

Vibration excitation and control of a pedestrian walkway by individuals and crowds

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Abstract

As part of a continuing study on effects of humans on loading and dynamic response of footbridges, a steel frame walkway has been the subject of studies on the effects of multiple pedestrians with respect to loading and response mitigation. Following finite element modeling and experimental modal analysis to identify the low frequency vibration modes likely to be excited by normal walking, the variation of response with pedestrian density and of system damping and natural frequency with occupancy by stationary pedestrians were both studied. The potentially mitigating effect of stationary occupants is still not well understood and the study included direct measurement of damping forces and absorbed energy using a force plate. The various tests showed that energy dissipation measured directly was consistent with the observed change in damping, that vertical and lateral response both varied approximately with square root of number of pedestrians, and that the simple model of a human as a single mass-spring-damper system may need to be refined to fit observed changes in modal parameters with a crowd of humans presence. *Modal parameter changes with moving pedestrians were small compared to those with stationary pedestrians indicating that within limits, modal parameters for the empty structure could be used in analysis.*

Background

Structural design normally focuses on ultimate limit states, with passing attention to serviceability through limiting deflections. Except in the case of facilities such as wafer fabrication plants where vibration control is the prime concern, vibration serviceability is often overlooked or mishandled. In recent years a number of high profile structures, such as London Millennium Bridge and Cardiff Millennium Stadium, perfectly capable of sustaining the static load of a large crowd, have 'failed' without collapse: they were found to be unable to sustain the dynamic loads of large number of pedestrians without inducing significant discomfort, and retrofit or restrictions on use have been required.

Footbridges are frequently lively structures and serviceability requirements such as in the British bridge design code BD37/01[1] have provided reasonably well for limiting unacceptable response, but a number of recent developments have shown a need to revise serviceability requirement. Specifically, the phenomenon of synchronous lateral excitation (SLE) is not confined to the Millennium Bridge[2]; SLE has been observed in at least two other new structures and the only mitigation possible for SLE is crowd control or enhanced damping. For vertical response, the code requirement of a maximum allowable acceleration induced by a single perfect pedestrian is either over-conservative in the case of a real (and imperfect) pedestrian or inappropriate in the case of a large crowd where effects such as synchronization of movement and enhanced damping due to stationary pedestrians are not covered. For SLE it is claimed that the problem is understood and it is possible to predict if a problem will occur and if so, to then estimate the limiting number of pedestrians. On the other hand, the beneficial effects of human damping are not well understood and all that is known of crowd loading is that response seems to go approximately with the square root of number of pedestrians[3].

This paper describes a comprehensive study on an unusual pedestrian bridge. It is a steel framed elevated walkway that is cantilevered at one end, hence the vertical mode frequency is quite low, at 5Hz, and damping is also quite low. It is heavily used by students between classes, frequently empty, accessible and relatively easy to model analytically.

Finite element modeling and vibration testing aimed at identifying the modal parameters is first described. Modal mass is particularly important for study of vibration serviceability yet it is surprisingly difficult to measure or predict with a degree of accuracy. As well as estimating from a finite element model and from the modal constant derived by hammer testing, a force plate was used to measure directly contact force from a jumping student which, by fitting to a model with known frequency and damping, provided probably the most reliable estimate.

In the spirit of BD37/01, response to a single perfect pedestrian was then computed and compared to response for a range of pedestrian densities to add to the body of evidence on the limitations of the approach for multiple pedestrians.

Finally, the ability of stationary humans to dissipate energy was measured directly using a force plate and the capability of up to 21 stationary pedestrians (students) to increase damping capacity (to as much as 10%) was demonstrated, a result of particular value to researchers of grandstand dynamics.

Pedestrian bridge at Singapore Polytechnic

The 46m long steel pedestrian bridge linking teaching Block T15 and Engineering Block T12A in Singapore Polytechnic was constructed in 1999. The bridge is a steel space truss and frame system while the deck floor comprises timber planks with a layer of stone tiling. The end at Block T15 is not structurally connected to the block except by a narrow elastomeric expansion joint, whereas at the other end, top chords and bottom chords are all bolted to the RC wall of Engineering Block T12A.

Finite Element Modal

Mode shapes, natural frequencies and modal masses of the bridge were estimated from dynamic analysis using GT-STRUDL analysis software.

The bridge was modeled by incorporating all structural details according to the design concept. The four steel stanchions providing a clearance of 6m above ground are assumed rigidly connected to the foundation while the top chords and bottom chords of the bridge are assumed to be pinned to the wall of Block T12A. Within the bridge, member connections are taken as rigid or pinned based on its original design assumptions. Fig(1) shows the isometric view of finite element model of the bridge.

The frequencies for 1st lateral, vertical and torsional natural frequencies were estimated to be:

L1: 0.87 Hz,

V1: 4.72 Hz

T1: 10.23 Hz

The corresponding mode shapes are displayed in Fig(2).

Analysis of vertical ground reaction forces during walking [4,5] shows that vertical forces are concentrated at the fundamental frequency of the pacing with diminishing components at harmonics or multiples of this frequency. Lateral forces during walking are harder to measure directly, but it is clear that due to the oscillating forces required to keep balance during a cycle of two footsteps, lateral forces are concentrated at half the pacing rate, then at third and possibly fifth harmonics. Hence based on the FEM analysis it may be expected that L1 (at 0.87Hz) could be excited by normal walking at 1.74Hz and V1 by the second harmonic of fast walking at 2.36Hz or third harmonic of slow walking at 1.57Hz. Predictions of response in either mode require an accurate estimate of modal mass, which for the purposes of these studies would use unit normalization of mode shape at the free end with maximum response. This means that the ratio of response to excitation at the free end fixes a value for modal mass; for mode V1, modal mass was estimated from the FEM to be 8500kg, about 14% of total mass of the bridge.

Modal testing

Fig(3) shows averaged auto-spectral density of lateral and vertical response at the free end of the bridge to crowd loading on a busy day. Strong vertical response is seen around 5Hz with strong lateral response just under 2Hz and also around 4.5Hz. Since the range of typical walking footfall frequencies has a mean of approximately 2Hz [5], relatively strong non-resonant dynamic response might be expected in the lateral direction around 1Hz and in the vertical direction around 2Hz. The observed peaks do not fit these values hence they are likely to represent vibration modes excited by wind or by higher harmonics of the footfall forces.

To check the identity of these modes and obtain reliable modal parameter estimates, a forced vibration test was carried out using an instrumented hammer to excite vertical or lateral vibrations at the free end; Fig(4) shows estimates of mode shape and frequency for V1 and T1 from combination of the measurements.

Single degree of circle fit[6] was used to extract modal parameters from the frequency response functions (FRFs). For vertical response, best quality FRFs were obtained for excitation at the free end and response at

the midspan of the cantilever rather than at driving point hence modal mass values are not available directly corresponding to the free end of the bridge:

Mode V1:	f=4.992Hz	$\zeta=0.85\%$
Mode T1:	f=11.13Hz	$\zeta=1.45\%$

For lateral response only three modes could be identified from circle fit of driving point FRF from which modal masses could be estimated. Other modes are estimated from the ambient vibration response shown in Fig(3), but as there is no information about forcing function, no modal mass values are available. The estimated lateral modes are summarized in Table 1.

The lateral modes have high modal mass as they involve sway of the whole bridge. Modes with higher damping are less reliable and mode L1 is believed to be the fundamental lateral vibration mode.

The FEM predictions for V1 and T1 frequencies, at 4.72Hz and 10.23Hz are quite close to the measured values at 4.99Hz and 11.12Hz respectively while the measured L1 frequency at 1.96Hz is much higher than the predicted value of 0.87Hz, which would be due mainly to ignoring lateral stiffness from bridge wooden deck and steel roof systems. Judging from Fig(3), mode V1 is the most important mode for consideration of effects of crowd loading, and is now studied in more detail.

Detailed identification of mode V1 with human factors

The aim of the investigation was to study all three components of the generic relationship $X=HF$ where X and F represent the response and loading (in time, frequency or other domains) and H represents the structural system. For the study of vertical response of the bridge in mode n to vertical pedestrian forces, H can be represented by a single degree of freedom system with mass m_n , viscous damping coefficient c_n and stiffness k_n so that, for example, in Laplace domain

$$X(s) = \frac{F(s)}{m_n(s^2 + 2\zeta_n\omega_n + \omega_n^2)} + \frac{\dot{x}_0 + x_0}{m_n(s^2 + 2\zeta_n\omega_n + \omega_n^2)} \quad (1)$$

where $\zeta_n = c_n/2\sqrt{k_n m_n}$, $\omega_n = \sqrt{k_n/m_n}$ are damping ratio and natural frequency (in radians per second) of the oscillator.

Solution of equation (1) requires accurate values of m_n , ζ_n , ω_n . Although the frequency and damping estimates from the FRF circle fit were repeatable for the low response levels, the most reliable estimates of frequency and damping are generally obtained from the single-mode free vibration decay, which can also track non-linearity through amplitude dependence of these parameters. Also, estimates of modal mass varied between measurements and were not consistent the FEM estimate so alternative estimation procedures were used.

The solution of equation (1) for initial velocity and no forcing is a decaying sinusoid:

$$\ddot{y}(t) = \ddot{y}_{\max} \sin\left(\omega_n t \sqrt{1 - \zeta_n^2}\right) e^{-\omega_n \zeta_n t} \quad (2)$$

and it is possible to estimate ζ_n , ω_n from curve fitting to all or part of the free decay. Usually damping and frequency vary with amplitude so piecewise fitting using a moving window of, say eight cycles, is used to estimate parameters throughout the decay.

Frequency and damping ratio without human occupant

Based on the method of curve fitting to equation (2), Fig(5) shows variation of mode V1 damping and frequency with amplitude merged from analysis of ten recordings of free vibration in V1. For each recording the bridge was excited by a sequence of six jumps timed by a metronome running at 148 beats per minute (BPM). On the last jump the jumper jumped off the bridge (onto the adjacent landing).

As well as showing repeatability of the estimates and an inverse relationship of frequency with amplitude, the tri-partite relationship of damping with amplitude and the peak at 1.3% is remarkable. The behaviour of the expansion joint must be in part responsible for the strange damping characteristic; for small oscillations it is hardly mobilized, the character is friction-like in the mid range, and at higher amplitudes of +/-2mm, corresponding to 2m/sec^2 , the bridge is moving so much that construction joints are beginning to creak. Response of V1 to normal pedestrian traffic has amplitudes in the range 0.02 to 0.15m/sec^2 , where damping averages at 1.2% and frequency around 5.0Hz.

Frequency and damping ratio with human occupant

The same exercise used to generate Fig(5) was repeated to obtain parameter estimates with a single 87kg human remaining on the bridge and standing perfectly still on the bridge after the jumping sequence. The results in Fig(6) show several features compared to Fig(5), whose piecewise best-fit trend-lines are given in Fig(6) for comparison.

As well as a drop in frequency of approximately 0.02Hz at all amplitudes, damping has increased, by approximately 0.4% at amplitudes greater than 0.04m/sec^2 , although the values show considerable scatter.

Modal mass

As the estimates of modal mass via hammer testing were not reliable and disagreed with FEM predictions, a force plate was used to estimate mass via measurement of force together with response at a single due to a human jumping. The top plot of Fig(7) shows measured acceleration response due to an 80kg male jumping at the free end of the bridge at 2.5Hz to build up a resonant response through the second harmonic of the jumping frequency. The middle plot shows the input force recorded using a single axis force plate. The data up to 62 seconds were used to estimate modal mass as follows.

The measured force signal was used as input to a simulation using equation (1) for $n=V1$ with mass $m=1000\text{kg}$, $\omega = 2\pi \times 5 \text{ Hz}$ and $\zeta = 1.3\%$. The bottom plot of Fig(7) shows the result. For the initial build up, before the effect of damping becomes significant, the increment of acceleration is inversely proportional to mass, hence the modal mass can be estimated by comparing acceleration increment per cycle between measurement (top plot, 0.08m/sec^2) and simulation (bottom plot, 0.59m/sec^2), leading to an estimate of modal mass $m_{V1}=7,380\text{kg}$.

Mode V1 energy input and extraction due to single occupant

The data of Fig(7) after 62 seconds reveal the effect of the occupant on damping and frequency of the occupied bridge. The same process of time domain curve-fitting to free decay that was used to obtain the data points in Fig(6) was used on last few seconds of data shown in Fig(7) to demonstrate that the presence of this test subject at the free end of the bridge increases damping by approximately 0.3%. It is certain that none of the extra damping is due to the force plate itself.

Close examination of the top two plots of Fig(7) shows that during the free decay part of this jumping sequence (and other similar measurements) when the jumper is stationary, the phase angle between the component of force at the bridge frequency switches from leading response by 90° to lagging response by approximately 155° .

The observed differences with and without a human occupant are consistent with the accepted notion that a stationary human acts as a passive damper. While jumping, the effect is as an external force, and the direct measurement of contact force isolates any influence of jumper dynamics, hence the modal mass estimate should be reliable. In the decay phase, since the contact force has been recorded, the mechanism of the enhanced damping could be studied. In fact it should be possible to estimate energy input (before 62 seconds) and energy extraction (after 62 seconds).

Figs(8) and (9) show the power and its time integral (work done or energy) by the jumper during the forcing and decay phases of Fig(7), i.e. before and after 62 seconds. The power is obtained from the product $F_{jumper} \times v_{plate}$ where v_{plate} is the plate velocity derived by integration of acceleration data shown in the top plot of Fig(7). In the forcing phase the power averages 5Joules/sec (Watts) and when the response reaches a maximum or steady state it could be argued that 5Watts must also equal the energy dissipation of the bridge.

With this line of argument, since the 5W rate of energy dissipation (power) due to bridge damping in mode V1 is $2\zeta_{V1}\omega_{V1} \times m_{V1} \times v_{plate}^2$, an estimate of mode V1 structural damping $\zeta_{V1} \approx 1.3\%$ can be obtained given values of m_{V1} , ω_{V1} , $\sigma^2(v_{plate})$ where σ^2 represents variance.

The bottom plot of Fig(9) shows the kinetic energy of the bridge as the vibration decays; the maxima of each cycle represent the total of potential and kinetic which is slowly reduced by internal and external damping losses during the decay cycles. Of the 1.8 Joules that has been dissipated, the middle plot shows that 0.3Joules is due to the stationary human. The human contribution of 0.3 Joules to a total energy loss of 1.8 Joules matches quite well the observed 0.3% increase in damping to 1.6%.

Mode V1 pedestrian-induced vibration control by stationary humans

The ability of a stationary human to dissipate energy has been demonstrated clearly and it should be worthwhile to investigate the effect of different stationary humans in mitigating vibrations induced by pedestrians. Because different subjects (human dampers) were used and because a perfectly repeatable loading could be generated, the additional damping was studied from the ratio of measured energy loss in a cycle to total energy.

The effective viscous damping is defined via the ratio of energy loss per cycle of oscillation, ΔE , to total stored energy E i.e.

$$\zeta = \Delta E / 4\pi E . \quad (3)$$

Work done (hence energy loss) by a human damper on a force plate during one cycle of harmonic oscillation lasting T seconds is given by

$$\Delta E = \int_0^T F_{plate} v_{plate} dt = (T/2) \widehat{F}_{plate} (\widehat{a}_{plate} / \omega) \sin \phi \quad (4)$$

where \widehat{F}_{plate} , \widehat{a}_{plate} are amplitudes of force and acceleration, ϕ is phase angle between acceleration and force measured on the force plate, and total energy is peak kinetic energy defined as

$$E = (1/2) m_{v1} (\widehat{a} / \omega)^2 .$$

Since cycle duration $T = 2\pi / \omega$, the additional human contribution to damping ratio is given by

$$\zeta_{h_add} = \frac{(\pi / \omega) \widehat{F}_{plate} (\widehat{a}_{plate} / \omega) \sin \phi}{4\pi (1/2) m_{v1} (\widehat{a}_{plate} / \omega)^2} = \frac{\widehat{F}_{plate} \sin \phi}{2m_{v1} \widehat{a}_{plate}} . \quad (5)$$

A number of individuals were asked to stand on a force plate (located at the free end of the bridge) while recording the response to a pedestrian walking normally, as shown in Fig(10). Due to the normalization with respect to kinetic energy, the actual values of response and force are less important than having a steady state for long enough to identify the force, acceleration and phase angles. From the walking experiment, taking $m_{v1}=7,380\text{kg}$, values of $\widehat{F}_{plate} / \widehat{a}_{plate}$, ϕ and body mass m_h for seven individuals are given in Table 2. There is considerable variation of damping potential among individuals but the coefficient of variation (COV) shows this is reduced with normalization with respect to body mass.

Crowd effects: variations of loading response, damping and frequency with multiple human occupants

Variation of V1 and L1 response and modal parameters with crowd numbers

The response at the free end of the bridge in modes V1 and L1 was tracked during a 136-minute period when the bridge was heavily used during an exhibition set up close to one end of the bridge. The number of people N passing the free end of the bridge during 30-second periods was tracked with a moving window shifting 3 seconds. N does not exactly represent the number of people on the bridge, but it is an indication, and 30 seconds is the estimated time taken to cross the whole bridge at a leisurely pace. Fig(11) shows variation of RMS response within each 30-second period with corresponding value of N . The dependence on square root of N is clear, being consistent with previous observations and models of recently developed models of crowd loading for vertical response[2,4].

Vertical response is almost entirely in mode V1 whereas lateral response is spread evenly across the first four lateral modes L1 ...L4 identified in Table 1. Fig(11) thus not only provides useful calibration for the models of vertical loading but provides evidence for similar models of lateral loading yet to be developed.

It has been suggested that the presence of a large number of pedestrians modifies the modal properties of the bridge. This is certainly true to some degree even though it is generally taken that moving pedestrians act as load generators, in which case the effect cannot be modeled simply using the SDOF human models proposed for stationary humans. Fig(12) shows estimates of frequency and damping ratio corresponding to the data points of Fig(11). The only way possible to obtain such values was from 'output-only' parameter estimation techniques, in this case by curve fit to the auto power spectra the 30-second response samples. There are unavoidable flaws in this approach, not least due to the non-stationarity and variable frequency content of the forcing function and even with inclusion of only those records having good quality of fit, the absolute accuracy of the values should not be relied on. Nevertheless what is clear is that mode V1 frequency values are generally consistent with values for the empty bridge at relatively low amplitude with

minimal reduction due to increasing response. For damping the scatter does not allow for any trend to be detected. More reliable observations from a test on another much large bridge [7] indicate that lateral vibration mode parameters are barely affected even with large numbers of pedestrians, while moving pedestrians enhance damping slightly, but less so than when pedestrians stand still. This effect is still being researched.

The peak vertical response of the bridge was $\pm 0.3\text{m/sec}^2$. Using the BD37/01 criterion, assuming a pedestrian moving at 2m/sec with a footfall rate of 5Hz and a reduction of 70% , the predicted peak response is 0.105m/sec^2 , a value exceeded frequently during the measurement.

Variation of mode V1 damping and frequency with stationary pedestrians

The final experiment involved a study of variation of natural frequency and damping ratio of the bridge as the number of human occupants increased from one to 21 then back to one. As occupants were added or removed, the free decay of the bridge was studied using a heel drop, having the aim to investigate the effect of occupants on a range of bridge modes. The bridge occupants, or a subset of them provided the heel drop, and free decay response beginning with amplitude 0.15m/sec^2 was (for consistency) used to extract damping and frequency estimates.

Fig(13) shows the variation of estimates, which became difficult to obtain as damping increased. The slope for additional damping is 0.26% (absolute) per person, and for reduction in frequency is -0.26% (relative to unloaded frequency) per person.

Modelling the human as a SDOF system

An attempt was made to explain the variation of modal parameters observed in Fig(13) using an established model of the human dynamic system.

The approach was to model the group of humans as individual or combined SDOF systems attached to the bridge, and determine the parameters of the combined two degree of freedom system. For this purpose, values of human damping coefficient and stiffness would be needed.

The most comprehensive study of the effect of human damping has been by Sachse[8] who identified modal properties for sitting humans. Sitting and standing humans have similar effect[9], and for average mass of a student ($m_h=58\text{kg}$), Sachse's human model 'A' with effective human mass $m_h=\alpha m$, $\alpha=0.9$, $f_h=5.87$ and $\zeta_h=33\%$ should have $k_h=71\text{kN/m}$ and $c_h=1270\text{Ns/m}$ but these last two parameters are only indicative. Other experimental data provide similar values: the human natural frequency is in the range 5-6Hz, damping from 30-40%.

The original (unoccupied) mode V1 frequency and damping are taken as 5Hz and 1.3%, but the modal mass is assumed not to be known precisely. For unit mode shape at the tip of the free end that value is $m_{V1}=7,380\text{kg}$ from the force plate, (FP) a result which should be reliable, or $m_{V1}=8,500\text{kg}$ from the FEM, a result which may better fit the results. For the location where the students were grouped the effective modal mass would be scaled up by squared inverse of mode shape $\psi \approx 0.9$, providing effective masses at the centre of the group as either 8,930kg or 10,500kg.

For the two values of modal mass and using the indicative human parameters as starting values, Newton's method was used to solve for parameters k_h and c_h that best fitted the frequency and damping change per person of Fig(13).

The human parameter that fit the data of Fig(13) are given in Table 3 and compared with published data for human parameters from three different sources[8,10,11].

The last row is for the data of Figs(5,6) for one 87kg male. Again, a reduced body mass and increased bridge mass is required for a good fit, and in this case damping is measured at the point of unit mode shape.

Compared with published data it would seem that reasonable agreement can only be obtained using larger modal mass with smaller human mass. One possible explanation for the discrepancy is due to considering the bridge as a SDOF system, which may be acceptable if added mass is small compared to modal mass. As the total human mass exceeds 1000kg, a distributed mass model should work better, yet the same analysis still shows human parameters that still struggle to match published values.

Discussion and conclusions

The study has provided the opportunity to address a number of contentious issues relating to vibration serviceability of footbridges.

First, the ability of a human to act as a damper, mentioned in many researches and code provisions, has been identified directly using measurements of the human force that damps vibration during free decay and transit of other pedestrians. The effect on damping matches observed changes in damping values.

Second, the variation of response with crowd numbers appears to go with the square root of number of pedestrians, as mentioned by other researchers but this exercise has demonstrated the correlation for lateral as well as vertical response. Usually the code provision based on perfect first harmonic forcing is adequate but this case shows much stronger levels can be achieved by second (or third) harmonic forcing. Definitive guidance on loading due to multiple pedestrians is still lacking in bridge codes.

Thirdly, the increase in damping and decrease in frequency with crowd size has been demonstrated, yet it is curious that the extracted stiffness and damping parameters do not fit well to published models which represent the human system as single degree of freedom mass-spring-dashpot system. This shows that such model may not be too simplistic and suggests the need for extra research for the case of large crowds.

The damping and frequency changes observed with varying numbers of moving pedestrians were indistinct but clearly far smaller than those observed due to stationary pedestrians. This supports existing understanding that response prediction models may give reasonable results if moving pedestrian pedestrians are taken purely as load generators. Also, while presence of stationary pedestrians cannot be relied on, the probable numbers of stationary pedestrians would increase with crowd numbers, particularly if movement is restricted, leading to a self-limiting mechanism for vertical vibrations with high crowd density.

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FIGURE CAPTIONS

Figure 1 Isometric View of Finite Element Model of the Bridge

Figure 2 Fundamental mode shapes (top to bottom) V1, L1 and T1 obtained from finite element analysis

Figure 3 Auto-spectral density of vertical (upper) and lateral (lower) acceleration response due to heavy pedestrian activity

Figure 4 Estimates of frequency and mode shape of two modes recovered from hammer testing in vertical direction, corresponding to V1 and T1

Figure 5 Frequency (+) and damping (o) estimates from free decay of mode V1 without human occupation

Figure 6 Frequency (+) and damping (o) estimates from free decay of mode V1 with single human occupant. Values for empty bridge (from Figure 5) are shown as solid lines.

Figure 7 80kg test subject jumping at 2.5Hz.
top: measured acceleration response
middle: measured contact force
Bottom: simulated acceleration response for 1000kg mass

Figure 8 Upper: Power supplied to bridge derived from contact force and acceleration.
Lower: Time integral of power; energy supplied to bridge

Figure 9 Top: Energy dissipation rate due to human occupant
Lower: Time integral
Bottom: Kinetic energy

Figure 10 Measurement of mitigating effect of stationary occupant on walking-induced response

Figure 11 Variation of RMS modal response with crowd numbers

Figure 12 Estimates of vertical mode frequency and damping with pedestrians walking across bridge

Figure 13 Absolute changes in % damping and % changes in frequency due to human occupation

Table 1 Lateral modes estimated from *ambient* and forced vibration testing

mode	Frequency f /Hz	Damping ζ /%	Modal mass /kg
L1	1.962	1.96	
L2	2.35	3.7	
L3	4.414	1.08	35,200
L4	6.06	1.16	80,000
L5	6.88	3.23	
L6	13.62	1.79	40,000

Table 2 Damping and energy dissipation parameters for seven students

Student	m_h /kg	$\hat{F}_{plate} / \hat{a}_{plate}$ /kg	ϕ	ζ_{h_add} /%	ζ_{h_add} / m_h
#1	53	95.5	-22.5	0.24	46e-6
#2	56	104	-21	0.25	44e-6
#3	69	136	-12.5	0.20	29e-6
#4	42	86	-15	0.16	39e-6
#5	49	96	-11	0.13	27e-6
#6	87	163	-21.5	0.40	46e-6
#7	80	141	-14.7	0.24	29e-6
COV (%)				37.5	23

Table 3 Human parameters from literature and as identified from crowd test

Mass parameters		Fitted human parameters			
Human mass	m_{vj} /kg	k_h (kN/m)	c_h (Ns/m)	f_h (Hz)	ζ_h (%)
0.9×58 (Sachse[8])	-	71	1270	5.87	33
80 (Brownjohn[10])	-	82	1950	5.27	36
various (Zheng[11])	-	-	-	5.24±0.4	39±5
0.9×58	7380 (FP)	44.3	1790	4.64	59
0.8×58	7380 (FP)	45.2	1440	4.97	50
0.9×58	8500 (FEM)	52	1590	5.02	48
0.8×58	8500 (FEM)	50	1240	5.23	41
0.8×87	8500 (FEM)	76	1810	5.25	39

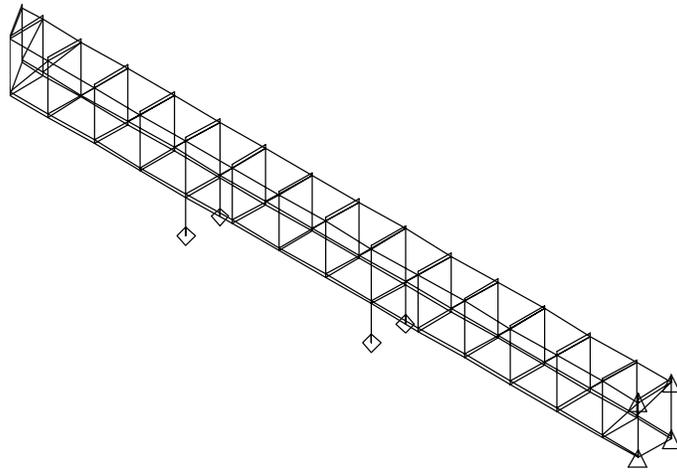
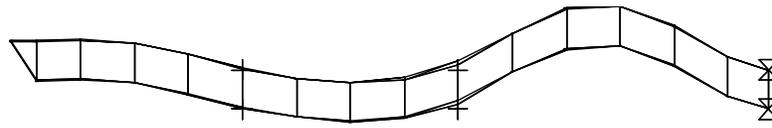
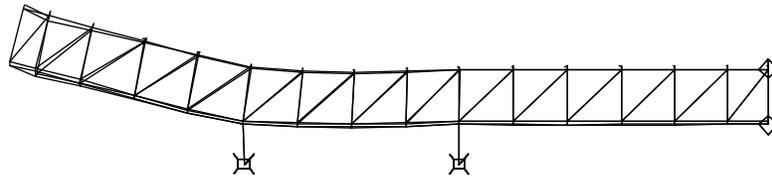


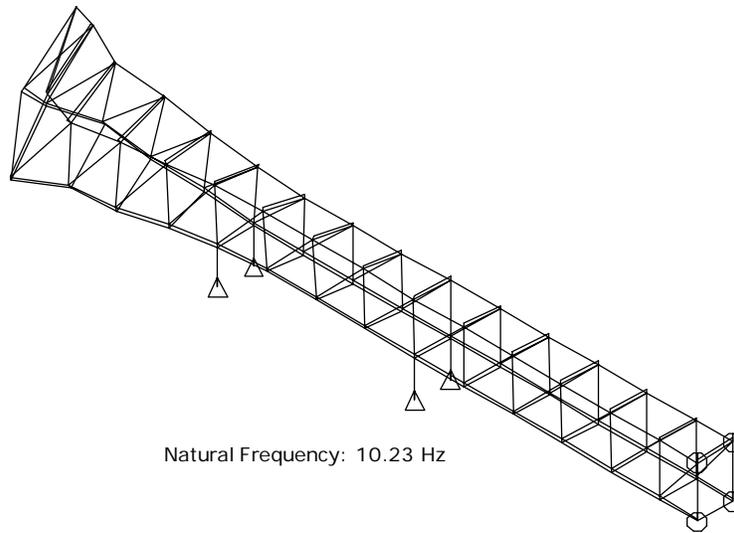
Figure 1 Isometric View of Finite Element Model of the Bridge



Natural Frequency: 0.87 Hz



Natural Frequency: 4.72 Hz



Natural Frequency: 10.23 Hz

Figure 2 Fundamental mode shapes (top to bottom) V1, L1 and T1 obtained from finite element analysis

Figure 3 Auto-spectral density of vertical (upper) and lateral (lower) acceleration response mainly due to heavy pedestrian activity

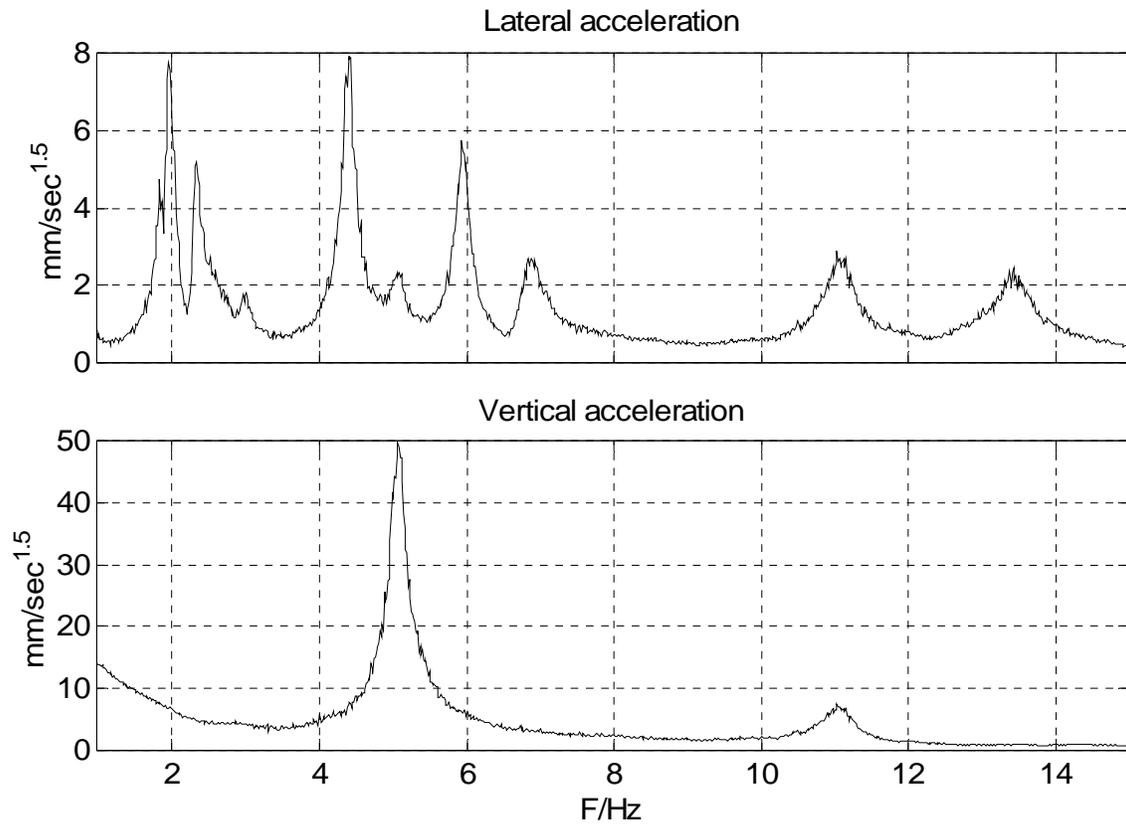
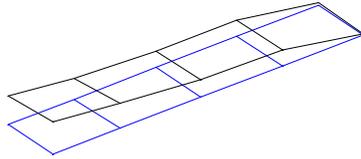


Figure 4 Estimates of frequency and mode shape of two modes recovered from hammer testing in vertical direction, corresponding to V1 and T1

mode: 1 $f=5.033\text{Hz}$



mode: 2 $f=11.12\text{Hz}$

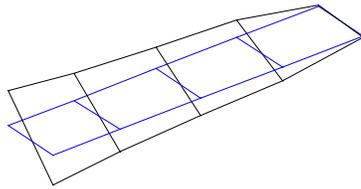


Figure 5 Frequency (+) and damping (o) estimates from free decay of mode V1 without human occupation

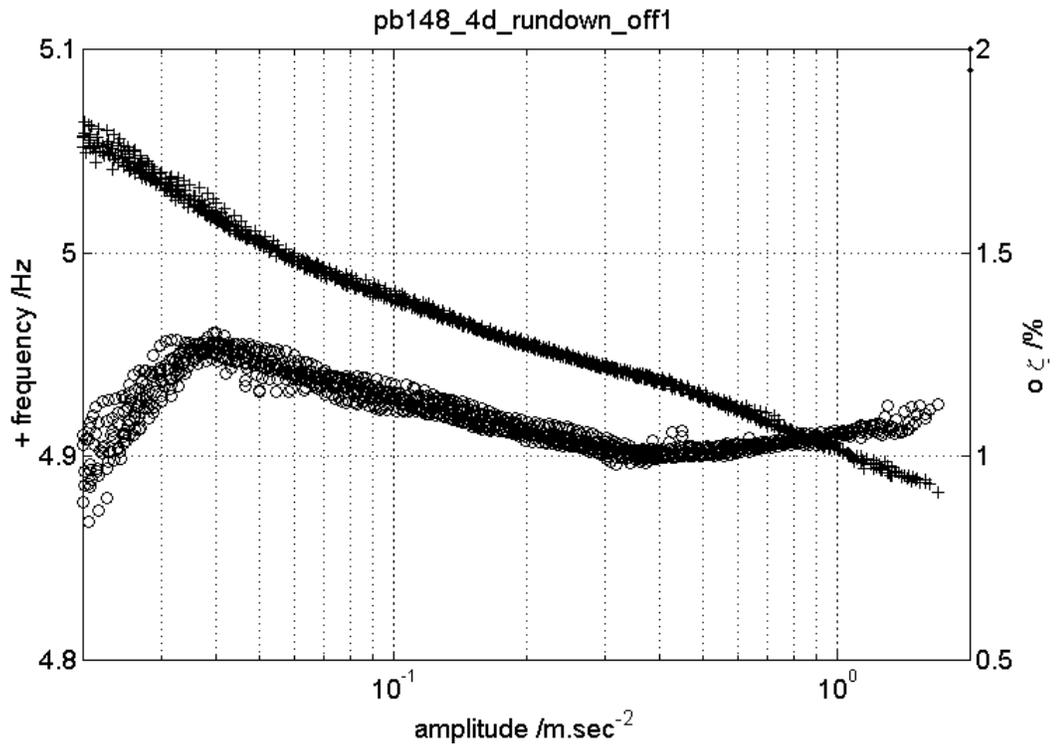


Figure 6 Frequency (+) and damping (o) estimates from free decay of mode V1 with single human occupant. Values for empty bridge (from Figure 5) are shown as solid lines

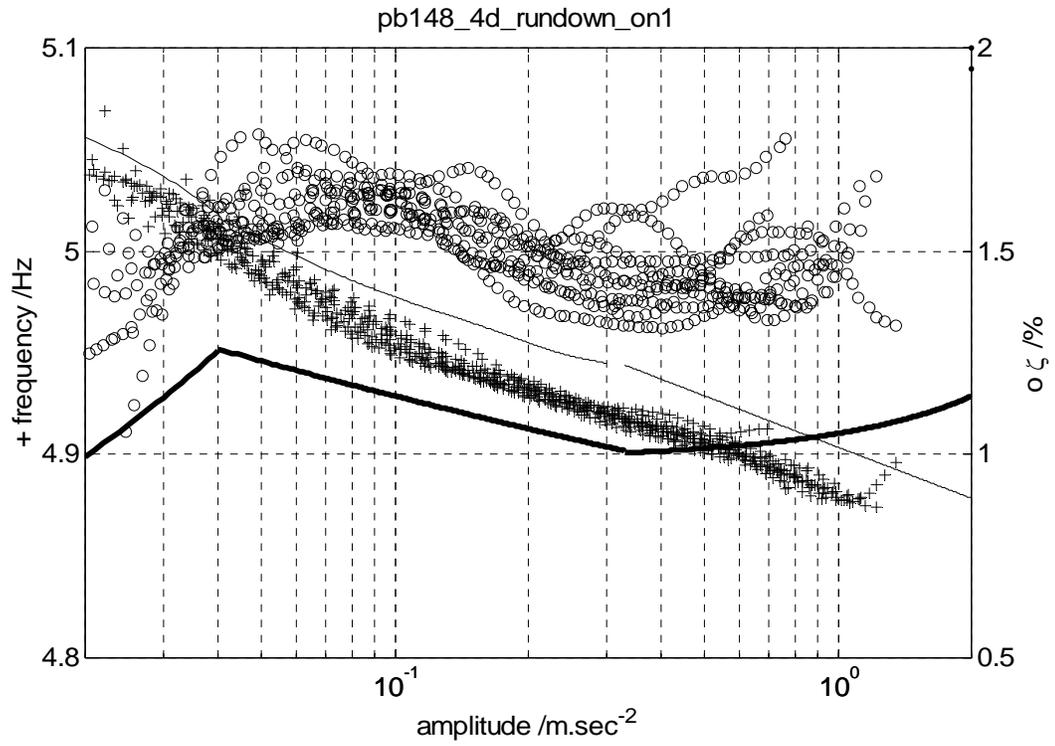


Figure 7 80kg test subject jumping at 2.5Hz.

top: measured acceleration response

middle: measured contact force

Bottom: simulated acceleration response for 1000kg mass

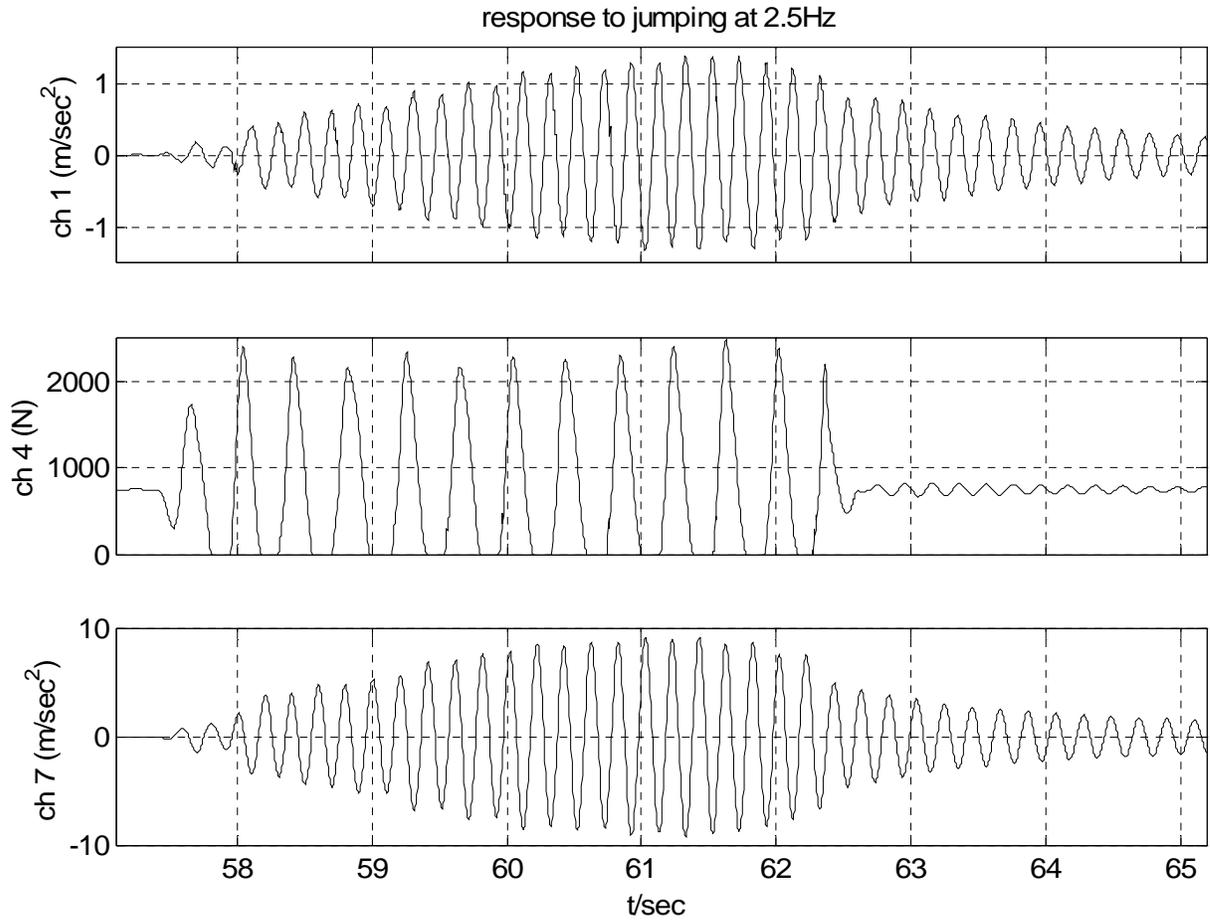


Figure 8

Upper: Power supplied to bridge derived from contact force and acceleration.
Lower: Time integral of power; energy supplied to bridge

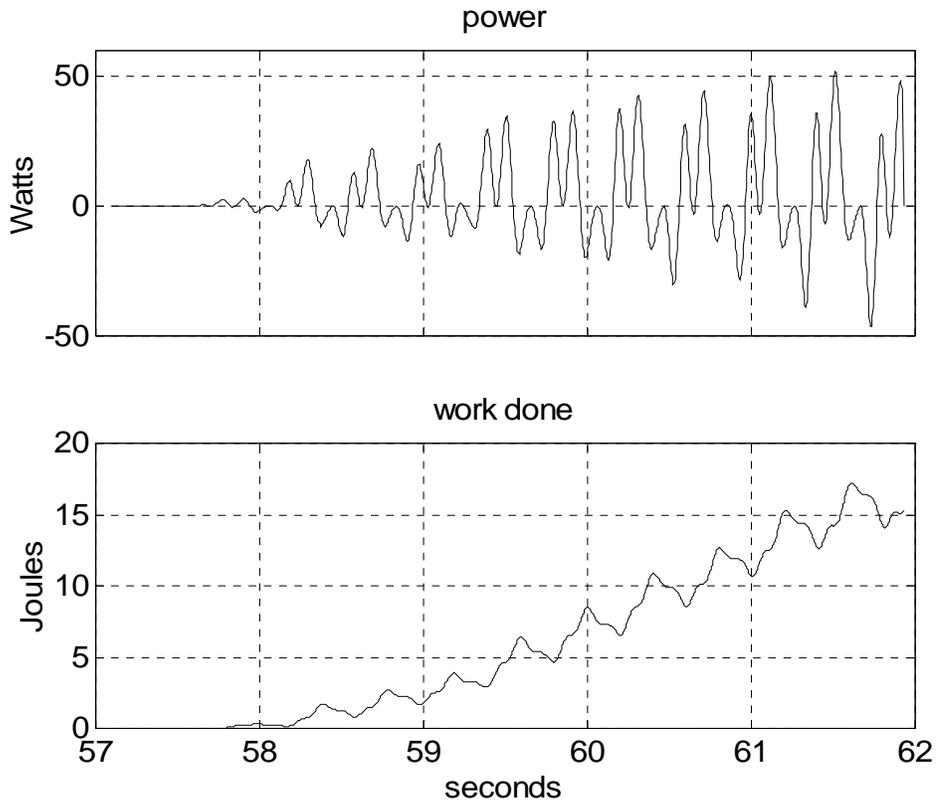


Figure 9 Top: Energy dissipation rate due to human occupant
Lower: Time integral
Bottom: Kinetic energy

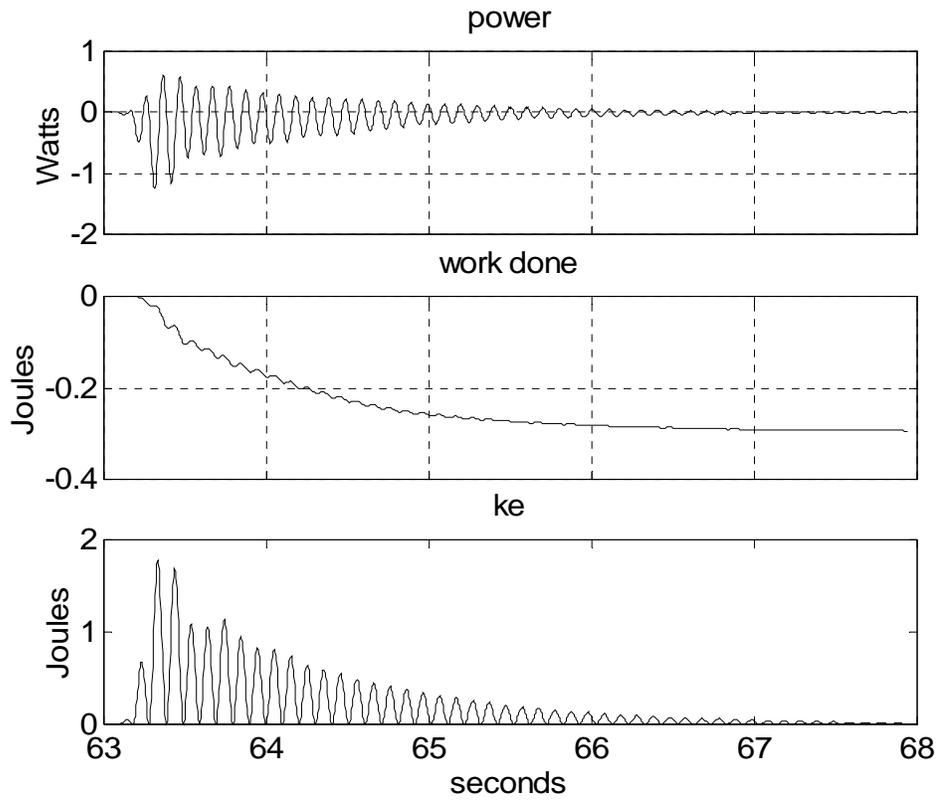


Figure 10 Measurement of mitigating effect of stationary occupant on walking-induced response



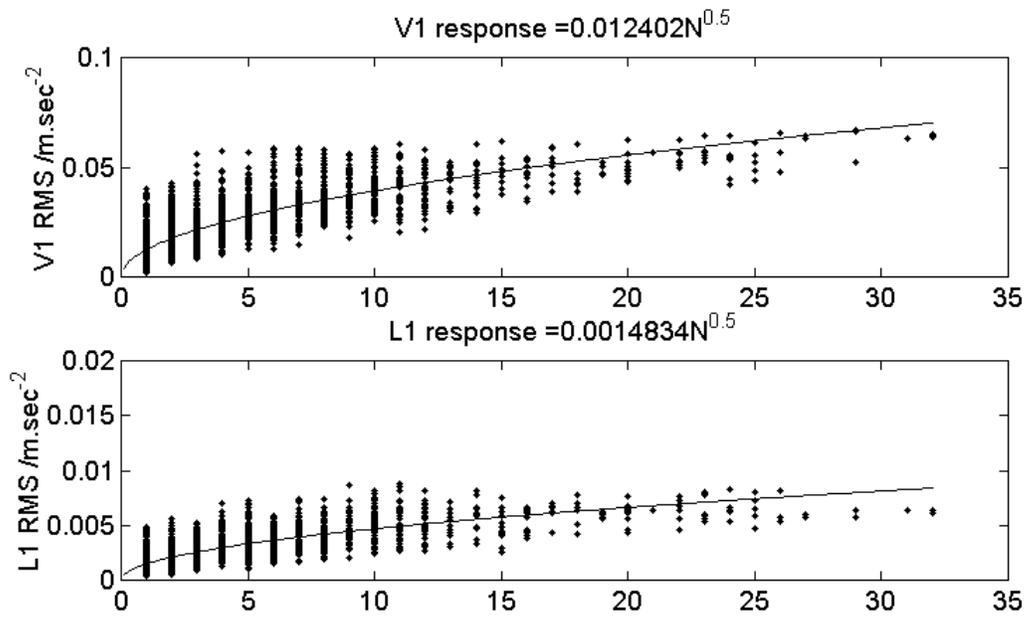


Figure 11 Variation of RMS modal response with crowd numbers

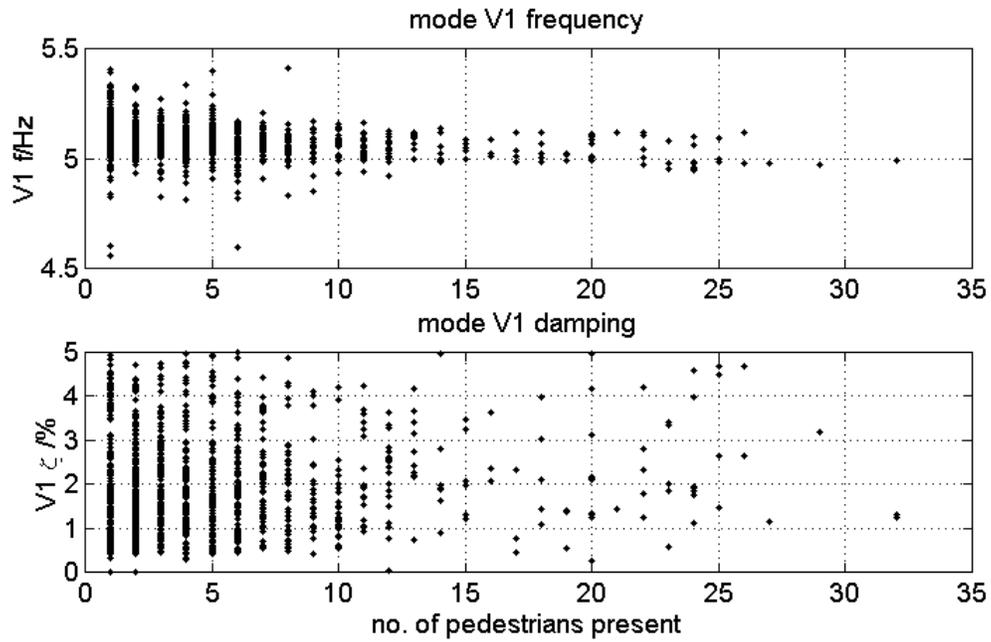


Figure 12 Estimates of vertical mode frequency and damping with pedestrians walking across bridge

Figure 13 Absolute changes in % damping and % changes in frequency due to human occupation

