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Preserving duration-intensity correlation on synthetically generated water demand pulses

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Abstract

This paper proposes the application of three different methods for preserving the correlation between duration and intensity of synthetically generated water demand pulses. The first two methods, i.e., the Iman and Canover (1982) method and the Gaussian copula (Nelsen, 1999) respectively, are derived from the known statistical approaches, though they had never been applied to the context of demand pulse generation. The third is a novel methodology developed in this work and is a variation in the Gaussian cupola approach. Applications carried out to reproduce the demand pulses measured in one household prove that the three methods are effective and applicable under general conditions.

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Keywords: Water demand, demand pulses, intensity, duration, correlation, Iman-Canover, Gaussian copula

Nomenclature

C	correlation matrix of the independently generated pulse duration and intensities
Cp	correlation matrix to be preserved
D	daily demand
i	index
I	pulse intensity

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k	parameter in the gamma distribution
$\mathbf{L}$	Choleski decomposition of matrix $\mathbf{C}$
$\mathbf{L_p}$	Choleski decomposition of matrix $\mathbf{C_p}$
n	number of pulses in a generic sequence
T	pulse duration
V	pulse volume
$\mathbf{X}$	matrix of the independently generated pulse duration and intensities
$\mathbf{X_1}$	auxiliary matrix in the Iman Canover procedure
$\mathbf{X_2}$	matrix of the correlated generated pulse duration and intensities
x	generic random variable
z	number of pulses per unit interval of time
α	parameter in the beta distribution
B	beta function
β	parameter in the beta distribution
Γ	gamma function
Δt	interval of time
θ	parameter in the gamma distribution
λ	parameter for Poisson distribution
μ	parameter in the bivariate normal distribution
ρ	correlation between pulse duration and intensity
$\dot{\rho}$	correlation in the bivariate normal distribution
ρ_{ep}	correlation between pulse duration and intensity to be preserved
σ	parameter in the bivariate normal distribution
τ	generic time

1. Introduction

In the last two decades, the issue of residential water demand generation has been extensively investigated (Buchberger and Wu, 1995; Buchberger and Wells, 1996; Guercio et al., 2001; Alvisi et al., 2003; Garcia et al., 2004; Buchberger et al., 2003; Alcocer et al., 2006; Blokker et al., 2010; Alcocer-Yamanaka and Tzatchkov, 2012; Alvisi et al., 2014; Creaco et al., 2015). A possible approach to modelling of residential water demand is based on the use of Poisson pulse models (Buchberger and Wu, 1995; Buchberger and Wells, 1996; Guercio et al., 2001; Buchberger et al., 2003; Garcia et al., 2004; Alcocer et al., 2006; Creaco et al., 2015). Inside these models, pulse durations and intensities can be generated using suitable probability distributions. In most cases (Buchberger and Wu, 1995; Buchberger and Wells, 1996; Guercio et al., 2001; Buchberger et al., 2003; Garcia et al., 2004; Alcocer et al., 2006), pulse duration and intensity were considered to be independent random variables. However, Creaco et al. (2015) have recently shown that a non-negligible positive correlation exists between the two variables. The same authors then postulated that this has to be considered in order to obtain synthetic water demand pulses that are more consistent in terms of overall daily water demand volumes, while respecting statistical properties of measured demand pulses. In particular, their method is based on the use of a bivariate normal distribution. They also showed that the model can be effectively used when either the intensity or the duration of the pulse is well represented by either the normal or the lognormal distribution. However, the issue of how correlation can be preserved in the case of pulse duration and intensity being represented by other probability distributions has not dealt with yet.

In this paper, three methods are described that can be applied to obtain correlated pulse intensities and durations for any marginal distribution used to represent the two variables as independent random variables.

In the following sections, first the methodologies are described, then they are applied to a literature case study and a comparison with the method of Creaco et al. (2015) is also provided. Finally, results are analyzed and conclusions are drawn.

2. Methodology

Hereinafter, first the typical Poisson model with no correlation between pulse intensity and duration is described. Then, the methods used to preserve correlation are described, followed by the model parameter estimation.

2.1. Poisson model

Inside the model, the time axis is scanned with a certain time resolution Δt. The probability of having z generated pulses in the time interval Δt that follows the generic time τ is described by the Poisson distribution (Buchberger and Wu, 1995):

$$P(z) = \frac{e^{-\lambda \Delta t} (\lambda \Delta t)^z}{z!} \quad \text{with } z = 0,1,... \tag{1}$$

where rate parameter λ represents the expected number of "events" or "arrivals" that occur per unit time.

For each pulse generated, the associate duration T and intensity I are generated using suitable probability distributions. As an example, the density functions of the beta and gamma distributions (Johnson and Bhattacharyya, 1992) are provided in eqs. (2) and (3) respectively:

$$f(x) = \frac{1}{B} \frac{\left(\frac{x - x_{\min}}{x_{\max} - x_{\min}} \right)^{\alpha-1}}{\left(1 - \frac{x - x_{\min}}{x_{\max} - x_{\min}} \right)^{\beta-1}} \tag{2}$$

$$f(x) = \frac{x^{k-1} e^{-\frac{x}{\theta}}}{\theta^k \Gamma} \tag{3}$$

where x is the random variable, equal to T or I, depending on which variable has to be generated; α and β are the parameters of the beta distribution and $B=B(\alpha, \beta)$ is the beta function; k and θ are the parameters of the gamma distribution and $\Gamma=\Gamma(k)$ is the gamma function. Whereas the gamma distribution is defined on the interval [0,+∞], the beta distribution is defined on the interval $[x_{\min}, x_{\max}]$. Therefore, in order for the latter to be used for the generation of either the duration or the intensity, the interval $[x_{\min}, x_{\max}]$ has to be defined. In any case, the cumulative distribution function F that ranges from 0 to 1 can be obtained as (Johnson and Bhattacharyya, 1992):

$$F(x) = \int_0^x f(x)\, dx \tag{4}$$

After distribution parameters values have been fixed, values of the generic random variable can be sampled by generating for F random numbers in the range [0,1] and then deriving the corresponding elements of x by inverting eq. (4).

If a Poisson model is used with constant parameter values to generate water demand pulses for a certain time duration, a sequence of n pulses, each of which featuring its own time arrival τ_i, duration T_i and intensity I_i, would be obtained, as shown in **Error! Reference source not found.**.

Table 1– Arrival time τ, duration T and intensity I of the pulses generated by the Poisson model

τ (s)	T (s)	I (L/s)
τ_1	T_1	I_1
τ_2	T_2	I_2
…	…	…
τ_i	T_i	I_i
…	…	…
τ_{n-1}	T_{n-1}	I_{n-1}
τ_n	T_n	I_n

Variables T and I, as they appear in columns 2 and 3 of Table 1, are independent random variables; the corresponding correlation matrix **C** (see eq. 5) should thus feature an expected value of ρ, Pearson correlation coefficient out of the diagonal, equal to 0.

$$\mathbf{C} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \tag{5}$$

2.2. Preserving correlation

Method 1: Iman and Canover (IC) (1982)

The Iman and Canover (1982) procedure is made up of two steps. In step 1, variables T and I are generated as independent random variables, as is described in section 2.1. This results in the matrix **X**, which is made up of columns 2 and 3 of **Error! Reference source not found.**. The corresponding correlation matrix **C** is given by eq. (5). Step 2 is then applied to find a new pairing for these variables, which enables the desired/observed correlation ρ_{ep} value to be preserved between the variables.

In step 2 the following matrix operations are performed, which first entail constructing matrix $\mathbf{C_p}$ related to the desired/observed correlation ρ_{ep} to be preserved:

$$\mathbf{C_p} = \begin{pmatrix} 1 & \rho_{ep} \\ \rho_{ep} & 1 \end{pmatrix} \tag{6}$$

Then, matrices **L** and $\mathbf{L_p}$ can be obtained as lower triangular matrices from the Cholesky decomposition (Press et al., 1990) of matrices **C** and $\mathbf{C_p}$ respectively.

The new matrix $\mathbf{X_1}$, which features a correlation matrix equal to $\mathbf{C_p}$ in eq. (6), has to be calculated as:

$$\mathbf{X_1} = \mathbf{X} \cdot \left(\mathbf{L_p} \cdot \mathbf{L}^{-1}\right)^T \tag{7}$$

The elements of each column of matrix **X** have to be reordered in order to have the same sorting as the elements of the corresponding column of matrix $\mathbf{X_1}$, thus producing the matrix $\mathbf{X_2}$. In this manner, the matrices $\mathbf{X_2}$ and $\mathbf{X_1}$ will have the same rank correlation matrix, and, consequently, similar (Pearson) correlation matrices. Since the application of step 2 simply modifies the sorting of the T and I values and does not change the values themselves, it preserves the exact form of the marginal distributions on these variables, as it comes from step 1.

Method 2: Gaussian copula

Unlike method 1 that is applied to pulse durations and intensities that have already been generated, method 2 precedes the generation. A further difference lies in the fact that method 2 does not entail matrix operations.

Method 2 is known as the method of the Gaussian copula (Nelsen, 1999). It is based on generating *n* couples of auxiliary random variables y_1 and y_2 with average values $\mu_1 = \mu_2 = 0$ and standard deviations $\sigma_1 = \sigma_2 = 1$ through the bivariate normal distribution (eqs. 8 and 9):

$$f(y_1, y_2) = \frac{1}{\sqrt{(2\pi)^2|\boldsymbol{\Sigma}|}} \mathrm{e}^{\left\{-\frac{1}{2(1-\dot{\rho})}\left[\left(\frac{y_1-\mu_1}{\sigma_1}\right)^2+\left(\frac{y_2-\mu_2}{\sigma_2}\right)^2-\frac{2\dot{\rho}(y_1-\mu_1)(y_2-\mu_2)}{\sigma_1\sigma_2}\right]\right\}}, \tag{8}$$

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \dot{\rho}\sigma_1\sigma_2 \\ \dot{\rho}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \tag{9}$$

where $\dot{\rho}$ represents correlation between y_1 and y_2.

For each of the *n* values generated for y_1, the value F_1 of the cumulative probability of the marginal distribution can be calculated. In a similar way, for each of the *n* values generated for y_2, the value F_2 of the cumulative probability of the marginal distribution can also be calculated.

The *n* values of F_1 and F_2 can be used to sample the probability distributions chosen for pulse durations and intensities (eq. 4) and then to obtain *n* couples of *T* and *I*. As a result of correlation $\dot{\rho}$ imposed between y_1 and y_2, a certain degree of correlation is also imposed on *T* and *I*. In particular, the resulting correlation between *T* and *I* is a monotonous function of $\dot{\rho}$. Iterative methods can then be applied in order to determine the suitable value of $\dot{\rho}$ that yields the expected correlation ρ_{ep} to be preserved between *T* and *I*.

Method 3

Method 3 is a modified version of method 2. Similar to the original method, it does not require matrix operations and is based on the use of the bivariate normal distribution (eqs. 8 and 9). However, following from method 1, it is applied after a preliminary first step, in which *n* uncorrelated couples of *T* and *I* are generated.

Then, *n* couples of y_1 and y_2 with correlation equal to $\dot{\rho}$ are also generated. As in method 2, the corresponding values of F_1 and F_2 can be obtained. *T* and *I* can be reordered using the same sorting as F_1 and F_2 respectively. As a result of correlation $\dot{\rho}$ imposed between y_1 and y_2, a certain degree of correlation is also imposed on *T* and *I*. Iterative methods can be applied in order to determine the suitable value of $\dot{\rho}$ that yields the expected correlation ρ_{ep} to be preserved between *T* and *I*.

2.3. Parameter estimation method

The set of parameters of a Poisson model for pulse generation, in which *T* and *I* are generated as independent random variables, by using either the beta (eq. 2) or the gamma distribution (eq. 3), or using any kind of 2 parameter probability distribution, is 5: one parameter (λ) for pulse arrival and two parameters for either of *T* or *I*. If correlation between *T* and *I* needs to be accounted for, the number of parameter increases to 6 and correlation ρ_{ep} is the sixth parameter of the model.

As it was done by some authors (Alvisi et al., 2003; Buchberger et al., 2003; Creaco et al., 2015), the generic day of the month can be subdivided into a certain number of time slots (e.g., 12 bihourly) for parameter estimation. Robust models can then be obtained by allowing pulse arrival-related parameter λ to take on a different value in each daily time slot. Each of the other parameters (in this case, the parameters related to *T* and *I* and correlation ρ_{ep}), instead, is allowed to take on a single value valid for all the time slots.

The method of the moments (Hall, 2004) is considered in this work for estimating the parameters of the Poisson model.

3. Applications

3.1. Case study

By making calculations on data collected during an experimental campaign in some households in Milford, Buchberger et al. (2003) were able to reconstruct, with one second time step resolution, the water demand pulses which were taking place in these households in the period from April to October 1997. The data made available by the Authors concern pulse duration T, intensity I and volume $V=T\cdot I$.

As case study in this work, the indoor water demand pulses recorded in one of the households, i.e. household 2, in the month of April were selected. This case study has already been chosen by Creaco et al. (2015) on the basis of the regularity of the daily water consumption. The basic statistical parameters of measured water consumption variables z, T, I, V, total daily demand D and ρ are reported in Table 2.

The modelling framework of this paper is aimed at investigating the extent to which the methods described in this paper for preserving correlation lend themselves to being used inside Poisson models.

Overall, four Poisson models were constructed and compared with two benchmark Poisson models, hereinafter indicated as models A and B and drawn from the work of Creaco et al. (2015). In particular, model A features pulse durations and intensities being generated through the bivariate lognormal distribution. This enables correlation to be obtained between the two variables in this model. In model A parameters are estimated with the method of the moments. Model B differs from model A in that pulse durations and intensities are generated as independent (uncorrelated) variables by making use of the lognormal distribution. The four models constructed in this work are models C-1, C-2, C-3 and D. Models C-1, C-2 and C-3 differ from model A in the way correlation is preserved. In fact, unlike model A, these models feature adoption of one of the three methods described in section 2.2 (methods 1, 2 and 3 respectively). Model D differs from model C-3 in the generation of pulse durations and intensities, which take place through the beta (eq. 2) and gamma (eq. 3) distributions respectively, instead of the lognormal distribution.

The features of all the models used in this work are summarized in Table 3.

Overall, applications consisted in 3 phases for each test:

phase 1 – parameter assessment;

phase 2 – generation of synthetic water demand pulses;

phase 3 – analysis of the results of the models and comparison with the observed data.

Table 2 – Basic statistical parameters of water consumption variables z, T, I, V, D and ρ derived from the measured pulses and from the pulses generated by the models.

	mean (z) [s^{-1}]	mean (T) [sec]	var (T) [sec^2]	mean (I) [L/s]	var (I) [L^2/s^2]	mean (V) [L]	mean (D) [L/day]	cov (I,T) [L^2]	ρ [-]
measured	0.00054	56	12743	0.106	0.00742	9.45	442	3.50	0.36
model A	0.00054	56	11822	0.106	0.00730	9.39	438	3.48	0.37
model B	0.00054	56	12321	0.106	0.00737	5.95	278	0.03	0.00
model C-1	0.00054	56	12321	0.106	0.00737	9.26	431	3.34	0.35
model C-2	0.00054	56	12049	0.105	0.00712	9.17	427	3.27	0.35
model C-3	0.00054	56	12321	0.106	0.00737	9.27	432	3.36	0.35
model D	0.00054	56	12723	0.106	0.00744	9.44	440	3.50	0.36

Table 3 – Features of the models used in this work.

	distribution for pulses		correlation
	duration	intensity	method
model A	lognormal	lognormal	no correlation
model B	bivariate lognormal distribution		Creaco et al. (2015)
model C-1	lognormal	lognormal	method 1
model C-2	lognormal	lognormal	method 2
model C-3	lognormal	lognormal	method 3
model D	beta	gamma	method 3

3.2. Results

Phase 1

The results of phase 1 for models C-1, C-2, C-3 and D are reported in Tables 4 and 5. The results for models A and B, instead, can be found in the work of Creaco et al. (2015). For the analysis of these tables, it has to be recalled that only λ is parametrized in 12 daily time slots; the other parameters, instead, are assigned a single daily value. The data reported in Tables 4 and 5, related to models C-1, C-2, C-3 and D, were obtained by applying the method of the moments. In model D, in which the beta distribution is used to generate pulse durations, the interval $[x_{\min}, x_{\max}]$ was set to [0,850] following analysis of the experimental pulses.

Phase 2

The models calibrated in phase 1 were then applied in order to create synthetic demand pulses for one month for each test. In order to account for the influence of the random seed, each month long pulse generation was repeated 100 times.

In each of the models which use one of the methods described in section 2.2 to preserve correlation (models C-1, C-2, C-3 and D), the method was applied once at the end of each of the 100 monthly generations of pulses.

As an example, Fig. 1 shows the single realization of the simulated total demand for a typical day at the scale of 1 sec, obtained by using model C. As expected, the figure shows a higher concentration of the pulses in the morning and in the late afternoon, when the household occupants usually get up and get back home after work. Very few pulses are instead generated at nighttime.

Table 4 – Calibrated parameters for models C-1, C-2 and C-3.

	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-24
λ	0.000065	0.000028	0.001056	0.000991	0.000579	0.000356	0.000130	0.000333	0.001023	0.001032	0.000745	0.000157
μ_{lnT}	3.22											
σ_{lnT}	1.27											
μ_{lnI}	-2.50											
σ_{lnI}	0.71											
ρ_{ep}	0.36											

Table 5 – Calibrated parameters for model D.

	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-24
λ	0.000065	0.000028	0.001056	0.000991	0.000579	0.000356	0.000130	0.000333	0.001023	0.001032	0.000745	0.000157
α						0.17						
β						2.33						
θ						0.07						
κ						1.51						
ρ_{ep}						0.36						

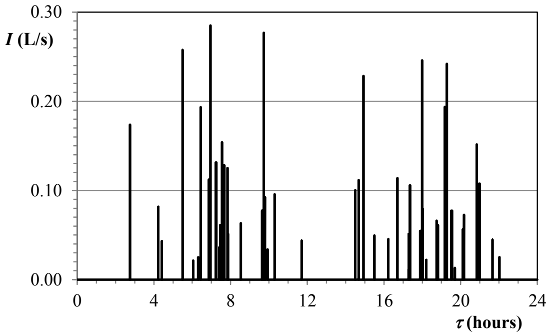

Fig. 1. Model C - single realization of the simulated total demand for a typical day at the scale of 1 sec.

Phase 3

A first analysis was made concerning the basic statistical parameters of water consumption variables z, T, I, V and ρ derived from the pulses generated by models A-D, in comparison with those of the measured pulses (see Table 1). This table shows that all the models reproduce well mean (z), mean (T), var (T), mean (I) and var (I). This is a direct consequence of the goodness of the method of the moments for parameter calibration. Mean(V), mean (D), ρ and cov(T,I) are well reproduced only by the models that consider correlation (i.e. all the models except for model B). In particular, with regard to ρ, the analysis of Table 1 proves also that the methods described in section 2.2 and adopted in models C-1, C-2, C-3 and D have similar effects to the use of the bivariate distribution in model A. The advantage for these methods of being also applicable in the cases (see model D) where no bivariate distribution is available, that is when T and I are represented by two different kinds of marginal probability distributions, must also be highlighted.

Another test was then carried out to analyse the consistency of the synthetic water demand pulses generated by means of the models with the measured water demand pulses in terms of overall daily water demand volume D. In particular, the total synthetic water demand volume D was calculated for each day in the generic one-month long pulse generation of each test. Then, the cumulative frequency curve was constructed reporting, for each value of D, the Weibull cumulative frequency F of days in the month which feature an overall daily water demand volume lower than or equal to D. Since each model application comprises 100 one-month long pulse generations, a band of synthetic cumulative frequencies was then obtained for each test. For each test, the band upper envelope (BUE), lower envelope (BLE) and mean value (BMV) of the 100 cumulative frequency curves were determined for all the models. The cumulative frequency of the measured daily water demand volume (ECF) was also calculated.

The graphs in Figure 2 report BUE, BLE and BMV obtained using the various models as well as ECF. Analysis of the graphs shows that, as already highlighted by Creaco et al. (2015), the BMV obtained with model A (model that takes account of the mutual dependence of pulse intensity and duration by means of the bivariate distribution) follows ECF much more closely than that obtained with model B (model that neglects the mutual dependence of pulse intensity and duration). Furthermore, all the data points of ECF lie inside the band of cumulative frequency obtained with model A. Only a few ECF data points, instead, are found inside the band of cumulative frequency obtained with model B. This attests to the better capability of model A to generate water demand pulses which are consistent with the observed demand pulses in terms of overall daily water demand volume. Figure 2 also shows that models C-1, C-2 and C-3, which use the methods described in this paper to preserve correlation, have an almost identical performance to model A. The change in the distributions used to represent the pulse durations and intensities does not affect results significantly (see results of model D with method 3).

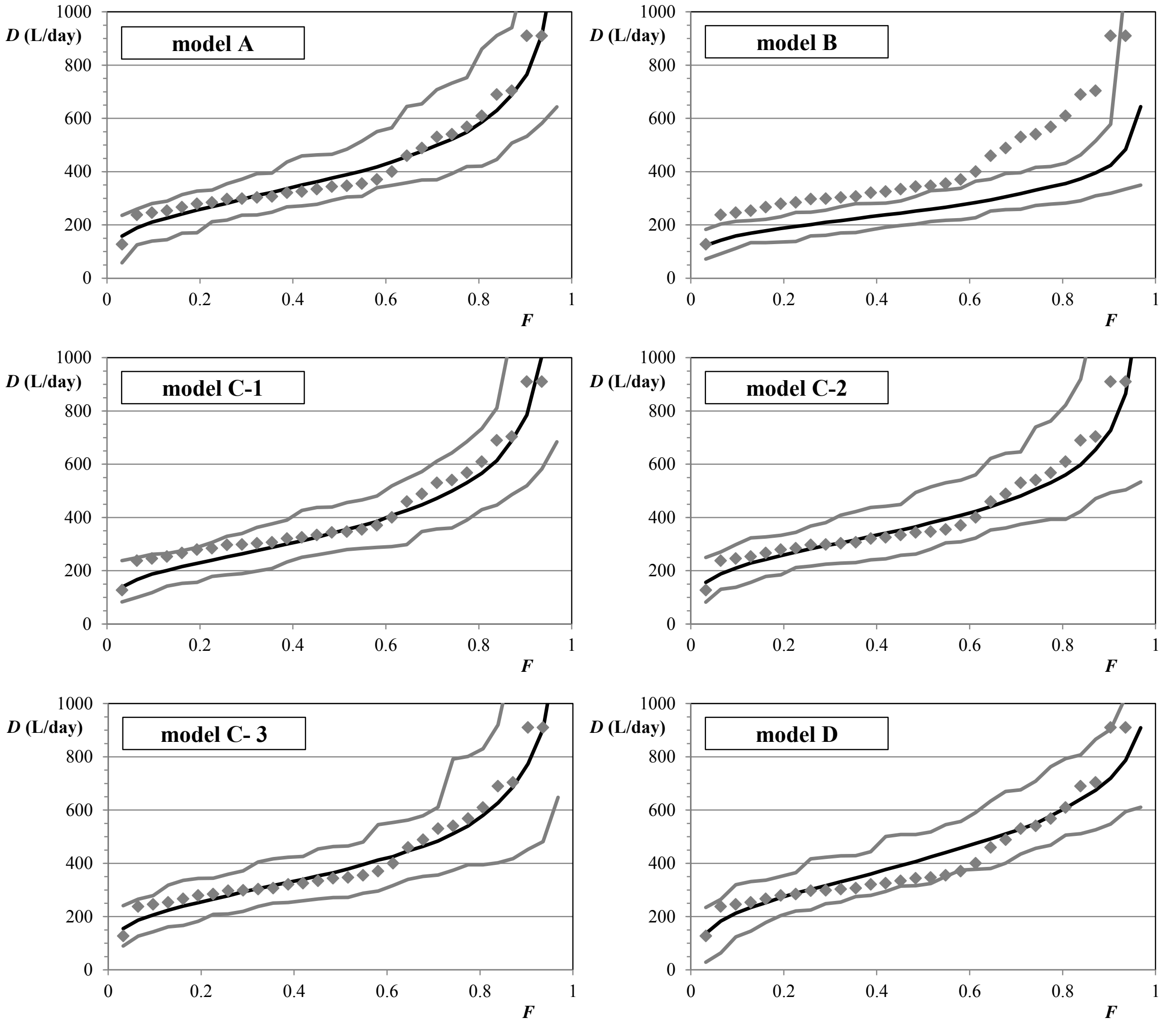

Fig. 2. Upper and lower envelopes (grey lines) and mean value (black line) of the band of Weibull cumulative frequencies F of daily water demand D produced by models A, B, C-1, C-2, C-3 and D in comparison with the daily water demand cumulative frequency calculated starting from the measured data (dots).

4. Conclusions

This paper presented the application of three different methods that can be used to preserve correlation between duration and intensity of water demand pulses. Whereas the first two methods are derived from the known statistical approaches, the third was newly developed in this work. Applications showed that the three method yield similar results to those previously reported by Creaco et al. (2015) with the advantage of being applicable with any marginal distributions to represent the duration and the intensity.

In light of the results of the paper, which confirmed the findings of Creaco et al. (2015), it seems that taking the correlation into account in water demand pulses is an essential requirement for producing synthetic pulses which are consistent with the measured pulses in terms of statistical properties while determining consistent daily demand values.

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References

Alcocer-Yamanaka, V.H., Tzatchkov, V., and Buchberger, S.G., 2006. Instantaneous Water Demand Parameter Estimation from Coarse Meter Readings. In: Proceedings of the Water Distribution Systems Analysis Symposium 2006, pp. 1-14.

Alcocer-Yamanaka, V.H., and Tzatchkov, V.G., 2012. Modeling of Drinking Water Distribution Networks Using Stochastic Demand. Water Resources Management, 26, 1779-1792.

Alvisi, S., Franchini, M., and Marinelli, A., 2003. A stochastic model for representing drinking water demand at residential level. Water Resources Management, 17 (3), 197–222.

Alvisi, S., Ansaloni, N., Franchini, M. 2014. Generation of synthetic water demand time series at different temporal and spatial aggregation levels. Urban Water Journal, 11 (4), 297-310.

Blokker, E.J.M., Vreeburg, J.H.G., and van Dijk, J.C., 2010. Simulating residential water demand with a stochastic enduse model. Journal of Water Resources Planning and Management, 136 (1), 375–382.

Buchberger, S.G., Carter, J.T., Lee, Y.H., and Schade, T.G., 2003. Random Demands, Travel Times and Water Quality in Dead-ends, prepared for American Water Works Association Research Foundation, AwwaRF Report No. 294, 2003, 470 pages.

Buchberger, S.G., and Wells, G.J., 1996. Intensity, duration and frequency of residential water demands. Journal of Water Resources Planning and Management, 122 (1), 11–19.

Buchberger, S.G., and Wu, L., 1995. Model for instantaneous residential water demands. Journal of Hydraulic Engineering, 121 (3), 232–246.

Creaco, E. , Farmani, R., and Kapelan, Z., Vamvakeridou-Lyroudia, L., Savic, D., 2015. Considering the mutual dependence of pulse duration and intensity in models for generating residential water demand. Accepted for publication on Journal of Water Resources Planning and Management.

Garcıa, V.J., Garcıa-Bartual, R., Cabrera, E., Arregui, F., and Garcıa-Serra, J. 2004. Stochastic model to evaluate residential water demands. Journal of Water Resources Planning and Management, 130 (5), 386–394.

Guercio, R., Magini, R., and Pallavicini, I., 2001. Instantaneous residential water demand as stochastic point process. In: Water resources management, C. A. Brebbia et al. eds., WIT Press, Southampton, U.K., 129-138.

Hall, A.R., 2004. Generalized Method of Moments. Advanced Texts in Econometrics, OUP Oxford, ISBN: 0198775202, 9780198775201.

Iman, R.L., and Conover, W.J., 1982. A distribution-free approach to inducing rank correlation among input variables. Communication in Statistics, 11, 311–334.

Johnson, R.A. and Bhattacharyya, G.K., 1992. Statistics: Principles and Methods. 2nd Edition. Wiley, 1992.

Nelsen, R.B., 1999. An Introduction to Copulas. Springer, New York, ISBN 0-387-98623-5.

Press, W.H., Flannery, B.P., Teukolsky, S.A., and Vetterling, W.T., 1990. Numerical recipes: the art of scientific computing. New York: Cambridge University Press.