Metacognition in the mathematics classroom: an exploration of the perceptions of teachers and students in secondary schools in Saudi Arabia

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

Signature ………………………………………………………………………………

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Abstract

This study aimed to explore teachers' and students' perceptions of metacognition in relation to mathematics teaching and learning in secondary schools in Saudi Arabia. This research adopted an interpretive paradigm. This meant that a socio-cultural perspective was central to examining perceptions of metacognition in relation to mathematics among secondary students and their teachers in Saudi Arabia. The use of case studies was a methodical means to achieve elaborate data and to shed light on issues facing the study. The instruments used for data collection were semi-structured interviews, group discussions and classroom observation. The participants consisted of two case study classes from secondary schools in Saudi Arabia. There were three stages of the study's fieldwork: the pilot study and the two subsequent stages which comprise the main body of fieldwork. These last two stages were carried out in order to enable the formulation of a clearer and more complete picture of mathematics teaching and learning through metacognition in Saudi Arabia, before and after the implementation of the IMPROVE programme, regardless of improvements in specific strategy or any boost to students' achievement.

Several findings were drawn from the data, the first of these being that the traditional method can hinder mathematics teaching and learning through metacognition. Secondly, although metacognitive mathematics instruction should be planned, the strategy that is introduced should be directly targeted at improving the monitoring and regulation of students' thought when dealing with mathematics problems. Thirdly, metacognition should be given priority to improve students' consciousness of the learning processes. This is because conscious reflection enables students to develop an ability to choose the most appropriate strategies for learning concepts and solving mathematics problems. The findings underlined the importance of the student's role in learning through metacognition. The study presented a perspective for dealing with metacognition along with a practice-based model of metacognitive mathematics teaching and learning. These are in the educational context of Saudi Arabia and are set out after the implementation of the IMPROVE programme. In addition, this study asserts that metacognition can be enhanced through the creation of a suitable socio-cultural context that encourages the social interaction represented through cooperative learning.

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1 Introduction

This chapter explains my interest in metacognition and mathematics, provides a review of the context of the current study within the education of the Kingdom of Saudi Arabia (KSA) and gives the rationale for undertaking the study along with its aim and associated questions. It also presents operational definitions and potential difficulties. The chapter concludes with an outline of the thesis.

My interest in the subject of metacognition began at a time when I pondered my method of thinking a lot, and how I could improve this to achieve good decisions in different situations of my life in general. My favourite subject since primary school has been mathematics, and in my methods to solve problems I would also ponder how I could improve my way of thinking when it came to dealing with mathematics problems. I would often discuss this matter with my friends, as I did not have any background education to help me in answering such questions. I also did not know that this subject could be classified under metacognition, of which Flavell (1976) spoke when he wished to expand the benefit of metacognition from inside the classroom to the field of daily life. When I became a teacher, I taught mathematics for nearly ten years at the primary and secondary levels. I noticed that students face difficulties in mathematics learning, even though many of them possess good mathematical knowledge. They could not link their knowledge and new mathematical concepts, nor could they employ previously learnt information correctly in solving new and different problems. I thought that simply using teaching strategies in my teaching method would be sufficient in improving the performance of students in learning mathematics, and this is what I did in my Master's research. However, I noticed that the mere deployment of a limited strategy for teaching may help students in boosting performance in the subject pertaining to the strategy, but did not effect change in them in their way of thinking which could help them deal with numerous mathematical problems in an effective and positive manner.

After my transfer to the university, specifically in programmes to train teachers, I noticed that student teachers limited their interests to transferring mathematical information to the students. This came at the expense of interest in improving students' way of thinking in solving mathematics problems, which means that the traditional method remains dominant. This is represented by the teacher being a mere conveyor of knowledge, rather than an assistant in improving students' thought methods in mathematics learning. Through my discussion of this subject with a group of teachers and education specialists, one of professors at the university I worked at suggested that I research the subject of metacognition and its role in mathematics learning. This is where my academic journey with this research began, which seeks to employ metacognition in mathematics teaching in the educational context of the Kingdom of Saudi Arabia. Thus, the next section will give the context of the current study, and based upon this, the rationale for the study.

1.1 Context of the study

The Kingdom of Saudi Arabia possesses a centralised education system (Alfares, 2014; Alnesyan, 2012; Alsaeed, 2012) in which the Ministry of Education oversees education policy for the entire country. It manages the construction and equipping of educational facilities, along with the content and distribution of all textbooks, which are standardized throughout the Kingdom. The education system is divided into five levels, with kindergarten stage non-compulsory, six years of primary school, middle and high schools of three years each (secondary spans from 15 to 18 years old), and a separate tertiary education system. All five levels are overseen by the Ministry. The academic year tends to consist of two 18-week terms, with two weeks set aside for examinations. Each class period usually lasts 45 minutes, with the total number of periods weekly ranging from 26 to 33 periods, depending on grade and subject emphasis. Mathematics is a key subject whereby students are obliged to study the subject for five periods per week. While the education system is sex-segregated, both genders receive the same quality of education, with almost identical subjects and school stages, although there may be slight contrasts based on differing needs.

The goals and policies of education in the Kingdom of Saudi Arabia were built on a group of foundations, as published on the Ministry of Education's website. Among these was the enabling of the student to possess the skills of continuous learning. In order to achieve such goals, the Ministry seeks to improve academic curricula, teaching methods and evaluation processes, which will reflect positively on students' learning. One of the most important practical steps taken to achieve educational goals in the Kingdom is the King Abdullah Project for the Development of Education – named *‘Tatweer’* (‘Development’) in Arabic (TATWEER). This project began in 2008 and seeks to present educational services through projects and programmes to elevate the educational process and to develop and improve pedagogy.

The project’s philosophy is based on a group of principles such as student-centric learning, cooperative learning, active learning based on discovery and investigation, developing thinking skills, developing decision-making skills and linking learning with real-life contexts. One of the programmes involved in this project was the development of Science, Technical, Engineering and Mathematical education (STEM). This initiative seeks to improve students’ acquisition of thinking skills in practical ways along with improving their academic attainment. However, STEM currently focuses on programmes of vocational development through multinational companies, with leading organizations and with universities in mathematics and science instruction. It also seeks to establish scientific centres and to build supporting digital content for learning and teaching.

Emerging from this was a partnership with the company Obeikan Education. This company was contracted to design the curriculum for all the stages of general education in the subjects of mathematics, sciences and the English and Arabic languages. It is also responsible for providing the expertise required in classrooms, with this being done to the highest global standards, as claimed by the company. The services would also be carried out according to the recommendations of current research in the field of vocational education and curriculum development. Obeikan Education mentioned that they had partnered with McGraw Hill Education, which provided the Saudi Ministry of Education with comprehensive education solutions for all the stages of general education. These efforts drew upon support and programmes for all those concerned with pedagogy, be they students or teachers both within and outside the Kingdom. In this regard, Almazroa and Al-shamrani (2015) pointed out that American mathematics and science textbooks had been modified and translated to suit the Saudi context. However, even with adjustment, importing learning materials from abroad has disadvantages; for example, some curriculum content may be difficult to relate to in a contrasting cultural setting (Alshammari, 2014) . The other issue is the shortcomings of teacher training provided in Saudi Arabia, meaning teachers struggle with the unfamiliar curriculum (Almazroa & Al-shamrani, 2015).

The establishment of the Public Education Evaluation Commission (PEEC) is noteworthy in examining the structure of the education system in the Kingdom. This commission was established in 2013, in accordance with a decree of the Council of Ministers, which determined its structure and goals. This decree proposed that the commission serve as a public organization with an autonomous corporate identity. Its target was to administer the evaluation of schools in the Kingdom, both private and public.

Within the mandate of this authority were several concepts which enhance the importance of activating thought improvement in public education. It stated, 'Oral education is not limited to conversation, but includes feedback or responses that increase thinking and push for the development of ideas and the sharing of these' (PEEC, 2013). Regarding the studies concerned with the subject of thinking in education within the Saudi context and the extent of the effectiveness of these policies in pedagogical reality, Alnesyan's (2012) study was among the prominent works. He highlighted a dearth of research into the teaching and learning of thinking skills in the Kingdom. In evidencing this, one of the few public commitments made by the government to encourage thinking skills was the Ministry of Education's offer to apply thinking skills alongside its projects in the education system. As part of this, an initiative named 'Development of Thinking Skills' was the main reference for development in this area. In support of this project, the Ministry published the first issue of 'Teacher's Guide for the Development of Thinking Skills' in 2004 in a bid to assist head teachers. Following this the Ministry published a second issue in 2007 with real-life examples of methods to develop thinking skills. However, since then, no further issues have been published. The efforts of the Ministry were extended to textbooks in 2007-8, modifying content related to thinking skills and adopting an infusive approach in several subjects. In the following year, the approach was employed in mathematics and science materials as a key aspect in the Developing Science and Mathematics Curriculum Project. Another dimension of these policies was the Ministry's recent overhaul of the school system's components, i.e. curriculum, teacher training, research, school structure, and technology, which sought to build a constructive learning environment that fosters the development of general thinking skills, and in particular, critical thinking (Alwadai, 2014).

The Ministry's efforts to encourage thinking skills have certainly not been immune to criticism. Alnesyan (2012) conceded that education specialists were indeed convinced of the importance of implementing thinking skills, but this conviction had not extended into practical steps to achieve this. He claimed that the Ministry's efforts have concentrated more on providing resources to inform the approach to thinking skills (such as the infusion method) at the expense of other key aspects in the successful development of thinking skills, such as preparing instructors to adopt the infusion approach. This partial nature of implementation poses an obstacle to achieving the goals of policymakers. Alnesyan (2012) pinpointed the general issue as lying in the authorities' focus on curriculum content and neglecting other issues such as teacher training. Alnesyan's (2012) view was consistent with Alwadai's (2014) assertion that, while some teacher training has taken on aspects of thinking skills development, the programmes fell short of preparing the teachers themselves for instructing students, and instead focused on curriculum. Trainee teachers on secondment from university are not sufficiently informed or trained in regards to teaching thinking skills – these are not gained from their training at university nor their practical experience from teaching in the education system. Hence, teacher training for methods to encourage student thinking (e.g. utilizing challenging questions and specialized strategies in teaching) are still lacking.

The overall reasons for the priority being placed on curriculum development, rather than other aspects, are manifold. However two reasons are prominent in explaining this phenomenon, according to Alnesyan (2012). The first relates to a difference in governing structures, as in Saudi Arabia there are separate ministries for general and tertiary education. A lack of communication between the two is key in understanding weak results, as universities may not be aware, willing or able to train teachers in these skills. However, this reason is no longer present, as on 29th January 2015 a Royal Decree was published ordering the combination of the two ministries. It is hoped that this will integrate the teachers' practical, field training in schools with theoretical, pedagogical training at university.

This should also aid in mitigating another issue, that of poor interaction between the ministries and teachers, as now communication is centralized. In Alnesyan's (2012) view the development of thinking skills requires cooperation between the two parties, with teachers' opinions being taken into account and them playing an active role in the curriculum process. There is plenty of room for improvement in this regard, for example when the infusion approach was inserted into materials, the Ministry should have readied teachers to ensure that the adjustments and reforms were consistent with student and teacher needs. As the Ministry is the primary financier of the school system, it is their responsibility to provide training courses on thinking skills, but instead teachers were given information booklets. This stresses the need for clearer and more effective means of communication, which could go a long way towards lessening communication issues.

However, this claim too has been taken into account by the Ministry, which recently launched the 'Be Our Partner' slogan as part of the King Abdullah Project for the Development of Education. The website of the programme stated it to be based on the vision of the Tatweer project, which is a national initiative seeking to present educational services through projects and programmes (TATWEER, 2013),

> Including the strategy to develop general education in Saudi Arabia. It seeks to elevate the educational process and to develop and improve pedagogy in a way that is consistent with the vision of the wise leadership of education in the Kingdom of Saudi Arabia. It realizes the importance of establishing companies with individuals and institutions with interests in working towards the achievement of our mutual goals. Thus, we invite you to acquaint yourselves with the programs and projects of Tatweer. In addition, we seek to build a partnership in this field, just as your registration on our database (Partners of Tatweer) pleases us.

In addition, the Public Education Evaluation Commission (PEEC) has established 'Our Teacher's Platform' service. This is an interactive electronic service for society which is flexible in its registration process and allows the student, teacher and parents/guardians to present their opinions and link them with the criteria for an 'ideal teacher', and share them through social media (Facebook, Twitter), in an interactive way in public view. Despite this, the need to legally codify the communication between employees in the field and those in the Ministry remains present.

As for the reality of mathematics teaching in the Saudi educational context, determining the reality of the educational process is an important starting point in terms of educational reform. The first steps of this reform process are represented by the conducting of evaluative studies at the national and international levels, because they provide quantitative and qualitative indications of performance levels and measure the impact of several related factors. These evaluations have a role in presenting a clear picture of the education systems' results. They also assist decision-makers in directing educational policies and taking necessary measures to reform the educational establishment by developing educational inputs, such as the curriculum, learning and teaching environments, methods of teaching and evaluation. This has a positive impact on the results of the education system, as seen through the achievements and skills of the students. In this context, the Ministry of Education is eager to participate in the Trends in International Mathematics and Science Study (TIMSS), which is considered to be one of the most important international evaluation systems. It provides a wide and varied database which assists in creating education policy and developing the quality of education.

Following the results of TIMSS, the Ministry has appealed for the enhancement of mathematics education. TIMSS evaluates mathematics and science for 4th and 8th grade students, and has been undertaken once every four years since 1995.This process seeks to highlight international trends in mathematics and science. This is achieved by comparing pupils' performance in mathematics education across an array of cultural, economic and social backgrounds. While measuring achievement, the test also seeks to determine the impact of various factors in relation to performance. The test is quality controlled and overseen by International Commission for the Assessment of Educational Achievement (IEA). It has emerged that Saudi students have not performed on par with their international peers. In fact, the study demonstrated that in 2007 and 2011, Saudi students scored among the lowest in the rankings. While this situation had somewhat improved in 2011, the performance of Saudi pupils remained far below average. Such results certainly demonstrate the lack of well-developed planning and teaching strategies for mathematics education in Saudi Arabia.

The results of these surveys can be found in detail at http://www.iea.nl/home.html.

However, it would be unfair to deny the existence of outstanding Saudi students who have attained advanced capability in the subject. This is demonstrated by their strong performance in both school and university contexts, and the creativity exhibited by entrants into the national mathematics competition. For an instance of this, from the Foundation website of The King Abdulaziz and His Companions for Giftedness (Mawhiba), the following table presents some examples of these achievements.

Table 1.1: Achievements examples of outstanding Saudi students who have attained capability in mathematics

Competition name	Host Country	Year	Medals won by Saudi Arabia		
			Gold	Sliver	Bronze
International Math- Olympiad (IMO)	Thailand	2015		1	3
Balkan Math- Olympiad	Greece	2015			4
Balkan Math- Olympiad Joiner	Serbia	2015	1	1	4
Gulf Math Olympiad	Kuwait	2015	2	4	
European Girls' Mathematical Olympiad	Belarus	2015			1
European Girls' Mathematical Olympiad	Romania	2016	1		2
Gulf Math Olympiad	Saudi Arabia	2016	4	2	

Nevertheless, the results of TIMSS have rendered mathematics an area of heightened attention, which has led to sustained pressure for improvement in mathematics education. Recommendations were made in the wake of the 2007 and 2011 (TIMSS) reports for Saudi students to enhance their strategic capabilities and adaptive reasoning, which would enable them to better solve non-routine problems, similar to those present in the TIMSS tests.

Alsaeed (2012) cited the direct nature of teaching that results in students imitating problem-solving strategies, thus hindering creative thinking and the independent generation of solutions. Using non-routine problems would be a means to encourage students to cultivate diverse problem-solving strategies, allowing them to easily adapt to unfamiliar mathematical scenarios, similar to those on international tests. Alsaeed (2012) demonstrated in his research that stimulating students' thinking and involving them in complex problem-solving procedures (in which the solution is unexpected) did not feature in Saudi teachers' practices. The greater the cognitive challenge posed by the problem, the more helpful it is in fostering students' conceptual grasp. Therefore, the educators of Saudi Arabia should consider exposing students to more complex thinking processes, which could assist them in creating independent approaches to solving and gaining greater comprehension of concepts. Alnather (2009) suggested greater concentration on skills mastery in addition to creating strategies and methods for critical and creative thinking, as well as for metacognitive skills for mathematics teaching methods. A key finding from numerous studies into the Saudi educational context noted that the students observed often had poor elevated thinking skills, and also pointed to the use of traditional teaching methods as a factor in the lack of encouragement for the enhancement of thinking, perception and awareness (Althbaiti, 2012; Alwhhaba, 2008). In the face of such criticisms, the Ministry was then compelled to rapidly exert efforts to develop standardized mathematics curricula.

In response to the previous studies, several further studies were conducted into the Saudi educational context relating to thinking, critical thinking and metacognition. These studies concluded that there is still a necessity in education to activate thinking skills and enhance critical thinking, relevant to the age group that each study was conducted on. The following sections will present an overview of these kind of studies. The reason behind presenting studies into thinking and critical thinking is that, as will be shown in the literature review (see 2.2 and 2.2.2), metacognition is related to cognition, as Flavell (1979) and Brown (1987) explained. In addition, metacognition has a correlation with critical thinking, with metacognition as an essential prelude to achieving the critical thinking (Halpern, 1998; Magno, 2010; Schön, 1983) . Alternatively the link between critical thinking and metacognitive skills may be multidirectional with the two being interrelated, as Veenman (2015) asserted.

1.1.1 Studies into thinking

One prominent study into thinking was conducted by Alfares (2014), who examined the key areas in which thinking skills (TS) were encouraged for English as a Foreign Language (EFL) students in Saudi Arabia, looking particularly at textbooks and teachers. It was revealed that 36.71% of textbook exercises could potentially be utilized to encourage TS. Therefore the majority (63.29%) of the books' exercises held no potential in this regard. While it was noted that some teachers failed to exhibit teaching conducive to TS, those who did hold potential to do so encouraged students to interact, and students were seen to have greater involvement in tasks than in classes led by teachers lacking in this aspect. Based on the results of teacher observation, it was concluded that textbooks which could potentially promote TS are not sufficient in themselves (in this context), and the teachers' conduct greatly influenced the actual productivity of a task.

In another study examining the relationship between teaching and thinking-based learning Alwehaibi (2012) cited previous literature in underlining the significance of providing instructors with diverse methods and strategies for this type of teaching. Consequently, he highlighted the need for teacher training programmes to match the expectations resulting from an increased demand for enhancing students' thinking skills. He called for the effective inclusion of such skills into curricula, which would require training teachers to use thinking skills before their entry into service, which would also extend to teachers already working in schools. There would be an integrative framework for a comprehensive and sophisticated training programme, seeking to build the knowledge, skills, and experience required for training teachers to employ thinking methods in EFL classes. Another outcome of this research was the creation of a checklist for EFL observation, looking to identify actions that encourage, sustain and develop thinking. It was suggested that this checklist be implemented as a criteria for assessing teacher performance. As demonstrated by the results of the current study, the programme was an effective tool in boosting the teachers' thinking skills in several ways. The strategies allowed students to undertake thinking processes and respond thoughtfully. The role of the teacher in the classroom was transformed to a facilitator of learning. Due to this, teachers came to realize that concentrating on questioning and utilizing thinking-based exercises which encourage learners to think independently and propel themselves in their learning were useful strategies.

Alnesyan's study (2012) sought to comprehend teaching and learning thinking skills at the primary level, which was achieved by examining the experiences of teachers and students. The study arrived at four distinct outcomes. Firstly, it was noted that of the most common techniques used by teachers was the infusion approach which inserted thinking skills into subject matter. Secondly, aspects of classroom interaction being complementary was underlined in its importance. Thirdly, students' and teachers' enthusiasm drawn from spiritual/cultural principles had a significant impact on their learning and teaching of thinking skills. Fourthly, the centrality of student and teacher identities was revealed as influencing performance in thinking skills. The instruction and acquisition of thinking skills appeared to be in harmony with the topical areas where the skills were implemented using the infusion approach. The teachers surveyed were convinced by the significant influence of the infusion approach on both themselves and their students. This was because they perceived it as a motivating force in providing opportunities for encouraging thinking skills in students.

1.1.2 Studies into critical thinking

Allamnakhrah (2013) sought to display the perspectives of Saudi teachers in training towards critical thinking-focused education. The study took a qualitative approach and was undertaken at King Abdulaziz University and the Arab Open University. Based on the findings, the necessity for educational reform targeted at critical thinking was identified. Through the interviews conducted, a key shared perspective from participants was that the skill was perceived as being very important. Yet, many participants also pointed to the fact that critical thinking did not form the foundation of pre-service teacher training at either of the universities surveyed.

Following this study Alwadai (2014) published research probing the perspectives of Islamic Studies teachers towards the encouragement of critical thinking skills. This study was conducted at the primary school level in the South Western province of Saudi Arabia. The study investigated the various factors that may influence the use of critical thinking in teaching. One of the findings of the study was that Saudi teachers were not implementing teaching targeted at critical thinking skills, with the reason for this being cited as their own unfamiliarity with critical thinking skills. Alwadai outlined seven key barriers to critical thinking: student ability, teaching methods, classroom structure, socio-cultural factors and the school community, pre-service training for teachers, professional development for in-service teachers, and the Islamic Studies curriculum.

Indeed pre-service training seems to feature a lot in studies targeted at examining the Saudi education system's performance. Gashan (2015) looked specifically at the knowledge of teachers in training regarding the general principles, skills and teaching methods related to critical thinking. The surveyed group in this quantitative study was 29 male teachers-in-training enrolled at King Saud University. Similar to the aforementioned research, in this study too it was found that students at the College of Education were lacking in knowledge regarding critical thinking skills. However, teachers at this institution also held positive perspectives towards the benefits of utilizing critical thinking in teaching, yet were unconfident about whether they possessed the skills required for stimulating critical thinking among their future students.

1.1.3 Studies into metacognition

The studies related to metacognition that were conducted in Saudi Arabia followed quantitative research methods to measure the impact of using metacognitive strategies, be this on academic attainment, attitude, or creative thinking. All the studies confirmed the effectiveness of using metacognitive strategies in learning. Examples of these studies include:

Al-zhrane (2013) sought to identify the effect of employing metacognitive strategy on attainment and the enhancement of creative thinking and to contrast this with the effect of the traditional methods. The group surveyed were science students in the third intermediary grade in Alqrayyat province. Based on the findings of the study it was suggested that more research be conducted into the effectiveness of metacognitive strategies at all age groups and across all subjects. Althbaiti (2012) investigated the effectiveness of the metacognitive learning cycle model in teaching mathematics for developing the creativity and achievement of primary fourth graders. An experimental design was used and the results showed a tangible effect resulting from using the metacognitive strategy for developing mathematical creativity and academic achievement.

Alharthi (2008) investigated the effectiveness of reciprocal teaching in further developing metacognitive reading skills in secondary schools. Aljeladei (2009) also surveyed effectiveness, by looking at the role of a certain metacognitive strategy in enhancing the skills needed for literacy tests. Almalki (2013) looked at the links between metacognition, creative thinking and coping strategies for stress among secondary school educators in Laith. Alghamdi (2012) took effectiveness into account when examining teaching in light of the social constructivist theory, particularly in enhancing numerous learning processes, and metacognitive skills in addition to the attainment levels of female Biology students at secondary schools in Al-Baha District. Ali (2014) sought to measure metacognitive awareness in students at Princess Noura University, and the link between this awareness and other educational variables. Almetari (2014) sought to determine the impact of utilizing metacognitive strategies for boosting English reading comprehension among Year Two secondary students in Jeddah. Similarly, Ismail (2014) sought to assess the effectiveness of metacognitive reading strategies instruction (MRSI) on Taif University EFL students who had achieved low results in reading.

These studies, which note the importance of focusing on the teaching of thinking, critical thinking, and metacognitive skills, recommend further research into measuring the impact of using metacognitive strategies on the attainment of students. Among these was, for example: Gashan (2015) who suggested that pre-service teacher training programmes should be reformed so as to include specialized courses in critical thinking. Trainee teachers' knowledge on the subject should be developed to allow them to self-evaluate their fulfilment of the required skills for their future teaching careers. Alwadai (2014) also presented recommendations for further research that would incorporate qualitative studies, interviews with students, and classroom observation. These would be conducted to gauge student perspectives towards the teaching of critical thinking, as well as obstacles to its further improvement. As for the discourse on the means to activate metacognition in the pedagogical field, this has not been an area of interest or special attention in educational research in the Saudi context. This study seeks to amend this, as mentioned in this review of the context of the current study in addition to the following mention regarding the rationale of this study.

1.2 Rationale for the study

Studies have discussed mathematics teaching and learning in terms of metacognition (see 2.5 for more details). One of the main conclusions from these studies is that students are having difficulties in their mathematics and problem-solving tasks because they are ignoring a wide range of cognitive or metacognitive processes (Cardelle-Elawar, 1992; Grizzle-Martin, 2014; Tok, 2013; Wolf, Brush, & Saye, 2003). The second conclusion is that mathematical performance is significantly improved by applying metacognitive strategies (Bernard & Bachu, 2015; Desoete, 2007; Gillies & Richard Bailey, 1995; Goos, 1993; Grant, 2014; la Barra et al., 1998; Sahin & Kendir, 2013; Schoenfeld, 1987). Despite the results of the aforementioned studies, three significant recommendations in this context should be presented. Firstly, there is a need to study mathematics learning and the role of metacognition from a practical perspective in order to understand how students employ metacognition in enhancing their capability to solve problems, in addition to how teachers can employ metacognition in mathematics instruction effectively with respect to classroom activities. Secondly, it is crucial to study the subject of mathematics teaching/learning and the role of metacognition within the social context. A third subject of study concerns the various methodologies used in metacognition research.

Regarding the first issue, Schudmak (2014) remarked on the need for further research to gain understanding of how behaviours involving metacognition appear during mathematical problem-solving. Education professionals in the mathematics field should inform students about metacognition and assist them to improve their cognizance of metacognitive processes involved in problem-solving. This is in line with Kramarski and Mevarech (2003), Martinez (2006), and Schraw (1998) who all urge teachers to promote general awareness of metacognition in their students by modelling metacognitive skills during instruction. Eldar and Miedijensky (2015) asserted that their study is in agreement with that of Zohar and Barzilai (2013), who suggested that educators should comprehend the meaning of metacognition and deploy it in practice in the classroom. They should also be able to clearly explain metacognitive knowledge and practise metacognitive skills during science (and mathematics) classroom activities.

In terms of the second recommendation, Thomas (2012) explained that as metacognition should assist students to achieve goals in their wider life context, then it is crucial to adapt metacognition in its application to varying realities. Metacognition should be seen as a result of the surrounding environment in which students gain reasoning skills, instead of perceiving it as intuitive. Hence some strategies for implementation may only be suited to certain contexts and models - proposing a broad and ‘one size-fits-all’ nature should be treated with scepticism due to the risks involved. Despite this, according to Thomas (2012), given thoughtfully planned implementation and adjusting classroom environments to facilitate metacognition has allowed for flexibility in developing the concept. Iiskala, Vauras, Lehtinen, and Salonen (2011) point out that despite that much problem-solving and learning occurs in social situations, previous research has mostly neglected to consider metacognition from the social point of view. Therefore, Iiskala et al. (2011) propose that socially shared metacognition is a useful concept which should be added to the conceptual tools of learning research. They recommend that more research is needed on the effects of socially shared metacognition on the quality of problem-solving and learning. Overall, such research ascribes mutual, social metacognition as a significant feature of collaborative problem-solving approaches. Yet an in-depth explanation of what gives metacognition a social and mutual aspect is still uncommon and further efforts are required to understand the social and shared features along with their significance in the problem-solving process.

As for the third recommendation concerning methodology , Whitebread et al. (2009) highlight that much research is dependent on self-reporting or interview-based methodologies in terms of metacognitive and self-regulated performance, rather than a more diverse set of observation strategies. Veenman and Spaans (2005), after evaluating and appraising the various methodologies used in metacognition research, pointed out the need for the enhancement of the observation process so as to better explore the concept. In assessing his own study, Grizzle-Martin (2014) identified that the lack of formal observation led to a scarcity of knowledge about the strength of group communication in differing circumstances. Winne and Perry (2000) pointed out numerous benefits that stem from the use of observation: it provides a real-time record of participants’ actions, rather than their recollections and perspectives of this, and it facilitates the drawing of connections between participant behaviour and the context of the activity. Furthermore, the realistic education context recorded through observation allows for the recording of social interactions relating to metacognitive development. As previously discussed, there is a sizeable body of previous theoretical and empirical research, in line with the Vygotskian, socio-cultural tradition, suggesting that social interaction plays a key role in this regard. Hurme, Järvelä, Merenluoto, and Salonen (2015) remark that in-depth studies on putting metacognition in collaborative learning into practice are few and far between. Furthering understanding of participative metacognition in education and the procedures that develop the use of metacognition from an individual to a group concern should be more explicitly defined.

1.3 Research aims

Based on the previous two sections, there are notably no previous studies in the Saudi educational context (see 1.1.3) focusing on exploring the perceptions of teachers and students towards metacognition in relation to mathematics in secondary students in Saudi Arabia, exploring what if any metacognitive manifestations can be observed in mathematics classrooms, how secondary students and their teachers perceive metacognition in mathematics teaching and learning and what the experiences of secondary students and their teachers are in Saudi Arabia of metacognition in relation to mathematics.

On the basis of the previously mentioned theoretical elements and important recommendations from studies in metacognition and the mathematics field compared with the reality of mathematics learning and teaching in Saudi Arabia, this study sought to identify what is lacking in both mathematics learning and teaching in the classroom regarding metacognition. How does metacognition (if it is used) play a central role in mathematics learning and teaching and why? What are the main benefits and difficulties experienced by students and teachers wishing to improve their mathematical performance through metacognition? Which characteristics that seemed to enhance the positive effects of the interventions were indicated by analysing the beneficial effects of the metacognitive training with students?

1.4 Operational definitions

The concept of metacognition has been defined in different ways. Despite this, it can be concluded, according to Flavell (1979), Brown (1987) and Kluwe (1982), (for more details see 2.2) that metacognition from an educational standpoint refers to a student’s knowledge and the monitoring and control of their own systematic cognitive activity, which requires certain metacognitive skills such as planning and evaluation in order to identify to what extent he/she follows the right approach to achieve his/her goal.

Metacognitive knowledge: it can be considered as knowledge gained about the cognitive processes that govern cognitive activities (Flavell, 1979), which consist of declarative, procedural and conditional knowledge (Cross & Paris, 1988). Declarative knowledge can be understood as knowledge about oneself in a learning context and the factors that could potentially influence performance. Procedural knowledge refers to knowledge about the 'how' to conduct cognitive activities. Conditional knowledge refers to the question of when and why a certain strategy or procedure was used (Schraw, Olafson, Weibel, & Sewing, 2012).

Metacognitive skills: they are in turn related to the range of procedural knowledge used for monitoring and control of a person's cognitive processes (Veenman, 2015) such as planning and evaluation skills.

Metacognitive Strategy: Flavell (1979) used the concept of metacognitive strategy to describe the executive process of monitoring one's cognition. In the same vein, Schraw and Gutierrez (2015) used the same concept to describe the training interventions which can improve the processes of monitoring and control of one's cognition. Using the concept of metacognitive strategy in this study is targeted at assisting students in monitoring and adjusting their thought when dealing with mathematics problems.

1.5 Potential difficulties

There were some difficulties faced in conducting this study. Firstly, metacognition is an unclear term and is not much used in the educational context in Saudi Arabia. Therefore, one difficulty was the need to explain and clarify this concept in practice in the current study to enable the participants, whether teachers or students, to understand it. Secondly, there were difficulties in obtaining permission from the Ministry of Education in Saudi Arabia to conduct this study due to the bureaucratic system in this institution, and also relating to dealing with school principals, as some preferred not to take part in the research because they were already committed to strict lesson plans. Thus, I tried to clarify the objectives and importance of this study to them all, to help in conducting the study and in the achievement of its objectives. Thirdly, further difficulties were encountered in conducting observations in the classroom; it was not easy to convince mathematics teachers to participate in this study as many preferred not to be placed under observation, or felt that the research subject required more effort from them. Therefore, I initially explained the objectives and importance of the study in order to obtain teachers' support. Fourthly, it was unclear whether or not the time available for the collection of data would be sufficient. Therefore, taking advantage of this available time was very important. Fifthly, difficulties emerged related to the time available during the school day for participation, whether for interviews or group discussion. Thus, arranging appointments with participants was not easy during fieldwork. Consequently, it was very important to find adequate support from the Ministry of Education, school principals, teachers and students in implementing this study, and to take advantage of every opportunity. One of the difficulties was that the manifestations of metacognition in classroom mathematics teaching are limited. Furthermore, sufficient data were not provided for this study, and thus there was a pressing need to implement a programme based on metacognition and mathematics learning which would also assist in achieving the goals of this study. Further discussion of this programme and the goal behind its use will be presented later (see 2.5.5 and 3.3.5).

1.6 Structure of the thesis

This thesis comprises six chapters with the following structure:

1) Chapter One consists of the research introduction and background, my interest in this topic, a review of the context of the current study, rationale for the study, research aim, operational definitions, potential difficulties, and structure of the thesis.
2) A literature review forms the second chapter, which provides theories and models of metacognition which include the concept of metacognition and components of metacognition, metacognition and socio-cultural context, assessment of metacognition. Following this is a review of research into the role of metacognition and learning in mathematics, which includes the possibility of improving metacognitive skills and strategies in mathematics learning and teaching through metacognitive training, methodological considerations, metacognition and cooperative learning. The chapter concludes with a review of mathematics

intervention programmes specifically the IMPROVE programme. Consequently, this study will present a summary regarding the nature of the relationship between metacognition and mathematics, in addition to the research aim and its associated questions.

3) The third chapter in this research is the methodology chapter, which includes theoretical and philosophical assumptions (ontological and epistemology), social construction, research design (which includes methods for collecting, selection of participants, general procedure of data collection, pilot and main study, trustworthiness, and generalization from the case study), ethical considerations and data analysis
4) Chapter Four presents the thematic findings.
5) Chapter Five provides discussion and interpretation of the findings of this study which include teacher and student perceptions of metacognition in light of the literature of the study, teaching and learning of mathematics according to metacognition, based on the implementation of the IMPROVE programme, and cooperative learning and metacognition
6) Chapter Six presents the overview of the study, research limitations, implications of the study for the mathematics teacher, students, educational supervision and the school administration and policymakers, suggestions for future research and the final conclusion.

2 Literature review

Previous studies into metacognition draw on a diverse range of theoretical frameworks which suggest key concepts in addressing its nature and components (Peña-Ayala & Cárdenas, 2015). This chapter begins with a comparison of the three major theories of metacognition Flavell (1979), Brown (1987), and Kluwe (1982). Following this is a review of research into the role of metacognition and learning in mathematics, including discussion of whether students can improve their mathematics' learning through metacognitive training, methodological considerations, and metacognition and cooperative learning. The chapter concludes with a review of mathematics intervention programmes, most notably the IMPROVE programme.

2.1 Introduction

Flavell (1979) indicated that young children have thinking limitations of cognitive enterprises. Therefore, researching of cognitive monitoring and cognitive regulation is important in developing these kinds of activities for children and adults alike. The term 'metacognition', coined by Flavell, emerged from this research area (Flavell, 1979). Use of the term 'metacognition', according to Brown (1987), began in psychological literature within two different research areas: knowledge about cognition, and regulation of cognition. The former refers to one's knowledge concerning thinking processes, whereas the latter refers to the regulation and monitoring of one's course of thinking. Similarly, Kluwe (1982) claimed that research relating to metacognition is based on distinguishing between one's own knowledge about cognition and the executive processes of cognition. The former refers to one's own knowledge about features of one's cognition and that of others, whereas the latter refers to the monitoring of cognitive activity, its application, and its effects on problem solving strategies, in addition to the regulation of the course of cognition.

Despite these premises, to present a certain definition of the metacognition concept is still difficult. Adding to this difficulty in definition, differentiating between cognition and metacognition has proven to be another issue. Efklides and Misailidi (2010) underscored this by explaining that the differentiation between cognition and metacognition is challenging and that the wide range of metacognitive phenomena would indicate that there is no single word to define the complex processes involved in metacognition. Peña-Ayala and Cárdenas (2015) explained that cognition means to know, and elaborated on this by suggesting cognition involves an individual's perception and comprehension of the world, and how he/she behaves in that context. According to them the process of cognition covers acquisition, development, and exploitation of a range of knowledge-based and cognitive functions. Whereas, knowledge itself, consists of memories which have been shaped by the manipulation and integration of 'raw input' – or rather information processed through one of the five senses or resulting from cognitive functions such as thought, reasoning, recall, learning and experiences. Forming a key part of cognition is the way in which we organize our knowledge through association or categorization. Knowledge can come in many forms – for example, facts, beliefs and symbols (such as & or $), which are then used to gather and combine more intricate associations. Knowledge is then used to guide or adjust actions towards targets – thus forming the basis of cognitive activity.

According to Peña-Ayala and Cárdenas (2015), the challenge of definition appears, as cognitive abilities cannot necessarily be distinguished from one another, they can overlap. Hence, cognition has been divided into wider cognitive abilities, for example, perception, attention, reasoning, speaking, planning, learning… There is a difference between the metacognitive and cognitive processes, as pointed out by Kuhn (2000) who further explained that cognitive processes are involved in doing, while the metacognitive processes are involved in choosing and planning what is required and monitoring what is being done.

Taking all these arguments into account, the presentation of a definition for metacognition does not mean that there is unanimous agreement about the borders of the concept. This is due to the fact that, over time, the scope of definition has grown in tandem with metacognition becoming a multifaceted concept (Buratti & Allwood, 2015). Despite this, a need for theoretical clarity is certainly present. This would include improved definitions and descriptions of the numerous components of the concept (Azevedo & Aleven, 2013). Hence, the following section will cover three original models of metacognition that further our comprehension of the nature of metacognition, its components, and their relationships to one another.

2.2 Theories and models of metacognition

This section covers three essential models of metacognition which in turn help us to clarify the concept. These three models will be those presented by Flavell (1979), Kluwe (1982) and Brown (1987). There are three reasons behind choosing these models, which in turn assisted in the undertaking of this study: they provide a theoretical framework for metacognition instead of others which concentrate on specific aspects of metacognition (Gama, 2004). Secondly, they significantly distinguish between different classifications of metacognition - knowledge and regulation of cognition (Gama, 2004). Thirdly, they have the most relevance for education. As a result, the practical definition of metacognition and its components will be included at the end this section.

The concept of metacognition was explained by Flavell, Brown and Kluwe. Flavell (1979, p. 1232) referred to metacognition as “one’s knowledge concerning one’s own cognitive processes and products or anything related to them”, and as:

> The active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or date on which they bear, usually in the service of some concrete goal or objective. (p. 1232)

Brown (1987) referred to it as someone’s knowledge and control of their own cognitive system. Similarly, Kluwe (1982) emphasized that,

> There are general attributes which are common to these activities referred to as ‘metacognitive’: a) the thinking subject has some knowledge about his own thinking and that of others; b) the thinking subject may monitor and regulate the course of his own thinking. (p. 202)

On the basis of this premise, it can be said that the concept of metacognition contains two major elements; firstly, knowledge of cognition, secondly, monitoring and regulating ones’ own cognition, which can be called executive processes, as described by Kluwe (1982).

Kluwe (1982) distinguished between a general knowledge which refers to one’s beliefs about information processes systems, and specific knowledge, which refers to one’s knowledge and belief about features and traits of this cognition and others, such as differences between individuals. Flavell (1979) talked about this kind of knowledge when he split it into knowledge regarding variables which act and interact in various ways to influence the course and results of cognitive enterprise, these variables being person, task and strategy, and described it as metacognitive knowledge.

Flavell (1979) provides an explanation of these variables. In terms of the ‘person’ variable, it can be said that all perceptions concerning the nature of one’s thinking as well as others’ are included in this category, i.e. perception about individual differences or universals of cognition: for example, when you think that you can learn better by listening than reading. When someone realizes that they can perfectly understand something now which they could not understand previously, they ponder how they will understand in the future. The second variable, according to Flavell (1979), relates to knowledge that one already uses during a cognitive activity in order to determine whether a cognitive activity can be managed to accomplish an objective. In addition, a child’s perception of distinguishing between difficult and easy cognitive activities can be included in this category. The third category is strategy. There is a great deal of information concerning which strategies can be effective in achieving either the subsidiary or main aim of cognitive enterprise.

Flavell (1979) believes that this kind of knowledge has an important influence on both a child’s and an adult’s cognitive activity because it enables them to select, revise, assess, adjust or even omit a cognitive task, object and strategy in the light of their relationships with others and their ability and interest in the cognitive activity. Similarly, it can lead them to have metacognitive experiences regarding person, task and strategy and help them as well to interpret the meaning and behavioural application of metacognitive experiences.

On the basis of these arguments it can be concluded that the first aspect of metacognition refers to one’s own knowledge or beliefs about features of one’s cognition, as the above authors agreed; knowledge about the information processing system, as Brown (1987) added: and knowledge about three categories, person, task and strategy variables, as Flavell (1979) illustrated.

The second aspect of metacognition refers to the monitoring and regulation of cognitive enterprise. Flavell (1979) used the concept of metacognitive strategy to describe the executive process through monitoring one's cognition. Brown (1987), on the other hand, described both as metacognitive skills, which are assumed to both monitor and regulate one's systematic cognitive activity. Kluwe (1982) asserted that the executive process has two main functions aiming directly at gaining knowledge about one's cognitive processes: monitoring these processes and regulating one's cognitive activity. The executive process, according to Kluwe (1982), refers to four elements that are included in executive monitoring: Identification (what am I doing?), checking (did I succeed? did I make progress?), evaluation (is my plan good? are there better alternatives?) and prediction (what could I do? what will the result be?).

According to Brown (1987), the second aspect of metacognition includes activities such as planning, monitoring and evaluation. Planning activities requires predicting the results, planning strategies, choosing alternative trails, etc. Monitoring activities requires testing, rescheduling and revising learning strategies. An evaluation outcome requires evaluating the use of effects in the light of the criteria of effectiveness and efficiency. Flavell (1979) explained that monitoring of an extensive variety of cognitive enterprises occurs through the interaction of four categories: metacognitive knowledge; metacognitive experience; goal or task; action or strategy. Flavell (1979) presented an example of these four categories: when a task needs to be completed, knowing the degree of difficulty or ease of this task can be considered as metacognitive experience. This metacognitive experience can be gathered from previous metacognitive knowledge to logically select suitable cognitive strategies. These in turn allow learners to gain more metacognitive experience. This forces learners to concentrate on thought processes and further enhances metacognitive knowledge. This discovery of strategies occurs in light of metacognitive knowledge and metacognitive experience, demonstrating the interaction between metacognitive knowledge and experience. According to Flavell (1979), metacognitive experience occurs before, during or even after cognitive enterprise and occurs as a reaction to stimulating situations of awareness and careful thought, or a task which requires explicit thinking or several important steps which have to be scheduled before assessment, and afterwards, when making a risky decision. Some metacognitive experiences are considered to be a kind of metacognitive knowledge which interacts with the consciousness. For example, when a wide gap is perceived by the learner between him and achieving his aim, this is not considered to be metacognitive knowledge. However this feeling and required actions are informed by metacognitive knowledge; thus there is an obvious overlap between metacognitive knowledge and metacognitive experience because some experience contains knowledge and some does not. On the other hand, some knowledge, as well, can become conscious and include experiences.

According to Flavell (1979), metacognitive experience has many benefits. Firstly, it has an essential impact on cognitive goals, cognitive or metacognitive tasks, cognitive activities and strategies. Secondly, it can lead to the establishing of a new aim or change or even to omit a previous aim. Thirdly, it impacts on metacognitive knowledge by adding or refining or even deleting it. Fourthly, it influences activation-directed strategies to gain cognitive or metacognitive goals. For example, when you know that you cannot perfectly understand certain information needed to pass an examination (metacognitive experience) this information will be revised (cognitive strategy) so as to improve knowledge (cognitive goal). In addition, if it is known sufficient revision has been conducted for an exam, certain introspective questions will be asked to test if the information can be remembered (metacognitive strategy) to achieve a cognitive goal of passing the examination. This in turn creates a new metacognitive experience. On the basis of these premises, learners engage with cognitive strategies to progress cognitively, with metacognitive strategies being employed to survey this procedure. It is clear that a strategy can be used to achieve both aims at the same time. For instance, self-questioning can be conducted either to improve knowledge about something or to monitor this process. Therefore, metacognitive knowledge can include knowing about either cognitive or metacognitive strategies. To conclude, according to Flavell (1979), monitoring cognitive enterprises is carried out through the interaction of a variety of categories: metacognitive knowledge, metacognitive experience, goals (task) and actions (strategies).

This idea was expressed by Kluwe (1982) when he explained that,

> It is important that human beings understand themselves as agents of their own thinking. Our thinking is not just happening, like a reflex,

> it is caused by the thinking person, it can be monitored and regulated deliberately, and it is under the control of the thinking person. (p.222)

On the basis of these arguments, it can be concluded that the second aspect of metacognition refers to the monitoring and regulation of cognitive enterprise. Despite these premises, it is difficult to distinguish between which activity is 'meta' and which is not. Distinguishing between both depending on its function is one approach, according to Flavell (1979). Flavell's example is when a question is asked by the reader regarding the improvement of knowledge after reading a paragraph; this is cognitive, while a question concerning monitoring this improvement might be metacognitive. Similarly, it is difficult to differentiate between cognitive and metacognitive strategies. Some strategies are themselves cognitive, such as when looking for the main idea in a text whereas monitoring and evaluating this process is considered to be metacognitive (Brown, 1987). Both Brown (1987) and Kluwe (1982) discussed skills used to control one's cognition such as planning, prediction, identification, checking, monitoring and evaluation.

Taking all these arguments into account, a need for theoretical clarity is certainly present. This would include improved definitions and descriptions of the numerous components of the concept (Azevedo & Aleven, 2013). With regards to improved definitions, it can be concluded that metacognition from an educational standpoint refers to one's knowledge and the monitoring and control of one's own systematic cognitive activity which requires certain metacognitive skills such as planning and evaluation. Noteworthy in the context of discussing the concept of metacognition, the important issue remains determining the basic subject of the concept of metacognition. Particularly since Brown (1987) mentioned that the concept of self-monitoring and control method is essential in the growing field of metacognition and Kluwe's view (1982, p. 220) being that "the subject of metacognition is regulation of one's own information processing".

2.2.1 Components of metacognition

With regards to descriptions of the numerous components of metacognition, the two main aspects of metacognition are one's knowledge and the monitoring and control of one's own systematic cognitive activity. The following sections present these components in some detail.

2.2.1.1 Metacognitive knowledge

Metacognitive knowledge can be considered as a deeper understanding of cognitive processes (Flavell, 1976). Flavell (1979) stated that metacognitive knowledge is knowledge gained about the cognitive processes that govern cognitive activities. Hence, having metacognitive knowledge means being aware of the strong and weak points present in our cognitive resources, strategies and abilities, especially related to the performance of certain cognitive tasks.

Cross and Paris (1988) and Jacobs and Paris (1987) considered declarative knowledge, procedural knowledge and conditional knowledge as important. Among these, being conscious of the thinking process (Jacobs & Paris, 1987) is termed 'procedural metacognitive knowledge'. This also includes the knowledge of the means by which goals and aims can be attained. Conditional metacognitive knowledge is defined as being aware of the circumstances and environment that have an impact on learning, for example, why tactics and certain approaches are successful and when they need to be implemented (Jacobs & Paris, 1987). Surat, Rahman, Mahamod, and Kummin (2014) explained that declarative knowledge (facts and information) is "knowledge about" or "knowledge concerning". This could involve a whole range of thought and information, from fact to beliefs, opinions generalizations, theories, hypotheses and attitudes towards objects or other individuals, or even oneself. According to Surat et al. (2014, p. 213) declarative knowledge can be facilitated through the following questions; "(i) What do I want to know? (ii) What keywords and information can be obtained? (iii) What is already known by me? And (iv) what information should I seek?" In contrast to declarative knowledge, procedural knowledge which pertains more to the 'how' aspects of cognitive activities. It assists us in controlling various factors when examining or appraising a phenomenon (e.g., a set of steps taken in solving a problem). Conditional knowledge spans across the 'when' and the 'why' aspects in regards to the choice of strategy. In a similar manner Schraw et al. (2012) ascribed knowledge of cognition as the information we possess about our own cognition. It generally includes three sub-components. The first, declarative knowledge, encompasses knowledge and awareness about oneself in a learning context and the factors that could potentially influence performance. As opposed to this, procedural knowledge involves knowledge about strategies and procedures. The last of the sub-components was conditional knowledge, which involved knowledge used to decide why and when to use a specific strategy.

2.2.1.2 Monitoring and control of one's cognition

As mentioned previously, Flavell (1979) used the concept of metacognitive strategy to describe the executive process through monitoring one's cognition and he used the concept of metacognitive experience to describe the control or regulation of one's cognition. Brown (1987), on the other hand, described both of these as metacognitive skills, or executive process as Kluwe (1982) stated, which are assumed to both monitor and regulate one's systematic cognitive activity. In this regard, Nelson and Narens (1990) stressed the role of monitoring and control by explaining that their model of metacognition was based on three precedents: firstly that cognitive processes are divided into related levels – the meta and the object. Secondly, within the meta-level lies a 'dynamic model' (e.g. a mental simulation) of the object-level. Thirdly, two dominance relations exist, these being 'control' and 'monitoring'. These relations are determined by the direction in which information flows between the aforementioned levels. The two features of division into levels and dominance relations have been explained as follows. The concept of control can be depicted through speaking into a telephone handset. In this example, the information is directed from meta-level to the object-level and either transforms the status object-level process or transfers the object-level process itself. The resulting action at object level could be either initiation, continuation or termination. As control does not produce information at the object level, monitoring (independent of control) becomes relevant. Similar to control, the telephone analogy is relevant, yet in this case, listening instead of speaking represents the meta-level being alerted by the object-level. This influences the monitoring level's modelling of a situation, even including 'no change in state'.

Building on this discussion about the components of metacognition, it can be concluded that the two main aspects of metacognition are one's knowledge and the monitor and control of one's own systematic cognitive activity. The former contains all knowledge concerning the nature of one's thinking as well as that of others'. This could be knowledge that a learner already uses during a cognitive activity in order to address how a cognitive activity can be managed so that he/she can accomplish his/her objective. It is also knowledge that enables learners to distinguish between difficult and easy cognitive activities, and knowledge regarding which strategies can be effective in achieving either the subsidiary or main aim of the cognitive enterprise and knowledge about information process systems. The latter contains the monitoring and regulating of systematic cognitive activities, which requires some skills including the ability to use metacognition in such as planning and evaluation (see 2.2).

As far as these skills are concerned, it was found that these identified skills helped in the development of the necessary procedural knowledge that is needed to control and regulate a person's learning actions (Veenman & Spaans, 2005).There has been a large amount of data gathered on the four skills which help in the development of the necessary procedural knowledge that is needed to control and regulate a person's learning actions, namely orientation, planning, monitoring and evaluation (Lucangeli & Cornoldi, 1997; Lucangeli, Cornoldi, & Tellarini, 1998). Orientation skills, which can also be termed as potential prediction skills, are the determining factor behind the slow accomplishment of new or complex tasks and the fast accomplishment of easy or familiar tasks, and these help an individual to consider the learning purpose, its features and the time given. Learners use prediction skills to assess the difficulty level of any task, then use this evaluation in its accomplishment. According to Garrett, Mazzocco, and Baker (2006) they can decide which task is easy or difficult, which requires time, skill and effort, all of which students possessing strong predictive skills can successfully accomplish. Prediction skills also enables learners to understand how one problem is associated with another while developing the intuitive knowledge as to what conditions are required to carry out a task (Desoete, 2009).

Planning is done intentionally to set certain sub-goals, the aim of which is to ensure the smooth completion of any task (Winne, 1997). Desoete, Roeyers, and Buysse (2001) suggested that, in classroom settings, planning involves going through problem solving by examining a question, determining its type and then working out the manner in which this question will be solved before executing it. With planning skills, students can reflect on their experience to determine why, how and when to do something to reach their objective by working through a chain of events that will help them solve the problem successfully.

Monitoring skills are implemented at the time of the ongoing activity so as to identify any difficulties and improve conditions (Brown, 1987; Sigmund Tobias & Everson, 1996). Typically, according to Desoete et al. (2001), monitoring is associated with learning questions and context. Questions such as ‘Is this plan working?’ and ‘Is the plan being followed?’ are part of the process. It can be said that monitoring skills are the control aspects of cognitive skills used to not only identify problems but also to make quick adjustments to the plan of action. Monitoring is a person’s awareness level of the problem solving and learning strategies in use. This also includes being able to make use of other strategies to avoid errors and improve understanding.

The evaluation skill is concerned with reflections which are carried out after the task is done (Brown, 1987). Here learners review the tactics used, their success rate and their result; they consider the problem, the plan’s suitability, how the solution technique was achieved and whether the solution was sufficient in contrast to the problem (Garofalo & Lester Jr, 1985; Vermeer, 2000). These evaluation skills are important for the students since they assist them in considering the solution and in identifying any errors they might have made. Weak evaluation skills lead to weak monitoring skills and so it will not be an easy task for the students to decide if their plan is appropriate or the solution was the correct one (Garrett et al., 2006).

Schraw and Gutierrez (2015) propose an additional skill, so presented a total of five skills in this area, these being planning, organizing, monitoring, debugging and evaluation. In clarifying these, they state that panning strategies involve the creation of goals and readying oneself for an effective learning experience. Meanwhile, organizing includes applying strategies for information management. Monitoring uses on-line learning assessment, while debugging involves strategies targeted at reducing and eliminating performance errors or false assumptions about task and strategy. Finally, evaluation involves retrospective analysis of both performance and the effectiveness of strategies employed.

Examining the available literature, there is a vast array of research e.g. (Corliss, 2005; Fowler, 2004; Gama, 2004; Kumar, 1998; Schraw & Brooks, 2011) which defined procedurally the major metacognitive skills as planning, monitoring and evaluation. Sáiz-Manzanares and Montero-García (2015) identified the importance of the consistent usage of such skills from start to end of problem solving. They also explained that the skills rely on the intentional control over one's own cognition. These researchers sequentially ran through the steps of this process, beginning with the suggestion that the learner should examine the task, recall previously learnt information, set goals and implement planning for problem solving and strategies for regulating the cognitive process. Following this, and during the progression of the task, the learner can now systematically pursue the plan, this being done through constant monitoring of process – thus guiding and regulating task completion. Finally, when the task is completed, the learner may now evaluate the process used to tackle the problem, as well as the solution that he/she has arrived at. This enables the student to learn from the experience and this knowledge can then be used to solve similar problems in the future.

The disagreement surrounding the number of metacognitive skills raises questions as to the source of this disagreement. In this context, the following reasons can be cited. Firstly, the disagreement over the number of metacognitive skills is based on whether or not metacognition is a broad concept and inclusive of all fields. Furthermore, whether or not there are defined skills for metacognition, or if metacognition differs across the fields in which it is being utilized, with the according skills differing based on that. One view in support of the notion of a universal set of skills is that of Schraw (2002), who states that metacognitive skills can manifest in numerous and diverse areas, and that students can build knowledge in numerous areas while constructing overarching knowledge, for example understanding the constraints of memory and regulatory skills such as strategy selection. Schraw (2002) notes that this difference plays out across age groups, in the sense that students from older age groups can attain generalized metacognitive skills suited for a variety of tasks, allowing them to operate in multiple domains. Overall, Schraw's views point to the notion that there exists a universal set of metacognitive skills and knowledge that manifests across subject areas and age groups. However, in contrast to Schraw's view, Veenman (2013) states that metacognitive skills can be customized and adjusted depending on the tasks as production rules become subject to task constraints. Veenman (2013) also points out a certain flexibility and adaptability to these skills, in that they are dependent on external factors. He (2013) highlights that competence with metacognitive skills can be seen as an aptitude, which is a fixed point of reference in interacting with learning environments. This is not to say that the skills themselves are fixed, as learning experiences, teaching and training may influence such skills. Veenman does not entirely disagree with Schraw in explaining that there does indeed exist a generalized set of metacognitive skills, but these must be adjusted depending on specific tasks. In this context, Veenman (2013) points out that learners have quite a consistent and stable set of skills which they draw upon in encountering new learning situations. Activating this set of skills will be done through adjusting them to specific task demands, along with other contextual factors.

Secondly, another area of disagreement surrounds the origin of the meaning of metacognitive skills, thus Desoete (2007:712) states that 'areas of non-agreement were discussed with reference to the definitions of the skills and were resolved through mutual consent'. If it is supposed that these skills hold a defined meaning, as Veenman (2013) states that metacognitive skills relate to the procedural knowledge needed for regulating and controlling cognitive activity, then researchers differ on terminology in the context of discussing these skills, variously describing them as procedures, strategies, or activities targeting the control of thinking. For example, Schraw (2002: 4) in one instance uses the phrase 'a set of activities' in stating that 'regulation of cognition refers to a set of activities that help students control their learning'. In another instance, he (2002: 4) uses 'regulatory skills' in explaining that 'although a number of regulatory skills have been described in the literature, three essential skills are included in all accounts: planning, monitoring, and evaluating'. In contrast, Veenman (2013: 157) uses the term 'metacognitive skills'. Based on the differences in researchers' use of these terms, the metacognitive skills noted in their studies will differ accordingly.

Thirdly, Desoete indicates that there is another reason for disagreement related to the linking of these skills with age groups, which creates a variety in these skills depending on the age at which the skills are learnt. Desoete (2007:717) states, ‘Metacognitive skills may be age-dependent and still maturing. Evaluating metacognitive skills in young learners may shed light on areas of weakness and their function in learning and development.’ Desoete (2002: 122) states that ‘metacognitive skills have been found to be maturing until adolescence’. As already mentioned Schraw (2002) notes that differences can be found across age groups, in the sense that students from older age groups can attain generalized metacognitive skills suited for a variety of tasks, allowing them to operate in multiple domains.

Fourthly, the disagreement over the number of metacognitive skills may stem from the fact that there are researchers who place a skill under the classification of another. For example Schraw (2002: 5) places the skill of predictions under the skill of planning. Indeed, he states that ‘planning involves the selection of appropriate strategies and the allocation of resources that affect performance. Examples include making predictions before reading, strategy sequencing, and allocating time or attention selectively before beginning a task.’

Based on this discussion, the current study has discussed these skills in the context of mathematics teaching within the Saudi context, as will be noted in the discussion chapter (see 5.1).

Despite that it can be said that metacognitive knowledge and the monitoring and control of one’s own systematic cognitive activity are the two main aspects of metacognition, others add metacognitive beliefs as a main aspect of metacognition (Simons, 1996). Hence the following section discusses Self-Beliefs in some details.

2.2.1.3 Metacognition and self-beliefs

In addition to metacognitive knowledge, monitoring and control processes, Kuhn (2000) includes metacognitive beliefs in the model of metacognition. Metacognitive beliefs, according to Schoenfeld (1992); revolve around an individual’s perceptions and insights, such as the ideas a person generates when doing mathematics and how this changes the manner in which he does it. Similarly, Cook, Salmon, Dunn, and Fisher (2014) discussed metacognition as the knowledge, beliefs and cognitive processes employed throughout the course of monitoring, control and evaluating cognition. Two areas of metacognitive beliefs hold relevance: positive and negative. The positive set of beliefs involve confidence in the benefit of strategies in dealing with negative emotions and thought processes (e.g. the belief that stress is a helpful coping mechanism). The negative beliefs involve the perceived overpowering and threatening nature of such preservative thinking (e.g. the stress is spinning out of control). Schudmak (2014) held views consistent with this, in which he identified two aspects of metacognition: knowledge and beliefs about cognitive phenomena, and the regulation and control of cognitive actions.

Despite that, metacognitive beliefs can be seen as a distinct aspect of metacognition, some authors termed these metacognitive beliefs non-metacognitive but affecting and motivating factors (Boekaerts, 1999; Garcia & Pintrich, 1994; Masui & De Corte, 1999; McLeod, 1992; Vermunt, 1996). This idea is supported by Lucangeli and Cornoldi (1997) and Lucangeli et al. (1998) who do not regard metacognitive beliefs as a distinct component of metacognition but rather associated with metacognitive knowledge only. Heyman and Dweck (1996) discussed beliefs as the driving and determining factor behind behaviour. They can be regarded as a tool for implementing metacognitive knowledge and skills (Boekaerts, 1999). Attributions are what one believes to be the cause of success and failure and they are important to the individual's set goals (Vermeer, 2000). To illustrate this aspect practically, Schoenfeld (1992) claimed that a number of students consider that mathematics and problem solving do not relate to their real life and they have a strong belief that only highly intelligent individuals can learn mathematics. According to Lester (1994) researchers believe that research should clarify the role and impact of metacognitive beliefs in problem solving.

2.2.1.4 Studying the components of metacognition

Noteworthy in the context of our discussion regarding the components of metacognition is studying the relationship between the components of metacognition, knowledge, and the monitoring and control of one's own systematic cognitive activity. Secondly, it is important to study the improvement of metacognition through using strategies designed in this field. For example, regarding the first issue, Veenman, Hesselink, Sleeuwaegen, Liem, and Van Haaren (2014) claimed that possessing declarative knowledge does not ensure its use in regulating learning. This type of metacognitive knowledge may have errors or may not be comprehensive, as such the learner may not succeed in identifying areas to potentially apply such knowledge, or may even lack the necessary skills to do so. In order to complement this knowledge, Schraw and Gutierrez (2015) highlighted heightened monitoring and control as a means to improve learners' information-gathering, and consequently, metacognitive awareness. Due to an enhanced monitoring-control process, learners should be able to elevate their conditional knowledge (e.g., implementing effective strategies in the right context while acquiring predicated on-task demands). To elaborate further on the interrelation between components, an issue in the literature was that most models simply highlight and describe aspects, rather than discussing how they interact (Peña-Ayala & Cárdenas, 2015).

As for the second issue of improving metacognitive skills through specially designed strategies, Schraw and Gutierrez (2015) report that there is a dearth of strategy-instruction programmes centred on conveying monitoring and control skills to students. Four strategies were suggested by Schraw and Gutierrez (2015), these being targeted specifically at monitoring and control. The first is training all students in the automatic and rapid recall of a range of strategies, using the most up-to-date institutional practices and covering the five main categories (planning strategies – creating goals and preparing for effective learning; organizing – applying information management strategies; monitoring – on-line assessment for learning, performance or strategy use; debugging - reducing and eliminating performance errors or false assumptions about tasks, and strategy; evaluation - retrospective analysis for both performance and the effectiveness of strategies employed. A second strategy is to improve teachers' use of strategy through training so that they can teach the strategies and conditional knowledge regarding these. Thirdly, it was suggested that clear monitoring and control training could be employed for older students, with feedback after this. Lastly, students can be assisted in learning how to build mental structures of the task. This will help educators to evaluate their understanding and will allow students to self-regulate. Previous research on metacognitive strategy training has argued that training interventions, even if short and minimal, can improve the processes of monitoring and control, (Schraw & Gutierrez, 2015). Through strategies gained in training, students can become more aware and focused on a task by decelerating cognitive processing, thus assisting in introspection relating to internal cognitive workings. The strategies they learn during training subsequently encourage students to collect information about a task in a more precise and efficient manner (Schraw & Gutierrez, 2015).

2.2.2 Metacognition, critical thinking and self-regulation

In this section, certain assertions will be presented, since there are a number of terms that are linked to metacognition such as critical thinking and self-regulation, and it is important to realize that each term has different definitions. For instance, Halpern (1998) proposed a definition for critical thinking as conscious, purposeful, and goal-directed reasoning to achieve a sought result when tackling complex problems, inferring, analysing assumptions, estimating probabilities, and decision making. Thus, critical thinking draws on cognitive strategies and skills for reasoning, and on metacognitive skills to monitor and control the reasoning process. Halpern (1998) also discussed the term of critical thinking as involving the deployment of cognitive skills or strategies that boost the likelihood of successfully reaching a desirable result as a long-term process. Halpern (1998) highlighted the relevance of metacognitive planning, monitoring, and evaluation activities in critical thinking. Hence a wide range of these higher order cognitive skills can lead to critical thinking processes. In this regard, Schön (1983) maintained that skills related to critical thinking were improved through metacognition. Similarly, Magno (2010) asserted that metacognition created and resulted in critical thinking. In clarifying this, Hogan, Dwyer, Harney, Noone, and Conway (2015) claim that reflective judgement is a key component of metacognition, which is employed in the area of critical thinking to make reflective judgements and decisions. According to Veenman (2015), numerous quantitative surveys have examined the link between metacognitive and critical thinking skills. The general finding was that enhanced metacognitive skills translated to a corresponding effect on critical thinking. These studies arrived at the conclusion that metacognitive skills act as a prerequisite to satisfactory critical thinking performance. Veenman (2015) added that critical thinking processes may refine metacognitive knowledge and skills and boost conditional knowledge when deciding which thinking skills to apply and how to boost effectiveness of metacognitive skills when conducting those of critical thinking.

Hence the link between critical thinking and metacognitive skills is multidirectional with the two being interrelated.

Another instance is the concept of self-regulation. Vohs and Baumeister (2011) examined it as one of the essential concepts in the field of metacognition, which encompasses planning, organizing, self-instruction, self-monitoring, and self-evaluation. On the other hand, Whitebread et al. (2009) pointed out that self-regulation was a more general umbrella term which encompassed the monitoring and control of the human condition, which involves emotional, social and motivational facets. In this regard, Larkin (2010) claimed that gaining the capability of monitoring and control of learning will result in more effective time management and self-awareness in learning. This understanding of metacognition ties in with the concept of self-regulation. Larkin (2010) identified self-regulation as an overarching concept that encompasses metacognition. Models of self-regulation encompass a wider range of concepts than metacognition; these range from emotions, motivation and context to cognitive monitoring and control processes. Yet metacognitive awareness is a fundamental necessity for learners seeking to self-regulate.

2.3 Metacognition and sociocultural context

There is evidence, according to Brown (1987), that a great deal of learning happens through interactions between the learner and others. Thus, a teacher who is interested can improve a child’s active metacognition by providing him/her with related experiences regarding regulation skills which are created within a social situation. According to Larkin (2010), a large body of research on metacognition has surrounded information processing models and cognitive psychology since the 1970s. Another significant area of study has been concerned with understanding the way in which metacognition assists in ‘wise and thoughtful life decisions’ as Flavell (1979, p. 910) put it. The concept of agency among social psychologists has also been of great importance, looking into how individuals act purposefully though monitoring and evaluating behaviour. The way in which we ‘think about thinking’ and develop metacognition of self, other, tasks and strategies is dependent on the sociocultural context (Larkin, 2010).

In this regard, Thomas (2012) highlighted two beliefs surrounding metacognition that should be questioned: that metacognition in all its forms is a positive influence, or that only one type of metacognition is beneficial. These premises do not take into account the influence of the context in which students operate. As metacognition should assist students to achieve goals in their wider life context, then it is crucial to adapt metacognition in its application to varying realities. Metacognition should be seen as a result of the surrounding environment in which students gain reasoning skills, instead of perceiving it as intuitive. The way in which cultures evaluate effective thought and consequently metacognition differs greatly across the globe. Hence some strategies for implementation may only be suited to certain contexts and models - proposing a broad and 'one size-fits-all' nature should be treated with scepticism due to the risks involved. Despite this, according to Thomas (2012), given thoughtfully planned implementation and adjusting classroom environments to facilitate metacognition has allowed for flexibility in developing the concept. Thus, it is essential to accept varying opinions and create a unified but multifaceted theory in defining metacognition, its assessment and enhancement throughout everyday learning contexts, and this should be given priority in guiding future reform efforts. Based on these premises, Larkin (2010) concluded that a theory of metacognition which boosts a process of reflection and self-criticism, encourages individuals to discuss education, considers the needs of specific groups in specific contexts, and allows for introspection on issues such as the student-teacher relationship, would be a theory that can be employed in order to build a more socially representative education establishment. In this regard, Larkin (2015) highlighted the sociocultural theory of metacognition in identifying the interrelated social, psychological and cultural aspects of education and the development of metacognition.

2.4 Assessment of metacognition

Veenman, Van Hout-Wolters, and Afflerbach (2006) claimed that,

> The evolution in understanding metacognition is paralleled by an evolution in our understanding of assessments that are suitable for measuring and describing metacognition. (p. 8)

Borkowski, Chan, and Muthukrishna (2000) stated that there are three phases of metacognition assessment. The first phase relies on introspective reports about memory states and processes, particularly on the fact that children can accurately report their knowledge about memory processes as they relate to a variety of tasks, circumstances, and strategies. The second phase relies on interconnections between memory knowledge and memory performance. However some conceptual and methodological problems hinder the search for reliable and valid measures of metacognition that continue to affect contemporary research, such as a lack of consistent definitions for each metacognitive construct and a lack of thoroughly analysed tasks that permit the separation of process and performance measurements. The third phase focuses on the issues of monitoring and control which refer to executive functions of metacognition and their correlation to a variety of motivational variables.

The literature on metacognition provides many methods for the assessment of metacognition. However, Larkin (2005) asserts that inferences from classroom performance, interviews of students questioned concerning their knowledge and cognitive processing, and analysis of 'think aloud' protocols are typically the grounds of assessments of metacognition. Obviously, each one of these assessment methods has both advantages and disadvantages. For instance, a questionnaire is more suitable for large groups, while thinking-aloud protocols are better suited to individual assessments (Veenman et al., 2006). Some instruments of metacognition assessment relate to certain subjects such as problem solving or reading text. For instance, an example of problem solving is a version of the Kreutzer, Leonard and Flavell questionnaire, which focuses on person, task and strategy variables. This can be used to research the correlation between metacognitive knowledge and aptitude in problem-solving (Kreutzer, Leonard, Flavell, & Hagen, 1975). An example of a reading text, the Reading Strategy Use (RSU) is a self-report measure, developed by Pereira-Laird and Deane (1997) to assess adolescent students' perceptions in the use of cognitive and metacognitive strategies of reading. Schmitt (1990), as another example, developed a multiple-choice questionnaire to measure elementary students' awareness of strategic reading processes, while Miholic (1994) developed a multiple-choice inventory to measure students' metacognitive awareness of reading strategies. Despite these instruments seeming to have limitations for use in research, they are aimed at increasing student and teacher awareness of metacognition in reading rather than just measurement of metacognitive or other reading strategies. Similarly, the Metacognitive Awareness of Reading Strategies Inventory (MARSI) is a self-report measure, designed by Mokhtari and Reichard (2002) to be used as a comprehensive gauge of students' comprehension monitoring capabilities. This instrument enables students to evaluate themselves compared with other readers. This idea, according to Mokhtari and Reichard (2002), is based on transferring responsibility for monitoring learning from teachers to students themselves, and promoting positive perceptions and motivation among students.

In the study of mathematics, several attempts to measure metacognition have been made. One of these was that of Fortunato, Hecht, Tittle, and Alvarez (1991), who tasked students with solving an irregular problem and presented them with 21 statements about thought processes throughout problem solving, to which they would respond. This was done to gauge metacognitive ability in relation to problem-solving performance. Schraw and Dennison (1994) created a Likert scale consisting of 52 items allowing adults to self-report (MAI). This was targeted at measuring knowledge and awareness of cognition. Following this, Sperling, Howard, Miller, and Murphy (2002) developed the idea of the MAI inventory and created two further inventories for younger age groups, known as the Jr. MAI. Panaoura, Philippou, and Christou (2003) also created an inventory derived from the idea of MAI (1994), Jr MAI (2002) and the Fortunato et al. (1991) questionnaire – this was done to assess metacognitive ability among mathematics students at primary school.

According to Tobias (1995), the metacognitive evaluation procedure used in many studies is intended to determine students' accuracy in certain skills or strategies concerning their cognitive activities. Therefore, the positive findings relating to enhanced accuracy of specific metacognitive skills or strategies should not be generalized to the rest of the metacognitive skills or strategies.

Although there are many methods for assessing metacognition, this subject presents some difficulties. The reliability and validity of the means of measuring is the main difficulty for measuring metacognition due to the lack of theoretical consensus regarding this concept (Larkin, 2005). Similarly, Mokhtari and

Reichard (2002) state that the main critical issues regarding measures of metacognitive awareness,

> The use of scales with a small number of items, limited psychometric properties, evidence of reliability and validity, or an uncertain characterization of the construct of metacognition in particular. (p. 3)

Larkin (2005) added that measuring one individual's metacognitive knowledge in isolation is still difficult because several overlapping aspects are involved in metacognitive knowledge, such as range of application, level of awareness, coherence, ease of access, etc. Adding to these shortcomings, Sadeghi, Hassani, and Rahmatkhah (2014) claimed that assessing metacognition was fundamentally hindered by the fact that it was not an explicitly outward behaviour. Yet metacognition being an internal procedure alone was not the only issue; another problem highlighted was that many individuals may not even be aware of instances when they are practising it.

On the basis of these premises, three key points can be made as to the necessity of evaluating metacognition in the first place. Firstly, it is important to accurately determine which metacognitive knowledge or metacognitive skill should be assessed by which technique (Veenman et al., 2006). Secondly, it is important to decide which stage of cognitive activity should be focused on in order to measure a particular metacognitive skill or strategy. For example, off-line assessments of metacognition encompasses several skill areas, such as prediction and evaluation, which are used either before or after task performance, while on-line assessments including skills such as monitoring are used during task performance (Veenman et al., 2006). Finally, although the findings of many studies of metacognition assessment indicate that the technique used to determine students' metacognitive knowledge, skill or strategy seems useful for further investigations of metacognition as Sigmund Tobias (1995) mentioned, metacognition should not be regarded as a final objective for curriculum or instruction. Instead, it should be regarded as an opportunity to enable students to manage their own thinking for active learning (Mokhtari & Reichard, 2002).

2.5 Metacognition and mathematics

The current section discusses the nature of the relationship between metacognition and mathematics. Subsequently it will discuss the significance of metacognition as a key factor behind students' academic performance in the field of mathematics, the difference between the cognitive and metacognitive processes of mathematics, and the possibility of improving metacognitive skills and strategies in mathematics learning and teaching through metacognitive training, methodological considerations, metacognition and cooperative learning, and the IMPROVE programme. Consequently, it will present a conclusion regarding the nature of the relationship between metacognition and mathematics.

2.5.1 The nature of the relationship between metacognition and mathematics

Zhang (2014) indicated that problem solving was an essential foundation of mathematics learning. After establishing this he proposed a correlation between metacognition and problem solving; he actually identified the study of metacognition as holding answers to many of our questions about the decision making process in problem solving. It was underlined that due to our current lack of understanding of metacognition and its functioning, the development of a theory for mathematics problem solving was less likely. In this regard, Cardelle-Elawar (1992) highlighted that a wide range of cognitive or metacognitive strategies are being ignored by children while solving mathematical problems, thus making it difficult for them to solve these problems, as indicated by the research in the field of mathematics and metacognition. However, this could lead to the assumption that struggling students are lacking in crucial metacognition (Coles, 2013). According to Schoenfeld (1987) and Goos (1993), mathematical concepts are better understood if children are taught to think and reflect metacognitively. To affect the mathematical thinking, metacognition plays a central role, which will eventually affect children's general academic performance and specifically their mathematical performance (Panaoura & Philippou, 2005; Schoenfeld, 1992). According to Yimer (2004), the student's inability to perform the required regulation and monitoring processes for learning is the main factor behind poor performance in mathematical problem solving and it is not due to a lack of mathematical knowledge. Adding to those premises, Tok (2013) highlighted a deficiency in mathematics teaching as it mostly focuses on knowledge rather than the role of metacognition in problem solving. Grant (2014) provided further evidence to this end, as students possessing heightened metacognitive skills tended to attain better grades in mathematics. Based on this, he discussed the increasing prominence of standardized testing, which has now led to the need for students to build on knowledge with more profound thought on how thinking is conducted. This would entail not only improved thinking skills, but effective self-assessment and control of cognition. Thus, it may be important to evaluate the metacognitive skills with respect to the complicated nature of the mathematical domain, to focus on metacognitive training and to enhance its potential to affect mathematical performance (Desoete, 2007).

The effectiveness of using metacognitive strategies was again demonstrated by Gama (2004) who claimed that research showed that students who used metacognitive processes performed better than their classmates who did not. One such study by Mevarech (1999) showed increased mathematical problem solving capacities by those who received training in metacognition over those who did not. Gillies and Richard Bailey (1995) conducted their study on a total of thirty-nine fifth grade students from two classes at a primary school in New South Wales. They claimed that the performance in mathematical problem solving and mathematical achievements can be enhanced by applying metacognitive strategies. In the same vein, la Barra et al. (1998) concluded that the quality of mathematical thinking and problem solving was enhanced through metacognitive strategies in addition to developing a student's achievement. Likewise, enhancing mathematical thought among learners involves the enhancement of metacognition (Coles, 2013). In a further study, Sahin and Kendir (2013) sought to determine the impact of metacognitive strategies for problem solving in a fifth grade geometry class. They examined achievement, metacognitive skills and attitudes towards the strategies. It appeared that the students had a more positive outlook towards geometry and mathematics in general, which could be due to an enhanced self-confidence. Another factor was that students could now comprehend the significance of problem solving and better understand problems. It was also noted that they had developed an awareness of the importance of planned study, along with controlling their problem solving. Their increasingly positive attitude to geometry and the study of mathematics was matched by an improvement in achievement levels. Grant's (2014) study included the research question, "how can a metacognitive-based tutoring programme improve math abilities of rural high school students?" In the hopes of achieving this outcome, Polya's (1965/1981) schema was called upon. This laid out four stages in the problem solving process, and was used to understand strengths and gaps in students' mathematical knowledge as well as their awareness of solving methods. Students were asked to complete a daily journal answering the following questions:

> (a) What did I learn today? (b) When and how do I plan to use my work? (c) When should I switch to another strategy? Or what should I try next? and (d) Does this solution make sense? Why or Why not? Explain." (Grant, 2014, p. 33).

Grant (2014) touched upon such improvements, explaining that the use of metacognitive processes to enhance mathematics capabilities allowed students to reflect on previous knowledge as well as to implement strategies gained in training sessions to summon previous mathematics experience. Bernard and Bachu (2015) highlighted the importance of understanding the basis of each problem in mathematics, what it requires, analysing it and evaluating a range of solutions – this would help in enhancing metacognitive abilities.

Based on these premises, Moseley, Elliott, Gregson, and Higgins (2005) asserted that research on metacognitive skills indicate that the future mathematics learning of students is strengthened in those who practice metacognitive skills and build on their knowledge through the process of reflection. The question that arises, however is, why are metacognitive processes important in the performance of mathematics? As multiple mental activities are included in metacognitive thinking, it is a significant factor. The effectiveness of a problem solving process will increase when a student is able to monitor his own learning. This fact was endorsed by Schoenfeld (1987), and (Venezky & Bregar, 1988), in their studies. According to Stacey (1990), more metacognitive skills are demonstrated by students who excel in solving problems, as they are aware of the processes of learning and mathematics. In presenting a practical implication of such strategies Cetin, Sendurur, and Sendurur (2014) revealed that students in the control group would frequently ask classmates for solutions whereas most of those in the experimental group who had been introduced to metacognition favoured diverse strategies when faced with difficult problems. (A sample of 28 male and 23 female 2nd-year university students with an average age of 21 from the department of Computer Education and Instructional Technology in Turkey were involved in this study). They also tended to examine varying sources to fully comprehend the problem. They mostly consulted classmates when trying to understand conceptual issues or to share ideas. This is consistent with the conclusions set forth by Teong's (2003) research of forty 11-12-year-old low achievers in mathematics, which stated that making choices derived from metacognitive strategies indicates more sophisticated problem solving. Hence, metacognitive teaching may have played a role in encouraging the students to ponder the question rather than the solution first.

Another issue to be taken into consideration is the impact of strategies on students' metacognitive skills in the context of mathematics learning. One study revealing such effects was that of Tok (2013), who discussed the impact of K-W-L on the metacognitive skills of 6th grade students (KWL are an acronym for the instruction steps that comprise, in the course of a lesson, what students already know, want to know, and ultimately learn). The sample cohort was a group of 55 6th grade students enrolled in government elementary schools. While a control group was instructed in mathematics using traditional methods, the aforementioned K-W-L strategy was introduced to an experimental group. Subsequent results indicated that the usage of such strategies enabled the development of metacognitive skills. In a contradictory study conducted by Yang and Lee (2013), a group of 9th grade students in Taiwan was studied to reveal the impact of cognitive and metacognitive strategies on general metacognitive ability. Metacognitive-Strategy Worksheets (MSW) were distributed to encourage the use of relevant strategies during the problem solving process. However, the study's findings highlighted that such forms of instruction and resources did not appear to have a wide-reaching impact on the students' metacognitive abilities. The reasons for this were not clarified in the study – perhaps the programme used was not suited to the study, or furthermore, the method of implementation in the study may have been deficient in some regards. However, after reviewing this study, it appears that it did not focus on cooperative learning in discussing ideas. Besides this the nature of the programme in this study did not appear to contain the features of metacognition in a clear manner. Moreover, the allotted time for the strategies' implementation was a mere 15 minutes, and this is not in line with the recommendations of the study of Sahin and Kendir (2013) who emphasized that sufficient time should be provided for the problem solving process, and rather than rushing students to finish, it was recommended that they be reassured about taking a slower and more cautious approach.

Another area to be discussed is the difference between cognitive and metacognitive processes in the subject of mathematics, which poses a particular challenge (Garofalo & Lester, 1985; Goos & Galbraith, 1996). In order to systemize and organize the study of metacognition, various proposals for frameworks linked to problem solving performance have been made, according to Yimer and Ellerton (2006). The metacognitive processes were differentiated from the cognitive processes in the framework proposed by Artz and Armour-Thomas (1992) by stating that the actual processing is expressed verbally or non-verbally in the cognitive processes, whereas problem solving is expressed through statements made about a problem solving process in the metacognitive processes. For instance, when I come to realize that the problem is more complex than I had earlier presumed while solving a mathematical problem, this is a metacognitive process. When I decide to read the problem again to fully understand the problem, I have applied the cognitive process, after applying the metacognitive process of when I decided that the best option would be to start all over again. The progress of the problem may be monitored (metacognitive) while the problem is solved. I would keep moving forward with my objective (cognitive), if I am satisfied with the progress. I would however look for another option (metacognitive), if the task was not solving. Therefore, the actual processing of the word problem will define a cognitive behaviour, whereas the knowledge of and awareness of a word-problem solving strategies and the use, control and regulation of these strategies to regulate the performance of the problem solving will be deemed as metacognitive behaviours. To this end, Desoete et al. (2001) stated that cognition is supervised by metacognition and Veenman et al. (2006, p. 5) said 'metacognition draws on cognition'.

2.5.2 Can mathematics learning be improved through metacognitive training?

Grizzle-Martin (2014) pointed out that numerous academics have proposed theories on mathematical problem solving. These have generally concluded that students struggle to attain competency in mathematics without the aid of metacognitive processes, proving them to be crucial in a student's success. Other theories discussed the concept of strategy instruction, which has been found to be an effective method to address mathematical problem solving, elevating thought skills (Grizzle-Martin, 2014). This is in line with Naglieri and Johnson's claim (Naglieri & Johnson, 2000) that instructions can improve mathematical performance, and increase awareness of cognitive activities for children. Students are able to consider issues in a broader context while solving problems through problem solving strategies. In the same vein, Moga (2012) concluded that mathematical performance was improved through metacognitive development which was induced after metacognition was taught in classrooms at both the elementary and secondary school levels. Naglieri and Johnson (2000) suggested that with the provision of metacognitive instruction, students' performance in mathematics can be further boosted, hence drawing attention to the significance of planning to gain desired results. Thus, Desoete (2009) highlighted the fact that metacognition needs to be taught explicitly so that the mathematical and problem solving skills of the students can be developed. In the same vein, Grizzle-Martin (2014) discussed metacognitive instruction, and strongly recommended clear teaching that concentrates on cognitive and metacognitive strategies. Many studies have displayed the positive outcomes of teaching metacognitive strategies, and identify improved self-regulation, self-direction and achievement as results Raoofi, Chan, Mukundan, and Rashid (2013). A further benefit is metacognition's impact in creating more introspective thinking. Grizzle-Martin (2014) detailed this by explaining that once students could clearly understand their own comprehension, they would then be better equipped to reflect on their learning.

The research literature related to the field of improvement of metacognition was summarized by Hartman (2001). The four main approaches were presented by them as models introduced by the teachers to encourage general awareness, improve metacognitive knowledge, improve metacognitive skills and foster learning environments. It was observed that modifications could be introduced in the metacognitive skills through explicit training which also added value to the mathematical problem solving strategies. According to Mevarech and Kramarski (1997), the ability to transfer knowledge and improve mathematical performance can be developed by training the students to differentiate between the similarities and differences of problems. The students are able to plan their solving strategies and monitor their performance to change their selected strategy, if required, when they are provided with explicit instructions and metacognitive strategies (Schoenfeld, 1987).

With the growing body of research surrounding metacognition and its impact on learning, Carr (2010) recommended that the mathematics curriculum should include metacognitive instruction so as to improve the pace and quality of learning. Schudmak (2014) stressed the need for clear instruction on how to express thinking and sufficient time for students to do this in order to collect information about students' mathematical thought processes. Hence, it is significant that Larkin (2000) recommended that teachers themselves begin to reflect and question metacognitively on the means to improve metacognition in students. This should come in tandem with professional training, but the teacher should have a stake in the theory itself as a genuine belief in its importance for learning will assist in effecting change in others. Teachers will not be able to perform this while they are not provided with sufficient training in this field, as stressed by the study of Sahin and Kendir (2013), who established that teachers themselves should be given on-the-job training for problem solving and metacognitive association in order to enable them to implement such processes soundly. Thus, teachers first and foremost should be educated about instructing students on this so they can fully engage their students in gaining such strategies. In this regard, Thomas (2012), suggested that while the importance of metacognition is widely accepted, its implementation into practice among teachers falls short. The enhancement of metacognition will demand the aforementioned capabilities, but the extent to which they are already practised is not particularly evident. Hence further studies into how teachers practice metacognition could lead to an improvement in assisting them to bring metacognition into science (or mathematics) learning contexts. In the same vein, Coles (2013) pointed out an absence of studies investigating metacognitive requirements placed on the teachers seeking to enhance this type of skill among their students. This absence is concerning, considering the numerous studies demonstrating that an instructor's conceptualization of mathematics and student learning has an impact on classroom discourse.

2.5.3 Methodological considerations

According to Thomas (2012), after conducting a literature review, it was found that interventions seeking to boost metacognition tend to fall into one of two categories. Firstly, a focus extensively on the use of heuristics features (such as concept maps, reading charts, Venn diagrams) and learning strategies which are known commonly as metacognitive activities. In this regard, despite it being a challenge to achieve the goal of rendering thinking more evident and boosting activation and transmission of learning in order to improve learning (McGregor, 2007), numerous studies have recommended further research on learning through metacognition within education. Further research was also called for on models to ensure effectiveness in the development of metacognition through teaching and learning in the classroom (Moseley et al., 2005). This approach, according to Thomas (2012), has several advantages: first, as students attend class with the goal of learning science (or mathematics), adding metacognitive training to the mix of typical science (or mathematics) tuition improves attendance for such activities, in turn increasing the opportunity for reflection on these activities. Hence, it can be assumed to an extent that learners metacognitive knowledge would be enhanced from this embedding of metacognitive activities into typical instruction. Yet certain doubts have been expressed as to the suitability of using this approach for enhancing metacognition, according to Thomas (2011). If students are not consciously reflecting on the newer tasks introduced to the classroom and the impact on learning, then the development of metacognition can be questioned. Thus, Thomas (2011) concluded that conscious reflection on the efficiency of learning is essential for the development of metacognition. However, data confirming the presence of this in science (and mathematics) learning contexts remains weak.

In this regards, Larkin (2010) distinguishes between the conscious performance of metacognition and the classification of certain automatic processes as metacognitive activity, pointing out the ongoing debate surrounding this. Automatic processing is regarded as quick, easy and controlled by the actor, but also occurs unconsciously. This poses a challenge to theorizing metacognition, as tasks which were previously conscious acts may become automated and no longer within the realm of conscious thought and voluntary control. However the benefits of this process are clear as it renders a greater proportion of memory available for use in other areas.

An alternative to the metacognitive strategies approach was reported by Thomas (2012) and he mentioned that this approach comes in line with recommendations provided by Schraw (1998), in that metacognitive knowledge is perceived as multidimensional, domain-general and can be taught. This involves tasking students to reflect on the nature of learning, in this case with science (or mathematics), in line with constructivist epistemology, which advocates entering learners into unfamiliar contexts, where their preconceived conceptual frameworks can be challenged and they would be required to ponder unfamiliar perspectives of scientific (or mathematics) concepts. In asking students to reflect on how science (or mathematics) learning can best be perused and evaluated, it is conducive to conditions in which scientific conceptual change and metacognition - especially metacognitive knowledge - occur. Metacognitive experiences are embodied when students are tasked with reflecting on the feasibility of existing science (or mathematics) learning conventions and urged to implement and evaluate processes in line with newer conceptions. In performing the aforementioned steps, students are taking conscious decisions related to their learning (Thomas, 1999). In planning to deploy metacognition for mathematics instruction, Coles (2013) explained that it is incorrect to understand metacognition through any set of knowledge and skills, declarative or procedural. These skills alone are not sufficient in addressing problem solving heuristics. He highlighted the need for clearer criteria for the appropriate time to use these skills. In conclusion, Larkin (2010) mentioned that conscious introspection into automated cognition is not equivalent to being aware of cognition in the moment it occurs. Hence, metacognition should involve consciousness of current cognition, and monitoring and controlling this. Perhaps when certain cognitive skills have been practised extensively in varying contexts, an automated metacognition occurs. Yet in dealing with young students who are inexperienced and yet to understand metacognitive behaviour, clarity and the modelling of conscious metacognitive behaviour is crucial.

2.5.4 Metacognition and cooperative learning

Research has shown that metacognition can be developed through co-operative or collaborative learning (Bernard & Bachu, 2015; Hurme et al., 2015 and Kramarski & Mevarech, 2003). However a distinction needs to be made between these two terms as they refer to different types of learning although both are founded on constructivist learning theory (Bernard & Bachu, 2015). In order to understand them, the intended meaning behind these terms will be clarified, along with the context of their usage. Following this, it will be summarized into a provisional definition to be used in this study.

According to Chinn (2010), the two terms may be used interchangeably in everyday and even academic language. The rationale behind this is that student participation through small groups predominates in both situations (whereas passive lecture-based teaching, or the 'traditional method' as referred to in this study, favours the completion of particular tasks). Both strategies also fundamentally support a discovery-based method of learning (Chinn, 2010). It has been suggested that this misconception has emerged due to the overlap in both the concepts themselves and the use of the terms (Pannitz, 1996). According to Pannitz (1996), while collaboration is a belief system or even a philosophy held in terms of lifestyle, cooperation is a structure for interaction targeted at a defined goal related to the content. Hence it is more structured and guided than collaborative learning, with the teacher playing a role in control of the interaction. In cooperative learning, groups are focused on advancement towards a teacher-set goal, rendering the group more structured. In contrast, with collaborative learning groups differ depending on group members rather than a goal (Panitz, 1999).

Rockwood (1995) identifies the contrast also, yet he states that specific tasks exist in both methods and notes that comparisons among groups regarding method and conclusion are present in both contexts. However, he identified that the key difference was in the type of knowledge that the strategies dealt with. In this he concluded that cooperative dealt with traditional (canonical) knowledge whereas collaborative was more social-constructivist. Despite existing examples of collaborative learning being implemented in primary school such as CASE@KS1 (Adey, Robertson, & Venville, 2001), Panitz (1999) suggested that cooperative learning be applied mostly at the primary school level but this could extend to secondary. This was because this age group required structure in order to achieve targets and maintain focus. He claimed that collaborative learning was more suitable for tertiary education, where foundational knowledge was already well developed and non-foundational knowledge should be focused on, or where concepts may require conference and exchange of ideas without a set answer. The group can operate as a feedback mechanism in such a context, where unanimous agreement is not the end result. Rockwood (1995) concurred with this view by stating that cooperative learning was a useful means to reach mastery of fundamental knowledge, and only then would students become ready to converse, discuss and assess.

Based on these premises, it is difficult to determine which specific definition of group work to apply with all of its characteristics in mathematics learning through metacognition at the secondary stage. However, it can be said that group work in mathematics learning in secondary schools in the Saudi educational context takes some advantages from both methods. This is because this style of learning generally has some cooperative characteristics, as highlighted earlier; however, the teacher's role is not central and is supposed to remain supervisory. This is specifically seen when solving problems as a step in the context of dealing with mathematics problems (see 4.1.1.4 and 4.2.1.4). There should be innovation to generate solutions and methods to solve problems or to understand new mathematical concepts. Furthermore, the teacher should not, at this stage, be guiding this process but rather supervising it. If it is suggested for the student to be the centre of the learning process in each stage of solving the mathematics problem, than this is confirmed at the stage of finding a strategy to solve (as discussed in 5.3.4). Hence it is challenging to classify the systematic learning of mathematics for this age group in the characteristics of a single framework from among the types of group learning. This is because mathematics learning at secondary school holds characteristics of both types. As a result, clarifying the provisional definition for the nature of group work remains the important aspect, whether it is named as cooperative or collaborative learning.

Artzt and Newman (1997) outlined necessary ingredients for a group working towards a common goal. Firstly, there must be a perception of teamwork and a common goal. Secondly, groups must realize that the problem is shared and the benefits of success or burdens of failure are equally shared among all members. Thirdly, to achieve this goal, members must interact with one another and discuss all problems. Lastly, it should be obvious that all individuals' work has a direct impact on the success or failure of a group. Adding to this, in the context of the current study, group work is focused on certain prepared activities which had a previously defined goal dependent on the steps of the IMPROVE programme. Finally, despite the benefits of group work, this is not to say that other methods are invalid, as the importance of teacher instruction and individual work remains, as Blatchford, Kutnick, Baines, and Galton (2003) asserted.

In terms of how researchers regard metacognition and cooperative learning, there is uncertainty towards cooperative learning's effectiveness in improving the effects of metacognitive training. Hinsz (2004) explored the improvement of one's understanding of cognitive processes through metacognition in a team setting. A comparative study was conducted by Desoete (2007) among students who had all undergone metacognitive training, yet were divided by those who had done so individually and those who had done so in small groups. The study indicated that the individually trained students improved more than the students trained in groups. This is because there are no external stimuli in the individual sessions to distract the students when they are analysing the task, building connections between the new and old knowledge and solving problems through strategies. A study conducted by Moga (2012) showed that both the individual and group training showed improved results. The study conducted on seventh grade students showed that students in the individual training programme showed better results of improvement in prediction skills compared to the group training session. He justified his study's result by claiming that the Romanian education system does not support cooperative learning and hence students are not familiar with learning in a group environment, so the results obtained were expected as per the conditions. The results of Kramarski and Mevarech (2003) are contradicted by these studies, as they endorsed the concept that cooperative learning combined with metacognitive training seemed to yield much better results than the individual ones. In the same vein, Goos and Galbraith (1996) concluded that interaction within groups could either harm or encourage metacognitive decision making during problem solving. The deciding factor in this was the students' capacity and willingness to share metacognitive training. Artz and Armour-Thomas (1992) expressed more definitively that problem solving in small groups may encourage metacognitive behaviours, thus assisting students to find sound solutions. Bernard and Bachu (2015) concurred with this view by explaining that collaborative learning has assisted students in problem solving by encouraging metacognition. Hurme et al. (2015) presented findings indicating that when pairs worked on computer assisted problem solving, metacognition was a mutual process and encouraged peer thinking. Yet participants of a group must all participate in the monitoring and control of collective problem solving to effectively build knowledge. According to Hartman (2015), while metacognition has previously been theorized as self-reflection on thought, pairs and groups can also collectively be involved in metacognitive activities. Coles (2013) called for further research into the idea of co-regulation in group settings to determine similarities and dissimilarities in cognitive processes, the influence of this on self-regulation, and the effectiveness in arriving at learning outcomes.

Hartman (2015) employed the term metacognitive group activities to describe groups of 3-4 students, whereas Hogan et al. (2015) employed the term metacognitive collaboration. This involves a process of group members pondering and reflecting on their collective information processing, and attitudes towards work. According to Hogan et al. (2015, p. 90) various features need to be present to bring about effective metacognitive collaboration. These are:

> effective facilitation, feedback and instruction for the collaborative process and goals; fostering improved team functioning in the collaborative context, including the encouragement of cooperative, investigative discourse; and the use of tools and methodologies which facilitate group coherence, and the management of complexity and group problem-solving. (p. 90)

Hurme et al. (2015) described the role of metacognition in collaborative learning contexts, where metacognition was perceived as a mutual social dynamic. This shared social metacognition is both the monitoring and regulation of cognitive processes on the interpersonal level. Overall, such research ascribes mutual, social metacognition as a significant feature of collaborative problem solving approaches. Yet an in-depth explanation of what gives metacognition a social and mutual aspect is still uncommon and further efforts are required to understand the social and shared features, along with their significance in the problem solving process.

2.5.5 IMPROVE programme

Shifting from theoretical to practical domains in this field, one example of training for mathematics teaching is the IMPROVE programme presented by Mevarech and Kramarski (1997). The programme is centred on the belief that learning is not a rote process but rather one of interpretation, as many constructivists would argue. With the occurrence of new information, learners try to link this to previously learnt information and their own experiences. In doing this, students build meaningful relationships between new and previous knowledge, thus leading to the assertion that this is a process of construction rather than recording and memorization. In the context of mathematics, students try to make sense of linking between new and previous knowledge. During this process, learners will attempt to analyse the problem and its nature, invoking existing strategies, tactics or principles and making comparisons with previously encountered similar problems. Due to the fact that previous knowledge is so relevant in knowledge construction, small group settings are optimal because previous knowledge is often varied. This diversity in knowledge is useful as it can be exploited in agglomerating the input of all the group members so as to provide a wide knowledge bank for learners to draw from in the knowledge building process.

One method is to create and answer questions that revolve around information processing procedures. Mevarech and Kramarski (1997) asserted that the decisions taken relating to the ‘when’, ‘why’ and ‘how’ aspects of problem solving, planning, monitoring and evaluation contain a form of control and regulation. Hence, it can be suggested, according to Mevarech and Kramarski (1997), that encouraging students to create specific types of questions could lead to more intricate justifications of when, why, and how to use strategies/tactics/principles; and inferences about the introduced concepts; along with fresh perspectives towards some areas of previous knowledge. Such questions should be concerned with (a) the structure of the problem, (b) links between the new and previous knowledge, and (c) specific strategies/tactics/principles that are suitable for solving the new problem. The IMPROVE programme encompasses three interrelated components (Mevarech & Kramarski, 1997):

> (a) Facilitating both strategy acquisition and metacognitive processes; (b) Learning in cooperative team[s] so four students with different prior knowledge: one high, two middle, and one low-achieving student; and (c) Provision of feedback-corrective-enrichment that focuses on lower and higher cognitive processes. (p. 369)

IMPROVE is an acronym for the instruction steps that comprise the method: Introducing new concepts, Metacognitive questioning, Practising, Reviewing and reducing difficulties, Obtaining mastery, Verification, and Enrichment. This is designed for implementation in smaller groups which include four students of diverse capabilities, particularly after a concept has been introduced to a class. Students pose three forms of metacognitive questions, these being categorized as comprehension, strategic and connection questions.

The IMPROVE method has proven to have a sizeable positive impact on mathematical performance in problem solving for seventh-grade students (Mevarech and Kramarski, 1997). In one instance, Kramarski and Mevarech (2003) examined junior high school students of diverse achievement levels in mathematics so as assess IMPROVE's influence on performance. The sample included a control group who were taught with traditional methods throughout the school year and an experimental group who were instructed through IMPROVE. The performance of problem solving is significantly affected by the use of metacognitive processes among the experimental group in relation to the control group, this being assessed through various measures of mathematics performance. Studying a younger cohort of 91 7th grade students, Kramarski, Mevarech, and Arami (2002) decided to split the group via metacognitive and non-metacognitive learning methods. The first group underwent metacognitive learning in a cooperative setting. The second also learnt in a cooperative setting, yet this time without the metacognitive element to teaching. The tasks set were the same, and the study was conducted over a six week period. The cooperative learners who were also provided with metacognitive instruction significantly outperformed their classmates who were not instructed in this way on various measures of mathematics achievement, both in standardized and authentic tasks.

A further study performed by Mevarech and Amrany (2008) looked at Israeli high school students studying for university enrolment exams. The group was again split among those who had been exposed to IMPROVE instruction and those who had not. As in the aforementioned studies, the results showed that students who had studied using IMPROVE attained higher results in the entrance exams than their classmates who had not. It is noteworthy that Mevarech and Amrany (2008) emphasized the need for students to be trained in cognitively regulating their learning so as to achieve the desired results. In a more recent study, Kramarski and Michalsky (2013) claimed there were significant advantages to IMPROVE questioning prompts, in regards to the long-term shift to unfamiliar tasks for students with either advanced or weak previous knowledge. In a similar manner Grizzle-Martin (2014) assessed the effectiveness of IMPROVE in enhancing the performance of students with Maths Learning Difficulties (MLD). The 2013 Georgia Criterion-Referenced Competency Test (CRCT) for total mathematics scale scores was used and it was found that students who had been exposed to IMPROVE significantly outperformed those who had not. Thus Grizzle-Martin (2014) specifically attributed this to the mathematical teaching components of IMPROVE which boost comprehension through cognition and metacognition.

A study undertaken by Cetin et al. (2014) also showed that an IMPROVE-based intervention resulted in enhanced results for learning the basic concepts of computer programming and was thus an effective tool. This study observed that the implementation of IMPROVE at the start was difficult, yet with practice became easier. Cetin et al. (2014) stated that in the study of computer programming, the task of problem solving may pose difficulty for beginners, due to a lack of previous exposure to general concepts, problems and solutions. The metacognitive training provided to the participants who included a total of 1072 students (322 male and 750 female) at the Canakkale Onsekiz Mart University sought to direct approaches to problems and the means of progressing past these. A ‘laboratory sheet’ was provided in the study, and included four reflection questions for each programming task derived from the IMPROVE model to guide learners. A second observation in implementing this study was that some students in the group surveyed were reluctant to engage in reflection questions throughout the duration of the study. This was due to a variety of factors: students did not perceive any benefit, due to time constraints, or the fact that it did not gain them their desired grades. Yet despite this resistance, they still appeared to reap some of the benefits of the exercise (Cetin et al., 2014).

2.5.6 Conclusion regarding the nature of the relationship between metacognition and mathematics

In summary, there are several essential dimensions regarding the nature of the relationship between metacognition and mathematics, which in turn provide this study with important points of discussion.

Firstly, a key finding in the literature was that students perceive difficulties in mathematics and problem solving tasks because they are neglecting a wide range of cognitive or metacognitive processes (Cardelle-Elawar, 1992; Grizzle-Martin, 2014; Tok, 2013; Wolf et al., 2003). However, this could lead to the assumption that struggling students are lacking in crucial metacognition (Coles, 2013). Secondly, many studies asserted that mathematical performance is significantly and positively affected by applying metacognitive strategies (Bernard & Bachu, 2015; Desoete, 2007; Gillies & Richard Bailey, 1995; Goos, 1993; Grant, 2014; Sahin & Kendir, 2013; Schoenfeld, 1987). Hence, metacognition plays a central role in the learning process, which ultimately affects the student’s academic performance at school generally and their mathematical performance specifically (Almeqdad, 2008; Grizzle-Martin, 2014; Panaoura & Philippou, 2005; Schoenfeld, 1992). Thirdly, and more specifically, the student’s inability to perform the required monitoring and controlling process in their learning is the factor behind low performance in mathematics, rather than a lack of mathematical knowledge (Grant, 2014; Tok, 2013; Yimer, 2004). Hence, the effectiveness of a problem solving process will increase when a student becomes capable of monitoring and controlling his/her own learning processes (Grant, 2014; Sahin & Kendir, 2013; Schoenfeld, 1987). Fourthly, students can be trained to improve mathematical performance through metacognitive skills such as monitoring or regulation (Grant, 2014; la Barra et al., 1998; Sahin & Kendir, 2013). Fifth, teachers need to explicitly instruct their students to monitor and subsequently control their cognition in order to become more self-directed in their mathematical performance (Desoete, 2007, 2009; Grizzle-Martin, 2014; Raoofi et al., 2013; Schoenfeld, 1987). Sixthly, it is important that teachers themselves begin to reflect metacognitively on the means to improve metacognition in students. This should come in tandem with professional training, but the teacher should have a stake in the theory itself, as a genuine belief in its importance for learning will assist in effecting change in others (Larkin, 2000). Teachers will not be able to perform this as long as they are not provided with sufficient training in this field, as stressed by the study of Sahin and Kendir (2013). Thus, teachers first and foremost should be educated about instructing students on this so they can fully engage their students in gaining such strategies. Coles (2013) pointed out an absence of studies investigating metacognitive requirements placed on the teacher seeking to enhance this type of skill among their students. This absence is concerning, considering the numerous studies demonstrating that an instructor's conceptualization of mathematics and student learning has an impact on classroom discourse. In terms of subject-specific metacognition, Larkin (2010) asserted that the process pertains to the nature of the task at hand along with specialized skills needed for specific subjects. Therefore, the use of metacognition, particularly in mathematics teaching, will remain a wide area of inquiry, requiring further research.

Finally, many studies such as Kramarski and Mevarech (2003), Bernard and Bachu (2015), Hurme et al. (2015) claim that cooperative learning appeared to be effective in heightening the positive impact of metacognitive training: students in a group training session showed greater improvement in metacognitive skills when compared to those in an individual training programme. In contrast, Desoete (2007) affirmed that individually trained students improved more than those trained in group settings. In further disagreement, Moga's study (2012) displayed that both individual and group training at both the elementary and secondary level showed improved results; however, students in individual training sessions exhibited better results in prediction skills compared with the group training programme. Regarding this dimension, the current study sought to explain the nature of the correlation between cooperative learning and an improvement in metacognition in the mathematics classroom.

Based on these several essential dimensions regarding the nature of the relationship between metacognition and mathematics, in light of the reality of mathematics learning and teaching in Saudi Arabia, this study - which are notably absent in the educational context of this country - sought to identify teachers' and students' perspectives regarding the use of metacognition in the mathematics classroom. This study sought to identify inadequacies in mathematics learning and teaching in the classroom regarding metacognition in that country. How does the use of metacognition (if used at all) play a central role in mathematics learning and teaching, and why? What are the main encouraging signals and difficulties perceived by students and teachers wishing to improve their mathematical performance through metacognition? What are the characteristics that seemed to enhance the positive effects of employing metacognitive processes by analysing the beneficial effects of metacognitive training with students?

2.6 Research aim and questions

Based on theoretical notions of metacognition in light of the reality of mathematics learning and teaching in Saudi Arabia, this study aimed to explore teachers' and students' perceptions of metacognition in relation to mathematics teaching and learning in secondary schools in Saudi Arabia. Consequently, this study sought to respond to three questions:

1) How do secondary students and their teachers perceive metacognition in mathematics teaching and learning?

2) What, if any, indications of metacognition can be observed in the mathematics teaching and learning classroom?

3) What are the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics before and after the implementation of the IMPROVE programme, regardless of improvements in specific strategy or the aim to boost students' achievement?

3 Methodology

This chapter presents the theoretical assumptions and methodological approach adopted throughout the research. The main aim of this study is to explore teachers' and students' perceptions of metacognition in relation to mathematics teaching and learning in a secondary school in Saudi Arabia. Therefore the research questions under investigation in this study are as follows: what, if any, indications of metacognition can be observed in the mathematics teaching and learning classroom? And what are the experiences of secondary students and their teachers of metacognition in relation to mathematics? This includes the main perceived opportunities encouraging teaching and learning in mathematics through metacognition, in addition to the possible challenges facing teaching and learning through metacognition in the mathematics classroom and how these challenges can be met.

3.1 Theoretical and philosophical assumptions

Certain theoretical and philosophical assumptions inform every piece of educational research. According to Wegerif (2008, p. 395) "there are always theoretical assumptions involved in research determining which phenomena are visible and which are invisible". Therefore, it is important to clarify the assumptions of any piece of research. These assumptions are commonly referred to in educational research as paradigms, which according to Carr and Kemmis (1986), embody the particular theoretical framework through which the community of researchers operates and a particular interpretation of reality is generated. The notion of paradigm also incorporates models of research, standards, rules of enquiry and a set of techniques and methods, all of which ensure that any theoretical knowledge that is produced is consistent with the view of reality that the paradigm supports.

On the basis of its aims and research questions, the study was informed by an interpretivist approach to illuminate the issues under examination. Interpretivist approaches aim to understand "the world of human experience" (Cohen, Manion & Morrison, 2007, p. 36). In interpretive research and its related approaches, social situations are examined with the goal of comprehending and building meanings based on the views of participants, who describe and infer meaning from a given phenomenon. Hence, this reality is a construct, and is multifaceted and intricate (Cohen, Manion & Morrison, 2007; Wellington, 2000). This approach can be implemented when examining social activities or documents by employing qualitative approaches. Furthermore, this paradigm underlines that the researcher himself is the central tool for gathering data (Wellington, 2000). A possible shortcoming of this paradigm may be partiality of the frame of inquiry along with the results gained from it. For Radnor (2001), based on the idea that reality is multiple and socially constructed, it is fair to say that people will differ in the way they perceive this reality. Therefore, the interpretivist researcher depends on the "participants' views of the situation being studied" (Creswell, 2003, p. 8).

On the basis of these premises, my assumptions as a researcher are not based on a realist conception. This point can be seen from the purpose of this study which aims neither to discover facts nor to change the research reality but to explore the perceptions of teachers and students towards metacognition. In addition, the research did not start with a set of predetermined hypotheses as positivist researchers do. With regards to positivist research, Grix (2010) states that prediction is the real purpose of explanation within this paradigm, which assumes that reality is one of cause and effect. This study did not seek to discover the causes and effects of the implementation of metacognition in mathematics teaching and learning. Rather, the expected goal of this study is to gain an in-depth understanding of mathematics teaching and learning through metacognition in the Kingdom of Saudi Arabia in terms of theory and practice. This study endeavours to describe and contextualize the various perspectives that manifest within the phenomenon of metacognition in the Kingdom of Saudi Arabia. This study applies a sociocultural approach that encompasses components such as the perception of actual occurrences, social realities and identities - all of which are subject to varying interpretations contingent on the social and cultural background of the participants (Packer & Goicoechea, 2000). It is imperative that social factors, and how these factors engage and are engaged, are accounted for in the study. Therefore participants must be at ease and empowered to vocalize their life experiences (Packer & Goicoechea, 2000). As experiences are a product of human cognition, this study relies on the use of the interpretative paradigm. Therefore the researcher must remain immersed in the educational environment of study to fully conceptualize the roles and experiences of the participants. This strategy aids in illuminating the ‘mathematics’ behind the perspectives of both those receiving and providing instruction through metacognition. The objectives of explicating the ‘mathematics’ is actualized by the interpretive researcher through a nuanced and detailed evaluation of shared meanings that considers and dissects all aspects of the participants’ socially constructed realities (Wellington, 2000). The study applies a methodology that incorporates the diverse constructions and experiences from both the interpretative level of the student and the instructor. Based on the above premises, this research is based on a set of three fundamental interconnected elements: ontological assumptions, epistemological assumptions and methodological considerations, as explained in the following sections.

3.1.1 Ontological assumptions

Even though Jenkins (2002, p. 91) states that the concept of ontology has become “shrouded in mystery”, Crotty (2010) claims that ontology can be defined as the study of being. Ontology, according to Grix (2010, p. 62), refers to “a system of categories that make up a particular vision of the world”.

In terms of the ontological assumptions of this research, the mathematics class was considered as a world reflecting various realities that are constructed by the students’ subjective accounts of their learning metacognitively and those of their mathematics teachers. The variety of subjective understandings of the students and their teachers constituted the many different realities of the mathematics class. This is the ontological position of the interpretive paradigm, which portrays the world as a construction of many multiple realities reflecting the variety and multiplicity of individuals (Pring, 2005).

3.1.2 Epistemological assumptions

According to Crotty (2010, p. 8), epistemology can be considered as a way of understanding and explaining "how we know what we know”. For Pring (2005), epistemology is concerned with varying foundational theories of clarification, fact and authentication. Grix (2010, p. 63) claimed that “if ontology is about what we may know, then epistemology is about how we come to know what we know”. Moreover, he further explains the importance of these two notions as follows:

> It is of paramount importance that students understand how a particular view of the world affects the whole research process, by setting out clearly the interrelationship between what a researcher thinks can be researched (her ontological position), linking it to what we can know about it (her epistemological position), and how to go about acquiring it (her methodological approach). (Grix, 2010 p. 66)

Based on these premises, the focus of this thesis is on how students' and teachers' perceptions about the learning process metacognitively were constructed. These perceptions can be formed by and can influence the perceptions of other students and teachers through social interactions. Thus the knowledge constructed from the interaction of human beings is the basis for the epistemological stance of the interpretive paradigm (Pring, 2005). According to Crotty (2010), conversation plays an essential role in helping researchers become aware of the perceptions of the research participants. Therefore, it was important to interact with the teachers and students who were the focus of my research through various activities such as listening, observing and discussing, making myself a human instrument for collecting and analysing data.

3.2 Social construction

Lowenthal and Muth (2008) explained that the foundations of the constructivist approaches can plausibly be attributed to Goodman, Rousseau, Kant, Dewey, and Vygotsky. However, the concept of social cultural or constructivism emerged initially from Vygotsky's (1962) work. This notion includes three fundamental premises according to Vygotsky (1962). Firstly, there is the interaction and interdependence between the individual level and the social level. Vygotsky (1978) states:

> Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. (p. 57)

Secondly, the social context cannot be separated from individual actions. Vygotsky (1978, p. 90) argued, "Learning is a necessary and universal aspect of the process of developing culturally organized, specifically human, psychological functions." (1978, p. 90). Thirdly, learners' organized cognitive activity becomes enhanced by social interaction, as Vygotsky (1962: 12) states:

"Directed thought is social. As it develops, it is increasingly influenced by the laws of experience and of logic proper". Consequently, the interactions in society between individuals become internalized by each individual. A number of authors such as Rogoff and Chavajay (1995) and John-Steiner and Mahn (1996) cite the importance of Vygotsky's work as stemming from the concept of dynamic interdependence, by which the individual cannot be isolated from the social context. Thus, learning occurs through sharing knowledge which means that it is not simply a process of transmission from the teacher to the learner. Following this idea, Costa (2006) posits the following view:

> Meaning making is not just an individual operation. The individual interacts with others to constructed shared knowledge. There is a cycle of internalization of what is socially constructed as shared meaning, which is then externalized to affect the leaner's social participation. (p. 64)

Moreover, one general argument of Rogoff (2003) is the elucidating nature of cultural practices and community circumstances in regard to the adaption of the individual within the social context and hierarchy. Thus, putting instructional processes into a social context enables us to recognise how students change and develop as participants in the classroom learning environment.

According to Cross (2010), one fundamental assertion that can be derived from the work of Vygotsky is that socially constructed meanings are helpful guides for understanding the development of human consciousness. In addition, foundational features of this consciousness are awareness and the deliberate control of mental activity. On the basis of this premise, the social and communicative aspect of human life should be taken into account in this study when dealing with the concept of metacognition (see 2.3 in literature review) in the process of teaching and learning.

Based on these premises, a sociocultural perspective is suitable for this study based on several factors. There is a clear correlation between the instruction of thinking skills (of which metacognition is a part) and the social context (Vygotsky, 1962), as this does not occur in a vacuum nor is it immune from a community's history and circumstances. Thinking takes place within social interaction, and thus can be directly or indirectly constrained or encouraged in divergent contexts (Moseley et al., 2005). Another significant factor pertains to perspectives of thinking skills adopted in this study (emphasising both the social and individual nature of thought) – this underlines the centrality of social interaction in the wider scope of human development. These same factors apply when examining the specific cultural context in this study, as it too affects the instruction of thinking.

3.3 Research design

According to Crotty (2010), methodology can be seen as the design of the research which regulates our use of methods and links them to our ontological and epistemological assumptions. McMillan and Schumacher (2006) claimed that our theoretical point of view is grounded and enhanced in the paradigm and the ultimate purpose of the research equally influences our methodology. Based on the above, a qualitative approach was followed in this study, which is usually associated with an ontological conception of multiple realities and a socially-constructed epistemological stance (Merriam, 1998). This approach is therefore consistent with the philosophical underpinnings of this study. The qualitative research approach can fulfil research needs in terms of understanding how humans make sense of the world they experience and live in (Merriam, 1998). To help achieve this, Stake (1995) asserts that the researcher-as-interpreter should observe the situations under investigation in a subjective manner in order to recognise what is happening and, at the same time, examine, revise or verify the co-constructed meanings of the participants. Thus a subjective, qualitative approach was helpful to interpret the perceptions of the students and teachers towards metacognition in mathematics teaching and learning. Consequently it was of paramount importance to physically attend the mathematics classes in order to interact with all the participants and co-construct knowledge together.

A growing body of literature highlights that the fundamental aim of case studies is to gain better understanding of a phenomenon (e.g. Bell, 2005; Wellington, 2000; Yin, 2014). Yin (2014:16) explains the case study as ‘an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context, especially when the boundaries between phenomenon and context may not be clearly evident’. Since this study aims to explore the perceptions of teachers and students towards metacognition in a specific context (certain mathematics classrooms in secondary schools in Saudi Arabia), a case study design was favoured in order to suit the research agenda and meet its aims.

There are further reasons behind the utilization of case study as a design for the current study. Firstly, a case study design was appropriate for the current study (which was informed by an interpretivist approach) to illuminate the issues under examination. In interpretive research and its related approaches, social situations are examined with the goal of comprehending and building meanings based on the views of participants, who describe and infer meaning from a given phenomenon (see section 3.1). Therefore, the interpretivist researcher depends on the 'participants' views of the situation being studied' (Creswell, 2003:8). In this regard, Yin (2014:17) asserts that 'case study research also can excel in accommodating a relativist perspective - acknowledging multiple realities having multiple meanings, with findings that are observer dependent… A case study may very well concern the way that you will capture the perspective of different participants, and how and why you believe their different meanings will illuminate your topic of study.'

Secondly, the main research question of the current study relates to how secondary students and their teachers perceive metacognition in mathematics teaching and learning. This kind of research question is often consistent with case study design, according to Yin (2014). Yin (2014) claims that if 'how' or 'why' questions are asked and when the research focus is on a present phenomenon within a particular real-life context, a case study is the most appropriate approach. Such questions tackle operational links which are monitored over time, instead of examining only frequency or incidence. Although two research questions in the current study began with 'what' (see section 2.6), these two research questions aimed to explore the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics before and after the implementation of the IMPROVE programme, regardless of any improvements in specific strategy or any boost to students' achievement. These kinds of research questions are also consistent with case study design, according to Yin (2014). He (2014:10) asserts that 'some types of 'what' question are a justifiable rationale for conducting an exploratory study, the goal being to develop pertinent hypotheses and proposition for further inquiry'.

Thirdly, the current study focussed on a phenomenon present in certain contexts in which the relevant behaviours cannot be regulated. The researcher has little or no influence over behaviours and external influences. In addition, when direct observation of events and interviews with those involved in such events are introduced, a case study design is favoured. According to Yin (2014: 12), ‘The case study is preferred when examining contemporary events, but when the relevant behaviours cannot be manipulated. The case study relies on many of the same techniques as a history, but it adds two sources of evidence not usually available as part of the historian’s repertoire: direct observation of the events being studied and interviews of the persons involved in the events.’ Fourthly, the use of case studies allows for the deployment of numerous research tools and sources of information (Yin, 2014). These provide greater insight into study questions, as well as boosting the trustworthiness of the data (see section). Fifthly, the case study is suitable for an individual study as it permits depth of study within limited time (Bell, 2005). This is consistent with conducting this research under the limitations of the study (see section 6.2). Sixth, the deployment of case studies is not only conducive to exploring and describing the data in real-life settings, but additionally serves to explain the complexities of such settings which may not be sufficiently illustrated through other approaches, for example experimental or survey research.

Based on the purpose of a study, Yin (2014) identifies three types of case study: exploratory, explanatory and evaluative. The basic differences between these three approaches have been explained by Thomas (2011). An *exploratory* case study is suitable when researchers possess minimal knowledge of a specific complex issue they are faced with and wish to gain further information about it. If a researcher wishes to gain depth rather than breadth of understanding in regard to a specific issue and provide explanations based on this in-depth understanding, an *explanatory* case study is appropriate. An *evaluative* case study is adopted when researchers seek to evaluate certain changes that occurred in a particular setting and find out what these changes have led to. In this study, given the research aims, objectives and questions, an explanatory approach was adopted. There were two case studies from a secondary school and each class was considered as a single case.

The study was carried out over two stages. In the second stage, the IMPROVE programme (Mevarech & Kramarski, 1997) was used whereas in the first one it was not (see Table 3.2). The IMPROVE programme is an acronym for the instruction steps that comprise the method: introducing new concepts, metacognitive questioning, practising, reviewing and reducing difficulties, obtaining mastery, verification, and enrichment. These two stages were undertaken in order to enable the researcher to respond to two questions. The first of these was what, if any, indications of metacognition can be observed in the nature of mathematics teaching and classroom learning. The second question was then: what are the experiences of secondary students and their teachers of metacognition in relation to mathematics before and after the implementation of the IMPROVE programme, regardless of improvements in specific strategy or the aim to boost students' achievement?

There are some reasons that the IMPROVE programme was chosen to be implemented in this study. Firstly, the IMPROVE programme uses the metacognitive perspective and how it can be activated in mathematics teaching and learning. Secondly, the programme is centred on the belief that learning is not a rote process but rather one of interpretation, as many constructivists would argue. In doing this, students build meaningful relationships between new and previous knowledge, thus leading to the assertion that this is a process of construction rather than recording and memorization. This conforms to the current study which was engaged in the socio-cultural perspective (see section3.2). Thirdly, the programme includes cooperative learning which in turn helps in understanding metacognition and mathematics within the socio-cultural context as it was presented in relevant section of the current study. Fourthly, the IMPROVE method has proven to have a sizeable positive impact on mathematical performance in problem solving across several age groups. (Cetin et al., 2014; Grizzle-Martin, 2014; Kramarski and Mevarech 2003; Kramarski, Mevarech, and Arami, 2002; Kramarski and Michalsky, 2013; Mevarech and Amrany, 2008; Mevarech and Kramarski, 1997). Despite all these reasons, it is important to assert that the IMPROVE programme was carried out in order to enable the formulation of a clearer and more complete picture of mathematics teaching and learning through metacognition in Saudi Arabia, rather than seeking to improve a specific strategy or to measure students' achievement.

3.3.1 Selection of participants

According to Creswell (2013), there are several techniques for selecting participants for qualitative researchers, such as convenience sampling or purposive sampling or theoretical sampling. Since this study does not seek to generalize its results but to understand 'what is happening' and 'the relations linking the events, purposive sampling was used as the method of selecting the sample (Merriam, 1998). According to Merriam (1998), using a non-probability purposive sample is generally related to the qualitative research approach whereby the researcher seeks to understand what happens in the research condition. In addition, a smaller sample size is often used in qualitative studies compared to quantitative studies (Fraenkel & Wallen, 2006). The reason behind this, according to Barbour (2001, p. 1115) lies in "rather than aspiring to statistical generalisability or representativeness, qualitative research usually aims to reflect the diversity within a given population." The participants were chosen based on a purposive sampling technique. Firstly, I consulted an experienced Saudi university researcher who specialises in education in order to choose a suitable school in which to carry out this research. He has many contacts in Saudi Arabia and agreed to assist me in finding suitable participants for this project. He contacted several schools but I was surprised to learn that, unfortunately, all these schools had over 40 students per class and there was no pre-existing practice of cooperative mathematics teaching or learning. Then I decided to move to another small city which might be a more suitable environment to fulfil the following requirement criteria: there should be a pre-existing practice of cooperative mathematics learning among students and teachers, and teachers should be cooperative and enthusiastic to implement the idea of metacognitive teaching. I also sought a school principal willing to support and provide school facilities and resources and the use of school computers and office equipment. Since in the Saudi Arabian education system the concept of metacognition in mathematics teaching and learning is unfamiliar, considering these criteria to find a suitable environment might help me to focus on the main subject of the study, particularly the IMPROVE programme based on cooperative learning. At the suggestion of the municipal government of the city, I visited three schools in order to search for the most favourable environment in which to undertake the study. The school I selected was the most suitable environment, as it best fulfilled the previously cited criteria. There were two classes at the secondary school; each class is considered a case study. Each case study contains 30 students and their teacher. The teaching staff involved in this research were, Case 1: Mr Fallatah (pseudonyms are used throughout). Subsequent to gaining his undergraduate degree in mathematics at King Abdulaziz University in 1998 he commenced his teaching career, which spans 15 years in two provinces of the country. The participating students in Mr Fallatah's class were Mohammed, Ziyad, Ragab, Omar, Mazen, Qusay and Fadul (all pseudonyms). All the participating students were 17 years old and lived in the same area of the city. Case 2 participating teacher was Mr Hatem who also received an undergraduate degree in mathematics from King Abdulaziz University, in 2002. He then proceeded to complete a Master's degree in Education at the Madinah University. Following that, he taught Mathematics for 11 years in Yanbu city. The participating students in Mr Hatem's class were Asaad, Babseal, Nawaf, Fares, Abdullah, Sultan and Ammar (pseudonyms). All these participating students were 16 years old and lived in the same area of city.

3.3.2 Methods for collecting data

According to Robson (2002), a case study can be defined as an approach where the concentration is on a phenomenon in context and multiple methods of data collection, such as interview and observation, are typically utilised in this situation. In collecting the qualitative data for this research, the methods used are individual and focus group interviews and participant observation. The major purpose in the observation element was to explore whether any indications of metacognition can be observed in mathematics classrooms of secondary schools in Saudi Arabia. The major purpose of the individual and focus group interviews was to explore how participants perceive metacognition in relation to mathematics teaching methods, and what opportunities and challenges they encounter in developing mathematics learning through metacognition, and how they meet these challenges. The following table (3.1) indicates how each method of data collection was linked to the research questions.

Table 3.1: Research questions and methods

Research question	Method	Participants
B. What, if any, indications of metacognition can be observed in the mathematics teaching and learning classroom?	observation	30 students from one class, along with their mathematics teacher (before IMPROVE programme implemented)
C. What are the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics before and after the implementation of the IMPROVE programme?		30 students from one class, along with their mathematics teacher (through IMPROVE implementation).
A. How do secondary students and their teachers perceive metacognition in mathematics teaching and learning? C. What are the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics before and after the implementation of the IMPROVE programme?	interview	Seven students from one class in secondary school to participate (before IMPROVE implemented)
		Their mathematics teacher (before IMPROVE implemented).
		Seven students from one class to participate (after IMPROVE implementation).
		Their mathematics teacher (after IMPROVE implementation).
	Focus group discussion	Seven students from one class in secondary school to participate (before IMPROVE implemented).
		Seven students from one class to participate (through IMPROVE implementation).

3.3.2.1 Interviews

According to Janesick (2010), interviews can be seen as a meeting of two persons to exchange information and ideas through questions and responses, resulting in communication and joint construction of meaning about a particular topic. Robson (2002) states that there are three types of interview design. Firstly, a *fully structured* interview, which has predetermined questions with a certain wording, generally in a predetermined order. Secondly, a *semi-structured* interview, which has predetermined questions, but their order could be altered based upon the interviewer's perceptions of what appears most appropriate. The interviewer can modify question wording or omit inappropriate questions or even include additional questions with certain interviewees. Thirdly, an *unstructured* interview, which can be considered informal, whereby the interviewer has a general area around which to develop the conversation of interest and inquiry.

The second, semi-structured, design was used in this study, and it includes some open-ended questions. This is due to several reasons, the first of which being that in a semi-structured interview the researcher anticipates that the interviewees' thoughts are more likely to be elicited in a flexibly designed interview condition than in other interview designs, according to Flick (2006). Secondly, a semi-structured interview keeps the dialogue comfortable and enables the interviewer to decide the appropriate time to raise the questions in such a way that the structure of the interview remains coherent, according to Radnor (2001). Thirdly, a semi-structured interview enables the interviewer to collect equivalent data through different interviews in order to achieve the aim of the research, while still enabling each interviewee to define what they consider to be a priority in their own situation, according to Radnor (2001). The application of this method is covered in further detail in 3.3.5 section.

3.3.2.2 Focus groups

In terms of focus group interviews, Fontana and Frey (1994, p. 365) state that a focus group can be seen as a 'formal group interview', where the role of the interviewer is directive and the question design that they use is semi-structured. A focus group interview is an open-ended group discussion on a certain issue which is guided by the interviewer (Robson, 2002). This context can be adjusted to gain understanding of the most important issues regarding participants' experiences of the study through the lens of social interaction, undoubtedly a key feature of sociocultural perspective. Thus, the essential purpose of group discussion is to allow a free-flowing exchange of information by means of interaction between students, which aided me in gaining more profound and insightful conclusions. The discussion also opens up the floor for each student to express themselves freely on the subject of learning through metacognition in the classroom. An additional advantage is that the group discussion promotes variation in communication, revealing their attitudes, feelings, beliefs and experiences. This is particularly important for the part of the study relating to sociocultural context. I found these interactions helped me gain a more profound grasp of certain events than those gleaned from the one-to-one interviews. It has been stated that group discussions support a more open forum, providing a platform that makes participants feel at ease when expressing feelings, which may or may not be underdeveloped or neglected from mention during an interview (Kitzinger, 1994). A potential disadvantage of group discussions is the possible dominance of the conversation by a smaller number of students (Wellington, 2000). In order to prevent the conversation from being affected by this it was crucial for me to manage the conversation carefully, with the added responsibility of bringing the conversation back to the original topic should it get side-tracked by an individual; this aided in improving the quality of data. Both the interview and focus group methods sought responses to two questions: how do secondary students and their teachers perceive metacognition in mathematics teaching and learning; and what are the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics?

The agendas for interviews and focus groups contained open-ended questions designed to shed light on various issues and spark detailed statements by participants. This was in order to gain further understanding of their viewpoints and experiences in regards to mathematics learning and teaching through metacognition (see Appendices 2, 3 and 4). These questions were planned and formulated through a literature review and after earlier interactions with teachers throughout the pilot stage of this research. The planning for interviews and focus groups was composed in English, and later translated into Arabic to suit the local context. At the commencement of each interview, the key goals of the research were clarified to participants.

3.3.2.3 Observations

With regard to observations, these help a researcher to see things that might not be freely spoken about in interview conditions, such as pedagogic styles or curricula, and enables them to understand the context in depth (Cohen, Manion, & Morrison, 2007). In this study I used semi-structured observations focusing on a number of set areas but also aiming to generate relevant data in a less structured manner (Cohen et al., 2007). Participant observation and non-participant observation are two basic types of observation (Grix, 2010). In the former, the observer participates in the events which are observed, while in the latter the observer purposely attempts to be as inconspicuous as possible (Cohen et al., 2007; Wellington, 2000) Besides this, the basic aim of observation is to understand whether or not participants behave in the way they claim to behave (Bell, 2014). Since my original role was to observe any indications of metacognition in relation to mathematics learning and teaching in the classroom as well as activities and interaction, either between students or with their teachers, participant observation was used in the current research. Classroom observation enabled me to build on my understanding of the process of teaching and learning through metacognition in the classroom and the contexts in which it occurs. In addition, the observation enabled me to identify the key challenges facing both teachers and students when they are teaching and learning through metacognition in the mathematics classroom. In my role as a participant observer, I attempted to create a friendly relationship with participants so as to gain a positive rapport with them to the greatest possible extent, this being done in order to gain a deeper understanding of their teaching and learning through metacognition. This kind of relaxed and friendly environment encouraged participants to be at ease and act and speak in the way they would without the presence of an observer. Through the use of several observation periods, the researcher sought to become better acquainted with participants so that he would be seen less as a ‘stranger’, as more time was spent with them in the classroom. The observation process was undertaken by the researcher himself, without assistance. A session of eight class periods was decided on for the observation of each case study class.

Each observation was 45 minutes in duration (with the length of a class period being 45 minutes). Instead of only observing, I preferred to take the role of participant observer, as this aided me in obtaining more in-depth information relating to mathematics teaching and learning through metacognition. The application of this method is discussed in further detail in section3.3.5.

I used these three methods to collect the data by giving students and the teachers the opportunity to reflect upon the reality of the mathematics class in which they had actively participated. This in turn provides rich descriptions of the activities of the mathematics classes which helps me understand the actions of the students and their teachers.

3.3.3 General Procedure of Data Collection

In the current study, the research in its entirety (including the pilot phase) lasted 11 weeks from 2nd February 2013 to 24th April 2013 (see Table 1.1 and Table 3.1) I was pleasantly surprised with the optimism and enthusiasm of the teachers to deal with the study, despite the perseverance and hard work required for such a process. I was also impressed by the eagerness with which the students approached this study. This is not to say that difficulties were not encountered in the undertaking of the research; these included it being an entirely new concept to deal with in the teaching and learning of mathematics in Saudi Arabia. Mathematics teaching and learning through metacognition requires greater time and effort than traditional methods, and also calls for a certain cooperative environment to pervade in the classroom, the likes of which had not previously existed to a sufficient degree. Teachers and students found themselves bound to the completion of a pre-existing curriculum, in which they were expected to prepare for tests using traditional methods of study, which did not include metacognitive strategies. The official evaluation criteria of teachers do not take into account any aspects relating to metacognition. These aspects will be touched upon further in the chapter four pertaining to findings.

There were three phases of my study fieldwork, the pilot study and the two subsequent phases which comprise the main body of fieldwork. These two phases aimed to observe metacognition in the nature of teaching and learning mathematics in the classroom and the implementation of the IMPROVE programme (see 3.3.5.2). Both phases were carried out in order to enable me to formulate a clearer and more complete picture of mathematics teaching and learning through metacognition in Saudi Arabia, instead of improving a specific strategy or seeking to measure students' achievement.

3.3.4 Pilot study

According to Robson (2002, p. 185), a pilot study can be considered as "a small-scale version of the real thing, a try-out of what you propose so its feasibility can be checked". Yin (2014) states that use of a pilot study helps the researcher to refine their plan to collect data with consideration of the content of the data or the procedures to collect these data. The main tools of research were initially created through the period of the literature review's reading. I found and then considered many forms of the interview, focus group and observation which are related to metacognition and mathematics. I then built the main tools according to the study aim and questions. These main tools of research were discussed with and agreed by my supervisors, colleagues from Exeter University in the United Kingdom, and two mathematics teachers in secondary school in Saudi Arabia. This process was carried out in order to receive feedback, helping me to verify the clarity and pertinence of the instruments to the scheduled targets of the study. After this process, the research tools were modified and adapted, with the end product being the finalised interview, observation and group discussion schedules. These revised versions were then reviewed by my supervisors for further feedback, particularly to confirm that the make-up of items was suitable to the respondents and that they would be easily understood and responded.

Following the extensive feedback process a period of two weeks was set aside for the pilot study. By this point I had organised two mathematics teachers with two each of their students in two separate classrooms. I commenced my research with a pilot study, the duration of which was two weeks. This pilot involved observation, interviews of students and teachers and a group discussion – all with the goal of improving my research tools, specifically to verify the practicalities of such methods. At the pilot stage, interviews were held with each class teacher and two of their students. The interviews were intended to be undertaken in an informal manner, with questions being used more to provoke discussion than elicit direct answers. This way, a large amount of data could be collected despite the fact that participants were uninitiated in the area of metacognition. Hence, the pilot study proved to be instrumental in demonstrating the feasibility and clarity of my research tools. Each teacher interview continued for 40 minutes in a meeting room on school premises.

Another research tool was observation, which was carried out six times on various classes in the pilot stage, including the two classes which were observed in the main study. This aimed to gather information to enhance and improve the observation plan. These observations were carried out in the pilot phase across the entirety of the class period, lasting 45 minutes. In addition, one group discussion was hosted for each class, lasting half an hour.

Overall the pilot stage was designed to gain feedback on the intelligibility and pertinence of the interview and observation strategies, in particular relating to terms regarding metacognition. The phase was also useful in providing a clearer vision for time management within the three research tools, for example estimating the length of interviews in the main phase of the research. Minor modifications were made to the interview schedule including omissions or additions of certain items, but overall the interview schedule seemed appropriate to yield relevant data and answer the research questions. For instance, the structure of a question on obstacles to the application of metacognitive teaching in the mathematics classroom in the first teacher's interview schedule was vague (see Appendix 1). Thus, after the pilot study this question in the final schedule was divided into sub-questions which were: what are the obstacles to the application of metacognitive teaching in the mathematics classroom in terms of teachers; students; and the school environment? (see Appendix 2). In another example in this regards, there was a question of the most important aspects of metacognition to emphasise in the mathematics classroom in the first teachers' interview schedule (see Appendix 1). This question was difficult to answer without intensive training to teach metacognitively. Thus, it was omitted in the final schedule (see Appendix 2). Generally speaking, observation schedules seemed well designed to collect relevant information in terms of the teaching and learning through metacognition employed in the classroom. Only two items were inserted in the observation schedule. These were related to student-student interaction and teacher-student interaction (see Appendix).

3.3.5 Main Study

3.3.5.1 First stage

Following the pilot study I proceeded to the first stage of my fieldwork in which I dealt with two classes as independent case studies, in order to observe each case in isolation. My task was to observe, based on the observation schedule, occurrences in the classroom relating to teaching and learning through metacognition in order to determine the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics without implementing any metacognitive intervention. Throughout the weeks I observed each class six times, with each observation session lasting 45 minutes.

Throughout each observation I sat with one of the work groups in the class solving the activity presented by the teacher. After gaining student consent I audio recorded their conversations as they solved the problem. I also observed their way of discussing and examined the steps of their work according to the metacognitive questions found in IMPROVE programme. These were questions relating to understanding the question, the solving strategy, and linking previously and newly learnt information (see details of IMPROVE programme in 2.5.5). Furthermore, I observed the obstacles facing their solving of mathematics problems and the restrictions of the circumstances of the situation in which they worked. They were asked when necessary about the items of the observation schedule prepared previously (see Appendix). When the teacher presented the next activity I moved to another group, and undertook the same work in addition to noting the observations of the teacher's method of instruction. This was based on the items of the observation schedule, and in the appendices there are examples of some of these observations (see Appendix).

I individually interviewed the two class teachers and seven students from each class. These students were chosen through co-ordination with the teachers in order to determine which students were best able to express themselves on their opinions and feelings, with these students being of various educational achievement levels. Each teacher's interview lasted 45 minutes, with each student's interview lasting approximately 30 minutes. I also interviewed the two class teachers together with the same students from each class in the context of a group discussion. Seven students were chosen as an optimum number for interaction to occur within a focus group. Each group discussion lasted 30 minutes. All these interviews and groups discussion took place in the library or the meeting room of the school. Both the interview and focus group methods that were used sought to respond to two questions: how do secondary students and their teachers perceive metacognition in mathematics teaching and learning, and what are the experiences of secondary students and their teachers in Saudi Arabia of metacognition, in relation to mathematics? At the end of the first stage I transcribed the data that I had collected throughout this stage. These transcriptions provided me with preliminary theoretical and practical aspects regarding mathematics teaching and learning through metacognition as they are presented in the Findings chapter.

3.3.5.2 Second stage

Before I began the second stage I met the teachers twice; each time the meetings lasted one hour. These meetings were scheduled in order to discuss the IMPROVE programme and how the teachers could implement it in the maths classroom context. The aim of doing the entire stage was to enable me to formulate a clearer and more complete picture of mathematics teaching and learning through metacognition rather than evaluating the IMPROVE programme, or improving a specific strategy, or even seeking to measure students' achievement. I gave the teachers the freedom to choose appropriate situations in which to apply the IMPROVE programme, based on the content of the lesson and the preparedness of the students. Through this particular period I bore in mind that it was planned for each teacher to present eight lessons over a period of seven weeks in which the theory would be applied. The timing of this study fulfils Schraw and Gutierrez's suggestion (2015) which explains that programmes ranging from six weeks to several months tend to be more effective. This is because longer-term programmes enable students to model, practice and automate strategies, while also enhancing conditional knowledge. Furthermore, another benefit is that instructors themselves improve their teaching and modelling of strategies over a lengthier period of time.

As a result of the discussion surrounding the IMPROVE programme (see for more details 2.5.5), it was underlined that this programme encompasses three interrelated components (Mevarech & Kramarski, 1997):

> (a) Facilitating both strategy acquisition and metacognitive processes, (b) Learning in cooperative team so four students with different prior knowledge: one high, two middle, and one low-achieving student and (c) provision of feedback-corrective-enrichment that focuses on lower and higher cognitive processes. (p.369)

Based on these three elements, each teacher prepared the following:

1) Work groups consisting of four students of differing academic attainment, based on previous reports of the teachers. The number of students in each class was 30, so there were five groups containing four students, and two groups containing five students each. It is noteworthy that the nature of work in this school is originally based on cooperative learning, which facilitated the implementation of this study.
2) Mathematical problems suitable for learning according to the metacognitive questions as stated in the IMPROVE programme. These were questions relating to understanding the question, the solving strategy, and linking previously and newly learned information.
3) Worksheets for the student groups to solve the problems chosen in (2) above (see Appendix as an example).
4) The steps which should be considered by the teacher during instruction, as noted in IMPROVE, which were: introducing new concepts, metacognitive questioning, practising, reviewing and reducing difficulties, obtaining mastery, verification, and enrichment. IMPROVE is an acronym for the instruction steps that comprise the method.

Since IMPROVE assumes that cooperative-mastery learning based on peer interaction and the systematic provision of corrective feedback enhances mathematical thinking, students learnt in teams consisting of four students, as follows:

- Each session began with the teacher's short presentation (about 10 minutes) of the new materials to the whole class using the question-answering technique.
- Following the introduction, students started to work in small groups using the materials the teacher had designed. Students took turns in asking and answering three kinds of metacognitive questions: (a) Comprehension

question: What's in the problem? (b) Connection question: What are the differences between the problem you are working on and the previous problems? (c) Strategic question: What is the strategy/tactic/principle appropriate for solving the problem? Whenever there was no consensus, the team discussed the issue until the disagreement was resolved (see Appendix as an example of an activity).

- Talking about the problem, explaining it to one another, comparing it to what was already known, approaching it from different perspectives, balancing the perspectives against one another, and proceeding according to what seems to be the best option at the time, students actually used the diversity in their own prior knowledge to self-regulate their learning. When all team members agreed on a solution, they wrote it down on their answer sheets. Students' answers included the final solution, mathematical explanations, and a sample of metacognitive responses (e.g., "This is a problem about ...," "The difference between this problem and the previous problem is ...," "The mathematical principle appropriate for solving the problem is . . . because . . . .").
- When none of the team members knew how to solve a problem, they asked for teacher assistance.
- At the end of the lesson, the teacher reviewed the main ideas of the lesson with the entire class.
- When common difficulties were observed, the teacher provided additional explanations to the whole class.
- When students worked in small groups, the teacher joined one team for 10 minutes and worked with them as an additional team member.
- When the teacher's turn arrived, he modelled the use of the metacognitive questioning in solving the problems. The teacher read the problem aloud, used the metacognitive questions, and explained each step of the solution. Teachers listened to how students coped with the problems and provided assistance when need. Teachers worked with each team at least once a week.

The third component of the IMPROVE programme is the provision of feedback-corrective-enrichment. However, since this study is not concerned with the assessment students’ achievement, at the end of each lesson the teacher provided certain activity in order to enable me to observe students' metacognition while they solve these activities.

All of these applications were observed. Throughout each observation I sat with one of the work groups in the class solving the activity presented by the teacher. After gaining student consent I audio recorded their conversations as they solved the problem. (See Appendix for one full example of this conversation). I also observed their way of discussing and examined the steps of their work according to the metacognitive questions found in IMPROVE. These were questions relating to understanding the question, the solving strategy, and linking previously and newly learn information. Furthermore, I observed the obstacles facing their solving of mathematics problems and the restrictions of the circumstances of the situation in which they worked. They were asked when necessary about the items of the observation schedule prepared previously. When the teacher presented the next activity I moved to another group, and undertook the same work, in addition to noting the observations of the teacher's method of instruction. This was based on the items of the observation schedule, and in the appendices there are examples of some of these observations (see Appendix)

The Observations method used aimed to outline occurrences in the classroom relating to teaching and learning through implementation of the IMPROVE programme regardless of improving a specific strategy or seeking to measure students' achievement.

I also hosted a focus group for each class, consisting of seven discussed items that had been identified previously with the related to the students (see

Appendix). Furthermore, we discussed items related to the steps that should be followed by students in solving mathematics problems according to metacognitive questions. These questions were found in IMPROVE and relate to understanding the question, the solving strategy, and linking previously and newly learn information. These focus groups were set up to gain understanding of what students found difficult, easy, beneficial and practical in relation to metacognition during the implementation of the programme. At the conclusion of this period I conducted semi-structured interviews with each teacher and the previously mentioned fourteen students (seven from each class). Both interview and focus group methods sought to respond to two questions: how do secondary students and their teachers perceive metacognition in mathematics teaching and learning? And what are the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics after implementation of the IMPROVE programme?

Table 3.2: Number of data collection instruments for two cases (each case has the same number of instruments of data collection)

Month 2014	Week	Number of instruments of data collection			
		Number of interviews		Number of group discussions (Students)	Number of classroom observations
		Teacher	Student		
February "Pilot study"	Week 1	1	2	1	3
	Week 2	1	2	1	3
February "First Phase"	Week 3	1	7	1	6
	Week 4	1	7	1	6
March "Second Phase: IMPROVE Implementation"	Week 1	-	-	2	2
	Week 2	-	-	2	2
	Week 3	-	-	2	3
	Week 4	-	-	2	2
April "Second Phase: IMPROVE Implementation"	Week 1	-	-	2	3
	Week 2	1	7	2	2
	Week 3	1	7	2	2
Total	11	6	32	18	34

3.3.6 Trustworthiness

Trustworthiness is an important feature that must be addressed in all research into education. This is done to convince readers that the study's findings are worthy of note (Lincoln Yvonna & Guba Egon, 1985). In this regard, two essential principles are often cited in academic research, these being validity and reliability. Guba and Lincoln (1989) substituted the terms validity and reliability with the parallel concept of 'trustworthiness' in order to establish rigour. However, as these principles are not always mentioned in interpretive studies, substitute terms are used for these circumstances. The first of these is credibility, which often relates to validity. The concept of qualitative validity (credibility) is relevant, involving confirmation of the findings' accuracy ensuring that the participants' contributions were accurately conveyed (Creswell, 2013). This comes alongside qualitative reliability (consistency), involving a steady and unchanging approach that is implemented by different individuals throughout the research study (Creswell, 2013).

There were a number of processes adopted in this study to enhance the trustworthiness of the study and the credibility of the research. To begin with, I concentrated on interaction and consistent field observation so as to establish a good rapport and trust with participants. This involved gaining an understanding of the school and the society's cultural norms. I favoured the use of an open-ended questions format for interviews and focus groups, to ensure that participants could comfortably express themselves, which also assisted me in understanding contextual circumstances as I was able to embed myself in their interaction. I identified the importance of Creswell's (2012) concept of credibility (validity) at an early stage to be a clear advantage of qualitative research, due to the extensive interaction and familiarity between researcher and participants, which added to our study's credibility.

Secondly, the variety of data collection tools used was another asset to the credibility of the study. This is despite the fact that we used different means to collect it and that it was collected at two levels, the first being the students and the second the teachers. Creswell (2013) noted that data should be corroborated among different sources by bringing together evidence from all of them and harnessing it to build a comprehensive reasoning for themes. If agreement on themes is established using complimentary data resources or participant views, then these procedures can convincingly add to the research's value.

Thirdly, despite the lack of discrepancies in data presented by participants – be they students or teachers – this does not mean that there were no differences in opinions among participants. Indeed, the data clarified that there were students who were not convinced by the importance of learning mathematics according to metacognition. Creswell (2013) stressed that demonstrating conflicting evidence would portray the study as realistic and valid as it would be representative of real-life contexts in which differing opinions are often expressed.

Fourthly, interview results were reviewed with colleagues to ensure the correct and transparent coding of the transcript data on two occasions. The first of these meetings was with a fellow PhD candidate at the college of Education (he worked as an English teacher in British secondary schools) and the second occasion was with my supervisors. These consultations sought to ensure the consistency and uniformity in code definitions and to prevent any potential distortion of these during coding. This checking of the coding is another addition to the accuracy and validity of the study, as explained by Creswell (2013), as it cross checks the researcher's interpretation with neutral third parties.

Furthermore, the relatively lengthy period of time spent in the field (three months) also improves the validity of research. It allowed me to gain a detailed comprehension of the subject of study, and hence enhances the validity of the data. Creswell (2013) pointed out the importance of time spent in the field, linking the duration of one's presence with participants in the study's context to the enhanced validity and accuracy of data.

A sixth distinction of this data was that in-depth descriptions of findings were provided to help readers to envisage the context and allow them to participate in the experience. Creswell (2013) explained that this too can enhance the validity of research. Lastly, the presence of independent third parties (supervisors) to review the accuracy of data, its relevance to research questions, and the depth of its analysis was greatly beneficial in terms of value and consistency of the study, also bringing the study in line with another one of Creswell's (2013) suggestions in this regard.

3.3.7 Generalisation from the case study

Although it has been argued that the use of generalizations are shortcomings for case studies, according to Yin (2014), Creswell (2013) points out that one of the advantages of qualitative research is found in the use of themes identified in the context of the case study. In fact, Yin (2014) suggested that findings from such studies can be generalized into wider theories. Researchers may generalize when they move to other cases and transfer previous generalizations from old cases to apply them in the new context. Hence, it is suggested that further studies be carried out in this regard in the Saudi context, especially since there are many similarities among secondary schools in Saudi Arabia in that none of them is entirely independent in relation to decision-making because they are all managed by the Ministry of Education. In addition, all teachers in secondary schools in Saudi Arabia follow the same regulations under the Ministry of Education. As a result they are often exposed to the same educational influences. Consequently it can be claimed that the findings of this case study could be transferred to similar cases in the educational context of secondary schools in Saudi Arabia.

3.3.8 Ethical considerations

Much research highlights the importance of creating and complying with clear ethical guidelines. Wellington (2000) emphasises that the main criterion for any educational research should be ethical. Ethical issues relate to what is appropriate and what is not at the many stages of any research, such as formulation of research questions, data collection and analysis (Creswell, 2013). As a result of this, several organisations have become increasingly important, such as the British Educational Research Association (BERA) and the American Educational Research Association (AERA). Pring (2005, p. 142) emphasises that the crucial issue in this context is "the meaning and justification of moral considerations which underlie research".

This study follows the University of Exeter’s and BERA’s (2011) ethical guidelines for research. This study succeeded in obtaining Ethical Approval from the Graduate School of Education at the University of Exeter (see Appendix). For consideration of these principles I requested permission for conducting this study in the school from the municipal government in the city.

After obtaining this permission, I met with the Principal after which I explained to him the purpose of the study. I requested him to obtain permission from the parents and guardians of the students and to permit me to meet with two mathematics teachers. I obtained the parents'/guardians' acquiescence for all participating students in addition to permission from the teachers (see Appendix). Subsequently, I met with the students and explained to them the goals of the study, while also obtaining their personal signed consent on separate forms (see Appendix). I also obtained consent from the participants to conduct this research and provide them with information about about the aims of the study. According to the BERA (2011) ethical guidelines, "researchers must take the steps necessary to ensure that all participants in the research understand and agree to the process in which they are to be engaged, including why their participation is important and how it will be used" (p. 5) . In addition, I gave the participants the right to see the results and comment on them before making them more widely known. According to Wighting, Nisbet, and Tindall (2005, p. 93), "interpretations and conclusions were taken back to a representative of the participants to comment on the accuracy and credibility of the account". In terms of the participants' rights, anonymity and confidentiality are assured in this research. Following the BERA (2011) guidelines, anonymity and confidentiality of the participants were applied to every aspect of this project. Pseudonyms were used for participants' names and workplaces, and personal details relating to the students and their environment were concealed to ensure that no output will provide information which might allow any student, teacher and school to be identified from names, data, contextual information, or a combination of these. Radnor (2001, p. 39) mentioned that: "The principle of ethics-in-action focuses centrally on the need for the researcher to show respect for the participants". In addition, in accordance with BERA (2011) guidelines, participants were requested to give permission for their recording in interviews, to which all complied. It was also explained to them that they were free to exit the study should they wish, and in which case, their data would be deleted immediately. However, no participant exercised this right. Records of the data collected (including transcripts and any audio recordings) were stored in a secure box in my home. Electronic information was only accessed by me as a researcher with my username and password. This information was coded to ensure anonymity and stored on a secure system with recognised virus protection. This will remain anonymous in the write-up of the research. Collected written information will be destroyed by shredding and audio recordings will be disposed of digitally after final submission of the research. As far as I was aware, this study did not come into conflict with any ideological or religious beliefs in the Saudi Arabian context.

In the course of this study, ethical issues were taken into consideration at every step of the research. This is not to say that ethical issues did not arise, as these can often be circumstantial, as such is the nature of social enquiry. These involved power dynamics, and uneasy student-teacher relationships or between myself and the participants. To amend this, I attempted to reduce the sway of power dynamics, by either direct or indirect means. Such steps included notifying participants of their ability to withdraw their participation at any point, taking steps to alleviate a sense of discomfort, and mitigating any distress felt during the implementation of this study.

3.3.9 Data analysis

This section provides an insight into how the data were gradually developed and refined from their 'raw form' into codes and subsequently grouped into categories and themes, using both an inductive and theoretical approach. The analysis was initially guided, but not limited, by the presence of anticipated ideas. In addition, the analysis sought to discover new ideas, categories and themes that would emerge from the data. Moreover, the data analysis attempted to reflect the experiences, motivations and difficulties relating to the concept of metacognition in relation to the teaching and learning of mathematics in the Saudi educational context.

Thematic data analysis has been discussed by Braun and Clarke (2006) who explain that data themes can be categorised as inductive ('bottom up'), or theoretical ('top down'). With the inductive approach comes the assumption that themes are closely related to the data itself. Thus, inductive analysis involves coding data without trying to make it fit within a pre-assumed coding framework. In that respect, although in this study the three initial research questions guided the process of analysis, the work was not restricted to these questions and all the information provided by the participants' responses was left open to interpretation whereby themes or categories could arise.

In contrast, ‘theoretical’ analysis is more motivated by the researcher’s interest in the data in relation to a particular theory instead of providing a detailed description of the entire data. This form of analysis tends to provide less of a rich description of the data, and more of a detailed analysis of certain aspects of the data. Therefore, with this approach, coding is conducted for a specific research question (Braun & Clarke, 2006; Radnor, 2001). One of the drawbacks of this approach is that early reading could narrow the researcher’s thought through a focus on particular aspects at an early stage, which could lead to neglecting other possible important aspects within the data; however, prior reading can enrich analysis as it may make one aware of the more nuanced and subtle characteristics of the data.

Therefore, the method employed in this study involved elements of both approaches; inductive then theoretical thematic analysis. Furthermore, throughout this process the research questions were left open to adjustment and addition. Overall, coding reflected information that had been expected to be found before the study, but also surprising and unforeseen data were collected in the field, along with other significant and pertinent information relating to the study such as some issues regarding cooperative learning and metacognition.

3.3.9.1 Data analysis procedures

First, I immersed myself in the data through intensive reading of the interview transcripts which involved searching for meanings, patterns and themes, while making initial notes for coding that could be reviewed later. The individual and group interviews, along with the observations (see the implementation processes of these methods in 3.3.5) were conducted in Arabic, and transcribed and analysed in that language to preserve the meanings. After acquainting myself with the data and having formulated some general ideas about the notable features within it, I then began to generate preliminary coding by assigning a ‘code’ to specific content using a software called MAXQDA (MAXQDA is professional software for qualitative data analysis that organises and categorises data, retrieves results and creates illustrations and reports). In order to do this I uploaded the transcript to the software and assigned a code to a highlighted segment of text. After the entire transcript had been coded, I had a long list of codes that were assigned to extracts (see an example from teacher interview Appendix). I then examined each coded extract and organised these codes into groupings that I called ‘categories’. These categories were checked by a colleague (who holds a doctoral degree in Education) who agreed with the logical aspect of these groupings after extensive discussion. This phase involved sorting these different codes into potential categories, and collating all the relevant coded data extracts within these categories using the software. I then read through the ‘code system’ (as it is called in the software) and pondered how much each code agreed with the category (see an example from teacher interview Appendix). Then I created themes that were inferred based on the link between the different categories. To this end, the following section presents a more detailed example to explain these processes.

3.3.9.2 Examples of data analysis

This section provides two examples to illustrate how I assigned a code to the data then linked it with a suitable category and a relevant theme. During the course of the interview the teacher was asked about his perception of metacognition and answered: “the work was done within logical steps, in order to resolve problems”. I thought this quote could be coded as ‘metacognition as a systematic logical procedure’. Similarly, the teacher added: “another important element of the method is the existence of a logical ‘thinking map’”. This led me to code this extract as ‘metacognition contains a logical thinking map’. Consequently, it seemed to me that these codes could be grouped under one category that I called ‘Conception of metacognition’. In addition, the teacher said that “the metacognitive method supports students’ thinking and their abilities, which enables them to evaluate their thinking”; this quote was coded as ‘metacognition helps students evaluate their thinking’. Furthermore, when the teacher said “Another important element of the method is the existence of a logical ‘Thinking Map’, which in turn aided students in time management for dealing with solving problems”, I thought this extract could be coded as ‘Metacognition helps students to manage time in solving problems’. Consequently, all these codes seemed to be related to a broader category that I called ‘Function of metacognition’. The two above-mentioned categories (‘Conception of metacognition’ and ‘Function of metacognition’) were related to a broader theme that was named ‘Teachers’ understanding of metacognition’ (see an example from teacher interview Appendix). This theme was inferred based on the fact that the teacher seemed to understand the notion of metacognition from two different angles; hence metacognition understood as a function or as a concept.

With regards to the second example to illustrate the process of data analysis, in the course of the interview, the same teacher was asked about the possible challenges to the implementation of metacognition in mathematics within the school, and replied: “I am convinced that after implementing this method, I really found that I had the motivation to teach with it, as this conviction was very influential to me in teaching”. This extract was coded as ‘Teacher’s conviction about metacognition affects motivation’ because it seemed evident that his conviction has an effect on his motivation to teach metacognitively. Likewise, the teacher added that “some teachers need to see in front of them the positive results of implementing the method in order for them to interact with it positively”. This quote was coded as ‘seeing the positive results of the implementation of metacognition as a source of motivation’. Overall, these two codes seem to relate to the factors that promote motivation; therefore they were grouped under a category called ‘What promotes motivation’. In addition, when asked about the obstacles to the implementation of metacognition, the teacher talked about “a deadlock in the discussion and dialogue surrounding the practical development of education among the public in general and more specifically among teachers”. I coded this extract as ‘deadlock in discussion about practical development of education’. Moreover, the teacher mentioned “The school administration being unconvinced because its focus is on the direct academic attainment of students and completion of the syllabus”. This extract was coded as: ‘Focus on completion of syllabi’. Consequently, these codes seemed to point to what inhibits motivation and were grouped under the category, ‘What inhibits motivation’. Then a theme was created based on the link between these two categories and how both relate to ‘Teacher’s motivation in implementing metacognition’.

To explain how the theoretical ‘top down’ approach was conducted in this study, the third example was presented. It was discussed in the literature review of the study that metacognition contains two main components, being knowledge and regularity skills which include planning, monitoring and evaluation. This perspective of metacognition guided me to find any related data to help me respond to the research questions in this regard. The research question is how secondary students and their teachers perceive metacognition in mathematics teaching and learning With regards to the third example, in the course of the interview, Mr Fallatah was asked about his perception of metacognition, and he replied: “We were in need of the embodiment of the concept of thinking monitoring”. This extract was coded as ‘Need to transfer the concept of thinking monitoring from theory to practice’. Likewise, the teacher added that “I will also pay attention to evaluation”. This quote was coded as ‘Importance of evaluation skills’. In addition, Mr. Fallatah explained more about these skills and said “This can be done by introducing an ‘ideal example’ approach to deal with the problem [and] at the same time, trying to highlight the skill of planning and monitoring”. This quote was coded as ‘The importance of planning and monitoring skill in metacognition’. Overall, these three codes seem to relate clearly to metacognitive skills; therefore they were grouped under a category called ‘metacognitive skills’ which was connected to the main theme ‘Teachers’ understanding of metacognition’.

Some themes and categories needed to be reformulated as some did not make sense, or some overlapped while others were too general. For instance, education system, resources and equipment, the school Principal, supervision issues, training in implementing and traditional methods are categories that were first grouped under a theme called ‘implementation of metacognition’. This theme seemed too general and far too vague; therefore, it was reshaped and reformulated as ‘challenges to the implementation of metacognition’ which seemed more precise and appropriate to the categories. Appendix illustrates how the data were coded and grouped into categories and themes.

4 Findings

This chapter presents the findings of the data analysis. There were two case studies from the same school. Each case examined one mathematics teacher along with their students using the tools of interview, focus groups and observation. Each case was dealt with in two stages to observe metacognition in the teaching and learning of mathematics both before and after the implementation of the IMPROVE programme. Both stages were carried out in order to enable the formulation of a clearer and more complete picture of mathematics teaching and learning through metacognition in Saudi Arabia, rather than improving specific strategies or seeking to boost students' achievement. In this chapter, the findings are presented to avoid any repetitions, or data unrelated to the study's subject. The findings appear in the following order: all themes regarding stage one for Case One, then all themes regarding stage two for Case One, for both the teacher and their student. Similarly, this was done with Case Two. (Appendix 13 presented all themes that arose across data analysis)

4.1 Case study one: Mr. Fallatah's class

The students of this class sat in groups consisting of five or six students at tables that were shaped as trapezoids, which facilitated their arrangement into a circular configuration. Students worked in small groups. The classroom was also equipped with a smart board, a projector and an internet connection. The classroom was spacious and the number of students in the class did not exceed 30 (see **Error! Reference source not found.** for more details under selection of participants).

4.1.1 Thematic findings of observations

This section demonstrates the observation findings of the classroom reality of mathematics teaching and learning. The observation was designed and planned with the intent of observing the reality of metacognitive mathematics teaching and learning, in the aim of answering the research questions. This was undertaken with regards to both teachers' instruction and students' learning, and the extent of this reality's consistency with learning through metacognition.

The observation data collected from Mr. Fallatah and his students were categorized into the following themes: teaching strategies related to metacognition, mathematics learning strategy of the students related to metacognition, and teacher-students interaction.

4.1.1.1 Mathematics teaching strategies related to metacognition before the implementation of the IMPROVE programme

This section displays the teaching methods in mathematics related to metacognition that are followed by the teacher. As will be shown by the interview data (see 4.1.2.1) Mr. Fallatah had not heard of the term or even the concept of metacognition, and thus his teaching method in mathematics was generally unrelated to it or approaching it and it was not an intended procedure. Observation data clarified the following categories of teaching methods:

1) Focus on understanding the mathematics problem: The teacher concentrated on understanding the given problem to a good degree. This was the first step in the approach to the mathematics problem. For example, it was observed that the teacher pointed to the importance of the mind map in solving mathematics problems. In doing this he stated, “There are many mathematics rules in this topic, and you need to use a specific strategy to distinguish between the uses of these rules.” On another occasion, in a lesson concerning vectors on a coordinate plan: In a walking race, Ahmed progressed 120 metres in an easterly direction, then 80 metres in a N50 direction; how far is Ahmed from the starting line, and what is the angle of the quarterly direction? The teacher asked the students to illustrate the problem using a geometric diagram for the givens, in order to provide a preliminary mental visualization to the problem, to understand it and to determine the unknown. Then the teacher asked them to solve the problem. In another session, he teacher asked one of the students to read the mathematics problem and then define the given precisely and predict a preliminary solution before beginning to solve.
2) Dividing the solving method: The teacher occasionally split the solving of the problem into two parts to be dealt with separately, then to be combined to find the overall solution. On one occasion it was noted that the teacher asked the students to find each unknown from the problem: “Find the length of direction AB, the starting point of which is (-4.2) and finishing point is (-

5.3)", so that they could complete it. They then substituted the suitable rule to find the solution.

3) Presenting multiple solutions: The teacher displayed more than one solution to a problem. On one occasion it was observed that Mr. Fallatah displayed two methods to solve the same problem, with each of these being correct. Despite this he did not compare between the two and neither appeared to be related to metacognition.
4) Linking between previous and new information: As observed, the teacher also clarified new concepts by explaining the extent of the relationship between the previous and new information, and then linking them clearly. He also used a mind map, but this was only delivered verbally for the concepts being clarified.

4.1.1.2 Mathematics teaching strategies related to metacognition within the implementation of the IMPROVE programme

This section displays mathematics teaching methods related to metacognition that were followed by the teacher within the implementation of the IMPROVE programme.

The observations of Mr. Fallatah's teaching have been categorized as follows:

1) Following the steps of the IMPROVE programme: It was clear in observation that the teacher had taken to following logical steps in his approach to mathematics problems, be it in presenting concepts or solving problems. The steps of the IMPROVE programme are: presenting the new concept, metacognitive questions, practice, review and reducing difficulties, mastery, verification and improvement. For example, in one of the observations of a lesson regarding polar coordinates, the teacher presented a new concept with the discussion of students. He then presented a mind map to show several different concepts, then he told the students to review previous information, so that the new concept could be built upon it. The teacher then set a pre-prepared activity (see Appendix). After this he chose a group to display their solution to all of the students and discussed the group's solution with them in order to reduce difficulties and master this solution. The teacher determined the difficulties faced by students in their solving strategy during the equation finding phase. Following this, he clarified the method of solution

verification, which had already been carried out by one of the groups, to them all (for more details see section 3.3.5.2). Indeed, during the observation Mr. Fallatah demonstrated his belief in the importance of showing students a methodology to deal with the problem by following the steps of the IMPROVE programme, and I observed that he tried to highlight the four metacognitive skills. On another occasion Mr. Fallatah presented a new concept regarding the polar and Cartesian forms of equation through the use of Power Point, and then set an activity directly after so that it could be applied. Following this he distributed a worksheet containing a mathematics activity suited to metacognition, and set aside sufficient time. In doing this he asked students to use the IMPROVE strategy. After each group solved the activity, the teacher explained the solution on the whiteboard, and students discussed it. He explained the most obvious difficulties that they had faced in solving, doing this through his supervision of group work. He presented another activity to verify students' understanding and gave less time to solve it, then chose one of the groups to present their solution, then chose another with a different one, so students could compare between the two ways of solving.

2) Managing work in lesson time: at the beginning of the IMPROVE implementation, the teacher began to administer the timing of the lesson in an ineffective manner. For instance, the teacher planned to discuss two prepared activities in the first lesson but he presented just one of them. But with practice, he administered the lesson time more effectively, trying to complete what he had planned, using most of the steps noted in the IMPROVE programme. In one of the observation sessions the teacher divided the class into groups of no more than four with the goal of their members being diverse in academic attainment. He then presented the new concept based on previous knowledge, and linked between them in a clear and brief manner (polar coordinate system and complex number). He did this at the start of the lesson, using whiteboard diagrams to support himself. He then gave the groups a worksheet designed for metacognitive questions. These questions are: questions to understand the problem, the solving strategy, and about linkage. He then asked them to begin solving the first activity. In this exercise, student interaction was strong, as they pondered

and justified each step. Among the questions they asked were: Why are we doing that? Why do not we do this? The teacher then discussed with each group and tried to determine the difficulties that they were facing. Five of the seven groups presented a correct solution, then the teacher displayed the correct solution to the class.

3) Preparation of suitable activities: The teacher's choice of activities (see Appendix) to present to students in class was very appropriate for metacognitive teaching (see 3.3.5.2). This was due to several reasons. Firstly, the activity contained a new idea that created a sense of challenge for the students. Secondly, the design of the worksheets encouraged students to work effectively. Thirdly, the teacher presented the activity then asked the students solve it on the worksheet without having solved any examples for them. This helps students to more easily solve mathematical problems. This was constructive learning to a degree. It became clear through observation that Mr. Fallatah's initial notions (see the finding of his interview in 4.1.2.2) of metacognition were consistent with his teaching performance in general.

4.1.1.3 Mathematics learning strategy of the students related to metacognition before the implementation of the IMPROVE programme

As emerged from the student interviews data (see 4.1.3.3), they were not aware of the term or concept of metacognition, and they had not experienced learning through metacognition. Thus there was no hint of any clear strategy being followed in mathematics learning that related to metacognition.

The observation data clarified that the students' focus was entirely on solving the mathematics problem directly and in the quickest way possible. It also emerged that the majority of the students solved mathematics problems using the method of identifying the demanded value and the givens. Then they would find rules to link between the givens and the demanded. To do that many students relied on matching their solutions with previous ones which had been presented to them by the teacher or were in the textbook, and thus it was found that students would often ask about this. It was also observed that the students were uninterested in discussing their method of thinking in solving mathematics problems, but rather they only discussed the solution to a given problem. They were generally uninterested in generating multiple solutions, but instead were satisfied with simply solving the problem using the solution given by the teacher or found in the textbook.

4.1.1.4 Mathematics learning strategy of the students related to metacognition within the implementation of the IMPROVE programme

This observation was conducted on students during lesson time and aimed to investigate their learning in relation to metacognition. To achieve this aim, this section provides some examples of group conversations in order to demonstrate learning according to the IMPROVE steps in practice. Consequently, this section is concluded by a summary of how the observation data showed that the method of mathematics learning improved. This observation, undertaken after the implementation of IMPROVE, confirmed that at the beginning of IMPROVE implementation, the students' interest was upon solving the mathematics problem directly, regardless of metacognition. There was no clear methodology for work, and not even for effective dialogue between students. The following conversation from one of the groups demonstrates this:

> Student 1: (reads the question) 'Find the coordinates of the unknown that match the following conditions: P1= (5, 125°), P2= (2, θ), P1 P2=4, 0≤θ≤180°'.
> Student 2: There is a rule that can be applied to this to solve the problem. I will look for it in the book.
> Student 1: You will not find this in the book.
> (Teacher asks all the students, 'Please do not limit yourselves to today's lesson, and try to invoke knowledge from the whole topic.')
> Student 1: There are two givens, and the question says that multiplying them by each other will equal four.
> Student 2: (reads the question again, aloud) There is a rule that can be applied to this problem.
> Student 1: Ah, the length and the sum of their multiplication equal four.
> Student 2: What exactly is the unknown?
> Student 3: OK, why don't we write down the original problem?
> Student 1: Find the theta.
> Student 2: What is the method to solve it?
> Student 1: It can be solved by finding the theta through an equation.
> Student 3: Guys, we are not asked for the distance but rather to find the coordinates of the unknown.
> Student 1: It's done, we have already found it.

(The student writes the solution steps.)
Student 2: Why don’t you write with a pencil, so if we make a mistake you can erase it?
Student 3: You have now found the root, do you put this here and that there?
Student 2: We can divide it up.
Student 3: Now we need to find the theta, what do you think, shall we divide it?
Student 1: R is an unknown and not the angle.
Student 2: We want to find this theta.
Student 1: The theta is cosine 25.
Student 2: No, 25 is subtracted from the theta.
Student 3: I have an idea, why don’t we solve with alpha?
Student 2: How is it cosine 25 when this tells you that it’s cosine *x* minus 25?
Student 1: Ok, are we saying that we subtract this from this and multiply the result by 9 to equal 16?
Student 3: It says that it’s smaller than 180 and bigger than…
(The teacher says, Students, do not forget to categorize the question. What topic is it under?)
Student 2: Cosine what gave us 16?
Teacher: What’s important is for the method of solving to be correct, not only the final solution.
(A student asks where the attendance sheet for students is.)
Teacher: OK students, time’s up.

This is an example of there being no clear methodology for work, and not even for effective dialogue between students, as mentioned above. Furthermore, not all group members contributed, as shown by Student 4. In another IMPROVE implementation after this, the teacher tried to comply with all seven steps of the programme. However, in the reality of the first implementation, the teacher presented the new concept at the start of class time, using whiteboard drawings to assist him. He presented the new concept based on past knowledge and they were linked together clearly (Rule of distance between two points in a space).

Following this, the teacher gave the first worksheet to the groups. This was designed according to the steps of metacognitive questions, with these being questions to understand the problem, the solving strategy, and of linkage. What follows is an example of the dialogue within the different work groups. (See sections 3.3.5.1 and 3.3.5.2 for more details about how observation was conducted including several observations of different groups. This meant that Student 1, for example, was not always the same person in the findings):

> Student 1: Of course, this problem relates to the lesson we had the previous time.
> Student 2: Let's read the question – 'draw the following on a Cartesian diagram'.
> Student 3: We need to determine the givens and the demanded.
> Student 1: We are required to transform the equation.
> Student 3: I think that a strategy to solve that we could follow is one related to this problem.
> Student 2: We have here R equalling… We can multiply both sides by R and it will become…
> Student 1: Why did you do that, like that the Sin will equal…
> Student 3: This is instead substituting it for Y, and it becomes…
> Student 1: The solution is just like that, or what?
> The group says to the teacher 'we have got to this solution, shall we complete it or stop?'
> The teacher participates with the students and tries to help them by saying: You also made a mistake in R, because you multiplied both sides by it. This means you are multiplying it in the numerator and not the denominator, so how did R get into the dominator?
> The students subsequently tried again after the guidance of the teacher.
> The teacher said in a loud voice: 'There are six minutes left students, and the time set aside for solving is up!'
> Student 3: It seems to me that the problem now, after this step, is how we can substitute in the rule.
> Student 3: I don't want the teacher to choose me to explain the solution because I don't understand how I got here.

The teacher calls upon the student Basel and asks him to bring his solution sheet with him. He asks him to demonstrate his group's solution and then asks the others to identify the errors that the group made.

It was observed that the interaction of the students was better than before. They pondered and justified each step, and the questions 'why have we done this' and 'why haven't we done this' were constantly repeated.

On this occasion of IMPROVE's implementation, it was observed that the groups were organized well. Perhaps this was a reason behind their extensive cooperation. Also, the teacher's choice of the worksheets activity was very appropriate for metacognitive teaching, for several reasons. Firstly, the activity contained a new idea that created a sense of challenge for the students. Secondly, the design of the worksheets encouraged them to work effectively. Thirdly, the teacher presented the activity then asked the students to solve it on the work sheet without having solved anything for them. This makes students enthusiastic to solve, and after the teacher had corrected them, their understanding of the problem was complete. This was constructive learning to a degree because it encouraged students to search for knowledge by themselves.

On the other hand, the teacher was not able to display the difficulties faced by students and then discuss them, due to time constraints. So the teacher set a homework activity to check students' understanding, although this was supposed to take place in the same lesson. The teacher gave general corrective evaluation, but he was not precise in evaluating the type of mathematical thinking. It was clear that the students faced a greater difficulty in understanding the problem.

Following this, and after further practice, the students began to have a thinking method for dealing with mathematics problems in accordance with the metacognitive questions in the IMPROVE programme. These are: questions to understand the problem, the solving strategy and questions of linkage. To this end, the following example of dialogue from one of the groups illustrates this issue:

> Student 1: Let's determine the givens and the demanded clearly.
> Student 2: This is good, now what is the relationship that links the givens and the demanded.
> Student 3: How will relationship benefit us?
> Student 1: It is a key to the solution.
> Student 4: What will we do after that?
> Student 2: We need to transform the root to a number.
> Student 3: How will we do that, I mean we want to know what the solving strategy is.
> Student 1: We transform the Cartesian diagram to a polarity one, then we will try to find the demanded value.
> Student 4: We can't do that because we need to find R and then substitute in the general diagram.
> The teacher: Who has found the demanded number? You have one minute left, and I need completed worksheets.
> Another example from students' dialogue to illustrate how students deal with mathematics problems in accordance with IMPROVE's metacognitive questions:
> Student 1: What is demanded in this activity?
> Student 2: It's…
> Student 3: It's important that we categorize this problem under a topic.

> Student 1: Since he [the teacher] has asked us to find the angle, then we must use the angle rule [solving strategy].
> Student 4: Correct, so we need to write the correct relationship between the sides.
> Student 1: Yes, then we find the value of the angle so then we can find the final demanded value.

(The students find the solution, but one of the students asks, 'Does this solution mean that the two sides are perpendicular?')

Another example of practising learning according to the IMPROVE steps:

The teacher says 'We have ten minutes to solve this problem, and the sheet in front of you is blank. I will be evaluating you on how well you follow the steps that we have learnt on previous occasions.'

> Student 1: Let's read the problem.
> Student 2: To begin with, we have to determine the givens, the demanded and the strategy to solve.
> Student 1: We need to find the first root then the second, etc.
> Student 3: But how can we find the first root?
> Student 4: I have an idea, let's substitute the rule, then it will give us the first root.
> Student 3: How do you know this will give us the first root?
> Student 4: Because it says that's the quantity of the first root.
> Student 1: So he doesn't want us to find the value of the first root?
> Student 2: We haven't learnt these sorts of problems before, let's transform it to a Cartesian diagram and see.
> Student 1: It tells us that theta is over N, but in the second root we add, because it says it increases. So this means that it is related to the second root. Let's try.
> Student 3: So far we have used all the givens, so it remains for us to find the first root.
> Student 2: Are we moving ahead correctly?
> Student 1: Yes, yes
> Student 4: One moment, now how much is this number cosine?
> Student 3: The first theta is over 2 and we add to the second one K over 2.
> Student 1: Now, it's in a polarity diagram, so we will change it to a Cartesian diagram, and the value of cosine multiplied by this number equals….

The observation data showed that the method of mathematics learning changed from a complete reliance on the explanation and solution of the teacher to efforts by the student to search for the knowledge and build upon it.

In summary, the observation data demonstrate that the students' learning method at the beginning IMPROVE's implementation (similar to stage one) was based on solving the mathematics problems directly and this was done regardless of any thinking method. However afterwards, with practice, the students took on a thinking method for the mathematics problems in accordance with the metacognitive questions. These were set out in the IMPROVE programme and they were: questions to understand the problem, the solving strategy, and questions of linkage. The students' learning method in mathematics transformed from a complete reliance on the explanation and solving of the teacher to them making efforts themselves to search for knowledge and build upon it.

4.1.1.5 Teacher and Student relationship before the implementation of the IMPROVE programme

This section reveals the nature of the teacher-student relationship in mathematics learning inside the classroom. It also presents the extent of compatibility of this relationship with the use of metacognition. Observations showed that the teacher tried to stimulate cooperative learning methods, in which students would sit in small work groups. After he explained a new concept using the whiteboard, he distributed worksheets to students for them to solve collaboratively. However, because the activity presented was simple [Find the length of the direction which has a starting point of (-7, - 2) and (6, 1)?] and the steps to solve it were relatively straightforward, the level of student cooperation was minimal. In addition, the extent of their interaction with the teacher was also minimal.

Another strand of observation data shows that students feel at ease when asking the teacher questions, or vice versa. For instance, one of the students mentioned some previously learnt knowledge to the teacher and asked about its link with the new knowledge. Another student asked the teacher, “if … happens, can I do …?” A different student asked, “when … happens, is it … that has changed? Can I then do…?” In all these situations, the teacher's reception of this participation and the questions from students indicated that there was a positive and constructive relationship between them.

Furthermore the teacher approached errors made by students in a positive manner. In one instance, the teacher clarified an error made by a student during their cooperative work in drawing the diagram related to the problem. It was also noticed that some students used a diverse range of methods. Their acceptance of some other students' suggestions was good, but they did not discuss which of the two methods was easier, to then be decided upon.

It was also noticed that the teacher discussed ideas with students when they presented answers and asked them to justify their points of view. The teacher often repeated, "The product doesn't interest me, what does interest me is how you got it". The teacher also accepted new perspectives shown by the students. For example, a student explained a method, and the teacher responded that it was corrected but was lengthy, so another student suggested a different, easier solution. In doing this he stated,

> It is better for me to have a student solve one problem in different ways than for him to solve 20 problems with the same method. This is because the student gains multiple numerical and geometrical skills and a quick intuition.

Despite these positive notes on the teacher's interaction with students, he remained concerned with the correction of errors the students made when solving mathematics problems. This was instead of a relationship based upon adjusting thought methods in dealing with such problems. Thus it was not observed that the teacher discussed students' thinking with them when they were learning mathematics, which would have educated them in metacognition.

The observation data highlights that the teacher used a cooperative leaning strategy, which in turn encourages metacognitive mathematics learning. This involves discipline in timing, distribution of groups, managing class activities well, as well as presenting new concepts, solving mathematics problems and correcting student errors. The teacher's method was that he would usually distribute worksheets to students after explaining a concept. They would then solve these cooperatively, with the teacher asking if the problem was clear in what it demanded, and by asking which methods could help students and be used to arrive at this demanded value. However, due to the activities presented by the teacher being occasionally simple in the steps to solve them and being clear in idea, the students' cooperation was weak.

The observation data highlights that the degree of cooperation between the students was not large, with students solving problems alone rather than cooperatively. One reason for poor cooperation was that some problems did not require cooperation to begin with, due to them lacking aspects which could stimulate active thought. It was also observed that most of the students' discussions revolved around correcting each other's mistakes when solving the mathematics problems, and not around discussing their way of thinking when solving such problems.

4.1.1.6 Teacher and Student relationship within the implementation of the IMPROVE programme

Mr. Fallatah's relationship with his students was characterized by calm dialogue. For example, when he presented a new concept, a student was observed asking, "Does this concept mean that…?" The teacher confirmed that the student's idea was correct. The student also asked, "Is it possible for me to solve the problem by….?" The teacher then decided on the correctness of the student's method. It was noted that the teacher participated in the students' work during their solving of mathematics problems. For example, the teacher intervened in one group's discussions, and asked the students a question. The answer to this question was a key to solving the problem, yet students could not do so because they had forgotten important previous knowledge for solving. On another occasion, one of the students suggested an alternative solution to the teacher, and the teacher discussed this in front of the class, and asked them to compare, then students arrived at the opinion that the initial solution was easier.

The aforementioned actions by the teacher were consistent with his views; expressing these he said that "metacognitive teaching is important, because it makes the student the centre of the educational process. It also encourages him to search for information and stimulates his thinking and abilities, and gives them the tools of self and thought evaluation, particularly with serious students". This was also displayed by him stating that "the presence of a logical thought map helped in this. Also the allocation of time to the different stages of dealing with a problem. The important point here is that this method helped students to arrive at the knowledge by themselves, and that learning has become constructive."

In summary, the observation data shows that the teacher was able to create a cooperative environment, which assists learning through metacognition. One of the important reasons for this was his management of class time, as has been observed. Another reason was the dividing of students into diverse groups, and now ensuring that the number of students in each of these should not exceed four. In addition he prepared class activities in order for them to be consistent with the IMPROVE programme. Furthermore the manner of dialogue between the teacher and the students, as well as among the students themselves, improved in accordance with the metacognitive questions. As outlined in IMPROVE, these were: questions to understand the problem, the solving strategy and questions of linkage. The student Ragab said, “This method clearly encouraged cooperative learning, because each group was asked to display its work and there was a map to follow.”

4.1.2 Thematic findings of Interview

The interview data collected from Mr. Fallatah were categorized into the several themes as following.

4.1.2.1 Teacher’s understanding of metacognition before the implementation of the IMPROVE programme

The teacher Mr. Fallatah – the first faculty member involved in the study - was asked about his concept of metacognition. He began to see that the reason for many of the problems in mathematics learning for students lies in the method of thinking when dealing with concepts and problems. Speaking on this he said:

> I think that the reason for many of the problems in learning mathematics is our method of thinking about the action, and not about a specific problem in how we did that action.

However, when he was asked about his conception of metacognition, he said “This is the first time I have heard of this term”. He then asked for a brief explanatory overview, after which it appeared he did indeed have a little background knowledge of the subject, it was merely a matter of terminology. So he was able to state, “As far as I am concerned, the concept of metacognition is new to me, however I can tell that I do know certain concepts related to metacognition”. He included three ideas in his conception of metacognition, which can be categorized as follows:

1) Corrective purpose: Metacognitive concept was one of correcting the students' train of thought in linking mathematical concepts. He stated:

> Some of the students match specific problems, which are not comparable to begin with, and some of them continue to do this – which proves that they cannot distinguish between problems. Through observation of these students, it has also become clear that the students have a problem in linking between concepts. The role of the teacher is to correct this mistake in the course of thought.

2) The thought process: The second point was that the concept of metacognition involved re-guiding the course of a student's thought, before he or she arrives at an incorrect solution. This was seen by the teacher as "the process of monitoring the logic of the thought sequence in the mind of a person".
3) The improvement of thinking: The third idea was that metacognition was the skill of improving and developing thinking when dealing with mathematics problems; the teacher was aware that this process involved the honing of particular skills, rather than a general and vague body of knowledge. During his interview he said that "this concept involves certain skills, which cannot be gained without previous planning".

It emerged from the data that the teacher did not have a specific conception of metacognition, however, he could summarize it at least in principle with three concepts. Not only did the teacher hold a notion as to the issues that metacognition sought to correct, but also the methods which that process involves. This background knowledge proved to be a useful base upon which to build further, more detailed knowledge of the concept, and would allow him to be a constructive participant in its implementation.

4.1.2.2 Teachers' understanding of metacognition after the implementation of the IMPROVE programme

After the implementation of the IMPROVE programme, Mr Fallatah was asked about his perception of metacognition. His answer contained four categories as follows:

1) Concept of metacognition: Mr. Fallatah believed that metacognition contained a logical thinking map. In explaining the nature of this map, he said: "Another important element of the method is the existence of a logical

thinking map, which in turn aids students in time management for dealing with solving problems". In addition, he asserted that metacognition was a systematic logical procedure. Thus, he expressed his conception of metacognition by saying, "The work was done within logical steps, in order to resolve problems".

2) Metacognitive skills: In his interview Mr. Fallatah was asked about some of the skills of metacognition, these being planning, management, monitoring and evaluation. He emphasized the importance of practising these metacognitive skills, saying: "A large amount of practice of the four skills strengthens this aspect". In particular, Mr. Fallatah stipulated that two skills were very important to learn metacognitively, these being monitoring and evaluation. In terms of evaluation, he said: "I will also pay attention to evaluation" and, "We are still in need of greater efforts to deal with the issue of evaluation and assessment". Mr Fallatah asserted that:

   > The most important thing is to create a class atmosphere. This could involve reducing the number of students in the class to facilitate group and individual evaluation.

   Mr Fallatah explained how this could encourage students to monitor their thinking by saying, "This can be done by introducing an ideal example approach to deal with the problem; at the same time, trying to highlight the skill of monitoring".

3) The function of metacognition: Mr. Fallatah also discussed metacognition from another angle. This aspect was the function of metacognition in learning. In doing this he mentioned several such functions. Firstly, he thought that metacognition would help him to discuss students' thinking rather than simply discussing solving methods. When Mr. Fallatah was asked "Do you discuss with your students about how they think when they learn mathematics?" He answered, "This method helped me in doing this and that is what I hope for in the future". Secondly, metacognition enhances students' expertise in maths. He said; "This method provides students with expertise in dealing with maths". Thirdly, metacognitive teaching encourages students to participate in a constructive learning process. He said:

   > The importance of metacognition in teaching is great, due to it making students at the core of the education process, with it being

> them who search for information in order to encourage constructive learning.

Finally, his statement was based on his view that metacognition helps students evaluate their thinking. He mentioned that “this method supports students’ thinking and their abilities which enables the student to evaluate their thinking”.

4) Academic achievement: After implementing the IMPROVE programme: Mr. Fallatah felt that low academic achievers benefitted less from metacognition than other students. He said:

> I believe that students of weak academic achievement are the ones who benefit least from this teaching style, the reason for this being a smaller body of prior knowledge and experiences to draw on.

He explained this point by saying:

> A low academic achievement of the student in maths is specifically because they cannot participate with their classmates in the discussion and work to the solution of the problems, and also because mathematics requires prior knowledge.

In addition, he suggested that “

> Weak students must be taken into account if we want to benefit from implementing metacognition, in that the school must examine how it can deal with them to address this weakness.

4.1.2.3 Mathematics teaching technique before the implementation of the IMPROVE programme

The teacher was questioned about his mathematics teaching methods and the extent of compatibility between these and teaching according to metacognition. His answer to this involved three aspects, which can be classified as: compliance with the traditional method, discussing some of the important skills in maths learning, and speaking of strategy in mathematics teaching.

1) The traditional method: The traditional method of teaching mathematics featured prominently in his answer, which was presenting a new concept, then discussing its direct application by relying on what is asked for and how to arrive at it. He clarified this by stating,

> After explaining the concept, I move on to its application in solving mathematics problems, I then ask students if what is demanded by the problem is clear, what are the things we could use to help us in arriving at an answer and how we move from these givens to the solution of the problem by using our past knowledge from rules, etc. Then we check our solution by using substitution, for example.

The teacher also noted that he encourages students to create new solutions for a mathematics problem, but he did not have a clear method as to how this was done. He said:

> I try to tell the students to not stick to a specific method for the solution and I reward any student who comes up with a new way to solve, and we tell the student you must get accustomed to searching for alternate ways to solve.

2) Skills for learning mathematics: The teacher noted a group of essential skills in learning mathematics: planning, estimation, monitoring and evaluation, despite him not explaining a clear application for these skills or how one could help a student in mastering them. Regarding the skill of planning, Mr. Fallatah saw that this was important in improving a student in his thinking, as he said “Metacognition is skills that relate to thinking, and skills are not gained without prior planning for this”. In terms of estimation, the teacher saw that this skill helps the students in their method of thinking as he said:

> Anticipating and estimating the solution, even if it’s only close and doesn’t give the correct solution, then this improves the student’s method of thinking.

In terms of the skill of monitoring, the teacher did not go into detail about this, but he spoke about monitoring the sequence of steps to solving a maths problem. He explained, “I monitor my method in solving the mathematics problem and this helps me to arrive at the correct solution”. On the skill of evaluation, the teacher felt that it had a large role in correcting the thought methods of students, but he limited the skill of evaluation to only being part of a teacher’s performance and did not mention that it was one of the skills that students must master. He discussed this by saying, “Through evaluating a student’s work, participation and questions one can identify his way of thinking”, further clarifying that “correcting a test shows you common errors that students make, which demonstrates that there is an error in the method of thinking which has led to this mistake or problem”. Mr. Fallatah highlighted that evaluation could involve the entirety of a student’s performance or it could be undertaken during their solving of a maths problem. He explained this by saying:

> I believe that there is an evaluation for performance after the student has finished and there is evaluation for the course and sequence of thoughts, step by step, before a student finishes solving a problem.

3) Teaching strategy: The teacher mentioned a strategy that he occasionally used, which was thinking aloud in order to train the students to explore the ways of dealing with a mathematics problem. Describing this process, he stated: “I can practise thinking out loud in front of the students in order to train them for this method”. He emphasized the importance of this method by stating that:

> He who uses this method does so as if he is addressing his mind when solving the problem and criticizing himself and his steps; it’s as if he monitors his thinking in his own specific way.

4.1.2.4 Challenges to the implementation of metacognition after the implementation of the IMPROVE programme

As for the challenges that might confront teaching mathematics metacognitively, Mr. Fallatah saw these as revolving around five categories. These categories merged from the data drawn from his individual interview.

1) Traditional methods: This issue was seen as a major challenge to implementing metacognition by Mr. Fallatah. He said:

> The students have the ability, but we are the ones who undermine their potential with using traditional styles of teaching, as well as our focus on grades from tests.

He explained his idea by saying, “Unfortunately, until now, we were not able to focus on evaluating these aspects, as our focus was placed on evaluating the results of their problem-solving”. Mr. Fallatah said:

> The greatest difficulty is that we have become accustomed, over time to a particular way of teaching mathematics, involving the use of the traditional courses and the investment of resources. This makes it difficult for change to be accepted.

He described the difficulty of departing from traditional methods: "We were not able to free ourselves of the old methods, despite our knowledge of many important aspects in the subject of evaluating thinking in itself". One of the main reasons behind that was that "some students feel that their own methods are more beneficial to them, and get them good grades, and thus there is no need for them to try other ways", Mr. Fallatah claimed.

The term 'traditional method' in this context refers to the presentation of mathematical concepts in a direct manner – i.e. without linking it to other concepts or how such concepts really work, so the students are aware of how to imitate but they do not know why; essentially a process of rote learning. This method is not conducive to shaping mathematical thinking to deal with varying problems using differing methods in changing contexts. Mr. Fallatah was asked about his method of teaching mathematics; his answer was based on following the traditional method in teaching, through which he presented a mathematical concept then clarified the application of this for solving mathematics problems. On this subject he said:

> After explaining the concept, I move on to its application in solving mathematics problems, I then ask students if what is demanded by the problem is clear, what are the things we could use to help us in arriving at an answer and how we move from these givens to the solution of the problem by using our past knowledge from rules, etc. Then we check our solution by using substitution, for example.

2) Training: Mr. Fallatah pointed out that there was an absence of preparation and training for teaching through metacognition, be it at university or during a teacher's service in education. This was considered to be one of the challenges confronting instruction though metacognition Mr. Fallatah said, "There was no previous preparation for this concept, neither in university, nor during my work in teaching, so how can I teach with this method?" Indeed, he pointed to the importance of training for this style of teaching and of limiting educational oversight concerned with this matter. He said, "I think that the educational supervisory authorities should focus on training and development of teachers in the field, not only in a training room".
3) The education system: In citing issues relating to the education system, Mr. Fallatah said:

> The general lack of previous adoption of metacognition in education is one obstacle. In addition, the lack of pursuit of the question of how we can implement metacognition in reality.

Providing greater detail on this matter, Mr. Fallatah stated:

> If the educational system does not give importance to metacognition, there will be inaction on the part of the teacher to research, inform himself and try the method.

Mr. Fallatah hoped that there would be a partnership between the education system and educational research centres. He expressed disappointment at the fact that there was an “absence of partnership with research centres supporting and activating an educational environment in the school”. He suggested:

> There must be specific agencies for new methods in teaching which the teacher can communicate with to develop his performance in teaching fundamentally and in application.

Regarding the nature of such centres, he specified that Saudi Arabia had a “need for specialised institutions that offer metacognitive teaching”. In addition, he requested for there to be “additional incentives for teachers who apply metacognition”.

4) Educational supervisors: Supervision issues are one of the challenges for implementing metacognitive teaching, according to Mr. Fallatah. He stated:

> I see that educational supervision has a role in spreading new methods, with that being done through seminars presented by them and visits they undertake. Regardless of any belief that educational supervisors might have in a method that improves the educational process, they haven’t presented anything relating to this subject.

An absence of teacher evaluation criteria for using metacognition and a focus on superficial issues when evaluating teachers were both identified and explained as obstacles to the implementation of the method. He said:

> The principal or supervisor's evaluation of a teacher does not include any criteria pertaining to the application of this method; instead, there is a focus on how much scheduled material one has completed.

Mr. Fallatah claimed that "educational supervision has a very important role in promoting metacognitive methods".

The educational supervisor is a technical expert in teaching methods of a certain subject. His main role is to provide teachers with technical services for improving teaching methods, and help them solve educational problems that meet them in the teaching domain.

5) The school Principal: The school Principal himself was considered as a challenge facing the implantation of metacognition. Mr. Fallatah explained, saying that:

> The Principal's adoption of this concept is a strong motivation for its implementation, but I do not see that he is interested in metacognitive teaching with this method, which forms a difficulty facing metacognitive teaching.

Mr. Fallatah claimed that:

> The conviction of the school Principal has a very important role in promoting methods such as these, because this is what convinces the Principal or educational supervisor to potentially convey the method to the teachers.

In other words, he said "The school administration remains unconvinced because its focus is on the direct academic attainment of students and the completion of curricula".

4.1.2.5 The teacher's requirements for the implementation of metacognition

Based on the interview with Mr. Fallatah, several issues were mentioned regarding teachers' needs and requirements for metacognition to be successfully implemented.

1) Evaluation skills are required to implement metacognitive teaching in mathematics, Mr. Fallatah said:

> The most important thing is to create a class environment. This could involve reducing the number of students in the class to facilitate the issue of group and individual evaluation.

In addition, he said, "We are still in need of greater efforts in dealing with the issue of evaluation". To explain how teachers can perform better in this area, Mr. Fallatah said, "Taking the worksheets back and looking over them reveals to the teachers a lot about flaws in thinking in a certain manner." Moreover, he added, "I will also pay attention to evaluation, be it in reviewing each group's work or displaying group worksheets and discussing with students". Despite the aforementioned suggestions, Mr. Fallatah said:

> Unfortunately, until now we were not able to focus on evaluating these aspects, as our focus was placed on evaluating the results on their problem-solving.

In addition he said. "We were not able to free ourselves of the old methods, despite our knowledge of many important aspects in the subject of evaluating thinking in itself". Finally, Mr. Fallatah asserted that '"the teacher needs to transfer the concept of monitoring thought from theory to practice".

2) Learning materials: some issues about the design of materials were raised. Mr. Fallatah thought that "the activities should involve indirect solutions, previous experience, hold new ideas, and should be challenging". Nevertheless, he still saw value in the presently used resources, which he said lay in "the syllabus containing activities which are compatible with metacognition", and he suggested the creation of "enriched books to support the curriculum".
3) Low achieving students: According to Mr. Fallatah, for the implementation of metacognition it is important to deal with students' low achievements carefully. He said, "Weak students must be taken into account when implementing metacognition in order for the school to examine how it can deal with them to address this weakness". He justified his opinion by saying that this was:

> Because they cannot participate with their classmates in the discussion and working to the solution to the problems and also because mathematics requires previously gained knowledge".

4) Techniques: Mr. Fallatah mentioned some issues regarding the techniques that would be needed to implement metacognition. He claimed that students lacked thinking strategies by citing "problems at a time when many students possess only prior knowledge, which does not involve the strategy of thinking." Mr. Fallatah used many techniques to implement metacognition; one of them was "displaying the solutions of the different groups to the class

is important". In addition he said, "We can implement a brainstorm strategy". He added, "Also the display of outstanding examples and the use of a camera for documentation are very important". He also mentioned, "This can be done by introducing an ideal example approach to deal with the problem at the same time, trying to highlight the skill of monitoring". Moreover, he said:

> As for metacognition in itself, the division of students into varying educational achievement groups proved to be valuable in aiding cooperation. Another important element of the method is the existence of a logical 'thinking map', which in turn aided students in time management for dealing with solving problems.

Mr. Fallatah sought to "present new concepts metacognitively". Finally, Mr. Fallatah asserted that time management was a skill needed to implement metacognition. He said "Metacognition calls for several skills; one of these is time management. This will ensure that we benefit from every minute of the tight class sessions."

5) Initial phase of implementation: Mr. Fallatah mentioned some aspects regarding the preliminary phase of implementation. He said:

> In the beginning I was concerned about the reaction of some students and the difficulties they might have found, however implementing it smoothly with the provision of incentives is better.

In the same regard, he said "At the beginning of the process there were concerns but in the end, when the students practised this theory, it turned out well".

6) Motivation to implement: Mr. Fallatah was very keen to encourage students to seek knowledge. He said "We began a certain process of change in making our efforts for the students greater, these efforts being targeted at obtaining knowledge". Thus, he felt satisfied because he "saw the benefit for the students. Despite the greater efforts required". Mr. Fallatah asserted two things; "The qualified teacher in learning strategies and its theory will have the motivation to engage with this method" and "A large amount of practice for the four skills is needed to strengthen this aspect [monitoring student's thinking]".

7) Characteristics: From the students' point of view, the teacher possessing certain characteristics for the practice of metacognitive teaching was seen as important by several students. One such characteristic was for the teacher to have knowledge of various styles of thinking in dealing with mathematics problems. Based on the findings of student interviews, Mohammed explained, "It is important for the teacher be informed about the students' thinking styles in mathematics learning. This is so he can be well versed in metacognitive teaching". Likewise, Asam pointed out,

> The readiness of the teacher to deal with this methodology is important to reap the benefit of teaching with metacognition. I noticed that the teacher changed in his enthusiasm and even his methods as time went on and the method was practised.

Ziyad added, it was necessary "for the teacher to have absorbed the method of metacognitive teaching".

4.1.2.6 Cooperative learning and metacognition before the implementation of the IMPROVE programme

Despite Mr. Fallatah not having a specific and clear notion regarding metacognition, he did see a link between cooperative and metacognitive learning in that the former assisted learning through the latter. He stated, "It is possible that cooperative learning could help learning through metacognition". He clarified this by saying that cooperation helps to generate multiple solutions, which can be criticized through the participation of students, which aids the improvement of thinking. He explained this:

> Collaborative learning is one of the methods for generating multiple solutions which can then be displayed in front of all the students to know which of the solving methods are easier or more difficult and why – all of this improves the students' thought process.

The teacher pointed out that students had a weakness when it came to the skill of communicating with others, which reduces the interaction of collaborative learning, in addition to interaction in metacognition. Mr. Fallatah highlighted this by stating,

> In order to evaluate the thought of the student, he must show you how he thinks, however when you ask students to express what they

> are thinking, many of them are shy to do that. This is because they are not accustomed to presenting their ideas and discussing solutions with others.

Thus, the teacher stressed the importance of distributing the students into work groups in a manner appropriate for metacognitive learning, as he said that “the seating configuration of students and their distribution in the classroom is important for the application of metacognitive teaching”. This could be done by strategically equally distributing the more vocal and adept communicators among the groups to promote discussion. As mentioned by Mr. Fallatah, the seating configuration is also important as tables should be arranged in a manner conducive to an all-inclusive discussion, rather than having one student lead it.

4.1.2.7 Metacognition and cooperative learning after the implementation of the IMPROVE programme

In terms of the relationship between cooperative learning and metacognition, Mr. Fallatah thought that a strong connection existed between metacognition and cooperative learning. When speaking of cooperative learning, he considered low academic achievers as being unable to participate with their classmates in the discussion. He said:

> Students with low academic achievement benefit less than other students because they cannot participate with their classmates in the discussion and working towards the solution of the problems and also because mathematics requires previously gained knowledge.

On the other hand, he said: “If students are outstanding students (in terms of grades), this could increase their enthusiasm for engaging in cooperative learning with other students of lesser ability”. Based on these premises, he suggested that:

> The activities should involve indirect solutions, previous experience, hold new ideas, and should be challenging. This encourages students to interact more with the subject, the teacher, and among themselves.

Hence, the importance of monitoring the cooperation of each work group was raised in the interview. He said: “Looking at the worksheets to evaluate students’ work reveals to the teachers many aspects of the groups’ cooperation". (These worksheets had been designed according to the IMPROVE programme.) In addition he considers that

> Every student should try to present what would help the other group members with solving problems. As for this method in itself, the division of students into varying educational achievement groups proved to be valuable in aiding cooperation.

4.1.3 Thematic findings of student interviews and focus group (Mr. Fallatah's students)

The major purpose of conducting the individual and focus group interviews (see Table 3.1: Research questions and methods) was to explore how secondary students and their teachers perceive metacognition in relation to mathematics teaching and learning, and to discover the experiences of metacognition in relation to mathematics of secondary students and their teachers in Saudi Arabia. Consequentially, reporting the findings from the interviews and the focus groups together should help to achieve the major purpose of using these methods. Interviews and focus group discussions were conducted with students participating in the study before the implementation of the IMPROVE programme. The data collected from these participant interviews and focus groups have been categorized under the following themes.

4.1.3.1 Students' understanding of metacognition before the implementation of the IMPROVE programme

Students were asked about their notion of the concept of metacognition. It became clear that the term 'metacognition' was new to them, as the students Mohammed, Mazen, Qusay and Ziyad all stated, "No, I have not heard of the term metacognition before, and I have not studied metacognitively". Ziyad elaborated on this by adding that "the term metacognition is interesting, because most of the time the word 'thinking' by itself is sufficient, but 'thinking about thinking' is something new". After the term was explained to them, it appeared they did have a little background knowledge of the subject. Their initial notions of metacognition can be summarized into a few aspects.

1) The concept of metacognition: Some students said that metacognition meant knowing the courses of thought. Mohammed expressed this idea by stating

> It seems to be that the concept of metacognition is a student knowing about the courses of thought. In this, if a student only has one course of thought, then it will be difficult for him to know the errors in this method.

He also added that:

> Having numerous styles of thought enables a student to choose a method or style of thought first. Second, it enables you to compare your thought with other methods and to adapt and improve thought methods in the future.

Mohammed also explained, “I think that if every student understood how he thought and arrived at this stage, it would be a very positive thing”. Ziyad’s notion of metacognition resembled that of Mohammed’s, as he articulated,

> My concept of metacognition is that it is about how you discover the error in your thought, and how to recognize the weaknesses and strengths in it, and being able to adjust it. However, if you are just taking information without this method then it means that you will continue in a way of thinking that could be wrong.

Despite students having been given an introduction to the research topic, Qusay still felt that “it is not easy to determine thinking styles; since it relates to thinking it is a hard subject”.

2) Function of metacognition: In the interviews, certain students expressed views about metacognition within the context of its function in learning. One of these functions was the improvement of thinking through identifying strong and weak points in thought methods. The student Mohammed said, “I try to search for my errors in thinking and their reasons; this is so I can avoid such mistakes”. Mohammed also added that

> When everyone thinks in a certain way, they believe that this is the correct way to think, but if they are able to identify the positives and negatives in their thought, they would be enabled to correct and develop it.

3) Metacognitive skills: In their interviews students were asked about some of the skills of metacognition, these being planning, management, monitoring and evaluation. The answers of participants reflected their views of metacognition from the skills perspective. For example, Mohammed said,

> Planning is easy, I can plan my course of thought. As far as managing it is concerned, most of the time people are under the influence of one style of thinking when managing their thought. As for monitoring thought, I think that it cannot be done until finishing the work, and then after that comes evaluation.

Ziyad held the view that:

> If I knew how one thought, then I could monitor, correct and adjust it. This means that you have to have a certain way of thinking, which you can monitor in new situations, and it is then possible for you to adjust it.

Qusay added to this discussion by saying, “I think that evaluation is the hardest of these skills because it needs criteria”. Fadul also mentioned that “I don’t have knowledge about the skills of metacognition”.

4.1.3.2 Students' understanding of metacognition after the implementation of the IMPROVE programme

After the implementation of the IMPROVE programme, students were asked about their notions of metacognition. The data gathered from the participating students in their interviews can be divided into three categories:

1) The Concept of metacognition: Mohammed remarked that the concept of metacognition was a student understanding his own thinking, and possessing the ability to judge and adjust its course in order to improve it when dealing with mathematics problems. He presented this view as,

   > My conception of metacognition is that it is about knowing my thinking and being able to judge its course in a positive way. This is done in order to arrive at a sound result when solving mathematics problems.

   In another quote, he used the phrase ‘monitoring thought and adjusting it’. His original words were,

   > It is a method of organizing your way of thinking in solving mathematics problems. In doing this, the solution will be closer to the correct one as the student deals with the problem step by step. In addition, we can monitor our thinking and adjust it.

   Mazen’s notion of metacognition resembled that of Mohammed, as he articulated, “My notion of metacognition is that it is a strategy that helps to fix understanding by thinking about my method of thought”. The students Ziyad and Ragab included the term ‘planning’ in their statements. Ziyad noted:

> My conception of metacognition is that it is a student organizing his thought when solving mathematics problems, and this is done by planning according to a certain thought methodology when dealing with problems.

Ragab added to this by stating: “I conceive metacognition as planning to solve mathematics problems in a particular style of thinking; monitoring this thinking improves it”.

2) The function of metacognition: Some students spoke of their conception of metacognition within the context of its function in learning.

- Adjusting thought: Some students remarked that metacognition was used to organize a method of thought. For instance, Omar said, “metacognition arranges your thoughts and makes you reconsider them in order to discover your errors and amend them”.
- Evaluation of thought: Several students mentioned that metacognition helped them in evaluating thought. Mohammed stated:

> With this method I can diagnose aspects of weakness in my way of thinking. After that I can try to improve this area, so I can gain a better way of thinking.

Ziyad explained that “metacognition provides a lot of space for a student to think and discover his errors and then amend them”.

- Understanding mathematical concepts: some students also discussed how metacognition could help students in understanding mathematics problems and concepts in a better manner. Mazen said,

> Metacognition helps all the students in understanding. Even for students who do not understand, they are still close to understanding. The evidence for this was seen when I noticed one of my friends changing in his way of discussion, as we began to discuss specific points. This is because this method draws a method to deal with problems as well as a method for discussion between us.

Mazen recounted his previous experiences,

> Prior to this, the teacher would give us a problem and ask us to solve it directly, and I might stop in the middle of solving it and be unable to

> complete it or find out where the problem lies. However, now this method has made it easier for us to discover our mistakes.

- Another assertion was that metacognition aided students in thinking logically when dealing with mathematics problems. Ragab said:

> This method is a logical way to solve, relying on four skills according to cooperative learning and diverse groups. The biggest role is played by the students to a large extent, which has an effect in consolidating mathematics concepts in a student's mind.

Fadul discussed this further by stating that:

> After I studied with this method, I found that it developed the student in his thinking. The task of the student is no longer just copying and pasting [remembering] mathematics concepts, but rather it helps the student to develop his thinking so that it can be logical.

3) Metacognition as a set of skills: A group of participating students viewed metacognition within the context of metacognitive skills. The skills of monitoring and adjustment arose in the answers of interviewees. For instance, Mohammed commented:

> Metacognition is the student following a structured way of thinking that enables him to monitor his thoughts. He then can adjust it, as working in groups improves the ability to monitor thought.

Some students made a connection between the skills of monitoring and planning thought. For example, Qusay said, "It is possible to monitor my thinking but there must be planned thinking to begin with". This was confirmed by Fadul, who emphasized, "Yes, thinking can be monitored if the thinking is planned to begin with". The skills of planning, management of planning, and evaluation were all emphasized by the participants as being important in learning through metacognition. Mohammed said,

> I see that all the four skills and working cooperatively improve its application, particularly monitoring. However, it is important that students understand these skills and have them clarified to them so that they can pay attention to it and evaluate their performance through it.

Qusay added. “The skill of planning was clear, and I feel that the skills of management and monitoring are linked to each other”.

4.1.3.3 Learning techniques used in mathematics before the implementation of the IMPROVE programme

Various pieces of data collected from the interviews and focus group with students participating in the study can be categorized under the theme ‘Learning techniques for mathematics students and the extent of its relation with metacognitive learning’. Most of the students dealt with the mathematics problems - such as, if we want to form a committee of ten students from the 1st and 2nd grades at school, what is the probability that the committee will contain seven students from the 1st grade, bearing in mind that students are chosen at random - in a singular manner, which was identifying the givens, defining what is asked of them and then attempting to find the solution. The student Ziyad gave his views on this method by saying:

> I deal with the mathematics problem by understanding exactly what is required of me, then finding the givens and later finding a rule that connects between the givens and what is asked.

Other students, namely Omar and Qusay, followed the same method, but they tried to compare the given problem to another resembling it. Omar explained that,

> I compare the question to a previous one that I have solved, I then identify the difference and similarity, and finally I solve it. I do this by defining the givens and what is asked, and sometimes finding what is required needs a multi-stage approach, the phases of which need to be linked through a rule or a formula.

As for Qusay, he revealed that “I identify what is asked so I can know the solving method and I compare my solution with that of the teacher, a classmate or the textbook”.

Mohammed had a different method of dealing with mathematics problems, as he identified precisely what was asked of him, then searched for givens which could bring him to the solution. Speaking on this method, he said,

> Firstly I try to understand specifically what is required of me or what is it that I want to find out. Secondly I try to identify what are the

> things that can help me to arrive at a solution, meaning that I start with a reverse method by defining what the final demand of the question is, then I find what things are asked of me to reach the final demand. Then I search for these demands in the problem and I will find that some of these demands require further questions, on which I continue to search. This method forces me to avoid the things that could hinder me, like some of the givens that don't relate to the problem. This is because the presence of irrelevant givens in the question causes a barrier to solving the mathematics problem.

Some students relied on memorizing mathematics rules and tried to apply them directly. For example, the student Fadul recounted, "Ever since I started learning maths, I have relied on one method, and that is to memorize the rule then apply any mathematics problem to this rule only". Linking between new and previously learnt knowledge is a technique used by some of the students to solve mathematics problems. The student Mohammed explained this approach as:

> One can deal with mathematics problems in several styles or models, such as identifying what is current knowledge and what is previously learnt and how can one link between the two sets of information to arrive at the solution.

Based upon what has been revealed, it cannot be stated that there is a clear learning through metacognition. Instead the reality is that the focus is primarily on the method to solve a mathematics problem, rather than on improving thought methods in dealing with such problems. (See 4.1.1.3 for mathematics learning strategy of the students related to metacognition).

4.1.3.4 Challenges to the implementation of metacognition in mathematics learning

Through the presentation of student quotes gathered from interviews and focus group, it became clear that there were several obstacles and difficulties that might hinder the implementation of metacognitive mathematics learning. These were divided into five categories:

1) The domination of the traditional method over mathematics learning: Within this area was the focus of students on direct solutions to mathematics problems, without considering the improvement of thought methods when dealing with such questions. Qusay spoke on this first by stating, "I always

focus on solving maths problems as quickly as I can. I do not have time to think about my thinking." Fadul felt that the teacher only focused on dealing with direct errors in solving mathematics problems and not on the style of thinking when working with such problems. He described this as, "The teacher concentrates on the mistakes students make in solving the problem directly, there is no focus on the method to solve". Mohammed spoke on this point, saying that "previous traditional habits are an obstacle to learning through metacognition". Ziyad expressed a similar view by commenting, "The student is unaccustomed to it, because for many long years he has become used to the traditional method". Another issue was the teacher's attachment to the solutions of the book instead of assisting students in their methods of thinking in learning mathematics. Mazen discussed this by saying, "What I see is that a lot of the teachers are attached to the solution of the textbook only, rather than being concerned about metacognition".

2) Qualifications of the teacher: A weakness in the teachers' qualifications was seen as one of the barriers to learning though metacognition. Omar provided insight into the matter by stating, "The teacher must be active in teaching with metacognition, because it will combine conveying the concepts of the syllabus and training students for this method". Qusay commented:

> Learning through metacognition will require the teacher to be knowledgeable about several methods of thinking so that students can absorb them. The teacher also needs to be able to prepare suitable activities for learning with this method.

   Omar added to this by saying that "the teacher must have a motivation for doing this". As for Mazen, he felt that "the teacher's language must fit metacognitive teaching".

3) Number of students: An increase in the number of students in a class would be considered a hindrance to metacognition which would not allow individual and collective evaluation to a greater extent. Thus, many students called for a reduction in the number of students in general, and for the number of students in one group to not exceed four. This was highlighted by Mazen, Ziyad and Asam. Asam mentioned that

> It is important for the number of students in the class to be appropriate for learning through metacognition. Also, the number of

individuals in a group to be no greater than four students, which will allow individual and collective evaluation to a greater extent.

4) Limitation in thinking style: Students being limited to a single way of thinking was also discussed as a potential obstacle. This issue was raised by Omar, who said,

> The disadvantage here is seen in the student being limited to one style of thinking as well as specific planning, with there being no alternatives created by the students. This forms a challenge to learning through metacognition.

5) Time required: It was noted that learning through metacognition required more time in the lesson. This is what was highlighted by Mohammed, Mazen, Qusay, Omar, Fadul, and Ziyad. Qusay said:

> One of the obstacles to learning through metacognition is that it requires more time, which is an issue, particularly with the limitations of lesson time and the great amount of content in the textbook.

4.1.3.5 Students' requirements for the implementation of metacognition

Certain pieces of data from the student interviews and focus group can be included within the theme of requirements of students to learn mathematics through metacognition.

1) Role of the student: One of the major requirements highlighted was for the role of the student to be in searching for and building knowledge, rather than simply receiving knowledge by the method of memorization from the teacher. The student Mohammed explained his views on this subject by stating:

> I can tell you that the way of learning was different, and that something changed in this, because the student became the one who builds knowledge, instead of just memorizing it from the mouth of the teacher.

Ragab’s notion of this role resembled that of Mohammed, as he mentioned his own personal experience:

> I have spent 11 years using one style and method of thinking. The role in learning and gaining knowledge has been on the teacher entirely. Now I find that students are the ones who work more than

> the teacher does and learning groups have become more effective, whereas students would be inefficient and used to just wait for the teacher to finish the lesson. I was one of those students. Now the problems are different from the past and are more challenging and the work method is different, and so is the way of dealing with these problems. Students are also more serious.

The further building of metacognition in learning was discussed by Mazen when he was told in an interview that it would be his last meeting with the researcher in this study. He stated:

> Metacognition will probably stop being implemented after you leave, but it will not stop being implemented with me as a way of learning, and I will try to teach my brothers this technique.

2) Mind maps: Another need expressed was for students to have a work map for dealing with mathematics problems. This would enable them to monitor their thinking and help in its adjustment and its improvement. This was stated by Mohammed:

> It is necessary to follow a systematic thought method in learning mathematics metacognitively. This will allow the students to monitor their thought as well as adjusting it, and working in groups improves the ability to monitor.

Therefore it can be asserted that work maps for dealing with mathematics problems really do enable students to monitor their thought process and help them to improve it. Such a thought map was seen as essential by Mohammed, Omar and Ragab. Omar contributed:

> Metacognition has benefitted group work, this is because there was an organized work map for thinking in dealing with problems. This helped me to find a mechanism for dialogue with members of the group instead of dealing with the problems directly as we used to do.

3) Practice: students also discussed how practising for metacognition was necessary for them to benefit from it; this was expressed by several students in their interviews. Ragab said:

> Through practice, the benefit of learning to solve mathematics problems metacognitively was clear, and there was an improvement seen between the start and finish of implementation.

Based on this, students were able to perceive resistance to the approach at the start of the implementation of IMPROVE. Qusay remarked:

> At first I did not accept metacognition, but after a period of time and the corrections and evaluation of the teacher and then going back to our work, I found that this methodology is very useful in avoiding the repetition of mistakes.

An important point added by Ragab was that:

> The student should continue practising metacognition so that it is part of his culture, and not simply an application. Therefore, it would be useful to apply metacognition in more than one subject.

4) Comprehension of mathematical concepts: The need for metacognitive mathematics learning to include understanding of new concepts in addition to problem solving was outlined. For instance, Omar said,

> Learning mathematics through metacognition is useful, especially if it is related to the presentation of concepts rather than just problem solving. This is because it would better understanding and would reinforce knowledge to a greater extent, as this is constructive learning, rather than rote learning.

This was emphasized by Fadul, Mazen and Ziyad, with Mazen's stating, "I think that using metacognition with new concepts is more beneficial, but it needs more time because it helps us to understand better". Fadul commented, "I think that using metacognition in presenting a new concept is more beneficial than simply solving maths problems".

5) Evaluation: another need expressed was for students to have a role in evaluating their method of thinking, with this being done with a mental work map for dealing with mathematical problems. Fadul discussed evaluation, as:

> Following this method will really help the student in identifying the weak points in his thought method when dealing with mathematics problems, without the help of the teacher, as I discover the errors by myself.

Mohammed expressed a similar view by saying that "with this method I can diagnose aspects of weakness in my way of thinking. After that I can try to improve this area, so I can gain a better way of thinking." Assisting this process is the training of students to practise metacognitive skills, as described by Mohammed, who said, "It is very important that students understand the metacognitive skills so that they can pay attention to them and evaluate their performance through them".

4.1.3.6 Student-student relationship before the implementation of the IMPROVE programme

Regarding the subject of communication between students and to what extent it can facilitate metacognition, it arose from interviews and the focus group with students that – in general – there was a weakness in skills involving communication with others. For example, a group of study participants stated that they did not wish to speak with others, be it about methods of thinking or even in their wider learning. The student Fadul asserted,

> I don't like to discuss with others about my way of thinking, because I am rather weak in mathematics, so I don't want to speak about this weakness in front of others. Another aspect is that some students just have a better thought method than me, so I am somewhat shy to talk to them.

This was similarly expressed by the student Mazen, who said: "I don't discuss how I think with my class partner, which is because I think that is a personal matter, relating only to me".

Despite this, when the participants were asked whether or not they would accept their classmates' corrections in mathematics learning, their answers differed between those who did and didn't accept this, and yet every participant mentioned that such correction is a positive influence. For example, the student Mohammed stated,

> My classmates' correcting my mistakes is something more positive than negative. It's an advantage because I can correct my own errors, but is also negative as when the classmate correcting me is at a much higher mathematics level than me, his correction indicates that I have a big flaw in my thought method, which is something that embarrasses me.

The student Fadul explained that, "The downside is that when my classmates correct my mistakes during mathematics learning, I doubt my self-confidence. The positive thing is that I can learn from my mistakes." Expressing a noteworthy view, Ziyad articulated that “the negative aspect is that the person correcting you is still presenting you with ready knowledge about a certain mistake, and doesn’t teach you how you learn”. During the interview the participants generally did not raise anything indicating the presence of skills in dialogue with others, which would hinder learning through metacognition.

4.1.3.7 Metacognition and cooperative learning after the implementation of the IMPROVE programme

Several participants in the interviews and focus group presented data pointing to a relationship between cooperative learning and metacognition. Mohammed highlighted that “metacognition really benefitted cooperative learning because it provoked thinking in an organized way, and gave a greater opportunity for all students to participate”. He added:

> Following an organized thought method enables the student to monitor his thought and then amend it. Therefore I see that working in groups improves the students’ ability to monitor their thinking.

The student also mentioned that:

> Through learning in a small group, I can know which students have ways of thinking parallel to mine and which differ from my method of thought; to do this I draw a link in my mind between the person and his type of thinking.

In this regard, Mohammed underlined the importance of “dividing the class into groups in a way suitable to benefit from metacognition”. Mazen added to this by stating,

> Really, cooperative learning enhances metacognition. Metacognition is also beneficial in creating a cooperative atmosphere between group members in how they monitor each other, and how they evaluate their method of thinking in dealing with mathematics problems, and the teacher is more able to discover their methods of thinking.

The student also described how learning through metacognition would require good communication between learners and not with the textbook. Speaking on this point, he said, “I think that learning through metacognition is one that exists between learners and cannot be between the student and the textbook”. Ragab built on this by explaining how metacognition encouraged cooperative learning. He commented, “Metacognition helps you to arrange your ideas and helps group members cooperate better than before. The reason for that is because it is a technique for thinking and also dialogue.” On the same topic, the student Ziyad thought that the lack of implementation of cooperative learning was a barrier to learning through metacognition. He stated, “One of the obstacles to learning with metacognition is that the school has not used cooperative learning in a practical or correct way”.

As for learning groups, it emerged from the interviews and focus group that students saw the importance of such groups including no more than four students. They also felt that group members should be of differing academic attainment levels. The students also held the view that each group should contain a student with the traits of leadership to manage the group’s work. Mohammed said,

> Dividing the class into groups in a way suitable to benefit from metacognition is important. The number of students in each group should be no greater than four and they should be of varying academic attainment levels to enable them to benefit from each other.

This was also emphasized by Ziyad, Qusay, Ragab and Mazen.

4.1.4 Summary of the Case Study One findings

As shown by interview data Mr. Fallatah was unfamiliar with the term or even the concept of metacognition. However, he did possess some background knowledge of the subject; it was merely a matter of terminology. He included three ideas in his conception of metacognition, correcting the students’ train of thought in linking mathematics concepts, the process of monitoring the logic of the thought sequence in the mind of a person, and that this concept involves certain skills which cannot be gained without previous planning. When asked about how compatible his teaching strategies were with metacognition, in general the answer involved compliance with the traditional method, discussing some of the important skills in maths learning, e.g. planning, estimation, monitoring and evaluation, despite him not explaining a clear application for these skills or how one could help a student in mastering them. Despite these teacher's claims, observations showed that his teaching method in mathematics was generally unrelated to metacognition, or approaching it was unintentional.

Post-IMPROVE Mr. Fallatah was again questioned on his perception, in which he explained the presence of logical thinking, and the importance of practising planning, management, monitoring and evaluation. Mr. Fallatah also discussed the function of metacognition in learning. He thought that metacognition would help him to discuss students' thinking rather than simply discussing solving methods. Metacognitive teaching encourages students to participate in a constructive learning process. Overall, Mr. Fallatah considered that metacognitive instruction was a positive experience for teachers.

In the initial stages of IMPROVE implementation, lessons were delivered ineffectively. Through practice this improved, as the seven steps of IMPROVE were better adhered to. The teacher's choice of activities was more appropriate for metacognitive teaching. This made students enthusiastic to solve problems, and after correction, they fully understood the problem.

In terms of students, the term 'metacognition' was new to them. After the term was clarified, it appeared they did have some background knowledge. Their initial notions of metacognition can be summarized into, knowing the courses of thought, the improvement of thinking through identifying strong and weak points in thought methods, and possession certain skills to improve their thought. As shown by the interviews, most of the students dealt with the mathematics problem in a singular manner, which was identifying the givens, defining what is asked of them and then attempting to find the solution to the problem. Some students relied on memorizing mathematics rules and tried to apply them directly. The observation data showed that the students' focus was entirely on solving the mathematics problem directly and in the quickest way possible. They were generally uninterested in generating multiple solutions.

Post-IMPROVE student perspectives of metacognition can be summarized as follows: the concept of metacognition was perceived as an awareness of thought and being able to judge its course in a positive way. In discussing its function in learning, several points were discussed, including evaluation and adjusting thought, helping students in understanding mathematics problems and concepts, and thinking logically when dealing with mathematics problems.

Metacognitive skills were also emphasized as significant, these being planning, management of planning, and evaluation. Observation demonstrated that the students' learning method at the beginning of IMPROVE's implementation is similar to stage one, which centred on direct solutions and this was done regardless of any thought method. However, with practice, the students took on a thought method for the mathematics problem in accordance with the metacognitive questions. As outlined in IMPROVE, these were: questions to understand the problem, the solving strategy and questions of linkage. The students' learning method in mathematics transformed from a complete reliance on the explanation and solving of the teacher to them making efforts to search for knowledge and building upon it.

Regarding the teacher-student relationship in mathematics learning in the classroom, it was noticed that the teachers approached errors made by students in a positive manner. However, the relationship between teacher and their students remained concerned with the correction of student errors instead of it being based upon adjusting thought methods in dealing with such problems. Regarding weak communication skills, participants were generally reluctant to speak on the subject. Observation demonstrated that cooperation was largely ineffective, with students solving alone or being shy in participation. Most of the students' discussions revolved around correcting each other's mistakes rather than thought in the context of solving.

Post-IMPROVE, interviewees and focus group members pointed to a relationship between cooperative learning and metacognition. Students highlighted that metacognition benefitted cooperative learning because it provoked thinking in an organized way, and gave a greater opportunity for all students to participate. The students identified the need for strong communication. As for group size it emerged from interviews that the perception was that this should be no more than four students of differing academic attainment levels, and should be led by students with leadership traits. Mr. Fallatah thought that a strong connection existed between metacognition and cooperative learning. As shown by the observation data, his actions were consistent with his views. He said: "[Metacognitive teaching] makes the student the centre of the educational process. It also encourages him to search for information and stimulates his thinking and abilities, and gives him the tools of self and thought evaluation." The teacher was able to create a cooperative environment, due to his management of class time. Another factor of success was the creation of diverse groups not exceeding four students. In addition he prepared class activities well, in order for them to be consistent with the IMPROVE programme.

Mr Fallatah expressed requirements for the successful implementation of metacognitive learning. The activities should involve indirect solutions, previous experience, hold new ideas, and should be challenging. Sensitive handling of student weakness as well as practice were also identified as key requirements. Students outlined a set of characteristics to be embodied in the teacher: he or she should hold knowledge of various styles of thinking in dealing with mathematics problems. Readiness, evaluation skills and the setting of suitable activities were also identified as important factors. Another of the major requirements was for the role of the student to be in searching for and building knowledge, rather than simply receiving knowledge by the method of memorization from the teacher. The creation of work maps for dealing with mathematics problems would enable them to monitor their thinking and help in its adjustment and its improvement. Students also felt the need to be well prepared and trained for full benefit. Another need expressed was for students to have a role in evaluating their method of thinking, with this being done with a mental work map for dealing with mathematical problems.

Obstacles identified in interviews included the domination of the traditional method over mathematics learning, lack of teacher readiness, and students being limited to a single way of thinking. Syllabus and textbook content was highlighted as being too large. It was noted that learning through metacognition required more lesson time. As for the challenges that might confront teaching mathematics metacognitively, Mr. Fallatah saw these as revolving around five issues. Firstly, the teacher being long-accustomed to teaching mathematics in a particular way, requiring traditional courses and investment of resources. Secondly, the absence of preparation and training for teaching through metacognition, be it at university or during a teacher's service in education. Thirdly, the general lack of previous adoption of metacognition in education is an obstacle and the lack of pursuit of how we can implement metacognition in reality. Fourthly, the school administration remains unconvinced because its focus is on the direct academic attainment of students and the completion of curricula. Finally, an absence of teacher evaluation criteria for using metacognition and a focus on superficial issues when evaluating teachers were both identified and explained as obstacles to the implementation of the method.

4.2 Case study two: Mr. Hatem's class

The students of this class sat in groups consisting of five or six students at tables that were shaped as trapezoids, which facilitates their arrangement into a circular configuration. For this, students worked in small groups. The classroom was also equipped with a smart board, a projector and an internet connection. The classroom was spacious and the number of students in the class did not exceed 30 (see **Error! Reference source not found.** for more details in the selection of participants).

4.2.1 Thematic findings of observations (Mr. Hatem's class)

Through classroom observations of teacher and student performance before the implementation of the IMPROVE programme, the following data collected analysed into the following themes: mathematics teaching strategies related to metacognition, mathematics learning strategy of the students related to metacognition, and teacher-student relationship.

4.2.1.1 Mathematics teaching strategies related to metacognition before the implementation of the IMPROVE programme

Analysis of interview data indicates that (see 4.2.2.1) Mr. Hatem had not heard of the term or concept of metacognition, thus his method for teaching mathematics was not related directly to it. These teaching methods have been categorized as follows aspects:

1) Presentation of steps to solve mathematics problems: on one occasion Mr. Hatem presented these steps to solve a mathematics problem to the students. 'Write a formula for the n^{th} term for the numerical sequence -6, 3, 12' In doing this, he stated, "These steps are to master the solution for a mathematics problem on the topic of variants". He also rephrased this by saying, "These steps are for thinking about solving problems related to the topic of variants". The observation indicated that these were not steps for thinking but rather steps to directly solve the problem.

2) Urging students to think critically: On another occasion the teacher urged students to employ critical thinking. This would be done by criticizing the solution of an activity in the textbook. For this, the teacher instructed:

    > I want you to criticize the solution displayed in the textbook because it has presented a lengthy solution, and it seems to me that it could provide another, more brief solution.

    The teacher encouraged students to learn through the practice of critical thinking with expressions when he said, “When we are learning mathematical concepts, we want to add to what is included in the syllabus and we want to exceed this with greater knowledge”.
3) Mind maps: Presenting mind maps to link between several mathematics concepts was another technique used. It was observed in one of the lessons regarding ‘Arithmetic Sequences and Series’ that the teacher presented a summarized map of concepts. This was done to give an initial notion of the lesson and to distinguish between various situations which include a certain mathematical concept.
4) Determining the givens and the demanded: On another occasion, the teacher provided students with the following question, ‘Write a formula for the Nth term for the numerical sequence in which d=8 and a6=16?’

He emphasised that students must first determine the givens and the demanded in dealing with a mathematics problem so as to understand it. For example, he said to one of the groups, “Define the given and the demanded with all accuracy, and try to identify how you can arrive at the solution, without concentrating too much on the product”.

5) Linking previous and current knowledge: During the presentation of new concepts or solutions to problems, the teacher would often mention previous knowledge. He would then link this to the new information. The teacher would repeatedly ask ‘Why?’ and ‘How did you do that?’ The teacher occasionally asked for justifications for any answer and would clarify that to all the groups.
6) Correction of errors: The teacher paid attention to the errors made by students and then corrected them continuously on a both individual and collective basis using the whiteboard. He was also concerned with sharing

his criticism of one group with all the groups. For instance, when the teacher presented a new activity, he asked each group to present varied answers, in accordance with their way of thinking in solving the problem. He sometimes asked each group to think of the solution of another group and how that group had arrived at that solution. Also the teacher gave time for each student to correct their classmates in some activities.

The teacher emphasized to the students on one occasion that “ready knowledge doesn’t develop a student’s level in mathematics learning”. Learning the method of gaining knowledge is the most important, despite it not being tangible that the teacher built upon what he said through his methods of teaching. These methods involved a heavy reliance on presenting mathematics concepts in a direct manner. He also presented mathematics problems and exercises to students that relied on the direct application of the determined mathematics concepts and rules. The observation data also showed that Mr. Hatem tried to motivate students by turning the activity into a race by asking which would be the first group to solve. It was indeed noticed that this method meant only the outstanding students participated while the rest of the students did not, neither in solving nor even in attempting to solve.

The observation data also showed that Mr. Hatem provided his students with an activity that presented two different solutions to a single problem. He asked the students to determine which of the two solutions was correct. This would help the students to observe various types of thinking in their dealing with mathematics problems. This emerged in one of the observations, in which the activity presented by the teacher suited this description. It is preferable for these activities to include a challenging idea. This will enable students to be more challenged to think about how they deal with it.

Based on the above observations, the teacher focused more on the steps to directly solve a problem, rather than focusing on the thought methods involved in solving problems. This meant the relationship between the teacher and his students was not participatory or constructive but rather one of monitoring errors made by students in their problem solving, with this being done in order to correct them. Thus, when the teacher’s role is limited to being a conveyor of information, it hinders any observance of manifestations and indicators of metacognition.

4.2.1.2 Mathematics teaching strategies related to metacognition within the implementation of the IMPROVE programme

Data analysis of interviews showed (see 4.2.2.4) that the teacher's view changed in regards to the reliance on traditional teaching and that this would emerge due to the existence of a clear methodology to solve mathematics problems. In this regard, he stated, "I say that there is a change in teaching method, even if only slightly. This is because there was a clear method to solve, identify difficulties and compare problems." Thus, it was important to observe Mr. Hatem's performance in order to illustrate the compatibility of his teaching with metacognition. In the observations there were indications of metacognitive teaching after the implementation of the IMPROVE programme. On one occasion regarding Arithmetic Sequences and Series, the teacher followed some of the steps outlined in IMPROVE, with these being: presentation of the concept, metacognitive questions, practice and reduction of difficulties. However, he did not present corrective evaluation for students' thinking in dealing with mathematics problems. In another observation session the teacher tried to follow the steps of IMPROVE, yet he was more focused on discussing the differences and similarities between problems with students, to which they were responsive. Those problems were in the Arithmetic Sequences and Series subject, such as:

> If A is the 3rd term in a numerical sequence, B the 5th and C the 11th, express C through the use of A & B', and 'Write a numerical sequence with eight terms with its total equalling 324?.

Analysis of interviews data indicates that the teacher realized the importance of following logical steps to solve problems, even if the students did not arrive at the final solution. It was noted that in the initial phase of IMPROVE implementation, the teacher would display a mathematics problem related to a daily life situation and would ask students to discuss the problem in accordance to the metacognitive questions. These were set out in accordance with the IMPROVE programme and were: questions to understand the problem, the solving strategy and questions of linkage. However, the teacher did not provide students with enough time to solve the activity according to these metacognitive questions. It was also noticed that students focused on the steps to solve the problem, without concentrating on discussion of their understanding of the problem and describing it correctly. Due to this, one of the groups made an error, from which stemmed an incorrect answer, and the students' interaction was minimal.

In another observation session, the teacher followed the steps of the IMPROVE programme (these being presentation of concept, metacognitive questions, practice and reduction of difficulties). The teacher presented the new concept with student discussion regarding Infinite Geometric Series; he then set an activity containing the new concept, and asked groups to cooperate in solving it according to the indicated steps. This activity was:

> If Saeed is on a swing, and launches from the starting point, without any push from him, and the distance swung begins to decrease by 10% on each swing – find the total distance that Saeed has covered when the swing eventually ceases to move?.

The students took on a thought method for dealing with mathematics problems in accordance with the metacognitive questions, with work being carried out in groups of four. As outlined in IMPROVE, these were: questions to understand the problem, the solving strategy and questions of linkage. Despite this the teacher did not deal with reducing difficulties to a sufficient degree, as he spoke about these difficulties without the participation of the students. As for checking, the teacher had presented a quick activity which was:

> Saad swings the pendulum, the span of its swing decreases in each swing by 15%. If the first span of the pendulum's swing is10 inch, find the total distance swung by the pendulum by the time it stops moving',

And there were groups able to solve this, but the teacher did not perform corrective evaluation for their method of thinking in dealing with mathematics problems on that occasion.

After IMPROVE's implementation, the teacher's method was more constructive. For example, he presented a new concept regarding the 'Binomial Theorem' while discussing with the students. He then gave them an activity and asked them to work in groups to solve the mathematics problem according to the steps specified by IMPROVE. This activity was:

> If we want to form a committee of eight students from the 1st and 2nd grades at school, what is the probability that the committee will contain six students from the 1st grade and two students from the 2nd, bearing in mind that students are chosen at random?

The students began to converse about defining the givens and generating further possible givens from the problem, as well as defining the demanded value. Then the students created a strategy to solve it which could only be done by returning to their past knowledge. After having discussed all of these aspects, they began on the steps to solve it while justifying each of these steps until they arrived at the solution. Following this, the students checked the solution to the problem, then the teacher discussed with the students about their method of verification, by saying: "If we check the smaller components of the solution, then we can check the entire solution because the nature of the question defines how you check it".

After this the teacher asked one of the groups that was experiencing difficulty in their solving to display their solution to the class. This was so that the difficulty they were facing could be discussed as a way to evaluate their method of thinking in solving the mathematics problem. Mr. Hatem subsequently spoke about the characteristics of the problem and compared it to other previously encountered ones, such as:

> If we want to form a committee of ten students from the 1st and 2nd grades at school, what is the probability that the committee will contain seven students from the 1st grade, bearing in mind that students are chosen at random?

The application of the IMPROVE steps was better than in previous lessons, as the teacher was more of an administrator of the work taking place according to the programme, and was able to encourage students in this to a great extent. In a more practical manner, the teacher presented the new concept to the students and accurately explained the givens and what was demanded of them. The students then discussed the formula that combined the givens and what was required. He talked to them about the significance of this step so that students could choose the appropriate rule or formula to solve problems; then students discussed justifications for their choice of rule and then the method of solving the problem. The teacher discussed with students the difficulties they faced in dealing with mathematics problems, as well as discussing the similarities and differences between two problems, to which students responded well.

Data analysis of observations after the implementation of IMPROVE revealed that the teacher did change his teaching method in order to relate to metacognition, but not to a great extent. This change saw teaching methods develop from the mere presentation of ready knowledge to the student along with some applications, to a methodology for dealing with mathematics problems with a focus on thinking. This method had the aim of controlling and adjusting thought, and thus it became noticeable that the teacher had begun to follow logical steps in his approach to mathematics problems. These steps were taken in accordance with the metacognitive questions, with these being: questions to understand the problem, the solving strategy and questions of linkage. This was also carried out in accordance with the wider steps provided by the IMPROVE programme: presentation of the concept, metacognitive questions, practice and reduction of difficulties.

Although data analysis of observations after the implementation of IMPROVE revealed that the teacher did improve his teaching method in order to relate to metacognition, it was noted that this improvement was not to a great extent. There were difficulties relating to the teacher that might hinder the implementation of metacognition in mathematics learning. One of the important obstacles relating to the teacher was him only partially adopting metacognitive teaching. An example of this was that the teacher did not adopt the method unless in the presence of the researcher (see 4.2.1.4), and his enthusiasm being placed on the completion of the syllabus meant that he was not giving students sufficient time to solve problems according to the metacognitive questions. Consequently, the students felt that there were differing goals between the teacher and the researcher (see4.2.3.4). The observation data collected after the implementation of IMPROVE revealed that the teacher did not use the IMPROVE approach in the understanding of new concepts, but instead exclusively used them in solving problems. Furthermore, the teacher did not deal with the reduction of difficulties to a sufficient degree, and he was the only one who spoke about these difficulties. In addition, he did not perform corrective evaluation for the method of thinking in solving mathematics problem.

Thus, it was observed that the teacher's role in the learning process had not changed greatly, as he spoke a lot at the expense of student interaction.

4.2.1.3 Learning strategy of students related to metacognition before the implementation of the IMPROVE programme

The observations data display the learning strategies used by students that are related to metacognition before the implementation of the IMPROVE programme.

Analysis of interviews data indicates that (see 4.2.3.1) the students were unaware of the term or concept of metacognition and had not experienced learning through it. As well as this, there was no hint of a clear strategy for mathematics learning that was related to metacognition.

As the students worked in small groups, it was observed that there was a partial manifestation of the skill of monitoring, but not of the style and method of thinking. Rather this was performed on the steps of direct problem solving.

For example, in one of the observations,

Student 1 stated: There is something incorrect in our solution.

Student 2: Yes, this is true, therefore we have deviated from the right solution.

Student 1 then discovers the error and indeed, the students arrive at a solution.

On another occasion of the observation different group: Student 1 states:

We made a mistake because we reversed the equation.

Student 2 agrees: Yes, this is true, we are far from the solution.

The teacher urges them to do one thing instead of another.

Student 2 then matches the current problem to the solution of a previous one and then the group solves it.

There is a constant effort from students to define the given and demanded values in each problem. However, this effort does not extend to basic steps in the learning through metacognition, but rather steps to solve the mathematics problem directly. It was also noticed that students did not justify their suggestions for solving in their discussions and noted that students were not focused on monitoring their thought and adjusting it, but instead relied on matching their solution to previous ones, be it from the teacher, the textbook or a classmate.

4.2.1.4 Mathematics learning strategies for students relating to metacognition within the implementation of the IMPROVE programme

This section displays the student's learning method for mathematics that is related to metacognition within the implementation of the IMPROVE programme. At the start of implementation, difficulty was observed in students' attempts to change their traditional method of learning mathematics. Students focused on the steps to solve a problem directly, without concentrating on discussion of their understanding of the problem and describing it correctly. Thus their interaction was minimal. In one of the observations, this appeared clearly in the students' discussion about the given and the demanded value (see section 3.3.5.1 and 3.3.5.2 for more details about how observation was conducted including several different groups observations. This meant that Student 1 for example, is not always the same person in the findings).

> Student 1: What can we create from these givens?
> Student 2: We can write the relation an=a1 + (n-1) d, which might help us in solving.
> Student 3: Yes, that's true, but what about after that?
> Student 1: We will try to experiment several times so we can get to the demanded value.
> Student 2: What do you think about substitute 'a' with 'a3'?
> Student 1: That's good, then we can multiply the first equation in … so we can get rid of the similar parts.

However, with practice, the students began to interact with the steps of the IMPROVE programme in a better way, and this was noticed in their learning and discussion method. It was noticed that they were focusing more on understanding in a more accurate manner, be it in identifying the givens and demanded values or in classifying the problem. For example, in one of the observations:

> Student 1: For us to understand the problem, how can we classify it?
> Student 2: Since the problem mentions a numerical sequence, then we can classify it under numerical sequences.
> Student 3: Now how can we solve the problem?
> Student 2: There is a rule that can help us in doing that.

> Student 1: But we have two unknowns, and not just one. So how are we going to solve it now?
> Student 3: Yes, the problem is that there are two parts, so I think using these givens we can get the missing part by…

It was also noted that students dealt with mathematics problems according to a specific logical methodology. In the last session observed, it was noticed that the students' method of learning was more constructive. The teacher presented the new concept for discussion with the students, and then gave them an activity and asked them to work in groups to solve the mathematics problem. The students began to converse about defining the givens and generating further possible givens from the problem, as well as defining the demanded value. Then the students created a strategy to solve it which could only be done by returning to their past knowledge. After having discussed all of these aspects, they began on the steps to solve the problem while justifying each of these steps until they arrived at the solution. Following this, the students checked the solution to the problem, then the teacher discussed with the students about their method of verification by saying, "If we check the smaller components of the solution, then we can check the entire solution, because the nature of the question defines how you check it". Subsequently, one of the groups displayed the difficulties it had faced in solving the problem and discussed this with the rest of the class as a way to evaluate their method of thinking in solving the mathematics problem. With that, the teaching method had transformed from presenting knowledge in a direct manner to a thought process in building it.

4.2.1.5 Teacher-student relationships before the implementation of the IMPROVE programme

Analysis of the observations data indicates that Mr. Hatem tried to encourage implicitly the students to develop their critical thinking on one occasion. This was done through criticizing the solution of an activity presented in the textbook, as it had presented a lengthy solution. In addition, he motivated the students by saying, "We want to add to what is included in the syllabus and we want to exceed this with greater knowledge". Another time when the teacher presented a new activity, he urged each group to present varied answers, in accordance with their way of thinking in solving the problem. He then asked each group to think about the solution from another group and how that group arrived at that solution, rather than merely looking at the solution itself.

The teacher was concerned with and focused on errors made by students and then corrected them continually. This was done on both an individual and collective basis. For this, the teacher focused more on the steps to directly solve a problem, rather than focusing on the solving method. This meant the relationship between the teacher and his students was not participatory or constructive but rather one of monitoring errors made by students in their problem solving, with this being done in order to correct them. Thus, when the teacher‘s role is limited to being a conveyor of information, it hinders any observance of manifestations and indicators of metacognition. It was also observed that there was a constant haste in the teacher’s delivery of lesson concepts and in solving problems. This could be because the teacher is required to finish all of the syllabus units in a limited time, regardless of anything relating to metacognition.

The aforementioned points have clarified that the relationship between teacher and student is not based on participation in building knowledge, but is instead essentially based on conveying mathematical concepts in direct ways. This is then followed by monitoring errors made by students in their solving of mathematical problems, which makes the student-teacher relationship minimal. The reasons for this minimal relationship are numerous, one of which could be the overbearing supervision of the teacher on students’ errors. Another reason could be his characteristic of rushing, which was noticed when he was conveying concepts or solving problems, which confuses students a lot (see 4.2.3.4).

4.2.1.6 Teacher-student relationship within the implementation of the IMPROVE programme

This section discusses the relationship between the teacher and his students in mathematics learning in the classroom within the implantation of the IMPROVE programme and examines whether the nature of this relationship is conducive to learning through metacognition.

It is natural that the teacher-student relationship at the start of the programme’s implementation did not differ greatly from the past. It was noticed in one instance that the teacher accounted for much of the speaking, even when it came to dealing with difficulties faced by the students. However, later on in the process, the teacher-student relationship improved. This was because the mathematics learning method became more systematic and methodological than before. The teacher now discussed specific issues, be they in lessening difficulties faced by students when solving, checking the method pursued in solving, or comparing problems to others. All this was done to expand the students' awareness of their thinking and their ability to perform metacognition. On one occasion, the teacher set an activity for the students using a worksheet, and the students' interaction was good in solving the problem, as questions such as 'Why have we done this?' increased. There was a group that faced difficulty in solving the problem, so they tried repeatedly with multiple methods to arrive at a solution, then they discussed the challenges they faced in doing this. On another occasion the teacher discussed the similarities and differences between two problems with his students. This assists in expanding the students' thought in their approach to mathematics problems. A clear interaction between the teacher and the students was noticed. Another time the teacher discussed the method of verification by saying, "If we check the smaller components of the solution, then we can check the entire solution because the nature of the question defines how you check it". In addition, the teacher stressed the importance of seriousness, discipline and good listening by the students in order for the results of cooperative work and IMPROVE's implementation to be beneficial.

4.2.2 Thematic findings of interview

The interview data collected from Mr. Hatem were categorised into the following themes.

4.2.2.1 Teachers' understanding of metacognition before the implementation of the IMPROVE programme

Through interviewing and questioning the teacher Mr. Hatem on his concept of metacognition, it became clear that he did not have a clear notion of the term 'metacognition' itself. However, his background knowledge of education provided him with a more general conception on students' thought processes and how they learn. Consequently, his conception was based on this background knowledge and can be outlined in three areas: evaluating thinking with the goal of its improvement, the function of this concept in the student's life, and the link between the concept and the age of the student.

1) Evaluation: Mr. Hatem saw that metacognition was a student's evaluation of his thought when dealing with a mathematics problem. He explained this as:

> I would imagine that the concept of metacognition is the student's criticism of his thought; this criticism involves examining both the positive and negative aspects of thinking by the student in solving mathematics problems.

2) The function of metacognition: Mr. Hatem felt that metacognition had a role to play within several domains, ranging from the mathematics classroom to the student's wider life outside the school environment. Discussing this, he said:

> I expect that after the stages of implementing this method, it will have a positive effect on improving the students' style of thinking, be it in mathematics learning or even on their thought methods outside the classroom.

He added that learning through this method would shift the largest role in learning from the teacher to the student, as the task of the student becomes searching for knowledge, while the task of the teacher becomes supervisory and corrective. Speaking on this matter, he stated it would result in

> A lightening of the teaching burden because if students are trained in this method, the greatest role is played by the students. So the task of the teacher becomes supervisory and corrective, while the task of the student becomes searching for knowledge. Thus you will observe the students presenting entire lessons by themselves.

4.2.2.2 Teachers' understanding of metacognition after the implementation of the IMPROVE programme

After having implemented the IMROVE programme, the teacher Mr. Hatem was then asked again about his notion of metacognition. His answer involved three key categories, these being the concept of metacognition itself, its function in learning, and beneficiaries of this method.

1) The concept of metacognition: Mr. Hatem discussed the concept itself as being "evaluation of thought with the goal of its adjustment and improvement," or as he also phrased it, "criticism of thought methods with the goal of adjusting them". He expressed his thoughts on the concept in a practical manner that reflected a more informed and experienced perspective: "The following of logical steps to solve even if a final solution is not reached".
2) The function of metacognition in learning: the teacher mentioned that metacognition had a function in learning, which he emphasized as being a diversification in approach, away from a rigid traditional method. Expressing his thoughts on this, he explained the function as, "Following this method would assist students in finding a strategy to solve, which would in turn generate multiple solutions for the same problem". He then developed this idea by stating:

> This is one of the methods that would help the teacher to diagnose the issues and identify strengths in a student's thought when dealing with mathematics problems.

   He also reiterated previously held views that the value of metacognition for a student extends beyond the school environment, by stating: "The benefit of metacognitive learning could extend to the students wider life".
3) Beneficiaries of metacognitive learning: Mr Hatem was asked who he felt reaped the greatest benefit from the introduction of metacognitive methods to the classroom environment – he felt that it was useful to types of students, yet he singled out students of average achievement as being the greatest beneficiaries of the method. Clarifying this view, he stated:

> I see that this method suits those with average attainment levels, as outstanding students rush through this method in their heads and do not favour drawing mind maps or discussing them, whereas the weaker ones already suffer from a lack of previously held knowledge, impeding the application of methods to new situations.

   Finally, he also discussed the importance of academic attainment in students when it came to them reaping the benefit of metacognition:

> The previously attained knowledge of a student plays an essential role when it comes to their success in utilizing this method, as it provides them with a foundation on which new knowledge is built.

4.2.2.3 Mathematics teaching technique before the implementation of the IMPROVE programme

Mr. Hatem was asked about his methods of mathematics teaching in order to clarify the extent of how suitable the currently used methods were in regards to metacognitive teaching. This would involve the compatibility of previously used methods and whether or not they merely had to be readjusted, or if they had to be reformulated altogether. Mr. Hatem mentioned that he was interested that the students would have a new methodology for dealing with mathematics problems, saying it would be “the acclimatization of the students to how problems are solved more than simply giving a solution to the problem”. He clarified this by saying, “The overlap between mathematics problems demands that students draw a mind map to solve different types of similar problems”. Thus, the teacher saw his own task as being to “attempt to build a notion or mind map for students for solving all the applications presented to them, however they differ or however many they number”. This would be done through “their participation in the learning process and providing them with the opportunity to understand or arrive at the solution based on the syllabus’s methodology or another chosen methodology”.

Throughout the interviews, Mr. Hatem stressed the importance of evaluating and monitoring errors made by students, about which he said, “I see that this is the basis upon which the role of the teacher in the class is built, which is urging students to think, with him correcting their errors”. He also felt that the goal of this was to work with students to reduce the errors they faced and then arrive at mastery of solving mathematics problems, which he described as

> The attempt of the teacher to monitor the errors of the groups in their solutions, and then displaying these to the rest of the students. This enhances learning through the method of metacognition.

With that, students can observe their method of thinking: “Through correcting the students’ errors in solving problems, they can observe their thinking”.

Mr. Hatem also emphasized in his interview the importance of practising learning through metacognition, saying

> This responsiveness grows [over time], because at the start one may notice that students want ready-made knowledge and do not favour learning through thinking methods that help them [in understanding] how they think. However, over time students develop to a point where they will not even notice when they are practising this method in learning.

Consequently, the teacher expected there to be a resulting benefit from practising learning through metacognition, by stating "I expect that after the stages of application for this method, it will improve students' style of thinking, even outside the classroom, in addition to improve the teaching process".

4.2.2.4 Challenges of implementing metacognition after the implementation of the IMPROVE programme

Mr. Hatem was asked about the difficulties and the obstacles that might challenge the implementation of metacognition in teaching. His answer involved:

1) The domination of the traditional method: Mr. Hatem pointed out that students preferred to learn with the traditional method, by which they could solve mathematics problems in the quickest time; conversely they did not favour learning new styles of thinking, and he said, "The students prefer to take ready-made information instead of learning methods of thinking for solving problems". Mr. Hatem mentioned that his focus was on finishing the prescribed material of the syllabus, so as to enable the students to prepare for examinations, he did not focus on improving the students' way of thought when it came to dealing with mathematics problems. Discussing this point, he stated:

   > I might not be able to practise this method in [my] teaching to a sufficient degree due to the lack of adequate time provided coupled with the pressures of the syllabus content and the necessity of finishing it so [students can] prepare for examinations at the end of term.

When Mr. Hatem was asked if he discussed students' thinking methods with them, he said "Rarely do I discuss the students' thinking method when dealing with mathematics problems, and that is because of the lack of time in a class session". Nevertheless, he held the view that teaching with metacognition would lead to a change in the reliance on traditional teaching and that this would emerge due to the existence of a clear methodology to solve mathematics problems. To this end, he stated: "This method involves a change from the monotony of the traditional method which fails to inspire today's generation of students". Mr. Hatem clarified this statement by adding:

> I say that there is a change in teaching method, even if only slightly, for example through clear methodology like this one to solve math problems and identify difficulties and compare problems.

Mr. Hatem felt that the need to comply with the traditional methods was an obstacle in the way of metacognitive teaching. He said,

> The focus on completion of academic content within a specific timeframe is the controlling factor in mathematics learning methods, and is also an obstacle to the implementation of metacognitive learning.

2) The lack of training in metacognitive teaching: Mr. Hatem mentioned that he had not had any training relating to metacognition. He confirmed, "I have not come across any training programmes for this concept". He also mentioned that there was a weakness of teacher expertise regarding metacognition, which negatively impacted on his adoption of teaching methods. Speaking on this matter, he alleged "A weakness in knowledge about experiences in this field affects the teachers' willingness to adopt methods like this". Mr. Hatem felt that the absence of this knowledge was a challenge that confronted teaching with metacognition.

> We are in need of specific training for metacognition, as I currently cannot deal with the metacognitive concept without having undergone preparation for it.

For this process to occur, Mr. Hatem hoped that "supervision would play a role in training teachers for this method based on the true reality [of our situation] and would have a good concept of this method", which would be done "through a genuine collaborative participation in teacher training during the school day and examination of his performance in lessons". He added that "I think it would be useful for them to prepare examples for the implementation of metacognition; this would greatly facilitate teaching with it".

3) The Principal's current lack of adoption of metacognition: Mr. Hatem reasoned that the principal's embrace of metacognitive teaching would be a great catalyst in its implementation:

> The principal's adoption of metacognition would be very important, but at the same time it should not be compulsory for each session – this would leave the teacher with the choice of how and when to apply it.

He added that another obstacle related to the principal would be

> Occupying teachers with additional duties, which are unrelated to mathematics teaching, like competitions; this would be an issue as metacognition requires extensive planning and effort.

Another obstacle mentioned was that the principal demanded teacher compliance to the timeframe of the syllabus units – regardless of any utilization of metacognition. He said "The principal demands that the teacher complies with the general teaching plan, without taking into account the implementation of metacognition".

4) The scepticism of the educational supervisor regarding metacognitive teaching: Mr. Hatem highlighted that one of the obstacles to metacognitive teaching was the practices of the educational supervisor. He discussed the problem as lying in the fact that

> The supervisor plays no role when it comes to supporting metacognitive teaching, and I see the extent of the supervisor's conviction as being quite important.

He added,

> The educational supervisor focuses on superficial issues such as the notebook for preparing lessons, and does not focus on the teaching

> practices much, which is considered an obstacle to metacognitive teaching.

Mr. Hatem added that the supervisor should have knowledge and experience with the concept of metacognition and how it can be implemented in the teaching context, stating, “The supervisor must be well versed to begin with [before overseeing] this method, and he must have a good concept of it so he will be able to convey it to the teachers”. Due to this, Mr. Hatem recommended that teachers would present a teaching plan, in which they would clarify how one can teach mathematics metacognitively. He added

> I do not think that it should be part of a teacher’s assessment, because I believe that this method is one of instruction, rather one of evaluation. This is so it can become a teaching culture dealt with by the teacher, rather than a set of criteria that a teacher himself is assessed by.

5) Disadvantages in the education system: Mr. Hatem highlighted that there were disadvantages relating to the education system and that these could be obstacles or challenges that might hinder metacognitive teaching. Of these issues, he mentioned that the pressure of the syllabus contents along with the requirement that the teacher finishes all of this before the exams is considered to be a great obstacle in applying metacognitive teaching. He remarked,

> I might not be able to practise this method in [my] teaching to a sufficient degree due to the lack of adequate time provided coupled with the pressures of the syllabus content and the necessity of finishing it so [students can] enter into exams at the end of term.

The increasing number of students in a class is a disadvantage relating to the education system. He pointed out,

> The increased numbers of students would have a negative impact on learning with metacognition, because when using it a teacher requires individual and direct interaction with each student, which of course cannot be done with a growth in student numbers.

He also was of the opinion that another one of the obstacles related to the education system was the length of class sessions, on which he said:

> The shortage of time within the class session is an obstacle to metacognitive teaching, because this requires a greater length of time, be it in class or time a teacher spends with the students.

Lastly, he noted that metacognitive teaching required the presence of a teaching assistant in order to monitor the work of groups in class, on which he said,

> One of the key obstacles to the application of metacognition in classrooms is the absence of teaching assistants to follow the groups' work during their attempts to solve mathematics problems.

4.2.2.5 The requirements of the teacher to implement of metacognitive teaching strategies

Another theme covered in the interviews was the requirements relating to the teacher that would enable its implementation in the teaching context. These included:

1) The clarity of the concept of metacognition in the eyes of the teacher: this matter was discussed particularly in terms of its clarity in the practical implementation of metacognition. As stated by Mr. Hatem, "It is very important that the concept of metacognition itself is clear for the teacher; this will enable him to teach with it in a complete and correct manner".
2) The ability of the teacher to individually and collectively evaluate a method of thinking: Mr. Hatem spoke about a teacher's capability to implement metacognition, mentioning the mainly external factor of time. This was termed as,

> I feel that if a teacher really commits himself to metacognitive teaching exclusively, then he simply will not have the time to do any other work, as the greatest share of his time will be taken up by students and their discussions. Then having to evaluate their thinking method in mathematics learning on an individual or group basis, as well as distributing his time among students to discuss their thinking, will all result in exceeding the time limits of the lesson.

He also pointed out the necessity of there being reports specific to all students that would evaluate their thought method when dealing with mathematics problems. Putting forth the idea, he said:

This is the importance of special evaluative reports for each student's method of thinking when solving mathematics problems and their role in developing the standards of students.

3) Design of materials: as mentioned by the teacher, it is necessary to prepare activities that are appropriate for metacognitive teaching. Regarding this, he said "Impressing metacognition on students in their learning will depend largely on the selection of appropriate activities for this method". He stressed the importance of syllabus content corresponding with metacognitive teaching, by saying: "The content within the syllabus could enable the teacher to teach with metacognition as some content focuses primarily on methods rather than solutions".

4) Characteristics: From the students' point of view, it was seen as important for the teacher to possess certain characteristics in order to practise metacognitive teaching. One of these was for the teacher to be knowledgeable on the use of learning through metacognition in dealing with mathematics problems. The student Abdulelah said, "Learning through metacognition requires the teacher to be aware of this methodology and for him to believe in its concept". This was underlined by Fares, who said "for the teacher to adopt learning through metacognition and for him to be enthusiastic about it". Nawaf and Babseal also mentioned this point.

6) Constancy of metacognitive teaching: From the students' point of view, another need expressed was for metacognition to become a constant, and not only used in some lessons. This is in order for students to benefit from it to the greatest extent. Nawaf remarked, "For metacognitive teaching to become a habit for the teacher in most of the lessons, this will enable students to benefit from the method".

4.2.2.6 Cooperative learning and metacognition before the implementation of the IMPROVE programme

Mr. Hatem's background knowledge of education provided him with a conviction of interrelation between cooperative learning and metacognition, and justified this with a number of ways that students can benefit from learning cooperatively. These methods can be summarized as follows:

1) Multiple solutions: Cooperative learning aided students in presenting multiple solutions for a specific mathematics problem, after which each workgroup evaluated the work of other groups, which helped to improve their thought process. Mr. Hatem explained this:

> Cooperative learning in groups generates multiple solutions to mathematics problems, so each group presents an answer which could be different from that of another group.

2) Self-correction: A student's errors could be corrected in front of the rest of their classmates, which reflects the student's way of thinking to them. Mr. Hatem said,

> Students learning through cooperative methods and correcting the errors of their groups in front of the class is a type of metacognition, as this provides a reflection of their thinking.

   He added to this by stating, "The teacher tries to monitor the errors of the groups in their solution, and displays these to the students, which enhances learning through this method".

3) Comparing and contrasting solutions: Cooperative learning aids the student in comparing the solutions of each group with all the other groups, or with the solution from the teacher or the textbook. This helps to improve the method of thinking in solving mathematics problems, as stated by Mr. Hatem:

> I ask the student to compare his solution with that of his classmate or [I ask] a group to do this with another group, or with the solution of the book or the teacher. Through this, groups can confer about these solutions and discuss their own solutions, which assists in improving the thought process.

4.2.2.7 Metacognition and cooperative learning after the implementation of the IMPROVE programme

Data analysis of interviews after implementation of the IMPROVE programme presented Mr. Hatem's assertion that there was a close relationship between cooperative learning and learning through metacognition. He remarked,

> Cooperative learning is an essential component of the learning through metacognition, and its value should not be underestimated by those advocating more traditional methods.

He also thought that cooperative learning assisted in the implementation of metacognition, as he asserted “There is great significance for cooperative learning, as it will assist in the creation of the appropriate atmosphere for the implementation of metacognition”. However, he also indicated that “students with weak motivation to learn mathematics in general, let alone through metacognition, will delay their classmates’ success in learning through this method”. In addition, Mr. Hatem stipulated that these students should be of varying academic achievement levels, and that they should be diverse in regards to their grades. This “distributing students into groups that reflect varying academic achievement levels because that would be helpful if we were to teach with this method”. As well as this, he was of the opinion that “coordinating with some of the students as leaders of workgroups could be done in order to facilitate teaching with this method”. Mr. Hatem emphasized that cooperative learning alongside learning through metacognition delegates the largest role in the learning process to the student, as he searches for and builds knowledge by himself. As such, the role of the teacher becomes supervisory and corrective. He stated,

> Engaging students in the learning process and providing them with the opportunity to understand and arrive at the solution according to the methodology in the syllabus or another chosen methodology is a necessary process.

He believed that it was important for a teacher to continue with his students for a longer period of time so he could train them to learn with this concept. He asserted,

> Cooperative learning lightens the burden from explaining everything by oneself to supervising the students’ learning. Also, discussion between students motivates them to learn, thus I think that the continuation of the same teacher for a suitable period of time is useful in learning through this method and I feel that the suitable period would be four academic terms.

4.2.3 Thematic findings of interviews and focus group (Mr. Hatem's students)

The data collected from these participant interviews and focus group have been categorised under the following themes;

4.2.3.1 Students' understanding of metacognition before the implementation of the IMPROVE programme

In gathering interview and focus group data from participants, they were asked about their notions of the term 'metacognition'. It became clear that it was new to them, with Babseal, Abdulelah and Asaad all saying, "I have not heard of the term metacognition before". After clarification of the term was provided to them, it appears that they had an understanding of it from their general education. These initial conceptions on metacognition can be summarized into two categories:

1) The Concept of metacognition: Babseal speculated, "I imagine that the concept of metacognition is knowing how a student thinks". Abdulelah explained, "My conception of it is that it is a correction, revision or re-ordering process of the thinking which you conduct". A similar view was provided by Babseal, who remarked, "I think that it relates to how one can change their way of thinking in the most exemplary way".
2) The Function of metacognition: Some students expressed their views about metacognition by discussing the function of this concept in learning. Asaad felt that metacognition would help in learning mathematics, as he noted:

> It is a useful method that would help to convey concepts in a better and more participatory manner, rather than just taking ready information. This would help us to consolidate it and know how to apply it in new situations.

4.2.3.2 Students' understanding of metacognition after implementation of IMPROVE programme

After the implementation of the IMPROVE programme, students were asked about their notions of metacognition. The data gathered from the participating students can be divided into the following categories.

1) Metacognition as a concept: Asaad began by stating that "the concept of metacognition is the criticism of your thought or method of thought in order

to correct it". Ammar remarked "It is the arranging of thought in accordance with logical steps to deal with mathematics problems and then reviewing and evaluating them for amendment". Several students linked metacognition with solving mathematics problems, such as Fares who said,

> My conception is that metacognition is following certain steps which oblige us to think, and then solve a mathematics problem. Then one thinks about this style of thinking, evaluates it and then amends it.

2) Function of metacognition: Some students spoke about their notions of metacognition in the context of its function within learning. They discussed metacognition as being an organizer for the method of thought. Abdulelah expressed this view as "a new way to learn and making solving organized, and rethinking these systematic steps with the goal of adjusting them". Fares then clarified how metacognition could help a student in organizing thought, which he explained as,

   > An organized thought method for solving and a way to evaluate this method; this is done through identifying the difficulties in the thought method so one can adjust and develop it.

   Metacognition also aided students in understanding mathematical concepts to a greater extent. Sultan noted that "learning through metacognition helps you to understand the problem properly and following the methodology will help you to get closer to the solution of the problem". He added that,

   > Learning through metacognition provokes thought and discussion between students in a good way. It has really pushed us to analyse the problem and understand it more precisely, particularly in situational problems.

   Perhaps one of the reasons behind this is that learning through metacognition motivates students to review previous knowledge and link it with new information. Fares mentioned this as "metacognition makes you review previous knowledge more and makes you more familiar with the rules than before".
3) Metacognition as skills: several students participating in the study viewed metacognition from the perspective of the skills it involves. The skills of planning, management of planning, monitoring and evaluation, followed by

control or adjustment were all raised in the discussion of several students. For example, Abdulelah mentioned:

> I found that learning through metacognition was useful, and the reason is that there is a thought methodology for solving problems. This facilitates dealing with such problems and is beneficial for cooperative learning, and it highlighted the four skills of planning, management of planning, monitoring and evaluation.

Asaad added that:

> These four skills were present, but evaluation was weak because sufficient time was not given for that. I understand that this method relies on the extent to which you have mastered these four skills in learning.

However, Fares saw that the skill of monitoring required more practice in learning because students were weak in it. He said,

> I think that these four skills are present, but perhaps monitoring is not present to a good level because of the difficulty of carrying out this skill. This is due to the group's individuals all being in agreement on the solution and then fixating on it, so it was difficult for them to monitor each other's thinking.

The student stressed the importance of practice in gaining these skills:

> These four skills were present but evaluation was weak because sufficient time was not given for that. I think that these skills became more apparent with practice. We really felt that bringing out these skills benefitted our performance in learning mathematics.

4.2.3.3 Mathematics learning technique before the implementation of the IMPROVE programme

Certain strands of data provided in interviews and focus group with participating students can be categorized under the theme of learning techniques for mathematics students and the extent of these techniques' relation to metacognitive learning. Most of the students dealt with maths problems by defining what was asked of them, then trying to find the solution to it. Asaad described his method as,

> I determine the givens and then try to invoke the suitable rule to solve it, then I substitute to find the product and occasionally check to see if my solution is correct.

Some students follow the same method, but they try to compare the given problem with similar ones, as Fares describes “I identify my errors by comparing my solution with a previous one from a test we have done for example, or the solution in the book or the teacher”. Babseal presented his method as, “I directly link it with the method explained in the lesson, or the method of the teacher while solving a previous problem”. Some students relied on memorizing mathematics rules and tried to directly apply them. An example of this was given by Babseal, who noted,

> I rely on the idea of the lesson and directly apply it. Sometimes when I have understood this idea from my perspective, it is the first thing I apply when solving problems.

Based on what has been revealed through the interviews, it cannot be observed that there is a clear manifestation of metacognition. Instead, the reality is that there is much focus on the method of solving a problem, rather than improving the method of thinking when dealing with mathematics problems. (See thematic findings of observations 4.2.1.4 - Learning strategy of students related to metacognition).

4.2.3.4 Challenges to the implementation of metacognition in mathematics learning

During interviews and focus groups with participating students after the implementation of the IMPROVE programme, it became apparent that there were several challenges that could hinder the application of learning through metacognition. These issues have been categorized as follows:

1) Domination of traditional method: This issue involved the focus of the student being predominantly on directly solving mathematics problems, without taking into account the improvement of the thought method in dealing with such problems. Asaad identified this issue by explaining,

> It is clear that our method in learning mathematics is to focus on the direct solution to the mathematics problem, rather than improving our method of thinking.

He also mentioned, “The focus is on how we use specific rules and the means of applying them in a direct manner”. Fares felt that their habituation to a direct technique of solving problems - focusing on the speed of solution - was another challenge confronting learning through metacognition. He said

> The teacher’s method sometimes is not suitable for learning through metacognition. With him, we have become accustomed to solving quickly and directly according to a specific technique. This is because the idea of finishing the syllabus and quickly achieving things controls the teacher, and his focus is not on the improvement of students’ ways of thinking.

Based on this data, it has been clarified that the focus of both teacher and student is on correcting errors, rather than dealing with their way of thinking in solving mathematics problems. An effect of learning through the traditional method was a constant awareness of the time available to learn all the units of the syllabus. Another impact appeared to be a continual worry about exams, with both of these forming obstacles to learning though metacognition. On this point, Babseal said: “Learning by metacognition is difficult because of the time factor, as it is demanded that I solve the question rapidly, so I cannot find the time to spend on thinking about my thinking”. Another problem identified was the lack of discussion by students regarding their way of thinking. Asaad was asked if his teacher had discussed thinking when teaching mathematics with him so that he could learn about metacognition. His replied in the negative and said, “Rarely do we discuss how we think when learning mathematics; most of the focus surrounds correcting errors in solving mathematics problems”.

2) Time needed: metacognitive learning requires more time. This is what was explained by Sultan, Abdulelah and Fares. Ammar commented that “learning through metacognition takes time and some students are only interested in solving the problem faster”. Abdulelah discussed this matter as well: “The greatest obstacle is the lack of time”. Expanding on this idea, he said, “Time management is an important aspect of learning through metacognition”.
3) Absence of important teacher characteristics for metacognition: The teacher not possessing certain important characteristics was seen as an obstacle to teaching with metacognition by some of the participants. Asaad said that “The teacher must deal with students fairly and must be cheerful, to create

an atmosphere conducive to metacognition". Speaking on the same matter, he remarked,

> The teacher must deal with students equally, as some teachers treat outstanding mathematics students better, and might marginalize everyone else in their group.

Abdulelah stated that "the teacher must be suited to metacognitive teaching". He added, "The personality of the teacher should be suitable for metacognitive teaching". Discussing teachers further, Abdulelah noted that learning through metacognition required "good supervision, of both groups and the individuals within those groups".

4) Number of students in class: The students suggested reducing the number of students in the class generally, and for the number of students in an individual group not to exceed four students. Abdulelah underlined this theme by stating,

> It is essential to reduce the number of individuals in a single group, because increasing the number is a challenge to learning through metacognition. This is due to it hindering discussion between students as well as impeding the teacher's evaluation of each group's work.

5) Indifference of teacher: The teacher's indifference in implementing learning through metacognition was seen as an additional challenge. Nawaf recounted,

> I did not see a large difference in the performance of the teacher after the implementation of this programme compared with how he was in the past. Perhaps it was this that meant some students did not change, and could also be the reason for the teacher not practising learning through metacognition except for in the presence of the researcher.

This was confirmed by Ammar, who said,

> I didn't notice a change in the teacher's methods, which played a role in students not being receptive. Also, the teacher did not use learning through metacognition in the understanding of new concepts, but rather used it to solve problems.

Sultan added that,

> Learning through metacognition was not applied unless you were here; it was clear that the teacher was instead keen to finish the syllabus. So we felt that there were different goals for you two.

The teacher's concentration on completing the syllabus content rather than stimulating learning through metacognition was also cited by Fares as a challenge. The student said,

> At first the teacher was not adopting metacognitive teaching to a sufficient degree. This might have had an effect on students not taking it seriously, because I did not observe the teacher's method changing, but instead I only noticed the worksheets and simple things with the teacher changing at the end of the period. This might be because the teacher was focused on finishing the syllabus.

4.2.3.5 Students' requirements for the implementation of metacognition

Data gathered from the student interviews and focus groups can be categorized under the theme of 'Students' needs for the Implementation of metacognition'.

1) The role of the student: For the role of the student to be searching for and building information and not simply receiving it via the traditional method from the teacher. Sultan stated in clarification of this idea, "Metacognitive learning demands that the student searches for knowledge by himself instead of the teacher presenting ready-made knowledge to him". He added to this by commenting,

> Metacognitive learning transforms the role of the student from receiving ready information from the teacher to a group searching for a solution. It really made dealing with situational problems easier; previously we did not expect that we would be able to deal with such problems.

2) Mind map: For the student to have a mind map to deal with mathematics problems. This would enable him to monitor his thinking and help in its adjustment and improvement. Fares commented,

> Metacognitive learning is useful in mapping the approach to problems and changing the work focus from being on a single outstanding student to being shared by all the group members.

Asaad underlined this by adding,

> With difficult problems, the only student solving it would be the outstanding one, but now everyone participates in arriving at the solution. This is due to there being a work map for thinking.

3) Practice: practice for learning through metacognition was seen as necessary in order to benefit from this method. This was expressed by several students. Abdulelah recounted,

   > When the programme was first implemented, our interaction within the work group was not great, but we tried to apply it so that we wouldn't find it hard. Then we noticed its benefit as a methodology to solve, even if we did not arrive at the solution.

   This was underlined by Asaad, who noted,

   > Learning through metacognition gave me a clear mechanism to deal with mathematics problems. I had pictured myself as being unconcerned with thinking, but rather with memorization and recalling only. However, after practising learning through metacognition I felt the desire to tackle the difficult problems.

   Ammar suggested "for learning through metacognition to be followed by the teacher in its continuation so that students can become accustomed to it".

4) Culture of Interaction among students: Mr. Hatem highlighted how students should be educated in the 'culture of dialogue', which would serve to assist them in discussing their methods of thought during the solving of mathematics problems. He stated, "Educating students to have a discussion among themselves could be used to facilitate metacognitive learning". Some of the issues that needed to be addressed in such education were respect for the group leader and students' mastery of communication skills with their workgroup. Mr. Hatem identified these issues by saying, "Respecting leadership and knowing how to deal with a group (communication skills) are all areas that could be developed to facilitate teaching metacognitively". He also rephrased his statement as, "Students mastering communication skills would help them to learn through the concept of metacognition".

5) Academic attainment: The importance of academic achievement in terms of its relation to metacognition was also discussed, with the teacher seeing it as an essential component to making metacognitive learning a success. On

this point Mr. Hatem said, “The previously learnt knowledge of the students is an important factor in practising metacognition”.

4.2.3.6 Student-student relationship before the implementation of the IMPROVE programme

Regarding the subject of communication between students and to what extent it can facilitate metacognition, it arose from the interviews and focus groups with students that, in general, there was a weakness in the skills of communicating with others. For example, a group of study participants stated that they did not wish to speak with others, be it about methods of thinking or even in their wider learning. The student Babseal asserted,

> I do not discuss with others my way of thinking, because I do not expect them to be able to understand my way of thinking, as it is quick and I cannot speak about it as I am unable to explain it.

The student Abdulelah also expressed dislike for the method by stating, “I do not like to speak to others about my way of thinking, but if it was demanded of me I would”. Babseal continued,

> With some problems, I like to solve them by myself, but with others I participate with others. This depends on the nature of the mathematics problem and if I find it difficult of not, but I participate in the discussion with my classmates If solving the problem is hard.

Asaad held a more favourable view, saying, “Yes, I like to speak with other students about my thoughts and my way of thinking in learning mathematics because that acts as a correction for my thinking”.

Participants were asked if they accepted the corrections of their classmates in learning mathematics. The students’ answers varied between those who accepted and those who didn’t, however all of them mentioned that their classmates’ corrections when learning mathematics was a positive thing. For instance, Asaad explained that “the advantage of my classmates correcting my errors in class when learning mathematics is that it develops my skills. However, the disadvantage is represented in the inappropriate method used by some students to correct others, but this is very rare.” Babseal expressed,

> I feel that it is a positive thing and I do not see my classmates’ correcting me as them being critical, but rather they are reminding

> me of knowledge and consolidating it in my mind and they want me to understand. Yes, I rarely make mistakes in front of my classmates, so I have no problem with them correcting me on a few occasions because I have probably corrected them a lot in the past

Abdulelah noted a disadvantage in his classmate's corrections of him by stating,

> The downside is that my classmate will criticize the solution and not criticize my method of thinking, so he is also presenting ready knowledge to me, so it will not be consolidated. On the other hand, if he criticized my way of thinking, and I went back to solve it again by myself, then it would be better.

Babseal explained how he would not accept it if a student of lower academic achievement corrected him: "The negative is that if someone of a low academic standard corrects me then I will not accept it". However, participants did not raise anything indicating the presence of skills in dialogue with others. Confirming this is a quote by Asaad who said, "Some students do not interact in cooperative learning in a positive way. This is because of the weakness of their skills in communicating with others in a dialogue."

4.2.3.7 Metacognition and cooperative learning after the implementation of the IMPROVE programme

Several students highlighted that there was a relationship between cooperative learning and metacognition. Nawaf said,

> Really, learning through metacognition benefitted collaborative learning because the map for dealing with problems was clear, and changed the focus from being on a single outstanding student to being shared by all the group members.

He added,

> Because of learning through metacognition, cooperation got better and encouraged the low achieving students, as well as us to understand together. So we worked with team spirit rather than one student in the group doing all the work.

Sultan contributed by saying,

> Learning through metacognition provokes thought and dialogue between students to a great extent, and really motivates us to analyse the mathematics problem and understand it more accurately.

Babseal presented his view as, “I think that the application of cooperative learning along with the implementation of metacognition will help students in solving mathematics problems in a better way”.

In terms of learning groups, it became apparent through interviews and focus groups with students the importance of learning in small groups in order to make metacognition possible. It was important that the number of students in each group did not exceed four. They also stipulated that these students should be of varying academic achievement levels, and that they should be diverse in regards to their grades. This was emphasized by several students, including Nawaf, Asaad and Fares. Fares discussed this point by stating that

> It is important to distribute students in a way designed so that there will be interaction between the members of a group. In doing this, the members of a given group must not be of the same academic achievement level, which will allow students to confer about ideas.

4.2.4 Summary of the case study two findings

At the start, Mr. Hatem did not have a clear notion of the term ‘metacognition’ itself. However, his background knowledge of education provided him with a more general conception of students’ thought processes and how they learn. His conception can be outlined in two areas: self-critique of a student’s thought with the goal of its improvement and the function of this concept in the student’s life. Students also did not have a clear notion of metacognition when questioned on this point. After clarification was provided, it appeared that they had an understanding of it from their general education. In their view, metacognition was knowing how a student thinks, or it was a revision, criticizing and correction of thinking. Interview responses indicated that students did not have a clear concept of metacognitive skills.

Mr. Hatem was asked about his methods of mathematics teaching. He stressed the importance of evaluating and monitoring errors made by students. He felt that the goal of this was to work with students to reduce the errors they faced and then arrive at mastery of solving mathematics problems. He also mentioned that students want ready-made knowledge and do not favour learning through thinking methods that help them in understanding how they think. Mr Hatem’s teaching methods were not related directly to metacognition, and dealing with it was not an intended process. The teacher paid attention to the errors made by students and then corrected them continuously on a both individual and collective basis. The teacher was also concerned with sharing his criticism of one group with all the groups. He also presented mathematics problems and exercises to students that relied on the direct application of the concepts and rules he had taught them previously. The teacher tried to motivate students by turning the activity into a race. This meant that only the outstanding students participated with the teacher, while the rest neither solved problems nor attempted to. The teacher focused more on the steps to directly solve a problem, rather than the thought methods involved in solving. This meant the relationship between the teacher and his students was not participatory or constructive but rather one of monitoring errors. His role was limited to being a conveyor of information, it hindered the appearance manifestations and indicators of metacognition. In terms of mathematics learning technique, most of the students dealt with maths problems by defining what was asked of them, then trying to find the solution to it. Some students would take additional steps, such as linking the problem to previous ones, and applying memorized mathematics rules. Based on what has been revealed through the interviews, it cannot be said that there was a clear manifestation of metacognition. Instead, there was much focus on the method of solving a problem, rather than improving the method of thinking.

Post-IMPROVE, Mr. Hatem was again asked about his conception of metacognition. His answer involved three key categories: the concept of metacognition itself, its function in learning, and beneficiaries of this method. He discussed the concept in itself as being the evaluation of thought with the goal of its adjustment and improvement. He mentioned that metacognition had a function in learning, which he emphasized as being a diversification in approach, away from a rigid traditional method. He also reiterated previously held views that the value of metacognition for a student extends beyond the school environment. Mr Hatem singled out students of average achievement as being the greatest beneficiaries of the method as outstanding students rush through this method in their heads and do not favour drawing mind maps or discussing them, whereas the weaker ones already suffer from a lack of previously held knowledge, impeding the application of methods to new situations.

Post-IMPROVE observation data also displayed improvement in the teaching methods. However, it was noted that this improvement was not to a great extent. Teaching methods developed from the mere presentation of ready knowledge to the students along with some applications, to a methodology for dealing with mathematics problems with a focus on thinking. It became noticeable that the teacher had begun to follow logical steps in the approach to mathematics problems. These steps were taken in accordance with the IMPROVE programme, and these were: presentation of the concept, metacognitive questions, practice, and reduction of difficulties.

Post-IMPROVE, students were questioned on their notions of metacognition. Ideas suggested included: thought criticism for correcting, arranging thought through logical steps in problem solving, and reviewing and evaluating this for amendment. In discussing metacognition's function in learning, some students mentioned that it could help them in organizing thought. Metacognition was also perceived to assist in understanding mathematical concepts to a greater extent. Several students identified the skills involved as planning, management of planning, monitoring and evaluation, followed by control or adjustment.

Post-IMPROVE observation demonstrated difficulty in students' attempts to change their traditional method of learning mathematics. Students focused on the steps to solve a problem directly, without concentrating on discussion of their understanding of the problem and describing it correctly. However, with practice, the students began to interact more with the steps of the IMPROVE programme, and this was noticed in their learning and discussion. It was noticed that students were focusing more on understanding and following logical methodologies. With that, the teaching method had transformed from presenting knowledge in a direct manner to a thought process in building it.

In terms of metacognition and cooperative learning, Mr. Hatem was convinced of an interrelation between cooperative working and learning through metacognition, and justified this with a number of ways that students can benefit from learning cooperatively, including: presenting multiple solutions, after which each workgroup evaluated the work of other groups, which helped to improve the thought process. Students' errors could be corrected in front of the rest of their classmates. Regarding communication between students and to what extent it can facilitate metacognition, it arose from the interviews and focus group with students that, in general, there was a weakness in communication skills. For example, some did not wish to speak with others, be it about methods of thinking or even in their wider learning. Observation demonstrated that the relationship between teacher and student is not based on participation in building knowledge, but is instead essentially based on conveying mathematical concepts in direct ways. This is then followed by monitoring errors made by students in their solving of mathematical problems, which makes the student-teacher relationship tense.

Post-IMPROVE observation revealed that the teacher-student relationship at the start of the programme's implementation did not differ greatly from the past. However, later on in the process, the teacher-student relationship improved. This was because the mathematics learning method became more systematic and methodological than before. The teacher now discussed specific issues, be they in lessening difficulties faced by students when solving problems, checking the method pursued in solving, or comparing problems to others. All this was done to expand the students' awareness of their thinking and their ability to demonstrate metacognition.

Mr Hatem realized the importance of cooperative learning in relation to metacognition, as it created a suitable atmosphere for implementing metacognition. Mr. Hatem emphasized that cooperative learning alongside learning through metacognition delegates the largest role in the learning process to the student. Students too highlighted the use of cooperative learning in this context, and stated that the map for dealing with problems was clear, and changed the focus from being on a single outstanding student to being shared by all the group members. The method of learning in small groups was also mentioned as an important factor in the success of metacognitive learning. They also stipulated that these students should be of varying academic achievement levels.

In identifying the requirements of teachers for the implementation of metacognition, these were the clarity of understanding of the concept of metacognition by the teacher, the teacher's ability to individually and collectively evaluate a method of thinking, and to prepare activities that are appropriate for metacognitive teaching. In terms of practice, students stated that teachers should be knowledgeable about the use of learning through metacognition in dealing with mathematics problems, and for metacognition to become a habit. The students also highlighted their own needs, such as for the role of the student to be searching for and building information, training for learning through metacognition, and practice. Mr. Hatem highlighted that students should be educated in the 'culture of dialogue', which would serve to assist them in discussing their methods of thought during the solving of mathematics problems.

In terms of challenges of implementing metacognition, post-IMPROVE interview and focus group data showed that there were several challenges that could hinder the application of metacognition. One was the domination of traditional teaching methods. Another issue was that metacognitive learning requires more time and some students are only interested in solving the problem quickly. The teacher not possessing certain important characteristics was seen as an obstacle to teaching with metacognition by some of the participants. The students suggested reducing the number of students in the class generally, and for the number of students in an individual group not to exceed four students due to it hindering discussion between students as well as impeding the teacher's evaluation of each group's work. Additional obstacles were revealed, such as the absence of appropriate training, as well as disadvantages in the education system and the negative role played by the educational supervisor and the principal of the school.

While post-IMPROVE observation data noted some improvement in the teaching method, teacher-related difficulties were also evident as hindering the programme's implementation. One of the important obstacles was the partial adoption of metacognitive teaching; for example, the teacher only adopting the method in the presence of the researcher, and his enthusiasm to complete the syllabus on time. Consequently the students felt sometime that there were differing goals between the teacher and the researcher. Therefore the teacher's method did not change to a great extent. The observation data also showed that the teacher did not use this method in the understanding of new concepts, but instead exclusively used it in solving problems. Furthermore, the teacher did not deal with the reduction of difficulties to a sufficient degree, and he was the only one who spoke about these difficulties.

5 Discussion of the main findings

This chapter deals with the central thematic findings that have arisen from analysis of the qualitative data within the wider context of the existing literature. This incorporates the research questions and provides a more detailed view of the findings drawn from semi-structured interviews, classroom observations and group discussions.

In the first part of this chapter, the perceptions of maths teachers and their students regarding metacognition are discussed. In addition, a perspective for dealing with metacognition in the educational context of Saudi Arabia in mathematics learning is presented according to the findings of this study and in light of the literature based on the models of Flavell (1976), Brown (1987) and Kluwe (1982) for metacognition. It is hoped that the presented conceptualization represents a foundation for future interaction with the subject of metacognition and mathematics – at least among the Arab research community – which aspires to utilize this concept in effective and practical teaching.

In the second section of this discussion chapter, the topic of teaching mathematics metacognitively according to the IMPROVE programme is discussed. The discussion in this section revolves around the basic components of the IMPROVE programme and is grounded in the findings of this study. Subsequently, discussion will turn to the link between learning through metacognition and the role of the students – which is represented in the processes they undertake in order to obtain new information built upon that which had been previously learnt. Based on the discussion of these aspects, a basic practice model of metacognitive mathematics teaching and learning is drawn out. This basic practice model will emphasize the metacognitive skills which were discussed in the literature chapter. In doing so I focus on the components of the IMPROVE programme in light of the socio-cultural context which was also revealed in the literature of this study.

In the third section of this chapter, metacognition and cooperative learning regarding mathematics is discussed (see 2.5.4 for some details regarding cooperative and collaborative learning). This is the second component of the IMPROVE programme (1997) and will be discussed separately due to the importance of the subject. All three sections are linked with the goals of this study, along with the extent of its potential contribution to the realization of these goals.

5.1 Teachers' and students' perceptions of metacognition

The findings initially revealed that there was not a clear notion of 'metacognition' among teachers and students in both case studies (see 4.1.2.1, 4.1.3.1, 4.2.2.1 and 4.2.3.1). However, their experience of the education system provided them with a wider conception of thought and learning techniques such as thinking skills. The findings showed that there was an awareness of the importance of the monitoring aspect of metacognition, particularly in monitoring the logic of the thought process. The findings also highlighted that participants were aware of thinking skills and the importance of planning effectively to impart them.

As for the students, their perception of metacognition in both case studies indicated the importance of understanding one's course of thought. The conceptions of the two sets of interviewees regarding metacognition reflected the belief that it involved evaluating one's course of thought, with the goal of improving thought when dealing with mathematic problems. It was observed that certain key words were used by teachers and students when expressing their conceptions, a feature evidenced in the literature surrounding metacognition (see 2.2.1). These words are: evaluation, discovery of errors in thinking (or 'monitoring'), course of thought, improvement of thought, and logical thought. However, these words do not express metacognition in the way it was described in the literature of this study. Since interviewees in both case studies were unfamiliar with the term metacognition, terms such as these might be discussed as a result of the introduction to the study given to students and teachers at the beginning of its implementation. This interpretation clarifies how the teachers and students presented a more accurate conception after the implementation of the IMPROVE programme. The participants' conceptions and the extent of their consistency with those described by Flavell (1979), Brown (1987) and Kluwe (1982) will now be discussed.

After the implementation of the IMPROVE programme, the findings in both case studies showed that metacognition was a concept that required skills and had a function (see 4.1.2.2, 4.1.3.2, 4.2.2.2 and 4.2.3.2). In terms of its concept, it was defined as a knowing of thinking and monitoring of thought procedures to enable its adjustment so as to improve it. For example, the participants see metacognition as a systematic and logical procedure for solving problems. The concept's role as a tool for thought evaluation was also identified as being targeted at adjustment and improvement. Findings also stressed the importance of judging one's thought in a positive rather than strictly critical manner, as these may be habits practised by students and teachers in the traditional learning context in Saudi Arabia. The findings highlighted that the participants' conceptions of metacognition in both case studies lacked a comprehensive vision of the concept due to an absence of the individual's set of metacognitive knowledge (see 4.1.2.2, 4.1.3.2, 4.2.2.2 and 4.2.3.2). These findings contrast with the theory of metacognition developed by Flavell (1979), Brown (1987) and Kluwe (1982) which involves the individual's set of knowledge regarding their own cognition. Brown (1987) developed this by explaining it was an awareness about the 'information processing system', with Flavell (1979) adding that this awareness encompassed three categories: person, task and strategy variables. Furthermore, metacognition was not activated in mathematics teaching as set out in the metacognition framework which includes knowledge about cognition and the regulation of cognition, with this instead being carried out based on the teacher's own conception. This highlights the need for research to be conducted in the Saudi Arabian context, which would seek to explain this absence, along with the importance of this component which is knowledge about cognition and how it can be emphasized in mathematics teaching, and more generally in educational culture. Moreover, this absence underlines the aforementioned importance of this study due to the relatively new approach of metacognition within the Saudi educational environment and its shortcomings in terms of research in this field.

As for the conception of metacognition within the context of certain skills, which can be defined as the ability to use metacognition, participants in both case studies did indicate this awareness. The findings indicated an awareness of planning, monitoring and evaluation in relation to using metacognition, with monitoring and evaluation which aim to regulate the processes in relation to the cognitive objects, being particularly emphasized (see 4.1.2.2, 4.1.3.2, 4.2.2.2 and 4.2.3.2). This is perhaps due to these aspects being implicitly given more attention during the implementation of the IMPROVE programme, which focused on the practical aspects such as metacognitive questions relating to understanding the question, solving strategy, linking previously and newly learnt information to a greater degree than the first component of metacognition. Yet this emphasis on the prominent component of metacognition falls short of the vision of Flavell, who combined monitoring and regulation when he (1979, p. 1232) referred to metacognition as "the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects". Brown (1987) followed suit by designating both as skills needed to manage cognition.

According to Nelson and Narens (1990), monitoring and regulation are considered to be to foundations of metacognition stressed (see 2.2.1.2 in the literature review chapter). Yet the data of this study did not indicate that the teachers in both case studies held a complete conception regarding these two foundations, neither in the theoretical nor applied sense - rather, their focus was on monitoring more than regulation. Hence the need for a study taking into account this subject in the Saudi Arabian educational context, particularly considering the assertions of Brown and Kluwe that regulation is a key topic in metacognition. Brown (1987, p. 95) noted, "The notion of self-regulatory mechanisms has a central place in the emergent field of metacognition", following on from Kluwe's (1982, p. 220) conviction that "the subject of metacognition is regulation of one's own information processing".

The third aspect of the findings in both case studies was the suggestion that metacognition would serve as an aid in discussing students' thinking, instead of simply evaluating problem solving methods. Other findings aligned with this, as metacognition could serve to assist both teachers in identifying issues and strengths in students' thought during mathematics class. This is consistent with Grizzle's study result (2014), in which introducing teaching for problem solving drawing on researched models, such as the IMPROVE programme, could enhance the teacher's ability to boost the progress of students and allow them to reach their full potential. Metacognition held value as an aid to thought evaluation as well, with the findings demonstrating that it could play such a role in future implementations too. This raises the question of which metacognitive skills were in evidence in this context, based on the teachers' conceptions in comparison with those noted in the literature.

Three skills that comprise the ability to use metacognition, emerged through the findings in both case studies, and these are planning, monitoring and evaluation (see 4.1.2.2, 4.1.3.2 and 4.2.3.2). This comes as expected in a wide range of studies e.g. (Corliss, 2005; Fowler, 2004; Gama, 2004; Kumar, 1998; Schraw & Brooks, 2011). These specified the key metacognitive skills as being the aforementioned three. Planning is described as being undertaken in order to create sub-goals, so as to ensure the smooth working of tasks (Winne, 1997). Monitoring is an ongoing process: monitoring thinking while engaged on a task in order to locate obstacles and improve procedure (Brown, 1987; Sigmund Tobias & Everson, 1996). Finally, evaluation works as a post-completion reflection on performance (Brown, 1987). This comes with knowledge of a large body of data which portrays orientation as a metacognitive skill (Lucangeli & Cornoldi, 1997; Lucangeli et al., 1998). Orientation can be defined as the skill of prediction, and is the determining factor in the speed at which new or complex tasks are carried out. This is due to the fact that familiar tasks can be carried out at a quicker pace, with the skill of orientation assisting the learner to contemplate purpose, features and allotted time (Garrett et al., 2006). Despite this, this study's finding in both case studies did not demonstrate the presence of orientation as a basic skill included in those pertaining to use metacognition (see 4.1.2.2, 4.1.3.2 and 4.2.3.2). Instead, another skill in the mathematics teaching/learning through metacognition of no lesser importance was discussed, this being the management of planned thought. Several participants in both case studies remarked on this skill in the context of thought methodologies (see 4.1.3.2 and 4.2.3.2). Ziyad stated, "I think that the skill of managing your solving in accordance to a thought methodology, and then evaluating that, is the most practical thing I can do in my learning". Adding to this, Fadul and Qusay also touched upon the importance of solution management in accordance with planning (see 4.1.3.2). The supporting arguments for the significance of this skill can be found in the study's literature. For example, in a scale named the Metacognitive Awareness Inventory (MAI) Schraw and Dennison (1994), as well as in Schraw et al. (2012), management strategies were discussed. This involved the assertion that skills targeted at heightened efficiency in information skills could be described as metacognitive skills.

Based on the previous sections, I summarize the four skills which comprise the ability to use metacognition related to mathematics as planning, management, monitoring and evaluation. Learners perceive that this group of skills will help them to adjust the course of their thinking. This was exemplified by the study's data, with interviewees stressing in both case studies the importance of all of these skills in learning metacognitively and how implementation must come in tandem with the four skills and cooperative work. The relationship between the metacognitive skills arose through the findings from case one (see 4.1.2.2 and 4.1.3.2). Findings showed a link between the skills of monitoring and evaluation when it came to observing student interaction and evaluating work outcomes. This link between monitoring and evaluation was confirmed by the study of Garrett et al. (2006). Their study explained that deficiencies in evaluation would result in weaknesses in monitoring, thus hindering students' ability to judge the suitability of a plan or validity of a solution. Another link that emerged through this study's findings was that between monitoring and planning, a link indicated by several student interviewees. For example, findings showed that participants were confident about monitoring their thought as long as there was prior planning involved. Indeed, the findings also showed that planning was a clear skill area, with management and monitoring being linked to one another. What emerged from the study's results was that there was a close relationship between the four skills, as evaluation requires monitoring as a precursor, which in turn cannot take place without the management of planning. Overall, these four skills are targeted at the adjustment of the learner's course of thought in mathematics learning. Studying the relationship between metacognitive skills and their role in reaping the benefits of metacognition in learning generally, and more specifically mathematics learning, is of great importance. This is because it is a practical aspect that assists the learners in transforming metacognition from theory to application. These results are consistent with those of Peña-Ayala & Cárdenas (2015), which pointed out that most models merely highlight and describe aspects, rather than discussing how they interact.

One interesting and exciting aspect of the study and one of the prominent results that emerged from it was the conception of the teachers regarding the extension of metacognition's function from the classroom to general life. This is because metacognition plays a role in formulating thought methods in a valid manner to deal with problems in life. This was evidenced from findings in both case studies which highlighted a perception of metacognition extending benefits to the student's wider life (see 4.1.2.2 and 4.2.2.2). Metacognition was perceived as being related to lifestyles to a certain extent, as it could help overcome problems in a systematic and procedural manner. Findings were consistent with one of Flavell's (1979) important studies in this field. Flavell (1979) hoped that metacognition would extend to students monitoring their thought in daily life situations. This was so they could make wise and mature decisions, similar to those they made in the classroom. In the same vein, Larkin (2010, p. 26) stated that a significant area of study on metacognition has been concerned with understanding the way in which metacognition assists in 'wise and thoughtful life decisions' as Flavell (1979, p. 910) put it. Furthermore, this is in line with Grizzle's (2014) findings, which describe how IMPROVE can boost learner's knowledge of strategies, specifically for those who are lacking in this regard. He also noted that problem solving skills are transferrable, and can be applied in the area of social skills for students dealing with real life issues. This form of student engagement in mathematics will assist them in actively engaging with the world around them. It was also suggested that strategy instruction programmes like IMPROVE could serve not only to facilitate learning but also facilitated the retention of basic information and the use of higher-order thinking skills (Grizzle-Martin, 2014).

Based on the above discussion - which dealt with the findings of the study in the Saudi educational context and in light of the theoretical framework for metacognition – this study asserted that the concept of metacognition is founded on two principles relating to thought. The first of these is an awareness of thought, and the second is regulating systematic cognitive activities which require some skills, which is the ability to use metacognition. The first principle encompasses, according to Jacobs and Paris (1987), declarative knowledge, which means knowledge about the self and strategy; procedural knowledge which involves knowledge about how to use a strategy; and conditional knowledge which relates to knowledge about when and why to use a strategy. The second principle includes the skills of planning, managing the implementation of such plans, monitoring and evaluation. The supporting arguments for this understanding of metacognition can be found in the studies of Schraw and Dennison’s study (1994) and Schraw et al. (2012). These studies highlight that previous research into metacognition has differentiated between knowledge about cognition and the regulation of cognition. Awareness of cognition involves three processes that ease the reflective area of metacognition. These are declarative, procedural and conditional knowledge. Regulating cognition also involves processes that ease the control area of learning. These involve skills that have been covered in depth: planning, information management strategy, debugging strategy and evaluation. Buratti and Allwood (2015) stated that metacognition has evolved into a multidimensional concept, with definitions and components differing greatly. Thus, suggesting a perspective of metacognition is essential as this research seeks to present a vision for dealing with this concept in the educational context of Saudi Arabia. This vision could be in mathematics learning or other subject areas particularly in light of the fact that such comprehensive conceptions for metacognition are relatively lacking in the Arab countries. Such a perspective on constituents of metacognition is consistent with Peña-Ayala and Cárdenas’ (2015) request to highlight alternative models. This stance can be categorized as the description of constituent parts or organizing the processes for metacognition. This is what leads us to shift our discussion of the other two components, which will be dealt with in the following section. These are the teaching of mathematics according to metacognition based on the implementation of the IMPROVE programme and cooperative learning and metacognition.

5.2 Metacognition, mathematics and the IMPROVE Programme

After having discussed the concept of metacognition theoretically in the first part of the discussion chapter, this section discusses the concept of metacognition practically in teaching and learning mathematics. This is done with the goal of building a comprehensive depiction of the features and characteristics of teaching and learning mathematics according to metacognition in the Saudi educational context. In order to achieve this goal, the IMPROVE programme was implemented. This was not done to study its impact on attainment or to direct students towards learning mathematics, nor to evaluate the IMPROVE programme itself or its suitability in application within mathematics teaching. Instead, it was implemented as an assistive instrument to understanding metacognition in a practical regard for mathematics learning and teaching. The need in the Saudi educational system is not based on investigating the effectiveness of IMPROVE in teaching mathematics, or in building metacognitive skills, or even in improving the attitude towards learning mathematics, as there are numerous studies demonstrating the effectiveness of this (Cetin et al., 2014; Grizzle-Martin, 2014; Mevarech and Amrany, 2008; Kramarski and Mevarech, 2003;, Kramarski and Michalsky, 2013) (see 2.5.5). Instead, there is an urgent need to understand a full conceptualization to assist in implementing metacognition in the Arab education context in general, and the Saudi one specifically, for teaching and learning mathematics (see the context and the rationale of the study 1.1 and 1.2).

In this second section of the discussion chapter, the subject of teaching and learning mathematics metacognitively grounded in the implementation of the IMPROVE programme (Mevarech & Kramarski, 1997) is discussed. The basic interdependent components of the IMPROVE programme are as follows:

> (a) Facilitating both strategy acquisition and metacognitive processes; (b) learning in cooperative teams of four students with different prior knowledge: one high, two middle, and one low-achieving student; and (c) provision of feedback-corrective-enrichment that focuses on lower and higher cognitive processes' (1997, p. 369)

The discussion will then turn to the link between learning through metacognition and the role of the students – which is represented in the processes they undertake in order to obtain new information based on pre-existing knowledge. Building on the discussion of these aspects, a basic practice model of metacognitive mathematics teaching and learning is drawn out. This will include the strategy for dealing with mathematics problems and teaching and learning steps. This model will highlight the aforementioned metacognitive skills – in doing so the components of the IMPROVE programme are cited in light of the socio-cultural context, which was also discussed in the literature of this study (see 2.5.3 and 3.2).

As a prelude to our discussion of the first component, three important issues will be pointed out. Firstly, domination of the traditional method that pervaded the reality of teaching methods in mathematics before the implementation of IMPROVE. Secondly, mathematical knowledge deficiency and metacognition. Thirdly, metacognitive mathematics teaching as a planned procedure.

5.2.1 Dominance of the traditional method in mathematics teaching prior to the implementation of the IMPROVE programme

As for the Traditional Method, the term in this context refers to the presentation of mathematical concepts in a direct manner, i.e. without linking to other concepts or explaining how such concepts really work, so the students are aware of how to imitate but they do not know why they are doing things. This is a process of rote learning. This method is not conducive to shaping mathematical thinking to deal with varying problems using differing methods in changing contexts.

Drawing on the findings in both case studies, certain methods pursued by the teachers could be described as being indirectly related to metacognition, but this was neither intentional nor systematic (see 4.1.1.1,4.1.2.3,4.2.1.1 and 4.2.2.3). One of these was a focus on understanding the problem through commencing by defining the given and demanded values of such a question. Another feature was splitting the process of solving into two parts, each dealt with independently and then later amalgamated to arrive at a final solution. An additional observation in both case studies (see 4.1.1.1 and 4.2.1.1) was that teachers did indeed present multiple solutions but stopped short of comparing them because they were interested and focused on solving the mathematics problem, rather than being concerned with monitoring and regulating the thought of the students. This is consistent with the study of Artz and Armour-Thomas (1992), who attributed student weakness in problem solving to the lack of monitoring that occurs surrounding their cognitive procedure throughout problem solving exercises. Another technique was an attempt to link between previous and new information, which was also occasionally done through the use of mind maps to illustrate connections between numerous mathematics concepts. Participating teachers in both case studies also strived to correct the errors that students made, both for individual students and to the class as a whole using the whiteboard. Yet despite this, these efforts were mostly not pursued to target metacognition. This is because metacognitive practices were often limited in their scope to the direct correction of errors in solving mathematics problems. This is consistent with the study of Artzt and Armour-Thomas (1998), in which a lack of monitoring and regulation was seen as a frequent shortcoming in the numerous mathematics classes observed in their research. Truelove (2013) conducted research into the phases of the problem solving process, and concluded that participants lacked persistence in this and neglected the reflection aspect at the end of this process. Indeed, the reality of classroom teaching and learning in the current study in both case studies is clearly dominated by the traditional method, as answers alluding to it featured extensively in the answers of participants. This usually involved the presentation of a new concept, then discussing its direct application by looking at the demanded value and the method to obtain it.

This issue extends to the students and their focus on direct problem solving in their approach to mathematics. This is in agreement with Schudmak's (2014) research, which explained that study participants (11-year-old pupils) felt that the reflective process was unfamiliar and they did not encounter it during regular school days. This comes at the cost of improvement in systematic thought when interacting with such problems. The findings demonstrated that students in both case studies were not cognisant of the term or wider concept of metacognition, nor had they perceived its presence in specific learning experiences (see 4.1.1.3 and 4.2.1.3). Adding to the focus on direct solving, students in both case studies also sought to solve problems in the quickest manner possible. The present study's findings are consistent with those of Sahin and Kendir (2013), who identified the rushed nature of problem solving among students along with the lack of time allotted for arriving at correct solutions. The findings in both case studies also indicated a tendency to depend on linking a presented solution to ones which had been previously encountered, a point of great interest to students which was often asked about (see 4.1.1.34.2.1.3). They also seemed to lack focus when tasked with thought monitoring or adjustment, and rather preferred to link current to previous solutions. This link was not of the reflective, metacognitive sort, but instead was a form of rote learning and imitation.

The findings in both case studies demonstrate that the traditional method can be considered as an obstacle to learning through metacognition (see 4.1.2.4, 4.1.3.4, 4.2.2.4 and 4.2.3.4). Participants particularly mentioned that despite awareness as to the benefits of thought evaluation, the traditional method continued to be used in their habitual and routine methods in their mathematics lessons. The findings suggest that the presence of this obstacle to learning through metacognition has two sources: firstly, the notion that the traditional method focuses on a single direct strategy for student learning, so it does not account for metacognitive knowledge in its three forms. The first of these forms is declarative knowledge, which involves knowledge about things. Procedural knowledge encompasses a method or 'how to' of doing certain things. Conditional knowledge is that which relates to the 'why' and 'when' of things – which is variable rather than constant (Mokos & Kafoussi, 2013). King (1991) cited numerous studies claiming that improving procedural knowledge has knock-on positive effects on problem solving skills. It is more probable that individuals with a more developed procedural knowledge effectively employ sequence strategies (Pressley, Borkowski, & Schneider, 1987). Furthermore, Carr (2010) suggested that declarative knowledge enhances student's comprehension of a task and the choice of strategy to solve it. In addition, declarative knowledge enhances procedural knowledge and strategy formation. Secondly, monitoring and modification are notably missing from traditional methods. This absence is unhelpful in motivating students to learn through these techniques as revealed by the findings of this study in both case studies (see 4.1.1.3 and 4.2.1.3). This is consistent with the study of Schoenfeld (1985), who advanced a theory on the interaction between cognitive and metacognitive procedures which occur during a student's mathematical problem solving process. Schoenfeld (1985) delineated four areas of knowledge and behaviour. These were sources (mathematical knowledge), heuristics (mathematical problem solving method), control (metacognition) and beliefs (attitudes). Contemporary teaching is usually weighted towards the former two areas, whereas student failure in problem solving actually pertains to the weaknesses in the two latter aspects. Hence, according to Schoenfeld (1985), while students may process the requisite mathematical knowledge, they fail to fully exploit its potential as they lack the skills of control and monitoring.

At this point, it can be added that the teacher's position as conductor of the learning process (as occurs through the traditional method of mathematics teaching) and the conveyer of knowledge served as an obstacle in observing metacognitive characteristics in learning. This was consistent with Larkin's (2006) study, which identified a lack of sufficient opportunities for students to cooperate on a higher cognitive level as a key obstacle. This was evidenced by the findings of this study, in which in both case studies steps to solve problems directly were focused on, rather than the thought methods involved in that process. Hence, the teacher-student relationship was neither participatory nor constructive but rather one in which monitoring errors was the norm. When the teacher serves as the central point of the learning process and his role does not extend beyond the delivery of information, it results in a hindered manifestation of metacognition. In a similar vein, Hurme et al. (2015) concluded that, in regards to problem solving in mathematics, student groups were neglectful of the analysis aspect. In addition, they failed to monitor and regulate workflow, which is a key component of metacognition. Therefore, neglecting analysis and verification weakened the use of metacognition and the full realization of its potential. In the event that such important areas are side-lined or ignored, previously gained knowledge is not being fully utilized to formulate and implement appropriate strategies.

5.2.2 Mathematical knowledge deficiency and metacognition

The findings of this study demonstrate that students in both case studies tended to use mathematical knowledge rather than monitoring and regulating their thinking when it came to problem solving in mathematics (see 4.1.1.4, 4.1.3.5, 4.2.1.24.2.3.5). This was consistent in several works from the study's literature. For instance, Tok (2013) pointed out that teaching is often centred on mathematical knowledge, but lacks indication of the role played by metacognition in problem solving. Yimer (2004) emphasized that failure in regulating and monitoring procedures was a key factor in an overall weak performance in mathematical problem solving, rather than simply being due to the absence of mathematical knowledge. Hence, metacognition needs to be given greater prominence in order to strengthen students' awareness of learning processes. Schoenfeld's theory (1985) of interaction between cognitive and metacognitive operation during mathematics exercises, highlighted several key aspects of knowledge and behaviour; among these were control (metacognition) and beliefs (attitudes). The failure of the students to solve problems seems to occur due to weaknesses in these two second aspects. The findings of Schoenfeld's study (1985) showed that students' weakness in the metacognitive skills of planning, mentoring and evaluation were reasons for their weakness in the subject of mathematics. Mr. Fallatah stated that the reason for many of the problems in mathematics learning for students lies in the method of thinking when dealing with mathematics concepts and problems. He claimed that students lacked thinking strategies by citing "problems at a time when many students possess only prior knowledge, which does not involve the strategy of thinking." The findings of this study demonstrate that many potential metacognitive skills were neglected by students in both case studies when it comes to problem solving in mathematics (see 4.1.1.14.1.1.34.2.1.34.2.3.3). This hinders their ability to problem-solve, a claim which is consistent with other research in the field of mathematics and metacognition (Cardelle-Elawar, 1992; Grizzle-Martin, 2014; Tok, 2013). The findings of the present study in both case studies showed that mathematics concepts in general can be better comprehended if students are able to reflect on these through metacognition (see 4.1.3.2 and 4.2.3.2). This means that students become capable of monitoring and regulating their own thought; the effectiveness of their problem solving is enhanced, as was recognized by the study of Schoenfeld (1987). Based on this idea, one could point to Hogan et al. (2015) who discussed that a key issue of contemporary education is not only enhancing area-specific knowledge but also the development of metacognitive capabilities and methods of enquiry that encourage learners to reflect on thought.

5.2.3 Metacognitive mathematics teaching as a planned procedure

When the IMPROVE programme had been implemented, the manifestation of indicators of metacognition and their extent in mathematics learning were observed. Based on the study's findings in both case studies, there were indeed many signs of metacognitive mathematics learning, as noted in the finding chapter (see 4.1.1.2, 4.1.1.4, 4.2.1.2 and 4.2.1.4). This highlights that the process of teaching mathematics metacognitively is one that should be planned and intentional, which is consistent with the assertions of literature. For example, the study of Naglieri and Johnson (2000) indicated that the provision of explicit metacognitive strategies can further enhance students' performance in mathematics – displaying the importance of planning to ensure effectiveness. Adding to this, Grizzle-Martin (2014) recommended the use of clear teaching that concentrates on cognitive and metacognitive strategies. Teachers should instruct students to monitor and subsequently control learning processes so as to assist them in gaining a more autonomous approach to problem solving (Desoete, 2007). An ability to plan solving strategies and monitor performance and the aforementioned autonomy would assist in changing that strategy if needed. In line with previous assertions, Schoenfeld (1987) added that this can occur with the provision of explicit instructions and metacognitive strategies.

The findings of this study in both case studies demonstrated the importance of metacognitive mathematics teaching as a planned and intentional process (see 4.1.2.5, 4.1.3.5, 4.2.2.54.2.3.5). To this end, the provision of a model to assist both teachers and students in achieving this type of learning is essential. This is consistent with Hartman (2001) who outlined studies surrounding the development of metacognitive practice. She summarized these as containing four main approaches, targeted at: raising general awareness through teacher-presented models, enhancing metacognitive knowledge, improving metacognitive skills, and developing learning environments. A wide body of research has suggested that teaching the use of metacognitive strategies assists students to regulate and direct themselves along with improving their performance overall (Raoofi et al., 2013). Hence, if learners are capable of discerning how they understand concepts, they are enabled to think introspectively and furthermore analyse how knowledge and its meanings are built through metacognition (Grizzle-Martin, 2014). Carr (2010) highlighted that mathematics syllabi should contain metacognitive learning as it would serve to boost the quality of learning. Moreover, Grizzle-Martin (2014) expressed the view that IMPROVE is an explicit form of teaching. This is because teachers direct and guide learners during problem solving, but eventually seek to enhance their abilities as independent learners. Adjustments could be made to the IMPROVE programme by instructing metacognitive skills through explicit training of both teachers and students. These adjustments would lend greater value and utility to the problem solving strategies. This underlines the importance of constructing a model for metacognitive mathematics teaching to assist both teachers and students in achieving this type of mathematics learning through metacognition. This will be dealt with at greater length in the following section and will be called upon when discussing the desired outcome of this study along with the conclusion of the discussion.

5.2.4 Teaching mathematics metacognitively according to the IMPROVE programme

The IMPROVE programme is centred on three basic components: facilitating both strategy acquisition and metacognitive processes; provision of feedback-corrective-enrichment that focuses on lower and higher cognitive processes; and learning in cooperative teams of fours students with differing prior knowledge (Mevarech & Kramarski, 1997). Thus, it is important to discuss these three components through the findings of the study and in light of the literature.

5.2.4.1 Facilitating both strategy acquisition and metacognitive processes

The choice of an appropriate strategy for learning through metacognition plays an important role in mathematics learning. This was displayed by the study's findings in both case studies and the strategy can be considered as a mind map (see 4.1.3.5 and 4.2.3.5). This has a role in learning as the presence of a strategy being represented as a mental map for the student in dealing with mathematics problems would assist them in monitoring and adjusting their thinking for its enhancement. The findings in both case studies point to the strategy's systematic nature in pointing out its utility as a method to monitor and adjust thought (see 4.1.3.54.2.3.5). It was also highlighted that this method helped in identifying and locating errors, before remedying them.

The IMPROVE programme is based upon the processing of new information based on pre-existing information. This is done through metacognitive questions, the formulation and answering of which are targeted at processing such information, according to Mevarech and Kramarski (1997). This is because a key feature of control and regulation is “the decisions one makes concerning when, why, and how one should explore a problem, plan a course of action, monitor one’s actions, and evaluate one’s progress.” (Lester, Garofalo & Kroll, 1989, p.1). According to Mevarech and Kramarski (1997), there is a clear case for instructing students to create questions that may result in rich and elaborate explanations. These explanations involve numerous facets, such as justifications of timing, purpose and method in using strategies and principles; inferences about the introduced concepts; and new perspectives on some aspects of the existing knowledge. Such questions are targeted at (a) the structure of the problem, (b) connections between the new and existing knowledge, and (c) specific strategies and principles that are appropriate for solving the new problem.

The IMPROVE programme presents a useful vision in the field of metacognitive mathematics teaching in this area. However, it is essential in this context that the use of a strategy is targeted at assisting students in monitoring and adjusting their thought when dealing with mathematics problems, which was underlined by the findings of this study in both case studies (see4.1.2.24.1.3.2 and 4.2.3.2). With such strategies, it is important that the suggested ones are utilized not only in problem solving but also in understanding new mathematical concepts. This was revealed by the study’s findings in case one (see 4.1.3.5) which highlighted the IMPROVE programme improved understanding and reinforced new mathematical concepts to a greater extent. Despite the importance of a clear strategy for learning mathematics - be it in problem solving or understanding new mathematical concepts - it is also essential not to limit students to a single strategy. Doing so would not be consistent with conscious reflection on the efficiency or learning for the development of metacognition (Thomas, 2012). Hence, limiting students to dealing with a single strategy in mathematics learning was seen not to help the students in both case studies (see 4.1.1.4,4.2.1.4 and 4.1.3.4) in creating and innovating with new strategies, which would enable students to develop an ability to choose the most appropriate strategies for learning concepts and solving numerous mathematics problems - the absence of which means an absence of learning through metacognition. This is consistent with Thomas's (2012) assertion that if students are not consciously reflecting on the newer tasks introduced to the classroom and the impact on learning then the development of metacognition can be questioned. However, the problem of confining students to certain strategies can be alleviated through distinguishing between a general and limited suggested strategy such as a mind map for dealing with math problems that students can be trained in, and a specific strategy to illustrate the key to solving the mathematics problem. A specific strategy which is a key to solving a mathematics problem must not be confined to a specific pattern. Therefore, the presence of a general strategy such as a mind map for dealing with mathematics problems helps in creating multiple methods and strategies for solving.

Despite Moga's (2012) claim that enhancing students' knowledge and metacognitive skills was the goal of the IMPROVE programme, an important issue that remains is coupling the training of students in a strategy to deal with mathematics problems with the aspect of metacognitive skills. This aspect was not given great prominence explicitly in the IMPROVE programme, and many researchers concentrated on the link between mathematical problem solving and the use of metacognitive skills. The literature asserts that metacognition can enhance students' problem solving skills (Fortunato et al., 1991; Kapa, 2001; Mevarech & Kramarski, 1997; Mohini & Nai, 2005). The greater the monitoring and control of strategies by the students, the greater the gain of problem solving abilities (Kapa, 2001; Mevarech & Fridkin, 2006; Schoenfeld, 1992). Hence, metacognition supports the cognitive level, through the activation of the monitoring and control functions during mathematical problem solving. Sahin and Kendir (2013) discussed the impact of this by explaining that if such skills are successfully absorbed by students, it will often enhance their ability to solve problems correctly.

5.2.4.2 Provision of feedback-corrective-enrichment that focuses on lower and higher cognitive processes

The findings demonstrated that evaluating students' thinking in dealing with mathematics problems can be considered a fundamental pillar in learning through metacognition in both case studies (see 4.1.2.5, 4.1.3.5 and 4.2.2.5). Findings pointed to a need for greater efforts in the approach to evaluation. One specific example of this was the provision of metacognitive activities, on which students could then be evaluated by the teacher in discovering their thought methods. It was also clear from the findings in both case studies that the teacher's role should focus on evaluation and supervising the lesson, rather than reverting to traditional methods of rote teaching. This is consistent with Hogan et al. (2015) who outlined that the type of feedback, while just giving the student the right answer, fails to prompt them or suggest appropriate strategies for future problems. Hence, this traditional feedback style may be insufficient in assisting the learner to monitor, adjust or even to be aware of learning strategies and their effectiveness. In contrast, prompting is targeted at directing the learner as to when and why to apply a given strategy.

Based on the findings, teachers can be better evaluators by:

- Individual student reports targeted at assessing their thought method for dealing with mathematics problems
- Displaying the group's worksheets to the class and discussing these with them
- Reviewing the worksheets, which reveals the level of cooperation occurring in a group as well as issues with their thinking
- Monitoring cooperation and interaction among students throughout cooperative problem solving tasks and evaluating outcomes
- Groups can also demonstrate their strategy and method by filming their problem solving, after which a teacher can correct their thought process.

This view concurs with that of Mutekwe (2014) who discussed the need for cognition among students undertaking tasks, but added to this by pointing out that metacognitive skills often help students to understand how tasks are performed. Therefore providing a quality feedback structure is essential, as it helps to regulate, monitor and direct students. In this regard, the findings in case one underlined the importance of evaluating students' thinking in dealing with mathematics problems from a peer (see 4.1.3.5). This is consistent with Mutekwe (2014) who mentioned rapid yet flawed feedback could be more effective than better thought-out feedback provided by an instructor at a much later stage.

This evaluation for students in their dealing with mathematics problems cannot be undertaken successfully unless there is a prominent role for the student in the process of learning through metacognition. Thus, it is difficult to create learning based on metacognition when the student’s role is limited to receiving information without participating in the search for it. A confirmation of the importance of the student’s role in learning through metacognition was underlined by the findings of this study in both case studies (see 4.1.3.5 and 4.2.3.5). They revealed that the onus was upon students as a significant aspect in implementation. After all, metacognition itself is targeted at bringing students to the centre of the learning process, and giving them the responsibility to search for information so as to create a constructive learning atmosphere. This decentralization of teaching responsibilities encapsulates the difference between the traditional and metacognitive methods, and this was highlighted by interviewees as the new approach made students think, in contrast to memorizing and solving a problem by direct application.

The role of the students in this context is represented in the processes that they undertake in gaining new knowledge based on previously learnt knowledge. This was indicated by several studies. Kramarski and Mevarech (1997), described learning as being undertaken not to record or absorb but rather to interpret information. Students who are capable of differentiating between new and old information hold greater potential in reviewing and learning the new information (Tobias & Everson, 1998). Using metacognition assists such students in learning, understanding and recognizing knowledge gained both in the classroom and in daily life (Kramarski et al., 2004), giving them greater autonomy when faced with unfamiliar situations. Kramarski and Mevarech (1997) specifically discussed mathematics, in which students also draw links between new and existing information. During this process, students determine the nature of the problem and refer to particular strategies, tactics or rules that they are already aware of and hence can associate problems with ones previously encountered. Drawing on previous information lends group learning a heightened effectiveness as the variety in students’ previous knowledge is exploited to provide a wide range of reference points and foundations for knowledge construction.

Two other important requirements emerged from the findings in both case studies regarding learning through metacognition. These fall into the context of evaluating the thought of students when dealing with mathematics problems. The requirements include the provision of sufficient time to practise and provision thorough preparation for mathematics activities – both regarding learning through metacognition. This is consistent with Sahin et al.’s (2013) research, which highlighted the importance of providing sufficient time for problem solving. They explained that students should be urged to take their time and be cautious in problem solving. Regarding long-term time allotment, Schraw and Gutierrez (2015) highlighted that programmes ranging from six weeks to several months tended to be more effective. This is because long-term programmes enable students to model, practise and automate strategies, while also enhancing conditional knowledge. Furthermore, another benefit is that instructors themselves improve their teaching and modelling of strategies over a lengthier period of time.

On the subject of practice, the findings in both case studies showed that responsiveness to metacognitive techniques improved over time, which was attributed to providing a sufficient period for their implementation and practice (see 4.1.3.5, 4.2.3.5 and 4.1.2.5). Students required much time to practise the four metacognitive skills, as well as creating solution strategies. The findings of this research are consistent with those of Grant (2014), as after intervention it was found that many students needed more time to absorb and enhance schemata after a new mathematics concept is presented to them. Some students in both case studies were initially reluctant to use the method, but after witnessing benefits such as better understanding of mathematical concepts they realized its utility. A lack of familiarity was highlighted as an obstacle, yet participants explained that this faded with greater practice, as it was absorbed into their mathematics learning ‘culture'. It was also underlined that metacognition should be maintained in practice for it to become a permanent feature in mathematics learning, rather than one applied in controlled circumstances, with one suggestion being its introduction into other subjects.

Furthermore, the findings in both case studies demonstrated the importance of metacognitive instruction becoming permanent for the teacher (see 4.1.2.54.2.2.5). It was suggested that the technique should be used in more lessons so students could reap the maximum benefit. The findings in the second case demonstrated that Mr. Hatem's partial adoption of metacognitive teaching was one of the important obstacles (see 4.2.3.4); for example, the teacher only adopted the method in the presence of the researcher, and his enthusiasm to complete the syllabus on time. Consequently the students felt sometimes that there were differing goals between the teacher and the researcher. Therefore the teacher's method did not change to a great extent. Data analysis of observations also showed that the teacher did not use this method in the understanding of new concepts, but instead exclusively used it in solving problems. Furthermore, Mr. Hatem did not deal with the reduction of difficulties to a sufficient degree, and he was the only one who spoke about these difficulties. Meanwhile, in the first case, the results demonstrated that Mr. Fallatah tried to implement the IMPROVE programme to a greater extent, and this was not limited to the occasions in which I was present. Thus, the importance of metacognitive instruction becoming permanent for the teacher was demonstrated. The study's findings indicate that there is a disparity between Mr. Fallatah and Mr. Hatem as to the extent of implementing the IMPROVE programme. This could be attributable to their differing beliefs in teaching through metacognition. For example, Mr. Fallatah emphasized this issue: "The qualified teacher in learning strategies and its theory will have the motivation to engage with metacognitive teaching". In contrast, the findings show that Mr. Hatem did not teach according to metacognition to a greater extent, due to the lack of time in which to do so. He stated "I might not be able to practise this method in [my] teaching to a sufficient degree due to the lack of adequate time provided, coupled with the pressures of the syllabus content and the necessity of finishing it so [students can] prepare for examinations at the end of term". And when Mr. Hatem was asked if he discussed students' thinking methods with them, he said "Rarely do I discuss the students' thinking method when dealing with mathematics problems, and that is because of the lack of time in a class session".

Therefore, the role of 'belief' is highlighted in teaching mathematics according to metacognition. This comes in agreement with the study of Schoenfeld, (1992). He states that 'belief', in this context, revolves around an individual's perceptions and insights, such as the ideas a person generates when doing mathematics and how this changes the manner in which he does it. This issue highlights the need for research to be conducted in the Saudi Arabian context, which would seek to explain the correlation between beliefs and metacognitive teaching and learning.

The findings highlighted that since learning through metacognition had the aim of mentoring and adjusting students' thought, the teachers in both case studies began to follow logical steps in their approach to mathematical problems solving whether in his speech or work (see4.1.1.2, 4.2.1.2 and 4.2.2.5). These steps were taken in accordance with the metacognitive questions, with these being: questions to understand the problem, the solving strategy and questions of linkage. The logical steps in the teachers' approach to mathematical problems solving was also carried out in accordance with the wider steps provided by the IMPROVE programme, and these were: presentation of the concept, metacognitive questions, practice, and reduction of difficulties. This concurs with Grizzle's (2014) view that the use of metacognitive questioning encourages students to actively contribute to the learning process and furthers their conceptual understanding.

Teachers' thorough preparation for mathematics activities is also required with metacognitive teaching. This assists in the process of evaluation for learning mathematics metacognitively – which the findings demonstrated in both case studies (see 4.1.2.54.2.2.5). Preparing suitable activities for metacognitive teaching is essential in leaving a lasting impact on students regarding the method. Syllabus content should be consistent with metacognitive teaching, which would be method rather than solution-oriented. These findings are supported by a study carried out by Simons (1996), which explained that certain features that improved intervention became clear through analysing the beneficial impact of metacognitive training. First was the formulation of tasks relevant to students' experiences both within and outside the school environment. Activities which suggest that achieving mastery in school is possible tend to encourage students to perform better, particularly in tasks overseen and evaluated by teachers or parents. This involves a suitable difficulty level, as a task too easy may negate the purpose of monitoring and regulation, as students will simply invoke routine processes. In contrast, a task too difficult may discourage low-achieving students while high performers may persevere despite failures.

In terms of the current study, dealing with such activities designed with preparation to suit metacognition calls for the implementation of the IMPROVE programme used in this context. Through the IMPROVE programme, teachers presented new concepts to the entire class. The class then, worked in groups made up of members with diverse achievement levels. Students took turns in answering metacognitive questions, which included comprehension questions, strategic questions, and connection questions. Comprehension questions encouraged students to: express the key demand of the question, categorize the problem and expand on new concepts. Strategic questions were targeted at strategies suitable for problem solving. If a unit revolves around particular mathematics principles, students should select one, justify this choice and discuss its application to the given problem. If the unit concentrates on algebra and word problems, students should be encouraged to employ diagrams and tables. Connection questions draw parallels and contrasts between the problem being faced and those previously seen.

Based on this foundation, and through the findings of the study (see 4.1.1.2, 4.1.1.4, 4.2.1.2 and 4.2.1.4), the steps to this strategy can be arranged to deal with mathematics problems in a clearer and more effective way. These steps are as follows: comprehension questions, strategic questions, justifying solution steps, verification, connection questions.

IMPROVE is an acronym for the teaching steps that are combined to form the method. These are Introducing new concepts, Metacognitive questioning, Practising, Reviewing and reducing difficulties, Obtaining mastery, Verification, and Enrichment. Indeed the findings of this study revealed through observing teaching, learning, interviews and discussions that there was an overlap between these steps. This is due to the fact that there are steps related to the student in his dealing with mathematics problems such as metacognitive questions and practice, along with other steps relating to the role of the teacher in helping students learn metacognitively, such as reducing difficulties and introducing new concepts. An example is in the metacognitive questions, which are questions to understand the problem, the solving strategy and questions of linkage. These questions are some of the most important subjects for discussion - which should occur between students in a group. This assists students of varying academic achievement to understand advanced mathematics concepts. It reveals to students the beginnings of their thought method in dealing with mathematics problems. Emphasizing the importance of metacognitive questions, this stage must be related to students in a clear manner in the implementation of IMPROVE. Elsewhere the teacher, in the step of reducing difficulties, holds great responsibility in surveying obstacles faced by work groups in their method of thinking when dealing with mathematics problems. Following this is an attempt at discussion by students regarding assisting factors in overcoming these obstacles. This highlights the importance of there being the step of difficulty reduction related to the performance of the teacher in all clarity. This study suggested steps for the teacher to implement IMPROVE in the Saudi educational context. These steps involved presenting new mathematics concepts; also supervising group work and observing the difficulties faced by students in their thought method for dealing with mathematics problems. This is important so that the difficulties can be discussed later with the students with the goal of overcoming them. Then, the presentation of corrective evaluation helps students improve their thought method for dealing with mathematical problems. This study presented also steps for student work groups, including metacognitive questions to be represented and related to understanding within discussions, categorizing and comparing the problem. Furthermore, students' finding a strategy to solve the problem and justifying each step are the steps to discovering a strategy to solve and confirming the validity of the solution. Then, the problem will be compared with others to find areas of similarity and differences.

Some of the most important aims of Kramarski and Mevarech's (1997) study included the creation of an instructional intervention plan targeted at metacognitive training, cooperative learning, and the provision of corrective/enrichment feedback that can be implemented throughout the mathematics curriculum. Hence, the following practice-based model was set out. It is founded on three basic components. The first is the steps to deal with mathematical problems, which the student should follow. The second component relates to the steps to be followed by the teacher. Finally, this model underpins the metacognitive skills that have been discussed. All of this is in light of the socio-cultural context which has been previously discussed in the methodology and literature review (see 3.2 and 2.5.3). The suggested basic practice model was demonstrated to and discussed with a group of educational supervisors and teachers in Saudi Arabia, and their opinions were considered (see Appendix 12

The themes that arose across data analysis:

Method	Participants	Themes
Thematic findings of observations data	Teachers and students from both cases	Mathematics teaching strategies related to metacognition before the implementation of the IMPROVE programme
		Mathematics teaching strategies related to metacognition within the implementation of the IMPROVE programme
		Mathematics learning strategy of the students related to metacognition before the implementation of the IMPROVE programme
		Mathematics learning strategy of the students related to metacognition within the implementation of the IMPROVE programme
		Teacher and Student relationship before the implementation of the IMPROVE programme
		Teacher and Student relationship within the implementation of the IMPROVE programme
Thematic findings of teachers' interview	Teachers from both cases	Themes
		Teacher's understanding of metacognition before the implementation of the IMPROVE programme
		Teachers' understanding of metacognition after the implementation of the IMPROVE programme
		Mathematics teaching technique before the

		implementation of the IMPROVE programme
		Challenges to the implementation of metacognition after the implementation of the IMPROVE programme
		The teacher's requirements for the implementation of metacognition
		Cooperative learning and metacognition before the implementation of the IMPROVE programme
		Metacognition and cooperative learning after the implementation of the IMPROVE programme
Thematic findings of student interviews and focus group	Students from both cases	Theme
		Students' understanding of metacognition before the implementation of the IMPROVE programme
		Students' understanding of metacognition after the implementation of the IMPROVE programme
		Learning techniques used in mathematics before the implementation of the IMPROVE programme
		Challenges to the implementation of metacognition in mathematics learning
		Students' requirements for the implementation of metacognition
		Student-student relationship before the implementation of the IMPROVE programme
		Metacognition and cooperative learning after the implementation of the IMPROVE programme

Appendix 1). In the current and previous section, the discussion agrees with the assertions of Eldar and Miedijensky (2015) and Zohar and Barzilai (2013) that educators need to comprehend the meaning of metacognition and utilize it themselves in the classroom environment. They should be able to present and explain metacognitive knowledge and practice the skills during class.

5.3 Cooperative learning and metacognition

A further aspect of the IMPROVE programme was cooperative learning in groups of four – which consisted of one student with a strong body of prior knowledge, two with average knowledge and one at the lower end of the spectrum (Mevarech & Kramarski, 1997). This final aspect was discussed along with the need for cooperative learning in an environment suitable for learning through metacognition.

Moga's (2012) study indicated that the link between metacognition and cooperative learning had not been given sufficient attention. Thus, the relationships between metacognition and cooperative learning needs more research. In order to become acquainted with the nature of cooperative learning as practised by the mathematics teachers and their students who participated in this study, it is important to discuss the nature of the teacher-student relationship. Furthermore, it is essential to explore the student-student relationships as seen prior to the implementation of the IMPROVE programme so as to discuss the features of progress in this context after the programme's implementation. Moreover, the extent of the connection between the concept of cooperative learning and metacognition will be discussed in this section.

5.3.1 Pre-IMPROVE Teacher-Student Interaction

The findings of the current study in both case studies underlined the presence of the required skills among participating teachers for cooperative learning strategy (see 4.1.1.5, 4.1.2.6, 4.1.3.6, 4.2.1.5, 4.2.2.6 and 4.2.3.6). Such skills included commitment and discipline to timing, group distribution, activity management, presentation of concepts, mathematics problem solving and the correction of student errors. Adding to this, the teachers in both case studies excelled at communicating with students about issues in class and were open to their suggestions. The findings revealed an openness to new and unorthodox methods, with teachers in both case studies encouraging quality over quantity of solutions. Specifically, this meant a preference for multiple solutions of the same problem, rather than the solution of a greater number of problems. However, it cannot be said that the teacher-student relationship was entirely conducive to the implementation of IMPROVE. This is because the learning in both case studies largely revolved around the direct delivery of mathematical concepts. This came in contrast to a more suitable participatory atmosphere targeting knowledge construction as well as the necessary adjustment of students' thought in dealing with such problems. This also involved the monitoring and highlighting of errors made by students in their problem-solving, leading to tension in the relationship. This tension originated from several sources, one of which may have been the overbearing nature of such supervision, or the haste with which concepts were delivered and problems solved, which only served to confuse students. It would be unfair to hold teachers solely responsible for the existence of time constraints – they are required to complete all the units in the curriculum by the end of term – regardless of whether or not extra time is needed to employ metacognition. Hence, the teacher's position in both case studies as conductor of the learning process and the conveyer of knowledge served as an obstacle in observing metacognitive characteristics in learning. This was consistent with Larkin's (2006) study, which identified a lack of sufficient opportunities for students to cooperate on a higher cognitive level as a key obstacle.

5.3.2 Pre-IMPROVE Students' Interactions

Within small groups consisting of five or six students, the findings displayed that students' group interaction in both case studies focused on mutual correction of solutions rather than individual ways of thinking used to arrive at such solutions (see 4.1.2.6, 4.1.3.6, 4.2.2.6 and 4.2.3.6). It is important to discuss the extent to which student-student communication can facilitate metacognition. The findings demonstrated that communication skills in both case studies were generally weak in participants hampering the interaction required for cooperative learning, let alone that which would be needed for productive learning through metacognition. This confirms the importance of communication for effective group work, as explained by Larkin (2006). She explained that communication skills such as listening, contribution and sharing were enhanced by collaborative work. Furthermore she stated that such working arrangements would impact on students' individual ways of thinking.

The findings demonstrated various reasons for a weak cooperative environment (see 4.1.2.6, 4.1.3.6, 4.2.2.64.2.3.6). Firstly, a reluctance on the part of several students to participate in group work, either about individual ways of thinking or learning in general. This stems from a number of beliefs held by some students, one of which was the perceived difficulty of expressing their method of thinking. Another was the perception that thought methods were a personal matter that did not require expression. A further reason was that students felt insecure about revealing their errors in front of others as it would demonstrate their weakness in the subject of mathematics, which they felt would have a negative impact on their self-confidence. There were some students who did not accept criticisms from others who were weaker academically. Furthermore, many did not see the benefit of criticisms as their solutions rather than methods were being scrutinized, which they correctly pointed out was a form of ready knowledge rather than a constructive comment.

The second important reason for the reluctance of students to discuss was a weakness in communication skills, which was clarified in this study as participants did not contribute in a way that demonstrated the presence of such skills. One of the manifestations of this weakness was a shyness to participate, which was raised by the findings of this study. This could be due to the lack of students' familiarity with presenting ideas and discussing solutions; this undoubtedly hindered the evaluation of their thought and perhaps the presentation of a full and clear picture of their thought process.

Another reason for weakness in this regard was the nature of the activities presented to the students. The findings showed that activities were overly simplified in their steps to solving and explicit as to the ideas behind them. Some problems did not even require any form of cooperation as they lacked features that might stimulate thought. Participants were often able to distinguish between problems that required group work and those that didn't. According to them, this depended on the nature of the problem and its difficulty. Another reason for poor communication may have been weak academic achievement in mathematics. Students with low levels either felt embarrassed to participate or may have been discouraged from doing so by other students and instead preferred to leave the process to students with greater capabilities.

5.3.3 Post-IMPROVE teacher-student Interaction

In the early stages of IMPROVE's implementation, it was naturally difficult to perceive a significant shift in the teacher-student relationship in both case studies (see 4.1.1.6 and 4.2.1.6). Both teachers continued to dominate much of the discussion, notably so in confronting obstacles faced by students. Further along in the programme's implementation, discussions became more systematic and interactive. This is due to the fact that learning methods themselves took on these same characteristics. Teachers began to discuss problems in greater depth and detail, which encompassed the reduction of difficulties, checking solving methods and comparison of problems. This was targeted at the development of student awareness of thinking and building confidence in their abilities to learn through metacognition. Yet findings continued to demonstrate the significance of student commitment, discipline and listening skills in order to obtain the desired results of IMPROVE's implementation.

The findings in both case studies displayed an improvement in the participation of students in the learning process after the programme's implementation. Another finding of this study was that the reasons for this progress lie in the intended preparation of activities presented to students (see 4.1.2.5 and 4.2.2.5). A second reason was distributing the groups in a manner that encouraged cooperative learning (see 4.1.3.44.1.3.7, 4.2.3.4 and 4.2.3.7) .The findings showed that preparation of activities was essential in motivating students to work cooperatively and metacognitively, which is in line with the study of Larkin (2006). Her study highlighted that the task itself was crucial to the success of collaborative group work. The findings present some characteristics of such activities, such as employing indirect solutions and previous experience while containing new concepts and challenging students. Such activities push students to engage head-on with the subject, its teacher and each other. The worksheets presented were designed in line with the IMPROVE programme (see 5.2.3 in the discussion chapter).

The findings highlighted the importance of the make up of the small groups for learning through metacognition. This involved dispersing the more confident and skilful communicators into separate groups, so as to initiate discussions. Such students often also held leadership skills, and if they did not others were sought out – this was so that group work could be managed and participation by every group member encouraged. Meanwhile, seating arrangements were a significant aspect in offsetting the domination of more vocal students, as they were organized in a more inclusive manner so as to prevent the dominance of a single student. It was crucial that group members had diverse academic achievement levels, hence encouraging conference and exchange of ideas allowing for a quality discussion. In addition, it was preferable that such groups did not exceed four students so as to allow a greater share of speaking time to individuals.

5.3.4 After the implementation of the IMPROVE programme

The findings in both case studies displayed that metacognition and cooperative learning were closely intertwined (see 4.1.1.6, 4.1.2.7, 4.1.3.7, 4.2.1.6, 4.2.2.7 and 4.2.3.7). Cooperative learning is crucial in bringing about a suitable environment for learning through metacognition, as students are made capable of monitoring and evaluating each other's method of thinking at close quarters in the mathematics classroom. This is consistent with a number of studies, such as Desoete (2007); Kramarski and Mevarech (2003). These studies affirmed that cooperative learning seemed to be an effective way to further the impact of metacognitive instruction. In that context, students placed in cooperative groupings during training sessions showed greater development in their metacognitive skills than those being trained individually. In the present study, the findings in both case studies also highlighted that success in cooperative learning can be attributed to the utilization of work maps, which are of significant assistance when problem solving and communicating with other students. Specifically, they helped to shift the group's centre of gravity from a dominant outstanding student to one which was more equally dispersed.

On the other hand, the findings in both case studies showed that metacognition assisted cooperative learning, and hence the relationship between the two is one of mutual benefit as metacognition contributed a more organized thought method, relating back to the use of work maps. This helped in administrating group dialogues in an effective and useful manner. This characteristic was noted in Moga's (2012) research, in which the significance of metacognition in cooperative learning was described as lying in its capacity to harness the capabilities of stronger students in a constructive manner. More specifically the aforementioned study explained that students with better developed metacognitive abilities would hold greater awareness of learning requirements and hence could contribute more in cooperative groups.

As cooperative learning combined with metacognition bestows students with the central role in the learning process, they are tasked with knowledge construction, which enhances their ability to solve mathematical problems. This was consistent with Mokos and Kafoussi's (2013) study which claimed that students' performance in mathematical problem solving was boosted by working in small groups. This was due to the fact that such arrangements created a socially interactive atmosphere which was grounded in metacognitive questioning for a more systematic and structured process.

The fact that cooperative learning can serve as an aid to learning through metacognition was stressed by a participant in both case studies who detailed specific methods which could be used to maximise the benefits of this combination (see 4.1.2.7, 4.1.3.7, 4.2.2.7 and 4.2.3.7). The first of these was the use of multiple solutions, with groups collectively evaluating the solutions of other groups, enhancing the thought process. Another method discussed was that of self-correction, after which students would present their errors and amendments in front of classmates, providing a window into their thought process and allowing other students to reflect on this. The strategy of comparing and contrasting solution was seen as critical, be it within a group or with other groups, the teacher and the textbook. This enhances introspection and allows the learner to discern his errors rather than being told them. This is supported by Moga's (2012) research which suggested that students should note down their solutions, discuss with a classmate and subsequently present it to the class. The benefit of this stems from an obligation to discuss ways of thinking, reflecting on their position and expressing their opinion. Therefore, students can simultaneously evaluate themselves and gain knowledge from their classmates. It would also allow the teacher to evaluate students collectively, checking for true understanding by examining the confidence with which they communicated and presented. Mr. Hatem hoped for a period of time longer than the academic year so that students could be fully acclimatized to learning with metacognition (see4.2.2.7). Not only did it allow for the teacher to evaluate all the students collectively, it also relieved the teachers of the need to explain all the material, and hence their roles became more supervisory and corrective than explanatory. This is consistent with Larkin's (2006) study, which mentioned that teachers would initially be the key motivators of group work. However, as time progressed teachers were able to withdraw more, and rather than driving group work would gently guide the group, thus allowing for greater awareness of thought among students.

The findings of this study demonstrated these conclusions in several ways (see 4.1.3.5 and 4.2.3.5). Participants themselves in both case studies alluded to the significant shift in responsibilities that comes as a consequence of metacognitive practice. They explained that students shifting to the centre of the educational process stimulated a search for knowledge or an intellectual curiosity. It also developed students' thinking abilities and bestowed them with the necessary tools to evaluate themselves, particularly in a way pertaining to thought. These impacts were heightened among the more serious students. This does not mean to say that the teacher's role in learning through metacognition diminishes; rather, it is reformulated to transform from one which merely transfers knowledge to one which constructs it. It transforms to one targeted at assisting and enabling students to assess their way of thinking in order to improve it in their learning of mathematics. This confirms the importance of the teacher's role in the cooperative context and is in line with the study of Mokos and Kafoussi (2013). In this regard, the study's findings clarified that the educational context in Saudi Arabia was lacking when it came to adopting cooperative learning, as well as metacognition (see 4.1.2.4 and 4.2.2.4). This explains to us the difficulty that teachers and students face in adopting mathematics learning through metacognition. This context is represented by the school's administration and in the educational supervisors within the wider education system. Each of these bodies will now be discussed in detail.

5.3.5 Socio-cultural aspects of metacognition

Metacognition was an entirely new concept being adopted into the education system in Saudi Arabia, hence there was no material or advice to guide its implementation. Teachers in both case studies had not taken the initiative to inform themselves about metacognition and thus there had been no previous attempts to implement it. An additional cause of complaint by participants was the ever-present pressure to complete the syllabus prior to exam season. This of course served as an obstacle in the IMPROVE programme. The findings portray the possibility of a future partnership between educational authorities and research centres which could serve to create a suitable environment for the future implementation of metacognition. The creation of agencies tasked with innovating new teaching methods was also suggested by Mr. Fallatah (see 4.1.2.4). The teacher would then be able to communicate with such bodies to enhance their performance in teaching with the method. Therefore, the Ministry of Education in Saudi Arabia should supervise the provision of training courses for all teachers. The teachers surveyed wanted more assistance in defining their roles, which they felt would help them in metacognitive teaching successfully. Alnesyan's (2012) finding supports this, and indicated that the socio-cultural environment in Saudi Arabia influenced teachers' ability to develop thinking skills among students. Yet a general lack of societal awareness overbears potential changes to the system, as genuine belief in the potential of the method is yet to be created. There were calls by both teachers (see 4.1.2.44.2.2.4) for the establishment of school-based or partnered media to spread awareness not only to students but society as a whole.

With regards to the educational supervision, a key obstacle identified was the lack of criteria for assessing the implementation of metacognitive issues. Instead, it was noted that such supervision tended to focus on more perfunctory matters, where again, syllabus completion served as the omnipresent benchmark for teaching performance. Supervisors should enter the educational context armed with the necessary knowledge and experience of metacognition and its implementation, so as to encourage teachers to implement the method, rather than assessing them on it. The findings showed that this may allow for the emergence of a teaching culture surrounding metacognition, rather than there being a set of criteria with which teachers are evaluated.

The school principal plays an important role in the adoption of new teaching techniques. This was demonstrated by findings, as neglect towards the adoption of metacognition was identified as a key obstacle to the implementation of the method. It is crucial that such individuals are fully convinced and committed to the promotion of new methods. If this can be achieved, methods such as metacognition can then be passed on to the teachers for implementation. Therefore if school administration is disinterested or sceptical, this will greatly hinder the implementation of metacognitive teaching. In my study, this was because of a preoccupation with academic achievement and timely completion of the curriculum. Thus IMPROVE was not taken into consideration and teachers were expected to fulfil the same obligations as they were prior to the study. Due to this distraction and the school vying for prestige, teachers were overburdened with extra duties, such as preparing students for municipal and regional competitions. Consequently, a supportive principal is a key ingredient to any future success of the method. A principal can support teachers though a variety of means, such as reducing the number of students in classes where the method would be applied. Furthermore, material and professional incentives along with advocating teacher attendance at conferences could go a long way in ensuring the success of future implementations. This is supported by Alghamdi’s (2012) study in the Saudi context, which suggested that teachers should be supported by the school principal and the educational supervision, which would allow them to self-evaluate their fulfilment of the required skills for their future metacognitive teaching.

Based on this, metacognition can be assisted through the creation of a suitable socio-cultural context. This study served to clarify this point, and agrees with the study of Sandi-Urena, Cooper, and Stevens (2012). This work stressed that examining the effects of social interaction on learning could benefit metacognition and problem solving. These researchers used qualitative analysis to investigate the enhancement of metacognition in contexts that are already well-developed in terms of social skills such as reflective discussion, verbalization, thinking aloud, group planning, monitoring and evaluating.

These premises clarify the importance of creating an educational context that encourages social interaction in learning. This has a role in motivating the establishment of metacognition, as the absence of this social interaction would impede this type of learning. This was set out in the findings and was consistent with the conclusions of Larkin (2006). Her study underlined the fact that metacognition is susceptible to change though social persuasion, explaining that just as metacognition is adaptable it can also be the opposite. Larkin (2006) went on to state the following:

> Unless students are given the opportunities to interact with others at a substantive cognitive level it may be difficult for them to practice or elaborate on metacognitive strategies or to gain feedback about their own cognitive processing. (p. 25)

This is an important result of this section of the discussion. It is complimented by the understanding the subject of metacognition both in theory and practice in teaching mathematics in the educational context of the Kingdom of Saudi Arabia. This was summarized in the conclusion of this thesis.

6 Conclusion

This chapter presents an overview of the study by explaining its aims and how these were realized. It presents research limitations, in addition to the implications of the findings for teachers, students, the educational supervision and the school administration and policy-makers, possibly encouraging teaching and learning through metacognition in Saudi Arabia. The chapter concludes with suggestions for further studies in the field of metacognition.

6.1 Overview of the study

On the basis of many factors informing the literature of metacognition, it is important to move beyond the mere assessment of metacognition and its effects on achievement towards concentrating on the quality of metacognition education. It is important, as well, to focus on cognitive processes more than final outcomes, in order to enhance learning performance. I believe that creating an optimal classroom atmosphere is one of the main challenges which faces educational systems in many countries. Thus, it is important to reformulate teaching and learning in the classroom to become more metacognitive in order to improve the quality of mathematics teaching and learning in Saudi Arabia. Moreover, it can be noted that most research findings regarding metacognition have emerged from the western world. Therefore, presenting a different perspective, focusing on Saudi Arabia, a country with a different background and culture, provides the Saudi education system or perhaps other Middle Eastern countries with an important contribution to the literature of metacognition within mathematics learning. To meet these needs it was important to take four essential elements into account: the concept of metacognition and its components as provided by Flavell (1979), Brown (1987), and Kluwe (1982); the main recommendations regarding metacognition and mathematics (see 2.5.6); the socio-cultural theory as the foundation of this study (see 3.2) which in turn adapts to the Saudi educational context; and certain practical frameworks of metacognition in relation to mathematics, in this case the IMPROVE programme (Mevarech and Kramarski, 1997). In this regard, the study aimed to explore teachers' and students' perceptions of metacognition in relation to mathematics teaching and learning in secondary schools in Saudi Arabia. Consequently, this study sought to respond to three questions:

1) How do secondary students and their teachers perceive metacognition in mathematics teaching and learning?

2) What, if any, indications of metacognition can be observed in the mathematics teaching and learning classroom?

3) What are the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics before and after the implementation of the IMPROVE programme, regardless of improvements in specific strategy or any boost students' achievement?

Since the study aimed to explore teachers' and students' perceptions of metacognition in relation to mathematics teaching and learning in secondary schools in Saudi Arabia, the data were not gathered through tests and surveys but rather through gaining participants' perspectives through interviews, focus groups, and observation.

Answering the research questions enabled me to present a vision for dealing with metacognition in the educational context of Saudi Arabia in mathematics teaching and learning. It also enabled me to determine practice for activating this kind of teaching and learning, and to understand the degree of difficulty of performing this in the mathematics classroom. Accordingly, the perspective of metacognition presented by this study and the suggested basic practice model of metacognitive mathematics learning in the Saudi Arabian context are set out after the implementation of the IMPROVE programme.

In terms of the understanding of metacognition presented by this study, it was conceived on the premise that metacognition was founded on two concepts relating to thought: one's knowledge, and the monitoring and control of one's own systematic cognitive activity. The former encompasses declarative knowledge, which entails knowledge about the self and strategy; procedural knowledge which involves knowledge about how to use a strategy; and conditional knowledge which relates to knowledge about when and why to use a strategy. The latter involves monitoring and regulating systematic cognitive activities which require some skills, which is the ability to use metacognition, such as planning, managing the implementation of such plans, monitoring and evaluation.

With respect to the suggested basic practice model of metacognitive mathematics teaching and learning, it is based on three basic components. The first is the steps to deal with mathematical problems, which the student should follow. The second is the steps to implement IMPROVE, which the teacher should follow. In addition, this model underpins the metacognitive skills such as planning, managing the implementation of such plans, monitoring and evaluation. Based on this, the steps required of the teacher to implement IMPROVE in the Saudi educational context involve presenting new mathematics concepts. Moreover, supervising group work and observing the difficulties faced by students in their thought methods used for dealing with mathematics problems is another aspect. This is significant, as it allows the difficulties to be discussed at a later stage with the students, with the goal of overcoming these difficulties. Then, the presentation of corrective evaluation assists students to improve their thought method for dealing with mathematical problems. As for student work groups, metacognitive questions are represented and related to understanding, categorizing and comparing the problem within group discussions. Furthermore, students find a strategy to solve the problem and justify each step to discovering this strategy, and then confirm the validity of the solution. Then, the problem is compared with others to find areas of similarity and difference. Thus, it is difficult to create learning based on metacognition when the student's role is limited to receiving information without participating in the search for it. In this regard, the study asserts that metacognition can be enhanced through the creation of a suitable socio-cultural context that encourages the social interaction represented in cooperative learning and the importance of the student's role in learning through metacognition. This is consistent with Larkin's (2010) assertion that a theory of metacognition which includes reflection and self-criticism encourages individuals to discuss education, considers the needs of specific groups in specific contexts, and allows for introspection on issues such as the student-teacher relationship, would be a theory that can be employed in order to build a more socially representative education establishment.

The findings of this project suggest that the understanding metacognition both in theory and practice in teaching mathematics is complemented by creating an educational context that encourages social interaction in learning. This has a role in motivating the establishment of metacognition, as the absence of this social interaction impedes this type of learning.

The findings suggest that the traditional method of teaching mathematics can be considered as an obstacle to mathematics learning through metacognition. The teacher's position as conductor of the learning process and the conveyor of knowledge serve as an obstacle in observing metacognitive characteristics in learning. Hence, the teacher's role in implementing metacognitive teaching should be adjusted, in order to reinforce students' consciousness of the learning processes. Secondly, despite that the process of teaching mathematics metacognitively is one that needs to be planned and intentional, it is essential in this context that the use of a strategy is targeted at assisting students in monitoring and adjusting their thought when dealing with mathematics problems. This, according to Larkin (2010), poses a challenge to theorizing metacognition, as tasks which were previously conscious acts may become automated and no longer within the realm of conscious thought and voluntary control. In this regard, it should be emphasized that conscious reflection on the efficiency of learning is essential for the development of metacognition, which can be questioned if students are not consciously reflecting on the newer tasks introduced to the classroom, and the subsequent impact on their learning (Thomas, 2012). In addition, limiting students to dealing with a single strategy in mathematics learning does not help students in creating and innovating with new strategies. Conscious reflection enables students to develop an ability to choose the most appropriate strategies for learning concepts and solving numerous mathematics problems - the absence of which ability suggests an absence of learning through metacognition. Thirdly, evaluating students' thinking in dealing with mathematics problems can be considered a fundamental pillar in learning through metacognition. Hence, a need for greater efforts in the approach to evaluation is evident. The findings underline as well the importance of evaluating students' thinking in dealing with mathematics problems from a peer. This evaluation cannot be undertaken successfully unless there is a prominent role for the student in the process of learning through metacognition. Two other important requirements can be added in the context of evaluating the thought of students when dealing with mathematics problems: the provision of sufficient time to practice, and of thorough preparation for mathematics activities.

6.2 Research Limitations

There are three limitations in this study. Firstly, there exists a limitation of position: the position of the current study is in a secondary school in Saudi Arabia. Secondly, there is a limitation of time: the study was conducted during the second term of the 2014-2015 study year. Thirdly, there is a limitation of subject domain: the subject domain of this study is a focus on the perceptions of teachers and students towards metacognition in relation to mathematics teaching and learning in secondary schools in Saudi Arabia. This domain subject was researched theoretically according to three models of metacognition which were presented by Flavell (1979), Kluwe (1982) and Brown (1987), and practically according to the IMPROVE programme, which was created by Mevarech and Kramarski (1997).

6.3 Implications of the study

The study aimed to explore teachers' and students' perceptions of metacognition in relation to mathematics teaching and learning in secondary schools in Saudi Arabia. Consequently, this study sought to respond to three questions. Firstly, how do secondary students and their teachers perceive metacognition in mathematics teaching and learning? Secondly, what, if any, indications of metacognition can be observed in the mathematics teaching and learning classroom? Thirdly, what are the experiences of secondary students and their teachers in Saudi Arabia of metacognition in relation to mathematics before and after the implementation of the IMPROVE programme, regardless of improvements in specific strategy or any boost to students' achievement?

Accordingly, this study sought to identify what is lacking in both mathematics learning and teaching in the classroom regarding metacognition in the educational context of Saudi Arabia. How does using metacognition play a central role in mathematics learning and teaching, and why? What are the main facilitating factors and difficulties experienced by students and teachers wishing to improve their mathematical performance through metacognition? What are the characteristics that seemed to enhance the positive effects of the interventions that emerged from analysing the beneficial effects of the metacognitive training with students? The current study sought to explain the nature of the relationship between cooperative learning and an improvement in metacognition in the mathematics classroom. This kind of research is notably absent in the educational context of Saudi Arabia. Thus, the theoretical and practical significance of conducting the current study stems from its potential contributions to the following aspects:

6.3.1 Implications for mathematics teachers

The study's findings suggest that the mathematics teacher should allocate more time for serving as a role model to students in learning through metacognition. Furthermore, it sought to benefit teachers in developing their mastery of metacognitive teaching in mathematics. The study presents the suggested implications for the practice of metacognitive mathematics learning, and is based on three basic components. One of these components is the strategy which the teacher should follow when their students deal with mathematical problems. Based on this, the steps required of the teacher in implementing IMPROVE in the Saudi educational context involve the presentation of new mathematics concepts. Moreover the teacher should supervise group work and observe the difficulties faced by students in their thought method for dealing with mathematics problems. This is important so that the difficulties can be discussed later with the students with the goal of overcoming these difficulties. Then, the presentation of corrective evaluation assists students in improving their thought method for dealing with mathematical problems.

Based on the study's findings, several issues are mentioned regarding teachers' needs and requirements for metacognition to be successfully implemented. The clarity of the concept of metacognition in the eyes of the teacher is vital. The ability of teachers to individually and collectively evaluate a method of thinking in problem-solving is required to implement metacognitive teaching in mathematics. It is necessary to prepare activities that are appropriate for metacognitive teaching. These activities should involve indirect solutions, previous experience, new ideas, and should be challenging. The readiness of the teacher to teach metacognitively is an important factor in the implementation of metacognitive teaching in mathematics. Consistency of metacognitive teaching is important in order for students to benefit to the greatest extent. In addition, it is important to deal with students' low achievement levels carefully. This is because low achievers cannot sufficiently participate with their classmates in discussion and working towards the solution to the problems and also because mathematics requires previously gained knowledge.

6.3.2 Implications for students

The study's findings indicate the importance of students becoming more aware of learning through metacognitive perception. This is in order to provide such students with a more conducive creative atmosphere, empower them to freely express their ideas and solutions without any embarrassment, and to prompt the four main skills in metacognition: planning, management, monitoring and evaluation. These skills in turn help students to improve their performance in the mathematics classroom. The study suggests the basic practice model of metacognitive mathematics learning, which includes the steps which the student should follow to deal with mathematical problems. In student work groups, metacognitive questions are represented and related to understanding, categorizing and comparing the problem within group discussions. Secondly, students find a strategy to solve the problem and then justify each decision; these are the steps to discovering a strategy to solve and confirming the validity of the solution. Thirdly, the problem is compared with others to find areas of similarity and difference.

One of the major requirements highlighted is for the role of the student in searching for and building knowledge, rather than simply receiving knowledge by the method of rote learning. There is also a need for students to have varied work maps for dealing with mathematics problems. These enable them to monitor their thinking and help in its adjustment and improvement. The need for metacognitive mathematics learning to include understanding of new concepts in addition to problem-solving is outlined. Practising and training for metacognition is necessary in order for students to benefit from learning through metacognition. The implication is that the student should continue practising metacognition so that it is part of his culture, and not simply an application. A further need is for students to have a role in evaluating their method of thinking, with this being done using a mental work map for dealing with mathematical problems.

6.3.3 Implications for educational supervision and the school administration

The current study has implications for the support of the educational supervision and the school administration. One aspect of this is the need for more time for teachers to serve as role models for students in learning through metacognition. A key obstacle identified is the lack of criteria for assessing the implementation of metacognitive issues. Instead, it was noted that such supervision tends to focus on more perfunctory matters, where again, syllabus completion serves as the omnipresent benchmark for teaching performance. Supervisors should enter the educational context armed with the necessary knowledge and experience of metacognition and its implementation, so as to encourage teachers to implement the method. The findings show that this may allow for the emergence of a teaching culture surrounding metacognition.

In terms of the school administration, the study’s findings indicate that the school principal plays an important role in the adoption of new teaching techniques. Neglect by the principal towards the adoption of metacognition was identified as a key obstacle to the implementation of the method. It is crucial that such individuals are fully convinced and committed to the promotion of new methods. If this can be achieved, methods such as metacognition can then be passed on to the teachers for implementation. Therefore if the school administration is disinterested or sceptical, this will greatly hinder the implementation of metacognitive teaching. In this study, this was because of a preoccupation with academic achievement and timely completion of the curriculum. Due to this distraction and the school vying for prestige, teachers were overburdened with extra duties, such as preparing students for municipal and regional competitions. Consequently, a supportive principal is a key ingredient to any future success of the method. A principal can support teachers though a variety of means, such as reducing the number of students in classes where the method would be applied. Furthermore, material and professional incentives along with advocating teacher attendance at conferences could go a long way in ensuring the success of future implementations.

6.3.4 Implications for policy-makers

This study has implications on the investigation of additional procedures targeted at the enhancement of metacognition's application in mathematics, seeking to alert policy-makers to this. The study clarifies the importance of creating an educational context that encourages social interaction in learning. This has a role in motivating the establishment of metacognition, as the absence of this social interaction would impede this type of learning. Unfortunately, the study's findings clarify that the educational context in Saudi Arabia is lacking when it comes to adopting cooperative learning, as well as metacognition. This explains to us the difficulty that teachers and students face in adopting mathematics learning through metacognition. This context is represented by the school's administration and the educational supervisors within the wider education system.

Based on the study's findings, there is an absence of preparation and training for teaching through metacognition, be it at university or during a teacher's service in education. This is considered to be one of the challenges confronting instruction though metacognition. The findings portray the possibility of a future partnership between educational authorities and research centres, which could serve to create a suitable environment for future implementation of metacognition. The creation of agencies tasked with innovating new teaching methods is also suggested. The teacher would then be able to communicate with such bodies to enhance their performance in teaching with the method. Yet a general lack of societal awareness makes potential changes to the system difficult, as genuine belief in the potential of the method is yet to be created. There are calls for the establishment of school-based or partnered media to build awareness not only to students but to society as a whole.

6.4 Suggestions for future research

It is hoped that this research will serve as a motivating factor and may inspire further studies into metacognition and mathematics learning in Saudi Arabia.

The findings highlight that the participants' conceptions surrounding metacognition lacked a comprehensive vision of the concept, due to an absence of the individuals' metacognitive knowledge. Furthermore, it was not activated in mathematics teaching as was set out in the framework, with this instead being carried out based on the teachers' own conceptions. This highlights the need for research to be conducted in the Saudi Arabian context, which would seek to explain this absence, along with the importance of this component and how it can be emphasized in mathematics teaching, and more generally in educational culture.

Based on the literature (see 2.2.1.2), monitoring and regulation are considered to be two foundations of metacognition. Yet the data of this study does not indicate that the teachers hold a complete conception regarding these two foundations, neither in the theoretical nor applied sense - rather, their focus is on monitoring more than regulation. Hence there is a need for a study taking into account this subject in the Saudi Arabian educational context, particularly considering the assertions of Brown (1987) and Kluwe (1982) that regulation is a key topic in metacognition

The findings highlight the importance of the four skills of metacognition related to mathematics learning as being planning, management, monitoring and evaluation. This group of skills is what enables and assists the learner to implement adjustment to their course of thought. What emerges from the study's results is that there is a close relationship between the four skills, as evaluation requires monitoring as a precursor, which in turn cannot take place without the management of planning. Overall, these four skills are targeted at the adjustment of the learner's course of thought in mathematics learning. The findings show a link between the skills of monitoring and evaluation when it comes to observing student interaction and evaluating work outcomes. This study explains that deficiencies in evaluation result in weaknesses in monitoring, thus hindering students' ability to judge the suitability of a plan or validity of a solution. Another link that emerges through this study's findings is of that between monitoring and planning. For example, findings show that participants were confident about monitoring their thought as long as there was prior planning involved. Indeed, the findings also show that planning is a clear skill area, with management and monitoring being linked to one another. Studying the relationship between metacognitive skills and their role in reaping the benefits of metacognition in learning generally, and more specifically mathematics learning, is of great importance. This is because it is a practical aspect that assists the learners in transforming metacognition from theory to application.

The study presents a practice-based model of metacognitive mathematics learning, which was founded on three basic components. The first is the steps to deal with mathematical problems, which the student should follow, along with the steps to be followed by the teacher. Secondly, this model underpins the metacognitive skills that have been discussed. Thirdly, all of this is in light of the socio-cultural context which is discussed in the study's literature. Yet there is a pressing need for further research to determine the impact of using the practice-based model on performance in mathematics classes. Such studies could analyse other variables, such as motivation and attitude in mathematics.

An interesting and exciting aspect of this study and one of its prominent results is the conception of the teachers and students regarding the extension of metacognition's function from the classroom to general life. This is because metacognition plays a role in formulating thought methods in a valid manner to deal with problems in life. Findings are consistent with one of Flavell's (1979) important studies in this field. Flavell (1979) hoped that metacognition would extend to students monitoring their thought in daily life situations. This was so they could make wise and mature decisions, similar to those they made in the classroom. Hence the need for a study taking into account this subject is of great relevance.

The study's findings clarify the importance of creating an educational context that encourages social interaction in learning. This has a role in motivating the establishment of metacognition, as the absence of this social interaction impedes this type of learning. In addition, the study's findings clarify that the educational context in Saudi Arabia is lacking when it comes to adopting cooperative learning, as well as metacognition. This explains to us the difficulty that teachers and students face in adopting mathematics learning through metacognition. A research study could be implemented to explore the reasons behind this lack. Such a study would be useful for the Ministry of Education in Saudi Arabia as that body is responsible for the planning and organizing of professional procedures of education development.

6.5 Final remarks

This research study provided me with the opportunity to advance an understanding for approaching metacognition within the educational context of Saudi Arabia, both in the theoretical and practical domains. After conducting the IMPROVE programme, a basic practice model and potential implications of metacognitive mathematics teaching and learning are suggested. This is bolstered by encouraging a learning environment where interactions (manifesting in cooperative learning) and the centrality of the students' role in learning are encouraged. This has a positive influence on the establishment of metacognition, as an environment without the social interactive aspect would act as an obstacle. It would also be challenging to create metacognitive learning if a student does not play an active role in the search for information and merely receives it in a ready form.

Several findings are drawn from the data, the first of these being that the traditional method can hinder mathematics learning through metacognition. In this regard, it can also be said that the teacher's role as conveyor of knowledge is an obstacle in itself to observing metacognitive characteristics in learning. Therefore, metacognition should be given priority in order to improve students' consciousness of the learning processes.

A second finding is that although metacognitive mathematics instruction should be planned, the strategy that is introduced should be directly targeted at improving the monitoring and regulation of students' thought when dealing with mathematics problems. Larkin (2010) identified this as a challenge in the ongoing theorization of metacognition, as some tasks may become routine and automated and their categorization of conscious thought and voluntary control could be called into question. Yet the advantages of this process remain clear as it makes a greater proportion of memory available for conscious tasks. If students fail to reflect on unfamiliar tasks, then this has a negative influence on learning efficiency and the wider development of metacognition (Thomas, 2012).

At the conclusion of this journey of learning and research, my knowledge and perspective of the subject of research has undergone drastic change – it has expanded in size and depth. My beliefs surrounding the importance of creating a learning atmosphere suitable for metacognition have transformed into a strongly held conviction. I now feel it is crucial to expand our focus outside of the teaching and learning processes, and to take an all-encompassing view of the classroom features that facilitate the use of metacognition; by doing this we can work to build and foster such environments. A key part of this will be enhancing the knowledge and attitudes of those involved, to avoid obstacles in creating this environment. These efforts should bring about a strong approach to metacognitive mathematics education in the Saudi learning environment, and will influence the creation of thought processes that can assist students in life's general challenges.

Appendix 1

Teachers' semi-structured interview schedule (before piloting)

In this interview I would like to concentrate on the concept of metacognition. I am interested in your thoughts and opinions on this, it is not a test to see how much you know about the theory. I am particularly interested in examples and stories you may have from your own classroom.

With your permission I would like to tape record the interview, because it is more accurate than taking notes. The comments you make will be in total confidence and your name or the name of your school will not appear on any typed transcripts. I hope to be able to provide you with a summary report of my research when all the data has been analysed and written up.

Before we begin is there anything you would like to ask me about the interview procedure itself?

How do mathematics teachers in Saudi Arabia perceive metacognition?

1. What is your perception about metacognition (thinking about thinking)?
2. What kind of preparation related to metacognition did you experience during your study at university/college?
3. Have you had in-service training/short courses/workshops about metacognition after you became a teacher? Do you feel it is important to be trained in this? Why?
4. What do you think are the most important aspects of metacognition to emphasise in the mathematics classroom?
5. Have you any stories or examples of metacognitive teaching from students in mathematics?
6. Could you give me examples of the questions you ask to facilitate metacognition?
7. How important do you think metacognitive teaching is in mathematics?
8. Do you think development of students' metacognition is reflected in determining your actions in the classroom? Why?

Application of metacognitive teaching of mathematics:

9. How do you see the importance of metacognitive strategies for mathematics teaching?
10. What metacognitive strategies do you use to teach mathematics?
11. What obstacles do you find when you use metacognitive strategy to teach mathematics?
12. What do you think about your students' reaction to the metacognitive strategy that you have used in classroom?

13. How can you encourage students to generate multiple methods before he decides which is the best one to solve a mathematics problem?
14. How can you encourage student to self-evaluate their performance in problem-solving?
15. How can you encourage a student to monitor his thinking when he solves mathematic problem?
16. Do you try to support students to identify their errors in the thought process when they make mistakes during the problem solving process? How?
17. What are some ways you help your students to be metacognitive learner?
18. How do you know when metacognitive learning is occurring in your classroom?
19. Do you discuss with your students about how they think when they learn mathematics?
20. Do you discuss with your students about their difficulties in thinking while learning mathematics?
21. What do you feel when you are teaching metacognitively?
    6)

What are the perceptions towards the encouragement of secondary teachers regarding metacognition in mathematics?

22. Do you motivate yourself to become a metacognitive teacher? Why / why not?
23. What do you think about the relationship between beliefs and metacognitive mathematics teaching?
24. How are your beliefs related to your perception of metacognitive mathematics teaching?
25. Do you see any external factors that may have a link with your perception of metacognitive teaching in mathematics?
26. Are there any local cultural factors that may play an important part in teaching mathematics metacognitively?
27. What are the key aspects within the school environment that play a role in metacognitive teaching of mathematics?
28. Do you think student's parents can play an essential part in the metacognitive learning of mathematics?
29. What do you think should be the primary role of educational authorities in supervising the metacognitive teaching of mathematics?
30. What do you think the authorities should do to make it easy for you to apply metacognition in your classroom instruction?
31. In general, what methods should be pursued to facilitate metacognitive teaching?

What are the main perceived challenges facing secondary teachers in implementing a metacognitive approach to mathematics?

32. What do you perceive as the main obstacles to metacognitive teaching?

33. What are the obstacles to the application of metacognitive teaching in the mathematics classroom?
34. How do you deal with these obstacles?
35. What do you think of the attitude of the student population to metacognition? Do you consider the student population as an obstacle? Why?
36. What are the most serious issues surrounding school facilities that create barriers when you teach mathematics metacognitively?
37. In general, what other obstacles might make metacognitive teaching of mathematics difficult?

Thank you very much for agreeing to the interview.

Appendix 2

Teachers' semi-structured interview schedule

In this interview I would like to concentrate on the concept of metacognition. I am interested in your thoughts and opinions on this, it is not a test to see how much you know about the theory. I am particularly interested in examples and stories you may have from your own classroom.
With your permission I would like to tape record the interview, because it is more accurate than taking notes. The comments you make will be in total confidence and your name or the name of your school will not appear on any typed transcripts. I hope to be able to provide you with a summary report of my research when all the data has been analysed and written up.
Before we begin is there anything you would like to ask me about the interview procedure itself?

How do mathematics teachers in Saudi Arabia perceive metacognition?

38. What is your perception about metacognition (thinking about thinking)?
39. What kind of preparation related to metacognition did you experience during your study at university/college?
40. Have you had in-service training/short courses/workshops about metacognition after you became a teacher? Do you feel it is important to be trained in this? Why?
41. Have you any stories or examples of metacognitive teaching from students in mathematics?
42. Could you give me examples of the questions you ask to facilitate metacognition?
43. How important do you think metacognitive teaching is in mathematics?
44. Do you think development of students' metacognition is reflected in determining your actions in the classroom? Why?

Application of metacognitive teaching of mathematics

1. What metacognitive strategies do you use to teach mathematics?
2. What do you think about your students' reaction to the metacognitive strategy that you have used in classroom?
3. How can you encourage students to generate multiple methods before he decides which is the best one to solve a mathematics problem?
4. How can you encourage a student to monitor his thinking when he solves mathematic problem?
5. Do you try to support students to identify their errors in the thought process when they make mistakes during the problem solving process? How?
6. What are some ways you help your students to be metacognitive learner?
7. How do you know when metacognitive learning is occurring in your classroom?
8. Do you discuss with your students about how they think when they learn mathematics?
9. Do you discuss with your students about their difficulties in thinking while learning mathematics?
10. What do you feel when you are teaching metacognitively?

What are the perceptions towards the encouragement of secondary teachers regarding metacognition in mathematics?

1. Do you motivate yourself to become a metacognitive teacher? Why / why not?
2. What do you think about the relationship between beliefs and metacognitive mathematics teaching?
3. Do you see any external factors that may have a link with your perception of metacognitive teaching in mathematics?
4. What are the key aspects within the school environment that play a role in metacognitive teaching of mathematics?
5. Do you think student's parents can play an essential part in the metacognitive learning of mathematics?
6. What do you think should be the primary role of educational authorities in supervising the metacognitive teaching of mathematics?
7. What do you think the authorities should do to make it easy for you to apply metacognition in your classroom instruction?
8. In general, what methods should be pursued to facilitate metacognitive teaching?

What are the main perceived challenges facing secondary teachers in implementing a metacognitive approach to mathematics?

1. What are the obstacles to the application of metacognitive teaching in the mathematics classroom in terms of teacher?
2. What are the obstacles to the application of metacognitive teaching in the mathematics classroom in terms of student?
3. What are the obstacles to the application of metacognitive teaching in the mathematics classroom in terms of school environment?
4. What are the most serious issues outside the school that create barriers when you teach mathematics metacognitively?
5. What do you think of the attitude of the student population to metacognition? Do you consider the student population as an obstacle? Why?
6. How do you deal with these obstacles?
7. In general, what other obstacles might make metacognitive teaching of mathematics difficult?

Thank you very much for agreeing to the interview.

Appendix 3

Students' semi-structured interview schedule

In this students' semi-structured interview, I am interested in what you think about metacognitive learning. It is not a test to see how much you know about the theory. I am particularly interested in examples and stories you may have from your own classroom. So if you do not understand any questions please ask me to clarify them. You do not have to answer the questions but please tell me if you do not want to answer any specific question.
With your permission I would like to tape record this event because it is more accurate than taking notes. I hope to be able to provide you with a summary report of my research when all the data has been analysed and written up.
Before we begin is there anything you would like to ask me about the interview procedure itself.

How do mathematics students in Saudi Arabia perceive metacognition?

1. Have you heard of metacognition (thinking about thinking)?
2. Have you ever been taught or learnt metacognitively? What is that like?
3. What do you think about metacognitive learning?
4. How do you find metacognitive learning?
5. Do you motivate yourself to become a metacognitive learner? Why and how?
6. Do you like talking about your ideas/thinking with the other students in mathematics class? Why? How can you activate this strategy?
7. What are the advantages/disadvantages of your friends correcting your mistakes in mathematics class?

Application of metacognitive learning of mathematics:

8. How can you monitor your thinking when you solve mathematics problems? Is there any example?
9. Do you generate multiple ideas before you deciding on the best one to solve a mathematic problem? Why?
10. How can you deal with using strategy while you learn mathematics? (Do you find yourself thinking about the usefulness of strategies while you studied? Do you summarize (put into my own words) what you've learned after you finish? Do you create your own examples to make information more meaningful? Do you draw pictures or diagrams to help yourself understand while learning? What do you ask yourself when you learn mathematics)?
11. Do you self-evaluate your performance? Why and how?
12. Do you try to identify errors in thinking when you make mistakes during the problem solving process? How?
13. Do you discuss with your partner how you think when you learn mathematics? Why? (e.g. how you think about different ways of learning and how well you are learning mathematics).
14. Does your teacher discuss with you about thinking while learning mathematics in order to become a metacognitive learner? Is there any example?
15. Does your teacher discuss with you about how to evaluate difficulties and weaknesses when learning mathematics? Is there any example?
16. Does your teacher encourage you to try new ways of learning mathematics? Is there any example?

What are the main perceived opportunities and challenges in encouraging a metacognitive approach to mathematics?

17. How was the interaction with the strategy that your teacher used today?
18. How difficult were the mathematics problems you saw today?
19. How well did you plan/monitor/manage/evaluate your thinking processes of the problems you solved?
20. Which metacognitive skill you found easy to do? And which one you found difficult? Why?
21. Did you find it useful to learn/teach mathematics metacognitively? Why?
22. Can you tell me how your experience of using metacognitive skills now was?
23. What more could be done to facilitate metacognitive learning?
24. What are the obstacles in terms of teacher to learning metacognitively?
25. What are the obstacles in terms of student to learning metacognitively?
26. What are the obstacles in terms of school environment to learning metacognitively?
27. How do you deal with these obstacles?

In general:

28. Is there anything you want to add?
29. Is there anything you want to ask me?

Thank you very much for agreeing to the interview.

Appendix 4

Students' semi-structured Focus group schedule

In this students' semi-structured focus group, I am interested in what you think about metacognitive learning. It is not a test to see how much you know about the theory. I am particularly interested in examples and stories you may have from your own classroom. So if you do not understand any questions please ask me to clarify them. You do not have to answer the questions but please tell me if you do not want to answer any specific question.
With your permission I would like to tape record this event because it is more accurate than taking notes. I hope to be able to provide you with a summary report of my research when all the data has been analysed and written up.
Before we begin is there anything you would like to ask me about the interview procedure itself.

1. How did you find the teaching strategy used today?
2. How difficult were the mathematics problems you saw today?
3. How well did you plan/monitor/manage/evaluate your thinking processes when dealing with the problems?
4. Which metacognitive skill did you find easy to use? And which one did you find difficult? Why?
5. If you can, please complete this sentence regarding metacognitive skills: "Today I learnt how to …."?
6. Did you find it useful to learn mathematics metacognitively? Why?
7. Did you find difficulty with any particular component of the thought process today? Which one? Why?
8. What more could be done to facilitate metacognitive learning?
9. What are the obstacles to learning metacognitively?
10. How would you deal with these obstacles?
11. Do you want to say anything else? Do you have any other comments?
12. Is there anything you want to ask me?

Thank you very much for agreeing to the group discussion.

Appendix 5

Teachers' semi-structured interview schedule

Mr. Fallatah second interview 23/04/2014

How do mathematics teachers in Saudi Arabia perceive metacognition?

1. **What is your perception about metacognition (thinking about thinking)?**

The work was done within logical steps, in order to resolve problems. Four skills were used to carry out these logical steps; these were planning, administration, evaluation and monitoring.

2. **What kind of preparation related to metacognition did you experience during your study at university/college?**

No.

3. **Have you had in-service training/short courses/workshops about metacognition after you became a teacher? Do you feel it is important to be trained in this? Why?**

There is training involved with active & cooperative learning. As for 'thinking about thinking' there is no pre-established training program to follow.

4. **Have you any stories or examples of metacognitive teaching from students in mathematics?**
5. **What do you think are the most important aspects of metacognition to emphasise in the mathematics classroom?**

As far as I was concerned as a teacher, it was a fascinating experience, as it broke with traditional routine, as well as the boredom that often occurs in classrooms. There was a feeling of renewal in my teaching. It was an excellent experience for the students, with the reason for this being that they were put in the position to obtain information for themselves.

6. **Could you give me examples of the questions you ask to facilitate metacognition?**
7. **How important do you think metacognitive teaching is in mathematics?**

As for this method in itself, the division of students into varying educational achievement groups, proved to be valuable in aiding cooperation. Another important element of the method is the existence of a 'Thinking Map', which in turn aided students in time management for dealing with solving problems. Thus, I think after seeing this method, a strong connection existed between metacognition and cooperative learning.

The importance of metacognition in teaching is great, due to it being in accordance with modern theory which seeks to make students the main convenors of the education process, it being them who search for information in order to encourage constructive learning. This method supports students' thinking and their abilities which enables the student to evaluate their thinking.

8. **Do you think development of students' metacognition is reflected in determining your actions in the classroom? Why?**

The activities should involve indirect solutions, previous experience, hold new ideas, and should be challenging. This encourages students to interact more with the subject, the teacher, and among themselves. Every student should try to present what would help the other group members with solving problems.

Application of metacognitive teaching of mathematics:

1. **What metacognitive strategies do you use to teach mathematics?**

We applied the method, and achieved good results; however we are still in need of greater efforts in deal with the issue of evaluation and estimation. In addition to this, we were in need of the embodiment of the concept of thought monitoring. Despite the fact that we are now ready to present a clear conception to the teacher, as to how he can implement this theory in the classroom.

2. **What do you think about your students' reaction to the metacognitive strategy that you have used in classroom?**

At the beginning of the process, there were concerns but in the end, when the students practiced this theory, it turned out well. They began to feel the benefits and their optimism greater, and thus, as soon as I told them that the method will be applied next lesson, they appeared to be happy, and with that most of the students seemed to be optimistic. In the other hand, there were two reasons for the lack of enthusiasm from some students.

1) Their lack of belief in the benefits of using metacognitive thinking, whether it was in their academic achievement or on future studies

2) The nature of the students, if they were outstanding students (in terms of grades), this could affect their enthusiasm for cooperative learning with others of lesser ability.

In addition, in the beginning I was concerned about the reaction of some students and the difficulties they might have found, however implementing it smoothly with the provision of incentives is better. I also thing that implementing it at a young age is preferable.

3. **How can you encourage a student to monitor his thinking when he solves mathematic problem?**

This can be done by introducing an ideal example approach to deal with the problem at the same time, trying to highlight the skill of monitoring.

A large amount of practice for the four Skills strengthens this aspect.

The step of Solution Strategy is what highlights the answer to this question, and here we can implement brainstorm strategy.

Displaying the solutions of the different groups to the class is important.

4. **Do you try to support students to identify their errors in the thought process when they make mistakes during the problem solving process? How?**

Most of our efforts involved the solution and obtaining results.
However, taking the worksheets back and looking over them, reveals to the teachers many aspects of the groups cooperative harmony. It also reveals a lot about flaws in thinking in a certain manner.
Yet, unfortunately, until now, we were not able to focus on evaluating these aspects, as our focus was placed on evaluating the results to their problem solving.
We were not able to free ourselves of the old methods, despite our knowledge of many important aspects in the subject of evaluating thinking in itself.

5. **What are some ways you help your students to be metacognitive learner?**

This method provides students with experience in dealing with maths, problems at a time where many students' poses only previously learnt knowledge which does not involve the strategy of thinking.
I seek to present new concepts with this method.
I will also pay attention to evaluation, be it in review of each groups work or displaying group worksheets and the discussion of students within the groups. Also the display of outstanding examples and the use of a camera for documentation is very important.

6. **How do you know when metacognitive learning is occurring in your classroom?**

This is done by monitoring the students' cooperation and interaction when problem solving and with reviewing worksheets and evaluating outcomes of the work.

7. **Do you discuss with your students about how they think when they learn mathematics?**

This method helped me in doing this and that is what I hope for in the future.

8. **Do you discuss with your students about their difficulties they face in their thinking when they deal with mathematics? Tell me more please?**

The students have the ability, but we are who undermine their potential with using traditional styles of teaching, as well as our focus on grades from tests.
I believe that students of weak academic achievement are the ones who benefit least from this teaching style, the reason for this being a smaller body of previously learned knowledge and experiences to draw on. This issue could be specific to mathematics, because it requires these skills, as opposed to other subjects. Thus I view students of average achievement to be benefiting the most from these skills.

9. **What do you feel when you are teaching metacognitively?**

We began to a certain process of change in making our efforts for the students greater, these efforts being targeted at obtaining knowledge. This is what happened with me with the last two implementations, and it is what I had hoped for to begin with. I felt satisfied when I saw the benefit for the students. Despite the greater efforts.

What are the perceptions towards the encouragement of secondary teachers regarding metacognition in mathematics?

1. **Do you motivate yourself to become a metacognitive teacher? Why / why not?**

Definitely, after the experience, I gained a great desire to teach with this method alongside the methods which we already work with. This is because the experience proved that students felt they had a role in the education process despite there being difficulties in the content.

2. **What do you think about the relationship between beliefs and metacognitive mathematics teaching?**

I am convinced that after implementing this method, I really found that I had the motivation to teach with it, as this conviction was very influential to me in teaching. However, more generally, the education of the teacher is an influential factor in accepting this teaching method in the first place. Thus, the educated and well-versed teacher in learning strategies and its theory will have the motivation to engage with this method.

3. **Do you see any external factors that may have a link with your perception of metacognitive teaching in mathematics?**

7) The principal or supervisor's evaluation of a teacher does not include any criteria pertaining to the application of this method, instead, there is focus on how much scheduled material one has completed.

4. **What are the key aspects within the school environment that play a role in metacognitive teaching of mathematics?**

If an environment that gives importance to methods such as these does not exist, there will be inaction on the part of the teacher to research, inform himself and try the method. Therefore, some teachers need to see in front of them the positive results of implementing the method in order for them to interact with it positively.

5. **Do you think student's parents can play an essential part in the metacognitive learning of mathematics?**

In this age group, the role of parents in generally absent, so I suggest that teachers guide books for these matters be sent to them so that they can help their children.

6. **What do you think should be the primary role of educational authorities in supervising the metacognitive teaching of mathematics?**

It has a very important role in promoting methods such as these, because this is what convinces the principal or educational supervisor to potentially convey the method to the teachers.

7. **What do you think the authorities should do to make it easy for you to apply metacognition in your classroom instruction?**

- We could choose a teacher in each school to be trained, after which he would undertake the role of trainer / instructor within his school
- Provision of sources which display the subject
- Provision of a documentation camera
- The supervision must be keen to train and develop teachers in the field, not only in the context of training.

8. **In general, what methods should be pursued to facilitate metacognitive teaching?**

The most important thing is to create a class environment. This could involve reducing the number of students in the class to facilitate the issue of collective and individual evaluation. Another feature could be the provision of a camera to document the class, the provision of resources to better illustrate the method, and books containing activities which are compatible with the method (enriched books to support the curriculum).

What are the main perceived challenges facing secondary teachers in implementing a metacognitive approach to mathematics?

8. **What are the obstacles to the application of metacognitive teaching in the mathematics classroom in terms of teacher?**

The greatest difficulty is, the typicality that we have become accustomed to over a long time, requiring courses and sources. I have definitely seen that with the teaching of this method comes various big differences to the reality of teaching mathematics here. This is because this method has created something new, and thus we need it to be applied in the best possible manner to provide training courses for teachers.

9. **What are the obstacles to the application of metacognitive teaching in the mathematics classroom in terms of student?**

8) First of all, a low academic achievement of the student in maths is one of them, specifically because they cannot participate with their classmates in the discussion and working to the solution to the problems and also because mathematics requires previously gained knowledge.
Secondly, a student's failing to realize the importance of this method, with their thinking and focus instead being on obtaining good grades in tests.
Thirdly, some students feel that their own methods are more beneficial to them, and get them good grades, and thus there is no need for them to try other ways.

10. **What are the obstacles to the application of metacognitive teaching in the mathematics classroom in terms of school environment?**

Equipped classrooms, an appropriate number of students for the size of the classroom, internet, sound and video recording equipment and computers, data show, optical cameras, and in general a complete re-equipping of classrooms.

The school administration being unconvinced because its focus is on the direct academic attainment of students and completion of curricula.

11. **What are the most serious issues surrounding school facilities that create barriers when you teach mathematics metacognitively?**

- General lack of previous adoption of methods such as this in education is one obstacle. In addition, lack of pursuit of the question of how we can implement this method in reality
- Absence of partnership with research centres supporting and activating an educational environment in the school
- There is a deadlock in the discussion and dialogue surrounding the practical development of education among the public in general and more specifically among teachers.

12. **What do you think of the attitude of the student population to metacognition? Do you consider the student population as an obstacle? Why?**

13. **How do you deal with these obstacles?**

In terms of teachers: there must be specific agencies for new methods in teaching which the teacher can communicate with to develop his performance in teaching fundamentally and in application. For there to be additional incentives for teachers who apply such methods.

In terms of Students: Weak students must be taken into account when implementing a method like this in order for the school to examine how it can deal with them to address this weakness.

14. **In general, what other obstacles might make metacognitive teaching of mathematics difficult?**

An important aspect is for the teacher to feel and see the benefit for the students, this pushes the teacher to continue in methods such as this and to focus on developing thinking.

Thank you very much for agreeing to the interview.

Appendix 6

Observation schedule

Date: / / 2014 **Case study:** (…….)

Context: Metacognition in Mathematics classroom. **Page number**: (……)

Time	Input	Observations	Reflection
	T		
	S		
MK: Metacognitive Knowledge, MS: Metacognitive Skills, MST: Metacognitive Strategy, MQ: Metacognitive Question, SSI: Student to Student Interaction, TSI: Teacher Student Interaction.			

Appendix 7

The class observation

Date: 10/02/2014 **Case study:** (2) Context**:** Metacognition in Mathematics classroom.

Time	Input	Observations	General notes
	T	•The teacher presents a 'concepts map' to give a conceptualization of the lesson and to distinguish between cases (MST). •The teacher asks one of the groups to define with accuracy the given task and what is demanded with it (MQ). He tries to clarify to the students how they can infer a solution regardless of the output (MST). •The teachers asks one of the groups to identify a mistake that they made. •The teacher reminds the students of a common mistake, which is not writing the mathematics rule until it is substituted in the correct manner. •The teacher stresses to the students that getting knowledge without constructional effort (rote-learnt knowledge/ready-made knowledge) (MK), does not develop the students' level, and that understanding the method of access to knowledge is the most important thing. •An activity in the book presents two solutions for the same problem, and asks students to identify which of the two is correct (MST). The teacher focuses on this exercise (25/44)	•The importance of there being a leader for each group means that communication skills are improved. There should be prior coordination with leaders and for them to have incentives for the work (SSI). •There is a difference between criticizing an individual's idea and criticizing his way of thinking. •It is important for there to be activities to be prepared in advance that present numerous correct solutions. •There is nothing that clearly confirms the existence of a step of validating a solution.
	S	The students make relate and link the new problem to the previous one, they then solve it. Student 1: In our solution, there is something incorrect (MS). Student 2: Yes, that is true, therefore we have deviated from the correct solution (MS). Student 1 discovers the error, and indeed the correct solution is arrived at. Student 1 compares his solution with that of his classmate (SSI), and explains to him the steps of his process. Student 2 notes that the solution of is classmate is in fact correct (SSI), despite the fact that his teacher said there was an error in it (TSI) The students define what is given or what is missing (MQ) and try to complete the data in order to arrive at the solution Students are clearly making links and parallels with similar, previously given problems to solve the current one.	
MK: Metacognitive Knowledge, MS: Metacognitive Skills, MST: Metacognitive Strategy, MQ: Metacognitive Question, SSI: Student to Student Interaction, TSI: Teacher Student Interaction.			

The class observation

Date: 24/2/2014 **Case study:** (1) **Context:** Metacognition in Mathematics classroom.

Time	Input	Observations	General notes
	T	The new concept was presented with the aid of a PowerPoint Presentation and activities were directly presented to implement the new concept The teacher presents a worksheet containing math's problems suited to the metacognitive method (MST) and sufficient time was allocated. Commitment to the IMPROVE steps was requested from the students (MST). After each group had attempted to solve the problems, the teacher explained the solution on the whiteboard and the students discussed the solution and its explanation. The most prominent difficulties that students faced in their problem solving – these were identified by overseeing the work of each group. The greatest difficulty in this exercise was in the solution strategy (the main key to the solution) (MK). The activity presented on this occasion did not involve verbal activity, but it did involve a new idea. The teacher presents another activity to verify the students' understanding. He gives less time for finding solutions, then chooses one of the groups to present its solution (MST). He then choses another group, which had a different solution (MST), this was done so students could compare between the two solving methods (MQ).	The teacher was more administrative of the shared time. The students were also more active, and their administration was better. The concept of planning and the administration of planning were distinct in this application (MS) but the monitoring and evaluation aspects were not part of the required task (MS). In this application, one of the difficulties which students found was represented in the solution strategy. Also, the teacher did not present corrections for the metacognitive method and limited his evaluations to the steps of problem solving (MS).
	S	The student asks when the teacher is presenting the new concept 'does this concept mean that...'. The teacher then explains the students idea and states it is correct. The student asks 'Can I solve the question by...' (MST). The teacher establishes that the student's method is correct (TSI). Example conversation from one of the groups: Student 1: What is the problem? (MQ) Student 2: The problem here is that... Student 1: Under what category can we put it? (MQ) Student 2: Inner product Student 3: Let us try and do a rough drawing to understand the problem Student 1: This is good Student 3: What if we also created something else … Student 2: I think that this will not help us to solve the problem The teacher intervenes and asks a question	

		(TSI), its answer is considered a key to solving the problem, but the students cannot solve it, because they lacked important previously learnt knowledge. Thus, they tried to find another way to solve the problem. Another example of the students' conversation 1: What is required? (Understanding the problem) (MQ) 2: Required is... (Understanding the problem) 3: What is the category of this problem? (MQ) (Understanding the problem) 1: Finding an angle, therefore we must use the angle rules (solution strategy) (MQ). 2: Correct, so we need to write the relationship between the sides (MQ) 1: Yes, and then we must find the value of the angle, at which point we can find the final answer The students created a solution.	
MK: Metacognitive Knowledge, MS: Metacognitive Skills, MST: Metacognitive Strategy, MQ: Metacognitive Question, SSI: Student to Student Interaction, TSI: Teacher Student Interaction.			

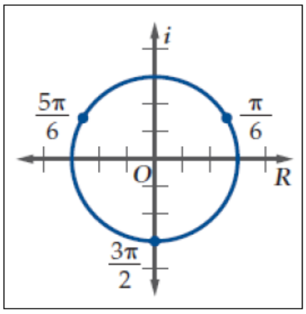
Appendix 8

Worksheet based on the IMPROVE programme

المجموعة (Group):	(Worksheet:7) ورقة عمل	مقرر
.1 .2 .3 .4	عنوان الدرس (Lesson title) : الاحداثيات القطبية والأعداد المركبة The polar coordinates and the complex number	الرياضيات 6 Math 6

أوجد الجذور المحددة على الشكل المقابل وعبر عنها بالصورة القطبية ، ثم عين العدد المركب الذي له هذه الجذور :

Find the roots which are determined on the next figure in the polar coordinate, then find out the complex number which has the following roots



1) وصف المشكلة : Understanding and categorizing the problem

[تحديد المعطيات والمطلوب ، وتصنيف المسألة تحت أي درس]

المعطيات Given / مستوى قطبي ممثل به ثلاثة نقاط هي :

Polar coordinate represented in three points

$\left(3, \frac{\pi}{6}\right)$, $\left(3, \frac{5\pi}{6}\right)$, $\left(3, \frac{3\pi}{2}\right)$

المطلوب Demand / التعبير عن الأعداد الثلاث بالصورة القطبية .

Express the three roots in the polar coordinates

/ تعيين العدد المركب الذي له هذه الجذور الثلاث .

Determine the complex number which has the three roots

تصنيف المشكلة : Categorizing the problem

This problem was classified under of the complex number and De Moivre's Theorem

2) استراتيجية الحل (A strategy to solve the problem)

ا) استخدام الصيغة الأساسية للعدد المركب في الصورة القطبية $r(\cos\theta + i\sin\theta)$ للتعبير عن الأعداد

ب) استخدام قوانين ضرب الأعداد المركبة وقوى الأعداد المركبة (نظرية ديموافر) لإيجاد العدد المعطاة جذوره .

Use the rules of complex number multiplication and the power of the complex number (De Moivre's Theorem) to get the number which has the roots

3) الحل : (the solution)

التبرير :justifying each	الخطوة :(Steps of solution)
الصورة القطبية للعدد المركب	العدد الأول $\left(3, \frac{\pi}{6}\right)$ بالصورة القطبية يساوي : $3(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$ In the complex number form is
الصورة القطبية للعدد المركب	العدد الثاني $\left(3, \frac{5\pi}{6}\right)$ بالصورة القطبية يساوي : $3(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$
الصورة القطبية للعدد المركب The complex number in the polar coordinate	العدد الثالث $\left(3, \frac{3\pi}{2}\right)$ بالصورة القطبية يساوي : $3(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})$
	لإيجاد العدد المطلوب والذي جذوره الأعداد السابقة نرفع كل عدد منها للأس الثالث : By using the power of three, we get
	$\left[3(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})\right]^3 = 3^3(\cos 3\times\frac{\pi}{6} + i\sin 3\times\frac{\pi}{6})$
	$= 27(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$
	$= 27(0 + i(-1)) = -27i$
	إذا العدد الذي جذوره الأعداد المعطاة هو : $-27i$

4) التحقق من صحة الحل (Confirming the validity of the solution)

للتحقق من صحة الحل نأخذ عدد آخر ونرفعه للأس الثالث

$$\left[3(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})\right]^3 = 3^3(\cos 3\times\frac{3\pi}{2} + i\sin 3\times\frac{3\pi}{2})$$

$$= 27(\cos\frac{9\pi}{2} + i\sin\frac{9\pi}{2})$$

$$= 27(0 + i(-1)) = -27i$$

5) مقارنة المسألة بمسائل أخرى :Comparison this problem with others

6) تلخيص ابرز الصعوبات في المسألة :What are the most obstacles for this problem?

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أوجد العدد المركب الذي أحد جذوره التكعيبية العدد $-1+\sqrt{3}\,i$ Find the complex number, which has a cubic root? $-1+\sqrt{3}\,i$

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Appendix 9

Ethical approval from the Graduate School of Education at the University of Exeter.

Certificate of ethical research approval

MSc, PhD, EdD & DEdPsych theses

To activate this certificate you need to first sign it yourself, and then have it signed by your supervisor and finally by the Chair of the School's Ethics Committee.

For further information on ethical educational research access the guidelines on the BERA web site: http://www.bera.ac.uk/publications and view the School's Policy online.

READ THIS FORM CAREFULLY AND THEN COMPLETE IT ON YOUR COMPUTER (the form will expand to contain the text you enter). **DO NOT COMPLETE BY HAND**

Your name: Khalid Saleh Alzahrani

Your student no: 590043284

Return address for this certificate: 30 Richmond Court EX4 3RA

Degree/Programme of Study: PhD

Project Supervisor(s): Dr. Shirley Larkin and Dr. Nigel Skinner

Your email address: ksaa201@exeter.ac.uk and kz111@hotmail.com

Tel: 07788488038 and in Saudi Arabia 00966555720059

I hereby certify that I will abide by the details given overleaf and that I undertake in my thesis to respect the dignity and privacy of those participating in this research.

I confirm that if my research should change radically, I will complete a further form.

Signed:..date:.........13/12/2013...........

Chair of the School's Ethics Committee
updated: March 2013

Certificate of ethical research approval

TITLE OF RESEARCH PROJECT:

An exploration of teachers' and students' perceptions of metacognition in relation to mathematics teaching and learning in secondary schools in Saudi Arabia

1. Brief description of your research project:

Many researchers assert that children are having difficulties with mathematics because they are ignoring a wide range of cognitive or metacognitive strategies. Thus, they claim that metacognition plays a central role, which will ultimately affect the student's academic performance at school generally and their mathematical performance specifically. The effectiveness of a problem-solving process will improve when students are able to monitor and control their own learning processes. On the basis of these premises, this study will explore students' and teachers' perceptions of metacognition in mathematics classes by gaining a deeper understanding of the particular research situation. Consequently, this study will present how the sociocultural approach is essential in exploring interactions among different participants in a sociocultural context in Saudi Arabia. In addition, reformulating teaching and learning in the classroom to become metacognitive is the essential aim of this study in order to enhance the quality of mathematics teaching and learning in Saudi Arabia. The study will contribute to the broader knowledge of metacognition and aims to further enhance mathematics education in the Saudi Arabian context. .

In order to achieve the aim of the study, an interpretive approach will be used. This will enable me to interpret the perceptions of the students and their teachers toward metacognition in mathematics. The study also adopts a socio-cultural perspective in its exploration of these perceptions. Since its aim is to explore perceptions in a particular contemporary context (certain mathematics classrooms in Saudi Arabia); a case study is adopted in this research as an appropriate methodology that suits the agenda and meets the aims of this study. Therefore, because the research is founded on a case study methodology, this necessitates the utilization of qualitative methods in order to understand these perspectives and experiences.

In the current research, there will be two case studies within a secondary school. Each case contains thirty students and their teacher in which they will be will be observed in a classroom environment. Also, the IMPROVE intervention, which constitutes the following elements of instruction: Introducing new concepts, Metacognitive questioning, Practicing, Reviewing and reducing difficulties, Obtaining mastery, Verification, and Enrichment. This will be introduced with the consent of the teacher and the administrators. This strategy will be thoroughly explained to the teacher before its implementation in the classroom environment. This will enable me to study the reality of metacognition within the mathematics classroom. Subsequently, six students and their teacher from each class will be interviewed individually. Following this I will conduct a group discussion with these students.

2. Give details of the participants in this research (giving ages of any children and/or young people involved):

The study will be conducted among sixty Saudi Arabian secondary school students aged between 13 and 17 years old along with two of their mathematics teachers in order to identify issues that might provide direct experience of the research situation. The students are part of two separate classes which will be observed. Twelve students amongst these sixty students will be interviewed face to face and taking part in focus groups, the mathematics teachers of the two classes will also be interviewed. In order to recruit the participants, I will first liaise with an experienced Saudi university researcher who specialises in Education in order to choose a suitable school in which to carry out this research. He has many contacts in Saudi Arabia and has agreed to assist me in finding suitable participants for this project. He has already contacted several schools. Once I have chosen the middle school, I will arrange to meet with the school administration in order to fully explain to them the purpose of my research and also to seek their consent. Teacher participants will be recruited based on the recommendation of the Saudi university researcher I will also arrange to contact and meet the proposed teachers. Contacts can be made by emails or telephone. As for the students, I will meet the mathematics teachers with their students in their classroom in order to explain to them the aim of my research study as well as the importance of their participation. I will ask for students to volunteer to participate and then contact their parents by mail in order to seek their approval. The sample will be purposefully selected in order to best suit the aim and objectives of the study among.

Give details (with special reference to any children or those with special needs) regarding the ethical issues of:

I will be following the Code of Ethics and Conduct set out by the British Educational Research Association (BERA, 2011). Issues regarding respect, confidentiality, informed consent, safe guarding will be carefully considered as detailed below.

Respect: The views of students and teachers will be paramount in this study. I will ensure that these are listened to, respected, represented and acted upon. I will also endeavor to respect individual, cultural and role differences, including those involving age, disability, education, ethnicity, national origin, race, sexual orientation, and socio-economic status.

Safe guarding: In the exceptional event that there is evidence to raise serious concern about the safety of participants or other people, information will be passed on to relevant bodies.

3. Informed consent: Where children in schools are involved this includes both headteachers and parents). Copy (ies) of your consent form(s) you will be using must accompany this document. A blank consent form can be downloaded from the GSE student access on-line documents: **Each consent form MUST be personalized with your contact details.**

The basis of the research will be explained to participants including the approximate time needed for the interview (30 minutes for teacher and 15 minutes for student). It will be essential to obtain informed consent from parents, both for student participation in classroom observation, semi-structured and focus group interviews. In order to obtain consent from all the participants, I will first liaise with an experienced Saudi university researcher who specialises in Education in order to choose a suitable school in which to carry out this research. He has supervised many research studies in Saudi Arabian schools and has agreed to assist me in finding teachers and headteacher who would be willing to cooperate with me. The school administration will arrange to inform parents of the nature of the study prior to seeking approval for the participation of the students. Details of the date, signatory and any comments will be recorded. . In the unlikely event that a parent would be unwilling to give consent, no data will be collected from these students. Participants will be made aware of how the research findings will be used. Copies of the information sheets as well as consent forms to be signed by teachers/students' parents, to voluntarily participate in the study, are attached. Participants will be reminded that they have the right to withdraw from the research at any given time and that data related to them will be destroyed.

4. Anonymity and confidentiality

Pseudonyms will be used for participants' names and workplaces, and personal details relating to the students and their environment will be concealed to ensure that no output will provide information which might allow any student, teacher and school to be identified from names, data, contextual information or a combination of these.
Records of the data collected (including transcripts and any audio recordings) will be stored in a secure box in my home. Electronic information will only be accessed by me as a researcher with my username and password. This information will be stored on a secure system with recognised virus protection. Electronic and printed information will be locked in a secure building. Information will also be coded to ensure anonymity. This will remain anonymous in the write up of the research. Collected written information will be destroyed by shredding and audio recordings will be disposed of digitally after final submission of the research.

5. **Give details of the methods to be used for data collection and analysis and how you would ensure they do not cause any harm, detriment or unreasonable stress:**

Classroom observations will be applied in a sample comprising of sixty students from a secondary school in Saudi Arabia, along with two of their mathematics teachers. This will be done in order to identify incidents of metacognition as well as identify issues that might provide direct experience of the research situation.
The second step is to collect data through the use of separate interviews with students and teachers, a semi-structured interview with twelve students along with two of their mathematics teachers will be used to develop their own views about the situation they experienced. Interviews with the students will focus on their perception of the metacognitive learning process, whereas interviews with the teachers will focus on the metacognitive teaching process.
The use of focus group interviews will be the third step in collecting data. Two focus groups will participate in the research. The two groups will contain six students for each class and me as researcher.
Before collecting the data permission must be requested from the Ministry of Education in Saudi Arabia, local authority and school management to conduct the fieldwork and take videos. Then consent forms will be signed by teachers and students' parents (and approved by Head Teacher) who agree to participate voluntarily after informing them of the nature and purpose of the research. They will receive an information sheet/pamphlet on the study nature and aims, with the consent forms. It will be explained that there is no obligation to be involved in the study and that they can withdraw at any for/without any reason at any time (BERA, 2011: 6).
After collecting the data, interviewees' records and transcripts, and observation sheets and videos will be held in confidence and kept in a locked storage cabinet. They will not be used other than for the purposes described above. A copy of interview transcriptions will be sent by email to any participant who expresses his interest, so that he can comment on and edit it as he sees fit.

In terms of data analysis, all interview data will be fully transcribed. Although the interviews will be conducted and transcribed in Arabic, I will arrange for an official translation into English, providing a certificate of accuracy from a reputable organization. Following this, thematic coding will be performed, that is, I will read all transcripts and assign a code to every sentence. Coding processes will also be followed for the observation data (All qualitative information will be transcribed and uploaded to NVivo). Some reflections about this evidence will also be kept in 'memo' format. This process will progressively lead to the identification of some themes in the data. These are connected sets of statements, reflecting the findings and conclusions of the study. Results and findings will be shared with the participants (who express their interest) and their views sought on findings.

6. **Give details of any other ethical issues which may arise from this project - e.g. secure storage of videos/recorded interviews/photos/completed questionnaires, or**

During the data collection, data analysis and write up, data (audio recordings, observation records, interview data and individual data) will be securely stored in a locked cabinet in a secure building. Electronic information will only be accessed by the researcher with their username and password. Electronic information will also be stored on a secure system, within a locked building with recognised virus protection. It will be destroyed when it is no longer required immediately upon completion of the project.

7. **Special arrangements made for participants with special needs etc.**

I will ask the teachers to be made aware of any special needs students, if any, in the selected sample. This will be taken into account upon collecting and analysing data.

5. Give details of any exceptional factors, which may raise ethical issues (e.g. potential political or ideological conflicts which may pose danger or harm to participants):

As far as I am aware, this study does not come into conflict with any ideological or religious beliefs in the Saudi Arabian context.

This form should now be printed out*, signed by you on the first page and sent to your supervisor to sign. Your supervisor will forward this document to the School's* ***Research Support Office*** *for the Chair of the School's Ethics Committee to countersign. A unique approval reference will be added and this certificate will be returned to you to be included at the back of your dissertation/thesis.*

N.B. You should not start the fieldwork part of the project until you have the signature of your supervisor

This project has been approved for the period: Jan 2014 **until Aug 2015:**

By (above mentioned supervisor's signature): S. M. Lolli .... **date:** 13/12/2013

N.B. To Supervisor: *Please ensure that ethical issues are addressed annually in your report and if any changes in the research occur a further form is completed.*

Chair of the School's Ethics Committee
updated: March 2013

GSE unique approval reference:......D/13/14/19......................

Signed:......P. L. D[signature]......................................**date:**.....13/12/14......
Chair of the School's Ethics Committee

Chair of the School's Ethics Committee
updated: March 2013

UNIVERSITY OF EXETER

GRADUATE SCHOOL OF EDUCATION

Title of Research Project: An exploration of teachers' and students' perceptions of metacognition in relation to mathematics teaching and learning in secondary schools in Saudi Arabia.

CONSENT FORM OF PARTICIPANT

I have been fully informed about the aims and purposes of the project.

I understand that:

There is no compulsion for me to participate in this research project and, if I do choose to participate, I may at any stage withdraw my participation and may also request that my data be destroyed

I have the right to refuse permission for the publication of any information about me

Any information which I give will be used solely for the purposes of this research project, which may include publications or academic conference or seminar presentations

All information I give will be treated as confidential

The researcher will make every effort to preserve my anonymity

....
(Signature of participant) (Date)

....
(Printed name of participant)

One copy of this form will be kept by the participant; a second copy will be kept by the researcher

Contact phone number of researcher: 00447788488038

If you have any concerns about the project that you would like to discuss, please contact:

Khalid on ksaa201@exeter.ac.uk or kz111@hotmail.com
OR
Dr Shirley Larkin on +44(0) 1392 724814 or email at S.Larkin@exeter.ac.uk

* When research takes place in a school, the right to withdraw from the research does NOT usually mean that pupils or students may withdraw from lessons in which the research takes place

Data Protection Act: The University of Exeter is a data collector and is registered with the Office of the Data Protection Commissioner as required to do under the Data Protection Act 1998. The information you provide will be used for research purposes and will be processed in accordance with the University's registration and current data protection legislation. Data will be confidential to the researcher(s) and will not be disclosed to any unauthorised third parties without further agreement by the participant. Reports based on the data will be in anonymised form.

Revised March 2013

UNIVERSITY OF EXETER

GRADUATE SCHOOL OF EDUCATION

Title of Research Project: An exploration of teachers' and students' perceptions of metacognition in relation to mathematics teaching and learning in secondary schools in Saudi Arabia.

<u>CONSENT FORM: participants' parents / guardians</u>

I have been fully informed about the aims and purposes of the project.

I understand that:

There is no compulsion for my son to participate in this research project and, if he does choose to participate, he may at any stage withdraw his participation

I have the right to refuse permission for the publication of any information about my son

I have the right to refuse participation
Any information which my son gives will be used solely for the purposes of this research project, which may include publications or academic conference or seminar presentations

The information, which my son gives, may be shared between any of the other researcher participating in this project in an anonymised form

All information my son gives will be treated as confidential

The researcher will make every effort to preserve my son's anonymity

....
(Signature of parent / guardian) (Date)

....
(Printed name of parent / guardian) (Printed name of participant)

One copy of this form will be kept by the participants' parent or guardian; a second copy will be kept by the researcher

Contact phone number of researcher: 00447788488038

If you have any concerns about the project that you would like to discuss, please contact:

Khalid on ksaa201@exeter.ac.uk or kz111@hotmail.com OR

Dr Shirley Larkin on +44(0) 1392 724814 or email at S.Larkin@exeter.ac.uk

UNIVERSITY OF
EXETER

Graduate School of Education

Dear parent(s) or guardian(s)

I am writing to introduce my self and ask permission for your child to participate in a research project I am undertaking at the current time. My name is Khalid Alzahrani and I am a lecturer at Taif University and currently a student researcher at the Graduate School of Education, University of Exeter, England. A part of this research involves looking at understanding of the concept of metacognition, and the experience of mathematics teachers and their students with metacognition in Saudi Secondary schools.

I have kindly been given permission to do this at your child's school. And I am currently spending some time in the school getting to know the children and what they do in the daily basis.

In consultation with a class teacher, I would like now to talk to a small number of students about their experience at school. This will involve short individual interviews of no more than 15 to 20 minutes, and this will take place outside lesson time. I would like to ask permission to interview your child and include a tear-off slip for you to complete at the end of this letter. I can also assure you that you child will also be asked if he is happy to be interviewed and he can withdraw from the interview at any time.

The interview will be recorded, with your permission. I can assure you that the content of the interview will be kept absolutely confidential and not shared with the school, and neither you nor your child will be identified by name in my thesis or any publication. The result of the interview will be contributed to furthering our knowledge about metacognition.

If you have any concerns or queries do feel free to contact me at the school or by email at: ksaa201@exeter.ac.uk . Also if you require any further information about specific aspect of this research please contact my research supervisors Dr Shirley Larkin on +44(0) 1392 722874.

Many thanks for considering my request. And I look forward for hearing from you.

Yours sincerely,

Khalid Alzahrani

I give permission for my child__________________________ to be interviewed and participated with focus group.

Parent/ Guardian ________________________

Signature_________________ **Date:-____________________**

Appendix 10

List of codes assigned to extracts from teacher interview.

Document	Code	line	quote
Translated Teacher's semi-structured interview schedule	absence of concern for the implementation of M within the system	81	General lack of previous adoption of methods such as this in education is one obstacle. In addition, lack of pursuit of the question of how we can implement this method in reality.
	absence of adoption of M methods in education system	81	General lack of previous adoption of methods such as this in education is one obstacle. In addition, lack of pursuit of the question of how we can implement this method in reality
	absence of incentive for teachers	88	For there to be additional incentives for teachers who apply such methods.
	absent of partnership with research centres	82	Absence of partnership with research centres supporting and activating an educational environment in the school
	needs to specialised institutions that offer M methods	87	There must be specific agencies for new methods in teaching which the teacher can communicate with to develop his performance in teaching fundamentally and in application
	syllabus content and materials' activities	69	Books containing activities which are compatible with the method (enriched books to support the curriculum).
	provision of M resources	65	Provision of sources which display the subject
	provision of M resources	69	the provision of resources to better illustrate the method
	providing 'document camera' supports using M	39	The use of a camera for documentation is very important.
	providing 'document camera' supports using M	66	Provision of a documentation camera
	providing 'document camera' supports using M	69	Another feature could be the provision of a camera to document the class,
	needs to courses and resources	72	The greatest difficulty is, the typicality that we have become accustomed to over a long time, requiring courses and resources.
	classroom equipment	78	Equipped classrooms, an appropriate number of students for the size of the classroom, internet, sound and video recording equipment and computers, data show, optical cameras, and in general a complete re-equipping of classrooms.
	class sizes	69	The most important thing is to create a class environment. This could involve reducing the number of students in the class to facilitate the issue of group and individual evaluation.
	class sizes	78	an appropriate number of students for the size of the classroom,
	the role of educational supervision in promoting M	62	It has a very important role in promoting metacognitive methods,
	focus on superficial issues when evaluate teacher	55	The principal or supervisor's evaluation of a teacher does not include any criteria pertaining to the application of this method, instead, there is focus on how much scheduled material one has completed.
	absence of teacher evaluation criteria in using M	55	The principal or supervisor's evaluation of a teacher does not include any criteria pertaining to the application of this method, instead, there is focus on how much scheduled material one

			has completed.
	principal focuses on covering the syllabus	79	The school administration being unconvinced because its focus is on the direct academic attainment of students and completion of curricula.
	conviction of the school principal	62	It has a very important role in promoting metacognitive methods, because this is what convinces the principal or educational supervisor to potentially convey the method to the teachers.
	conviction of school principal	79	The school administration being unconvinced because its focus is on the direct academic attainment of students and completion of curricula.
	Understanding of metacognition	27	This can be done by introducing an ideal example approach to deal with the problem at the same time, trying to highlight the skill of monitoring.
	Average level students benefit the most from M	46	I view students of average achievement to be benefiting the most from these skills.
	the importance of monitoring skill in M	27	This can be done by introducing an ideal example approach to deal with the problem at the same time, trying to highlight the skill of monitoring.
	M helps students to manage time in solving problem	14	The existence of a logical 'Thinking Map', which in turn aided students in time management for dealing with solving problems.
	metacognition and age	25	I also think that implementing it at a young age is preferable.
	group and individual evaluation	69	The most important thing is to create a class environment. This could involve reducing the number of students in the class to facilitate the issue of group and individual evaluation.
	efforts needed in dealing with assessment issues	20	We are still in need of greater efforts in deal with the issue of evaluation and estimation.
	a low academic achievement	74	A low academic achievement of the student in maths is one of them, specifically because they cannot participate with their classmates in the discussion and working to the solution to the problems and also because mathematics requires prior knowledge.
	low academic achievers benefit less from M than other student	46	I believe that students of weak academic achievement are the ones who benefit least from this teaching style, the reason for this being a smaller body of prior knowledge and experiences to draw on.
	M is concerned with discussion students' thinking	42	Do you discuss with your students about how they think when they learn mathematics? This method helped me in doing this and that is what I hope for in the future.
	M is concerned with evaluating students' outcomes	41	This is done by monitoring the students' cooperation and interaction when problem solving and with reviewing worksheets and evaluating outcomes of the work.
	importance of evaluation skill	39	I will also pay attention to evaluation,
	M enhances students' expertise in math	37	This method provides students with experience in dealing with math
	practice on metacognitive skills	28	A large amount of practice is needed for the four Skills strengthens this aspect.

	needs to transfer the concept of thinking monitoring	20	We were in need of the embodiment of the concept of thought monitoring.
	M encourages students to be part in constructive learning	15	The importance of metacognition in teaching is great, due to it being in accordance with modern theory which seeks to make students at the core of the education process, it being them who search for information in order to encourage constructive learning.
	M supports students' thinking	15	This method supports students' thinking and their abilities which enables the student to evaluate their thinking.
	M helps students evaluate their thinking	15	This method supports students' thinking and their abilities which enables the student to evaluate their thinking.
	metacognition contains a logical thinking map	14	Another important element of the method is the existence of a logical 'Thinking Map', which in turn aided students in time management for dealing with solving problems.
	M as a positive experience for teachers	11	it was a fascinating experience,
	M as a positive experience for students	11	It was an excellent experience for the students,
	M as a systematic logical procedure	4	The work was done within logical steps, in order to resolve problems.
	metacognition contains four skills	4	Four skills were used to carry out these logical steps; these were planning, administration, evaluation and monitoring.
	particular care needs to be taken with low achievers when using	89	Weak students must be taken into account when implementing a method like this in order for the school to examine how it can deal with them to address this weakness.
	it is hard to be free yourself from traditional methods	35	We were not able to free ourselves of the old methods, despite our knowledge of many important aspects in the subject of evaluating thinking in itself.
	teachers undermine the students' potential by using traditional methods	45	The students have the ability, but we are who undermine their potential with using traditional styles of teaching, as well as our focus on grades from tests.
	Gaps between M and traditional teaching	72	I have definitely seen that with the teaching of this method comes various big differences to the reality of teaching mathematics here.
	traditional methods as strongly established in teaching and lea	72	The greatest difficulty is, the typicality that we have become accustomed to over a long time, requiring courses and resources.
	students' dislike for non-traditional routine of teaching	76	Thirdly, some students feel that their own methods are more beneficial to them, and get them good grades, and thus there is no need for them to try other ways.
	implement M broke traditional routine of teaching	11	It broke with traditional routine, as well as the boredom that often occurs in classrooms.
	focus on test result more than M	32	Most of our efforts involved the solution and obtaining results.
	focus on test result more than M	34	Unfortunately, until now, we were not able to focus on evaluating these aspects, as our focus was placed on evaluating the results to their problem solving.
	focus on test result more than M	45	The students have the ability, but we are who undermine their potential with using traditional styles of teaching, as well as our focus on

			grades from tests.
	focus on test result more than M	75	Secondly, a student's failing to realise the importance of this method, with their thinking and focus instead being on obtaining good grades in tests.
	teachers' training taking place in the classroom	67	
	trainers with a teaching background	64	We could choose a teacher in each school to be trained, after which he would undertake the role of trainer / instructor within his school
	conviction of principal and supervisor regarding metacognition	62	It has a very important role in promoting metacognitive methods, because this is what convinces the principal or educational supervisor to potentially convey the method to the teachers.
	teachers' capability to explain M to colleagues	20	Despite the fact that we are now ready to present a clear conception to other teachers as to how they can implement M in the classroom.
	absent of M teacher training	8	There is training involved with active & cooperative learning. As for 'thinking about thinking' there is no pre-established training programme to follow.
	The kind of activities encourage students' interaction with other	17	The activities should involve indirect solutions, previous experience, hold new ideas, and should be challenging. This encourages students to interact more with the subject, the teacher, and among themselves.
	Students' achievement and enthusiasm to for engaging in cooperative learning	24	If they are outstanding students (in terms of grades), this could increase their enthusiasm for engaging in cooperative learning with other students of lesser ability.
	Mix abilities groups encourage students' cooperative	14	As for this method in itself, the division of students into varying educational achievement groups, proved to be valuable in aiding cooperation. Another important element of the method is the existence of a logical 'Thinking Map', which in turn aided students in time management for dealing with solving problems. Thus, I think after seeing this method, a strong connection existed between metacognition and cooperative learning.
	a low academic achiever cannot participate with other	74	A low academic achievement of the student in maths is one of them, specifically because they cannot participate with their classmates in the discussion and working to the solution to the problems and also because mathematics requires prior knowledge.
	monitoring cooperative learning	41	This is done by monitoring the students' cooperation and interaction when problem solving and with reviewing worksheets and evaluating outcomes of the work.
	worksheets as an indication of group harmony	33	.
	students' cooperation in problem solving	17	Every student should try to present what would help the other group members with solving problems.
	relationship between metacognition and cooperative learning	14	A strong connection existed between metacognition and cooperative learning.
	Student's role	28	A large amount of practice is needed for the four

			Skills strengthens this aspect.
	Student's role	29	The step of Solution Strategy is what highlights the answer to this question, and here we can implement brainstorming strategy.
	more practice on M skills	28	A large amount of practice is needed for the four Skills strengthens this aspect.
	more practice on solution strategy	29	The step of Solution Strategy is what highlights the answer to this question,
	more practice on brainstorm	29	The step of Solution Strategy is what highlights the answer to this question, and here we can implement brainstorming strategy.
	students are knowledgeable but inexperienced	37	Problems at a time where many students' poses only prior knowledge which does not involve the strategy of thinking.
	student seeks to knowledge	11	They were put in the position to obtain information for themselves.
	student-centred effort	48	We initiated a certain change by having students making more efforts in order to obtain knowledge.
	student as seeker of knowledge	15	The importance of metacognition in teaching is great, due to it being in accordance with modern theory which seeks to make students at the core of the education process, it being them who search for information in order to encourage constructive learning.
	student's role in M learning	51	This is because the experience proved that students felt they had a role in the education process despite there being difficulties in the content.
	student's thinking and evaluating of it	15	This method supports students' thinking and their abilities which enables the student to evaluate their thinking.
	logical steps	4	The work was done within logical steps, in order to resolve problems.
	metacognitive skills	4	Four skills were used to carry out these logical steps; these were planning, administration, evaluation and monitoring.
	need to more practice for the metacognitive skills	28	A large amount of practice is needed for the four Skills strengthens this aspect.
	metacognitive teaching involves more efforts	48	I felt satisfied when I saw the benefit for the students. Despite the greater efforts.
	need to encourage student seeks to knowledge	48	We initiated a certain change by having students making more efforts in order to obtain knowledge.
	implement M needs educated teacher	53	The qualified teacher in learning strategies and its theory will have the motivation to engage with this method.
	concern about beginning of implement	22	At the beginning of the process, there were concerns but in the end, when the students practiced this theory, it turned out well.
	reaction of student toward beginning of implement	25	At the beginning I was concerned about the reaction of some students and the difficulties they might have found, however implementing it smoothly with the provision of incentives is better. I also think that implementing it at a young age is preferable.
	presenting a new concept by using M	38	I seek to present new concepts metacognitively.
	introducing an ideal example to deal with the problem metacognition	27	This can be done by introducing an ideal example approach to deal with the problem at the same time, trying to highlight the skill of

			monitoring.
	displaying ideal example for student	39	Also the display of outstanding examples and the use of a camera for documentation is very important.
	time management as skill to implement M	14	As for this method in itself, the division of students into varying educational achievement groups, proved to be valuable in aiding cooperation. Another important element of the method is the existence of a logical 'Thinking Map', which in turn aided students in time management for dealing with solving problems.
	solution strategy of solving problem as difficult	29	The step of Solution Strategy is what highlights the answer to this question,
	brainstorm strategy as skills	29	We can implement brainstorming strategy.
	displaying the solution of the different group to the class	30	Displaying the solutions of the different groups to the class is important.
	students lack strategies of thinking	37	Problems at a time where many students' poses only prior knowledge which does not involve the strategy of thinking.
	weak academic achiever benefits less from M than other students	46	I believe that students of weak academic achievement are the ones who benefit least from this teaching style, the reason for this being a smaller body of prior knowledge and experiences to draw on.
	a low academic achievement	74	A low academic achievement of the student in maths is one of them, specifically because they cannot participate with their classmates in the discussion and working to the solution to the problems and also because mathematics requires prior knowledge.
	a low achiever should be taken into account	89	Weak students must be taken into account when implementing a method like this in order for the school to examine how it can deal with them to address this weakness.
	activity includes challenges	17	The activities should involve indirect solutions, previous experience, hold new ideas, and should be challenging.
	activity includes a new idea	17	The activities should involve indirect solutions, previous experience, hold new ideas,
	activity includes previous experience	17	The activities should involve indirect solutions, previous experience,
	activity includes an indirect solution	17	The activities should involve indirect solutions,
	contents of the syllabus and M	69	Books containing activities which are compatible with the method (enriched books to support the curriculum).
	teacher needs to transfer the concept of thinking monitoring	20	We were in need of the embodiment of the concept of thought monitoring.
	discussing students based on looking over the worksheet	39	Be it in reviewing of each groups work or displaying group worksheets and discussing with students. Also the display of outstanding examples and the use of a camera for documentation is very important.
	evaluating students' outcomes by looking over the worksheet	41	This is done by monitoring the students' cooperation and interaction when problem solving and with reviewing worksheets and evaluating outcomes of the work.

	monitoring students' interaction	41	This is done by monitoring the students' cooperation and interaction when problem solving and with reviewing worksheets and evaluating outcomes of the work.
	teacher needs to deal with the evaluation and assessment	20	We are still in need of greater efforts in deal with the issue of evaluation and estimation.
	evaluating student's thinking	35	We were not able to free ourselves of the old methods, despite our knowledge of many important aspects in the subject of evaluating thinking in itself.
	revealing the flaws thinking by looking over the worksheets	33	It also reveals a lot about flaws in thinking in a certain manner.
	evaluation the result instead of thinking	34	Unfortunately, until now, we were not able to focus on evaluating these aspects, as our focus was placed on evaluating the results to their problem solving.
	teacher needs to evaluate a group and individual	69	The most important thing is to create a class environment. This could involve reducing the number of students in the class to facilitate the issue of group and individual evaluation.
	metacognitive skills to solve problem	4	Four skills were used to carry out these logical steps; these were planning, administration, evaluation and monitoring.
	metacognitive logical steps to resolve problem	4	The work was done within logical steps, in order to resolve problems.
	reason for students' lack of enthusiasm	22	In the other hand, there were two reasons for the lack of enthusiasm from some students. 1) Their lack of belief in the benefits of using metacognitive thinking, whether it was in their academic achievement or on future studies 2) The nature of the students, if they are outstanding students (in terms of grades), this could increase their enthusiasm for engaging in cooperative learning with other students of lesser ability.
	motivation and students' level achievement	46	I believe that students of weak academic achievement are the ones who benefit least from this teaching style, the reason for this being a smaller body of prior knowledge and experiences to draw on.
	focus on result more than M	76	Thirdly, some students feel that their own methods are more beneficial to them, and get them good grades, and thus there is no need for them to try other ways.
	seeing benefits of using metacognition	22	They began to feel the benefits and their optimism increased, and thus, as soon as I told them that the method will be applied next lesson, they appeared to be happy, and with that most of the students seemed to be optimistic.
	lack of belief in benefits of using M about future	23	Their lack of belief in the benefits of using metacognitive thinking, whether it was in their academic achievement or on future studies
	lack of belief in the benefit of using M about achievement	23	Their lack of belief in the benefits of using metacognitive thinking, whether it was in their academic achievement or on future studies
	seeing the benefits of using M for students	92	An important aspect is for the teacher to feel and see the benefit for the students, this pushes the teacher to continue in methods such as this

			and to focus on developing thinking.
	the level of education of teacher affects motivation	53	The education of the teacher is an influential factor in accepting this teaching method in the first place. Thus, the qualified teacher in learning strategies and its theory will have the motivation to engage with this method.
	teacher's conviction about M affects motivation	53	I am convinced that after implementing this method, I really found that I had the motivation to teach with it, as this conviction was very influential to me in teaching.
	experience in using M affects motivation	53	I am convinced that after implementing this method, I really found that I had the motivation to teach with it, as this conviction was very influential to me in teaching.
	school principal focuses on completion of syllabi	79	The school administration being unconvinced because its focus is on the direct academic attainment of students and completion of curricula.
	lack of adoption of M methods in education system	81	General lack of previous adoption of methods such as this in education is one obstacle. In addition, lack of pursuit of the question of how we can implement this method in reality
	absent of partnership with research centre implement	82	Absence of partnership with research centres supporting and activating an educational environment in the school
	deadlock in discussion about practical development of education	83	There is a deadlock in the discussion and dialogue surrounding the practical development of education among the public in general and more specifically among teachers.
	absence of incentives to motive teachers	88	For there to be additional incentives for teachers who apply such methods.
	teacher is satisfied when he sees the benefits of M for student	48	I felt satisfied when I saw the benefit for the students. Despite the greater efforts.
	seeing the positive results of implement as source of motivation	57	Therefore, some teachers need to see in front of them the positive results of implementing the method in order for them to interact with it positively.
	importance for the teacher to see the benefit of M	92	An important aspect is for the teacher to feel and see the benefit for the students, this pushes the teacher to continue in methods such as this and to focus on developing thinking.

Appendix 11

‘Code system’ from one teacher interview.

Teachers’ understanding of metacognition
M: metacognition

Theme	category	code		line	Quotation
Teachers’ understanding of metacognition	Conception of metacognition	metacognition contains a logical thinking map		14	Another important element of the method is the existence of a logical ‘Thinking Map’, which in turn aided students in time management for dealing with solving problems.
		M as a systematic logical procedure		4	The work was done within logical steps, in order to resolve problems.
	Metacognitive skills	metacognition contains four skills		4	Four skills were used to carry out these logical steps; these were planning, administration, evaluation and monitoring.
		The importance of practice of metacognitive skills		28	A large amount of practice of the four Skills strengthens this aspect. (encouraging a student to monitor his thinking when he solves mathematic problem)
		evaluation	group and individual evaluation	69	The most important thing is to create a class environment. This could involve reducing the number of students in the class to facilitate the issue of group and individual evaluation.
			efforts needed in dealing with assessment issues	20	We are still in need of greater efforts to deal with the issue of evaluation and assessment. (Evaluation is part of assessment. Assessment includes evaluation, feedback, questions, formative assessment, summative assessment…)
			M is concerned with evaluating students’ outcomes	41	This is done by monitoring the students' cooperation and interaction when problem solving and with reviewing worksheets and evaluating outcomes of the work.
			importance of evaluation skill	39	I will also pay attention to evaluation,
		monitoring	The importance of monitoring and planning skill in M	27	This can be done by introducing an ideal example approach to deal with the problem at the same time, trying to highlight the skill of planning and monitoring. (encouraging a student to monitor his thinking when he solves mathematic problem)
			Need to transfer the concept of thinking monitoring from theory to practice	20	We were in need of the embodiment of the concept of thinking monitoring.
	Function of M	M is concerned with discussion students’		42	Do you discuss with your students about how they think when they learn mathematics? This method helped me in doing this and that is

		thinking		what I hope for in the future.
		M enhances students' expertise in math	37	This method provides students with expertise in dealing with math
		M encourages students to be part in constructive learning	15	The importance of metacognition in teaching is great, due to it being in accordance with modern theory which seeks to make students at the core of the education process, it being them who search for information in order to encourage constructive learning.
		M supports students' thinking	15	This method supports students' thinking and their abilities which enables the student to evaluate their thinking.
		M helps students evaluate their thinking	15	This method supports students' thinking and their abilities which enables the student to evaluate their thinking.
		M helps students to manage time in solving problems	14	Another important element of the method is the existence of a logical 'Thinking Map', which in turn aided students in time management for dealing with solving problems.
	academic achievement of the student and M	Low academic achievers benefit less from M than other students	46	I believe that students of weak academic achievement are the ones who benefit least from this teaching style, the reason for this being a smaller body of prior knowledge and experiences to draw on.
		A low academic achievement	74	A low academic achievement of the student in maths is one of them, specifically because they cannot participate with their classmates in the discussion and working to the solution to the problems and also because mathematics requires prior knowledge.
		Average level students benefit the most from M	46	Thus I view students of average achievement to be benefiting the most from these skills.[relationship between academic achievement and the benefit of using M]
		Particular care needs to be taken with low achievers when using M	90	Weak students must be taken into account if we want to benefit from implementing a method like this in order for the school to examine how it can deal with them to address this weakness.
		M as a positive experience for teachers	11	it was a fascinating experience,
		M as a positive experience for students	11	It was an excellent experience for the students,
		metacognition and age	25	I also think that implementing it at a young age is preferable.

metacognition and Cooperative learning (CL) M: metacognition				
Theme	category	Code	line	Quotation

M and Cooperative learning (CL)		relationship between metacognition and CL	14	A strong connection existed between metacognition and cooperative learning.
	Relationship between low academic achievement CL	low academic achiever cannot participate with their classmates in the discussion	74	A low academic achievement of the student in maths is one of them, specifically because they cannot participate with their classmates in the discussion and working to the solution to the problems and also because mathematics requires previously gained knowledge. (the obstacles to the application of metacognitive teaching in the mathematics classroom)
		Students' achievement and enthusiasm to for engaging in cooperative learning	24	If students are outstanding students (in terms of grades), this could increase their enthusiasm for engaging in cooperative learning with other students of lesser ability.
	monitoring in CL	monitoring cooperative learning	41	This is done by monitoring the students' cooperation and interaction when problem solving and with reviewing worksheets and evaluating outcomes of the work. (How do you know when learning through metacognition is occurring in your classroom?)
		Worksheet as an indication of the group's harmony	33	Looking at the worksheets to evaluate students' work reveals to the teachers many aspects of the groups' cooperative harmony. (these worksheets have been designed according to the IMPROVE programme)
	encouraging students' interaction with others	The kind of activities encourage students' interaction with others	16	The activities should involve indirect solutions, previous experience, hold new ideas, and should be challenging. This encourages students to interact more with the subject, the teacher, and among themselves.
		Students' cooperation in problem solving	17	Every student should try to present what would help the other group members with solving problems.
		Mixed abilities groups encourage students' cooperation	14	As for this method in itself, the division of students into varying educational achievement groups, proved to be valuable in aiding cooperation.

The Teacher's requirements to the implementation metacognition M: metacognition				
theme	category	Code	line	Quotation
The Teacher's requirements to implement M	Perception of M	metacognitive logical steps in problem solving	4	The work was done within logical steps, in order to resolve problems.
		metacognitive skills used in problem solving	4	Four skills were used to carry out these logical steps; these were planning, administration, evaluation and monitoring.
	Evaluation skills	Group and individual evaluation	69	The most important thing is to create a class environment. This could involve reducing the number of students in the class to facilitate the issue of group and individual evaluation.
		Focus on evaluating results rather than thinking	34	Unfortunately, until now, we were not able to focus on evaluating these aspects, as our focus was placed on evaluating the results to

				their problem solving.
		Worksheet evaluation reveals flaws in thinking modes	33	Taking the worksheets back and looking over them, reveals to the teachers many aspects of the groups cooperative harmony. It also reveals a lot about flaws in thinking in a certain manner.
		evaluating student's thinking	35	We were not able to free ourselves of the old methods, despite our knowledge of many important aspects in the subject of evaluating thinking in itself.
		teacher needs to deal with the evaluation and assessment	20	We are still in need of greater efforts in deal with the issue of evaluation and estimation.
		monitoring students' interactions	41	This is done by monitoring the students' cooperation and interaction when problem solving and with reviewing worksheets and evaluating outcomes of the work.
		evaluating students' outcomes by looking over the worksheet	41	This is done by monitoring the students' cooperation and interaction when problem solving and with reviewing worksheets and evaluating outcomes of the work.
		Teacher discusses with students by looking over their worksheet	39	I will also pay attention to evaluation, be it in reviewing each group's work or displaying group worksheets and discussing with students
		teacher needs to transfer the concept of thinking monitoring from theory to practice	20	We were in need of the embodiment of the concept of thought monitoring.
	Materials'' design	syllabus content and M	69	The syllabus containing activities which are compatible with the method (enriched books to support the curriculum).
		activity should include an indirect solution	17	The activities should involve indirect solutions,
		activity should include previous experience	17	The activities should involve indirect solutions, previous experience,
		Activity should include a new idea	17	The activities should involve indirect solutions, previous experience, hold new ideas,
		activity should include challenges	17	The activities should involve indirect solutions, previous experience, hold new ideas, and should be challenging.
	Dealing with students' levels	Students' low achievements should be taken into account	90	Weak students must be taken into account when implementing a method like this in order for the school to examine how it can deal with them to address this weakness.
		low academic achievement	74	A low academic achievement of the student in maths is one of them, specifically because they cannot participate with their classmates in the discussion and working to the solution to the problems and also because mathematics requires previously gained knowledge.
		low academic achievers benefit less from M than other students	46	I believe that students of weak academic achievement are the ones who benefit least from this teaching style, the reason for this being a smaller body of previously learned knowledge and experiences to draw on.

	technique to implement M	students lack thinking strategies	37	Problems at a time where many students' poses only prior knowledge which does not involve the strategy of thinking.
		revealing the problem's solution to other groups	30	Displaying the solutions of the different groups to the class is important.
		solution strategy of problem solving as difficult	29	The step of Solution Strategy is what highlights the answer to this question,
		brainstorming strategy is a skill	29	We can implement brainstorm strategy.
		time management as a skill to implement M	14	As for this method in itself, the division of students into varying educational achievement groups, proved to be valuable in aiding cooperation. Another important element of the method is the existence of a logical 'Thinking Map', which in turn aided students in time management for dealing with solving problems.
		Teachers need to show ideal solutions to students	39	Also the display of outstanding examples and the use of a camera for documentation is very important.
		introducing an ideal example to deal with the problem	27	This can be done by introducing an ideal example approach to deal with the problem at the same time, trying to highlight the skill of monitoring.
		presenting a new concept by using M	38	I seek to present new concepts metacognitively.
	beginning of implementation	Careful implementation of M for the first time	25	In the beginning I was concerned about the reaction of some students and the difficulties they might have found, however implementing it smoothly with the provision of incentives is better.
		concern when implementing M for the first time	22	At the beginning of the process, there were concerns but in the end, when the students practiced this theory, it turned out well.
		Need to encourage student to seek knowledge	48	We began to a certain process of change in making our efforts for the students greater, these efforts being targeted at obtaining knowledge.
		M teaching requires qualified teacher	53	The qualified teacher in learning strategies and its theory will have the motivation to engage with this method.
		metacognitive teaching involves more efforts	48	I felt satisfied when I saw the benefit for the students. Despite the greater efforts.
		Need more practice for the metacognitive skills	28	A large amount of practice for the four Skills is needed to strengthen this aspect [monitoring student's thinking].

The student's requirements to the implementation of metacognition
M: metacognition

theme	category	Code	line	Quotation
The student's requirement	Perception of M	logical steps to solve problems	4	The work was done within logical steps, in order to resolve problems.
		metacognitive skills to solve problems	4	Four skills were used to carry out these logical steps; these were planning, administration, evaluation and monitoring.

		Students' thinking and its evaluation	15	This method supports students' thinking and their abilities which enables the student to evaluate their thinking.
	Student-centred Effort	Students' role in M Learning	51	This is because the experience proved that students felt they had a role in the education process despite there being difficulties in the content.
		Student-centred efforts	48	We initiated a certain change by having students making more efforts in order to obtain knowledge.
		Student as seeker of knowledge	15	The importance of metacognition in teaching is great, due to it being in accordance with modern theory which seeks to make students the main convenors of the education process, it being them who search for information in order to encourage constructive learning.
		Student seeks knowledge	11	They were put in the position to obtain information for themselves.
	practice	Students Knowledgeable but Inexperienced	37	Problems at a time where many students' poses only previously learnt knowledge which does not involve the strategy of thinking.
		Students should have more practice in solution strategy	29	The step of Solution Strategy is what highlights the answer to this question,
		Students should have practice in brainstorming strategy	29	We can implement brainstorming strategy.
		Need of more practice in the metacognitive skills	28	A large amount of practice for the four Skills strengthens this aspect.

Challenges to the implementation of metacognition M: metacognition				
theme	category	code	line	Quotation
Challenges to the implementation of M	Education system	Absence of adoption of M methods in the education system	81	General lack of previous adoption of methods such as this in education is one obstacle. In addition, lack of pursuit of the question of how we can implement this method in reality
		Absence of concern for the implementation of M within the education system concern	81	lack of pursuit of the question of how we can implement this method in reality
			57	If the educational system does not give importance to methods, there will be inaction on the part of the teacher to research, inform himself and try the method.
		absence of partnership with research centres	82	Absence of partnership with research centres supporting and activating an educational environment in the school
		Absence of incentive for teachers	88	For there to be additional incentives for teachers who apply such methods.
		Need of specialised institutions that offer M methods in teaching	87	There must be specific agencies for new methods in teaching which the teacher can communicate with to develop his performance in teaching fundamentally and in application
	Resources and equipment	Class sizes	69	The most important thing is to create a class environment. This could involve reducing the number of students in the class to facilitate the issue of collective and individual

				evaluation.
			78	an appropriate number of students for the sise of the classroom,
		Classroom equipment	78	Equipped classrooms, an appropriate number of students for the size of the classroom, internet, sound and video recording equipment and computers, data show, optical cameras, and in general a complete re-equipping of classrooms.
		Need of courses and resources	72	The greatest difficulty is, the typicality that we have become accustomed to over a long time, requiring courses and resources.
		providing a 'document camera' supports using M	39	The use of a camera for documentation is very important.
			69	Another feature could be the provision of a camera to document the class,
			66	Provision of a documentation camera
		provision of M resources	69	the provision of resources to better illustrate the method
			65	Provision of resources which display the subject
		Syllabus content and materials' activities	51	This is because the experience proved that students felt they had a role in the education process despite there being difficulties in the content.
			69	Books containing activities which are compatible with the method.
	The school principal	conviction of the school principal	62	It has a very important role in promoting methods such as these, because this is what convinces the principal or educational supervisor to potentially convey the method to the teachers.
			79	The school administration being unconvinced because its focus is on the direct academic attainment of students and completion of curricula.
		The principal's focus on covering the syllabus	79	The school administration being unconvinced because its focus is on the direct academic attainment of students and completion of curricula.
	Supervision issues	Absence of teacher evaluation criteria in using M	55	The principal or supervisor's evaluation of a teacher does not include any criteria pertaining to the application of this method, instead, there is focus on how much scheduled material one has completed.
		focus on superficial issues when evaluating teacher as a challenge	55	The principal or supervisor's evaluation of a teacher does not include any criteria pertaining to the application of this method, instead, there is focus on how much scheduled material one has completed. focus on superficial issues when evaluate teacher as a challenge
		the role of educational supervision in	62	It has a very important role in promoting metacognitive methods.

		promoting M			
	Training in implementing M	Teacher training taking place in the classroom		67	The supervision must focus on training and developing teachers in the field, not only in a training room.
		Trainers with a teaching background		64	We could choose a teacher in each school to be trained, after which he would undertake the role of trainer / instructor within his school
		conviction of principal and supervisor regarding metacognition		62	It has a very important role in promoting metacognitive methods, because this is what convinces the principal or educational supervisor to potentially convey the method to the teachers.
		Teachers' capability to explain M to colleagues		20	We are now ready to present a clear conception to other teachers as to how they can implement M in the classroom.
		absence of M teacher training		8	There is training involved with active & cooperative learning. As for 'thinking about thinking' there is no pre-established training programme to follow.
	Traditional methods	Focus on test results more than M		75	Secondly, a student's failing to realise the importance of this method, with their thinking and focus instead being on obtaining good grades in tests.
				34	Unfortunately, until now, we were not able to focus on evaluating these aspects, as our focus was placed on evaluating the results to their problem solving.
				32	Most of our efforts involved the solution and obtaining results.
				45	The students have the ability, but we are who undermine their potential with using traditional styles of teaching, as well as our focus on grades from tests.
		Long lasting typical methods as a difficulty	Traditional methods as strongly established in teaching and learning	72	The greatest difficulty is, the typicality that we have become accustomed to over a long time, requiring courses and resources.
			Gaps between M and traditional teaching	72	I have definitely seen that with the teaching of this method comes various big differences to the reality of teaching mathematics here.
			Teachers undermine the students' potential by using traditional methods	45	The students have the ability, but we are who undermine their potential with using traditional styles of teaching, as well as our focus on grades from tests.
			it is hard free yourself from traditional methods	35	We were not able to free ourselves of the old methods, despite our knowledge of many important aspects in the subject of evaluating thinking in itself.
			Students'	76	Thirdly, some students feel that their own

			dislike for non-traditional methods		methods are more beneficial to them, and get them good grades, and thus there is no need for them to try other ways.
			Implementing M broke traditional routine of teaching	11	It broke with traditional routine, as well as the boredom that often occurs in classrooms.

Teacher's motivation in implementing metacognition
M: metacognition

theme	category	Code	line	Quotation
Teacher's motivation in implementing M:	What inhibits motivation	Absence of incentives to motivate teachers	88	For there to be additional incentives for teachers who apply such methods.
		deadlock in discussion about practical development of education	83	There is a deadlock in the discussion and dialogue surrounding the practical development of education among the public in general and more specifically among teachers.
		absence of partnership with research centre	82	Absence of partnership with research centres supporting and activating an educational environment in the school
		lack of adoption of M methods in the education system	81	General lack of previous adoption of methods such as this in education is one obstacle. In addition, lack of pursuit of the question of how we can implement this method in reality
		School principal focuses on completion of syllabi	79	The school administration being unconvinced because its focus is on the direct academic attainment of students and completion of curricula.
	What promotes motivation	experience in using M affects motivation	53	I am convinced that after implementing this method, I really found that I had the motivation to teach with it, as this conviction was very influential to me in teaching.
		Teacher's conviction about M affects motivation	53	I am convinced that after implementing this method, I really found that I had the motivation to teach with it, as this conviction was very influential to me in teaching.
		the level of education of teacher affects motivation	53	The education of the teacher is an influential factor in accepting this teaching method in the first place. Thus, the educated and well-versed teacher in learning strategies and its theory will have the motivation to engage with this method.
		seeing the positive results of the implementation of M as a source of motivation	57	Some teachers need to see in front of them the positive results of implementing the method in order for them to interact with it positively.
			92	An important aspect is for the teacher to feel and see the benefit for the students, this pushes the teacher to continue in methods such as this and to focus on developing thinking. (In general, what other obstacles might make metacognitive teaching of mathematics difficult?)
			48	I felt satisfied when I saw the benefit for the students. Despite the greater efforts.

Students’ motives for implementing metacognition M: metacognition				
theme	category	Code	line	Quotation
Students’ motives for implementing M:	belief in benefits of using M	seeing the benefits for students in using M	92	An important aspect is for the teacher to feel and see the benefit for the students, this pushes the teacher to continue in methods such as this and to focus on developing thinking.
		lack of belief in the benefit of using M in terms of achievement	23	Their lack of belief in the benefits of using metacognitive thinking, whether it was in their academic achievement or on future studies
		lack of belief in benefits of using M in terms of future studies	23	Their lack of belief in the benefits of using metacognitive thinking, whether it was in their academic achievement or on future studies
		Teachers sees benefits of using M	22	They began to feel the benefits and their optimism increased, and thus, as soon as I told them that the method will be applied next lesson, they appeared to be happy, and with that most of the students seemed to be optimistic.
	motivation and students' achievement level	focus on results more than M	76	Some students feel that their own methods are more beneficial to them, and get them good grades, and thus there is no need for them to try other ways.
		motivation and students' achievement level	46	I believe that students of weak academic achievement are the ones who benefit least from this teaching style, the reason for this being a smaller body of prior knowledge and experiences to draw on.
		Reasons for students’ lack of enthusiasm	22	The reason for the lack of enthusiasm from some students that if they were outstanding students (in terms of grades), this could affect their enthusiasm for cooperative learning with others of lesser ability.

Appendix 12

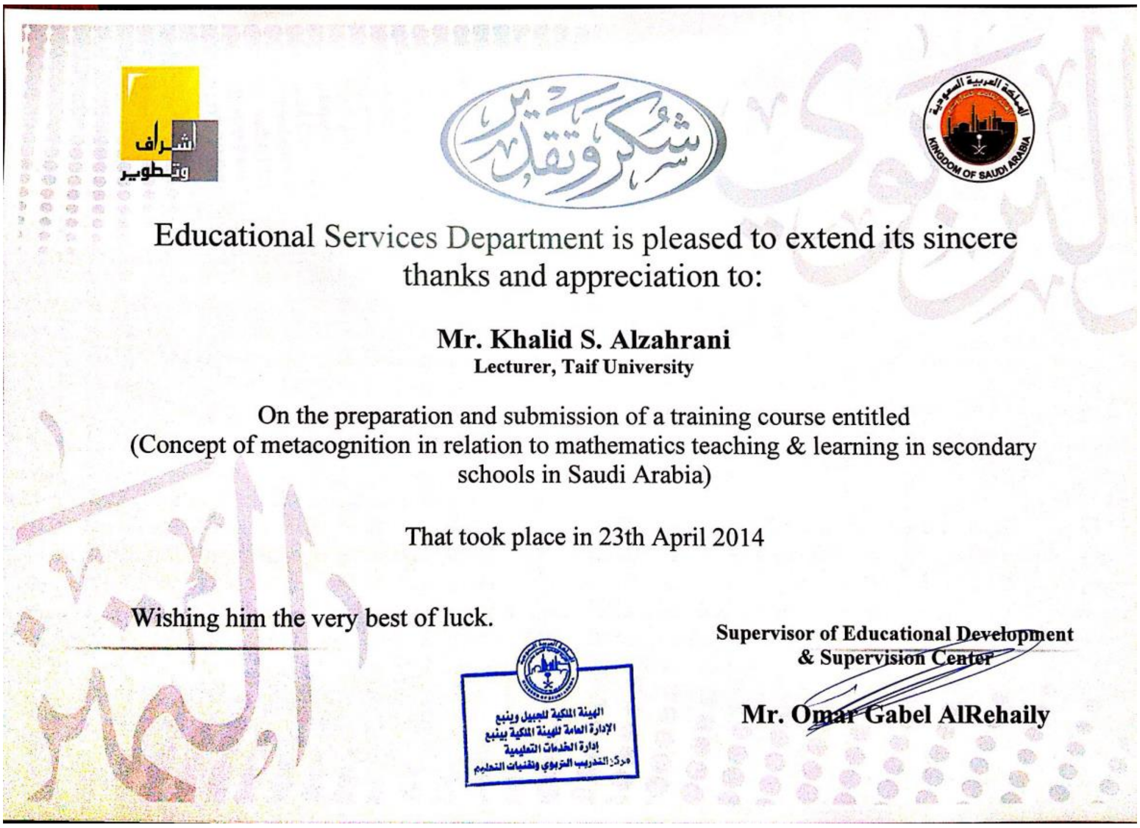
The themes that arose across data analysis:

Method	Participants	Themes
Thematic findings of observations data	Teachers and students from both cases	Mathematics teaching strategies related to metacognition before the implementation of the IMPROVE programme
		Mathematics teaching strategies related to metacognition within the implementation of the IMPROVE programme
		Mathematics learning strategy of the students related to metacognition before the implementation of the IMPROVE programme
		Mathematics learning strategy of the students related to metacognition within the implementation of the IMPROVE programme
		Teacher and Student relationship before the implementation of the IMPROVE programme
		Teacher and Student relationship within the implementation of the IMPROVE programme
Thematic findings of teachers' interview	Teachers from both cases	Themes
		Teacher's understanding of metacognition before the implementation of the IMPROVE programme
		Teachers' understanding of metacognition after the implementation of the IMPROVE programme
		Mathematics teaching technique before the implementation of the IMPROVE programme
		Challenges to the implementation of metacognition after the implementation of the IMPROVE programme
		The teacher's requirements for the implementation of metacognition
		Cooperative learning and metacognition before the

		implementation of the IMPROVE programme
		Metacognition and cooperative learning after the implementation of the IMPROVE programme
Thematic findings of student interviews and focus group	Students from both cases	Theme
		Students' understanding of metacognition before the implementation of the IMPROVE programme
		Students' understanding of metacognition after the implementation of the IMPROVE programme
		Learning techniques used in mathematics before the implementation of the IMPROVE programme
		Challenges to the implementation of metacognition in mathematics learning
		Students' requirements for the implementation of metacognition
		Student-student relationship before the implementation of the IMPROVE programme
		Metacognition and cooperative learning after the implementation of the IMPROVE programme

Appendix 13

Educational training certificate



إشراف وتطوير

شكر وتقدير

KINGDOM OF SAUDI ARABIA

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Mr. Khalid S. Alzahrani
Lecturer, Taif University

On the preparation and submission of a training course entitled (Concept of metacognition in relation to mathematics teaching & learning in secondary schools in Saudi Arabia)

That took place in 23th April 2014

Wishing him the very best of luck.

Supervisor of Educational Development & Supervision Center

Mr. Omar Gabel AlRehaily

الهيئة الملكية للجبيل وينبع
الإدارة العامة للهيئة الملكية بينبع
إدارة الخدمات التعليمية
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