Loop Antennas v. Ferrite Rods: A Case Study

*An induction radio's transmitting antenna may be an air-cored loop or a ferrite-cored solenoid. Because there are so many variables involved it is difficult, in the general case, to compare the two types of antenna and it is not possible to state, unequivocally, which type of antenna will perform better. However, in specific practical situations, a comparison is possible. In this article **David Gibson** describes one such case.*

Recently, a company approached me to ask for advice concerning a commercial product of theirs, which utilised an induction loop transmitter. They needed to replace the air-cored loop with a more compact design, and were proposing a solution based on a ferrite rod. I was asked for my comments.

The Original Air-cored Loop
Specific Aperture

The loop that was originally in use comprised two turns of 16/0.2 mm equipment wire, wound on an 850 mm diameter circular loop, and was driven with a current of 420 mA rms at 122 kHz.

The first step, with all such designs, is to calculate the specific aperture, which is a figure of merit that relates magnetic moment to power dissipation and is, for many purposes, more useful and instructive than a consideration of the number of turns. In summary, an explanation of specific aperture is as follows. (Further detail is given in [Gibson, 1999]). The magnetic dipole moment is given[1] by

$$m_d = NIA \qquad (1)$$

where μ'_r is the *effective relative permeability*, which I will discuss later. (For an air-cored loop $\mu'_r = 1$ of course). Instead of using the current we can write the formula in terms of the power dissipation in the antenna's resistance, to give

$$m_d = \mu'_r \cdot \frac{NA}{\sqrt{R}} \cdot \sqrt{P} \qquad (2)$$

and we can choose to define

$$\Phi = \frac{NA}{\sqrt{R}} \qquad (3)$$

allowing us to write the magnetic moment as

$$m_d = \mu'_r \cdot \Phi\sqrt{P} \qquad (4)$$

The parameter Φ is the specific aperture. Although it appears, from the above definition, to be a function of the number of turns

N we can also define it in terms of the mass of the antenna, as

$$\Phi = \tfrac{1}{2}r\sqrt{M\frac{\sigma}{\rho}} \qquad (5)$$

To calculate Φ for the antenna under consideration, we need to work out the effective resistance of the wire. This is a function of the skin depth, which is a figure of merit that describes how the current density falls off with distance in a conductor. The skin depth in copper[2] at 122 kHz is around 0.2 mm; found from

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \qquad (6)$$

In the current situation, the conductor has a diameter of around 0.8 mm and the skin depth is sufficiently smaller than this that the attenuation will be more-or-less exponential[3], and the total current will therefore[4] be the same as the d.c. current that would flow in an annulus of depth δ and circumference πa. This means that the *apparent* diameter of the wire is the quantity a' in

$$\pi a'^2/4 \approx \pi a\delta \qquad (7)$$

which, in this case, is 0.8 mm. This means that we are justified in using the diameter of the wire in the following calculation of the wire resistance, which is, of course, simply

$$R = \frac{length}{area \times conductivity} = \frac{2\pi rN}{\tfrac{1}{4}\pi a^2 \sigma} \qquad (8)$$

giving a value for the loop of 180 mΩ. The resistance now allows us to calculate, from (3), the specific aperture to be 2.6 m²/√Ω.

The current was given as 420 mA, so the magnetic moment, from (1), is 480 mAm² which allows us to deduce the power dissipation, from (4), to be just 34 mW.

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2   $\sigma = 58 \times 10^6$ S/m

3   If the wire were any thinner, relative to δ, the conventional view of exponential attenuation would not hold. Instead, the current would follow a hyperbolic function. Formulas for use in such conditions can be found in text books.

4   As an integration will demonstrate.

The above results were calculated from a spreadsheet, the output of which is listed in the table below.

Loop diameter	**850**	mm
Turns	**2**	
Current	**420**	mA
Wire diameter	**0.8**	mm
Frequency	**122**	kHz
Conductivity	**58**	MS/m
Mass density	**8930**	kg/m^3
Skin depth	0.189	mm
Eff. Wire diam.	0.778	mm
Eff. Wire CSA	0.476	mm^2
Wire length	5.341	m
Resistance	0.194	ohm
Eff. Wire vol.	2.540	cm^3
Eff. Wire mass	22.679	g
Loop area	0.567	m^2
SA	2.579	m^2/rt-ohm
Moment	0.477	Am^2
Power	34.159	mW

Table 1 – Original antenna parameters
Figures in bold are inputs to the calculations

This is a surprisingly small loop, with a modest specific aperture and, compared with what cavers might be used to, a low power. I will discuss the reasons for this later.

Q-factor

The second parameter we need to know, in order to consider alternative designs, is the ratio of the loop's reactance to resistance, otherwise known as the Q-factor. In some situations Q may be more important than the individual values of inductance and resistance. Ultimately, we can use a matching transformer to adjust the antenna's R and L to match the amplifier's expectation, provided they are in the correct ratio.

The inductance of a loop (or of any electric circuit) is not always easy to calculate. Inductance is the ratio of magnetic flux linkage to the current that generated it. The difficulty is that magnetic flux is not confined to well-defined paths and so calculating the total flux due to, say, a loop of wire, involves many approximations. An often-quoted formula for loop inductance is

$$L = N^2 \mu_0 r \ln\left(\frac{8d}{w} - 2\right) \qquad (9)$$

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1   The symbols used in the equations are listed in a glossary at the end of this article.

which is derived in many text books, e.g. [Ramo, Whinnery and Van Duzer, 1984]. For a single-turn loop, w is the width of the wire (with $w \ll d$) and it is commonly supposed that this is also true of a multi-turn antenna. In fact, this is incorrect, and it is more accurate to state that w is the width of the entire winding. (See **Box: Inductance of a Loop**, below). Using (3) and (9) Q is

$$Q = \frac{\omega L}{R} = \omega \frac{\mu_0}{\pi^2} \frac{\Phi^2}{r^3} \ln\left(\frac{8d}{w} - 2\right) \quad (10)$$

This shows (as summarised in the table below) that, if the two turns are spaced 3mm apart, the Q-factor of the original antenna is around 65 and its inductance is around 17 μH. If the turns are spaced at only 0.5 mm L and Q rise by 23% and if they are spaced by 50 mm the values fall by 37%. This is a useful result to bear in mind when the L or Q-factor of an antenna is critical.

Inductance of a Loop

The formula for the inductance of a loop, given in the main text as equation 9, is complicated to derive. One standard method proceeds by arguing that the external self-inductance of the loop is approximated by the mutual inductance between a filament of current flowing in the centre of the wire, at radius ½d and a filament on the inside edge of the wire, a distance ½w away. The mutual inductance is then calculated, resulting in the well-known formula *

Understanding this derivation can lead to some interesting deductions. For example, consider two concentric single-turn loops connected in parallel, as shown below.
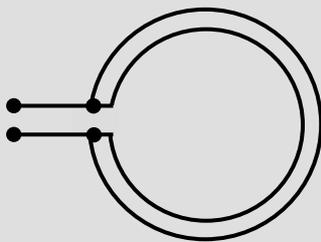
Figure 1 – concentric single-turn loops connected in parallel

Clearly, the same derivation applies to this topology as to the single wire loop, and so the same formula applies. The concept can be used to argue that, to some degree of approximation, the parameter w in the formula for a wire loop refers to the overall width or extent of the winding.

This is an important finding. In many instances, the inductance of a loop is a problem, and we would like to find ways of reducing it. Here is a clear indication that they way to lower the inductance is to space the turns further apart. If this is difficult (e.g. suppose we are utilising ribbon cable, or we have only a single turn antenna) then we should be able to achieve a similar effect by connecting two or more spaced windings in parallel.

* In my PhD thesis and for many years afterwards, I have quoted this formula incorrectly, using 8r / w, without anybody commenting. Fortunately, the mistake makes little difference to practical calculations.

U0	**1.257**	uH/m
Frequency	**122**	kHz
extent, w	**3**	mm
log term	7.725	
Specific L	4.126	uH/turn^2
L for given turns	16.5032	uH
Q-factor	65.328	
extent, w	**0.5**	mm
log term	9.518	
rise in L & Q	23%	
extent, w	**50**	mm
log term	4.898	
rise in L & Q	-37%	

Table 2 – Q-factor of original antenna

Inductance of a Solenoid

It is likely that the loop antenna could be replaced by a solenoid, although whether this is ferrite or air-cored remains to be discussed. For a long thin solenoid, with $\ell \gg r$,

$$L = N^2 \mu_0 \mu'_r \frac{A}{\ell} \quad (11)$$

where μ'_r is the *effective relative permeability*, which I will discuss later. For air, μ'_r is unity, of course. If the solenoid is shorter than $\ell \gg r$ then a standard practical formula (e.g. as quoted in [Ramo *et al.*, 1984]) is

$$L \approx N^2 \mu_0 \mu'_r \frac{A}{\ell + 0.9r} \quad (12)$$

which is valid $\ell > 0.8\,r$. One of the difficulties in making comparisons between the antennas is knowing which of (9), (11) and (12) to apply. Unfortunately, in practical situations, none is completely accurate and the inductance of an antenna must be physically measured at its operating frequency, if it is desired to know the value.

Often, during a theoretical study we will not know the number of turns, because this is not a significant factor in the design at this stage, but the above equations can be rewritten in terms of the specific inductance or inductance factor. For (11) this is

$$A_L = \frac{L}{N^2} = \mu_0 \mu'_r \frac{A}{\ell} \quad (13)$$

Similarly, the specific resistance can be defined from (3) to be

$$A_R = \frac{R}{N^2} = \left(\frac{A}{\Phi}\right)^2 \quad (14)$$

which results in the Q-factor being

$$Q = \omega \frac{A_L}{A_R} = \omega \frac{\mu_0}{\mu'_r} \frac{(\mu'_r \Phi)^2}{A \cdot \ell} \quad (15)$$

where I have written it as a function of $\mu_r'\Phi$, because this is the appropriate figure of merit for a ferrite-cored solenoid.

An Initial Comparison

We could suppose that the replacement antenna should have the same values of $\mu'_r\Phi$ and Q as the original. This may well be a good approach in some situations – perhaps when the loop is part of a tuned antenna – but I will show that in this particular case a direct replacement is not possible, and we will need to take a closer look at the power amplifier. First, though, a couple of examples.

Air-Cored Solenoid

Suppose the replacement antenna is to be an air-cored solenoid of diameter 85 mm. This is a tenth of the diameter of the original antenna and so, from (5), we know that it must have 100 times the mass. The mass of the wire was originally 23 g so its replacement must utilise 2.3 kg of wire. If the winding were to have a width of 0.8 mm, this would be a cylinder of copper some 1.2 m long[5]. Whilst in some situations this may be a more practical shape than the original, it is certainly not compact, and it may suffer from an increased inter-winding capacitance and therefore a problem with the self-resonant frequency. A spreadsheet for the above results is shown below.

A moment's thought shows that, for the same Φ, the length must vary as the inverse cube of the diameter so another example could be an antenna that was 1/5th the diameter with 125 times the mass, i.e. 570 g. This would have a length of 150 mm, so this is looking like a possible contender. But using (12) its Q-factor comes out to be rather high. I will discuss this later, after a consideration of a ferrite antenna.

ratio of diams	**10**	
new solenoid dia	85	mm
winding width	**0.8**	mm
winding CSA	213.628	mm^2
wire vol for =SA	253.960	cm^3
solenoid length	1.189	m
wire mass	2.268	kg
ratio of diams	**5**	
new solenoid dia	170	mm
winding width	**0.8**	mm
winding CSA	427.257	mm^2
wire vol for =SA	63.490	cm^3
solenoid length	0.149	m
wire mass	0.567	kg
U0	**1.257**	uH/m
solenoid CSA	22698.0	mm^2
L > 0.8R?	yes	
Specific L	126.714	nH/turn^2
Specific R	77.458	u-ohm/turn^2
Frequency	**122**	kHz
Q factor	1254	

Table 3 – An air-cored solenoid

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5 In this initial comparison I am ignoring the packing factor of the winding.

Ferrite-cored Solenoid

Introducing a ferrite core gives rise to a number of difficulties with the simple algebraic modelling I have outlined above and there comes a point when the approach has to be abandoned in favour of a 'try it and see' spreadsheet-based calculation. However, the danger of using a spreadsheet to begin with is that we can miss any insights that might arise from an algebraic analysis.

Consider a ferrite-cored solenoid of diameter 8.5 mm. This is 1/100th of the original diameter. The difficulty is now that there is little scope for a massive copper winding. If the winding was 0.8 mm wide and 120 mm long, it would have a mass[6] of about 23 g – i.e. the same as the original antenna and so the specific aperture will be 1/100th of the original value. This dictates, from (4), that we need $\mu'_r = 100$.

At this point, I need to introduce a discussion of how μ'_r depends on the shape factor of the ferrite rod. This is discussed in the **Box: Magnetic Materials**, to the right. To obtain $\mu'_r = 100$ from a high permeability material we need a shape factor of about 16, and so the ferrite rod must have a length of 16×8.5 mm = 136 mm. In practice, as noted in the Box, the rod should be two or three times the length of the solenoid; so perhaps 240 mm. The results are summarised in the table below.

So far, this looks like a good, practical antenna. The ferrite rod – 240 mm × 8.5 mm∅ – will not be difficult to obtain[7]; the winding is a 'sensible' size, and it should – in theory – have the same overall $\mu'_r \Phi$ figure as the original. However, we have yet to examine its inductance and Q-factor.

ratio of diams	*100*	
new solenoid dia	8.5	mm
winding width	*0.8*	mm
winding CSA	21.363	mm^2
original mass	*22.679*	g
winding volume	2.540	cm^3
sol length	*118.879*	mm
sig/rho	80.591	[etc]
SA	0.026	m^2/rt-ohm
reqd Urel	100	
1/demag factor	**100**	[solver target]
ferrite length	**132.833**	mm
shape factor	15.627	

Table 4 – A ferrite-cored solenoid
Figures in bold are inputs to the calculations. The spreadsheet's 'solver' algorithm has been applied to the two parameters shown bold/underlined.

We can calculate the Q-factor from (15) which, at 122 kHz, is almost 9500. This raises the question of the validity of the result, since Q-factors that large simply do

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6   In this initial comparison I am ignoring the packing factor of the winding.
7   8 mm rods are available; the additional 0.5 mm allows for the winding thickness.

Magnetic Materials

Magnetic materials have an 'intrinsic' or 'bulk' permeability that is determined by the atomic structure of the material. However, in practice, this permeability is not observed due to what is termed the de-magnetisation factor. In simple terms, magnetic materials need to be long and thin in order to make best use of their properties. (For example, it can be shown that a sphere of a material with a very high intrinsic permeability actually has an effective relative permeability μ'_r of only 3).

With several approximations and *caveats* we can write the effective permeability as

$$\mu'_r \approx \frac{m^2}{\ln(2m)-1} \qquad (16)$$

where *m* is the shape factor, defined by

$$m = \frac{z}{2r} \qquad (17)$$

with *z* being the length of the ferrite rod. For these formulas to be reasonably accurate we need the conditions $m > 6$ and $\mu'_r \ll \mu_r$.

The formula was derived for a prolate ellipsoid, magnetised about its long axis. A cylindrical rod is only an approximation to this and therefore further limits the accuracy of the result. A prolate ellipsoid has the characteristic that the flux density is constant along its axis. This is not the case for a cylindrical rod, and so it is sometimes suggested that the winding on a cylindrical rod should be concentrated on the middle third to half of the length. It is for this reason that *z* is not the same quantity as ℓ in the formulas in this article.

For a further discussion see [Gibson, 2010].

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not occur in practice. Additionally, of course, the value is nowhere close to that of the original antenna.

U0	*1.257*	uH/m
solenoid CSA	56.745	mm^2
Specific L	59.983	nH/turn^2
Specific R	4.841	u-ohm/turn^2
Frequency	*122*	kHz
Q factor	9498	

Table 5 – Q-factor of the solenoid in Table 4

Both this result and the previous one support the general principle – evident from the equations – that a physically small antenna has a greater Q-factor than a larger one. This could lead us to consider using a larger ferrite rod. This, and the use of a bobbin to increase the effective permeability, is explored in the **Box: Large Ferrite Antennas** below.

Results of Initial Comparison

The results outlined above suggest that we can match the new antenna for specific aperture or for Q-factor but not both at the same time. As a demonstration of this, suppose that we decide not to use a matching transformer, meaning that we must maintain

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the $\mu'_r NIA$ product of the original antenna. This dictates that the new antenna must have 200 turns and an inductance of 2.4 mH.

Alternatively, we can reduce the number of turns and utilise a matching transformer. However, because of the need to maintain the specific aperture, using fewer turns would require the use of a larger diameter wire, metal tape or a multi-filar winding with the turns connected in parallel. The latter two options might result in a problem with the self-resonant frequency because the inter-winding capacitance will be potentially high. The only way to ensure that the SRF is not a problem (other than by a long series of practical experiments) is to consider using a smaller amount of copper. But this means that the shape factor must be increased, by using a longer rod.

As an example of the use of a matching transformer, consider the use of a 4:1 step-down transformer, which will result in four times the current in the antenna and so we will need only 50 turns. *L* is now a 16th of 2.4 mH, i.e. 150 μH but, of course, the values of *L* and *R* as 'seen' by the amplifier are unaltered, at 2.4mH and 190 mΩ.

We can see that it is not possible to maintain Φ and, at the same time, make the amplifier see a load other than 2.4 mH. This inductance is far higher than the original antenna (see Table 2, above) and is likely to prove a problem.

The converse should be obvious at this point – if we *do* attempt to maintain the original inductance then we cannot achieve the required specific aperture.

Although I have demonstrated this by means of an example, it is also possible to derive the result using a formula. By equating the loop antenna's Q and Φ (from 10) with a solenoid's Q and $\mu'_r \Phi$ (from 14) and making a very broad generalisation that the log term in (10) might be approximately 2π, we can deduce the rough comparison that

$$2\mu'_r r^2 \ell \Big|_{solenoid} \approx r^3 \Big|_{loop} \qquad (18)$$

which, unfortunately, does not have any practical solution, thus suggesting the impossibility of *any* alternative design.

Power Amplifier Considerations

To find a solution to the problem, it is necessary to ask further questions about the power amplifier. Given that we cannot provide an antenna that matches the original load, what other load will the PA accept? In particular, why does the original antenna comprise only two turns of wire and why is it operated with only a few mW of power? The answer is that it was intended to be used with

Large Ferrite Antennas

In the main text, a small ferrite antenna with $\mu'_r = 100$ was discussed. Now consider a larger rod with twice the diameter, i.e. 17 mm. Using the same design process as before – maintaining the mass and calculating the required μ_r – suggests that the length of the winding should be halved, to 60 mm and we need $\mu'_r = 50$. This dictates a shape factor of ≈ 10, and so a ferrite length of 10×17 mm = 170 mm.

This is also a contender for a replacement antenna. Instead of a 17 mm rod, we could utilise three standard 8 mm rods bundled together. The less-than-100% fill factor would reduce the permeability, but we have already made the assumption that it is 'very high', and μ'_r is therefore shape-factor-limited; so a reduction in μ_r should not affect μ'_r much. However, the Q-factor of this antenna is still rather large.

Use of a Ferrite Bobbin

There is one further aspect of ferrite antennas to consider and that is a 'bobbin'. A bobbin comprises a thin rod with two larger end-plates, made from flat sheets of a ferrite material, as depicted below.
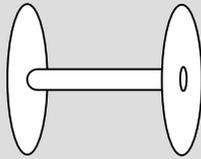

Figure 2 – A ferrite bobbin antenna

If the parameters r and ℓ refer to the rod itself and the radius of the plates is r_b then, from a consideration of the probable flux path, the effective relative permeability is

$$\mu'_r \approx \left(\frac{r_b}{r}\right)^2 \qquad (19)$$

which is more likely to be accurate if $\ell < r$.

This is an interesting and potentially significant result because it provides a method of increasing the permeability and reducing the mass of the ferrite and the winding. Clearly a solid ferrite rod of the same dimensions would have a very low μ'_r indeed, if the condition $\ell < r$ were true.

To illustrate how we could utilise such a device, consider how the example in the main text might be adapted. The solenoid was 8.5 mm$\varnothing \times 120$ mm but the ferrite rod had to be 130 mm long to achieve the shape factor of 10, and possibly as long as 240 mm to accommodate the 120 mm winding. Suppose we *shorten* the solenoid to 60 mm whilst keeping the mass of the winding unaltered. Now, instead of using a 240 mm ferrite rod to support the winding, we can add two end plates that have an area 100 times that of the rod; that is, a diameter of 85 mm.

Thus we have changed the size from 8.5mm$\varnothing \times 240$ mm to 85 mm$\varnothing \times 60$ mm. Whether that is a useful alteration will depend on the circumstances in which we wish to use it but, clearly, the use of a bobbin can change the aspect ratio of the antenna significantly – especially if the shape factor dictates a very long rod.

dissipation[10] is (from 20) 3.4 W.

$$\cos\phi = \frac{1}{\sqrt{1+Q^2}} \qquad (21)$$

A consideration of the U_L / U_P term would need to introduce the saturation voltage and, if appropriate, the minimum on-resistance of the output transistors, together with an appreciation of how the PA output resistance is matched to the load. It is convenient to ignore all this and to assume that $U_L / U_P \approx 1$.

Writing the inductance of the antenna as $L = N^2 A_L$ and using (3), we obtain

$$Q = \frac{\omega A_L \Phi^2}{A^2} \qquad (22)$$

and, using this, together with $U_L / U_P \approx 1$ and $Q \gg 1$ allows us to write the moment as

$$m_d = \mu'_r \, \Phi \sqrt{P_L}$$
$$\approx \mu'_r \, \Phi \sqrt{P_S} \cdot \sqrt{\frac{\pi}{4} \cos\phi} \qquad (23)$$
$$\approx \mu'_r \, A \sqrt{P_S} \cdot \sqrt{\frac{1}{8 f A_L}}$$

which appears to be an extraordinary result, suggesting not only that the moment is independent of the specific aperture but that it is independent of the number of turns too. However, since a higher Φ implies a higher Q and therefore a lower PA efficiency, it can be appreciated that the effects cancel out.

Using the data from Table 1 and Table 2 we can check that the relationship makes sense. The moment given in Table 1 is 91% of the value derived from (23), which is due to our failing to take into consideration the ratio $\sqrt{(U_L / U_P)}$ which is also 91%.

Frequency	**122**	kHz
Antenna power	**34.159**	mW
Q-factor	**65.328**	
AL	**4.126**	uH/turn^2
UL from I.XL	7.514	V
Rail voltage	**9**	V
PA power	3.403	W
Loop area	**0.567**	m^2
Moment	0.522	Am^2
Orig. moment	**0.477**	Am^2
ratio of answers	0.91372	

Table 6 – Verification of equation 23
Figures in bold are inputs or from earlier tables

Because the number of turns of wire is apparently now unimportant, we should try to choose it to match the expectation of the existing PA – that is, for a reactance of 13 Ω (5.3 V$_{rms} \div 420$ mA) although this is not essential, because we could use a matching transformer. What is clear, though, is that the earlier conflict between trying to maintain

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a very wide signal bandwidth, which usually[8] suggests the use of an untuned antenna. Driving such an antenna is problematic and results in a low antenna power dissipation.

Driving an Untuned Antenna

There are two reasons why an untuned antenna is difficult to drive. The first is to do with the voltage 'headroom' of the power amplifier. The load presents a large reactance and so, for a given PA output voltage U_L the available current is (for a high-Q inductor) U_L / X_L rather than U_L / R. The current is therefore reduced by a factor of X_L / R which is, of course, Q.

This is why the antenna in this case study has such a low inductance. With a load current of 420 mA rms at 122 kHz, the voltage across the 16.5 µH inductor is 5.3 V$_{rms}$ or 7.5 V$_{peak}$, and it is being used with a ± 9 V supply. You may wonder why we cannot simply use a step-up transformer to boost the output voltage but this does not actually solve the problem because the same result could be achieved by altering the number of turns on the antenna.

The second problem associated with an untuned antenna is that if it is driven from a class B analogue driver (represented in a stylised form in the diagram below) then a further inefficiency of $1/Q$ arises, due to the phenomenon of node-forcing.
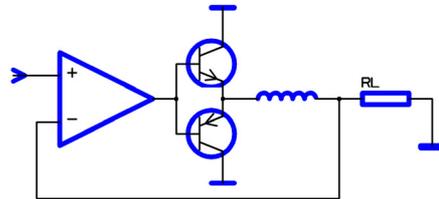

Figure 3 – Stylised class-B driver output
Using current feedback for stabilisation

Node forcing arises because the load current and load voltage are out of phase and it results in an increase in power dissipation in the output transistors. I analysed it in detail in [Gibson, 2010] and in earlier CREG articles and showed[9] that the ratio of the load power to the source power is

$$\frac{P_L}{P_S} = \frac{\pi}{4} \frac{U_L}{U_P} \cos\phi \qquad (20)$$

Thus, for maximum efficiency, the peak output voltage U_L should be as close to the power rail U_P as possible, and the phase angle ϕ should be as low as possible. The phase angle is related to the Q-factor by (21) below, so if $Q \gg 1$, the efficiency is proportional to $1/Q$. In the present study, with $Q = 65$, $P = 34$ mW and $U_P = 9$ V, the PA

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8   But see my later observations on tuned and damped antennas.

9   Or, rather, confirmed what was already well-known by experts in amplifier design.

10   This assumes that the load is driven with a sine wave at the carrier frequency.

both the specific aperture and the Q-factor was erroneous, because we had not taken into consideration the efficiency of the driver.

Another curiosity that arises from (23) is that because Φ does not appear we could wind the antenna from very thin wire – or use thick wire and add a series resistor to match the PA's load requirement. This 'damping' principle also arises in the following discussion on tuned antennas.

To further investigate (23) we can substitute for A_L. Using (12) we can write

$$m_d{}^2 = \frac{\mu'_r}{\mu_0}\,\frac{A\,(\ell + 0.9r)}{8f}\cdot P_S \qquad (24)$$

or if (9) applies, then

$$m_d{}^2 = \frac{1}{\mu_0}\,\frac{\pi^2 r^3}{8f}\,\frac{1}{\ln\!\left(\dfrac{8d}{w}-2\right)}\cdot P_S \qquad (25)$$

From (25) we can see that, for a loop, we must raise the source power as the inverse cube of the radius – so a 170 mm diameter loop will require 400 W. And from (24) it can be deduced that an air-cored solenoid is not going to be suitable either. However, provided we use a fairly large volume of ferrite (24) should give rise to possible contenders for a viable antenna.

For a ferrite-cored solenoid with $\ell \gg r$ we can use (16) and (17) to write (24) as

$$m_d{}^2 \approx \frac{z^2 \ell}{(\ln(\ell/r)-1)}\cdot \frac{\pi}{32f\,\mu_0}\cdot P_S \qquad (26)$$

showing a further salient point which is that the moment does not depend 'much' on the diameter of the ferrite rod – only on its length, and on the length of the winding. This is the key to the solution, and I will discuss some examples in a follow-up article.

Using a Tuned Antenna

We have seen that use of an untuned antenna presents particular problems and, as (23) shows, the magnetic moment is limited, and is independent of the specific aperture.

However, if the antenna can be tuned to resonance, with its inductive reactance cancelled by a series capacitor, the current will rise by Q times. Additionally, the PA efficiency will rise by Q times. The original antenna cannot be resonated because its Q-factor would result in a bandwidth that was far too low, which is why it was being driven untuned, of course. However, I discussed, some time ago [Gibson, 1995] how, given a particular perspective on the problem, the bandwidth of a loop antenna did not actually affect its performance, and this point is now worth investigating further.

We know that Q is related to the 3 dB bandwidth and to f_0, so (22) can be written

$$B = \frac{A^2}{2\pi A_L \Phi^2} \qquad (27)$$

That is, for a tuned antenna, the bandwidth is *inextricably* related to the specific aperture. For the antenna we are studying, this indicates that, if we tuned it to resonance, its 3 dB bandwidth would be just 1.9 kHz – a fact we knew anyway, via the Q-factor of 65.

But suppose we reduce the specific aperture by nine times? The bandwidth will rise by 81 times to the required 154 kHz of this case study. To compensate for the reduction in Φ we can utilise 81 times the power. This is 'almost' possible, given the absence of the inefficiency of driving an untuned antenna.

Additionally, because we are tuning the antenna, its inductance does not matter – which was one of the problems of the earlier analysis (although, clearly, we want to end up with a sensible value of tuning capacitor). Expressing the above in an equation by combining (23) with (27) (and noting $\phi = 0$ at resonance); and then applying (12) we get

$$m_d{}^2 = \mu'_r{}^2\,\frac{A^2}{8A_L B}\,P_S \qquad (28)$$

and

$$m_d{}^2 = \frac{\mu'_r}{\mu_0}\,\frac{A\,(\ell + 0.9r)}{8B}\,P_S \qquad (29)$$

Hence we can deduce, by comparing with (24) that, whilst tuning and damping the transmitter is an interesting solution, it is advantageous only when $B < f$, that is, $Q > 1$, which is not true in this case study.

Concluding Remarks

I have looked at the use of smaller loops, and of air-cored and ferrite-cored solenoids but it does not appear possible to design a physically smaller replacement antenna that is electrically similar to the original. However, that is to disregard the effect of the untuned class-B driver. When this is taken into account, some possible contenders for an alternative antenna arise. I will discuss this further in a follow-up article – **Wideband Loop Antennas: A Case Study**.

Glossary
Abbreviations and Definitions

Demagnetisation factor
  A factor that leads to a reduction in the effective value of magnetic permeability in a sample of a magnetic material that has a poor shape factor.

Effective permeability
  The magnetic permeability of a material after allowing for its reduction due to the shape factor

PA – Power amplifier

Q-factor
  The ratio of reactance to resistance for a single component or the ratio of 3 dB bandwidth to resonant frequency in a tuned circuit.

Shape factor
  The ratio of length to diameter for a ferrite rod.

Skin depth
  A figure of merit describing the rate of attenuation of fields within a conductor.

Solenoid
  A helix of wire where, generally the length is greater than the radius. A solenoid does not have to be wound on a ferrite rod, hence the distinction, in the equations in this article, between the length ℓ of the solenoid and the length z of the rod on which it *may* be wound.

Specific aperture
  A figure of merit describing how magnetic moment is related to power dissipation for an induction loop antenna.

SRF – Self-resonant frequency

Symbols used in equations

Φ	Specific aperture	$m^2/\sqrt{\Omega}$
σ	Conductivity of winding material	S/m
ρ	Mass density of winding material	kg/m^3
ω	Angular frequency	rad/s
μ	Magnetic permeability	H/m
δ	Skin depth	m
μ_0	Permeability of free space	H/m
μ_r	Relative magnetic permeability	–
μ'_r	Effective relative permeability	–
A	Area of loop or solenoid	m^2
a	Diameter of wire	m
a'	Effective diameter of wire	m
A_L	Specific inductance	$H/turn^2$
A_R	Specific resistance	$\Omega/turn^2$
B	Bandwidth; 3 dB bandwidth	Hz
d	Diameter of loop	m
I	Current	A
L	Inductance	H
ℓ	Length of solenoid	m
M	Mass of winding	kg
m	Shape factor	–
m_d	Magnetic dipole moment	Am^2
N	Number of turns	–
P	Power dissipation in antenna	W
P_L	Power dissipated in PA load	W
P_S	Power drawn from power supply	W
Q	Q-factor	–
r	Radius of loop or solenoid	m
r_b	Radius of the end-plate of a bobbin	m
R	Resistance of winding	Ω
U_L	Peak load voltage of power amp.	V
U_P	Rail voltage $\pm U_P$ of power amp.	V
w	Width or extent of loop winding	m
X_L	Inductive reactance	Ω
z	Length of ferrite rod	m

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