Quantum Electrodynamics in the Presence of Moving Bodies

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Abstract—The vacuum state of the electromagnetic field close to a dielectric body is not the same in all reference frames. Here we show that this has some surprising consequences. For instance, if we attempt to quantize electromagnetism interacting with a uniformly moving medium (properly including effects of dispersion and dissipation), then the resulting Hamiltonian lacks a lower bound. Consequently, a detector placed close to a moving surface can be excited by the vacuum! It is shown that when the dielectric is allowed to change velocity then the Hamiltonian again becomes bounded from below, thus demonstrating that the source of the energy exciting the detector is the kinetic energy of the dielectric.

1. INTRODUCTION

It is shown in Landau and Lifshitz’ first book on statistical physics, that the state of maximum entropy for any system of moving macroscopic bodies is when those bodies are at relative rest [1]. However, it is not shown how the bodies get into this situation. At finite temperature, it seems that radiation pressure can do the job: each body emits an isotropic field in its rest frame which appears anisotropic when the body moves. This anisotropy causes an asymmetry in the radiation pressure between the bodies, which acts to bring the bodies to relative rest (for details see [2, 3]). However, the limiting behaviour of this interaction as the temperature of is reduced to zero is not so immediately clear, and a variety of results have been obtained using different approaches (e.g., [4–6]). To obtain a clear understanding of this limit requires an exact canonical treatment of quantum electrodynamics in the presence of moving dielectric bodies. Here we provide the details of such a treatment, illustrating some surprising features.

2. QUANTIZATION

It seems that a correct quantum mechanical description of a system must follow from an action principle. We look for an action that reproduces the classical macroscopic Maxwell equations for electromagnetism interacting with a moving dielectric, and properly includes the phenomena of dispersion and dissipation (the Kramers-Kröning relations). Huttner and Barnett [7] and Philbin [8] previously showed how to do this for stationary objects. In [9, 10] this was extended to objects in motion, where the Lagrangian density was found to be of the form,

\[ \mathcal{L} = \mathcal{L}_F + \mathcal{L}_{INT} + \mathcal{L}_R \]  

(1)

The first term is the Lagrangian density associated with the free electromagnetic field, \( \mathcal{L}_F = \epsilon_0 \left[ E^2 - c^2 B^2 \right]/2 \), and the final term,

\[ \mathcal{L}_R = \frac{1}{2} \int_0^\infty d\omega \left[ \left( \frac{\partial X_\omega(x,t)}{\partial t} + V \frac{\partial X_\omega(x,t)}{\partial x} \right)^2 - \omega^2 X_\omega(x,t)^2 \right] \]  

(2)

represents a field (a reservoir) of simple harmonic oscillators, \( X_\omega \) that mimic the linear response of the moving dielectric to the field (velocity \( V = V\hat{x} \)). The interaction between the two is characterized by an interaction Lagrangian,

\[ \mathcal{L}_{INT} = (E + V\hat{x} \times B) \cdot \int_0^\infty \sqrt{\frac{2\omega \text{Im} [\epsilon(x\parallel, \omega)]}{\pi}} X_\omega d\omega \]  

(3)

where for simplicity of presentation it has here been assumed that the dielectric is without magnetic response (\( \mu = 1 \)), and that the medium is only inhomogeneous in the plane normal to the direction of motion, with coordinates in this plane given by the two component vector, \( x\parallel \). The classical equations of motion that follow from (1) are the macroscopic Maxwell equations for a dielectric slab in uniform motion along \( \hat{x} \) so long as the permittivity, \( \epsilon \)— defined in the rest frame— satisfies the Kramers-Kröning relations. The Hamiltonian operator constructed from (1) is quadratic in
the fields and can thus be diagonalized into a continuum of normal modes. In [10], the required
transformation was found, and the diagonalized Hamiltonian is,

\[ H = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \int d^2x_\parallel \hbar \omega C_\omega^\dagger (k, x_\parallel) \cdot C_\omega (k, x_\parallel) \]  

(4)

where \( \omega_+ = \omega +Vk \). The \( \hat{C}_\omega \) operators appearing in (4) obey bosonic commutation relations
and are combinations of both the electromagnetic field and reservoir operators. This is a very
strange Hamiltonian, for the normal mode frequencies range from \(-\infty \) to \( \infty \), rather than beginning
at zero. However (4) seems to be correct, for when \( V = 0 \) it reduces to the result of [7, 8],
and it is therefore clear that the diagonalization is simply telling us that the normal modes are
Doppler shifted when the dielectric moves, which could have been guessed without following the
full quantization procedure. In [10, 11] it is shown that if the velocity of the dielectric is included
as a dynamical variable, then the Hamiltonian becomes bounded from below. One can therefore
interpret these negative frequency normal modes as an accounting device for the fact that once it
interacts with another system which is in relative motion, some external input is required to keep
a medium moving at a constant velocity.

3. A DETECTOR CLOSE TO A MOVING SURFACE

If the vacuum is defined as the zero particle state, \( \hat{C}_\omega |0\rangle = 0 \), then (4) tells us that it is not the
lowest energy state of the system of field plus dielectric. Therefore energy can be extracted from the
vacuum electromagnetic field outside of a moving dielectric when, for example, a two level system
at rest and in its ground state is brought close to the surface (see Figure 1). In [10] it is found that
the transition rate of this ‘detector’, \( \Gamma_{0\rightarrow1} \) is given by the expression,

\[ \Gamma_{0\rightarrow1} = \frac{2\omega^2}{\hbar} \int_{\omega/V}^{\infty} d\omega_0 \cdot \text{Im}[ G(-k, x_\parallel, x_\parallel, -\omega_0) ] \cdot d\omega_0 \frac{dk}{2\pi} \]  

(5)

where \( \omega \) is the transition frequency of the detector, \( x_\parallel |0\rangle \) is the position of the detector in the plane
normal to the velocity, \( d_0 \) characterises the strength of the interaction between field and detector,
and \( G \) is the electromagnetic Green function. It is shown in [10] that explicitly evaluating (5) in
terms of reflection coefficients gives expressions which are analogous to those of [5], which were
used to calculate the rate of dissipation of energy between two dielectric plates in relative motion.
Given that \( V \ll c \), (5) shows that the excitation of the detector is caused by a sum over waves
which exponentially decay from the surface into free space (e.g., plasmons, as considered in [5, 12]).
Indeed unless the frequency of the detector is low (e.g., GHz), then the transition rate will be
extremely small for reasonable distances from the dielectric, due to the fact that one is summing
over extremely tightly bound surface waves. Nevertheless it may be measurable if one can operate
at low enough temperatures and find a detector that operates at such low frequencies (perhaps flux
qubits would be suitable for this purpose [13]).

Figure 1: A dielectric slab moves up the x-axis at \( V = V\hat{x} \). If a detector is positioned a distance \( z_0 \) away
from the surface, with both field and detector initially in the ground state.
4. CONCLUSION

An outline has been given for the procedure necessary to quantize the electromagnetic field in the presence of moving bodies. It was found that if one fixes the velocity of a dielectric body by some external means, then the system of field plus dielectric must be described using a Hamiltonian that is not bounded from below. A detector placed outside such a moving surface can be excited by the vacuum field at a rate given by a sum over all waves bound to the surface of the medium.

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REFERENCES