EFFICIENT DYNAMIC PERFORMANCE ASSESSMENT OF A FOOTBRIDGE

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Abstract

Dynamic performance of footbridges has become a significant concern in recent years, resulting in increasing demand for assessment of the performance of new and existing footbridges subject to pedestrian loading scenarios far more complex than the existing code provisions. Performance assessment typically involves predictions based on numerical simulations using finite element representations and pedestrian load models, possibly followed by experimental assessment under normal and limiting load conditions. Since dynamic performance is strongly related to all the modal parameters of the bridge i.e. modal frequency, damping, shape and mass, their reliable identification for modes to be involved in pedestrian-induced response is central to assessment. The paper describes an efficient procedure for assessing dynamic characteristics and performance using a combination of visual survey, finite element modeling and brief and unobtrusive dynamic testing, followed by short observation of pedestrian-induced response and finite element model updating for evaluating other loading scenarios. The procedure involves minimal effort for maximum effect, generating a reliable analytical representation for response simulation and checking the serviceability of the bridge. The procedure has been tested using an existing non-problematic bridge, as described here, before being used on new structures.

Introduction

Footbridge dynamic response studies have been long been a favourite choice for student project work[1], and lively footbridges have often been viewed as curiosities. 'Curious' bridge behaviour is becoming less acceptable and authorities responsible for new and existing bridges now find it necessary to be fore-warned and take measures to mitigate lively response of their charges. For an existing bridge, checks may need to be made to predict and prevent unsatisfactory performance for extreme pedestrian loads e.g. during an evacuation or major public event. For a new bridge, performance even under normal loadings is a concern, moreover public openings of showcase structures are likely to generate the largest pedestrian loads in the lifetime of the bridge. Lively vertical response tends to be a problem with lighter flexible structures (e.g. suspension footbridges) whereas lateral response problems can occur in any bridge having lateral response frequencies below approximately 1Hz. Excessive lateral sway due to pedestrian crowds has been observed on existing major road bridges such as Auckland Harbour Bridge, Brooklyn Bridge and Bogazici Bridge (Istanbul) but structural remedial measures for cure or prevention are more feasible for pedestrian bridges e.g. London Millennium Bridge[2] and Changi Mezzanine Bridge[3]

For a new design, in the first instance the performance will be simulated analytically using finite element models and pedestrian loading scenarios. Potential design blunders can be checked more effectively as understanding of pedestrian loading improves but there are still large gaps in understanding of pedestrian loading with very limited code guidance, and numerical models are also error prone. Hence some kind of testing of an as-built structure is strongly recommended and now more frequently practiced. Such testing can involve checking of modal parameters or may go as far as proof testing in the presence of large numbers of pedestrian[2,3].

For an existing design, prediction of response during extreme loading will require reliable estimates of all modal parameters, with particular emphasis on damping and modal mass.

Analytical methods for assessment

Footbridge assessment is treated differently depending on whether the concern is due to lateral or vertical pedestrian effects and the effects they have in inducing excessive response in lateral vibration modes (for lateral forcing) or vertical/torsional response. In each case reliable estimates of modal parameters are required.

For lateral loading effects, synchronous lateral excitation is the concern. According to research related to London Millennium Bridge[3], there is a critical number *N* of pedestrians needed to bring about the situation where feedback of lateral forces cancels out the positive damping of the structure resulting in unbounded growth of response.

The number *N* is estimated from

$$N = \frac{8\pi f_r \zeta_r m_r}{k} \quad 1)$$

where f_r, ζ_r and m_r are (lateral) mode *r* frequency, damping and unit-normalised modal mass and *k* is an empirical constant estimated as 300Ns/m. The result has been validated on at least one other structure. As this is an instability criterion, there is no corresponding allowable vibration limit.

There has also been a suggestion[4] that synchronisation and unbounded response can occur in the vertical direction. This is not yet clearly proven, and has to be set against the proven positive damping capability of stationary pedestrians for vertical response, but an equivalent of the Scruton number used in wind engineering has been proposed[4] to provide a form of reduced damping that can be applied to footbridges. The form used for footbridges would be

$$S_{cp} = 2\zeta M / m \quad 2)$$

where S_{cp} = pedestrian Scruton number, ζ = modal damping ratio, M = bridge mass per unit length, m = pedestrian mass per unit length. A higher value is better and the suggestion is that Scruton number should exceed some value less than unity, as low as 0.27.

For vertical loading effects the traditional UK assessment method has been to use the bridge code BD37/01[5]. This prescribes a sinusoidal load

$$F_1 = 180 \sin 2\pi f_o t \quad 3)$$

corresponding to perfect generation of first harmonic of pacing rate by a pedestrian walking at exactly the frequency of the bridge, f_o,. Naturally, because such perfection is never achieved with real people, the result is invariably conservative for a single pedestrian. In fact this conservatism provides a significant but unquantifiable reserve to deal with response to multiple pedestrians, a result frequently observed[3,6] but risky to formalize.

For crowd loading a frequency domain approach would be appropriate, with harmonic forcing amplitude

$$P = \left(\alpha \times N \times |F_1|\right) \quad 4)$$

but apart from a few studies on the random response due pedestrians[7-9] there is so far no accepted guidance on the crowd factor α . For vertical loading with bounded response, acceptable vibration limits are prescribed.

The possibility of unacceptable response to pedestrian loading can be checked via these formulae and the modal parameter estimates estimated during the design phase, but confirmation and adjustment should be made on the (as-built) prototype by experimental assessment. Given the opportunity, crowd loading tests can be used to check the conservatism of the BD37/01 serviceability assessment and even, given enough volunteers, the accuracy of the instability parameter calculations.

Experimental assessment is usually regarded as a costly exercise requiring closure of the structure to pedestrian access and use of forced vibration test equipment such as shaker(s) or instrumented hammer. This paper shows how quick, simple and unobtrusive measurements can generate reliable parameter estimates for modes likely to be a problem. The method can also be extended so that a prior finite element model can be systematically adjusted so as to provide a reliable basis for simulating other loading scenarios. The example used to illustrate the procedure is a walkway in Plymouth.

Western Approach Footbridge, Plymouth

The bridge, connecting a large car park and leisure complex to the town shopping centre is shown in Figure 1. It resembles a variety of Warren truss with vertical supports, constructed from hollow steel sections, supporting a concrete slab deck and carrying a inverted U-shaped shelter that apparently adds little to the bending resistance of the bridge frame. The 27.6m span is simply supported on concrete piers and isolated from the short sidespans. With no drawings available, all dimensions, member sizes and slab thickness were estimated from a visual survey, and a finite element model was constructed using ANSYS finite element analysis code before any experimental investigation.

An approximate estimate of first vertical vibration mode frequency was obtained by modeling the bridge as a simple beam using the equation

$$\omega_n = \frac{\pi^2}{L^2}\sqrt{\left(\frac{EI}{m}\right)} \qquad 5)$$

for span L, taking mass m as 1304 kg/m, second moment of area I as 0.0438m^4 from estimated member sizes and spacing, and the modulus of elasticity E as 205 kN/mm^2. This predicts a first mode frequency of 5.4Hz (compared with 4.5Hz from the ANSYS model) and having half-sine mode shape with modal mass of 17,710kg for unity mode shape normalisation.

The bridge is heavily used during busy shopping periods, and an experimental study was conducted on two days of a weekend close to Christmas 2004. There was no possibility to close the bridge, pedestrian flow could not be impeded and time was limited for measurement due to the battery life of the recording equipment. Hence the bare minimum of equipment was used: three QA700 servo-accelerometers, a simple signal conditioner and a PC-based 16-bit digital recorder. An instrumented hammer was available but proved inefficient due to heavy usage of the bridge and resulting poor signal to noise ratios.

Measurement procedure day 1: modal survey

The bridge was tested briefly on two separate days. The measurements on the first day (a Friday evening with moderate pedestrian usage) used only two accelerometers and served to identify the modes by a short program of free-vibration or ‘output only’ measurements at a combination of key locations on the bridge i.e.:

Measurement T1: 1/6th span and 5/6th span on same side

Measurement T2: 1/6th span both sides

Measurement T3 1/6th span and 1/2 span same side.

For each measurement, 200Hz acquisition rate was used and each measurement lasted no more than six minutes. The intention of this set of measurements was to identify the lowest few vibration modes including those readily excited by pedestrian movement. The recordings were sufficient to identify the following vibration modes and estimate their frequencies and damping ratios, obtained with varying numbers of pedestrians in transit across the bridge:

first (symmetric) vertical mode	V1:	4.55Hz	1.74%
first (symmetric) torsional mode	T1:	8.83Hz	0.8%
second (anti-symmetric) vertical mode	V2:	12.72Hz	1.3%

third (symmetric) mode	V3:	20.24Hz	3.1%

The natural excitation technique/eigensystem realization algorithm (NExT/ERA[10,11]) was used to recover the modes from combination of the three sets of measurements. To begin, time series were divided into 2048 point blocks and three cross-power matrices having dimension 2x2x1024 were computed using the Welch algorithm[12]. These were then divided by the reference channel auto-power to normalize the cross powers with respect to the response at the 1/6th span position. The three sets of normalized cross-powers for each of the three alternate rover accelerometer positions (1/2 and 5/6th span on same side and 1/6th span on other side) were assembled together with the reference auto-power averaged over the three measurements to form a 4x1024 point vector. This was then multiplied by the averaged reference auto-power and the inverse Fourier transform obtained.

Figure 2 shows the resulting set of four cross-powers and Figure 3 shows their inverse Fourier transforms. It can be shown[10] that for a flat spectrum excitation force, these traces are scaled equivalents of impulse response functions that contain all the information necessary to extract modal parameters by a number of techniques including ERA[11]. The impulse response functions can also be derived from time series using cross-correlation functions and there is a growing array of output only identification procedures becoming available for this type of work, capable of providing reasonable estimates of damping and true mode shapes rather than operational deflection shapes, removing the need for artificial forcing. The only information missing is an experimental estimate of modal mass.

Measurement procedure day2: response to pedestrian dynamic loads.

For the second set of measurements (on the following Saturday morning), three accelerometers were used, measuring vertically at 1/6th span and 1/2 span locations and horizontally at 1/6th span location as follows:

Measurement T5: heavy pedestrian activity, duration 43 minutes.

Measurement T7: brief walking tests during empty periods, duration 8 minutes

Measurement T8: jumping test, duration 80 seconds.

T5 was carried out as shoppers walked across the bridge into town from the car park, so that the number of pedestrians on the bridge varied from zero to 24; Figure 4 shows variation of vibration strength in the 0-10Hz band together with the number of pedestrians over the 43 minute period. The correlation of response levels with pedestrian numbers is quantified in Figure 5, which shows variation of RMS acceleration response with a narrow band pass filter of 4-5Hz around mode V1 frequency. There is a clear but non-linear dependence superimposed on a background of response that is partly due to light winds and partly due to the approximate method of estimating pedestrian numbers by recording a single digital pulse on the recorder every time someone was observed to pass the 1/2 span position.

In Figure 3 a peak in the power spectra appears at 2Hz. While this looks like a vibration mode, ERA fails to identify it as such and there is no evidence of any such low frequency vibration mode from the finite element modeling so it has to be concluded that it results from fundamental frequencies of the varying pacing rates as a quasi-static forcing, well away from resonance. Figure 6 shows variation of the 2Hz RMS response levels with pedestrian numbers, a different relationship from Figure 5.

Assuming a modal mass of 17,700kg (from the initial FE model) hence modal stiffness of 14.4MN/m, and that static deflected shape resembles first mode shape, acceleration response due to the fundamental component of walking force would, using the BD37/01 180N force at 2Hz result in acceleration of 2mm/sec^2. This is the same order as the observed response. The point here is that occasionally, significant response amplitudes result from time varying loads without any help from resonance, and can be misinterpreted.

The lateral response shows a peak at 4.26Hz which is likely to be a lateral mode of the deck (and probably piers). There is also relatively strong lateral response at mode T1 frequency.

In T7, attempts were made to force bridge vibration in modes V1 and T1 by walking at a range of pacing rates during the few brief periods when the bridge was practically empty. The strongest response was obtained by jumping at half 135 jumps per minute, corresponding to half of mode V1 frequency. Hence in T8, a single sequence of jumping at 135bpm at 1/2 span was used to excite the bridge in mode V1. Figure 7 shows the 1/2 span response during the build up and the subsequent free vibration decay, from which it was possible to obtain estimates of credible estimates of modal mass and damping.

Jumping forces can be approximately represented by a sum of sinusoids, and for jumping at 2.25Hz the second harmonic forces the response in mode V1. The first part of the build up due to such forcing is relatively insensitive to either damping or to imperfections in the timing of the jumps so the first few cycles can provide modal mass estimates if the characteristics of the jumper are known.

Since the jumper force characteristic i.e. the strength of the harmonic components has been recorded previously on a force plate, the value of mass that is consistent with observed response due to second harmonic forcing at 4.5Hz should provide an estimate of modal mass. The method was applied here and provided a modal mass estimate of 18,000kg for mode V1.

This method has the advantage, over hammer testing, of a high signal to noise ratio and (over shaker testing) of portability. Estimation errors are comparable to those with either mechanical excitation method.

Curve fitting to the complete decay curve provided frequency and damping estimates of 4.44Hz and 1.2%. Piecewise curve fitting to groups of six successive cycles of the decay showed frequency variation from 4.4Hz at the largest amplitude ($0.7 m/sec^2$) to 4.5Hz for the weakest part of the responses. It was not possible to identify any consistent variation of modal parameters due to variations in pedestrian numbers or the response levels induced. Effects of passive damping of stationary or even moving pedestrians have been observed on other structures and a subject of continuing investigation.

Finite element model updating and tuning

The original ANSYS model represented the bridge as a combination of two vertical and one horizontal grillages of beam elements, with diagonal truss elements, supporting an integral 100mm slab pinned at each end, and excluding any kind of contribution from the shelter. This a-priori model predicted a set of frequencies given in column 3 of Table 1 and compared with the ERA experimental estimates:

Table 1 Modal parameter estimates from measurement and finite element modeling

Mode	ERA Frequency	ANSYS a-priori	ANSYS updated	ANSYS tuned
V1:	4.55Hz	4.51Hz	4.58Hz	4.55Hz
T1:	8.83Hz	6.68Hz	8.17Hz	8.05Hz
V2:	12.72Hz	13.74Hz	12.93Hz	12.71Hz
V3:	20.24Hz	-	20.56Hz	20.38Hz

Within the first 20 modes of the preliminary (a-priori) model, many modes with lateral flapping of the upright frames were generated, but not the third vertical mode V3. After the testing the model was adjusted to exclude the flapping modes from the solution. Initially truss elements were used to close the U-shaped main structure into a box, but this was not fully effective so the trusses were changed to frames elements with areas approximating to the area of the shelter member sizes. This manual adjustment or updating led to an improvement in the ANSYS torsional mode (T1) frequency. The 'updated' frequency values are given in the fourth column of Table 1.

These frequency values are already close enough to the experimental values to satisfy many analysts, but further improvement can be obtained by a process of systematic updating called tuning. The procedure uses sensitivity analysis to minimize differences between selected analytical response parameters $\mathbf{r}_i$ and their experimental counterparts $\mathbf{r}_\mathrm{e}$ by adjusting the selected structural parameters $\mathbf{p}_i$. The procedure attempts to find roots of

$$\mathbf{r}_i\left(\mathbf{p}_i\right)-\mathbf{r}_e=\mathbf{0} \qquad 6)$$

by iteration using the scheme

$$\mathbf{p}_{i+1}=\mathbf{p}_i-\mathbf{T}^{-1}\left(\mathbf{r}_i-\mathbf{r}_e\right) \qquad 7)$$

The matrix $\mathbf{T}^{-1}$ is the pseudo-inverse of the Jacobian matrix $\left[\partial r_j / \partial p_k\right]$ that expresses the variation of the j^{th} response parameter with respect to changes in the k^{th} structural parameter. The Jacobian can be evaluated via finite differences by re-running the FE solution and perturbing each of the parameters *k*, but in principle terms can be evaluated analytically.

Response parameters that are typically chosen are the modal frequencies but mode shapes are also used either in the form of modal assurance criteria ('MAC' which is a correlation coefficient between mode shape ordinates of experimental and corresponding analytical mode degrees of freedom) or coordinate modal assurance criteria ('COMAC' which expresses how modal ordinates correlate across all modes for a specific degree of freedom).

In practice zeroes of Equation 6 cannot be found unless all the features of the real structure are present in the FE model, so values of **p** are found to obtain a minimum. In the desirable case where the number of response *Nr* exceeds the number of structural parameters *Np*, Equation 7 involves a least squares solution, otherwise (unless *Nr*=*Np*) the values of **p** are found, rather arbitrarily, as those among the infinite number of solutions having the smallest absolute values. Since this makes no physical sense it is advisable to ensure that *Nr*<=*Np*.

A well conditioned problem and fast convergence is likely to be found if the chosen responses are sensitive to the chosen parameters. Hence for this example the values of normalized sensitivities $\left[\partial r_j / \partial p_k\right] \times \left[p_k / r_j\right]$ were examined and six parameters were chosen for which the mode frequencies and MAC values were most sensitive:

Parameter 1: diagonal bracing Young's modulus

Parameter 2: upright and top chord Young's modulus

Parameter 3: diagonal bracing area

Parameter 4: upright and top chord area

Parameter 5: upright and top chord second moment of area

Parameter 6: slab thickness

Response values chosen were the four modal frequencies and corresponding MAC values. Nine iterations of Equation 7 were run using FEMTools[13] model updating software interfaced to ANSYS and the final values of modal frequencies are given in the fifth column of Table 1.

Figure 8 shows the convergence of response parameters illustrating the large initial error in the second response parameter (Mode T1 frequency) and the already low error in mode V1 frequency. Parameters 5-7 and 12-14 are dummy (zero) values but parameters 9-11, which are MAC values, also demonstrate low initial errors which improve very slightly.

Figure 10 shows the trends in the structural parameters; Parameter 2 (upright/top chord Young's modulus) has increased 20%, parameter 6 has increased 14% and parameters 3-5 have decreased 8 to 9%. Subsequent to the updating exercise, a previous study of a neighboring bridge made with identical member sizes was found showing that slab thickness was in fact 125mm.

Figure 10 shows the almost perfect matching of the FE mode shapes from the final model with the few experimental data points; the matching of mode shapes was already good for vertical modes in the earlier models. The sparsity of experimental data points used to identify the modes worked in this case, but in more complex structures a finer grid would be needed involving more measurement setups or more accelerometers (or both). The aim in this case was to do to the minimum, quickly, to obtain the necessary identification.

The physical significance of an indicated 20% increase in Young's modulus is worth considering along with a reduced area and increased second moment of area increased. The density parameter was not used and it is not known for sure what the hollow section thickness is: typical sizes are used since only external dimensions are known. Physical bounds can be put on the structural parameters but the tuning is still a mathematical procedure (not quite a black box) where results have to be viewed with caution. Since only a few accelerometer locations were used, higher order modes could not be identified, restricting the number of structural parameters to be considered.

This type of model updating is not restricted to specialized software. It can be organised using any finite element software, repeated runs with perturbed parameters to generate finite difference sensitivities and some limited mathematical capability. Such an exercise has previously been conducted on a suspension footbridge[14.] Complete control of the process is possible, with appreciation of the physical significance of parameter variations and intimate acquaintance with the mathematical issues.

Response scenarios

With reliable modal parameter estimates and an understanding of the (in this case very simple) performance mechanisms, serviceability assessments under unusual loading scenarios can be made.

For vertical response, the final values of mode V1 parameters are:

Modal frequency: 4.4Hz	(based on experiment, for large amplitude response):
Modal damping: 1.2%	(based on experiment for large amplitude response
Mode shape: half-sine	(from validated finite element model)
Modal mass: 18,035kg	(from validated finite element model)

Based on BD37/01, as mode V1 frequency exceeds 4Hz, using 88% of the 180N load of Equation 3 for a pedestrian moving over the bridge at a rather fast 3.96m/sec and crossing in 7 seconds, peak acceleration is 0.279m/sec^2. This compares to peak of 0.32m/sec recorded due to multiple pedestrian loads over the total frequency band of response or 0.23m/sec^2 around mode V1 frequency and the 1.05m/sec^2 acceptance limit from BD37/01.

From Equation 2, 24 people provide a pedestrian Scruton number of approximately 0.5 (i.e. ‘safe’). In any case the response is due to second harmonic of pedestrian loads and there is no known experience of synchronization of modes with frequencies as multiples of pacing rates so this may not be a relevant result. Likewise for lateral response, the first lateral mode frequency is well in excess of 1.3Hz so instability due to lateral forces is not an issue.

Discussion

The bridge studied was chosen for convenience of apparent simplicity rather than due to any known problems. In fact the bridge behaves very well under pedestrian loading. The point of the exercise has been to evaluate a combined analytical/experimental assessment process that can be applied to other new and existing footbridges.

A simple assessment procedure using a pair of accelerometers will in many cases be all that is needed to identify critical modal parameters. Visual inspection can provide a first approximation to a finite element (FE) model that can be updated based on experimental modes. Updating or validation of a FE model can provide confidence to use it in scenario simulation and as a cross-check on modal mass values. A few simple walking and jumping experiments can, if conducted appropriately and analysed correctly provide a wealth of information regarding footbridge serviceability. There is no inconvenience to users due to the efficiency of the measurement techniques and the use of pedestrians themselves to excite the bridge.

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Figure 1 Western Approach Footbridge

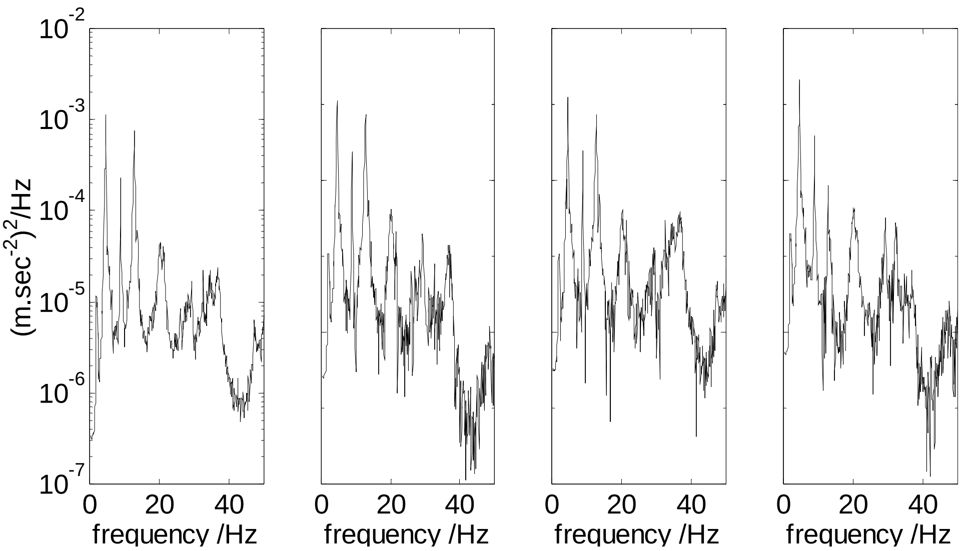

Figure 2 Cross-power spectra of a set of four vertical response measurements

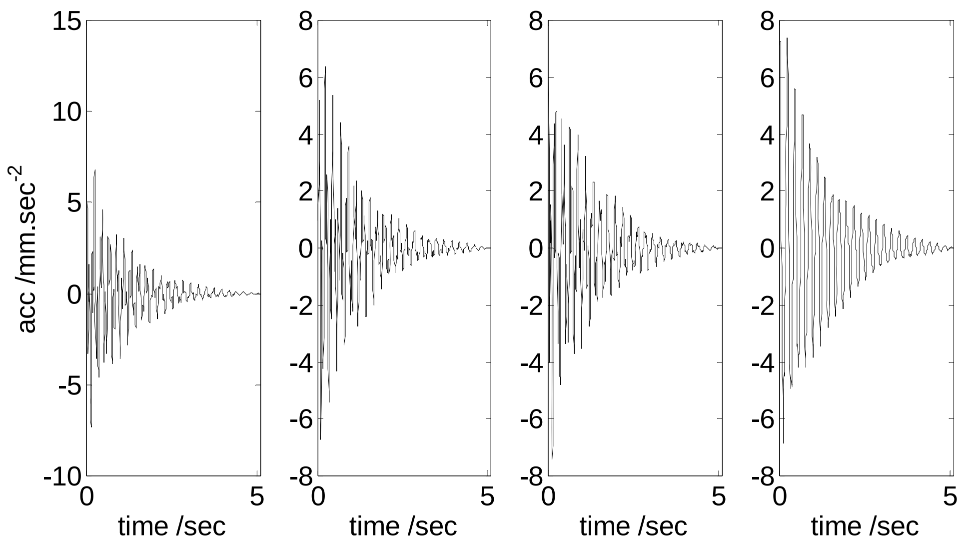

Figure 3 Impulse response functions resulting from cross-powers of Figure 2

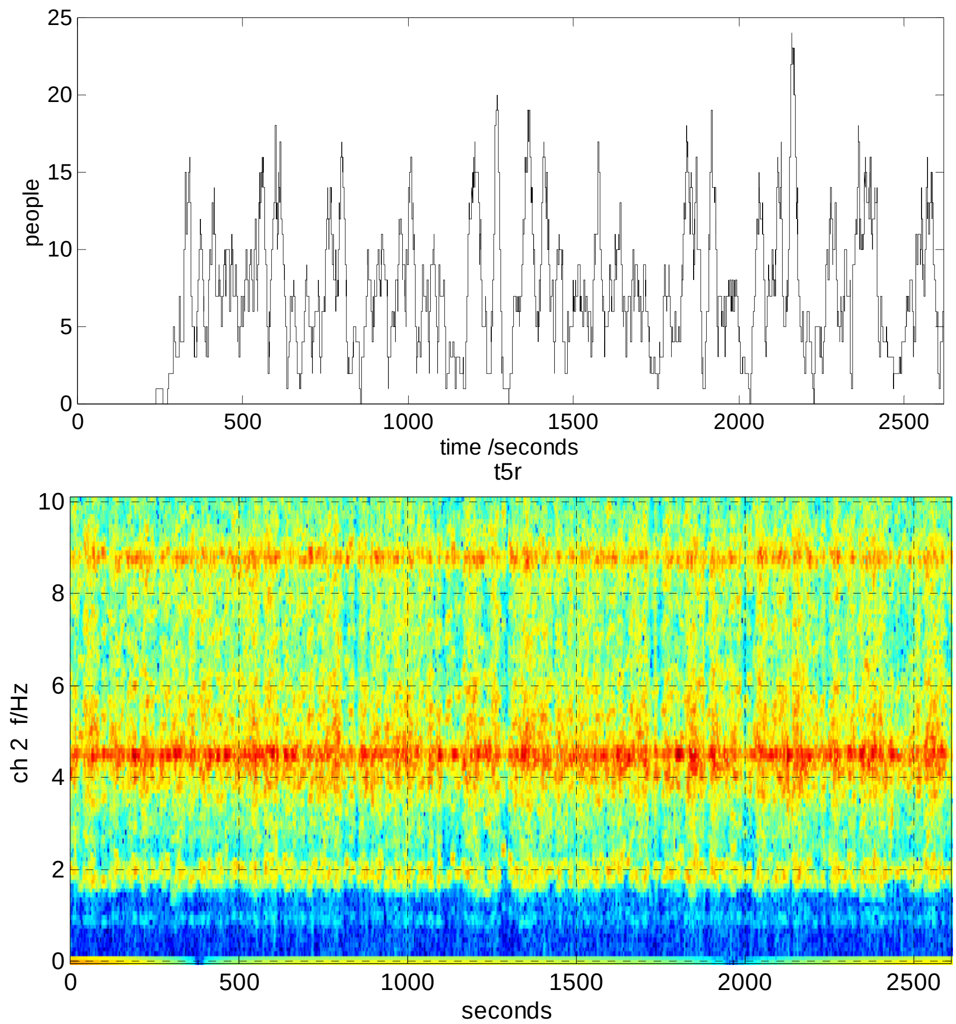

Figure 4 Pedestrian numbers and 1/2 span vertical response spectrogram

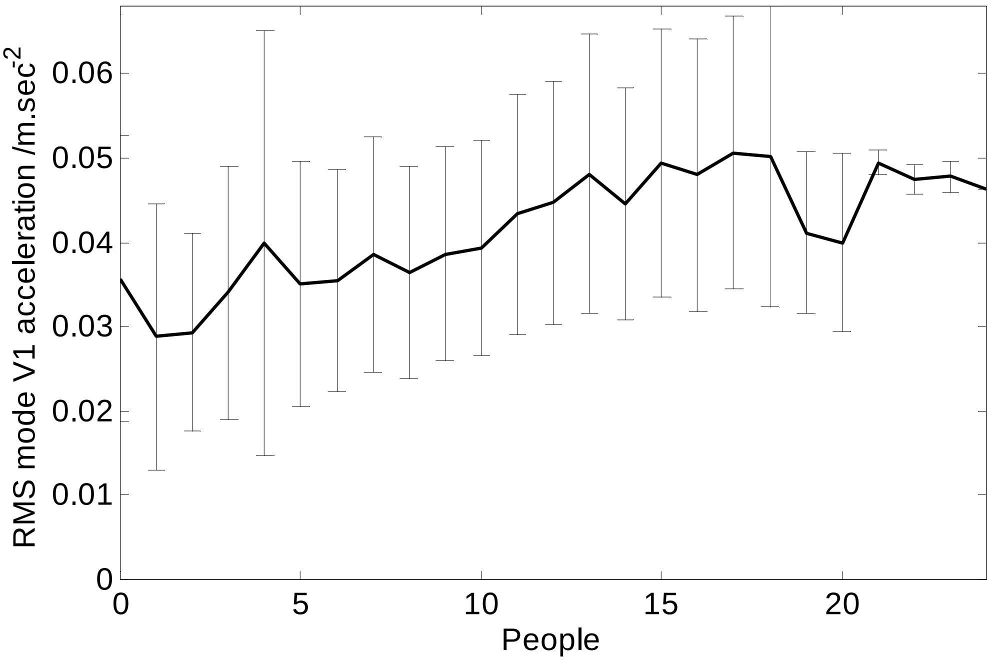

Figure 5 Variation of mode V1 response with pedestrian numbers

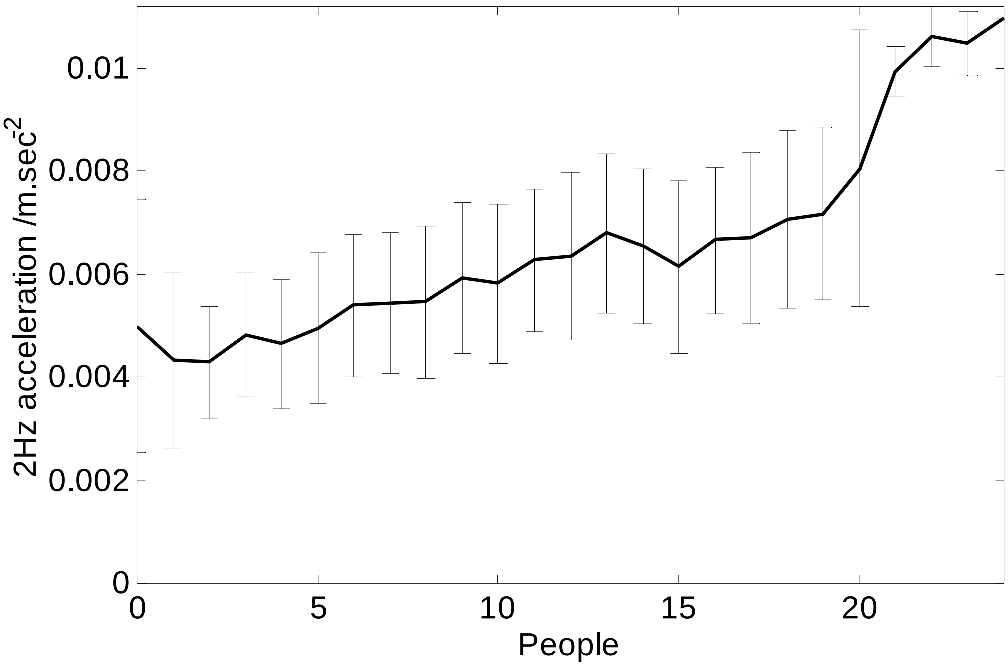

Figure 6 Variation of quasi-dynamic (2Hz) response with pedestrian numbers

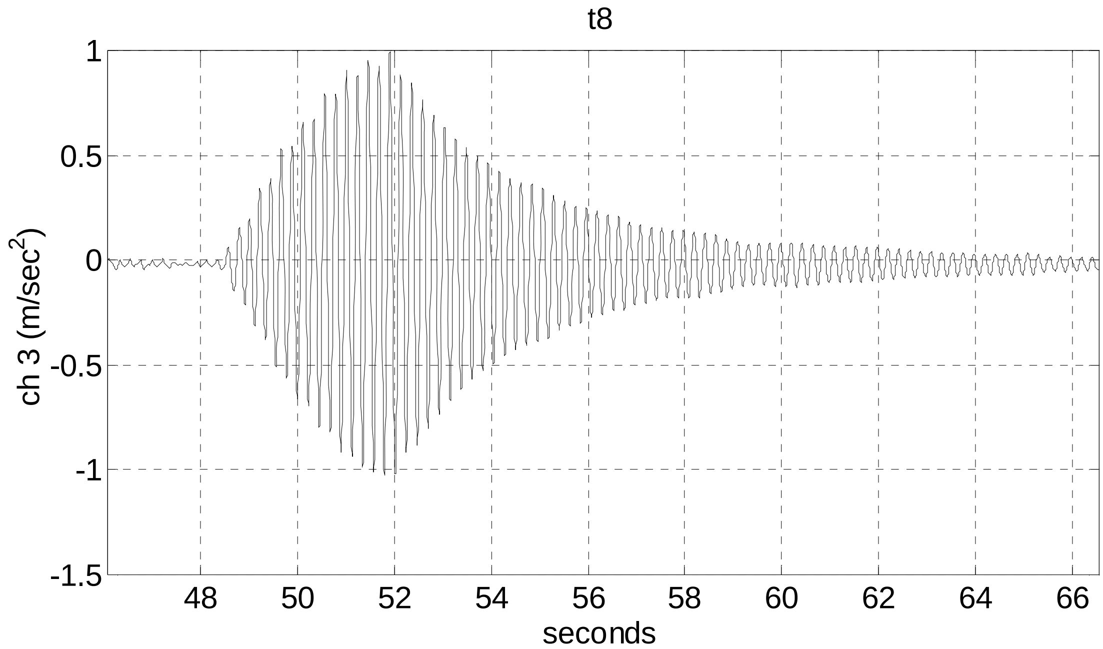

Figure 7 1/2 span build-up and decay due to jumping at 2.25Hz

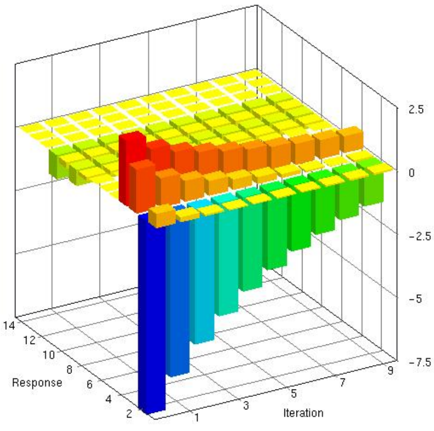

Figure 8 Converge of analytical response parameters (frequencies and mode shapes) to target experimental values during model tuning

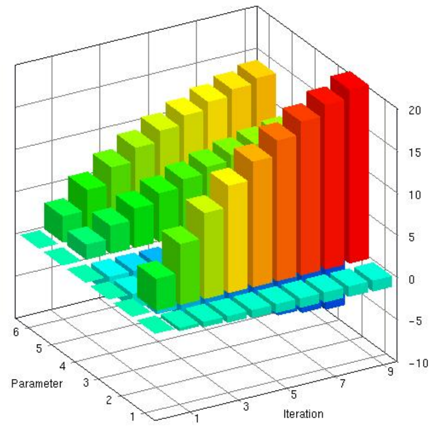

Figure 9 Convergence of analytical structural parameters during model updating

Figure 10 Comparison of experimental mode shapes (red dots) with analytical modes of final model. Clockwise from top left: V1, T1, V3, V2