Statistical Analysis of a Fast Cellular Automata-based Sewer Flood Model

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Abstract

The Block Cellular Automata 1D (BCA1D), a fast conceptual model, is developed to simulate the flow in sewers. It uses simple empirical equations including a downstream boundary condition. Statistical analyses show a good agreement between BCA1D and benchmarks.

INTRODUCTION

Each year storm events occur causing flooding which damage the environment and vital infrastructure which negatively affects the economy. As the climate changes the frequency of storm events resulting in flooding is increasing. This increases the need for a fast and accurate sewer simulator to be used for real time control of sewer networks as well as to be used in dual-drainage models to allow the complete system to be modelled during a storm event and behaviour of the flooding to be predicted.

METHODOLOGY

Cellular Automata (CA) can simulate a process or event using a simple set of rules and the neighbourhood of cells. The neighbourhood of cells affecting the simulation varies between different CA models. The rules are applied and they change the state of the cell depending on its current condition and the state of those neighbouring.

BCA1D moves the flow in the form of blocks each time step. An integer number of blocks is moved and it is determined by one of the 2 transition rules. The neighbourhood of a manhole is comprised of all manholes connected to it. The movement of the blocks is shown in Figure 1.

\[ T = M_M A R^{2/3} \sqrt{S_f} \]  \hspace{1cm} (1)  
\[ T = M_M S_f^{0.54} \]  \hspace{1cm} (2)

SIMULATION

BCA1D was used to simulate the Stockbridge area of Keghley in Yorkshire along with the recognised benchmarks InfoWorks, SIPSON and SWMM. The models all simulated the network during a 3 hour storm event where 0.016m of rainfall occurs. A small base flow also enters at the most upstream manhole of each branch in the network.

STATISTICAL ANALYSIS

To determine if BCA1D is producing results in agreement with the benchmarks. The results where analysed using a number of statistical measures. The measures used were the Nash Sutcliffe Efficiency (NSE) (3), Root Mean Square Error (RMSE) (4), Peak Percentage error (PPE) and (5) the Index of Agreement (IoA). These where calculated for the discharge through each pipe.

\[ RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (P_i - O_i)^2} \]  \hspace{1cm} (3)  
\[ PPE = \frac{(\max(P_i) - \max(O_i))}{\max(P_i)} \times 100 \]  \hspace{1cm} (4)  
\[ IoA = 1 - \left( \frac{\sum_{i=1}^{N} (P_i - O_i)^2}{\sum_{i=1}^{N} (P_i - \bar{O})^2 + \sum_{i=1}^{N} (O_i - \bar{O})^2} \right) \]  \hspace{1cm} (5)  
\[ NSE = 1 - \frac{\sum_{i=1}^{N} (O_i - P_i)^2}{\sum_{i=1}^{N} (O_i - \bar{O})^2} \]  \hspace{1cm} (6)

DISCUSSIONS

For the RMSE and PPE the nearer the value is to 0 the better the results are. The RMSE is shown in Figure 2.

\[ \text{RMSE (m/s)} = \begin{cases} 0.03 & \text{SWMM5} \\ 0.02 & \text{SIPSON} \\ 0.01 & \text{INFORMS} \end{cases} \]

Figure 3. Average PPE

The PPE is overestimated on average when BCA1D is compared to the benchmarks SWMM5 and InfoWorks but underestimated when compared to SIPSON. Overall PPE is always within 0-10%

For the NSE and IoA, the nearer the value is to 1, the more similar are the hydrographs being compared. These are shown in Figure 4 and 5, respectively.

\[ \text{NSE} = \begin{cases} 1 & \text{SWMM5} \\ 0.8 & \text{SIPSON} \\ 0.6 & \text{INFORMS} \end{cases} \]

Figure 4. Average NSE

\[ \text{IoA} = \begin{cases} 1 & \text{SWMM5} \\ 0.8 & \text{SIPSON} \\ 0.6 & \text{INFORMS} \end{cases} \]

Figure 5. Average IoA

The NSE values between the benchmarks SWMM5 and InfoWorks and BCA1D are close to 1. The NSE between SIPSON and BCA1D however is slightly lower being close to 0.8. This is still high enough that they can be considered to be in agreement. The IoA values are approximately 1 when BCA1D is compared to all the benchmarks.

CONCLUSION

From this analysis BCA1D is clearly in agreement with all three benchmarks. Thus the model has sufficient accuracy to simulate sewer networks.

The highest PPE and RMSE as well as lowest IoA and NSE occur between BCA1D and SIPSON suggesting that BCA1D is in least agreement with this model. However, agreement between these models is still indicated by the measures.

The greatest level of agreement is suggested to occur between InfoWorks and BCA1D with only slightly less agreement between BCA1D and SWMM5.