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New perspectives on the aggregated risk of extratropical cyclones

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In this study the dependence between the frequency and intensity of extratropical cyclones over the North Atlantic is investigated. A cyclone track database of extended October to March winters was obtained from the NCEP-NCAR reanalysis. Large positive correlation is found between winter cyclone counts and local sample mean vorticity over the exit region of the North Atlantic storm track in this cyclone track database. Conversely, negative correlation is found over the Gulf stream. Possible causes for the dependence are investigated by regressing winter cyclone counts and local sample mean vorticity on teleconnection indices with Poisson and linear models. The indices for the Scandinavian pattern, North Atlantic Oscillation and East Atlantic Pattern are able to account for most of the observed positive correlation over the North Atlantic. To consider the implications of frequency intensity dependence for the insurance industry an aggregate risk metric was used as a proxy for the annual aggregate insured loss. Here the aggregate risk is defined as the sum of the intensities of all events occurring within a season. Assuming independence between the frequency and intensity results in large biases in the variance and the extremes of the aggregate risk, especially over Scandinavia. Therefore including frequency intensity dependence in extratropical cyclone loss models is necessary to model the risk of extreme losses.

Key Words: clustering; extratropical cyclone; aggregate risk, NCEP-NCAR reanalysis

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1. Introduction

In Europe extratropical cyclones have caused tens of billions of Euros in insured losses since 1990, and quantifying the risk of further losses has been identified as being of the highest priority for the global reinsurance industry*. Modelling the risk of multiple events within a season (referred to here as the aggregate risk) of extratropical cyclones is of particular interest due to

*http://www.willisresearchnetwork.com/research-and-impact/natural-hazard-and-risk/european-windstorm.html the temporal clustering of storms in the Northern Hemisphere (Mailier *et al.* 2006). Clusters of extratropical cyclones can result in economic losses comparable to those of a U.S. hurricane, and due to the structure of reinsurance contracts a cluster of events can cost more than a single event with the same total loss (Vitolo *et al.* 2009). Climate models have been shown to underestimate clustering (Kvamstø *et al.* 2008) and the physical drivers of clustering remains an active area of ongoing research (e.g. Hanley and Caballero (2012); Pinto *et al.* (2013); Neu *et al.* (2013); Blender *et al.* (2015)). Catastrophe modelling firms have recently attempted to include clustering into their windstorm models but as the models are not open to scrutiny it is difficult to assess how effectively this has been accomplished†.

Both (Mailier *et al.* 2006) and (Vitolo *et al.* 2009) have considered the implications of clustering for modelling the counts of extratropical cyclones. This study extends this to include the relation between cyclone counts and intensity. The frequency and intensity are investigated within a broader aggregate risk framework. Here the aggregate risk refers to the distribution of total intensity from the sum of all cyclone intensities in a season or year. Previous studies on hazard counts within a season, such as Pinto *et al.* (2013); Mailier *et al.* (2006); Katz (2002), are a special case of the aggregate risk where the intensity is unity for each event. Aggregate risk is the main focus due to its importance to the insurance industry for estimating the total claims that can occur in a season.

The aim of this paper is to develop a flexible framework which can be used to quantify and understand the aggregate risk of extratropical cyclones. The framework is used to investigate the sensitivity of the aggregate risk to different modelling assumptions. This investigation will address the following main questions:

- Is there dependence between the frequency and mean intensity of extratropical cyclones within a season/year?
- What are the physical drivers for any dependence between the frequency and intensity?
- How does frequency intensity dependence affect the distribution of aggregate losses (aggregate risk)?

2. How to model the aggregate risk

To fully quantify the aggregate risk from extratropical cyclones one requires a measure of cyclone activity which includes both the frequency and intensity. Such a metric exists in the risk management community called the annual aggregate loss (AAL) and is defined as the sum of the intensities (losses) for all events in a year. The distribution of the AAL can be investigated using a collective risk model (Prabhu 1961). In this section the basic formulation of a collective risk model is described and applied to extratropical cyclones.

Collective risk theory has its roots in the actuarial literature, dating back to the mid-20th century (Houston 1960). In the collective risk model formulation both the number of claims and size of individual claims are assumed random. The AAL is therefore modelled as the sum of a random number of random variables and so it is sometimes also called the random sum model (McNeil *et al.* 2005). Collective risk theory was developed by Filip Lundberg between 1909-1939, however it was not widely adopted by the actuarial community as the relatively large amount of computational power required to apply the theory made it of little practical use (Borch 1967). Increases in computer power allowing the implementation of techniques such as Monte Carlo simulation have resulted in collective risk theory becoming widely adopted by the insurance industry over the latter half of the 20th century, and collective risk models are now widely used in the insurance industry (Embrechts *et al.* 1997). Collective risk theory also has the potential to be used in climate science, for example to model annual U.S. hurricane losses (Katz 2002) or total monthly precipitation (Katz and Parlange 1998).

2.1. *Frequency and intensity*

Extremes in a single meteorological variable at a specific location can be modelled as a marked point process (Stephenson 2008). Events occur at irregular times T_i with variable intensities X_i (see Fig. 1). For natural hazards the occurrence of events is typically modelled as a Poisson process, where $N(t)$ denotes the number of occurrences in a time interval $[0, t]$ (McNeil *et al.* 2005). Each

† http://www.air-worldwide.com/Publications/AIR-Currents/2010/European-Windstorms-Implications-of-Storm-Clustering-on-Definitions-of-Occurrence-Losses/

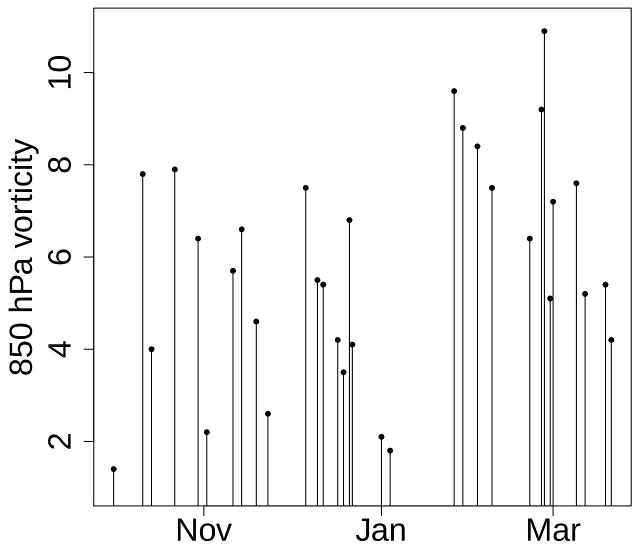

Figure 1. Time series of cyclone transits and corresponding relative vorticity (ζ_{850}) passing near Gothenburg [$12.5°$E, $57.5°$N] between October 1989 and March 1990.

individual occurrence has a mark or intensity[‡] $X_1, ..X_N$. The number of occurrences N is a non negative integer valued random variable, while the intensities X_i are real valued random variables. The AAL, referred to here as the aggregate risk, S is the aggregate total intensity of the N events that occur in a given time period (e.g. over a season or year);

$$S = X_1 + ... + X_N = \sum_{i=1}^{N} X_i.$$

The mean expected aggregate risk can be expressed using the law of total expectation by conditioning on the number of events N

$$E[S] = E_N[\sum_{i=1}^{N} E[X_i|N]] = E[N]E[Y] + Cov(N,Y), \qquad (1)$$

where Y is the mean intensity; ($Y = \sum_{i=1}^{N} X_i/N$). The variance of the aggregate risk from the law of total variance is

$$\begin{aligned} Var(S) &= E_N[(\sum_{i=1}^{N} X_i|N)] + Var_N(E[\sum_{i=1}^{N} X_i|N]) \\ &= Cov(N^2, Y^2) - [Cov(N,Y)]^2 - 2Cov(N,Y)E[N]E[Y] \\ &+ Var(N)E[Y]^2 + Var(Y)E[N^2] \end{aligned} \qquad (2)$$

see Frishman (1971); McNeil *et al.* (2005).

‡Intensity is often used to refer to the rate parameter of the Poisson distribution. In this paper intensity refers to the mark size X.

3. Aggregate risk of extratropical cyclones

In this section the database of storm tracks used throughout the paper is introduced. The climatology of the aggregate risk for extratropical cyclones is then shown.

3.1. Data

The cyclone tracks considered here were obtained from the 6-hourly reanalyzes of the extended October-March winters between October 1950 and March 2003, which was produced jointly by the National Centers for Environmental Prediction and the National Center for Atmospheric Research (NCEP-NCAR reanalysis) (Kalnay *et al.* 1996; Kistler *et al.* 2001). The mean sea level pressure (MSLP) and the zonal and meridional 850mb wind components were extracted. This dataset has been widely used in previous extratropical cyclone studies e.g. Mailier *et al.* (2006); Vitolo *et al.* (2009); Zhang *et al.* (2004).

An objective tracking algorithm was used on the data extracted from the NCEP-NCAR reanalysis to provide storm tracks defined at 6-hourly intervals, from October 1950 to March 2003 (Hodges 1994; Hodges *et al.* 1995; Hodges 1999). The tracking algorithm uses the following intensity variables: vorticity, sea level pressure and max wind speed. In this investigation relative vorticity ζ_{850} is used as an intensity measure, which is less influenced by the background state of the atmosphere than MSLP as it focuses on smaller spatial scales. The vorticity has also been used as the cyclone intensity measure in previous studies on extratropical cyclone risk (Mailier *et al.* 2006; Vitolo *et al.* 2009).

As in Vitolo *et al.* (2009) a spatial grid covering the North Atlantic and Western Europe between $[125W, 40E]$ in longitude and $[20N, 80N]$ in latitude was used to provide a set of reference points. Here the spatial resolution was $2.5°$ in both longitude and latitude. At each grid point the vorticity of cyclones as they passed within $\pm 10°$ was recorded (Fig 1). Time series of the total winter counts and winter local sample mean vorticity could then be constructed for all grid points for each winter. Previous studies have typically used aggregation periods of one or three months (e.g. Vitolo *et al.* (2009)). Here the extended winter (six month) aggregation period is used instead as it reflects the aggregation period of losses which would be used by an insurer.

The results of any analysis of cyclone tracks will be sensitive to the database, tracking algorithm and cyclone intensity measure used Ulbrich *et al.* (2009); Neu *et al.* (2013); Raible *et al.* (2008); Hodges *et al.* (2003). Alternative tracking methods and intensity measures were not considered here, however an investigation into extratropical cyclones using an alternative tracking method and intensity measure was found to produce qualitatively similar results to alternate studies which used Hodges algorithm and vorticity as an intensity measure (Pinto *et al.* 2013). The spectral resolution of the NCEP-NCAR reanalysis is T62, which has been truncated to a total wavenumber of T42 for use with Hodges algorithm, thus a filtering takes place. This filtering was investigated in Mailier (2007), Sec 5.7.2, where it was only found to have a noticeable impact on the cyclone counts statistic over central Asia, which is not considered in this study.

3.2. *Climatology of the aggregate risk*

The mean cyclone counts $\bar{n}$ (see Fig. 2 a) show the location of the North Atlantic storm track, agreeing with that shown in Hoskins and Hodges (2002). Areas of high cyclone activity can also be seen in the lee of the Rockies in North America. The sample variance in cyclone counts s_n^2 (Fig. 2b) is greatest over the storm track with the maximum towards the exit region of the storm track. These findings agree qualitatively with those of the mean and variance of monthly counts in Mailier *et al.* (2006) and the 3 monthly counts in Vitolo *et al.* (2009), where both studies considered winter storm tracks from the NCEP-NCAR reanalysis.

Maxima of the sample local mean vorticity $\bar{y}$ can be noted over the North Atlantic storm track (Fig. 2 c). The variance in winter sample local mean vorticity is greatest along the north/south edges of the storm track (Fig. 2 d).

The sample mean aggregate risk for the 6 month winter, $\bar{s}$, and variance s_s^2, shows a broadly similar pattern to the sample mean and variance of the cyclone counts (Fig. 2 e,f). This suggests that regional variation in the mean and variance of the aggregate loss might be largely accounted for by regional variation in cyclone counts. To quantify sources of variation in the aggregate risk, the sample variance of s is expressed in terms of y, as in Eqn. 2, as $s_s^2 = V_n + V_y + V_c$, where

$$\begin{aligned} V_n &= s_n^2 \bar{y}^2 \\ V_y &= s_y^2 \bar{n^2} \\ V_c &= cov(n^2, y^2) - cov(n, y)^2 - 2cov(n, y)\bar{y}\bar{n} \end{aligned} \tag{3}$$

and $cov(.)$ is the sample covariance between n and y (see Appendix for details). The terms V_n, V_y are non-negative as the sample mean and variance of the counts and vorticity is non-negative. The V_c can be negative if there is negative covariance between n and y. From Eqns. 2 and 3, positive covariance will increase the variance of the aggregate loss (both sample and population variance), conversely negative covariance results in lower s_s^2 and $Var[S]$. The component due to variance in counts, V_n, accounts for a large proportion ($50 - 80\%$) of the variance in the aggregate loss of extratropical cyclones over the North Atlantic storm track (see Fig. 3 a). Over the Gulf Stream, which is the primary region of cyclogenesis for the storm track, $V_n > s_s^2$ (therefore $V_c < 0$) and over North Western Europe V_n accounts for less than half of s_s^2. The variance component due to variance in intensity, V_y, (Fig. 3b) accounts for less of the variance in s than V_n however still makes a significant contribution over the Gulf Stream. The variance component due to covariance between counts and intensity, V_c, (Fig. 3 c) accounts for the least of the variance in s along much of the storm tracks but is non-negligible and positive (negative) over the cyclo-lysis(cyclogenesis) regions for the storm tracks. Including covariance between frequency and intensity is thus necessary for accurately modelling the variance in the aggregate risk of extratropical cyclones in these regions.

4. Understanding the frequency-intensity dependency and its impact on aggregate risk

It is of interest to further diagnose the magnitude and extent of correlation between the frequency and intensity. In this section, the correlation between frequency and intensity is quantified both for the original and detrended time series of counts and sample mean vorticity. A collective risk model is then proposed to investigate the impact of various modelling assumptions on the aggregate risk distribution.

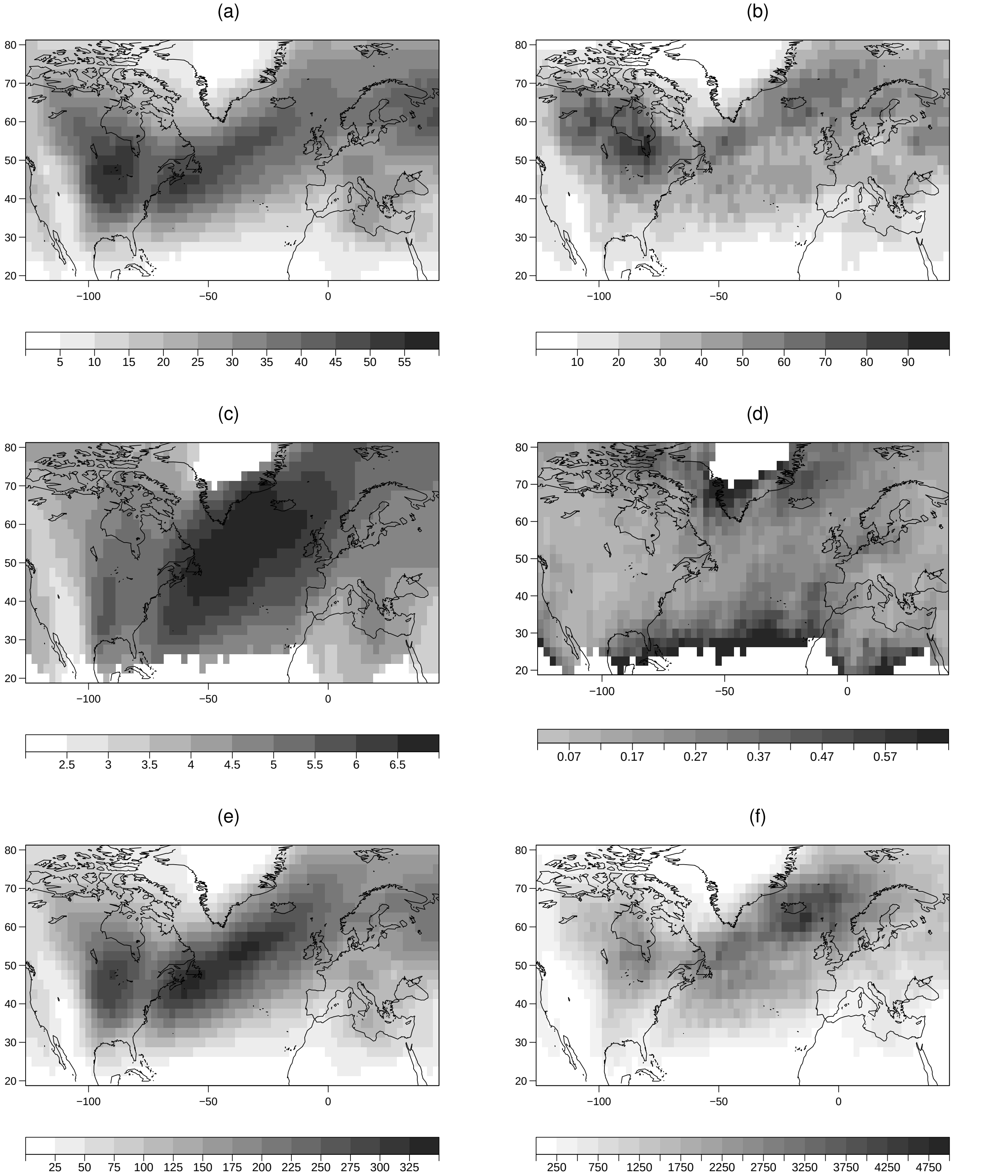

Figure 2. (a) Sample mean $\bar{n}$ cyclone counts per 6 month winter and (b) sample variance s_n^2 of winter cyclone counts, (c) sample winter sample local mean vorticity $\bar{y}$, (d) sample variance s_y^2 , (e) sample winter mean $\bar{s}$ of the aggregate risk and (f) sample variance s_s^2

4.1. Sample correlation

Figure 4 shows a map of the (Pearson's) sample correlation r between n and y. Positive correlation between the frequency and mean intensity of extratropical cyclones along the North Atlantic storms track can be seen over Scandinavia, Northern Germany and the Benelux countries ($r = 0.2 - 0.6$), as well as negative correlation over the Gulf Stream ($r = -0.3$).

Previous studies have shown that there are increasing trends in intense cyclone counts for the NCEP NCAR reanalysis between 1950-2003 (e.g. Mailier *et al.* (2006); Vitolo *et al.* (2009)). The

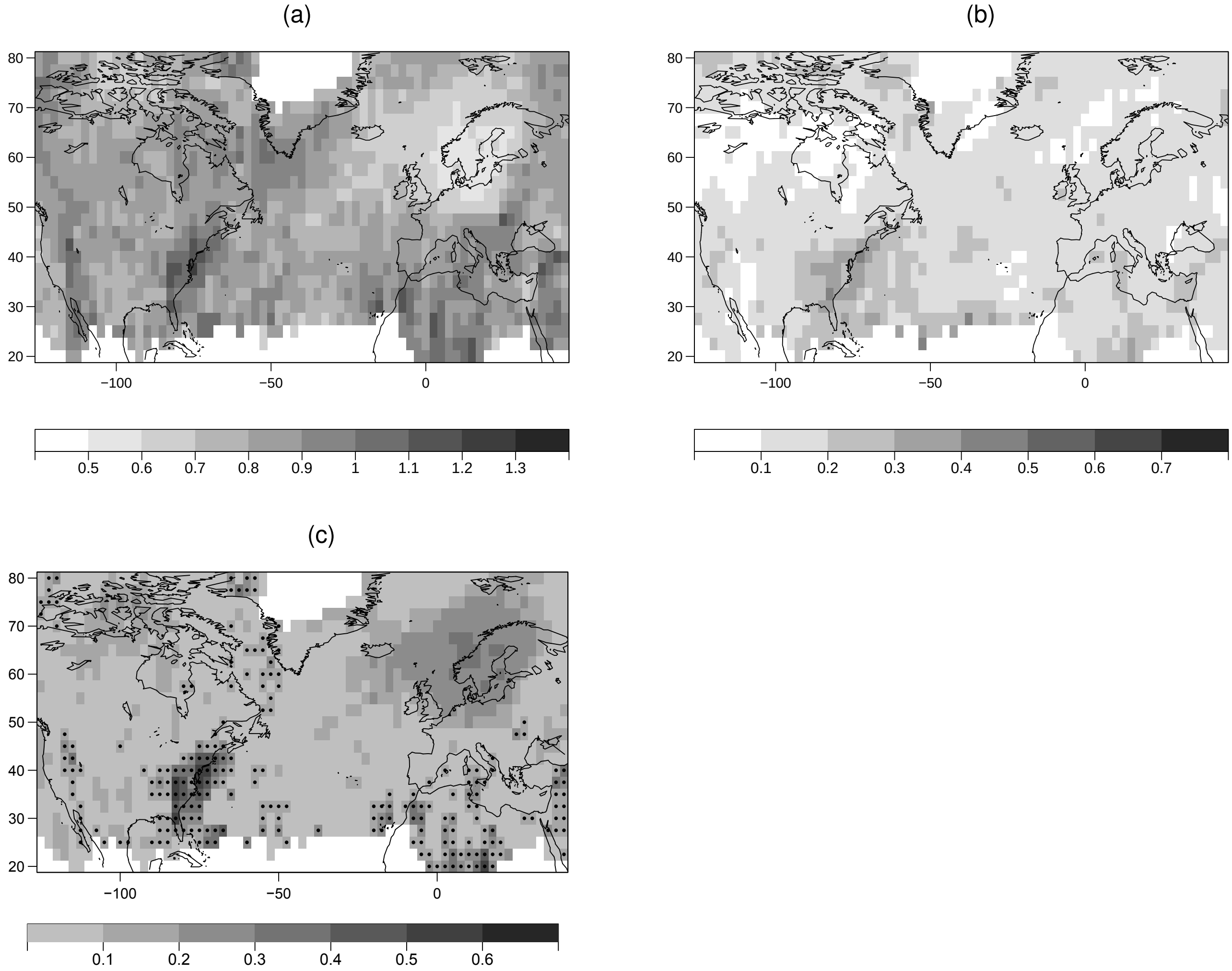

Figure 3. Fraction of variance in s accounted for by (a) counts V_n/s_s^2 (b) cyclone intensities V_y/s_s^2 and (c) covariance between the frequency and intensity V_c/s_s^2. Stipling indicates negative values for the covariance.

time series of n and y at each grid point was detrended using a first order differencing method to assess if the observed correlation was due to trends in both the counts and sample local mean vorticity. Here we define Δn and Δy as

$$\begin{aligned} \Delta y_t &= y_t - y_{t-1}, \\ \Delta n_t &= n_t - n_{t-1}, \end{aligned}$$

where t is the extended winter. Figure 4 b shows the map of the correlation between Δn and Δy. For the North Atlantic storm track the magnitude and sign of the correlation between Δn and Δy is roughly equal to the correlation between n, y. The statistical significance of the correlation was assessed using the $cor.test$ function in R, and values of $cor(n, y)$ and $cor(\Delta n, \Delta y)$ which were significant at the 5% level are shown in Fig. 4 a,b. The positive correlation over northern Europe and negative correlation over the Gulf stream can both be seen to be statistically significant at the 5% level. From this we can infer that the correlation between frequency and intensity of Northern European extratropical cyclones is not primarily due to trends in the data.

The sensitivity of the sample correlation to barrier width and cyclone intensity was briefly considered. The use of the $\pm 10^\circ$ barrier results in a convergence of the meridians. Using a larger barrier of $\pm 20^\circ$ (not shown) results in an extension of the region of positive correlation further north above Scandinavia, but otherwise the map of the sample correlation is unchanged. The sample correlation statistic for the subset of the 50% most intense events was also investigated (not shown). The location and magnitude of the positive correlation over Scandinavia remains robust, while the negative correlation over the Gulf stream largely disappears. This suggests the negative correlation is a feature of the weaker systems.

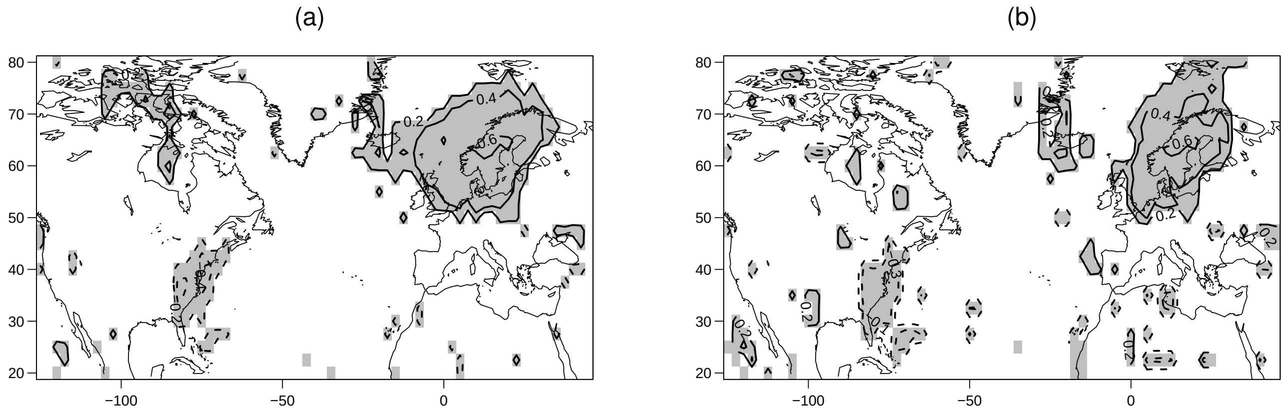

Figure 4. Map of the correlation between a) n and y and b)$\Delta n, \Delta y$ which is significant at the 5% level determined using the $cor.test$ function in R. Solid contours denote positive correlation and dashed negative.

4.2. Covariance between the frequency and intensity

Three parametrizations of a collective risk model are proposed here for the mean and variance of S (see Appendix for detailed derivations). The purpose of these models is to test assumptions such as N and X are independent, and X_i and X_j independent for $i \neq j$. The first parameterization, M_1, assumes there is linear dependence between N and X, and only allows for covariance between consecutive cyclones. Covariance is considered between consecutive cyclones as there may be some dependence due to secondary cyclogenesis, however there is no reason to assume non-neighboring cyclones will be related. The first model assumes that:

$$\mu_X|N = \beta_0 + \beta_1 N$$

$$\sigma_{XX}|N = \begin{cases} \sigma_X^2 & \text{for} \quad i = j \\ \rho\sigma_X^2 & \text{for} \quad i = j \pm 1 \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where ρ,σ_{XX} is the correlation and covariance between consecutive cyclone intensities respectively. The mean and variance of S can then be shown to be given by

$$\begin{aligned} \mu_S &= \beta_0\mu_N + \beta_1(\sigma_N^2 + \mu_N^2) \\ \sigma_S^2 &= \sigma_X^2\mu_N + 2(\mu_N - 1)\rho\sigma_X^2 + \beta_0^2\sigma_N^2 + \beta_1^2\sigma_N^2 \\ &+ 2\beta_0\beta_1(\mu_{N^3} - \mu_N\mu_{N^2}) \end{aligned} \tag{5}$$

(see Appendix). Sample estimates for σ_X^2 (s_x^2), ρ ($cor(x_i, x_j)$) were calculated from the dataset at each grid point, along with maximum likelihood estimates and standard errors for $\hat{\beta}_0, \hat{\beta}_1$ which were calculated using the lm function in R (not shown). Figure 5 shows the modelled variance σ_S^2 of the aggregate risk as well as the ratio of the modelled variance to the sample variance (σ_S^2/s_s^2). From Figure 5 a,b σ_S^2 can be seen to provide a reasonable approximation to the sample variance s_s^2, as it is within $\pm 5\%$ for most grid points.

For the second model parameterization, M_2, X and N are assumed independent and Eqn. 4 becomes $E[X_i] = \hat{\beta}_0 = \bar{x}$. Figure 5 c,d shows σ_S^2 underestimates s_s^2 by between $10 - 50\%$ over the storm tracks, with the greatest discrepancy over northern Europe. For regions of cyclogensis over the Gulf Stream, σ_S^2 is greater than the sample variance s_s^2 by up to 50% (see Fig 5 c,d).

For the third model parametrization, M_3, X and N are again assumed linearly related but X_i, X_j are now assumed independent ($\rho = 0$). The modelled variance σ_S^2 (Fig. 5 e,f) provides a reasonable approximation for s_s^2 although there is some under estimation to the east of Scandinavia. The collective risk model is able to account for variance in the aggregate risk S, under suitable modelling assumptions. When X and N are assumed independent the model underestimates the variance of S over the exit of the storm track, and overestimates variance over the Gulf stream. Assuming independence between the intensities of consecutive cyclones does not significantly effect the modelled variance.

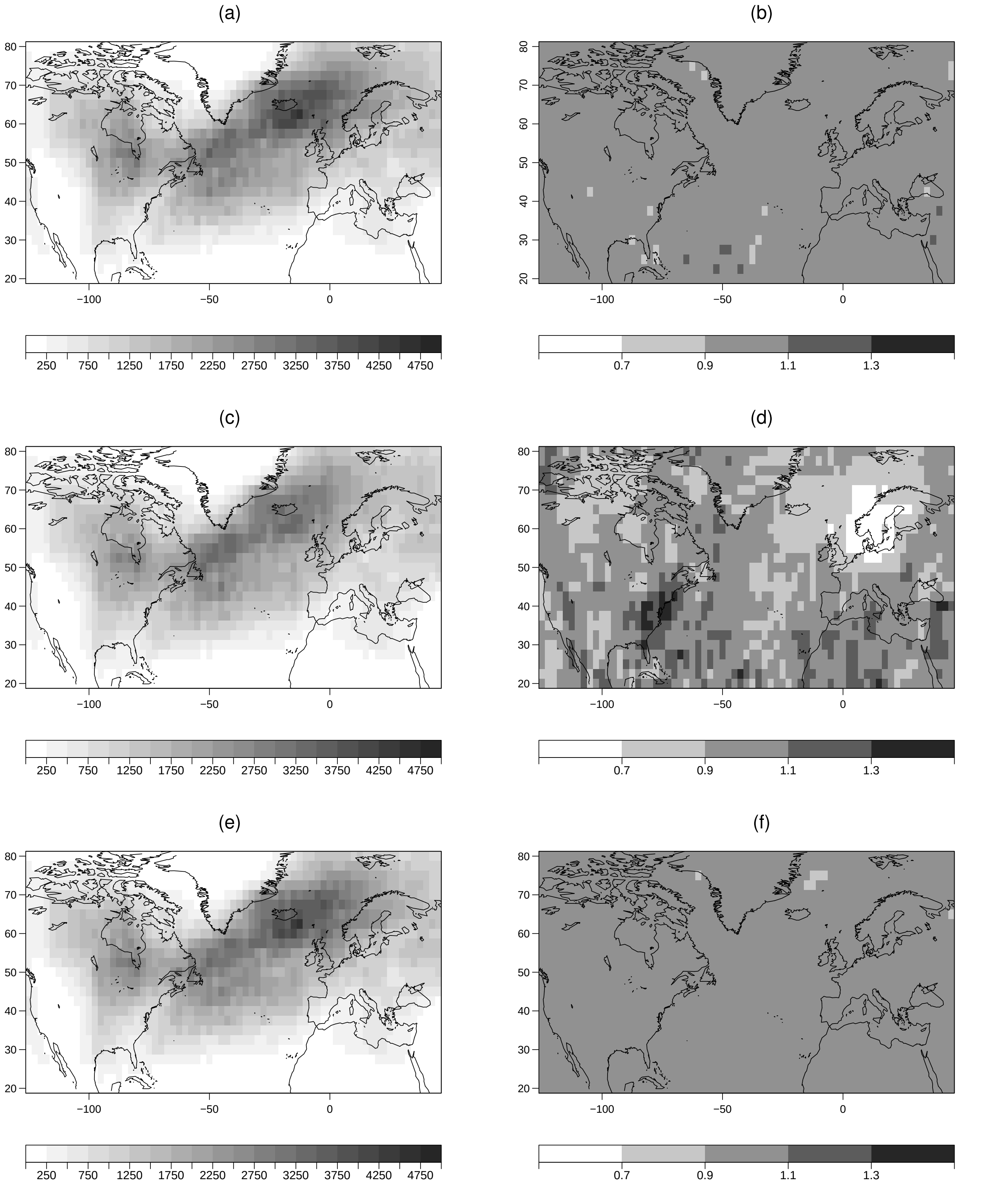

Figure 5. Left column; modelled variances σ_S^2 , right column; ratio of the modelled variance to the sample variance σ_S^2/s_s^2 . a,b) M_1 c,d) M_2 e,f) M_3.

5. Can climate modes explain the frequency-intensity dependence?

This section investigated whether correlation between frequency and intensity could be due to joint forcing by underlying large-scale flow patterns (see Fig. 6). To test this hypothesis, winter cyclone counts and sample local mean vorticity are both regressed on the same set of climate indices as explanatory variables. Similar approaches have been successful in previous studies for explaining the clustering of extratropical cyclones (Mailier *et al.* 2006; Vitolo *et al.* 2009).

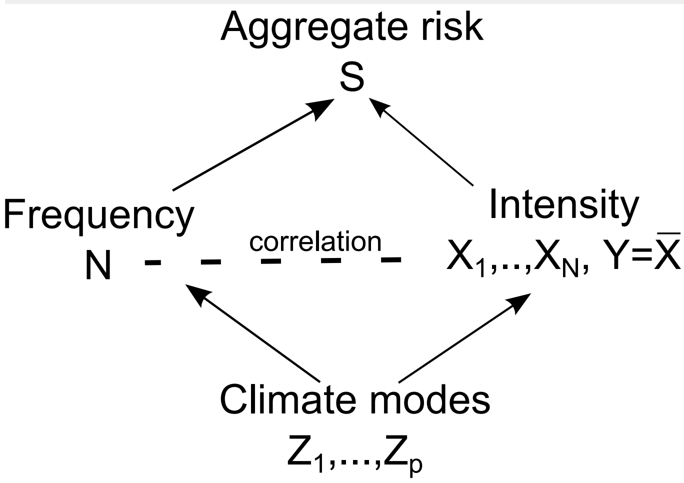

Figure 6. Schematic showing the suggested relationship between large scale flow patterns, frequency and intensity and the aggregate risk.

5.1. *Large scale flow patterns*

Barnston and Livezey (1987) identified 10 teleconnection patterns which describe the state of the large scale flow for the Northern Hemisphere. Monthly indices for these teleconnection patterns between 1950-2003 were obtained from the Climate Prediction Center[§]. The indices were calculated using Rotated Principal Component analysis applied to monthly mean 700-mb geopotential height anomalies between January 1950 - July 2003, then for every month, 10 leading orthogonal functions (EOFs) are selected and the amplitudes are standardized to zero mean and unit variance. The teleconnection indices are mutually uncorrelated by definition, making them useful as a basis of explanatory variables in regression models. Given the results of (Mailier *et al.* 2006) only the first 5 EOFs are considered here (in order); the North Atlantic Oscillation (NAO), the East Atlantic Pattern (EAP), the Scandinavian Pattern (SCP), the East Atlantic/West Russian Pattern (EWP) and the Polar/Eurasian Pattern (POL).

The NAO, the leading mode of climate variability in the Northern Hemisphere, is characterized by a meridional dipole of pressure anomalies of opposite sign located over Iceland (low) and the Azores (high). The positive phase, which corresponds to below normal pressure over Iceland, has already been linked to increased cyclone activity over the North Atlantic in previous studies, e.g. (Pinto *et al.* 2009; Rogers 1997; Trigo 2006; Hurrell and Van Loon 1997). The EAP and SCP are also important modes of variability in the winter months, and describe changes in pressure and in the position and speed of the North Atlantic jet stream which can influence cyclone activity (Woollings *et al.* 2010; Bueh and Nakamura 2007).

Extratropical cyclones passing within $\pm 10^\circ$ north or south of the grid point nearest Gothenburg (Sweden) [12.5°E,57.5° N] were analyzed in detail as this location exhibits the strongest positive correlation between the frequency and intensity ($r = 0.47$). Cyclones passing the grid point closest to Barcelona (Spain) [2.5°E, 40° N] were also investigated as this is a location which has low negative correlation between n and y ($r = -0.10$). Correlation maps of n, y and the $700mb$ stream function were used to identify possible teleconnection patterns driving both frequency and mean intensity of extratropical cyclones. The $700mb$ stream function was chosen as it had been used in a previous study investigating the relation between large-scale flow and extratropical cyclone activity in the same region (Bueh and Nakamura (2007)).

Correlation maps for n and y with Ψ_{700} at Gothenburg (Fig. 7a,b) show a broadly similar pattern, which shows a strong resemblance to the SCP, with a centre of action over Scandinavia and another of opposite sign over western Europe. The correlation of $\bar{\Psi}_{700}$ and n for Barcelona shows a centre of action centred over central Europe, with two other centres; one of opposite sign over west Russia/Kazakhstan, and another of the same sign located over the Gulf Stream (Fig. 7c). The map of the correlation between the mean intensity y and Ψ_{700} also shows a tripole pattern, except the centre of action over Central Europe is now the weakest of the three, and the location of the other two centres has been shifted northwards (Fig. 7d). These figures suggest that there may be different physical mechanisms for the steering and intensification of cyclones near Barcelona.

5.2. *Regression modelling of frequency and intensity*

Regression models were developed for the frequency N and mean intensity Y to formally assess the association with large-scale flow patterns. The regression models for N and Y used

§http://www.cpc.ncep.noaa.gov/data/teledoc/telecontents.shtml

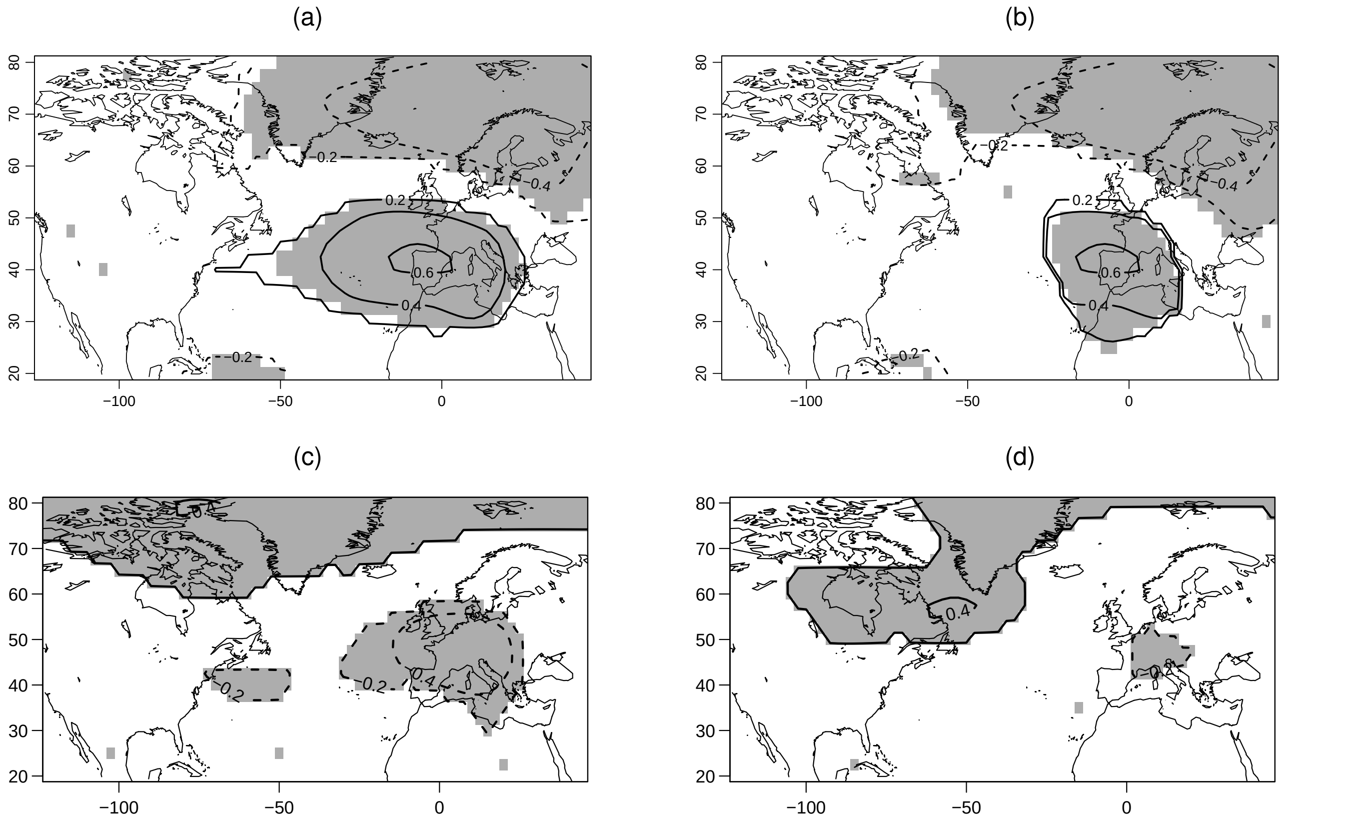

Figure 7. Plots of the correlation between the $700mb$ stream function Ψ_{700} and (a) Gothenburg storm counts, (b) Gothenburg mean storm vorticity, (c) Barcelona storm counts, (d) Barcelona sample mean vorticity. Grey shading indicated the correlation is significantly different from zero at the 5% level level according to a t test. Solid contours denote positive correlation and dashed negative.

teleconnection patterns considered particularly relevant for the North Atlantic region as explanatory variables. As well as the three North Atlantic teleconnection patterns discussed above, there is the East Atlantic/West Russian (EWP) pattern and the Polar/Eurasian (POL) pattern active in the region for some winter months.

The occurrence of natural hazards is often modelled using a Poisson distribution (e.g. Mailier *et al.* (2006); Katz (2002)). The winter cyclone counts N for Gothenburg and Barcelona were modelled as Poisson distributed with rate parameter λ_n (see e.g. Aitkin *et al.* (2009) for more on Poisson regression in R). The mean number of cyclone counts was related to the winter means of the teleconnection patterns using the following generalised linear model:

$$N \sim Poisson(\lambda_n)$$
$$log(\lambda_{n,t}) = \beta_0 + \sum_{k=1}^{6} \beta_k z_{k,t},$$

see e.g. Cameron and Trivedi (2013) Section 2.3. Here $k = 1, \ldots, 6$ and $t = 1, \ldots, 53$ is the year and $z_{2,t}, ,..., z_{6,t}$ are the values of the extended winter means of the teleconnection indices for the North Atlantic in year t. The Polar/Eurasian pattern $z_{6,t}$ is inactive during October, November and March and is set to zero for these months. The coefficient β_1 accounts for any linear time trend, and $\beta_2, ..\beta_6$ are the dependence of the cyclone counts on the teleconnection patterns.

Following Vitolo *et al.* (2009), a Lagrange multiplier test is used to formally assess if there is overdispersion/underdispersion not accounted for by the Poisson regression. This is done by testing for overdispersion against the Katz system where the test statistic is

$$TLM = 0.5 \sum_{i=1}^{m} [(n_i - \mu_i)^2 - n_i] / \sqrt{0.5 \sum_{i=1}^{m} \mu_i^2},$$

see Cameron and Trivedi (2013) Sec. 5.4.1. At Gothenburg, there was found to be some (residual) underdispersion of counts, but it is not significant at the 5% level. From this and the residual analysis the Poisson GLM is concluded to be an appropriate model for the winter cyclone counts.

Maximum likelihood estimates for the coefficients ($\hat{\beta_k}$) for the winter count models for Gothenburg and Barcelona are given

Table 1. The regression coefficient estimates in the Poisson regression of cyclone counts over Gothenburg and Barcelona. Standard errors are in brackets, estimators with $p \leq 0.05$ are in bold.

Indices	z_k	Gothenburg $\hat{\beta_k}$	Barcelona $\hat{\beta_k}$
Time	z_1	0.17 (0.14)	-0.23(0.19)
NAO	z_2	0.09 (0.06)	**-0.20** (0.09)
EAP	z_3	0.02 (0.05)	0.11 (0.07)
SCP	z_4	**-0.19** (0.06)	-0.04 (0.09)
EWP	z_5	0.03 (0.08)	-0.09 (0.11)
POL	z_6	-0.03 (0.08)	-0.21 (0.11)

Table 2. The regression coefficient estimates in the linear regression of cyclone sample local mean vorticity over Gothenburg and Barcelona. Standard errors are in brackets, estimators with $p \leq 0.05$ are in bold.

Indices	z_k	Gothenburg $\hat{\alpha_k}$	Barcelona $\hat{\alpha_k}$
Time	z_1	**0.91** (0.29)	0.04 (0.38)
NAO	z_2	-0.12 (0.29)	0.08 (0.17)
EAP	z_3	-0.04 (0.10)	-0.23 (0.14)
SCP	z_4	**-0.58** (0.14)	0.18 (0.18)
EWP	z_5	0.09 (0.17)	0.22 (0.22)
POL	z_6	-0.10 (0.17)	-0.39 (0.22)

in Table 1. For the grid point closest to Gothenburg, only the SCP showed a significant (according to a t-test at the 5% level) relationship with the winter cyclone counts. The SCP is negatively associated with the number of cyclones passing near Gothenburg. For extratropical cyclones passing near Barcelona only the NAO shows a significant (negative) relationship with counts. These findings are consistent with those of Vitolo *et al.* (2009); Mailier *et al.* (2006), where the SCP slope estimate was significant over most of Scandinavia, including the Gothenburg grid cell.

Normal linear regression was found to be suitable for modelling winter sample local mean vorticity Y, with the modelled intensity regressed against the same teleconnection indices as the modelled counts. The extended winter sample local mean vorticity is then

$$Y \sim N(\mu_y, \sigma)$$
$$\mu_{y,t} = \alpha_0 + \sum_{k=1}^{6} \alpha_k z_{k,t}.$$

The $z_{k,t}$ is the same as for the Poisson model, and the regression coefficient estimates α_k have the same interpretations as above except they are (linearly) related to winter sample local mean vorticity instead of cyclone counts.

Maximum likelihood estimates for the regression coefficients for Gothenburg and Barcelona sample mean vorticity ($\hat{\alpha_k}$) are given in Table 2. The estimate for the time trend coefficient $\hat{\alpha_1}$ is highly significant over Gothenburg suggesting non-stationarity in the winter mean intensity. This is consistent with Vitolo *et al.* (2009) where non-stationarity was found for the counts of intense cyclones over the same region, but not for all cyclones. In Vitolo *et al.* (2009) it was suggested the increase in the rate of intense cyclones could be due to either climatic change or inhomogeneities in the reanalysis dataset. The SCP coefficient ($\hat{\alpha_4}$) is also highly significant for Gothenburg winter sample mean vorticity, suggesting this may be a driver of cyclone intensity. The model of Gothenburg sample mean vorticity has an R^2 value of 0.49 meaning just under half the variance in sample local mean vorticity is explained by the model. For Barcelona none of the teleconnection indices are significant at the 5% level, although the EAP and POL are significant at the 10% level. The Barcelona model has a lower R^2 value of 0.22, suggesting large scale flow patterns are of less use in explaining winter mean intensity over this region.

5.3. *Modelled covariance*

The modelled covariance between $\hat{N}$ and $\hat{Y}$ for Gothenburg can be expressed as

$$\begin{aligned} Cov(\hat{N},\hat{Y}) &= cov(\alpha_0 + \alpha_1 z_1 + ...\alpha_6 z_6, e^{\beta_0+\beta_1 z_1+...\beta_6 k z_6}) \\ &= cov(\alpha_1 z_1, e^{\beta_1 z_1}) + cov(\alpha_2 z_2, e^{\beta_2 z_2}) + cov(\alpha_3 z_3, e^{\beta_3 z_3}) \\ &+ cov(\alpha_4 z_4, e^{\beta_4 z_4}) + cov(\alpha_5 z_5, e^{\beta_5 z_5}) + cov(\alpha_6 z_6, e^{\beta_6 z_6}). \end{aligned} \tag{6}$$

since the teleconnection indices are uncorrelated by definition and so $cov(z_i, z_j) = 0$ when $i \neq j$. From Eqn 6 and the modelled standard deviations $\sigma_{\hat{N}}, \sigma_{\hat{Y}}$ the modelled correlation can be estimated for Gothenburg, $Cor(\hat{N},\hat{Y}) = 0.32$ (compared to the observed correlation $Cor(n, y) = 0.48$). The regression models using teleconnection indices as explanatory variables account for two thirds of the correlation over Gothenburg. Using the same method for Barcelona the modelled correlation is $Cor(\hat{N},\hat{Y}) = -0.02$. Regression models using teleconnection indices as explanatory variables are thus suitable for reproducing the positive correlation between N and Y over northern Europe. For cyclones passing near Barcelona teleconnection indices are possible drivers for the cyclone counts but not for the sample mean vorticity.

6. Regression models for all grid points

The analysis conducted at Gothenburg and Barcelona was repeated for all Northern Hemisphere grid points. The regression coefficients for the linear trend term, the NAO, SCP and EAP were significant for the models for N and Y over much of the North Atlantic and Europe. Reduced models with only these 4 explanatory covariates are now assessed.

The maximum likelihood estimates of the North Atlantic regression parameters for cyclone counts and sample local mean vorticity, $\hat{\beta_k}$ and $\hat{\alpha_k}$, $k = 1, .., 4$ are shown in Fig. 8 and 9 respectively. Statistical significance was determined with a t test at the 5% level. There is a clear relationship between large scale flow patterns and both winter cyclone counts and winter sample local mean vorticity (Figs. 8, 9). The NAO parameter for counts is statistically significant over much of the North Atlantic (Fig. 9b). The positive phase of the NAO is associated with an increase in extratropical cyclones across Canada, Greenland and Iceland as well as north Great Britain, as well as with a decrease in cyclones over the North West coast of Africa. Although the NAO appears to be the single greatest driver in cyclone counts of the 3 teleconnection indices considered here, it is not significant for the modelled winter cyclone counts over the region of Europe (Scandinavia, North Germany, Great Britain, Benelux) where the positive correlation was observed (Fig. 4a). The NAO parameter for sample local mean vorticity is significant over Iceland and most of the Norwegian sea. There appears to be very few grid points where the NAO parameter is significant for *both* the frequency and intensity. From Fig. 8c, the SCP coefficient for counts is significant over a region extending from the east coast of Greenland, over Scandinavia and into eastern Europe. The SCP coefficient estimate for sample local mean vorticity (Fig. 9c) is significant over Scandinavia, north Germany and the Benelux countries. From these plots it would seem likely that, as with Gothenburg, the SCP coefficients account for much of the correlation because of its importance for explaining both counts and mean intensity in many locations.

The EAP coefficient is significant for cyclone counts over East coast of the United States across the Atlantic up to the Iberian Peninsula (Fig. 8 d) . The EAP coefficient is also significant for sample local mean vorticity over some grid points in the eastern United States and to the south east of Greenland (Fig. 9 d). There are few grid point where the EAP coefficient is significant for both counts and intensity. The time trend coefficient for counts is significant for part of the United States eastern seaboard and for a few grid point over and around the north of Great Britain. For sample local mean vorticity the time trend coefficient is significant for a large region of North Europe and over Canada. This agrees with the findings in Vitolo *et al.* (2009) where a linear time trend was not found to be significant for Poisson regression of 3 monthly counts over most grid points, but was found to be significant over Canada and north western Europe.

The coefficient estimates for the SCP are significant for both counts and mean intensity over much of the Scandinavian peninsula, as well as parts of Northern Germany and the Benelux countries. The plot of the modelled correlation (Fig. 10) shows that the large scale flow patterns account for much of the observed positive correlation for the Atlantic region.

7. Discussion about possible physical mechanisms

The Scandinavian pattern modulates both cyclone frequency and mean intensity, and thereby induces positive correlation between the frequency and the intensity. Possible physical mechanisms are discussed here for how the Scandinavian pattern interacts with extratropical cyclones. The negative correlation observed over the Gulf stream is also briefly considered.

7.1. Positive correlation

In the previous section it was shown that negative phases of the Scandinavian pattern are associated with increased cyclone activity; more occurrences with higher mean intensity. It is important to distinguish between cause and effect, as increased cyclone activity may also result in persistent negative SCP index values. The potential for synoptic scale activity, such as cyclones to influence the state of the background flow has been discussed in the literature, such as in Pinto *et al.* (2009) where it explains that cyclones themselves may play a major role in steering the phase of the NAO. However, in Whitaker and Sardeshmukh (1998) it was shown that while transient eddies/cyclones can affect the background upper tropospheric circulation, the latter

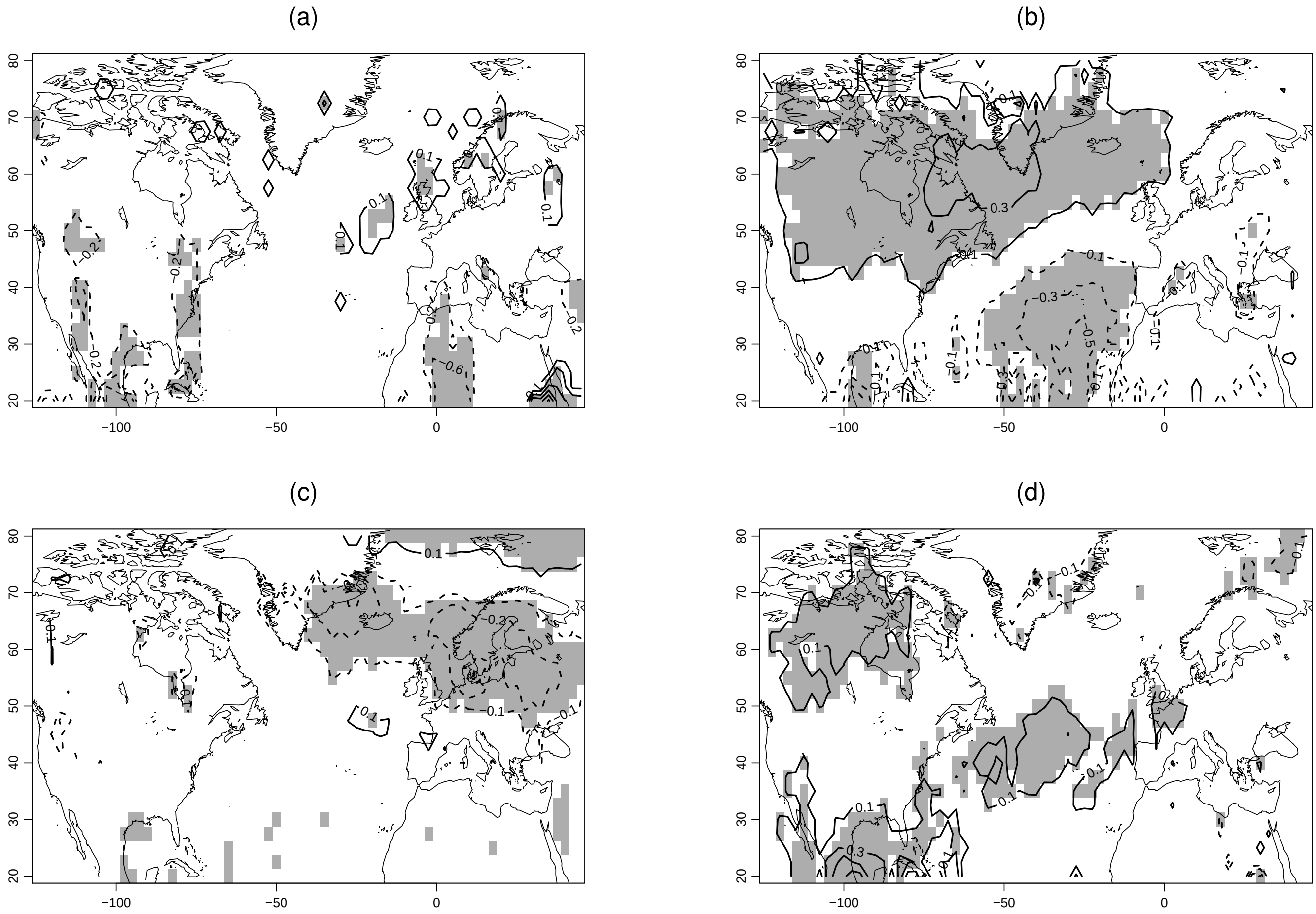

Figure 8. Slope estimates from Poisson regression of storm counts on (a) linear time trend (b) North Atlantic Oscillation (c) Scandinavian pattern (d) East Atlantic Pattern. Solid (dashed) lines indicate positive (negative) values and grey shading means the coefficient is significantly different from zero at the 5% level level according to a t-test.

is more important in initiating eddy formation and controlling intensification.

Four key environmental factors which control cyclone intensification were considered in Pinto *et al.* (2009): latent energy (equivalent potential energy 850 hPa), upper air baroclinicity, horizontal divergence and jet stream strength. The growth of extreme cyclones was shown to be related to these four explanatory variables, with the jet stream location and velocity in particular showing a clear connection to extreme cyclone intensification. The major (i.e. NAO, EAP, SCP) extratropical teleconnections essentially describe jet stream variability over the ocean basins (Woollings *et al.* 2010).

In Raible (2007) the occurrence of extreme intensified cyclones in Northern Europe are linked to a rotated NAO like pattern, which corresponds to the SCP as identified here and discussed in Section 5.1. In Hanley and Caballero (2012) it was also shown that intense European extratropical cyclones occurred during an eastward shifted NAO-like pattern, which is again qualitatively similar to a negative phase of the Scandinavian pattern. The large scale low-pressure system located over the Scandinavian peninsula, associated with negative values of the SCP index, helps steer extratropical cyclones into Northern Europe, as well as generating an intense baroclinic jet streak which acts to intensify cyclones. In particular, a case study of extreme storm Daria was conducted which showed that the background atmospheric conditions *preceded* Daria's birth (Hanley and Caballero 2012) .

Most of the studies cited above have investigated the link between environmental factors and cyclone clustering over synoptic time scales. Synoptic variability over the North Atlantic and Europe is related to the NAO and other teleconnection patterns, which in turn has been related to the occurrence and development of cyclones over synoptic time scales (Pinto *et al.* 2009). However this study has considered aggregate cyclone activity over a longer 6 month extended winter period. The same arguments put forward for linking teleconnection patterns and extratropical cyclone activity for shorter time periods remain

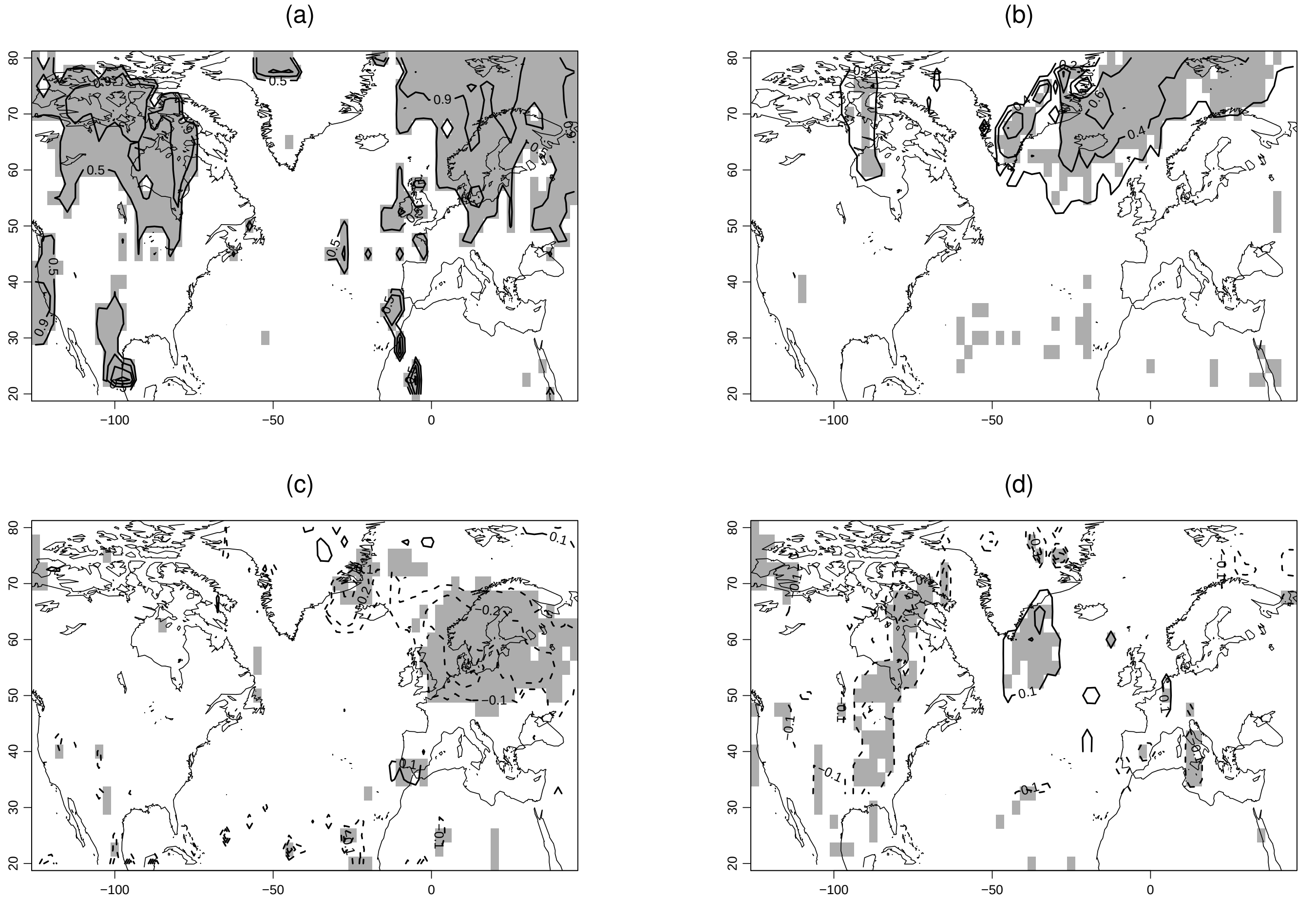

Figure 9. Slope estimates from multiple linear regression of sample mean vorticity on (a) linear time trend (b) North Atlantic Oscillation (c) Scandinavian pattern (d) East Atlantic Pattern. Solid (dashed) lines indicate positive (negative) values and grey shading means the coefficient is significantly different from zero at the 5% level level according to a t-test.

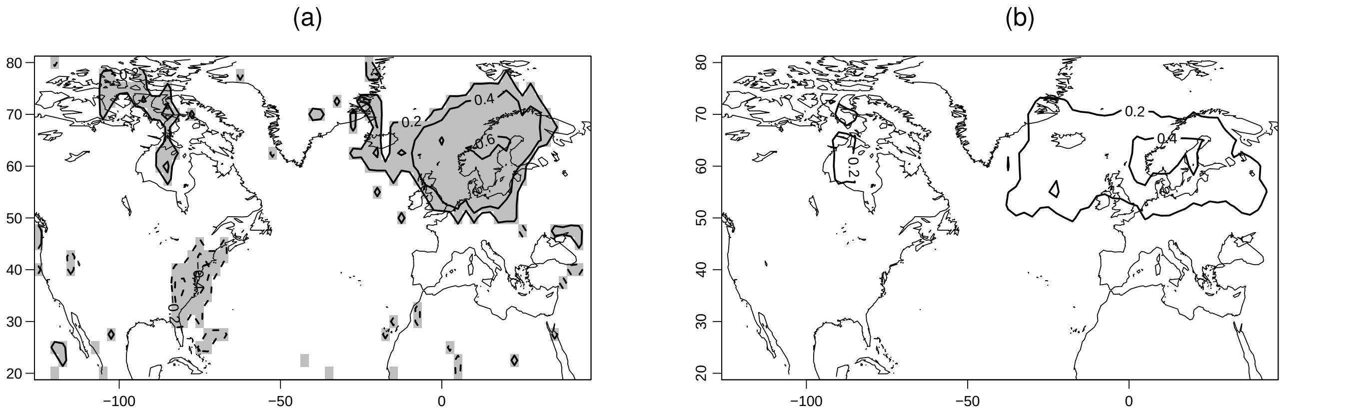

Figure 10. Plots of the (a) sample correlation and (b) the modelled correlation between the cyclone counts n and sample mean vorticity y. Grey shading indicated the correlation is significantly different from zero at the 5% level level according to a t test. Solid contours denote positive correlation and dashed negative.

valid for the 6 month aggregated period. In particular the winter mean NAO has been linked to inter annual variability in the mid Tropospheric baroclinicity (Baldwin *et al.* 1994). The baroclinicity in the mid Troposphere is related to surface temperature gradients which in turn influence cyclone activity (Raible 2007).

The relation between the Scandinavian pattern and its climatic impact, as well as possible forcing mechanisms is discussed in Bueh and Nakamura (2007). For negative phases of the SCP the 200mb zonal wind anomalies are observed over Northern Europe over the same region positive correlation is observed (see Fig. 3 in Bueh and Nakamura (2007)), as well as increased baroclinicity over the same area. It was shown in Bueh and Nakamura (2007) that there is positive feedback over the exit region of the North Atlantic storm track between the Scandinavian pattern and passing cyclones. This positive feedback between extratropical cyclones and the background atmospheric flow is a possible mechanism for the frequency-intensity dependence found here, as well as the observed clustering of intense cyclones shown in Vitolo *et al.* (2009); favourable environmental conditions result in increased cyclone activity through enhanced steering and intensification, which in turn help maintain these conditions.

This work is of particular relevance to the insurance industry. Whilst many catastrophe reinsurance contracts begin on January 1st, others begin at different dates. For example from April 1st or from July 1st. Therefore a reinsurance contract may cover 2 halves of different consecutive winter seasons (January 1st renewal) or 1 whole winter season. If frequency intensity dependence results in increased financial risk from extra-tropical cyclones for the entire winter season there will also be an impact, albeit smaller, from the combination of 2 independent half-seasons.

7.2. Negative correlation

The negative correlation discovered over the Gulf stream is also possibly due to some interaction between the background atmospheric flow and extratropical cyclones. However none of the regression coefficients from either the North Atlantic or North Pacific sample local mean vorticity models are significant over the Gulf stream. One possible mechanism is the velocity and position of the North Atlantic subtropical and/or eddy driven jet stream. In Pinto *et al.* (2009) factors contributing to the development of extreme North Atlantic cyclones was considered. It was shown that during strongly positive phases of the NAO the jet stream over North America is enhanced, resulting in increased cyclogenesis and more extreme storms over the North Atlantic. However extratropical cyclones originating from the West Atlantic/North American east coast typically develop slowly, not reaching maximum intensification until further East into the Atlantic (Dacre and Gray 2009). In Pinto *et al.* (2009) it can be seen that during strongly negative NAO phases, although there are fewer extreme cyclones, they reach their point of maximum intensification closer to the eastern U.S.

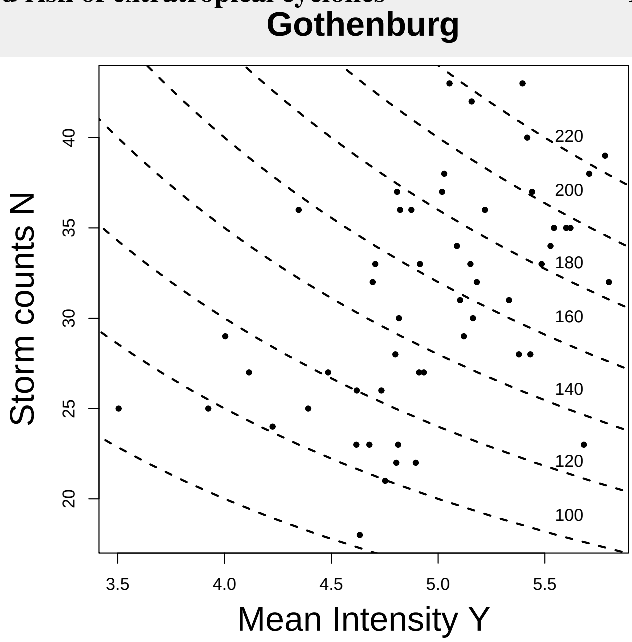

Figure 11. Plot of sample mean vorticity Y against storms counts N for extratropical cyclones passing near Gothenburg. Contours of S have been added for $S = 80, 100, 120, 140, 160, 180, 200$

8. Extremes of the aggregate risk

Extremes in the aggregate risk are due to either an above average number of occurrences or the occurrence of one or more high intensity events, or a combination of both (see Fig 11). As discussed in Section 2 analytic results for the distribution of S; F_S, are generally not available except under certain restrictive conditions. Uncertainty would be large when trying to estimate the distribution of extremes through simulation, so instead bootstrapped confidence intervals and Cantelli bounds are used to investigate upper limits on the quantiles of the aggregate risk distribution.

8.1. Bootstrap confidence intervals

Bootstrap confidence intervals can be constructed to estimate upper bounds for the return levels of the aggregate risk. By assuming independence between the frequency and intensity a block boostrapping method can be used to construct confidence intervals for the return level at T years (and thus the exceedance probabilities $p = 1 - 1/T$) as follows.

For r in $1, ..., R$;

1. Construct a resampled (with replacement) time series of cyclone counts $n^*_{1,r}, ..., n^*_{m,r}$.
2. Construct a resampled (with replacement) time series of cyclone sample local mean vorticity $y^*_{1,r}, ..., y^*_{m,r}$.

3. Calculate the corresponding time series of aggregate losses $s^*_{1,r}, ..., s^*_{m,r}$ for each r, from the resampled counts and sample local mean vorticity.
4. Calculate the new resampled return levels; $q^*_{p,r}(s)$. Here $q^*_{p,r}(s)$ is the empirical quantile estimate for the rth resampled time series of aggregate losses s.

Then the 90% confidence intervals for the pth quantile are the $5th$ and $95th$ percentiles of the resampled $q^*_{p,r}(s)$. As n and y are assumed independent in this bootstrapping algorithm, if the empirical return level plot for s diverges outside of the confidence intervals this would provide an indicator that inclusion of frequency-intensity dependence is necessary to model extremes of the aggregate loss.

8.2. *Cantelli bounds*

Upper bounds for return levels can be calculated from Cantelli's inequality (Royden 1953). This states that for a positive real random variable S with mean μ_s and variance σ_S^2

$$Pr(S \geq \mu_S + k\sigma_S) \leq \frac{1}{1+k^2} = \frac{1}{T} \tag{7}$$

where $k \geq 0$, and T is the return time. Cantelli bounds can be used to consider the effect of covariance between the frequency and intensity on the aggregate loss S for exceedance probabilities beyond that which have been observed. To do this two cases are considered, in the first the frequency and intensity are considered independent, and μ_S, s_S^2 are estimated as,

$$\begin{aligned} \bar{s} &= \bar{n}\bar{y} \\ s_s^2 &= V_n + V_y, \end{aligned}$$

as in Section 2. In the second case, the the sample mean and variance of s are

$$\begin{aligned} \bar{s} &= \bar{n}\bar{y} + cov(n, y) \\ s_s^2 &= V_n + V_y + V_c, \end{aligned}$$

which will result in increased Cantelli bounds in Eqn. 7, when $cov(n, y) > 0$, and reduced Cantelli bounds when $cov(n, y) < 0$.

8.3. *Results*

The empirical return level plots for s at Gothenburg and Barcelona are shown in Fig. 12. The bootstrap intervals assume the sample counts n and sample local mean vorticity y are independent. For Gothenburg (Fig. 12 a) this assumption is not valid, and the empirical AEP curve is outside of the bootstrapped intervals for the upper and lower tails, suggesting extremes of the aggregate loss (both high and low) are sensitive to frequency-intensity dependence. The Barcelona sample aggregate loss is contained within the intervals, suggesting that the small amount of negative dependence at this location does not significantly effect the extremes.

The Cantelli bounds provide a (high) upper bound for the return levels. At Gothenburg the Cantelli bounds with and without dependence diverge with increasing return periods, where the upper bound for s is greater when dependence is included (Fig. 12 a). At Barcelona the relation is reversed; the Cantelli bound with dependence has lower return levels than the bound without dependence, although the difference between bounds is less than at Gothenburg (Fig. 12 b). As with the bootstrapped confidence intervals this suggests that positive dependence between the frequency and intensity results in an increase in the extremes of the aggregate loss. Conversely negative dependence may result in a decrease.

The percentage change in the Cantelli bounds for $T = 200$ in Eqn 7) with the inclusion of dependence was $+13.0\%$ at Gothenburg and -5.0% at Barcelona. The ratio for the 1 in 10 year return levels with dependence/without dependence and 1 in 200 year return levels with dependence/without dependence were calculated for all Northern Hemisphere grid points (Fig. 13). Similar conclusions can be drawn as from Gothenburg and Barcelona; locations with positive (negative) dependence show an increase (decrease) in the upper bound for the return levels of the aggregate risk when dependence is included and the difference between the bounds increases with greater return periods.

9. Conclusions

This paper introduces a framework for quantifying the aggregate risk of extratropical cyclones. This framework is then applied to

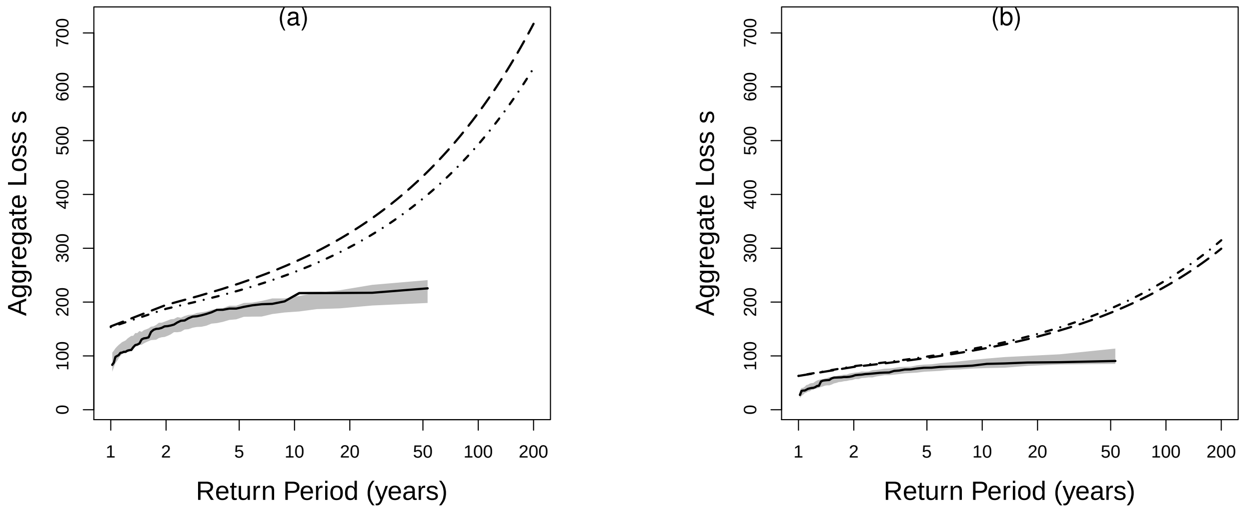

Figure 12. Empirical return level of the aggregate loss for extratropical cyclones s (solid lines) with 90% bootstrap confidence intervals (grey shading), and Cantelli bounds (dashed line X,N non i.i.d, dashed-dotted line X,N i.i.d) against the return period in years at a) Gothenburg and b) Barcelona

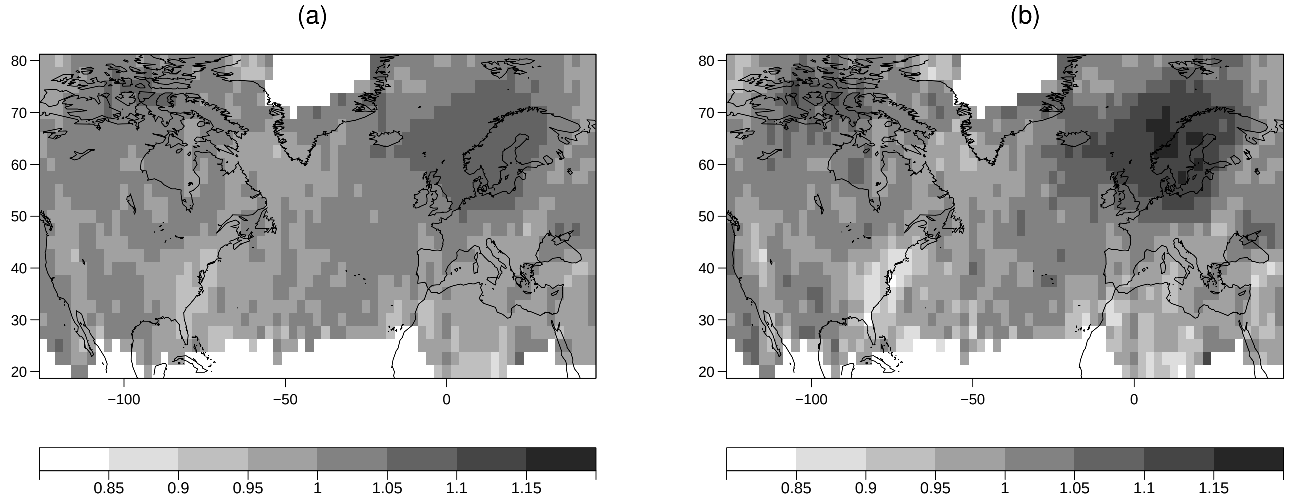

Figure 13. Plots of the ratio of the Cantelli bounds S_D/S_I for (a) the 10 year return level (b) the 200 year return level

extratropical cyclones using a database of tracks for the Northern Hemisphere (covering October-March winters from 1950-2003). Statistical models were used to investigate the sensitivity of the variance of the aggregate risk to dependence between the frequency and intensity of cyclones as well as dependence between successive events.

Statistically significant correlation was found between the frequency and intensity of extratropical cyclones over parts of northern Europe including Scandinavia, Germany and Great Britain as well as the eastern end of the North Atlantic storm track. The findings for extended winter cyclones counts agreed with those of Vitolo *et al.* (2009), concerning linear trends in intense cyclones over Scandinavia, and the effect of large scale flow patterns on cyclone counts.

Joint modulation by large-scale flow patterns is shown to be responsible for generating the covariance between cyclone frequency and mean intensity. The Scandinavian pattern in particular is strongly negatively correlated with both counts and sample local mean vorticity over much of northern Europe. Regressing the counts and sample local mean vorticity on the Scandinavian pattern index is able to reproduce most of the observed correlation. Other important teleconnection indices for both frequency and intensity are the North Atlantic Oscillation and the East Atlantic pattern.

Non-parametric Cantelli bounds and bootstrap confidence intervals were used to investigate the effect of frequency-intensity dependence on the extremes of the aggregate risk distribution for extratropical cyclones passing near Gothenburg and Barcelona. Positive (negative) dependence was shown to result in a increase (decrease) in the exceedance levels. Therefore any statistical model for extratropical cyclone risk which (falsely) assumes frequency-intensity independence will understimate the return periods for extreme events.

The framework presented here is relevant to other natural hazards which have been shown to cluster, e.g. floods and hurricanes (Villarini *et al.* 2013; Mumby *et al.* 2011). . The aggregate risk for any type of meteorological event where the dependency between frequency and intensity is not properly modelled could be underestimated.

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Appendix

Analysis of modelling assumptions

To investigated the effect of modelling assumptions, such as N and X independent, a collective risk model was proposed where,

$$\mu_X|N = \beta_0 + \beta_1 N$$

$$\sigma_{XX}|N = \begin{cases} \sigma_X^2 & \text{for} \quad i = j \\ \rho\sigma_X^2 & \text{for} \quad i = j \pm 1 \\ 0 & \text{otherwise,} \end{cases}$$

where σ_{XX} is the covariance and ρ the correlation between X_i and X_j. This gives

$$\begin{aligned} \mu_S = & \quad E_N\left[\beta_0 N + \beta_1 N^2\right] = \beta_0\mu_N + \beta_1\left(\sigma_N^2 + \mu_N^2\right) \\ \sigma_S^2 = & \quad E_N[N\sigma^2 + 2N(N-1)\rho\sigma^2] + Var_N\left(\beta_0 N + \beta_1 N^2\right) \\ = & \quad \sigma^2\mu_N + 2\left(\mu_N - 1\right)\rho\sigma^2 \\ + & \quad \beta_0^2\sigma_N^2 + \beta_1^2\sigma_{N^2}^2 + 2\beta_0\beta_1\left(\mu_{N^3} - \mu_N\mu_{N^2}\right) \end{aligned} \tag{8}$$

since $Var\left(\beta_0 N + \beta_1 N^2\right) = \beta_0^2 Var[N] + \beta_1^2 Var[N^2]^2 + 2\beta_0\beta_1 Cov[N, N^2]$

and Cov$[N, N^2] = \mu_{N^3} + \mu_N\mu_{N^2}$

Sample estimators

Consider a dataset with years $t = 1, 2, ..., T$. Each year there are n_t events, and for each event there is a severity measure $x_{1,t}, x_{2,t}...x_{n_t,t}$. The estimators for the sample mean and variance of variable n are denoted $\bar{n}, s_n^2$,

$$\begin{aligned} \bar{n} &= \frac{1}{T}\sum_{t=1}^{T} n_t \\ s_n^2 &= \frac{1}{T-1}(\sum_{t=1}^{T} n_t - \bar{n})^2. \end{aligned}$$

The sample mean vorticity y_t in year t is

$$\begin{aligned} y_t &= \frac{1}{n_t}\sum_{i=1}^{n_t} x_{i,t} \\ \bar{y} &= \frac{1}{T}\sum_{t=1}^{T} y_t. \\ s_y^2 &= \frac{1}{T-1}(\sum_{t=1}^{T} y_t - \bar{y})^2 \end{aligned}$$

The sample estimator for the covariance between n and y is

$$cov(n, y) = \frac{1}{T-1}\sum_{t=1}^{T}(n_t - \bar{n})(y_t - \bar{y}).$$

The sample aggregate risk is

$$\begin{aligned} s_t &= \sum_{i=1}^{n_t} x_{i,t} \\ \bar{s} &= \frac{1}{T}\sum_{t=1}^{T} s_t. \\ s_s^2 &= \frac{1}{T-1}(\sum_{t=1}^{T} s_t - \bar{s})^2 \end{aligned}$$

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