Omni-directional Isotropic Surface Wave Cloaks

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This paper presents a general approach to design surface wave cloaks which circumvent the problems associated with superluminal phase velocities and anisotropy. The cloaks proposed in this paper are very thin, just a fraction of a wavelength in thickness, yet can cloak electrically large objects. This solution combines curved, rotationally symmetric geometries with isotropic radially dependent refractive index profiles to achieve perfect cloaking. Three refractive index profiles are obtained: an anti-fish eye, a conical cloak and a cosine cloak, and each is simulated using a full-wave solver. The performance is analysed using dielectric filled waveguide geometries, and the curvature of the surface is shown to be rendered invisible, hiding any object positioned underneath. For the cosine cloak, a transformation of the required dielectric slab permittivity was performed for surface waves propagation, demonstrating the practical applicability of this technique.

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Using a combination of metamaterials and the theory of transformation optics, has been the focus of much attention since the idea was proposed in 2006 [1,2]. Whilst the advances that have been made show much promise for this goal, many limitations have been demonstrated to restrict the performance of such devices. These include the necessity of dispersive materials due to the required superluminal phase velocities, which vastly restricts the bandwidth [3]; the losses associated with the metamaterials required, which produce shadowing [4]; and the anisotropy requirements which create the need for highly complex structures to implement the cloaks [5].

More recently, designs for cloaking devices applied to surface waves have been investigated [6-12]. The Pendry’s cloak design [1] is one of the examples, however, it requires anisotropy, where some of the axial components of the permittivity and permeability tensors are below unity [6-8], which creates complexity in the implementation. Another proposal is to utilise a carpet cloak design [6,7,9,10], but the drawback here is that the cloaks are electrically large, necessarily larger than the cloaked object, and include approximations limiting the accuracy of the cloak. In addition, some of these designs for surface wave cloaks have the disadvantage of being uni-directional, which limits the use in a practical application.

This paper presents a solution to the problem of surface wave cloaking which utilises curved geometries combined with graded index media to create perfect, isotropic, omni-directional cloaks which can be used to cover perturbations on surfaces, to hide objects placed underneath. These cloaks could be applied to the design of surface wave-based antenna devices [13] which are retro-fitted to vehicles or airborne platforms, where perturbations of the surface are necessary for structural or aerodynamic reasons, but create scattering of the surface waves which is detrimental to their performance.

The index profile necessary to make a rotationally symmetric curved surface invisible to waves confined to a surface can be derived using the same technique that was used to determine the Maxwell-fish eye profile from a sphere [14,15], but in the converse sense. Rather than utilising a graded index on a flat plane that emulates the ray behaviour on a sphere, here it is the ray behaviour on a flat homogeneous plane that is being reproduced by a curved surface with a graded index. The latter is calculated by equating the optical

![FIG. 1 (color online). A diagram illustrating the orthogonal ray paths that are equated in order to derive the appropriate index distribution to cloak a particular surface, of radius $a$ and height $b$. The blue lines are the radial geometric path lengths ($l_1$ and $l_2$) and the red lines denote the circular geometric paths ($s_1$ and $s_2$), in the curved space and flat space. The green dashed line denotes the length from the origin to the surface, $R(\theta)$.](image-url)
path length of a ray as it traverses across a flat plane with homogeneous refractive index, to the optical path on the rotationally symmetric curved plane with a refractive index that depends on the angle for two orthogonal paths. The first is a circular path (of fixed radius), illustrated in Fig. 1 in red, where the geometric paths are equal, but the optical paths are different. The second is a radial path (of fixed angle) which connects the outer radius of the device, \( r = a \), to the central point, \( r = 0 \), shown in blue in Fig. 1.

The optical path length of a ray travelling the two radial geometric paths, \( l_1 \) and \( l_2 \), are equated to give

\[
\int_0^{\pi/2} n(\theta)\sqrt{R(\theta)^2 + R'(\theta)^2} \, d\theta = \int_0^a dr
\]

and the optical path length of a ray following the two circular geometric paths, \( s_1 \) and \( s_2 \), are equated to give

\[
2\pi R(\theta)\sin(\theta)n(\theta) = 2\pi r
\]

Equations (1) and (2) can be combined, and after some manipulation, take the form

\[
n'(\theta) = \frac{\sqrt{R(\theta)^2 + R'(\theta)^2} - R(\theta)\cos\theta}{R(\theta)\sin\theta} - R(\theta)\frac{n(\theta)}{n'(\theta)}
\]

This general approach can be applied to any rotationally symmetric surface, provided that the curvature of the surface, \( dz/dr \), does not change sign. Three surface cloaks are demonstrated in this letter. The first case study is the anti-fish eye cloak, so named because while the Maxwell fish eye lens is the mapping of a hemisphere onto a flat plane, this index distribution is calculated with a mapping of a flat plane onto a hemisphere. This is a relatively simple distribution to derive, since the radius of the surface does not depend on the angle. The analytical solution to equation (3) is \( n(\theta) = \tan(\theta/2)/\sin(\theta) \), and is valid for any radius, \( a \), of the hemispherical surface. Although the index distribution here perfectly corrects the phase difference of the waves traversing the sphere, to equal those that only propagate across the flat plane, this system will produce reflections at the boundary of the surface due to the 90° transition between the flat and the spherical surfaces. For this reason, the second design was chosen to have a conical shape, and this also has an analytical solution of the form

\[
n(\theta) = ay\left(\int_0^{\pi/2} \frac{\gamma\beta}{R(\theta)\sin\theta} d\theta\right)R(\theta)\sin\theta,
\]

where \( \beta = \sqrt{R(\theta)^2 + R'(\theta)^2} \) and \( \gamma = \exp\left(\int \frac{\beta}{R(\theta)\sin\theta} d\theta\right) \), valid when \( b < a \). This index distribution reaches asymptotically to zero at \( r = 0 \), but to a good approximation, this can be truncated to a quarter of the maximum. Whilst the conical cloak reduces the reflections from the boundary, it does not remove then completely, particularly when \( b \rightarrow a \). The final shape solves the problem of reflections entirely, and is the form of a cosine function. However, in this case, equation (3) does not have an analytical solution, and therefore a numerical approach must be employed to calculate the value of \( n(\theta) \).

This cloaking behaviour is only exhibited if the rays or electromagnetic waves are confined to the surfaces under consideration. Two techniques can be employed to achieve this confinement. The first is to fill a thin, curved waveguide [16] with the index distribution, and the second is to utilise surface wave propagation. Surface wave supporting structures, with a variable refractive index, can be achieved with metasurfaces [17] or dielectric slabs (varying thickness or permittivity).

The first quantification is performed using waveguide structures. Fig. 2 shows the simulations which demonstrate the performance of all three of the aforementioned surface cloaks. Fig. 2(a-c) illustrate the propagation of a plane wave incident from the left in a homogeneous dielectric filled waveguide of thickness \( \lambda_m/10 \) where \( \lambda_m \) is the wavelength in the medium for the hemispherical (a), conical (b) and cosine (c) waveguides. The radius of the curved part of the guide, \( a \), is \( 4\lambda_m \). It is clear that the plane wave fronts are severely distorted by the curvature in all the cases. Fig. 2(d-f) are simulations of the same three waveguide structures, when the dielectric filling the guide is replaced with the appropriate index for the curvature in the waveguide to be undetectable in the limit of geometrical optics. The refractive index distributions are shown in the insets. The values of refractive index shown are all normalised to have a lower boundary of unity, but they are valid for normalisation to any positive minimum value, provided the whole distribution is scaled by the same value. All three cases: (d) is the anti-fish eye cloak, (e) is the conical cloak and (f) is the cosine cloak; can be seen to correct the phase difference, and reconstruct the plane wave fronts, although they suffer from varying degrees of shadowing due to reflections in the transition between the flat plane and the curved shape. As mentioned before, this is most severe in the hemispherical geometry, and it is only totally removed for the cosine geometry, which maintains the amplitude of the wave, effectively rendering the curvature in the waveguide completely invisible. Although a plane wave was selected for this demonstration, the cloak would also perform perfectly for any other form of excitation, such as a point source or a narrow beam.
FIG. 2 (color online). Simulations of the wave propagation in waveguides with curved geometries, where the incident wave is a plane wave propagating from left to right. The insets illustrate the waveguide structure under investigation, with the refractive index, \( n \), of the medium filling the guide indicated. (a)-(c) Hemispherical, cone-shaped and cosine-shaped waveguides filled with homogeneous material; (d)-(f) Hemispherical, cone-shaped and cosine-shaped waveguides filled with the appropriate graded refractive index medium for them to become invisible to the incident plane wave.

In order for these cloaks to be utilised for a practical application, a surface wave implementation is investigated using a graded dielectric slab above a ground plane. Any other surface wave implementation, such as metasurfaces, could be employed, however, a graded dielectric implementation is simple to fabricate with curved geometries. To achieve the appropriate propagation characteristics, it is necessary to alter the permittivity of the materials accordingly, for a given slab thickness, so that the effective refractive index achieves the required value.

Fig. 3 illustrates the required transformation of the dielectric constant of a 4.5mm slab over a ground plane to achieve the equivalent refractive index at 10GHz. The insets show the mode shape and confinement to the surface for an homogeneous slab with dielectric constants, \( \varepsilon_r \), of 2, 7, 11 and 15, which were calculated using an eigenmode solver. The confinement is an important consideration for cloaking applications, to ensure minimal radiation from the surface when curved surfaces are employed. The relationship shown in Fig. 3 was used to calculate the required permittivity profile to cloak a cosine surface with a maximum height, \( h \), equal to half the radius, \( a/2 \). This cloak shape has a requirement of permittivity range of the order 1:1.7. The permittivities were chosen to range from 9-15 so that the upper end of the Fig. 3 can be used where the confinement of the field to the surface is high. In order to evaluate the practical performance of this technique, a discretisation process was performed on the continuous distribution to give a very simple seven

FIG. 3 (color online). Illustration of the required epsilon of a 4.5mm slab against the obtained refractive index at 10GHz. The insets are the \( |E_z| \) field component and illustrate the confinement of the modes to the slab with \( \varepsilon_r=2, 7, 11 \) and 15.
FIG. 4 (color online). Radial cross-section of the discretised cloak design with seven distinct dielectric layers above a cosine shaped ground plane.

layered structure. Both the surface and the dielectric slab have linear boundaries, and the slab has a constant thickness perpendicular to the surface of the metallic ground plane, as illustrated in Fig.4.

Fig.5 shows the perpendicular component of the electric field at the surface of the dielectric slab, which is positioned above the metallic, cosine-shaped ground plane. Fig.5(a) illustrates the perturbation to the plane surface wave when a homogenous slab is used. Fig.5(b) has the seven layer cloak covering the bump in the ground plane, and it can be seen that the plane wave incident on the left is reconstructed on the right. This crude discretisation of the required profile demonstrates the robustness of this cloak design, and suggests that small deviations arising during the manufacturing process would not seriously influence the performance. Fig.5(c-d) show the frequency scanning performance for 9 and 11GHz, respectively. Although the surface wave implementation was designed to operate at 10GHz, this study demonstrates a bandwidth of operation at which the performance is shown to be accurate in the reconstruction of the plane wave fronts.

In conclusion, we have demonstrated the first practical proposal of an electromagnetic cloak applicable for surface waves. A robust method for designing surface wave cloaks that are rotationally symmetric, omni-directional and isotropic, without the requirement of superluminal propagation, is presented. This method can cloak objects that are electrically very large, yet the cloaks themselves are only a fraction of a wavelength in thickness. First, waveguide geometries are utilised to show the performance of three different cloak designs, although only the cosine shape is reflectionless. The latter was then transformed into a surface wave implementation, after calculating the equivalent refractive index for the surface mode in dielectric slabs over a metallic ground plane. The performance is demonstrated to reconstruct the plane waves to a good accuracy, even for a coarse discretisation of the index distribution, and for a range of frequencies, demonstrating the robustness of this technique. The presented results can be applied in the design of antennas and devices which rely upon surface waves which find practical applications in both microwave and THz regimes.

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