risk premium, and dividend yield. We denote them as RFD, DTRP, DCRP, and DY, respectively. We construct these variables using exactly the same methods and data sources as in the BDL paper. Precisely, the risk free rate is the 1 month Treasury Bill obtained from Kenneth R. French's online database. The term structure risk premium is calculated as the difference between the yield of 10 year Treasury Note (obtained from the database of the

Board of Governors of the Federal Reserve System) and the 1 month Treasury Bill. The credit risk premium is measured as the yield of MOODY's BAA corporate bond minus that of MOODY's AAA corporate bond (also obtained from the Board of Governors of Federal Reserve System database). The dividend yield is constructed by the method of Fama and French (1988), using the distribution-included value-weighted CRSP index return and the distribution-excluded one (which is available from the CRSP database).

Additionally, we also consider some additional macroeconomic variables that have been shown in the literature to be important determinants of aggregate equity returns, namely Industrial Production, Monetary Base, Inflation, and Oil price (see, for example, Chen et al., 1986; Kaul, 1990; Anoruo, 2011; Aburachis and Taylor, 2012). Specifically, we use the monthly series of the yearly growth rate of industrial production constructed using the same method as Chen et al. (1986), the monthly growth rate of M2 constructed as logarithmic changes in M2, the monthly change in inflation, and the monthly change in oil price. The industrial production and monetary supply data are obtained from the Board of Governors of the Federal Reserve System database, while the inflation rate and oil price (WPU0561 series) are obtained from the Bureau of Labour Statistics database. It is widely accepted that these variables influence stock returns and their absence in the BDL explanatory variable set could be argued to bias the regression results. Further, the state-dependent investigation of relationships without incorporating important state variables is essentially inconsistent.

3.3. Empirical results

3.3.1. Extreme downside risk in different states of the market

We first examine the tail risk-return relationship in different states of the market using the MS-BDL model given by function (3.3). Table 3.2 presents the estimated coefficients and t-statistics for each of the states, the variance in each state and the duration of each state, using the estimates of VaR employed by BDL: raw non-parametric VaR, raw Skewed Student-t VaR, AR4 non-parametric VaR and AR4 Skewed Student-t VaR. We also carry out similar investigations and report the results for parametric VaR estimated using Gaussian and Student-t distribution since these two distributions are among the most popular ones in the literature. All measures are estimated using daily returns over the previous one month (21 trading days). As BDL use estimation samples of different lengths up to six months, we also estimate the model using two to six month sample VaR. This yields similar results to those reported here. Due to limited space, we only report the results using two month measures in the Appendix section of this chapter. The results for longer estimation samples will be available upon request.

It is clear that we can identify two distinct states of the market: a relatively frequent calm state of low volatility and a relatively infrequent turbulent state of high volatility. The volatility in the turbulent state is about two to three times the level in the calm state. The expected duration of the calm state is double that of the turbulent state. Figure 3.1 plots the market realised daily volatility of every month over the sample period (Panel A), the smoothed probabilities from the Markov switching model (Panel B) and the state transitions (Panel C). This illustrated Markov switching result is obtained from the AR4 Skewed Student-t model and the patterns for other models are very similar. From these figures we could clearly see that the turbulent state covers a number of periods of market distress, including the 1973-1974 oil crisis, the October 1987 crash, the burst of the dot-com bubble in the early 2000s and the recent financial crisis.

In the low volatility state of all models, the coefficient on tail risk is positive and highly significant. Thus it appears that in relatively calm states of the market, there is a strong relationship between returns and tail risk, as implied by asset pricing theory. However, in contrast, in the high volatility state, the coefficient on VaR is significantly negative for all VaR measures. In other words, in turbulent states of the market, it appears that an increase in tail risk leads to lower returns in expectation.

[Table 3.2]

[Figure 3.1]

In order to shed further light on this finding, we estimate the original BDL model (without Markov switching) using three samples: the original sample used by

BDL (July 1962 to December 2005), the new sample (January 2006 to June 2013) and the full sample (July 1962 to June 2013). Similar to the previous investigation, we report the results for one-month measures in Table 3.3, while similar results for two-month measures are available in the Appendix. With the original BDL sample (Panel A), we obtain results that are very close to those reported by BDL. For measures used in the original BDL studies, namely non-parametric VaR and Skewed Student-t VaR, the estimated coefficient on the tail risk measure is significantly positive, suggesting that high tail risk is associated with high returns. However, for Gaussian VaR and student-t VaR, the coefficient is not significant in this period. We will show later that this is purely due to variable omission bias, which could be fixed by using a more complete set of state variables.

The more problematic result is in the new sample (Panel B), where the coefficient on tail risk is, in all four cases, insignificant or even negative, suggesting a breakdown in the tail risk-return relation identified by BDL. As a result, using the full sample (Panel C), the coefficient on tail risk is not significant using any of the four measures. These sub-sample results suggest that the absence of a significant tail risk-return relation in the high volatility state of the Markov switching model may be attributable to a failure of the BDL model during the recent financial crisis. This is a surprising finding since it is during episodes such as this that tail risk could reasonably be expected to be more relevant.

[Table 3.3]

One possible explanation for the failure of the tail risk-return relation to hold in all volatility states is that it reflects a bias arising from the omission of state variables that are correlated with the tail risk measure. BDL include four control variables (the de-trended risk free rate, the change in the term structure risk premium, the change in the credit risk premium and the dividend yield), but it could be argued that these are insufficient to capture the full dynamics of the economic cycle during crisis periods. This is perhaps suggested by the fact that the BDL control variables, while significant in the original sample, are insignificant in the new sample. We therefore expand the set of state variables used by BDL to include four additional macro-variables that are commonly used in the asset pricing literature: growth in industrial production, growth in the monetary base, the change in inflation and the change in oil price.

The estimation results including the expanded set of variables are reported in Tables 3.4 and 3.5 for the MS-BDL and BDL regressions, respectively. In Table 3.4 it is clear that the distress period inconsistency still persists, where the coefficient of the tail risk measure is negative in most of the models. However, we observe decreases in the state variances as compared to MS-BDL models using original BDL variables, especially in the second state. This suggests the ability of our new added state variables in improving the goodness of fit of the model. In fact, we observe the general improvement of log likelihood function and AIC statistics in the Markov switching estimation using the extended variable set compared to those of the original set. In Table 3.5, the additional state variables clearly improve the overall fit of the BDL model, both in the original sample and the new sample. In particular, the R-squared coefficient increases very substantially (five times that of models using the original variable set), and in the new sample, the model explains as much as 30 percent of the variation in returns. The inclusion of the additional state variables serves to increase the magnitude and significance of the coefficient on VaR in the original sample and, consequently, it is now positive and significant in the full sample. However, in the new sample, it remains insignificant or negative. It should be noted that Gaussian and Student-t measures now have a similar performance to non-parametric and Skewed Student-t measures, demonstrating our explanation for their poor performance in the original sample period in the previous examination. Thus, the results of both BDL and MS-BDL regressions support the use of the extended variable set in any BDL-framework-related investigation to yield more precise, unbiased results. Accordingly, we apply this variable set consistently throughout all the other investigations of the chapter.

[Table 3.4]

[Table 3.5]

A second possible explanation for the failure of the risk-return relation to hold in all states is that the estimators of tail risk employed by BDL are based on the unconditional distribution of returns, and therefore implicitly assume that returns are *iid*. Ignoring the true dependence structure in returns, such as autocorrelation and volatility clustering, is likely to reduce the power of the regression based tests used to identify the risk-return relation. We therefore relax the *iid* assumption and estimate tail risk using a location-scale VaR model, in which VaR is estimated using the standardised residuals of an AR(1)-GARCH(1,1) model for daily market returns (see, for example, Berkowitz and O'Brien, 2002; Kuester et al., 2006). Specifically, to estimate market VaR for day d, we first estimate the location-scale model using information up to day $d-1$ as:

$$r_d = \mu_d + \varepsilon_d = \mu_d + \sigma_d z_d \tag{3.4}$$

$$\mu_d = a_0 + a_1 r_{d-1} \tag{3.5}$$

$$\sigma_d^2 = c_0 + c_1 \sigma_{d-1}^2 + c_2 \varepsilon_{d-1}^2 \tag{3.6}$$

We then estimate the quantile of the standardised residual $z_d = \varepsilon_d / \sigma_d$, which is then transformed into an estimate of VaR using the one-step ahead forecast of the mean and volatility of returns for day d.[1] After obtaining non-*iid*-return-robust VaR estimates for every trading day, we take the average of these daily VaRs within a period (one month to six months) to be the raw risk measures (hereafter raw NIID measures). This corresponds to the one-month to six-month raw VaRs in the original BDL model. We also fit these raw measures into an AR(4) process to estimate the corresponding AR4 NIID measures. In comparison with these NIID measures, we name the raw and AR4 measures constructed assuming *iid* returns in original BDL framework IID measures. We estimate the AR(1)-GARCH(1,1) models using a five-year rolling window (1260 daily observations), and employ the Gaussian, Student-t, and Skewed Student-t distributions for the residuals. Since we must specify a distribution for the error

[1] As a robustness check, we also employ the asymmetric GJR GARCH model of Glosten et al. (1993).

term in the location-scale estimation, we do not have an NIID version of the non-parametric measure.

The estimation results using the NIID VaR measures are reported in Table 3.6. Allowing for the dependence structure of returns in the estimation of VaR generally leads to an improvement in capturing the tail risk-return relationship in the MS-BDL framework. The coefficient of the tail risk measure is now more positive and, in the case of raw NIID Skewed Student-t, it becomes marginally significant. Similarly, in Table 3.7 with the results of BDL regressions, NIID measures also improve performance in all subsamples. In the original sample, the coefficient on VaR is in most cases larger and more significant than when using the IID measures. Moreover, in the new sample, the coefficient is positive, although not significant at conventional levels. Thus, this confirms our conjecture that accounting for non-*iid* features improves the ability to capture tail risk and its relationship with returns. However, the distress period inconsistency still persists and this implies more subtle underlying rationales.

[Table 3.6]

[Table 3.7]

3.3.2. The persistence of leverage and volatility feedback effects in turbulent markets

We conjecture that the poor performance of the BDL framework in the turbulent state is due to the leverage and volatility feedback effects. We argue that the use of the expected risk measures instead of the realised ones in the BDL framework essentially filters out these effects. However, the underlying assumption here is that the leverage and volatility feedback effects last for a month, which is the timescale for returns in BDL. This might be true in calm periods but it is unlikely in turbulent periods. The prolonged calm periods of the US market in the last century explain the good performance of the BDL framework in this sample. However, in turbulent periods, high volatility induces volatility clustering, which in turn causes leverage and volatility feedback effects to impact on several consecutive periods.

To substantiate this claim, in Table 3.8 we regress the product of the market standard deviation of month $t+1$ and month $t+2$, which represents volatility clustering, on the variance of the market return in month t, with and without the control variables used in our framework. The variance (standard deviation) is calculated using the realised variance (standard deviation) formula similar to that of the BDL paper, which will be discussed in a later part of this chapter. In all investigations, the variance coefficient is always positive and highly significant, implying that high volatility induces volatility clustering. In other words, we typically observe turbulent periods that last for several months. Further evidence for this is that in almost all of our MS-BDL investigations, the expected duration of the turbulent state is longer than two months. The only case where it is shorter than two months is in the AR4 NIID Skewed Student-t measure, and its duration is still over 1.9 months. As leverage and volatility feedback effects are correlated with high volatility, this implies that these effects would also last for several months during turbulent times. As a result, the autoregression-based expected risk measures in the BDL framework tend to negatively correlate with returns just like any realised risk measure.

[Table 3.8]

This argument leads us to propose a simple but effective modification of the BDL risk measures to account for the persistence of the leverage and volatility feedback effects. Better filtering out of the leverage and volatility feedback effects will ultimately reconcile the turbulent period inconsistency regarding the extreme risk-return relationship. We suggest calculating expected risk measures using the two-month ahead expectation of VaR, lagged by two months. Specifically, we model the expected tail risk measure as:

$$E_t(VaR_{t+1}) = \theta_0 + \theta_1 E_{t-1}(VaR_t) + \sum_{i=2}^{p} \theta_i VaR_{t-i} \qquad (3.7)$$

$$E_{t-1}(VaR_t) = \theta_0 + \sum_{i=1}^{p} \theta_i VaR_{t-1-i}$$

where θ_i (i = 0,1,…,p) are the estimated coefficients of the AR(p) model of the VaR series. Thus, these measures are similar to the AR(p) expected measures in the BDL framework except that the first observation VaR_t is replaced by its time-$t-1$ expected value. We name them first-observation-skipped-autoregressive (FOSA) measures. To compare with the standard autoregressive expected VaRs in the BDL models, we estimate our new expected VaRs using the AR(4) model.

Table 3.9 shows that using this new type of expected risk measures, we can now capture the significantly positive extreme risk-return relationship in any state of the market. Accordingly, the t-statistics of the tail risk's coefficient is positive even in the turbulent state at very low significance levels (mostly lower than 1 percent). In these Markov switching estimations, a change in the estimated state separation should be noticed. Specifically, the turbulent period becomes more popular and, in some cases, it even has a longer expected duration than the calm period. Indeed, this is not surprising because our use of two period expectations partially reduces the explainability of the risk measure with respect to returns. This causes the residual to be more volatile in some periods and therefore increases the probability of turbulent state in the estimation. However, on average, this loss of information due to taking two period expectation is minor. In fact, we observe an overall improvement in the log likelihood function and the AIC statistics of models using FOSA as compared to those using raw and AR4 measures. Similarly, in BDL regression analysis (Table 3.10), in addition to resurrecting the significantly positive risk-return relationship in the new sample, FOSA measures also slightly improve the R-squares in all subsamples.

[Table 3.9]

[Table 3.10]

In fact, a signal for our strategy to solve the problem through filtering leverage and volatility feedback effects is embedded in the performance of BDL and MS-BDL regressions of original measures. From the results of measures with different estimation sample lengths, we notice a pattern that measures of longer estimation samples tend to perform better than one-month-sample measures, and some are statistically significant in the new sample period or in the turbulent state of Markov switching estimation. These measures could not consistently solve the problem since they still include information of the last month and, therefore, are still influenced by the spread out of the leverage and volatility feedback effects in turbulent times. However, the improvement when using more data from the past illustrates how better filtering out these effects will lead to better results. Additionally, in Tables A3.9 and A3.10 of the Appendix, measures similar to FOSA but skipping more lags also have good performance in many examinations. However, their performances are not consistent since skipping too many lags causes them not quite related to the actual tail risk. Thus, this further evidence all supports our argument regarding the underlying reason for the distress period inconsistency, and the success of our FOSA measure is based on an actual economic pattern rather than the product of "fishing".

Economic interpretation for the new risk measures

The improved performance of the BDL framework using FOSA measures reveals an underlying mechanism of the tail risk-return relationship. Specifically, assets with a highly uncertain future should have high expected returns to compensate investors for the loss that could be inflicted on them. This is exactly what the BDL framework captures, which is a positive relationship between next month's market excess return (R_{t+1}) and the corresponding expected tail risk $E_t(VaR_{t+1})$. However, in turbulent periods, the heightened leverage and volatility feedback effects cause fire sales over multiple periods, distorting or even reversing the positive extreme risk-returns relationship. As a result, it takes more than one period for this relationship to be reinstated. Our analysis suggests that, on average, during turbulent times the risk-return trade-off will be reinstated at period $t+2$. Specifically, we observe a significantly positive relationship between R_{t+2} and $E_t(VaR_{t+2})$, which is the underlying rationale of our FOSA measures. This suggests that continuous fire sales generally do not last longer than two months before the market recovers. Although this might be just a short-term recovery (within a long-lasting bear market), its implication for practitioners is still worthwhile, as it provides them with a useful guideline to time the market.

3.4. Robustness checks

3.4.1. Alternative extreme downside risk measures

3.4.1.1. Asymmetric GARCH models for NIID extreme downside risk measures

As argued in the literature review section, asymmetric GARCH has the ability to capture the different effects of positive and negative shocks on volatility and, thus, is more preferred by researchers in conditional volatility modelling. Therefore, in this section we examine the performance of our framework using the popular asymmetric GJR GARCH model. Corresponding to our standard AR(1)-GARCH(1,1) in the main analysis, we use the asymmetric AR(1)-GJR GARCH(1,1) in our analytic framework.

Tables 3.11 to 3.12 show the results for the MS-BDL analysis and BDL regression sub-sample analysis employing the GJR GARCH measures. Accordingly, the raw and AR4 measures explain returns in the correct manner only in calm periods. On the other hand, the FOSA measures are significantly positively correlated to returns in both states. These results are similar to those of symmetric GARCH measures. Thus, our inferences are robust regardless of whether simple or advanced conditional volatility models are employed.

[Table 3.11]

[Table 3.12]

3.4.1.2. Expected Tail Loss

Artzner et al. (1999) argue that ETL is a coherent measure of risk. Thus, we investigate whether the inferences drawn from ETL measures are similar to those of VaR. Given the similarity of the results of different parametric distributions in the BDL framework, we employ the Gaussian distribution to measure ETL. Formally, under the assumption of Normal distribution for daily market returns $r \sim N(\mu, \sigma^2)$, the ETL at 100α percent level of significance for r is determined as:

$$ETL_\alpha = \frac{1}{\alpha}\varphi\left(\Phi^{-1}(\alpha)\right)\sigma - \mu \qquad (3.8)$$

where $\Phi^{-1}(\alpha)$ and $\varphi(\cdot)$ are the α-quantile and the Standard Normal probability density function. Tables 3.13 to 3.14 present the results of the main models using the ETL risk measures. These tables show that the positive extreme downside risk-return relationship also holds when risk is measured by ETL. However, while all measures perform well in calm periods, only FOSA measures are robust during turbulent times. This result is consistent with Bali et al. (2009), where ETL- and VaR- measures perform similarly.

[Table 3.13]

[Table 3.14]

3.4.1.3. Different significance levels of VaR measures

In addition to the significance level of 1 percent for all parametric VaR calculations, we conduct robustness checks for other significance levels such as 2.5 percent and 5 percent and obtain similar results in all cases. Thus, our inferences are robust with respect to the level of extreme tail risk. As an illustration, Tables 3.15 to 3.16 summarise the main results of the sub-sample analysis and MS-BDL analysis for the one-month-sample Skewed Student-t measures under 5 percent significance levels. The results for the two-month-sample measures are available in the Appendix, while similar results for the longer estimation samples as well as other significance levels are available upon request.

[Table 3.15]

[Table 3.16]

3.4.1.4. Right tail measures: extreme downside risk or extreme movement risk?

In the BDL framework, the focus is on extreme downside risk, which is captured by VaR-based measures. As the two main components of the extreme downside risk are the negative-skewness risk and fat-tail risk, we are interested in examining which one dominates. The general market return is well-known to exhibit slightly negative skewness but considerable excess kurtosis (see, for example, Campbell et al., 1997). Thus, we conjecture that the fat-tail component is the main driver of extreme downside risk, at least under the BDL framework as it is to deal with the risk at general market level. By utilising VaR as the extreme downside risk measure, the BDL framework provides us with a simple way to investigate this question. Specifically, VaR-based measures could be easily modified to capture the right-tail-fatness by specifying VaR significant level to $100\% - \alpha$ instead of α as in our analytic framework so far. For example, 99 percent VaR could be used instead of 1 percent VaR. We could then examine whether these right-tail measures perform similarly to the extreme downside risk measures in the BDL framework. If their performances are very similar, we could infer that fat-tail risk is the main component of the extreme downside risk measures captured in the BDL framework.

Tables 3.17 and 3.18 show the results of this investigation. Specifically, we report the results for parametric Skewed Student-t 99-percent VaR measures. We use the Skewed Student-t measures to emphasise the role of the skewness risk of market returns (if any). It is clear that all the results of the 99 percent measures are very similar to those of the 1 percent VaR measures. Thus, this confirms our conjecture. In other words, the dominating component of the extreme downside risk in the BDL framework is extreme movement risk (or fat-tail risk), which is consistent with the non-normality features in the shape of the distribution of market return.

[Table 3.17]

[Table 3.18]

3.4.2. Accounting for realised variance

A common investigation in any tail risk study is to examine whether tail risk contains additional information in excess of volatility risk. Thus, an important robustness check in the BDL framework is to investigate the performance of VaR measures controlling for volatility. In this section, we show that our FOSA measures are valid in any state of the market even after accounting for volatility. Following BDL's suggestion, we use the French et al. (1987) realised variance definition which accounts for autocorrelation in returns:

$$\tilde{\sigma}_t^2 = \sum_{w=1}^{D} r_w^2 + 2\sum_{w=2}^{D} r_w\, r_{w-1} \qquad (3.9)$$

where $\tilde{\sigma}_t^2$ is the realised variance of month t, D is the number of trading days in the month (assumed to be 21), and r_w is the return of the market return on day w. The results of MS-BDL analysis (Table 3.19) and sub-sample analysis (Table 3.20) confirm that our measures are robust to the inclusion of variance.

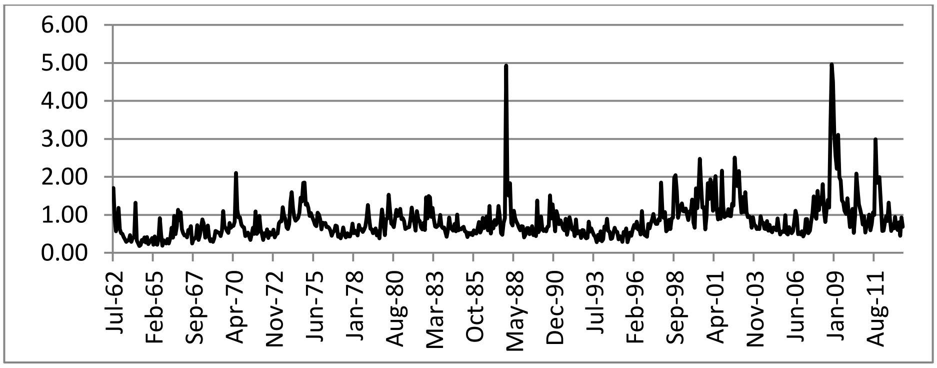
[Table 3.21]

[Table 3.22]

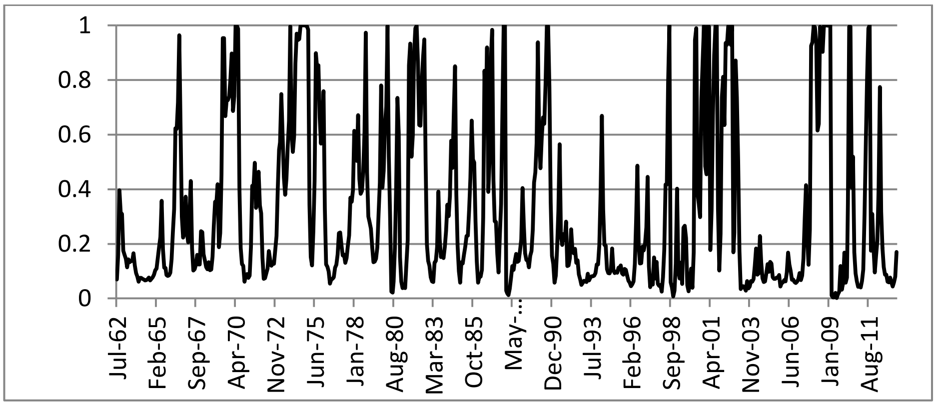
3.5. Conclusion

In this chapter, using Markov switching analysis for the Bali et al. (2009) framework, we document conflicting evidence regarding the dynamic relationship between extreme downside risk and returns. The tail risk-return relationship is insignificant or even negative during turbulent periods when the tail risk is most prominent. We show that this result is robust to a range of modifications to the BDL framework, including expanding the set of state variables and accounting for non-*iid* returns. We argue that the underlying reason for this finding is the heightened leverage and volatility feedback effects during turbulent periods and propose a new measure that better filters out these effects and reinstates the positive extreme downside risk-return relationship. The success of our new measure underlines an interesting pattern of investors' behaviour regarding the time span of their fire sales when facing a crisis. Specifically, financial asset fire sales expand in turbulent periods but tend to be over after two months. Finally, our new measures suggest that investigating the dynamic relationship between tail risk and returns at both market-level as well as asset-level would yield important insights.

Panel A: Volatility of daily market returns in a month over time



Panel B: Smoothed probability of turbulent state



Panel C: Markov switching state timing

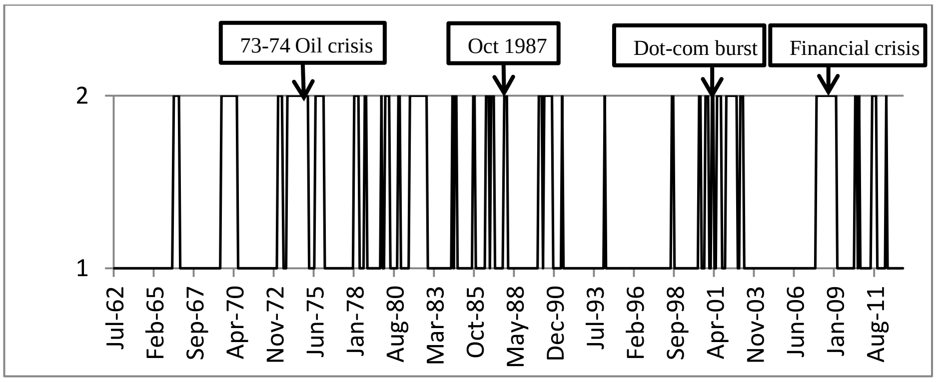


Figure 3.1: Market volatility and estimated states over time. The figure on the top shows the volatility of daily market returns for every month in the sample period (July 1962 – Jun 2013). The other two figures show the smoothed probability of the turbulent state and the corresponding state timing (comparing the smoothed probability of states with the 0.5 threshold) estimated in MS-BDL model using AR4 Skewed Student-t tail risk measure.

Table 3.1: Summary statistics for market returns and tail-risk-related risk measures

This table gives the summary statistics for the CRSP all stock index's value weighted monthly excess return, along with some common tail-risk related risk measures, including standard deviation, mean absolute deviation, skewness, kurtosis, and the BDL non-parametric VaR (which is calculated as -1 times the minimum return of the estimation period). These risk measures are calculated using estimation sample of one month daily returns, which is also the month used to calculate the Monthly Excess returns of the market. t-statistics of AR(4) coefficients are in brackets. The sample period is July 1962 – June 2013.

	Monthly Excess return	Standard deviation	MAD	Skewness	Kurtosis	Non-parametric VaR
Panel A: Basic statistics						
Mean	0.50	0.84	0.64	-0.05	3.06	1.63
Median	0.86	0.70	0.54	-0.06	2.82	1.36
Standard deviation	4.49	0.52	0.39	0.58	1.13	1.27
Minimum	-23.14	0.18	0.14	-2.88	1.63	0.18
Maximum	16.05	4.96	3.79	2.51	11.71	17.13
Panel B: Cross correlation						
Monthly Excess return	1.00	-0.31	-0.30	0.08	-0.02	-0.44
Standard deviation	-0.31	1.00	0.99	0.02	0.08	0.90
MAD	-0.30	0.99	1.00	0.05	-0.03	0.85
Skewness	0.08	0.02	0.05	1.00	-0.17	-0.27
Kurtosis	-0.02	0.08	-0.03	-0.17	1.00	0.27
Non-parametric VaR	-0.44	0.90	0.85	-0.27	0.27	1.00
Panel C: Lags' coefficients in AR(4)						
Lag 1	0.09	0.56	0.61	0.07	-0.01	0.31
(t-statistic)	(2.397)	(33.964)	(30.304)	(1.889)	(-0.240)	(13.773)
Lag 2	-0.05	0.12	0.14	0.06	0.02	0.17
(t-statistic)	(-1.274)	(3.239)	(4.127)	(1.683)	(0.448)	(5.519)
Lag 3	0.03	0.11	0.03	0.14	0.11	0.22
(t-statistic)	(0.818)	(2.216)	(0.778)	(4.105)	(3.018)	(4.666)
Lag 4	0.01	-0.03	0.01	0.05	-0.01	-0.06
(t-statistic)	(0.343)	(-0.982)	(0.197)	(1.059)	(-0.197)	(-1.525)

Table 3.2: Extreme downside risk-return relationship under Markov switching mechanism – One month measures

This table shows the results of the MS-BDL framework for main extreme downside risk measures in BDL papers (calculated using estimation sample of one month daily returns). Specifically, monthly market excess return at time $t+1$ is regressed on $E_t(VaR_{t+1})$ and other control variables at time t. The control variables are those in BDL original framework, including lagged monthly market excess return, October 1987 dummy variable, de-trended risk free rate (RFD), change in term structure risk premium (DTRP), changes in credit risk premium (DCRP), and dividend yield (DY). Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). All parametric VaRs are at 1% level of significance. The sample period is July 1962 – June 2013.

State	Const	$E_t(VaR_{t+1})$	Lagged return	RFD	DTRP	DCRP	DY	State variance	Expected Duration
				Raw Non-parametric VaR					
1	0.058	1.028	-0.012	-0.529	0.038	2.705	-0.008	9.399	11.531
	(0.199)	(5.934)	(-0.273)	(-2.740)	(0.170)	(1.442)	(-0.134)		
2	-2.331	-0.706	-0.019	-0.330	-1.953	5.126	0.648	29.624	5.347
	(-1.467)	(-2.318)	(-0.217)	(-0.754)	(-3.258)	(1.634)	(1.595)		
				AR4 Non-parametric VaR					
1	-2.117	2.151	-0.156	-0.600	-0.763	4.665	0.305	9.401	5.642
	(-2.464)	(6.068)	(-3.622)	(-3.113)	(-2.822)	(2.758)	(1.841)		
2	-1.725	-1.202	0.144	-0.468	-0.793	2.617	0.252	20.111	2.369
	(-0.853)	(-1.623)	(1.327)	(-1.179)	(-1.354)	(0.914)	(0.614)		
				Raw Gaussian VaR					
1	-0.316	1.221	-0.015	-0.519	0.119	2.542	-0.021	8.777	8.901
	(-0.488)	(5.868)	(-0.266)	(-2.478)	(0.410)	(1.311)	(-0.141)		
2	-1.759	-0.797	-0.029	-0.318	-1.937	6.379	0.588	28.116	4.505
	(-1.031)	(-2.145)	(-0.287)	(-0.752)	(-3.139)	(1.921)	(1.419)		
				AR4 Gaussian VaR					
1	-1.980	1.862	-0.121	-0.541	-0.605	4.251	0.292	9.229	5.778
	(-2.406)	(6.421)	(-2.631)	(-2.695)	(-2.101)	(2.446)	(1.723)		
2	-0.736	-1.246	0.057	-0.445	-0.824	3.940	0.150	20.436	2.633
	(-0.426)	(-2.570)	(0.538)	(-1.117)	(-1.430)	(1.318)	(0.378)		

(Continued)

Table 3.2: Continued

State	Const	$E_t(VaR_{t+1})$	Lagged return	RFD	DTRP	DCRP	DY	State variance	Expected Duration
				Raw Student-t VaR					
1	-0.227	1.060	-0.024	-0.571	0.055	3.052	-0.022	9.160	10.930
	(-0.302)	(5.754)	(-0.461)	(-2.724)	(0.178)	(1.575)	(-0.116)		
2	-1.818	-0.724	-0.037	-0.312	-1.920	6.362	0.589	28.954	5.187
	(-1.057)	(-2.198)	(-0.348)	(-0.723)	(-3.179)	(1.902)	(1.404)		
				AR4 Student-t VaR					
1	-1.550	1.748	-0.064	-0.569	0.044	3.251	-0.008	8.853	10.188
	(-1.856)	(6.201)	(-1.249)	(-2.776)	(0.163)	(1.735)	(-0.046)		
2	-1.106	-1.062	0.008	-0.378	-1.928	5.685	0.567	28.989	4.934
	(-0.544)	(-1.650)	(0.066)	(-0.851)	(-3.192)	(1.726)	(1.361)		
				Raw Skewed Student-t VaR					
1	-0.118	1.019	-0.018	-0.524	0.074	2.772	-0.011	8.999	7.954
	(-0.239)	(5.777)	(-0.355)	(-2.506)	(0.273)	(1.508)	(-0.122)		
2	-2.423	-0.803	-0.011	-0.382	-1.998	5.714	0.752	27.304	3.817
	(-1.514)	(-2.822)	(-0.150)	(-0.877)	(-3.358)	(1.825)	(1.840)		
				AR4 Skewed Student-t VaR					
1	-1.452	1.732	-0.076	-0.551	-0.008	2.877	0.011	8.723	7.938
	(-2.013)	(6.109)	(-1.609)	(-2.777)	(-0.066)	(1.591)	(0.090)		
2	-1.582	-1.161	0.054	-0.467	-1.965	5.171	0.683	28.068	3.788
	(-0.780)	(-1.732)	(0.569)	(-1.030)	(-3.227)	(1.605)	(1.658)		

Table 3.3: BDL regressions in different periods – One month measures

This table shows the results of BDL regressions in 3 periods: the Original sample of BDL paper (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on one-month-sample tail risk measure $E_t(VaR_{t+1})$ and other control variables at time t. The control variables are those in BDL original framework. They are lagged monthly market excess return, October 1987 dummy variable, de-trended risk free rate (RFD), change in term structure risk premium (DTRP), changes in credit risk premium (DCRP), and dividend yield (DY). Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). All parametric VaRs are at 1% level of significance.

	Const	$E_t(VaR_{t+1})$	Lagged return	Dummy	RFD	DTRP	DCRP	DY	Adjusted R^2
				Panel A: Original sample July 1962 - December 2005					
Raw NonPara VaR	-1.067	0.472	0.049	-14.684	-0.460	-0.722	3.694	0.267	3.44%
	(-1.529)	(2.098)	(1.030)	(-4.625)	(-2.500)	(-2.349)	(2.249)	(1.592)	
AR4 NonPara VaR	-2.103	1.078	0.032	-12.888	-0.454	-0.736	3.492	0.292	3.82%
	(-2.372)	(2.790)	(0.742)	(-6.398)	(-2.466)	(-2.425)	(2.118)	(1.691)	
Raw Gaussian VaR	-0.697	0.244	0.030	-10.353	-0.486	-0.728	3.795	0.242	2.93%
	(-1.001)	(1.023)	(0.656)	(-4.109)	(-2.668)	(-2.375)	(2.232)	(1.459)	
AR4 Gaussian VaR	-1.062	0.428	0.029	-10.362	-0.479	-0.729	3.709	0.254	3.02%
	(-1.285)	(1.257)	(0.658)	(-4.752)	(-2.636)	(-2.381)	(2.180)	(1.510)	
Raw Student-t VaR	-0.565	0.176	0.024	-10.200	-0.491	-0.727	3.863	0.231	2.87%
	(-0.822)	(0.770)	(0.536)	(-3.489)	(-2.682)	(-2.367)	(2.296)	(1.404)	
AR4 Student-t VaR	-1.164	0.452	0.027	-10.710	-0.477	-0.725	3.697	0.253	3.07%
	(-1.341)	(1.289)	(0.617)	(-4.588)	(-2.614)	(-2.365)	(2.192)	(1.510)	
Raw Skewed Student-t VaR	-1.068	0.413	0.047	-12.754	-0.466	-0.727	3.746	0.261	3.35%
	(-1.475)	(1.896)	(0.996)	(-4.871)	(-2.525)	(-2.360)	(2.268)	(1.546)	
AR4 Skewed Student-t VaR	-1.905	0.830	0.034	-11.621	-0.464	-0.740	3.524	0.282	3.64%
	(-2.094)	(2.386)	(0.784)	(-6.246)	(-2.500)	(-2.425)	(2.136)	(1.629)	

(Continued)

Table 3.3: Continued

	Const	$E_t(VaR_{t+1})$	Lagged return	Dummy	RFD	DTRP	DCRP	DY	Adjusted R^2
Panel B: New sample January 2006 - June 2013									
Raw NonPara VaR	-5.724	-0.539	0.055		0.443	-0.076	-0.912	3.371	7.16%
	(-1.417)	(-0.949)	(0.442)		(0.852)	(-0.052)	(-0.329)	(1.803)	
AR4 NonPara VaR	-6.430	0.218	0.133		0.995	0.286	-2.197	2.967	5.59%
	(-1.878)	(0.277)	(1.306)		(1.332)	(0.184)	(-0.995)	(1.983)	
Raw Gaussian VaR	-6.137	-0.108	0.110		0.762	0.121	-1.540	3.144	5.60%
	(-1.719)	(-0.227)	(0.827)		(1.209)	(0.080)	(-0.548)	(1.945)	
AR4 Gaussian VaR	-6.112	-0.073	0.121		0.811	0.160	-1.794	3.087	5.54%
	(-1.736)	(-0.126)	(1.080)		(1.241)	(0.106)	(-0.702)	(1.939)	
Raw Student-t VaR	-6.133	-0.092	0.112		0.779	0.147	-1.592	3.133	5.59%
	(-1.724)	(-0.219)	(0.864)		(1.258)	(0.097)	(-0.575)	(1.950)	
AR4 Student-t VaR	-6.053	-0.111	0.119		0.784	0.144	-1.730	3.107	5.56%
	(-1.704)	(-0.205)	(1.125)		(1.212)	(0.095)	(-0.678)	(1.945)	
Raw Skewed Student-t VaR	-5.960	-0.180	0.096		0.690	0.100	-1.535	3.159	5.79%
	(-1.602)	(-0.438)	(0.762)		(1.042)	(0.065)	(-0.664)	(1.949)	
AR4 Skewed Student-t VaR	-6.767	0.423	0.149		1.206	0.413	-2.637	2.862	6.02%
	(-2.012)	(0.840)	(1.398)		(1.419)	(0.254)	(-1.303)	(2.069)	
Panel C: Full sample July 1962 - June 2013									
Raw NonPara VaR	-0.328	0.056	0.065	-8.081	-0.326	-0.689	0.715	0.228	1.44%
	(-0.451)	(0.166)	(1.449)	(-1.590)	(-1.812)	(-2.308)	(0.414)	(1.529)	
AR4 NonPara VaR	-1.183	0.496	0.073	-9.435	-0.285	-0.679	0.399	0.275	1.72%
	(-1.165)	(0.895)	(1.724)	(-3.106)	(-1.566)	(-2.279)	(0.228)	(1.813)	
Raw Gaussian VaR	-0.039	-0.063	0.052	-6.787	-0.350	-0.699	0.933	0.203	1.45%
	(-0.055)	(-0.332)	(1.127)	(-1.397)	(-1.976)	(-2.688)	(0.585)	(1.244)	
AR4 Gaussian VaR	-0.176	-0.011	0.059	-7.267	-0.338	-0.694	0.807	0.216	1.43%
	(-0.192)	(-0.025)	(1.399)	(-2.671)	(-1.905)	(-2.348)	(0.460)	(1.435)	
Raw Student-t VaR	-0.051	-0.054	0.053	-6.716	-0.349	-0.699	0.919	0.204	1.45%
	(-0.073)	(-0.204)	(1.191)	(-1.927)	(-1.929)	(-2.355)	(0.517)	(1.375)	
AR4 Student-t VaR	-0.293	0.037	0.061	-7.516	-0.331	-0.691	0.741	0.223	1.43%
	(-0.342)	(0.135)	(1.403)	(-1.560)	(-1.867)	(-2.656)	(0.469)	(1.365)	
Raw Skewed Student-t VaR	-0.316	0.044	0.065	-7.783	-0.327	-0.690	0.721	0.227	1.44%
	(-0.442)	(0.155)	(1.444)	(-2.197)	(-1.808)	(-2.308)	(0.410)	(1.518)	
AR4 Skewed Student-t VaR	-0.923	0.305	0.072	-8.523	-0.296	-0.682	0.450	0.262	1.61%
	(-0.950)	(0.690)	(1.718)	(-3.460)	(-1.622)	(-2.284)	(0.257)	(1.723)	

Table 3.4: MS-BDL investigation: Expanding state variable set – One month measures

This table shows the results of the MS-BDL framework for one-month-sample risk measures using the expanded set of state variables. In each Markov switching regression, monthly market excess return at time $t+1$ is regressed on $E_t(VaR_{t+1})$ and other control variables at time t. The first line of each regression shows the value of regression coefficients, while the second line shows their t-statistics (in brackets). All parametric VaRs are at 1% significance level. The sample period is July 1962–June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw Nonparam	1	-0.657	1.198	0.018	-0.223	0.253	2.187	-0.028	17.152	-47.616	-1.693	0.030	9.276	5.345
		(-0.820)	(5.130)	(0.349)	(-0.942)	(0.937)	(1.059)	(-0.157)	(2.927)	(-0.852)	(-1.722)	(0.990)		
	2	-3.116	-0.605	-0.042	-0.359	-2.246	11.306	0.941	26.852	-129.475	-0.902	0.102	24.557	2.454
		(-1.886)	(-1.883)	(-0.377)	(-0.660)	(-3.331)	(2.929)	(2.150)	(2.662)	(-0.882)	(-0.531)	(2.168)		
AR4 Nonparam	1	-2.547	2.413	-0.054	-0.165	0.211	2.821	-0.053	18.665	-32.656	-0.656	0.003	8.982	5.531
		(-2.333)	(5.233)	(-1.108)	(-0.629)	(0.787)	(1.370)	(-0.225)	(3.037)	(-0.586)	(-0.882)	(0.092)		
	2	-4.589	0.110	0.060	-0.749	-2.310	9.277	1.055	28.866	-165.632	-2.792	0.123	25.667	2.498
		(-2.076)	(0.132)	(0.601)	(-1.336)	(-3.089)	(2.471)	(2.239)	(2.813)	(-1.187)	(-1.426)	(2.693)		
Raw Gaussian	1	-1.512	1.551	0.029	-0.160	0.367	2.848	-0.017	23.458	-71.813	-0.613	0.005	8.630	5.220
		(-2.222)	(6.907)	(0.588)	(-0.718)	(1.422)	(1.432)	(-0.137)	(4.341)	(-1.389)	(-0.729)	(0.197)		
	2	-3.161	-0.457	-0.016	-0.536	-2.348	10.515	0.936	23.823	-125.970	-2.330	0.102	24.761	2.532
		(-1.911)	(-1.299)	(-0.191)	(-0.965)	(-3.362)	(2.597)	(2.341)	(2.339)	(-0.928)	(-1.026)	(2.273)		
AR4 Gaussian	1	-2.720	2.266	-0.011	-0.145	0.363	3.062	-0.043	24.230	-65.942	-0.521	0.003	8.279	4.973
		(-2.901)	(6.918)	(-0.237)	(-0.600)	(1.416)	(1.482)	(-0.207)	(4.088)	(-1.261)	(-0.715)	(0.117)		
	2	-3.619	-0.286	0.029	-0.595	-2.267	9.277	0.989	25.191	-141.637	-2.524	0.112	24.639	2.567
		(-1.937)	(-0.515)	(0.307)	(-1.145)	(-3.413)	(2.553)	(2.414)	(2.496)	(-1.070)	(-1.343)	(2.586)		

(Continued)

Table 3.4: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw Student-t	1	-1.401	1.332	0.015	-0.202	0.312	3.173	-0.040	22.611	-51.237	-1.004	0.012	9.076	5.954
		(-1.750)	(6.445)	(0.307)	(-0.888)	(1.166)	(1.687)	(-0.189)	(4.398)	(-0.934)	(-0.693)	(0.407)		
	2	-2.898	-0.528	-0.044	-0.433	-2.339	11.828	0.958	24.529	-155.466	-1.575	0.091	25.302	2.610
		(-1.625)	(-1.573)	(-0.378)	(-0.644)	(-3.128)	(2.472)	(2.212)	(2.260)	(-1.001)	(-0.477)	(1.759)		
AR4 Student-t	1	-3.047	2.166	-0.031	-0.135	0.308	3.423	-0.038	23.897	-46.107	-0.678	0.005	8.871	5.855
		(-3.389)	(6.963)	(-0.676)	(-0.612)	(1.229)	(1.850)	(-0.192)	(4.480)	(-0.876)	(-0.932)	(0.233)		
	2	-3.713	-0.323	0.033	-0.703	-2.491	10.159	1.054	25.635	-168.270	-2.476	0.110	25.551	2.514
		(-1.757)	(-0.560)	(0.321)	(-1.193)	(-3.394)	(2.537)	(2.353)	(2.359)	(-1.185)	(-1.175)	(2.407)		
Raw Skewed Student-t	1	-0.768	1.271	0.040	-0.174	0.337	2.278	-0.118	17.512	-48.281	-1.560	0.017	8.188	4.357
		(-0.883)	(5.363)	(0.722)	(-0.544)	(1.190)	(1.041)	(-0.547)	(2.270)	(-0.869)	(-1.698)	(0.614)		
	2	-2.882	-0.484	-0.031	-0.484	-1.903	10.281	0.933	25.447	-122.962	-0.929	0.092	24.193	2.639
		(-1.959)	(-1.549)	(-0.324)	(-1.075)	(-3.176)	(3.014)	(2.368)	(2.797)	(-0.981)	(-0.638)	(2.205)		
AR4 Skewed Student-t	1	-2.513	2.338	-0.020	0.032	0.296	1.442	-0.229	22.097	-40.308	-0.502	-0.005	7.412	3.451
		(-2.013)	(4.900)	(-0.378)	(0.067)	(0.959)	(0.600)	(-0.963)	(2.347)	(-0.713)	(-0.521)	(-0.136)		
	2	-4.271	-0.064	0.053	-0.705	-1.802	8.574	1.113	26.788	-131.447	-2.317	0.115	24.055	2.295
		(-1.865)	(-0.069)	(0.576)	(-1.502)	(-2.848)	(2.591)	(2.789)	(2.559)	(-0.962)	(-1.218)	(2.705)		

Table 3.5: BDL regression in different periods: expanded set of state variables – One month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on one-month-sample risk measure $E_t(VaR_{t+1})$ and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). Parametric VaRs are at 1% level of significance.

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
				Panel A: Original sample July 1962 - December 2005									
Raw	-2.418	0.848	0.022	-21.416	-0.161	-0.455	5.093	0.378	27.867	-71.021	-0.891	-0.003	8.67%
NonPara VaR	(-3.142)	(3.409)	(0.460)	(-3.827)	(-0.863)	(-1.778)	(2.796)	(2.228)	(5.657)	(-1.214)	(-1.206)	(-0.087)	
AR4	-4.396	1.962	-0.013	-18.408	-0.122	-0.459	4.898	0.424	30.194	-69.371	-0.837	-0.007	9.82%
NonPara VaR	(-4.526)	(5.031)	(-0.339)	(-8.921)	(-0.688)	(-1.575)	(3.224)	(2.611)	(6.199)	(-1.364)	(-1.128)	(-0.202)	
Raw	-2.619	0.807	0.018	-16.991	-0.154	-0.435	4.949	0.401	29.647	-79.069	-0.770	-0.008	8.35%
Gaussian VaR	(-3.419)	(3.375)	(0.427)	(-6.408)	(-0.907)	(-1.452)	(3.152)	(2.523)	(6.037)	(-1.547)	(-1.051)	(-0.226)	
AR4	-3.778	1.379	0.008	-16.912	-0.120	-0.421	4.814	0.439	31.442	-85.878	-0.783	-0.008	8.96%
Gaussian VaR	(-4.197)	(4.189)	(0.212)	(-7.504)	(-0.701)	(-1.422)	(3.064)	(2.720)	(6.246)	(-1.689)	(-1.069)	(-0.237)	
Raw	-2.216	0.600	0.005	-16.622	-0.178	-0.445	5.060	0.359	28.135	-67.648	-0.812	-0.005	7.88%
Student-t VaR	(-3.075)	(2.836)	(0.107)	(-5.956)	(-1.036)	(-1.483)	(3.198)	(2.299)	(6.025)	(-1.327)	(-1.102)	(-0.145)	
AR4	-3.761	1.300	-0.003	-17.135	-0.127	-0.419	4.871	0.414	30.697	-77.113	-0.821	-0.006	8.87%
Student-t VaR	(-4.267)	(4.203)	(-0.077)	(-7.795)	(-0.735)	(-1.420)	(3.082)	(2.603)	(6.397)	(-1.529)	(-1.113)	(-0.168)	
Raw Skewed	-2.553	0.799	0.022	-18.583	-0.162	-0.457	5.189	0.376	28.369	-72.647	-0.789	-0.006	8.67%
Student-t VaR	(-3.605)	(3.855)	(0.495)	(-7.593)	(-0.934)	(-1.513)	(3.321)	(2.444)	(6.084)	(-1.388)	(-1.038)	(-0.185)	
AR4 Skewed	-4.358	1.664	-0.006	-16.750	-0.128	-0.458	4.903	0.424	30.829	-74.918	-0.740	-0.010	9.73%
Student-t VaR	(-4.763)	(5.239)	(-0.148)	(-9.583)	(-0.708)	(-1.549)	(3.187)	(2.628)	(6.209)	(-1.465)	(-0.993)	(-0.312)	

(Continued)

Table 3.5: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
					Panel B: New sample January 2006 - June 2013								
Raw	1.112	-0.759	-0.106		-2.034	-1.738	6.393	0.618	44.513	-192.284	-3.242	0.111	29.20%
NonPara VaR	(0.256)	(-1.294)	(-1.004)		(-1.560)	(-2.062)	(2.064)	(0.367)	(2.363)	(-1.245)	(-2.334)	(4.262)	
AR4	-1.894	0.018	-0.002		-0.872	-1.324	4.897	1.259	32.885	-184.180	-3.330	0.118	26.62%
NonPara VaR	(-0.411)	(0.019)	(-0.019)		(-0.662)	(-1.488)	(1.717)	(0.771)	(1.886)	(-1.214)	(-2.669)	(4.626)	
Raw	-0.857	-0.277	-0.048		-1.385	-1.546	5.968	1.082	37.689	-181.475	-3.373	0.117	27.00%
Gaussian VaR	(-0.221)	(-0.521)	(-0.429)		(-1.053)	(-1.662)	(1.616)	(0.702)	(2.115)	(-1.164)	(-2.429)	(4.428)	
AR4	-1.317	-0.164	-0.016		-1.117	-1.429	5.294	1.169	35.061	-184.081	-3.309	0.117	26.68%
Gaussian VaR	(-0.329)	(-0.234)	(-0.169)		(-0.859)	(-1.530)	(1.636)	(0.765)	(2.063)	(-1.191)	(-2.575)	(4.549)	
Raw	-1.079	-0.203	-0.036		-1.253	-1.466	5.721	1.136	36.448	-185.150	-3.339	0.117	26.87%
Student-t VaR	(-0.294)	(-0.441)	(-0.338)		(-1.046)	(-1.600)	(1.612)	(0.762)	(2.203)	(-1.194)	(-2.470)	(4.411)	
AR4	-1.431	-0.120	-0.011		-1.050	-1.398	5.174	1.191	34.441	-185.653	-3.299	0.117	26.65%
Student-t VaR	(-0.365)	(-0.187)	(-0.115)		(-0.887)	(-1.504)	(1.718)	(0.793)	(2.199)	(-1.209)	(-2.607)	(4.505)	
Raw Skewed	0.029	-0.360	-0.066		-1.588	-1.577	5.745	0.819	39.989	-195.006	-3.346	0.114	27.41%
Student-t VaR	(0.007)	(-0.809)	(-0.618)		(-1.269)	(-1.811)	(1.956)	(0.507)	(2.284)	(-1.256)	(-2.417)	(4.403)	
AR4 Skewed	-3.286	0.348	0.018		-0.389	-1.146	4.294	1.552	28.294	-178.398	-3.372	0.121	26.84%
Student-t VaR	(-0.721)	(0.504)	(0.187)		(-0.305)	(-1.227)	(1.587)	(0.944)	(1.724)	(-1.192)	(-2.647)	(4.481)	

(Continued)

Table 3.5: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel C: Full sample July 1962 - June 2013							
Raw	-1.448	0.465	0.023	-15.397	-0.237	-0.527	3.893	0.319	26.687	-97.023	-1.601	0.092	10.23%
NonPara VaR	(-2.114)	(1.839)	(0.567)	(-4.112)	(-1.584)	(-1.805)	(3.021)	(2.219)	(7.884)	(-1.890)	(-2.128)	(4.073)	
AR4	-3.152	1.374	0.008	-15.106	-0.173	-0.507	3.660	0.384	28.847	-99.912	-1.624	0.094	11.50%
NonPara VaR	(-3.509)	(3.747)	(0.197)	(-3.249)	(-0.994)	(-2.045)	(2.345)	(2.400)	(7.009)	(-1.942)	(-2.486)	(4.740)	
Raw	-1.545	0.443	0.024	-12.913	-0.231	-0.516	3.630	0.332	27.547	-103.767	-1.530	0.092	10.21%
Gaussian VaR	(-2.314)	(2.014)	(0.617)	(-5.271)	(-1.572)	(-1.775)	(2.718)	(2.261)	(7.903)	(-2.011)	(-2.045)	(3.930)	
AR4	-2.301	0.800	0.020	-13.101	-0.199	-0.499	3.587	0.362	28.685	-108.164	-1.575	0.094	10.64%
Gaussian VaR	(-2.987)	(2.748)	(0.544)	(-6.327)	(-1.355)	(-1.726)	(2.748)	(2.465)	(8.095)	(-2.110)	(-2.057)	(3.834)	
Raw	-1.431	0.371	0.018	-13.239	-0.240	-0.519	3.707	0.316	27.127	-98.383	-1.544	0.092	10.10%
Student-t VaR	(-2.010)	(2.123)	(0.410)	(-2.745)	(-1.367)	(-2.069)	(2.341)	(1.957)	(6.508)	(-1.883)	(-2.344)	(4.584)	
AR4	-2.482	0.827	0.014	-13.693	-0.194	-0.492	3.632	0.360	28.727	-103.725	-1.605	0.094	10.74%
Student-t VaR	(-3.197)	(3.002)	(0.379)	(-6.589)	(-1.310)	(-1.703)	(2.793)	(2.456)	(8.197)	(-2.024)	(-2.082)	(3.819)	
Raw Skewed	-1.551	0.435	0.025	-13.753	-0.230	-0.526	3.895	0.323	27.067	-97.488	-1.546	0.093	10.33%
Student-t VaR	(-2.322)	(2.087)	(0.625)	(-5.291)	(-1.522)	(-1.793)	(3.021)	(2.243)	(7.888)	(-1.883)	(-2.044)	(3.973)	
AR4 Skewed	-2.904	1.047	0.013	-13.372	-0.179	-0.509	3.622	0.380	29.010	-102.933	-1.567	0.094	11.34%
Student-t VaR	(-3.351)	(3.604)	(0.321)	(-2.956)	(-1.027)	(-2.048)	(2.317)	(2.367)	(6.998)	(-1.995)	(-2.396)	(4.753)	

Table 3.6: MS-BDL Investigation: NIID measures – One month measures

This table shows the results of the MS-BDL framework for one-month-sample NIID measures using the expanded set of state variables. In each Markov switching regression, monthly market excess return at time $t+1$ is regressed on $E_t(VaR_{t+1})$ and other control variables at time t. The first line of each regression shows the value of regression coefficients, while the second line shows their t-statistics (in brackets). All parametric VaRs are at 1% significance level. The sample period is July 1962–June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw Gaussian	1	-2.993	1.822	-0.172	-0.225	-0.674	4.177	0.393	20.865	-32.986	-0.697	0.007	9.434	3.905
		(-3.446)	(7.416)	(-3.837)	(-0.977)	(-2.249)	(2.014)	(2.025)	(4.144)	(-0.595)	(-0.996)	(0.284)		
	2	-4.160	0.256	0.188	-0.346	-0.307	5.413	0.410	38.188	-134.880	-2.454	0.081	17.851	1.885
		(-2.537)	(0.613)	(1.901)	(-0.859)	(-0.500)	(1.738)	(1.045)	(3.998)	(-1.116)	(-1.666)	(1.936)		
AR4 Gaussian	1	-3.661	2.197	-0.173	-0.228	-0.650	4.620	0.412	20.895	-42.688	-0.724	0.009	9.303	3.716
		(-3.875)	(7.419)	(-3.842)	(-0.986)	(-2.160)	(2.222)	(2.120)	(4.060)	(-0.768)	(-1.027)	(0.363)		
	2	-3.930	0.206	0.183	-0.329	-0.321	5.330	0.368	36.941	-120.655	-2.386	0.078	17.617	1.908
		(-2.295)	(0.411)	(1.867)	(-0.857)	(-0.551)	(1.768)	(0.967)	(4.041)	(-1.018)	(-1.662)	(1.932)		
Raw Student-t	1	-2.909	1.640	-0.173	-0.202	-0.699	4.202	0.393	20.420	-24.443	-0.676	0.007	9.364	3.780
		(-3.367)	(7.298)	(-3.834)	(-0.845)	(-2.299)	(1.989)	(1.971)	(4.069)	(-0.436)	(-0.958)	(0.261)		
	2	-4.382	0.301	0.187	-0.336	-0.259	5.270	0.456	38.627	-144.302	-2.492	0.085	17.724	1.873
		(-2.655)	(0.764)	(1.926)	(-0.824)	(-0.437)	(1.709)	(1.144)	(4.043)	(-1.184)	(-1.696)	(2.002)		
AR4 Student-t	1	-3.431	1.919	-0.171	-0.202	-0.697	4.451	0.403	20.319	-34.621	-0.704	0.009	9.336	3.649
		(-3.693)	(7.338)	(-3.762)	(-0.825)	(-2.277)	(2.097)	(2.008)	(3.984)	(-0.617)	(-0.991)	(0.390)		
	2	-4.082	0.227	0.187	-0.336	-0.260	5.331	0.404	37.253	-126.462	-2.420	0.080	17.582	1.885
		(-2.392)	(0.508)	(1.925)	(-0.870)	(-0.461)	(1.747)	(1.042)	(4.050)	(-1.062)	(-1.683)	(1.954)		

(Continued)

Table 3.6: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw Skewed Student-t	1	-2.184	1.452	-0.094	-0.154	0.196	3.448	0.059	22.813	-32.724	-0.472	0.002	8.941	6.446
		(-2.631)	(7.023)	(-2.055)	(-0.769)	(0.783)	(1.835)	(0.289)	(4.442)	(-0.641)	(-0.647)	(0.074)		
	2	-6.011	0.511	0.082	-0.639	-2.410	8.070	1.239	31.891	-230.519	-3.201	0.139	25.952	2.585
		(-2.924)	(1.121)	(0.860)	(-1.086)	(-3.078)	(2.003)	(2.599)	(2.667)	(-1.533)	(-1.713)	(2.951)		
AR4 Skewed Student-t	1	-3.428	1.805	-0.176	-0.218	-0.717	4.459	0.454	19.948	-36.494	-0.669	0.008	9.261	3.530
		(-3.622)	(7.211)	(-3.859)	(-0.886)	(-2.335)	(2.056)	(2.239)	(3.873)	(-0.634)	(-0.938)	(0.314)		
	2	-4.052	0.224	0.187	-0.312	-0.251	5.305	0.404	37.040	-124.934	-2.384	0.081	17.390	1.913
		(-2.398)	(0.538)	(1.953)	(-0.838)	(-0.442)	(1.788)	(1.060)	(4.117)	(-1.072)	(-1.700)	(2.017)		

Table 3.7: BDL regression in different periods: NIID risk measures – One month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on one-month-sample NIID measure $E_t(VaR_{t+1})$ and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). Parametric VaRs are at 1% level of significance.

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel 1: Original sample July 1962 - December 2005							
Raw	-3.796	1.265	-0.030	-16.162	-0.084	-0.413	4.709	0.474	33.014	-82.834	-0.794	-0.010	9.92%
Gaussian VaR	(-4.581)	(4.664)	(-0.785)	(-8.297)	(-0.492)	(-1.427)	(2.937)	(2.923)	(6.626)	(-1.683)	(-1.100)	(-0.293)	
AR4	-4.325	1.545	-0.027	-16.605	-0.074	-0.392	4.730	0.481	33.238	-87.929	-0.819	-0.010	9.92%
Gaussian VaR	(-4.698)	(4.647)	(-0.718)	(-8.126)	(-0.431)	(-1.370)	(2.966)	(2.944)	(6.594)	(-1.772)	(-1.128)	(-0.299)	
Raw	-3.856	1.172	-0.033	-14.544	-0.073	-0.416	4.729	0.488	32.995	-78.945	-0.798	-0.008	9.98%
Student-t VaR	(-4.621)	(4.718)	(-0.870)	(-8.747)	(-0.428)	(-1.433)	(2.954)	(2.997)	(6.649)	(-1.603)	(-1.104)	(-0.241)	
AR4	-4.250	1.375	-0.028	-14.957	-0.070	-0.399	4.740	0.488	32.930	-83.698	-0.816	-0.008	9.89%
Student-t VaR	(-4.676)	(4.610)	(-0.732)	(-8.482)	(-0.407)	(-1.385)	(2.974)	(2.996)	(6.600)	(-1.687)	(-1.123)	(-0.244)	
Raw Skewed	-3.893	1.121	-0.034	-14.168	-0.074	-0.414	4.742	0.509	33.080	-79.693	-0.793	-0.008	9.88%
Student-t VaR	(-4.537)	(4.556)	(-0.879)	(-8.709)	(-0.430)	(-1.430)	(2.999)	(3.106)	(6.677)	(-1.625)	(-1.094)	(-0.237)	
AR4 Skewed	-4.254	1.302	-0.029	-14.532	-0.072	-0.397	4.756	0.509	32.981	-84.179	-0.809	-0.008	9.78%
Student-t VaR	(-4.562)	(4.439)	(-0.747)	(-8.453)	(-0.414)	(-1.384)	(3.021)	(3.095)	(6.622)	(-1.701)	(-1.112)	(-0.238)	

(Continued)

Table 3.7: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel 2: New sample January 2006 - June 2013													
Raw	-4.150	0.639	0.025		0.339	-0.894	2.376	1.672	20.711	-194.716	-3.462	0.125	28.14%
Gaussian VaR	(-1.134)	(1.350)	(0.258)		(0.295)	(-0.959)	(0.834)	(1.137)	(1.548)	(-1.361)	(-2.649)	(4.269)	
AR4	-3.844	0.642	0.022		0.117	-0.973	2.992	1.547	23.052	-196.623	-3.422	0.124	27.69%
Gaussian VaR	(-1.063)	(1.095)	(0.227)		(0.103)	(-1.046)	(1.040)	(1.066)	(1.710)	(-1.352)	(-2.645)	(4.341)	
Raw	-4.072	0.554	0.025		0.348	-0.875	2.404	1.678	20.601	-194.372	-3.457	0.125	28.10%
Student-t VaR	(-1.113)	(1.322)	(0.264)		(0.297)	(-0.928)	(0.839)	(1.136)	(1.518)	(-1.355)	(-2.644)	(4.259)	
AR4	-3.595	0.512	0.023		0.048	-0.984	3.086	1.522	23.659	-197.062	-3.381	0.123	27.55%
Student-t VaR	(-1.003)	(0.995)	(0.235)		(0.042)	(-1.050)	(1.032)	(1.047)	(1.709)	(-1.347)	(-2.648)	(4.355)	
Raw Skewed	-3.963	0.499	0.024		0.310	-0.883	2.477	1.655	20.947	-194.021	-3.445	0.124	27.98%
Student-t VaR	(-1.079)	(1.254)	(0.249)		(0.263)	(-0.929)	(0.855)	(1.116)	(1.522)	(-1.345)	(-2.635)	(4.249)	
AR4 Skewed	-3.494	0.457	0.021		0.016	-0.992	3.143	1.505	23.951	-196.565	-3.374	0.123	27.46%
Student-t VaR	(-0.972)	(0.942)	(0.220)		(0.014)	(-1.051)	(1.040)	(1.032)	(1.708)	(-1.337)	(-2.640)	(4.343)	

(Continued)

Table 3.7: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel 3: Full sample July 1962 - June 2013							
Raw	-2.518	0.806	-0.002	-13.054	-0.167	-0.491	3.258	0.411	29.718	-112.698	-1.612	0.099	11.64%
Gaussian VaR	(-3.878)	(4.344)	(-0.055)	(-8.145)	(-1.128)	(-1.722)	(2.428)	(2.771)	(8.341)	(-2.181)	(-2.069)	(3.711)	
AR4	-2.807	0.964	-0.001	-13.242	-0.165	-0.480	3.357	0.411	29.699	-114.942	-1.621	0.099	11.57%
Gaussian VaR	(-3.954)	(4.188)	(-0.023)	(-7.922)	(-1.110)	(-1.694)	(2.512)	(2.776)	(8.308)	(-2.225)	(-2.086)	(3.760)	
Raw	-2.465	0.713	-0.004	-11.868	-0.162	-0.493	3.268	0.413	29.518	-110.277	-1.619	0.100	11.58%
Student-t VaR	(-3.802)	(4.261)	(-0.100)	(-8.416)	(-1.096)	(-1.727)	(2.439)	(2.791)	(8.363)	(-2.130)	(-2.073)	(3.748)	
AR4	-2.660	0.821	-0.001	-12.053	-0.166	-0.485	3.344	0.409	29.355	-112.647	-1.612	0.099	11.45%
Student-t VaR	(-3.781)	(3.983)	(-0.022)	(-8.135)	(-1.115)	(-1.710)	(2.494)	(2.774)	(8.312)	(-2.177)	(-2.085)	(3.816)	
Raw Skewed	-2.432	0.658	-0.005	-11.546	-0.166	-0.492	3.300	0.422	29.429	-110.023	-1.616	0.099	11.47%
Student-t VaR	(-3.703)	(4.109)	(-0.121)	(-8.443)	(-1.119)	(-1.729)	(2.485)	(2.846)	(8.375)	(-2.124)	(-2.068)	(3.742)	
AR4 Skewed	-2.603	0.751	-0.002	-11.706	-0.170	-0.485	3.374	0.419	29.261	-112.198	-1.607	0.099	11.34%
Student-t VaR	(-3.664)	(3.834)	(-0.049)	(-8.186)	(-1.139)	(-1.713)	(2.540)	(2.822)	(8.320)	(-2.166)	(-2.079)	(3.809)	

Table 3.8: Volatility clustering in turbulent periods

This table shows how volatility induces volatility clustering. Volatility clustering is represented by the product of daily market standard deviations in month $t+1$ and $t+2$, which is then regressed on daily market variance of month t, with or without the state variables at different timings. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). The sample period is July 1962 – June 2013.

	Const	$\tilde{\sigma}_t$	Market return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
$\tilde{\sigma}_{t+1}\tilde{\sigma}_{t+2}$ with no state variable	14.475	0.252											14.22%
	(6.917)	(3.380)											
$\tilde{\sigma}_{t+1}\tilde{\sigma}_{t+2}$with state variables at *t*	25.224	0.225	-0.941	-116.136	-2.758	-0.806	-12.172	-2.127	-136.915	175.848	6.836	-0.571	28.80%
	(4.098)	(2.593)	(-3.062)	(-2.287)	(-2.036)	(-0.529)	(-1.424)	(-1.765)	(-2.616)	(0.517)	(1.497)	(-1.481)	
$\tilde{\sigma}_{t+1}\tilde{\sigma}_{t+2}$with state variables at *t*+1	23.983	0.149	-1.670	176.291	-2.805	-1.100	39.926	-2.823	-100.513	892.815	-4.014	-0.565	47.15%
	(5.735)	(2.315)	(-2.951)	(16.590)	(-2.258)	(-0.651)	(1.374)	(-2.799)	(-3.777)	(3.208)	(-1.377)	(-1.726)	
$\tilde{\sigma}_{t+1}\tilde{\sigma}_{t+2}$with state variables at *t*+2	22.796	0.219	-0.774	97.327	-2.621	1.683	53.255	-2.333	-85.371	412.735	-7.147	-0.496	35.74%
	(5.740)	(3.704)	(-2.243)	(14.365)	(-2.140)	(0.955)	(2.610)	(-2.592)	(-3.157)	(1.607)	(-1.410)	(-1.483)	

Table 3.9: MS-BDL Investigation: FOSA measures – One month measures

This table shows the results of the MS-BDL framework for one-month-sample FOSA measures using the expanded set of state variables. In each Markov switching regression, monthly market excess return at time $t+1$ is regressed on $E_t(VaR_{t+1})_{FOSA}$ and other control variables at time t. The first line of each regression shows the value of regression coefficients, while the second line shows the corresponding t-statistics (in brackets). All parametric VaRs are at 1% significance level. The sample period is July 1962–June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
	1	-4.415	4.144	-0.222	-0.527	0.048	4.853	0.231	5.669	-57.487	0.200	-0.015	5.433	4.127
IID		(-3.206)	(6.144)	(-4.184)	(-1.928)	(0.126)	(2.543)	(1.113)	(0.777)	(-0.999)	(0.308)	(-0.652)		
Nonparam	2	-6.279	1.905	-0.044	-0.316	-0.959	5.125	0.701	32.335	-130.926	-2.757	0.135	20.893	4.917
		(-4.251)	(3.152)	(-0.704)	(-1.072)	(-2.454)	(2.111)	(2.460)	(5.309)	(-1.481)	(-2.361)	(4.307)		
	1	-2.543	2.462	-0.173	-0.513	0.196	4.300	0.025	10.262	-36.031	0.045	-0.005	7.220	6.016
IID		(-2.055)	(5.172)	(-3.237)	(-2.332)	(0.665)	(2.186)	(0.101)	(1.290)	(-0.679)	(0.084)	(-0.188)		
Gaussian	2	-7.494	1.942	-0.028	-0.217	-1.489	5.365	0.950	33.080	-175.401	-3.428	0.155	22.670	4.584
		(-4.252)	(3.610)	(-0.425)	(-0.598)	(-2.870)	(1.932)	(2.772)	(4.776)	(-1.679)	(-2.554)	(4.439)		
	1	-3.499	2.759	-0.181	-0.539	0.117	4.707	0.063	9.695	-31.377	0.072	-0.006	6.975	5.920
IID		(-2.534)	(5.116)	(-3.354)	(-2.368)	(0.401)	(2.388)	(0.258)	(1.246)	(-0.565)	(0.142)	(-0.251)		
Student-t	2	-7.539	1.882	-0.041	-0.218	-1.311	5.640	0.910	33.179	-167.117	-3.408	0.152	22.462	4.816
		(-4.354)	(3.501)	(-0.591)	(-0.598)	(-2.644)	(2.056)	(2.716)	(4.700)	(-1.621)	(-2.596)	(4.413)		
IID	1	-3.084	2.728	-0.203	-0.501	0.096	4.940	0.188	5.691	-34.138	0.150	-0.013	5.812	4.579
Skewed		(-2.411)	(5.365)	(-3.849)	(-1.792)	(0.281)	(2.504)	(0.890)	(0.725)	(-0.594)	(0.210)	(-0.551)		
Student-t	2	-5.875	1.454	-0.037	-0.370	-1.023	4.717	0.711	31.800	-147.208	-2.872	0.136	21.618	5.167
		(-3.912)	(2.824)	(-0.585)	(-1.252)	(-2.605)	(1.920)	(2.458)	(5.123)	(-1.633)	(-2.407)	(4.332)		

(Continued)

Table 3.9: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
	1	-2.296	2.025	-0.149	-0.390	0.176	5.101	-0.075	17.361	-11.217	-0.217	-0.005	8.292	5.488
NIID		(-1.325)	(5.046)	(-2.135)	(-1.801)	(0.636)	(2.590)	(-0.204)	(2.475)	(-0.162)	(-0.276)	(-0.127)		
Gaussian	2	-7.539	1.567	0.007	-0.208	-1.823	7.507	1.145	35.537	-216.337	-3.674	0.142	23.647	3.081
		(-2.589)	(2.202)	(0.018)	(-0.420)	(-2.863)	(2.326)	(1.956)	(3.080)	(-1.637)	(-2.193)	(2.673)		
	1	-2.108	1.809	-0.148	-0.376	0.214	5.037	-0.085	17.021	-9.258	-0.228	-0.005	8.086	5.189
NIID		(-2.150)	(5.797)	(-3.049)	(-1.682)	(0.787)	(2.556)	(-0.352)	(2.451)	(-0.125)	(-0.283)	(-0.196)		
Student-t	2	-7.223	1.300	0.011	-0.208	-1.834	7.386	1.143	34.633	-206.870	-3.506	0.140	23.485	3.026
		(-3.746)	(2.430)	(0.119)	(-0.441)	(-2.978)	(2.315)	(2.870)	(3.536)	(-1.663)	(-2.137)	(3.398)		
NIID	1	-1.965	1.683	-0.148	-0.392	0.236	5.002	-0.064	16.194	-12.016	-0.186	-0.006	7.897	5.032
Skewed		(-2.062)	(5.695)	(-3.037)	(-1.827)	(0.868)	(2.496)	(-0.306)	(2.225)	(-0.244)	(-0.293)	(-0.263)		
Student-t	2	-7.063	1.194	0.008	-0.196	-1.793	7.237	1.137	34.347	-200.391	-3.477	0.139	23.240	3.115
		(-3.774)	(2.445)	(0.119)	(-0.456)	(-3.100)	(2.348)	(2.933)	(3.668)	(-1.677)	(-2.255)	(3.554)		

Table 3.10: BDL regression in different periods: FOSA measures – One month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on one-month-sample FOSA measure $E_t(VaR_{t+1})_{FOSA}$ and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). Parametric VaRs are at 1% level of significance.

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Returns	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
				Panel 1: Original sample July 1962 - December 2005									
IID	-4.380	1.992	-0.067	-10.199	-0.155	-0.503	4.891	0.380	29.300	-53.901	-0.850	-0.006	9.51%
NonPara	(-4.071)	(4.051)	(-1.481)	(-2.340)	(-0.840)	(-1.981)	(2.694)	(2.269)	(5.924)	(-0.936)	(-1.155)	(-0.167)	
IID	-4.627	1.803	-0.064	-10.475	-0.120	-0.496	4.619	0.435	32.299	-72.287	-0.806	-0.005	9.87%
Gaussian	(-4.842)	(4.897)	(-1.676)	(-9.288)	(-0.692)	(-1.688)	(2.890)	(2.659)	(6.410)	(-1.474)	(-1.105)	(-0.144)	
IID	-4.988	1.872	-0.066	-10.329	-0.113	-0.492	4.687	0.424	31.958	-68.157	-0.861	-0.003	10.22%
Student-t	(-4.946)	(4.964)	(-1.739)	(-9.367)	(-0.648)	(-1.691)	(2.949)	(2.588)	(6.475)	(-1.400)	(-1.164)	(-0.087)	
IID Skewed	-4.211	1.620	-0.064	-10.206	-0.167	-0.501	4.873	0.381	29.820	-59.054	-0.808	-0.009	9.26%
Student-t	(-4.142)	(4.407)	(-1.695)	(-9.435)	(-0.911)	(-1.718)	(3.251)	(2.313)	(5.897)	(-1.203)	(-1.097)	(-0.286)	
NIID	-4.298	1.572	-0.068	-10.440	-0.094	-0.467	5.162	0.423	31.831	-64.447	-0.842	-0.007	9.09%
Gaussian	(-4.354)	(4.116)	(-1.804)	(-9.205)	(-0.542)	(-1.599)	(3.250)	(2.648)	(6.479)	(-1.326)	(-1.146)	(-0.203)	
NIID	-4.309	1.434	-0.069	-10.514	-0.085	-0.453	5.254	0.440	31.894	-63.212	-0.837	-0.006	9.07%
Student-t	(-4.264)	(4.050)	(-1.850)	(-9.224)	(-0.485)	(-1.554)	(3.327)	(2.727)	(6.471)	(-1.301)	(-1.138)	(-0.196)	
NIID Skewed	-4.325	1.363	-0.069	-10.377	-0.083	-0.452	5.268	0.461	32.037	-64.012	-0.837	-0.006	9.02%
Student-t	(-4.173)	(3.928)	(-1.830)	(-9.169)	(-0.478)	(-1.553)	(3.371)	(2.837)	(6.466)	(-1.321)	(-1.138)	(-0.194)	

(Continued)

Table 3.10: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Returns	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel 2: New sample January 2006 - June 2013													
IID	-8.188	2.214	-0.022		1.115	-0.565	2.736	2.333	15.185	-146.955	-3.818	0.126	30.46%
NonPara	(-1.799)	(2.358)	(-0.255)		(0.998)	(-0.593)	(1.388)	(1.468)	(1.369)	(-1.150)	(-2.538)	(4.010)	
IID	-7.345	1.766	-0.007		1.388	-0.623	2.339	2.103	14.102	-156.840	-4.022	0.131	32.28%
Gaussian	(-1.823)	(2.874)	(-0.077)		(1.194)	(-0.664)	(1.204)	(1.397)	(1.294)	(-1.292)	(-2.531)	(3.982)	
IID	-6.624	1.462	-0.011		0.877	-0.651	3.300	1.929	18.820	-151.139	-3.958	0.130	30.04%
Student-t	(-1.604)	(2.179)	(-0.122)		(0.771)	(-0.651)	(1.682)	(1.294)	(1.717)	(-1.171)	(-2.470)	(3.982)	
IID Skewed	-7.727	1.569	-0.015		1.178	-0.542	2.490	2.426	14.412	-165.336	-3.778	0.124	30.46%
Student-t	(-1.688)	(2.271)	(-0.173)		(1.046)	(-0.593)	(1.240)	(1.477)	(1.225)	(-1.283)	(-2.471)	(3.928)	
NIID	-5.708	1.315	0.000		1.009	-0.687	3.630	1.689	15.748	-156.174	-3.995	0.127	30.67%
Gaussian	(-1.508)	(2.525)	(-0.003)		(0.889)	(-0.693)	(1.864)	(1.153)	(1.331)	(-1.226)	(-2.565)	(4.003)	
NIID	-5.578	1.158	0.001		1.086	-0.657	3.287	1.683	14.804	-152.218	-3.935	0.125	30.97%
Student-t	(-1.484)	(2.621)	(0.015)		(0.943)	(-0.661)	(1.648)	(1.145)	(1.219)	(-1.192)	(-2.569)	(3.943)	
NIID Skewed	-5.391	1.053	0.001		1.061	-0.652	3.250	1.645	15.011	-152.257	-3.917	0.125	30.78%
Student-t	(-1.437)	(2.555)	(0.014)		(0.922)	(-0.653)	(1.613)	(1.116)	(1.226)	(-1.184)	(-2.557)	(3.926)	

(Continued)

Table 3.10: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Returns	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel 3: Full sample July 1962 - June 2013							
IID	-3.804	1.763	-0.037	-9.559	-0.169	-0.528	3.624	0.377	29.324	-91.413	-1.656	0.092	12.26%
NonPara	(-4.611)	(5.171)	(-1.002)	(-9.071)	(-1.079)	(-1.864)	(2.906)	(2.459)	(7.847)	(-1.792)	(-2.045)	(3.438)	
IID	-3.704	1.449	-0.031	-9.700	-0.143	-0.523	3.484	0.407	31.013	-103.589	-1.678	0.096	12.59%
Gaussian	(-4.869)	(5.303)	(-0.840)	(-8.873)	(-0.954)	(-1.812)	(2.723)	(2.689)	(8.375)	(-2.060)	(-2.020)	(3.485)	
IID	-3.982	1.488	-0.034	-9.596	-0.139	-0.515	3.661	0.399	30.792	-98.581	-1.726	0.096	12.65%
Student-t	(-4.872)	(5.176)	(-0.914)	(-8.914)	(-0.919)	(-1.799)	(2.891)	(2.647)	(8.278)	(-1.957)	(-2.060)	(3.516)	
IID Skewed	-3.347	1.279	-0.031	-9.487	-0.182	-0.523	3.576	0.373	29.461	-97.979	-1.640	0.091	11.86%
Student-t	(-4.259)	(4.777)	(-0.852)	(-8.944)	(-1.165)	(-1.836)	(2.882)	(2.445)	(7.948)	(-1.911)	(-2.006)	(3.373)	
NIID	-3.233	1.171	-0.033	-9.600	-0.145	-0.506	4.109	0.388	29.807	-94.775	-1.711	0.092	11.86%
Gaussian	(-4.136)	(4.290)	(-0.905)	(-8.813)	(-0.961)	(-1.747)	(3.276)	(2.585)	(8.116)	(-1.867)	(-2.058)	(3.479)	
NIID	-3.146	1.030	-0.033	-9.621	-0.139	-0.497	4.096	0.395	29.657	-93.474	-1.697	0.092	11.84%
Student-t	(-4.080)	(4.300)	(-0.899)	(-8.819)	(-0.922)	(-1.718)	(3.263)	(2.619)	(8.125)	(-1.840)	(-2.050)	(3.460)	
NIID Skewed	-3.092	0.949	-0.032	-9.509	-0.141	-0.497	4.109	0.408	29.620	-93.607	-1.696	0.092	11.77%
Student-t	(-4.005)	(4.224)	(-0.880)	(-8.778)	(-0.938)	(-1.720)	(3.295)	(2.692)	(8.143)	(-1.841)	(-2.050)	(3.452)	

Table 3.11: MS-BDL Investigation: GJR GARCH NIID measures – One month measures

This table shows the results of the MS-BDL framework for one-month-sample GJR GARCH NIID tail risk measures using the expanded set of state variables. Specifically, monthly market excess return at time $t+1$ is regressed on a GJR GARCH NIID tail risk measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). All parametric VaRs are at 1% level of significance. The sample period is July 1962 – June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel A: Gaussian measures							
	1	-2.496	1.748	-0.036	-0.156	0.204	3.473	0.059	24.408	-42.259	-0.497	0.007	9.315	7.041
Raw		(-3.019)	(6.967)	(-0.793)	(-0.805)	(0.825)	(1.997)	(0.321)	(4.917)	(-0.822)	(-0.747)	(0.301)		
Gaussian	2	-5.388	0.333	0.098	-0.778	-2.712	9.212	1.162	28.760	-209.532	-3.165	0.132	26.818	2.531
		(-2.517)	(0.606)	(0.925)	(-1.182)	(-3.237)	(2.134)	(2.367)	(2.330)	(-1.344)	(-1.605)	(2.755)		
	1	-4.015	2.361	-0.130	-0.262	-0.641	4.947	0.458	21.179	-49.879	-0.678	0.011	9.170	3.294
AR4		(-3.760)	(6.883)	(-2.721)	(-1.040)	(-2.038)	(2.289)	(2.245)	(3.888)	(-0.855)	(-0.944)	(0.439)		
Gaussian	2	-3.518	0.171	0.183	-0.326	-0.302	5.020	0.284	35.788	-105.678	-2.391	0.080	17.480	1.926
		(-2.065)	(0.333)	(1.868)	(-0.863)	(-0.533)	(1.718)	(0.781)	(4.130)	(-0.937)	(-1.734)	(2.062)		
	1	-2.736	2.231	-0.150	-0.414	0.171	4.832	-0.038	17.764	-13.475	-0.155	-0.005	8.223	5.748
FOSA		(-2.501)	(5.369)	(-3.008)	(-1.770)	(0.583)	(2.405)	(-0.145)	(2.244)	(-0.218)	(-0.200)	(-0.219)		
Gaussian	2	-7.792	1.716	-0.003	-0.238	-1.762	6.937	1.114	34.737	-199.483	-3.754	0.145	23.575	3.341
		(-3.958)	(2.712)	(-0.067)	(-0.537)	(-3.069)	(2.181)	(2.759)	(3.708)	(-1.640)	(-2.355)	(3.613)		

(Continued)

Table 3.11: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel B: Studen-t measures														
	1	-2.341	1.616	-0.040	-0.137	0.236	3.507	0.037	24.369	-40.051	-0.474	0.005	9.114	6.661
Raw		(-2.793)	(6.976)	(-0.888)	(-0.703)	(0.942)	(1.983)	(0.190)	(4.824)	(-0.785)	(-0.722)	(0.225)		
Student-t	2	-5.573	0.370	0.093	-0.741	-2.638	8.754	1.190	29.172	-210.666	-3.188	0.134	26.364	2.536
		(-2.629)	(0.714)	(0.914)	(-1.174)	(-3.259)	(2.069)	(2.467)	(2.419)	(-1.373)	(-1.659)	(2.820)		
	1	-3.057	2.006	-0.036	-0.130	0.281	3.833	0.030	24.591	-45.595	-0.483	0.006	8.957	6.408
AR4		(-3.437)	(7.061)	(-0.787)	(-0.652)	(1.123)	(2.136)	(0.165)	(4.715)	(-0.882)	(-0.724)	(0.262)		
Student-t	2	-5.267	0.312	0.076	-0.726	-2.581	8.862	1.132	28.738	-193.813	-3.202	0.132	26.034	2.611
		(-2.456)	(0.516)	(0.767)	(-1.205)	(-3.294)	(2.172)	(2.463)	(2.477)	(-1.291)	(-1.712)	(2.874)		
	1	-2.531	2.073	-0.148	-0.393	0.229	4.847	-0.079	18.047	-12.606	-0.150	-0.006	7.892	5.430
FOSA		(-2.372)	(5.491)	(-2.964)	(-1.724)	(0.809)	(2.403)	(-0.318)	(2.178)	(-0.238)	(-0.198)	(-0.206)		
Student-t	2	-7.426	1.452	-0.003	-0.233	-1.751	6.780	1.119	33.453	-192.043	-3.558	0.142	23.400	3.373
		(-3.947)	(2.605)	(-0.088)	(-0.544)	(-3.144)	(2.187)	(2.867)	(3.713)	(-1.618)	(-2.291)	(3.641)		

(Continued)

Table 3.11: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel C: Skewed Student-t measures														
Raw	1	-2.297	1.518	-0.043	-0.152	0.288	3.442	0.072	24.013	-44.800	-0.424	0.002	8.817	5.984
Skewed		(-2.720)	(6.928)	(-0.938)	(-0.735)	(1.136)	(1.827)	(0.366)	(4.494)	(-0.877)	(-0.646)	(0.123)		
Student-t	2	-5.414	0.335	0.085	-0.654	-2.487	8.274	1.156	29.143	-196.573	-3.150	0.133	25.612	2.552
		(-2.710)	(0.737)	(0.887)	(-1.127)	(-3.242)	(2.103)	(2.516)	(2.544)	(-1.347)	(-1.716)	(2.879)		
AR4	1	-2.936	1.858	-0.041	-0.158	0.337	3.845	0.069	24.056	-50.548	-0.421	0.003	8.618	5.721
Skewed		(-3.181)	(6.930)	(-0.874)	(-0.729)	(1.312)	(1.985)	(0.341)	(4.313)	(-0.961)	(-0.618)	(0.109)		
Student-t	2	-5.097	0.291	0.069	-0.623	-2.402	8.236	1.087	28.767	-180.199	-3.161	0.131	25.219	2.645
		(-2.483)	(0.548)	(0.738)	(-1.144)	(-3.292)	(2.179)	(2.451)	(2.618)	(-1.264)	(-1.777)	(2.934)		
FOSA	1	-2.355	1.898	-0.147	-0.408	0.252	4.804	-0.049	17.169	-16.194	-0.108	-0.006	7.655	5.240
Skewed		(-2.210)	(5.546)	(-2.987)	(-1.804)	(0.899)	(2.376)	(-0.194)	(1.922)	(-0.312)	(-0.164)	(-0.262)		
Student-t	2	-7.129	1.267	-0.004	-0.228	-1.723	6.670	1.114	32.903	-185.564	-3.499	0.141	23.183	3.466
		(-3.807)	(2.623)	(-0.074)	(-0.553)	(-3.077)	(2.226)	(2.925)	(3.887)	(-1.600)	(-2.346)	(3.794)		

Table 3.12: BDL regression in different periods: GJR GARCH NIID measures – One month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods. In each regression, the monthly market excess return at time $t+1$ is regressed on two-month-sample GJR GARCH NIID measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets).

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
				Panel A: Original sample July 1962 - December 2005									
Raw	-3.939	1.332	0.004	-16.539	-0.083	-0.390	4.771	0.478	33.103	-82.410	-0.749	-0.012	9.93%
Gaussian	(-4.179)	(4.345)	(0.091)	(-3.568)	(-0.443)	(-1.526)	(2.632)	(2.771)	(6.342)	(-1.415)	(-1.020)	(-0.309)	
AR4	-4.653	1.700	0.003	-16.637	-0.069	-0.377	4.813	0.487	33.531	-85.834	-0.796	-0.011	9.92%
Gaussian	(-4.704)	(4.853)	(0.089)	(-8.432)	(-0.395)	(-1.316)	(3.073)	(2.895)	(6.574)	(-1.701)	(-1.097)	(-0.326)	
FOSA	-4.815	1.807	-0.070	-10.458	-0.098	-0.468	4.944	0.437	32.746	-60.566	-0.835	-0.010	9.45%
Gaussian	(-4.588)	(4.393)	(-1.833)	(-9.181)	(-0.553)	(-1.614)	(3.102)	(2.658)	(6.441)	(-1.245)	(-1.136)	(-0.295)	
Raw	-3.978	1.263	-0.001	-15.647	-0.068	-0.385	4.782	0.482	33.363	-79.957	-0.755	-0.012	10.04%
Student-t	(-4.580)	(4.920)	(-0.021)	(-8.715)	(-0.393)	(-1.325)	(3.052)	(2.873)	(6.639)	(-1.596)	(-1.048)	(-0.360)	
AR4	-4.612	1.575	0.002	-15.942	-0.056	-0.371	4.818	0.490	33.653	-84.351	-0.795	-0.011	10.01%
Student-t	(-4.735)	(4.927)	(0.061)	(-8.666)	(-0.320)	(-1.289)	(3.092)	(2.906)	(6.628)	(-1.670)	(-1.095)	(-0.322)	
FOSA	-4.851	1.705	-0.071	-10.581	-0.082	-0.455	5.021	0.446	33.095	-59.652	-0.830	-0.010	9.54%
Student-t	(-4.629)	(4.490)	(-1.877)	(-9.209)	(-0.462)	(-1.568)	(3.176)	(2.694)	(6.493)	(-1.227)	(-1.128)	(-0.306)	
Raw Skewed	-3.997	1.181	-0.002	-14.999	-0.072	-0.385	4.796	0.514	33.334	-81.262	-0.748	-0.012	9.90%
Student-t	(-4.495)	(4.731)	(-0.055)	(-8.704)	(-0.415)	(-1.330)	(3.096)	(3.024)	(6.672)	(-1.627)	(-1.036)	(-0.357)	
AR4 Skewed	-4.580	1.454	0.001	-15.244	-0.061	-0.372	4.831	0.521	33.602	-85.489	-0.786	-0.010	9.86%
Student-t	(-4.609)	(4.721)	(0.021)	(-8.642)	(-0.347)	(-1.295)	(3.138)	(3.052)	(6.650)	(-1.696)	(-1.081)	(-0.317)	
FOSA Skewed	-4.832	1.582	-0.071	-10.426	-0.082	-0.452	5.040	0.480	33.214	-61.790	-0.828	-0.010	9.46%
Student-t	(-4.516)	(4.346)	(-1.859)	(-9.154)	(-0.463)	(-1.565)	(3.227)	(2.870)	(6.516)	(-1.276)	(-1.124)	(-0.313)	

(Continued)

Table 3.12: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel B: New sample January 2006 - June 2013													
Raw	-4.083	0.619	0.043		0.222	-0.880	2.604	1.663	21.937	-198.757	-3.425	0.125	27.84%
Gaussian	(-1.036)	(1.074)	(0.404)		(0.181)	(-0.905)	(0.850)	(1.104)	(1.543)	(-1.386)	(-2.614)	(4.234)	
AR4	-3.641	0.560	0.028		-0.080	-0.987	3.448	1.534	25.165	-195.872	-3.450	0.124	27.27%
Gaussian	(-0.913)	(0.771)	(0.266)		(-0.066)	(-1.000)	(1.164)	(1.033)	(1.770)	(-1.337)	(-2.582)	(4.235)	
FOSA	-6.069	1.388	0.001		0.978	-0.704	3.456	1.812	16.322	-164.880	-3.983	0.127	30.63%
Gaussian	(-1.558)	(2.570)	(0.017)		(0.916)	(-0.721)	(1.704)	(1.221)	(1.433)	(-1.285)	(-2.526)	(3.967)	
Raw	-3.975	0.518	0.041		0.182	-0.869	2.742	1.683	22.191	-198.499	-3.406	0.124	27.71%
Student-t	(-1.005)	(1.014)	(0.386)		(0.146)	(-0.873)	(0.895)	(1.104)	(1.533)	(-1.375)	(-2.605)	(4.229)	
AR4	-3.320	0.409	0.025		-0.205	-1.021	3.666	1.509	26.250	-195.371	-3.404	0.122	27.09%
Student-t	(-0.834)	(0.643)	(0.237)		(-0.167)	(-1.011)	(1.198)	(1.007)	(1.788)	(-1.321)	(-2.586)	(4.255)	
FOSA	-6.012	1.228	0.004		1.030	-0.686	3.176	1.858	15.424	-162.596	-3.907	0.125	30.85%
Student-t	(-1.545)	(2.644)	(0.046)		(0.960)	(-0.701)	(1.530)	(1.236)	(1.320)	(-1.264)	(-2.510)	(3.901)	
Raw Skewed	-3.781	0.447	0.039		0.125	-0.893	2.845	1.632	22.749	-197.616	-3.397	0.123	27.57%
Student-t	(-0.960)	(0.939)	(0.364)		(0.099)	(-0.890)	(0.910)	(1.073)	(1.541)	(-1.361)	(-2.601)	(4.228)	
AR4 Skewed	-3.109	0.332	0.022		-0.273	-1.052	3.783	1.467	26.916	-194.174	-3.394	0.122	26.99%
Student-t	(-0.789)	(0.569)	(0.211)		(-0.221)	(-1.034)	(1.214)	(0.980)	(1.802)	(-1.306)	(-2.581)	(4.253)	
FOSA Skewed	-5.760	1.093	0.004		1.026	-0.678	3.112	1.802	15.440	-162.308	-3.889	0.124	30.73%
Student-t	(-1.492)	(2.593)	(0.051)		(0.951)	(-0.689)	(1.485)	(1.199)	(1.308)	(-1.254)	(-2.501)	(3.887)	

(Continued)

Table 3.12: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel C: Full sample July 1962 - June 2013							
Raw	-2.552	0.822	0.020	-13.127	-0.171	-0.477	3.288	0.412	29.692	-113.579	-1.589	0.099	11.50%
Gaussian	(-3.297)	(3.748)	(0.471)	(-2.922)	(-0.981)	(-1.917)	(2.090)	(2.538)	(7.095)	(-2.187)	(-2.433)	(4.957)	
AR4	-2.887	1.005	0.017	-13.026	-0.169	-0.471	3.446	0.409	29.699	-113.875	-1.635	0.099	11.37%
Gaussian	(-3.703)	(3.880)	(0.447)	(-7.859)	(-1.122)	(-1.659)	(2.619)	(2.725)	(8.230)	(-2.192)	(-2.079)	(3.810)	
FOSA	-3.472	1.281	-0.033	-9.560	-0.153	-0.507	3.907	0.395	30.285	-94.092	-1.720	0.092	11.95%
Gaussian	(-4.085)	(4.113)	(-0.886)	(-8.755)	(-1.002)	(-1.765)	(3.090)	(2.603)	(8.207)	(-1.849)	(-2.050)	(3.456)	
Raw	-2.451	0.731	0.016	-12.316	-0.168	-0.476	3.299	0.407	29.533	-111.859	-1.598	0.099	11.42%
Student-t	(-3.539)	(3.932)	(0.427)	(-8.148)	(-1.119)	(-1.667)	(2.492)	(2.710)	(8.286)	(-2.147)	(-2.045)	(3.770)	
AR4	-2.703	0.867	0.015	-12.328	-0.169	-0.471	3.441	0.402	29.428	-112.727	-1.630	0.099	11.25%
Student-t	(-3.236)	(3.516)	(0.366)	(-2.762)	(-0.964)	(-1.887)	(2.190)	(2.474)	(7.014)	(-2.166)	(-2.491)	(4.933)	
FOSA	-3.374	1.154	-0.032	-9.601	-0.144	-0.500	3.894	0.396	30.238	-93.286	-1.707	0.092	11.98%
Student-t	(-4.076)	(4.174)	(-0.874)	(-8.768)	(-0.948)	(-1.739)	(3.085)	(2.611)	(8.268)	(-1.832)	(-2.038)	(3.422)	
Raw Skewed	-2.391	0.653	0.014	-11.803	-0.174	-0.479	3.337	0.420	29.353	-111.519	-1.594	0.099	11.28%
Student-t	(-3.418)	(3.741)	(0.374)	(-8.207)	(-1.158)	(-1.677)	(2.540)	(2.784)	(8.290)	(-2.142)	(-2.042)	(3.771)	
AR4 Skewed	-2.597	0.762	0.013	-11.799	-0.176	-0.475	3.477	0.414	29.230	-112.125	-1.625	0.099	11.11%
Student-t	(-3.342)	(3.445)	(0.349)	(-8.008)	(-1.162)	(-1.669)	(2.667)	(2.752)	(8.207)	(-2.157)	(-2.076)	(3.843)	
FOSA Skewed	-3.270	1.030	-0.031	-9.476	-0.148	-0.499	3.914	0.416	30.134	-94.122	-1.703	0.092	11.88%
Student-t	(-3.964)	(4.065)	(-0.851)	(-8.728)	(-0.972)	(-1.740)	(3.124)	(2.722)	(8.291)	(-1.849)	(-2.037)	(3.415)	

Table 3.13: MS-BDL Investigation: ETL measures – one month measures

This table shows the results of the MS-BDL framework for one-month-sample Gaussian ETL tail risk measures using the expanded set of state variables. Specifically, monthly market excess return at time $t+1$ is regressed on Gaussian ETL tail risk measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). All parametric VaRs are at 1% level of significance. The sample period is July 1962 – June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel A: IID measures														
	1	-1.501	1.354	0.020	-0.160	0.369	2.827	-0.020	23.441	-72.255	-0.608	0.004	8.612	5.193
Raw IID		(-1.916)	(6.829)	(0.396)	(-0.704)	(1.424)	(1.409)	(-0.112)	(4.253)	(-1.381)	(-0.668)	(0.163)		
	2	-3.165	-0.395	-0.013	-0.535	-2.340	10.466	0.935	23.827	-124.926	-2.335	0.102	24.736	2.536
		(-1.930)	(-1.267)	(-0.122)	(-0.957)	(-3.478)	(2.633)	(2.329)	(2.313)	(-0.911)	(-1.007)	(2.245)		
	1	-2.700	1.970	-0.018	-0.145	0.365	3.039	-0.046	24.208	-66.155	-0.520	0.003	8.261	4.944
AR4		(-2.963)	(6.962)	(-0.396)	(-0.605)	(1.417)	(1.461)	(-0.231)	(4.120)	(-1.249)	(-0.709)	(0.118)		
IID	2	-3.624	-0.245	0.030	-0.590	-2.260	9.249	0.988	25.185	-141.181	-2.516	0.111	24.624	2.565
		(-1.943)	(-0.504)	(0.307)	(-1.132)	(-3.385)	(2.533)	(2.466)	(2.509)	(-1.071)	(-1.326)	(2.587)		
	1	-2.503	2.129	-0.172	-0.508	0.201	4.322	0.018	10.370	-36.329	0.039	-0.005	7.220	5.931
FOSA		(-2.233)	(5.233)	(-3.289)	(-2.308)	(0.702)	(2.199)	(0.098)	(1.303)	(-0.682)	(0.106)	(-0.253)		
IID	2	-7.450	1.669	-0.027	-0.215	-1.495	5.401	0.953	33.060	-176.433	-3.428	0.154	22.673	4.515
		(-4.275)	(3.587)	(-0.385)	(-0.602)	(-2.911)	(1.955)	(2.841)	(4.755)	(-1.672)	(-2.572)	(4.449)		

(Continued)

Table 3.13: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel B: NIID measures														
	1	-3.008	1.590	-0.173	-0.222	-0.672	4.170	0.392	20.874	-32.781	-0.695	0.007	9.449	3.924
Raw		(-3.468)	(7.411)	(-3.865)	(-0.969)	(-2.231)	(2.017)	(2.013)	(4.167)	(-0.592)	(-1.012)	(0.301)		
NIID	2	-4.169	0.223	0.188	-0.349	-0.311	5.420	0.412	38.241	-135.354	-2.459	0.081	17.872	1.884
		(-2.528)	(0.605)	(1.892)	(-0.871)	(-0.507)	(1.745)	(1.045)	(3.989)	(-1.112)	(-1.661)	(1.939)		
	1	-3.674	1.915	-0.175	-0.226	-0.647	4.607	0.411	20.902	-42.473	-0.722	0.009	9.318	3.733
AR(4)		(-3.877)	(7.400)	(-3.872)	(-0.977)	(-2.140)	(2.219)	(2.105)	(4.059)	(-0.756)	(-1.026)	(0.363)		
NIID	2	-3.937	0.179	0.182	-0.333	-0.324	5.338	0.370	36.985	-121.012	-2.390	0.078	17.638	1.907
		(-2.302)	(0.414)	(1.853)	(-0.857)	(-0.540)	(1.764)	(0.973)	(4.038)	(-1.020)	(-1.660)	(1.948)		
	1	-2.308	1.766	-0.149	-0.387	0.177	5.100	-0.076	17.374	-11.172	-0.220	-0.005	8.295	5.485
FOSA		(-2.255)	(5.504)	(-3.019)	(-1.822)	(0.655)	(2.592)	(-0.327)	(2.504)	(-0.198)	(-0.314)	(-0.179)		
NIID	2	-7.530	1.359	0.007	-0.210	-1.827	7.508	1.146	35.483	-216.296	-3.675	0.141	23.668	3.077
		(-3.797)	(2.544)	(0.091)	(-0.441)	(-3.036)	(2.328)	(2.850)	(3.583)	(-1.733)	(-2.265)	(3.468)		

Table 3.14: BDL regression in different periods: ETL risk measures – One month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on one-month-sample Gaussian ETL measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). Parametric VaRs are at 1% level of significance.

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
				Panel A: Original sample July 1962 - December 2005									
Raw IID	-2.620	0.705	0.013	-16.983	-0.154	-0.435	4.949	0.401	29.661	-79.151	-0.770	-0.008	8.35%
	(-3.419)	(3.370)	(0.319)	(-6.400)	(-0.907)	(-1.452)	(3.153)	(2.520)	(6.037)	(-1.549)	(-1.051)	(-0.224)	
AR4 IID	-3.768	1.197	0.004	-16.902	-0.120	-0.421	4.818	0.438	31.429	-85.882	-0.781	-0.008	8.96%
	(-4.187)	(4.173)	(0.104)	(-7.486)	(-0.700)	(-1.422)	(3.067)	(2.716)	(6.246)	(-1.689)	(-1.067)	(-0.236)	
FOSA IID	-4.594	1.558	-0.064	-10.474	-0.119	-0.495	4.637	0.433	32.191	-72.266	-0.811	-0.005	9.83%
	(-4.835)	(4.885)	(-1.671)	(-9.295)	(-0.686)	(-1.686)	(2.901)	(2.657)	(6.412)	(-1.474)	(-1.111)	(-0.140)	
Raw NIID	-3.804	1.103	-0.031	-16.154	-0.084	-0.414	4.707	0.474	32.995	-82.756	-0.793	-0.010	9.91%
	(-4.583)	(4.662)	(-0.822)	(-8.291)	(-0.490)	(-1.427)	(2.935)	(2.925)	(6.624)	(-1.681)	(-1.098)	(-0.295)	
AR4 NIID	-4.334	1.346	-0.029	-16.599	-0.074	-0.392	4.727	0.481	33.220	-87.844	-0.817	-0.010	9.92%
	(-4.697)	(4.644)	(-0.758)	(-8.118)	(-0.429)	(-1.370)	(2.964)	(2.945)	(6.590)	(-1.771)	(-1.125)	(-0.301)	
FOSA NIID	-4.292	1.364	-0.068	-10.437	-0.094	-0.467	5.168	0.423	31.770	-64.397	-0.846	-0.007	9.07%
	(-4.345)	(4.103)	(-1.803)	(-9.209)	(-0.541)	(-1.599)	(3.255)	(2.648)	(6.478)	(-1.325)	(-1.151)	(-0.200)	

(Continued)

Table 3.14: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel B: New sample January 2006 - June 2013													
Raw IID	-0.843	-0.244	-0.047		-1.392	-1.548	5.988	1.079	37.761	-181.217	-3.372	0.117	27.01%
	(-0.217)	(-0.528)	(-0.427)		(-1.057)	(-1.664)	(1.617)	(0.699)	(2.116)	(-1.161)	(-2.430)	(4.429)	
AR4 IID	-1.341	-0.135	-0.015		-1.107	-1.424	5.282	1.172	34.981	-184.034	-3.311	0.117	26.68%
	(-0.335)	(-0.224)	(-0.159)		(-0.852)	(-1.524)	(1.628)	(0.766)	(2.057)	(-1.191)	(-2.575)	(4.548)	
FOSA IID	-7.324	1.522	-0.006		1.379	-0.630	2.347	2.108	14.054	-156.388	-4.023	0.131	32.22%
	(-1.815)	(2.858)	(-0.071)		(1.186)	(-0.672)	(1.210)	(1.398)	(1.286)	(-1.289)	(-2.530)	(3.974)	
Raw NIID	-4.153	0.558	0.025		0.339	-0.894	2.376	1.671	20.715	-194.761	-3.462	0.125	28.14%
	(-1.134)	(1.347)	(0.260)		(0.294)	(-0.959)	(0.833)	(1.137)	(1.547)	(-1.361)	(-2.649)	(4.268)	
AR4 NIID	-3.842	0.559	0.022		0.114	-0.974	2.997	1.546	23.082	-196.642	-3.422	0.124	27.68%
	(-1.061)	(1.090)	(0.228)		(0.101)	(-1.047)	(1.041)	(1.065)	(1.711)	(-1.352)	(-2.644)	(4.340)	
FOSA NIID	-5.718	1.148	0.000		1.011	-0.686	3.625	1.689	15.736	-156.261	-3.995	0.127	30.68%
	(-1.509)	(2.526)	(-0.002)		(0.890)	(-0.692)	(1.860)	(1.153)	(1.329)	(-1.227)	(-2.565)	(4.003)	

(Continued)

Table 3.14: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel C: Full sample January 2006 - June 2013													
Raw IID	-1.541	0.385	0.021	-12.894	-0.231	-0.516	3.629	0.331	27.542	-103.812	-1.531	0.092	10.20%
	(-2.310)	(2.009)	(0.552)	(-5.271)	(-1.573)	(-1.776)	(2.715)	(2.257)	(7.899)	(-2.011)	(-2.046)	(3.933)	
AR4 IID	-2.294	0.694	0.018	-13.092	-0.199	-0.499	3.581	0.362	28.674	-108.252	-1.574	0.094	10.64%
	(-2.987)	(2.749)	(0.477)	(-6.331)	(-1.355)	(-1.727)	(2.743)	(2.463)	(8.094)	(-2.111)	(-2.055)	(3.835)	
FOSA IID	-3.676	1.250	-0.031	-9.699	-0.142	-0.523	3.496	0.406	30.924	-103.491	-1.681	0.096	12.56%
	(-4.857)	(5.292)	(-0.834)	(-8.878)	(-0.950)	(-1.811)	(2.732)	(2.685)	(8.369)	(-2.058)	(-2.025)	(3.483)	
Raw NIID	-2.522	0.702	-0.003	-13.043	-0.167	-0.491	3.256	0.411	29.706	-112.677	-1.612	0.099	11.63%
	(-3.877)	(4.333)	(-0.075)	(-8.134)	(-1.126)	(-1.722)	(2.425)	(2.773)	(8.340)	(-2.181)	(-2.068)	(3.711)	
AR4 NIID	-2.811	0.839	-0.002	-13.231	-0.165	-0.480	3.354	0.411	29.688	-114.916	-1.620	0.099	11.56%
	(-3.948)	(4.172)	(-0.043)	(-7.906)	(-1.109)	(-1.694)	(2.509)	(2.777)	(8.306)	(-2.225)	(-2.085)	(3.760)	
FOSA NIID	-3.234	1.018	-0.033	-9.600	-0.145	-0.506	4.110	0.388	29.782	-94.816	-1.712	0.092	11.84%
	(-4.135)	(4.288)	(-0.904)	(-8.816)	(-0.958)	(-1.747)	(3.277)	(2.586)	(8.116)	(-1.867)	(-2.061)	(3.482)	

Table 3.15: MS-BDL Investigation: 5 percent Skewed Student-t measures – One month measures

This table shows the results of the MS-BDL framework for one-month-sample 5-percent Skewed Student-t tail risk measures using the expanded set of state variables. In each model, monthly market excess return at time $t+1$ is regressed on a tail risk measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). The sample period is July 1962 – June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel A: IID measures							
	1	-1.162	2.160	0.077	-0.082	0.417	2.319	-0.080	21.394	-70.681	-0.598	0.006	8.136	4.126
Raw IID		(-1.334)	(6.374)	(1.428)	(-0.294)	(1.532)	(1.034)	(-0.363)	(3.142)	(-1.277)	(-0.561)	(0.214)		
	2	-3.248	-0.634	-0.007	-0.632	-2.122	9.535	0.960	24.150	-101.546	-2.237	0.102	24.025	2.418
		(-2.098)	(-1.281)	(-0.110)	(-1.254)	(-3.256)	(2.548)	(2.443)	(2.587)	(-0.790)	(-0.908)	(2.257)		
	1	-0.778	3.328	0.080	-0.169	0.343	-0.410	-0.749	21.898	-26.518	-1.453	0.031	4.423	2.355
AR4		(-0.846)	(8.281)	(1.672)	(-0.726)	(1.308)	(-0.227)	(-3.358)	(3.996)	(-0.530)	(-1.904)	(1.329)		
IID	2	-3.369	-0.212	0.003	-0.374	-1.158	7.498	1.036	26.294	-124.415	-0.796	0.090	22.176	3.010
		(-2.549)	(-0.344)	(0.115)	(-1.324)	(-2.646)	(2.916)	(3.874)	(3.892)	(-1.376)	(-0.764)	(2.714)		
	1	-2.371	3.512	-0.195	-0.534	0.164	4.766	0.176	5.218	-38.131	0.176	-0.008	6.143	5.135
FOSA		(-1.852)	(4.845)	(-3.503)	(-2.199)	(0.514)	(2.445)	(0.785)	(0.685)	(-0.640)	(0.273)	(-0.348)		
IID	2	-6.297	2.413	-0.042	-0.326	-1.107	4.399	0.757	32.640	-158.650	-2.994	0.143	21.906	5.387
		(-4.099)	(3.275)	(-0.662)	(-1.070)	(-2.696)	(1.779)	(2.563)	(5.148)	(-1.722)	(-2.487)	(4.552)		

(Continued)

Table 3.15: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel B: NIID measures														
	1	-2.042	2.302	-0.091	-0.208	0.174	3.600	0.056	23.163	-47.983	-0.423	-0.001	8.944	6.464
Raw		(-2.485)	(6.979)	(-1.972)	(-1.044)	(0.688)	(1.923)	(0.262)	(4.480)	(-0.939)	(-0.668)	(-0.064)		
NIID	2	-5.913	0.857	0.085	-0.618	-2.379	8.000	1.191	32.095	-232.726	-3.234	0.142	25.947	2.609
		(-2.956)	(1.177)	(0.879)	(-1.062)	(-3.024)	(1.996)	(2.553)	(2.684)	(-1.567)	(-1.752)	(3.037)		
	1	-2.499	2.647	-0.081	-0.198	0.187	3.765	0.058	23.080	-54.480	-0.441	0.001	9.057	6.657
AR4		(-2.992)	(7.065)	(-1.752)	(-1.009)	(0.723)	(2.058)	(0.291)	(4.527)	(-1.053)	(-0.700)	(0.068)		
NIID	2	-5.621	0.722	0.081	-0.657	-2.450	8.532	1.144	31.025	-217.650	-3.195	0.137	26.214	2.625
		(-2.590)	(0.802)	(0.823)	(-1.108)	(-3.065)	(2.093)	(2.428)	(2.588)	(-1.428)	(-1.720)	(2.905)		
	1	-1.868	2.674	-0.153	-0.444	0.198	5.116	-0.056	16.228	-24.178	-0.143	-0.010	7.948	5.101
FOSA		(-1.928)	(5.765)	(-3.081)	(-2.061)	(0.710)	(2.581)	(-0.223)	(2.098)	(-0.429)	(-0.185)	(-0.376)		
NIID	2	-6.943	1.927	0.010	-0.187	-1.783	7.308	1.087	34.586	-201.353	-3.485	0.142	23.246	3.115
		(-3.645)	(2.395)	(0.097)	(-0.411)	(-2.980)	(2.353)	(2.737)	(3.661)	(-1.667)	(-2.215)	(3.464)		

Table 3.16: BDL regression in different periods: 5 percent Skewed Student-t measures – One month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on one-month-sample 5-percent Skewed Student-t measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets).

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel A: Original sample July 1962 - December 2005							
Raw IID	-2.918	1.387	0.050	-16.141	-0.137	-0.431	5.056	0.420	29.903	-83.462	-0.734	-0.009	8.99%
	(-3.855)	(4.178)	(1.095)	(-8.055)	(-0.802)	(-1.421)	(3.250)	(2.651)	(6.151)	(-1.604)	(-0.982)	(-0.281)	
AR4 IID	-4.306	2.388	0.026	-15.452	-0.111	-0.432	4.838	0.461	31.990	-88.980	-0.723	-0.011	9.65%
	(-4.722)	(5.020)	(0.633)	(-9.037)	(-0.638)	(-1.443)	(3.114)	(2.837)	(6.282)	(-1.733)	(-0.977)	(-0.325)	
FOSA IID	-4.426	2.513	-0.067	-10.412	-0.153	-0.491	4.723	0.424	31.659	-70.940	-0.767	-0.010	9.39%
	(-4.507)	(4.780)	(-1.753)	(-9.363)	(-0.854)	(-1.677)	(3.089)	(2.555)	(6.085)	(-1.448)	(-1.050)	(-0.312)	
Raw NIID	-3.649	1.732	-0.028	-13.955	-0.105	-0.422	4.752	0.479	32.993	-90.224	-0.778	-0.008	9.72%
	(-4.422)	(4.357)	(-0.742)	(-8.611)	(-0.617)	(-1.460)	(2.971)	(2.958)	(6.585)	(-1.825)	(-1.081)	(-0.255)	
AR4 NIID	-3.988	2.008	-0.023	-14.298	-0.102	-0.406	4.770	0.479	32.868	-94.642	-0.797	-0.008	9.59%
	(-4.444)	(4.235)	(-0.598)	(-8.344)	(-0.597)	(-1.415)	(2.994)	(2.946)	(6.530)	(-1.891)	(-1.104)	(-0.252)	
FOSA NIID	-4.089	2.116	-0.068	-10.348	-0.112	-0.461	5.239	0.434	32.024	-73.490	-0.807	-0.008	8.96%
	(-4.112)	(3.778)	(-1.789)	(-9.110)	(-0.650)	(-1.583)	(3.318)	(2.707)	(6.383)	(-1.506)	(-1.099)	(-0.244)	

(Continued)

Table 3.16: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel B: New sample January 2006 - June 2013													
Raw IID	-0.055	-0.593	-0.084		-1.673	-1.666	6.133	0.881	40.584	-188.486	-3.421	0.115	27.58%
	(-0.014)	(-0.845)	(-0.711)		(-1.280)	(-1.852)	(1.898)	(0.554)	(2.256)	(-1.214)	(-2.403)	(4.434)	
AR4 IID	-1.683	-0.060	-0.006		-0.953	-1.356	5.005	1.220	33.631	-184.810	-3.324	0.118	26.63%
	(-0.396)	(-0.061)	(-0.061)		(-0.722)	(-1.462)	(1.663)	(0.772)	(1.951)	(-1.208)	(-2.578)	(4.565)	
FOSA IID	-7.572	2.447	-0.007		1.458	-0.588	1.929	2.311	12.531	-171.890	-3.848	0.127	32.07%
	(-1.777)	(2.680)	(-0.077)		(1.265)	(-0.644)	(0.963)	(1.460)	(1.095)	(-1.395)	(-2.476)	(3.950)	
Raw NIID	-4.119	0.847	0.024		0.356	-0.882	2.376	1.684	20.838	-193.781	-3.472	0.125	28.20%
	(-1.137)	(1.388)	(0.255)		(0.311)	(-0.946)	(0.847)	(1.146)	(1.583)	(-1.359)	(-2.651)	(4.276)	
AR4 NIID	-3.671	0.804	0.023		0.082	-0.980	3.002	1.530	23.560	-197.088	-3.389	0.124	27.67%
	(-1.036)	(1.076)	(0.238)		(0.074)	(-1.057)	(1.025)	(1.057)	(1.753)	(-1.354)	(-2.656)	(4.378)	
FOSA NIID	-5.506	1.711	0.001		1.055	-0.671	3.344	1.668	15.575	-150.853	-3.949	0.126	30.99%
	(-1.488)	(2.666)	(0.007)		(0.933)	(-0.680)	(1.692)	(1.144)	(1.318)	(-1.185)	(-2.585)	(3.976)	

(Continued)

Table 3.16: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel C: Full sample January 2006 - June 2013							
Raw IID	-1.592	0.666	0.036	-11.962	-0.225	-0.517	3.769	0.334	27.391	-102.007	-1.521	0.093	10.31%
	(-2.295)	(2.004)	(0.861)	(-5.981)	(-1.510)	(-1.769)	(2.890)	(2.296)	(7.923)	(-1.985)	(-2.014)	(3.971)	
AR4 IID	-2.537	1.305	0.030	-12.020	-0.188	-0.503	3.619	0.377	28.892	-107.809	-1.552	0.095	10.95%
	(-3.206)	(3.032)	(0.782)	(-7.342)	(-1.263)	(-1.732)	(2.832)	(2.566)	(8.171)	(-2.109)	(-2.002)	(3.769)	
FOSA IID	-3.369	1.893	-0.031	-9.583	-0.173	-0.520	3.456	0.397	30.396	-105.355	-1.630	0.093	12.04%
	(-4.426)	(4.962)	(-0.839)	(-8.855)	(-1.134)	(-1.811)	(2.749)	(2.587)	(8.176)	(-2.074)	(-1.983)	(3.400)	
Raw NIID	-2.371	1.054	-0.002	-11.537	-0.178	-0.495	3.314	0.412	29.706	-115.768	-1.606	0.099	11.49%
	(-3.732)	(4.160)	(-0.055)	(-8.455)	(-1.213)	(-1.742)	(2.485)	(2.786)	(8.419)	(-2.248)	(-2.059)	(3.729)	
AR4 NIID	-2.545	1.205	0.001	-11.701	-0.181	-0.488	3.389	0.408	29.529	-117.948	-1.597	0.098	11.35%
	(-3.725)	(3.912)	(0.028)	(-8.218)	(-1.229)	(-1.725)	(2.539)	(2.766)	(8.366)	(-2.288)	(-2.072)	(3.798)	
FOSA NIID	-3.010	1.512	-0.032	-9.511	-0.156	-0.502	4.115	0.395	29.903	-99.471	-1.678	0.092	11.79%
	(-3.993)	(4.166)	(-0.876)	(-8.750)	(-1.043)	(-1.737)	(3.286)	(2.625)	(8.160)	(-1.967)	(-2.025)	(3.416)	

Table 3.17: MS-BDL Investigation: 99 percent Skewed Student-t measures – One month measures

This table shows the results of the MS-BDL framework for one-month-sample 99 percent Skewed Student-t tail risk measures using the expanded set of state variables. Specifically, monthly market excess return at time $t+1$ is regressed on a tail risk measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). The sample period is July 1962 – June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel 1: IID measures							
	1	-1.225	1.316	-0.246	-0.492	-0.565	4.756	0.370	18.367	-86.781	-0.526	0.004	8.730	2.960
Raw IID		(-1.360)	(5.951)	(-4.973)	(-2.018)	(-1.342)	(1.729)	(1.619)	(3.025)	(-1.347)	(-0.686)	(0.124)		
	2	-2.222	-0.282	0.099	-0.303	-0.531	4.964	0.236	30.404	-67.980	-2.175	0.075	17.279	2.292
		(-1.783)	(-1.045)	(1.085)	(-0.902)	(-0.993)	(1.632)	(0.697)	(4.072)	(-0.657)	(-1.661)	(2.141)		
	1	-3.521	2.315	-0.239	-0.365	-0.666	4.795	0.418	20.045	-64.520	-0.779	0.012	8.259	2.916
AR4		(-3.529)	(7.494)	(-5.160)	(-1.610)	(-2.080)	(2.174)	(2.091)	(3.659)	(-1.118)	(-1.087)	(0.478)		
IID	2	-2.662	-0.193	0.130	-0.274	-0.439	4.922	0.270	31.811	-91.217	-1.892	0.072	16.666	2.074
		(-1.786)	(-0.452)	(1.607)	(-0.842)	(-0.938)	(1.875)	(0.831)	(4.110)	(-0.863)	(-1.506)	(2.101)		
	1	-2.191	2.060	-0.142	-0.427	0.176	4.969	-0.089	12.709	-27.307	-0.215	0.005	7.824	5.369
FOSA		(-2.211)	(5.791)	(-2.811)	(-2.033)	(0.636)	(2.628)	(-0.384)	(1.550)	(-0.510)	(-0.295)	(0.199)		
IID	2	-7.190	1.581		-0.153	-1.750	6.967	1.017	35.626	-203.637	-3.740	0.148	23.405	3.378
		(-3.789)	(2.707)	(0.084)	(-0.348)	(-3.065)	(2.241)	(2.617)	(3.841)	(-1.649)	(-2.385)	(3.710)		

(Continued)

Table 3.17: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel 2: NIID measures							
	1	-3.036	1.746	-0.189	-0.157	-0.646	4.183	0.314	20.728	-19.188	-0.676	0.009	9.620	4.143
Raw		(-3.478)	(7.184)	(-4.209)	(-0.652)	(-2.064)	(2.041)	(1.566)	(4.180)	(-0.349)	(-0.955)	(0.372)		
NIID	2	-4.427	0.296	0.175	-0.398	-0.316	5.406	0.469	39.615	-153.686	-2.527	0.083	18.377	1.871
		(-2.496)	(0.665)	(1.753)	(-0.928)	(-0.502)	(1.668)	(1.106)	(3.886)	(-1.199)	(-1.631)	(1.883)		
	1	-2.958	1.896	-0.104	-0.116	0.142	3.585	-0.001	23.309	-31.380	-0.569	0.009	9.441	7.783
AR4		(-3.391)	(6.674)	(-2.346)	(-0.344)	(0.254)	(2.099)	(-0.014)	(4.389)	(-0.642)	(-0.500)	(0.201)		
NIID	2	-5.785	0.451	0.079	-0.758	-2.660	9.477	1.218	30.067	-236.964	-3.005	0.132	27.541	2.601
		-2.276	0.620	0.708	-1.050	-2.954	2.011	2.362	2.285	-1.272	-1.205	2.569		
	1	-2.405	1.955	-0.143	-0.326	0.204	5.090	-0.133	17.687	-6.417	-0.310	-0.001	8.316	5.312
FOSA		(-2.424)	(5.801)	(-2.979)	(-1.531)	(0.761)	(2.632)	(-0.576)	(2.691)	(-0.116)	(-0.397)	(-0.077)		
NIID	2	(-7.055)	1.262	0.017	-0.256	-1.911	7.505	1.119	33.967	-208.175	-3.531	0.137	24.042	2.928
		(-3.403)	(2.013)	(0.190)	(-0.530)	(-3.028)	(2.273)	(2.786)	(3.395)	(-1.642)	(-2.096)	(3.240)		

Table 3.18: BDL regression in different periods: 99 percent Skewed Student-t measures – One month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on two-month-sample 99-percent Skewed Student-t measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets).

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel A: Original sample July 1962 - December 2005							
Raw IID	-1.680	0.378	-0.051	-12.544	-0.219	-0.470	5.152	0.323	27.097	-62.363	-0.850	-0.003	7.24%
	(-2.224)	(1.749)	(-1.328)	(-5.601)	(-1.273)	(-1.598)	(3.273)	(2.027)	(5.411)	(-1.237)	(-1.158)	(-0.098)	
AR4 IID	-3.085	0.993	-0.058	-13.355	-0.148	-0.438	4.885	0.391	30.138	-77.972	-0.845	-0.002	8.19%
	(-3.453)	(3.040)	(-1.502)	(-7.207)	(-0.875)	(-1.503)	(3.036)	(2.483)	(6.027)	(-1.563)	(-1.158)	(-0.074)	
FOSA IID	-4.224	1.518	-0.059	-10.368	-0.084	-0.472	4.910	0.411	31.166	-75.621	-0.948	0.004	9.37%
	(-4.546)	(4.102)	(-1.604)	(-9.280)	(-0.507)	(-1.633)	(2.927)	(2.680)	(6.619)	(-1.527)	(-1.281)	(0.114)	
Raw NIID	-3.763	1.169	-0.054	-14.802	-0.074	-0.427	4.716	0.448	32.262	-74.845	-0.788	-0.008	9.84%
	(-4.096)	(4.286)	(-1.202)	(-3.276)	(-0.392)	(-1.678)	(2.598)	(2.624)	(6.261)	(-1.291)	(-1.072)	(-0.219)	
AR4 NIID	-4.222	1.396	-0.051	-15.267	-0.070	-0.407	4.714	0.452	32.356	-79.563	-0.788	-0.009	9.87%
	(-4.648)	(4.615)	(-1.333)	(-8.274)	(-0.402)	(-1.401)	(2.919)	(2.782)	(6.422)	(-1.603)	(-1.076)	(-0.273)	
FOSA NIID	-4.010	1.347	-0.067	-10.564	-0.091	-0.459	5.331	0.393	30.507	-60.281	-0.896	-0.005	8.66%
	(-4.114)	(3.872)	(-1.825)	(-9.351)	(-0.522)	(-1.565)	(3.346)	(2.487)	(6.368)	(-1.234)	(-1.222)	(-0.152)	

(Continued)

Table 3.18: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
					Panel B: New sample January 2006 - June 2013								
Raw IID	-1.714	-0.052	-0.006		-0.981	-1.377	5.208	1.242	33.893	-182.022	-3.333	0.118	26.64%
	(-0.522)	(-0.145)	(-0.064)		(-0.914)	(-1.480)	(1.460)	(0.858)	(2.263)	(-1.177)	(-2.516)	(4.375)	
AR4 IID	-2.708	0.342	0.007		-0.428	-1.142	3.995	1.311	29.009	-187.964	-3.417	0.121	26.96%
	(-0.816)	(0.735)	(0.082)		(-0.421)	(-1.262)	(1.472)	(0.925)	(2.175)	(-1.272)	(-2.658)	(4.450)	
FOSA IID	-5.469	1.295	-0.002		0.721	-0.910	4.030	1.563	19.712	-147.149	-4.151	0.134	30.45%
	(-1.501)	(2.262)	(-0.020)		(0.666)	(-0.947)	(2.183)	(1.112)	(1.873)	(-1.173)	(-2.552)	(4.077)	
Raw NIID	-4.390	0.652	0.029		0.399	-0.867	2.287	1.733	20.169	-194.835	-3.459	0.125	28.23%
	(-1.176)	(1.393)	(0.296)		(0.340)	(-0.929)	(0.802)	(1.172)	(1.496)	(-1.368)	(-2.653)	(4.272)	
AR4 NIID	-3.869	0.601	0.025		0.082	-0.979	3.039	1.565	23.401	-197.355	-3.393	0.124	27.61%
	(-1.052)	(1.039)	(0.256)		(0.072)	(-1.052)	(1.028)	(1.077)	(1.701)	(-1.355)	(-2.651)	(4.363)	
FOSA NIID	-6.129	1.336	0.000		1.131	-0.655	3.370	1.774	14.648	-153.853	-3.974	0.126	31.13%
	(-1.585)	(2.643)	(0.004)		(0.973)	(-0.659)	(1.706)	(1.203)	(1.220)	(-1.218)	(-2.572)	(3.976)	

(Continued)

Table 3.18: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel C: Full sample January 2006 - June 2013													
Raw IID	-1.078	0.250	-0.018	-10.897	-0.272	-0.533	3.725	0.288	26.564	-98.112	-1.555	0.090	9.80%
	(-1.816)	(1.468)	(-0.499)	(-5.856)	(-1.856)	(-1.854)	(2.693)	(1.949)	(7.411)	(-1.870)	(-2.098)	(3.943)	
AR4 IID	-2.191	0.719	-0.022	-11.631	-0.206	-0.502	3.554	0.348	28.588	-108.478	-1.609	0.094	10.62%
	(-3.192)	(2.999)	(-0.593)	(-7.386)	(-1.424)	(-1.745)	(2.659)	(2.366)	(7.850)	(-2.102)	(-2.097)	(3.839)	
FOSA IID	-3.474	1.272	-0.030	-9.680	-0.123	-0.511	4.020	0.380	29.916	-99.834	-1.799	0.097	12.28%
	(-4.542)	(4.624)	(-0.831)	(-8.894)	(-0.834)	(-1.772)	(3.113)	(2.594)	(8.019)	(-1.991)	(-2.165)	(3.709)	
Raw NIID	-2.532	0.768	-0.014	-12.229	-0.155	-0.497	3.213	0.394	29.461	-109.390	-1.606	0.099	11.58%
	(-3.890)	(4.278)	(-0.379)	(-8.247)	(-1.046)	(-1.728)	(2.369)	(2.676)	(8.281)	(-2.115)	(-2.057)	(3.747)	
AR4 NIID	-2.775	0.895	-0.013	-12.435	-0.158	-0.487	3.286	0.392	29.368	-111.880	-1.597	0.099	11.50%
	(-3.863)	(3.989)	(-0.330)	(-7.891)	(-1.060)	(-1.705)	(2.427)	(2.669)	(8.222)	(-2.164)	(-2.055)	(3.802)	
FOSA NIID	-3.188	1.082	-0.034	-9.747	-0.132	-0.499	4.120	0.370	29.322	-93.601	-1.720	0.093	11.66%
	(-4.146)	(4.363)	(-0.919)	(-8.923)	(-0.870)	(-1.714)	(3.249)	(2.478)	(8.051)	(-1.843)	(-2.107)	(3.529)	

Table 3.19: MS-BDL Investigation: FOSA measures with Realized Variance – One month measures

This table shows the results of the MS-BDL framework where both one-month-sample FOSA measures and Realized Variance are included. Specifically, monthly market excess return at time $t+1$ is regressed on a FOSA measure, last month Realized Variance and other time-t expanded control variables. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). All parametric VaRs are at 1% level of significance. The sample period is July 1962 – June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Realized Variance	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
IID	1	-4.657	4.400	-0.007	-0.232	-0.559	0.018	4.968	0.242	5.788	-61.015	0.225	-0.016	5.286	3.951
Nonparam		(-3.081)	(4.981)	(-0.434)	(-3.819)	(-2.076)	(0.116)	(2.562)	(1.167)	(0.805)	(-1.091)	(0.358)	(-0.676)		
	2	-6.464	2.135	-0.009	-0.079	-0.295	-0.940	5.862	0.699	32.672	-126.236	-2.967	0.127	20.542	4.857
		(-4.415)	(3.386)	(-1.438)	(-1.159)	(-0.996)	(-2.525)	(2.367)	(2.446)	(5.408)	(-1.428)	(-2.576)	(4.018)		
	1	-2.872	2.857	-0.016	-0.193	-0.613	0.125	4.721	0.032	9.102	-32.965	0.127	-0.007	6.982	5.785
IID		(-2.361)	(4.565)	(-1.020)	(-3.267)	(-2.732)	(0.420)	(2.395)	(0.172)	(1.129)	(-0.608)	(0.223)	(-0.302)		
Gaussian	2	-7.841	2.271	-0.011	-0.087	-0.120	-1.420	6.275	0.958	34.110	-175.478	-3.789	0.147	21.956	4.731
		(-4.729)	(3.997)	(-1.573)	(-1.164)	(-0.356)	(-3.002)	(2.181)	(2.944)	(4.912)	(-1.723)	(-2.902)	(4.335)		
	1	-3.841	3.131	-0.016	-0.199	-0.640	0.056	5.083	0.056	8.843	-28.302	0.144	-0.008	6.780	5.671
IID		(-2.738)	(4.789)	(-0.878)	(-3.332)	(-2.764)	(0.192)	(2.548)	(0.245)	(1.120)	(-0.527)	(0.227)	(-0.313)		
Student-t	2	-7.906	2.170	-0.010	-0.092	-0.127	-1.288	6.582	0.928	34.115	-168.159	-3.764	0.145	21.808	4.869
		(-4.630)	(3.890)	(-1.386)	(-1.238)	(-0.340)	(-2.794)	(2.296)	(2.863)	(4.912)	(-1.671)	(-2.910)	(4.221)		
IID	1	-3.127	2.783	-0.002	-0.205	-0.517	0.101	4.964	0.196	5.734	-38.462	0.179	-0.013	5.743	4.523
Skewed		(-2.326)	(4.489)	(-0.146)	(-3.432)	(-1.829)	(0.312)	(2.524)	(0.930)	(0.738)	(-0.666)	(0.279)	(-0.554)		
Student-t	2	-6.083	1.661	-0.008	-0.072	-0.349	-1.017	5.351	0.706	32.112	-141.873	-3.096	0.129	21.384	5.207
		(-4.046)	(3.086)	(-1.340)	(-1.060)	(-1.161)	(-2.651)	(2.137)	(2.446)	(5.205)	(-1.576)	(-2.639)	(4.120)		

(Continued)

Table 3.19: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Realized Variance	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
	1	-2.180	2.156	-0.015	-0.146	-0.491	0.142	5.544	-0.083	13.844	0.083	-0.062	-0.008	8.164	5.618
NIID		(-2.146)	(5.229)	(-3.042)	(-2.615)	(-2.252)	(0.496)	(2.787)	(-0.359)	(1.634)	(0.113)	(-0.128)	(-0.358)		
Gaussian	2	-7.202	1.478	0.006	-0.048	-0.126	-1.892	5.885	1.074	36.404	-199.947	-3.825	0.154	23.026	3.434
		(-3.557)	(2.203)	(0.424)	(-0.530)	(-0.298)	(-3.272)	(1.637)	(2.545)	(3.708)	(-1.657)	(-2.473)	(3.683)		
	1	-1.702	1.573	0.013	-0.123	-0.369	0.286	4.717	-0.124	17.356	-20.895	-0.153	-0.001	7.833	5.306
NIID		(-1.713)	(3.892)	(0.872)	(-2.266)	(-1.696)	(1.040)	(2.329)	(-0.554)	(2.259)	(-0.403)	(-0.265)	(-0.096)		
Student-t	2	-7.661	1.641	-0.012	-0.071	-0.094	-1.755	8.649	1.184	34.671	-202.999	-3.984	0.134	22.742	3.496
		(-4.021)	(3.100)	(-1.749)	(-0.838)	(-0.235)	(-3.063)	(2.665)	(3.103)	(3.748)	(-1.706)	(-2.582)	(3.542)		
NIID	1	-1.494	1.744	-0.016	-0.163	-0.601	0.214	5.418	-0.036	7.033	-4.711	0.076	-0.013	7.308	5.469
Skewed		(-1.440)	(4.758)	(-3.173)	(-2.625)	(-2.551)	(0.731)	(2.521)	(-0.117)	(0.691)	(-0.076)	(0.079)	(-0.406)		
Student-t	2	-6.306	1.131	0.006	-0.047	-0.134	-1.726	5.234	0.947	33.460	-165.261	-3.511	0.154	22.378	4.310
		(-3.557)	(2.254)	(0.440)	(-0.590)	(-0.300)	(-3.248)	(1.595)	(2.506)	(4.422)	(-1.521)	(-2.443)	(3.937)		

Table 3.20: BDL regression in different periods: FOSA measures in the presence of realized variance – One month measures

This table shows the regression results regarding how the one-month-sample FOSA tail risk measures explain market excess return given the presence of last month realized variance and other control variables in different sub-sample periods. In each regression, the monthly market excess return at time $t+1$ is regressed on FOSA measure, last month Realized Variance, and other expanded control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets).

Tail risk measure	Const	$E_t(VaR_{t+1})$	Realized Variance	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel A: Original sample July 1962 - December 2005														
IID Nonparam	-4.336	1.879	0.006	-0.061	-13.774	-0.145	-0.496	4.754	0.391	29.903	-60.436	-0.823	-0.007	9.38%
	(-4.029)	(3.568)	(0.498)	(-1.610)	(-1.843)	(-0.797)	(-1.702)	(3.023)	(2.318)	(5.860)	(-1.177)	(-1.118)	(-0.200)	
IID Gaussian	-4.638	1.816	-0.001	-0.065	-10.091	-0.120	-0.497	4.631	0.434	32.262	-71.732	-0.809	-0.005	9.69%
	(-4.770)	(3.885)	(-0.049)	(-1.656)	(-1.246)	(-0.682)	(-1.692)	(2.832)	(2.613)	(6.320)	(-1.420)	(-1.114)	(-0.142)	
IID Student-t	-5.001	1.886	-0.001	-0.066	-9.878	-0.114	-0.493	4.702	0.423	31.906	-67.450	-0.865	-0.003	10.04%
	(-4.871)	(4.063)	(-0.058)	(-1.731)	(-1.230)	(-0.640)	(-1.695)	(2.882)	(2.528)	(6.352)	(-1.335)	(-1.175)	(-0.085)	
IID Skewed	-4.157	1.520	0.006	-0.058	-13.745	-0.157	-0.494	4.742	0.391	30.352	-65.128	-0.785	-0.009	9.14%
Student-t	(-4.099)	(3.644)	(0.485)	(-1.522)	(-1.800)	(-0.847)	(-1.682)	(3.034)	(2.319)	(5.817)	(-1.276)	(-1.069)	(-0.284)	
NIID Gaussian	-4.223	1.471	0.005	-0.062	-13.800	-0.090	-0.462	5.022	0.430	32.175	-69.794	-0.819	-0.007	8.96%
	(-4.160)	(3.074)	(0.432)	(-1.577)	(-1.723)	(-0.518)	(-1.568)	(3.066)	(2.654)	(6.471)	(-1.395)	(-1.117)	(-0.204)	
NIID Student-t	-4.230	1.341	0.005	-0.064	-13.867	-0.081	-0.449	5.108	0.445	32.228	-68.620	-0.814	-0.007	8.94%
	(-4.063)	(3.021)	(0.431)	(-1.616)	(-1.730)	(-0.466)	(-1.527)	(3.125)	(2.733)	(6.469)	(-1.370)	(-1.109)	(-0.198)	
NIID Skewed	-4.240	1.267	0.006	-0.062	-14.045	-0.080	-0.447	5.107	0.466	32.397	-69.869	-0.812	-0.006	8.90%
Student-t	(-3.977)	(2.954)	(0.471)	(-1.588)	(-1.744)	(-0.458)	(-1.524)	(3.151)	(2.843)	(6.468)	(-1.400)	(-1.106)	(-0.196)	

(Continued)

Table 3.20: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Realized Variance	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel B: New sample January 2006 - June 2013								
IID Nonparam	-8.738	2.540	-0.006	-0.054		1.132	-0.337	3.715	2.395	16.332	-145.431	-4.011	0.126	29.73%
	(-1.747)	(2.074)	(-0.437)	(-0.424)		(1.001)	(-0.294)	(1.149)	(1.447)	(1.396)	(-1.124)	(-2.414)	(3.877)	
IID Gaussian	-7.734	2.080	-0.009	-0.050		1.367	-0.320	3.866	2.107	16.508	-158.042	-4.331	0.131	31.80%
	(-1.822)	(2.861)	(-0.742)	(-0.440)		(1.158)	(-0.292)	(1.249)	(1.368)	(1.399)	(-1.278)	(-2.463)	(3.813)	
IID Student-t	-6.797	1.580	-0.003	-0.027		0.866	-0.533	3.884	1.930	19.752	-150.658	-4.075	0.130	29.20%
	(-1.568)	(1.930)	(-0.248)	(-0.218)		(0.753)	(-0.460)	(1.154)	(1.278)	(1.639)	(-1.155)	(-2.358)	(3.911)	
IID Skewed	-7.970	1.696	-0.004	-0.033		1.177	-0.407	3.083	2.463	15.196	-166.215	-3.888	0.124	29.64%
Student-t	(-1.670)	(2.262)	(-0.306)	(-0.283)		(1.035)	(-0.403)	(1.011)	(1.457)	(1.220)	(-1.270)	(-2.433)	(3.810)	
NIID Gaussian	-5.754	1.396	-0.003	-0.014		0.986	-0.588	4.205	1.667	16.578	-156.435	-4.097	0.126	29.83%
	(-1.487)	(2.125)	(-0.229)	(-0.123)		(0.865)	(-0.517)	(1.237)	(1.136)	(1.332)	(-1.212)	(-2.506)	(3.896)	
NIID Student-t	-5.609	1.236	-0.004	-0.015		1.060	-0.545	3.923	1.656	15.739	-152.335	-4.046	0.125	30.15%
	(-1.466)	(2.241)	(-0.265)	(-0.128)		(0.918)	(-0.476)	(1.187)	(1.124)	(1.243)	(-1.177)	(-2.522)	(3.813)	
NIID Skewed	-5.408	1.110	-0.003	-0.012		1.039	-0.562	3.760	1.622	15.771	-152.343	-4.006	0.124	29.93%
Student-t	(-1.420)	(2.198)	(-0.216)	(-0.103)		(0.899)	(-0.491)	(1.148)	(1.099)	(1.234)	(-1.169)	(-2.511)	(3.806)	

(Continued)

Table 3.20: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Realized Variance	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel C: Full sample July 1962 - June 2013								
IID Nonparam	-3.879	1.847	-0.003	-0.042	-7.898	-0.172	-0.526	3.799	0.374	29.234	-88.826	-1.682	0.090	12.14%
	(-4.506)	(4.393)	(-0.395)	(-1.119)	(-1.765)	(-1.086)	(-1.857)	(2.809)	(2.415)	(7.816)	(-1.709)	(-2.081)	(3.330)	
IID Gaussian	-3.926	1.647	-0.007	-0.045	-5.347	-0.148	-0.516	3.920	0.404	31.062	-98.551	-1.751	0.092	12.58%
	(-4.736)	(4.253)	(-0.939)	(-1.154)	(-1.114)	(-0.961)	(-1.797)	(2.897)	(2.645)	(8.333)	(-1.912)	(-2.088)	(3.306)	
IID Student-t	-4.165	1.641	-0.006	-0.045	-6.098	-0.143	-0.509	4.032	0.395	30.762	-93.878	-1.789	0.092	12.59%
	(-4.679)	(4.194)	(-0.768)	(-1.156)	(-1.291)	(-0.929)	(-1.784)	(2.963)	(2.592)	(8.233)	(-1.810)	(-2.133)	(3.332)	
IID Skewed	-3.409	1.339	-0.002	-0.036	-8.048	-0.185	-0.521	3.721	0.371	29.417	-96.106	-1.662	0.090	11.73%
Student-t	(-4.190)	(3.995)	(-0.331)	(-0.948)	(-1.740)	(-1.166)	(-1.825)	(2.753)	(2.412)	(7.919)	(-1.838)	(-2.047)	(3.238)	
NIID Gaussian	-3.294	1.228	-0.002	-0.038	-8.101	-0.147	-0.502	4.287	0.386	29.780	-92.663	-1.738	0.091	11.73%
	(-3.902)	(3.297)	(-0.325)	(-0.967)	(-1.693)	(-0.960)	(-1.732)	(3.109)	(2.568)	(8.114)	(-1.797)	(-2.106)	(3.317)	
NIID Student-t	-3.217	1.091	-0.003	-0.039	-7.875	-0.140	-0.493	4.304	0.394	29.625	-90.957	-1.727	0.091	11.71%
	(-3.862)	(3.293)	(-0.375)	(-0.987)	(-1.625)	(-0.919)	(-1.697)	(3.118)	(2.606)	(8.122)	(-1.761)	(-2.095)	(3.306)	
NIID Skewed	-3.151	0.998	-0.002	-0.037	-8.004	-0.142	-0.492	4.288	0.408	29.593	-91.457	-1.722	0.091	11.64%
Student-t	(-3.800)	(3.246)	(-0.323)	(-0.951)	(-1.645)	(-0.936)	(-1.699)	(3.121)	(2.685)	(8.145)	(-1.772)	(-2.093)	(3.304)	

APPENDIX

Table A3.1: Extreme downside risk-return relationship under Markov switching mechanism – Two month measures

This table shows the results of the MS-BDL framework for main extreme downside risk measures in BDL papers (calculated using estimation sample of two month daily returns). Specifically, monthly market excess return at time $t+1$ is regressed on $E_t(VaR_{t+1})$ and other control variables at time t. The control variables are those in BDL original framework, including lagged monthly market excess return, October 1987 dummy variable, de-trended risk free rate (RFD), change in term structure risk premium (DTRP), changes in credit risk premium (DCRP), and dividend yield (DY). Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). All parametric VaRs are at 1% level of significance. The sample period is July 1962 – June 2013.

State	Const	$E_t(VaR_{t+1})$	Lagged return	RFD	DTRP	DCRP	DY	State variance	Expected Duration
				Raw Non-parametric VaR					
1	0.422	0.682	-0.064	-0.575	-0.086	2.237	-0.055	9.801	11.667
	(0.631)	(5.169)	(-1.307)	(-2.773)	(-0.298)	(1.156)	(-0.294)		
2	-3.340	-0.514	0.043	-0.381	-2.078	4.923	0.848	30.199	4.620
	(-1.884)	(-1.529)	(0.432)	(-0.787)	(-2.979)	(1.469)	(1.762)		
				AR4 Non-parametric VaR					
1	-0.079	0.980	-0.060	-0.564	-0.054	2.259	-0.077	9.630	11.928
	(-0.153)	(5.355)	(-1.170)	(-2.816)	(-0.199)	(1.146)	(-0.526)		
2	-2.710	-0.590	0.045	-0.364	-2.010	4.397	0.768	30.394	5.222
	(-1.567)	(-1.581)	(0.476)	(-0.801)	(-3.077)	(1.373)	(1.744)		
				Raw Gaussian VaR					
1	-1.003	1.233	-0.138	-0.569	-0.754	3.272	0.285	9.442	6.550
	(-1.490)	(6.929)	(-3.226)	(-2.997)	(-2.798)	(1.904)	(1.741)		
2	-2.232	-0.679	0.109	-0.447	-0.728	3.990	0.195	20.757	2.525
	(-1.346)	(-2.082)	(1.076)	(-1.073)	(-1.249)	(1.292)	(0.462)		
				AR4 Gaussian VaR					
1	-1.535	1.563	-0.112	-0.535	-0.683	3.455	0.259	9.462	6.691
	(-2.083)	(6.820)	(-2.465)	(-2.741)	(-2.445)	(1.944)	(1.549)		
2	-1.759	-0.866	0.062	-0.377	-0.770	4.308	0.224	21.213	2.709
	(-1.060)	(-2.379)	(0.591)	(-0.907)	(-1.284)	(1.391)	(0.529)		

(Continued)

Table A3.1: Continued

State	Const	$E_t(VaR_{t+1})$	Lagged return	RFD	DTRP	DCRP	DY	State variance	Expected Duration
				Raw Student-t VaR					
1	-0.840	1.072	-0.143	-0.593	-0.806	3.121	0.268	9.612	6.589
	(-1.269)	(6.751)	(-3.366)	(-3.147)	(-3.020)	(1.790)	(1.652)		
2	-1.814	-0.772	0.095	-0.496	-0.759	4.722	0.132	20.334	2.446
	(-1.014)	(-2.538)	(0.923)	(-1.183)	(-1.286)	(1.519)	(0.285)		
				AR4 Student-t VaR					
1	-1.149	1.278	-0.141	-0.594	-0.806	3.433	0.241	9.726	6.366
	(-1.581)	(6.224)	(-3.220)	(-3.085)	(-2.976)	(1.929)	(1.468)		
2	-1.176	-1.005	0.077	-0.453	-0.766	4.615	0.123	20.126	2.491
	(-0.674)	(-2.792)	(0.733)	(-1.093)	(-1.310)	(1.508)	(0.287)		
				Raw Skewed Student-t VaR					
1	-0.114	0.957	-0.069	-0.585	-0.093	2.063	-0.021	9.195	9.506
	(-0.231)	(6.141)	(-1.406)	(-2.631)	(-0.309)	(1.060)	(-0.151)		
2	-1.547	-1.008	0.017	-0.486	-2.043	6.742	0.635	28.215	4.196
	(-0.852)	(-2.407)	(0.149)	(-1.011)	(-3.185)	(1.963)	(1.388)		
				AR4 Skewed Student-t VaR					
1	-0.441	1.164	-0.058	-0.578	-0.082	2.073	-0.052	9.242	9.305
	(-0.530)	(5.678)	(-1.173)	(-2.600)	(-0.263)	(1.050)	(-0.246)		
2	-0.919	-1.239	0.011	-0.447	-2.077	6.693	0.615	27.937	4.264
	(-0.473)	(-2.641)	(0.121)	(-0.965)	(-3.042)	(2.001)	(1.325)		

Table A3.2: BDL regressions in different periods – Two month measures

This table shows the results of BDL regressions in 3 periods: the Original sample of BDL paper (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on two-month-sample tail risk measure $E_t(VaR_{t+1})$ and other control variables at time t. The control variables are those in BDL original framework. They are lagged monthly market excess return, October 1987 dummy variable, de-trended risk free rate (RFD), change in term structure risk premium (DTRP), changes in credit risk premium (DCRP), and dividend yield (DY). Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). All parametric VaRs are at 1% level of significance.

	Const	$E_t(VaR_{t+1})$	Lagged return	Dummy	RFD	DTRP	DCRP	DY	Adjusted R^2
				Panel A: Original sample July 1962 - December 2005					
Raw NonPara VaR	-1.333	0.497	0.037	-15.246	-0.453	-0.747	3.266	0.283	4.47%
	(-2.016)	(4.070)	(0.851)	(-7.875)	(-2.508)	(-2.434)	(1.993)	(1.661)	
AR4 NonPara VaR	-1.754	0.711	0.042	-17.066	-0.446	-0.747	3.268	0.281	4.32%
	(-2.395)	(3.542)	(0.935)	(-6.558)	(-2.465)	(-2.457)	(1.992)	(1.667)	
Raw Gaussian VaR	-1.096	0.405	0.028	-10.711	-0.479	-0.752	3.456	0.273	3.35%
	(-1.581)	(1.752)	(0.638)	(-5.687)	(-2.669)	(-2.437)	(2.009)	(1.627)	
AR4 Gaussian VaR	-1.108	0.425	0.033	-11.310	-0.479	-0.744	3.531	0.262	3.18%
	(-1.502)	(1.481)	(0.740)	(-4.520)	(-2.657)	(-2.415)	(2.044)	(1.589)	
Raw Student-t VaR	-1.001	0.346	0.027	-9.997	-0.481	-0.758	3.464	0.262	3.32%
	(-1.492)	(1.631)	(0.619)	(-6.215)	(-2.684)	(-2.467)	(2.024)	(1.577)	
AR4 Student-t VaR	-1.005	0.362	0.027	-10.004	-0.482	-0.755	3.546	0.250	3.14%
	(-1.383)	(1.332)	(0.618)	(-5.489)	(-2.675)	(-2.461)	(2.068)	(1.525)	
Raw Skewed Student-t VaR	-1.236	0.460	0.032	-11.002	-0.481	-0.758	3.402	0.273	3.61%
	(-1.747)	(2.301)	(0.723)	(-6.640)	(-2.620)	(-2.485)	(2.036)	(1.608)	
AR4 Skewed Student-t VaR	-1.439	0.564	0.037	-11.541	-0.478	-0.757	3.402	0.270	3.57%
	(-1.723)	(2.102)	(0.780)	(-2.424)	(-2.702)	(-2.936)	(1.831)	(1.631)	

(Continued)

Table A3.2: Continued

	Const	$E_t(VaR_{t+1})$	Lagged return	Dummy	RFD	DTRP	DCRP	DY	Adjusted R^2
Panel B: New sample January 2006 - June 2013									
Raw NonPara VaR	-5.931	-0.253	0.100		0.573	-0.084	-1.286	3.229	6.02%
	(-1.648)	(-0.568)	(1.228)		(0.852)	(-0.061)	(-0.462)	(1.778)	
AR4 NonPara VaR	-4.883	-0.773	0.077		0.204	-0.570	-0.818	3.300	8.22%
	(-1.285)	(-1.573)	(0.858)		(0.390)	(-0.481)	(-0.314)	(1.741)	
Raw Gaussian VaR	-6.252	0.398	0.161		1.368	0.533	-3.691	2.624	6.62%
	(-2.017)	(1.220)	(1.556)		(1.513)	(0.323)	(-1.580)	(1.962)	
AR4 Gaussian VaR	-6.198	0.050	0.133		0.921	0.243	-2.125	2.995	5.54%
	(-1.827)	(0.111)	(1.222)		(1.374)	(0.162)	(-0.806)	(1.933)	
Raw Student-t VaR	-6.187	0.048	0.131		0.933	0.260	-2.194	2.984	5.54%
	(-1.835)	(0.144)	(1.295)		(1.310)	(0.171)	(-0.804)	(1.910)	
AR4 Student-t VaR	-5.902	-0.355	0.101		0.509	-0.162	-0.729	3.341	6.25%
	(-1.564)	(-0.709)	(0.979)		(0.913)	(-0.116)	(-0.227)	(1.864)	
Raw Skewed Student-t VaR	-6.069	-0.101	0.118		0.739	0.097	-1.634	3.116	5.60%
	(-1.743)	(-0.225)	(1.308)		(1.005)	(0.066)	(-0.579)	(1.857)	
AR4 Skewed Student-t VaR	-5.457	-0.474	0.092		0.370	-0.290	-0.840	3.265	6.58%
	(-1.449)	(-0.839)	(0.972)		(0.615)	(-0.220)	(-0.284)	(1.813)	
Panel C: Full sample July 1962 - June 2013									
Raw NonPara VaR	-0.850	0.238	0.078	-10.619	-0.283	-0.681	0.275	0.272	1.88%
	(-1.287)	(1.014)	(1.842)	(-2.739)	(-1.570)	(-2.272)	(0.161)	(1.824)	
AR4 NonPara VaR	-0.758	0.223	0.072	-10.057	-0.300	-0.684	0.482	0.253	1.62%
	(-0.884)	(0.611)	(1.742)	(-1.995)	(-1.675)	(-2.293)	(0.287)	(1.698)	
Raw Gaussian VaR	-0.617	0.155	0.070	-8.240	-0.305	-0.690	0.385	0.254	1.56%
	(-0.883)	(0.571)	(1.661)	(-3.529)	(-1.704)	(-2.296)	(0.217)	(1.703)	
AR4 Gaussian VaR	-0.332	0.052	0.063	-7.680	-0.327	-0.692	0.682	0.227	1.44%
	(-0.409)	(0.146)	(1.512)	(-2.461)	(-1.847)	(-2.321)	(0.388)	(1.538)	
Raw Student-t VaR	-0.459	0.090	0.066	-7.761	-0.317	-0.692	0.532	0.240	1.48%
	(-0.665)	(0.350)	(1.573)	(-3.870)	(-1.787)	(-2.308)	(0.304)	(1.628)	
AR4 Student-t VaR	-0.169	-0.012	0.059	-7.261	-0.339	-0.694	0.817	0.215	1.43%
	(-0.205)	(-0.036)	(1.398)	(-3.193)	(-1.907)	(-2.324)	(0.465)	(1.458)	
Raw Skewed Student-t VaR	-0.494	0.105	0.067	-7.929	-0.316	-0.690	0.551	0.242	1.49%
	(-0.639)	(0.356)	(1.619)	(-3.281)	(-1.769)	(-2.311)	(0.324)	(1.616)	
AR4 Skewed Student-t VaR	-0.335	0.051	0.063	-7.609	-0.328	-0.692	0.701	0.227	1.44%
	(-0.365)	(0.132)	(1.520)	(-2.680)	(-1.849)	(-2.323)	(0.413)	(1.520)	

Table A3.3: MS-BDL investigation: Expanding state variable set – Two month measures

This table shows the results of the MS-BDL framework for two-month-sample risk measures using the expanded set of state variables. In each Markov switching regression, monthly market excess return at time $t+1$ is regressed on $E_t(VaR_{t+1})$ and other control variables at time t. The first line of each regression shows the value of regression coefficients, while the second line shows their t-statistics (in brackets). All parametric VaRs are at 1% significance level. The sample period is July 1962–June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw Nonparam	1	-0.362	0.751	-0.043	-0.156	0.034	2.118	-0.092	18.724	-31.621	-0.691	0.023	10.411	7.525
		(-0.489)	(5.128)	(-0.958)	(-0.781)	(0.145)	(1.100)	(-0.473)	(4.033)	(-0.585)	(-0.989)	(0.938)		
	2	-5.220	-0.246	0.067	-1.200	-3.206	11.177	1.346	26.928	-130.355	-2.819	0.124	27.597	2.159
		(-2.336)	(-0.472)	(0.525)	(-1.532)	(-3.083)	(2.217)	(2.165)	(2.096)	(-0.672)	(-1.207)	(2.215)		
AR4 Nonparam	1	-0.809	1.117	-0.039	-0.150	0.086	1.844	-0.132	17.538	-38.535	-0.779	0.018	9.975	6.455
		(-1.021)	(5.072)	(-0.809)	(-0.673)	(0.339)	(0.902)	(-0.664)	(3.459)	(-0.720)	(-1.089)	(0.726)		
	2	-4.492	-0.311	0.044	-0.983	-2.809	10.434	1.205	28.021	-118.401	-2.847	0.126	26.436	2.321
		(-2.266)	(-0.789)	(0.403)	(-1.509)	(-3.318)	(2.395)	(2.285)	(2.515)	(-0.750)	(-1.371)	(2.546)		
Raw Gaussian	1	-1.697	1.648	-0.058	-0.204	0.310	2.708	-0.056	24.959	-54.687	-0.322	-0.002	8.029	5.157
		(-1.980)	(7.108)	(-1.258)	(-0.978)	(1.156)	(1.416)	(-0.219)	(4.422)	(-0.992)	(-0.368)	(-0.070)		
	2	-5.002	0.351	0.065	-0.456	-1.992	8.388	1.151	29.042	-214.051	-2.473	0.127	25.235	2.710
		(-3.056)	(0.981)	(0.720)	(-0.905)	(-3.120)	(2.291)	(2.756)	(2.913)	(-1.634)	(-1.283)	(2.809)		
AR4 Gaussian	1	-2.139	1.840	-0.012	-0.162	0.186	2.407	-0.081	24.316	-53.065	-0.609	0.003	8.777	5.756
		(-2.609)	(7.433)	(-0.234)	(-0.763)	(0.716)	(1.313)	(-0.396)	(4.845)	(-1.022)	(-0.771)	(0.108)		
	2	-3.883	-0.344	0.038	-0.616	-2.449	10.695	1.106	23.807	-143.676	-2.406	0.106	25.350	2.455
		(-2.056)	(-0.723)	(0.335)	(-1.016)	(-3.281)	(2.604)	(2.502)	(2.127)	(-0.958)	(-1.117)	(2.234)		

(Continued)

Table A3.3: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw Student-t	1	-1.841	1.515	-0.068	-0.221	0.252	2.769	-0.034	24.834	-44.089	-0.474	0.000	8.360	5.818
		(-2.006)	(7.675)	(-1.520)	(-1.119)	(0.946)	(1.526)	(-0.115)	(4.641)	(-0.824)	(-0.649)	(0.036)		
	2	-4.833	0.237	0.057	-0.486	-2.074	9.169	1.162	29.209	-230.857	-2.285	0.121	26.065	2.715
		(-2.715)	(0.699)	(0.602)	(-0.906)	(-3.059)	(2.269)	(2.508)	(2.803)	(-1.693)	(-1.077)	(2.600)		
AR4 Student-t	1	-2.223	1.753	-0.040	-0.189	0.180	2.484	-0.093	23.606	-40.530	-0.674	0.004	8.675	6.162
		(-2.391)	(5.956)	(-0.856)	(-0.843)	(0.661)	(1.329)	(-0.445)	(4.449)	(-0.744)	(-0.615)	(0.139)		
	2	-4.157	-0.041	0.018	-0.565	-1.971	10.659	1.140	28.745	-214.703	-1.933	0.114	26.405	2.838
		(-1.624)	(-0.061)	(0.146)	(-0.926)	(-2.631)	(2.273)	(2.545)	(2.650)	(-1.473)	(-0.632)	(1.932)		
Raw Skewed Student-t	1	-0.786	1.096	-0.033	-0.083	0.077	1.126	-0.161	21.243	-37.740	-0.609	0.016	9.306	5.043
		(-1.013)	(6.143)	(-0.703)	(-0.343)	(0.297)	(0.483)	(-0.760)	(3.717)	(-0.678)	(-0.767)	(0.566)		
	2	-3.957	-0.610	0.049	-0.975	-2.643	11.010	1.228	23.038	-81.369	-1.961	0.102	25.507	2.011
		(-1.917)	(-0.938)	(0.459)	(-1.599)	(-3.179)	(2.407)	(2.324)	(1.951)	(-0.467)	(-0.748)	(1.894)		
AR4 Skewed Student-t	1	-0.532	1.536	0.027	-0.006	0.042	-1.688	-0.508	19.111	-22.910	-1.906	0.045	6.877	2.588
		(-0.612)	(7.210)	(0.536)	(-0.106)	(0.165)	(-0.870)	(-2.225)	(3.449)	(-0.422)	(-2.285)	(1.830)		
	2	-2.905	-0.750	0.013	-0.556	-1.495	9.871	1.140	25.783	-78.466	-0.013	0.089	22.701	2.070
		(-1.998)	(-1.838)	(0.193)	(-1.662)	(-2.722)	(3.222)	(3.411)	(3.145)	(-0.704)	(-0.082)	(2.332)		

Table A3.4: BDL regression in different periods: expanded set of state variables – Two month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on two-month-sample risk measure $E_t(VaR_{t+1})$ and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). Parametric VaRs are at 1% level of significance.

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
					Panel A: Original sample July 1962 - December 2005								
Raw	-2.368	0.660	-0.011	-19.066	-0.172	-0.503	4.715	0.354	27.061	-52.751	-0.784	-0.004	9.56%
NonPara VaR	(-3.450)	(4.657)	(-0.266)	(-8.381)	(-1.005)	(-1.665)	(3.053)	(2.252)	(5.976)	(-1.063)	(-1.057)	(-0.131)	
AR4	-3.004	0.987	-0.004	-22.024	-0.164	-0.504	4.645	0.364	27.319	-62.092	-0.912	0.001	9.53%
NonPara VaR	(-4.130)	(5.355)	(-0.093)	(-9.103)	(-0.980)	(-1.699)	(3.018)	(2.377)	(6.192)	(-1.240)	(-1.201)	(0.016)	
Raw	-3.022	0.929	-0.012	-15.652	-0.147	-0.488	4.595	0.425	30.879	-73.806	-0.699	-0.008	9.32%
Gaussian VaR	(-4.192)	(4.664)	(-0.304)	(-8.483)	(-0.882)	(-1.610)	(2.870)	(2.674)	(6.409)	(-1.500)	(-0.967)	(-0.240)	
AR4	-3.219	1.065	0.006	-17.655	-0.151	-0.475	4.591	0.420	30.372	-80.573	-0.733	-0.006	8.94%
Gaussian VaR	(-4.189)	(4.154)	(0.145)	(-7.288)	(-0.912)	(-1.578)	(2.876)	(2.686)	(6.386)	(-1.624)	(-1.001)	(-0.179)	
Raw	-2.598	0.719	-0.014	-13.541	-0.169	-0.514	4.646	0.378	29.190	-63.584	-0.734	-0.006	8.84%
Student-t VaR	(-3.895)	(4.184)	(-0.355)	(-8.939)	(-1.021)	(-1.703)	(2.904)	(2.440)	(6.366)	(-1.289)	(-1.013)	(-0.184)	
AR4	-2.763	0.828	-0.009	-13.911	-0.176	-0.516	4.647	0.368	28.641	-66.610	-0.769	-0.004	8.43%
Student-t VaR	(-3.893)	(3.728)	(-0.214)	(-8.284)	(-1.057)	(-1.709)	(2.914)	(2.398)	(6.304)	(-1.341)	(-1.055)	(-0.123)	
Raw Skewed	-2.688	0.799	-0.011	-14.646	-0.188	-0.511	4.699	0.379	28.489	-64.900	-0.703	-0.010	9.02%
Student-t VaR	(-3.785)	(4.785)	(-0.278)	(-9.742)	(-1.069)	(-1.713)	(2.998)	(2.399)	(6.117)	(-1.301)	(-0.958)	(-0.314)	
AR4 Skewed	-3.035	0.986	-0.003	-15.603	-0.188	-0.512	4.664	0.378	28.302	-69.655	-0.742	-0.009	8.91%
Student-t VaR	(-4.004)	(4.555)	(-0.063)	(-9.154)	(-1.081)	(-1.733)	(2.956)	(2.426)	(6.128)	(-1.384)	(-0.994)	(-0.253)	

(Continued)

Table A3.4: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel B: New sample January 2006 - June 2013							
Raw	-1.013	-0.158	-0.018		-1.231	-1.485	5.362	1.040	36.486	-183.376	-3.180	0.115	26.74%
NonPara VaR	(-0.215)	(-0.290)	(-0.214)		(-0.765)	(-1.605)	(1.863)	(0.567)	(1.715)	(-1.165)	(-3.057)	(5.058)	
AR4	1.381	-0.712	-0.044		-1.956	-1.931	5.875	0.489	43.231	-174.419	-2.858	0.108	28.11%
NonPara VaR	(0.291)	(-1.136)	(-0.530)		(-1.403)	(-2.068)	(2.327)	(0.265)	(2.278)	(-1.073)	(-2.558)	(4.804)	
Raw	-4.304	0.598	0.051		0.405	-0.868	2.259	1.763	21.098	-185.846	-3.510	0.125	28.32%
Gaussian VaR	(-1.166)	(1.589)	(0.538)		(0.357)	(-0.959)	(0.836)	(1.169)	(1.526)	(-1.339)	(-2.710)	(4.365)	
AR4	-2.410	0.161	0.016		-0.622	-1.227	4.400	1.354	30.541	-187.563	-3.367	0.120	26.71%
Gaussian VaR	(-0.635)	(0.306)	(0.167)		(-0.512)	(-1.349)	(1.381)	(0.894)	(1.882)	(-1.243)	(-2.688)	(4.546)	
Raw	-2.773	0.220	0.016		-0.414	-1.108	3.887	1.428	28.735	-187.670	-3.423	0.122	26.91%
Student-t VaR	(-0.776)	(0.592)	(0.182)		(-0.363)	(-1.233)	(1.264)	(0.957)	(1.893)	(-1.244)	(-2.763)	(4.672)	
AR4	-1.358	-0.133	-0.011		-1.110	-1.446	5.331	1.172	34.970	-180.721	-3.271	0.116	26.70%
Student-t VaR	(-0.369)	(-0.269)	(-0.136)		(-0.986)	(-1.594)	(1.664)	(0.781)	(2.248)	(-1.172)	(-2.598)	(4.584)	
Raw Skewed	-1.635	-0.040	-0.006		-0.981	-1.370	5.055	1.201	33.897	-184.112	-3.302	0.118	26.63%
Student-t VaR	(-0.363)	(-0.077)	(-0.069)		(-0.639)	(-1.457)	(1.609)	(0.691)	(1.741)	(-1.190)	(-2.840)	(4.820)	
AR4 Skewed	0.073	-0.410	-0.031		-1.618	-1.703	5.865	0.809	39.624	-178.829	-3.091	0.113	27.12%
Student-t VaR	(0.015)	(-0.579)	(-0.371)		(-1.051)	(-1.761)	(1.863)	(0.441)	(2.021)	(-1.110)	(-2.664)	(4.747)	

(Continued)

Table A3.4: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel C: Full sample July 1962 - June 2013													
Raw	-1.734	0.482	0.011	-15.845	-0.214	-0.534	3.530	0.337	27.028	-95.477	-1.646	0.094	11.20%
NonPara VaR	(-3.035)	(3.512)	(0.295)	(-6.903)	(-1.453)	(-1.821)	(2.768)	(2.361)	(7.944)	(-1.848)	(-2.053)	(3.892)	
AR4	-1.994	0.636	0.010	-16.982	-0.222	-0.533	3.678	0.331	26.949	-99.494	-1.712	0.095	10.85%
NonPara VaR	(-2.726)	(3.099)	(0.236)	(-3.347)	(-1.277)	(-2.143)	(2.344)	(2.089)	(6.668)	(-1.924)	(-2.607)	(4.746)	
Raw	-2.167	0.655	0.013	-13.130	-0.189	-0.530	3.165	0.385	29.331	-108.415	-1.550	0.096	11.41%
Gaussian VaR	(-3.622)	(4.081)	(0.343)	(-7.912)	(-1.310)	(-1.802)	(2.393)	(2.627)	(8.390)	(-2.124)	(-1.975)	(3.672)	
AR4	-2.080	0.663	0.022	-13.855	-0.210	-0.526	3.354	0.363	28.456	-110.187	-1.570	0.095	10.77%
Gaussian VaR	(-3.100)	(2.881)	(0.589)	(-6.187)	(-1.460)	(-1.797)	(2.535)	(2.512)	(8.280)	(-2.146)	(-2.030)	(3.824)	
Raw	-1.843	0.497	0.007	-11.652	-0.215	-0.546	3.314	0.353	28.295	-102.709	-1.571	0.095	10.89%
Student-t VaR	(-3.135)	(3.201)	(0.200)	(-7.856)	(-1.494)	(-1.863)	(2.511)	(2.463)	(8.316)	(-1.979)	(-2.033)	(3.837)	
AR4	-1.770	0.506	0.006	-11.595	-0.236	-0.549	3.533	0.330	27.565	-102.157	-1.594	0.094	10.38%
Student-t VaR	(-2.622)	(2.311)	(0.173)	(-6.876)	(-1.613)	(-1.877)	(2.672)	(2.323)	(8.144)	(-1.977)	(-2.082)	(3.979)	
Raw Skewed	-1.769	0.491	0.008	-12.014	-0.228	-0.540	3.520	0.344	27.718	-101.398	-1.589	0.093	10.70%
Student-t VaR	(-2.743)	(2.717)	(0.205)	(-7.307)	(-1.518)	(-1.863)	(2.746)	(2.364)	(8.109)	(-1.969)	(-1.994)	(3.762)	
AR4 Skewed	-1.849	0.555	0.009	-12.321	-0.237	-0.540	3.622	0.333	27.387	-102.535	-1.617	0.093	10.45%
Student-t VaR	(-2.506)	(2.239)	(0.239)	(-6.322)	(-1.583)	(-1.875)	(2.816)	(2.301)	(7.981)	(-1.980)	(-2.037)	(3.854)	

Table A3.5: MS-BDL Investigation: NIID measures – Two month measures

This table shows the results of the MS-BDL framework for two-month-sample NIID measures using the expanded set of state variables. In each Markov switching regression, monthly market excess return at time $t+1$ is regressed on $E_t(VaR_{t+1})$ and other control variables at time t. The first line of each regression shows the value of regression coefficients, while the second line shows their t-statistics (in brackets). All parametric VaRs are at 1% significance level. The sample period is July 1962–June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw Gaussian	1	-2.117	1.686	-0.120	-0.217	0.167	4.110	-0.005	22.858	-20.649	-0.403	-0.001	8.792	6.316
		(-2.825)	(6.601)	(-2.659)	(-1.047)	(0.646)	(2.165)	(-0.031)	(4.155)	(-0.372)	(-0.624)	(-0.059)		
	2	-6.535	0.876	0.064	-0.508	-2.243	7.780	1.237	33.716	-242.248	-3.546	0.145	25.405	2.714
		(-3.326)	(1.816)	(0.669)	(-0.883)	(-2.899)	(2.065)	(2.881)	(2.804)	(-1.708)	(-1.988)	(3.304)		
AR4 Gaussian	1	-2.677	1.882	-0.099	-0.163	0.108	3.445	0.026	23.802	-29.259	-0.468	0.001	9.307	7.160
		(-3.370)	(7.414)	(-2.285)	(-0.897)	(0.442)	(2.011)	(0.158)	(5.034)	(-0.590)	(-0.757)	(0.095)		
	2	-6.096	0.560	0.113	-0.721	-2.671	9.147	1.250	30.478	-233.873	-3.188	0.133	26.545	2.461
		(-2.753)	(0.920)	(1.098)	(-1.098)	(-3.161)	(2.134)	(2.551)	(2.359)	(-1.475)	(-1.617)	(2.761)		
Raw Student-t	1	-1.954	1.544	-0.122	-0.222	0.224	4.246	-0.020	22.118	-19.558	-0.377	-0.001	8.391	5.727
		(-2.668)	(6.490)	(-2.679)	(-1.056)	(0.885)	(2.165)	(-0.141)	(3.878)	(-0.392)	(-0.580)	(-0.095)		
	2	-6.507	0.813	0.050	-0.425	-2.115	7.357	1.238	33.685	-234.599	-3.483	0.145	24.690	2.763
		(-3.548)	(1.918)	(0.568)	(-0.813)	(-3.084)	(2.096)	(3.009)	(3.081)	(-1.769)	(-2.041)	(3.393)		
AR4 Student-t	1	-2.507	1.672	-0.100	-0.156	0.146	3.517	0.027	23.103	-25.729	-0.424	0.000	9.176	6.859
		(-3.309)	(7.237)	(-2.295)	(-0.839)	(0.595)	(2.005)	(0.179)	(4.856)	(-0.517)	(-0.689)	(0.093)		
	2	-6.395	0.638	0.104	-0.669	-2.564	8.577	1.285	32.002	-246.628	-3.207	0.137	26.218	2.486
		(-2.937)	(1.162)	(1.042)	(-1.059)	(-3.131)	(2.056)	(2.696)	(2.537)	(-1.583)	(-1.670)	(2.875)		

(Continued)

Table A3.5: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Raw Skewed Student-t	1	-1.842	1.470	-0.123	-0.251	0.274	4.237	-0.007	21.382	-23.291	-0.316	-0.002	8.001	5.217
		(-2.641)	(6.372)	(-2.679)	(-1.170)	(1.049)	(2.030)	(-0.114)	(3.588)	(-0.472)	(-0.458)	(-0.105)		
	2	-6.233	0.728	0.041	-0.361	-1.986	7.042	1.201	33.136	-220.253	-3.367	0.143	24.098	2.855
		(-3.596)	(1.888)	(0.497)	(-0.766)	(-3.150)	(2.122)	(3.160)	(3.263)	(-1.747)	(-2.063)	(3.425)		
AR4 Skewed Student-t	1	-2.408	1.575	-0.103	-0.170	0.188	3.431	0.042	22.701	-28.498	-0.387	-0.002	8.946	6.253
		(-2.749)	(7.058)	(-2.315)	(-0.858)	(0.743)	(1.810)	(0.197)	(4.452)	(-0.559)	(-0.593)	(-0.052)		
	2	-6.223	0.570	0.096	-0.601	-2.435	8.262	1.264	31.820	-230.372	-3.177	0.136	25.564	2.503
		(-2.781)	(1.085)	(0.977)	(-1.024)	(-3.136)	(2.094)	(2.571)	(2.640)	(-1.538)	(-1.710)	(2.809)		

Table A3.6: BDL regression in different periods: NIID risk measures – Two month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t + 1$ is regressed on two-month-sample NIID measure $E_t(VaR_{t+1})$ and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). Parametric VaRs are at 1% level of significance.

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel 1: Original sample July 1962 - December 2005							
Raw	-3.892	1.295	-0.051	-13.505	-0.072	-0.440	4.822	0.471	33.388	-74.597	-0.810	-0.008	9.90%
Gaussian VaR	(-4.651)	(4.749)	(-1.361)	(-9.158)	(-0.419)	(-1.510)	(3.001)	(2.902)	(6.683)	(-1.532)	(-1.117)	(-0.257)	
AR4	-3.901	1.335	-0.036	-15.142	-0.094	-0.439	4.771	0.462	32.626	-79.548	-0.753	-0.009	9.65%
Gaussian VaR	(-4.587)	(4.366)	(-0.945)	(-8.264)	(-0.555)	(-1.494)	(2.956)	(2.883)	(6.677)	(-1.629)	(-1.037)	(-0.270)	
Raw	-4.014	1.222	-0.054	-12.757	-0.056	-0.430	4.864	0.492	33.600	-73.136	-0.823	-0.007	10.02%
Student-t VaR	(-4.678)	(4.824)	(-1.447)	(-9.346)	(-0.321)	(-1.482)	(3.040)	(3.005)	(6.715)	(-1.498)	(-1.131)	(-0.216)	
AR4	-3.884	1.206	-0.038	-13.872	-0.087	-0.433	4.829	0.472	32.471	-76.164	-0.747	-0.008	9.59%
Student-t VaR	(-4.510)	(4.310)	(-1.007)	(-8.592)	(-0.515)	(-1.473)	(3.005)	(2.936)	(6.662)	(-1.558)	(-1.028)	(-0.241)	
Raw Skewed	-4.071	1.175	-0.054	-12.493	-0.055	-0.427	4.878	0.517	33.746	-74.189	-0.820	-0.007	9.94%
Student-t VaR	(-4.598)	(4.664)	(-1.438)	(-9.277)	(-0.317)	(-1.478)	(3.088)	(3.129)	(6.730)	(-1.527)	(-1.125)	(-0.213)	
AR4 Skewed	-3.906	1.148	-0.039	-13.532	-0.088	-0.430	4.844	0.492	32.538	-76.790	-0.744	-0.008	9.50%
Student-t VaR	(-4.422)	(4.175)	(-1.009)	(-8.584)	(-0.517)	(-1.471)	(3.047)	(3.042)	(6.682)	(-1.576)	(-1.022)	(-0.236)	

(Continued)

Table A3.6: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
					Panel 2: New sample January 2006 - June 2013								
Raw	-4.816	0.788	0.014		0.715	-0.778	2.570	1.799	17.619	-178.881	-3.788	0.127	29.14%
Gaussian VaR	(-1.288)	(1.900)	(0.150)		(0.613)	(-0.810)	(1.157)	(1.207)	(1.395)	(-1.324)	(-2.579)	(4.119)	
AR4	-4.621	0.853	0.039		0.603	-0.815	1.953	1.679	17.979	-193.509	-3.486	0.124	28.90%
Gaussian VaR	(-1.253)	(1.760)	(0.412)		(0.512)	(-0.880)	(0.721)	(1.140)	(1.338)	(-1.398)	(-2.634)	(4.222)	
Raw	-4.694	0.681	0.014		0.719	-0.763	2.574	1.796	17.543	-178.430	-3.773	0.127	29.07%
Student-t VaR	(-1.259)	(1.859)	(0.154)		(0.609)	(-0.788)	(1.149)	(1.199)	(1.370)	(-1.314)	(-2.572)	(4.108)	
AR4	-4.476	0.730	0.042		0.599	-0.796	1.925	1.679	17.908	-194.413	-3.452	0.123	28.83%
Student-t VaR	(-1.218)	(1.710)	(0.439)		(0.501)	(-0.852)	(0.694)	(1.135)	(1.307)	(-1.398)	(-2.627)	(4.213)	
Raw Skewed	-4.571	0.621	0.013		0.684	-0.766	2.601	1.765	17.841	-178.354	-3.752	0.126	28.91%
Student-t VaR	(-1.225)	(1.789)	(0.146)		(0.578)	(-0.787)	(1.148)	(1.176)	(1.378)	(-1.305)	(-2.562)	(4.097)	
AR4 Skewed	-4.363	0.662	0.039		0.567	-0.799	1.981	1.658	18.196	-194.036	-3.442	0.123	28.67%
Student-t VaR	(-1.182)	(1.632)	(0.419)		(0.470)	(-0.848)	(0.704)	(1.116)	(1.307)	(-1.386)	(-2.619)	(4.199)	

(Continued)

Table A3.6: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel 3: Full sample July 1962 - June 2013													
Raw	-2.634	0.843	-0.018	-11.459	-0.150	-0.499	3.554	0.410	30.069	-104.658	-1.688	0.097	11.84%
Gaussian VaR	(-3.974)	(4.415)	(-0.475)	(-8.759)	(-1.008)	(-1.737)	(2.733)	(2.749)	(8.350)	(-2.041)	(-2.068)	(3.594)	
AR4	-2.680	0.891	-0.004	-12.543	-0.165	-0.503	3.310	0.408	29.693	-111.113	-1.585	0.097	11.67%
Gaussian VaR	(-4.030)	(4.296)	(-0.114)	(-8.232)	(-1.125)	(-1.742)	(2.461)	(2.761)	(8.416)	(-2.161)	(-2.012)	(3.644)	
Raw	-2.597	0.753	-0.019	-10.851	-0.143	-0.493	3.573	0.415	29.916	-103.290	-1.699	0.098	11.81%
Student-t VaR	(-3.887)	(4.321)	(-0.504)	(-8.801)	(-0.961)	(-1.722)	(2.751)	(2.779)	(8.368)	(-2.010)	(-2.079)	(3.620)	
AR4	-2.564	0.767	-0.005	-11.542	-0.164	-0.500	3.323	0.407	29.396	-109.050	-1.581	0.097	11.54%
Student-t VaR	(-3.866)	(4.130)	(-0.135)	(-8.393)	(-1.119)	(-1.732)	(2.469)	(2.760)	(8.411)	(-2.116)	(-2.011)	(3.679)	
Raw Skewed	-2.572	0.698	-0.019	-10.628	-0.147	-0.493	3.597	0.426	29.838	-103.281	-1.695	0.097	11.71%
Student-t VaR	(-3.808)	(4.207)	(-0.502)	(-8.795)	(-0.983)	(-1.724)	(2.793)	(2.844)	(8.384)	(-2.009)	(-2.076)	(3.613)	
AR4 Skewed	-2.524	0.707	-0.006	-11.250	-0.167	-0.499	3.352	0.417	29.316	-108.818	-1.579	0.097	11.44%
Student-t VaR	(-3.772)	(4.009)	(-0.150)	(-8.428)	(-1.140)	(-1.735)	(2.513)	(2.817)	(8.424)	(-2.110)	(-2.009)	(3.674)	

Table A3.7: MS-BDL Investigation: FOSA measures – Two month measures

This table shows the results of the MS-BDL framework for two-month-sample FOSA measures using the expanded set of state variables. In each Markov switching regression, monthly market excess return at time $t+1$ is regressed on $E_t(VaR_{t+1})_{FOSA}$ and other control variables at time t. The first line of each regression shows the value of regression coefficients, while the second line shows the corresponding t-statistics (in brackets). All parametric VaRs are at 1% significance level. The sample period is July 1962–June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
	1	-3.655	3.156	-0.230	-0.685	0.042	3.801	0.213	-0.023	-86.183	0.323	-0.009	5.486	3.735
IID		(-2.584)	(5.093)	(-3.881)	(-2.844)	(0.129)	(2.048)	(0.986)	(-0.111)	(-1.450)	(0.461)	(-0.400)		
Nonparam	2	-5.205	1.221	-0.029	-0.312	-0.980	5.045	0.595	31.134	-116.174	-2.985	0.133	20.607	4.964
		(-4.038)	(3.154)	(-0.492)	(-1.085)	(-2.565)	(2.126)	(2.199)	(5.399)	(-1.372)	(-2.659)	(4.468)		
	1	-1.934	2.063	-0.172	-0.540	0.166	4.112	0.056	8.129	-35.156	0.088	-0.001	6.997	5.481
IID		(-1.589)	(4.724)	(-3.135)	(-2.342)	(0.541)	(2.011)	(0.231)	(0.942)	(-0.621)	(0.083)	(-0.048)		
Gaussian	2	-5.817	1.276	-0.023	-0.270	-1.371	5.510	0.810	32.326	-170.821	-3.140	0.144	22.875	4.553
		(-3.750)	(2.827)	(-0.313)	(-0.734)	(-2.700)	(1.916)	(2.345)	(4.564)	(-1.639)	(-2.160)	(4.154)		
	1	-2.530	2.164	-0.173	-0.531	0.120	4.677	0.037	9.222	-25.279	0.022	-0.006	7.033	5.953
IID		(-2.229)	(5.390)	(-3.296)	(-2.407)	(0.427)	(2.333)	(0.163)	(1.158)	(-0.460)	(0.048)	(-0.296)		
Student-t	2	-5.985	1.254	-0.024	-0.234	-1.295	5.730	0.847	32.147	-182.256	-3.004	0.136	23.365	4.731
		(-3.660)	(2.827)	(-0.352)	(-0.642)	(-2.684)	(2.024)	(2.494)	(4.314)	(-1.714)	(-2.210)	(3.851)		
IID	1	-2.606	2.335	-0.196	-0.513	0.049	4.180	0.174	4.881	-37.808	0.147	-0.010	5.744	4.311
Skewed		(-2.298)	(5.881)	(-3.703)	(-1.787)	(0.169)	(2.164)	(0.795)	(0.637)	(-0.674)	(0.206)	(-0.411)		
Student-t	2	-4.861	0.974	-0.024	-0.397	-1.031	5.274	0.653	31.412	-143.262	-2.863	0.130	22.062	4.757
		(-3.398)	(2.158)	(-0.383)	(-1.284)	(-2.601)	(2.066)	(2.205)	(4.990)	(-1.553)	(-2.345)	(4.032)		

(Continued)

Table A3.7: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
	1	-1.362	1.646	-0.144	-0.481	0.220	5.110	-0.064	12.075	-11.069	-0.117	-0.001	7.803	5.507
NIID		(-1.349)	(4.636)	(-2.687)	(-2.060)	(0.750)	(2.397)	(-0.262)	(1.319)	(-0.141)	(-0.114)	(-0.045)		
Gaussian	2	-6.300	1.296	-0.014	-0.221	-1.657	6.509	0.946	33.656	-183.228	-3.496	0.143	23.629	3.762
		(-3.497)	(2.537)	(-0.179)	(-0.534)	(-2.979)	(2.171)	(2.421)	(3.887)	(-1.466)	(-2.175)	(3.607)		
	1	-1.350	1.519	-0.141	-0.448	0.245	5.085	-0.078	13.015	-9.458	-0.168	0.000	7.702	5.322
NIID		(-1.251)	(5.152)	(-2.640)	(-1.929)	(0.838)	(2.379)	(-0.253)	(1.293)	(-0.149)	(-0.211)	(0.036)		
Student-t	2	-6.329	1.134	-0.010	-0.218	-1.714	6.480	0.992	33.369	-182.178	-3.467	0.143	23.532	3.611
		(-3.347)	(2.407)	(-0.083)	(-0.499)	(-3.052)	(2.035)	(2.290)	(3.729)	(-1.563)	(-2.165)	(3.496)		
NIID	1	-1.205	1.410	-0.141	-0.473	0.265	5.036	-0.051	11.317	-12.053	-0.109	-0.002	7.496	5.322
Skewed		(-1.252)	(5.017)	(-2.655)	(-2.014)	(0.929)	(2.312)	(-0.210)	(1.177)	(-0.200)	(-0.171)	(-0.096)		
Student-t	2	-6.172	1.058	-0.014	-0.215	-1.670	6.301	0.972	32.919	-173.369	-3.460	0.144	23.325	3.861
		(-3.579)	(2.599)	(-0.173)	(-0.540)	(-3.096)	(2.182)	(2.526)	(3.939)	(-1.521)	(-2.389)	(3.834)		

Table A3.8: BDL regression in different periods: FOSA measures – Two month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on two-month-sample FOSA measure $E_t(VaR_{t+1})_{FOSA}$ and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). Parametric VaRs are at 1% level of significance.

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Returns	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel 1: Original sample July 1962 - December 2005							
IID	-3.475	1.234	-0.062	-9.767	-0.185	-0.526	4.828	0.329	27.019	-47.904	-0.957	0.001	8.95%
NonPara	(-3.654)	(3.718)	(-1.711)	(-9.335)	(-1.042)	(-1.805)	(3.162)	(2.077)	(5.882)	(-0.980)	(-1.269)	(0.031)	
IID	-3.617	1.272	-0.060	-10.157	-0.154	-0.509	4.832	0.391	30.300	-62.321	-0.802	-0.005	9.06%
Gaussian	(-4.102)	(4.141)	(-1.578)	(-9.171)	(-0.904)	(-1.725)	(3.031)	(2.440)	(6.228)	(-1.278)	(-1.092)	(-0.150)	
IID	-3.949	1.348	-0.063	-10.106	-0.128	-0.464	4.992	0.393	30.706	-64.835	-0.827	-0.006	9.33%
Student-t	(-4.092)	(4.039)	(-1.710)	(-9.340)	(-0.755)	(-1.605)	(3.143)	(2.453)	(6.328)	(-1.340)	(-1.119)	(-0.189)	
IID Skewed	-3.654	1.273	-0.064	-10.034	-0.187	-0.511	4.835	0.366	28.975	-56.556	-0.848	-0.007	9.04%
Student-t	(-3.778)	(4.057)	(-1.738)	(-9.449)	(-1.033)	(-1.743)	(3.209)	(2.253)	(5.988)	(-1.154)	(-1.147)	(-0.211)	
NIID	-3.628	1.252	-0.065	-10.300	-0.115	-0.462	5.295	0.399	30.805	-59.039	-0.869	-0.006	8.64%
Gaussian	(-4.032)	(3.934)	(-1.730)	(-9.198)	(-0.663)	(-1.584)	(3.346)	(2.527)	(6.282)	(-1.216)	(-1.185)	(-0.191)	
NIID	-3.799	1.205	-0.067	-10.398	-0.099	-0.455	5.333	0.421	31.188	-58.345	-0.884	-0.005	8.84%
Student-t	(-4.170)	(4.150)	(-1.780)	(-9.273)	(-0.569)	(-1.566)	(3.374)	(2.650)	(6.347)	(-1.201)	(-1.205)	(-0.165)	
NIID Skewed	-3.830	1.151	-0.066	-10.276	-0.098	-0.454	5.346	0.441	31.329	-59.117	-0.884	-0.005	8.80%
Student-t	(-3.608)	(3.516)	(-1.457)	(-2.348)	(-0.514)	(-1.774)	(2.944)	(2.529)	(5.960)	(-1.019)	(-1.198)	(-0.138)	

(Continued)

Table A3.8: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Returns	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel 2: New sample January 2006 - June 2013							
IID	-6.628	1.419	0.011		0.558	-0.849	3.305	2.024	18.883	-160.625	-3.988	0.125	29.27%
NonPara	(-1.631)	(1.898)	(0.126)		(0.569)	(-0.921)	(1.651)	(1.350)	(1.762)	(-1.286)	(-2.473)	(4.084)	
IID	-6.422	1.366	0.008		1.156	-0.736	2.467	2.005	15.531	-163.003	-3.965	0.130	31.85%
Gaussian	(-1.668)	(2.709)	(0.093)		(1.030)	(-0.807)	(1.208)	(1.339)	(1.389)	(-1.368)	(-2.464)	(4.079)	
IID	-6.104	1.273	0.022		0.990	-0.756	2.668	1.802	16.220	-153.471	-3.814	0.123	31.26%
Student-t	(-1.619)	(2.526)	(0.259)		(0.858)	(-0.806)	(1.292)	(1.226)	(1.341)	(-1.299)	(-2.432)	(3.948)	
IID Skewed	-5.787	1.080	0.017		0.681	-0.900	3.164	1.920	18.938	-165.772	-3.779	0.123	29.43%
Student-t	(-1.454)	(2.150)	(0.185)		(0.639)	(-1.007)	(1.450)	(1.244)	(1.530)	(-1.273)	(-2.574)	(4.128)	
NIID	-5.756	1.199	-0.012		1.032	-0.691	3.465	1.792	15.699	-147.146	-4.010	0.129	31.00%
Gaussian	(-1.550)	(2.646)	(-0.141)		(0.942)	(-0.702)	(1.792)	(1.224)	(1.391)	(-1.145)	(-2.535)	(4.043)	
NIID	-5.597	1.022	-0.013		1.041	-0.688	3.184	1.807	15.437	-144.181	-3.970	0.129	31.05%
Student-t	(-1.514)	(2.666)	(-0.153)		(0.950)	(-0.697)	(1.619)	(1.229)	(1.349)	(-1.110)	(-2.536)	(4.007)	
NIID Skewed	-5.428	0.932	-0.013		1.017	-0.684	3.162	1.768	15.631	-144.428	-3.952	0.128	30.86%
Student-t	(-1.471)	(2.611)	(-0.150)		(0.933)	(-0.691)	(1.595)	(1.199)	(1.358)	(-1.104)	(-2.525)	(3.991)	

(Continued)

Table A3.8: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Returns	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel 3: Full sample July 1962 - June 2013													
IID	-3.116	1.165	-0.031	-9.134	-0.199	-0.552	3.572	0.329	27.530	-88.083	-1.754	0.092	11.76%
NonPara	(-4.057)	(4.380)	(-0.867)	(-8.909)	(-1.287)	(-1.923)	(2.817)	(2.211)	(7.633)	(-1.741)	(-2.149)	(3.691)	
IID	-2.957	1.060	-0.027	-9.430	-0.173	-0.537	3.560	0.374	29.777	-98.739	-1.653	0.095	11.94%
Gaussian	(-4.174)	(4.635)	(-0.724)	(-8.748)	(-1.164)	(-1.852)	(2.747)	(2.497)	(8.163)	(-1.963)	(-1.996)	(3.587)	
IID	-3.240	1.108	-0.028	-9.335	-0.151	-0.501	3.704	0.381	29.888	-98.134	-1.649	0.092	12.20%
Student-t	(-3.895)	(4.358)	(-0.687)	(-2.152)	(-0.867)	(-2.029)	(2.387)	(2.412)	(7.249)	(-1.919)	(-2.534)	(4.665)	
IID Skewed	-2.907	1.010	-0.028	-9.275	-0.197	-0.536	3.598	0.356	28.844	-94.615	-1.667	0.091	11.63%
Student-t	(-3.760)	(4.077)	(-0.752)	(-8.772)	(-1.279)	(-1.864)	(2.857)	(2.378)	(8.047)	(-1.872)	(-2.052)	(3.467)	
NIID	-2.865	0.991	-0.033	-9.569	-0.156	-0.503	4.130	0.376	29.511	-91.340	-1.711	0.093	11.62%
Gaussian	(-4.063)	(4.578)	(-0.905)	(-8.835)	(-1.028)	(-1.740)	(3.280)	(2.508)	(7.963)	(-1.794)	(-2.082)	(3.558)	
NIID	-2.860	0.897	-0.033	-9.608	-0.147	-0.500	4.096	0.386	29.486	-90.058	-1.722	0.094	11.72%
Student-t	(-4.070)	(4.666)	(-0.906)	(-8.865)	(-0.968)	(-1.734)	(3.250)	(2.565)	(8.007)	(-1.766)	(-2.095)	(3.554)	
NIID Skewed	-2.827	0.832	-0.033	-9.505	-0.148	-0.499	4.110	0.398	29.461	-90.261	-1.721	0.094	11.66%
Student-t	(-4.000)	(4.578)	(-0.889)	(-8.818)	(-0.980)	(-1.735)	(3.284)	(2.637)	(8.030)	(-1.770)	(-2.094)	(3.546)	

Table A3.9: MS-BDL Investigation: measures skipping 2 lags (TOSA) – One month measures

This table shows the results of the MS-BDL framework for one-month-sample TOSA (two-lag-skipped-autoregressive) measures using the expanded set of state variables. In each Markov switching regression, monthly market excess return at time $t+1$ is regressed on $E_t(VaR_{t+1})_{TOSA}$ and other control variables at time t. The first line of each regression shows the value of regression coefficients, while the second line shows the corresponding t-statistics (in brackets). All parametric VaRs are at 1% significance level. The sample period is July 1962–June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
	1	-4.468	4.059	-0.195	-0.587	-0.133	4.887	0.145	1.540	-26.155	-0.008	-0.011	6.856	5.704
IID		(-2.494)	(4.538)	(-3.284)	(-2.283)	(-0.329)	(2.455)	(0.624)	(0.248)	(-0.463)	(-0.062)	(-0.506)		
Nonparam	2	-5.075	1.345	-0.036	-0.422	-1.068	6.934	0.595	32.530	-143.968	-3.081	0.133	23.067	4.058
		(-2.763)	(1.832)	(-0.490)	(-1.180)	(-2.099)	(2.505)	(1.786)	(4.734)	(-1.399)	(-2.344)	(3.897)		
	1	-3.201	2.811	-0.194	-0.478	0.105	5.252	0.176	3.544	-33.216	-0.031	-0.006	6.139	4.507
IID		(-2.299)	(4.934)	(-3.466)	(-1.714)	(0.330)	(2.548)	(0.773)	(0.542)	(-0.556)	(-0.083)	(-0.246)		
Gaussian	2	-4.773	1.149	-0.032	-0.369	-1.006	6.209	0.597	33.595	-153.076	-3.028	0.130	22.486	4.776
		(-2.789)	(1.868)	(-0.503)	(-1.161)	(-2.324)	(2.471)	(1.959)	(4.994)	(-1.593)	(-2.370)	(3.957)		
	1	-3.718	2.869	-0.194	-0.512	0.117	5.263	0.198	2.301	-33.697	0.131	-0.009	5.925	4.411
IID		(-2.341)	(4.361)	(-3.356)	(-1.632)	(0.322)	(2.461)	(0.839)	(0.315)	(-0.480)	(0.156)	(-0.316)		
Student-t	2	-4.881	1.197	-0.036	-0.365	-1.007	6.134	0.559	33.727	-145.363	-3.177	0.132	22.273	4.984
		(-2.803)	(1.794)	(-0.524)	(-1.137)	(-2.353)	(2.469)	(1.858)	(5.104)	(-1.557)	(-2.378)	(3.926)		
IID	1	-3.586	3.025	-0.213	-0.547	-0.078	5.327	0.162	1.307	-13.023	0.033	-0.013	5.850	4.184
Skewed		(-2.473)	(5.224)	(-3.691)	(-1.749)	(-0.238)	(2.668)	(0.748)	(0.194)	(-0.238)	(0.121)	(-0.569)		
Student-t	2	-4.457	1.022	-0.021	-0.402	-0.944	6.108	0.542	31.622	-133.011	-2.944	0.127	21.906	4.940
		(-2.678)	(1.701)	(-0.327)	(-1.322)	(-2.276)	(2.459)	(1.878)	(5.062)	(-1.468)	(-2.450)	(4.094)		

(Continued)

Table A3.9: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
	1	-1.127	1.754	-0.153	-0.631	0.293	5.241	-0.067	3.191	-18.434	0.005	-0.004	7.053	5.240
NIID		(-0.961)	(3.891)	(-2.614)	(-2.544)	(1.002)	(2.214)	(-0.288)	(0.439)	(-0.330)	(0.077)	(-0.145)		
Gaussian	2	-5.587	1.375	-0.024	-0.205	-1.434	6.795	0.737	32.372	-157.041	-3.505	0.135	22.974	4.747
		(-2.951)	(2.143)	(-0.361)	(-0.542)	(-2.858)	(2.602)	(2.243)	(4.642)	(-1.547)	(-2.678)	(4.118)		
	1	-1.142	1.654	-0.154	-0.617	0.308	5.280	-0.065	3.070	-19.910	-0.008	-0.004	6.878	5.143
NIID		(-0.984)	(4.288)	(-2.612)	(-2.476)	(1.004)	(2.266)	(-0.257)	(0.437)	(-0.356)	(-0.055)	(-0.175)		
Student-t	2	-5.544	1.207	-0.028	-0.204	-1.418	6.797	0.759	32.124	-153.944	-3.483	0.136	22.837	4.765
		(-2.960)	(2.152)	(-0.409)	(-0.575)	(-2.860)	(2.626)	(2.237)	(4.648)	(-1.539)	(-2.684)	(4.126)		
NIID	1	-1.034	1.534	-0.153	-0.623	0.325	5.268	-0.046	2.805	-23.262	0.005	-0.005	6.814	5.115
Skewed		(-0.960)	(4.276)	(-2.624)	(-2.549)	(1.119)	(2.299)	(-0.211)	(0.391)	(-0.417)	(0.101)	(-0.216)		
Student-t	2	-5.485	1.114	-0.027	-0.203	-1.427	6.770	0.773	32.105	-152.156	-3.476	0.135	22.768	4.814
		(-3.004)	(2.161)	(-0.412)	(-0.589)	(-2.922)	(2.626)	(2.339)	(4.680)	(-1.535)	(-2.691)	(4.181)		

Table A3.10: BDL regression in different periods – measures skipping 2 lags (TOSA) – One month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on two-month-sample TOSA (two-lag-skipped-autoregressive) measure $E_t(VaR_{t+1})_{TOSA}$ and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). Parametric VaRs are at 1% level of significance.

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Returns	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel 1: Original sample July 1962 - December 2005							
IID	-3.418	1.515	-0.066	-10.009	-0.192	-0.485	5.425	0.328	27.413	-54.804	-0.965	-0.002	7.93%
NonPara	(-3.105)	(3.008)	(-1.790)	(-9.322)	(-1.078)	(-1.698)	(3.544)	(2.052)	(5.636)	(-1.103)	(-1.274)	(-0.066)	
IID	-4.199	1.670	-0.071	-10.127	-0.129	-0.436	5.207	0.388	30.585	-72.586	-0.999	-0.003	8.63%
Gaussian	(-3.726)	(3.717)	(-1.913)	(-9.402)	(-0.724)	(-1.557)	(3.336)	(2.359)	(5.881)	(-1.441)	(-1.324)	(-0.106)	
IID	-4.385	1.663	-0.073	-10.095	-0.133	-0.438	5.243	0.375	30.103	-68.591	-1.024	-0.002	8.79%
Student-t	(-3.735)	(3.794)	(-1.983)	(-9.408)	(-0.744)	(-1.557)	(3.367)	(2.287)	(5.957)	(-1.384)	(-1.345)	(-0.069)	
IID Skewed	-3.858	1.508	-0.066	-10.126	-0.181	-0.474	5.422	0.345	28.566	-61.357	-0.986	-0.004	8.21%
Student-t	(-3.289)	(3.163)	(-1.807)	(-9.364)	(-1.004)	(-1.670)	(3.573)	(2.155)	(5.645)	(-1.234)	(-1.291)	(-0.106)	
NIID	-4.274	1.627	-0.066	-10.025	-0.090	-0.406	5.455	0.399	30.848	-70.793	-1.005	-0.006	8.53%
Gaussian	(-3.774)	(3.687)	(-1.779)	(-9.199)	(-0.491)	(-1.466)	(3.514)	(2.438)	(5.981)	(-1.420)	(-1.336)	(-0.173)	
NIID	-4.510	1.583	-0.068	-10.127	-0.069	-0.398	5.452	0.425	31.304	-71.832	-1.029	-0.005	8.73%
Student-t	(-3.945)	(3.905)	(-1.802)	(-9.216)	(-0.374)	(-1.438)	(3.525)	(2.562)	(6.048)	(-1.440)	(-1.367)	(-0.140)	
NIID Skewed	-4.495	1.492	-0.067	-10.001	-0.068	-0.397	5.459	0.446	31.439	-72.557	-1.028	-0.004	8.67%
Student-t	(-3.882)	(3.818)	(-1.775)	(-9.162)	(-0.369)	(-1.436)	(3.565)	(2.653)	(6.038)	(-1.453)	(-1.362)	(-0.138)	

(Continued)

Table A3.10: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Returns	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel 2: New sample January 2006 - June 2013													
IID	-4.369	0.989	-0.022		-0.167	-1.034	4.821	1.608	27.362	-170.104	-3.659	0.123	27.22%
NonPara	(-1.089)	(0.979)	(-0.242)		(-0.176)	(-1.065)	(2.331)	(1.087)	(2.506)	(-1.190)	(-2.457)	(4.139)	
IID	-3.339	0.574	-0.014		-0.287	-1.066	5.008	1.409	28.809	-178.906	-3.658	0.123	27.09%
Gaussian	(-0.991)	(0.916)	(-0.155)		(-0.306)	(-1.118)	(2.371)	(0.988)	(2.581)	(-1.239)	(-2.479)	(4.289)	
IID	-3.027	0.420	-0.010		-0.478	-1.135	5.025	1.373	30.174	-178.333	-3.570	0.122	26.84%
Student-t	(-0.849)	(0.641)	(-0.112)		(-0.518)	(-1.198)	(2.316)	(0.951)	(2.674)	(-1.222)	(-2.379)	(4.174)	
IID Skewed	-3.212	0.445	-0.011		-0.435	-1.128	4.905	1.447	29.544	-176.322	-3.536	0.121	26.87%
Student-t	(-0.882)	(0.711)	(-0.122)		(-0.462)	(-1.201)	(2.280)	(0.987)	(2.523)	(-1.189)	(-2.521)	(4.236)	
NIID	-3.652	0.828	-0.014		0.045	-0.996	5.511	1.271	25.539	-177.123	-3.783	0.125	27.77%
Gaussian	(-1.134)	(1.404)	(-0.148)		(0.052)	(-1.071)	(2.584)	(0.870)	(2.400)	(-1.256)	(-2.475)	(4.364)	
NIID	-3.400	0.662	-0.016		-0.008	-1.011	5.390	1.262	25.948	-176.695	-3.750	0.124	27.60%
Student-t	(-1.059)	(1.274)	(-0.167)		(-0.009)	(-1.088)	(2.535)	(0.863)	(2.410)	(-1.244)	(-2.463)	(4.352)	
NIID Skewed	-3.250	0.585	-0.015		-0.041	-1.019	5.337	1.241	26.213	-176.886	-3.725	0.124	27.52%
Student-t	(-1.013)	(1.217)	(-0.162)		(-0.046)	(-1.097)	(2.509)	(0.845)	(2.421)	(-1.241)	(-2.456)	(4.338)	

(Continued)

Table A3.10: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Returns	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel 3: Full sample July 1962 - June 2013													
IID	-2.953	1.371	-0.040	-9.476	-0.215	-0.517	4.315	0.319	27.883	-90.752	-1.733	0.090	10.74%
NonPara	(-3.321)	(3.429)	(-1.091)	(-8.987)	(-1.394)	(-1.834)	(3.438)	(2.131)	(7.408)	(-1.776)	(-2.154)	(3.681)	
IID	-3.012	1.197	-0.040	-9.462	-0.182	-0.482	4.378	0.344	29.206	-99.619	-1.811	0.091	11.02%
Gaussian	(-3.436)	(3.476)	(-1.068)	(-8.877)	(-1.188)	(-1.718)	(3.487)	(2.293)	(7.655)	(-1.950)	(-2.197)	(3.693)	
IID	-3.244	1.231	-0.041	-9.437	-0.182	-0.480	4.409	0.341	29.074	-96.913	-1.830	0.092	11.10%
Student-t	(-3.486)	(3.572)	(-1.115)	(-8.870)	(-1.179)	(-1.712)	(3.502)	(2.274)	(7.663)	(-1.906)	(-2.211)	(3.719)	
IID Skewed	-2.897	1.130	-0.037	-9.480	-0.212	-0.507	4.385	0.323	28.275	-93.420	-1.763	0.089	10.75%
Student-t	(-3.193)	(3.184)	(-1.000)	(-8.905)	(-1.371)	(-1.807)	(3.503)	(2.176)	(7.538)	(-1.827)	(-2.169)	(3.613)	
NIID	-3.082	1.161	-0.037	-9.374	-0.161	-0.465	4.761	0.357	28.934	-96.053	-1.809	0.089	11.18%
Gaussian	(-3.734)	(3.890)	(-0.995)	(-8.784)	(-1.037)	(-1.665)	(3.797)	(2.383)	(7.709)	(-1.882)	(-2.195)	(3.717)	
NIID	-3.054	1.046	-0.037	-9.437	-0.154	-0.463	4.769	0.365	28.876	-95.042	-1.825	0.090	11.22%
Student-t	(-3.735)	(3.925)	(-1.002)	(-8.793)	(-0.989)	(-1.658)	(3.815)	(2.433)	(7.736)	(-1.859)	(-2.210)	(3.731)	
NIID Skewed	-2.975	0.955	-0.037	-9.348	-0.157	-0.465	4.769	0.377	28.836	-94.964	-1.818	0.090	11.15%
Student-t	(-3.660)	(3.849)	(-0.981)	(-8.757)	(-1.011)	(-1.664)	(3.828)	(2.493)	(7.746)	(-1.856)	(-2.203)	(3.726)	

Table A3.11: MS-BDL Investigation: GJR GARCH NIID measures – Two month measures

This table shows the results of the MS-BDL framework for two-month-sample GJR GARCH NIID tail risk measures using the expanded set of state variables. Specifically, monthly market excess return at time $t+1$ is regressed on a GJR GARCH NIID tail risk measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). All parametric VaRs are at 1% level of significance. The sample period is July 1962 – June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel A: Gaussian measures							
	1	-2.338	1.724	-0.090	-0.203	0.152	3.768	0.018	23.991	-21.409	-0.385	0.001	9.076	6.853
Raw		(-2.961)	(6.592)	(-1.957)	(-0.996)	(0.610)	(2.059)	(0.101)	(4.562)	(-0.399)	(-0.568)	(0.053)		
Gaussian	2	-6.788	0.910	0.078	-0.593	-2.370	7.836	1.271	33.023	-241.706	-3.562	0.147	25.908	2.702
		(-3.277)	(1.704)	(0.824)	(-0.974)	(-3.028)	(1.989)	(2.708)	(2.708)	(-1.619)	(-1.924)	(3.201)		
	1	-2.869	1.967	-0.057	-0.174	0.144	3.377	0.036	24.539	-34.613	-0.418	0.003	9.325	6.993
AR4		(-3.217)	(7.083)	(-1.282)	(-0.910)	(0.582)	(1.924)	(0.175)	(4.999)	(-0.651)	(-0.657)	(0.122)		
Gaussian	2	-6.156	0.573	0.117	-0.736	-2.678	9.017	1.258	29.761	-225.911	-3.213	0.134	26.366	2.471
		(-2.743)	(0.907)	(1.116)	(-1.102)	(-3.169)	(2.128)	(2.527)	(2.347)	(-1.440)	(-1.645)	(2.789)		
	1	-1.682	1.810	-0.142	-0.516	0.230	4.780	-0.035	11.453	-10.689	-0.016	-0.001	7.706	5.995
FOSA		(-1.444)	(4.324)	(-2.627)	(-2.202)	(0.785)	(2.199)	(-0.126)	(1.368)	(-0.154)	(-0.050)	(-0.075)		
Gaussian	2	-6.408	1.420	-0.023	-0.238	-1.607	5.990	0.899	32.698	-167.687	-3.558	0.145	23.585	4.298
		(-3.513)	(2.769)	(-0.322)	(-0.602)	(-2.925)	(2.099)	(2.486)	(4.301)	(-1.540)	(-2.516)	(4.060)		

(Continued)

Table A3.11: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel B: Studen-t measures														
	1	-2.132	1.629	-0.093	-0.203	0.236	3.976	-0.019	23.852	-22.369	-0.337	-0.001	8.527	5.997
Raw		(-2.327)	(6.636)	(-2.013)	(-0.945)	(0.921)	(2.057)	(-0.075)	(4.141)	(-0.434)	(-0.439)	(-0.069)		
Student-t	2	-6.493	0.799	0.061	-0.499	-2.199	7.281	1.238	32.122	-224.744	-3.432	0.145	25.057	2.786
		(-3.432)	(1.697)	(0.661)	(-0.916)	(-3.092)	(1.944)	(2.768)	(2.869)	(-1.657)	(-1.900)	(3.242)		
	1	-2.665	1.808	-0.058	-0.158	0.203	3.395	0.013	24.460	-34.394	-0.385	0.001	9.038	6.403
AR4		(-3.259)	(7.091)	(-1.296)	(-0.817)	(0.814)	(1.868)	(0.085)	(4.834)	(-0.693)	(-0.626)	(0.068)		
Student-t	2	-6.050	0.516	0.106	-0.675	-2.582	8.644	1.250	29.611	-217.470	-3.168	0.133	25.853	2.474
		(-2.848)	(0.916)	(1.054)	(-1.088)	(-3.238)	(2.101)	(2.726)	(2.443)	(-1.442)	(-1.691)	(2.903)		
	1	-1.569	1.694	-0.139	-0.493	0.275	4.790	-0.064	11.949	-10.867	-0.023	-0.001	7.514	5.914
FOSA		(-1.384)	(4.691)	(-2.533)	(-2.046)	(0.932)	(2.230)	(-0.210)	(1.201)	(-0.179)	(-0.124)	(-0.081)		
Student-t	2	-6.337	1.264	-0.024	-0.235	-1.633	5.868	0.927	32.070	-164.110	-3.519	0.145	23.463	4.350
		(-3.657)	(2.758)	(-0.328)	(-0.611)	(-3.127)	(2.036)	(2.512)	(4.099)	(-1.504)	(-2.553)	(4.040)		

(Continued)

Table A3.11: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel C: Skewed Student-t measures														
Raw	1	-2.035	1.528	-0.095	-0.239	0.299	4.058	0.016	23.069	-27.999	-0.247	-0.005	8.080	5.424
Skewed		(-0.947)	(6.690)	(-1.864)	(-0.993)	(1.100)	(1.849)	(0.017)	(3.052)	(-0.348)	(-0.338)	(-0.198)		
Student-t	2	-6.164	0.708	0.051	-0.417	-2.052	6.813	1.183	31.522	-207.893	-3.345	0.144	24.340	2.913
		(-3.231)	(1.532)	(0.405)	(-0.856)	(-2.827)	(1.871)	(1.883)	(2.627)	(-1.583)	(-1.963)	(3.281)		
AR4	1	-2.584	1.690	-0.061	-0.172	0.259	3.304	0.046	24.137	-39.393	-0.334	-0.002	8.709	5.719
Skewed		(-2.964)	(6.959)	(-1.354)	(-0.837)	(1.027)	(1.720)	(0.226)	(4.514)	(-0.770)	(-0.513)	(-0.111)		
Student-t	2	-5.839	0.452	0.096	-0.595	-2.433	8.229	1.216	29.428	-201.209	-3.118	0.132	25.108	2.495
		(-2.894)	(0.901)	(1.005)	(-1.046)	(-3.249)	(2.140)	(2.717)	(2.579)	(-1.404)	(-1.723)	(2.946)		
FOSA	1	-1.507	1.567	-0.137	-0.490	0.293	4.761	-0.035	11.864	-14.423	-0.007	-0.002	7.406	5.755
Skewed		(-1.510)	(4.934)	(-2.562)	(-2.056)	(1.028)	(2.216)	(-0.147)	(1.297)	(-0.184)	(-0.072)	(-0.088)		
Student-t	2	-6.274	1.137	-0.023	-0.237	-1.647	5.880	0.956	32.014	-163.386	-3.517	0.144	23.322	4.313
		(-3.577)	(2.773)	(-0.320)	(-0.634)	(-3.065)	(2.089)	(2.650)	(4.257)	(-1.525)	(-2.571)	(4.100)		

Table A3.12: BDL regression in different periods: GJR GARCH NIID measures – Two month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods. In each regression, the monthly market excess return at time $t+1$ is regressed on two-month-sample GJR GARCH NIID measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets).

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
				Panel A: Original sample July 1962 - December 2005									
Raw	-4.032	1.354	-0.034	-13.692	-0.083	-0.431	4.741	0.473	33.624	-70.718	-0.759	-0.012	10.01%
Gaussian	(-4.620)	(4.772)	(-0.900)	(-9.056)	(-0.474)	(-1.478)	(2.969)	(2.833)	(6.586)	(-1.442)	(-1.049)	(-0.371)	
AR4	-4.207	1.476	-0.009	-15.401	-0.089	-0.420	4.698	0.472	33.049	-79.212	-0.695	-0.012	9.89%
Gaussian	(-4.704)	(4.748)	(-0.227)	(-8.641)	(-0.519)	(-1.437)	(2.957)	(2.837)	(6.671)	(-1.599)	(-0.956)	(-0.350)	
FOSA	-3.917	1.384	-0.067	-10.302	-0.124	-0.462	5.139	0.404	31.352	-55.262	-0.863	-0.009	8.77%
Gaussian	(-4.064)	(3.873)	(-1.768)	(-9.113)	(-0.701)	(-1.593)	(3.237)	(2.513)	(6.215)	(-1.140)	(-1.176)	(-0.271)	
Raw	-4.111	1.299	-0.038	-13.322	-0.064	-0.419	4.796	0.482	34.030	-69.531	-0.772	-0.012	10.13%
Student-t	(-4.664)	(4.888)	(-0.996)	(-9.192)	(-0.364)	(-1.441)	(3.026)	(2.867)	(6.640)	(-1.416)	(-1.065)	(-0.365)	
AR4	-4.197	1.378	-0.012	-14.771	-0.076	-0.408	4.745	0.475	33.218	-77.277	-0.693	-0.012	9.92%
Student-t	(-4.692)	(4.781)	(-0.312)	(-8.823)	(-0.442)	(-1.394)	(3.006)	(2.845)	(6.700)	(-1.560)	(-0.954)	(-0.371)	
FOSA	-4.028	1.339	-0.069	-10.431	-0.108	-0.455	5.189	0.415	31.806	-54.339	-0.876	-0.008	8.94%
Student-t	(-4.199)	(4.088)	(-1.813)	(-9.165)	(-0.609)	(-1.569)	(3.288)	(2.568)	(6.286)	(-1.122)	(-1.192)	(-0.258)	
Raw Skewed	-4.164	1.226	-0.038	-12.925	-0.066	-0.417	4.812	0.518	34.105	-71.631	-0.767	-0.012	10.03%
Student-t	(-4.585)	(4.719)	(-0.996)	(-9.156)	(-0.373)	(-1.440)	(3.075)	(3.047)	(6.678)	(-1.465)	(-1.057)	(-0.368)	
AR4 Skewed	-4.201	1.284	-0.013	-14.215	-0.079	-0.407	4.762	0.506	33.210	-78.697	-0.687	-0.012	9.79%
Student-t	(-4.598)	(4.617)	(-0.334)	(-8.825)	(-0.461)	(-1.396)	(3.049)	(3.002)	(6.734)	(-1.592)	(-0.945)	(-0.370)	
FOSA Skewed	-4.039	1.251	-0.068	-10.302	-0.108	-0.454	5.204	0.444	31.918	-56.192	-0.876	-0.009	8.88%
Student-t	(-4.127)	(3.985)	(-1.795)	(-9.119)	(-0.610)	(-1.569)	(3.336)	(2.726)	(6.308)	(-1.163)	(-1.190)	(-0.264)	

(Continued)

Table A3.12: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
					Panel B: New sample January 2006 - June 2013								
Raw	-5.363	0.915	0.033		0.893	-0.692	2.029	1.924	15.940	-183.532	-3.740	0.128	29.65%
Gaussian	(-1.357)	(2.019)	(0.357)		(0.756)	(-0.721)	(0.882)	(1.257)	(1.261)	(-1.375)	(-2.551)	(4.064)	
AR4	-4.767	0.883	0.061		0.544	-0.802	2.105	1.724	18.611	-200.082	-3.462	0.124	28.67%
Gaussian	(-1.214)	(1.585)	(0.602)		(0.452)	(-0.846)	(0.760)	(1.143)	(1.350)	(-1.453)	(-2.574)	(4.209)	
FOSA	-6.346	1.344	-0.003		1.123	-0.695	3.158	1.941	15.030	-153.193	-4.007	0.129	31.76%
Gaussian	(-1.667)	(2.957)	(-0.038)		(1.093)	(-0.728)	(1.558)	(1.308)	(1.392)	(-1.201)	(-2.511)	(4.000)	
Raw	-5.246	0.784	0.032		0.861	-0.673	2.095	1.953	16.003	-183.461	-3.699	0.127	29.46%
Student-t	(-1.327)	(1.937)	(0.348)		(0.727)	(-0.691)	(0.902)	(1.261)	(1.245)	(-1.359)	(-2.536)	(4.045)	
AR4	-4.625	0.742	0.061		0.501	-0.781	2.184	1.753	18.773	-200.713	-3.418	0.123	28.50%
Student-t	(-1.174)	(1.498)	(0.601)		(0.410)	(-0.809)	(0.774)	(1.147)	(1.325)	(-1.444)	(-2.560)	(4.194)	
FOSA	-6.232	1.154	-0.004		1.116	-0.696	2.903	1.997	14.676	-151.220	-3.937	0.127	31.76%
Student-t	(-1.641)	(2.972)	(-0.045)		(1.101)	(-0.727)	(1.410)	(1.330)	(1.340)	(-1.170)	(-2.495)	(3.952)	
Raw Skewed	-5.064	0.704	0.032		0.837	-0.678	2.092	1.896	16.203	-183.187	-3.680	0.126	29.31%
Student-t	(-1.285)	(1.866)	(0.346)		(0.701)	(-0.692)	(0.885)	(1.227)	(1.242)	(-1.349)	(-2.530)	(4.036)	
AR4 Skewed	-4.408	0.649	0.059		0.453	-0.799	2.262	1.700	19.248	-200.050	-3.408	0.122	28.32%
Student-t	(-1.123)	(1.412)	(0.577)		(0.366)	(-0.822)	(0.786)	(1.114)	(1.331)	(-1.430)	(-2.558)	(4.192)	
FOSA Skewed	-6.018	1.038	-0.003		1.121	-0.685	2.835	1.941	14.608	-150.613	-3.926	0.127	31.66%
Student-t	(-1.593)	(2.934)	(-0.038)		(1.104)	(-0.713)	(1.367)	(1.293)	(1.325)	(-1.159)	(-2.488)	(3.937)	

(Continued)

Table A3.12: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel C: Full sample July 1962 - June 2013							
Raw	-2.729	0.880	-0.005	-11.539	-0.156	-0.493	3.403	0.415	30.319	-104.596	-1.647	0.097	11.88%
Gaussian	(-3.957)	(4.382)	(-0.133)	(-8.675)	(-1.040)	(-1.718)	(2.605)	(2.747)	(8.372)	(-2.032)	(-2.019)	(3.576)	
AR4	-2.806	0.949	0.015	-12.531	-0.167	-0.491	3.264	0.413	29.837	-112.208	-1.563	0.097	11.67%
Gaussian	(-3.900)	(4.134)	(0.393)	(-8.177)	(-1.123)	(-1.707)	(2.450)	(2.749)	(8.430)	(-2.174)	(-1.963)	(3.667)	
FOSA	-3.046	1.075	-0.033	-9.515	-0.163	-0.504	3.939	0.380	29.930	-90.668	-1.711	0.093	11.71%
Gaussian	(-4.094)	(4.405)	(-0.882)	(-8.750)	(-1.067)	(-1.752)	(3.093)	(2.521)	(8.052)	(-1.780)	(-2.069)	(3.523)	
Raw	-2.650	0.792	-0.007	-11.132	-0.150	-0.488	3.428	0.413	30.207	-103.608	-1.657	0.097	11.84%
Student-t	(-3.870)	(4.336)	(-0.183)	(-8.729)	(-0.996)	(-1.701)	(2.638)	(2.738)	(8.405)	(-2.007)	(-2.030)	(3.583)	
AR4	-2.653	0.828	0.012	-11.880	-0.165	-0.486	3.281	0.407	29.599	-110.804	-1.563	0.097	11.54%
Student-t	(-3.734)	(3.982)	(0.333)	(-8.278)	(-1.112)	(-1.689)	(2.471)	(2.715)	(8.428)	(-2.141)	(-1.969)	(3.678)	
FOSA	-2.995	0.980	-0.033	-9.575	-0.154	-0.500	3.915	0.383	29.940	-89.720	-1.714	0.094	11.78%
Student-t	(-4.108)	(4.542)	(-0.881)	(-8.782)	(-1.011)	(-1.742)	(3.080)	(2.538)	(8.122)	(-1.759)	(-2.075)	(3.510)	
Raw Skewed	-2.607	0.715	-0.007	-10.802	-0.154	-0.488	3.457	0.430	30.056	-104.030	-1.653	0.097	11.71%
Student-t	(-3.770)	(4.188)	(-0.193)	(-8.725)	(-1.028)	(-1.706)	(2.681)	(2.839)	(8.426)	(-2.016)	(-2.029)	(3.581)	
AR4 Skewed	-2.578	0.737	0.011	-11.425	-0.170	-0.488	3.319	0.421	29.427	-110.630	-1.562	0.097	11.39%
Student-t	(-3.605)	(3.809)	(0.290)	(-8.324)	(-1.150)	(-1.697)	(2.519)	(2.796)	(8.439)	(-2.139)	(-1.969)	(3.681)	
FOSA Skewed	-2.936	0.885	-0.032	-9.464	-0.156	-0.500	3.931	0.402	29.869	-90.565	-1.713	0.094	11.71%
Student-t	(-4.017)	(4.443)	(-0.858)	(-8.737)	(-1.027)	(-1.745)	(3.116)	(2.651)	(8.154)	(-1.776)	(-2.074)	(3.501)	

Table A3.13: MS-BDL Investigation: ETL measures – two month measures

This table shows the results of the MS-BDL framework for two-month-sample Gaussian ETL tail risk measures using the expanded set of state variables. Specifically, monthly market excess return at time $t+1$ is regressed on Gaussian ETL tail risk measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). All parametric VaRs are at 1% level of significance. The sample period is July 1962 – June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel A: IID measures														
	1	-1.501	1.354	0.020	-0.160	0.369	2.827	-0.020	23.441	-72.255	-0.608	0.004	8.612	5.193
Raw IID		(-1.916)	(6.829)	(0.396)	(-0.704)	(1.424)	(1.409)	(-0.112)	(4.253)	(-1.381)	(-0.668)	(0.163)		
	2	-3.165	-0.395	-0.013	-0.535	-2.340	10.466	0.935	23.827	-124.926	-2.335	0.102	24.736	2.536
		(-1.930)	(-1.267)	(-0.122)	(-0.957)	(-3.478)	(2.633)	(2.329)	(2.313)	(-0.911)	(-1.007)	(2.245)		
	1	-2.700	1.970	-0.018	-0.145	0.365	3.039	-0.046	24.208	-66.155	-0.520	0.003	8.261	4.944
AR4		(-2.963)	(6.962)	(-0.396)	(-0.605)	(1.417)	(1.461)	(-0.231)	(4.120)	(-1.249)	(-0.709)	(0.118)		
IID	2	-3.624	-0.245	0.030	-0.590	-2.260	9.249	0.988	25.185	-141.181	-2.516	0.111	24.624	2.565
		(-1.943)	(-0.504)	(0.307)	(-1.132)	(-3.385)	(2.533)	(2.466)	(2.509)	(-1.071)	(-1.326)	(2.587)		
	1	-2.503	2.129	-0.172	-0.508	0.201	4.322	0.018	10.370	-36.329	0.039	-0.005	7.220	5.931
FOSA		(-2.233)	(5.233)	(-3.289)	(-2.308)	(0.702)	(2.199)	(0.098)	(1.303)	(-0.682)	(0.106)	(-0.253)		
IID	2	-7.450	1.669	-0.027	-0.215	-1.495	5.401	0.953	33.060	-176.433	-3.428	0.154	22.673	4.515
		(-4.275)	(3.587)	(-0.385)	(-0.602)	(-2.911)	(1.955)	(2.841)	(4.755)	(-1.672)	(-2.572)	(4.449)		

(Continued)

Table A3.13: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel B: NIID measures														
	1	-3.008	1.590	-0.173	-0.222	-0.672	4.170	0.392	20.874	-32.781	-0.695	0.007	9.449	3.924
Raw		(-3.468)	(7.411)	(-3.865)	(-0.969)	(-2.231)	(2.017)	(2.013)	(4.167)	(-0.592)	(-1.012)	(0.301)		
NIID	2	-4.169	0.223	0.188	-0.349	-0.311	5.420	0.412	38.241	-135.354	-2.459	0.081	17.872	1.884
		(-2.528)	(0.605)	(1.892)	(-0.871)	(-0.507)	(1.745)	(1.045)	(3.989)	(-1.112)	(-1.661)	(1.939)		
	1	-3.674	1.915	-0.175	-0.226	-0.647	4.607	0.411	20.902	-42.473	-0.722	0.009	9.318	3.733
AR4		(-3.877)	(7.400)	(-3.872)	(-0.977)	(-2.140)	(2.219)	(2.105)	(4.059)	(-0.756)	(-1.026)	(0.363)		
NIID	2	-3.937	0.179	0.182	-0.333	-0.324	5.338	0.370	36.985	-121.012	-2.390	0.078	17.638	1.907
		(-2.302)	(0.414)	(1.853)	(-0.857)	(-0.540)	(1.764)	(0.973)	(4.038)	(-1.020)	(-1.660)	(1.948)		
	1	-2.308	1.766	-0.149	-0.387	0.177	5.100	-0.076	17.374	-11.172	-0.220	-0.005	8.295	5.485
FOSA		(-2.255)	(5.504)	(-3.019)	(-1.822)	(0.655)	(2.592)	(-0.327)	(2.504)	(-0.198)	(-0.314)	(-0.179)		
NIID	2	-7.530	1.359	0.007	-0.210	-1.827	7.508	1.146	35.483	-216.296	-3.675	0.141	23.668	3.077
		(-3.797)	(2.544)	(0.091)	(-0.441)	(-3.036)	(2.328)	(2.850)	(3.583)	(-1.733)	(-2.265)	(3.468)		

Table A3.14: BDL regression in different periods: ETL risk measures – Two month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t + 1$ is regressed on two-month-sample Gaussian ETL measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets). Parametric VaRs are at 1% level of significance.

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel A: Original sample July 1962 - December 2005							
Raw IID	-3.030	0.814	-0.015	-15.673	-0.146	-0.488	4.602	0.425	30.879	-74.135	-0.705	-0.008	9.31%
	(-4.204)	(4.650)	(-0.373)	(-8.473)	(-0.874)	(-1.610)	(2.872)	(2.678)	(6.417)	(-1.507)	(-0.974)	(-0.234)	
AR4 IID	-3.213	0.927	0.002	-17.643	-0.151	-0.476	4.599	0.419	30.367	-80.650	-0.737	-0.006	8.93%
	(-4.189)	(4.140)	(0.042)	(-7.278)	(-0.912)	(-1.579)	(2.880)	(2.684)	(6.386)	(-1.624)	(-1.007)	(-0.178)	
FOSA IID	-3.628	1.113	-0.060	-10.160	-0.152	-0.507	4.842	0.391	30.292	-62.648	-0.806	-0.005	9.06%
	(-3.716)	(3.723)	(-1.332)	(-2.326)	(-0.816)	(-1.992)	(2.656)	(2.311)	(5.953)	(-1.080)	(-1.093)	(-0.122)	
Raw NIID	-3.898	1.128	-0.052	-13.499	-0.072	-0.440	4.824	0.471	33.354	-74.599	-0.812	-0.008	9.89%
	(-4.649)	(4.740)	(-1.380)	(-9.155)	(-0.415)	(-1.510)	(3.002)	(2.904)	(6.684)	(-1.532)	(-1.119)	(-0.256)	
AR4 NIID	-3.909	1.164	-0.037	-15.136	-0.094	-0.439	4.769	0.462	32.608	-79.515	-0.754	-0.009	9.65%
	(-4.590)	(4.365)	(-0.976)	(-8.261)	(-0.552)	(-1.494)	(2.954)	(2.887)	(6.677)	(-1.629)	(-1.038)	(-0.271)	
FOSA NIID	-3.627	1.089	-0.065	-10.299	-0.115	-0.462	5.299	0.399	30.761	-59.047	-0.871	-0.006	8.63%
	(-4.028)	(3.926)	(-1.730)	(-9.203)	(-0.661)	(-1.584)	(3.349)	(2.529)	(6.284)	(-1.216)	(-1.187)	(-0.189)	

(Continued)

Table A3.14: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
					Panel B: New sample January 2006 - June 2013								
Raw IID	-4.272	0.515	0.048		0.385	-0.877	2.315	1.755	21.258	-185.525	-3.512	0.125	28.26%
	(-1.157)	(1.561)	(0.515)		(0.339)	(-0.969)	(0.856)	(1.164)	(1.531)	(-1.335)	(-2.712)	(4.365)	
AR4 IID	-2.421	0.143	0.016		-0.616	-1.225	4.388	1.355	30.487	-187.536	-3.369	0.120	26.71%
	(-0.637)	(0.310)	(0.166)		(-0.506)	(-1.346)	(1.374)	(0.895)	(1.876)	(-1.243)	(-2.690)	(4.543)	
FOSA IID	-6.392	1.177	0.008		1.140	-0.742	2.524	2.003	15.608	-162.585	-3.961	0.130	31.72%
	(-1.659)	(2.679)	(0.088)		(1.014)	(-0.813)	(1.238)	(1.337)	(1.391)	(-1.360)	(-2.463)	(4.079)	
Raw NIID	-4.823	0.689	0.014		0.716	-0.777	2.565	1.798	17.605	-178.948	-3.787	0.127	29.14%
	(-1.289)	(1.900)	(0.152)		(0.614)	(-0.809)	(1.155)	(1.206)	(1.394)	(-1.325)	(-2.579)	(4.118)	
AR4 NIID	-4.625	0.744	0.039		0.602	-0.815	1.954	1.678	17.984	-193.601	-3.486	0.124	28.90%
	(-1.253)	(1.758)	(0.414)		(0.511)	(-0.879)	(0.721)	(1.140)	(1.338)	(-1.399)	(-2.634)	(4.221)	
FOSA NIID	-5.767	1.047	-0.012		1.035	-0.690	3.460	1.792	15.678	-147.202	-4.012	0.129	31.01%
	(-1.552)	(2.649)	(-0.141)		(0.944)	(-0.701)	(1.789)	(1.224)	(1.389)	(-1.146)	(-2.535)	(4.043)	

(Continued)

Table A3.14: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel C: Full sample January 2006 - June 2013													
Raw IID	-2.167	0.572	0.011	-13.131	-0.188	-0.530	3.176	0.385	29.308	-108.429	-1.554	0.096	11.39%
	(-3.622)	(4.058)	(0.289)	(-7.895)	(-1.306)	(-1.802)	(2.402)	(2.626)	(8.393)	(-2.124)	(-1.981)	(3.674)	
AR4 IID	-2.076	0.577	0.019	-13.844	-0.210	-0.527	3.358	0.362	28.447	-110.148	-1.572	0.095	10.76%
	(-3.098)	(2.873)	(0.520)	(-6.184)	(-1.461)	(-1.798)	(2.537)	(2.509)	(8.277)	(-2.145)	(-2.034)	(3.822)	
FOSA IID	-2.959	0.924	-0.027	-9.432	-0.171	-0.536	3.577	0.374	29.748	-98.759	-1.656	0.095	11.94%
	(-4.178)	(4.633)	(-0.725)	(-8.757)	(-1.151)	(-1.848)	(2.762)	(2.500)	(8.169)	(-1.964)	(-2.000)	(3.587)	
Raw NIID	-2.638	0.734	-0.018	-11.454	-0.150	-0.499	3.554	0.410	30.053	-104.693	-1.688	0.097	11.83%
	(-3.482)	(4.050)	(-0.443)	(-2.610)	(-0.857)	(-2.014)	(2.279)	(2.550)	(7.202)	(-2.036)	(-2.587)	(4.888)	
AR4 NIID	-2.684	0.776	-0.005	-12.533	-0.165	-0.503	3.308	0.408	29.680	-111.113	-1.585	0.097	11.67%
	(-4.029)	(4.286)	(-0.131)	(-8.225)	(-1.123)	(-1.742)	(2.459)	(2.763)	(8.416)	(-2.161)	(-2.013)	(3.645)	
FOSA NIID	-2.869	0.863	-0.033	-9.569	-0.155	-0.503	4.131	0.376	29.493	-91.402	-1.712	0.093	11.62%
	(-4.065)	(4.576)	(-0.905)	(-8.838)	(-1.024)	(-1.739)	(3.281)	(2.509)	(7.964)	(-1.795)	(-2.084)	(3.560)	

Table A3.15: MS-BDL Investigation: 5 percent Skewed Student-t measures – Two month measures

This table shows the results of the MS-BDL framework for two-month-sample 5-percent Skewed Student-t tail risk measures using the expanded set of state variables. In each model, monthly market excess return at time $t+1$ is regressed on a tail risk measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). The sample period is July 1962 – June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel A: IID measures							
	1	-1.089	2.068	-0.023	-0.141	0.258	1.955	-0.135	22.938	-46.761	-0.251	-0.003	8.026	4.488
Raw IID		(-1.310)	(5.843)	(-0.446)	(-0.440)	(0.937)	(0.784)	(-0.539)	(3.470)	(-0.868)	(-0.341)	(-0.102)		
	2	-4.853	0.331	0.060	-0.617	-2.018	8.203	1.153	27.618	-176.461	-2.690	0.126	24.931	2.538
		(-2.730)	(0.423)	(0.686)	(-1.173)	(-3.111)	(2.223)	(2.675)	(2.645)	(-1.313)	(-1.518)	(2.792)		
	1	-1.204	2.784	0.061	0.041	0.087	-0.636	-0.433	22.713	-25.754	-1.601	0.025	5.826	2.495
AR4		(-0.997)	(6.682)	(1.142)	(0.118)	(0.169)	(-0.286)	(-1.402)	(3.369)	(-0.468)	(-1.502)	(0.750)		
IID	2	-3.033	-0.514	0.018	-0.513	-1.172	8.681	1.007	25.415	-122.176	-0.516	0.083	23.120	2.437
		(-1.962)	(-0.782)	(0.222)	(-1.442)	(-1.839)	(2.858)	(2.953)	(2.971)	(-1.139)	(-0.410)	(2.044)		
	1	-1.786	2.940	-0.180	-0.558	0.147	4.222	0.152	4.770	-37.758	0.187	-0.004	6.259	5.026
FOSA		(-1.530)	(5.124)	(-3.317)	(-2.241)	(0.463)	(2.265)	(0.672)	(0.608)	(-0.648)	(0.269)	(-0.124)		
IID	2	-5.072	1.576	-0.035	-0.358	-1.134	5.221	0.682	31.870	-148.538	-3.009	0.140	22.483	5.025
		(-3.436)	(2.420)	(-0.530)	(-1.126)	(-2.671)	(2.009)	(2.243)	(4.888)	(-1.575)	(-2.416)	(4.251)		

(Continued)

Table A3.15: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel B: NIID measures							
	1	-1.729	2.315	-0.124	-0.305	0.232	4.405	-0.003	21.880	-37.927	-0.277	-0.006	8.124	5.343
Raw		(-2.584)	(6.356)	(-2.682)	(-1.467)	(0.889)	(2.189)	(-0.070)	(3.619)	(-0.751)	(-0.412)	(-0.265)		
NIID	2	-6.180	1.188	0.046	-0.360	-2.004	7.119	1.167	33.414	-223.374	-3.434	0.146	24.199	2.833
		(-3.569)	(1.904)	(0.535)	(-0.748)	(-3.109)	(2.124)	(3.068)	(3.249)	(-1.752)	(-2.090)	(3.437)		
	1	-3.061	2.693	-0.185	-0.304	-0.803	4.342	0.435	19.908	-34.733	-0.506	-0.003	9.053	3.400
AR4		(-3.373)	(7.325)	(-4.031)	(-1.329)	(-2.540)	(2.005)	(2.200)	(3.846)	(-0.626)	(-0.711)	(-0.150)		
NIID	2	-4.032	0.447	0.185	-0.251	-0.211	5.307	0.406	37.344	-139.293	-2.286	0.085	17.402	1.941
		(-2.541)	(0.727)	(1.966)	(-0.690)	(-0.395)	(1.852)	(1.116)	(4.228)	(-1.209)	(-1.647)	(2.160)		
	1	-1.129	2.236	-0.147	-0.518	0.228	5.150	-0.040	11.349	-22.089	-0.089	-0.005	7.555	5.332
FOSA		(-1.199)	(5.114)	(-2.793)	(-2.261)	(0.796)	(2.417)	(-0.169)	(1.228)	(-0.401)	(-0.111)	(-0.202)		
NIID	2	-6.016	1.679	-0.011	-0.213	-1.670	6.376	0.923	33.225	-175.302	-3.450	0.146	23.337	3.801
		(-3.534)	(2.617)	(-0.157)	(-0.534)	(-3.116)	(2.188)	(2.416)	(4.064)	(-1.535)	(-2.315)	(3.863)		

Table A3.16: BDL regression in different periods: 5 percent Skewed Student-t measures – Two month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on two-month-sample 5-percent Skewed Student-t measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets).

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel A: Original sample July 1962 - December 2005													
Raw IID	-3.138	1.443	-0.002	-12.964	-0.162	-0.476	4.646	0.436	30.862	-77.678	-0.632	-0.012	9.38%
	(-4.094)	(4.942)	(-0.046)	(-9.336)	(-0.938)	(-1.592)	(2.957)	(2.682)	(6.265)	(-1.566)	(-0.864)	(-0.369)	
AR4 IID	-3.321	1.629	0.019	-13.587	-0.163	-0.475	4.647	0.426	30.110	-80.990	-0.650	-0.010	9.09%
	(-4.041)	(4.362)	(0.461)	(-8.718)	(-0.953)	(-1.598)	(2.937)	(2.660)	(6.203)	(-1.621)	(-0.872)	(-0.297)	
FOSA IID	-3.802	1.974	-0.067	-10.360	-0.177	-0.494	4.838	0.411	30.912	-66.498	-0.782	-0.010	9.11%
	(-3.774)	(3.760)	(-1.484)	(-2.371)	(-0.957)	(-1.941)	(2.655)	(2.409)	(6.011)	(-1.145)	(-1.059)	(-0.256)	
Raw NIID	-3.796	1.806	-0.050	-12.351	-0.090	-0.438	4.865	0.484	33.617	-84.898	-0.795	-0.008	9.79%
	(-4.504)	(4.487)	(-1.345)	(-9.214)	(-0.524)	(-1.515)	(3.042)	(2.971)	(6.632)	(-1.738)	(-1.096)	(-0.251)	
AR4 NIID	-3.656	1.767	-0.034	-13.339	-0.118	-0.439	4.858	0.463	32.423	-86.422	-0.726	-0.008	9.35%
	(-4.292)	(3.958)	(-0.888)	(-8.464)	(-0.698)	(-1.501)	(3.027)	(2.892)	(6.583)	(-1.758)	(-1.000)	(-0.254)	
FOSA NIID	-3.598	1.774	-0.065	-10.246	-0.125	-0.463	5.318	0.416	31.263	-67.524	-0.861	-0.007	8.72%
	(-4.031)	(3.880)	(-1.723)	(-9.154)	(-0.725)	(-1.592)	(3.368)	(2.623)	(6.258)	(-1.386)	(-1.176)	(-0.212)	

(Continued)

Table A3.16: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel B: New sample January 2006 - June 2013													
Raw IID	-4.077	0.717	0.048		0.239	-0.959	2.976	1.772	22.673	-189.300	-3.506	0.123	27.76%
	(-1.027)	(1.229)	(0.504)		(0.197)	(-1.030)	(1.124)	(1.115)	(1.522)	(-1.359)	(-2.609)	(4.329)	
AR4 IID	-2.740	0.316	0.023		-0.490	-1.182	4.256	1.450	29.281	-188.792	-3.391	0.120	26.78%
	(-0.648)	(0.396)	(0.243)		(-0.376)	(-1.277)	(1.456)	(0.883)	(1.725)	(-1.254)	(-2.650)	(4.484)	
FOSA IID	-5.475	1.545	0.000		0.813	-0.855	3.033	1.875	18.983	-171.681	-3.815	0.127	30.18%
	(-1.436)	(2.304)	(0.004)		(0.764)	(-0.949)	(1.374)	(1.234)	(1.617)	(-1.321)	(-2.581)	(4.202)	
Raw NIID	-4.689	1.018	0.013		0.693	-0.779	2.623	1.791	18.180	-177.535	-3.792	0.127	29.10%
	(-1.275)	(1.912)	(0.142)		(0.604)	(-0.813)	(1.188)	(1.206)	(1.466)	(-1.314)	(-2.583)	(4.130)	
AR4 NIID	-4.487	1.101	0.040		0.592	-0.809	1.916	1.676	18.359	-193.431	-3.469	0.124	28.92%
	(-1.236)	(1.771)	(0.423)		(0.507)	(-0.876)	(0.701)	(1.140)	(1.379)	(-1.399)	(-2.638)	(4.234)	
FOSA NIID	-5.526	1.509	-0.014		1.005	-0.701	3.256	1.790	16.245	-143.056	-3.983	0.129	31.02%
	(-1.515)	(2.691)	(-0.160)		(0.931)	(-0.715)	(1.670)	(1.227)	(1.453)	(-1.104)	(-2.548)	(4.035)	

(Continued)

Table A3.16: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel C: Full sample January 2006 - June 2013													
Raw IID	-2.083	0.909	0.017	-10.978	-0.202	-0.521	3.320	0.382	29.051	-109.480	-1.539	0.095	11.17%
	(-3.244)	(3.585)	(0.459)	(-8.319)	(-1.368)	(-1.794)	(2.566)	(2.563)	(8.271)	(-2.149)	(-1.912)	(3.616)	
AR4 IID	-2.084	0.964	0.028	-11.211	-0.214	-0.522	3.420	0.366	28.338	-110.441	-1.547	0.094	10.78%
	(-2.955)	(2.853)	(0.742)	(-7.604)	(-1.454)	(-1.805)	(2.630)	(2.476)	(8.146)	(-2.145)	(-1.931)	(3.713)	
FOSA IID	-2.782	1.427	-0.030	-9.509	-0.197	-0.527	3.630	0.374	29.750	-100.424	-1.646	0.093	11.60%
	(-3.679)	(3.965)	(-0.802)	(-8.708)	(-1.299)	(-1.820)	(2.860)	(2.468)	(8.131)	(-1.999)	(-2.018)	(3.556)	
Raw NIID	-2.490	1.109	-0.017	-10.616	-0.162	-0.498	3.608	0.413	30.107	-109.126	-1.683	0.097	11.71%
	(-3.797)	(4.179)	(-0.468)	(-8.769)	(-1.093)	(-1.741)	(2.788)	(2.772)	(8.410)	(-2.138)	(-2.060)	(3.590)	
AR4 NIID	-2.461	1.131	-0.003	-11.244	-0.180	-0.503	3.368	0.407	29.584	-114.396	-1.566	0.097	11.45%
	(-3.785)	(4.015)	(-0.092)	(-8.415)	(-1.234)	(-1.748)	(2.514)	(2.754)	(8.466)	(-2.230)	(-1.993)	(3.659)	
FOSA NIID	-2.736	1.316	-0.033	-9.504	-0.163	-0.505	4.114	0.385	29.698	-95.661	-1.706	0.093	11.65%
	(-3.974)	(4.516)	(-0.884)	(-8.791)	(-1.086)	(-1.752)	(3.273)	(2.566)	(8.027)	(-1.884)	(-2.077)	(3.516)	

Table A3.17: MS-BDL Investigation: 99 percent Skewed Student-t measures – Two month measures

This table shows the results of the MS-BDL framework for two-month-sample 99 percent Skewed Student-t tail risk measures using the expanded set of state variables. Specifically, monthly market excess return at time $t+1$ is regressed on a tail risk measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). The sample period is July 1962 – June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel 1: IID measures														
	1	-1.802	1.528	-0.151	-0.170	0.180	3.045	-0.070	23.924	-32.248	-0.666	0.003	8.588	6.410
Raw IID		(-2.514)	(8.132)	(-3.526)	(-0.953)	(0.738)	(1.818)	(-0.373)	(5.110)	(-0.637)	(-1.016)	(0.128)		
	2	-5.113	0.229	0.080	-0.551	-2.440	9.378	1.182	29.142	-224.699	-2.530	0.119	25.988	2.570
		(-2.643)	(0.530)	(0.828)	(-0.953)	(-3.224)	(2.281)	(2.632)	(2.432)	(-1.500)	(-1.291)	(2.487)		
	1	-2.240	1.899	-0.153	-0.222	0.265	3.350	-0.130	25.086	-47.591	-0.681	0.000	7.945	5.475
AR4		(-2.832)	(8.574)	(-3.517)	(-1.191)	(1.057)	(1.932)	(-0.631)	(5.277)	(-0.927)	(-1.089)	(-0.034)		
IID	2	-4.062	-0.115	0.051	-0.432	-2.175	9.701	1.061	25.298	-166.521	-2.411	0.110	24.837	2.666
		(-2.256)	(-0.258)	(0.508)	(-0.858)	(-3.223)	(2.600)	(2.651)	(2.366)	(-1.206)	(-1.319)	(2.504)		
	1	-1.201	1.609	-0.152	-0.459	0.238	4.885	-0.104	9.127	-27.461	-0.075	-0.001	7.620	5.196
FOSA		(-1.271)	(4.932)	(-2.932)	(-2.155)	(0.854)	(2.264)	(-0.488)	(1.256)	(-0.523)	(-0.172)	(-0.158)		
IID	2	-6.728	1.397	-0.005	-0.169	-1.611	6.451	0.992	35.051	-199.898	-3.508	0.146	23.024	3.713
		(-3.754)	(2.893)	(-0.122)	(-0.433)	(-2.954)	(2.271)	(2.752)	(4.402)	(-1.755)	(-2.486)	(4.074)		

(Continued)

Table A3.17: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel 2: NIID measures														
	1	-2.257	1.644	-0.129	-0.157	0.167	4.191	-0.048	22.736	-14.337	-0.460	0.003	8.927	6.599
Raw		(-2.582)	(6.670)	(-2.832)	(-0.772)	(0.667)	(2.308)	(-0.230)	(4.360)	(-0.255)	(-0.647)	(0.069)		
NIID	2	-6.696	0.857	0.056	-0.536	-2.341	7.870	1.262	33.770	-251.571	-3.578	0.145	25.800	2.683
		(-3.190)	(1.665)	(0.582)	(-0.892)	(-3.077)	(1.994)	(2.741)	(2.819)	(-1.743)	(-1.920)	(3.104)		
	1	-2.790	1.817	-0.117	-0.126	0.126	3.525	-0.009	23.432	-22.220	-0.460	0.003	9.346	7.407
AR4		(-3.834)	(7.311)	(-2.708)	(-0.699)	(0.504)	(2.100)	(-0.082)	(5.094)	(-0.436)	(-0.712)	(0.133)		
NIID	2	-6.292	0.615	0.098	-0.713	-2.670	8.963	1.270	31.275	-251.547	-3.154	0.134	26.979	2.485
		-2.685	0.971	0.941	-1.048	-3.122	2.017	2.589	2.364	-1.527	-1.556	2.699		
	1	-1.564	1.635	-0.135	-0.399	0.233	5.134	-0.140	14.225	-6.225	-0.244	0.003	7.951	5.280
FOSA		(-1.629)	(5.302)	(-2.668)	(-1.853)	(0.854)	(2.463)	(-0.611)	(1.826)	(-0.161)	(-0.368)	(0.164)		
NIID	2	(-6.410)	1.155	-0.004	-0.235	-1.763	6.639	1.008	33.487	-188.593	-3.497	0.140	23.894	3.379
		(-3.472)	(2.349)	(-0.136)	(-0.556)	(-2.976)	(2.155)	(2.629)	(3.962)	(-1.604)	(-2.248)	(3.698)		

Table A3.18: BDL regression in different periods: 99 percent Skewed Student-t measures – Two month measures

This table shows the BDL regressions using the expanded set of state variables in the 3 sub-sample periods: the Original sample (July 1962 – December 2005), the New sample (January 2006 – June 2013), and the Full sample (July 1962 – June 2013). In each regression, the monthly market excess return at time $t+1$ is regressed on two-month-sample 99-percent Skewed Student-t measure and other control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets).

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel A: Original sample July 1962 - December 2005							
Raw IID	-2.559	0.695	-0.055	-12.455	-0.147	-0.485	4.823	0.384	29.762	-68.687	-0.841	0.000	8.42%
	(-3.772)	(3.323)	(-1.428)	(-8.478)	(-0.909)	(-1.616)	(2.970)	(2.569)	(6.448)	(-1.405)	(-1.167)	(-0.014)	
AR4 IID	-2.598	0.744	-0.056	-12.633	-0.165	-0.485	4.855	0.370	29.169	-69.704	-0.854	-0.001	8.08%
	(-3.652)	(3.076)	(-1.420)	(-8.024)	(-1.018)	(-1.591)	(3.031)	(2.452)	(6.290)	(-1.405)	(-1.188)	(-0.023)	
FOSA IID	-3.698	1.239	-0.061	-10.195	-0.089	-0.439	5.215	0.397	30.396	-74.334	-0.898	-0.001	8.86%
	(-4.228)	(3.933)	(-1.650)	(-9.387)	(-0.535)	(-1.558)	(3.217)	(2.619)	(6.490)	(-1.530)	(-1.198)	(-0.024)	
Raw NIID	-3.849	1.198	-0.064	-12.890	-0.056	-0.436	4.922	0.446	32.506	-69.831	-0.852	-0.007	9.71%
	(-4.602)	(4.707)	(-1.710)	(-9.341)	(-0.321)	(-1.496)	(3.042)	(2.770)	(6.609)	(-1.427)	(-1.172)	(-0.201)	
AR4 NIID	-3.816	1.212	-0.056	-14.088	-0.086	-0.441	4.819	0.435	31.802	-72.773	-0.764	-0.008	9.47%
	(-4.508)	(4.283)	(-1.465)	(-8.471)	(-0.504)	(-1.484)	(2.956)	(2.739)	(6.527)	(-1.486)	(-1.055)	(-0.246)	
FOSA NIID	-3.605	1.161	-0.065	-10.464	-0.101	-0.458	5.388	0.382	30.071	-56.633	-0.921	-0.004	8.55%
	(-4.074)	(4.026)	(-1.762)	(-9.411)	(-0.577)	(-1.570)	(3.379)	(2.444)	(6.296)	(-1.161)	(-1.252)	(-0.126)	

(Continued)

Table A3.18: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
					Panel B: New sample January 2006 - June 2013								
Raw IID	-3.875	0.557	0.021		0.255	-0.903	2.049	1.544	22.202	-173.374	-3.508	0.124	28.68%
	(-1.175)	(2.050)	(0.232)		(0.262)	(-1.045)	(0.802)	(1.094)	(1.827)	(-1.229)	(-2.826)	(4.371)	
AR4 IID	-3.106	0.450	0.025		-0.176	-1.073	3.174	1.344	26.313	-185.711	-3.456	0.122	27.62%
	(-0.944)	(1.250)	(0.276)		(-0.173)	(-1.222)	(1.090)	(0.948)	(1.957)	(-1.271)	(-2.845)	(4.430)	
FOSA IID	-5.528	1.134	-0.006		0.772	-0.813	3.844	1.692	18.653	-148.774	-3.955	0.131	30.51%
	(-1.477)	(2.108)	(-0.072)		(0.671)	(-0.828)	(2.069)	(1.175)	(1.690)	(-1.212)	(-2.400)	(4.100)	
Raw NIID	-5.082	0.797	0.015		0.780	-0.753	2.490	1.869	17.092	-178.904	-3.790	0.127	29.26%
	(-1.334)	(1.934)	(0.166)		(0.655)	(-0.782)	(1.117)	(1.244)	(1.344)	(-1.330)	(-2.580)	(4.117)	
AR4 NIID	-4.855	0.851	0.044		0.646	-0.792	1.859	1.741	17.555	-194.920	-3.459	0.124	28.97%
	(-1.296)	(1.790)	(0.466)		(0.541)	(-0.855)	(0.679)	(1.177)	(1.296)	(-1.414)	(-2.629)	(4.225)	
FOSA NIID	-6.116	1.194	-0.014		1.108	-0.677	3.226	1.887	15.057	-145.176	-4.009	0.129	31.34%
	(-1.617)	(2.729)	(-0.157)		(0.999)	(-0.688)	(1.653)	(1.281)	(1.328)	(-1.132)	(-2.541)	(4.020)	

(Continued)

Table A3.18: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
							Panel C: Full sample January 2006 - June 2013						
Raw IID	-2.017	0.567	-0.020	-11.234	-0.187	-0.533	3.265	0.364	28.877	-103.702	-1.596	0.097	11.15%
	(-3.661)	(3.808)	(-0.529)	(-8.434)	(-1.326)	(-1.806)	(2.399)	(2.544)	(8.254)	(-2.025)	(-2.120)	(3.774)	
AR4 IID	-1.968	0.583	-0.019	-11.247	-0.210	-0.534	3.408	0.345	28.289	-104.779	-1.603	0.095	10.72%
	(-3.339)	(3.078)	(-0.496)	(-7.876)	(-1.493)	(-1.793)	(2.524)	(2.426)	(8.219)	(-2.042)	(-2.153)	(3.861)	
FOSA IID	-3.085	1.053	-0.031	-9.536	-0.126	-0.484	4.138	0.376	29.478	-101.018	-1.718	0.095	11.91%
	(-4.309)	(4.655)	(-0.864)	(-8.997)	(-0.851)	(-1.712)	(3.236)	(2.570)	(7.905)	(-2.013)	(-2.062)	(3.647)	
Raw NIID	-2.657	0.808	-0.024	-11.106	-0.133	-0.495	3.556	0.394	29.764	-103.037	-1.702	0.098	11.76%
	(-4.010)	(4.447)	(-0.658)	(-8.821)	(-0.891)	(-1.719)	(2.708)	(2.659)	(8.312)	(-2.008)	(-2.099)	(3.642)	
AR4 NIID	-2.644	0.828	-0.015	-11.851	-0.157	-0.503	3.285	0.389	29.336	-108.279	-1.582	0.097	11.53%
	(-3.968)	(4.145)	(-0.396)	(-8.272)	(-1.066)	(-1.729)	(2.413)	(2.653)	(8.333)	(-2.104)	(-2.022)	(3.689)	
FOSA NIID	-2.928	0.958	-0.034	-9.731	-0.137	-0.499	4.117	0.365	29.222	-90.553	-1.736	0.094	11.62%
	(-4.160)	(4.749)	(-0.921)	(-8.984)	(-0.897)	(-1.724)	(3.237)	(2.445)	(7.962)	(-1.779)	(-2.128)	(3.597)	

Table A3.19: MS-BDL Investigation: FOSA measures with Realized Variance – Two month measures

This table shows the results of the MS-BDL framework where both two-month-sample FOSA measures and Realized Variance are included. Specifically, monthly market excess return at time $t+1$ is regressed on a FOSA measure, last month Realized Variance and other time-t expanded control variables. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding t-statistics (in brackets). All parametric VaRs are at 1% level of significance. The sample period is July 1962 – June 2013.

Measure	State	Const	$E_t(VaR_{t+1})$	Realized Variance	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
IID	1	-3.463	2.996	0.006	-0.211	-0.640	0.077	3.616	0.197	0.709	-86.545	0.288	-0.006	5.539	3.806
Nonparam		(-2.095)	(3.755)	(0.347)	(-2.899)	(-2.209)	(0.224)	(1.905)	(0.882)	(0.109)	(-1.448)	(0.397)	(-0.244)		
	2	-5.288	1.319	-0.006	-0.059	-0.310	-0.991	5.637	0.593	30.989	-110.305	-3.136	0.128	20.674	4.959
		(-4.098)	(3.241)	(-1.029)	(-0.888)	(-1.052)	(-2.606)	(2.301)	(2.163)	(5.269)	(-1.287)	(-2.723)	(4.104)		
	1	-1.676	1.893	0.004	-0.152	-0.504	0.253	3.900	-0.004	10.816	-41.674	0.079	0.001	7.260	5.454
IID		(-1.518)	(2.985)	(0.219)	(-2.498)	(-2.118)	(0.695)	(1.908)	(-0.112)	(1.320)	(-0.785)	(0.164)	(0.116)		
Gaussian	2	-6.479	1.574	-0.011	-0.067	-0.192	-1.547	6.861	0.905	33.344	-174.794	-3.661	0.140	22.694	4.181
		(-3.811)	(2.957)	(-1.525)	(-0.874)	(-0.507)	(-2.681)	(2.297)	(2.771)	(4.625)	(-1.638)	(-2.549)	(3.866)		
	1	-2.591	2.260	-0.005	-0.175	-0.560	0.110	4.762	0.021	9.587	-26.066	0.056	-0.006	6.973	5.802
IID		(-2.184)	(4.513)	(-0.287)	(-2.938)	(-2.430)	(0.343)	(2.349)	(0.100)	(1.161)	(-0.481)	(0.066)	(-0.221)		
Student-t	2	-6.287	1.465	-0.010	-0.070	-0.160	-1.283	6.756	0.874	32.964	-183.456	-3.325	0.129	22.983	4.669
		(-3.768)	(3.110)	(-1.394)	(-0.921)	(-0.423)	(-2.700)	(2.243)	(2.572)	(4.390)	(-1.721)	(-2.382)	(3.585)		
IID	1	-2.600	2.351	-0.001	-0.194	-0.525	0.057	4.193	0.178	4.876	-41.237	0.174	-0.009	5.704	4.282
Skewed		(-2.089)	(3.963)	(-0.046)	(-3.045)	(-1.817)	(0.177)	(2.135)	(0.813)	(0.646)	(-0.722)	(0.255)	(-0.394)		
Student-t	2	-4.996	1.122	-0.007	-0.056	-0.377	-1.028	5.879	0.645	31.630	-137.220	-3.077	0.124	21.880	4.785
		(-3.486)	(2.415)	(-1.159)	(-0.797)	(-1.205)	(-2.605)	(2.249)	(2.170)	(5.016)	(-1.462)	(-2.532)	(3.836)		

(Continued)

Table A3.19: Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Realized Variance	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
	1	-1.377	1.336	0.021	-0.101	-0.309	0.282	4.320	-0.104	18.775	-27.836	-0.288	0.006	8.337	5.600
NIID		(-1.507)	(3.483)	(1.543)	(-1.908)	(-1.386)	(1.060)	(2.102)	(-0.485)	(2.913)	(-0.523)	(-0.411)	(0.277)		
Gaussian	2	-7.561	1.616	-0.012	-0.066	-0.188	-1.961	9.084	1.207	35.809	-210.045	-4.196	0.135	24.009	3.012
		(-3.870)	(2.774)	(-1.589)	(-0.697)	(-0.367)	(-3.137)	(2.586)	(2.955)	(3.399)	(-1.643)	(-2.460)	(3.250)		
	1	-1.354	1.292	0.018	-0.105	-0.318	0.293	4.472	-0.115	18.267	-23.291	-0.292	0.006	8.055	5.339
NIID		(-1.484)	(3.802)	(1.305)	(-2.015)	(-1.423)	(1.095)	(2.163)	(-0.497)	(2.732)	(-0.416)	(-0.394)	(0.249)		
Student-t	2	-7.311	1.395	-0.012	-0.068	-0.168	-1.908	8.742	1.200	35.034	-201.387	-4.062	0.135	23.715	3.066
		(-3.861)	(2.768)	(-1.617)	(-0.721)	(-0.342)	(-3.134)	(2.560)	(2.966)	(3.479)	(-1.609)	(-2.406)	(3.282)		
NIID	1	-0.769	1.199	0.013	-0.117	-0.492	0.352	4.499	-0.073	8.132	-17.971	0.017	-0.001	7.009	6.305
Skewed		(-0.802)	(3.466)	(0.765)	(-1.884)	(-1.935)	(1.213)	(1.952)	(-0.327)	(0.878)	(-0.336)	(0.051)	(-0.103)		
Student-t	2	-6.035	1.259	-0.010	-0.075	-0.160	-1.525	6.808	0.904	31.625	-144.333	-3.730	0.139	22.654	5.624
		(-3.616)	(3.219)	(-1.479)	(-1.020)	(-0.448)	(-3.127)	(2.452)	(2.619)	(4.621)	(-1.436)	(-2.787)	(4.169)		

Table A3.20: BDL regression in different periods: FOSA measures in the presence of realized variance – Two month measures

This table shows the regression results regarding how the two-month-sample FOSA tail risk measures explain market excess return given the presence of last month realized variance and other control variables in different sub-sample periods. In each regression, the monthly market excess return at time $t+1$ is regressed on FOSA measure, last month Realized Variance, and other expanded control variables at time t. Within each regression, the first line shows the value of regression coefficients, while the second line shows their corresponding HAC t-statistics (in brackets).

Tail risk measure	Const	$E_t(VaR_{t+1})$	Realized Variance	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
						Panel A: Original sample July 1962 - December 2005								
IID Nonparam	-3.507	1.123	0.010	-0.053	-15.974	-0.164	-0.511	4.592	0.354	28.309	-59.823	-0.901	0.000	8.93%
	(-3.711)	(3.176)	(0.871)	(-1.412)	(-2.148)	(-0.920)	(-1.726)	(2.913)	(2.160)	(5.817)	(-1.170)	(-1.204)	(0.001)	
IID Gaussian	-3.580	1.197	0.005	-0.055	-13.089	-0.148	-0.503	4.731	0.398	30.662	-67.009	-0.784	-0.005	8.91%
	(-4.052)	(3.141)	(0.374)	(-1.420)	(-1.617)	(-0.852)	(-1.689)	(2.896)	(2.436)	(6.171)	(-1.320)	(-1.071)	(-0.154)	
IID Student-t	-3.913	1.279	0.005	-0.059	-12.935	-0.123	-0.460	4.883	0.401	31.073	-69.348	-0.808	-0.006	9.19%
	(-4.022)	(3.126)	(0.357)	(-1.541)	(-1.580)	(-0.713)	(-1.578)	(2.980)	(2.450)	(6.294)	(-1.378)	(-1.095)	(-0.192)	
IID Skewed	-3.634	1.171	0.007	-0.056	-14.715	-0.172	-0.501	4.664	0.382	29.755	-64.802	-0.814	-0.007	8.95%
Student-t	(-3.768)	(3.327)	(0.645)	(-1.516)	(-1.933)	(-0.941)	(-1.693)	(2.986)	(2.283)	(5.905)	(-1.267)	(-1.106)	(-0.214)	
NIID Gaussian	-3.570	1.126	0.008	-0.056	-15.452	-0.107	-0.455	5.068	0.411	31.412	-67.696	-0.830	-0.006	8.57%
	(-3.925)	(2.867)	(0.670)	(-1.429)	(-1.950)	(-0.618)	(-1.540)	(3.083)	(2.561)	(6.311)	(-1.346)	(-1.134)	(-0.195)	
NIID Student-t	-3.742	1.101	0.007	-0.059	-14.966	-0.093	-0.450	5.127	0.430	31.717	-66.153	-0.848	-0.006	8.75%
	(-4.054)	(3.067)	(0.597)	(-1.502)	(-1.894)	(-0.534)	(-1.529)	(3.113)	(2.668)	(6.361)	(-1.315)	(-1.158)	(-0.171)	
NIID Skewed	-3.769	1.047	0.008	-0.058	-15.106	-0.091	-0.448	5.128	0.449	31.881	-67.300	-0.847	-0.006	8.72%
Student-t	(-3.980)	(3.007)	(0.629)	(-1.477)	(-1.903)	(-0.524)	(-1.525)	(3.140)	(2.770)	(6.366)	(-1.343)	(-1.155)	(-0.170)	

(Continued)

Table A3.20: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Realized Variance	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel B: New sample January 2006 - June 2013														
IID Nonparam	-6.592	1.396	0.001	0.014		0.566	-0.870	3.185	2.022	18.691	-160.568	-3.963	0.125	28.36%
	(-1.600)	(1.801)	(0.057)	(0.117)		(0.555)	(-0.902)	(0.972)	(1.337)	(1.561)	(-1.276)	(-2.446)	(4.043)	
IID Gaussian	-6.571	1.536	-0.007	-0.021		1.107	-0.536	3.578	1.994	17.471	-164.694	-4.177	0.129	31.18%
	(-1.654)	(2.548)	(-0.514)	(-0.184)		(0.978)	(-0.510)	(1.153)	(1.316)	(1.458)	(-1.361)	(-2.422)	(3.936)	
IID Student-t	-6.241	1.426	-0.006	-0.003		0.945	-0.575	3.659	1.775	17.867	-153.637	-3.989	0.122	30.54%
	(-1.611)	(2.386)	(-0.440)	(-0.025)		(0.814)	(-0.531)	(1.144)	(1.200)	(1.397)	(-1.284)	(-2.432)	(3.721)	
IID Skewed	-5.830	1.111	-0.001	0.012		0.675	-0.866	3.353	1.922	19.226	-165.968	-3.814	0.122	28.53%
Student-t	(-1.406)	(1.750)	(-0.080)	(0.097)		(0.631)	(-0.857)	(1.047)	(1.231)	(1.517)	(-1.263)	(-2.602)	(4.004)	
NIID Gaussian	-5.785	1.265	-0.003	-0.027		1.003	-0.599	4.019	1.775	16.580	-147.027	-4.107	0.129	30.16%
	(-1.533)	(2.351)	(-0.240)	(-0.224)		(0.905)	(-0.543)	(1.223)	(1.207)	(1.359)	(-1.129)	(-2.444)	(3.965)	
NIID Student-t	-5.610	1.072	-0.003	-0.026		1.013	-0.605	3.674	1.791	16.246	-143.977	-4.055	0.128	30.21%
	(-1.499)	(2.396)	(-0.222)	(-0.223)		(0.914)	(-0.549)	(1.148)	(1.213)	(1.318)	(-1.094)	(-2.448)	(3.929)	
NIID Skewed	-5.433	0.968	-0.002	-0.023		0.994	-0.620	3.538	1.755	16.257	-144.271	-4.017	0.128	30.00%
Student-t	(-1.457)	(2.352)	(-0.172)	(-0.195)		(0.900)	(-0.564)	(1.111)	(1.186)	(1.311)	(-1.090)	(-2.440)	(3.922)	

(Continued)

Table A3.20: Continued

Tail risk measure	Const	$E_t(VaR_{t+1})$	Realized Variance	Lagged Return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
Panel C: Full sample July 1962 - June 2013														
IID Nonparam	-3.100	1.139	0.001	-0.028	-9.993	-0.196	-0.553	3.482	0.332	27.625	-89.499	-1.738	0.093	11.62%
	(-4.042)	(3.886)	(0.210)	(-0.769)	(-2.275)	(-1.259)	(-1.918)	(2.533)	(2.209)	(7.678)	(-1.749)	(-2.168)	(3.578)	
IID Gaussian	-3.032	1.140	-0.004	-0.034	-7.188	-0.176	-0.534	3.784	0.371	29.745	-95.923	-1.689	0.093	11.83%
	(-4.063)	(3.646)	(-0.481)	(-0.868)	(-1.477)	(-1.168)	(-1.844)	(2.757)	(2.460)	(8.130)	(-1.874)	(-2.039)	(3.450)	
IID Student-t	-3.352	1.212	-0.005	-0.037	-6.417	-0.154	-0.494	4.012	0.377	29.833	-94.347	-1.695	0.089	12.12%
	(-4.174)	(3.708)	(-0.605)	(-0.962)	(-1.276)	(-1.024)	(-1.727)	(2.895)	(2.514)	(8.156)	(-1.850)	(-2.035)	(3.238)	
IID Skewed	-2.932	1.037	-0.001	-0.030	-8.484	-0.199	-0.535	3.678	0.355	28.808	-93.518	-1.680	0.090	11.49%
Student-t	(-3.672)	(3.354)	(-0.176)	(-0.794)	(-1.771)	(-1.275)	(-1.856)	(2.721)	(2.352)	(8.045)	(-1.817)	(-2.095)	(3.325)	
NIID Gaussian	-2.883	1.011	-0.001	-0.035	-8.956	-0.157	-0.502	4.203	0.375	29.496	-90.412	-1.722	0.093	11.48%
	(-3.923)	(3.503)	(-0.138)	(-0.903)	(-1.927)	(-1.025)	(-1.727)	(3.014)	(2.496)	(7.974)	(-1.753)	(-2.111)	(3.463)	
NIID Student-t	-2.891	0.928	-0.002	-0.037	-8.575	-0.147	-0.497	4.219	0.385	29.459	-88.446	-1.741	0.093	11.58%
	(-3.951)	(3.596)	(-0.233)	(-0.942)	(-1.837)	(-0.966)	(-1.717)	(3.029)	(2.553)	(8.018)	(-1.711)	(-2.127)	(3.463)	
NIID Skewed	-2.851	0.855	-0.001	-0.035	-8.669	-0.149	-0.497	4.209	0.398	29.439	-88.969	-1.736	0.093	11.52%
Student-t	(-3.884)	(3.539)	(-0.188)	(-0.910)	(-1.847)	(-0.979)	(-1.718)	(3.038)	(2.630)	(8.044)	(-1.721)	(-2.124)	(3.457)	

CHAPTER 4: THE IMPACT OF EXTREME DOWNSIDE RISK ON STOCK RETURNS: STOCK LEVEL ANALYSIS

4.1. Introduction

The risk of asset returns experiencing some extreme movement is a topic that receives much interest in asset pricing literature. Many studies have documented the non-trivial impact of this type of risk on expected returns, which serves as one of the main explanations for the well-known Mehra and Prescott's (1985) equity premium puzzle. An important strand of the literature regarding extreme downside risk originates from Rietz (1988) and was subsequently developed by Barro (2006), Gabaix (2012), and Wachter (2013), among others, where they relate the risk of economic disaster to asset returns. However, the room for the development and applicability of this approach is limited due to the unavailability of rare event observations. On the other hand, an alternative approach for investigating extreme downside risk is to rely directly on the distribution of asset returns, which are well recognised to be skewed and tail-fatted. Utilising the availability of return data, many tail-risk-related measures have been proposed and demonstrated to have significant asset pricing implications. Measures such as VaR and the tail index of Generalised Extreme Value Distribution directly capture tail risk. Other related measures deal with components of tail risk including downside beta risk (Bawa and Lindenberg, 1977; Ang et al*.,* 2006a, among others) and higher moment risks (Kraus and Litzenberger, 1976; Harvey and Siddique, 2000; Dittmar, 2002; among others) and their idiosyncratic versions (Mitton and Vorkink, 2007; Boyer et al., 2010; among others).

In order to investigate the asset pricing implication of tail risk, a version of systematic tail risk measure is required. In fact, under the canonical perspective of equilibrium asset pricing models, a systematic ('co-') version of risk measures is more preferable because it recognises the view of investors towards an asset under portfolio perspective rather than in isolation. However, almost every 'co-tail-risk' measure in the literature is essentially an indirect measure which only captures a component of tail risk, such as skewness, kurtosis, or higher moments. The direct measures such as VaR and tail index are only used in total format rather than 'co-' format. Alternatively, they are directly applied in the general market rather than at a cross-sectional level to bypass the need for such a systematic version for individual assets (see, for example, Bali et al., 2009). To the best of our knowledge, the newly proposed Left Tail Dependence measure of Ruenzi and Weigert (2013) is the only direct systematic measure that is associated with a significant tail risk premium. However, as we have analysed in Chapter 2, this copula-based measure is still subjected to certain limitations, such as the intensive data requirement and the ignorance of the actual performance of assets in distress periods.

In this chapter, we propose several new systematic extreme downside risk measures and analyse their relationship to returns. These measures could be classified into two groups. The first group is constructed by shifting the standard beta and comoment measures to the tail of return distribution utilising the approach of downside beta. The second group is based on the performance of a stock return when market crash risk varies. It directly relies on the hedging need of investors against extreme downside risk. We demonstrate that our measures are significantly related to returns and are robust under different settings.

We first investigate the systematic extreme downside risk-return relationship by sorting all stocks in NYSE, AMEX, NASDAQ, and ARCA into quintiles based on the proposed 'co-tail-risk' measures. We observe a monotonic increase of average returns from the lowest risk quintile to the highest risk quintile. This, along with the significantly positive alphas of long-short strategies on these portfolios, confirms the economic significance of the systematic tail risk premium. We further carry out two-way sorting to take into account additional risk measures such as size, downside beta, and systematic skewness. Given these related measures, extreme downside risk still significantly explains returns in the expected direction.

In order to filter out the pure relationship between extreme downside risk and returns, we then carry out the Fama and MacBeth (1973) cross-sectional analysis, controlling for a large set of explanatory variables including downside beta, upside beta, size, Book-to-Market, volatility, past return, coskewness, cokurtosis, and the systematic tail risk measures. We show that even taking into account other risk variables, extreme downside risk still exhibits a significant and positive relationship to returns. Further, other risk-return relationships including the size effect, the Book-to-market effect, the downside beta effect, and the leverage and volatility feedback effects also exhibit their acknowledged behaviours. Among the controlling risks, systematic skewness is remarkably affected by extreme downside risk, which is not surprising since they are both used to capture downside risk. However, the robustness of extreme downside risk over coskewness risk suggests that investors tend to put more weight on the extreme part of the distribution.

We carry out different robustness checks for our results. We first demonstrate the need to use the postformation setting in the portfolio sorting and cross-sectional regression similar to those in Ang et al*.* (2006a) by showing the low persistency property of extreme downside risk measures. We then alter different settings in our framework including the tail threshold, the VaR and ELT models of market tail risk, and different lengths of the estimation period for risk measures. These tests confirm our conjecture regarding the inconsistency of the first measure group in low threshold setting due to the lack of tail event observations. On the other hand, they also show the desirable performance of our second measure group, which is robust under any set-up, even at a low extreme threshold and monthly sampling. Finally, we demonstrate that our measures contain significant information in excess of the related systematic volatility risk measure of Ang et al*.* (2006b).

We make the following contributions to the literature. First, we create some new systematic tail risk measures which are simple but effective in capturing tail risk and its relation to returns. Particularly, our proposed measures are the natural developments of canonical systematic measures and therefore enrich the corresponding literature. Second, given the limited evidence in the literature regarding systematic tail risk premium, we demonstrate a significantly positive premium across a large universe of stocks, confirming the risk-return relationship postulated by asset pricing theory. Thirdly, the empirical results in our studies provide further evidence for a number of established risk-return relationships, including beta risk, size risk, Book-over-Market risk, coskewness risk, cokurtosis risk, as well as leverage and volatility feedback effects. Finally, our measures solve one of the most troublesome obstacles in tail risk studies, helping to construct tail risk measures using a short sample setting so that a large number of tail risk observations could be generated without the need for high frequency data. This is not only essential to the extension of research data to a far history, but also important for the introduction of more complicated econometric models in tail risk tests.

The remainder of this chapter is organised as follows. Section 4.2 introduces our new systematic risk measures along with their behaviours over time. Sections 4.3 and 4.4 utilise these proposed measures to investigate the relationship between extreme downside risk and expected returns using portfolio sorting analysis and Fama and MacBeth (1973) cross-sectional analysis respectively. Section 4.5 provides different robustness checks for the validity of our inferences. Section 4.6 summarises the main findings of the chapter and offers some concluding remarks.

4.2. Constructing new systematic tail risk measures

In this section, we propose two groups of systematic tail risk measure to capture the systematic tail risk of an asset. The first group is based on the expansion of comoment and downside-beta literature, while the second group is based on the argument that investors need to hedge against crashes. We also provide a statistic summary for the proposed measures, while their performance in asset pricing tests is presented in the next section.

4.2.1. Extreme downside beta and extreme downside comoment

The significant role of downside beta is supported in many studies in asset pricing literature, such as Bawa and Lindenberg (1977), Harlow and Rao (1989), Ang et al*.* (2006a), Estrada (2007), among others. For example, Bawa and Lindenberg (1977) construct a general framework of mean-lower partial moment for economic agents whose utility functions exhibit standard properties such as positive marginal utility and risk aversion. Using a similar approach to the CAPM, they derive a general asset pricing relationship between excess returns of an asset and its systematic lower partial moments. They also show that this relationship holds when returns follow different types of distribution including Normal, Student-t, and Stable distribution satisfying some criteria. When returns are normally distributed, the model reduces to the standard CAPM. Thus, they claim that the lower partial moment asset pricing model should be, at least, as good as CAPM in explaining empirical data. Similarly, Ang et al. (2006a) show evidence of a positive downside beta risk premium using a cross section of stock returns in the US market. They demonstrate that this premium is significant given the presence of a number of other risk factors including Size, Book-to-Market, volatility and higher comoments, among others. On the other hand, they find that the evidence for upside beta is weak, confirming the asymmetric preference of investors towards uncertainty. Recently, this measure is further demonstrated to explain the cross-sectional returns of investments in various asset classes, including currency, fixed income, commodity, equity, and equity index options in Lettau et al (2014). International evidence supporting downside beta is also well documented. For example, Estrada (2007) demonstrates that downside beta could explain returns significantly better than the canonical CAPM beta from a global portfolio perspective using the data of 50 developed and developing markets.

The development of downside beta from standard CAPM beta suggests some interesting ways to construct 'co-tail-risk' measures. Specifically, we could choose the thresholds of these downside measures to be some quantile other than the mean. Similarly, we can also construct the extreme-downside comoments from the standard comoment measures using the same approach. The formulas of commonly used downside betas in the literature, namely those of Bawa and Lindenberg (1977) (β_{BL}^{D}), Ang et al. (2006a) (β_{ACY}^{D}), and Estrada (2007) (β_{ES}^{D}), are given as:

$$\beta_{BL,i}^{D} = \frac{E\{(R_i-\mu_i)\times min(R_m-\mu_m,0)\}}{E\{min(R_m-\mu_m,0)^2\}} \quad (4.1)$$

$$\beta_{ACY,i}^{D} = \frac{E\{\tilde{R}_i^- \times \tilde{R}_m^-\}}{E\{\tilde{R}_m^{-\,2}\}} \quad (4.2)$$

$$\beta_{ES,i}^{D} = \frac{E\{min(R_i-\mu_i,0)\times min(R_m-\mu_m,0)\}}{E\{min(R_m-\mu_m,0)^2\}} \quad (4.3)$$

where R_i, R_m are excess returns of asset i and the stock market; μ_i, μ_m are the means of R_i, R_m; R_i^-, R_m^- are R_i, R_m when R_m is lower than its mean; and $\tilde{R}_i^-$, $\tilde{R}_m^-$ are the demeaned R_i^-, R_m^-. Changing the threshold from mean to some α-quantile (Q), we have the corresponding Extreme Downside Betas (hereafter EDB):

$$EDB_{BL,i} = \frac{E\{(R_i-\mu_i)\times(R_m-\mu_m)_Q\}}{E\left\{(R_m-\mu_m)_Q^2\right\}} \quad (4.4)$$

$$EDB_{ACY,i} = \frac{E\left\{\tilde{R}_i^Q\times\tilde{R}_m^Q\right\}}{E\left\{\tilde{R}_m^{Q\,2}\right\}} \quad (4.5)$$

$$EDB_{ES,i} = \frac{E\{(R_i-\mu_i)_Q\times(R_m-\mu_m)_Q\}}{E\left\{(R_m-\mu_m)_Q^2\right\}} \quad (4.6)$$

where $(R_i - \mu_i)_Q$ is equal to $(R_i - \mu_i)$ if $R_i < Q_i$ and 0 otherwise; and $(R_m - \mu_m)_Q$ is equal to $(R_m - \mu_m)$ i$R_m < Q_m$f and 0 otherwise; R_i^Q, R_m^Q are R_i, R_m conditional on $R_m < Q_m$; $\tilde{R}_i^Q$, $\tilde{R}_m^Q$ are the demeaned R_i^Q, R_m^Q; and Q_m, Q_i are the α-quantile of R_m, R_i.

On the other hand, we could also change the standard comoment measure to the lower tail to obtain Extreme Downside Comoment (hereafter EDC) measures. Specifically, starting from the formula of comoment (see, for example, Ang et al, 2006a; Guidolin and Timmermann, 2008):

$$k^{th} - comoment\ of\ asset\ i = \frac{E\left[(R_i-\mu_i)(R_m-\mu_m)^{k-1}\right]}{var(R_i)^{1/2}var(R_m)^{(k-1)/2}} \quad (4.7)$$

the corresponding Extreme Downside Comoment measures are given as:

$$EDC_{BL,i} = \frac{E\left[(R_i-\mu_i)(R_m-\mu_m)_Q\right]}{var(R_i)^{1/2}E\left((R_m-\mu_m)_Q^2\right)^{1/2}} \quad (4.8)$$

$$EDC_{ACY,i} = \frac{E\left[\tilde{R}_i^Q\times\tilde{R}_m^Q\right]}{var\left(\tilde{R}_i^Q\right)^{1/2}var\left(\tilde{R}_m^Q\right)^{1/2}} \quad (4.9)$$

$$EDC_{ES,i} = \frac{E\left[(R_i-\mu_i)_Q(R_m-\mu_m)_Q\right]}{E\left((R_i-\mu_i)_Q^2\right)^{1/2}E\left((R_m-\mu_m)_Q^2\right)^{1/2}} \quad (4.10)$$

where the notations are as in formulas (4) to (6). We use the moment order $k = 2$ since we are interested in how asset returns perform relative to the market returns in extreme market events.

Each of these extreme downside measures implies an aspect of the performance of an asset in distressed periods. Specifically, the Bawa and Lindenberg (1977) type measures (hereafter BL measures) show the tendency of an asset to offer worse returns when the market crashes, while the Estrada

(2007) type measures (hereafter ES measures) capture the tendency of an asset to crash in those times. ES measures appear to be somewhat similar to the Left Tail Dependence measure of Ruenzi and Weigert (2013), although they are much more straightforward and simple. On the other hand, Ang et al. (2006a) type measures (hereafter ACY measures) capture the tendency of an asset to move with the market in crash periods. Accordingly, we expect our measures to capture positive risk premiums since stocks with higher measures are undesirable to investors.

One potential problem with these 'co-tail-risk' measures is that they essentially rely on a very small number of observations. For instance, if we consider the conventional 5 percent quantile, a full year of 250 observations only produces 12 observations for the estimation of these measures. This may cause large errors in estimating these measures in cases of low crash thresholds such as 1 percent. Therefore, we only introduce these measures to gain some intuition about the influence of tail risk on returns. In the next sub-section, we introduce a more advanced measure which relies directly on the tail risk hedging argument. This new measure overcomes the observation-shortage problem and produces a desirable performance under any settings. Furthermore, among the three types of EDB and EDC measures, we expect the problem to be most serious for ACY measures. The reason for this is that they aim to capture how an asset moves with the market in crash times, while the market crash observations are quite separate. In fact, the market rarely experiences crashes on consecutive days.[2] Thus, we conjecture that ACY measures might not be reliable even in conventional crash thresholds.

4.2.2. Extreme Downside Hedge measures

The second group of systematic tail risk measures are the Extreme Downside Hedge measures (hereafter EDH), as they rely directly on the argument that investors like to hedge against extreme downside risk. Any asset that provides this kind of hedge would command a price premium. This measure can be constructed by simply regressing asset returns on a market tail risk measure. Since each observation of this measure is obtained from a regression, in order to generate enough observations of the measure over time, we need a very

[2] The correlation between two variables calculated from non-consecutive observations is invalid.

large number of observations for market tail risk. We opt out of the use of high frequency return data since we need a long history data to have sufficient observations of some risk measures like Book-to-Market. High frequency returns are only available for a short recent period and for a limited stock universe. Using daily return data, the only tail risk measure that can generate enough observations for the overtime 'co-tail-risk' regression is VaR. In fact, VaR is the only tail risk measure that produces daily observations from daily return input data. In the tail risk literature, VaR is one of the most commonly used tail risk measures (see Alexander, 2009; Bali et al., 2009; Adrian and Brunermeier, 2011; among others). Moreover, many studies have developed efficient models to estimate daily VaR with a high level of precision. A good review regarding VaR estimation can be found in Kuester et al. (2006).

Thus, we estimate market VaR for every trading day to capture the daily tail risk of the market, which is subsequently linked to the asset returns of corresponding days to obtain a 'co-tail-risk' measure. However, we do not directly link the market daily VaR of a day t (VaR_t) on day t return of an asset, since VaR_t is just the expected tail risk of day t estimated based on information of up to day $t-1$. Instead, we calculate change in market VaR from day t to day $t+1$, which is mainly influenced by information in the market in day t, to represent actual market tail risk on day t. Thus, we regress the asset returns on the innovation of VaR to capture its 'co-tail-risk'.[3] This approach is similar to that of Ang et al. (2006b) where they use change in market volatility to capture the systematic volatility risk of the market.

As the base framework in this study, we use AR(1)-EGARCH(1,1) location-scale filter to obtain *iid* residual in the VaR estimation. This filtering is essential for VaR models since the fitted distribution from which the quantile is used as VaR is estimated assuming *iid* observations. Furthermore, we assume the residual to follow Skewed Student-t distribution so that VaR further reflects any additional fat-tail and skewness of the return distribution after autocorrelation and volatility clustering are filtered. This format is well recognised as one of the most efficient and commonly used VaR estimation models (see, for example,

[3] We also examine 'co-tail-risk' calculated from regressing asset returns on the raw value of market VaR. However, this measure is not associated with any significant risk premium in any of our test.

Berkowitz and O'Brien, 2002; Bao et al., 2006; Kuester et al., 2006; among others). Additionally, we use 5 year rolling windows (1,250 daily observations) and 5 percent threshold for our VaR estimation models. In the robustness check section, we will examine the performance of our measures under alternative settings. We proxy the market excess return by the difference between the CRSP all stock index returns and the daily risk free rate obtained from Kenneth R. French's online database. The CRSP index represents the largest stock markets in the US, including NYSE, AMEX, NASDAQ, and ARCA.

After estimating the daily VaR for the market over time, the 'co-tail-risk' measure of an asset for a specific period is estimated by regressing its daily excess return with the corresponding change in the VaR of the market. Thus, the estimated coefficient, which is named EDH, directly shows how an asset performs as market risk changes.

$$R_{i,t} = \alpha_i + EDH_i \times \Delta VaR_{m,t+1} + \varepsilon_{i,t} \qquad (4.11)$$

where $R_{i,t}$ is excess return of stock i in day t; $\Delta VaR_{m,t+1}$ is the innovation of the daily VaR of the market from day t to day $t+1$; α_i and $\varepsilon_{i,t}$ are the intercept and error terms, respectively. In our setting, for convenience, we do not take the absolute of market VaR, so a lower VaR means a larger loss. Therefore, a lower VaR implies a higher risk, and a lower ΔVaR implies the risk is accelerating. Therefore, an asset with a high level of EDH offers poor returns when tail risk accelerates. Therefore, investors would require a price discount or return premium in order to hold it. On the other hand, an asset with a low level of EDH would offer a hedge when tail risk mounts up and, thus, can be sold at a price premium. Hence, the risk premium corresponding to this type of measure is expected to have positive sign.

4.2.3. Systematic tail risk over time

Figures 4.1 to 4.4 illustrate the evolutions of our measures over time, revealing some interesting patterns. Figure 4.1 shows the monthly average of innovations in the market tail risk (ΔVaR) from January 1973 to December 2012. It is clear that the market tail risk has changed significantly over time and clusters around some market distress periods such as the year 1987, the dot-com boom and bust of the early 2000s, and the recent 2007-2010 financial crisis. The lowest levels of this series, or equivalently the highest points of the market tail risk, are also located around these days.

The average 'co-tail-risk' of all stocks in the market measured by EDH is presented in Figure 4.2. For simplicity, we only calculate the equally weighted average of EDHs of all stocks in a year to represent the average 'co-tail-risk' of that year. The risk measure is winsorised using Ang et al*.'s* (2006a) method to reduce the effect of outliers. Specifically, we replace observations which are lower than the 1 percentile level or higher than the 99 percentile level of the distribution of the risk measure across all stocks with the 1 percentile and 99 percentile correspondingly. From this figure, the average systematic tail risk tends to increase over time, implying the market is becoming more and more crash sensitive. This is reasonable as investors are facing more and more turbulent market. Moreover, the 'co-tail-risk' appears to predict the systematic collapse in 2007-2008 as it surges significantly prior to the crisis, implying some individual crashes could trigger a serious collapse on a systematic scale. Specifically, it foretells a collapse spiral when a firm specific crash up to some level could lead to a market crash which, in turn, drives all stock prices to shrink sharply as the whole market is highly crash sensitive. This further amplifies the market index crash and then the crashes of companies' stock prices. This snowball phenomenon was exactly what happened in the last financial crisis. Indeed, the historically high level of this measure prior to the crisis does not say when the crisis will happen, but it does say that if the crisis happens, it will be serious and prolonged. Moreover, given that this high level could actually be observed using data prior to the crisis, it could be regarded as a practical warning rather than an ex-post indicator. If enough attention had been paid to this early warning of a systematic collapse and individuals had hedged their positions accordingly, they would have protected a significant part of their wealth during the last crisis.

Additionally, Figures 4.3 to 4.4 show the average 'co-tail-risk' measured by our EDB and EDC measures. They exhibit interesting patterns like jumps in 1987 or significant surges from the dot-com bubble to the current financial crisis. However, their high levels at the beginning of the sample period seem to be unreasonable. This is because these measures are developments of betas and downside betas, which also feature similar patterns as illustrated in Figure 4.5.

The abnormally high average beta levels in this period could be attributed to several factors. First, before 1982, the CRSP-all-stock market was mostly constituted from stocks in NYSE, and NASDAQ had not been included. Therefore, the market was highly concentrated at that time. Second, the peak of 1977-1979 might be due to the fact that the market experienced a rather 'quiet' period then as shown in the levels of annual volatility of the market index in Figure 4.6. As a result, the majority of stocks tend to move closely with the market and average beta approximate 1. Among these extreme downside measures, the patterns of ACY measures seem to be most dubious, which is consistent with our analysis above.

[Figure 4.1]

[Figure 4.2]

[Figure 4.3]

[Figure 4.4]

[Figure 4.5]

[Figure 4.6]

4.3. Portfolio sorting analysis

To examine the risk-return relationship, we first study whether portfolios sorted on 'co-tail-risk' measures earn significantly different average returns. By the construction of our measures, the portfolios of stocks with higher measures should earn higher returns on average. We follow Ang et al*.* (2006a) to sort stocks into portfolios using their postformation risk. In other words, stocks are sorted into portfolios based on the realisation of their risk during the period when the portfolio returns are calculated. We apply this setting because 'co-tail-risk' measures are not persistent over time. In the robustness checks, we show that our 'co-tail-risk' measures exhibit low persistency which is comparable to that of Ang et al.'s (2006a) downside beta, which are much less persistent than well-known preformation-suitable measures including size, Book-to-Market, and idiosyncratic volatility, among others. In fact, the commonly used preformation portfolio sorting is only suitable for measures that are highly persistent over time (see, for example, Daniel and Titman, 1997; Ang et al., 2006a; Ruenzi and

Weigert, 2013). As an illustration, Figure 4.7 shows that even CAPM beta fails to exhibit the expected relationship with return in a preformation setting and only works under a postformation setting. Moreover, as the main aim of any sorting scheme is to capture the asset pricing relationship of risk-return, postformation sorting naturally fits this purpose. On the other hand, asset pricing theory suggests that a forward looking risk measure should be used instead of a pure past period measure in preformation sorting (see Pastor and Stambaugh, 2003).

[Figure 4.7]

To carry out the postformation sorting for our risk measures, at the beginning of every year (from 1973 to 2012), we calculate our measures for all stocks in the NYSE, AMEX, NASDAQ, and ARCA markets using their daily data during that year.[4] We then sort them into quintiles based on the corresponding risk measures and calculate the equally weighted and value weighted excess returns of these quintiles for the concurrent year, which is the same year used to calculate the risk measures. The average returns of these portfolios over the entire sample period are reported in Tables 4.1 and 4.2. We further calculate the return of the long-short strategy which longs portfolio 5 (highest exposure) and shorts portfolio 1 (lowest exposure). We report the alphas of Fama-French's (1993) (hereafter FF) three factor model, and Carhart's (1997) four factor model in explaining the returns of this long-short strategy.

[Table 4.1]

[Table 4.2]

Table 4.1 shows a significant positive relationship between the systematic tail risk and returns with respect to all of our risk measures. The average excess returns increase monotonically from quintile 1 to quintile 5 and the returns from long-short strategies are highly positive, even after controlling for other systematic risks in the FF and Carhart models. The Newey-West t-statistics of the long-short strategy returns and of its alpha are significant at 1%. Moreover, this table also shows that stocks with higher systematic tail risks tend to have bigger sizes. The size patterns of our risk measures are consistent with an

[4] We eliminate stocks with less than half a year of observations (125 observations).

interpretation of systematic risk measures as ones reflecting the contribution of an asset to systematic risk (see, for example, Kraus and Litzenberger, 1976; Harvey and Siddique, 2000; Mitton and Vorkink, 2007; among others). On the other hand, this size pattern also suggests that if the average returns of quintile portfolios are measured in a value weighted manner, the influence of size effect on returns will weaken the systematic tail risk effect. This is because larger stocks have lower returns on average, which is opposite to the effect of systematic tail risk. Moreover, as argued by Ang et al. (2006a), the effects of downside risk would be stronger in small stocks compared to in large stocks[5] and using equally weighted returns would be more appropriate in examining downside risk. These may explain why the performance of our measures deteriorates when value weighting returns are examined, as shown in Table 4.2. However, they are still statistically significant in most cases.

Even if the performance of our measures is strong in the one-way sorting analysis, that may change if we account for other risk factors such as size, coskewness, and downside beta. The size effect may significantly affect tail risk given the size pattern of the 'co-tail-risk' mentioned above. On the other hand, since skewness risk is a component of the extreme downside risk, it is possible that skewness risk may account for the majority of the extreme downside risk effect. Furthermore, since many of our measures are constructed from downside beta, it is questionable whether they are robust given the presence of downside beta. Tables 4.3 to 4.16 provide results for the two-way sorting examinations of our risk measures. In the sorting exercise with downside beta, we report the results which use Ang et al.'s (2006a) downside beta as the sorting criteria. The results for other types of downside beta are similar and reported in the Appendix.

Tables 4.3 to 4.9 clearly show that the systematic tail risk measures still exhibit a strong influence on returns even after controlling for size, coskewness and downside beta factors. A monotonic upward trend of average excess returns corresponding to higher 'co-tail-risk' could be seen in almost every size, coskewness, and downside beta quintile. Similarly, our long-short strategy earns statistically significant abnormal returns in most cases. Moreover, the performances of our measures in size quintiles further demonstrate Ang et al.'s

[5] This is not contradicting the fact that larger stocks have higher risk measures.

(2006a) argument that downside risk exhibits a stronger impact in small firms. Specifically, the returns of the long-short strategy of small firms are much larger and more significant than those of larger firms. Furthermore, given that most of our measures are constructed using the downside beta approach, their strong performances even in the presence of this risk measure implies that extreme downside risk is not simply downside risk. Extreme downside risk should contain additional information regarding how investors valuate securities. Thus, we come up with similar inferences as those of Ruenzi and Weigert (2013). On the other hand, it should also be noted that common asset pricing consensus regarding the influences of the controlling risks still hold. Specifically, we observe the negative premiums for size and coskewness, and positive risk premium for downside beta within almost all systematic tail risk quintiles.

[Table 4.3]

[Table 4.4]

[Table 4.5]

[Table 4.6]

[Table 4.7]

[Table 4.8]

[Table 4.9]

However, the results for the value weighted returns (Tables 4.10 to 4.16) of the sorted portfolios are weaker than the corresponding equally weighted ones, except for the sorting by size. The similarity of the results between the two weighting schemes in the case of sorting by size is obvious since controlling for size would make these schemes converge. On the other hand, in the examinations controlling for other risk measures, the size pattern of tail risk mentioned previously explains the weakening of the tail risk effects, especially when our tail risk is accounted as the second sorting criteria.[6] Therefore, in order to truly reveal the net influence of extreme downside risk on returns in

[6] Berk (2000) and Chi-Hsiou Hung et al. (2004) argue about how a factor yields advantages or disadvantages when sorted as the first or second criteria.

excess of all the other risk factors, we turn to cross-sectional regression analysis.

[Table 4.10]

[Table 4.11]

[Table 4.12]

[Table 4.13]

[Table 4.14]

[Table 4.15]

[Table 4.16]

4.4. Cross-sectional analysis

In order to account for a large set of risk measures, given the large sample requirement in their estimations, we follow the approach of Ang et al*.* (2006a) to use overlapping yearly sample periods. Specifically, at the beginning of every month, we calculate the excess return of a security for the next one year over the T-bill rate over the same year. We then estimate its risk measures using its daily returns over the same year. These risk measures are referred to as realised risk measures as in Ang et al*.* (2006a). To be consistent with Ang et al*.* (2006a), we use the value available at the examining time for last year's return, size (natural logarithm of market capitalisation), and Book-to-Market rather than the valued realised over the next one year. We also follow their method to separate normal *Beta* into *Downside beta* and *Upside beta* in order to control for normal downside risk. Our full set of control variables includes *realised Downside beta, realised Upside beta, realised standard deviation, realised coskewness, realised cokurtosis, size, Book-to-Market, last year return,* and our *realised 'co-tail-risk' measures.*

The systematic risk premiums corresponding to the controlled risks are estimated by cross-sectionally regressing the realised next year's excess return on this set of risk factors using all stocks in the market. The time series of these estimated risk premiums, upon which the Fama-Macbeth's (1973) average risk premium is estimated, is obtained by repeating this procedure for every month.

Similar to Ang et al. (2006a), we use Newey-West's (1987) HAC with 12 lags in estimating the standard errors of the estimated risk premiums to account for the problem of overlapping estimations. We also apply Ang et al.'s (2006a) suggestion to winsorise all independent variables to reduce the effects of outliers. Similar to Ang et al. (2006a), we find that this practice is mainly for the benefit of Book-to-Market measure, while the performances of other measures are almost unchanged. Our total sample period is from 1973 to 2012, which provides 468 monthly cross-sectional regressions. Table 4.17 gives the results of this cross-sectional investigation.

[Table 4.17]

Before incorporating the new co-tail-risk measures, we first re-examine the performances of canonical risk measures in Models I to III. Accordingly, the results confirm the standard findings regarding these risk factors. For example, we capture a strongly positive downside beta risk premium and a much weaker (even negative) upside beta risk premium, which is similar to Ang et al. (2006a). Size and Book-over-Market significantly affect returns in a negative and positive direction respectively, which is consistent with other studies. We also reassure the existence of leverage effects and volatility feedback effects (see, for example, Black, 1976; Christie, 1982; Campbell and Hentschel, 1992; among others) by showing the significantly negative coefficient of realised volatility. Further, the risk premium of coskewness is negative and that of cokurtosis is positive. On the other hand, our finding regarding past returns tends to support the reversal effect rather than the momentum effect, although the effect is statistically insignificant.

In models IV to X, we incorporate the new systematic tail risk measures and capture interesting findings. First, our 'co-tail-risk' measures significantly positively explain returns at 1 percent significant level in most cases, except for ACY measures. The failure of ACY measures is, indeed, consistent with our expectation in the risk measure construction. On the other hand, regarding the performances of the other classical risk factors, introducing 'co-tail-risk' appears to only significantly change the performance of coskewness, leaving other risk factors relatively stable. This is not surprising since the coskewness and the systematic extreme downside measures both reflect downside risk, although at different levels. Thus, along with the portfolio sorting analysis of the previous part, these results further demonstrate that extreme downside risk contains additional important information in excess of the general downside risk. It implies that investors dislike downside risk and put more weight on the extreme part of the distribution.

4.5. Robustness checks

4.5.1. The persistency of risk measures

In order to support our choice to relate realised risk factors with concurrent returns in the above analysis, our first robustness check is to demonstrate the low persistence of downside risk and extreme downside risk over time. We first examine the common proportion of each quintile between any two consecutive years. This shows the tendency of a security to remain in the same quintile in the next year. Specifically, we take the number of securities belonging to a specific quintile between any two consecutive years divided by the average quintile size of the two years. The investigation is carried out every year and the average over our sample period from 1973 to 2012 is reported. We compare these statistics of our measures with those corresponding to some well-established risk measures commonly examined in the preformation framework including size, Book-to-Market, and idiosyncratic volatility of Ang et al*.* (2006b). We also use the corresponding statistic of Ang et al*.*'s (2006a) downside beta to be the benchmark for postformation-suitable risk measures. Hereafter, we denote preformation measures and postformation measures to refer to these two types. These statistics are given in Table 4.18. To be consistent with the standard approach in dealing with Book-to-Market, we choose the starting point of a year to be the 1st of July. Book-to-Market is calculated as the Equity Book Value of the last financial year over market capitalisation on the 31th December of last year. Size is the natural logarithm of market capitalisation on the 30th June, and all other risk measures are calculated using daily return over the defined year (July to June).

[Table 4.18]

According to this table, the persistence of systematic tail risk measures approximates that of Ang et al*.*'s (2006a) downside beta, which is much lower than that of the preformation measures. The differences between the persistence of preformation measures and postformation measures are most obvious for the lowest and highest quintiles. In these quintiles, the percentages of common stocks over the two consecutive years of postformation measures are normally 20% lower than those of preformation measures. Furthermore, less than half of the securities in the highest and lowest quintiles remain in these quintiles in the following year in all postformation measures. This is opposite to the much higher corresponding numbers of the preformation ones. This suggests that portfolio-sorting analysis using the preformation setting for the 'co-tail-risk' measures does not provide valid inferences. Thus, along with other arguments in previous sections, examining our risk measures in their relationship with concurrent returns is justified.

In the second investigation we follow the approach of Ruenzi and Weigert (2013) to explore how the risk level of a quintile changes over time. Specifically, for every year in our sample, we identify the constituent stocks of a quintile and calculate the equally weighted value of a risk measure for that quintile in that year as well as in the following four years. We apply this calculation for every year and average the five year pattern over time. This average five year pattern illustrates the persistence of the risk levels of stocks in different quintiles. Similar to our first investigation, we apply the same technique for Ang et al.'s (2006a) downside beta as the benchmark for postformation measures and Size, Book-to-Market, and idiosyncratic volatility as the examples of preformation risk measures. Figures 4.8 to 4.16 illustrate the patterns of these measures.

[Figure 4.8]

[Figure 4.9]

[Figure 4.10]

[Figure 4.11]

[Figure 4.12]

[Figure 4.13]

[Figure 4.14]

[Figure 4.15]

[Figure 4.16]

The above figures clearly demonstrate that our systematic extreme downside risk measures are much less persistent than the preformation measures. In cases of postformation measures, the differences in the risk levels between quintiles reduce significantly after just one year, while those of preformation measures stay relatively stable over time. Indeed, these patterns of our 'co-tail-risk' measures are similar to those of Ruenzi and Weigert's (2013) LTD measure, which is also a postformation one. Thus, this investigation is additional evidence against the use of the preformation format for analysing the asset pricing effects of the risks regarding the downside of return distribution.

4.5.2. Different extreme downside threshold levels

In this robustness check, we investigate whether altering the quantile levels in measuring 'co-tail-risk' would affect the performances and inferences of our measures. In our standard framework above, we utilise the 5 percent quantile of return distribution in all of our measures. In this examination, we replace it with the 10 percent and 1 percent thresholds. Table 4.19 shows how these recalculated measures work in the cross-sectional regression framework. The EDH measures show a consistent performance in all cases. Regarding the EDB and EDC measures, although they produce the expected performances in most cases, a clear pattern could be revealed. Specifically, except for the EDB-ES measure, other measures tend to perform worse when the quantile level is reduced. According to the results in this table as well as in Table 4.17 for the 5 percent threshold, the significance of these measures in explaining returns increases monotonically with alpha.

As explained in the 'co-tail-risk' measure construction, this behaviour is well-expected since these EDB and EDC measures rely on a small part of estimation sample. The actual number of observations used in the measurement reduces one-to-one with the level of quantile used. For example, in using one year data of daily returns, 10 percent quantile measures rely on a maximum of 25 observations, where the corresponding numbers of 5 percent and 1 percent quantile measures are 12 and 2. Thus, these measures are extremely sensitive to the chosen estimation window as well as the quantile level, and their monotonic decreasing power is reasonable. This is the major disadvantage of these types of systematic extreme downside risk measure. However, EDH measures do not suffer from this problem as they employ all observations in the estimation window. In a latter part, we further demonstrate that this advantage of EDH helps it to work well even in settings with a much shorter estimation window. Thus, it provides econometric tests such as the Fama-Macbeth (1973) cross sectional regression with a much larger number of observations over time. This benefit could not be achieved under any currently available downside risk measures.

[Table 4.19]

4.5.3. Different Value-at-Risk measures

We further check for the robustness of EDH measures by using different VaR models for the market daily tail risk. Specifically, we replace the Skewed Student-t with Gaussian for the distribution of residual terms in the AR(1)-EGARCH(1,1) model. We also use different estimation windows, namely 5-year window (1,250 days) and 2-year window (500 days). Moreover, we also calculate ETL as an alternative for VaR. According to Artzner et al. (1999), ETL is regarded as a "coherent" risk measure which is more complete than VaR. For simplicity, we apply Gaussian ETL, which is formulated as:

$$ETL_{\alpha} = \frac{1}{\alpha}\varphi\big(\Phi^{-1}(\alpha)\big)\sigma - \mu \qquad (4.12)$$

where the return is assumed to follow $N(\mu, \sigma)$; $\Phi^{-1}(\alpha)$ is the quantile of $N(0,1)$ corresponding to the level of significance α, and $\varphi(\cdot)$ is the probability density function of $N(0,1)$. The results of these examinations are summarised in Table 4.20. Accordingly, all EDH measures consistently positively explain returns at 1% level of significance. Thus, this type of systematic risk measures is highly robust under different settings.

[Table 4.20]

4.5.4. EDH performance in short-estimation-period setting

As previously argued, EDH measures experience an important advantage that all daily observations are utilised in their estimations. Therefore, it would be interesting to see whether this type of risk measures can work in shorter estimation period settings. For this purpose, we examine EDH measures estimated every month using stock returns and market daily VaR within that month. To the best of our knowledge, there is no currently available systematic extreme downside risk measure that could be calculated for a sample period as short as one month using daily data. In fact, they all need to rely on at least one year of daily data. As a result, researchers need to use overlapping regressions as in the cross-sectional analysis in order to obtain a large number of observations over time. Therefore, given the short-sample measurability, EDH measures might offer a solution.

Table 4.21 illustrates how EDH measures perform under this setting. Specifically, we run the cross sectional regression analysis for all stocks in the market in every month from January 1973 to December 2012 and report the mean and t-statistic of the estimated coefficients. This is similar to the Fama-Macbeth (1973) cross sectional framework we established in the previous section. However, we now use non-overlapping monthly regression instead of the overlapping yearly regression at monthly frequency. At the beginning of each month, we use the data of the next one month to calculate the dependent variable of monthly excess return, the independent variables of standard beta, coskewness, cokurtosis, EDH. We use standard beta instead of downside- and upside-beta due to the limited number of return observation in a month. We also include size and Book-to-Market observed at the beginning of the month and last month excess return as independent variables. However, we do not include realised volatility as an independent variable because when this measure is introduced, some well-known canonical risk-return relationships such as those of beta and size turn out to be paradoxical. Specifically, size strongly positively explains returns, while beta is significantly negatively related to returns.[7] Moreover, to be conservative, we still apply Newey-West standard error to account for autocorrelation and heteroskedasticity in calculating the t-statistics of the estimated risk premium series, despite the fact that our regressions now do not rely on overlapping samples.

The summarised results in Table 4.21 clearly illustrate the desirable performance of our short-term EDH measures, where they are all strongly positively related to returns at 1 percent significant levels. Thus, the performances of these small-sample EDH measures could confirm that they are

[7] However, it should be noted that our EDH measures still perform well (significantly positively explain returns) if realised volatility is included.

a solution for the dilemma in coping with data-extensive measures to capture extreme downside risk. Regarding other risk factors, beta tends to positively explain returns but only in an insignificant manner, while the size and Book-to-Market effects are still prominent. The effect of coskewness is weak and unstable, while cokurtosis even has the wrong sign in all cases. In fact, this is not surprising as all research evidence regarding the influences of coskewness and cokurtosis is only reported for large sample investigations.

[Table 4.21]

4.5.5. EDH and systematic volatility risk

By construction, our EDH measures are closely related to the systematic volatility risk measure proposed by Ang et al*.* (2006b). Specifically, a measure representing the sensitivity of a stock with respect to aggregate volatility risk is constructed by regressing excess stock returns on the changes of the Chicago Board Options Exchange's VIX index. Ang et al*.* (2006b) demonstrate that this systematic volatility risk measure significantly relates to returns, given other risk factors such as size, Book-to-Market and momentum. This risk is associated with a negative premium since a stock having higher sensitivity performs better when aggregate risk increases and, therefore, is more appealing to investors. Since VaR is significantly influenced by volatility, Ang et al.'s (2006b) risk measure should significantly cover the risk captured in EDH. The decisive question is whether extreme downside risk contains additional information above what is implied in volatility.

We confirm the close relationship between volatility and tail risk by the high correlation between Ang et al*.'s* (2006b) ΔVIX measure[8] and our ΔVaR measures, which is around 60 percent across all VaR models. However, it is interesting that this high correlation is between the change in VaR of day $t+1$ relative to day t and the change in VIX of day t relative to day $t-1$. Meanwhile, the correlation between ΔVIX and $\Delta VaRs$ of the same timing is very modest. This is not surprising because the information contained in the change of estimated VaR between day $t+1$ and t essentially reflects the change in the

[8] Similar to Ang et al. (2006b), we use the old index VXO to expand the data further to January 1986.

information set between day t and $t-1$. Thus, this correlation pattern between ΔVIX and ΔVaR supports our timing choice of VaR in the EDH construction.

In answering the question regarding the implication of extreme downside risk in excess of volatility, we first orthogonalise market tail risk against its volatility risk by obtaining the residuals of the regression of ΔVaR on ΔVIX. This residual term represents additional information of tail risk over volatility. We then construct the sensitivity measure of stock excess returns with respect to this residual using univariate regression similar to the way we construct EDH. We refer to this new measure as excess systematic extreme downside risk and denote it EDH_O. By construction, EDH_O is expected to have positive risk premium just like EDH. This is because a low level of the residual implies a low level of market (negative) VaR in excess of what is expected by the volatility level. Therefore, a stock with higher sensitivity coefficient will perform worse in risky environment, which is undesirable.

We include the new excess systematic extreme downside measure in our cross sectional regression to investigate its significance given other risk measures. In this framework, we also include Ang et al.'s (2006b) systematic volatility risk measure, which is denoted as SV. In Ang et al.'s (2006b) paper, SV is estimated via a multivariate regression of stock excess returns on market excess returns and change in the VIX index. However, since all measures in our framework, including betas (CAPM beta, downside beta, upside beta), are univariately estimated, we estimate SV using a univariate regression of stock excess returns on changes in VIX. Otherwise, the beta effect would be doubly accounted for.

Table 4.22 gives the results of the cross-sectional analysis with respect to different VaR models in our study. It should be noted that these results are for the period January 1986 to December 2012 due to the limitation of VIX data. This table confirms the significance of the systematic volatility risk premium where the corresponding estimated coefficient is highly significant and of correct sign. Importantly, our excess systematic extreme downside risk is also significant and the sign is consistent with what is theoretically postulated. Thus, similar to the findings in related studies, such as Bali et al. (2009) and our previous chapter, we show that extreme downside risk contains important information for investors in pricing assets, even in the presence of volatility. However, the incorporation of systematic volatility risk measure significantly reduces the importance of downside beta. This might not be that surprising since beta is essentially a systematic measure capturing how a stock contributes to the volatility of the market. On the other hand, upside beta seems to gain some significance. Therefore, the total influence of SV on beta should be evaluated through examining beta directly rather than breaking it into downside and upside versions. In fact, as shown in Tables 4.23 and 4.24, the CAPM beta is less significant in all cases when SV is included. Additionally, Book-to-Market also becomes insignificant when SV is included. However, as can be seen in Table 4.24, this is just a time-specific phenomenon rather than a consequence of the inclusion of SV. In other words, even without SV, Book-to-Market is still insignificant in the sample period from 1986 to 2012. Table 4.25 provides more supportive results where short-term (one month) risk measures are examined.

[Table 4.22]

[Table 4.23]

[Table 4.24]

[Table 4.25]

4.6. Conclusion

In this chapter, we have demonstrated the significantly positive relationship between extreme downside risk and returns by introducing new systematic extreme downside risk measures, classified into two main groups. The first group, including EDC and EDB measures, is a natural development of classical downside beta and comoment. On the other hand, the second group, which is EDH, is constructed based on investors' need to hedge against extreme downside risk. We successfully capture the positive risk premiums corresponding to these measures. Furthermore, the EDH measures can be estimated within a narrowed estimation sample as short as one month. This unique advantage provides a solution for the observation-shortage problem in every extreme downside risk investigation. Our EDH analytic framework further opens up ways to capture and investigate the implication of idiosyncratic extreme downside risk. For example, as mentioned in the review of related literature, the idiosyncratic risk framework suggested by Huang et al. (2012) ignores systematic tail risk from the construction of idiosyncratic tail risk measure. However, there is currently no possible solution to overcome this weakness. Fortunately, the success of our EDH framework suggests that one could include the daily innovation of market tail risk as a factor in addition to other systematic risk factors. As a result, the residual term could be regarded as being free of systematic extreme downside risk and subsequently used in their suggested idiosyncratic extreme downside risk calculations. Furthermore, the computability of EDH in a small sample period could enable a favourable working platform for the introduction of other new measures to capture this idiosyncratic risk in subsequent studies.

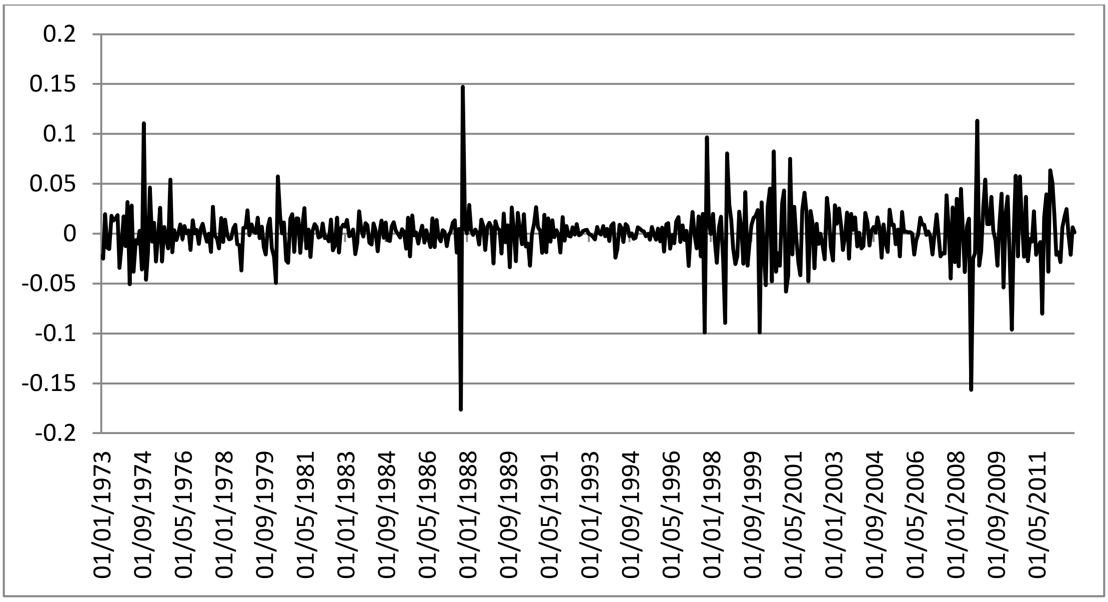


Figure 4.1: Innovation in Market Tail Risk. This figure displays the monthly average of daily changes in the market Value-at-Risk from January 1973 to December 2012. The market is represented by the CRSP index for all stocks in the NYSE, AMEX, NASDAQ, and ARCA markets.

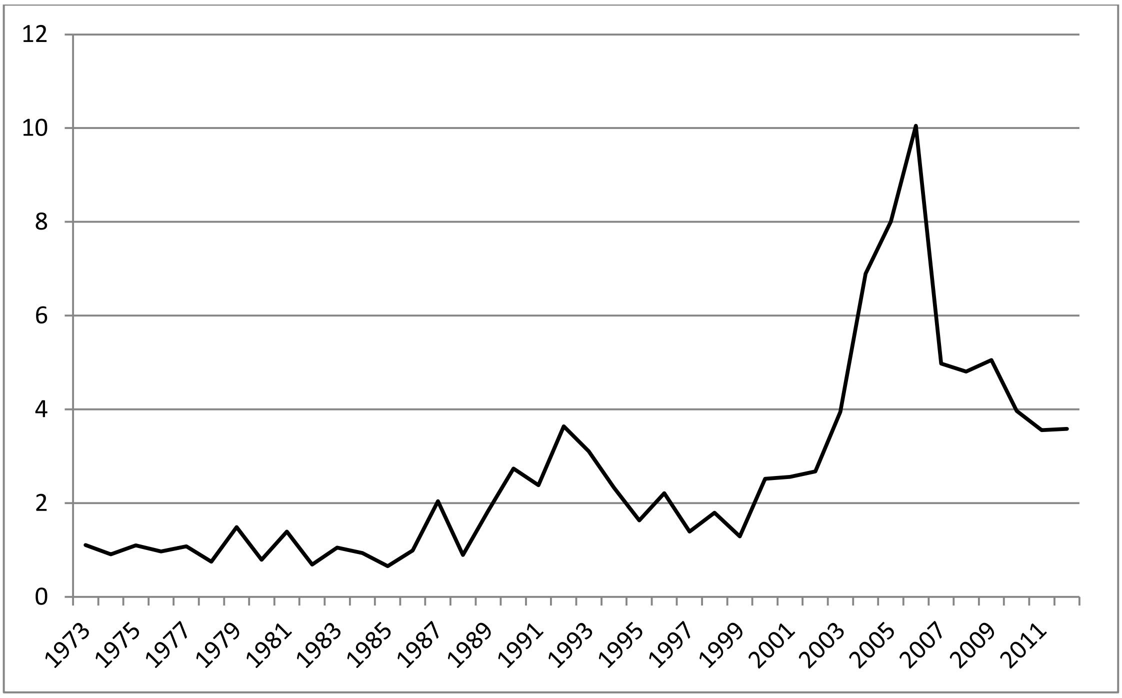
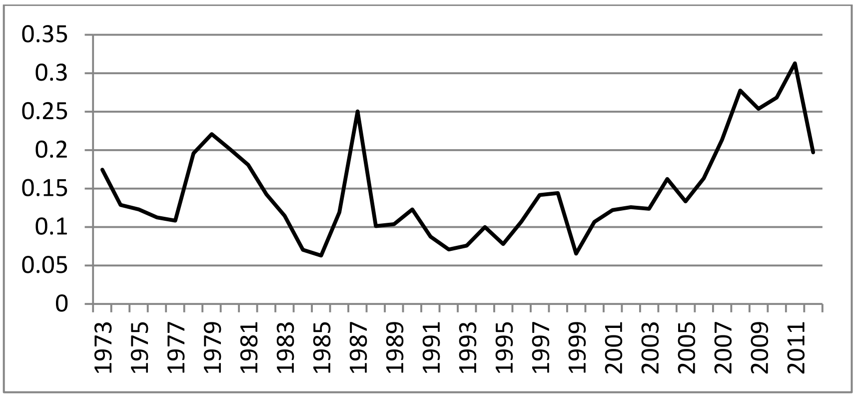
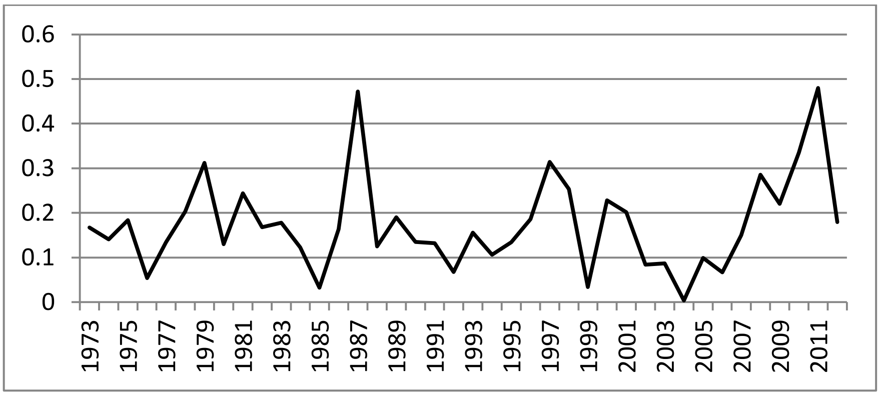


Figure 4.2: Average systematic tail risk over time: EDH measure. This figure displays the equally weighted average of yearly EDH for all stocks in the NYSE, AMEX, NASDAQ, and ARCA markets from 1973 to 2012. The value of EDH in a year is calculated using daily data of stock returns in that year.

Panel 1: EDC-BL



Panel 2: EDC-ACY



Panel 3: EDC-ES

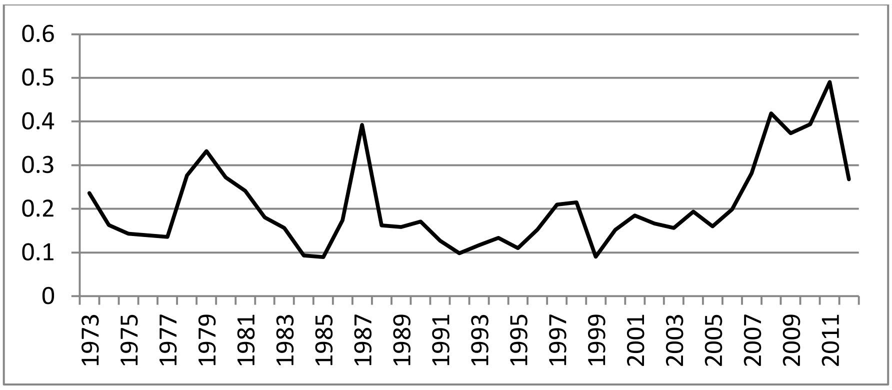
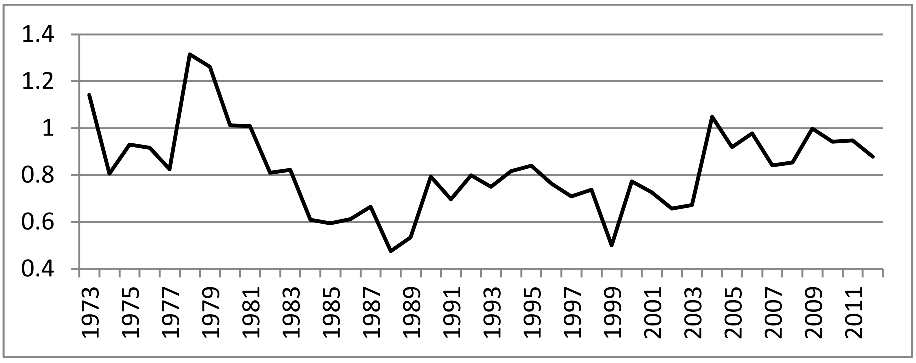
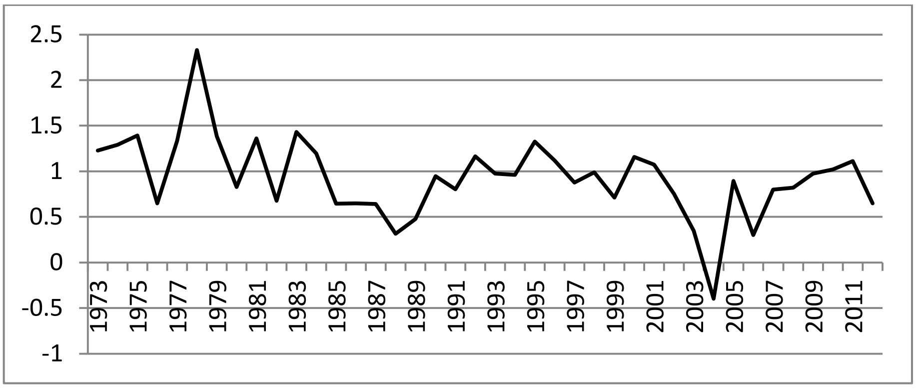


Figure 4.3: Average systematic tail risk over time: EDC measures. This figure displays the equally weighted average of yearly EDC-BL, EDC-ACY, and EDC-ES for all stocks in the NYSE, AMEX, NASDAQ, and ARCA markets from January 1973 to December 2012. The value of each measure in a year is calculated using daily data of stock returns in that year.

Panel 1: EDB-BL



Panel 2: EDB-ACY



Panel 3: EDB-ES

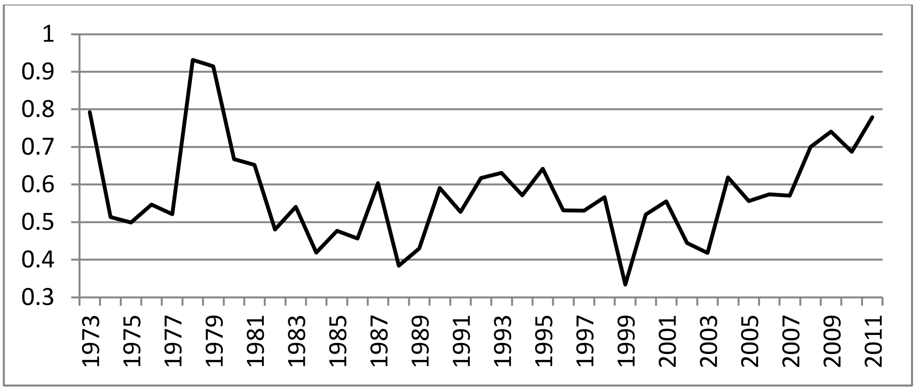
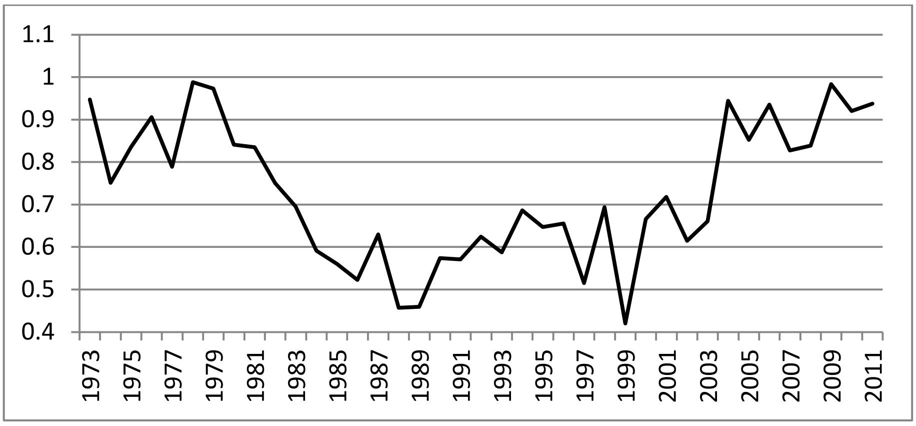


Figure 4.4: Average systematic tail risk over time: EDB measures. This figure displays the equally weighted average of yearly EDB-BL, EDB-ACY, and EDB-ES for all stocks in the NYSE, AMEX, NASDAQ, and ARCA markets from January 1973 to December 2012. The value of each measure in a year is calculated using daily data of stock returns in that year.

Panel 1: Beta



Panel 2: Downside Beta

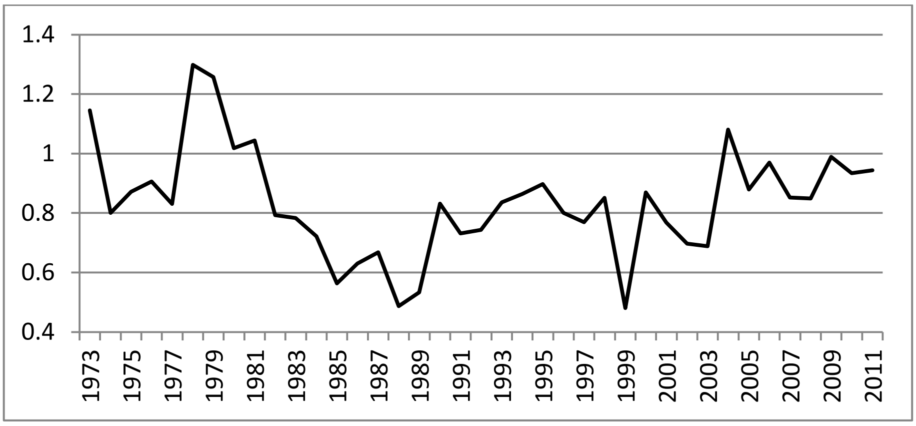


Figure 4.5: Average Betas & Downside betas over time. This figure displays the equally weighted average of yearly CAPM Beta & Ang et al.'s (2006a) downside beta for all stocks in the NYSE, AMEX, NASDAQ, and ARCA markets from January 1973 to December 2012. The values of Beta & Downside beta in a year are calculated using daily data of stock returns in that year.

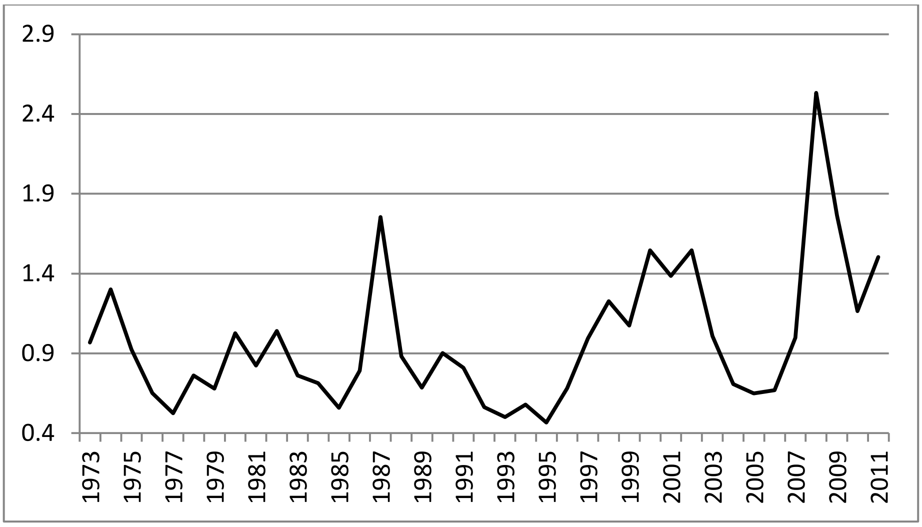


Figure 4.6: Market yearly volatility over time. This figure displays the yearly standard deviation calculated from daily returns of the US stock market from January 1973 to December 2012. The market is represented by the CRSP index for all stocks in the NYSE, AMEX, NASDAQ, and ARCA markets.

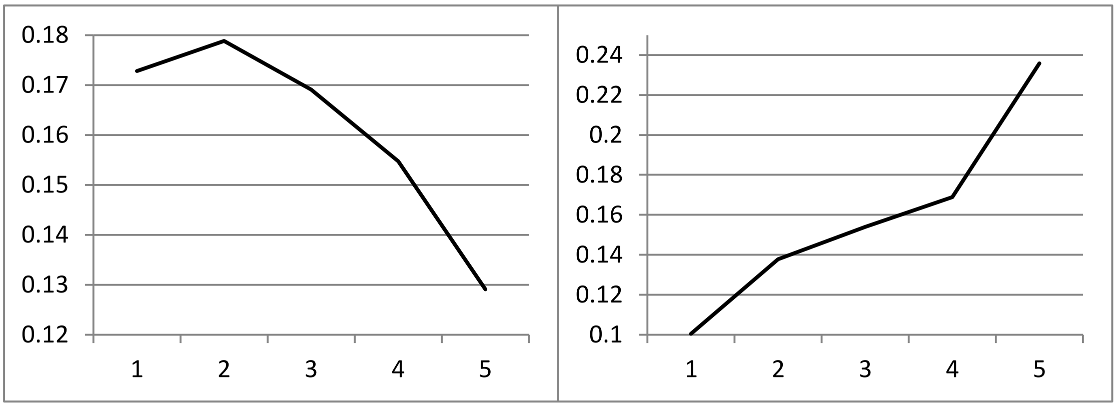


Figure 4.7: Average excess return of portfolios sorted on CAPM Beta. This figure shows the 1973-2012 average yearly excess return of the quintiles of all stocks in the NYSE, AMEX, NASDAQ, and ARCA markets sorted on CAPM Beta measure. In the left figure, the quintiles are formed by sorting preformation level of Beta, which are calculated using last 1 year daily data. In the right figure, the quintiles are formed by sorting postformation level of Beta, which are calculated using next 1 year daily data.

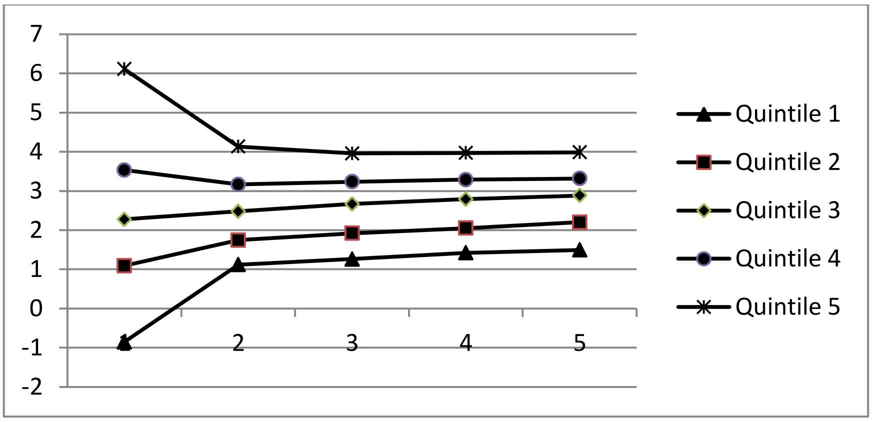


Figure 4.8: Persistency analysis-Average EDH measure of fixed quintiles over time. This figure displays the evolution over 5 years of the average EDH of the quintiles constructed at the beginning of the 5 year period by sorting all stocks in the NYSE, AMEX, NASDAQ, ARCA markets on their EDH of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

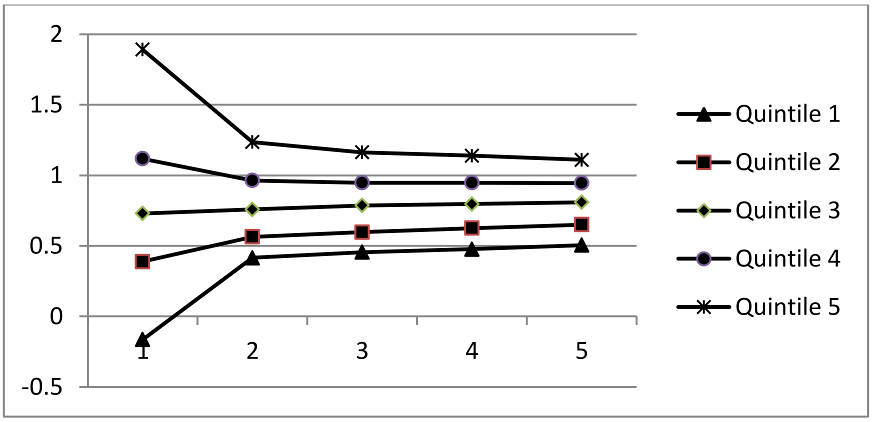


Figure 4.9: Persistency analysis-Average EDB-BL measure of fixed quintiles over time. This figure displays the evolution over 5 years of the average EDB-BL of the quintiles constructed at the beginning of the 5 year period by sorting all stocks in the NYSE, AMEX, NASDAQ, ARCA markets on their EDB-BL of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

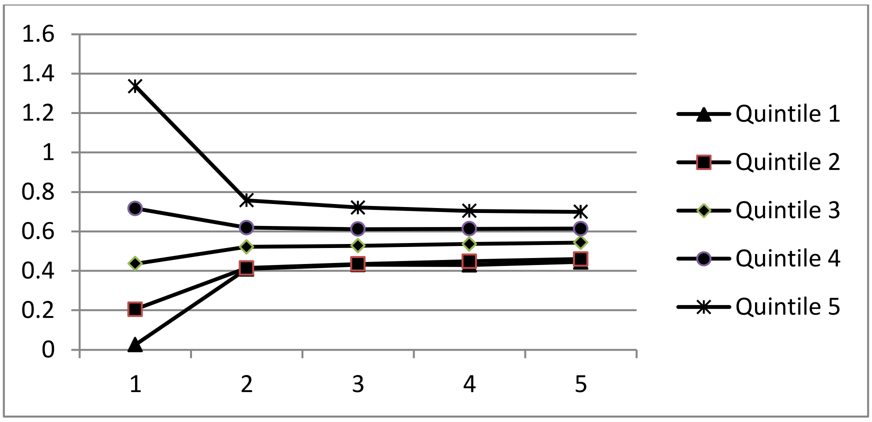


Figure 4.10: Persistency analysis-Average EDB-ES measure of fixed quintiles over time. This figure displays the evolution over 5 years of the average EDB-ES of the quintiles constructed at the beginning of the 5 year period by sorting all stocks in the NYSE, AMEX, NASDAQ, ARCA markets on their EDB-ES of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

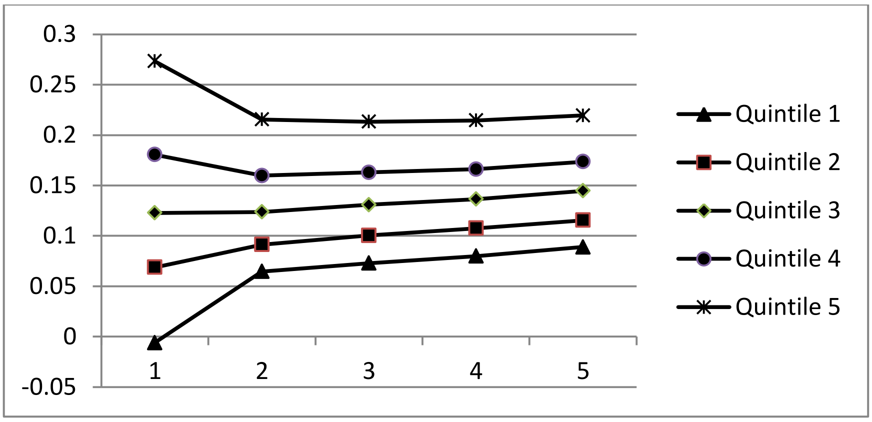


Figure 4.11: Persistency analysis-Average EDC-BL measure of fixed quintiles over time. This figure displays the evolution over 5 years of the average EDC-BL of the quintiles constructed at the beginning of the 5 year period by sorting all stocks in the NYSE, AMEX, NASDAQ, ARCA markets on their EDC-BL of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

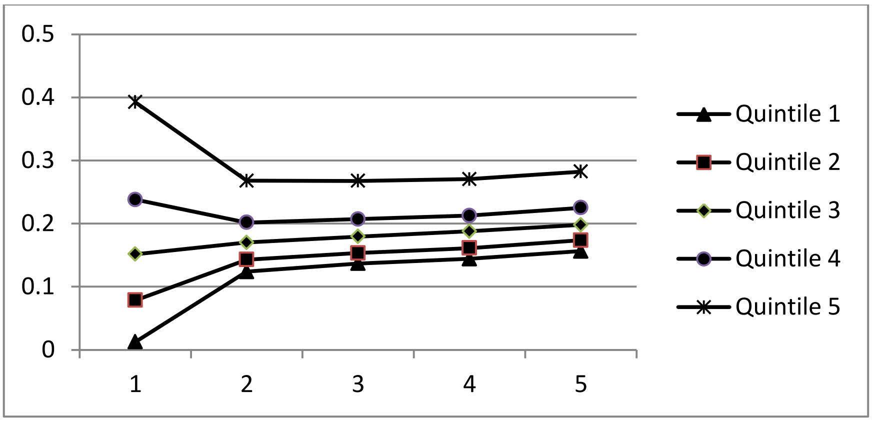


Figure 4.12: Persistency analysis-Average EDC-ES measure of fixed quintiles over time. This figure displays the evolution over 5 years of the average EDC-ES of the quintiles constructed at the beginning of the 5 year period by sorting all stocks in the NYSE, AMEX, NASDAQ, ARCA markets on their EDC-ES of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

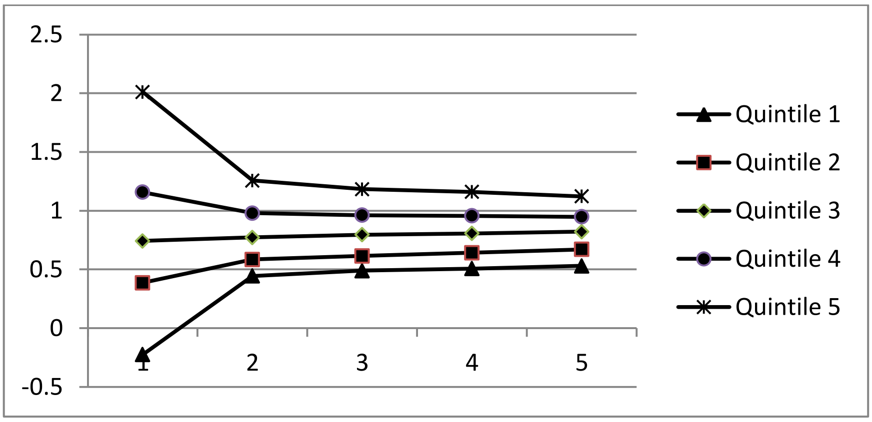


Figure 4.13: Persistency analysis-Average Downside Beta measure of fixed quintiles over time. This figure displays the evolution over 5 years of the average Ang et al. (2006a) Downside Beta of the quintiles constructed at the beginning of the 5 year period by sorting all stocks in the NYSE, AMEX, NASDAQ, ARCA markets on their Downside Beta of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

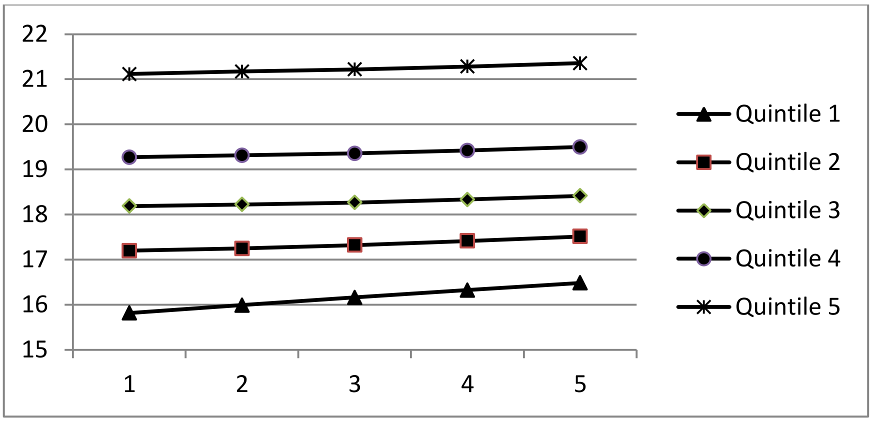


Figure 4.14: Persistency analysis-Average size of fixed quintiles over time. This figure displays the evolution over 5 years of the average size of the quintiles constructed at the beginning of the 5 year period by sorting all stocks in the NYSE, AMEX, NASDAQ, ARCA markets on their size at the beginning of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

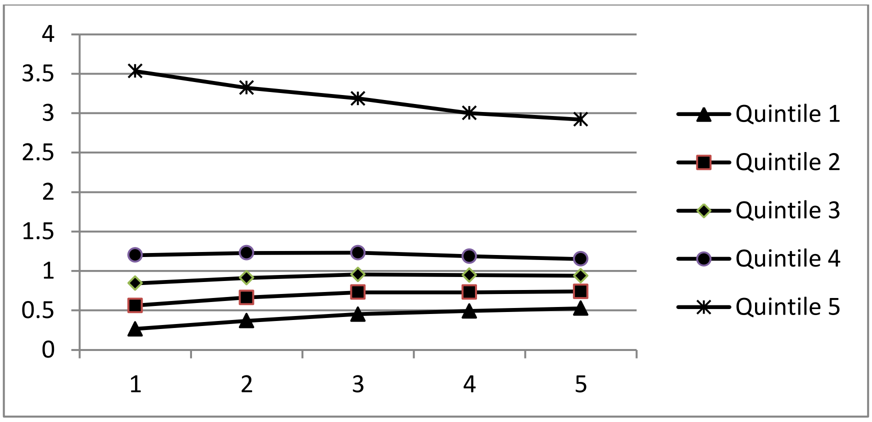


Figure 4.15: Persistency analysis-Average Book-to-Market measure of fixed quintiles over time. This figure displays the evolution over 5 years of the average Book-to-Market of the quintiles constructed at the beginning of the 5 year period by sorting all stocks in the NYSE, AMEX, NASDAQ, ARCA markets on their Book-to-Market at the beginning of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

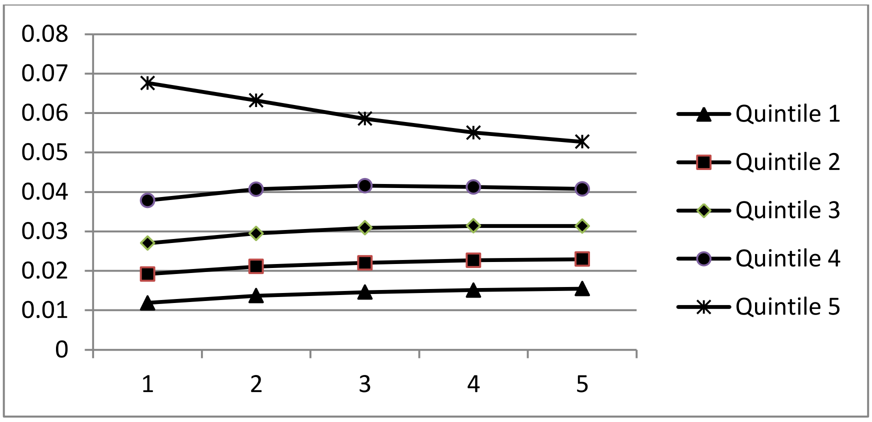


Figure 4.16: Persistency analysis-Average Idiosyncratic Volatility measure of fixed quintiles over time. This figure displays the evolution over 5 years of the average Ang *et al*.'s (2006b) Idiosyncratic Volatility of the quintiles constructed at the beginning of the 5 year period by sorting all stocks in the NYSE, AMEX, NASDAQ, ARCA markets on their Idiosyncratic Volatility of the first year. This evolution is averaged over 36 starting years from 1973 to 2008.

Table 4.1: Average excess returns of the equally weighted quintile portfolios sorting on systematic extreme downside risk measures

This table shows the average yearly excess returns and sizes of the equally weighted quintile portfolios sorted on different systematic extreme downside risk measures. These quintiles are sorted using 1 year postformation measures, which are concurrent with the quintile returns. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns on the corresponding first row. The last three columns are the average excess return of the long-short strategy which longs quintile 5 and shorts quintile 1, its alphas in Fama and French (1993) three factor model and Carhart (1997) four factor models. The overall sample period is from January 1973 to December 2012.

Quintiles	1	2	3	4	5	5 - 1	FF	Carhart
				EDH				
Average returns	5.289	8.436	9.967	12.016	17.112	11.823	10.988	8.456
t-statistics	(1.833)	(3.043)	(3.919)	(4.066)	(3.793)	(3.642)	(3.108)	(2.866)
Average size	17.346	18.333	18.830	18.897	18.536			
				EDB-BL				
Average returns	4.301	8.010	9.722	12.069	18.611	14.310	13.824	11.725
t-statistics	(1.429)	(3.175)	(3.785)	(3.949)	(4.038)	(4.035)	(3.736)	(3.366)
Average size	17.333	18.342	18.774	18.887	18.611			
				EDB-ACY				
Average returns	6.837	8.577	10.162	12.238	14.753	7.915	10.154	6.993
t-statistics	(1.879)	(3.498)	(4.118)	(4.457)	(3.686)	(3.156)	(4.254)	(3.506)
Average size	17.703	18.544	18.831	18.790	18.076			
				EDB-ES				
Average returns	4.604	7.573	10.237	12.555	17.586	12.982	12.672	11.186
t-statistics	(1.633)	(3.273)	(3.769)	(3.921)	(4.025)	(4.245)	(4.119)	(3.886)
Average size	17.532	18.453	18.691	18.756	18.511			
				EDC-BL				
Average returns	4.015	8.238	10.532	13.044	16.770	12.755	15.803	9.421
t-statistics	(1.164)	(2.419)	(3.430)	(4.740)	(5.191)	(3.784)	(6.993)	(4.717)
Average size	17.004	17.617	18.375	19.070	19.874			
				EDC-ACY				
Average returns	6.044	7.908	11.111	12.900	14.673	8.629	12.194	7.445
t-statistics	(1.813)	(2.540)	(3.587)	(4.609)	(4.902)	(3.282)	(6.268)	(4.127)
Average size	17.819	18.156	18.370	18.630	18.984			
				EDC-ES				
Average returns	2.680	5.242	10.141	13.920	20.523	17.843	20.563	14.900
t-statistics	(0.888)	(1.603)	(3.238)	(4.824)	(5.815)	(6.065)	(9.280)	(8.099)
Average size	17.341	17.821	18.359	18.871	19.551			

Table 4.2: Average excess returns of the value weighted quintile portfolios sorting on systematic extreme downside risk measures

This table shows the average yearly excess returns and sizes of the value-weighted quintile portfolios sorted on different systematic extreme downside risk measures. These quintiles are sorted using 1 year postformation measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the return in the corresponding first row. The last three columns are for the average excess return of the long-short strategy which longs quintile 5 and shorts quintile 1, its alphas in Fama and French (1993) three factor model and Carhart (1997) four factor models. The overall sample period is from January 1973 to December 2012.

Quintiles	1	2	3	4	5	5 - 1	FF	Carhart
				EDH				
Average returns	1.695	5.133	5.712	8.098	10.492	8.797	6.225	3.710
t-statistics	(0.908)	(3.088)	(3.195)	(3.223)	(2.552)	(1.999)	(1.581)	(0.947)
				EDB-BL				
Average returns	4.547	4.796	6.350	7.078	9.707	5.161	4.492	3.300
t-statistics	(2.566)	(3.065)	(3.551)	(2.647)	(2.467)	(1.339)	(1.247)	(0.865)
				EDB-ACY				
Average returns	2.219	4.554	6.019	8.439	7.442	5.223	7.284	3.884
t-statistics	(1.009)	(2.167)	(2.715)	(3.467)	(2.347)	(2.490)	(2.732)	(1.599)
				EDB-ES				
Average returns	0.638	4.505	5.304	7.878	10.803	10.165	9.584	8.417
t-statistics	(0.354)	(2.497)	(2.703)	(3.046)	(2.963)	(3.024)	(2.477)	(2.209)
				EDC-BL				
Average returns	2.217	0.186	1.452	3.498	8.725	6.508	9.121	7.139
t-statistics	(1.184)	(0.104)	(0.734)	(1.694)	(3.337)	(2.742)	(3.336)	(3.019)
				EDC-ACY				
Average returns	2.943	3.032	3.651	6.845	8.784	5.841	8.929	7.102
t-statistics	(1.414)	(1.343)	(1.673)	(3.181)	(3.568)	(3.965)	(4.629)	(3.674)
				EDC-ES				
Average returns	-3.755	-2.053	0.972	4.065	10.531	14.286	17.005	16.117
t-statistics	(-1.716)	(-1.052)	(0.476)	(1.795)	(4.189)	(7.803)	(8.646)	(8.696)

Table 4.3: Average excess return of the equally weighted quintile portfolios sorted on EDH and other risk measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDH as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which long quintile 5 and short quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					**Coskewness**					**Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	5.810	5.325	6.472	5.485	4.265	9.226	7.863	4.736	3.952	5.363	2.902	5.570	6.900	8.080	11.220
	(1.716)	(2.364)	(3.186)	(3.172)	(2.426)	(3.701)	(2.714)	(1.791)	(1.337)	(1.325)	(0.695)	(2.075)	(2.093)	(2.240)	(2.507)
2	8.704	7.128	6.960	7.082	4.315	11.374	10.132	9.906	7.349	6.719	5.317	6.904	10.206	10.935	14.500
	(2.808)	(3.084)	(3.881)	(3.853)	(2.407)	(4.610)	(3.566)	(3.723)	(2.691)	(2.250)	(2.098)	(2.888)	(4.161)	(3.856)	(4.012)
3	12.911	9.429	7.759	7.832	6.594	16.236	11.847	9.552	7.748	6.416	5.305	8.550	9.705	12.654	19.057
	(3.598)	(3.919)	(3.842)	(4.194)	(3.094)	(5.633)	(4.353)	(3.413)	(3.103)	(2.441)	(2.215)	(3.445)	(4.528)	(4.514)	(4.333)
4	15.021	9.804	8.751	8.346	7.593	18.867	14.582	10.882	8.461	5.666	6.406	9.282	10.280	12.882	21.201
	(3.782)	(3.154)	(2.838)	(3.303)	(2.898)	(4.989)	(4.597)	(3.505)	(2.767)	(1.959)	(2.275)	(3.713)	(4.006)	(4.615)	(4.293)
5	21.751	12.458	13.095	12.368	9.855	28.073	20.537	13.539	8.757	6.680	4.269	10.749	12.316	13.255	26.151
	(4.153)	(2.842)	(3.071)	(3.200)	(2.569)	(4.051)	(3.964)	(3.197)	(2.087)	(1.541)	(1.051)	(3.450)	(3.447)	(3.308)	(3.396)
5-1	15.941	7.133	6.623	6.882	5.589	18.846	12.674	8.804	4.805	1.317	1.367	5.179	5.416	5.175	14.931
	(5.028)	(1.987)	(1.621)	(1.797)	(1.528)	(3.429)	(3.804)	(3.550)	(1.743)	(0.377)	(0.799)	(4.484)	(3.186)	(1.616)	(2.299)

Table 4.4: Average excess return of the equally weighted quintile portfolios sorted on EDB-BL and other risk measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDB-BL as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					**Coskewness**					**Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	3.946	5.260	6.938	5.995	5.415	7.865	6.149	5.894	2.904	4.281	1.566	5.146	8.158	8.232	10.530
	(1.073)	(2.687)	(4.427)	(3.862)	(3.032)	(3.654)	(2.542)	(2.278)	(1.044)	(1.010)	(0.365)	(1.730)	(2.558)	(2.637)	(2.549)
2	8.405	7.741	7.344	6.975	4.987	12.089	11.030	9.626	7.123	5.862	3.857	7.640	8.967	11.492	16.059
	(2.939)	(3.499)	(3.949)	(3.960)	(2.823)	(4.954)	(4.140)	(3.599)	(2.827)	(1.827)	(1.311)	(3.365)	(3.989)	(4.001)	(4.414)
3	11.384	8.038	8.303	8.705	6.964	15.779	12.354	9.481	7.658	6.749	4.679	8.227	9.237	11.627	17.289
	(3.374)	(3.215)	(3.474)	(4.681)	(3.284)	(5.163)	(4.443)	(3.458)	(2.831)	(2.584)	(1.919)	(3.516)	(4.013)	(4.538)	(3.919)
4	15.667	9.223	7.540	8.061	7.084	19.865	15.167	10.342	8.861	5.571	7.365	8.793	11.092	12.622	20.619
	(3.836)	(2.758)	(2.501)	(3.140)	(2.815)	(5.066)	(4.172)	(3.401)	(2.953)	(2.015)	(2.924)	(3.616)	(4.202)	(4.043)	(4.375)
5	24.676	13.780	12.840	11.428	8.221	28.107	20.000	13.129	9.572	8.210	6.437	11.214	11.973	13.826	27.629
	(4.556)	(2.857)	(3.079)	(2.802)	(2.114)	(4.317)	(3.939)	(2.910)	(2.218)	(1.985)	(1.832)	(3.556)	(3.397)	(3.573)	(3.771)
5-1	20.730	8.520	5.903	5.434	2.806	20.242	13.851	7.235	6.668	3.928	4.871	6.068	3.814	5.595	17.099
	(6.140)	(1.911)	(1.499)	(1.360)	(0.774)	(3.763)	(3.804)	(2.300)	(2.296)	(1.016)	(2.829)	(4.789)	(2.690)	(2.933)	(3.418)

Table 4.5: Average excess return of the equally weighted quintile portfolios sorted on EDB-ACY and other risk measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDB-ACY as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					**Coskewness**					**Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	8.234	4.185	4.968	5.456	4.606	13.376	11.902	7.931	3.987	3.847	2.013	7.517	7.690	8.553	11.851
	(1.797)	(1.746)	(2.320)	(2.694)	(1.962)	(4.105)	(3.117)	(2.504)	(1.111)	(0.823)	(0.454)	(2.203)	(2.151)	(2.406)	(2.567)
2	10.691	8.276	8.128	8.554	5.718	14.366	11.160	9.290	7.111	6.916	5.757	8.207	10.137	12.223	19.542
	(3.262)	(3.792)	(3.894)	(4.628)	(2.967)	(5.775)	(4.610)	(3.844)	(2.817)	(2.745)	(2.106)	(3.549)	(4.454)	(4.140)	(4.586)
3	11.733	10.234	9.391	9.173	6.779	15.726	12.701	9.873	7.322	6.465	4.220	7.545	10.138	11.687	19.554
	(3.613)	(4.097)	(4.143)	(5.112)	(3.093)	(5.706)	(4.784)	(3.981)	(3.012)	(2.753)	(1.573)	(3.720)	(4.295)	(4.463)	(4.281)
4	15.880	10.717	10.325	8.231	7.666	18.444	13.918	11.486	8.748	5.423	5.358	8.029	9.996	12.584	21.492
	(4.355)	(3.613)	(4.260)	(3.629)	(3.408)	(4.458)	(4.172)	(3.800)	(3.270)	(2.025)	(2.144)	(3.533)	(4.227)	(4.350)	(4.259)
5	17.637	10.674	10.220	9.753	7.903	21.041	15.122	9.876	8.995	8.143	6.524	9.756	11.449	12.715	18.717
	(3.782)	(2.660)	(3.206)	(3.421)	(2.920)	(4.262)	(3.525)	(2.275)	(2.458)	(1.996)	(1.744)	(3.122)	(3.397)	(3.645)	(3.284)
5-1	9.403	6.489	5.252	4.298	3.297	7.665	3.220	1.945	5.008	4.297	4.512	2.239	3.759	4.161	6.866
	(3.760)	(2.717)	(2.325)	(2.295)	(2.259)	(2.992)	(1.325)	(0.738)	(3.525)	(1.978)	(1.989)	(1.517)	(3.943)	(3.211)	(1.910)

Table 4.6: Average excess return of the equally weighted quintile portfolios sorted on EDB-ES and other risk measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDB-ES as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					Coskewness					Downside beta				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	6.682	4.035	4.557	4.737	3.657	9.835	7.860	5.204	3.580	2.820	0.331	6.008	7.176	6.983	9.130
	(1.872)	(2.391)	(3.051)	(2.716)	(1.993)	(3.973)	(2.828)	(1.899)	(1.436)	(0.880)	(0.081)	(2.291)	(2.320)	(2.120)	(2.109)
2	7.122	7.157	6.972	6.424	4.977	12.661	9.785	9.112	4.934	4.639	4.455	7.370	8.816	11.721	17.592
	(2.493)	(3.000)	(3.251)	(3.423)	(2.687)	(5.201)	(3.863)	(3.813)	(1.876)	(1.692)	(1.706)	(3.130)	(3.972)	(4.007)	(4.462)
3	12.809	8.756	8.004	8.041	6.323	15.934	13.410	10.014	9.206	7.498	5.305	8.877	10.786	12.444	19.336
	(3.662)	(3.175)	(3.813)	(4.438)	(3.044)	(5.040)	(4.643)	(3.788)	(3.839)	(2.818)	(2.451)	(3.855)	(4.502)	(4.852)	(4.576)
4	15.665	10.619	9.823	9.187	8.217	19.347	15.127	11.866	9.190	7.716	8.187	8.730	11.362	12.361	21.247
	(3.668)	(3.483)	(3.213)	(3.708)	(3.509)	(5.333)	(4.180)	(3.439)	(3.031)	(2.424)	(3.023)	(3.690)	(4.180)	(4.133)	(4.565)
5	21.842	13.483	13.583	12.751	9.466	25.773	18.555	12.193	9.151	7.947	5.610	10.003	11.235	14.214	24.238
	(4.191)	(2.954)	(3.487)	(3.463)	(2.679)	(4.090)	(3.783)	(2.823)	(1.961)	(1.736)	(1.254)	(2.861)	(3.319)	(3.844)	(3.562)
5-1	15.160	9.448	9.026	8.014	5.808	15.938	10.695	6.988	5.571	5.128	5.278	3.995	4.059	7.231	15.108
	(5.656)	(2.589)	(2.667)	(2.370)	(2.060)	(3.388)	(3.296)	(2.441)	(1.880)	(1.808)	(3.169)	(2.969)	(3.693)	(5.241)	(3.411)

Table 4.7: Average excess return of the equally weighted quintile portfolios sorted on EDC-BL and other risk measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDC-BL as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					**Coskewness**					**Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	3.708	1.034	2.624	1.305	2.118	11.017	7.226	5.418	2.025	4.557	3.864	5.001	7.190	7.246	9.322
	(0.981)	(0.453)	(1.237)	(0.693)	(1.018)	(3.550)	(2.029)	(1.672)	(0.649)	(1.167)	(1.061)	(1.331)	(1.653)	(1.476)	(1.552)
2	8.317	4.054	4.089	5.195	3.161	16.869	12.752	8.231	5.470	5.590	2.926	9.344	10.842	11.245	15.850
	(2.248)	(1.479)	(1.597)	(2.044)	(1.471)	(4.301)	(3.339)	(2.498)	(1.689)	(1.421)	(0.921)	(2.982)	(3.057)	(3.331)	(3.238)
3	11.948	6.534	6.794	7.002	6.097	15.368	12.470	10.958	8.949	5.688	3.228	8.416	10.726	12.825	19.520
	(3.021)	(2.326)	(2.666)	(3.583)	(2.697)	(4.806)	(3.814)	(3.410)	(2.867)	(1.831)	(0.940)	(3.181)	(4.225)	(4.485)	(4.249)
4	16.200	12.907	10.730	11.304	8.857	19.019	15.667	10.231	9.063	6.410	6.419	8.836	10.931	13.574	22.435
	(4.156)	(4.294)	(4.399)	(4.835)	(3.664)	(5.273)	(5.199)	(3.976)	(3.155)	(2.215)	(2.513)	(4.254)	(5.202)	(5.033)	(4.695)
5	23.900	19.463	18.596	16.253	12.345	20.925	16.593	13.626	10.571	8.567	7.531	9.542	9.761	12.793	24.161
	(5.245)	(5.118)	(5.164)	(5.184)	(4.284)	(5.178)	(4.757)	(4.280)	(3.610)	(3.058)	(2.977)	(5.594)	(5.484)	(5.055)	(4.712)
5-1	20.193	18.429	15.971	14.947	10.226	9.909	9.367	8.208	8.546	4.011	3.667	4.541	2.570	5.547	14.839
	(5.517)	(6.044)	(4.692)	(4.951)	(4.340)	(3.095)	(2.902)	(3.950)	(4.162)	(1.329)	(1.909)	(1.682)	(0.736)	(1.423)	(2.535)

Table 4.8: Average excess return of the equally weighted quintile portfolios sorted on EDC-ACY and other risk measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDC-ACY as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					**Coskewness**					**Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	7.589	4.730	4.380	4.717	4.029	12.424	10.294	7.265	4.179	4.550	2.298	7.239	6.779	7.758	8.957
	(1.851)	(1.893)	(2.013)	(2.237)	(1.704)	(4.002)	(2.838)	(2.459)	(1.303)	(1.245)	(0.661)	(2.328)	(1.999)	(2.043)	(1.736)
2	9.394	5.483	6.878	6.899	4.932	14.909	11.711	8.215	5.621	5.567	3.719	6.603	9.419	11.366	18.333
	(2.420)	(2.337)	(2.796)	(3.087)	(2.280)	(4.758)	(3.368)	(2.648)	(1.771)	(1.792)	(1.218)	(2.415)	(3.212)	(3.216)	(3.822)
3	12.847	9.937	8.570	8.851	6.237	18.485	12.931	10.433	7.774	7.745	3.947	8.571	10.721	11.919	20.232
	(3.386)	(3.278)	(3.631)	(4.241)	(2.817)	(4.954)	(4.105)	(3.513)	(2.703)	(2.352)	(1.216)	(3.381)	(3.792)	(4.104)	(4.207)
4	15.478	10.362	9.709	9.596	7.868	19.072	14.229	11.564	8.945	6.277	6.826	9.247	11.656	12.869	20.990
	(4.268)	(3.667)	(3.785)	(4.704)	(3.325)	(5.128)	(4.592)	(3.701)	(2.930)	(2.024)	(2.303)	(3.896)	(4.664)	(4.563)	(4.208)
5	18.859	13.508	13.409	11.056	9.586	18.371	15.594	11.152	9.756	6.665	7.154	9.330	10.818	13.879	22.749
	(4.798)	(4.323)	(5.486)	(4.653)	(4.178)	(4.769)	(4.880)	(3.829)	(3.937)	(2.328)	(2.618)	(4.137)	(5.004)	(4.889)	(4.456)
5-1	11.271	8.778	9.028	6.339	5.557	5.947	5.300	3.887	5.577	2.115	4.856	2.091	4.040	6.121	13.792
	(4.164)	(4.875)	(5.569)	(4.033)	(3.858)	(2.940)	(2.221)	(2.232)	(3.198)	(1.038)	(2.450)	(1.451)	(2.199)	(2.380)	(2.765)

Table 4.9: Average excess return of the equally weighted quintile portfolios sorted on EDC-ES and other risk measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDC-ES as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					**Coskewness**					**Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	5.066	-1.491	-0.913	-1.170	-0.894	8.621	5.426	2.718	1.431	1.455	0.155	4.923	4.050	2.628	1.894
	(1.344)	(-0.652)	(-0.461)	(-0.515)	(-0.431)	(2.787)	(1.487)	(0.838)	(0.539)	(0.453)	(0.038)	(1.667)	(1.103)	(0.662)	(0.363)
2	4.820	2.768	3.407	4.000	3.027	11.909	9.652	5.828	2.063	2.284	2.769	5.161	7.841	9.657	13.311
	(1.288)	(1.061)	(1.387)	(1.781)	(1.300)	(3.845)	(2.637)	(1.925)	(0.683)	(0.712)	(0.925)	(1.680)	(2.427)	(2.618)	(2.855)
3	11.279	8.562	7.135	8.112	7.043	16.238	12.451	9.905	7.385	5.664	3.426	7.835	11.120	13.113	19.715
	(2.826)	(2.965)	(2.767)	(3.823)	(3.385)	(4.619)	(3.853)	(3.004)	(2.340)	(1.582)	(1.211)	(2.941)	(4.035)	(4.512)	(4.231)
4	16.896	12.917	12.570	12.204	10.084	21.327	15.205	13.237	10.479	9.515	6.734	10.706	12.744	15.392	27.749
	(4.315)	(4.459)	(4.813)	(5.569)	(4.291)	(5.455)	(5.010)	(4.732)	(3.147)	(2.896)	(2.227)	(4.045)	(5.047)	(5.468)	(5.096)
5	26.026	21.208	20.585	17.846	13.300	25.093	21.841	16.651	14.651	11.706	10.786	12.390	13.563	16.751	28.629
	(5.632)	(5.756)	(6.344)	(5.948)	(4.854)	(5.568)	(5.523)	(5.274)	(4.882)	(3.969)	(4.011)	(6.514)	(6.544)	(6.187)	(5.004)
5-1	20.961	22.700	21.499	19.015	14.194	16.472	16.415	13.934	13.220	10.251	10.631	7.467	9.513	14.123	26.735
	(7.297)	(9.964)	(8.488)	(5.951)	(7.176)	(4.563)	(4.406)	(6.413)	(9.450)	(4.917)	(4.212)	(4.338)	(3.560)	(4.957)	(4.821)

Table 4.10: Average excess return of the value weighted quintile portfolios sorted on EDH and other risk measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDH as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					Coskewness					Downside beta				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	1.738	5.376	6.266	5.489	3.896	2.917	5.419	2.304	-0.215	2.430	1.025	2.299	3.631	1.569	-0.996
	(0.554)	(2.439)	(3.045)	(3.129)	(2.202)	(1.474)	(2.633)	(1.064)	(-0.084)	(1.038)	(0.384)	(1.140)	(1.479)	(0.597)	(-0.235)
2	7.013	7.093	6.927	6.836	3.330	8.308	5.633	5.577	4.652	3.211	4.280	6.323	6.599	4.951	7.422
	(2.520)	(3.131)	(3.916)	(3.705)	(1.673)	(4.224)	(2.651)	(2.687)	(2.582)	(1.666)	(2.210)	(2.987)	(3.202)	(1.878)	(2.035)
3	9.645	9.389	7.661	7.714	6.132	11.666	6.438	6.151	4.833	4.351	5.153	5.330	6.899	9.211	8.280
	(3.342)	(3.952)	(3.855)	(4.200)	(2.482)	(5.192)	(2.745)	(3.097)	(2.739)	(2.166)	(2.912)	(3.125)	(4.019)	(3.484)	(2.095)
4	11.729	9.406	8.787	8.320	7.529	12.748	8.166	7.648	3.757	3.218	7.156	7.463	6.976	9.788	13.126
	(3.272)	(3.082)	(2.857)	(3.339)	(2.442)	(3.826)	(2.575)	(2.852)	(1.725)	(1.310)	(3.601)	(5.260)	(3.568)	(3.057)	(2.954)
5	17.569	12.683	12.967	12.372	8.468	16.829	10.920	6.482	2.433	2.608	1.572	4.436	9.178	10.435	15.822
	(3.487)	(2.805)	(3.082)	(3.157)	(2.005)	(3.451)	(2.560)	(1.595)	(0.622)	(0.639)	(0.475)	(2.211)	(2.987)	(2.691)	(2.529)
5-1	15.830	7.307	6.701	6.883	4.571	13.912	5.502	4.178	2.648	0.177	0.547	2.138	5.547	8.866	16.818
	(4.373)	(1.923)	(1.654)	(1.768)	(1.207)	(2.848)	(1.201)	(0.945)	(0.574)	(0.037)	(0.140)	(1.496)	(1.561)	(2.348)	(2.380)

Table 4.11: Average excess return of the value weighted quintile portfolios sorted on EDB-BL and other risk measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDB-BL as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					**Coskewness**					**Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	1.116	5.195	6.699	6.147	4.408	4.848	6.058	5.703	3.318	3.240	-1.183	3.119	4.955	4.832	2.340
	(0.358)	(2.675)	(4.349)	(4.001)	(2.315)	(2.800)	(3.542)	(3.882)	(1.636)	(0.867)	(-0.331)	(1.410)	(2.692)	(2.164)	(0.556)
2	5.994	7.714	7.470	6.980	3.955	7.739	7.172	5.864	4.723	4.417	5.351	5.347	5.033	5.336	7.642
	(2.420)	(3.477)	(4.022)	(4.005)	(2.069)	(4.199)	(3.190)	(3.531)	(2.735)	(2.855)	(2.911)	(2.877)	(3.222)	(1.989)	(1.947)
3	9.089	8.136	8.173	8.609	5.647	11.882	7.934	6.655	5.003	3.684	5.614	6.579	6.384	6.968	6.133
	(2.881)	(3.321)	(3.474)	(4.681)	(2.282)	(5.388)	(2.880)	(2.487)	(2.962)	(2.227)	(2.938)	(4.511)	(3.263)	(2.665)	(1.560)
4	11.607	9.404	7.327	7.838	6.879	12.177	8.696	7.089	3.882	2.496	7.569	4.557	7.268	8.922	12.392
	(3.225)	(2.810)	(2.448)	(3.112)	(2.356)	(3.581)	(2.989)	(3.118)	(1.862)	(0.974)	(4.688)	(2.529)	(3.357)	(2.716)	(2.568)
5	19.590	13.455	12.966	11.332	7.479	14.550	6.791	4.803	2.552	2.842	3.350	5.466	6.631	10.493	15.548
	(3.856)	(2.771)	(3.130)	(2.741)	(1.772)	(2.861)	(1.956)	(1.269)	(0.691)	(0.786)	(1.618)	(2.839)	(2.560)	(3.013)	(2.618)
5-1	18.474	8.261	6.267	5.185	3.070	9.702	0.732	-0.900	-0.766	-0.398	4.532	2.347	1.676	5.661	13.208
	(4.839)	(1.831)	(1.611)	(1.261)	(0.818)	(2.114)	(0.222)	(-0.257)	(-0.197)	(-0.078)	(1.555)	(1.278)	(0.821)	(1.873)	(2.525)

Table 4.12: Average excess return of the value weighted quintile portfolios sorted on EDB-ACY and other risk measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDB-ACY as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					Coskewness					Downside beta				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	4.060	4.320	4.965	5.316	3.875	6.168	5.948	2.775	-0.062	-2.762	-0.529	2.038	4.105	5.432	3.651
	(1.006)	(1.835)	(2.310)	(2.655)	(1.418)	(2.383)	(2.042)	(1.177)	(-0.029)	(-0.851)	(-0.155)	(0.899)	(1.605)	(1.865)	(0.926)
2	9.172	7.935	8.060	8.216	4.150	10.733	7.995	4.672	4.641	3.606	5.186	5.361	5.397	6.371	11.050
	(3.064)	(3.538)	(3.887)	(4.409)	(1.915)	(3.987)	(3.753)	(2.005)	(1.876)	(1.434)	(2.924)	(2.837)	(2.956)	(1.932)	(2.667)
3	9.952	10.414	9.318	9.228	5.355	10.113	7.151	6.422	4.404	2.449	5.227	5.613	7.283	9.219	10.240
	(3.471)	(4.284)	(4.182)	(5.140)	(2.093)	(3.359)	(3.058)	(2.685)	(2.490)	(0.932)	(2.678)	(3.538)	(3.176)	(3.655)	(2.196)
4	13.495	10.718	10.101	8.035	7.632	12.761	7.395	6.258	3.650	2.735	5.226	5.590	6.352	9.171	10.936
	(3.763)	(3.565)	(4.263)	(3.578)	(2.788)	(3.112)	(2.924)	(2.784)	(2.051)	(1.022)	(3.416)	(3.601)	(3.078)	(3.567)	(2.812)
5	13.114	10.574	10.198	10.161	6.483	11.211	6.239	2.974	2.654	3.381	2.563	5.754	6.268	8.209	6.399
	(3.239)	(2.589)	(3.199)	(3.497)	(2.214)	(3.184)	(2.308)	(0.786)	(0.994)	(1.369)	(1.232)	(3.157)	(3.461)	(3.145)	(1.420)
5-1	9.054	6.255	5.233	4.845	2.607	5.042	0.291	0.199	2.716	6.143	3.091	3.715	2.163	2.776	2.748
	(3.262)	(2.540)	(2.323)	(2.377)	(1.829)	(2.136)	(0.108)	(0.056)	(1.218)	(2.119)	(1.098)	(1.823)	(1.412)	(1.325)	(0.846)

Table 4.13: Average excess return of the value weighted quintile portfolios sorted on EDB-ES and other risk measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDB-ES as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					**Coskewness**					**Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	2.943	4.041	4.512	4.901	2.435	5.580	6.287	0.899	-0.076	-0.624	-4.963	2.226	2.476	0.543	0.248
	(0.917)	(2.402)	(3.031)	(2.851)	(1.096)	(2.881)	(3.127)	(0.446)	(-0.036)	(-0.342)	(-1.618)	(1.006)	(0.978)	(0.211)	(0.060)
2	4.674	7.142	6.818	6.469	3.743	8.350	5.848	4.924	5.114	2.554	3.985	3.246	4.700	5.364	6.979
	(1.734)	(3.020)	(3.219)	(3.486)	(1.773)	(4.319)	(2.705)	(2.531)	(2.366)	(1.181)	(2.438)	(1.735)	(2.485)	(1.799)	(1.778)
3	9.900	8.610	7.977	7.730	5.703	10.284	7.880	6.079	4.658	2.353	4.436	6.657	6.322	7.515	9.144
	(3.112)	(3.110)	(3.835)	(4.222)	(2.489)	(4.275)	(3.103)	(2.767)	(2.578)	(1.277)	(2.133)	(4.194)	(3.672)	(2.946)	(2.571)
4	12.361	10.496	9.753	9.032	7.726	13.768	7.890	8.466	4.393	2.845	5.979	5.356	7.646	8.066	12.242
	(3.343)	(3.489)	(3.236)	(3.722)	(2.831)	(3.944)	(3.103)	(3.038)	(2.200)	(1.048)	(3.344)	(3.620)	(3.767)	(2.958)	(2.698)
5	18.617	13.430	13.632	12.598	8.784	14.638	9.506	6.177	3.688	4.527	4.606	7.612	8.922	13.328	11.356
	(3.913)	(2.904)	(3.514)	(3.393)	(2.404)	(3.040)	(2.501)	(1.630)	(1.201)	(1.343)	(1.437)	(3.861)	(4.020)	(4.631)	(2.066)
5-1	15.675	9.388	9.120	7.697	6.349	9.058	3.220	5.279	3.764	5.151	9.569	5.386	6.447	12.785	11.108
	(5.451)	(2.472)	(2.691)	(2.225)	(2.267)	(2.065)	(0.900)	(1.593)	(1.115)	(1.842)	(3.123)	(3.442)	(3.354)	(5.634)	(2.983)

Table 4.14: Average excess return of the value weighted quintile portfolios sorted on EDC-BL and other risk measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDC-BL as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					Coskewness					Downside beta				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	0.054	1.047	2.534	1.472	0.925	2.335	3.080	2.523	1.106	4.362	4.531	-1.363	-3.311	-6.938	-15.901
	(0.016)	(0.456)	(1.217)	(0.801)	(0.415)	(0.989)	(1.591)	(1.205)	(0.443)	(1.388)	(2.100)	(-0.472)	(-0.809)	(-1.247)	(-2.464)
2	2.275	4.234	3.911	5.122	1.856	5.985	4.535	0.618	-0.294	2.415	4.823	3.509	1.673	-0.913	-4.160
	(0.691)	(1.531)	(1.527)	(2.068)	(0.751)	(2.675)	(1.655)	(0.228)	(-0.151)	(1.054)	(2.809)	(1.626)	(0.590)	(-0.268)	(-0.775)
3	5.370	6.183	6.699	6.655	4.012	6.407	2.899	0.742	-0.114	1.797	3.315	4.031	2.817	2.824	1.769
	(1.588)	(2.212)	(2.577)	(3.347)	(1.540)	(2.820)	(1.226)	(0.353)	(-0.058)	(1.020)	(1.602)	(1.997)	(1.438)	(0.821)	(0.406)
4	10.733	12.483	10.542	10.653	6.256	8.952	4.618	2.586	2.467	-0.515	3.196	4.452	5.960	6.489	8.052
	(3.165)	(4.242)	(4.388)	(4.628)	(2.357)	(4.208)	(2.151)	(1.209)	(1.022)	(-0.194)	(1.826)	(2.383)	(3.254)	(2.161)	(1.899)
5	21.361	19.118	18.240	16.130	10.937	12.401	7.852	7.038	4.764	3.478	5.058	6.084	7.484	10.234	12.666
	(5.126)	(4.974)	(5.249)	(5.036)	(3.511)	(3.770)	(2.730)	(2.567)	(2.288)	(1.442)	(3.206)	(4.417)	(3.753)	(3.799)	(3.142)
5-1	21.307	18.071	15.706	14.658	10.012	10.066	4.772	4.515	3.657	-0.884	0.527	7.446	10.795	17.172	28.567
	(7.528)	(5.712)	(4.798)	(4.638)	(4.598)	(2.659)	(1.469)	(1.727)	(1.427)	(-0.245)	(0.260)	(3.499)	(2.783)	(3.267)	(4.472)

Table 4.15: Average excess return of the value weighted quintile portfolios sorted on EDC-ACY and other risk measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDC-ACY as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					**Coskewness**					**Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	4.873	4.890	4.478	4.583	2.963	7.620	6.061	1.863	1.783	0.394	1.978	2.063	3.433	4.250	0.810
	(1.368)	(1.983)	(2.034)	(2.220)	(1.085)	(3.750)	(2.002)	(0.774)	(0.920)	(0.163)	(1.203)	(0.935)	(1.432)	(1.466)	(0.203)
2	6.271	5.329	6.639	6.666	3.063	7.478	4.995	4.434	4.316	2.905	4.665	4.979	5.095	3.918	4.938
	(1.758)	(2.292)	(2.746)	(3.035)	(1.309)	(2.841)	(1.981)	(2.014)	(1.548)	(1.132)	(2.637)	(2.777)	(2.449)	(1.286)	(1.171)
3	9.163	10.029	8.402	8.650	5.010	10.538	5.372	3.310	1.524	2.117	3.481	5.079	6.559	6.760	11.453
	(2.689)	(3.326)	(3.674)	(4.147)	(1.923)	(3.825)	(2.243)	(1.393)	(0.658)	(0.741)	(1.363)	(3.104)	(3.129)	(1.925)	(2.786)
4	11.515	10.284	9.624	9.809	7.033	11.146	7.218	6.649	3.899	-0.359	3.545	5.043	6.897	7.104	10.488
	(3.450)	(3.567)	(3.737)	(4.719)	(2.788)	(3.286)	(2.725)	(2.743)	(2.243)	(-0.160)	(2.229)	(2.781)	(4.068)	(2.884)	(2.516)
5	16.323	13.248	13.376	10.970	8.897	10.659	9.791	7.549	4.310	4.130	5.215	6.787	6.990	11.579	11.518
	(4.797)	(4.281)	(5.487)	(4.665)	(3.289)	(3.770)	(3.860)	(3.226)	(2.287)	(1.997)	(3.218)	(4.893)	(3.433)	(4.633)	(2.609)
5-1	11.449	8.358	8.897	6.387	5.934	3.039	3.729	5.686	2.526	3.736	3.237	4.723	3.557	7.329	10.708
	(4.825)	(4.579)	(5.575)	(4.204)	(3.934)	(1.324)	(2.056)	(3.299)	(1.612)	(2.184)	(2.099)	(2.532)	(2.371)	(2.844)	(2.331)

Table 4.16: Average excess return of the value weighted quintile portfolios sorted on EDC-ES and other risk measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Size/Coskewness/Downside beta as the first criterion and then on EDC-ES as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Size					**Coskewness**					**Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	0.015	-1.429	-1.042	-0.905	-1.713	0.699	-0.142	-5.762	-5.895	-2.143	-5.019	-1.797	-5.111	-9.520	-15.584
	(0.004)	(-0.621)	(-0.526)	(-0.408)	(-0.757)	(0.286)	(-0.049)	(-2.551)	(-2.586)	(-1.142)	(-1.632)	(-0.726)	(-1.817)	(-3.243)	(-3.428)
2	0.264	2.592	3.282	3.811	1.115	1.932	-0.188	-3.144	-3.253	-3.021	1.853	-0.548	-0.504	-2.179	-6.324
	(0.081)	(0.977)	(1.346)	(1.681)	(0.405)	(0.790)	(-0.077)	(-1.225)	(-1.689)	(-1.104)	(0.990)	(-0.244)	(-0.245)	(-0.580)	(-1.293)
3	6.704	8.535	6.932	7.816	4.242	6.610	1.770	1.631	-0.981	-1.092	0.033	4.431	4.947	2.954	2.387
	(1.887)	(2.960)	(2.706)	(3.707)	(1.891)	(2.753)	(0.734)	(0.795)	(-0.408)	(-0.438)	(0.015)	(2.644)	(2.184)	(0.882)	(0.598)
4	11.757	12.717	12.675	11.648	8.726	8.215	4.733	6.792	4.059	-1.268	4.892	5.150	7.072	7.260	11.439
	(3.529)	(4.447)	(4.883)	(5.287)	(3.182)	(3.330)	(1.971)	(3.048)	(1.739)	(-0.466)	(2.109)	(2.826)	(3.890)	(2.703)	(2.840)
5	23.199	20.804	20.173	17.690	11.558	15.468	10.997	8.929	7.168	6.150	9.928	8.054	8.849	12.219	14.040
	(5.647)	(5.592)	(6.402)	(5.681)	(4.029)	(4.947)	(4.029)	(3.352)	(3.414)	(2.630)	(4.902)	(6.496)	(4.641)	(4.833)	(3.229)
5-1	23.184	22.233	21.215	18.596	13.271	14.768	11.138	14.691	13.063	8.293	14.947	9.851	13.960	21.739	29.624
	(9.802)	(9.340)	(8.597)	(5.497)	(7.785)	(4.599)	(3.555)	(6.804)	(5.485)	(4.999)	(4.238)	(5.220)	(6.224)	(9.452)	(6.117)

Table 4.17: Cross-sectional analysis of systematic extreme downside risk

This table shows the Fama and MacBeth (1973) average risk premiums of standard risk measures and of the proposed systematic tail risk measures, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, yearly excess return of a stock is regressed against its one year realized risk measures of downside beta, upsize beta, volatility, coskewness, cokurtotis, and systematic tail risk; its past excess return of last year; and its size and Book-to-Market available at the time of the regression. The overall sample period is from January 1973 to December 2012 (468 monthly observations) and covers all stocks in NYSE, AMEX, NASDAQ, and ARCA markets.

Model	I	II	III	IV	V	VI	VII	VIII	IX	X
Intercept	0.047	0.525	1.030	1.033	1.032	1.046	1.026	1.058	1.045	1.033
	(2.032)	(3.571)	(8.175)	(8.152)	(8.205)	(8.232)	(8.135)	(8.312)	(8.173)	(8.309)
β-	0.059	0.076	0.039	0.017	0.010	0.037	0.031	0.034	0.034	0.045
	(3.412)	(3.777)	(1.897)	(1.120)	(0.754)	(1.827)	(1.605)	(1.704)	(1.732)	(2.307)
β+	-0.002	0.010	0.003	0.004	-0.001	0.004	0.000	0.000	0.004	-0.007
	(-0.250)	(1.270)	(0.295)	(0.471)	(-0.117)	(0.504)	(0.013)	(-0.009)	(0.445)	(-0.867)
Log-size		-0.026	-0.054	-0.055	-0.054	-0.056	-0.054	-0.056	-0.055	-0.055
		(-3.703)	(-8.911)	(-8.923)	(-8.899)	(-8.946)	(-8.782)	(-9.104)	(-8.860)	(-9.123)
B/M		0.019	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.012
		(3.138)	(2.421)	(2.526)	(2.464)	(2.389)	(2.425)	(2.378)	(2.407)	(2.336)
Past			-0.002	-0.005	-0.004	-0.002	-0.003	-0.003	-0.003	-0.001
Return			(-0.133)	(-0.267)	(-0.220)	(-0.129)	(-0.167)	(-0.154)	(-0.152)	(-0.037)
Volatility			-1.205	-1.302	-1.509	-1.130	-1.389	-1.147	-1.189	-1.295
			(-1.332)	(-1.448)	(-1.779)	(-1.247)	(-1.614)	(-1.278)	(-1.303)	(-1.464)
Coskew			-0.121	-0.125	-0.065	-0.157	-0.098	-0.036	-0.166	0.136
			(-2.432)	(-2.538)	(-1.159)	(-3.013)	(-1.938)	(-0.594)	(-3.127)	(2.814)
Cokurt			0.108	0.100	0.095	0.117	0.104	0.089	0.121	0.036
			(7.480)	(7.613)	(6.551)	(7.427)	(6.958)	(5.568)	(7.039)	(2.372)
EDH				0.018						
				(2.920)						
EDB − BL					0.052					
					(2.684)					
EDB − ACY						-0.006				
						(-2.451)				
EDB − ES							0.032			
							(2.151)			
EDC − BL								0.279		
								(2.969)		
EDC − ACY									-0.035	
									(-2.262)	
EDC − ES										0.581
										(12.376)

Table 4.18: Persistency analysis-Common fraction of quintiles in two consecutive years

This table shows the percentage of common fraction between two consecutive years of sorted quintiles based on different risk measures. These values are averaged over the whole sample period from 1973 to 2012.

Quintile	1	2	3	4	5	average
Panel 1: Postformation measures						
Downside beta	37.38%	28.13%	25.92%	27.36%	39.67%	31.69%
$\mathbf{EDH}$	37.55%	29.15%	26.27%	28.15%	39.11%	32.05%
$\mathbf{EDB-BL}$	39.27%	28.54%	26.32%	28.14%	41.22%	32.70%
$\mathbf{EDB-ES}$	30.20%	26.01%	22.44%	24.27%	32.89%	27.16%
$\mathbf{EDC-BL}$	36.36%	24.60%	22.92%	26.71%	47.84%	31.69%
$\mathbf{EDC-ES}$	28.39%	22.50%	20.28%	22.22%	38.31%	26.34%
Panel 2: Preformation measures						
Size	79.20%	60.44%	62.07%	70.69%	86.71%	71.82%
B/M	60.11%	42.08%	38.52%	42.25%	63.42%	49.28%
Idiosyncratic volatility	72.22%	50.60%	43.02%	43.08%	59.59%	53.70%

Table 4.19: Cross-sectional analysis of systematic extreme downside risk captured using different extreme downside thresholds

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures and of the systematic tail risk measures captured using different extreme downside thresholds, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, yearly excess return of a stock is regressed against its one year realized risk measure of downside beta, upsize beta, volatility, coskewness, cokurtotis, and systematic tail risk; its past excess return of last year; and its Book-to-Market available at the time of the regression. The overall sample period is from January 1973 to December 2012 and covers all stocks in NYSE, AMEX, NASDAQ, and ARCA markets.

Model	10 percent tail quantile					1 percent tail quantile				
	I	II	III	IV	V	I	II	III	IV	V
Intercept	1.030	1.057	1.029	1.109	1.053	1.040	1.031	1.018	1.041	1.022
	(8.131)	(8.353)	(8.218)	(8.349)	(8.511)	(8.183)	(8.174)	(8.119)	(8.262)	(8.171)
β-	0.023	-0.027	0.029	0.024	0.030	0.005	0.037	0.037	0.035	0.048
	(1.442)	(-2.316)	(1.591)	(1.215)	(1.564)	(0.347)	(1.865)	(1.825)	(1.727)	(2.351)
β+	0.004	-0.002	0.000	-0.002	-0.011	0.004	0.003	0.000	0.004	0.000
	(0.451)	(-0.253)	(0.052)	(-0.198)	(-1.357)	(0.520)	(0.350)	(0.028)	(0.474)	(-0.013)
Log-size	-0.055	-0.055	-0.054	-0.060	-0.060	-0.055	-0.055	-0.053	-0.055	-0.053
	(-8.911)	(-9.017)	(-8.868)	(-9.121)	(-9.963)	(-8.936)	(-8.912)	(-8.766)	(-9.079)	(-8.793)
B/M	0.013	0.013	0.013	0.013	0.012	0.013	0.013	0.013	0.013	0.013
	(2.515)	(2.527)	(2.446)	(2.347)	(2.288)	(2.544)	(2.443)	(2.455)	(2.459)	(2.435)
Past	-0.004	-0.006	-0.003	-0.003	0.000	-0.005	-0.003	-0.003	-0.003	-0.003
Return	(-0.244)	(-0.328)	(-0.167)	(-0.158)	(-0.013)	(-0.301)	(-0.168)	(-0.193)	(-0.174)	(-0.158)
Volatility	-1.264	-1.802	-1.500	-1.019	-1.153	-1.371	-1.234	-1.303	-1.149	-1.336
	(-1.404)	(-2.102)	(-1.822)	(-1.121)	(-1.306)	(-1.529)	(-1.394)	(-1.448)	(-1.266)	(-1.487)
Coskew	-0.125	-0.062	-0.104	0.065	0.267	-0.122	-0.123	-0.093	-0.179	-0.054
	(-2.542)	(-1.158)	(-2.022)	(1.123)	(5.729)	(-2.494)	(-2.274)	(-1.826)	(-3.114)	(-1.062)
Cokurt	0.102	0.084	0.104	0.048	-0.020	0.095	0.108	0.102	0.125	0.090
	(7.746)	(6.223)	(7.160)	(3.441)	(-1.255)	(7.245)	(7.110)	(7.093)	(6.972)	(6.219)
$\mathbf{EDH}$	0.014					0.030				
	(2.961)					(2.896)				
$\mathbf{EDB-BL}$		0.112					0.004			
		(3.522)					(0.601)			
$\mathbf{EDB-ES}$			0.033					0.029		
			(1.353)					(5.129)		
$\mathbf{EDC-BL}$				0.667					-0.137	
				(5.982)					(-2.117)	
$\mathbf{EDC-ES}$					1.053					0.147
					(15.212)					(8.738)

Table 4.20: Cross-sectional analysis of EDH measures captured using different models for market tail risk

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures and of the EDH measures captured using different market tail risk models, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, yearly excess return of a stock is regressed against its one year realized risk measures of downside beta, upsize beta, volatility, coskewness, cokurtotis, and EDH; its past excess return of last year; and its size and Book-to-Market available at the time of the regression. The overall sample period is from January 1973 to December 2012 and covers all stocks in NYSE, AMEX, NASDAQ, and ARCA markets. The name of each regression model specifies whether the corresponding EDH measure utilizes market VaR or ETL model with Gaussian (G) or Skewed Student-t (S) residual term distribution assumption; the estimation period is 5 years (1250 days) or 2 years (500 days); and tail threshold is 5 percent or 1 percent.

Model	VaR_G_ 1250_5	VaR_G_ 1250_1	VaR_S_ 500_5	VaR_S_ 500_1	VaR_G_ 500_5	VaR_G_ 500_1	ETL_ 1250_5	ETL_ 1250_1	ETL_ 500_5	ETL_ 500_1
Intercept	1.032	1.037	1.051	1.058	1.050	1.056	1.035	1.039	1.053	1.058
	(8.132)	(8.159)	(8.280)	(8.298)	(8.300)	(8.331)	(8.149)	(8.170)	(8.325)	(8.348)
β-	0.016	0.007	0.011	0.006	0.013	0.008	0.010	0.003	0.010	0.007
	(1.000)	(0.469)	(0.883)	(0.532)	(0.972)	(0.645)	(0.673)	(0.222)	(0.760)	(0.532)
β+	0.003	0.003	-0.002	-0.002	-0.002	-0.002	0.003	0.004	-0.002	-0.002
	(0.370)	(0.404)	(-0.173)	(-0.187)	(-0.187)	(-0.244)	(0.391)	(0.424)	(-0.202)	(-0.224)
Log-size	-0.055	-0.055	-0.055	-0.056	-0.055	-0.056	-0.055	-0.055	-0.055	-0.056
	(-8.888)	(-8.897)	(-8.984)	(-8.974)	(-9.016)	(-9.029)	(-8.894)	(-8.900)	(-9.031)	(-9.040)
B/M	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.013	0.013
	(2.529)	(2.544)	(2.557)	(2.568)	(2.542)	(2.545)	(2.538)	(2.550)	(2.546)	(2.546)
Past Return	-0.004	-0.005	-0.006	-0.006	-0.006	-0.006	-0.005	-0.005	-0.006	-0.006
	(-0.258)	(-0.282)	(-0.354)	(-0.364)	(-0.329)	(-0.338)	(-0.274)	(-0.291)	(-0.336)	(-0.341)
Volatility	-1.312	-1.364	-1.450	-1.509	-1.413	-1.456	-1.344	-1.387	-1.437	-1.468
	(-1.462)	(-1.521)	(-1.610)	(-1.675)	(-1.567)	(-1.617)	(-1.499)	(-1.547)	(-1.594)	(-1.631)
Coskew	-0.123	-0.122	-0.122	-0.118	-0.122	-0.121	-0.122	-0.121	-0.122	-0.121
	(-2.493)	(-2.480)	(-2.514)	(-2.418)	(-2.480)	(-2.457)	(-2.487)	(-2.469)	(-2.489)	(-2.471)
Cokurt	0.099	0.096	0.093	0.090	0.096	0.094	0.097	0.094	0.094	0.093
	(7.561)	(7.354)	(6.884)	(6.542)	(7.101)	(6.840)	(7.440)	(7.243)	(6.940)	(6.777)
EDH	0.023	0.033	0.023	0.036	0.026	0.037	0.029	0.038	0.032	0.041
	(3.533)	(3.512)	(2.955)	(2.906)	(3.129)	(3.043)	(3.524)	(3.494)	(3.053)	(2.999)

Table 4.21: Cross-sectional analysis of monthly EDH measures captured using different models for market tail risk

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures and of the EDH measures captured using different market tail risk models, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, monthly excess return of a stock is regressed against its one month realized risk measures of downside beta, upsize beta, coskewness, cokurtotis, and EDH; its past excess return of last month; and its size and Book-to-Market available at the time of the regression. The overall sample period is from January 1973 to December 2012 and covers all stocks in NYSE, AMEX, NASDAQ, and ARCA markets. The name of each regression model specifies whether the corresponding EDH measure utilizes market VaR or ETL model with Gaussian (G) or Skewed Student-t (S) residual term distribution assumption; estimation period is 5 years (1250 days) or 2 years (500 days); and tail threshold is 5 percent or 1 percent.

Model	VaR_S_ 1250_5	VaR_S_ 1250_1	VaR_G_ 1250_5	VaR_G_ 1250_1	VaR_S_ 500_5	VaR_S_ 500_1	VaR_G_ 500_5	VaR_G_ 500_1	ETL_ 1250_5	ETL_ 1250_1	ETL_ 500_5	ETL_ 500_1
Intercept	0.031	0.030	0.031	0.030	0.031	0.031	0.031	0.031	0.031	0.030	0.031	0.031
	(3.074)	(2.998)	(3.063)	(3.019)	(3.117)	(3.082)	(3.149)	(3.141)	(3.035)	(3.000)	(3.140)	(3.130)
β	0.003	-0.001	0.003	0.001	0.002	-0.001	0.003	0.002	0.002	0.000	0.002	0.001
	(1.088)	(-0.190)	(1.236)	(0.311)	(0.612)	(-0.161)	(0.855)	(0.467)	(0.643)	(-0.058)	(0.627)	(0.348)
Log-size	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
	(-2.921)	(-2.824)	(-2.905)	(-2.847)	(-2.963)	(-2.912)	(-3.007)	(-2.996)	(-2.867)	(-2.820)	(-2.996)	(-2.982)
B/M	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
	(3.706)	(3.718)	(3.706)	(3.715)	(3.701)	(3.706)	(3.701)	(3.701)	(3.712)	(3.718)	(3.702)	(3.702)
Past	-0.051	-0.051	-0.051	-0.051	-0.051	-0.051	-0.051	-0.051	-0.051	-0.051	-0.051	-0.051
Return	(-10.313)	(-10.323)	(-10.321)	(-10.322)	(-10.316)	(-10.330)	(-10.276)	(-10.272)	(-10.322)	(-10.322)	(-10.294)	(-10.286)
Coskew	0.000	0.005	0.000	0.003	-0.002	0.001	-0.001	0.000	0.002	0.005	0.000	0.001
	(0.116)	(1.889)	(-0.116)	(1.070)	(-0.898)	(0.271)	(-0.431)	(0.154)	(0.644)	(1.564)	(-0.106)	(0.294)
Cokurt	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
	(-4.277)	(-4.297)	(-4.285)	(-4.280)	(-4.219)	(-4.249)	(-4.158)	(-4.159)	(-4.284)	(-4.278)	(-4.157)	(-4.172)
EDH	0.001	0.002	0.001	0.002	0.002	0.004	0.001	0.002	0.001	0.002	0.002	0.003
	(4.356)	(5.239)	(4.487)	(5.380)	(2.896)	(3.197)	(2.508)	(2.730)	(5.135)	(5.594)	(2.608)	(2.768)

Table 4.22: Cross-sectional analysis of excess extreme downside measure

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures, SV, and EDH_O. In each cross-sectional regression, yearly excess return of a stock is regressed against SV, EDH_O, and other one year realized risk measures as well as last year excess return, size and Book-to-Market. The overall sample period is from January 1986 to December 2012 and covers all stocks in NYSE, AMEX, NASDAQ, and ARCA markets. The name of each regression model specifies whether the corresponding EDH_O measure utilizes market VaR or ETL model with Gaussian (G) or Skewed Student-t (S) residual term distribution assumption; the estimation period is 5 years (1250 days) or 2 years (500 days); and tail threshold is 5 percent or 1 percent.

Model	VaR_S_ 1250_5	VaR_S_ 1250_1	VaR_G_ 1250_5	VaR_G_ 1250_1	VaR_S_ 500_5	VaR_S_ 500_1	VaR_G_ 500_5	VaR_G_ 500_1	ETL_ 1250_5	ETL_ 1250_1	ETL_ 500_5	ETL_ 500_1
Intercept	1.123	1.123	1.124	1.124	1.138	1.141	1.138	1.141	1.124	1.124	1.139	1.141
	(7.226)	(7.218)	(7.226)	(7.216)	(7.340)	(7.336)	(7.354)	(7.364)	(7.219)	(7.213)	(7.365)	(7.368)
β-	-0.004	-0.015	-0.004	-0.010	-0.007	-0.011	-0.007	-0.010	-0.008	-0.013	-0.009	-0.011
	(-0.195)	(-0.759)	(-0.183)	(-0.490)	(-0.361)	(-0.583)	(-0.309)	(-0.499)	(-0.371)	(-0.638)	(-0.436)	(-0.574)
β+	-0.009	-0.006	-0.009	-0.008	-0.010	-0.008	-0.010	-0.010	-0.009	-0.007	-0.010	-0.009
	(-0.892)	(-0.646)	(-0.941)	(-0.786)	(-1.040)	(-0.856)	(-1.077)	(-1.009)	(-0.846)	(-0.708)	(-0.999)	(-0.946)
Log-size	-0.058	-0.058	-0.058	-0.058	-0.059	-0.059	-0.059	-0.059	-0.058	-0.058	-0.059	-0.059
	(-7.899)	(-7.884)	(-7.885)	(-7.867)	(-7.962)	(-7.946)	(-7.984)	(-7.985)	(-7.873)	(-7.859)	(-7.989)	(-7.984)
B/M	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
	(0.943)	(0.958)	(0.941)	(0.949)	(0.952)	(0.952)	(0.932)	(0.929)	(0.946)	(0.952)	(0.931)	(0.927)
Past	-0.035	-0.035	-0.034	-0.035	-0.036	-0.036	-0.035	-0.035	-0.035	-0.035	-0.035	-0.035
Return	(-1.625)	(-1.649)	(-1.619)	(-1.633)	(-1.690)	(-1.694)	(-1.658)	(-1.663)	(-1.628)	(-1.638)	(-1.664)	(-1.664)
Volatility	-0.972	-0.993	-0.972	-0.983	-1.063	-1.073	-1.022	-1.031	-0.979	-0.988	-1.025	-1.029
	(-0.835)	(-0.851)	(-0.838)	(-0.844)	(-0.908)	(-0.914)	(-0.872)	(-0.879)	(-0.842)	(-0.848)	(-0.874)	(-0.877)
Coskew	-0.079	-0.078	-0.075	-0.076	-0.071	-0.069	-0.073	-0.072	-0.076	-0.076	-0.073	-0.073
	(-1.184)	(-1.164)	(-1.130)	(-1.127)	(-1.092)	(-1.039)	(-1.113)	(-1.092)	(-1.131)	(-1.121)	(-1.116)	(-1.098)
Cokurt	0.087	0.083	0.087	0.084	0.084	0.082	0.086	0.084	0.085	0.083	0.085	0.084
	(6.045)	(5.753)	(6.063)	(5.872)	(5.739)	(5.552)	(5.842)	(5.677)	(5.946)	(5.777)	(5.751)	(5.647)
SV	-16.149	-17.566	-16.425	-17.293	-16.250	-16.448	-15.843	-16.073	-16.982	-17.649	-16.011	-16.176
	(-3.246)	(-3.355)	(-3.295)	(-3.353)	(-3.181)	(-3.146)	(-3.148)	(-3.149)	(-3.334)	(-3.373)	(-3.160)	(-3.164)
EDH_O	0.013	0.021	0.018	0.024	0.014	0.021	0.017	0.023	0.021	0.027	0.020	0.025
	(2.126)	(2.137)	(2.638)	(2.602)	(1.891)	(1.964)	(2.256)	(2.199)	(2.611)	(2.591)	(2.156)	(2.141)

Table 4.23: Cross-sectional analysis of excess extreme downside measure-CAPM beta as an regressor

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures, SV, and EHD_O, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, yearly excess return of a stock is regressed against its one month realized risk measures of CAPM beta, volatility, coskewness, cokurtotis, SV, and EDH_O; its last year excess return; and its size and Book-to-Market available at the time of the regression. The overall sample period is from January 1986 to December 2012 and covers all stocks in NYSE, AMEX, NASDAQ, and ARCA markets. The name of each regression model specifies whether the corresponding EDH_O measure utilizes market VaR or ETL model with Gaussian (G) or Skewed Student-t (S) residual term distribution assumption; the estimation period is 5 years (1250 days) or 2 years (500 days); and tail threshold is 5 or 1 percent.

Model	VaR_S_ 1250_5	VaR_S_ 1250_1	VaR_G_ 1250_5	VaR_G_ 1250_1	VaR_S_ 500_5	VaR_S_ 500_1	VaR_G_ 500_5	VaR_G_ 500_1	ETL_ 1250_5	ETL_ 1250_1	ETL_ 500_5	ETL_ 500_1
Intercept	1.127	1.124	1.128	1.127	1.134	1.135	1.135	1.135	1.127	1.127	1.135	1.136
	7.288	7.271	7.280	7.275	7.346	7.373	7.375	7.388	7.277	7.273	7.395	7.400
β	0.048	0.026	0.047	0.035	0.047	0.042	0.051	0.042	0.039	0.030	0.050	0.044
	(0.573)	(0.323)	(0.552)	(0.416)	(0.590)	(0.535)	(0.618)	(0.517)	(0.465)	(0.359)	(0.620)	(0.544)
Log-size	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058	-0.058
	(-7.860)	(-7.835)	(-7.834)	(-7.824)	(-7.881)	(-7.917)	(-7.908)	(-7.923)	(-7.828)	(-7.820)	(-7.930)	(-7.935)
B/M	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
	(1.011)	(1.019)	(1.009)	(1.013)	(0.995)	(0.989)	(0.984)	(0.983)	(1.012)	(1.014)	(0.984)	(0.981)
Past	-0.039	-0.039	-0.039	-0.039	-0.038	-0.038	-0.038	-0.038	-0.039	-0.039	-0.038	-0.038
Return	(-1.758)	(-1.754)	(-1.763)	(-1.757)	(-1.765)	(-1.761)	(-1.766)	(-1.759)	(-1.759)	(-1.754)	(-1.762)	(-1.757)
Volatility	-1.178	-1.191	-1.189	-1.192	-1.205	-1.192	-1.208	-1.197	-1.191	-1.195	-1.201	-1.194
	(-0.997)	(-1.006)	(-1.008)	(-1.008)	(-1.026)	(-1.011)	(-1.027)	(-1.017)	(-1.008)	(-1.010)	(-1.020)	(-1.014)
Coskew	-0.133	-0.103	-0.123	-0.108	-0.139	-0.133	-0.145	-0.142	-0.114	-0.102	-0.143	-0.140
	(-2.375)	(-1.876)	(-2.204)	(-1.936)	(-2.634)	(-2.557)	(-2.680)	(-2.661)	(-2.033)	(-1.825)	(-2.649)	(-2.621)
Cokurt	0.064	0.063	0.064	0.064	0.065	0.065	0.065	0.064	0.064	0.063	0.065	0.064
	(4.241)	(4.198)	(4.236)	(4.193)	(4.249)	(4.237)	(4.242)	(4.230)	(4.208)	(4.175)	(4.252)	(4.244)
SV	-10.490	-13.279	-10.765	-12.259	-11.086	-11.401	-10.156	-10.985	-11.725	-12.878	-10.045	-10.684
	(-1.510)	(-1.902)	(-1.512)	(-1.723)	(-1.564)	(-1.664)	(-1.389)	(-1.524)	(-1.649)	(-1.807)	(-1.410)	(-1.498)
EDH_O	0.009	0.015	0.012	0.016	0.008	0.013	0.009	0.011	0.014	0.018	0.010	0.012
	(2.905)	(3.266)	(3.626)	(3.794)	(1.772)	(1.911)	(2.199)	(2.074)	(3.736)	(3.835)	(1.965)	(1.890)

Table 4.24: Cross-sectional analysis of excess extreme downside measure-CAPM beta as an regressor

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures and of the extreme downside risk measure EDH, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, yearly excess return of a stock is regressed against its one year realized risk measures of CAPM beta, volatility, coskewness, cokurtotis, and EDH; its last year excess return; and its size and Book-to-Market available at the time of the regression. The overall sample period is from January 1986 to December 2012 and covers all stocks in NYSE, AMEX, NASDAQ, and ARCA markets. The name of each regression model specifies whether the corresponding EDH measure utilizes market VaR or ETL model with Gaussian (G) or Skewed Student-t (S) residual term distribution assumption; the estimation period is 5 years (1250 days) or 2 years (500 days); and tail threshold is 5 percent or 1 percent.

Model	VaR_S_ 1250_5	VaR_S_ 1250_1	VaR_G_ 1250_5	VaR_G_ 1250_1	VaR_S_ 500_5	VaR_S_ 500_1	VaR_G_ 500_5	VaR_G_ 500_1	ETL_ 1250_5	ETL_ 1250_1	ETL_ 500_5	ETL_ 500_1
Intercept	1.132	1.130	1.131	1.130	1.137	1.139	1.139	1.140	1.130	1.130	1.140	1.141
	(7.280)	(7.273)	(7.240)	(7.241)	(7.314)	(7.351)	(7.354)	(7.364)	(7.239)	(7.244)	(7.369)	(7.377)
β	0.060	0.032	0.056	0.036	0.054	0.050	0.063	0.054	0.043	0.028	0.062	0.056
	(1.054)	(0.558)	(0.981)	(0.628)	(1.078)	(0.951)	(1.211)	(0.984)	(0.755)	(0.492)	(1.163)	(0.994)
Log-size	-0.058	-0.058	-0.058	-0.058	-0.058	-0.059	-0.059	-0.059	-0.058	-0.058	-0.059	-0.059
	(-7.925)	(-7.897)	(-7.846)	(-7.829)	(-7.897)	(-7.937)	(-7.956)	(-7.961)	(-7.833)	(-7.825)	(-7.965)	(-7.968)
B/M	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
	(1.007)	(1.018)	(1.004)	(1.011)	(0.995)	(0.992)	(0.977)	(0.977)	(1.009)	(1.013)	(0.978)	(0.977)
Past	-0.038	-0.038	-0.038	-0.038	-0.037	-0.037	-0.038	-0.037	-0.038	-0.038	-0.037	-0.037
Return	(-1.751)	(-1.748)	(-1.751)	(-1.749)	(-1.747)	(-1.744)	(-1.747)	(-1.740)	(-1.750)	(-1.748)	(-1.741)	(-1.737)
Volatility	-1.096	-1.121	-1.114	-1.128	-1.118	-1.108	-1.108	-1.098	-1.123	-1.133	-1.105	-1.097
	(-0.918)	(-0.937)	(-0.934)	(-0.944)	(-0.943)	(-0.930)	(-0.932)	(-0.923)	(-0.940)	(-0.948)	(-0.929)	(-0.923)
Coskew	-0.132	-0.102	-0.125	-0.107	-0.144	-0.140	-0.151	-0.150	-0.114	-0.100	-0.150	-0.151
	(-2.451)	(-1.916)	(-2.323)	(-1.993)	(-2.851)	(-2.748)	(-2.861)	(-2.834)	(-2.113)	(-1.863)	(-2.820)	(-2.799)
Cokurt	0.066	0.065	0.066	0.065	0.066	0.066	0.066	0.066	0.065	0.065	0.066	0.066
	(4.242)	(4.222)	(4.210)	(4.174)	(4.243)	(4.222)	(4.237)	(4.228)	(4.186)	(4.162)	(4.237)	(4.230)
EDH	0.016	0.030	0.021	0.031	0.018	0.029	0.016	0.023	0.027	0.037	0.020	0.025
	(2.827)	(3.049)	(3.850)	(3.854)	(2.464)	(2.179)	(2.478)	(2.142)	(3.865)	(3.825)	(2.128)	(1.879)

Table 4.25: Cross-sectional analysis of monthly excess extreme downside measure

This table shows the Fama and Macbeth (1973) average risk premiums of standard risk measures, SV, and EDH_O, along with their corresponding Newey-West t-statistics (in brackets). In each cross-sectional regression, monthly excess return of a stock is regressed against its one month realized risk measures of CAPM beta, coskewness, cokurtotis, SV, and EDH_O; its last month excess return; and its size and Book-to-Market available at the time of the regression. The overall sample period is from January 1986 to December 2012 and covers all stocks in NYSE, AMEX, NASDAQ, and ARCA markets. The name of each regression model specifies whether the corresponding EDH_O measure utilizes market VaR or ETL model with Gaussian (G) or Skewed Student-t (S) residual term distribution assumption; the estimation period is 5 years (1250 days) or 2 years (500 days); and tail threshold is 5 percent or 1 percent.

Model	VaR_S_ 1250_5	VaR_S_ 1250_1	VaR_G_ 1250_5	VaR_G_ 1250_1	VaR_S_ 500_5	VaR_S_ 500_1	VaR_G_ 500_5	VaR_G_ 500_1	ETL_ 1250_5	ETL_ 1250_1	ETL_ 500_5	ETL_ 500_1
Intercept	0.040	0.039	0.040	0.039	0.040	0.040	0.041	0.040	0.040	0.039	0.040	0.040
	(3.476)	(3.410)	(3.455)	(3.419)	(3.549)	(3.522)	(3.555)	(3.545)	(3.432)	(3.402)	(3.551)	(3.544)
β	0.001	-0.001	0.001	0.000	0.000	-0.001	0.000	0.000	0.000	-0.001	0.000	-0.001
	(0.243)	(-0.366)	(0.386)	(-0.031)	(0.089)	(-0.222)	(0.041)	(-0.062)	(0.121)	(-0.210)	(-0.048)	(-0.119)
Log-size	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
	(-3.099)	(-3.017)	(-3.076)	(-3.029)	(-3.177)	(-3.136)	(-3.182)	(-3.169)	(-3.046)	(-3.007)	(-3.175)	(-3.165)
B/M	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	(2.566)	(2.581)	(2.566)	(2.573)	(2.557)	(2.567)	(2.574)	(2.577)	(2.571)	(2.577)	(2.572)	(2.571)
Past	-0.034	-0.035	-0.034	-0.035	-0.035	-0.035	-0.034	-0.034	-0.035	-0.035	-0.034	-0.034
Return	(-8.305)	(-8.316)	(-8.311)	(-8.310)	(-8.303)	(-8.313)	(-8.246)	(-8.233)	(-8.312)	(-8.311)	(-8.250)	(-8.238)
Coskew	-0.001	0.003	-0.002	0.001	-0.003	0.000	-0.002	0.000	0.000	0.002	-0.001	0.000
	(-0.206)	(0.985)	(-0.452)	(0.291)	(-0.858)	(-0.129)	(-0.436)	(-0.106)	(0.016)	(0.628)	(-0.278)	(-0.083)
Cokurt	-0.004	-0.004	-0.004	-0.004	-0.005	-0.005	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
	(-1.987)	(-1.975)	(-1.998)	(-1.984)	(-2.023)	(-2.043)	(-1.927)	(-1.899)	(-1.989)	(-1.979)	(-1.896)	(-1.889)
SV	-0.386	-0.665	-0.370	-0.598	-0.568	-0.803	-0.696	-0.756	-0.511	-0.703	-0.751	-0.795
	(-0.948)	(-1.466)	(-0.914)	(-1.337)	(-0.922)	(-1.192)	(-1.043)	(-1.104)	(-1.185)	(-1.507)	(-1.106)	(-1.147)
EDH_O	0.000	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	(3.828)	(5.252)	(3.467)	(4.836)	(2.832)	(2.969)	(2.736)	(2.861)	(4.376)	(5.256)	(2.789)	(2.823)

APPENDIX

Table A4.1: Average excess return of the equally weighted quintile portfolios sorted on EDH and other downside beta measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDH as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					Estrada (2007) Downside beta				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	2.088	6.796	6.190	8.318	13.173	2.699	7.049	4.428	5.237	8.423
	(0.472)	(2.212)	(1.887)	(2.421)	(2.990)	(1.007)	(2.327)	(1.259)	(1.137)	(1.227)
2	5.469	8.194	9.596	10.563	15.647	4.988	8.997	10.362	10.302	13.235
	(2.127)	(3.313)	(3.933)	(4.251)	(3.963)	(2.591)	(3.490)	(3.692)	(2.763)	(2.427)
3	4.193	8.890	10.761	12.321	18.978	6.170	9.286	10.007	12.729	17.093
	(1.784)	(3.649)	(4.598)	(4.553)	(4.405)	(3.144)	(4.048)	(4.316)	(3.924)	(3.339)
4	5.240	8.977	10.702	12.150	21.472	7.784	11.016	12.388	14.297	18.943
	(2.331)	(3.333)	(4.047)	(4.390)	(4.214)	(3.861)	(5.601)	(4.743)	(4.510)	(3.496)
5	3.110	9.426	10.643	13.739	28.186	8.470	10.425	13.222	14.150	23.199
	(0.959)	(2.563)	(3.004)	(3.547)	(3.456)	(4.237)	(4.641)	(4.584)	(3.298)	(3.011)
5-1	1.022	2.630	4.453	5.421	15.013	5.771	3.376	8.795	8.913	14.776
	(0.491)	(1.822)	(2.867)	(2.388)	(2.661)	(3.629)	(1.990)	(3.334)	(2.105)	(1.812)

Table A4.2: Average excess return of the equally weighted quintile portfolios sorted on EDB-BL and other downside beta measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDB-BL as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					**Estrada (2007) Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	1.552	6.006	7.382	8.105	13.717	1.676	6.481	5.767	2.839	5.717
	(0.373)	(1.738)	(2.197)	(2.618)	(3.107)	(0.545)	(1.855)	(1.609)	(0.590)	(0.832)
2	3.504	8.231	8.811	11.991	15.695	3.848	9.037	8.925	10.704	10.628
	(1.245)	(3.390)	(4.115)	(4.520)	(4.055)	(1.680)	(3.722)	(3.034)	(2.712)	(1.991)
3	3.977	9.236	10.036	11.864	17.842	7.484	10.728	11.617	12.957	16.455
	(1.688)	(3.951)	(4.294)	(4.560)	(3.835)	(3.916)	(5.047)	(4.665)	(3.965)	(3.038)
4	5.948	9.842	10.687	12.703	21.839	8.540	11.082	11.980	14.840	21.280
	(2.643)	(3.929)	(3.903)	(4.242)	(4.371)	(4.361)	(5.682)	(5.108)	(4.341)	(4.155)
5	4.947	8.994	10.922	12.360	28.367	8.538	9.366	12.039	15.241	26.866
	(1.608)	(2.418)	(3.018)	(3.060)	(3.900)	(4.763)	(4.500)	(4.704)	(4.160)	(3.731)
5-1	3.395	2.988	3.539	4.255	14.650	6.862	2.885	6.272	12.402	21.150
	(1.929)	(2.104)	(2.972)	(1.936)	(3.525)	(2.750)	(1.386)	(2.539)	(2.813)	(3.024)
FF	3.836	1.927	3.820	3.265	17.485	5.639	5.752	9.069	16.550	28.027
	(2.135)	(1.418)	(2.535)	(1.279)	(2.159)	(2.257)	(3.531)	(4.640)	(5.523)	(3.491)

Table A4.3: Average excess return of the equally weighted quintile portfolios sorted on EDB-ACY and other downside beta measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDB-ACY as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					Estrada (2007) Downside beta				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	0.699	6.906	8.249	9.616	13.074	4.175	7.624	6.855	6.049	6.511
	(0.180)	(1.824)	(2.139)	(2.700)	(2.815)	(1.608)	(2.471)	(1.965)	(1.286)	(1.013)
2	5.043	8.788	9.549	11.693	18.043	5.988	9.121	9.994	11.668	14.512
	(1.978)	(3.508)	(4.018)	(4.200)	(4.117)	(3.123)	(3.932)	(3.674)	(3.549)	(2.699)
3	4.028	8.830	10.388	11.183	20.802	5.143	10.000	10.752	12.700	21.632
	(1.580)	(3.943)	(4.572)	(4.446)	(4.216)	(2.248)	(4.814)	(4.367)	(3.583)	(3.773)
4	4.792	8.495	9.675	12.804	21.767	7.606	9.961	11.693	13.455	19.295
	(2.052)	(3.562)	(4.288)	(4.778)	(4.337)	(4.324)	(4.850)	(4.697)	(4.111)	(3.693)
5	5.389	9.266	9.961	11.728	22.925	7.253	9.924	11.025	12.701	17.979
	(1.526)	(2.650)	(2.829)	(3.139)	(3.564)	(3.859)	(4.358)	(4.219)	(3.539)	(2.950)
5-1	4.690	2.360	1.712	2.112	9.850	3.078	2.300	4.170	6.652	11.468
	(1.944)	(1.453)	(1.802)	(1.220)	(2.241)	(2.328)	(1.562)	(2.236)	(2.281)	(2.757)

Table A4.4: Average excess return of the equally weighted quintile portfolios sorted on EDB-ES and other downside beta measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDB-ES as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					**Estrada (2007) Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	-0.606	6.149	6.118	6.428	11.038	2.068	7.813	5.605	2.710	4.834
	(-0.175)	(2.123)	(2.018)	(2.164)	(2.467)	(0.758)	(2.649)	(1.725)	(0.673)	(0.873)
2	3.685	8.936	9.455	11.927	17.800	5.691	9.283	9.087	10.659	15.133
	(1.496)	(3.266)	(4.192)	(4.218)	(4.178)	(2.590)	(3.908)	(3.430)	(2.602)	(2.440)
3	5.688	9.847	10.600	12.956	20.234	7.884	10.455	11.422	13.404	17.747
	(2.505)	(4.368)	(4.235)	(5.051)	(4.758)	(4.142)	(4.738)	(4.424)	(4.101)	(3.512)
4	7.296	8.877	11.793	13.733	21.812	7.639	10.161	11.974	14.526	20.435
	(2.841)	(3.435)	(4.159)	(4.299)	(4.303)	(3.944)	(4.843)	(4.587)	(4.165)	(3.641)
5	3.832	8.387	9.742	11.933	25.975	6.707	8.920	12.101	15.117	22.051
	(0.926)	(2.199)	(2.738)	(3.319)	(3.745)	(3.305)	(3.952)	(4.640)	(4.140)	(3.450)
5-1	4.438	2.238	3.625	5.506	14.937	4.640	1.108	6.496	12.407	17.217
	(2.839)	(1.809)	(3.345)	(3.945)	(3.752)	(2.725)	(0.807)	(3.453)	(3.906)	(3.321)

Table A4.5: Average excess return of the equally weighted quintile portfolios sorted on EDC-BL and other downside beta measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDC-BL as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					Estrada (2007) Downside beta				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	3.614	6.053	6.997	7.938	13.279	2.637	6.695	5.798	2.569	5.379
	(1.008)	(1.427)	(1.475)	(1.663)	(2.061)	(0.918)	(1.871)	(1.583)	(0.541)	(0.756)
2	2.733	9.476	9.267	11.736	18.205	4.079	9.456	8.055	8.609	10.903
	(0.899)	(2.877)	(2.854)	(3.339)	(3.731)	(1.390)	(3.419)	(2.506)	(2.004)	(1.564)
3	2.009	9.079	10.563	11.862	19.179	7.831	10.570	10.714	11.462	15.228
	(0.640)	(3.247)	(4.098)	(4.303)	(4.170)	(4.031)	(4.798)	(4.104)	(3.411)	(2.884)
4	5.298	8.860	11.394	13.298	22.621	7.658	10.585	13.186	16.074	19.984
	(2.231)	(3.842)	(5.295)	(5.330)	(4.513)	(4.217)	(5.598)	(5.298)	(4.695)	(3.803)
5	6.342	8.956	9.740	12.156	23.125	7.860	9.433	12.534	17.819	29.234
	(2.821)	(4.832)	(5.274)	(5.300)	(4.696)	(4.842)	(5.248)	(5.442)	(4.843)	(4.282)
5-1	2.728	2.903	2.742	4.217	9.846	5.223	2.737	6.737	15.251	23.855
	(1.355)	(0.986)	(0.703)	(1.047)	(2.081)	(2.082)	(0.988)	(2.193)	(3.353)	(2.606)

Table A4.6: Average excess return of the equally weighted quintile portfolios sorted on EDC-ACY and other downside beta measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDC-ACY as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					Estrada (2007) Downside beta				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	2.047	6.449	7.854	8.716	11.071	3.455	7.959	7.310	5.811	7.106
	(0.612)	(2.015)	(2.067)	(2.403)	(2.111)	(1.265)	(2.642)	(2.143)	(1.259)	(1.098)
2	2.154	8.031	8.505	10.472	18.667	5.228	7.857	8.708	9.817	11.498
	(0.762)	(2.496)	(2.935)	(3.048)	(3.842)	(2.408)	(3.131)	(3.023)	(2.631)	(1.865)
3	3.956	8.015	10.460	12.511	21.508	6.353	9.985	10.976	11.508	17.619
	(1.372)	(2.993)	(3.931)	(4.281)	(3.967)	(3.070)	(4.282)	(4.104)	(3.235)	(3.066)
4	5.249	10.381	10.357	12.666	22.112	7.565	10.455	10.860	14.873	19.428
	(1.751)	(3.972)	(4.195)	(4.893)	(4.168)	(4.088)	(5.016)	(4.397)	(4.659)	(3.672)
5	6.566	9.365	10.665	12.724	23.202	7.517	10.388	12.516	14.725	24.505
	(2.736)	(3.689)	(4.611)	(4.651)	(4.451)	(4.543)	(5.414)	(5.214)	(4.096)	(4.007)
5-1	4.519	2.917	2.811	4.008	12.132	4.063	2.429	5.206	8.914	17.399
	(2.255)	(2.038)	(1.223)	(1.678)	(2.419)	(2.192)	(1.420)	(2.092)	(2.362)	(2.571)

Table A4.7: Average excess return of the equally weighted quintile portfolios sorted on EDC-ES and other downside beta measures

This table shows the average yearly excess returns of the equally weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDC-ES as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					**Estrada (2007) Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	-0.783	4.631	2.906	2.094	5.353	1.866	5.793	3.217	-0.814	1.120
	(-0.223)	(1.495)	(0.835)	(0.575)	(0.935)	(0.663)	(1.931)	(0.948)	(-0.196)	(0.191)
2	2.216	6.927	7.951	10.133	14.195	4.884	7.516	5.807	5.959	8.081
	(0.783)	(1.972)	(2.282)	(2.722)	(3.150)	(1.991)	(2.725)	(1.905)	(1.413)	(1.172)
3	3.955	8.276	11.476	13.220	22.157	6.421	9.419	11.627	12.298	13.617
	(1.418)	(2.870)	(4.007)	(4.182)	(4.449)	(2.969)	(3.710)	(4.402)	(3.550)	(2.527)
4	5.528	10.354	12.326	15.524	27.132	7.539	11.524	13.704	16.558	23.425
	(1.767)	(3.712)	(4.796)	(5.771)	(5.138)	(3.853)	(5.650)	(5.254)	(4.937)	(4.034)
5	8.919	12.129	13.051	15.857	27.688	9.258	12.369	15.783	22.319	34.347
	(4.012)	(5.857)	(6.338)	(6.296)	(4.952)	(5.864)	(6.715)	(6.288)	(5.609)	(4.787)
5-1	9.702	7.498	10.145	13.763	22.335	7.392	6.575	12.566	23.134	33.227
	(4.245)	(5.036)	(4.440)	(5.057)	(4.793)	(3.004)	(3.212)	(4.616)	(5.966)	(4.604)

Table A4.8: Average excess return of the value weighted quintile portfolios sorted on EDH and other downside beta measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDH as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					**Estrada (2007) Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	1.744	3.301	1.458	2.764	3.468	3.986	2.216	-2.493	-1.103	-15.496
	(0.557)	(1.524)	(0.710)	(1.346)	(1.031)	(2.178)	(0.712)	(-0.782)	(-0.192)	(-2.015)
2	3.452	7.040	6.025	6.055	10.274	5.882	3.880	3.579	1.757	-2.294
	(1.892)	(3.749)	(3.274)	(2.495)	(2.941)	(3.337)	(1.676)	(1.524)	(0.501)	(-0.444)
3	4.236	6.231	6.904	8.564	10.334	6.275	5.415	6.587	6.726	8.432
	(2.342)	(3.784)	(4.186)	(3.720)	(2.570)	(3.583)	(2.809)	(3.134)	(1.910)	(1.764)
4	5.834	7.462	7.674	7.303	12.440	8.196	7.403	9.099	8.027	10.767
	(3.281)	(4.131)	(4.340)	(2.678)	(2.728)	(4.639)	(4.175)	(3.593)	(2.281)	(2.160)
5	5.386	7.096	6.206	8.542	15.331	7.723	8.752	12.436	11.773	15.765
	(2.652)	(3.804)	(2.533)	(2.357)	(2.349)	(5.105)	(4.112)	(4.320)	(2.873)	(2.333)
5-1	3.642	3.795	4.748	5.778	11.863	3.737	6.536	14.929	12.876	31.260
	(1.128)	(2.335)	(1.975)	(2.174)	(2.384)	(2.141)	(2.143)	(4.192)	(1.896)	(3.351)

Table A4.9: Average excess return of the value weighted quintile portfolios sorted on EDB-BL and other downside beta measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDB-BL as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					**Estrada (2007) Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	-0.547	6.685	4.459	3.700	7.515	6.209	4.252	-1.578	-2.072	-17.220
	(-0.192)	(2.592)	(2.446)	(1.593)	(2.088)	(3.094)	(1.427)	(-0.649)	(-0.340)	(-2.001)
2	5.340	5.115	4.777	7.136	9.371	5.999	5.236	2.587	-0.893	-2.528
	(3.075)	(3.037)	(2.705)	(2.863)	(2.600)	(3.103)	(2.789)	(1.243)	(-0.225)	(-0.460)
3	5.799	7.265	6.862	7.563	7.448	7.047	5.388	7.387	5.048	1.963
	(2.657)	(4.628)	(3.736)	(3.338)	(1.885)	(3.632)	(3.089)	(3.790)	(1.394)	(0.478)
4	6.231	4.373	6.915	7.444	12.139	6.721	6.264	7.146	10.118	13.499
	(3.935)	(2.223)	(3.279)	(2.593)	(2.562)	(4.328)	(3.360)	(2.905)	(2.864)	(2.565)
5	5.244	7.950	4.272	5.637	15.672	7.649	7.002	10.510	9.506	15.750
	(2.245)	(3.579)	(1.931)	(1.947)	(2.666)	(5.178)	(3.295)	(4.265)	(2.798)	(2.588)
5-1	5.791	1.265	-0.188	1.937	8.157	1.440	2.750	12.089	11.578	32.970
	(3.511)	(0.607)	(-0.102)	(1.170)	(1.764)	(0.756)	(1.216)	(4.775)	(1.663)	(3.660)

Table A4.10: Average excess return of the value weighted quintile portfolios sorted on EDB-ACY and other downside beta measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDB-ACY as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					**Estrada (2007) Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	1.052	6.181	3.179	4.409	5.554	5.738	5.510	3.507	-0.602	-5.582
	(0.371)	(1.742)	(1.443)	(1.872)	(1.376)	(2.650)	(2.725)	(1.878)	(-0.169)	(-0.829)
2	6.104	6.265	5.436	5.566	9.831	7.015	3.923	7.262	7.869	5.808
	(3.135)	(3.205)	(3.001)	(2.105)	(2.493)	(4.485)	(1.929)	(2.806)	(2.237)	(1.240)
3	5.197	7.025	5.683	6.872	11.162	5.923	6.247	8.271	11.145	16.650
	(3.050)	(4.511)	(3.285)	(2.795)	(2.895)	(3.497)	(3.364)	(3.430)	(3.146)	(2.750)
4	5.715	5.894	6.529	8.942	9.835	7.735	6.454	8.526	9.503	13.080
	(3.198)	(3.473)	(3.325)	(4.463)	(2.299)	(4.642)	(2.818)	(3.836)	(2.690)	(2.455)
5	1.775	5.267	4.917	6.270	9.673	6.818	8.077	9.454	7.372	5.778
	(0.689)	(2.417)	(2.802)	(2.439)	(2.065)	(5.438)	(4.499)	(4.917)	(2.364)	(1.023)
5-1	0.723	-0.914	1.739	1.862	4.120	1.080	2.567	5.948	7.974	11.360
	(0.377)	(-0.242)	(1.161)	(0.871)	(1.345)	(0.659)	(1.958)	(4.142)	(2.744)	(2.071)

Table A4.11: Average excess return of the value weighted quintile portfolios sorted on EDB-ES and other downside beta measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDB-ES as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					**Estrada (2007) Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	-1.387	2.568	3.241	0.412	2.284	5.844	1.237	-0.414	-7.862	-11.380
	(-0.526)	(1.138)	(1.598)	(0.171)	(0.603)	(3.574)	(0.555)	(-0.166)	(-1.769)	(-2.050)
2	3.197	4.968	4.967	6.257	9.341	4.498	3.328	3.232	1.542	0.858
	(1.880)	(2.638)	(2.935)	(2.338)	(2.503)	(2.153)	(1.550)	(1.460)	(0.397)	(0.162)
3	4.635	7.349	5.063	8.211	9.653	7.385	6.452	5.885	7.720	8.011
	(2.336)	(4.844)	(2.873)	(3.527)	(2.687)	(3.805)	(3.915)	(2.661)	(2.421)	(1.813)
4	7.231	5.826	7.748	7.308	13.529	6.673	6.477	9.088	9.833	14.249
	(3.730)	(3.820)	(3.829)	(2.962)	(3.101)	(3.931)	(3.231)	(4.072)	(3.168)	(2.565)
5	3.326	9.060	7.000	8.149	11.264	8.095	7.840	11.044	10.923	13.284
	(1.152)	(3.992)	(3.325)	(3.593)	(2.050)	(6.491)	(3.856)	(4.259)	(2.978)	(2.318)
5-1	4.713	6.492	3.759	7.737	8.980	2.251	6.603	11.458	18.785	24.665
	(2.596)	(3.944)	(1.902)	(6.912)	(2.764)	(1.641)	(3.632)	(4.055)	(3.397)	(4.123)

Table A4.12: Average excess return of the value weighted quintile portfolios sorted on EDC-BL and other downside beta measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDC-BL as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					**Estrada (2007) Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	4.542	2.328	-2.659	-7.628	-13.206	6.679	4.423	-1.523	-6.739	-19.141
	(2.243)	(0.755)	(-0.879)	(-1.720)	(-2.332)	(3.375)	(1.176)	(-0.502)	(-1.174)	(-2.222)
2	4.778	4.734	2.568	0.634	-0.647	7.057	4.349	-2.180	-4.403	-14.635
	(2.823)	(2.151)	(0.979)	(0.223)	(-0.117)	(3.204)	(2.093)	(-0.796)	(-0.924)	(-2.019)
3	5.340	5.371	3.080	1.795	2.791	6.893	4.670	2.378	0.042	-9.121
	(2.251)	(2.902)	(1.733)	(0.623)	(0.665)	(3.502)	(2.257)	(1.025)	(0.010)	(-1.719)
4	4.628	4.601	6.029	5.865	8.428	6.649	5.681	7.063	5.243	1.473
	(2.575)	(2.313)	(3.676)	(2.425)	(2.060)	(3.435)	(3.401)	(3.187)	(1.455)	(0.291)
5	5.211	6.988	7.036	9.174	12.762	6.978	6.495	9.544	10.905	17.008
	(2.816)	(5.240)	(4.014)	(3.649)	(3.265)	(4.556)	(3.222)	(4.197)	(3.062)	(3.107)
5-1	0.668	4.660	9.694	16.802	25.968	0.300	2.072	11.067	17.644	36.149
	(0.365)	(1.838)	(3.769)	(4.733)	(4.400)	(0.158)	(0.660)	(3.346)	(2.859)	(3.814)

Table A4.13: Average excess return of the value weighted quintile portfolios sorted on EDC-ACY and other downside beta measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDC-ACY as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					**Estrada (2007) Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	4.357	4.885	3.103	4.016	4.838	6.433	5.613	3.339	1.857	-4.656
	(2.249)	(2.323)	(1.636)	(1.746)	(1.132)	(3.035)	(3.002)	(1.700)	(0.516)	(-0.791)
2	4.570	6.935	3.799	3.853	5.362	5.806	3.978	6.080	5.076	-1.497
	(2.872)	(2.736)	(2.115)	(1.453)	(1.315)	(3.710)	(2.025)	(2.484)	(1.401)	(-0.278)
3	3.350	4.494	6.374	5.666	8.260	5.997	5.528	6.457	4.802	7.291
	(1.698)	(2.574)	(3.273)	(2.044)	(2.265)	(3.216)	(2.739)	(2.625)	(1.477)	(1.495)
4	4.828	6.300	6.115	5.583	12.770	7.153	6.064	7.517	11.194	13.518
	(2.556)	(3.600)	(3.633)	(2.334)	(3.422)	(4.461)	(3.214)	(3.577)	(3.332)	(2.500)
5	4.779	7.073	6.381	9.627	10.873	7.368	7.194	10.811	10.050	14.973
	(2.612)	(5.145)	(3.555)	(4.548)	(2.511)	(5.696)	(3.314)	(5.192)	(2.730)	(2.677)
5-1	0.422	2.188	3.279	5.611	6.035	0.935	1.581	7.471	8.193	19.629
	(0.242)	(1.126)	(2.759)	(3.289)	(1.615)	(0.611)	(1.144)	(3.586)	(1.873)	(2.881)

Table A4.14: Average excess return of the value weighted quintile portfolios sorted on EDC-ES and other downside beta measures

This table shows the average yearly excess returns of the value weighted quintile portfolios sorted on Bawa and Lindenberg (1977) or Estrada (2007) Downside betas as the first criterion and then on EDC-ES as the second criterion. These quintiles are sorted using 1 year postformation risk measures, which are concurrent with the quintile return. The second row in each measure panel gives the value of the Newey-West t-statistics (in brackets) for the returns in the corresponding first row. The last two rows are for the average excess returns and t-statistics of the long-short strategy which longs quintile 5 and shorts quintile 1. The overall sample period is from January 1973 to December 2012.

	Bawa and Lindenberg (1977) Downside beta					**Estrada (2007) Downside beta**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	-1.243	-1.237	-3.834	-8.627	-13.250	4.849	-1.190	-4.555	-16.003	-23.139
	(-0.460)	(-0.490)	(-1.636)	(-2.601)	(-3.426)	(3.180)	(-0.569)	(-1.523)	(-4.583)	(-4.088)
2	1.908	0.784	0.229	-1.255	-3.026	3.509	-0.274	-4.248	-6.421	-13.769
	(0.985)	(0.369)	(0.121)	(-0.414)	(-0.640)	(1.633)	(-0.131)	(-1.714)	(-1.555)	(-1.894)
3	2.694	4.947	4.740	3.395	5.000	6.153	4.092	2.277	-1.193	-6.835
	(1.316)	(2.563)	(2.557)	(1.272)	(1.410)	(3.129)	(2.138)	(0.976)	(-0.263)	(-1.334)
4	6.894	6.539	6.885	7.842	11.875	6.574	6.769	7.209	6.347	8.614
	(3.001)	(3.355)	(3.908)	(3.721)	(2.859)	(3.482)	(3.642)	(3.501)	(1.992)	(1.713)
5	7.003	8.849	8.479	10.533	14.097	7.955	7.656	11.546	14.067	19.489
	(4.240)	(6.787)	(4.959)	(4.504)	(3.375)	(5.918)	(3.638)	(5.063)	(4.032)	(3.365)
5-1	8.245	10.086	12.313	19.161	27.348	3.107	8.846	16.101	30.070	42.628
	(3.126)	(4.802)	(6.232)	(7.814)	(6.427)	(2.663)	(4.787)	(5.502)	(7.380)	(8.224)

CHAPTER 5: THE IMPACT OF EXTREME DOWNSIDE RISK ON STOCK RETURNS: IMPLICATIONS FOR PORTFOLIO MANAGEMENT

5.1. Introduction

The non-trivial impact of extreme downside risk on asset returns, as demonstrated in many studies as well as in the previous two chapters, gives rise to the need to incorporate such type of risk into the portfolio management process of investors. The mean-variance optimisation of Markowitz (1952), while being the most popular framework in portfolio management, both theoretically and practically, only incorporates variance as the risk factor. The use of variance as a single risk objective potentially ignores information attributed to the tail. Importantly, as demonstrated in the previous two chapters, this variance-excessive information could significantly influence returns. Furthermore, it is the risk of suffering a loss that matters from the perspective of a practical portfolio manager rather than a simple dispersion risk. Therefore, numerous studies have proposed alternative portfolio optimisation frameworks that specifically account for downside risk, such as the mean-semivariance (Markowitz, 1959; Mao, 1970; Estrada, 2008; among others) and Safety-First criterion (Roy, 1952; Arzac and Bawa, 1977; Bawa, 1978; Jansen et al., 2000; among others).

Given the recent development of VaR as a measure for downside risk, as described in Chapter 1, the portfolio optimisation framework of mean and a VaR-related risk measure is receiving significant interest from researchers as well as practitioners. In fact, rather than using VaR as the risk measure in the optimisation, it has been demonstrated that ETL is more suitable for the problem. This is because VaR has some undesirable mathematical properties such as non-subadditivity and non-convexity (see, for example, Artzner, 1999; Rockafellar and Uryasev, 2000; Alexander, 2009). Consequently, mean-VaR optimisation is a non-convex programming problem which is mathematically intractable. On the other hand, ETL is a coherent risk measure and is therefore well-behave in an optimisation problem. Specifically, Rockafellar and Uryasev (2002) demonstrate that a general mean-ETL optimisation could reduce to a simple convex programming, helping to solve the optimisation problem for a large number of assets. The developments of Rockafellar and Uryasev's (2002) framework can be found in many subsequent studies, such as Krokhmal et al. (2002) with alternative constraints for the optimisation problem, Alexander et al. (2006) with an application for portfolios of derivatives, Quaranta and Zaffaroni (2008) with methods accounting for parameter uncertainty, and Kakouris and Rustem (2014) with the incorporation of copulas. Otherwise, different optimisation approaches for the mean-ETL problem could also be found in Shaw (2011) with a simple form optimisation problem when assets are multivariate Gaussian and multivariate Student-t distributed, or Abad and Iyengar (2014) with an alternative optimising algorithm.

Since mean-variance and mean-ELT focus on different types of risk measure, it is not surprising that they exhibit performances with distinguished features. Xiong and Idzorek (2011) use simulation to demonstrate how the non-normality features such as skewness and excess kurtosis lead to differences in asset allocations of mean-ETL from mean-variance. In fact, since these two optimisation methods aim to optimise different risk measures, their risk-return profile would not be comparable. Mean-variance would have a better variance risk profile but a worse downside risk profile than mean-ETL, which is confirmed in Roman et al. (2007). Thus, it is irrelevant to conclude about the superiority of one method over another and the suitability of a method should be considered according to the perspective of a specific investor.

However, from the finding in Chapter 4 regarding the impact of extreme downside risk on assets of different sizes, we conjecture about a case when a conclusive comparison could be obtained. Specifically, in Chapter 4, we show that extreme downside risk has a larger impact on small stocks than on large stocks. In fact, this is consistent with suggestions in Ang et al. (2006a) and the stylized fact that non-normality is more prominent in small assets. Therefore, it would be justified if a tail-risk focused method such as mean-ELT is consistently more effective than mean-variance when applied for small assets. Accordingly, in this Chapter, we will examine the size pattern of the comparative performance between mean-variance and mean-ELT. We expect the difference to be large and in the favour of mean-ELT when applied to small assets but to disappear in the case of large assets. An answer to this hypothesis is very important to practitioners since small stock investment is one of the main strategies for investment companies in the market, as shown in Chen et al. (2000) and Wermers (2000), among others. Furthermore, as investment in emerging market equity, where all firm sizes are generally small, is attracting a considerable amount of capital, recommendations regarding the choices of optimisation method for the small stock investment strategy are truly essential.

We choose to use the popular Fama and French's (1993) (hereafter FF) Size and Book-over-Market (hereafter BM) sorted portfolios to construct groups of assets with different sizes. This is because they are the most well-known groups of assets which are not only different in size, but also available with many years of data to support our data-intensive estimation framework. In the most basic investigation, we analyse the 25 FF Size-BM sorted portfolios where each size group of 5 sorted BM portfolios constitutes a tested stock universe. We apply mean-variance and mean-ELT for each of these universes and observe the pattern of their comparative performance across the universes. Interestingly, our size-pattern conjecture is confirmed as the difference is considerable for small stocks but almost disappears for large stocks. This result is robust even after we control for skewness as well as using a different set of investigated assets such as Size-Momentum sorted portfolios.

Additionally, given the distinguished risk measures, another major topic in this area of the literature is whether, and how, investors could effectively use some type of multiple-risk optimisation framework. In other words, is it possible to enhance performance by maximising returns while minimising multiple risk criteria? The seemingly straightforward solution to this question is to work directly on a multiple criteria optimisation problem, such as mean-variance-skewness (for example, Stone, 1973; Konno and Suzuki, 1995; Harvey et al., 2010; among others), mean-absolute deviation-skewness (Konno et al., 1993), and mean-variance-ELT (Roman et al., 2007, Gao et al., 2014). A common way to solve these multiple criteria optimisation problems is to maximise the expected returns or minimise one risk objective while constraining the others. For example, Konno and Suzuki (1995) maximise skewness and constrain mean and variance, while Roman et al. (2007) minimise variance and constrain mean and ELT.

However, as shown in Roman et al*.* (2007), the multiple criteria optimisation problem is normally Pareto efficient and one needs to scarify one (some) criterion (criteria) in order to improve another. In the empirical investigation, they show that mean-variance and mean-ELT still have the best variance and tail risk profile, respectively, among all the solutions of the mean-variance-ETL framework. However, they also have the worse tail risk and variance profile, respectively. Thus, they are nested in the mean-variance-ELT framework where other solutions have the risk profiles in between the two extremes. Accordingly, generally speaking, the multiple objective portfolio optimisation problem is just as good as the simple mean-variance or mean-ELT in proposing the optimal asset allocation. However, since the set of optimal solutions has expanded, the multiple objective approach does make an important contribution to the investment process. Specifically, investors who evaluate risk criteria differently in their utility functions could find their own better allocation in an expanded surface of optimal portfolios.

On the other hand, another promising solution for the question is to use a switching mechanism between optimisation problems. It is arguable that each portfolio optimisation scheme is suitable for a specific investment context. Therefore, instead of pooling all the risk criteria into a single complex problem that is applied consistently within the whole investment horizon, it might be more efficient to apply a single risk optimisation problem in each corresponding suitable context and switch between problems when the context changes. Theoretically, ignoring transaction cost, the switching method should be better than each individual optimisation method. This is because each of the constituent methods, such as mean-variance and mean-ELT, has a distinguished focus. While mean-variance focuses on the general dispersion risk, mean-ELT focuses on tail risk. Therefore, when the market is in distress state with a high extreme downside risk, mean-ELT is expected to perform better, while mean-variance is better in normal state. As a consequence, a switching method dependent on market states is expected to perform even better than both of the single methods.

In fact, the idea of using a switching mechanism to enhance portfolio performance is not new in the literature. Many studies, for example Ang and Bekaert (2004), Frauendorfer et al*.* (2007), Guidolin and Timmermann (2008), among others, have documented a great deal of supporting evidence. However, these research studies only incorporate Markov switching into a single optimisation method. They either apply Markov switching on a general market index to identify the switching indicator and state-dependent inputs for some standard optimisation problems such as mean-variance and mean-ETL, or propose and solve a comprehensive optimisation problem where Markov switching is applied directly to the constituent asset returns. Examples of the first type are Ang and Bekaert (2004) and Wang et al. (2012), while examples of the later one can be found in Frauendorfer et al. (2007) and Guidolin and Timmermann (2008). Thus, the literature lacks evidence regarding how portfolio optimisation performs when switching between optimisation methods. One of the very few studies investigating this question is Cain and Zurbruegg (2010), where evidence in favour of switching between mean-variance and mean-ELT is reported. However, their evidence is only strong and consistent in the in-sample investigation, while the out-of-sample results are mixed. Moreover, the switching mechanism applied in this study is simply statistical rather than related to a certain economic rationale.

Therefore, the second research question which we analyse in this chapter is whether a switching mechanism between the mean-variance method and the mean-ELT method in different market regimes would enhance performance. We apply the popular approach in the literature to determine market regimes by fitting a Markov switching model for market excess returns. We then propose a switching strategy to use mean-variance in calm state and mean-ELT in turbulent state and apply the strategy for all size groups of stocks. However, even under the assumption of perfect regime awareness, the switching method could not outperform the better method of the constituent ones. In other words, the switching method averages the performances of the two optimisation methods rather than enhancing them. Moreover, this result is still robust even when the switching indicator is statistically estimated directly from the actual outperforming time of each method using a large set of controlling variables. We demonstrate that the reason for this failure is the invalidity of the argument regarding the suitability of mean-ELT and mean-variance with respect to distress and normal periods. Thus, evidence in this chapter suggests the selection of portfolio optimisation method should be based on the specific risk perspective of each investor rather than being market-context dependent. This recommendation should be valid given that the general market information and investment strategy is applied consistently over a long horizon.

The remainder of this chapter is structured as follows. Section 5.2 summarises the analytical framework used in the chapter, including the portfolio optimisation methods and the multivariate conditional location-scale model to provide the input of the optimisation problem. Sections 5.3 and 5.4 provide the empirical results for the two research questions regarding the size pattern of the comparative performance between mean-variance and mean-ELT, and whether the switching mechanism can enhance the performance of single optimisation methods. Finally, section 5.5 concludes the chapter.

5.2. Portfolio optimisation framework

In our study, we investigate the performance of a portfolio with monthly rebalancing using the canonical mean-variance optimisation method or the mean-ETL optimisation method. We summarise the corresponding optimization problems as follows.

5.2.1. Mean-variance optimisation problem

The classical exercise to maximise portfolio returns while minimising its variance can be done by maximising the objective function of portfolio returns with a penalty for variance. Specifically,

$$min_{w \in W} \quad \lambda w' \Sigma w - w' \mu \tag{5.1}$$

$$subject\ to \quad \sum_{i=1}^{N} w_i = 1$$

$$0 \leq w_i \leq 1 \ \ for\ i = 1,2,\dots,N$$

where $w = (w_1, w_2, \dots, w_N)'$ is the weights of N assets in the portfolio; W is the set of all possible choices for w; $\mu = (\mu_1, \mu, \dots, \mu_N)'$ is the expected value and Σ is the variance-covariance matrix of these assets' returns; λ is the risk aversion coefficient. A more risk averse investor will have a higher λ, which is translated into the optimisation problem as a higher penalty for risk. In this study, we restrict short-selling for the sake of simplicity and faster convergence of the optimising algorithm. The mean-variance efficient frontier could be obtained by varying the risk aversion coefficient. This is the same efficient frontier that could be obtained if one specifies a slightly different version of the problem by minimising variance while constraining returns.

5.2.2. Mean-ETL optimisation problem

We follow the framework of Rockafellar and Uryasev (2002) to maximise portfolio returns while minimising the ETL. Specifically, they demonstrate that the ETL of a portfolio could be obtained by minimising the following function with respect to VaR:

$$F_{\alpha}(w, VaR) = VaR + \frac{1}{\alpha}\int_{r\in R^N}[-w'r - VaR]^{+}p(r)dr \qquad (5.2)$$

Where VaR is the corresponding Value-at-Risk of ETL; α is the significant level of the VaR, ETL; $w = (w_1, w_2, \ldots, w_N)'$ and $r = (r, r_2, \ldots, r)'$ are the weights and returns of N assets in the portfolio; these returns are random variables following some multivariate distribution with probability density function $p(\cdot)$; and $[t]^{+}$ is a function to take the value of t when $t > 0$ and 0 otherwise, $[t]^{+} = max\{t, 0\}$. Thus, Rockafeller and Uryasev (2002) demonstrate that

$$ETL_{\alpha}(w) = min_{VaR\in R}\, F_{\alpha}(w, VaR) \qquad (5.3)$$

It should be noted that VaR and ETL in this framework are defined as the value of loss, which is equal -1 times returns. This is to make them a true risk measure, which we would like to minimise in the optimisation framework. Rockafeller and Uryasev (2000, 2002) further demonstrate that minimising ETL with respect to w is equivalent to minimising $F_{\alpha}(w, VaR)$ over all (w, VaR):

$$min_{w\in W}\, ETL_{\alpha}(w) = min_{(w,VaR)\in W\times R}\, F_{\alpha}(w, VaR) \qquad (5.4)$$

Moreover, they prove that $F_{\alpha}(w, VaR)$ is convex with respect to (w, VaR) and $ETL_{\alpha}(w)$ is convex with respect to w. In a general setting, the set W is also a convex set. Therefore, minimising ETL is a convex programming problem. Thus, we could combine the risk minimisation objective with maximising returns to form the full mean-ETL optimisation problem as follows:

$$min_{(w,VaR)\in W\times R} \quad \lambda\left(VaR + \frac{1}{\alpha}\int_{r\in R^N}[-w'r - VaR]^{+}p(r)dr\right) - w'\mu, \quad (5.5)$$

$$subject\ to \qquad \textstyle\sum_{i=1}^{N} w_i = 1,$$

$$0 \leq w_i \leq 1 \;\; for\; i = 1,2, \dots, N.$$

Furthermore, Rockafeller and Uryasev (2002) suggest an approach to reduce this problem to a linear programming problem using discrete scenarios for constituent asset returns:

$$min_{(w,VaR)\in W\times R} \quad \lambda\left(VaR + \frac{1}{\alpha}\sum_{j=1}^{J}\pi_j z_j\right) - w'\mu, \qquad (5.6)$$

$$subject\ to \qquad \sum_{i=1}^{N} w_i = 1,$$

$$0 \leq w_i \leq 1 \;\; for\; i = 1,2, \dots, N$$

$$z_j \geq -w'r_j - VaR\; for\; j = 1,2, \dots, J$$

$$z_j \geq 0\; for\; j = 1,2, \dots, J$$

where r_j is the vector of constituent stock returns in scenario j; J is the set of all investigated scenarios, which could be generated by simulation; π_j is the probability of scenario j to occur; z_j are the dummy variables to replace the unsmooth $[\cdot]^+$ function. By varying risk aversion coefficient λ, we could obtain the efficient mean-ETL frontier. As demonstrated by Krokhmal et al. (2002), the same frontier could be obtained by using a different format of this problem, such as minimising ETL when the portfolio return is constrained, or maximising portfolio returns with a constrained ETL level.

5.2.3. Input data for the optimisation problem

In order to obtain the optimal allocation for next month, we need to provide the optimisation programmes with input data including expected returns, variance-covariance matrix, and return scenarios of constituent stocks. Since our portfolio is rebalanced monthly, we need those data for monthly returns. Therefore, we fit the multivariate conditional location-scale model of VAR-MGARCH on the monthly returns of our assets at the beginning of every month to estimate the conditional expected returns and the variance-covariance matrix of the coming month. More particularly, since our portfolio has 5 assets (5 BM portfolios within a size group) and the history of monthly data is narrow, we chose to use VAR(1) for the location model and Scalar BEKK GARCH(1,1) (Engle and Kroner, 1995) for the conditional volatility model. This helps our framework to limit the number of parameters to be estimated. Specifically, our estimated model is defined as:

$$r_t = A_0 + A_1 r_{t-1} + \varepsilon_t = A_0 + A_1 r_{t-1} + \Sigma_t^{1/2} e_t \quad e_t \sim multivariate\ N(0{,}1) \qquad (5.7)$$

$$\Sigma_t = B_0 B_0' + b_1^2 \varepsilon_{t-1} \varepsilon_{t-1}' + b_2^2 \Sigma_{t-1}$$

where $r_t$is the vector of returns of constituent assets at time t; ε_t and e_t are residual terms and standardised residual terms at time t, respectively; Σ_t is the conditional variance-covariance matrix of the constituent assets at time t; A_0 is $N \times 1$ vector of constant terms of the mean equation, with N is the number of assets; A_1 is $N \times N$ matrix of autoregressive coefficients; B_0 is $N \times N$ lower triangular matrix; b_1^2 and b_2^2 are scalars. To reduce the number of estimated parameters, we restrict A_1 to being a diagonal matrix so that the return of an asset is only affected by its own lagged return. Accordingly, we have 27 parameters to be estimated in the model.

At the beginning of each month, we use the monthly returns of assets over the last 30 years (360 observations) to estimate the VAR(1)-Scalar BEKK GARCH(1,1) model and to predict the expected value and variance-covariance matrix of the next month accordingly. These expected mean and variance-covariance matrices will be used as input for the mean-variance optimisation problem to obtain the corresponding optimal weights for the portfolio in the coming month. Regarding the mean-ETL problem, since we need return scenarios as input, we simulate 10,000 independent scenarios of asset returns using multivariate Normal distribution with the mean and covariance matrix predicted from the VAR(1)-Scalar BEKK GARCH(1,1) model.

5.3. The effect on portfolio optimisation of the size pattern in extreme downside risk impact

In order to compare the performance of mean-variance and mean-ETL across different size groups, we utilise the 5x5 FF Size-BM sorted portfolios available from Kenneth R. French’s website. We treat each size group of 5 BM sorted portfolios as a stock universe and apply the portfolio optimisation methods described in the previous section to this universe. After obtaining the optimal weights according to each method at the beginning of a month, we use the realised returns of these sorted portfolios during that month to calculate the realised returns of the corresponding optimal portfolio.

The performance of this optimal portfolio over the entire investigation period is summarised in Table 5.1. In this table, we report the final value of £1 investment at the beginning of 1965 obtained by the optimal portfolio at the end of 2013, and the Sharpe ratio of the optimal portfolio against the two risk measures of variance and ETL (using the average monthly T-bill over the investment horizon). Each column of the table shows the results corresponding to the optimal portfolio of a size group. We report the results of different risk aversion coefficients used in the optimisation problems, ranging from 1 to 10. The difference between the performances of mean-variance and mean-ETL methods are calculated as the ratio of mean-ETL over mean-variance for the final portfolio value to account for the fact that investments in small assets yield higher returns, and as mean-ETL minus mean-variance for Sharpe ratios since these ratios have taken risk-return trade-off into account. Additionally, it should be noted that these performances are calculated ignoring transaction costs. In fact, since we are interested in a theoretical question about the impact of extreme downside risk on portfolio optimisation, the investigated portfolios do not need to be investible and transaction costs could be ignored. Indeed, incorporating transaction costs could distort the investigated relationship.

[Table 5.1]

From this table, it is clear that our conjecture regarding the effectiveness of the mean-ETL method for small assets is confirmed. Specifically, the difference between mean-ETL and mean-variance tends to be higher for small size portfolios as compared to large size portfolios. While these two optimisation methods perform almost the same when applied to the fifth size group (large size), mean-ETL perform significantly better than mean-variance in the case of the first size group (small size). Additionally, we could point out some interesting patterns. Firstly, our results support common knowledge in investment practice regarding the relationship between riskier investment and higher return. Particularly, small stock investment yields better results and investors with lower risk aversion (lower lambda) have better performances. Moreover, we discover an interesting pattern that mean-ETL seems to consistently outperform mean-variance in all cases. As we will show later, this pattern is robust for other investigated assets.

The results of Table 5.1 could be subjected to some specific features of the investigated assets. Most obviously, one might argue that the reason that the difference is significantly higher in small size group is mainly due to the skewness of the examined portfolios. In fact, the skewness of small size portfolios is considerably higher than other portfolios. This can be seen in Figure 5.1 where we show the skewness of the returns of all 25 FF Size-BM portfolios in our sample (January 1965-December 2013). Therefore, portfolio optimisation schemes which focus on the downside of the distribution would be better than those that evaluate risk from the dispersion perspective. This is because the latter would limit potential large gains while limiting dispersion.

In order to check whether our inferences are simply due to skewness, we carry out investigations for groups of assets with similar skewness levels. Specifically, we divide 100 Fama-French 10 x 10 Size-BM sorted portfolios, which are also available from Kenneth R. French's website, into 4 groups of 25 portfolios from small size to large size (the first group contains the 25 smallest portfolios, the second group contains the next 25 smallest portfolios, and so on). We then pick from within each group 5 portfolios so that the skewness pattern of the 5 portfolios of a group is similar across all size groups (this practice is not possible if we allocate 100 portfolios into 5 groups of 20 because the ranges of skewness in some groups are not comparable to others). Figure 5.2 shows the skewness levels of all 100 Size-BM portfolios across 4 groups. In order to choose the best combination of portfolios whose skewness patterns across size groups are closest to each other, we use a programming algorithm to choose 5 portfolios in each group so that the pair-wise differences between the skewness of portfolios across groups are smallest. Table 5.2 shows the portfolio selection obtained from this algorithm. It is clear that the skewness patterns of portfolios across size groups are almost identical.

Table 5.3 shows the result of the investigation regarding the difference between optimisation methods using these chosen portfolios. It is clear that the differences between the performance of mean-variance and mean-ETL methods are still exposed to the size effect, where the difference of group 1 is considerably higher than that of group 5. In other words, the size pattern of extreme downside risk effect still significantly affects portfolio optimisation practice, even after controlling for skewness. This result further confirms our finding in Chapter 4 that extreme downside risk contains more information in excess of skewness risk.

[Figure 5.1]

[Figure 5.2]

[Table 5.2]

[Table 5.3]

We further check the robustness of this investigation using a different asset universe. Specifically, we carry out the same analysis for the 5x5 Size-Momentum sorted portfolios and the 10x10 Size-Momentum sorted portfolios. The 5x5 portfolios are available from Kenneth R. French's website; while the 10x10 portfolios are constructed using all stocks from the NYSE, AMEX, NASDAQ markets following the same method used for the 5x5 portfolios. Specifically, at the beginning of month t, stocks are sorted into 10x10 portfolios using their market capitalisation available at that time and their momentum, which is defined as the cumulative returns from month $t-12$ to month $t-2$. The stock returns of month t will be used to calculate the value weighted returns of these sorted portfolios in month t. Similar to the investigation of Size-BM sorted portfolios, we use Table 5.4 for the results of optimal portfolios constructed from 5x5 Size-Momentum portfolios, Figure 5.3 for the skewness of 10x10 Size-Momentum portfolios allocated into 4 size groups, Table 5.5 for the skewness of the chosen portfolios from 10x10 Size-Momentum portfolios, and Table 5.6 for the results of the optimal portfolios constructed from these chosen portfolios. These reported results are similar to the previous results of the Size-BM sorted portfolios. Thus, our conjecture regarding the influence on portfolio optimisation of the size pattern in tail risk impact is confirmed.

[Figure 5.3]

[Table 5.4]

[Table 5.5]

[Table 5.6]

5.4. Does switching between portfolio optimisation methods enhance performance?

In examining this hypothesis, we first analyse the monthly market excess return from January 1965 - December 2013 under Markov switching mechanism to determine the timing of calm and turbulent periods. Data on market excess return is obtained from Kenneth R. French's online database. The use of Markov switching estimation to signal the switch in portfolio optimisation is widely used in related literature (see, for example, Ang and Bekaert, 2004; Wang et al., 2012; Seidl, 2012; among others). We choose the most common setting in Markov switching analysis with the first order Markov process and two market states. Specifically, the market excess return is modelled as follows:

$$r_{mt} = \mu_{S_t} + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \sigma_{S_t}^2\right) \qquad (5.8)$$

where r_{mt} is monthly market excess return; μ_{S_t} and $\sigma_{S_t}^2$ are state-dependent mean and variance of market excess returns; and $S_t = \{1,2\}$ are the states of the market. Figure 5.4 shows the estimated timing of market states in the examined sample from January 1965 to December 2013. It is clear that the turbulent state timing covers all the major distress periods of the market (for example, the 1973-1974 oil crisis, the October 1987 crash, the dot-com burst, the last financial crisis, among others).

[Figure 5.4]

We utilise this Markov switching estimated timing to be the indicator of the switch between mean-variance and mean-ETL. At the beginning of a month, if the realised market state of last month is state 1, mean-variance will be used to obtain the optimal weights for the coming month. Otherwise, mean-ETL will be used. Thus, we assume investors have perfect knowledge about the realised state of the market. As we have argued, the focus of mean-ETL is to minimise extreme downside risk, which is suitable for turbulent period context. Therefore, it is expected that mean-ETL is more suitable for turbulent state while mean-variance could work well generally in normal state. Thus, one could conjecture that the switching method can outperform both single optimisation methods.

Table 5.7 shows the performance of the switching method against the single mean-variance and mean-ETL methods. From this table, we can see that the switching method, in general, does not improve performance compared to its constituent methods. Indeed, the performance of the switching method tends to lie between the performance of mean-variance and mean-ETL. It could only outperform both single methods in 4 out of 20 cases. Tables 5.8 to 5.10 show the results of similar investigations for the 10x10 Size-BM similar-skewness-selected portfolios, 5x5 Size-Momentum portfolios, and 10x10 Size-Momentum similar-skewness-selected portfolios. The results of these tables are similar to those of Table 5.7, further confirming the inability of the switching method to consistently outperform the single method approach.

[Table 5.7]

[Table 5.8]

[Table 5.9]

[Table 5.10]

The reason for the inability of the switching method to improve on the single method lies in its rationale. The rationale of the switching method is that mean-variance, as the most established method in practice as well as in literature, should work well in general while mean-ETL can performs specifically better in turbulent periods. However, our investigation shows that this is a wrong conjecture. Table 5.11 shows the percentage of months where the mean-ETL method is better than the mean-variance method in each state of the market, across all portfolio groups constructed from the 5x5 Size-BM sorted portfolios. According to this table, both mean-variance and mean-ETL have times to perform well under both states of the market. In other words, one cannot say that one method is better than another in a specific state.

Table 5.15 provides further evidence in rejecting the state-suitability of optimisation methods. It shows the results of probit regression of the binary signal of better method, which is equal to 1 if mean-ETL is better and 0 otherwise, on the regime indicator over time, controlling for other variables of contemporaneous time, including the monthly market returns, market conditional monthly volatility, market daily volatility, skewness and kurtosis based on the market daily returns of the month, change in credit risk premium, change in term risk premium, de-trended risk-free rate, dividend yield, industrial production growth, monetary supply growth, change in inflation rate and change in oil price. The included macro variables are the ones used in Chapter 3 of this thesis. According to this table, our regime indicator generated by Markov switching analysis could not, in general, significantly explain the better method over time. It even tends to point in the opposite direction, implying mean-ETL is better in calm states. Furthermore, the goodness of fit of these regressions is just slightly better than those of the pure mean-ETL method (the second row of the Goodness of fit). The goodness of fit of the pure mean-ETL method is simply how many percent of time that a naïve choice of using mean-ETL in all periods correctly specifies the better. This is essentially the percentage of time over the entire investigated period that mean-ETL is better than mean-variance. Thus, this implies the difficulty of explaining the best method at any point of time. Similar results for investigations using 10x10 Size-BM similar-skewness portfolios, 5x5 Size-Momentum portfolios, and 10x10 Size-Momentum similar-skewness portfolios are reported in Tables 5.12 to 5.14 and 5.16 to 5.18, further confirming our inferences.

[Table 5.11]

[Table 5.12]

[Table 5.13]

[Table 5.14]

[Table 5.15]

[Table 5.16]

[Table 5.17]

[Table 5.18]

Finally, we investigate whether it is possible to construct a switching method that could beat the single optimisation methods by using the predicted value of the predictive probit regression as the switch indicator. Specifically, we regress the realised indicator of the better method in the next period on explanatory variables in this period. We use the same set of explanatory variables as in the previous investigation. Thus, this new switch indicator takes into account a large amount of information on the explanatory variables rather than simply the Markov switching state indicator. Further, it is estimated using the ex-post observations of the best method over time (the dependent variable). In other words, it is an in-sample switching indicator. Thus, it could be considered as the most ideal switching indicator that an investor can have.

Tables 5.19 to 5.22 show the results of this investigation. Accordingly, since our switch indicator has improved, the performance of the switching method has improved compared to the case of the pure Markov-switching regime indicator. The switching method can beat both of the single methods in nearly 70 percent of the investigated cases. However, the improvement is very marginal. The relative improvement on the better method between the two constituent methods is over 1 percent in only a third of the successful cases. The relative improvement is over 3 percent in only 1 percent of the successful cases. There is no case when the relative improvement is higher than 5%. These calculations are based on Sharpe ratios as they account for both returns and risk. Thus, given that the switching indicator used is ideal and that using the switching method might be cumbersome in practice, switching between mean-variance and mean-ETL is not justified.

[Table 19]

[Table 20]

[Table 21]

[Table 22]

5.5. Conclusion

In this chapter, we report supporting evidence regarding the influence on portfolio optimisation of the size pattern of the tail risk impact. We show that tail-risk-focused optimisation methods such as mean-ETL are more effective when applied to small stock investments. Moreover, our results also support the use of mean-ETL consistently in optimising portfolio performance due to two main reasons. Firstly, we demonstrate the inability to use some sort of switching mechanism between the traditional mean-variance and the tail-risk-focused mean-ETL to enhance performance over the corresponding single methods.

This inability is caused by the fact that there is no consistent way to predict the better method within a specific investment context. Secondly, in choosing between mean-variance and mean-ETL, our results show the consistently better performance of the latter, at least in the context of our analysis. Moreover, since the practical risk for a risk-averse investor is downside risk rather than dispersion risk, the choice of mean-ETL is even more justifiable.

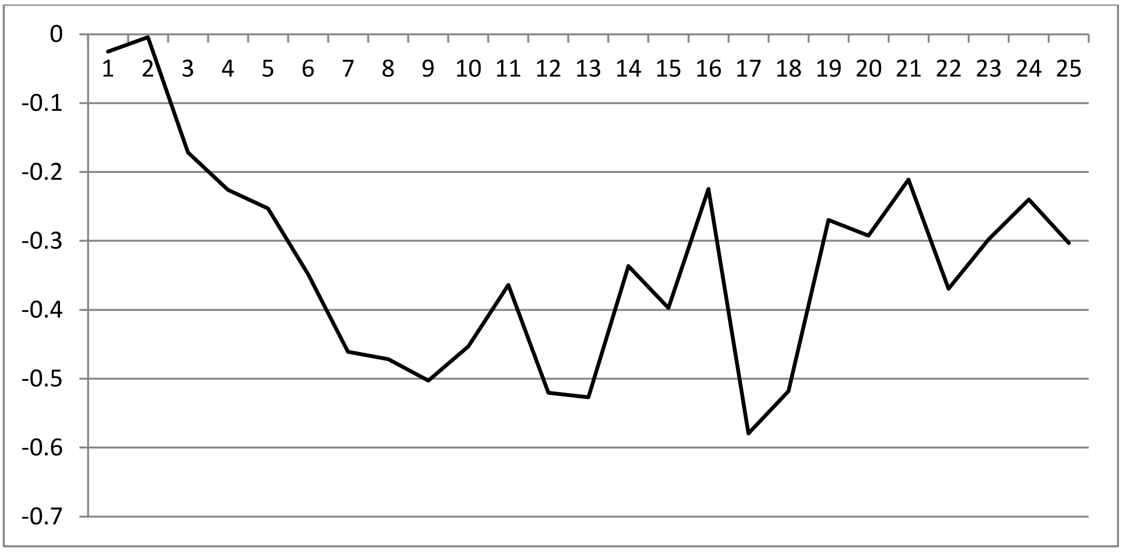


Figure 5.1: Skewness across 5x5 Size-BM sorted portfolios. This figure shows the skewness levels of 25 FF Size-BM sorted portfolios, calculated from their monthly returns from January 1965 to December 2013.

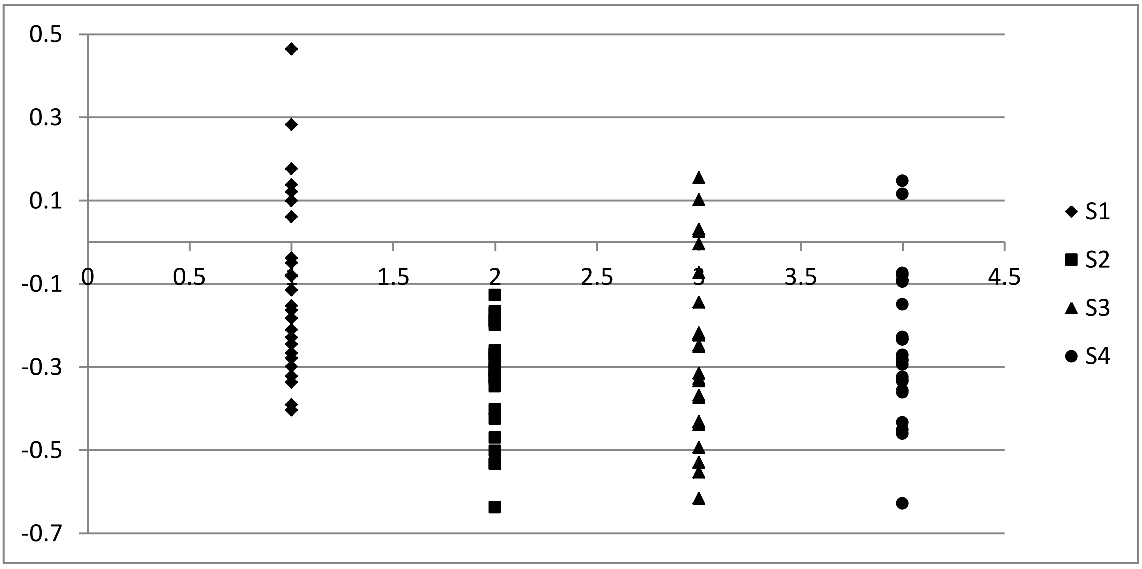


Figure 5.2: Skewness across 10x10 Size-BM sorted portfolios. This figure shows the skewness levels of 100 FF Size-BM sorted portfolios, calculated from their monthly returns from January 1965 to December 2013. These portfolios are divided into 4 size groups: S1 contains 25 smallest portfolios, S4 contains 25 largest portfolios.

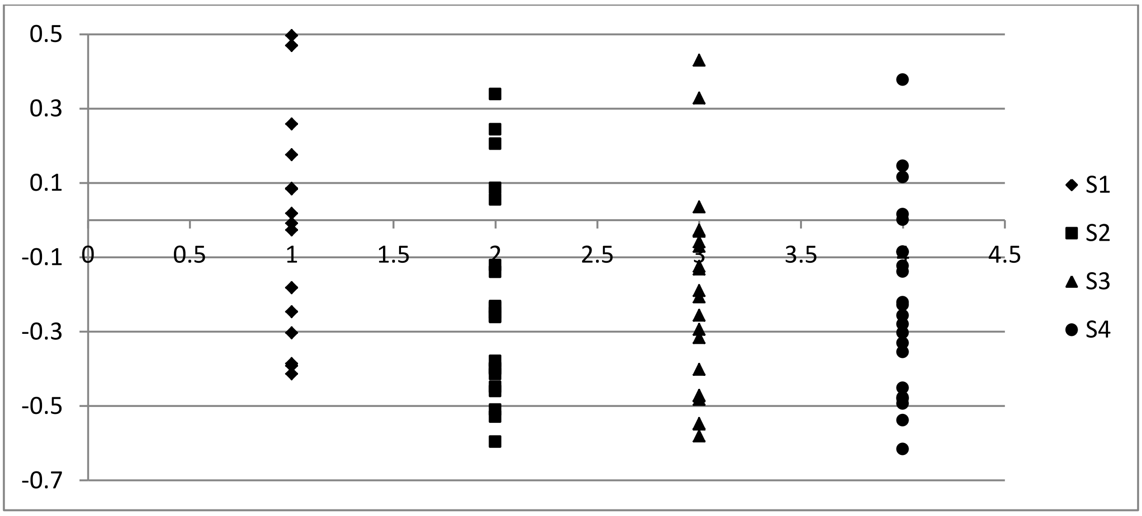


Figure 5.3: Skewness across 10x10 Size-Momentum sorted portfolios. This figure shows the skewness levels of 100 Size-Momentum sorted portfolios, calculated from their monthly returns from January 1965 to December 2013. These portfolios are divided into 4 size groups: S1 contains 25 smallest portfolios, S4 contains 25 largest portfolios.

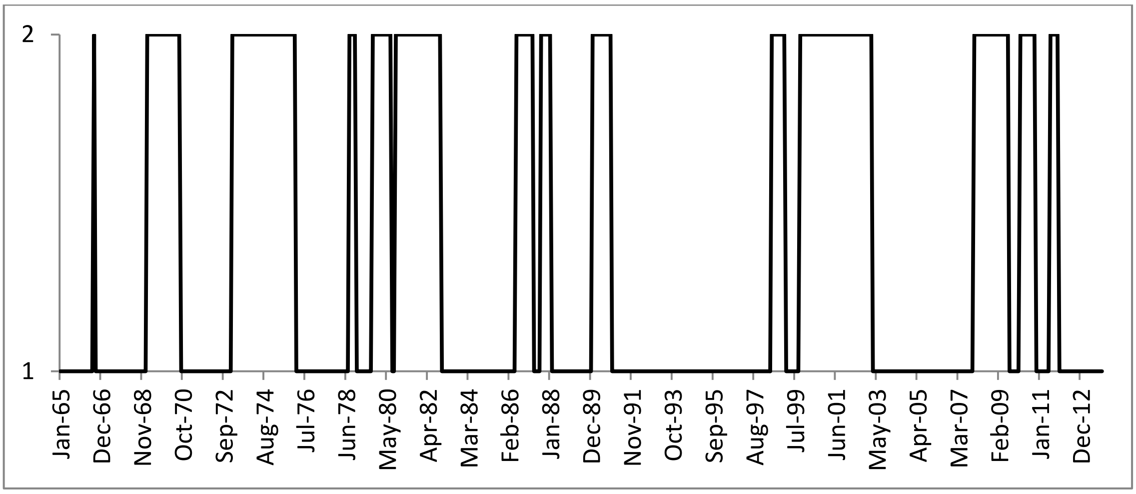


Figure 5.4: State timing under Markov switching analysis. This figure shows the timing of the market states estimated by Markov switchcing model for monthly market excess returns from January 1965 to December 2013. 1 is calm state and 2 is turbulent state.

Table 5.1: Performance of the mean-variance and the mean-ETL across size groups: 5x5 Size-BM sorted portfolios

This table shows the performance of the optimal portfolio constructed using the mean-variance and the mean-ETL optimization method across 5 size groups of 5x5 FF Size-BM sorted portfolios, using different risk aversion coefficients (lambdas), from January 1965 to December 2013. S1 to S5 are the size groups.

lambda		S1	S2	S3	S4	S5
		Final value of the portfolio				
1	Mean-variance	1627.312	1282.684	487.821	411.905	107.209
	Mean-ETL	2474.347	1354.038	592.932	489.231	115.727
	Difference	1.521	1.056	1.215	1.188	1.079
2.5	Mean-variance	1577.383	1272.780	479.430	404.572	107.599
	Mean-ETL	1883.250	1320.914	552.998	471.329	111.192
	Difference	1.194	1.038	1.153	1.165	1.033
5	Mean-variance	1560.913	1268.241	476.502	402.186	107.712
	Mean-ETL	1815.950	1302.836	539.671	467.095	109.661
	Difference	1.163	1.027	1.133	1.161	1.018
10	Mean-variance	1552.709	1265.890	475.025	400.955	107.768
	Mean-ETL	1789.086	1291.804	531.559	465.850	108.008
	Difference	1.152	1.020	1.119	1.162	1.002
		Sharpe ratio (based on variance)				
1	Mean-variance	0.177	0.178	0.155	0.151	0.113
	Mean-ETL	0.189	0.179	0.162	0.159	0.116
	Difference	0.012	0.001	0.007	0.008	0.003
2.5	Mean-variance	0.176	0.178	0.154	0.151	0.113
	Mean-ETL	0.181	0.178	0.159	0.157	0.115
	Difference	0.006	0.001	0.005	0.007	0.002
5	Mean-variance	0.175	0.177	0.154	0.150	0.113
	Mean-ETL	0.180	0.178	0.159	0.157	0.114
	Difference	0.005	0.001	0.005	0.006	0.001
10	Mean-variance	0.175	0.177	0.154	0.150	0.113
	Mean-ETL	0.180	0.178	0.158	0.157	0.114
	Difference	0.005	0.000	0.004	0.006	0.000
		Sharpe ratio (based on ETL)				
1	Mean-variance	0.082	0.081	0.070	0.069	0.053
	Mean-ETL	0.088	0.082	0.073	0.074	0.054
	Difference	0.006	0.001	0.003	0.005	0.002
2.5	Mean-variance	0.082	0.081	0.069	0.069	0.053
	Mean-ETL	0.085	0.081	0.072	0.073	0.054
	Difference	0.003	0.001	0.002	0.004	0.001
5	Mean-variance	0.082	0.081	0.069	0.068	0.053
	Mean-ETL	0.084	0.081	0.071	0.073	0.053
	Difference	0.003	0.001	0.002	0.004	0.001
10	Mean-variance	0.081	0.080	0.069	0.068	0.053
	Mean-ETL	0.084	0.081	0.071	0.072	0.053
	Difference	0.003	0.001	0.002	0.004	0.000

Table 5.2: Skewness of chosen portfolios across size groups of 10x10 Size-BM sorted portfolio

This Table shows the skewness levels of the chosen portfolios across 4 size groups constructed from 100 FF Size-BM sorted portfolios. Each row shows the skewness of the portfolios in a size group. The last row shows the standard deviation between the portfolios of the same skewness-order across groups.

	Skewness				
S1	-0.336	-0.322	-0.299	-0.266	-0.152
S2	-0.330	-0.323	-0.301	-0.268	-0.166
S3	-0.333	-0.328	-0.315	-0.251	-0.144
S4	-0.334	-0.324	-0.294	-0.271	-0.149
Standard deviation between skewness	0.002	0.002	0.008	0.008	0.008

Table 5.3: Performance of the mean-variance and the mean-ETL across size groups: similar-skewness portfolios from 10x10 Size-BM sorted portfolios

This table shows the performance of the optimal portfolio constructed using the mean-variance and the mean-ETL optimization method across 4 size groups of similar-skewness portfolios chosen from 10x10 FF Size-BM sorted portfolios, using different risk aversion coefficients (lambdas), from January 1965 to December 2013. S1 to S4 are the size groups.

lambda		S1	S2	S3	S4
	Final value of the portfolio				
	Mean-variance	1061.727	1170.487	193.264	159.057
1	Mean-ETL	3749.307	1244.711	267.332	183.201
	Difference	3.531	1.063	1.383	1.152
	Mean-variance	959.002	1154.250	188.431	156.745
2.5	Mean-ETL	2661.099	1236.873	245.045	181.296
	Difference	2.775	1.072	1.300	1.157
	Mean-variance	927.109	1149.170	186.932	156.159
5	Mean-ETL	2332.818	1226.153	237.474	180.001
	Difference	2.516	1.067	1.270	1.153
	Mean-variance	911.499	1146.674	186.142	155.903
10	Mean-ETL	2150.707	1227.298	234.490	179.863
	Difference	2.360	1.070	1.260	1.154
	Sharpe ratio (based on variance)				
	Mean-variance	0.164	0.176	0.121	0.129
1	Mean-ETL	0.196	0.174	0.133	0.134
	Difference	0.032	-0.001	0.012	0.005
	Mean-variance	0.161	0.175	0.120	0.128
2.5	Mean-ETL	0.188	0.175	0.130	0.134
	Difference	0.027	0.000	0.009	0.006
	Mean-variance	0.160	0.175	0.120	0.128
5	Mean-ETL	0.185	0.176	0.128	0.134
	Difference	0.025	0.001	0.008	0.006
	Mean-variance	0.160	0.175	0.120	0.128
10	Mean-ETL	0.183	0.176	0.128	0.134
	Difference	0.023	0.001	0.008	0.006
	Sharpe ratio (based on ETL)				
	Mean-variance	0.076	0.080	0.056	0.063
1	Mean-ETL	0.093	0.079	0.063	0.064
	Difference	0.017	-0.001	0.007	0.002
	Mean-variance	0.074	0.080	0.055	0.062
2.5	Mean-ETL	0.088	0.079	0.061	0.065
	Difference	0.014	0.000	0.005	0.002
	Mean-variance	0.074	0.080	0.055	0.062
5	Mean-ETL	0.087	0.080	0.060	0.065
	Difference	0.013	0.000	0.005	0.002
	Mean-variance	0.074	0.080	0.055	0.062
10	Mean-ETL	0.086	0.080	0.060	0.065
	Difference	0.012	0.000	0.005	0.003

Table 5.4: Performance of the mean-variance and the mean-ETL across size groups: 5x5 Size-Momentum sorted portfolios

This table shows the performance of the optimal portfolio constructed using the mean-variance and the mean-ETL optimization method across 5 size groups of 5x5 Size-Momentum sorted portfolios, using different risk aversion coefficients (lambdas), from January 1965 to December 2013. S1 to S5 are the size groups.

lambda		S1	S2	S3	S4	S5
		Final value of the portfolio				
1	Mean-variance	1087.336	587.864	313.579	277.595	76.791
	Mean-ETL	1319.995	887.540	447.661	372.834	84.913
	Difference	1.214	1.510	1.428	1.343	1.106
2.5	Mean-variance	1062.417	575.825	307.671	274.194	76.111
	Mean-ETL	1214.873	773.743	389.771	331.787	81.514
	Difference	1.143	1.344	1.267	1.210	1.071
5	Mean-variance	1053.287	572.288	305.582	273.156	75.923
	Mean-ETL	1195.906	732.939	373.290	318.777	80.340
	Difference	1.135	1.281	1.222	1.167	1.058
10	Mean-variance	1048.696	570.696	304.539	272.600	75.833
	Mean-ETL	1184.905	713.387	365.550	312.422	79.762
	Difference	1.130	1.250	1.200	1.146	1.052
		Sharpe ratio (based on variance)				
1	Mean-variance	0.172	0.154	0.140	0.138	0.099
	Mean-ETL	0.178	0.167	0.152	0.148	0.103
	Difference	0.006	0.013	0.012	0.011	0.004
2.5	Mean-variance	0.171	0.153	0.139	0.137	0.098
	Mean-ETL	0.176	0.163	0.148	0.144	0.101
	Difference	0.004	0.010	0.008	0.007	0.003
5	Mean-variance	0.171	0.153	0.139	0.137	0.098
	Mean-ETL	0.175	0.161	0.146	0.143	0.101
	Difference	0.004	0.008	0.007	0.006	0.002
10	Mean-variance	0.171	0.153	0.139	0.137	0.098
	Mean-ETL	0.175	0.160	0.145	0.142	0.100
	Difference	0.004	0.007	0.006	0.005	0.002
		Sharpe ratio (based on ETL)				
1	Mean-variance	0.080	0.070	0.063	0.066	0.048
	Mean-ETL	0.083	0.076	0.069	0.070	0.050
	Difference	0.003	0.006	0.006	0.005	0.002
2.5	Mean-variance	0.079	0.070	0.062	0.065	0.048
	Mean-ETL	0.082	0.074	0.067	0.069	0.049
	Difference	0.002	0.004	0.004	0.003	0.002
5	Mean-variance	0.079	0.070	0.062	0.065	0.048
	Mean-ETL	0.081	0.073	0.066	0.068	0.049
	Difference	0.002	0.003	0.004	0.003	0.001
10	Mean-variance	0.079	0.070	0.062	0.065	0.048
	Mean-ETL	0.081	0.073	0.066	0.068	0.049
	Difference	0.002	0.003	0.003	0.002	0.001

Table 5.5: Skewness of chosen portfolios across size groups of 10x10 Size-Momentum sorted portfolios

This Table shows the skewness levels of the chosen portfolios across 4 size groups constructed from 100 Size-Momentum sorted portfolios. Each row shows the skewness of the portfolios in a size group. The last row shows the standard deviation between the portfolios of the same skewness-order across groups.

			Skewness		
S1	-0.413	-0.385	-0.303	-0.246	0.019
S2	-0.448	-0.379	-0.261	-0.250	0.055
S3	-0.471	-0.401	-0.293	-0.256	0.036
S4	-0.451	-0.354	-0.303	-0.257	0.016
Standard deviation between skewness	0.021	0.017	0.017	0.004	0.016

Table 5.6: Performance of the mean-variance and the mean-ETL across size groups: similar-skewness portfolios from 10x10 Size-Momentum sorted portfolios

This table shows the performance of the optimal portfolio constructed using the mean-variance and the mean-ETL optimization method across 4 size groups of similar-skewness portfolios chosen from 10x10 FF Size-Momentum sorted portfolios, using different risk aversion coefficients (lambdas), from January 1965 to December 2013. S1 to S4 are the size groups.

lambda		S1	S2	S3	S4
	Final value of the portfolio				
1	Mean-variance	535.326	429.220	225.066	121.800
	Mean-ETL	985.563	846.298	281.879	115.705
	Difference	1.841	1.972	1.252	0.950
2.5	Mean-variance	531.441	412.697	219.428	122.197
	Mean-ETL	716.102	725.122	270.399	117.634
	Difference	1.347	1.757	1.232	0.963
5	Mean-variance	530.070	407.476	217.614	122.234
	Mean-ETL	660.706	682.866	269.024	118.798
	Difference	1.246	1.676	1.236	0.972
10	Mean-variance	529.431	404.791	216.713	122.240
	Mean-ETL	646.080	664.037	268.629	118.089
	Difference	1.220	1.640	1.240	0.966
	Sharpe ratio (based on variance)				
1	Mean-variance	0.149	0.148	0.131	0.113
	Mean-ETL	0.166	0.169	0.139	0.110
	Difference	0.017	0.022	0.008	-0.003
2.5	Mean-variance	0.149	0.146	0.130	0.113
	Mean-ETL	0.157	0.165	0.138	0.111
	Difference	0.009	0.018	0.008	-0.002
5	Mean-variance	0.149	0.146	0.130	0.113
	Mean-ETL	0.155	0.163	0.138	0.112
	Difference	0.006	0.017	0.008	-0.001
10	Mean-variance	0.149	0.146	0.129	0.113
	Mean-ETL	0.154	0.162	0.138	0.112
	Difference	0.006	0.016	0.008	-0.002
	Sharpe ratio (based on ETL)				
1	Mean-variance	0.070	0.068	0.061	0.054
	Mean-ETL	0.076	0.079	0.065	0.052
	Difference	0.007	0.011	0.004	-0.002
2.5	Mean-variance	0.070	0.068	0.061	0.054
	Mean-ETL	0.072	0.077	0.064	0.052
	Difference	0.003	0.009	0.003	-0.002
5	Mean-variance	0.070	0.067	0.061	0.054
	Mean-ETL	0.071	0.076	0.064	0.053
	Difference	0.002	0.008	0.003	-0.001
10	Mean-variance	0.070	0.067	0.061	0.054
	Mean-ETL	0.071	0.075	0.064	0.053
	Difference	0.002	0.008	0.003	-0.002

Table 5.7: Performance of Markov-based switching method: 5x5 Size-BM portfolios

This table shows the performance of the optimal portfolio constructed using the switching method between the mean-variance and the mean-ETL, where the switch indicator is estimated from Markov switching analysis of monthly market excess returns. The investigated assets are portfolios within each size groups of the 5x5 FF Size-BM sorted portfolios. Different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. S1 to S5 are the size groups. Cases when the switching method outperforms both single methods are bold.

lambda		S1	S2	S3	S4	S5
			Final value of the portfolio			
	Mean-variance	1627.312	1282.684	487.821	411.905	107.209
1	Mean-ETL	2474.347	1354.038	592.932	489.231	115.727
	Switch	1756.565	**1503.969**	520.443	483.809	112.093
	Mean-variance	1577.383	1272.780	479.430	404.572	107.599
2.5	Mean-ETL	1883.250	1320.914	552.998	471.329	111.192
	Switch	1579.276	**1396.797**	513.228	470.229	108.033
	Mean-variance	1560.913	1268.241	476.502	402.186	107.712
5	Mean-ETL	1815.950	1302.836	539.671	467.095	109.661
	Switch	1573.860	**1367.643**	507.369	465.038	106.840
	Mean-variance	1552.709	1265.890	475.025	400.955	107.768
10	Mean-ETL	1789.086	1291.804	531.559	465.850	108.008
	Switch	1572.114	**1351.122**	503.418	463.133	106.107
			Sharpe ratio (based on variance)			
	Mean-variance	0.177	0.178	0.155	0.151	0.113
1	Mean-ETL	0.189	0.179	0.162	0.159	0.116
	Switch	0.179	**0.183**	0.157	0.158	0.115
	Mean-variance	0.176	0.178	0.154	0.151	0.113
2.5	Mean-ETL	0.181	0.178	0.159	0.157	0.115
	Switch	0.176	**0.181**	0.157	0.157	0.113
	Mean-variance	0.175	0.177	0.154	0.150	0.113
5	Mean-ETL	0.180	0.178	0.159	0.157	0.114
	Switch	0.176	**0.180**	0.156	0.156	0.113
	Mean-variance	0.175	0.177	0.154	0.150	0.113
10	Mean-ETL	0.180	0.178	0.158	0.157	0.114
	Switch	0.176	**0.179**	0.156	0.156	0.113
			Sharpe ratio (based on ETL)			
	Mean-variance	0.082	0.081	0.070	0.069	0.053
1	Mean-ETL	0.088	0.082	0.073	0.074	0.054
	Switch	0.083	**0.083**	0.071	0.074	0.054
	Mean-variance	0.082	0.081	0.069	0.069	0.053
2.5	Mean-ETL	0.085	0.081	0.072	0.073	0.054
	Switch	0.082	**0.082**	0.071	0.073	0.053
	Mean-variance	0.082	0.081	0.069	0.068	0.053
5	Mean-ETL	0.084	0.081	0.071	0.073	0.053
	Switch	0.082	**0.082**	0.070	0.073	0.053
	Mean-variance	0.081	0.080	0.069	0.068	0.053
10	Mean-ETL	0.084	0.081	0.071	0.072	0.053
	Switch	0.082	**0.082**	0.070	0.072	0.053

Table 5.8: Performance of Markov-based switching method: similar-skewness portfolios from 10x10 Size-BM portfolios

This table shows the performance of the optimal portfolio constructed using the switching method between the mean-variance and the mean-ETL, where the switch indicator is estimated from Markov switching analysis of monthly market excess returns. S1 to S5 are the size groups constructed from the 10x10 FF Size-BM sorted portfolios. Each size group contains 5 similar-skewness portfolios. Different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. Cases when the switching method outperforms both single methods are bold.

lambda		S1	S2	S3	S4
		Final value of the portfolio			
1	Mean-variance	1061.727	1170.487	193.264	159.057
	Mean-ETL	3749.307	1244.711	267.332	183.201
	Switch	1444.914	1046.506	219.285	177.135
2.5	Mean-variance	959.002	1154.250	188.431	156.745
	Mean-ETL	2661.099	1236.873	245.045	181.296
	Switch	1216.634	1074.668	205.651	176.104
5	Mean-variance	927.109	1149.170	186.932	156.159
	Mean-ETL	2332.818	1226.153	237.474	180.001
	Switch	1151.818	1077.141	202.355	175.066
10	Mean-variance	911.499	1146.674	186.142	155.903
	Mean-ETL	2150.707	1227.298	234.490	179.863
	Switch	1119.520	1084.072	201.382	174.734
		Sharpe ratio (based on variance)			
1	Mean-variance	0.164	0.176	0.121	0.129
	Mean-ETL	0.196	0.174	0.133	0.134
	Switch	0.170	0.171	0.126	0.133
2.5	Mean-variance	0.161	0.175	0.120	0.128
	Mean-ETL	0.188	0.175	0.130	0.134
	Switch	0.166	0.173	0.124	0.133
5	Mean-variance	0.160	0.175	0.120	0.128
	Mean-ETL	0.185	0.176	0.128	0.134
	Switch	0.165	0.173	0.123	0.132
10	Mean-variance	0.160	0.175	0.120	0.128
	Mean-ETL	0.183	0.176	0.128	0.134
	Switch	0.164	0.173	0.123	0.132
		Sharpe ratio (based on ETL)			
1	Mean-variance	0.076	0.080	0.056	0.063
	Mean-ETL	0.093	0.079	0.063	0.064
	Switch	0.080	0.077	0.059	0.064
2.5	Mean-variance	0.074	0.080	0.055	0.062
	Mean-ETL	0.088	0.079	0.061	0.065
	Switch	0.077	0.078	0.058	0.064
5	Mean-variance	0.074	0.080	0.055	0.062
	Mean-ETL	0.087	0.080	0.060	0.065
	Switch	0.077	0.078	0.057	0.064
10	Mean-variance	0.074	0.080	0.055	0.062
	Mean-ETL	0.086	0.080	0.060	0.065
	Switch	0.077	0.078	0.057	0.064

Table 5.9: Performance of Markov-based switching method: 5x5 Size-Momentum portfolios

This table shows the performance of the optimal portfolio constructed using the switching method between the mean-variance and the mean-ETL, where the switch indicator is estimated from Markov switching analysis of monthly market excess returns. The investigated assets are portfolios within each size groups of the 5x5 Size-Momentum sorted portfolios. Different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. S1 to S5 are the size groups. Cases when the switching method outperforms both single methods are bold.

lambda		S1	S2	S3	S4	S5
				Final value of the portfolio		
	Mean-variance	1087.336	587.864	313.579	277.595	76.791
1	Mean-ETL	1319.995	887.540	447.661	372.834	84.913
	Switch	1147.100	699.798	390.211	332.519	83.544
	Mean-variance	1062.417	575.825	307.671	274.194	76.111
2.5	Mean-ETL	1214.873	773.743	389.771	331.787	81.514
	Switch	1083.601	646.220	356.627	312.109	81.397
	Mean-variance	1053.287	572.288	305.582	273.156	75.923
5	Mean-ETL	1195.906	732.939	373.290	318.777	80.340
	Switch	1075.155	626.768	346.524	304.300	**80.582**
	Mean-variance	1048.696	570.696	304.539	272.600	75.833
10	Mean-ETL	1184.905	713.387	365.550	312.422	79.762
	Switch	1071.612	618.601	341.175	301.132	**80.040**
				Sharpe ratio (based on variance)		
	Mean-variance	0.172	0.154	0.140	0.138	0.099
1	Mean-ETL	0.178	0.167	0.152	0.148	0.103
	Switch	0.174	0.160	0.147	0.145	0.102
	Mean-variance	0.171	0.153	0.139	0.137	0.098
2.5	Mean-ETL	0.176	0.163	0.148	0.144	0.101
	Switch	0.172	0.157	0.145	0.142	**0.101**
	Mean-variance	0.171	0.153	0.139	0.137	0.098
5	Mean-ETL	0.175	0.161	0.146	0.143	0.101
	Switch	0.172	0.156	0.144	0.141	**0.101**
	Mean-variance	0.171	0.153	0.139	0.137	0.098
10	Mean-ETL	0.175	0.160	0.145	0.142	0.100
	Switch	0.172	0.156	0.143	0.141	**0.100**
				Sharpe ratio (based on ETL)		
	Mean-variance	0.080	0.070	0.063	0.066	0.048
1	Mean-ETL	0.083	0.076	0.069	0.070	0.050
	Switch	0.081	0.072	0.067	0.069	0.050
	Mean-variance	0.079	0.070	0.062	0.065	0.048
2.5	Mean-ETL	0.082	0.074	0.067	0.069	0.049
	Switch	0.080	0.071	0.065	0.068	0.049
	Mean-variance	0.079	0.070	0.062	0.065	0.048
5	Mean-ETL	0.081	0.073	0.066	0.068	0.049
	Switch	0.080	0.071	0.065	0.068	**0.049**
	Mean-variance	0.079	0.070	0.062	0.065	0.048
10	Mean-ETL	0.081	0.073	0.066	0.068	0.049
	Switch	0.080	0.071	0.064	0.067	**0.049**

Table 5.10: Performance of Markov-based switching method: similar-skewness portfolios from 10x10 Size-Momentum portfolios

This table shows the performance of the optimal portfolio constructed using the switching method between the mean-variance and the mean-ETL, where the switch indicator is estimated from Markov switching analysis of monthly market excess returns. S1 to S5 are the size groups constructed from the 10x10 Size-Momentum sorted portfolios. Each size group contains 5 similar-skewness portfolios. Different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. Cases when the switching method outperforms both single methods are bold.

lambda		S1	S2	S3	S4
		Final value of the portfolio			
	Mean-variance	535.326	429.220	225.066	121.800
1	Mean-ETL	985.563	846.298	281.879	115.705
	Switch	692.799	433.091	234.887	112.803
	Mean-variance	531.441	412.697	219.428	122.197
2.5	Mean-ETL	716.102	725.122	270.399	117.634
	Switch	597.709	420.505	235.214	116.634
	Mean-variance	530.070	407.476	217.614	122.234
5	Mean-ETL	660.706	682.866	269.024	118.798
	Switch	568.428	412.860	236.572	117.636
	Mean-variance	529.431	404.791	216.713	122.240
10	Mean-ETL	646.080	664.037	268.629	118.089
	Switch	561.980	411.839	237.432	116.855
		Sharpe ratio (based on variance)			
	Mean-variance	0.149	0.148	0.131	0.113
1	Mean-ETL	0.166	0.169	0.139	0.110
	Switch	0.156	0.148	0.133	0.110
	Mean-variance	0.149	0.146	0.130	0.113
2.5	Mean-ETL	0.157	0.165	0.138	0.111
	Switch	0.152	0.147	0.133	0.111
	Mean-variance	0.149	0.146	0.130	0.113
5	Mean-ETL	0.155	0.163	0.138	0.112
	Switch	0.151	0.146	0.133	0.111
	Mean-variance	0.149	0.146	0.129	0.113
10	Mean-ETL	0.154	0.162	0.138	0.112
	Switch	0.150	0.146	0.133	0.111
		Sharpe ratio (based on ETL)			
	Mean-variance	0.070	0.068	0.061	0.054
1	Mean-ETL	0.076	0.079	0.065	0.052
	Switch	0.071	0.069	0.062	0.051
	Mean-variance	0.070	0.068	0.061	0.054
2.5	Mean-ETL	0.072	0.077	0.064	0.052
	Switch	0.070	0.068	0.062	0.052
	Mean-variance	0.070	0.067	0.061	0.054
5	Mean-ETL	0.071	0.076	0.064	0.053
	Switch	0.069	0.068	0.062	0.053
	Mean-variance	0.070	0.067	0.061	0.054
10	Mean-ETL	0.071	0.075	0.064	0.053
	Switch	0.069	0.068	0.062	0.052

Table 5.11: How the mean-ETL is better in different market states: 5x5 Size-BM sorted portfolios

This Table shows the percentages of time that the mean-ETL is better than the mean-variance in each state of the market, across different size groups and risk aversion coefficients (lambdas), from January 1965 to December 2013. Each size groups are the size group of 5x5 FF Size-BM sorted portfolios, containing 5 BM sorted portfolios.

		Lambda			
		1	2.5	5	10
			Size group 1		
State	1	59.32	57.74	55.38	58.27
	2	52.17	50.72	52.66	49.28
			Size group 2		
	1	49.08	49.87	48.29	49.87
	2	56.52	55.07	57.00	58.45
			Size group 3		
	1	52.49	53.28	50.39	53.02
	2	56.52	54.11	52.17	54.59
			Size group 4		
	1	49.87	46.19	46.19	49.61
	2	51.69	52.66	55.07	54.59
			Size group 5		
	1	53.54	53.28	51.97	53.02
	2	48.79	48.31	47.34	48.31

Table 5.12: How the mean-ETL is better in different market states: Similar-skewness portfolios from 10x10 Size-BM sorted portfolios

This Table shows the percentages of time that the mean-ETL is better than the mean-variance in each state of the market, across different size groups and risk aversion coefficients (lambdas), from January 1965 to December 2013. Each size groups are constructed from the 10x10 FF Size-BM sorted portfolios, containing 5 similar-skewness portfolios.

		Lambda			
		1	2.5	5	10
			Size group 1		
State	1	61.15	62.99	61.42	62.20
	2	54.11	52.66	51.21	51.21
			Size group 2		
	1	51.71	50.92	52.23	51.44
	2	52.66	56.04	53.62	53.62
			Size group 3		
	1	53.02	52.76	52.49	52.23
	2	52.17	54.11	55.56	55.07
			Size group 4		
	1	51.18	53.02	52.23	51.97
	2	55.56	57.49	56.52	57.00

Table 5.13: How the mean-ETL is better in different market states: 5x5 Size-Momentum sorted portfolios

This Table shows the percentages of time that the mean-ETL is better than the mean-variance in each state of the market, across different size groups and risk aversion coefficients (lambdas), from January 1965 to December 2013. Each size groups are the size group of 5x5 Size-Momentum sorted portfolios, containing 5 Momentum sorted portfolios.

		Lambda			
		1	2.5	5	10
		Size group 1			
State	1	58.27	56.17	56.17	55.12
	2	47.34	47.34	50.24	48.31
		Size group 2			
	1	56.96	57.22	55.38	56.69
	2	52.66	53.62	53.62	55.56
		Size group 3			
	1	51.71	51.44	50.13	49.34
	2	53.14	55.07	55.56	55.07
		Size group 4			
	1	54.07	53.54	51.97	51.97
	2	52.17	52.17	53.14	53.14
		Size group 5			
	1	49.08	48.03	48.29	48.03
	2	51.69	53.14	53.14	52.66

Table 5.14: How the mean-ETL is better in different market states: Similar-skewness portfolios from 10x10 Size-BM sorted portfolios

This Table shows the percentages of time that the mean-ETL is better than the mean-variance in each state of the market, across different size groups and risk aversion coefficients (lambdas), from January 1965 to December 2013. Each size groups are constructed from the 10x10 Size-Momentum sorted portfolios, containing 5 similar-skewness portfolios.

		Lambda			
		1	2.5	5	10
		Size group 1			
State	1	54.86	54.59	54.33	54.59
	2	54.59	52.66	52.17	52.66
		Size group 2			
	1	60.10	60.89	60.89	60.63
	2	50.24	53.62	54.59	54.11
		Size group 3			
	1	52.49	51.44	52.49	51.97
	2	48.31	52.17	53.14	52.66
		Size group 4			
	1	51.97	50.66	49.61	50.13
	2	43.00	44.44	44.93	44.44

Table 5.15: How regime indicator explains the best optimization method: 5x5 Size-BM sorted portfolios

This Table shows how regime indicator could explain the best optimization method, using the probit regression of the binary variable of the best method on the Markov switching estimated regime indicator and a set of other explanatory variables. The results for different size groups and different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. Each size groups are the size group of 5x5 FF Size-BM sorted portfolios, containing 5 BM sorted portfolios. Within the result of each size group, the first row is the estimated coefficient in the probit regression of the regime indicator, the second row is its corresponding t-statistics (in brackets), the third row is the goodness of fit of the corresponding probit regression (how many percent of the timing of the best method is correctly captured by the fitted value of the regression), the forth row is the goodness of fit if the mean-ETL is used in all periods (how many percent of the timing of the best method is correctly captured if the mean-ETL is considered to be the best method at all time)

	Lambda			
	1	**2.5**	**5**	**10**
	Size group 1			
Regime Indicator	-0.318	-0.230	-0.206	-0.365
	(-2.117)	(-1.538)	(-1.378)	(-2.430)
Goodness of fit	59.354	56.633	55.952	57.993
	56.803	55.272	54.422	55.102
	Size group 2			
Regime Indicator	0.184	0.149	0.270	0.247
	(1.227)	(1.000)	(1.806)	(1.651)
Goodness of fit	55.952	56.803	56.803	55.782
	51.701	51.701	51.361	52.891
	Size group 3			
Regime Indicator	0.035	-0.001	-0.070	-0.116
	(0.234)	(-0.010)	(-0.462)	(-0.769)
Goodness of fit	56.122	55.782	57.143	57.993
	53.912	53.571	51.020	53.571
	Size group 4			
Regime Indicator	-0.100	0.099	0.134	0.044
	(-0.666)	(0.660)	(0.892)	(0.297)
Goodness of fit	55.612	55.272	57.143	54.932
	50.510	48.469	49.320	51.361
	Size group 5			
Regime Indicator	-0.035	-0.084	0.011	-0.022
	(-0.235)	(-0.558)	(0.070)	(-0.150)
Goodness of fit	56.122	56.293	56.122	56.633
	51.871	51.531	50.340	51.361

Table 5.16: How regime indicator explains the best optimization method: similar-skewness portfolios from 10x10 Size-BM sorted portfolios

This Table shows how regime indicator could explain the best optimization method, using the probit regression of the binary variable of the best method on the Markov switching estimated regime indicator and a set of other explanatory variables. The results for different size groups and different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. Each size groups are constructed from the 10x10 FF Size-BM sorted portfolios, containing 5 similar-skewness portfolios. Within the result of each size group, the first row is the estimated coefficient in the probit regression of the regime indicator, the second row is its corresponding t-statistics (in brackets), the third row is the goodness of fit of the corresponding probit regression (how many percent of the timing of the best method is correctly captured by the fitted value of the regression), the forth row is the goodness of fit if the mean-ETL is used in all periods (how many percent of the timing of the best method is correctly captured if the mean-ETL is considered to be the best method at all time)

	Lambda			
	1	**2.5**	**5**	**10**
	Size group 1			
Regime Indicator	-0.170	-0.280	-0.291	-0.359
	(-1.125)	(-1.846)	(-1.934)	(-2.369)
Goodness of fit	61.565	62.925	62.075	60.884
	58.673	59.354	57.823	58.333
	Size group 2			
Regime Indicator	0.020	0.137	0.040	0.102
	(0.134)	(0.915)	(0.269)	(0.686)
Goodness of fit	51.190	55.102	51.531	51.701
	52.041	52.721	52.721	52.211
	Size group 3			
Regime Indicator	0.155	0.257	0.267	0.293
	(1.032)	(1.710)	(1.778)	(1.947)
Goodness of fit	54.932	56.633	55.442	54.932
	52.721	53.231	53.571	53.231
	Size group 4			
Regime Indicator	-0.001	0.013	0.055	0.083
	(-0.009)	(0.084)	(0.366)	(0.550)
Goodness of fit	55.612	56.803	57.143	55.612
	52.721	54.592	53.741	53.741

Table 5.17: How regime indicator explains the best optimization method: 5x5 Size-Momentum sorted portfolios

This Table shows how regime indicator could explain the best optimization method, using the probit regression of the binary variable of the best method on the Markov switching estimated regime indicator and a set of other explanatory variables. The results for different size groups and different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. Each size groups are the size group of 5x5 Size-Momentum sorted portfolios, containing 5 Momentum sorted portfolios. Within the result of each size group, the first row is the estimated coefficient in the probit regression of the regime indicator, the second row is its corresponding t-statistics (in brackets), the third row is the goodness of fit of the corresponding probit regression (how many percent of the timing of the best method is correctly captured by the fitted value of the regression), the forth row is the goodness of fit if the mean-ETL is used in all periods (how many percent of the timing of the best method is correctly captured if the mean-ETL is considered to be the best method at all time)

	Lambda			
	1	2.5	5	10
	Size group 1			
Regime Indicator	-0.174	-0.155	-0.106	-0.106
	(-1.161)	(-1.029)	(-0.703)	(-0.709)
Goodness of fit	57.143	58.503	55.782	57.483
	54.422	53.061	54.082	52.721
	Size group 2			
Regime Indicator	0.073	0.106	0.162	0.146
	(0.483)	(0.704)	(1.072)	(0.965)
Goodness of fit	57.653	57.823	55.442	57.313
	55.442	55.952	54.762	56.293
	Size group 3			
Regime Indicator	0.098	0.061	0.175	0.146
	(0.656)	(0.410)	(1.170)	(0.975)
Goodness of fit	55.102	56.633	57.823	56.293
	52.211	52.721	52.041	51.361
	Size group 4			
Regime Indicator	0.108	0.136	0.179	0.145
	(0.719)	(0.906)	(1.195)	(0.966)
Goodness of fit	56.463	55.612	55.102	54.592
	53.401	53.061	52.381	52.381
	Size group 5			
Regime Indicator	0.268	0.198	0.157	0.164
	(1.782)	(1.320)	(1.047)	(1.094)
Goodness of fit	55.612	54.422	54.762	54.082
	50.000	49.830	50.000	49.660

Table 5.18: How regime indicator explains the best optimization method: 10x10 similar-skewness portfolios from Size-Momentum sorted portfolios

This Table shows how regime indicator could explain the best optimization method, using the probit regression of the binary variable of the best method on the Markov switching estimated regime indicator and a set of other explanatory variables. The results for different size groups and different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. Each size groups are constructed from the 10x10 Size-Momentum sorted portfolios, containing 5 similar-skewness portfolios. Within the result of each size group, the first row is the estimated coefficient in the probit regression of the regime indicator, the second row is its corresponding t-statistics (in brackets), the third row is the goodness of fit of the corresponding probit regression (how many percent of the timing of the best method is correctly captured by the fitted value of the regression), the forth row is the goodness of fit if the mean-ETL is used in all periods (how many percent of the timing of the best method is correctly captured if the mean-ETL is considered to be the best method at all time)

	Lambda			
	1	**2.5**	**5**	**10**
	Size group 1			
Regime Indicator	0.104	0.161	0.184	0.153
	(0.693)	(1.069)	(1.222)	(1.017)
Goodness of fit	59.524	57.143	57.653	55.952
	54.762	53.912	53.571	53.912
	Size group 2			
Regime Indicator	-0.206	-0.160	-0.147	-0.151
	(-1.374)	(-1.068)	(-0.976)	(-1.004)
Goodness of fit	57.313	60.034	59.864	60.034
	56.633	58.333	58.673	58.333
	Size group 3			
Regime Indicator	-0.007	0.069	0.117	0.114
	(-0.047)	(0.459)	(0.778)	(0.756)
Goodness of fit	53.571	56.463	55.102	55.272
	51.020	51.701	52.721	52.211
	Size group 4			
Regime Indicator	-0.163	-0.084	-0.107	-0.125
	(-1.080)	(-0.553)	(-0.708)	(-0.832)
Goodness of fit	57.653	58.333	57.483	58.163
	48.810	48.469	47.959	48.129

Table 5.19: Performance of Probit-regression-based switching method: 5x5 Size-BM portfolios

This table shows the performance of the optimal portfolio constructed using the switching method between the mean-variance and the mean-ETL, where the switch indicator is estimated from the probit regression of the binary variable of the best method overtime on the Markov switching indicator and other explanatory variables. The investigated assets are portfolios within each size groups of the 5x5 FF Size-BM sorted portfolios. Different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. S1 to S5 are the size groups. Cases when the switching method outperforms both single methods are bold.

lambda		S1	S2	S3	S4	S5
		Final value of the portfolio				
	mean-variance	1627.312	1282.684	487.821	411.905	107.209
1	mean-ETL	2474.347	1354.038	592.932	489.231	115.727
	switch	**2565.323**	**1411.097**	583.620	488.148	112.486
	mean-variance	1577.383	1272.780	479.430	404.572	107.599
2.5	mean-ETL	1883.250	1320.914	552.998	471.329	111.192
	switch	**1963.090**	**1355.181**	532.417	458.780	**117.651**
	mean-variance	1560.913	1268.241	476.502	402.186	107.712
5	mean-ETL	1815.950	1302.836	539.671	467.095	109.661
	switch	**1864.949**	**1373.108**	**545.706**	**476.153**	**116.331**
	mean-variance	1552.709	1265.890	475.025	400.955	107.768
10	mean-ETL	1789.086	1291.804	531.559	465.850	108.008
	switch	**1830.739**	**1327.721**	523.131	456.531	**112.280**
		Sharpe ratio (based on variance)				
	mean-variance	0.177	0.178	0.155	0.151	0.113
1	mean-ETL	0.189	0.179	0.162	0.159	0.116
	switch	**0.190**	**0.180**	**0.162**	0.159	0.116
	mean-variance	0.176	0.178	0.154	0.151	0.113
2.5	mean-ETL	0.181	0.178	0.159	0.157	0.115
	switch	**0.182**	**0.179**	0.158	0.156	**0.117**
	mean-variance	0.175	0.177	0.154	0.150	0.113
5	mean-ETL	0.180	0.178	0.159	0.157	0.114
	switch	**0.181**	**0.180**	**0.159**	**0.157**	**0.117**
	mean-variance	0.175	0.177	0.154	0.150	0.113
10	mean-ETL	0.180	0.178	0.158	0.157	0.114
	switch	**0.180**	**0.179**	0.157	0.156	**0.115**
		Sharpe ratio (based on ETL)				
	mean-variance	0.082	0.081	0.070	0.069	0.053
1	mean-ETL	0.088	0.082	0.073	0.074	0.054
	switch	**0.089**	**0.082**	**0.073**	0.074	0.054
	mean-variance	0.082	0.081	0.069	0.069	0.053
2.5	mean-ETL	0.085	0.081	0.072	0.073	0.054
	switch	**0.086**	**0.082**	0.071	0.072	**0.055**
	mean-variance	0.082	0.081	0.069	0.068	0.053
5	mean-ETL	0.084	0.081	0.071	0.073	0.053
	switch	**0.085**	**0.082**	**0.072**	**0.073**	**0.055**
	mean-variance	0.081	0.080	0.069	0.068	0.053
10	mean-ETL	0.084	0.081	0.071	0.072	0.053
	switch	**0.085**	**0.081**	0.071	0.072	**0.054**

Table 5.20: Performance of Probit-regression-based switching method: similar-skewness portfolio from 10x10 Size-BM portfolios

This table shows the performance of the optimal portfolio constructed using the switching method between the mean-variance and the mean-ETL, where the switch indicator is estimated from the probit regression of the binary variable of the best method overtime on the Markov switching indicator and other explanatory variables. S1 to S5 are the size groups constructed from the 10x10 FF Size-BM sorted portfolios. Each size group contains 5 similar-skewness portfolios. Different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. S1 to S5 are the size groups. Cases when the switching method outperforms both single methods are bold.

lambda		S1	S2	S3	S4
			Final value of the portfolio		
	mean-variance	1061.727	1170.487	193.264	159.057
1	mean-ETL	3749.307	1244.711	267.332	183.201
	switch	3068.639	1226.916	**287.878**	**192.038**
	mean-variance	959.002	1154.250	188.431	156.745
2.5	mean-ETL	2661.099	1236.873	245.045	181.296
	switch	2324.534	1153.616	**257.101**	**190.406**
	mean-variance	927.109	1149.170	186.932	156.159
5	mean-ETL	2332.818	1226.153	237.474	180.001
	switch	2270.040	**1293.813**	**251.097**	179.029
	mean-variance	911.499	1146.674	186.142	155.903
10	mean-ETL	2150.707	1227.298	234.490	179.863
	switch	2064.196	**1289.461**	**240.835**	173.506
			Sharpe ratio (based on variance)		
	mean-variance	0.164	0.176	0.121	0.129
1	mean-ETL	0.196	0.174	0.133	0.134
	switch	0.192	0.175	**0.135**	**0.136**
	mean-variance	0.161	0.175	0.120	0.128
2.5	mean-ETL	0.188	0.175	0.130	0.134
	switch	0.184	0.174	**0.131**	**0.136**
	mean-variance	0.160	0.175	0.120	0.128
5	mean-ETL	0.185	0.176	0.128	0.134
	switch	**0.185**	**0.177**	**0.130**	0.133
	mean-variance	0.160	0.175	0.120	0.128
10	mean-ETL	0.183	0.176	0.128	0.134
	switch	0.183	**0.178**	**0.129**	0.132
			Sharpe ratio (based on ETL)		
	mean-variance	0.076	0.080	0.056	0.063
1	mean-ETL	0.093	0.079	0.063	0.064
	switch	0.089	0.079	**0.063**	**0.066**
	mean-variance	0.074	0.080	0.055	0.062
2.5	mean-ETL	0.088	0.079	0.061	0.065
	switch	0.086	0.078	**0.061**	**0.066**
	mean-variance	0.074	0.080	0.055	0.062
5	mean-ETL	0.087	0.080	0.060	0.065
	switch	**0.087**	**0.081**	**0.061**	**0.065**
	mean-variance	0.074	0.080	0.055	0.062
10	mean-ETL	0.086	0.080	0.060	0.065
	switch	**0.086**	**0.081**	**0.060**	0.064

Table 5.21: Performance of Probit-regression-based switching method: 5x5 Size-Momentum portfolios

This table shows the performance of the optimal portfolio constructed using the switching method between the mean-variance and the mean-ETL, where the switch indicator is estimated from the probit regression of the binary variable of the best method overtime on the Markov switching indicator and other explanatory variables. The investigated assets are portfolios within each size groups of the 5x5 Size-Momentum sorted portfolios. Different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. S1 to S5 are the size groups. Cases when the switching method outperforms both single methods are bold.

lambda		S1	S2	S3	S4	S5
		Final value of the portfolio				
	mean-variance	1087.336	587.864	313.579	277.595	76.791
1	mean-ETL	1319.995	887.540	447.661	372.834	84.913
	switch	**1451.876**	864.247	398.764	370.223	**89.110**
	mean-variance	1062.417	575.825	307.671	274.194	76.111
2.5	mean-ETL	1214.873	773.743	389.771	331.787	81.514
	switch	**1270.619**	734.521	378.063	**332.616**	**83.271**
	mean-variance	1053.287	572.288	305.582	273.156	75.923
5	mean-ETL	1195.906	732.939	373.290	318.777	80.340
	switch	**1262.628**	729.055	**373.547**	**327.334**	**84.482**
	mean-variance	1048.696	570.696	304.539	272.600	75.833
10	mean-ETL	1184.905	713.387	365.550	312.422	79.762
	switch	**1275.581**	704.006	354.232	**317.850**	**81.635**
		Sharpe ratio (based on variance)				
	mean-variance	0.172	0.154	0.140	0.138	0.099
1	mean-ETL	0.178	0.167	0.152	0.148	0.103
	switch	**0.181**	0.166	0.148	0.148	**0.105**
	mean-variance	0.171	0.153	0.139	0.137	0.098
2.5	mean-ETL	0.176	0.163	0.148	0.144	0.101
	switch	**0.177**	0.161	0.147	0.144	**0.102**
	mean-variance	0.171	0.153	0.139	0.137	0.098
5	mean-ETL	0.175	0.161	0.146	0.143	0.101
	switch	**0.177**	0.161	**0.147**	**0.144**	**0.103**
	mean-variance	0.171	0.153	0.139	0.137	0.098
10	mean-ETL	0.175	0.160	0.145	0.142	0.100
	switch	**0.177**	0.160	0.145	**0.143**	**0.101**
		Sharpe ratio (based on ETL)				
	mean-variance	0.080	0.070	0.063	0.066	0.048
1	mean-ETL	0.083	0.076	0.069	0.070	0.050
	switch	**0.084**	0.075	0.067	0.070	**0.051**
	mean-variance	0.079	0.070	0.062	0.065	0.048
2.5	mean-ETL	0.082	0.074	0.067	0.069	0.049
	switch	**0.082**	0.073	0.066	0.069	**0.050**
	mean-variance	0.079	0.070	0.062	0.065	0.048
5	mean-ETL	0.081	0.073	0.066	0.068	0.049
	switch	**0.082**	0.073	**0.066**	**0.069**	**0.050**
	mean-variance	0.079	0.070	0.062	0.065	0.048
10	mean-ETL	0.081	0.073	0.066	0.068	0.049
	switch	**0.082**	0.072	0.065	**0.068**	**0.050**

Table 5.22: Performance of Probit-regression-based switching method: similar-skewness portfolio from 10x10 Size-Momentum portfolios

This table shows the performance of the optimal portfolio constructed using the switching method between the mean-variance and the mean-ETL, where the switch indicator is estimated from the probit regression of the binary variable of the best method overtime on the Markov switching indicator and other explanatory variables. S1 to S5 are the size groups constructed from the 10x10 Size-Momentum sorted portfolios. Each size group contains 5 similar-skewness portfolios. Different risk aversion coefficients (lambdas) are reported. The sample period is from January 1965 to December 2013. S1 to S5 are the size groups. Cases when the switching method outperforms single methods are bold.

lambda		S1	S2	S3	S4
		Final value of the portfolio			
	mean-variance	535.326	429.220	225.066	121.800
1	mean-ETL	985.563	846.298	281.879	115.705
	switch	**995.541**	**937.596**	**325.294**	**140.391**
	mean-variance	531.441	412.697	219.428	122.197
2.5	mean-ETL	716.102	725.122	270.399	117.634
	switch	**748.548**	**749.214**	**308.926**	**135.568**
	mean-variance	530.070	407.476	217.614	122.234
5	mean-ETL	660.706	682.866	269.024	118.798
	switch	**684.382**	**688.319**	**301.123**	**140.850**
	mean-variance	529.431	404.791	216.713	122.240
10	mean-ETL	646.080	664.037	268.629	118.089
	switch	**673.324**	**664.561**	**289.941**	**128.506**
		Sharpe ratio (based on variance)			
	mean-variance	0.149	0.148	0.131	0.113
1	mean-ETL	0.166	0.169	0.139	0.110
	switch	0.166	**0.172**	**0.143**	**0.118**
	mean-variance	0.149	0.146	0.130	0.113
2.5	mean-ETL	0.157	0.165	0.138	0.111
	switch	**0.158**	**0.166**	**0.142**	**0.117**
	mean-variance	0.149	0.146	0.130	0.113
5	mean-ETL	0.155	0.163	0.138	0.112
	switch	**0.155**	**0.163**	**0.141**	**0.119**
	mean-variance	0.149	0.146	0.129	0.113
10	mean-ETL	0.154	0.162	0.138	0.112
	switch	**0.155**	0.162	**0.140**	**0.115**
		Sharpe ratio (based on ETL)			
	mean-variance	0.070	0.068	0.061	0.054
1	mean-ETL	0.076	0.079	0.065	0.052
	switch	**0.077**	**0.080**	**0.067**	**0.056**
	mean-variance	0.070	0.068	0.061	0.054
2.5	mean-ETL	0.072	0.077	0.064	0.052
	switch	**0.073**	**0.077**	**0.067**	**0.056**
	mean-variance	0.070	0.067	0.061	0.054
5	mean-ETL	0.071	0.076	0.064	0.053
	switch	**0.072**	0.076	**0.066**	**0.057**
	mean-variance	0.070	0.067	0.061	0.054
10	mean-ETL	0.071	0.075	0.064	0.053
	switch	**0.072**	0.075	**0.066**	**0.055**

CHAPTER 6: CONCLUSION

6.1. Conclusion

This thesis provides a broad analysis of the impact of extreme downside risk on stock returns. In fact, downside risk and extreme downside risk have long been recognised as the main concerns of investors and could significantly influence stock returns, as demonstrated in Bawa and Lindenberg (1977), Reitz (1988), Barro (2006), Ang et al. (2006a), among others. However, due to different limitations, including the scarcity of tail event observations and the work of leverage and volatility feedback effects, among others, evidence of a significant positive return premium corresponding directly to tail risk measures is limited in the literature. Having analysed the fundamental mechanisms regarding how different measures are used to capture tail risk as well as how their relationship with returns is influenced, we propose some new approaches to address the limitations and successfully capture new evidence about the robust impact of tail risk on returns. These evidences are reported from many perspectives, ranging from individual stocks, stock portfolios, to the general stock market. The evidence in all of our studies matches perfectly and the only reason for this is that extreme downside risk does affect asset returns in a broad and consistent manner. Therefore, this thesis provides a solid confirmation about the nontrivial relationship between returns and this type of risk.

Our research starts with a market level investigation about the tail risk-return relationship, where we discover an inconsistency that tail risk only significantly affects returns in calm periods rather than in turbulent periods. We demonstrate the reason for the inconsistency to be the leverage and volatility feedback effects and propose a simple and effective solution to the problem. By better filtering out these effects, we reconcile the conflicting evidence and report the significant impact of tail risk on market return in both states of the market. Our analysis further implies the average timeframe of stock fire sales, which is within one month in calm state and two months in turbulent state.

We then demonstrate that this systematic risk is factored into the price (returns) of individual securities. Using different approaches, we propose two new groups of systematic tail risk measures and attain significant return premium on the new measures. Our first group of measures is constructed using the same approach as downside beta, with the cut-off threshold shifted to the tail of return distribution. As a result, they are still subjected to the observation scarcity problem which is similar to other measures in the literature and they are not robust for low threshold settings. On the other hand, our second group is constructed from the sensitivity of stock returns on innovation of market tail risk. This type of measure possesses a unique feature that it performs consistently in extreme settings of short-sample and low tail thresholds. Thus, it solves the most challenging obstacle in the tail risk literature, clearing the way for related studies.

Finally, we show that extreme downside risk impact has a size pattern and this size pattern does influence the performance of the tail-risk-focused portfolio optimisation problem. Specifically, using standard mean-variance as a benchmark, we report evidence that the mean-ETL optimisation method is more effective when applied to smaller size stocks. Our results also suggest the consistent better performance of mean-ETL against mean-variance across different combinations of investigated assets and risk tolerance coefficients. Along with the practical tai-risk aversion of investors, these results support the choice of mean-ETL over standard mean-variance in portfolio optimisation practice. Moreover, this suggestion is further emphasised by our rejection of any systematic mechanism to switch between mean-ETL and mean-variance in the portfolio management process.

The studies in this thesis contribute to the literature in different aspects. In addition to the main contributions of new measures and the new evidence of the role of tail risk, we provide a critical view regarding other related evidence in the literature by revising them under different contexts or model modifications. Additionally, our studies confirm every canonical asset pricing relationship such as size effect, Book-over-Market effect, beta and downside beta effects, skewness and kurtosis effects, leverage and volatility feedback effects, among others. Our studies, therefore, could be used as references in the related literatures for any of those relationships. Finally, we identify some gaps in the literature that are still open for other research ideas. For example, our investigation about the difference between portfolio optimisation methods according to asset characteristics could be developed in a number of ways in future research studies.

6.2. Limitations of the research

Our studies are subjected to certain limitations. Firstly, all of our studies only use US stock data and therefore, any inference and implication should only be restricted within the US stock markets. Due to the large number of variables in the analytical framework, as well as their required long history data, the applicability of our models for markets other than the US market is limited. Secondly, the use of the stock index to represent the return of total assets should also be viewed with caution. Despite being the most widely used proxy for the total wealth of the representative agent, stock investment only accounts for part of the total wealth. Human capital is arguably an even more important part of wealth and therefore modelling the asset pricing relationship with the incorporation of human capital is desirable (see, for example, Jagannathan and Wang, 1996; Dittmar, 2002). However, as in any other tail risk study, our research requires extensive data, which it is impossible to attain with respect to human capital. Thirdly, our models assume static relationships between variables in the sense that their parameters are constant over time. This provides us with flexibility in handling our research. However, a dynamic model where parameters are time variant might be preferable. This is a good direction for future developments of our studies. Finally, we follow the canonical approach in asset pricing literature to examine only systematic tail risk in our models to capture the tail risk-return relationship. However, as argued in the literature review of Chapter 2, idiosyncratic tail risk should not be ignored. Developing our models to incorporate idiosyncratic tail risk would be an interesting future research.

6.3. Suggestions for future research

Since we have examined the impact of tail risk with respect to its systematic part at both market and individual stock levels, a nature development of our research will be towards idiosyncratic tail risk. In fact, as analysed in the literature review, the idiosyncratic tail risk of Huang et al. (2012) captures a significant tail risk premium but is not free of systematic tail risk. However, following our EDH construction and its success, we could incorporate the innovation of market tail risk in their factor model regression to obtain a residual that is free of systematic tail risk. This residual could then be used in Huang et al.'s (2012) framework to construct a truly idiosyncratic tail risk measure for stocks, which could be used to investigate whether this part of tail risk really affects returns.

A second direction for our future research could be the impact of tail risk on portfolio management. For example, incorporating tail risk into a multiple criteria optimisation problem could be promising since this problem has not been completely answered in the literature. Also, from the result of Chapter 5, it is interesting to continue the investigation regarding how the mean-ETL method performs across assets classified based on different characteristics so that we could have a complete picture about the suitability of this method across the whole stock universe.

Finally, another promising approach for future research could be combining tail risk investigations with other areas of the literature, such as corporate finance and behavioural finance. For instance, we could investigate the tail risk behaviour of stocks in merger and acquisition contexts. It could be expected that the stocks of companies within a merger and acquisition case should have related tail behaviours. It is also desirable to understand how their tail behaviour changes in different events during and after the completion of the case. Similarly, the extreme downside risk study also fits well with the behavioural finance area. Using order flow data, it might be possible to observe patterns in the trading activities of investors in the distress context. This understanding is valuable for all market participants, including investors, speculators, fund managers, dealers, and regulators.

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