Assessment of the Effect of Unusual Events Recorded by Long-Term Structural Health Monitoring Systems Using Box Jenkins Models and Wavelet Analysis.

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ABSTRACT

In-service civil infrastructure experience short-lived and transient changes in strain from time to time resulting for example from ground movements, development of cracks, heavy traffic, and accidents. With the advent of instrumented structural monitoring it is now possible to capture these events. In this paper an approach based on Box-Jenkins transfer functions is proposed for assessing the effect of these events on structural behaviour and performance. The analysis is based on strain data recorded by a structural health monitoring system installed in a major bridge at construction stage.

INTRODUCTION

Structural health monitoring (SHM) is defined here, in the context of a continuous long-term health monitoring system, as the continuous monitoring of a structure's response to the loading environment in order to build up a database of loading demand and to diagnose the onset of anomalies in structural behaviour and performance. Anomalies can include and are often defined as deterioration and damage resulting from changes in material properties, geometric properties, boundary conditions, system connectivity and the loading environment of the structure.

Identification and characterisation of anomalies is important from the point of view of infrastructure management as these events could have adverse effects on structural behaviour. In a long-term continuous monitoring system, these unusual random events often appear as abrupt or transient changes hidden in measurement data, and their detection requires systematic procedure. Once these events have been located it is essential to assess their effect on the structure since they may affect serviceability and long-term performance of structure.

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In this paper we use transfer functions of Box-Jenkins model type to assess the effect of unusual events observed in strain data recorded by a long-term health monitoring system. First a brief description of the procedure for identification of anomalous events using wavelet analysis is outlined. A background of the relevant theory on applying Box-Jenkins models to assess the effect of external events then follows. The proposed procedure is then used to assess the effect of some events identified from strain data recorded by a bridge health monitoring system.

IDENTIFICATION OF ANOMALOUS EVENTS

From experience gained with monitoring bridges in Singapore (Moyo and Brownjohn, 2002), identifying and locating abrupt and transient changes from health monitoring data such as stress, strain, accelerations, constitutes a significant portion of the data interpretation problem. To this end an approach based on wavelet analysis has been developed for identifying abrupt and transient changes from static strain data acquired by continuous structural health monitoring systems.

The basic idea of wavelet analysis is to represent general functions in terms of simpler fixed building blocks at different scales and positions. A theoretical treatment of wavelets and wavelet analysis can be found in Daubechies (1992). In most practical signal processing applications, data decomposition is carried out using the discrete wavelet transform. The discrete wavelet transform of a function Y_t consists of wavelet and scaling coefficients:

$$d_{j,k} = \int_{-\infty}^{\infty} Y_t\, \psi^*_{j,k}(t)dt\,, \quad c_{J,k} = \int_{-\infty}^{\infty} Y_t\, \phi^*_{J,k}(t)dt\,, \text{ j=J, J-1, …, 1} \qquad (1)$$

where,

$$J = \log_2(n)\,,\ n = \text{number of samples} \qquad (2)$$

$\psi_{j,k}(t)$ and $\phi_{j,k}(t)$ are known as the wavelet function and scaling function respectively given by:

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k) \qquad (3)$$

$$\phi_{j,k}(t) = 2^{-j/2}\phi(2^{-j}t - k) \qquad (4)$$

j is a scale or dilation variable and k represents time shift or translation.

In the frequency domain the wavelet coefficients $d_{j,k}$ are nominally associated with frequency bands $\left[2^{-(j+1)}, 2^{-j}\right]$ Daubichies(1992), i.e. the wavelet function approximates a band pass filter with band pass $\left[2^{-(j+1)}, 2^{-j}\right]$. Using the band pass properties of wavelet functions, it can be shown (Moyo and Brownjohn 2002) that the discrete wavelet transform decomposes strain data into components associated with creep effects, seasonal changes, diurnal temperature changes, and random changes as follows:

$$Y_k = C_k + T_k + R_k \qquad (5)$$

$$\mathrm{Y}_k \approx c_{J_c^l k}\phi_{J_c^l k}(t) + \left[A + \sum_{j=J_c^t}^{J_c^{t'}} d_{J_c^t}\psi_{J_c^t k}(t)]\right] + \sum_{j=J-3}^{J-4} d_{jk}\psi_{jk}(t) + \sum_{j=J}^{J-1} d_{jk}\psi_{jk}(t) \qquad (6)$$

where:

$$\mathrm{C}_k \equiv c_{J_c^l k}\phi_{J_c^l k}(t)\,,\ T_k \equiv \left[A + \sum_{j=J_c^t}^{J_c^{t'}} d_{J_c^t}\psi_{J_c^t k}(t)]\right] + \sum_{j=J-3}^{J-4} d_{jk}\psi_{jk}(t)\,,\ \ R_k \equiv \sum_{j=J}^{J-1} d_{jk}\psi_{jk}(t)$$

$c_{J_c^l k}$ =scaling coefficients associated with creep due to sustained loading

$d_{J_c^t}$ wavelet coefficients associated with strains due to temperature variations.

C_k represents time dependent strains due to creep and shrinkage of concrete, which appear as a trend in the observed strain values.

T_k represents strains induced by thermal dilation of concrete. This is largely due to diurnal and seasonal variations in temperature.

R_k represents changes in strain due to random events. These events could include transient events such as immediate settlement, change in weather, or abrupt changes such as, ground motions, accidents, short spells of weather changes, or heavy traffic. It is assumed that these events would appear in Y_t as uncorrelated data. Therefore by studying portions of the discrete wavelet transform of strain data associated with random events one can then identify and locate abrupt and transient changes. The identification process involves statistically extracting these events from noisy signals and then finding their occurrence in time (Moyo & Brownjohn 2002). Figure 1 shows an example of a random event identified from strain data recorded at hourly intervals.

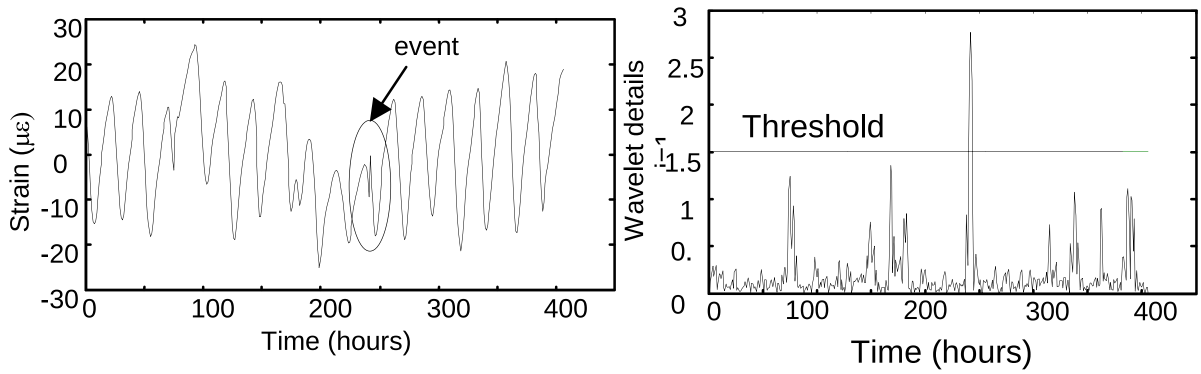


Fig.1: Identification of an unusual event using wavelet analysis

ASSESSMENT OF EFFECT OF UNUSUAL EVENTS ON STRUCTURE

Having identified the occurrence of an anomalous event, for example using wavelets, it is of interest to determine whether there is any evidence of change in structural behaviour and performance associated with the event. An approach to this problem is to consider components of strain in a simple rheological model, associated with anomalous events. The strain can be divided broadly into elastic strain and inelastic strain. Elastic strain is reversible, that is, it dies out once loading on a structure has been removed. On the other hand if the strain is inelastic the structure suffers an irreversible strain change, or permanent deformation. Irreversible strain can reduce the limit state capacity of a structure and may result in serviceability problems if strain change is large enough. In new healthy structures exposing the structure to certain levels of irreversible strains may not impair structural performance immediately, however accumulation of these strains may have a long-term effect. Therefore maintaining a database of these strains and their effect on the structure is essential for future condition assessment. Thus it is considered important here to get some indication of which of the components of instantaneous strain dominate strain change due to random effects that occur in a structure from time to time. For this impact assessment procedures proposed by Box & Tiao (1975), also known as intervention analysis, will be adopted.

BRIEF REVIEW OF INTERVENTION ANALYSIS

A Box-Jenkins model is a transfer function model of the type, (Ljung 1987):

$$y(t) = \frac{\omega(B)}{\delta(B)} x(t) + \frac{\theta(B)}{\phi(B)} e(t) \qquad (7)$$

where,
y(t) = output signal, *x(t)* = input signal, *e(t)* = external disturbance, B = back shift operator such that $By(t) = y(t-1)$, and ω, δ, θ and ϕ are the parameters of the transfer function model.

The Box-Jenkins model represents a family of transfer functions and time series processes. In the absence of an input for example the model reduces to:

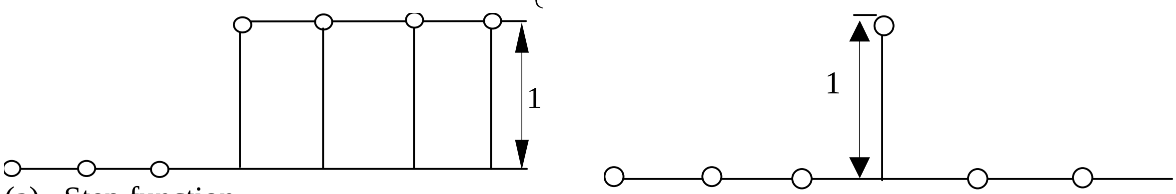
$$y(t) = \Psi(t) = \frac{\theta(B)}{\phi(B)} e(t) \qquad (8)$$

which, is a general form of an autoregressive-moving average ARMA time series model of order (*p,q*). On the other hand if e(t) is zero the model reduces to a noise free transfer function model $y(t) = \mathrm{X}(t) = \frac{\omega(B)}{\delta(B)} x(t)$. Intervention analysis takes advantage of this interaction between times series processes and transfer functions, proffered by the Box-Jenkins model.

The basic concept of intervention analysis is: Given a stationery time series process recorded at equal time intervals how does the onset of an exogenous event occurring at known time T affect the process? Using the Box-Jenkins model, the time series process can be represented by an ARMA model $\Psi(t) = \frac{\theta(B)}{\phi(B)} e(t)$ and the effect of an extraneous event can be modelled in terms of a deterministic input *x(t)* by a transfer function $\frac{\omega(B)}{\delta(B)} x(t)$. Therefore $\Psi(t)$ stands for normal behaviour of a system while the deterministic input series $x(t)$ serves to indicate the presence or absence of an external event and is often represented by a step function $W(t)$ or pulse function $U(t)$ taking the value of 0 or 1 (Figure 2).

$$W(t) = \begin{cases} 0 & t < T \\ 1 & t \geq T \end{cases} \qquad (9)$$

$$U(t) = \begin{cases} 0 & t \neq T \\ 1 & t = T \end{cases} \qquad (10)$$



(a) Step function (b) Pulse function
Figure 2: Input function

APPLICATION OF INTERVENTION ANALYSIS IN SHM

Consider a long-term structural health monitoring system logging strain data from a structure at equal time intervals. Under normal operational conditions of the structure, the resulting time series of strains, y(1), y(2),…, y(t), does not contain any unusual events and can be modelled as an ARMA model. Occasionally the structure may experience rapid changes of strains due to external inputs such as ground movement, traffic, accident effects, or immediate settlement of supports. Following the Box-Jenkins model given above, these external events can be represented as external inputs, using a step or pulse function. Thus, by applying the intervention analysis models represented by the Box-Jenkins model it is possible to assess the effect of an anomalous event on a structure. In the transfer function model, the magnitude of ω is proportional to the amount of change associated with a proposed model. In the model for an abrupt permanent change, for example, ω can be interpreted as irreversible or inelastic strain.

The first step of intervention analysis is building an ARMA model. Since the ARMA model describes stochastic behaviour of a time series process and intervention events are additional external disturbances, only the data before the onset of external events should be used in constructing $\Psi(t)$. It is also assumed that the form of the time series model for $\Psi(t)$ remains the same before and after the event. ARMA model building consists of selecting an appropriate order (p,q), for the model. There are a number of procedures for selecting the model order, for example partial auto-correlation analysis and Akaike's Information Criterion (AIC). Details of these procedures can be found in standard time series analysis books (Brockewell and Davis 1991, Box and Jenkins 1970). Here partial auto-correlation analysis will be used.

Once an ARMA model for the time series has been identified an appropriate Box-Jenkins model must be selected to assess the effect of the event. The model is selected as follows;
Estimate the parameter δ for the transfer function model using a pulse function as the input. If the estimated value of the rate of change parameter δ is close to unity, then strain change is largely permanent. Then estimate the parameter δ for a step function input. If the parameter is too small it implies the change was abrupt and permanent.

Intervention analysis procedures described above were used to assess the effect of some events identified during and after construction of the Singapore Malaysia Second Link (Figure 6). A SHM system was installed during construction in order to monitor the bridge's short-term and long-term behaviour and performance under construction loads, environmental loads, and vehicular loads. The SHM system includes a set of temperature sensors, stress cells, strain gauges and accelerometers distributed in three segments of the Singapore side of the bridge's main span and logging data once every hour.

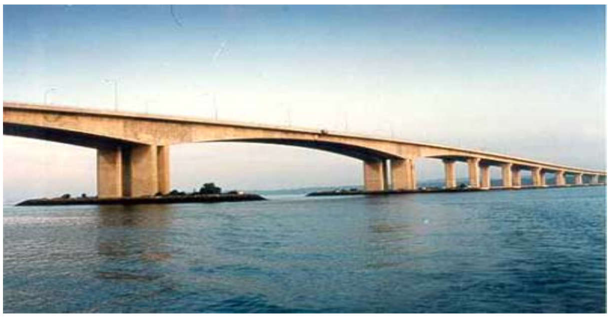


Figure 3: Second Link Bridge

In this study attention is paid to strain data recoded during and after construction of the bridge. The causes of some events that occurred during the construction phase are known and they are, post-tensioning, concreting of segments and shifting of form traveller. Figures 7 show some events associated with post-tensioning activities.

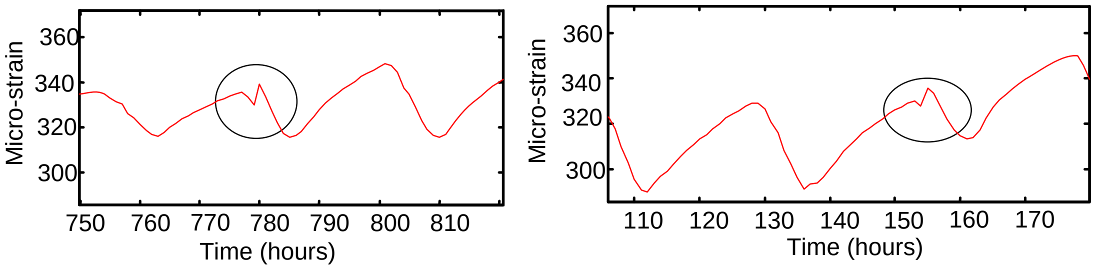


Figure 4: Post-Tensioning Events

Assessing the effect of post-tensioning

Figure 1 shows part of the strain time series that contains an abrupt change of strain that resulted from post-tensioning at a segment of the bridge. The event occurs at 241th record of strain.

The first step in intervention analysis is selection of an appropriate time series model for the data. Here it is assumed that the process is an Auto-regressive process of order *p*, AR(*p*) i.e. *q* is zero in the ARMA model. Such an assumption is valid since high order AR models can represent ARMA models well (Hannan and Kavaliers, 1984). The equation for $\Psi(t)$ then reduces to,

$$\phi(B)\Psi(t) = e(t), \text{ i.e.} \tag{11}$$

$$\Psi(t) = \sum_{i=1}^{p} \phi_i \Psi(t-i) + e(t) \tag{12}$$

For an AR process of order p, the estimated partial auto-correlations are approximately independent and normally distributed with zero mean and standard deviation σ_ϕ. The standard deviation can be estimated as $\sigma_\phi^2 = 1/n$ (Box and Jenkins 1970), in which n is the number of observations used in model identification. p is estimated from the partial auto-correlations as the lag at which the partial auto-correlations are less than $2\sigma_\phi$ (Box and Jenkins 1970). Further residuals of the AR(*p*) should be uncorrelated, i.e., all partial auto-correlation coefficients of the residuals should be less than $2\sigma_\phi$ and their power spectrum should be the same as white noise spectrum. For the data in Figure 10 only the first 240 observations will be used for AR model identification. Before identification of an AR(*p*) for the time series, low frequency components were filtered off the raw data to remove trends using wavelet analysis. An AR(25) was chosen and used in the Box-Jenkins model for effect assessment.

The step function for effect assessment takes the values of zero for the first 240 observations and one for subsequent observations. Similarly, the pulse function is zero everywhere except at observation 241 where it is one.
The parameters estimated for different effect models are shown in Table 1.

Event Date	Temporary Abrupt Effect Model Parameters		Gradual, Permanent, t Effect Model Parameters		Permanent, Abrupt Effect Model Parameters
	ω	δ	δ	ω	ω
31-05-1997	13.22	0.99	0.18	7.13	11.88

Table 1: Transfer function parameters for post-tensioning event.

The decay parameter for the temporary change is close to one [0.99], indicating that the effect of the event would not decay to pre-intervention level too quickly. This is confirmed by the rate of change parameter for the gradual permanent change, which is close to zero. From Therefore the change is abrupt and permanent with magnitude 11.88 $\mu\varepsilon$. This should be expected since post-tensioning force remains permanently after locking the strands at the ends.

Assessing the effect of post construction events

Figures 5 shows some unusual events identified from the data using procedures given in section of this paper.

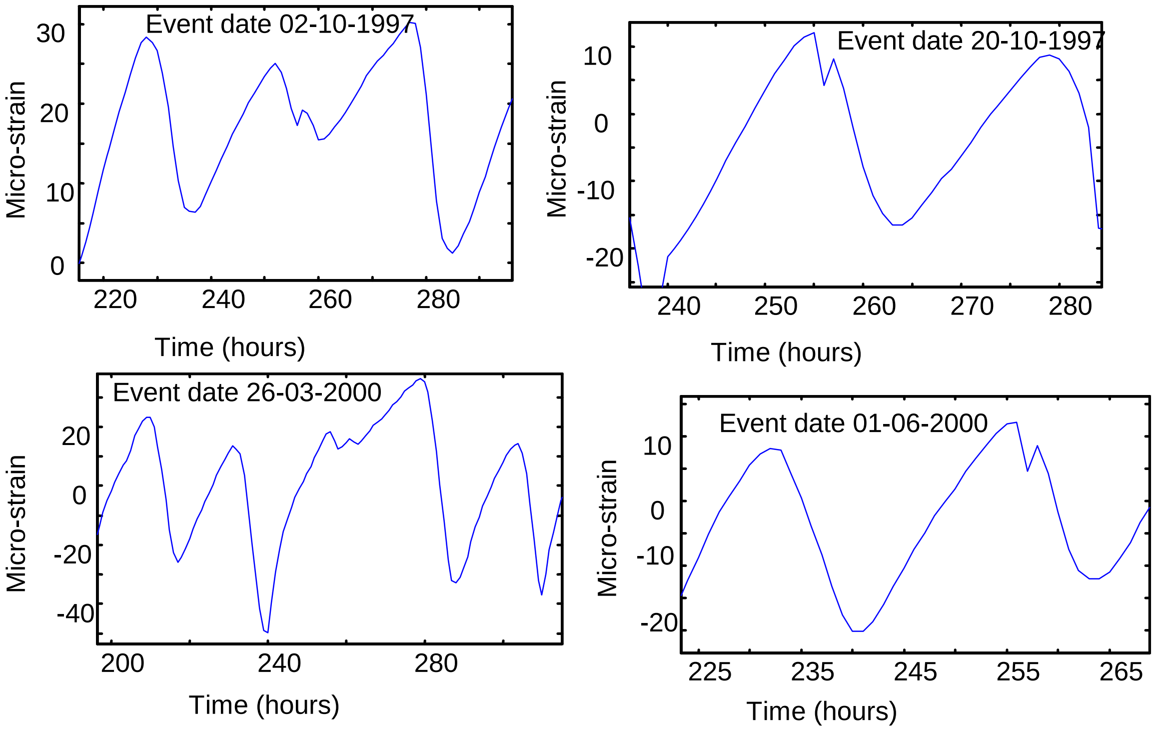


Fig. 5: Abrupt events identified after construction

Using the time series model identified above, an effect assessment gives listed in Table 2. Note that where the rate of change parameter for an abrupt temporary model is close to no further other model is tested as this strongly suggests a temporary effect.

Event Date	Temporary Abrupt Effect		Permanent Gradual Effect		Selected Model
	ω	δ	ω	δ	ω
02-10-1997	3.8	0.99	4.46	0.57	Gradual permanent effect, will eventually reach a value of 10.1 $\mu\varepsilon$
20-10-1997	-7.07	0.04			Temporary Abrupt Effect
26-03-2000	-2.30	0.46	-2.41	0.11	Temporay & Abrupt permanent effect. Effective drop of –2.52-(-2.30=0.22))
01-06-2000	6.48	0.009			Temporary Abrupt Effect

Table 2: Transfer function parameters for events in Figure 5.

Negative values of the parameter ω indicate loss of tension proportional to the magnitude of the parameter ω while positive values signify an increase in compressive strain. It is interesting to note that most of the tension effects are accompanied by significant recovery of strain. A plausible reason could be the effect of post tensioning which pulls the structure together.

DISCUSSION AND CONCLUDING REMARKS

Effect assessment of anomalous events observed in strain data begins with identification of an ARMA model for the data time series followed by formulation of transfer function models in which the input is represented by a step or pulse function. Three Box-Jenkins type transfer function models have been used to classify effects as temporary effects or permanent effects. The selection of an appropriate effect model for a particular event is based on the logical relationship between the effect models. The effectiveness of the approach was checked using post-tensioning events recorded during the construction of a bridge.

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