Sensitivity Analysis of Coupled Crowd-structure System dynamics to Walking Crowd Properties

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ABSTRACT: Increasing vibration serviceability problems of modern structures have drawn researchers' attention to the walking-induced vibration modelling and assessment of floors and footbridges. Changes of dynamic properties of structure due to presence of stationary people have been studied extensively in the literature. However, little is known about the similar effects of walking people, mainly due to the lack of experimental evidence and credible models capable of simulating human-structure dynamic interaction (HSI) in the vertical direction. This paper uses a single degree of freedom mass-spring-damper (MSD) model to simulate dynamics of walking crowd[1] on structure and investigates the sensitivity of the coupled crowd-structure system frequency and damping to properties of crowd model. Results of this study show that when the natural frequency of the crowd model is less than the natural frequency of the structure, both natural frequency and damping ratio of occupied structure are more sensitive to crowd's model stiffness. Similarly, when the natural frequency of the crowd model is greater than the natural frequency of the structure, both natural frequency and damping ratio of occupied structure are more sensitive to crowd's model mass. It also can be seen that natural frequency of the occupied structure has no sensitivity to damping of the crowd model while its damping ratio shows a limited sensitivity to the crowd's model damping with the maximum where both natural frequencies are equal.

1 INTRODUCTION

Recent increase in vibration serviceability problems of flexible structures under human dynamic loading have led to concerns about both safety and comfort of pedestrians using such structures. Hence, there is demand for more accurate and inclusive design methods (Shahabpoor & Pavic 2012, Zivanovic et al. 2010, Kasperski 1996, Reid et al. 1997, SCOSS 2001). As to the walking-induced dynamic loading, current design approaches, such as those in BS 5400 (2006) and Eurocode 5 (2004), use deterministic walking force models and simply ignore human-structure interaction effects and inherent stochasticity of the walking force (Shahabpoor & Pavic 2012, Zivanovic et al. 2010, Dougill 2006). Research (Shahabpoor & Pavic 2012, Zivanovic et al. 2010, Ellis & Ji 1997, Willford 2002, Brownjohn et al. 2004, Pavic 2011), mostly based on full-scale measurements, have indicated that interaction of the human body, as a bio-mechanical system, with the structure have significant effects on dynamic properties of the joint human-structure system and should be considered. It often leads to a considerable reduction in response and some change in the natural frequency of the structure (Shahabpoor & Pavic 2012, Zivanovic et al. 2010, Dougill 2006, Pavic 2011).

To address HSI, different types of mechanical or biomechanically-inspired models, such as single/ multiple degrees of freedom mass-spring dampers MSDs (Archbold 2004, 2008, 2011, Caprani et al. 2011) and single/bipedal inverted pendulum (Bocian 2011), are used to simulate kinematics of human motion for both vertical and horizontal directions and different postures and types of activities (Macdonald 2009, Matsumoto & Griffin 2003, Wei & Griffin 1998). This paper uses a classic single degree of freedom mass-spring-damper (S-MSD) model to simulate crowd-structure interaction during walking by using human body MSD properties. Although this S-MSD model may not be the best option for modeling human, the simplicity of its dynamics allows easy investigation of highly relevant coupled human-structure system dynamics under different loading conditions. The principal aim of this study is to improve understanding of the

[1] In this paper, 'crowd' is called to any group of people with more than a single person.

sensitivity of the occupied structure dynamic properties to each of the currently uncertain *crowd* parameters. This is done for a range of common structures and crowd occupation scenarios and should help dealing with large uncertainty when modeling crowds on structures during design process.

Section 1 of the paper presents a very short introduction into the subject and rationale of this research. Section 2 describes the proposed coupled S-MSD model and its formulation for the considered crowd occupation scenarios. Section 3 presents the analysis specifications and Section 4 illustrates the parameters used in the models. Section 5 presents the results of the parametric study and sensitivity analysis and finally Section 6 closes the paper by highlighting the important findings and making conclusions.

2 S-MSD MODEL DESCRIPTION

To simulate the problem, only the first mode of structural vibration is considered and is conceptualized using a SDOF oscillator with the corresponding modal properties (m_s, k_s and c_s). Considering only one structural mode does not affect the generality of the results as mode superposition principle applies to linear structures which is an acceptable assumption for this kind of problem.

In all simulations, a S-MSD model (m_c, k_c and c_c) is used to simulate the effects of the crowd's MSD properties on the structure. This model represents 'stationary' walking pedestrian -an imaginary case in which people are 'walking' on the 'anti-node' of the first mode of vibration but their location on the structure does not change[2]. Being stationary, such coupled crowd-structure system can be represented as a simple conventional two degrees of freedom system as illustrated in Figure 1, the behavior of which can be studied using closed form solutions of 2DOF equations of motion.

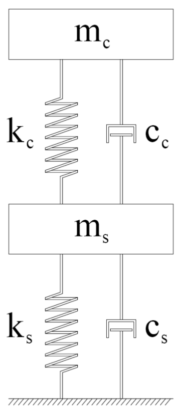


Figure 1. Conceptual 2DOF model of coupled crowd-structure system (stationary walking people)

[2] e.g. assume walking on a series of treadmills distributed over the length of a beam-like structure

More detailed discussion about the selected 2DOFs crowd-structure model and its equations of motion are presented by Shahabpoor et al. (2013).

3 ANALYSIS SPECIFICATIONS

The natural frequency f_{os} and damping ratio ζ_{os} of the occupied structure are chosen to represent dynamics of the occupied structure. In the first step, crowd's properties (m_c, k_c and c_c) are changed one at a time and the effect of this single parameter on natural frequency f_{os} and damping ratio ζ_{os} of the occupied structure are calculated.

In the next step, the *rates of change* of f_{os} and ζ_{os} of the occupied structure with respect to change of crowd's model properties m_c, k_c and c_c is considered as the sensitivity criteria. The chosen rate of change provides a measure of '*how fast'* the occupied structure properties f_{os} and ζ_{os} change as uncertain crowd's parameters m_c, k_c and c_c change.

To allow for comparison, as units of MSD parameters are different, a typical set of initial values for structure and crowd's parameters (m_{ci}, c_{ci}, $k_{ci,}$ f_{si} and $\zeta_{si)}$, are selected and unit-less ratios m_c / m_{ci}, c_c / c_{ci}, k_c / $k_{ci,}$ f_{os} / f_{si} and ζ_{os} / ζ_{si} are used for presentation. To ensure generality of results, the same analysis is repeated for several initial values and results are compared. The effects on f_{os} and ζ_{os} are considered for the changes of m_c (Case 1), k_c (Case 2), and c_c (Case 3).

In all three cases, the selected crowd parameter is varying over a certain range and other two parameters of the crowd model are kept constant and equal to the initial set of values.

4 MODEL PARAMETERS

The exact parameters used in the crowd - structure 2DOF model are described in this section. These parameters are selected to be realistic and to cover a range of possible values (in the case of the varying parameter).

4.1 Dynamic parameters of structure model

The dynamic parameters of the crowd and structure models that are used in simulations are presented in Table 1 for different analysis cases. An imaginary simply supported beam is selected as the structure and its first mode properties m_{si}, k_{si} and c_{si} are selected in a way to be both realistic and corresponding to a light weight structure. The latter is needed to highlight the crowd-structure interaction effects better. Three different natural frequencies (and therefore stiffnesses for the same mass m_{si} of 6500 kg) are selected for the structure to cover the scenarios in which the natural frequency of the structure is lower, close to and higher than the assumed natural frequency of the crowd model.

4.2 Dynamic properties of crowd model

The initial parameters of the crowd model m_{ci}, c_{ci} and k_{ci} are adopted from the results of studies done by the author to simulate crowd's dynamics on a real-life test structure. An extensive set of experiments were carried out on the Sheffield laboratory test structure (2 meters wide and 11 meters long pre-stressed concrete beam) with the groups of 2-15 pedestrians walking on it. An S-MSD crowd model was then fitted to each test scenario and the corresponding crowd's parameters were found. Properties corresponding to a group of 6 walking people are selected as the initial values for the crowd model. A six-people group represents a normal spatially-unconstrained crowd on the assumed test structure and is a very good starting point to study the effects of varying crowd parameters.

Table 1. Dynamic parameters of human and structure models used in different analysis cases

Analysis case	Initial values	Case 1	Case 2	Case 3	Unit
Structure model parameters					
Mass		6500			kg
Damping ratio		0.005			-
Natural frequency		2 - 4			Hz
Crowd model parameters					
Mass	168	8.4 - 462	168	168	kg
Stiffness	61698	61698	3085 - 169669	61698	N/m
Damping	1803	1803	1803	90 - 4958	N.s/m
Damping ratio	0.28	1.25 - 0.169	0.984 - 0.133	0.014- 0.770	-
Natural frequency	3.05	13.64 - 1.84	0.68 - 5.06	3.05	Hz

The ranges of possible crowd parameters m_c, c_c, k_c are adopted from the values reported by researchers for individuals and groups of people (Archbold et al. 2011, Zhang et al. 2000, Rapoport et al. 2003, Bertos et al. 2005, Lee & Farley 1998, Geyer et al. 1998) and are presented in Table 1.

5 ANALYSIS RESULTS

Distinction should be made between the results that are presented in Sections 5.1 and 5.2. The former provide a measure of '*how effective*' crowd parameters m_c, k_c and c_c are on occupied structure dynamics (represented by f_{os} and ζ_{os}) while the latter gives a measure of the sensitivity of f_{os} and ζ_{os} to crowd's uncertain parameters.

5.1 Parametric analysis

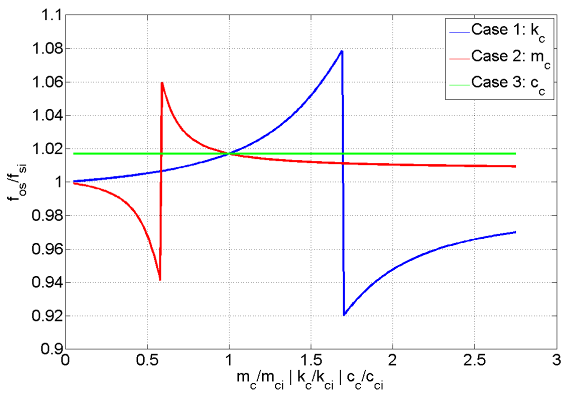
Figure 2 and Figure 3 present a typical set of results for f_{si} = 4 Hz. Results of analysis Cases 1, 2 and 3 corresponding to changing crowd's m_c, k_c and c_c are plotted on the same graph to be able to compare them. The horizontal axis presents the ratio of the crowd parameters to their initial values 'X', while the vertical axis presents the ratio of the changes in the occupied structure parameters 'Y'. These parameters are presented in equations 1 and 2, respectively.

$$X = {}^{x_c}/_{x_{ci}} \quad \therefore \quad (x_c = m_c, c_c, k_c) \qquad (1)$$

$$Y = {}^{y_{os}}/_{y_{si}} \quad \therefore \quad (y_{os} = f_{os}, \zeta_{os}) \qquad (2)$$

As the natural frequency of a SDOF is proportional to k/m, increase of stiffness or decrease of mass leads to the increase of the natural frequency. Keeping this in mind and knowing that f_{ci} = 3.05 Hz and f_{si} = 4 Hz in Figure 2 and Figure 3, increasing stiffness of the crowd model k_c (blue trace) or

Figure 2. Effects of the m_c, k_c and c_c on f_{os} (f_{si} =4 Hz)



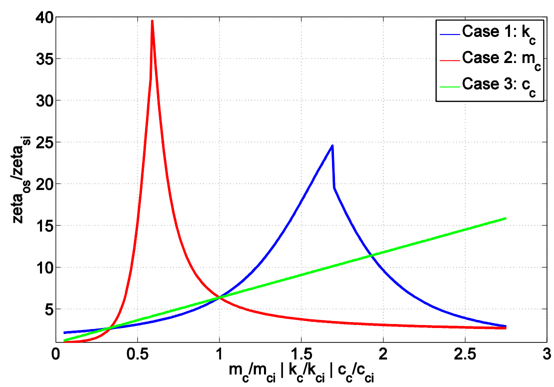


Figure 3. Effects of the m_c, k_c and c_c on ζ_{os} (f_{si} =4 Hz)

decreasing its mass m_c (red trace) leads into an increase of the crowd's natural frequency f_c and makes it closer to the structure's initial frequency f_{si}.

A closer look at Figure 2 and Figure 3 shows that the extreme values of m_c and k_c graphs (red and blue traces) represent the cases where natural frequencies of the crowd and initial structure model are equal ($f_{si}=f_c$). This means that m_c and k_c have maximum effects on f_{os} and ζ_{os} when $f_{si}=f_c$. It also can be seen that increasing ζ_c has no significant effects[3] on f_{os} but increases ζ_{os}. An extensive discussion of the observed trends in Figure 2 and Figure 3 is presented by Shahabpoor et al. (2013).

To compare the effects of the crowd's parameters m_c, c_c, k_c on the occupied structure's dynamics, a family of initial values is considered in which initial natural frequency of crowd model f_{ci} is 3.05 Hz and structure initial natural frequency f_{si} varies from 2 to 4 Hz. For each (f_{ci} , f_{si}) pairs, a set of X vs Y curves similar to the ones presented in Figure 2 and Figure 3, are plotted. Maximum absolute value of each of X vs Y graphs are then plotted against their corresponding f_{ci} /f_{si} (which is equal to 3.05/ f_{si}) and are presented for f_{os}/f_{si} and ζ_{os}/ζ_{si} in Figure 4 and Figure 5 accordingly.

As it can be seen in Figure 4 and Figure 5, as 3.05/f_{si} increases, maximum effects of m_c on f_{os} decrease and its effects on ζ_{os} increase. k_c has the opposite effects and as 3.05/f_{si} increases, its maximum effects on f_{os} increase and on ζ_{os} decrease. Maximum effects of crowd's damping c_c on both f_{os} and ζ_{os} is highest when $f_{ci}=f_{si}$.

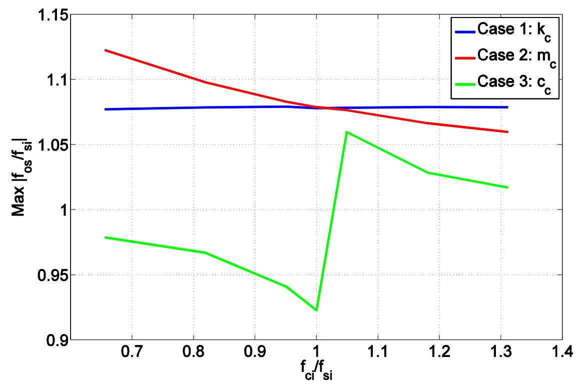


Figure 4: Maximum effects of the m_c, k_c and c_c on f_{os} for f_{ci}= 3.05 Hz and f_{si} varying from 2-4 Hz

[3] The case presented in Figure 2. Effects of the mc, kc and cc on fos (fsi =4 Hz) and Figure 3 is corresponding to the system with proportional damping matrix. In systems with non-proportional damping distribution, natural frequency is dependent on damping but its effects is not significant. Further discussion about details of this issue is beyond the scope of this paper.

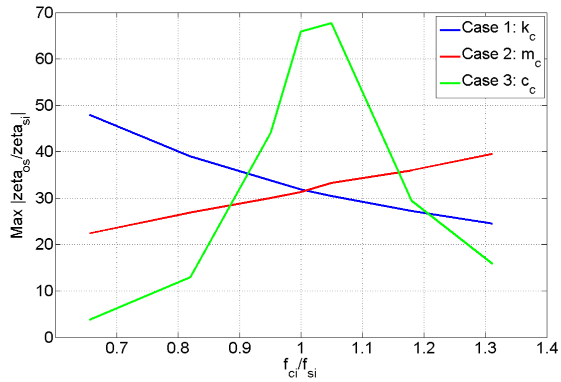


Figure 5: Maximum effects of the m_c, k_c and c_c on ζ_{os} for f_{ci}= 3.05 Hz and f_{si} varying from 2-4 Hz

5.2 *Sensitivity analysis*

Sensitivity here is defined as the rate of change of f_{os} and ζ_{os} to the changes in m_c, k_c and c_c. The sensitivity here is an indicator of '*how fast*' the effects of the crowd parameters on the occupied structure parameters change. Results of this section are presented in Figure 6,Figure 7, Figure 8

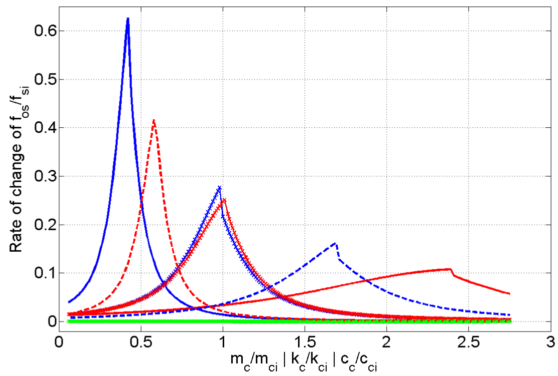


Figure 6: Sensitivity of f_{os} to m_c (red trace), k_c (blue trace) and c_c (green trace):

- Continues traces: f_{si} =2 Hz
- Crossed traces: f_{si} =3 Hz
- Dashed line: f_{si} =4 Hz

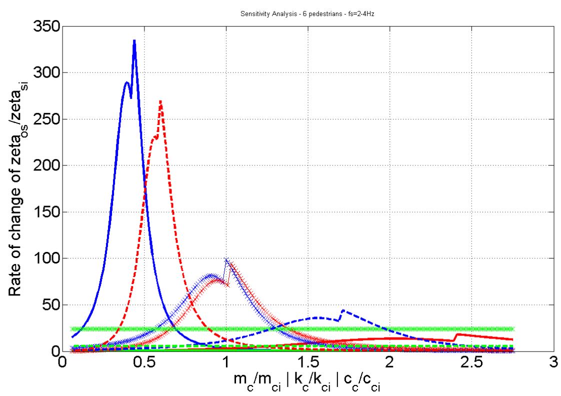


Figure 7: Sensitivity of ζ_{os} to m_c (red trace), k_c (blue trace) and c_c (green trace):

- Continues traces: f_{si} =2 Hz
- Crossed traces: f_{si} =3 Hz
- Dashed line: f_{si} =4 Hz

andFigure 9. In these figures, the horizontal axis presents the ratio of the crowd parameters X as is given in equation 1 and the vertical axis presents derivative of Y (given in equation 2) with regards to X.

Figure 6 displays the sensitivity of ζ_{os} and Figure 7 displays the sensitivity of f_{os} for two typical initial structural frequencies f_{si} = 2 and 4 Hz. Similar to the findings in Section 5.1, sensitivity of ζ_{os} and f_{os} to m_c and k_c increase significantly when frequency of the crowd and structure models are close in value.

For the case $f_{ci} < f_{si}$ (when f_{si} =4 Hz), as both Figure 6 and Figure 7 show, when k_c (dashed blue trace) increase from k_{ci} and mass m_c (dashed red trace) decrease from m_{ci}, their corresponding sensitivity curves show a peak. These peaks can be shown to correspond to $f_{si} = f_c$. The same applies when $f_{ci} > f_{si}$ (when f_{si} =2Hz) and the sensitivity curves show maximum when k_c (blue trace) decrease and m_c (red trace) increase. Also, as Figure 6 illustrates, rate of change of f_{os} is not sensitive to c_c while sensitivity of ζ_{os} is maximum when $f_{si} = f_{ci}$ (crossed green line in Figure 7).

To compare the sensitivity of f_{os} and ζ_{os} to m_c, k_c, and c_c, the same family of initial values that is described in the previous section is considered in which f_{ci} = 3.05 Hz and f_{si} varies from 2 to 4 Hz. For each (f_{ci} , f_{si}) pairs, then, a set of $X\ vs\ Y'$ curves similar to the ones presented in Figure 6 and Figure 7 are plotted. Maximum values of $X\ vs\ Y'$ graphs are then plotted against teir corresponding f_{ci} /f_{si} and are presented for f_{os} and ζ_{os} in Figure 8 and Figure 9 accordingly.

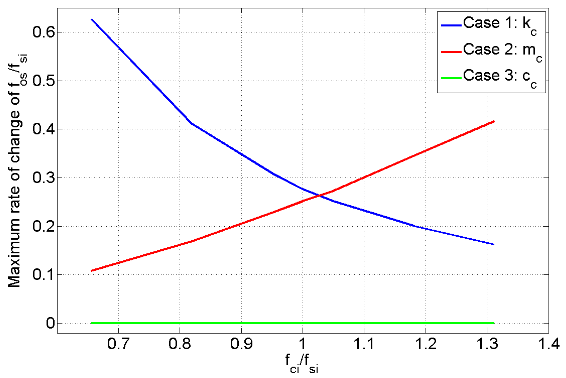


Figure 8: Maximum sensitivity of the f_{os} to m_c, k_c and c_c for f_{ci}= 3.05 Hz and f_{si} varying from 2-4 Hz

It can be seen in Figure 8 and Figure 9 that by increasing the f_{si}, maximum sensitivity of the f_{os} and ζ_{os} to k_c decrease (blue traces) and its maximum sensitivity to m_c increase (red traces). This means that when $f_{ci} < f_{si}$, both f_{os} and ζ_{os} are more sensitive to k_c, while when $f_{ci} > f_{si}$, both f_{os} and ζ_{os} are more sensitive to m_c. It also can be seen that f_{os} has no sensitivity to c_c while ζ_{os} shows a limited sensitivity to c_c with the maximum at $f_{ci} = f_{si}$.

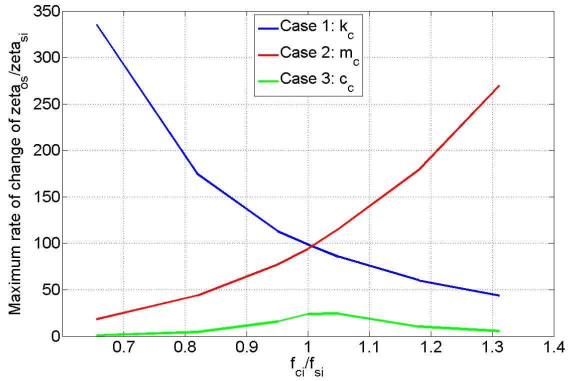


Figure 9: Maximum sensitivity of the ζ_{os} to m_c, k_c and c_c for f_{ci}= 3.05 Hz and f_{si} varying from 2-4 Hz

6 CONCLUSION

Modelling crowd dynamics on structures have always been a challenge due to uncertainty of human parameters. This paper combines results of the parametric study and sensitivity analysis that are done on a 2DOF mass-spring-damper human-structure model to describe effects and sensitivity of the occupied structure to the crowd model parameters m_c, k_c and c_c. Results of this analysis provide valuable insight for engineers to choose more realistic crowd properties during design process and researchers to understand better the human-structure interaction mechanisms.

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