The Origin and Limits of the Near Proportionality between Climate Warming and Cumulative CO₂ Emissions

ANDREW H. MACDOUGALL*
School of Earth and Ocean Sciences, University of Victoria, Victoria, British Columbia, Canada

PIERRE FRIEDLINGSTEIN
College of Engineering Mathematics and Physical Sciences, University of Exeter, Exeter, United Kingdom

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ABSTRACT

The transient climate response to cumulative CO₂ emissions (TCRE) is a useful metric of climate warming that directly relates the cause of climate change (cumulative carbon emissions) to the most used index of climate change (global mean near-surface temperature change). In this paper, analytical reasoning is used to investigate why TCRE is near constant over a range of cumulative emissions up to 2000 Pg of carbon. In addition, a climate model of intermediate complexity, forced with a constant flux of CO₂ emissions, is used to explore the effect of terrestrial carbon cycle feedback strength on TCRE. The analysis reveals that TCRE emerges from the diminishing radiative forcing from CO₂ per unit mass being compensated for by the diminishing ability of the ocean to take up heat and carbon. The relationship is maintained as long as the ocean uptake of carbon, which is simulated to be a function of the CO₂ emissions rate, dominates changes in the airborne fraction of carbon. Strong terrestrial carbon cycle feedbacks have a dependence on the rate of carbon emission and, when present, lead to TCRE becoming rate dependent. Despite these feedbacks, TCRE remains roughly constant over the range of the representative concentration pathways and therefore maintains its primary utility as a metric of climate change.

1. Introduction

The radiative forcing from an increase in atmospheric CO₂ concentration is close to a logarithmic function of concentration (Wigley 1987). The equilibrium surface temperature response to a change in radiative forcing has been shown to be approximately proportional to the magnitude of radiative forcing (e.g., Hansen et al. 1997). However, the response of the global average surface temperature to cumulative emissions of CO₂ has been shown to be roughly linear in simulations of historical and future climate (e.g., Matthews et al. 2009). A relationship that Earth system models suggest will hold until approximately 2000 Pg of carbon (Pg C) have been emitted to the atmosphere (e.g., Matthews et al. 2009; Allen et al. 2009; Zickfeld et al. 2012; Gillett et al. 2013; Allen and Stocker 2014). This linear relationship, the slope of which has been designated the transient climate response to cumulative CO₂ emissions (TCRE; Collins et al. 2013; Planton 2013), is a highly convenient metric for climate policy, as it establishes a simple relationship between the cause of warming (CO₂ emissions) and the expected magnitude of warming that is independent of when carbon is emitted (e.g., Matthews et al. 2009; Allen et al. 2009; Zickfeld et al. 2012; Gillett et al. 2013; Allen and Stocker 2014). In addition, the rate independence of warming (implicit in TCRE) allows for simple path-independent budgets of cumulative carbon emissions to be established for the climate system to stay below a chosen threshold of warming. For these reasons, TCRE features prominently in the Working Group 1 Fifth Assessment Report of the Intergovernmental Panel on Climate Change (IPCC AR5; IPCC 2013; Collins et al. 2013).

* Current affiliation: Institute for Atmospheric and Climate Science, ETH Zurich, Zurich, Switzerland.

Corresponding author address: Andrew H. MacDougall, Institute for Atmospheric and Climate Science, ETH Zurich, Universitätstrasse 16, CH-8092 Zurich, Switzerland. E-mail: andrew.macdougall@env.ethz.ch

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TCRE is closely related to the transient climate response (TCR), a metric of climate change that quantifies the change in the global mean surface temperature 70 yr into a model experiment where radiative forcing increases linearly from a 1% compounded yearly increase in atmospheric CO$_2$ concentration (e.g., Stocker et al. 2013). TCR has a wider application than its strict definition implies, as the metric describes the actual magnitude of warming expected when atmospheric CO$_2$ is doubled (e.g., Collins et al. 2013). TCR is smaller than the equilibrium response to a given radiative forcing because of the energy absorbed to warm the oceans, atmosphere, land surface, and cryosphere (e.g., Collins et al. 2013). Approximately 90% of this energy goes into warming the ocean (e.g., Rhein et al. 2013). TCR has long been shown to be approximately proportional to radiative forcing, as the heat absorbed by the ocean is, to a first order, proportional to surface temperature change (e.g., Hansen et al. 1997; Raper et al. 2002; Gregory and Forster 2008; Gregory et al. 2009). TCRE expands the quantification of transient response to anthropogenic emissions by amalgamating the notion of carbon sensitivity with TCR (Matthews et al. 2009). Carbon sensitivity describes the fraction of emitted carbon that remains in the atmosphere given the strength of carbon cycle feedbacks at a given point in time (Friedlingstein et al. 2006). By combining these two quantities, one can directly relate CO$_2$ emissions to near-surface temperature change.

TCRE relates CO$_2$ emissions to the transient change in global mean near-surface temperature and is approximately constant only while CO$_2$ emissions continue. Following cessation of emissions, other processes become dominant and Earth system model simulations suggest that global temperatures will either decline, stabilize (e.g., Matthews and Weaver 2010), or slowly continue to increase for centuries (e.g., MacDougall et al. 2013), contingent on behavior of physical and carbon cycle processes.

The origin of the near-constant nature of TCRE remains a subject of scientific interest (e.g., Matthews et al. 2009; Gregory et al. 2009; Raupach 2013). The leading hypothesis at present is that the decline in radiative forcing from a unit of CO$_2$ added to the atmosphere (because of the logarithmic nature of the forcing) is compensated by an increase in the airborne fraction of CO$_2$ (Matthews et al. 2009; Gregory et al. 2009). Why this cancelation should remain so precise over such a large range of cumulative carbon emissions remains poorly explained. Matthews et al. (2009) suggest that the link between ocean heat uptake and ocean uptake of carbon, which are driven by the same deep-ocean mixing processes on long time scales, is the origin of this precise cancelation of competing factors. However, Matthews et al. (2009) does not provide a detailed analysis to support this inference. Raupach (2013) suggests that the linearity of TCRE arises from a mix of CO$_2$ trajectory dependence, nonlinear effects in both carbon and climate, and non-CO$_2$ radiative forcing.

Here, a simple analytical model of the climate system is invoked to argue that TCRE largely arises from a cancelation of the diminishing radiative forcing from a unit of CO$_2$ added to the atmosphere and the diminishing flux of heat into the oceans. In addition, the property of the ocean whereby the ocean-borne fraction of carbon is a function of CO$_2$ emission rate is also necessary for stabilizing TCRE. This analytical analysis draws on Earth system model experiments where carbon is emitted to the atmosphere as a linear function of time. These model experiments allow examination of TCRE under condition of both strong and weak positive carbon cycle feedbacks from the terrestrial land surface. In cases where such feedbacks are strong, TCRE loses its rate independence and becomes near constant over a much narrower window of cumulative carbon emissions.

2. Modeling methods and experiment design

a. Model description

The University of Victoria Earth System Climate Model (UVic ESCM) is a coupled climate model of intermediate complexity with a full three-dimensional ocean general circulation model (Weaver et al. 2001), complex land surface (Meissner et al. 2003), thermodynamic–dynamic sea ice model, and simplified energy and moisture balance atmosphere (Weaver et al. 2001). The model has both a terrestrial and an oceanic carbon cycle. The terrestrial carbon cycle is simulated using the Top-down Representation of Interactive Foliage and Flora Including Dynamics (TRIFFID) dynamic vegetation model (Cox et al. 2002; Matthews et al. 2004). The inorganic ocean carbon cycle is simulated following the protocols of the ocean carbon cycle intercomparison project (Orr et al. 1999). Ocean biology is simulated using a nutrient–phytoplankton–zooplankton–detritus ecosystem model (Schmittner et al. 2008). Ocean sedimentary processes are simulated using an oxic-only model of sediment respiration (Archer 1996). The frozen ground version of the UVic ESCM is used for simulations described in this manuscript. This version of the UVic ESCM includes a deep soil column extending to 250-m depth, soil hydrology in the top 10 m of soil, full freeze–thaw physics, and a representation of the permafrost carbon pool (Avis et al. 2011; MacDougall et al. 2012).

b. Experiment design

The UVic ESCM was forced under scenarios where CO$_2$ was emitted as a linear function of time until a total
of 1920 Pg C was emitted to the atmosphere. Emission rates were set at 2, 4, 8, 12, and 16 Pg C a$^{-1}$. After the desired total of carbon had been emitted to the atmosphere, CO$_2$ emissions were set to zero for the remainder of the simulation. Simulations commenced from a preindustrial climate and were continued for 1000 model years. Forcing from non-CO$_2$ greenhouse gases, sulfate aerosols, land-use changes, and volcanic events were set to their preindustrial averages throughout these simulations. To explore the effect of carbon cycle feedback strength on the TCRE, the climate sensitivity of the UVic ESCM (to a doubling of CO$_2$ concentration) was varied to 2.0$^\circ$C, 3.2$^\circ$C, and 4.5$^\circ$C, following the method of Zickfeld et al. (2009). This method takes advantage of the simplified atmospheric model component such that outgoing longwave radiation to space is altered as a function of the change in global mean surface temperature to emulate variable strength of the physical climate feedbacks. These experiments are designated the linear emission experiments.

A second set of model experiments were conducted by forcing the UVic ESCM with the representative concentration pathways (RCPs) used in IPCC AR5 (e.g., Moss et al. 2010). The simulations are forced with prescribed atmospheric CO$_2$ concentrations, allowing carbon emissions compatible with each RCP to be diagnosed as the residual in the global carbon mass balance. These simulations allow the effect of the permafrost carbon feedback on the magnitude of the TCRE for each RCP to be examined. Simulations of the RCPs were conducted with the UVic ESCM’s inherent climate sensitivity of 3.2$^\circ$C (for a doubling of atmospheric CO$_2$ concentration).

### 3. Analytical analysis

For this analysis, two well established relationships are used: 1) that temperature change is proportional to the difference between radiative forcing and planetary heat uptake:

$$ F = \lambda \Delta T + \Delta Q $$

(e.g., Wigley and Schlesinger 1985), and 2) that TCRE is defined as follows:

$$ \Lambda = \frac{\Delta T}{E} $$

(Matthews et al. 2009), where $F$ is radiative forcing, $\lambda$ is the climate feedback parameter, $\Delta T$ is the change in surface air temperature, $\Delta Q$ is the planetary heat uptake (which is dominated by ocean heat uptake), $\Lambda$ is TCRE, and $E$ is the cumulative emitted carbon. For convenience, emission rate will initially be assumed to be constant; that is,

$$ E = rt, $$

where $r$ is the rate of CO$_2$ emissions and $t$ is time. In section 3a, we will alter this assumption and consider exponentially increasing emissions of CO$_2$. For the convenience of the reader, all constants are defined in text when they are first used as well as in Table 1.

In most analytical analyses of the climate system, $\Delta Q$ has been approximated as $\Delta Q = \kappa \Delta T$ (e.g., Hansen et al. 1997; Raper et al. 2002; Gregory et al. 2009). This relationship is mathematically convenient, as $\kappa$ can be treated as a negative feedback to climate warming and is directly comparable to $\lambda$ (e.g., Gregory et al. 2009). Despite this convenience, it is well understood that $\kappa$ is not constant in time and will diminish toward zero as the climate system comes into equilibrium with the imposed radiative forcing (e.g., Wigley and Schlesinger 1985; Hansen et al. 1997; Gregory et al. 2009). To take this effect into account, we will here use the approximation that ocean heat uptake can be parameterized as heat diffusion into a semi-infinite half space:

$$ \Delta Q = \frac{K \Delta T}{\sqrt{\pi kt}} $$

(Carslaw and Jaeger 1986), where $K$ is thermal conductivity, $k$ is thermal diffusivity, and $t$ is time. To reduce the number of constants, we define

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>5.3</td>
<td>W m$^{-2}$</td>
<td>Radiative forcing from e-fold increase in CO$_2$ concentration</td>
</tr>
<tr>
<td>$r$</td>
<td>4</td>
<td>Pg C a$^{-1}$</td>
<td>CO$_2$ emission rate</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1</td>
<td>W m$^{-2}$ K$^{-1}$</td>
<td>Climate feedback parameter</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.45</td>
<td>—</td>
<td>Airborne fraction of carbon</td>
</tr>
<tr>
<td>$C_o$</td>
<td>600</td>
<td>Pg C</td>
<td>Original carbon content of the atmosphere</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.0125</td>
<td>W$^{-2}$ m$^4$ K$^{-2}$ a$^{-1}$</td>
<td>Ocean heat uptake parameter</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.0133</td>
<td>a$^{-1}$</td>
<td>e-fold time for CO$_2$ emission rate</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1</td>
<td>Pg a$^{-1}$</td>
<td>Initial CO$_2$ emission rate</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0049</td>
<td>W m$^{-2}$ Pg$^{-1}$ C</td>
<td>Linearized radiative forcing from Gregory et al. (2009)</td>
</tr>
</tbody>
</table>

![Table 1. Constants used in this paper, their units, and the typical values of the parameters used in calculations.](image)
\[ \frac{1}{\sqrt{\phi}} = \frac{K}{\sqrt{\pi k}}. \tag{5} \]

This approximation of ocean heat flux has been used at least once before by Wigley and Schlesinger (1985) but fell out of use a generation ago. The approximation is derived for a step change in temperature at the surface of a semi-infinite half space and remains valid for a linear (or power law) increase in surface temperature. However, it is not applicable to an exponential increase in temperature at the surface; therefore, we caution against using a similar form of parameterization to approximate uptake of carbon by the ocean. Fitting this parameterization to a temperature and planetary heat uptake model output from phase 5 of the Climate Model Intercomparison Project (CMIP5; Taylor et al. 2012) yields a good fit in most cases, with correlation coefficients of between 0.74 and 0.94 (Table 2). The intermodel range of \( \phi \) is 0.011–0.137 W m\(^{-2}\) K\(^{-1}\); however, the value from the UVic ESCM (0.0125 W m\(^{-2}\) K\(^{-2}\) a\(^{-1}\)) will be used in calculations presented here. In the UVic ESCM, the value of \( \phi \) varies very little by model experiment or imposed model climate sensitivity.

Treating ocean heat uptake as a negative feedback to climate change has proved to be a useful way of conceptualizing the physics of the climate system (e.g., Hansen et al. 1997; Gregory et al. 2009). To this end, it is useful to retain the ocean heat uptake efficiency \( \kappa \), here defined as

\[ \kappa(t) = \frac{1}{\sqrt{\phi t}}. \tag{6} \]

The radiative forcing from CO\(_2\) is approximated using the classical logarithmic relationship:

\[ F = R \ln \left( 1 + \frac{\alpha t}{C_o} \right). \tag{7} \]

(Wigley 1987), where \( R \) is the radiative forcing for an \( e \)-fold increase in atmospheric CO\(_2\) concentration, \( \alpha \) is the airborne fraction of emitted carbon, and \( C_o \) is the original quantity of carbon in the atmosphere.

Combining the above equations, the relationship for the change in surface temperature can be shown to be the following:

\[ \Delta T = \frac{F}{\lambda} \left( \frac{1}{1 + \frac{1}{\sqrt{\phi \lambda^2 t}}} \right). \tag{8} \]

Substituting in Eq. (7) for \( F \), we find,

<table>
<thead>
<tr>
<th>Model name</th>
<th>( \phi )</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCC_CSM1.1</td>
<td>0.137</td>
<td>0.93</td>
</tr>
<tr>
<td>CanESM2</td>
<td>0.011</td>
<td>0.78</td>
</tr>
<tr>
<td>CCSM4</td>
<td>0.061</td>
<td>0.82</td>
</tr>
<tr>
<td>CNRM-CM5</td>
<td>0.186</td>
<td>0.94</td>
</tr>
<tr>
<td>CSMK3.6.0</td>
<td>0.100</td>
<td>0.89</td>
</tr>
<tr>
<td>GFDL CM3</td>
<td>0.072</td>
<td>0.94</td>
</tr>
<tr>
<td>INM-CM4.0</td>
<td>0.080</td>
<td>0.93</td>
</tr>
<tr>
<td>IPSL-CM5A-LR</td>
<td>0.029</td>
<td>0.74</td>
</tr>
<tr>
<td>IPSL-CM5A-MR</td>
<td>0.045</td>
<td>0.91</td>
</tr>
<tr>
<td>MIROC5</td>
<td>0.077</td>
<td>0.91</td>
</tr>
<tr>
<td>NorESM1-M</td>
<td>0.03</td>
<td>0.74</td>
</tr>
</tbody>
</table>

\[ \Delta T(t) = \frac{R}{\lambda} \ln \left( 1 + \frac{\alpha t}{C_o} \right) \left( \frac{1}{1 + \frac{1}{\sqrt{\phi \lambda^2 t}}} \right)^{\frac{1}{\alpha E}}. \tag{9} \]

Now, by substituting Eq. (9) into the definition of TCRE [Eq. (2)], we find the time dependent relationship for TCRE:

\[ \Lambda(t) = \frac{R \alpha}{\lambda C_o} \ln \left( 1 + \frac{\alpha t}{C_o} \right) \left( \frac{1}{1 + \frac{1}{\sqrt{\phi \lambda^2 t}}} \right)^{\frac{1}{\alpha E}}. \tag{10} \]

We can now replace time \( t \) with emissions \( E \) to find TCRE as a function of cumulative carbon emissions:

\[ \Lambda(E) = \frac{R \alpha}{\lambda C_o} \ln \left( 1 + \frac{\alpha E}{C_o} \right) \left( \frac{1}{1 + \frac{1}{\sqrt{\phi \lambda^2 E}}} \right)^{\frac{1}{\alpha E}}. \tag{11} \]

To understand how multiplying three functions of \( E \) together is yielding a near-constant value, an approximation for the natural logarithm is needed. The Taylor series based on the area hyperbolic tangent can be shown to be a good approximation for values of the natural logarithm relevant for the climate problem. Recall that this approximation takes the following form:

\[ \ln(z) = 2 \left[ \text{arctanh} \left( \frac{z - 1}{z + 1} \right) \right] \]

\[ = 2 \left[ \frac{z - 1}{z + 1} + \frac{1}{3} \frac{z - 1}{z + 1}^3 + \frac{1}{5} \frac{z - 1}{z + 1}^5 + \cdots \right] \tag{12} \]
(Gradshteyn and Ryzhik 2000). For the values in Table 1, the first term of this approximation can reproduce the logarithmic factor of Eq. (11) with a 3% error at $E = 1000$ Pg C and with a 7% error at $E = 2000$ Pg C. Invoking this approximation, Eq. (11) can be reduced too:

$$\Lambda(E) \approx \frac{R \alpha}{\lambda C_o} \left( \frac{2 \alpha E}{2 C_o + \alpha E} \right) \left( \frac{1}{1 + \frac{1}{2C_o} \left( \frac{1}{1 + \sqrt{\frac{r}{\phi \lambda^2 E}}} \right)} \right).$$

(13)

Combining the second and the third factor, this relationship simplifies to

$$\Lambda(E) \approx \frac{R \alpha}{\lambda C_o} \left( \frac{1}{1 + \frac{1}{2C_o} \left( \frac{1}{1 + \sqrt{\frac{r}{\phi \lambda^2 E}}} \right)} \right).$$

(14)

and therefore,

$$\Lambda(E) \approx \frac{R \alpha}{\lambda C_o} \left( \frac{1}{1 + \frac{r}{\phi \lambda^2 E} + \frac{\alpha E}{2C_o} \left( \frac{1}{1 + \sqrt{\frac{\alpha^2 r E}{4C_o^2 \phi \lambda^2}}} \right)} \right).$$

(15)

TCRE is approximately a constant over a large range of cumulative carbon emissions. This implies that the derivative of TCRE with respect to $E$ is near zero for values of $E$ up to 2000 Pg C. The derivative of TCRE with respect to $E$ is

$$\frac{d \Lambda(E)}{dE} = \frac{-R \alpha}{\lambda C_o} \left[ \frac{\alpha}{2C_o} - \sqrt{\frac{r}{4 \phi \lambda^2 E}} r^{1.5} + \frac{\alpha}{4C_o} \sqrt{\frac{r}{4 \phi \lambda^2 E}} \left( \frac{1}{1 + \frac{r}{\phi \lambda^2 E} + \frac{\alpha E}{2C_o} \left( \frac{1}{1 + \sqrt{\frac{\alpha^2 r E}{4C_o^2 \phi \lambda^2}}} \right)} \right)^2 \right].$$

(16)

This derivative (computed with the values in Table 1) is plotted in Fig. 1. The figure shows that, for the values in Table 1, $d \Lambda/dE$ reaches zero at 677 Pg C. For cumulative carbon emissions near this point, TCRE will be near constant.

Another way of expressing this is that the sum of the terms $\sqrt{r/\phi \lambda^2 E}$, $\alpha E/2C_o$, and $\sqrt{\alpha^2 r E/4C_o^2 \phi \lambda^2}$ from Eq. (15) is nearly constant over a large range of values. By comparing the first derivatives of these terms, we can see how the parameters compensate to create TCRE. As these derivatives need to nearly cancel one another out, we can set them as the similarity

$$\frac{\alpha}{\lambda + \kappa},$$

(18)

where $\theta$ is the linearized value for radiative forcing. Also shown in Fig. 2 is a parameterization where the logarithmic forcing for CO$_2$ [Eq. (7)] is substituted in for $\theta$ to demonstrate the effect of the nonlinearity of radiative forcing from CO$_2$ on the parameterization of Gregory et al. (2009). The figure shows that the

![Fig. 1. First derivative of TCRE ($\Lambda$) computed for values in Table 1. Note that the derivative reaches zero at 677 Pg C.](image-url)
parameterization derived above reproduces the near proportionality between temperature change and cumulative carbon emissions.

a. An exponential increase in emissions

The assumption that carbon is emitted linearly, used in the previous section, clearly does not represent the situation in the modern world, where emissions have been increasing approximately exponentially (e.g., Boden et al. 2011). Under the RCP scenarios, emissions are expected to continue to increase approximately exponentially until a peak emission value is reached, after which emissions are projected to decline (Fig. 3). To analytically explore the condition of exponentially increasing emissions, the above equations were resolved under the condition that

\[ E \approx \frac{\beta}{\epsilon} e^{\epsilon t}, \]  

(19)

where \( \epsilon \) is the \( e \)-folding time for emissions, and \( \beta \) is the initial rate of emissions. Substituting this into the appropriate equation, we find

\[ \Lambda(E) = \frac{Ra}{\lambda C_o} \left[ 1 + \frac{\epsilon}{\sqrt{\phi \lambda^2 \ln \left( \frac{E \epsilon}{\beta} \right)}} + \frac{1}{\frac{\alpha E}{2C_o} + \frac{\alpha E}{2C_o} \sqrt{\frac{\epsilon}{\phi \lambda^2 \ln \left( \frac{E \epsilon}{\beta} \right)}}} \right]. \]  

(20)

The exponential increase in emissions does not affect the radiative forcing part of this relationship and therefore does not have an effect on the accuracy of the approximation of the logarithm used to simplify the relationship.

From Figs. 2c and 2d, one can see that the ratio of temperature change to cumulative carbon emissions deviates significantly from a constant in the case of exponentially increasing CO\(_2\) emissions. Given that Earth system models expect TCRE to be nearly constant until
approximately 2000 Pg C have been emitted, there must be another factor changing to compensate for the decline in \( \ln(\frac{c}{C_{15}}) \), where the symbol \( c \) represents the concentration of carbon emissions.

The most intuitive suspect for that changing factor is the airborne fraction \( a \), which, in models, is known to increase as cumulative carbon emissions rise (e.g., Zickfeld et al. 2013). Results from the linear emissions experiments performed with the UVic ESCM help illuminate this problem. Shown in Figs. 4, 5, and 6 are the airborne, ocean-borne, and land-borne fractions of carbon for the linear emission experiments with the permafrost carbon pool of the UVic ESCM turned on and turned off. From the figures, it is clear that, in the UVic ESCM, ocean uptake of carbon is effectively a function of the rate of carbon emissions, as is the airborne fraction of carbon in experiments with weak terrestrial carbon cycle feedbacks. Taking into account that airborne fraction is a function of the rate of carbon emissions, we can recover the near-constant nature of TCRE (Figs. 2c,d). These results support the hypothesis of Matthews et al. (2009) that it is the ocean’s diminishing ability to absorb heat and carbon that creates TCRE.

b. The TCRE window

The derivatives in Eq. (17) only fully cancel one another at a single point:

\[
\frac{\alpha}{C_o} + \frac{\alpha}{C_o} \sqrt{\frac{r}{\phi \lambda^2}} \frac{1}{\sqrt{E}} - \sqrt{\frac{r}{\phi \lambda^2}} \frac{1}{E^{1.5}} = 0. \tag{21}
\]

For other values of cumulative carbon emissions, the cancelation between these terms is only approximate. Furthermore the further one gets from the cancelation value, the less constant TCRE becomes. Therefore, it may be useful to define a “TCRE window” where TCRE is approximately constant. The upper and lower bounds of this window are arbitrary but pragmatically define the window as the region where TCRE is within 95% of its peak value [which is where the terms in Eq. (21) sum to zero]. The cumulative carbon emissions consistent with the values of TCRE 95% of peak can be solved for numerically. For the parameter values in Table 1, the TCRE window is between 200 and 2100 Pg C of cumulative emissions, roughly reproducing the near-linear region inferred from Earth system model simulations.

The notion of a TCRE window also provides a means to examine the parameter sensitivity of the linearity of the temperature–cumulative carbon curve. Figure 7 shows the results of such an analysis. Each of the four parameters in Eq. (21) were varied through a plausible range of values, while all of the other parameters were held at their values from Table 1. The TCRE window becomes smaller as airborne fraction of carbon increases (Fig. 7a) and becomes larger as the rate of emissions is increased (Fig. 7b). That is, the TCRE is linear over a smaller range of cumulative carbon emissions when a larger fraction of carbon remains in the atmosphere. This intuitively makes sense, as the logarithmic form of the radiative forcing will become dominant at lower cumulative emission when a higher fraction of carbon remains in the atmosphere. The value of ocean heat uptake parameter \( f \), which varied over an order of magnitude between climate models (Table 2), also strongly affects the TCRE. Higher values of \( f \) correspond to a slower uptake of heat by the oceans and a narrower TCRE window (Fig. 7c).

The initial quantity of carbon in the atmosphere is not a variable, as its value is known (596 Pg C). However, examining variations in this quantity allows one to question whether the existence of a near-constant TCRE depends on what point in Earth’s history an industrial civilization happened to emerge. Varying the initial quantity of carbon in the atmosphere strongly affects the TCRE window, with both upper and lower bounds increasing as the initial quantity of carbon is increased (Fig. 7d). This suggests that, as long as the semi-infinite half-space parameterization for planetary heat uptake holds for a given climate state, there will
be a region where a temperature versus cumulative carbon curve will be nearly linear. That is, TCRE may be a general feature of the Earth system and not a fortuitous fluke of compensating parameter values.

4. Model results

a. Constant rate experiment

There are several systems within the carbon cycle, such as tropical forests and permafrost carbon reservoirs, which have the potential to disrupt the near-linear temperature–cumulative carbon emissions relationship by releasing large quantities of carbon into the atmosphere in a manner not linearly related to surface temperature change (e.g., Collins et al. 2013). If any of these terrestrial carbon cycle feedbacks turn out to be strong enough to disrupt the ocean-derived relationship between airborne fraction of carbon and rate of carbon emissions, then presumably the near-constant nature of TCRE could also break down. The UVic ESCM was recently modified to include a representation of the permafrost carbon pool (MacDougall et al. 2012). In the following experiments, the model was run with and without the permafrost carbon module turned on as a demonstration of the effect of a strong positive terrestrial carbon cycle feedback on the validity of the TCRE. However, we caution that substantial uncertainties exist in the magnitude, timing, and chemical species of the release of carbon from permafrost soils (e.g., Collins et al. 2013) and that, at present, UVic ESCM simulates the largest release of carbon from permafrost soils of any land surface model (MacDougall et al. 2012). Figure 8 displays temperature versus cumulative carbon emission curves for the linear emissions experiments for the three climate sensitivities used and for model runs with and without the permafrost carbon module turned on.
By comparing Fig. 8 and Fig. 6, one can see that the degree to which TCRE remains a constant is related to the strength and rate dependence of the terrestrial carbon cycle feedback to climate change. In simulations with a climate sensitivity of 2.0°C, the land-borne uptake of carbon (land-borne carbon fraction) is a rate-independent function of cumulative carbon emissions, stabilizing at about 0.3 and gradually declining to 0.2 at 2000 Pg C for the simulation without the permafrost carbon. For the simulation with permafrost carbon, land-borne uptake declines from about 0.3 to 0.1 at 2000 Pg C. The temperature versus cumulative carbon emissions curves for these two sets of simulations are rate independent with a near-constant TCRE.

Likewise, the temperature versus cumulative carbon emissions curves for these simulations is rate independent with a constant TCRE. For the simulations with permafrost carbon, the magnitude of the land-borne fraction of carbon becomes dependent on carbon emission rate, with steeper declines for slower emissions rates of carbon. This clearly affects the temperature versus cumulative carbon emissions curves, which in this set of simulations diverge by rate and have more variable TCRE at low emission rates. This rate dependence is likely a manifestation of the time lag inherent in the permafrost carbon feedback. That is, when a permafrost region is warmed, it takes time for soils to thaw and for carbon to decay so that there is a long time lag between forcing the system and carbon release from the system (e.g., Schuur et al. 2008). In the simulations with a slower rate of emission, more time has elapsed at a given quantity of cumulative carbon emissions, meaning that the permafrost soils are closer to

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**Fig. 5.** Ocean-borne fraction of carbon for each of the linear emission experiments carried out with the UVic ESCM. Note that ocean-borne fraction of carbon is approximately a function of carbon emission rate.
being in equilibrium with the new climate and that more carbon has been released from these soils.

For the simulations with a climate sensitivity of 4.5°C, strong rate-dependent terrestrial carbon cycle feedbacks are seen in simulations with and without permafrost carbon (these feedbacks are stronger in the simulations with permafrost carbon). This corresponds with nonlinear temperature versus cumulative carbon emissions curves for simulations both with and without permafrost carbon, especially for simulations with a low emissions rate.

An interesting feature of temperature versus cumulative carbon emissions curves is the vertical line where emissions cease. This represents the continued change in surface temperature after emissions cease. All emission rate curves end at the same point. That is, the simulations all end at the same equilibrium temperature (dependent on the climate sensitivity), even in simulations where there are large transient differences in temperature as a function of cumulative carbon emissions. This result in particular is expected to be model dependent, as the UVic ESCM does not exhibit some of the nonlinear climate system behavior that is contingent on the transient path of warming, such as a collapse in Atlantic meridional overturning circulation (e.g., Weaver et al. 2012).

b. A strong terrestrial carbon cycle feedback and the RCPs

MacDougall et al. (2012) described model experiments with the permafrost carbon version of the UVic ESCM forced with emissions derived from the RCPs. However, that study did not examine the effect of this irreversible long time scale release of terrestrially stored carbon on the TCRE of the RCPs. Compatible carbon emissions are calculated by prescribing atmospheric CO₂ concentration from a given RCP while using the UVic ESCM to simulate uptake of carbon by the oceans and the terrestrial land surface. This allows for anthropogenic emissions of
carbon to be solved for as the final unknown variable in the global carbon mass balance. Figure 9 shows temperature versus cumulative carbon curves for each of the four RCPs with and without the permafrost carbon component of the model turned on. In either case, the curves are roughly linear with a TCRE of 1.9 K Eg⁻¹ C⁻¹ (1 Eg = 1 × 10¹² kg) for the simulations without permafrost carbon and a TCRE of 2.2 K Eg⁻¹ C⁻¹ for the simulations with permafrost carbon. TCRE for the RCPs includes not only the effects of CO₂ from fossil fuels but also non-CO₂ radiative forcings and land-use changes. The most interesting feature to arise from the addition of the permafrost carbon pool in the RCP simulations is the “tail” features that the temperature curves gain for RCPs 2.6, 4.5, and 6.0. These tails represent the continued warming of the system once atmospheric CO₂ has been stabilized or, in the case of RCP2.6, begins to decline. The reverse slopes of the tails, most pronounced for RCP2.6, indicate that net negative emissions are necessary to achieve these scenarios, because of the continued emission of carbon from former permafrost soils after the cessation of anthropogenic emissions.

5. Discussion

As a metric of climate change, TCRE’s most important quality is that it provides a simple way of relating the cause of climate change (CO₂ emissions) to the most used index of climate change (global mean near-surface warming). Despite the rate dependence added by the inclusion of the permafrost carbon pool into the UVic ESCM, TCRE retains its simplicity for the RCP simulations and therefore retains its principle utility. From the analysis above, it appears that the hypothesis for the near-constant nature of TCRE proposed by Matthews et al. (2009) is largely correct. There is, in fact, a compensation between declining radiative forcing from a unit of CO₂ and diminishing uptake of heat and carbon by the ocean. These two oceanic effects, however, act in a surprisingly independent way. In the case of a constant rate of CO₂ emission, it is the diminishing rate of ocean heat uptake that compensates for the diminishing radiative forcing per unit atmospheric CO₂. In this case, the ocean carbon uptake is stable and therefore stabilizes the airborne fraction of carbon. In the more historically accurate case of exponentially increasing CO₂ emissions, a second compensation between airborne fraction of carbon and the CO₂ emission rate effect on ocean heat uptake becomes important. As the rate of CO₂ emissions increases, the ocean takes up a diminishing fraction of carbon. This leads to a stronger radiative forcing from CO₂ at given cumulative emissions but also increases the rate of ocean heat uptake, compensating in such a way as to maintain a near-constant TCRE.

The effects of non-CO₂ forcing on TCRE have not been addressed in the present paper. Many non-CO₂ greenhouse gases, such as CH₄ and N₂O, exhibit diminishing radiative forcing per unit mass at higher atmospheric concentrations (Houghton et al. 2001) and therefore, under certain conditions, may exhibit linear temperature versus emission rate or cumulative emission curves. However, for these gases there is no analogous relationship similar to that between CO₂ emission rate and ocean-borne fraction of carbon. Therefore, how to reconcile non-CO₂ greenhouse gases forcing and the negative forcing from aerosols with TCRE remains an opportunity for future research.

Here we have been seeking to explain the near-linear relationship between global mean temperature change and cumulative CO₂ emissions that is a feature of most climate model output. It should be noted, however, that in some climate models the relationship is evidently sublinear and more resembles a logarithmic curve than a linear curve [e.g., see Fig. 12.45 in Collins et al. (2013)]. This is particularly true of intermediate complexity climate models. Within the framework presented here, this sublinear behavior, before 2000 Pg C have been emitted, may be understood as such models having a particularly narrow TCRE window. However, it may also be the case that the semi-infinite half-space approximation of ocean heat uptake does not adequately approximate ocean heat uptake as simulated within these models.

The analysis presented here draws on model experiments conducted with the UVic ESCM. The results of
these simulations are, to some degree, model dependent, and caution should be used before generalizing these results to other Earth system models. In particular, differences in simulated physical feedbacks, carbon cycle processes, ocean heat uptake, and effective climate sensitivity are expected to alter how TCRE arises (or does not arise) in different Earth system models. The parameterization of ocean heat uptake as heat flow in a semi-infinite half space is a mathematically useful but ultimately a gross oversimplification of a complex process. The logarithmic approximation of the radiative forcing from CO$_2$ is also a simplification of complex physics (Wigley 1987), which, for
some of the analysis in this paper, we have here further simplified using an approximation of the logarithm. The results presented here should therefore be interpreted with appropriate caution.

6. Conclusions

The transient climate response to cumulative CO₂ emissions is a useful metric of climate warming that emerges as a near constant in Earth system model simulations of climate change up until approximately 2000 Pg C have been emitted to the atmosphere (Matthews et al. 2009; Collins et al. 2013). Here we have conducted an analytical analysis of the climate system using a parameterization of ocean heat uptake as heat flux into a semi-infinite half space to derive an analytical expression for TCRE. This analysis shows that TCRE is near constant if the change in the ocean heat uptake efficiency with respect to cumulative carbon emissions (relative to the climate feedback parameter) is the same order of magnitude as the sum of one-half of the airborne fraction of carbon (relative to the initial carbon content of the atmosphere) and the derivative of a cross term. This cancelation produces a window where TCRE is nearly constant. In the more historically accurate case of exponentially increasing emissions of CO₂, the property of the ocean whereby the ocean-borne carbon fraction is a declining function of emissions rate helps compensate for the increased ocean heat uptake (as a function of cumulative carbon emissions) at higher emission rates.

Supporting this analytical analysis are numerical modeling experiments conducted with the frozen ground version of the UVic ESCM. The model, when forced with linear emissions of CO₂ suggests that strong feedbacks from the terrestrial carbon cycle lead to rate dependence in the TCRE. These rate dependencies are of academic interest but do not result in significant changes in the linearity of temperature versus cumulative carbon emissions curves of the RCP scenarios. Overall we reason that the near proportionality between temperature and cumulative carbon emissions is a feature of the Earth system in its current configuration and is therefore a useful metric of climate warming.

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