How accurate is radio-location?

David GIBSON
Department of Electronic and Electrical Engineering, University of Leeds, LEEDS, LS2 9JT

Abstract: The accuracy of a conventional cave survey, constructed using compass, clinometer and tape, and the treatment of the associated surveying errors has been well-discussed. Such surveys are sometimes "corrected" by means of radio-location; but the accuracy of radio-location techniques has not been widely debated. Properly understood, radio-location errors can be subjected to the same treatments as other surveying results. As well as the measurement errors of distance and angle, radio-location accuracy may be affected by the use, in conditions where it is not valid, of the traditional 'quasi-static' model of the field lines.

INTRODUCTION

This paper reviews the sources of error inherent in the use of a radio-location beacon, but it will not attempt to quantify them. In that sense it will not directly answer the question posed in the title of the paper; instead, the intention is to bring the sources of error to the notice of cave surveyors and to encourage a theoretical and practical evaluation of radio-location errors which have, hitherto, not been widely discussed. The subject has received sparse attention in BCRA Transactions although Brooks and Ellis (1956) show that attempts in using radio-location for verifying cave surveys go back at least forty years. In the U.S.A. several cavers have done detailed practical studies but this work has not been widely published, nor widely disseminated in the UK.

RADIO-LOCATION TECHNIQUE

Radio-location using an induction loop is, by now, a standard procedure and need not be explained in detail here. A definitive description of the technique was given in Surveying Caves by Glover (1976). More recently, Bedford (1993) outlined the technique and presented (with circuit diagram and constructional notes) the electronic beacon previously designed by France and Mackin.

Essentially, a horizontal transmitter loop (vertical magnetic dipole) is placed underground and the point on the surface immediately above this is located using a receiver loop. At this "ground-zero" point the magnetic field lines from the transmitter are vertical so a vertical loop (i.e. with its axis horizontal) will pick up no signal because no field lines "cut" the loop. The ground-zero point is confirmed by holding the loop vertical, spinning it about a vertical axis, and confirming that there is no orientation where a signal can be detected. To locate ground-zero from another location the vertical receiver loop is rotated to give the direction of minimum signal, and a bearing taken along the plane of the loop. A series of at least three widely spaced bearings should, in theory, intersect exactly. In practice the bearings allow the surveyor to construct a "polygon of confusion" which describes a region, on the surface of the earth, in which ground-zero is likely to occur.

The depth of the underground point can be determined in two ways. With suitable equipment the most straightforward method is, perhaps, to measure the flux density (say \( B_2 \)) at ground zero and to compare this with the signal (\( B_1 \)) a short distance (\( y \)) above this. Using the ratio of these readings provides a convenient way of calculating the depth, \( d \), without needing to know the transmitter power or absolute gain of the receiver. The inverse cube law, which describes the change of flux density with distance leads directly to:

\[
y = \frac{B_2}{B_1} - 1
\]

This method, which could be termed 'depth by signal-strength' (DSS) is used in some commercial radio-location equipment, but most amateur designs have used a different method based on measurements of the field angle. The reasons for this are not entirely clear; it may be due to the nature of some amateur amplifier designs which make direct readings of field strength difficult to obtain; or it may be that a discrete design, based on the electronic components which were available 20 years ago, would have been complicated. The measurement of depth by signal-strength has been discussed in caving literature, but its dismissal may have been due to a lack of insight into ratiometric techniques. Using a ratiometric technique instead of trying to relate absolute signal strength to depth avoids the perceived problems in maintaining the transmitter power and the receiver gain. Currently available commercial equipment uses a micro-controller to allow direct "real-time" readings of depth to be obtained. Brian Pease (1995) is currently experimenting with a DSS device for cavers, but this does not use a ratiometric technique.

The more common method of depth determination is to measure the angle of the field lines. Away from ground-zero the magnetic field lines are not vertical. By measuring the angle of the field to the ground (\( \alpha \)), and knowing the distance to the ground-zero point (\( x \)), the depth of the transmitter (\( d \)) can be calculated (Fig. 1). This method assumes that the field lines obey the parametric equations for a traditional "bar magnet". The formula is:

\[
x = \frac{\sqrt{8 + 9 \tan^2 \alpha} - 3 \tan \alpha}{2}
\]
This result is straightforward to derive, and has been familiar to cavers for many years now. It is quoted by Mixon and Blenz (1964), also by Glover (1976) in *Surveying Caves*. For a simple derivation see, for example, Lee in an appendix to Glover (1973) or, more recently, see Gibson (1994).

A convenient technique for depth estimation is to find the distance $x$ at which the field lines lie at $45^\circ$ to the ground. The formula then indicates that $x/d = 0.56$, so the depth is approximately twice the distance $x$. Another technique would be to find the distance $x$ at which the field lines were at $18.4^\circ$, for which $x/d = 1$.

**MEASUREMENT ERRORS**

Clearly there is scope for errors of measurement to have a significant effect. Most of the sources of error affect the depth measurement more than they affect the location of ground-zero. The accuracy of a position fix also depends, of course, on the accuracy of the surface survey. Ideally several bearings would be taken, in order to locate ground-zero as accurately as possible. Then field-angle measurements at varying distances would be plotted, and used to obtain a best-fit curve from which the depth would be determined. In practice, cavers might only make one or two measurements but, if this is the case, the confidence of the result must be called into question. Glover (1976) demonstrated various graphical methods of converting $\alpha$ and $x$ into depth. His graphs show how small errors in reading can lead to large errors in depth. If $\alpha = 80^\circ$, for example, then a $1^\circ$ increase in $\alpha$ corresponds to an decrease in $x/d$ from 0.117 to 0.105, which is $10\%$. At $\alpha = 45^\circ$ the change is only $2.5\%$. Mixon and Blenz (1964) also discussed angular errors in their paper.

Measuring the angle of the field lines on the surface requires the surveyor to accurately sight on the ground-zero point. As he adjusts for the null position, by tilting the receiver loop, he must ensure that it remains pointing towards ground-zero. Obtaining an accurate null, and accurately measuring the angle of the loop are crucial aspects of the technique; and obtaining a good null is not always easy. There is a secondary field effect, to be described later, which builds up rapidly away from ground-zero and makes it increasingly difficult to get a deep null as the angle of the field lines, $\alpha$, decreases. Depth measurements should, ideally, be made with $\alpha$ from $40^\circ$ to $50^\circ$.

If a ratiometric DSS technique is used there is a limitation caused by how far above the ground the signal strength can be measured. Here, too, a secondary field effect can contribute to the error by causing the above-ground field to behave in a different way to the underground field.

The underground transmitter must be set up as accurately horizontal as possible. If the transmitter is only levelled to $5^\circ$ the axial field line will be displaced by $8.7\%$ of the depth (i.e. $\tan 5^\circ$). This field line will not be vertical but, from Figure 1 and equation 2, it can be shown that for a small tilt (say $< 10^\circ$) the field line which is vertical as it leaves the ground will be displaced by a third of the distance to the axial field line. Thus, with a $5^\circ$ tilt the apparent ground-zero point will move by about $3\%$ of the depth. Significantly, if the transmitter loop is not completely horizontal there will not be a field line which remains vertical as it leaves the ground. This could cause the null to be less sharp since there will always be some lines cutting the loop. In practice the loop can be levelled to better than $5^\circ$, but a spirit level is essential, as is a neatly wound induction loop. The terrain can be a source of measurement error because the ground-zero point may not coincide with the surface of the ground (Fig. 2). If the ground is sloping then equation 2 can still be applied, but $x$ must be the true horizontal distance to ground zero, and $d$ must be measured from the altitude of the field point. Distance $d'$ must be determined by surveying. Another source of error in addition to the obvious "sighting" errors is that it is possible to detect a false ground-zero in particularly rough terrain, especially if the estimation of the surface location is tenuous to begin with (Reid, 1990).

**FIELD LINE DISTORTION**

There is another source of error, potentially far more serious than the measurement errors described above. It is caused by the magnetic field lines departing from the supposed "bar magnet" shape. There are several reasons for this.

i) The receiver loop may be too close to the transmitter

Unless the receiver is far enough away for the transmitter to look like a point source, the field lines will not be of the simple "bar magnet" shape which is usually derived by considering a quasi-static field from a point-source dipole. In practice this means around five diameters, and this will not normally be a problem unless a large transmitter or receiver is used. For example, a 2m loop requires a depth of at least 10m in order to get an accurate reading. With a smaller loop, good results can probably be obtained closer than five diameters because a larger margin of error can be tolerated. Not only are the field lines distorted in the immediate vicinity of the transmitter loop, but the familiar inverse cube law breaks down too, so equation 1 cannot be used for depth estimation. It is possible to derive an expression for the field from a loop of finite extent but it is complicated and therefore of limited application. One procedure is to integrate the standard expression for 'retarded potential' over a suitably defined current density distribution. Mixon and Blenz quote a result; and show that it reduces to the simpler "bar magnet" field when the field point is at a large distance from the loop.

ii) The field lines will be distorted by magnetic rocks

The distortion of field lines is exploited by geophysicists and archaeologists, who use magnetometers as surveying tools. Unfortunately, unless a control grid of readings is correlated with an accurate compass and clinometer survey, the extent of the problem will not be known.

iii) Distortion by conductive rock – the "phase" problem

The absence of magnetic rocks and minerals does not imply that the field lines are undistorted because conductive, but non-magnetic, rock can also distort the field. Radio-location has to be used with care in areas where there is much mineralisation. This effect is well-known to geophysicists and archaeologists who utilise a magnetic gradiometer to induce a field in conductive rock; the field gradient is then a measure of the distortion of the field lines, and allows the structure of the ground to be determined.
iv) The “Transition Zone” problem

The field from an induction loop can be divided into two regions. The near-field (or induction field) predominates at distances less than $\lambda/2\pi$ (A is wavelength). The far-field (or radiation field) predominates at distances greater than this. The two fields have very different properties. For a large distance either side of $\lambda/2\pi$ there is a transition zone where the field gradually changes from the induction “bar magnet” shape to concentric circles which do not intersect the origin. The inference is obvious—within the transition zone the field lines will not be the simple “bar-magnet” shape which is predicted by the “quasi-static” model.

THE “POLARISATION” PROBLEM

The “transition zone” and “phase” problems can be discussed together as a “polarisation” problem. One or other of the effects have been observed by a number of cavers, though the effects are not always attributed to the correct causes. For example, a comment like “we could not find a null because the signal was so strong” (Williams and Todd in Caves and Caving, 35, Spring 1987) should probably be attributed to the predominance of the secondary field. The transition zone is centred on $\lambda/2\pi$ and this might be expected to be large at the low frequencies used for radio-location. However, the crucially important point is that the wavelength in the rock is much less than this. The transition zone moves inward to $\delta$ (and the wavelength to $2\delta$) where $\delta$ is the skin depth, given by:

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

Here $\omega$ is $2\pi$ x frequency [Hz], $\mu$ is magnetic permeability of the rock [H/m], and $\sigma$ is electrical conductivity [Ω m]. Strictly speaking this expression is only true for a “good” conductor, but it applies to most rock. (Rock is a “good” conductor by the mathematical definition of $\alpha/\sigma$ >> 1 unless it is very dry and the frequency is high). Note, that the skin depth does not describe a physical skin in which the signals are constrained to lie. The signals can and do penetrate further than the skin depth, which is simply a useful mathematical “figure of merit” for the rock. A derivation of the above result, with specific reference to “good” and “bad” conductors, was given by Gibson (1996).

Skin depth can vary from a metre or two to several hundred metres for the range of frequencies and rock types encountered by cavers. It is quite conceivable that a radio-location beacon could be operating at depths comparable with the skin depth and where the transition zone effects would be significant. At this distance, secondary fields would also be significant. Interestingly, the optimum depth for communications (as opposed to radio-location) may be around three skin depths (Gibson, 1993b, 1994).

The subject of radio wave propagation through rock has been well-studied, although the results have often been presented in a mathematical form which is not easy for non-mathematicians to interpret. Steven Shope (1991) has summarised some previous results and presented them graphically, showing how the direction of the field lines at the surface depends on the skin depth. One of these graphs was reproduced by Bedford (1993). Shope’s graphs are extremely significant because they show that in some circumstances the result given by (2) can be very much in error. It is intended that this will be the subject of further study by the author. It is worth pointing out that it is not only the field angle $\alpha$ which departs from simple “bar magnet” theory; the 1D rule for flux density also breaks down in the transition zone so, under these conditions, equation (1) cannot be used for depth determination either. There is some indication (Pease, pers. comm.) that, under these conditions, DSS gives rise to an over-estimation of depth, whereas field-angle measurement gives rise to an under-estimation.

ACCURACY OF RADIO-LOCATION

The measurement problems can be quantified and used to make an estimate of the accuracy, which could easily be 5-10% for depth, and several metres for ground zero. The polarisation problems are less easy to quantify. Depth determination starts to fail if a good null cannot be obtained, eventually failing completely. In these circumstances a ground-zero location can often still be performed. This only starts to fail if the rock is anisotropic, or if the ground is inhomogenous in a radially non-symmetric way (e.g. the antenna is located close to a fault-line or to one side of a large cavity).

AVOIDING THE PROBLEMS

The problem of the transition zone is lessened considerably by using a very low frequency, because the zone is further away, and because the secondary fields have a lower magnitude. The France/Mackin beacon operates at 874kHz; several US designs operate at 3496kHz. Radio-location at these frequencies is likely to be more successful than if it is done using carrier-based speech systems; common frequencies for which are around 27, 87, 115 and 185kHz; see Bedford (1994).

PRACTICAL MEASUREMENTS

Ian Drummond (1987b) has described some experiments which he, and others, performed in Lechuguilla Cave in New Mexico. Amongst them was a series of radio-locations along a passage at a depth of up to 210m. The purpose was to see if the magnetic field was well behaved, and if it diverged symmetrically from the null point. Plotting the data (and re-surveying part of the cave to check for errors) showed that the field was badly distorted in one area. This was attributed to mineralisation of a particular cross-rift. The experiments confirmed the wisdom of performing a series of locations to provide a control grid for a survey, rather than relying on one single point at the far end of the cave to check the survey. Drummond also found that the sharpness of the nulls depended on the orientation of the antenna. The precision of the location on the surface was much better along the passage than at right angles to it. This may well be a secondary field effect, but Drummond has noticed a similar effect on other occasions and suggests (pers. comm. quoted in Gibson 1993b) that it could be an anisotropic characteristic of the rock.

In the UK, members of the BCRA’s Cave Radio Group are currently resurveying Kingsdale Master Cave and performing a series of radio-location fixes. In addition to providing some simple tests of the accuracy of the radio-location, this will pave the way for a set of experiments, at different frequencies, which will attempt to verify Shope’s graphs.

An observation arising from experiments in several countries, is that the UK suffers comparatively badly from high rock conductivity and high levels of background interference. The inferences are that polarisation effects are likely to be worse, and that nulls are likely to be less sharp. The Cave Radio Group has demonstrated that radios which penetrate well in the US do not operate so well in the UK.

There will always be errors associated with the measurements made using radio-location beacons, and a proper understanding of them is essential. Cavers who have used radio-location beacons have sometimes misunderstood the operation of the device – Williams and Todd’s comment was quoted earlier. Other cavers have (pers. comm.) taken bearings of ground zero and, because the readings have intersected to give
a triangle of error, the cavers have deduced that the beacon “was not working properly”. Another common mistake is to assume, without justification, that the results obtained by radio-location are 100% accurate. Statements such as “we fixed the position by radio-location” suggest a misplaced confidence that the technique has an unfailing accuracy.

**SUMMARY**

Radio-location works best at very low frequencies (below a few kHz) and over distances which are short compared to the skin depth, but large compared to the size of the loop. Its accuracy is affected not only by measurement error, but by factors which are difficult to predict, such as distortion of the field lines. To use radio-location to best advantage users must understand the nature of the errors; they must know how to minimise them, and should know how to deal with uneven terrain.

This paper was intended to make users aware of the possible inaccuracies of radio-location, rather than to ascribe precise figures to the sources of error. Occasional tests of accuracy have been made but not widely reported; and it is hoped that this paper will encourage further discussion of both theoretical and practical aspects of the technique.

**ACKNOWLEDGEMENTS**

I would like to thank those members of the BCRA’s Cave Surveying Group (Oily Betts, Bryan Ellis, Stuart France and Wookgy) who discussed an early draft of this paper with me; also Brian Pease, who discussed the results of sophisticated signal measurements with a radio-location beacon; and especially Ian Drummond, who made many useful comments and reminded me of the work done by Bob Buecher, Frank Reid and others.

**REFERENCES**

The list of references contains material specifically oriented towards cavers, but the techniques are well-covered in geophysics and electromagnetics textbooks, as well as in various caving club publications.


