Combining Tidal Energy Yield Uncertainties

Sunny Shah#1, Hannah Buckland*2, Philipp R. Thies+3, Tom Bruce#4

#Industrial Doctorate Centre for Offshore Renewable Energy, University of Edinburgh, EH9 3JL, UK
1 S.Shah@ed.ac.uk
4 T.Bruce@ed.ac.uk

*Black & Veatch Ltd., 19 St Vincent Pl, Glasgow G1 2DT, UK
2 BucklandH@bv.com

+College of Engineering, Mathematics and Physical Sciences, University of Exeter, TR10 9FE, UK
3 P.R.Thies@exeter.ac.uk

Abstract— A robust understanding of the uncertainty in a yield estimate for a tidal energy project is a key investor requirement and a common barrier to the commercialisation of the nascent sector. The Root Sum Squared (RSS) method is commonly used to combine the uncertainty in site resource (i.e. velocity, m/s) with the uncertainty in plant performance and losses (i.e. energy, GWh). The validity of the assumptions underlying RSS has been questioned in literature, particularly for early stage projects. RSS assumes that all uncertainties are independent and normally distributed, that the relation between yield and velocity is linear for small variations and that the combined yield uncertainty is also normally distributed. Monte Carlo Analysis (MCA) is a competing method for uncertainty analysis which is not limited by the same assumptions. This study quantitatively compares the combined yield uncertainty for 4 realistic test cases derived using the two different methods with the same input uncertainty distributions. An excellent agreement is found for cases where the uncertainties are relatively small and where the site resource is low relative to the turbine rated velocity. Some divergence in results is shown for projects with higher uncertainties but it is noted that these projects are likely to be early stage with a higher tolerance for inaccuracy in the uncertainty estimate. RSS predicts a higher P90 yield than MCA but it is prudent to adopt the more conservative view. The point at which the divergence occurs is hard to define as it is a complex function of site resource, turbine rated velocity and project uncertainties. As such, the confidence in RSS results is somewhat compromised, particularly for early stage projects.

***Keywords*— Tidal Energy, Uncertainty Analysis, Annual Energy Production, Root Sum Squared, Monte Carlo Analysis**

I. Introduction

A key barrier to the commercialisation of the nascent tidal energy sector is the associated high investment risk. A significant risk component, affecting all tidal energy projects, is the uncertainty in the pre-construction resource and yield estimate. A robust understanding of those uncertainties will increase investor confidence [1].

It is important for the investor to know not only the most likely (P50) value for a project's yield (and therefore revenue) but also the conservative case (P90) in order to appraise the risk involved in the investment. A project with a lower P50 yield may in fact be more financeable than a similar project with a higher P50 yield if the P90/P50 ratio is higher, i.e. smaller likelihood of large deviation from the P50 (Figure 1).

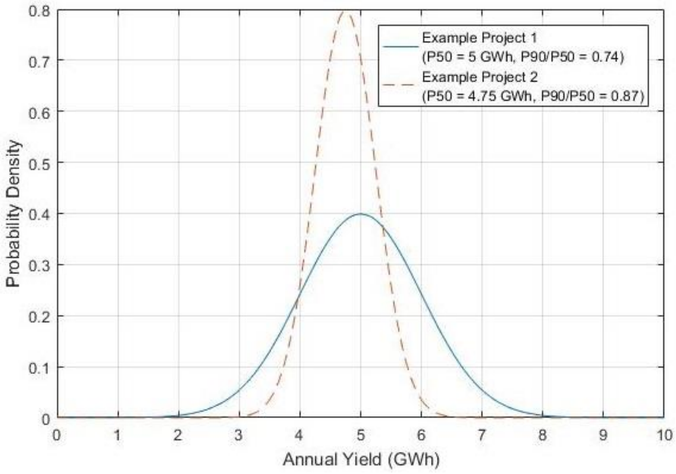

Figure 1: Annual yield uncertainty distributions for example projects

The distribution of annual yield uncertainty depends on:

1) Characteristics of the underlying individual uncertainties (magnitude, distribution shape and correlation)
2) Method used to combine the individual uncertainties

A standardised framework for the categorisation and quantification of marine energy yield uncertainties is proposed in [2] and [3] respectively. As such, the focus of this paper is on the methods used to combine the uncertainties once their characteristics have been quantified. The uncertainty categories are summarised in Table 1 as well as the values for case study projects described in *Section III.C*.

Reference [3] recommends using the Root Sum Squared (RSS) method for combining the individual uncertainties. However the validity of the assumptions implicit in such local methods when applied to the context of tidal energy has been questioned [1]. *Section II* discusses these assumptions and how a global method such as Monte Carlo Analysis (MCA) may be more suitable.

Confidence in the process used to combine the yield uncertainties is as important as confidence in the individual uncertainties themselves. The aim of this paper is to quantitatively assess the two competing methods for combining uncertainties to identify their benefits and limitations.

Table 1: *Taxonomy of yield uncertainty categories and uncertainty values used in this study* [2] [3]

Uncertainty Category	Sub-Category	Standard Uncertainty (%)	
		Project A1	Project A2
1. Site Measurement	1a. Instrument Accuracy	1%	1%
	1b. Measurement Interference	0%	0%
	1c. Short-term site data synthesis	0%	0%
	1d. Data quality and metadata	0%	0%
2. Temporal Variation	2a. Historic resource estimation	2%	2%
	2b. Future resource variability	0.2%	0.2%
	2c. Climate change	0%	0%
3. Spatial Variation	3a. Model inputs	0%	0%
	3b. Horizontal and vertical extrapolation	0%	11.2%
	3c. Other uncertainty	0%	0%
4. Plant Performance and Losses	4a. Availability	0%	1.2%
	4b. Resource-array interactions	0%	2.3%
	4c. Marine energy convertor performance	0%	5.5 %
	4d. Electrical losses	1%	1%
	4e. Performance degradation	0%	0%
	4f. Curtailment	0%	0%
	4g. Other losses	0%	0%
5. Other	5a. Other	0%	0%

Section II provides an overview of the theory, assumptions and limitations of RSS and MCA methods in the context of yield uncertainty analysis. The test case projects are defined in *Section III* as well as the common inputs such as resource and turbine data. The results from the RSS and MCA models using common input data are compared in *Section IV* and *V*, followed by a discussion and conclusion.

II. Theory

A. Root Sum Squared (RSS) method

The combined standard uncertainty in a variable can be expressed as a function of the individual uncertainties in the variables upon which it is dependent. If the individual uncertainties are independent and uncorrelated [4]:

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) \tag{1}$$

where f is the function between the individual variables x_i and the final variable y, $u(x_i)$ is the standard uncertainty in the individual variables and u_c is the combined standard uncertainty. The partial derivatives, called the sensitivity coefficients describe the sensitivity of y to changes in x_i.

Reference [1] presents a methodology for the application of the above generic equation to the context of tidal yield uncertainty calculation. A summary of the process is presented below for completeness.

$$u_c^2 = c_v^2 u_R^2 + u_4^2 \tag{2}$$

where u_c is the combined standard uncertainty in the annual yield and c_v is the sensitivity coefficient between mean velocity and yield. The standard uncertainty in the resource and plant performance and losses, u_R and u_4 respectively are:

$$u_R^2 = u_{1a}^2 + u_{1b}^2 + \cdots + u_{3b}^2 + u_{3c}^2 \tag{3}$$

$$u_4^2 = u_{4a}^2 + u_{4b}^2 + \cdots + u_{4g}^2 + u_{5a}^2 \tag{4}$$

Assuming that the relation between change in velocity (i.e. $dv = v_{pert} - v$) and the resultant change in annual yield (i.e. $dE = E_{pert} - E$) is linear for small variations in mean velocity, the c_v can be derived numerically:

$$c_v = \frac{(E_{pert}/E) - 1}{(v_{pert}/v) - 1} \tag{5}$$

where v_{pert} is a perturbed mean velocity and E_{pert} is the corresponding gross annual yield. The perturbed mean velocity is derived by multiplying the measured ADCP timeseries by a small perturbation factor. A value in the range of -3% to 3% is initially proposed in [1] and a perturbation of -0.5% is then shown to give results closest to the analytical solution for one particular case. Reference [3] implies the use of an average c_v from perturbations of ±5%.

As per the Central Limit Theorem (CLT), it may be assumed that the uncertainty in annual yield is normally distributed. The P90 yield can then be calculated from the P50 yield and combined standard uncertainty, u_c, using a coverage factor of 1.282[1]:

$$E_{P90} = E_{P50} - 1.282u_c \tag{6}$$

The implications of the major assumptions in this process are:

1) *Gaussian and independent uncertainties:* any skewed or correlated behaviour (e.g. increase in turbulence may have a correlated effect on Table 1 uncertainties 4a, 4b and 4c) is not modelled. The RSS method can account for correlated uncertainties but the derivation of c_v is more difficult.

2) *Linear approximation:* this assumption does not hold its validity when the uncertainty in the resource is relatively large. The deviation from assumed linearity will result in an inaccurate c_v calculation and in fact the value derived will be very dependent on the perturbation level used.
3) $u_R = f(u_v)$ *only:* implicit in Equations (2) and (3) is the assumption that resource uncertainty is wholly dependent on the uncertainty in velocity [1]. The uncertainty resulting from factors such as turbulence and flow direction is captured in the plant performance uncertainties.
4) *Normally distributed* u_c*:* the assumption of normally distributed yield uncertainty may be invalid if the individual uncertainties are not independent, uncorrelated and normally distributed [5]. The assumption is also invalidated in cases of significant non-linearity or if a few uncertainty components dominate the combined uncertainty

B. *Monte Carlo Analysis (MCA)*

Monte Carlo Analysis is a probabilistic method used to solve complex deterministic problems which are difficult to solve analytically. The function relating the input variables to the output parameter is solved repeatedly, with each repetition being seeded by a random value from each input distribution in Table 1 [6]. The MCA process in the context of tidal energy yield uncertainty assessment is shown in Figure 3.

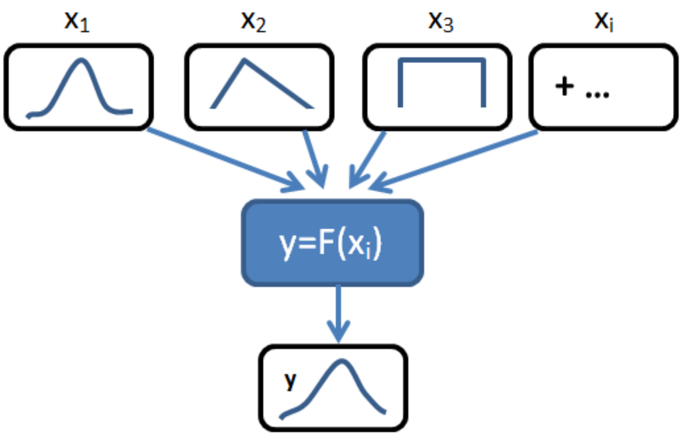

Figure 2: Generic Monte Carlo Analysis process flowchart

MCA overcomes each of the limitations with RSS highlighted earlier. Input uncertainty distributions that are non-Gaussian and correlated can be sampled easily. The power function is no longer assumed to be linear and is solved analytically (Equation 9) for each simulation. Therefore any non-linearity will naturally be accounted for and propagated through to the combined yield uncertainty distribution. Therefore large uncertainties can be processed validly. The distribution of the combined yield uncertainty is no longer assumed to be Gaussian as the individual uncertainty distributions are numerically propagated to calculate the combined uncertainty. However, MCA is more computationally onerous than RSS and the computational burden increases non-linearly with the number of simulations carried out.

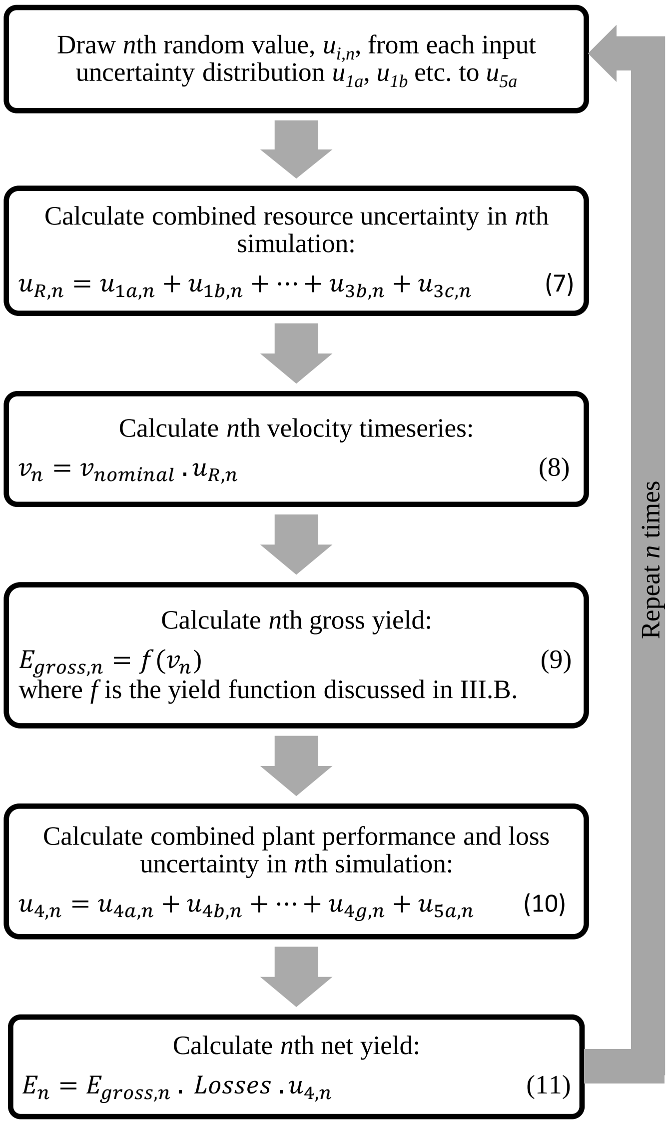

Figure 3: Monte Carlo Analysis applied to yield uncertainty analysis

III. Common Model Inputs

A. *ADCP Data*

Velocity timeseries data recorded by Acoustic Doppler Current Profilers (ADCP) at two sites in the North of Scotland are used in this study (Figure 4). The characteristics of the two datasets are summarised in Table 2.

Table 2: ADCP dataset specification

Dataset	Measurement Period	Total Days Recorded	Ensemble Period (mins)	Pings per Ensemble
ADCP 1	17/02/13 to 23/03/13	34	10	1200
ADCP 2	19/03/15 to 21/04/15	33	20	24

Figure 4: Location of the two ADCPs considered in this study

Both datasets are post processed by removing data from bins close to the seabed and sea surface to exclude data that is likely to be contaminated due to sidelobe ringing and surface wave interference [7]. The data in the remaining vertical bins is averaged to derive a depth averaged velocity timeseries for each site.

B. Annual Yield Calculation

A generic power curve representative of a variable pitch 1.3 MW turbine with a rated velocity of 2.7m/s is used throughout the study [8]. Figure 5 and Figure 6 show the power curve used in this analysis in relation to the prevalent resource at the two study sites. It is evident that the 'ADCP 1' experiences considerably higher velocities for a higher proportion of time. The velocity is above the turbine's rated velocity for a significantly larger period of time at 'ADCP 1' site.

Harmonic analysis using UTide [9] is used to synthesise an annual velocity timeseries for 2017 with a timestep of 10 minutes from ADCP1 and ADCP2 measured timeseries.

The yield is calculated using the time domain expression:

$$E_{gross} = \frac{t}{n}\sum_{i=1}^{n} f(v_i) \tag{12}$$

where E_{gross} is the gross yield, t is the hours in a year, n is the number of samples in the timeseries and $f(v_i)$ is the function defining power output for a given velocity v_i, as shown in Figure 5 and Figure 6.

Note that the power curve is defined in bin intervals of 0.1m/s. Linear interpolation is used to derive the power output for velocities between the bin edges. This approach is chosen over the frequency domain method of bins (Equation 13) to avoid introduction of numerical errors to the analysis which are not related to the topic of study.

$$E_{gross} = t\sum_{i=j}^{bins} f(v_j).p_j \tag{13}$$

where *bins* is the number of bins in the power curve and p_j is probability of velocity being in the jth bin.

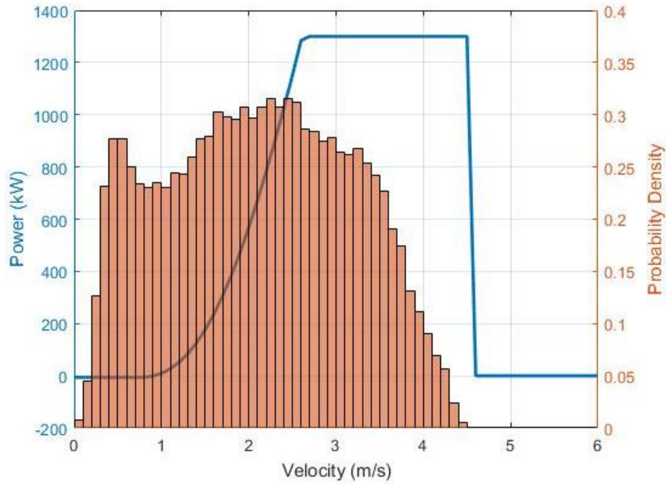

Figure 5: Power curve compared to 'ADCP 1' velocity frequency distribution

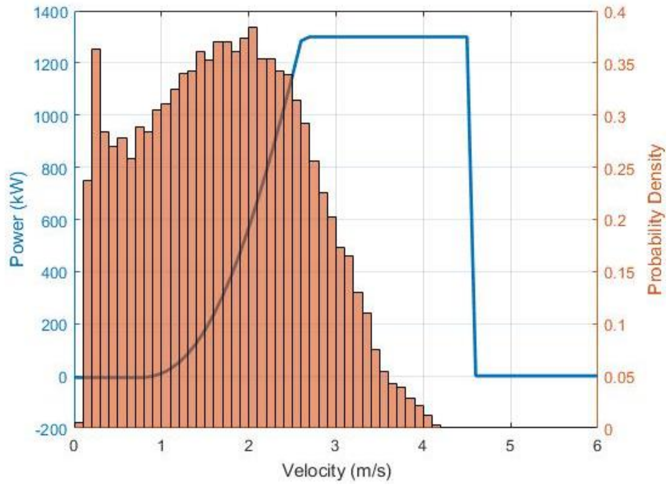

Figure 6: Power curve compared to 'ADCP 2' velocity frequency distribution

An arbitrary loss factor of 20% is applied to the gross yield to account for performance and availability losses.

C. Uncertainties

The yield uncertainty categories proposed in [2] are used in this study because it can be considered to be the industry standard. Reference [3] proposes representative values of standard uncertainties for each of the categories for a number of realistic, but hypothetical projects. The uncertainties proposed for Reference project A1 and A2 in [3] are used in this study and listed in Table 1. These projects are chosen as they represent realistic low and high uncertainty projects whilst being comparable to the ADCP data available for this study.

Reference Project A1 is a single turbine project with a 28 day ADCP measurement at the turbine location. The availability and performance is assumed to be warranted and therefore poses no additional uncertainty to the project yield. Project A2 is a 5 turbine project which uses a 2D hydrodynamic model to spatially extrapolate the resource from the 28 day ADCP measurement site. Only a minimum availability and performance level is warranted and therefore uncertainty on the plant performance is present. Both projects use harmonic analysis to extrapolate the measured resource temporally. It is assumed that all uncertainties are normally distributed and independent. Detailed justification of standard uncertainties listed in Table 1 can be found in [3] but is not described further as quantifying the uncertainties is not the focus here.

D. *Study test cases*

Therefore there are 4 projects being considered. Their defining characteristics in the context of yield uncertainty analysis can be summarised as:

Table 3: Summary of test case characteristics

	High uncertainties	**Low uncertainties**
High mean velocity	ADCP1 Project A2	ADCP1 Project A1
Low mean velocity	ADCP2 Project A2	ADCP2 Project A1

IV. Root Sum Squared (RSS) Model

A. *Combined Resource Uncertainty (u_R) and Performance and Loss Uncertainty (u_4)*

Equations (3) and (4) are used with the corresponding standard uncertainties in Table 1 to calculate the u_R and u_4 for the two projects:

Table 4: u_R and u_4 *values for study projects*

	Project A1	**Project A2**
$u_{R\,(\%)}$	2.24	11.42
$u_{4\,(\%)}$	1.00	6.16

B. *Sensitivity Coefficient (c_v)*

Sensitivity coefficients are calculated for a range of perturbations from -5% to 5% in increments of 0.5% to test the stability of the linearity assumption. Figure 7 shows that the linear assumption appears valid for ADCP2 but a slight deviation from the general linear trend is observed for positive perturbations greater than 2%. This can be explained as the linearity assumption is generally valid for velocities corresponding to the region in the power curve between the cut-in and rated velocities. The rated power and cut-out regions introduce severe non-linearity in the *dv-dE* relation. The ADCP1 site resource has a relatively large frequency of occurrence in the rated region in the nominal case (Figure 5). The larger positive perturbations push enough of the velocity timeseries into the rated region to have an apparent overall non-linear effect in the annual yield.

The sensitivity coefficient, c_v, is effectively the non-dimensionalised gradient of the lines in Figure 7 so any non-linearity in the data will result in instability in the derived c_v. Considering the linear line of best fit gradient as the most accurate derivation of c_v, Figure 8 shows the stability of the c_v dependent on the perturbation level used. It is clear that the value of c_v is extremely sensitivity to even a small non-linearity. This is demonstrated by a variation of up to ±4% in the c_v for ADCP2 despite the apparently linear relation observed in Figure 7. Similarly, the slight non-linearity for positive perturbations on ADCP1 observed in Figure 7 is exaggerated results in up to 10% difference from the line of best fit c_v. The average c_v of a positive and negative perturbation at a given level (e.g. average of ±5% c_v) provides a more stable value which is also closer to the line of best fit c_v.

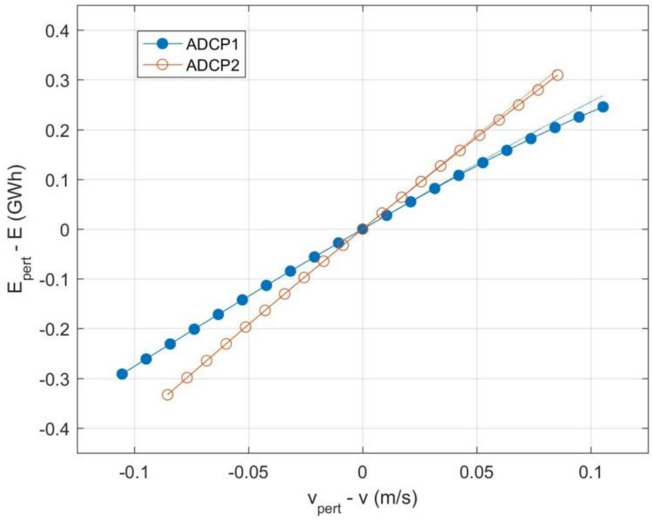

Figure 7: Variation in annual yield due to small perturbations in mean velocity (data shows perturbations from -5% to 5% in 0.5% increments)

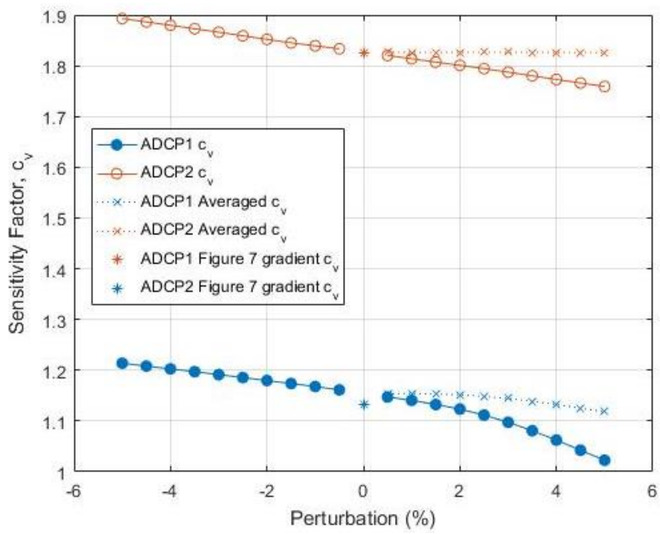

Figure 8: Variation in sensitivity coefficient with change in perturbation level

C. Combined and Expanded Yield Uncertainty

The c_v values calculated earlier can be used to calculate the P90 yield using Equation (13) and plot the combined yield uncertainty distribution. Figure 9 shows the P90/P50 ratios resulting for the 4 test cases resulting from the two ADCP sites and two projects being studied. The deviation in ADCP1 c_v for large positive perturbations results in a corresponding deviation in the P90/P50 ratios. However, this deviation is much more significant for project A2 because it has a much larger combined resource uncertainty, u_r, and the P90 is a function of the product of u_r and c_v. The small linear instability in c_v for ADCP2 is notable for project A2 but using the average c_v values gives a stable P90/P50 regardless for the perturbation level chosen.

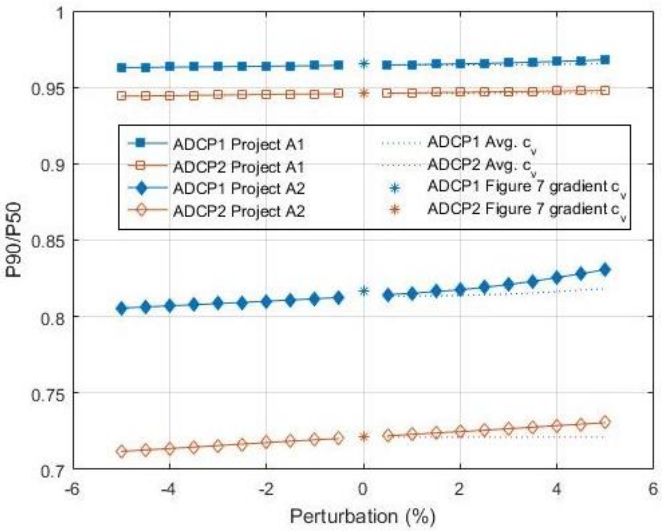

Figure 9: Variation on P90/P50 ratio dependent on perturbation level

Figure 10 shows the expanded yield uncertainty for ADCP1 Project A2 which has the worst case combination of the most instable c_v and largest individual uncertainties resulting in the biggest variance in P90/P50 depending on the perturbation level chosen. The difference in the exceedance probabilities for two extreme perturbations can be considered to be small.

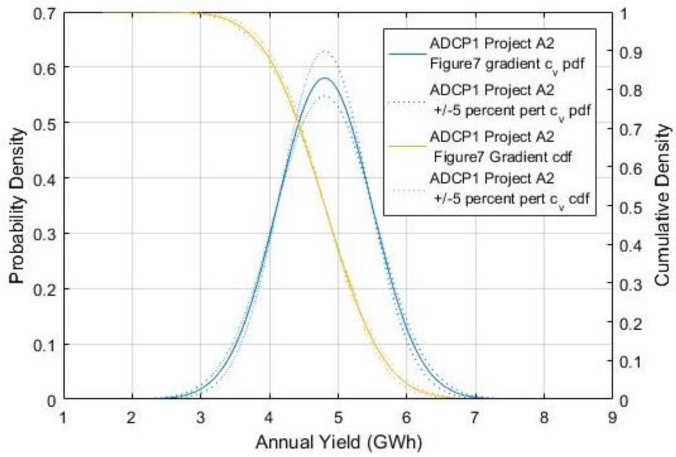

Figure 10: Probability and cumulative distribution functions for ADCP1 Project A2 combined annual yield uncertainty normalised per turbine

V. Monte Carlo Analysis (MCA) Model

All random samples for MCA are drawn using the Mersenne Twister random number generator with a fixed seed of '1' to allow reproducibility. A total of 10,000 simulations are carried out as it was found to provide a stable solution for a reasonable computational time of less than 2 minutes on a standard desktop PC using MathWorks MATLAB software.

A. Combined Resource Uncertainty (u_R) and Performance and Loss Uncertainty (u_4)

Equations 7 and 10 are used with the corresponding standard uncertainties in Table 1 to calculate the empirical distributions and standard deviations for u_R and u_4 for the two projects (Table 5). They are similar but not identical to the corresponding values in Table 4.

Table 5: u_R and u_4 *values for study projects*

	Project A1	Project A2
$u_{R\,(\%)}$	2.27	11.47
$u_{4\,(\%)}$	1.00	6.18

B. Combined Yield Uncertainty Distributions

The process described in Figure 3 is used to calculate the uncertainty distribution of annual yield for each of the test cases. The MCA distribution for Project A1 matches very well with the RSS results (Figure 11). A less good fit is observed in the comparison for Project A2. This is largely because the RSS assumption of combined uncertainty distribution being Gaussian is no longer valid due to the large uncertainties. The distribution is skewed towards the minimum as the turbine power output is not proportional to velocity in the rated region and large deviations from the mean velocity push sufficient velocities into this region to have a noticeable effect. The skew is more pronounced for ADCP1 which has a higher mean velocity in the nominal case. The effect of this on the P90/P50 ratio is shown in Table 6. Whilst there is excellent agreement for Project A1, approximately 2% difference is observed for the higher uncertainty Project A2.

Table 6: Comparison of P90/P50 ratios from RSS and MCA results

	RSS P90/P50	MCA P90/P50	Diff. (%)
ADCP1 Project A1	0.965	0.964	0.10
ADCP1 Project A2	0.816	0.801	1.87
ADCP2 Project A1	0.946	0.945	0.11
ADCP2 Project A2	0.721	0.706	2.12

VI. Discussion

Some dependence of c_v on the arbitrary choice of perturbation level has been shown. Whilst c_v is extremely sensitivity to small non-linearity in the *dE/dv* function, the effect on final P90/P50 is shown to be relatively small. Nonetheless, using the average of a negative and positive perturbation will provide a better result and such an approach is recommended if using RSS. As noted in [10], a turbine with a lower rated velocity will yield a lower c_v (and therefore u_c) than a higher rated turbine for a given resource. However, it will also increase the instability of c_v with perturbation level as shown in Figure 8. This is introducing an additional

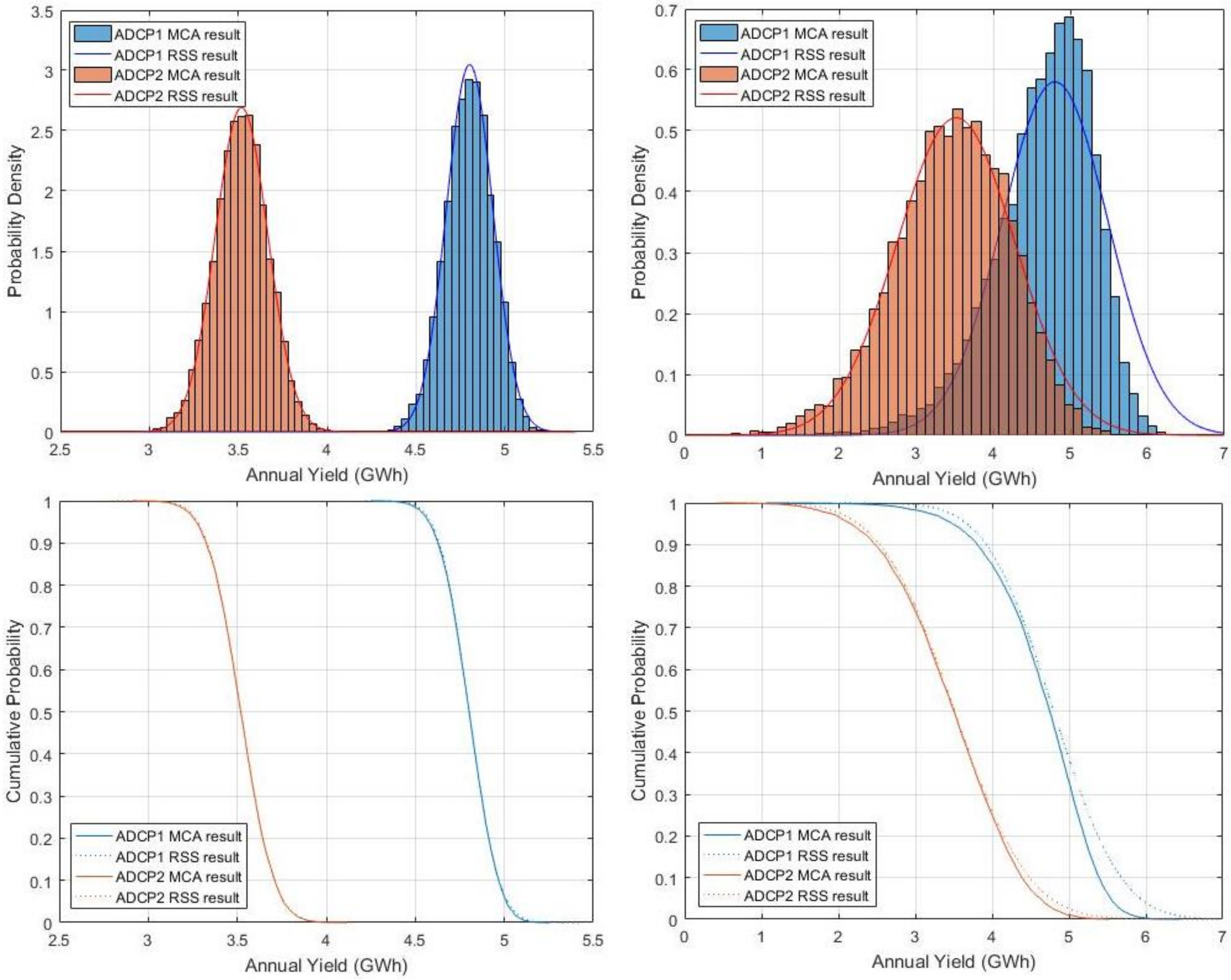

Figure 11: Annual yield distribution (top) and corresponding cumulative distribution functions (bottom) for resulting from RSS and MCA methods for Project A1 (left) and Project A2 (right). Note that the yield is normalised per turbine for Project A2 and RSS results are calculated using c_v derived from the gradient of lines in Figure 7

element of uncertainty to the results and must be included in the consideration of matching a turbine to a resource with respect to the uncertainty analysis.

MCA is shown to provide noticeably different results to RSS when the assumption of normally distributed u_c is not valid (and to a small extent also when the assumption of linearity is not valid). It is noted that whilst the difference is significant, a project with such a high uncertainty is likely to be early stage and therefore have a higher tolerance for inaccuracy in the risk metrics. However, it is noted that the RSS results predict a more optimistic P90 yield but it is prudent to adopt the more conservative view.

There is no easily defined point at which the RSS assumptions are invalidated as it is a complex function of the site velocity distribution, turbine power curve and magnitude of individual project uncertainties. As a general rule, the assumptions are pushed to the limit when a significant portion of the velocity frequency distribution occurs within and above the rated region of the power curve, and when the uncertainties are relatively high. In other words, RSS may be invalid for projects using turbines with a low rated velocity relative to the site resource and/or for early stage projects which have poor quality measurements, modelling and performance data. Nonetheless, the RSS results are shown to be approximately correct in the worst case tested here, which is representative of a realistic early stage project with low rated velocity turbine.

The RSS process can easily be carried out in a standard spreadsheet and a purpose built tool is freely available for this purpose [11]. MCA currently requires the analyst to build the model, although the publication of an equivalent MCA tool is quite possible. Nonetheless, licenses for specialist software and/or user expertise will be required and the computational time required is also higher.

VII. CONCLUSIONS AND FUTURE WORK

1) *Conclusions*

The known limitations of RSS are tested and compared to equivalent results from MCA, which is not restricted by the same limitations. Whilst good agreement is found in projects with relatively low uncertainty and site resource relative to turbine rated velocity, divergence in results is seen for the high resource, high uncertainty test case. Whilst there may be a higher tolerance for small inaccuracies in the yield estimate for a high uncertainty project, it is noted that inaccuracy results in a more optimistic P90 yield. It is difficult to define the limit at which the difference between RSS and MCA becomes significant. As such, the confidence in uncertainty estimates derived using RSS may be undermined. Confidence in the uncertainty analysis is key for investors and the use of MCA provides higher confidence, particularly for early stage projects where the RSS assumptions are stretched.

2) *Future Work*

The current work compares identical inputs and therefore limits MCA to the RSS assumption of normally distributed and independent input uncertainties. A study with some skewed and correlated uncertainty distributions will likely show further divergence in results. Analysing more projects with a large range in uncertainty and different power curves will provide more data points to more accurately define the transition region where MCA results become more accurate than RSS. The MCA method will be extended to include wave energy projects which have two parameters affecting the combined uncertainty rather than one as is the case for tidal energy.

ACKNOWLEDGMENT

The authors would like to thank the Energy Technology Institute and RCUK Energy programme for funding this research as part of the IDCORE programme (EP/J500847/1). ADCP data provided by the European Marine Energy Centre (EMEC) and by Prof. Lars Johanning and Mr. Jon Hardwick of the Marine Renewable Energy research group, University of Exeter are gratefully acknowledged. This work builds on [1] and therefore the indirect contribution of M. Cathain et al is acknowledged.

REFERENCES

[1] M. Ó. Catháin, T. Carnus, and C. Stock-Williams, "Wave & Tidal Energy Production Estimates : Converting Resource Uncertainties into Energy Uncertainties," in *European Wave and Tidal Energy Conference*, 2013, vol. 4.

[2] Offshore Renewable Energy Catapult, "Framework for the Categorisation of Losses and Uncertainty," no. July, 2015.

[3] Offshore Renewable Energy Catapult, "Wave and Tidal Energy Yield Uncertainty Reference Document," no. July, 2015.

[4] JCGM, "Evaluation of measurement data — Guide to the expression of uncertainty in measurement," no. September, 2008.

[5] Alan Derrick, "Uncertainty - The Classical Approach," in *AWEA Windpower Conference*, 2009.

[6] S. Raychaudhuri, "Introduction to monte carlo simulation," *Simul. Conf. 2008. WSC 2008.*, pp. 91–100, 2008.

[7] Teledyne RD Instruments, "Acoustic Doppler Current Profiler Principles of Operation A Practical Primer," vol. 00, no. January, 2011.

[8] A. S. Iyer, "IDCORE Marine Renewable Resource Assessment Course - Tidal Resource Assessment Coursework Prescription," p. 380, 2013.

[9] D. L. Codiga, "Unified Tidal Analysis and Prediction Using the UTide Matlab Functions," 2011.

[10] M. Ó. Catháin, "Towards increased confidence in wave and tidal energy production estimates," in *International Conferences on Ocean Energy*, 2012.

[11] Offshore Renewable Energy Catapult, "Wave and tidal energy yield uncertainty management tool." 2015.