A folded pendulum isolator for evaluating accelerometer performance

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ABSTRACT

This paper describes how a requirement to evaluate a new accelerometer led to the construction of a folded pendulum isolator as part of an undergraduate civil engineering student project. The exercise has some interesting lessons about the performance of accelerometers in low-vibration environments and the importance of paying attention to the detail of their mounting. It also demonstrates a fascinating mechanical device.

A folded pendulum is a compact mechanism comprising a positive pendulum a negative (or inverted) pendulum whose combined horizontal natural frequency can be made very small.

We decided to build one after searching for methods to estimate noise floors of the high-grade accelerometers used for civil structure dynamic assessment by isolating them from all mechanical excitation at their supports on the ground. As with other isolation devices, the folded pendulum can be built with a natural frequency low enough to provide significant attenuation with respect to ground motion in the operational range, but compared to other isolation devices it is relatively straightforward to construct.

With careful tuning the folded pendulum as constructed achieved a minimum natural frequency of 0.078Hz and proved capable of isolating the test accelerometer well enough to identify the instrument noise floor.

INTRODUCTION: LIMITATIONS ON LOW LEVEL, LOW FREQUENCY VIBRATION MEASUREMENTS

Dynamic response measurements of civil infrastructure pose several challenges that include problems of accessibility, low ranges of natural frequency, high levels of ambient noise and low levels of response even when artificial excitation can be used. In fact operational modal analysis using 'output only' response measurements is a growing research area because it is better to use ambient excitation than to try to avoid it. Experience with a range of civil structures including dams, long span bridges and tall buildings has shown that high quality but rugged and simple to use sensors are needed for ambient vibration testing. Seismometers, while having excellent low noise, low frequencies characteristics, are generally too fragile and troublesome for measurements in which small numbers of accelerometers are deployed over ranges of several hundred metres in different arrangements in a short time scale.

There are several popular choices of accelerometer for civil structure assessment, including the Kinemetrics Episensor and the seismic grade of PCB accelerometers, as well as the seismic grade Endevco 7754-1000 accelerometer (now obsolete). After years of experience, the Vibration Engineering Section at the University of Sheffield decided to use QA700 and QA750 servo accelerometers almost exclusively due to their versatility and superior noise performance across the whole range of relevant frequencies. Their ability to resolve accelerations as low as one micro-g has proved vital for testing of a long span suspension bridges [1], tall buildings [2] and dams [3].

Even lower levels of response are, by design, observed in structures housing equipment for manufacturing micro-chip wafers and other electronic products with features sizes smaller than one micron. Similarly, experimental facilities such as synchrotrons and lasers require extremely stable and low-vibration environments, and in many cases the vibration levels need to be measured experimentally in order to check that vibration serviceability limits are not exceeded [4]. In such cases the vibration levels may be at least an order of magnitude less than experienced in the buildings, bridges and dams tested in operation, so that the sensor noise and resolution characteristics may become serious limitations.

When using these sensors it is therefore necessary to know if the signal being recorded on a structure is due to structural response or internal noise from the measurement system including the accelerometer. Manufacturer specifications may be either misleading or provide insufficient information. For example, the quartz-flex accelerometers are supplied with no information about noise floor, only a value for ‘Resolution/Threshold’. Because of the importance of the measurements it is necessary to evaluate the noise floor independently to produce a noise floor specified in meaningful units.

MEASURES OF NOISE AND RESOLUTION CAPABILITY [5]

The standard measure of both accelerometer noise and vibration level is power spectral density. This is conventionally derived from the discrete Fourier transform (FFT) of a signal with record length *T* by squaring the FFT ordinates and dividing by 2/*T*, since (1/*T*) is the line spacing or width of each spectral line over which the density is calculated. Acceleration PSD will be reported in units such as g^2/Hz or $(m/sec^2)^2$/Hz. The mean square (MS) of the signal is the area under the PSD curve, and MS can also be determined by integrating between specific frequency limits.

More frequently, the ‘root PSD’ is quoted, in units such as g/√Hz. Note that specifying a signal strength or a resolution level in absolute units like m/sec^2 without reference to the bandwidth it occupies can be misleading as in principle a perfect harmonic can have PSD that is proportional to *T* and therefore theoretically infinite. These considerations are necessary for defining noise and resolution characteristics of accelerometers as well as the structural responses.

What really matters to the user of an accelerometer is the ability to resolve a structural vibration and to reveal it above the various sources of noise in the measurement chain. Apart from electrical noise introduced by cabling and amplifiers, there are fundamental limitations [6]. In a piezo-electric accelerometer, internal electrical resistance generates a broadband Voltage signal with PSD proportional to square root of absolute temperature and inversely proportional to frequency, while the mechanical components generate mechanical noise, converted to Voltage, that is proportional only to temperature. Hence there are concepts of resolution (of an acceleration above the mechanical noise) and of sensor electrical noise. Similar principles apply to other types of accelerometer.

Consistent with this, integrated circuit piezoelectric (ICP) accelerometers with built-in amplifiers and AC coupling to separate drive Voltage from the signal have characteristic increasing noise levels at low frequencies, rendering all but the very fragile and expensive models unusable for low-vibration, low frequency measurements. This is part of the reason why the Vibration Engineering Section (vibration.shef.ac.uk) now uses quartz-flex servo accelerometers exclusively for field testing. The Honeywell QA 750 and QA 700 units used quote 'Resolution/Threshold <1μg' but provide no information about noise levels. However, in a number of measurements made in very quiet environments, signals as low as 3μm/sec^2√Hz have been recorded. The responses observed in these cases appeared to be white noise, suggesting valid measures of accelerometer noise, but there always remained a suspicion that some of this could be genuine structural noise. One test for this possibility is to use two accelerometers to measure at the same location and compute the coherence function. This test shows that indeed such low levels are predominantly noise, but the ideal measurement environment to confirm the noise levels would require isolation from all mechanical inputs.

Because of the long lead time in acquiring quartz flex accelerometers, and because of increasing demand for permanent structural monitoring applications which are usually emergencies requiring very fast delivery of operational systems, alternative accelerometers may be required. One possible alternative to the quartz-flex accelerometers is the new Kistler 8330A accelerometer K-beam force-feedback capacitive accelerometer. Not trusting manufacturer noise specifications, a reliable method was needed to evaluate the sensor characteristics including noise floor.

CANDIDATE MECHANISMS FOR ESTABLISHING BACKGROUND NOISE LEVELS IN ACCELEROMETERS

The challenge was to find a means to isolate the sensitive axis of the new accelerometer from all inputs to its fixture, while also preventing electrical pickup or excitation from air currents. The main concern is mechanical isolation, which led back to the fundamental mechanics of base isolation and transmissibility functions.

Fig. 1 shows the classical transmissibility function (TF) for a single degree of freedom oscillator. The function indicates the proportion of ground/base motion transmitted to the sensor. Up to √2 times the natural frequency, TF≥1, while above this value the TF reduces approximately as ω^{-2}, with modest dependence on the damping ratio. Hence for an isolation system to transmit only 1% of ground motion at 1Hz requires an isolator frequency of 0.1Hz. Some research uncovered a range of exotic isolation devices that were evaluated for suitability.

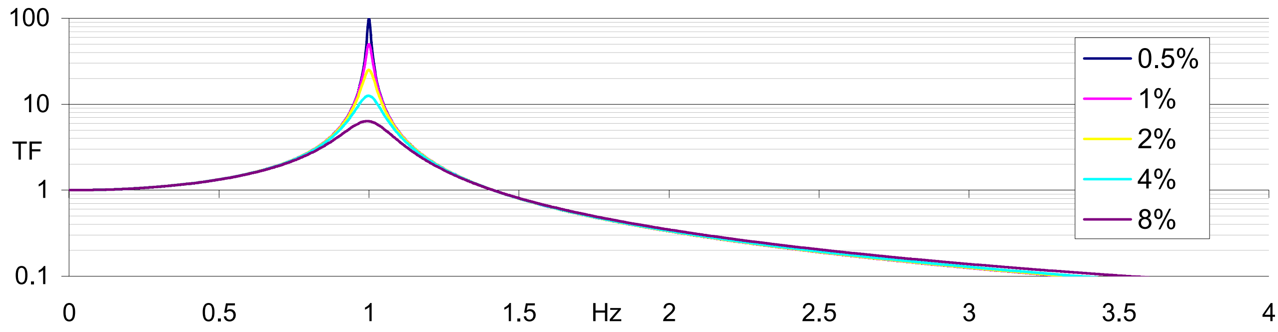


Figure 1 SDOF oscillator transmissibility function for a range of damping values

The simple pendulum and variants

For a simple pendulum with radius L the natural period is $T_p = 2\pi\sqrt{L/g}$. The range of civil structural response frequencies starts at 0.05Hz (for lateral modes of long span suspension bridges) and measuring sensor noise at such a low frequency requires isolation at less than half this value with a 0.025Hz pendulum requiring a radius of 400m. Alternatives that could reduce the height to manageable levels include the multistage pendulum, such as used for a tuned mass damper in the Yokohama Landmark Tower [7] or could use a fiction pendulum bearing [8], where the frequency depends on the radius of the bearing. More exotic solutions have been used to achieve isolation for low vibration measurements e.g. the LIGO stack [9] used to isolate sensing elements at the Laser Interferometer Gravitational Wave Observatory (LIGO) from ground borne excitation at frequencies exceeding 100Hz, Scott-Russell and Roberts linkages [10] and finally the folded pendulum [11].

Folded pendulum

The folded pendulum concept was created for low-frequency isolation of gravitational wave detectors since the peak frequency for micro-seismic noise [6] is approximately 0.2Hz. The folded pendulum comprises a platform attached at one end to a positive pendulum with grounded pivot at the top, and at the other to a negative pendulum with the grounded pivot at the bottom (and hence by itself unstable). A very low natural frequency is obtained with a small positive difference between the lengths of the positive and negative pendulum arms. The folded pendulum was chosen for this application because it can be made compact, there are no requirements for expensive machined suspension systems and it is conceptually very simple.

FOLDED PENDULUM (FIG. 2): FUNDAMENTALS

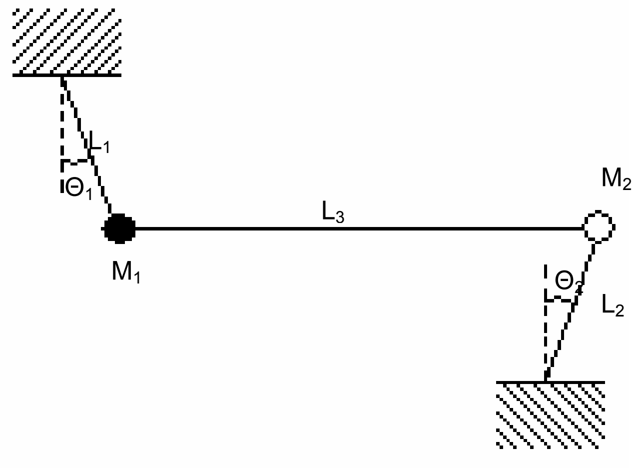


Fig. 2 Schematic arrangement of folded pendulum

The folded pendulum consists of a normal positive pendulum with mass M_1 connected to an unstable inverted pendulum with mass M_2 by a mass-less horizontal platform or deck. In reality, the pendulum arms and the deck will have distributed masses, but for theoretical analysis, M_1 and M_2 are calculated as half of the pendulum arm masses + a proportion of the deck weight.

For the left hand (positive) pendulum, the restoring force for small angles is

(1) $$F = M_1 g \theta_1$$

For the right hand (inverted), the restoring force is negative i.e.

(2) $$F = -M_2 g \theta_2$$

The total restoring force is then

(3) $$F = M_1 g\theta_1 - M_2 g\theta_2$$

For small horizontal displacement *x*, the two angles depend on the respective pendulum lengths L_1 and L_2 so that

(4) $$\left(\frac{M_1 g}{L_1} - \frac{M_2 g}{L_2}\right)x + \left(M_1 + M_2\right)\ddot{x} = 0$$

The natural frequency of the folded pendulum is then:

(5) $$f_n = \frac{1}{2\pi}\sqrt{\left(\frac{1}{\left(M_1 + M_2\right)}\left(\frac{M_1 g}{L_1} - \frac{M_2 g}{L_2}\right)\right)}$$

To keep the deck horizontal the pendulum arms should be equal, and for stability $M_1 > M_2$, while the smaller the mass difference, the lower the frequency that can be achieved. In practical terms adjustable masses would need to be attached to the deck, so that the effective centre of the platform mass could be shifted horizontally.

FOLDED PENDULUM COMPONENTS

The folded pendulum, Fig. 3, as constructed [12] has five main components, each with their own requirements.

<u>Hinges</u>: These connect pendulum arms, deck and supporting structure and must provide constraints against lateral motion while minimising friction. Clamped 0.1mm stainless steel shims were used to provide negligible stiffness in the horizontal direction, but provide lateral stability.

<u>Pendulum arms:</u> Figure 3 shows that the inverted pendulum arm needs to adopt a C-shape so that the shims can operate in tension.

<u>The deck:</u> This accommodates the payload and must also

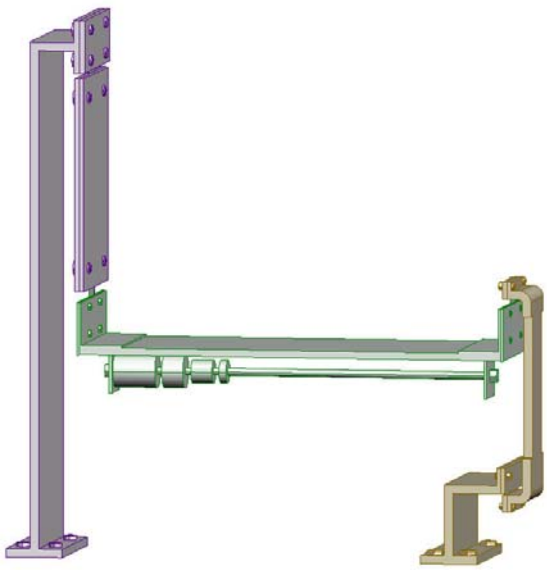


Figure 3 Folded pendulum design

support moveable masses to adjust M_1 and M_2.

The base and supports: The base plate may need to provide for adjustment of the pivot supports.

Casing: This needs to isolate the pendulum from air currents and support cables attached to the accelerometer.

Successful operation of the pendulum depends on careful adjustment of the masses of the moving parts. Additional masses with vertical height adjustment were attached to the pendulum arms. These are intended to allow for the centre of percussion of the pendulum arms to be set at the level of the deck, which minimises the transmissibility function, according to the equations of motion [13].

PERFORMANCE OF FOLDED PENDULUM

Experiments were done to evaluate both the performance of the pendulum and the noise characteristics of the new accelerometer. Fig. 4 shows one the test arrangements: the pendulum is attached to a wooden board screwed to the body of an APS400 electro-dynamic shaker set up in fixed-armature mode as a small shaking table (the specimen on the right is a model multi-storey building). The accelerometer was attached to the pendulum deck using a glue gun.

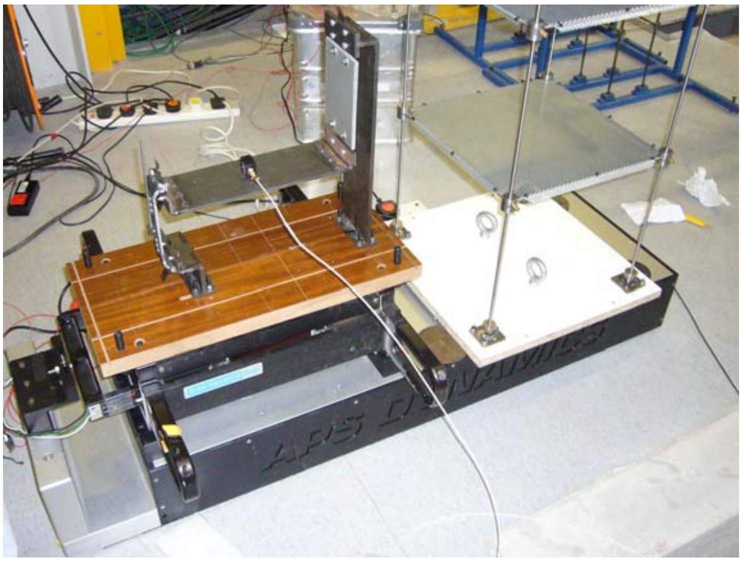


Figure 4 Folded pendulum performance test

For the first experiment the pendulum was not attached to the shaker, but placed on the concrete floor in the basement libratory. Free decay response of the pendulum was evaluated for various positions of the adjustable mass. Fig. 5 shows the recorded acceleration for one measurement. The transient is the initial gentle push and the linear oscillator that best fits the following oscillation has 0.16Hz natural frequency and 2.5% damping. The nonlinear character is expected at the deflection levels observed; for a 0.16Hz oscillator displacement and acceleration have almost the same numerical value so the pendulum arc (shown in Fig. 6) after 10 seconds would be up to 0.07 radian (4°) for the 150mm pendulum arm length. When observed over a much longer duration, the system decays to a response that perfectly matches a linear system.

The high frequency noise on the plot is believed to be due to one or more vibration modes of the accelerometer on the deck, via its glue fixing, predominantly at about 21Hz.

The lowest frequency achieved was 0.078Hz by careful adjustment of the movable masses under the deck. Damping ratios varied between 10% for the lowest frequency down to 2.1% for a frequency of 0.173Hz, suggesting the origin as various forms of friction.

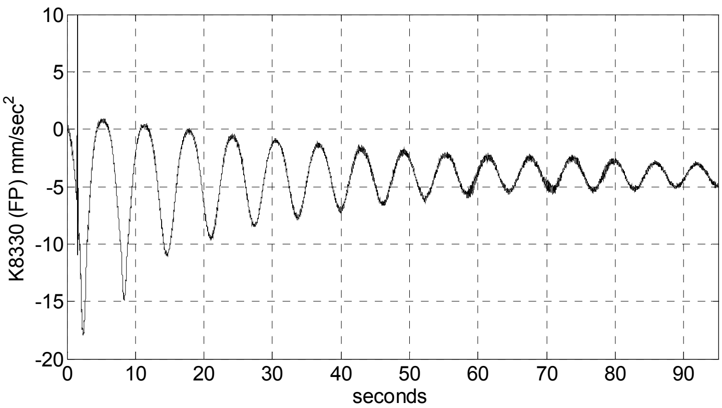


Figure 5

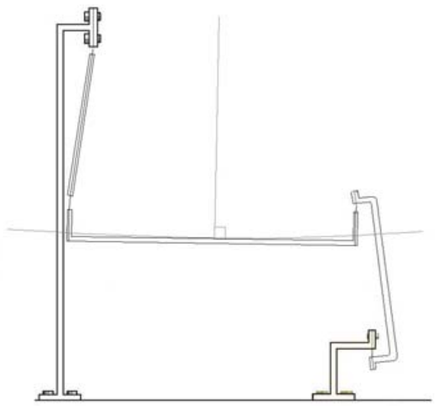


Figure 6

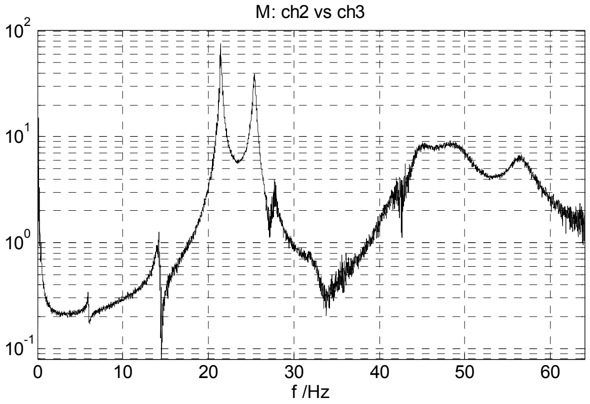
Fig.7 shows the transmissibility function for broadband random shaker excitations (Fig. 4) , using an Endevco 7754-1000 on the shaker body as reference. Below 18Hz, before the accelerometer fixture mode, the system is clearly effective at isolation.

Fig. 8 shows the isolation test repeated with the pendulum sat on the concrete floor in the basement laboratory, a vibration-quiet environment, with a QA700 and Endevco 7754-1000 on the concrete floor and the Kistler 8330 on the pendulum deck.

The QA700 has the superior noise performance, as low as 0.4μg/√Hz (4μm/sec^2/√Hz) at frequencies up to 4Hz, but displays some unusually strong response around 40Hz that is not seen in the other two accelerometers.

From 4Hz up to the fixture resonances, the response of the Kistler on the folded pendulum is the lowest, and coherence with the other accelerometers is negligible, suggesting 1μg/√Hz to be the noise floor of the accelerometer in this frequency band.

The Endevco has similar performance to the Kistler with increase noise at lower frequencies, again the best noise floor is 1μg/√Hz.



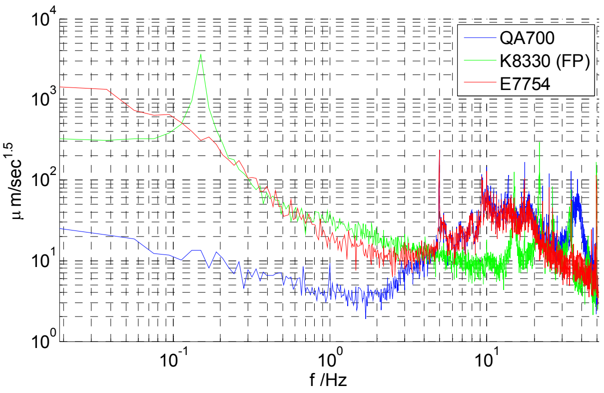


Figure 7 Transmissibility for shaker test

Figure 8 Ambient test

DISCUSSION

The folded pendulum is clearly capable of providing sufficient isolation to identify accelerometer noise floor, but there are some improvements that can be made, such as stiffer accelerometer fixing and experimentation with the centre of percussion adjustment. As it was built as part of a student project, these improvements and further testing will be incorporated in the next academic cycle of projects. The pendulum provides an excellent demonstration of isolation capability for students in the dynamics MSc courses ran in the department, as well as entertaining prospective undergraduates.

For the more serious application of the pendulum, it clearly demonstrated the characteristics of a new accelerometer, providing confidence for its application in a demanding field monitoring role.

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