Determining the Photographic Guide Number of an LED

*White LEDs are increasingly being used as sources of illumination for cave photography, both as modelling lamps and as alternatives to flashguns. Clearly, a simple way to determine the effectiveness of such illumination – the 'guide number' – is to calibrate the light source in a series of practical tests. However, we can also determine the guide number theoretically. **David Gibson** goes through the steps in this exercise, which brings together several aspects of physics and photometry that the reader may have forgotten.*

The easiest way of determining the guide number of an LED is probably to calibrate it by taking a series of photos with different exposures; but a theoretical exercise is useful for two reasons. Firstly, it puts the relationship on a firmer footing – we no longer have just a subjective impression, but the ability to apply the knowledge to future designs. Secondly, it is an interesting application of some principles of photometry, which might be educational for some people.

The calculation proceeds in stages, as we convert each physical measurement to the next one in the chain, starting with the electrical power of the LED and ending with the film speed. In summary, we must derive the following relationships...

1. Electrical power (W) to light power (lm)
2. Light power to beam intensity (cd)
3. Beam intensity to illumination (lux)
4. Scene illumination to luminance (cd/m^2)
5. Luminance to film illumination (lux)
6. Film illumination to film exposure (lux s)
7. Film exposure to film speed (ISO/ASA)

Stage 1 – Electrical Power to Light Power (lm)

Measuring light power is difficult and the only quantity that we can easily measure 'at home' is the power consumed by the LED. This by itself is not particularly helpful, and so we must hope that the LED has a specified output power in lumens.

The lumen is simply a measurement of power, like the watt, but it is weighted to the response of a typical human eye. At the eye's most sensitive wavelength, 555 nm (in the green region of the spectrum), a radiated power of 1 W corresponds to a luminous flux of 683 lm. In other words one lumen represents about 1.46 mW at this wavelength. (This is under bright light or *photopic* conditions. Dark-adapted or *scotopic* vision is said to peak at 1700 lm/W at 507 nm).

Writing the electrical power as P and the luminous flux as Φ, the relationship is

$$\Phi = \eta P \qquad (1)$$

where η is the ***luminous efficacy***.

The figure of 683 lm/W provides an upper limit on the luminous efficacy [1] of a monochromatic light source. For a non-monochromatic source, the changing sensitivity of the eye has to be taken into account. This brings the theoretical limit down to 260–300 lm/W, or about 38–44% of the monochromatic limit. [2], [3].

The most efficient LEDs commercially available have an efficacy of around 150 lm/W, although in February 2013, LED Magazine reported an announcement from the LED manufacturer Cree [4] that they had achieved 276 lm/W in a research and development environment. No significant further improvement would seem to be possible without an alteration of the colour temperature to something less 'white'. At 150 lm/W, some 22% of the electrical power is converted into visible light and the rest is converted into heat. This efficiency is higher than the best fluorescent and metal halide lamps and, of course, one to two hundred times more efficient than a low-voltage non-halogen tungsten-filament torch bulb!

Stage 2 – Light Power to Beam Intensity (cd)

The power of a light source, quoted in lumens, is not of direct relevance to us, for photographic purposes, for which we are more interested in its 'brightness'. Brightness is only a vague term; what we really mean is ***luminous intensity***, which is a measure of the degree of concentration of the light beam. Clearly if a light source is concentrated into a narrow beam, it will result in a brighter illumination, but over a smaller area. Luminous intensity is the power in lumens divided by the beam angle in steradians; measured in candelas (1 cd ≡ 1 lm/sr). The steradian (sr) is the solid (i.e. 3D) analogue of the radian. The relationship between solid angle Ω and conical angle θ is (see Box; Solid Angle)...

$$\Omega = 2\pi\left(1 - \cos\tfrac{1}{2}\theta\right) \qquad (2)$$

Using an expansion for $\cos\theta$ we obtain

$$\Omega \approx 0.00024\,\theta_d^{\,2} \qquad (3)$$

where the conical angle θ_d is in degrees. This approximation is good up to around 120°.

The salient point about the intensity is that it is unlikely to be constant over the beam width. Commonly, the beam width is measured at the point where the intensity drops to half of its value at the centre of the beam. However, this does not tell us how much power is within this beam, nor does it tell us the intensity at the centre of the beam.

Photographers have traditionally measured the on-axis intensity, referred to as the centre-beam candlepower or just the ***beam candlepower***, although it is now recognised that a mean, integrated figure is more useful.

If the intensity of the LED is not quoted, we must calculate it. Two methods are, 1) if we know that a reflector or lens is going to concentrate the power into a beam of solid angle Ω_b then, if the beam is uniform, we can write the centre-beam intensity as

$$I_b = R\frac{\Phi}{\Omega_b} \qquad (4)$$

where R is a coefficient representing the efficiency of the reflector or lens; or 2) if we have a datasheet that gives the spatial radiation pattern we can do a 'manual' integration by counting the squares under the graph, to give the centre-beam intensity as

$$I_b = \frac{\Delta A}{A}\frac{\Phi}{\Delta\Omega} \qquad (5)$$

where A is the total area under the graph, and ΔA is the area corresponding to a narrow beam of width $\Delta\Omega$.

Stage 3 – Beam Intensity to Scene Illumination (lux)

At a distance d, the beam of solid angle Ω will cover an area of Ωd^2 and we can define the scene illumination E as $\Phi/\Omega d^2$ lumen/m^2. This is more simply expressed in terms of the beam intensity I as

$$E = \frac{I}{d^2} \qquad (6)$$

Thus, E is the power flux density or ***illumination*** in lux (1 lux ≡ 1 lm/m^2) due to a source of intensity I cd at a distance d from a surface. If you are not used to photometric units, this may strike you as odd – surely if you divide candelas by square metres you should get cd/m^2 not lm/m^2 ? The confusion arises because angles are dimensionless.

Solid Angle

If we consider an arc of circle, with its two radius lines (length r) subtending an angle of α radians, the length of the circumference is simply αr and so there are 2π radians in a circle. Similarly, a solid angle of Ω steradians delimits a section of the surface area of a sphere of area Ωr^2 and so there are 4π steradians in a sphere.

For a sphere of radius r the surface area of a circular cap of height h (see figure) is given by $2\pi rh$. This is a standard result but can be proved with a little integration, if you need the exercise. Let this cap subtend a solid angle of Ω sr, such that

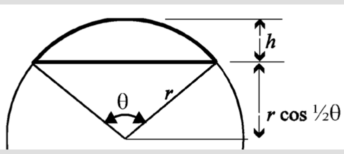
$$\Omega r^2 = 2\pi rh$$

If the cone bounding the cap has full-angle θ then, referring to the figure, we can write $r = r\cos½\theta + h$. From these two equations we can derive $\Omega = 2\pi(1 - \cos½\theta)$.

A further result can be derived by noting that for $\theta \ll 1$, $\cos\theta \approx 1 - ½\theta^2$. If, at the same time, we convert from radians to degrees, then Ω can be written in terms of a cone of full-angle θ_d degrees, as

$$\Omega \approx 0.000\,24\,\theta_d^{\,2}.$$

This is a reasonable approximation up to, say, $\theta_d = 120°$ for which it is about 9% low.



Stage 4 – Scene Illumination to Luminance (cd/m^2)

We now know the flux density of the light illuminating the photographic scene. We need to derive the reflected light power that is captured by the camera lens. This introduces one of the most difficult units to understand, that of luminance.

Obviously some parts of the scene will reflect more light than others. If the fraction of power reflected is ρ then we can define the ***luminous emittance*** as

$$M = \rho E \qquad (7)$$

Luminous emittance has the unit of lm/m^2 which, traditionally, is *not* referred to as the lux, which is reserved for illumination.

Simply knowing the luminous emittance is not sufficient, as we need to know how the flux is distributed spatially. The salient point is that most parts of the scene will give rise to a ***diffuse reflection***. For each point on the surface, a ray of incident light will give rise to a broad cone of reflected light. If this process is perfect then the surface will appear equally bright from all directions. This is a ***perfectly diffuse*** or ***Lambertian surface***.

Because we have a cone of light arising from the surface, the property to describe it is the candela. But, because the reflected light arises from an infinitesimal point, this property becomes the candela per square metre when used to describe an extended surface. This is the SI unit of ***luminance***. For a perfectly diffuse surface the luminance is related to the luminous emittance by $L = M/\pi$ (see Box; Luminance) and if the scene reflectance is the fraction ρ then, from (7),

$$L = \rho\frac{E}{\pi} \qquad (8)$$

Stage 5 – Scene Luminance to Film Illumination (lux)

We now need to calculate the light flux captured by the camera lens and relate this to the illumination of the film or CCD sensor. Two optical rules for simple lenses give rise, directly, to the two commonly known formulas for lenses. If f is the focal length of the lens, u and v are the subject and image distances and m is the magnification, then

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \qquad m=\frac{v}{u} \qquad (9)$$

We first consider a small element of the scene, on-axis and of area δS, which gives rise to a luminance of L cd/m^2. A beam of half-angle β is captured by the lens. Noting that it is the luminance and not the intensity that is constant over the beam width, we need to integrate I to find the total flux captured by the lens, or use equation (B) from the Box...

$$\Phi_{lens} = \pi L\delta S\sin^2\beta \qquad (10)$$

However, for most objects, distant from the lens, the beam is narrow enough that we *can* assume that the flux is evenly distributed and so the illumination of the camera lens, is (in line with the previous discussion of solid angle) simply given by $L\ \delta S/u^2$. To find the actual flux captured by the lens, we multiply this by the area of the lens, to give

$$\Phi_{lens} = \frac{L\delta S}{u^2}\tfrac{1}{4}\pi A^2 \qquad (11)$$

where A is the diameter or ***aperture*** of the lens. With $\beta \ll 1$ (10) and (11) are the same.

This flux is incident on a small area of the film, $m^2\ \delta S$. However, the flux does not all arrive from a normal direction and so we must apply (10) 'in reverse', with E_f being the illumination of the film in lux, and β′ being the image beam half-angle, to obtain

$$\Phi_{lens} = E_f\, m^2\delta S\sin^2\beta' \qquad (12)$$

Equating the two expressions then gives

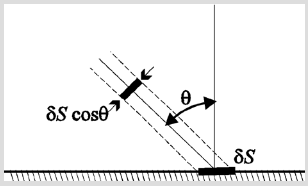
$$E_f = \tfrac{1}{4}\pi L\frac{A^2}{u^2m^2\sin^2\beta'} \qquad (13)$$

Luminance

If a surface has perfect reflectance then the luminous emittance (in lm/m^2) is equal to the illumination (in lm/m^2) – see equation (7). But how does emittance relate to luminance?

Consider a perfectly diffuse surface that emits light in all directions and – by definition – looks equally bright in all directions. The surface has luminous emittance M. Consider a small element of surface area δS for which the total reflected flux is $M\ \delta S$.

In a direction at an angle θ to the normal, the projected element of area (see figure below) will be δS cosθ.



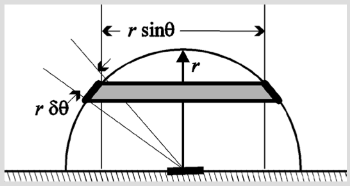
If the intensity in this direction is δI_θ then the luminance is *defined* as

$$L_\theta = \delta I_\theta\ /\ \delta S\cos\theta$$

Lambert's Law says that, for a perfectly diffuse surface, L is independent of direction (i.e. the intensity varies as $\delta I_\theta = \delta I_0\cos\theta$ where δI_0 is the intensity in the normal direction). Thus we can write the intensity as

$$\delta I_\theta = L\ \delta S\cos\theta$$

Now consider a thin cone of light emitted from this element of area at an angle θ to the normal, and incident on a hemisphere of radius r.



The radius of this cone is $r\sin\theta$, so its circumference is $2\pi r\sin\theta$. Its width is $r\,\delta\theta$. Therefore its area is $2\pi r^2\sin\theta\ \delta\theta$.

The intensity of the radiation in this direction is, as noted above, δI_θ. The luminous flux is therefore $\delta I_\theta/r^2$, which flows through the infinitesimal ring, described above. We integrate to obtain the flux in a beam half-angle β, which is

$$\Phi = \int_{\theta=0}^{\beta}\frac{\delta I_\theta}{r^2}\cdot 2\pi r^2\sin\theta\cdot d\theta$$

$$= 2\pi L\delta S\int_{\theta=0}^{\beta}\cos\theta\cdot\sin\theta\cdot d\theta$$

$$= \pi L\delta S\sin^2\beta \qquad \text{(B)}$$

Setting $\Phi = M\ \delta S$ and $\beta = \pi/2$ then gives us $M = \pi L$

That is, the luminance of a perfectly diffuse surface is – in SI units – equal to the luminous emittance in lm/m^2 divided by π; the unit of luminance being the cd/m^2.

Definition of the candela
Note that objects can also be self-luminant. The light output of a fluorescent lamp can be measured in cd/m^2, and it is the self-luminant property of a black-body radiator (60 cd/m^2 at the temperature of freezing platinum) that gave rise to an earlier definition of the candela. (Although the current SI definition defines the lumen directly, as 1/683 W of radiation at 540×10^{12} Hz (i.e. 555nm).

It is helpful to write u in terms of f and m, so

$$E_f = \tfrac{1}{4}\pi L \frac{1}{(f/A)^2}\cdot\frac{1}{(m+1)^2}\cdot\frac{1}{\sin^2\beta'} \qquad (14)$$

The term f/A is called the ***relative*** or ***numeric aperture*** or the ***f-number***. We usually write it as, e.g. *f*/8, *f*/11 to indicate that it is in a ratio to the focal length. Writing $N = f/A$ and noting that, for non-macro photography, m is essentially zero, we will arrive at the expression we are seeking. One final approximation needs to be made – we note that if $N >> ½$ (which it undoubtedly will be) then $\sin^2\beta' \approx 1$, so...

$$E_f = \tfrac{1}{4}\pi \frac{L}{N^2} \qquad (15)$$

which relates the film illumination to the scene luminance and the f-number.

This derivation assumed that the scene element was on-axis. Off-axis elements are further away from the lens, which introduces a $\cos^4\theta$ term; which explains one of the causes of the vignetting seen with wide-angle lenses. I also omitted to account for the light transmissibility of the lens and various other optical properties that do not concern us here.

Stage 6 – Film Illumination to Film Exposure (lux·s)

The effect we record depends on the incident energy density, and so we multiply the illumination (i.e. the power density) by the exposure time τ to get

$$H_f = E_f \cdot \tau \qquad (16)$$

Stage 7 – Film Exposure to Film Speed (ISO/ASA)

We now know how the power in the light source is related – via its distance to the scene – to the exposure of the photographic film. A definition of film speed provides the final link in the chain. The definition of film speed is complicated but we can gloss over many of the problems, as we are looking only for a broad answer. Ultimately, there must be a relationship between the exposure of the film in lux·s and the ISO film speed (on the numeric ASA scale or the logarithmic DIN scale). But the salient point is which part of the scene should we measure? The highlights will result in many times the exposure of the shadows. The full definition of film speed takes this into account; and there are several definitions based on different factors – such as whether the film is a monochrome negative, colour reversal or high-contrast lithographic film. The replacement of photographic film by CCD sensors introduces yet more ways of thinking about film speed.

We only need to consider a traditional black and white negative film, in order to observe the salient points. The important criterion for exposure is that even the darkest shadows should leave *some* indication on the film, so the speed of a black and white negative film is based on the illumination of the shadows. Of course, in a high-contrast scene, this might mean that the highlights are washed out, so we would under-expose such a scene and accept the fact that we lose detail in the shadows. But we are digressing, now, into a discussion of exposure meters, and of the contrast ratios of film (dependent on development process); and, in any case, digital sensors have superseded film.

The ISO definition of ASA film speed is

$$S_{\mathrm{ASA}} = \frac{0.8\,[\mathrm{lx\ s}]}{H_0} \qquad (17)$$

where H_0 is the film exposure in lux seconds that gives rise to a film density of 0.1 above 'base plus fog', when the film is developed to a specified contrast ratio. I am not going to explain those terms as they are not of relevance to the present discussion. They are discussed in photographic textbooks, e.g. [5] and, to a varying degree of reliability, on the Web. The salient point is that H_0 is the illumination of the darkest shadow that we want to record and that this, as I will explain below, is commonly taken to correspond to a reflectance of $\rho \approx 1.5\%$.

For a CCD, one definition – the Standard Output Sensitivity [6] – relates the film speed to the exposure needed to achieve 18% saturation of the recording scale when the image is recorded in a specified format, with

$$S_{\mathrm{SOS}} = \frac{10\,[\mathrm{lx\ s}]}{H_{18\%}} \qquad (18)$$

which is in agreement with the formula for ISO/ASA film speed given in (17).

Shadow Density

The definition of ISO/ASA film speed, given above, is – you might observe – clearly the one used by 'traditional' photographers, who would use a light meter to specifically measure shadow density. If you actually *measure* the illumination of the shadows then, clearly, it does not matter how deep they are. But, commonly, exposure meters measure the overall light reflected from a scene, so how does this equate to the 1.5% figure I quoted above?

Over the years, it has been established that a typical scene has an average reflectance of 18%, and it was considered that the darkest shadows would be about four stops down on this – at around 1.5%. But clearly this all depends on what is meant by a 'typical' scene. Much of the knowledge of scene exposure is probably now restricted to professional and expert photographers, since the advanced metering algorithms in digital cameras will take care of most problems. But traditionally, photographers would have been aware of high-contrast and low-contrast scenes and what this meant in terms of the ratio of average to shadow reflectance. Exposure meters were calibrated to cope with different scene contrasts. This is of relevance to cave photography because caves are not typical scenes and, as cave photographers know, guide numbers are not always a helpful guide, where the walls and ceilings do not have a high reflectance.

Combining the Results

We are now in a position to combine the elements discussed above. Starting from the brightness – or intensity – of our light source, combining equations 6, 8, 15, 16, and 17, and re-arranging the terms, we obtain

$$I\,\tau = 0.8\,[\mathrm{lx\ s}]\frac{4}{\rho_0}\frac{(Nd)^2}{S_{\mathrm{ASA}}} \qquad (19)$$

where ρ_0 is the shadow reflectance. If we assume $\rho_0 = 1.5\%$ this gives

$$I\,\tau \approx 220\,[\mathrm{cd\ s/m^2}]\frac{(Nd)^2}{S_{\mathrm{ASA}}} \qquad (20)$$

In this and the preceding equations, the notation in square brackets represents the units in which the constants are given since, clearly, they are not dimensionless.

The left side of (20) is the candela-second rating of the light source. Traditionally, this was taken to be the centre-beam value and was therefore quoted in beam-candlepower-seconds (BCPS). The quantity $N\ d$ is the guide number (GN) so this equation provides the sought-for link between the intensity of the light source, the exposure time and guide number.

Discussion of Guide Number

Equation (20) is quoted in many textbooks and datasheets although, unfortunately, not always without error. I have seen several misprints and confusions over units. The constant may vary from 200 to 250 but if you see it given as 20, that probably means the units are cd·s/ft^2 rather than cd·s/m^2.

You may also see it combined with (4) or (5) to express GN in terms of lumens and 'some sort of' beam-focussing constant but it is often difficult to establish the rationale or the validity of the formulas given.

The salient point – not adequately discussed in any reference I have read – is that this formula depends on some subjective reasoning. It excludes a number of factors (which are probably of minor consequence) but, importantly, it does not take into account the practical, rather than theoretical nature of the guide number. GN is determined by manufacturers, taking into account *all* the stray light from a flashgun, reflected from walls and ceilings when used in a typical environment. So, if (20) *is* correct, then the BCPS figure (as quoted by manufacturers of modelling lamps and studio flashguns) must *also* be subjective... which is perfectly possible, but there is certainly the inference that it is *not* subjective, because it is supposed to be the actual centre-beam intensity – something has to 'give' somewhere! In fact, it is recognised that centre-beam intensity is not an ideal figure to use, and that an integrated mean intensity provides a better figure.

I deduce – although I have not found a reference to confirm this – that the quoted intensity of a photographic lamp in BCPS is determined by measuring the reflectance from a typical scene using a spot photometer. This would validate equation (20), although it would not help us to convert an LED's luminous flux into a practical figure.

However, large errors in (20) give rise to only small exposure differences in 'stops', which are masked by automatic exposure meters. The point to remember is that (20) gives us as good a basis as we can expect for the relationship between beam intensity and guide number, which will allow us to compare LEDs with flashguns and bulbs.

Practical Example

These figures are based on a Luxeon LXR7-RW57 LED (colour 5700 K).

Operating current	2.1 A
Operating voltage	5.7 V
Light output	1000 lm
Half-power beam angle	± 60° (3.1 sr)
98% power beam angle	± 15° (0.2 sr)

From these figures, the power consumption is 6.8 W and so, from (1), the luminous efficacy is just under 150 lm/W. The beam is wide, so much of the light may be wasted (unless it is reflected from walls and ceilings) if we do not use a reflector to focus it.

The datasheet for this LED does not give any figures for intensity but, studying the graph for the spatial radiation pattern, I estimate that 24% of the power is radiated in the flat-topped 30° (0.2 sr) beam. So, putting $\Delta A / A = 0.25$ and $\Delta\Omega = 0.2$ sr into (5) gives a centre-beam intensity of around 1250 cd.

Similarly, the half-power beam contains around 88% of the power, so (5) indicates a mean intensity of 280 cd.

Or, if we focussed *all* the light into a 45° beam – (2) indicates this is 0.5 sr – and if the reflector was 90% efficient ($R = 0.9$) then, from (4), the intensity would be 1800 cd.

Which of these figures – 280, 1250 or 1800 cd – is the one to use? In a caving scene (i.e. little reflected stray light) then 1800 cd (with a reflector) or 1250 cd without, thus requiring a ½-stop increase in aperture, feels right. However, if we wish to *compare* the LED with the quoted GN of a flashgun we *need* to know how stray light from the beam edge contributes to the illumination, and we can only really guess at this. My guess is to say that if we use an LED without a reflector in a typical scene, then the calculated centre-beam intensity should perhaps be doubled to give an estimate of GN. If this is correct, then it would suggest that, for caving use, we need to reduce the quoted GN by $\sqrt{2}$.

A 'respectable' flashgun and an AG3B flashbulb both have a GN of around 40 m at 100 ASA. Equation (20) then suggests we need a source of ≈3500 cd·s, so our 2500 cd LED requires an exposure time of 1.4 s.

1.4 s and 6.8W is about 10 J. If we consider that a flashgun tube might have an efficacy of only a third to a half of the 150 lm/W of this LED, then we can equate this LED (with the 1.4 s exposure time) to a 20–30 J flashgun – which is in the right area.

Concluding Remarks

Equation (20) is the recognised relationship between the candela-second (or BCPS) rating of a light source and the photographic guide number. The BCPS rating can be derived from the power rating in lumens, provided the spatial radiation pattern is known. There are a number of *caveats*, as discussed, and we must remember that the guide number is, after all, only a 'guide'. However (20) allows us to produce a rough comparison between LEDs as light sources and flashguns or flashbulbs.

Glossary

Definitions

Beam Candlepower
The intensity of a light source at the beam centre. Somewhat outmoded as a means of rating a light source, in favour of a mean intensity.

Beam Candlepower Seconds (BCPS)
The product of beam candlepower and duration.

Diffuse reflection
Reflection where each incoming light ray could be considered to result in a point source radiating a broad cone of light.

Exposure
The product of illumination and time.

Film speed
A measure of the sensitivity of the photographic film or CCD sensor. In digital photography the term can be likened to a gain factor.

Guide number (GN)
A figure of merit for a flashgun. A subject will be correctly illuminated when the product of the flash-to-subject distance and the numeric aperture of the lens equals the guide number. For flashbulbs, GN is quoted for a particular reflector. For all uses, GN is highly dependent on the reflectivity of walls and ceilings and is quoted for use in an 'average' room.

Illuminance, Illumination
The luminous power density incident on a surface. Measured in lm/m^2, given the unit 'lux'.

Luminance
The luminous intensity of light reflected from a diffuse extended surface. Measured in cd/m^2.

Luminous v. Radiant values
'Luminous' indicates weighting for eye response.

Luminous Efficacy
The ratio of luminous to radiant power.

Luminous Emittance
The luminous power density radiated from a surface. Measured in lm/m^2.

Luminous Energy
The energy of a light source. Measured in lm s.

Luminous Flux
The power of a light source. Measured in lumens.

Luminous Intensity
The angular concentration of a light beam; colloquially, its 'brightness'. Measured in candelas.

Numeric or Relative Aperture, f-number
The ratio of the focal length of a lens to its aperture; often written as (e.g.) *f*/8, to indicate that the aperture is an eighth of the focal length.

Perfectly Diffuse or Lambertian Surface
A surface where the diffuse reflection appears equally bright from all directions.

Stop
The relative aperture of a lens expressed on a logarithmic scale. One stop corresponds to a halving or doubling of the exposure. If the numeric aperture goes from N_1 to N_2 the exposure goes up by the factor $(N_1 / N_2)^2$ which is an increase of $\log_2 (N_1 / N_2)^2$ stops.

Symbols used in equations

Φ	Luminous flux	lm
τ	Exposure time	s
Ω	Solid angle	sr
θ	Angle; beam full-angle	rad
ρ	Reflectivity	–
θ_d	Beam full-angle	degree
A	Aperture, i.e. lens diameter	m
d	Distance	m
E	Illuminance, Illumination	lux
H	Luminous exposure	lux·s
I	Luminous intensity	cd
L	Luminance	cd/m^2
M	Luminous emittance	lm/m^2
N	Numeric Aperture, f-number	–
Q	Luminous energy	lm·s
S	Film speed (on numeric scale)	–
u, v, f, m	Optical values, see eqn. (9)	
η	Luminous efficacy	–

References and Further Reading

This list includes a textbook and several Internet sources. However, the reader should remember that some Internet sources can be misleading, incomplete and, on occasion, simply wrong.

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