Simulating multimodal seasonality in extreme daily precipitation occurrence

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Highlights:

• The frequency of extreme daily precipitation occurrence displays a distinctive non-uniform seasonal pattern
• The seasonal distribution is simulated with Generalized Additive Models
• A strong dependence on atmospheric driving conditions is replicated.
• Statistical simulations indicate a future shift toward more frequent autumnal extreme precipitation.
Abstract

Floods pose multi-dimensional hazards to critical infrastructure and society and these hazards may increase under climate change. While flood conditions are dependent on catchment type and soil conditions, seasonal precipitation extremes also play an important role. The extreme precipitation events driving flood occurrence may arrive non-uniformly in time. In addition, their seasonal and inter-annual patterns may also cause sequences of several events and enhance likely flood responses.

Spatial and temporal patterns of extreme daily precipitation occurrence are characterized across the UK. Extreme and very heavy daily precipitation is not uniformly distributed throughout the year, but exhibits spatial differences, arising from the relative proximity to the North Atlantic Ocean or North Sea. Periods of weeks or months are identified during which extreme daily precipitation occurrences are most likely to occur, with some regions of the UK displaying multimodal seasonality.

A Generalized Additive Model is employed to simulate extreme daily precipitation occurrences over the UK from 1901-2010 and to allow robust statistical testing of temporal changes in the seasonal distribution. Simulations show that seasonality has the strongest correlation with intra-annual variations in extreme event occurrence, while Sea Surface Temperature (SST) and Mean Sea Level Pressure (MSLP) have the strongest correlation with inter-annual variations. The north and west of the UK are dominated by MSLP in the mid-North Atlantic and the south and east are dominated by local SST.

All regions now have a higher likelihood of autumnal extreme daily precipitation than earlier in the twentieth century. This equates to extreme daily precipitation occurring
earlier in the autumn in the north and west, and later in the autumn in the south and east.

The change in timing is accompanied by increases in the probability of extreme daily precipitation occurrences during the autumn, and in the number of days with a very high probability of an extreme event. These results indicate a higher probability of several extreme occurrences in succession and a potential increase in flooding.

**Key Words:** Precipitation, extreme, Generalized Additive Model, seasonality, UK

**Abbreviations:**

EA East Anglia (Extreme Region)
ES East Scotland (Extreme Region)
FOR Forth (Extreme Region)
GLM Generalized Linear Model
GAM Generalized Additive Model
HUM Humber (Extreme Region)
MSLP Mean Sea Level Pressure
MW Mid Wales (Extreme Region)
NAO North Atlantic Oscillation
NE North East (Extreme Region)
NHI North Highlands and Islands (Extreme Region)
NI Northern Ireland (Extreme Region)
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<td>SH South Highlands (Extreme Region)</td>
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<td>SOL Solway (Extreme Region)</td>
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<td>SST Sea Surface Temperature</td>
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<td>66</td>
<td>SW South West (Extreme Region)</td>
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<td>WC West Country (Extreme Region)</td>
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1 Introduction

Despite considerable evidence for increases in the number and intensity of extreme daily precipitation (e.g. Jones et al. 2014; Westra et al. 2014), there has been little focus on when and how often these events occur within the year (Pal et al., 2013; Dhakal et al., 2015). Yet, there is strong evidence that UK extreme and very heavy daily precipitation events do not occur uniformly. Instead, they follow a seasonal pattern that may differ substantially from seasonal cycles in monthly total, or mean wet day, precipitation (Rodda et al., 2009; Maraun et al., 2009). This article is about the occurrence of extreme daily precipitation exceeding a high threshold, and considers very heavy and extreme daily precipitation (Alexander et al., 2006) as “extreme”. Extreme daily precipitation in the UK tends to occur in the late summer and autumn (August-November), with very few occurrences during spring months (March-May) (Allan et al., 2015). Many parts of the UK exhibit more than one peak in the observed frequency of extreme daily precipitation during the year, or multi-modal seasonality (Rodda et al., 2009). There is also strong evidence of year-to-year variability in the number and time interval between extreme precipitation events, arising from fluctuations in atmospheric conditions (Tramblay et al., 2011).

While Zheng et al. (2015) focussed on seasonal changes in extreme precipitation intensities, they highlighted that the changes can differ at one location dependent on both the season and the duration of the event. Maraun et al. (2009) employed a seasonally varying statistical model to examine monthly block maxima of extreme daily precipitation intensity and frequency over multiple years, but did not explore the frequency distribution within the year. Changes in extreme precipitation occurrences do
not necessarily equate with changes in flooding (Stephens et al., 2015), as flooding is more dependent on the antecedent wetness conditions than extreme precipitation (Pui et al., 2011). There is evidence for increases in the duration of wet periods, particularly within autumn and winter, with associated increases in the intensity of longer wet spells (Zolina, 2014; Zolina et al. 2013). If seasonal extreme occurrences coincide with longer wet periods (e.g. autumn and winter) the potential for flooding increases. Changes in the time interval between extreme daily precipitation occurrence will have impacts on runoff response by either improving the recovery period (where the interval between maxima increases) or increasing flooding if the interval decreases (and increases catchment wetness). Unimodal sinusoidal patterns have often been adopted by previous analyses to replicate the annual frequency distribution (e.g. Rust et al., 2009).

This work demonstrates that simple sinusoidal forms do not adequately reflect the observed annual frequency distribution of extreme daily precipitation occurrence across the UK. Flooding also exhibits considerable interannual variability, with a tendency to cluster in “flood rich” and “flood poor” periods (Huntingford et al., 2014). In common with extreme precipitation frequency, these fluctuations respond to atmospheric oscillations such as the North Atlantic Oscillation or Arctic Oscillation (Bouwer et al., 2008; Hannaford & Marsh, 2010; Huntingford et al., 2014). Thus, identifying natural cycles and multi-year patterns is a crucial element in determining the current risks posed by extreme daily precipitation and can help identify vulnerabilities in risk management strategies (Wilby and Keenan, 2012). Statistical models play an essential role in identifying these changes and their significance.
Generalized Linear Models (GLM) have been widely applied to simulate seasonally varying daily precipitation occurrence and intensity (e.g. Chandler and Wheater, 2002; Verdin et al., 2014) and to examine long term responses to atmospheric and oceanic oscillations (Chandler, 2005; Sapiano et al., 2006). However, more flexibility is required for non-linear complex responses, such as the seasonal frequency distribution of extreme daily precipitation occurrence, to external drivers. Generalized Additive Models (GAM) are an enhancement to the GLM that employ smoothly varying non-linear functions (Hastie and Tibshirani, 1990) to achieve flexibility in the modelled response. Beuchat et al. (2012) demonstrated that explicit representation of temporally varying processes with a GAM is more efficient than examining each month separately (e.g. Maraun et al., 2009). Serinaldi and Kilsby (2014) employed vector GLMs and GAMs to successfully simulate precipitation processes over the Danube Basin on a 0.25° x 0.25° grid. However, their model includes seasonality as a sinusoidal measure and differs from the observed frequency of extreme daily precipitation occurrences.

GLMs and GAMs are useful for understanding long term changes in climatic variables (Chandler, 2005; Westra et al., 2013), and to obtain realistic estimates of future changes in precipitation through statistical downscaling (e.g. Kallache et al., 2011; Schindler et al., 2012). Other researchers have employed GAMs to assess the time- and atmospheric dependence in precipitation data (Hyndman and Grunwald, 2000) or trends in daily precipitation (Underwood, 2009). Likely changes in the future temporal distribution of sub-daily precipitation intensity were explored by Westra et al. (2013), premising a GAM on current responses to atmospheric variables. With a few exceptions (e.g. Chavez-Demoulin and Davison, 2005; Yee and Stephenson 2007; Maraun et al.,
2009; Westra et al., 2013; among others) there are not many examples of extreme daily precipitation response analyses using GLMs or GAMs.

In this paper we use GAMs to explore variability in the seasonal frequency distribution of extreme daily precipitation, evidence for long-term changes, and dependence on oceanic and atmospheric phenomena. Multiple simulations from a time-varying Binomial distribution are used to examine changes in the frequency distribution between 1901-2010 and to compare their spatial differences across the UK.

2 Data

2.1 Precipitation

Daily precipitation observations compiled and described by Jones et al. (2013, 2014) for 199 observation stations across the UK are used in this analysis and illustrated in Figure 1. All stations have >40 years observations, commencing between 1856 and 1961 and extending to 2010. Months were rejected when >3 days were missing, or years rejected when >10% observations were missing. Stations were removed from the analysis if >5 consecutive years were removed due to these criteria. Of the available 223 stations, only 199 had adequate consecutive observation years to be suitable for a peak-over-threshold analysis. Prior to 1961 and after 2000 there is considerable variability in the number of operational stations in each region. Model construction is based on the period where all observations are available (1961-2000) to remove the effects of this variability.

The analysis considers the upper tail of the wet day distribution, where wet days at all stations are those recording ≥1mm precipitation. Daily precipitation totals exceeding a high threshold, $u$, of wet days are considered “extreme”. The threshold, $u$, is defined as ≥ 95% of the wet day distribution (i.e. the wettest 5%), encompassing very
heavy and extreme daily precipitation (Alexander et al., 2006). The 95th percentile gives a
different value of $u$ at each station, but a similar mean annual frequency of extreme
occurrences enabling comparison across sites. Where extreme occurrences occur on two
consecutive days, the event with the highest precipitation total is selected as the day of
occurrence (Bernardara et al., 2011) to maintain the independence between events.
Excesses are recorded as 1, or 0 for no maxima, with no associated precipitation intensity
used in this analysis. For brevity, these excesses are referred to as “extreme occurrences”.

2.2 Air Temperature

Atmospheric moisture holding capacity is governed by temperature (e.g.
Trenberth, 2011), with recent changes in extreme daily precipitation associated with
enhanced water vapour availability (Westra et al., 2014). Evidently the relationship
between air temperature and extreme precipitation varies both by season and time of day,
as well as by the “extremity” of the precipitation (Wasko & Sharma, 2014). Changes in
extreme daily precipitation intensity and frequency are also associated with other
processes such as changes in event type (e.g. Berg et al. 2013, Prein et al., 2016) or
moisture transportation (Lenderink et al., 2008). The latter respond to large scale
atmospheric processes, represented by the covariates discussed below; while the former
may be well represented by air temperature and sea surface temperature (SST).

To maintain the assumption of identically distributed variables when considering
non-stationarity, covariates should vary on approximately the same time scale as the
model data. Precipitation occurrences appear to respond to monthly measures of air and
sea surface temperature (e.g. Philips & McGregor, 2002; King et al., 2014). As a result,
we examine the relationship between extreme occurrences and monthly maximum and
minimum air temperatures. Temperature data are from the CRU TS3.1 0.5° x 0.5° gridded global indices of land surface monthly climate variations over the period 1901-2011 (Harris et al., 2014). The selected 14 grid box time series correspond with the centroid of each extreme region (refer to Figure 1).

2.3 Sea Surface Temperature

Increasing North Atlantic SST will likely enhance the hydrological cycle, with increased oceanic evaporation generating positive moisture anomalies moving over the UK and Western Europe from the Atlantic (Wang and Dong, 2010). Moisture transported from the North Atlantic by wintertime extra-tropical cyclones provides a clear link between SST and winter extreme daily precipitation (Lavers et al., 2013a). Although convective precipitation is not directly related to North Atlantic evaporation (Lavers et al., 2013a), research indicates a strong relationship between lagged North Atlantic SST and summer precipitation in south and southeast England (Wilby et al., 2004; Neal and Phillips, 2009). Previous research has established stronger correlations between extreme precipitation and localised SST than with the wider North Atlantic basin (e.g. Phillips and McGregor, 2002). We examine the relationship between extreme occurrences, and concurrent and lagged monthly SST anomalies (from the mean annual cycle over 1880-2010) from the HadSST2 (Rayner et al., 2005) 5° x 5° gridded monthly averages. Nine grid boxes are used in this analysis, covering the UK and surrounding coastal waters.

2.4 Mean Sea Level Pressure

Both the North Atlantic Oscillation (NAO) and mean sea level pressure (MSLP) were considered as candidate variables for the statistical model and tested for the strongest relationship. The NAO is considered one of the major drivers of atmospheric moisture redistribution over Europe; its positive phase is strongly correlated with extreme
precipitation (Fowler and Kilsby, 2002). We present a brief comparison of the relationship between extreme occurrences and a principal components analysis of monthly values of the NAO index (Hurrell, 1995) in Section 3.

Connections between MSLP, or derived indices, and temperature and precipitation patterns are well established (e.g. Lavers et al. 2013a). Recent research suggests extreme precipitation has greater correlation with monthly MSLP than it does with the NAO index for Atlantic facing regions (Lavers et al., 2013a). Furthermore, normalised MSLP over the northern Atlantic is more reliably represented in climate models due to biases in climatology affecting the calculation of the NAO index (Flato et al., 2013). This connection to MSLP may facilitate the use of the current research for statistical downscaling analyses of future extreme daily precipitation occurrences.

Lavers et al. (2013a) found that MSLP centred over 63°-73°N, 10°W-5°E had the strongest significant correlation with precipitation across the UK in all seasons, despite fluctuations in the centre of action (i.e. the centre of a dipole in high and low pressure that does not necessarily coincide with the standard Azores-Iceland definition used for the NAO). Two measures of MSLP are examined here for their influence on UK extreme occurrences: a monthly time series for nine individual grid boxes covering the UK and adjacent oceans, and a monthly value averaged time series for the region over the larger ocean area (MSLPall). The Met Office Hadley Centre's MSLP data set, HadSLP2r from 1901 to the present day (Allan and Ansell, 2006) on a 5° x 5° global grid is used in this analysis. Nine grid boxes, corresponding with those for SST, are used here.
3 **Statistical Methods**

For a series of $t = 1, \ldots, T$ days, we consider an indicator variable $J_t = 1$ when the daily precipitation exceeds a high threshold, $u$; $J_t = 0$ otherwise. $u$ is defined individually for each station using the 95th percentile of the wet day distribution. The extreme occurrences are Binomially distributed $J_t \sim \text{Binomial}(n, \pi_t)$, with $n=1$ trial per day. $\pi_t = \text{Pr}(J_t=1)$ is the probability of a precipitation maximum, and each occurrence $J_t = 1$ is independent of previous maxima.

Time-varying extreme occurrences could be represented as a logistic regression model with covariates, derived from a Generalized Linear Model (GLM). That is, where the mean response variable, $E(Y_t) = \mu_t$, is described by a smooth monotonic link function, $g(\mu_t)$, of linear predictors $\beta_i x_i$ (Dobson, 2002):

$$g(\mu_t) = \eta_t = \beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n$$  \hspace{1cm} (1)

and

$$\eta_t = \text{logit}(\pi_t) = \log \left( \frac{\pi_t}{1 - \pi_t} \right)$$  \hspace{1cm} (2)

However, extreme occurrences are distributed non-homogeneously within the year and vary inter-annually, necessitating flexible statistical parameter estimates. Predictor terms, $g(\Phi_t)$, in the Generalized Additive Model (GAM; Hastie and Tibshirani, 1990) are represented with a summation of smooth flexible non-linear functions $f_y$ of the covariates, where $\eta_t$ is the logit function presented in Equation (2).

$$g(\Phi_t) = \eta_t = X_t \beta = \beta_0 + f_1(x_1) + \ldots + f_n(x_n)$$  \hspace{1cm} (3)
The relationship between $\beta_t$ and $X_t$ is constrained to be smooth by a non-negative smoothing parameter. Model smoothness functions, $f_i$, are transformed into individual linear models via linear basis functions, $b$, for each parameter of the explanatory variables ($\beta_1, ..., \beta_j$).

$$f(x) = \sum_{j=1}^{q} b_j(x)\beta_j$$ (4)

The simplest basis function is the polynomial basis (Equation 4). For example, a four-dimension polynomial ($q=4$ or $f(x) = \beta_1 + \beta_2x + \beta_3x^2 + \beta_4x^3$) is a GLM with four degrees of freedom and readily estimated by iterated re-weighted least squares (IRLS; Dobson, 2002). However polynomial bases are numerically unstable and have poor approximation theoretic properties so are often a poor choice for GAMs. In contrast, cubic spline bases are popular as they have good approximation theoretic properties and are easily reduced to a linear form (Wood and Augustin, 2002). In practice, cubic splines require knot locations to be specified which can lead to model over-fitting. Using penalized regression splines applies a “wiggliness” penalty so that a large number of knots can be used to reduce sensitivty to their selected location, while minimising over-fitting (Wood and Augustin, 2002; Wood, 2006). The penalty term is referred to as the smoothing parameter, $\lambda$. As $\lambda \to \infty$ the smooth will tend towards a straight line, while $\lambda \to 0$ leads to a fully interpolated spline. Penalized maximum likelihood estimation is an efficient method to optimise the model complexity where multiple dimensions and interactions are present (Wood, 2000).

There are several GAM fitting algorithms that are implemented in statistical software packages (e.g. Hastie and Tibshirani, 1990; Wood, 2006). The R statistical
software environment (R Core Team, 2014) and package mgcv (Wood, 2006) are used here. We use the default mgcv functions, except where there is strong motivation to choose an alternative (such as cyclic splines for calendar day), to remove the subjectivity in choosing an appropriate degree of model smoothness (Faraway, 2006). Variables are included in the GAMs in increasing combinations in the fitting process to determine their relative importance or redundancy as interactive terms. The aim is to achieve the best data representation with the greatest degree of parsimony tested by measures such as the deviance statistic, Generalized Cross Validation or Akaike Information Criterion (Akaike, 1974). Model simplifications can also be achieved through combinations of linear or quadratic terms in addition to flexible smoothness (Wood, 2006).

The Poisson distribution is a special case of the Binomial distribution, with $n \to \infty$ and $\pi_t \to 0$. As extreme occurrences are intermittent with a small sample per year, the annual frequency of extreme occurrences is Poisson distributed. Consequently, the statistical model can also be extended to examine long-term changes in the annual frequency of extreme occurrences. We make use of this relationship in Section 4.2 to identify potential covariates for the Binomial GAM using a log linear regression, or Poisson regression model. The Poisson regression takes the form of a GLM, Equation (1), to describe the log of the Poisson mean with nonlinear functions of the predictors. That is, $\ln(\mu_t) = \beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n$.

All extreme occurrence time series ($J_t = 1$ or 0) are collated into a single pooled time series for each extreme region (Jones et al., 2014) shown in Figure 1. The pooled time series improves estimates of the frequency distribution of extreme occurrences and extends the effective time series for model validation. As a large synoptic event could
generate extreme daily precipitation at several stations on the same day or over two days, repeated extremes are discarded leaving a single event per day. To maintain independence, and following the same criteria used for individual stations, the highest precipitation total of two consecutive days is considered as the day of occurrence.

4 Exploratory Analysis

Exploratory analyses presented here inform the statistical model development and are used to test the significance of any observed changes. The exploratory analyses examine whether there is a seasonal pattern in extreme occurrences, and the correlation with likely atmospheric and oceanic driving conditions. The remainder of this article focuses on the Southeast (SE) region, with the remaining results in the supplementary information.

4.1 Seasonality

Figure 2 shows frequency density distributions of regional extreme occurrences throughout the year for each extreme region. Frequency density is estimated for each region using the pooled record of extreme occurrences to obtain a summation of events per calendar day, which is then divided by the total number of events and a smooth spline applied. The extremes occur mostly in summer (June-August) or autumn (September-November) months with very few during spring (March-April). Rodda et al. (2009; Figures 5 and 6) presented a similar bimodal frequency distribution of UK extreme daily precipitation occurrences, peaking in mid-summer and late autumn. However, Rodda et al. (2009) included extreme occurrences from all UK observation stations in a single analysis, potentially creating an unrealistic multimodal distribution arising from geographically different extreme precipitation climatologies. The apparent multimodality may be also be
influenced by sampling variability, where natural fluctuations appear more important than the climatological response (Kundzewicz and Robson, 2000). However, it demonstrates that a simple sinusoidal form is unlikely to describe the annual frequency distribution of extreme daily precipitation occurrences in the UK adequately. Multimodal seasonality, with peaks in the frequency distribution in summer and again in late autumn, is most evident in regions along the UK east coast (e.g. NE and EA). In contrast, exposed western regions (notably NHI, NI, MW and SOL) exhibit a unimodal frequency distribution of extreme occurrences encompassing autumn and/or winter. These regional differences are representative of the dominant weather systems around the UK. Extreme daily precipitation in the northwest is mainly caused by westerly airflow from the Atlantic, while extreme precipitation in the southeast is dominated by easterly flows from the North Sea (Maraun et al., 2009).

The frequency distribution of extreme occurrences by day and year for South England (SE) is shown in Figure 3; a similar pattern is produced by all regions. Each colour band reflects the relative frequency density of extreme occurrences within a 15-day/15-year smooth. “Edge effects” are mitigated by plotting only the period where all SE region observation stations were effective (1950-1999). The highest frequency density of extreme occurrences is shown in pink/purple and is centred around July-November. This figure illustrates the multi-decadal fluctuations in both the annual distribution and total number of events, while highlighting the broader seasonal period when extreme occurrences are most likely to occur. There is a suggestion of longer-term changes in the duration and timing of the peak in extreme occurrences. This figure appears to show that most extreme events occurred earlier in the autumn during the 1990s than during the
1960s. The period when they are most likely is also several days shorter, i.e. an increasing number of extreme occurrences are more likely to be clustered in time.

4.2 Identification of potential covariates

Poisson regression models are used to examine the relationship between extreme occurrences and the candidate covariates. Relationships are tested at the 5% significance level against the $\chi^2$ distribution. We consider covariates to have field significance when the correlation between a variable and extreme occurrences is significant at >40 stations. Figure 4(a-d) presents the results of the most important correlations between extreme occurrences and the coincident monthly variables. These results are indicative only; the relationship between extreme occurrences and covariates is explored in greater detail with the GAM to identify the best model configuration.

While concurrent and lagged monthly SST anomalies for the nearest grid box are correlated with the annual number of extreme occurrences, the most significant correlations are found for the previous month (i.e. lag 1; SST1 henceforth). A strongly negative correlation across most of the UK is illustrated in Figure 4a; the few positive stations are only weakly positive, although the correlation is still significant. Monthly MSLP over the larger Atlantic region (63°-73°N, 10°W-5°E) has the most significant correlation with extreme occurrences (Figure 4b) and is the inverse of the monthly NAO correlation pattern (Figure 4c), i.e. positive MSLP in the northwest and negative MSLP in the southeast.

Monthly maximum and minimum air temperatures and maximum diurnal temperature range are also examined for correlation and have significant relationships. Although the correlation with extreme occurrences is not significant, monthly minimum
air temperature has the greatest correlation (Figure 4d) and is explored further with the
GAM. The correlation between the annual count of extreme occurrences and average
number of days (interval) between extreme occurrences presents a strong exponential
relationship (Figure 4e). Conditional statistical analyses require that the covariates and
the data are identically distributed and vary at the same scale. Using monthly variables
ensures that the data vary at the same time-scales. Confirming that both the distribution
of the extreme occurrences and their annual frequency are from the exponential family
validates the assumption of identical distributions.

Figure 5 presents pair plot correlations between the same atmospheric and oceanic
variables against the calendar day of extreme occurrences for South East England (SE).
All other extreme regions display similar patterns in their paired correlations (not shown).
SST1 and monthly minimum daily air temperature have the expected seasonal patterns
with minima in either April (SST1) or January (air temperature).

Monthly MSLP over the wider Atlantic domain (MSLPall) is linearly correlated
with the NAO index, confirming that only one of these measures is required in the final
GAM. Both measures of MSLP are better correlated with extreme occurrences than
NAO. However, monthly MSLPall, over the wider Atlantic domain, shows greater
coherence with extreme occurrences than individual grid box measures, and is selected as
a potential covariate. This is supported by recent research identifying that sea level
pressures to the west of Scotland explain more of the variance in precipitation (~80%) for
a larger proportion of the UK than the NAO index (~5.4%; van Oldenborgh et al., 2015).
As our aim is to identify a covariate that is reliably reproduced by most global circulation
models, the NAO index is not used in later analyses. Standard tests for multi-collinearity
in the remaining variables (e.g. Faraway, Ch9.4 2006) did not identify that other covariates should be removed.

5 Statistical Analysis of Long-term Changes

Non-stationary Binomial distributions, that is where the distribution parameters vary with time, are fitted to the regional time series of extreme occurrences. A subset of 1961-2000 is selected to fit the model, while the extended time series to 2010 is used to validate the statistical model. Finally, covariates from 1901-2011 are used to test for statistical evidence of longer-term changes in behaviour. The variables examined for importance in the regional GAMs are listed in Table 1. Explicit representation of seasonality is tested in addition to seasonal fluctuations represented by temperature and MSLP covariates.

5.1 Model construction

Three general methods are suggested by Wood (2006) for incorporating smooth functions of the covariates:

- All terms are additive of the form \( f_1(x) + f_2(y) \);
- Variable coefficients \( z + zf(x) \) where \( x \) varies smoothly, but its effect is modified by the constant covariate \( z \) (Hastie and Tibshirani, 1990);
- Bivariate smoothing \( f(x,y) \) where \( x \) and \( y \) vary jointly to represent a more complex interaction of covariates.

\( Mgcv \) has several fitting options for smoothing parameter functions, i.e. \( f \) from above, including cubic and polynomial regression splines; the default are thin plate
regression splines. We employ cubic regression splines fitted by penalized likelihood as these outperform the default option in efficiency and efficacy.

Covariates are tested individually in the Binomial family GAM to confirm their significance, before testing combinations of multiple covariates. Linear and non-linear functions of the smoothers are also examined for the best representation of extreme occurrences. Linear combinations would improve the parsimony and efficiency of the statistical model, but may not be fully representative. Each model combination is tested objectively using the AIC and deviance statistics, and subjectively with the distribution of the fitted model residuals using quantile plots and histograms (Wood, 2006). Covariates are included in the model in the order of greatest explanation of data variance, identified from the individual testing. To balance the overall model goodness of fit against parsimony a term is dropped from the model when all three of the following criteria are satisfied (Wood and Augustin, 2002):

- the term effective degree of freedom is close to its lower limit;
- the confidence region for the smooth contains zero everywhere;
- removing the term reduces the deviance statistic, or other relative comparison measure such as AIC.

Stepwise iteration to reject unnecessary covariates (Hastie and Tibshirani, 1990) is time consuming, particularly when many covariates are involved. A modified approach where only the significant terms from less complex models are included in later model iterations is used here.
The final Binomial distribution takes the form in Equation 5, with covariate smooth terms presented in Table S1 in the Supplement. All regional models follow this form, only the coefficients ($\beta_i$) and smoothing parameters ($f_i$) differ between regions.

$$\text{logit } \pi(t) = \beta_0 + f_1(d_t) + f_2(P_t) + f_3(\Theta_{Ni}) + f_4(ST1_t)$$  (5)

Figure 6 is an example of the GAM smoothing functions for the SE region; ±2 standard error bars are indicated by dashed lines. The day of occurrence ($d_t$) is the most important covariate in the SE region, followed by monthly minimum air temperature ($\Theta_{Ni}$). Seasonality arises from the interaction between $d_t$ and $\Theta_{Ni}$, modifying the sinusoidal shape of $d_t$ to develop the characteristic bimodal seasonality in extreme occurrences. There is a strong positive correlation with $\Theta_{Ni}$ related to summer convection that tails off at higher air temperatures, consistent with changes in Clausius-Clapeyron scaling (Westra et al., 2014). In contrast, both lagged monthly SST anomalies ($ST1_t$) and monthly MSLP ($P_t$) are negatively correlated; however the relationship with $ST1_t$ is not significant in SE.

Similar GAM smoothing functions were developed for all regions (not illustrated). $P_t$ and $d_t$, are the most important covariates for most regions and models, explaining between 34-89% of the variability in extreme occurrence frequency; the next most important variable is monthly minimum air temperature ($\Theta_{Ni}$). $ST1_t$ and $P_t$ differ in importance between regions, and display a north-south divide with the northwest UK dominated by MSLP and southeast by lagged SST anomalies. The relationship with $\Theta_{Ni}$ is also variable by region: in the north and west there is a strong negative correlation reflecting the dominance of winter frontal systems, with a reversal around the freezing point. In contrast, southern and eastern regions have a positive correlation similar to that
Air temperature could be a response to the precipitation rather than a driver, however the two are strongly interlinked and it is difficult to elicit which is the precursor.

5.2 Model validation

Each regional model is validated objectively using a leave-one-out cross validation to estimate the parameters. Median scale parameters from the cross validation are very close to the original model estimates (not shown). A more subjective test of model validity is to compare the observed extreme occurrence annual frequency distribution against model samples. Visual validation compares 1901-2010 observations against 100 samples per day from the regional Binomial distributions. Figure 7a illustrates the frequency density of extreme occurrences by calendar day for the observations in red and samples in grey. There is a good match between the observations and the sampled distributions, without excessive reproduction of day-to-day variability or implied over-fitting (Wood, 2006). Quantile-quantile plots (not shown) of the same samples also indicate good correspondence between the observed and simulated distribution quantiles for the Binomial distribution.

\[ N_{rel,i} = \frac{N_i}{S_i} \]  

The annual relative frequency of simulated extreme occurrences is also evaluated and shown in Figure 7b. Relative frequency is the number of extreme occurrences at one location, and calculated as the ratio of extreme occurrences \( N \) in the \( i^{th} \) year to the number of observation stations \( S \) in the same year (Equation 6). The simulations are derived from all available stations in 1961-2000, while observations may have fewer active stations prior to 1961 or after 2000. Results for all regions are included in
Supplemental Figures S1 and S2. There is generally good correspondence between the observed and simulated results.

5.3 Model results

Monthly observed covariate data (1901-2010) are applied to the regional Binomial distributions to predict the daily probability of an extreme occurrence for 109 years. We are most concerned with the period of consecutive days to weeks with the highest probability of extreme occurrences; that is the 0.95 quantile of the regional probability distribution. Figure 8 illustrates changes in extreme occurrences for South East England; the same figures for all regions are included in Supplemental Figures S3 to S6.

Figure 8a shows the approximate calendar day with the highest probability of an extreme occurrence (red squares) for 1901-2010 (i.e. ≥ 0.95 quantile of the distribution of probabilities). Confidence intervals are derived from the range of dates at which the highest probability ± standard errors exceeded the same threshold. Horizontal lines denote the first day of the month from July to February. A smoothed running mean (black dash) indicates that while the period with the highest likelihood of an extreme occurrence is variable, there is no discernible change in seasonality in the SE region. Similar responses can be seen for all regions, with nine regions also showing changes in the date when the probability of an extreme occurrence is greatest (Figure S3). Over the 1901-2010 period, SH, SOL and NW show changes to significantly earlier dates when the probability of an extreme precipitation occurrence is highest; NHI, NE and HUM all show significant changes to a later date (for the F-statistic at a 5% significance level). In
almost all cases, the changes equate to the highest probability of extreme occurrences between September and November.

Figure 8b illustrates the number of days where the daily probability of an extreme precipitation occurrence exceeds the 0.95 quantile of annual probabilities. We consider the total number of days as the duration of very high probability of an extreme occurrence. In the SE region the threshold probability is $p \geq 0.16$. Red circles indicate the most likely duration; confidence intervals are in grey, calculated as for the dates in Figure 8a. Zero duration indicates that the daily probability does not exceed $p=0.16$ during that year. The black dashed 5-year running mean displays an overall decrease in mean duration of very high probability of an extreme occurrence by about 1 week. Other regions (Figure S4) display more variability in the duration of very high probability, with seven others (NHI, ES, FOR, SOL, NW, MW, WC and HUM) showing significant (at 5% significance) increases in the duration.

Figure 8c shows the highest daily probability of an extreme occurrence each year, confidence intervals are derived from the distribution standard errors. Higher probabilities (e.g. 1920s, mid-1980s, and early 2000s) corroborate records of very wet periods with widespread flooding. A smoothed running mean (black dash) indicates a periodic fluctuation in the probability of extreme occurrences. The 0.95 quantile threshold of the distribution of probabilities for other regions lies in a range between 0.07 (Mid Wales) and 0.27 (Humber). Eight of the regions indicate increases in the highest daily probability of extreme occurrences (Figure S5) which, where combined with a longer duration of the highest probability, could lead to increased flooding. Considerable
variability indicates that the highest probability of extreme occurrences is most sensitive to multi-annual fluctuations in large-scale processes.

The mean daily probabilities of extreme occurrences over 20-year time-slices are compared in Figure 8d. Estimating probability over a longer timescale focuses on the climatological responses rather than inter-annual fluctuations. There is little change in the period of year with the lowest probability of extreme occurrences (F-M-A-M). However, the summer (J-A-S) and winter (N-D-J) bi-modal peak when extreme occurrences are most likely has shifted towards a single autumnal peak in the highest probability. A similar pattern of changes in the timing, and disappearance of the bi-modal peak, is apparent in all other regions (Figure S6). In some regions such as SH, the changes are very slight and difficult to discern; while in others, such as NHI and SE, they are more definitive. All regions indicate a change in the season with the highest probability of extreme occurrences toward the autumn (S-O-N).

Many patterns present in the observations are replicated by the simulation results, confirming that they did not arise solely through sampling variability. The time series of highest daily probability of extreme occurrences also confirms reported increases in the frequency of late summer and autumn extreme precipitation (Jones et al., 2013). Several devastating floods in the UK during the recent decade arose after a sequence of heavy precipitation days occurred in short succession (e.g. Lavers et al., 2011). The modelled results indicate that fluctuating years with shorter or longer durations of very high probability of extreme occurrences correspond with known natural fluctuations in wet and dry years. The simulated results also indicate a longer-term change in the time when extreme occurrences are most likely.
Discussion and Conclusions

This article examined the annual frequency distribution of extreme daily precipitation occurrences, or seasonal pattern, to characterize spatial and temporal differences across the UK. It was shown that extreme and very heavy daily precipitation does not occur uniformly throughout the year. There are also considerable regional differences, arising from the relative proximity to the North Atlantic Ocean or North Sea and dominating westerly or easterly air flows (Maraun et al., 2009). There are definite weeks or months during which extreme daily precipitation is most likely to occur, with some regions displaying multimodal seasonality. Oceanic and atmospheric variables were shown to have significant correlations with extreme occurrences and so were valuable proxy information for the probability of extremes.

The seasonality of the highest probability of extreme daily precipitation varies across the UK, contrasting with mean wet day occurrences which are more likely to occur over autumn and winter months for all regions (Rodda et al., 2009). The most intense events along the north and west of the UK are associated with large wintertime synoptic systems (multi-day), which generate both frequent wet days and high intensity precipitation. In contrast, short duration (sub-daily) summer convective storms are more common in the south and east of the UK (e.g. van Delden et al., 2001). Multimodal seasonality in event probability likely arises where both large-scale synoptic systems and convective systems regularly dominate regional weather. Recent research suggests that Atmospheric Rivers have a role in the development of winter floods over the UK but have little influence over summer extreme precipitation (Lavers et al., 2011, Champion et al., 2015). While such events usually extend over several days, and this analysis focussed
on single day extremes, this supports the likely cause of multimodal seasonality. Projections of future increases in North Atlantic winter atmospheric rivers (Lavers et al., 2013b) correspond with the shift towards late autumn extreme occurrences identified here.

Allowances for non-stationarity in regional extreme precipitation or flood estimates is not common practice (Jakob et al., 2011). Most analyses have focused on traditional extreme value analyses to estimate the likely return frequency of specific events (Ghil et al., 2011). Very few have considered the frequency distribution of extreme daily precipitation occurrences within the year. This article presented a statistical model that accommodates non-stationarity in the probability of extreme daily precipitation by allowing distribution parameters to vary in response to external covariates. Statistical model results indicated that the probability of several extreme events occurring in a few weeks has increased in most regions from 1901-2010.

A Generalized Additive Model (GAM) was constructed using observation data (1961-2000) and validated using all observation data (>1901-2010). Covariate data for 1901-2010 were used to test the changes in the daily probability of extreme occurrences. The statistical model reflected natural climate variability from year to year in the calendar days when the probability of an extreme occurrence is at its highest. Smoothing over several years to decades illustrated that all regions now have a higher likelihood of autumnal extreme precipitation. This equates to extreme occurrences earlier in the north and west (formerly winter dominated) and later in the south and east (formerly late summer and autumn). This change in timing was accompanied in eight regions with significant increases in the probability of extreme occurrences during the autumn. The same regions also had significant increases in the number of days with a very high
probability of extreme precipitation, or an extended duration of the peak season. These results indicate a higher probability of several extreme occurrences in succession, or a potential clustering of events. While intensity was not included in this analysis, the combination of a change in seasonality to coincide with the wetter period of the year and saturated ground conditions indicates a potential increase in flood likelihood.

Seasonality, represented in the model jointly by monthly minimum temperature and calendar day, is the principal driver of extreme daily precipitation event frequency in the UK. SST and MSLP were found to differ in importance between regions, displaying a north-south divide with the north and west of the UK dominated by MSLP and the south and east dominated by SST. Monthly minimum air temperature had a positive relationship with extreme occurrences; this decreased beyond a high temperature, similar to results from other research on changes in Clausius-Clapeyron scaling (e.g. Westra et al., 2014).

The statistical models presented here could be used in combination with covariates derived from climate model projections to improve future estimates of changes in the seasonal distribution of extreme daily precipitation, complementing other recent research (Schindler et al., 2012). Further refinements to the model could be made to simulate the sequences of extreme occurrences and their dependence structure. For instance, a Markov Chain or a Cox-Process model could be used to simulate the interval between extreme occurrences more effectively (e.g. Smith and Karr, 1986). Alternatively, an inhomogeneous Poisson point process model could enable examination of the intensity of extreme daily precipitation as part of the same analysis (e.g. Katz, 2010).
While the changes in the characteristics of extreme daily precipitation occurrences are significant, they are also fairly small. A potential follow on study could assess the strength of the link between changes in extreme occurrences to changes in flood characteristics. For example the extended model would examine the dependence of large floods on the monthly minimum air temperature, monthly MSLP and lagged SST.

Acknowledgments

The R analysis software (R Development Core Team, 2011) and packages mgcv (Wood, 2006), and plotrix (Lemon, 2006) were used for analyses and figures in this paper. Covariate data are available from the KNMI Climate Explorer portal http://www.climexp.knmi.nl and the NCAR Climate Data Guide https://www.climatedataguide.ucar.edu. R-scripts and extreme daily precipitation occurrence data can be made available on request for non-commercial use.

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References


Table 1: Terms used in the Generalized Additive Models

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day of the year</td>
<td>$d_t$</td>
<td>where $t$ is the event</td>
</tr>
<tr>
<td>Linear seasonality: sine</td>
<td>$S_{kt}$</td>
<td>$\sin(dt')$ $t' = \frac{2\pi t}{365.25}$</td>
</tr>
<tr>
<td>Linear seasonality: cosine</td>
<td>$C_{kt}$</td>
<td>$\cos(dt')$</td>
</tr>
<tr>
<td>Event occurrence</td>
<td>$y_t$</td>
<td>$y=[0,1]$ $y=1$ when the daily total exceeds a high threshold</td>
</tr>
<tr>
<td>Sea surface temperature</td>
<td>$ST1_t$</td>
<td>1 month lagged grid box average</td>
</tr>
<tr>
<td>Normalised mean sea level pressure</td>
<td>$P_t$</td>
<td>Monthly average over North Atlantic</td>
</tr>
<tr>
<td>Monthly min air temperature</td>
<td>$\Theta_{Nt}$</td>
<td>Grid box average minimum</td>
</tr>
</tbody>
</table>
Figure 1: Location of gauging stations in relation to the 14 extreme rainfall regions (Jones et al., 2014). North Highlands and Islands (NHI), East Scotland (ES), Forth (FOR), South Highlands (SH), North West (NW), North East (NE), North Ireland (NI), Solway (SOL), Humber (HU), South West (SW), Mid Wales (MW), West Country (WC), Southern England (SE), East Anglia (EA). Dashed grid lines indicate the 5°x5° boxes corresponding to SST and MSLP measurements; hatched boxes indicate 0.5°x0.5° corresponding to minimum monthly air measurements.
Figure 2: Frequency density distributions of 1-day extreme daily precipitation occurrences by month in each extreme rainfall region (Region names as Figure 1).
Figure 3: Smoothed relative frequency distribution of extreme occurrences per day of year each year for SE England. Data is regionally pooled extreme occurrences for 1950-1999. Shading indicates relative “density” of occurrences, pink/purple shades indicate more extreme occurrences in a 15 day/15 year smoothing window. Black lines indicate first day of month.
Figure 4: Significance (at 95%) of external covariates on Poisson regression model for annual counts of extreme occurrences from left to right top row: (a) Lagged Sea Surface Temperature for the 9 adjacent grid boxes; (b) Monthly Minimum Air Temperature per gauge in each extreme region; (c) Monthly NAO index; bottom row: (d) Sea Level Pressure over Atlantic Region; (e) Exponential distribution of annual count of extreme occurrences with respect to time between events. Colours relate to the extreme regions. Significant correlations are filled circles (can be positive or negative).
Figure 5: Pair plots of correlations between the calendar day of extreme occurrences (DAY) in South East England region and: monthly minimum air temperature (AIR); monthly lagged Sea Surface Temperature (SST1); concurrent monthly Sea Level Pressure in adjacent grid box (MSLP); concurrent monthly Sea Level Pressure over an Atlantic domain (MSLPall); North Atlantic Oscillation Index (NAO)
Figure 6: Smoothing parameters for South East England (SE) GAM showing: (a) seasonality as calendar day; (b) Monthly Sea Level Pressure over North Atlantic; (c) Monthly minimum daily air temperature in adjacent grid box; and (d) Lagged monthly sea surface temperature.
Figure 7: Comparison of observed and simulated Extreme occurrences for the validation data (1901-2010) for SE region for (a) occurrences per day; and (b) annual relative frequency extreme occurrences from 100 samples per day from the Binomial distribution. Relative frequency is the number of extreme occurrences per year at one location in the region.
Figure 8: Changes in the probability of extreme occurrences for SE region showing: (a) mean day of occurrence of highest 5% probability; (b) number of days per year with the highest 5% probability; (c) highest probability of extreme occurrences per year; and (d) 20 year mean daily probability. Confidence intervals for a-c shown in grey, derived from distribution standard errors, smoothed 5-year running mean in black dash, first day of month in yellow dash (a).