# Structural Vibration Serviceability: New Design Framework Featuring Human-Structure Interaction

**Erfan Shahabpoor[a], Aleksandar Pavic[b,c], Vitomir Racic[d,e]**

[a] INSIGNEO Institute for In-silico Medicine, The University of Sheffield, Department of Civil & Structural Engineering, Sir Frederick Mappin Building, Mappin Street, Sheffield, S1 3JD, UK.
[b] College of Engineering, Mathematics and Physical Sciences, Vibration Engineering Section, University of Exeter, North Park Road, Exeter EX4 4QF, UK.
[c] Full-scale Dynamics, ltd. Kay Building, North Park Road, Exeter EX4 4QF, UK.
[d] Department of Civil and Environmental Engineering, Politecnico di Milano, Piazza di Leonardo Da Vinci 32, Milano 20133, Italy.
[e] Department of Civil & Structural Engineering, The University of Sheffield, Sir Frederick Mappin Building, Mappin Street, Sheffield, S1 3JD, UK.

Corresponding author: Dr E. Shahabpoor
Department of Civil & Structural Engineering
University of Sheffield
Sir Frederick Mappin Building
Sheffield S1 3JD
E-mail: e.shahabpoor@sheffield.ac.uk
Tel: 0114-2225745
Fax: 0114-2225700

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## Abstract

Predicting the effect of walking traffic on structural vibrations is a great challenge to designers of pedestrian structures, such as footbridges and floors. This is mainly due to the lack of adequate design guidelines, which in turn can be blamed on poor research findings. Even the fundamental data are very rare and limited. This study proposes a new and more reliable method for serviceability assessment of the vertical vibrations induced by multi-pedestrian walking traffic. Key novelties include modelling the natural *variability* of the walking forces and the human bodies, as well as their individual *interaction* with the supporting structure at their *moving location*. Moreover, a novel approach to vibration serviceability assessment (VSA) is proposed based on the actual level of vibration experienced by each pedestrian, rather than the typical maximum vibration response at a fixed point. Application of this method on two full-scale footbridge structures have shown that, with a suitable calibration of human model parameters, the proposed method can predict the occupied structure modal frequency with less than 0.1% error and - more importantly - modal damping ratio with less than 1% error. The new method also estimated the structural responses with considerably less error (5-10%) compared to a selection of current design guidelines (200-500%). The proposed VSA method is not suitable for hand-based calculations. However, if coded and materialised as a user-friendly software, it can be incorporated into design guidelines and used by consultants in everyday engineering practice.



Keywords: human-induced vibration; walking human model; pedestrian traffic; footbridge;

# 1 Introduction

Models of pedestrian dynamic loading used in contemporary vibration serviceability assessment typically describe the vertical walking excitation as a vertical force that does not depend on structural vibrations [1]. The simplest models, such as those presented by FIB [2], ISO 10137 [3], French design guideline [4] and UK National Annex to Eurocode 1 [5], also assume that an individual walking force is periodic and presentable by a Fourier series. The frequency content of such a simple force model typically contains up to the first four dominant harmonics [1]. The design procedures usually require that one of the harmonics matches the frequency of a target vibration mode of the structure to create resonance, i.e. the worst case scenario yielding the maximum vibration response. To account for the imperfect synchronisation between individuals in a group or crowd, the walking force of a multi-pedestrian traffic is calculated by multiplying a sum of the individual forces with factor(s) which commonly depend only on the number of pedestrians on the structure [1].

A significant move towards a more realistic estimation of the vibration response was made only recently, by taking into account inter- and intra- subject variability of the pedestrians in statistical models of their walking force [6-12]. This has increased considerably the fidelity of the walking force models. Yet, these still do not account for human-structure interaction (HSI), despite its widely recognised importance to reliable prediction of the vibration response [13-15]. In the context of the present study, HSI refers to the effect of walking bodies on the dynamic properties of the occupied structure (i.e. modal mass, stiffness and damping).

The UK recommendations for the design of permanent grandstands [16] are the only guidelines that explicitly require taking into account the interaction of both passive and active people with the grandstand they occupy and excite by jumping or bouncing in the vertical direction. Based on the model proposed by Dougill, et al. [17], this guideline

suggests two single-degree-of-freedom (SDOF) systems attached to a SDOF model of the empty structure to simulate the aggregated effect of passive (mostly *sitting*) and active (mostly *jumping/bouncing*) people. Despite the satisfactory performance of this explicit modelling approach [18, 19], no other vibration serviceability design guideline has yet adopted a similar modelling concept to account for the HSI due to people *walking*.

The vibration serviceability assessment (VSA) method proposed in this paper (from now on referred to as *interaction-based VSA method*) has been developed to account for the following five main challenges when assessing the effects of walking people on structures:

1) The human-structure interaction;
2) Variability of the mass, stiffness and damping of the moving human body and the walking force due to inter- and intra-subject variability;
3) Variability of pedestrian traffic characteristics, such as traffic *volume* and *regime* (spatially unconstraint/constraint, group, etc.)
4) Varying *location* of each walking pedestrian on the structure, and
5) The actual level of vibration experienced by each pedestrian at their continuously moving location on the structure rather than the vibration response of the structure at a fixed point.

The detailed description of the proposed method is presented in Section 2. In Section 3, the sensitivity of the outputs of this method to uncertainties of its inputs is studied. Applications of the proposed *interaction-based VSA method* on two full-scale footbridge structures are described in Section 4, and the relevant response calculations are compared to a selection of current design guidelines. Finally, conclusions are presented in Section 5.

# 2 Description of assessment method

The proposed interaction-based VSA method involves four steps. In the first step, the effects of HSI are analysed by estimating the *occupied* structure modal properties: natural frequency $f_{os}$[Hz], modal damping ratio $\zeta_{os}$ [-] and modal mass $m_{os}$[kg]. In the second step, for each relevant mode of the occupied structure, the total modal force due to pedestrian traffic is calculated. This is done by scaling each individual's walking force by the amplitude of the corresponding mode shape, and superimposing such scaled walking forces of all pedestrians according to their arrival time on the structure. In the third step, the modal response of the structure is computed for each relevant mode of vibration, using the calculated modal walking force/s and the *occupied* structure modal properties. Finally, these modal vibration responses are used to calculate the physical vibration levels perceived by each pedestrian at their *continuously changing location* as they walk along the structure. This is deemed to be more appropriate and realistic than using the percentage of time that bridge response is within an acceptable range at a particular fixed location, which may or may not have a pedestrian on it.

It should be noted that the description of the interaction-based VSA method in this study is based on a uniformly distributed un-constrained traffic scenario. However, any traffic pattern/scenario can be simulated using this method by modifying the steps to reflect that pattern. For instance, a constrained walking due to heavy traffic can be simulated by reducing the average walking speed of the crowd, increasing the arrival rate and applying corresponding changes on the walking force and parameters of the SDOF walking human model.

## 2.1 Input parameters

The input parameters used in the interaction-based VSA method can be divided into four categories. The first category comprises the properties of mode 'j' of the *empty* structure:

modal mass $m_{es,j}$, frequency $f_{es,j}$ and damping ratio $\zeta_{es,j}$. In the second category are the parameters of the walking human SDOF model: mass $m_h$, natural frequency $f_h$ and damping ratio $\zeta_h$. The SDOF mass-spring-damper model of walking humans proposed by Shahabpoor, et al. [20] was used in this study (Fig. 1). The authors proposed normal distributions with mean and standard deviations of $\mu$=2.85Hz and $\sigma$= 0.34Hz for natural frequency $f_h$, and $\mu$=0.295 and $\sigma$= 0.047 for damping ratio $\zeta_h$ of the SDOF human model. Mass $m_h$ can either be generated using a statistical distribution for a certain human population, or assumed to be equal to the average mass of the occupants. Stiffness $k_h$ can be calculated using Equation (1):

$$k_h = m_h(2 \times \pi \times f_h)^2 \quad \text{(Eq. 1)}$$

The third category of the input parameters is related to the walking traffic. These parameters define the loading scenario in statistical terms. An appropriate traffic pattern first needs to be defined. For instance, it could be a stream of pedestrians with arrival rate $r_a$ [pedestrians/time unit] at the bridge and walking speed $v_w$ [m/s] defined by their corresponding statistical distributions.

The last category of inputs is individuals' walking force, which can be either measured or synthetically generated, as described in Section 2.3.

## 2.2 Step 1: Human-structure Interaction

Important effects of human-structure interaction on modal properties and vibration response of a structure are studied parametrically by the authors elsewhere [21]. The mass of a stationary human body accelerates when exposed to structural vibration, and applies interaction force on the structure [22]. The same applies to the moving body, in which case an additional ground reaction force is created due to the base vibration [23]. Similar to a tuned mass damper, these interaction forces manifest as changes in the modal frequency

(i.e. mass and/or stiffness) and damping of the structure. This is because the interaction forces have components proportional to acceleration, velocity and displacement, as well as components independent from the structural movement [1].

In reality, pedestrian locations on the structure and, therefore, their interaction with structure, are changing with time. The interaction-based VSA method uses a Monte-Carlo iterative process based on *sampling distribution* concept [24] to estimate the *average* effect of HSI on modal properties of the empty structure.

In statistics, a sampling distribution or finite-sample distribution is the probability distribution of a given statistic based on a random sample drawn from a larger data population [24]. According to the statistical inference theory, where the statistic is the sample mean and samples are uncorrelated, the standard deviation of the sampling distribution of a statistic, usually referred to as the standard error of that quantity, is inversely proportional to the square root of the number of samples *N* [24].

The interaction-based VSA method takes into account the HSI effects on the structure by replacing the empty structure modal properties ($f_{es}$, $\zeta_{es}$ and $m_{es}$) with the corresponding occupied structure modal properties ($f_{os}$, $\zeta_{os}$ and $m_{os}$). Based on the statistical inference theory, if the occupied structure modal properties $f_{os}$, $\zeta_{os}$ and $m_{os}$ (i.e. samples) are calculated for an increasing number of different walking traffic patterns, the average value of each of $f_{os}$, $\zeta_{os}$ and $m_{os}$ (i.e. statistics), gradually converges to their mean value i.e. the standard error of the statistics decreases.

The following steps describe the procedure to estimate the mean values of $f_{os}$, $\zeta_{os}$ and $m_{os}$:

Firstly (Step 1.1), the number of people walking on the structure is selected. This can be based on a statistical distribution of arrival rates and the average crossing time (i.e. the average time needed for a pedestrian to cross the structure). For instance, where the arrival

rate is 10 pedestrians per minute and the average crossing time is 2 minutes, under steady state conditions, there would be on average 20 people walking on the structure at any given time, assuming that their walking speeds are equal and constant.

Secondly (Step 1.2), a location must be assigned to each person, either randomly (e.g. assuming the uniform distribution), or based on a particular pattern that the loading scenario may require. The location assigned to each person is assumed constant (*stationary*) for that particular moment of time. This is the same as an imaginary case where people are walking on a series of treadmills installed at fixed locations on the structure, in which case their locations on the structure do not change while walking (Fig. 2).

The multi-degree of freedom (MDOF) model of a 'stationary' multi-pedestrian walking traffic-structure system is developed in Step 1.3. An SDOF model is used to simulate each walking individual on the structure (Fig. 1). Similarly, an SDOF model is used to simulate one mode of the empty structure at a time. The effects of the constant location of each person on the modal properties of the occupied structure are taken into account by using the structure mode shape ordinate at the location of each person [20].

By coupling a number of SDOF systems representing walking individuals and an SDOF system representing a mode of the structure, the proposed modelling approach essentially bridges the modal domain and the physical domain. Therefore, the modal properties of the structure and its mode shape have to have 'physically' meaningful values. To ensure that modal properties of the crowd-structure system are found with the same scaling as for the empty structure the *unity-normalised* mode shapes at the structural DOF must consistently be used throughout the calculations.

Being stationary in the current time-step, the walking traffic-structure system shown in Fig. 1 can be treated as a conventional multiple degree of freedom (MDOF) system [25].

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \quad \text{(Eq. 2)}$$

The following modified system of equations of motion (Eq. 3) can be used to account for the locations of the pedestrians:

$$\begin{bmatrix} \mathrm{m_{es,j}} & 0 & 0 & \cdots & 0 \\ 0 & \mathrm{m_{h1}} & 0 & \cdots & 0 \\ 0 & 0 & \mathrm{m_{h2}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \mathrm{m_{hn}} \end{bmatrix} \begin{bmatrix} \ddot{x}_{os,j}(t) \\ \ddot{x}_{h1}(t) \\ \ddot{x}_{h2}(t) \\ \vdots \\ \ddot{x}_{hn}(t) \end{bmatrix} +$$

$$\begin{bmatrix} \mathrm{c_{es,j}} + (\mathrm{c_{h1}} \times \emptyset_{1j}) + (\mathrm{c_{h2}} \times \emptyset_{2j}) + \cdots + (\mathrm{c_{hn}} \times \emptyset_{nj}) & -(\mathrm{c_{h1}} \times \emptyset_{1j}) & -(\mathrm{c_{h2}} \times \emptyset_{2j}) & \cdots & -(\mathrm{c_{hn}} \times \emptyset_{nj}) \\ -(\mathrm{c_{h1}} \times \emptyset_{1j}) & \mathrm{c_{h1}} & 0 & \cdots & 0 \\ -(\mathrm{c_{h2}} \times \emptyset_{2j}) & 0 & \mathrm{c_{h2}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -(\mathrm{c_{hn}} \times \emptyset_{nj}) & 0 & 0 & \cdots & \mathrm{c_{hn}} \end{bmatrix} \begin{bmatrix} \dot{x}_{os,j}(t) \\ \dot{x}_{h1}(t) \\ \dot{x}_{h2}(t) \\ \vdots \\ \dot{x}_{hn}(t) \end{bmatrix} +$$

$$\begin{bmatrix} \mathrm{k_{es,j}} + (\mathrm{k_{h1}} \times \emptyset_{1j}) + (\mathrm{k_{h2}} \times \emptyset_{2j}) + \cdots + (\mathrm{k_{hn}} \times \emptyset_{nj}) & -(\mathrm{k_{h1}} \times \emptyset_{1j}) & -(\mathrm{k_{h2}} \times \emptyset_{2j}) & \cdots & -(\mathrm{k_{hn}} \times \emptyset_{nj}) \\ -(\mathrm{k_{h1}} \times \emptyset_{1j}) & \mathrm{k_{h1}} & 0 & \cdots & 0 \\ -(\mathrm{k_{h2}} \times \emptyset_{2j}) & 0 & \mathrm{k_{h2}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -(\mathrm{k_{hn}} \times \emptyset_{nj}) & 0 & 0 & \cdots & \mathrm{k_{hn}} \end{bmatrix} \begin{bmatrix} x_{os,j}(t) \\ x_{h1}(t) \\ x_{h2}(t) \\ \vdots \\ x_{hn}(t) \end{bmatrix} =$$

$$\begin{bmatrix} F_{ex,j}(t) + (F_{h1}(t) \times \emptyset_{1j}) + (F_{h2}(t) \times \emptyset_{2j}) + \cdots + (F_{hn}(t) \times \emptyset_{nj}) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \text{(Eq. 3)}$$

where $m_{es,j}$ , $c_{es,j}$ and $k_{es,j}$ are the modal mass, damping and stiffness for the $j^{\text{th}}$ mode of the empty structure and $m_{hi}$ , $c_{hi}$ and $k_{hi}$ are those of the walking individuals. Viscous damping is assumed for SDOF walking human models. $\ddot{x}_{os,j}(t)$, $\dot{x}_{os,j}(t)$ and $x_{os,j}(t)$ are, respectively, the acceleration, velocity and displacement response of the occupied structure DOF. As one mode of the occupied structure ($j$) is simulated at a time, $\ddot{x}_{os,j}(t)$, $\dot{x}_{os,j}(t)$ and $x_{os,j}(t)$ represent the *modal* response of the occupied structure. Similarly, $\ddot{x}_{hi}(t)$, $\dot{x}_{hi}(t)$ and $x_{hi}(t)$ represent the acceleration, velocity and displacement of the $i^{\text{th}}$ walking person DOF. $\mathrm{F}_{ex,j}(t)$ is the mode '$j$' modal force (if any), due to an external force acting on the structural DOF, and $\mathrm{F}_{hi}(t)$ is a walking force of person '$i$' on a stiff surface. Meanwhile, $\emptyset_{ij}$ is the ordinate of the '$j^{\text{th}}$' mode shape of the structure at the location of person '$i$' in the current time-step.

The damping matrix in Equation (3) is normally not proportional. Therefore, the conventional formulation of the proportionally-damped eigenvalue problem [25] will not yield modal vectors (eigenvectors) that uncouple the equations of motion of the system. The state-space technique used here to overcome this problem was first documented by Frazer, et al. [26] and involves the reformulation of the original equations of motion, for an N-degree of freedom system, into an equivalent set of 2N first order differential equations.

In the first step, a new coordinate vector $\{y(t)\}$ containing displacement $\{x(t)\}$ and velocity $\{\dot{x}(t)\}$ is defined:

$$\{y(t)\} = \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix} \quad \text{(Eq. 4)}$$

Then Equation (2) is re-written into the following form for modal analysis:

$$\begin{bmatrix} [C] & [M] \\ [M] & [0] \end{bmatrix} \{\dot{y(t)}\} + \begin{bmatrix} [K] & [0] \\ [0] & [-M] \end{bmatrix} \{y(t)\} = \{0\} \quad \text{(Eq. 5)}$$

Where [M], [C] and [K] matrices are the mass, damping and stiffness matrices of the traffic-structure system, accordingly, as defined in Equation (3).

By defining **α** and **β** matrices as:

$$\boldsymbol{\alpha} = \begin{bmatrix} [C] & [M] \\ [M] & [0] \end{bmatrix}, \quad \text{(Eq. 6)}$$

$$\boldsymbol{\beta} = \begin{bmatrix} [K] & [0] \\ [0] & [-M] \end{bmatrix} \quad \text{(Eq. 7)}$$

Equation (5) leads to a standard eigenvalue problem in the form of Equation (8).

$$(\mathbf{s\alpha}+\mathbf{\beta})\boldsymbol{\psi}=0 \quad \text{(Eq. 8)}$$

The complex valued eigenvectors $\boldsymbol{\psi}$ (mode shapes) and real valued eigenvalues $\mathbf{s}$ (modal frequencies) in Equation (8) can be found by solving the corresponding characteristic polynomial equation:

$$\det(\mathbf{s\alpha}+\mathbf{\beta}) = 0 \quad \text{(Eq. 9)}$$

This yields natural frequencies, modal damping ratios and modal masses of the non-proportionally damped pedestrian traffic-structure MDOF system. Further discussion of modal analysis of systems with non-proportional damping is beyond the scope of this paper.

The MDOF system in Fig. 1 has n+1 modes of vibration. The *dominant mode* of vibration is defined as the mode with the maximum response at the 'structure' degree of freedom.

By repeating the process of eigenvalue extraction (Steps 1.1 – 1.3) for different combinations of pedestrian traffic parameters (number of pedestrians on the structure, their location, etc.) and calculating the average values of $f_{os}$, $\zeta_{os}$ and $m_{os}$ corresponding to increasing number of iterations, they each gradually converge to constant values. Fig. 3 illustrates the convergence of $f_{os}$ and $\zeta_{os}$ for a typical simulation involving 800 different locations of pedestrians. These converged modal properties of the occupied structure are then used in the response calculation instead of those of the empty structure. The calculation is not computationally demanding and can be completed within several seconds using a standard PC configuration.

## 2.3 Step 2: Generating modal force of multi-pedestrian walking traffic

The second step is to generate the modal force due to multi-pedestrian walking traffic. Most of the parameters of the walking traffic, such as arrival rate $r_a$, arrival time $t_a$, location, walking speed $v_w(t)$ and

261 walking force $F_w(t)$ of individuals, are time-varying and inherently stochastic. This makes
262 it impossible to predict the exact traffic force at any particular time. The way forward is to
263 treat it statistically. The step-by-step procedure for generating modal force due to walking
264 traffic is elaborated in the following paragraphs.

265 First (Step 2.1), the duration of the simulated vibration response is selected randomly. A
266 criterion is introduced in Step 4 (Section 2.5) to check whether the selected duration is
267 sufficiently long. In physical terms, this criterion ensures that the structure experiences
268 enough variations of the walking traffic loading necessary to assess the vibration
269 serviceability of the structure. In case the duration in Step 2.1 proves to be insufficient in
270 Step 4, it must be increased and Steps 2-4 repeated.

271 In Step 2.2, the number of people entering the structure needs to be selected, using a
272 statistical distribution of their arrival rate. Then, the arrival time is assigned randomly to
273 each pedestrian. For instance, assuming uniform distribution for an arrival rate of 4
274 pedestrians per minute, entering the structure between minute 12 and 13 of the simulation,
275 their random arrival times could be 12:03, 12:12, 12:38 and 12:51.

276 In Step 2.3, a constant walking speed ($v_w$) needs to be selected for each pedestrian, using a
277 statistical distribution, such as the one reported by Zivanovic [27]. It is assumed that $v_w$ is
278 constant for each pedestrian, but it varies between pedestrians. Having the walking speed
279 and the length of structure i.e. walking path, the duration of walking of each pedestrian (so
280 called 'crossing time') can be computed. For instance, if the pedestrian speed is $v_w$=1.8 m/s
281 and the structure length is 36 meters, it takes 20s for that person to cross.

282 A walking force needs to be assigned to each pedestrian in Step 2.4. The duration of the
283 walking force for each person should be equal to the crossing time of that person. As
284 previously mentioned, either an experimentally recorded [28] or a synthetically generated
285 walking force can be used in the simulation. If a walking force is to be generated artificially,

it is important to use a method that takes into account the inter- and intra-subject variability of the walking force and realistically simulates its frequency contents, such as those proposed by Zivanovic, et al. [7] and Racic and Brownjohn [6].

As pedestrians walk along the structure, their location and the level of interaction with it change. To account for this, the walking force of each individual '$i$' $F_{w,hi}(t)$, entering the structure at $t = t_{ai}$ and leaving it at $t = t_{bi}$ is scaled with $\emptyset_{ij}(t)$ which is the amplitude of the unity-scaled shape of mode '$j$' of the structure at the instantaneous location of the moving pedestrian '$i$' at time '$t$'. This yields the modal walking force of human '$i$' exciting mode '$j$' of the structure $F_{w,hi\rightarrow sj}(t)$:

$$F_{w,hi\rightarrow sj}(t) = \begin{cases} 0 & t < t_{ai} \\ \emptyset_{ij}(t) \times F_{w,hi}(t) & t_{ai} \le t \le t_{bi} \\ 0 & t_{bi} < t \end{cases}, \quad \text{(Eq. 10)}$$

Where $\emptyset_{ij}(t)$ is defined as:

$$\emptyset_{ij}(t) = \begin{cases} 0 & t < t_{ai} \\ \sin\left(\frac{t-t_{ai}}{t_{bi}-t_{ai}} j\pi\right) & t_{ai} \le t \le t_{bi}, \\ 0 & t_{bi} < t \end{cases} \quad \text{(Eq. 11)}$$

assuming that the structure mode shape is sinusoidal and the walking speed is constant. For other mode shapes, such as those calculated via FE analysis, a mode shape vector can be used in Equation (11).

Fig. 4a presents a typical walking force of an individual, scaled by the amplitude of the first unity-normalised mode shape of a simply supported beam structure. This person crosses the structure in 10.4s. It is assumed that the empty structure mode shape does not change when occupied by walking people [20].

Finally, in Step 2.6, the modal walking forces of all '$n$' pedestrians are superimposed, based on their arrival time on the structure, to generate the modal force of the walking traffic experienced by the mode '$j$' of the structure $F_{w,t\to sj}(t)$:

$$F_{w,t\to sj}(t) = F_{w,h1\to sj}(t) + F_{w,h2\to sj}(t) + \cdots + F_{w,hn\to sj}(t) \quad \text{(Eq. 12)}$$

Fig. 4 presents a typical superposition process, where modal forces due to walking of three individual pedestrians (Fig. 4a, b and c) are superimposed to generate the modal force of the walking traffic (Fig. 4d). Pedestrians 1, 2 and 3 arrive on the structure at $t_a$= 2, 6 and 8s respectively and each take 10.4s to cross the structure. Steps 2.5 and 2.6 need to be repeated for all relevant modes of structural vibration.

## 2.4 Step 3: Calculating structural modal response

Here, the modal force of the walking traffic $F_{w,t\to sj}(t)$, calculated in Step 2 (Section 2.3), is applied on the corresponding mode '$j$' of vibration ($m_{os,j}$, $\omega_{os,j}$ and $\zeta_{os,j}$) of the occupied structure, calculated in Step 1 (Section 2.2), to calculate the modal response. This can be done using a conventional closed form method such as convolution or numerical integration such as Newmark-beta and Runge-Kutta methods [25]. Step 3 is repeated for all modes and the resulting modal responses are calculated.

## 2.5 Step 4: Serviceability assessment

The Cumulative Distribution Function (CDF) of an acceleration response at a particular pre-defined fixed location on the structure - referred to as *fixed-location (FL)* CDF in this paper - is commonly used to assess vibration serviceability [7,8]. It provides a probability of non-exceedance for any particular response amplitude at a specific fixed location on the structure [29]. However, FL CDF is misleading in scenarios where traffic volume is not constant on the structure. Moreover, it does not take into account the location of pedestrians

on the structure and the changing level of vibration they actually experience while in motion.

To address these issues, the novel concept of *moving-location (ML)* CDF is introduced here and used in the interaction-based VSA method. ML CDF addresses the disadvantages of the FL CDF by taking into account the number and moving location of pedestrians on the structure at each moment of time. ML CDF further takes into account the level of acceleration response experienced by each pedestrian while moving over the structure rather than structural response at a fixed location, which may or may not be experienced by pedestrians.

To calculate the ML CDF, in the first step, the time-history of the vibration levels experienced by each pedestrian needs to be simulated. Fig. 5 shows the process of calculating the time-history of the acceleration response of structure $\ddot{x}_{s\to hi}(t)$ experienced by a typical pedestrian '$i$' crossing a beam-like simply supported structure. Only the first two vertical modes of structural vibration were considered relevant for this example. Each pedestrian's experience of vibration response created by each mode (blue traces in Fig. 5a and Fig. 5b) can be calculated by scaling the modal response $\ddot{x}_{os,j}(t)$ (grey traces in Fig. 5a and Fig. 5b) of the structure (pertinent to the time window during which the pedestrian is walking on the structure) by its corresponding $\emptyset_{ij}(t)$ (red traces in Fig. 5a and Fig. 5b). The total acceleration response of the structure experienced by moving pedestrian '$i$' $\ddot{x}_{s\to hi}(t)$ can be calculated by adding up his/her experience of vibration response due to all relevant modes, as shown in Equation (13):

$$\ddot{x}_{s\to hi}(t) = \begin{cases} 0 & t < t_{ai} \\ \sum_j \ [\emptyset_{ij}(t) \times \ddot{x}_{os,j}(t)] & t_{ai} \le t \le t_{bi} \\ 0 & t_{bi} < t \end{cases} \quad \text{(Eq. 13)}$$

For example, the pedestrian shown in Fig. 5 enters the structure at $t_a$=6s, and they need 10.4s to cross the beam structure. Fig. 5a and Fig. 5b show the time histories of the physical vibration (blue traces in Fig. 5) that the pedestrian experiences due to mode 1 and 2, respectively. The response of each mode '*j*' $\ddot{x}_{os,j}(t)$ (grey curve) is multiplied by the corresponding $\emptyset_{ij}(t)$ (red curve), starting at $t_a$=6s, with a duration of 10.4s to make this calculation. The two time histories are then superimposed in time to generate the time history of the total physical vibration experienced by the pedestrian (Fig. 5c).

If this process is repeated for all pedestrians crossing the structure and the time histories of their experiences are connected together in series, as shown in Fig. 5d, the time history containing levels of vibration that the pedestrian traffic experienced during their crossing $\ddot{x}_{s\to t}(t)$ is created. Such a method of calculating $\ddot{x}_{s\to t}(t)$ not only takes into account the actual level of vibration each pedestrian experiences based on their moving location on the structure, but also takes into account the duration in which each pedestrian is exposed to a certain level of vibration.

*ML CDF* is defined as the CDF of all samples in $\ddot{x}_{s\to t}(t)$ time history. For any particular amplitude of $\ddot{x}_{s\to t}(t)$, ML CDF ordinate provides the probability that a pedestrian does not experience a vibration level higher than the selected amplitude.

Using a typical example and assuming a single mode response, Fig. 6 compares the performance of the FL (at anti-node) and ML CDFs in the assessment of vibration serviceability. The acceleration response of the structure is given for 60 minutes for two loading scenarios A and B. As can be seen in Fig. 6a and Fig. 6c, the mean arrival rate in scenario A is constant (20 pedestrians / minute) whereas in scenario B it shows a 6 fold

increase from 10 peds/min to 70 peds/min in the last 10 minutes. A considerable difference between FL and ML CDFs is noticeable in both scenarios (Fig. 6b and Fig. 6d). In Scenario A, neglecting the location of people on the structure results in an over-estimation of the response in the FL CDF (Fig. 6b - blue trace). In Scenario B, the change of traffic volume amplifies the over-estimation problem (Fig. 6d - blue trace). For example, based on Fig. 6d, if 0.2 m/s$^2$ is selected arbitrarily as the maximum acceptable response, the maximum structural response at the fixed location would be acceptable for only 60% of the time (FL CDF) with, or more likely, without having any pedestrians experiencing that vibration. However, according to ML CDF the 0.2 m/s$^2$ response is acceptable for 80% of the total time, during which pedestrians experience vibrations while crossing the footbridge. There is a considerable difference between FL and ML CDF interpretations

As discussed in Section 2.3, since the pedestrian traffic on the structure is being treated statistically (walking speed, location, arrival time, etc.), the duration of the response simulation needs to be sufficiently long to ensure that the structure has experienced enough variations of the walking traffic loading necessary to assess its vibration serviceability. To check this sufficiency, the *sampling distribution* concept [24] is used again.

In statistical terms, the calculated time-history of traffic vibration experience $\ddot{x}_{s\to t}(t)$ and its corresponding ML CDF parameters is a finite sample from a larger population of possible vibration responses experienced by pedestrians. Assuming the statistic as the *mean* response amplitudes corresponding to 95%, 85%, 75% and 50% probability of non-exceedance ($a_{95\%}$, $a_{85\%}$, $a_{75\%}$ and $a_{50\%}$) corresponding to the CDF, the standard deviation (error) of these mean response amplitudes is inversely proportional to square root of data samples *N* (which is proportional to the duration of the response simulation) as *N* increases. In other words, the longer the duration of the response simulation, the $\ddot{x}_{s\to t}(t)$ contains the vibration experience of more pedestrians and therefore can represent more accurately and reliably the vibration response of the structure in probabilistic terms.

For the interaction-based VSA method suggested in this study, it is proposed to select the duration of the response simulation (Section 2.3) in a way to get the standard errors '$\bar{\sigma}$' less than 5% of the corresponding $a_{95\%}$, $a_{85\%}$, $a_{75\%}$ and $a_{50\%}$ mean values. The standard errors $\bar{\sigma}$ of the $a_{95\%}$, $a_{85\%}$, $a_{75\%}$ and $a_{50\%}$ values of the response ML CDF can be checked by monitoring their variations for increasing the length of the time window of the response being analysed. Fig. 7 presents a typical fluctuation of the mean $a_{95\%}$, $a_{85\%}$, $a_{75\%}$ and $a_{50\%}$ for up to 14 hours of the simulated response. The length of the time window of vibration response ($t_w$) (and therefore number of samples $N$) used for calculating $a_{95\%}$, $a_{75\%}$, $a_{75\%}$ and $a_{50\%}$ was increased in each iteration by 75s, yielding: $t_{w1}$=75s, $t_{w2}$=150s, $t_{w3}$=225s, etc. In the case of the response illustrated in Fig. 7, $\bar{\sigma}$ of the mean $a_{95\%}$, $a_{85\%}$, $a_{75\%}$ and $a_{50\%}$ reduced to below 5% of their mean value after fewer than 500 iterations. This is equivalent to 10h and 25mins of the simulated response, which a standard office PC can process in just a couple of minutes. . If $\bar{\sigma}$ values do not reduce to less than 5% of their mean value at the end of the simulation, the simulation duration determined in the Step 2 needs to be increased and Steps 2-4 repeated until $\bar{\sigma}$ meet the 5% error criteria.

# 3 Sensitivity analysis

As this is a new and untested methodology, this section examines the sensitivity of the outputs of the proposed method to its inputs. The human model parameters ($f_h$, $\zeta_h$ and $m_h$), mean arrival rate $r_a$ and walking speed $v_w$ were selected as input parameters. The selected outputs were the occupied structure modal parameters $f_{os}$ and $\zeta_{os}$, response amplitude with a 95% chance of non-exceedance $a_{95\%}$ and RMS of the total response time-history $a_{rms}$. The selected input parameters were varied by ±25% or ±30% and their effects on the outputs were analysed. In order to compare the sensitivity of each output parameter with different inputs, all parameters were normalised by the corresponding baseline (minimal) value. The baseline values were adopted from a real-world structure and a realistic traffic scenario as

follows: $f_{h,base}$=2.85Hz, $\zeta_{h,base}$ =0.295, $m_{h,base}$ =75kg, $r_{a,base}$=0.35peds/s, $v_{w,base}$=1.38m/s, $f_{os,base}$=2.03Hz, $\zeta_{os,base}$=0.007, $a_{95\%,base}$=0.341m/s$^2$ and $a_{rms,base}$=0.155m/s$^2$.

Fig. 8 presents sensitivity curves for each normalised output parameter: $f_{os}/f_{os,base}$, $\zeta_{os}/\zeta_{os,base}$, $a_{95\%}/a_{95\%,base}$ and $a_{rms}/a_{rms,base}$. The horizontal axis shows the normalised input parameters $f_h/f_{h,base}$ , $\zeta_h/\zeta_{h,base}$, $m_h/m_{h,base}$, $r_a/r_{a,base}$, and $v_w/v_{w,base}$. As can be seen in Fig. 8a, the natural frequency of the occupied structure, $f_{os,}$ shows low sensitivity to the variation of all input parameters. On the other hand, Fig. 8b shows that the occupied structure damping ratio $\zeta_{os}$ is highly sensitive to the human model natural frequency $f_h$ when $f_h$ is very close to the modal frequency of the empty structure $f_{es}$. For instance, when $f_h/f_{h,base}$ = 0.8 ($f_h$=2.28 Hz and relatively close to $f_{es}$ =2.04 Hz), $\zeta_{os}$ increases by 65% compared to its base value $\zeta_{os\ base}$ (i.e. $\zeta_{os}/\zeta_{os,base}$ =1.65). When $f_h$ and $f_{es}$ are not very close, $\zeta_{os}$ is not very sensitive to $f_h$. This also yields the high sensitivity of $a_{95\%}$ and $a_{rms}$ to $f_h$ (blue curve in Fig. 8c and d) when $f_h$ and $f_{es}$ are very close. Apart from the effects of $f_h$, a 30% variation in the rest of the input parameters ($m_h$, $\zeta_h$, $r_a$ and $v_w$) changed the response up to only 10%. In this sense, the method shows a high level of robustness to uncertain inputs.

# 4 Experimental verification

To examine the performance of the interaction-based VSA method, a set of tests was carried out on two full-scale footbridges: a post-tensioned concrete footbridge at the University of Sheffield (Fig. 9a) and a steel box girder footbridge located in Podgorica, capital of Montenegro (Fig. 9b) [30]. The modal frequency, damping ratio and modal mass of the first vertical mode of the Sheffield footbridge are: 4.44Hz, 0.6% and 7,128kg, respectively [20]. For the Podgorica footbridge these parameters are: 2.04Hz, 0.26% and 58,000kg, respectively [30]. Both structures are very lightly damped, and have natural frequencies in the range excitable by walking forces. Moreover, their natural frequencies are close to the natural frequency of the walking human SDOF model. This yields a high

level of interaction between pedestrians and structure, based on the analogies presented by Shahabpoor, et al. [21]. In this study, only the first vertical bending mode of vibration was considered for both footbridges.

### 4.1 Vibration monitoring

Three tests were carried out on the Sheffield footbridge with three (Test 1), six (Test 2) and 10 (Test 3) pedestrians walking in a closed-loop path along the full length of the footbridge [20]. The participants were asked to walk at their normal speed and they were free to pass each other. Each test was run for at least 120s. The body mass of each pedestrian was measured using a medical scale. Moreover, in a separate set of tests their walking forces on a stiff surface were recorded using an instrumented treadmill [6]. A pair of PeCo laser pedestrian counters [31], installed at both edges of the footbridge over the walkway, was used to record in real-time the location, walking direction and walking speed of each individual on the structure. Statistical parameters of pedestrian traffic corresponding to Tests 1-3 are presented in Table 1. A normal distribution was found suitable to describe the walking speed of pedestrians. For the average walking speed of 1.28 m/s, an average pedestrian needed 8.4 s to cross the 10.8m support-to-support length of the footbridge. Detailed descriptions of the tests and statistical analyses of traffic parameters are presented elsewhere [32].

Similar to the Sheffield tests, three monitoring tests, referred to as Tests 4, 5 and 6 and each lasting 44 minutes, were carried out on the Podgorica footbridge under normal pedestrian traffic. The flow of traffic was recorded using two video cameras located at both ends of the footbridge and synchronised with the recorded acceleration response. Pedestrians' crossing time, average speed and pacing frequency and the number of people on the structure at any particular moment were found using time-stamped video footage [27]. The statistical parameters of the pedestrian traffics for these three tests are adopted from

Zivanovic [27] and are presented in Table 1. A normal distribution was found suitable to describe the walking speed and number of people on the footbridge, while a Poisson distribution was used to describe the arrival rate. The average speed of the pedestrians was found to be 1.39 m/s. This means that, on average, a person needs about 75s to cross this 104m long bridge. Detailed descriptions of the tests and statistical analyses of traffic parameters are presented elsewhere [27].

In both the Sheffield and Podgorica tests, the acceleration response of the structure was recorded at mid-span (the anti-node of the mode 1). The statistical parameters of the structural response for all tests are presented in Table 2.

### 4.2 Vibration serviceability assessment

The interaction-based VSA method was used to assess the vibration serviceability of both structures for all six tests. The results were compared with the counterparts obtained from widely used design guidelines. Table 3 presents the input parameters used in the interaction-based VSA method to simulate traffic in Tests 1-6. For the Sheffield tests (Tests 1-3), the walking forces of the test subjects recorded separately with the instrumented treadmill were used in the simulations. However, in the Podgorica tests (Tests 4-6) such data were not available. Hence, the walking forces were randomly selected from the database of 1,200 force records [6] so that the average static component (i.e. body weight) of the recorded walking forces was equal to the average weight of the pedestrians in each test. The mass $m_h$, natural frequency $f_h$ and damping ratio $\zeta_h$ of the SDOF walking human model, were adapted from Shahabpoor, et al. [20].

To estimate the modal parameters of the occupied structures (Step 1), 800 iterations were carried out for each of the six tests, with a varying number of people and their locations on the structure. Such calculated modal parameters for both occupied structures are presented in Table 4 for all six tests.

For each of the Tests 1-3 on the Sheffield footbridge, an identical setup (same people, equipment setup, walking path, walking speed, etc.) was used in a forced FRF measurement test. In these tests, the structure was excited in resonance using an electrodynamic shaker, connected to the structure at the anti-node of the target mode while test subjects were walking on the structure. The resulting FRFs from each of these tests were curve-fitted to find the occupied structure (experimental) modal properties. These values are reported in Table 4. The detailed description of these FRF tests and the identification procedure of occupied structure modal properties are presented in [32].

As can be seen in Table 4, the interaction-based VSA method has estimated the occupied modal properties of the Sheffield footbridge with very high accuracy. The factors leading to such a good performance of the method are as follows. Firstly, the Sheffield footbridge is a clean beam-like structure with very straightforward dynamics and accurately measured modal properties. Secondly, the tests were carried out under controlled laboratory conditions, resulting in very accurate walking traffic parameters used for HSI simulation. Finally, the human model parameters proposed in [20] and used in this study for human-structure simulations are the results of extensive studies carried out on this particular footbridge. Although the data pertinent to the Tests 1-3 are not used as part of these studies, it is expected that the method will work better than average on this footbridge. However, as can be seen in Table 4, the interaction-based method also performs well in estimating the damping ratio of the occupied Podgorica footbridge where none of the above listed conditions apply.

In total, 15 hours of structural response was simulated in Step 3 (see Section 2.4) for each test to ensure the standard error $\bar{\sigma}$ values of the mean $a_{95\%}$, $a_{85\%}$, $a_{75\%}$ and $a_{50\%}$ are below 5% of their mean values. The duration of the available experimentally *measured* responses (2 minutes for Tests 1-3 and 44 minutes for Tests 4-6), however, were found insufficient to get the standard errors below 5%, as discussed in Section 2.5. Therefore, the CDF of the

524 measured responses could not be directly compared with the CDF of the simulated
525 response.

526 For such scenarios, it is proposed that a conclusion of *statistical inference* theory called
527 *interval estimation* be used. Interval estimation uses sample data to calculate an *interval* of
528 possible (or probable) values of an unknown *population* parameter so that, under repeated
529 sampling of such datasets, such intervals would contain the true parameter value with the
530 probability at the stated *confidence level* [33, 34].

531 For the purpose of the proposed Interaction-based VSA method, the population is defined
532 as the full length of the simulated response with $\bar{\sigma} < 0.05\mu$ and the sample is a random block
533 (window) of data from this response. The length of each sample block is taken to be equal
534 to the corresponding *measured* response. For instance, for each of the Tests 1-3, the
535 corresponding 15 hours of the simulated response is the population and any randomly
536 selected 2-minute block from these 15h responses is sample data. Similarly, for Tests 4-6,
537 any randomly selected 44-minute block of the corresponding 15h of the simulated
538 responses is sample data.

539 For each test, all possible sample data (2 minutes duration for Tests 1-3 and 44 minutes
540 duration for Tests 4-6) with a maximum 95% overlap were drawn from the corresponding
541 15h simulated structural response (population). The CDF of each of these sample data were
542 calculated. For each response value on the horizontal axis of the CDF, the confidence
543 interval $[\mu-2\sigma, \mu+2\sigma]$ is calculated using the corresponding values on all samples' CDFs.
544 The lower and upper limits of the confidence intervals form two new CDF curves (Fig. 10
545 - two dashed red curves) representing the corresponding lower and upper confidence limits
546 of the original CDF curves. Conceptually, this means that for any arbitrary 2-minutes
547 response measurement on the Sheffield footbridge and 44-minute measurements on the
548 Podgorica footbridge, the structural response CDF will be between the lower and upper

549 confidence limit CDFs (Fig. 10 - two dashed red curves) with approximately 95%
550 probability (assuming normal distribution of data points).

551 The results of these simulations are presented in Table 5 and Fig. 10 for Tests 1-6. As can
552 be seen in Fig. 10, the experimental CDF in all tests (blue curve) is within the predicted
553 confidence interval for the CDFs (two dashed red curves). In addition, it can be seen that
554 the experimental CDFs are closer but still above the lower confidence limit CDF. This
555 means that for any arbitrary response level, the probability of non-exceedance estimated by
556 the proposed model will be slightly lower than the actual value, resulting in a reasonably
557 conservative design.

558 To assess the significance of the HSI, identical simulations were repeated for each test
559 without taking into account the interaction effects. Here, empty structure modal properties
560 were used in simulations instead of the occupied structure modal properties. Everything
561 else was assumed to be the same. Fig. 10 demonstrates a significantly better performance
562 of the interaction-based VSA method (solid red curve) compared with its non-interactive
563 counterpart (green curve). It clearly shows the importance of the HSI in predicting response
564 levels and explains the frequent overestimation of responses due to the multi-pedestrian
565 excitation of footbridges when HSI is not taken into account.

566 ### 4.3 Comparison with design guidelines

567 The performance of the interaction-based VSA method was further compared with a
568 number of the relevant design guidelines: ISO 10137 standard [3], French road authorities
569 standard [4], UK National Annex to Eurocode 1 [5] and a method proposed by Butz [35].
570 For each test, the input parameters of the design guidelines were selected in a way to
571 simulate as best as possible (within the provision of the guidelines) the corresponding
572 walking traffic. The extensive discussion of the selected guidelines and their shortcomings

were presented by Shahabpoor and Pavic [15] and Zivanovic, et al. [14] and are not repeated here.

Setra and Butz methods use response amplitude, with 95% probability of non-exceedance ($a_{95\%}$) for assessment. ISO uses peak response and UK NA suggests a mean response plus 2.5 times standard deviation ($a_{2.5\sigma}$) for a serviceability assessment. The results of the interaction-based VSA method were calculated based on the FL CDF corresponding to the anti-node response to be able to compare them with the results of the selected guidelines. The interaction-based VSA method results were also compared with their non-interactive counterparts for all tests. As can be seen in Fig. 11, the accuracy of the interaction-based VSA method in predicting structural response is considerably higher than all other methods in all six tests. Comparing like with like, Setra, ISO, UK NA and Butz methods have a 300-700%, 200-500%, 100-400% and 50-100% error in estimating structural response, respectively. This error range is 100-200% for the non-interactive method. In comparison, the interaction-based VSA method results show a maximum 10% error in estimating $a_{95\%}$, $a_{2.5\sigma}$ and $a_{rms}$ and a maximum 30% error in estimating peak acceleration $a_{peak}$.

# 5 Conclusions

The interaction-based VSA method proposed in this paper addresses the most important shortcoming of the current vibration serviceability assessment guidelines for pedestrian structures: neglecting the HSI and inter- and intra-subject variability of the walking load and human body parameters. Similar to the successful modelling approach featured in the UK recommendation for the design of permanent grandstands [16], an SDOF mass-spring-damper model and the associated walking forces are used to describe each walking pedestrian on the structure. The key novelties of the method are:

1) It takes into account the individual interaction of pedestrians with the structure;

2) It takes into account the moving location of each pedestrian on the structure, making it possible to assess the actual level of vibration response experienced by each pedestrian while walking on the structure; and
3) It features a novel vibration assessment method based on this individualised experience of pedestrians from structural vibration. This is a considerable improvement compared with the conventional VSA methods that calculate structural vibration at a particular fixed point, which may or may not be experienced by the users.

The key limitations of the proposed model are as follows. Firstly, the properties of the walking human SDOF model are identified for free walking and do not consider the effects of the gait parameters such as walking speed and stride length. Secondly, other mechanisms of human-structure and human-human interactions such as synchronisation and lock-in are not considered in this model. Thirdly, the effects of stationary (not moving) people were not considered in this paper. Finally, autonomous simulation of the human-environment interaction is not included in the model.

The application of the proposed interaction-based VSA method to experimental data, from six vibration monitoring exercises on two full-scale footbridge structures under different walking traffic, demonstrated the superior performance of the new methodology. The interaction-based VSA method, together with a suitable calibration of human parameters, predicted the occupied structure modal frequency and damping ratio with less than 0.1% and 1% error, respectively. When compared with experimentally measured responses, the new method regularly overestimates the responses by only 5-10%. This is significantly less than 200-500% overestimation obtained when following popular international design guidelines that do not feature the HSI.

621 The method is not suitable for hand-based calculations, which is how VSA is traditionally
622 done. However, if coded and incorporated into a user-friendly software (e.g. with a
623 graphical user interface), it can be used effortlessly in everyday civil engineering practice.
624 Commonly required information for vibration serviceability assessment, such as mode
625 shapes and modal properties of the structure, is provided typically by FEM software or field
626 measurements that involve hardware. This fully computerised approach to VSE can be
627 carried out on a standard PC configuration within minutes.

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# Figures

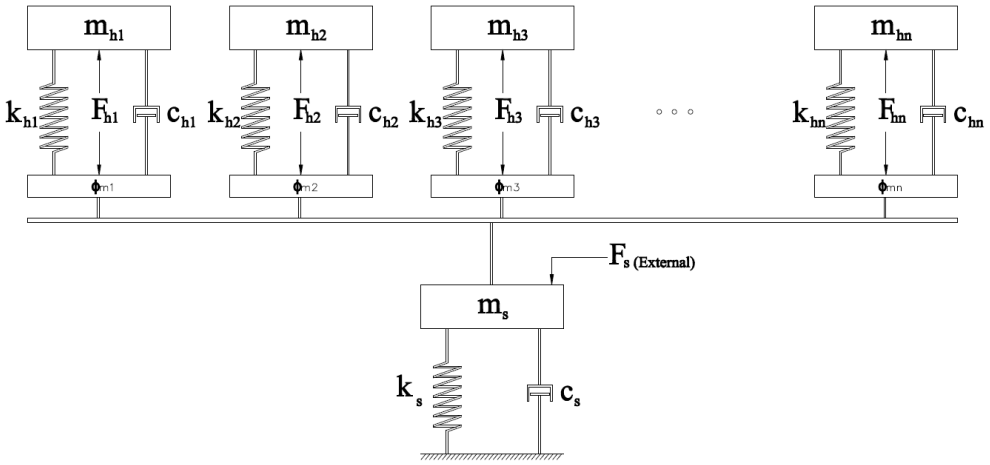



Fig. 1. Mass-spring-damper model of stationary walking traffic-structure system

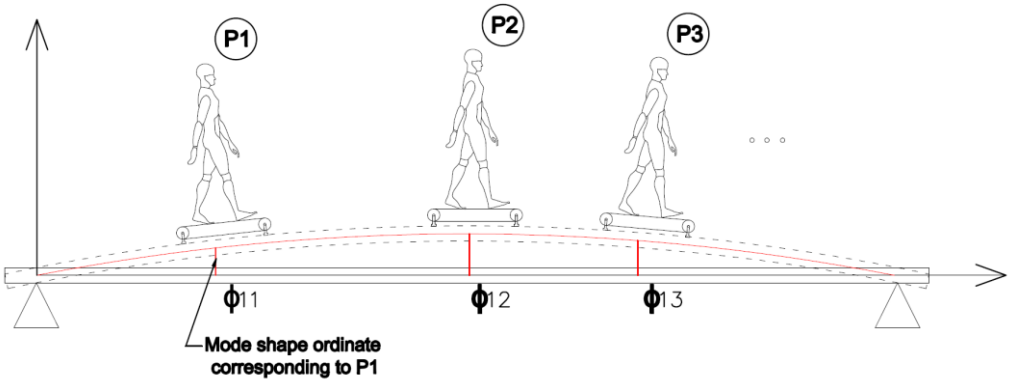



Fig. 2. A conceptual illustration of ‘stationary’ walking people. Ø represents the ordinate of the mode one shape at the location of each pedestrian.

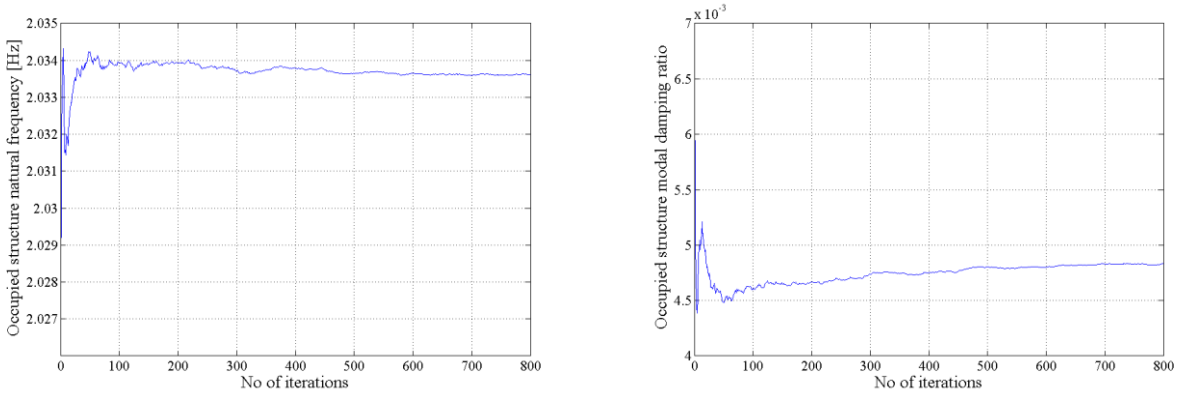



Fig. 3. A typical fluctuation of average occupied structure natural frequency $f_{os}$ and damping ratio $\zeta_{os}$

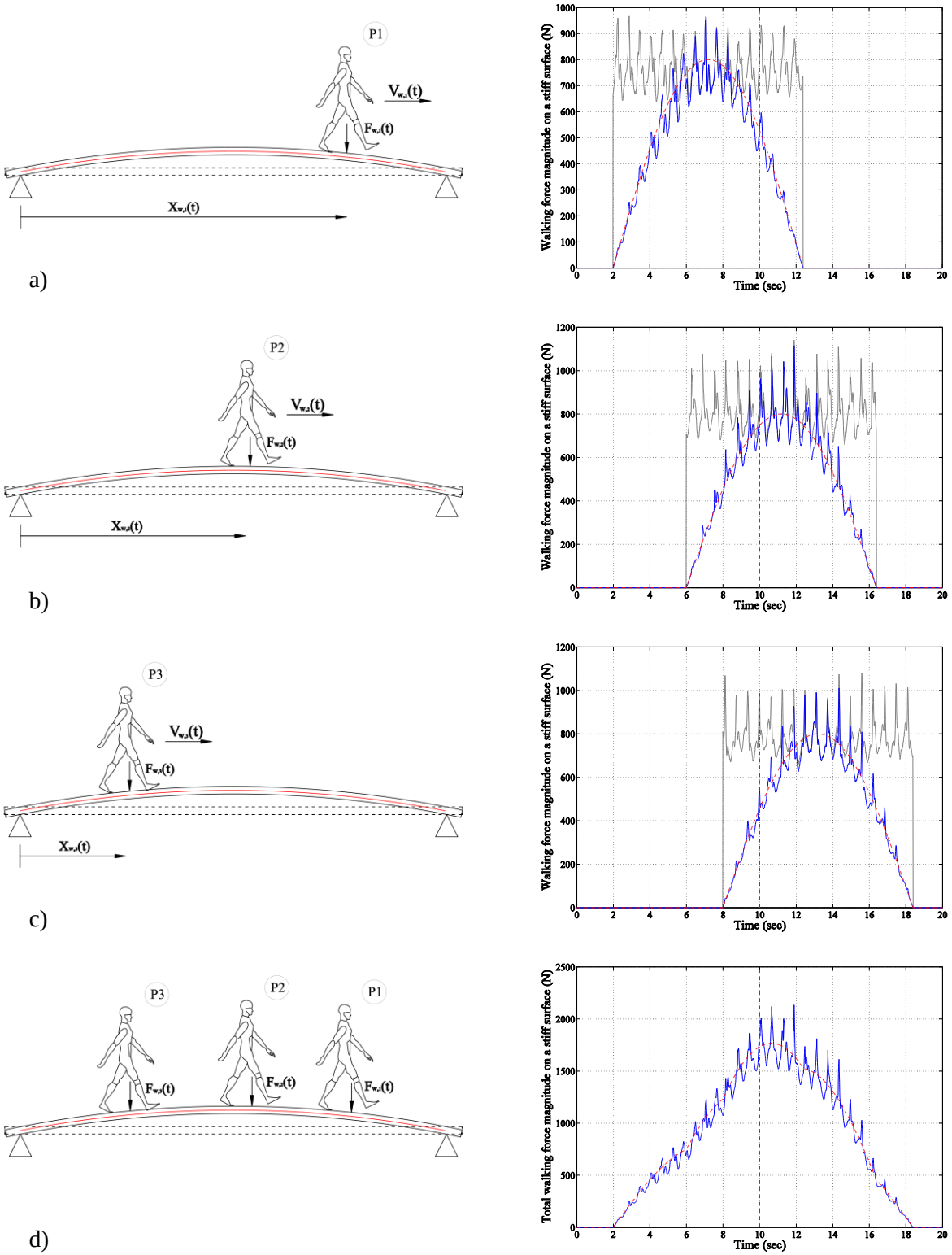
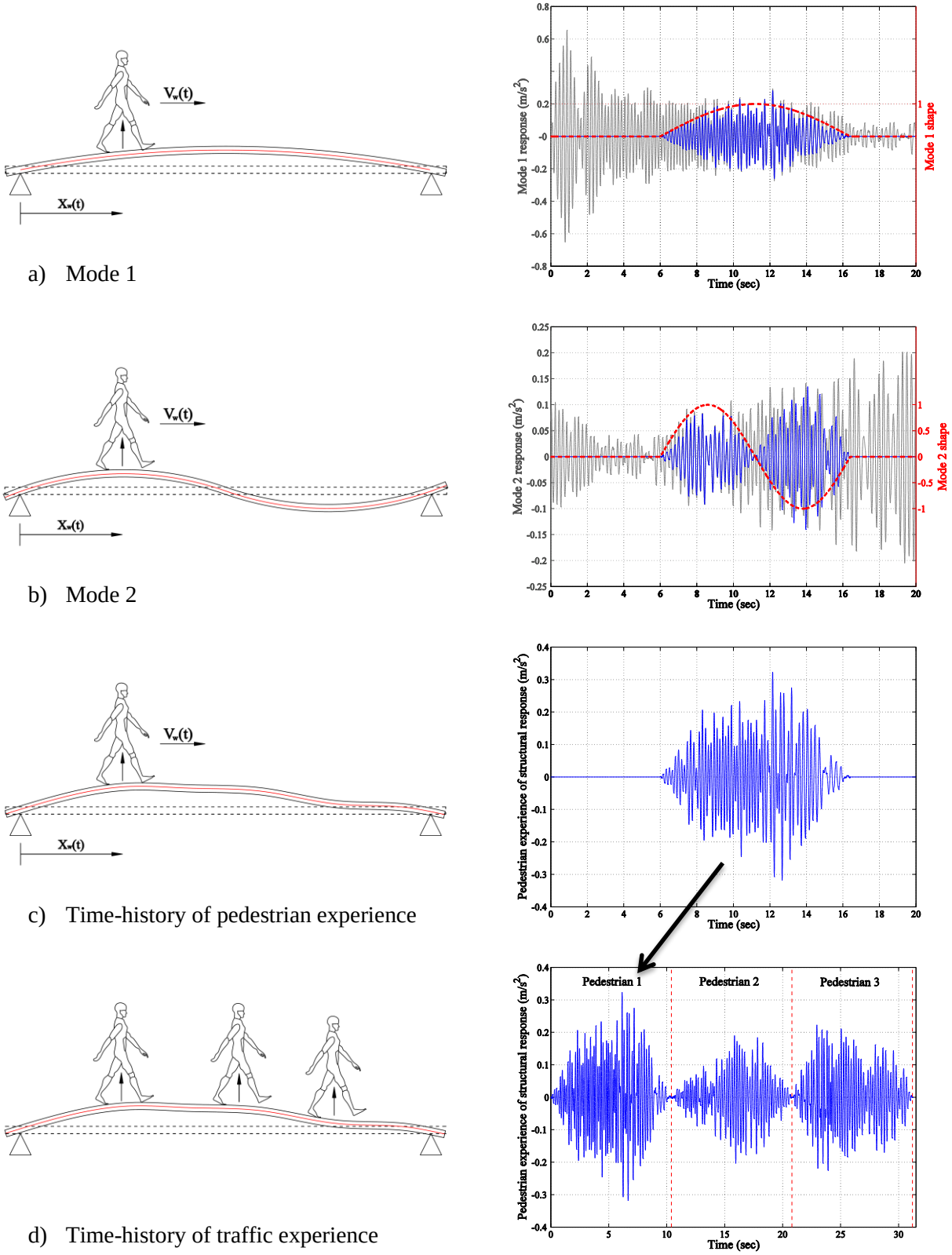



Fig. 4. Superposition of modal walking of three pedestrians (a, b and c) to generate modal force of walking traffic (d) – walking force (grey), modal walking force (blue) and moving average of modal walking force (red)



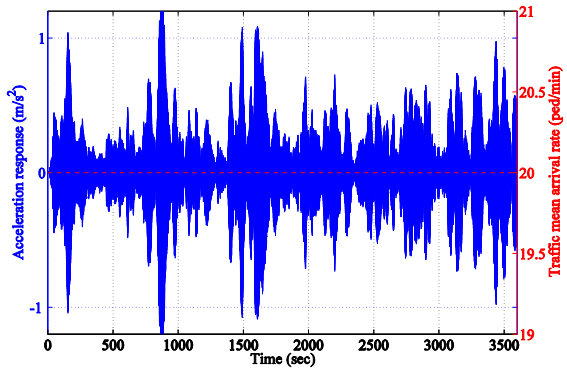

a) Mode 1

b) Mode 2

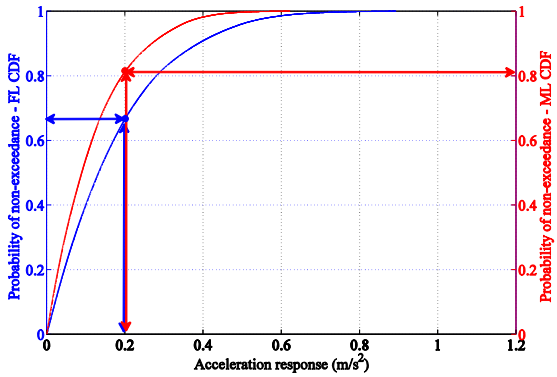
c) Time-history of pedestrian experience

d) Time-history of traffic experience

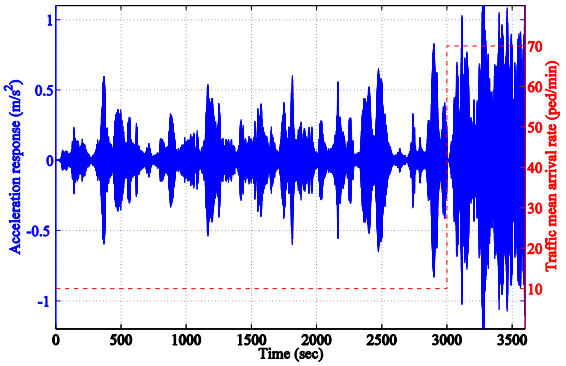
Fig. 5. Time-history of each pedestrian's experience as they walk along the structure



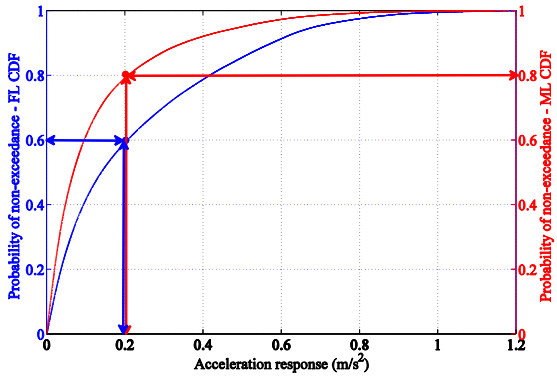

a) Scenario A: Acceleration response and mean arrival rate time-histories




b) Scenario A: Fixed-location (blue) and moving-location (red) CDFs




c) Scenario B: Acceleration response and mean arrival rate time-histories




d) Scenario B: Fixed-location (blue) and moving-location (red) CDFs

Fig. 6. Comparison of FL and ML CDFs

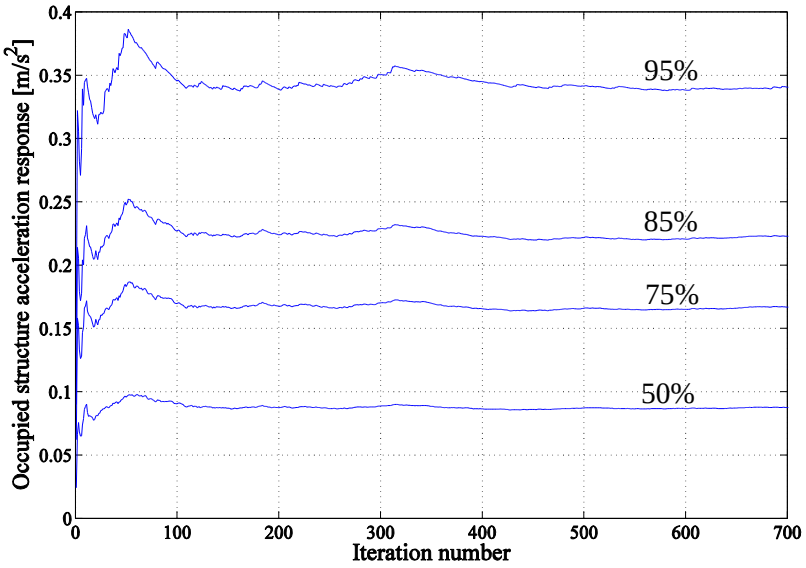
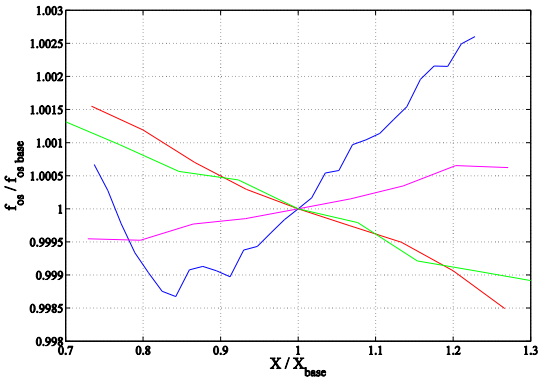
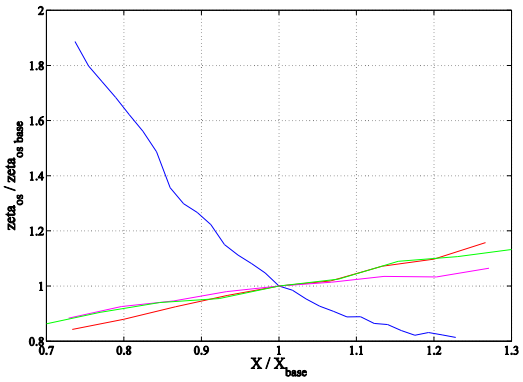


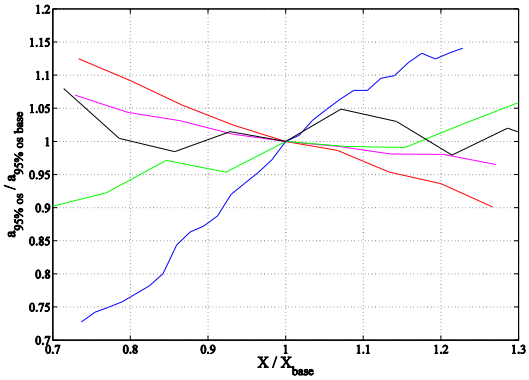

Fig. 7. Typical fluctuation of acceleration response with 95%, 85%, 75% and 50% probability of non-exceedance (from top to bottom)



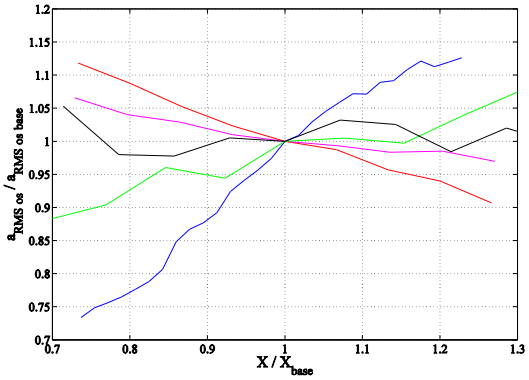

a) Sensitivity of the occupied structure modal frequency $f_{os}$ to input parameters




b) Sensitivity of the occupied structure modal damping ratio $\zeta_{os}$ to input parameters

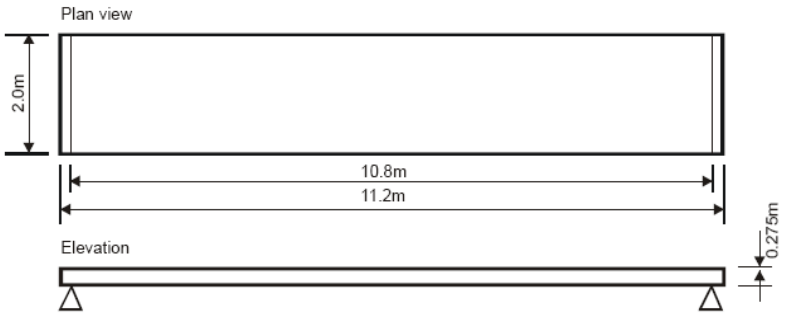



c) Sensitivity of acceleration response with 95% probability of non-exceedance $a_{95\%}$ to input parameters

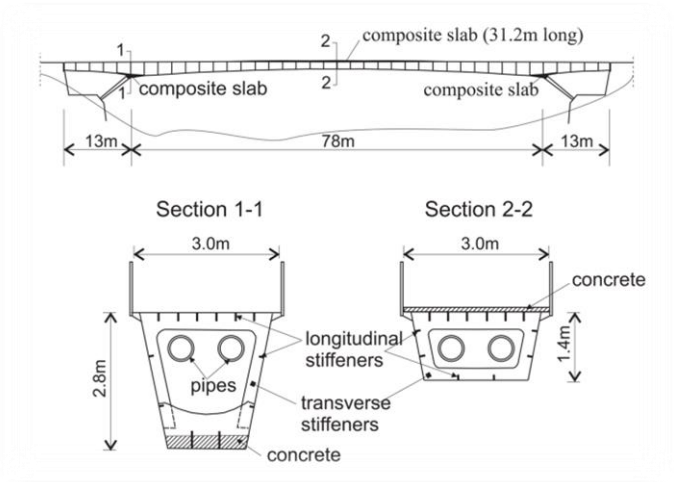



d) Sensitivity of acceleration response RMS $a_{rms}$ to input parameters

Fig. 8. Sensitivity of the interaction-based VSA method outputs $f_{os}$, $\zeta_{os}$, $a_{95\%}$ and $a_{rms}$ to input parameters ($x/x_{base}$): mean $f_h$ (blue), $m_h$ (red), $\zeta_h$ (pink), arrival rate $r_a$ (green) and walking speed $v_w$ (black)

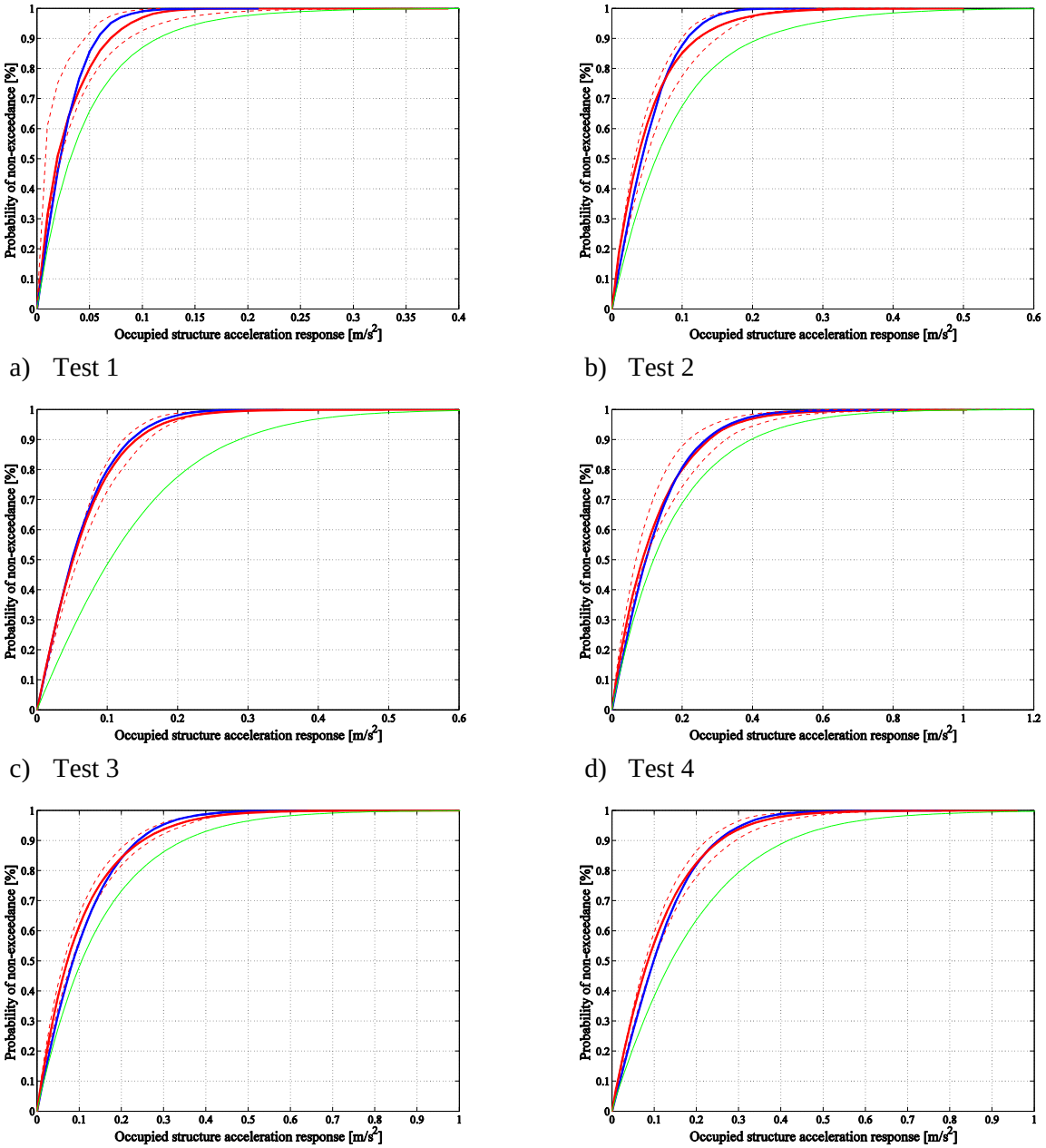



a) b)




c) d)

Fig. 9. Photo and schematics of the Sheffield University post-tensioned footbridge (a and b) and Podgorica footbridge (c and d)

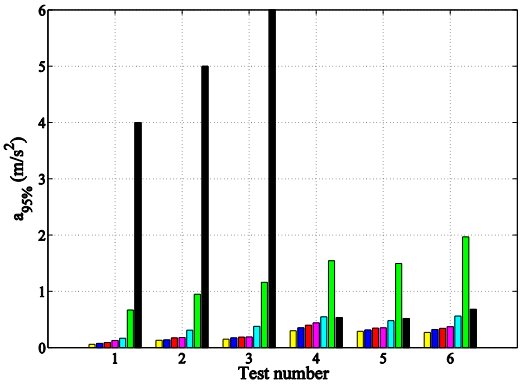



a) Test 1 b) Test 2

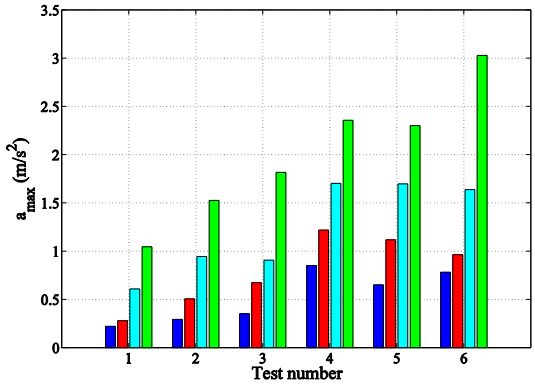
c) Test 3 d) Test 4

e) Test 5 f) Test 6

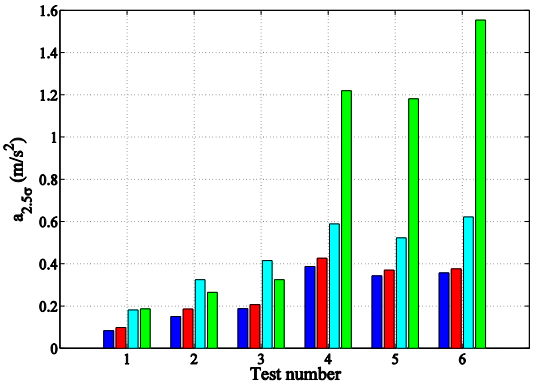
Fig. 10. Comparison of experimental and analytical CDFs. Experimental (blue), confidence interval CDFs (dashed red), the analytical CDF resulted from interaction based VSA method with (red) and without (green) taking into account the HSI effects



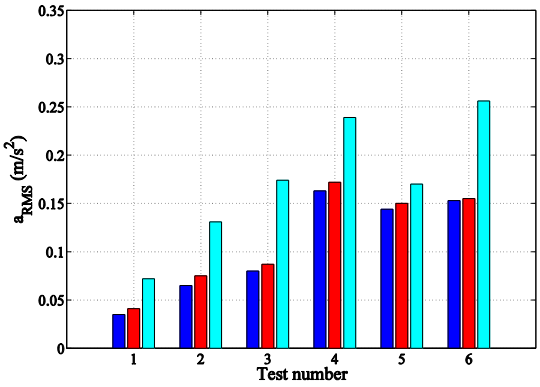

a) Comparison of acceleration response with 95% probability of non-exceedance – in order: experimental (blue), interactive (red), confidence intervals (yellow and magenta), non-interactive (cyan), Setra (green) and Butz (black)




b) Comparison of peak acceleration response - in order: experimental (blue), interactive (red), non-interactive (cyan) and ISO (green)




c) Comparison of acceleration response with $\mu+2.5\sigma$ probability of non-exceedance - in order: experimental (blue), interactive (red), non-interactive (cyan) and UK NA (green)




d) Comparison of acceleration response RMS - in order: experimental (blue), interactive (red), and non-interactive (cyan)

Fig. 11. Comparison of the performance of the interaction-based VSA method (red) with non-interactive (cyan), ISO, UK National Annex, Setra and Butz assessment methods

# Tables

Table 1. Traffic statistics of Sheffield and Podgorica footbridges tests

| **Sheffield footbridge** | | | | | | |
|---|---|---|---|---|---|---|
| *Parameter* | *Unit* | *Distribution* | *Test 1* | *Test 2* | *Test 3* | *Average* |
| Number of participants | peds | - | 3 | 6 | 10 | - |
| Mean arrival rate ($r_a$) | peds/s | - | 0.31 | 0.63 | 0.98 | - |
| Mean number of pedestrians on footbridge | peds | - | 2.5 | 4.9 | 7.86 | - |
| Mean walking speed ($v_w$) | m/s | Normal | 1.41 | 1.06 | 1.36 | 1.28 |
| Variance walking speed ($v_w$) | m/s | Normal | 0.06 | 0.04 | 0. 29 | 0.13 |
| Average crossing time ($t_c$) | s | - | 7.7 | 10.2 | 7.9 | 8.4 |
| Average body mass ($m_h$) | kg | - | 70 | 70 | 70 | 70 |
| **Podgorica footbridge*** | | | | | | |
| Mean arrival rate ($r_a$) | peds/s | Poisson | 0.21 | 0.20 | 0.35 | - |
| Mean number of pedestrians on footbridge | - | Normal | 14.9 | 15.7 | 26.1 | - |
| Variance - number of pedestrians on footbridge | - | Normal | 4.3 | 5.9 | 13.6 | - |
| Mean walking speed ($v_w$) | m/s | Normal | 1.42 | 1.38 | 1.38 | 1.39 |
| Variance walking speed ($v_w$) | m/s | Normal | 0.20 | 0.21 | 0.19 | 0.20 |
| Average crossing time ($t_c$) | s | - | 73.2 | 75.4 | 75.4 | 75 |
| Average body mass ($m_h$) | kg | - | 75 | 75 | 75 | 75 |

* Values adopted from [26]

Table 2. Statistics of the acceleration response of Sheffield and Podgorica footbridges

| *Test No.* | $a_{peak}\,(m/s^2)$ | $a_{95\%}\,(m/s^2)$ | $a_{2.5\sigma}{}^{*}\,(m/s^2)$ | $a_{rms}\,(m/s^2)$ |
|---|---|---|---|---|
| **Sheffield footbridge** | | | | |
| Test 1 | 0.220 | 0.074 | 0.083 | 0.035 |
| Test 2 | 0.292 | 0.133 | 0.150 | 0.065 |
| Test 3 | 0.352 | 0.172 | 0.188 | 0.080 |
| **Podgorica footbridge** | | | | |
| Test 4 | 0.801 | 0.352 | 0.387 | 0.163 |
| Test 5 | 0.649 | 0.312 | 0.343 | 0.144 |
| Test 6 | 0.780 | 0.321 | 0.357 | 0.153 |

* the response amplitude corresponding to 2.5 standard deviation away from mean value of structural response

Table 3. Input parameters of 6 tests used in the interaction-based VSA method

| Category | Parameters | Units | Distribution | Sheffield footbridge | | | Podgorica footbridge | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Test 1 | Test 2 | Test 3 | Test 4 | Test 5 | Test 6 |
| Empty Structure modal properties | $m_{es}$ | kg | - | | 7128 | | | 58000 | |
| | $f_{es}$ | Hz | - | | 4.44 | | | 2.04 | |
| | $\zeta_{es}$ | % | - | | 0.6 | | | 0.26 | |
| Walking human model parameters | $m_h$ mean | kg | - | | 70 | | | 75 | |
| | $f_h$ mean | Hz | Normal | | 2.85 | | | 2.85 | |
| | $f_h$ variance | Hz | Normal | | 0.34 | | | 0.34 | |
| | $\zeta_h$ mean | % | Normal | | 29.5 | | | 29.5 | |
| | $\zeta_h$ variance | % | Normal | | 4.7 | | | 4.7 | |
| Traffic parameters | $r_a$ mean | peds/s | Poisson | - | - | - | 0.21 | 0.20 | 0.35 |
| | | | - | 0.31 | 0.63 | 0.98 | - | - | - |
| | $v_w$ mean | m/s | Normal | 1.41 | 1.06 | 1.36 | 1.42 | 1.38 | 1.38 |
| | $v_w$ variance | m/s | Normal | 0.06 | 0.04 | 0.29 | 0.20 | 0.21 | 0.19 |
| Walking force | $F_w$ total | N | Recorded with treadmill on a stiff surface | | | | | | |

Table 4. Modal properties of the occupied structures

| Test Number | Experimental | | | Analytical | | |
|---|---|---|---|---|---|---|
| | $f_{os}$ (Hz) | $\zeta_{os}$(%) | $m_{os}$ (kg) | $f_{os}$ (Hz) | $\zeta_{os}$(%) | $m_{os}$ (kg) |
| **Sheffield footbridge** | | | | | | |
| Empty | 4.440 | 0.60 | 7128 | - | - | - |
| Test 1 | 4.445 | 1.10 | 7183 | 4.445 | 1.10 | 7183 |
| Test 2 | 4.465 | 1.65 | 7238 | 4.465 | 1.65 | 7238 |
| Test 3 | 4.475 | 2.30 | 7311 | 4.475 | 2.30 | 7311 |
| **Podgorica footbridge** | | | | | | |
| Empty | 2.04 | 0.26 | 58000 | - | - | - |
| Test 4 | -** | -** | -** | 2.034 | 0.49 | 58750 |
| Test 5 | -** | -** | -** | 2.034 | 0.49 | 58750 |
| Test 6 | -** | 0.67* | -** | 2.029 | 0.65 | 59300 |

* Value adopted from [14]

** Value not available

Table 5. Statistical features of the ‘interactive’ and ‘non-interactive’ responses

| | Interaction-based VSA method | | | | | | Non-interactive method | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $a_{peak}$ | $a_{95\%}$ | $a_{95\%\ min}$ | $a_{95\%\ max}$ | $a_{2.5\sigma}$ | $a_{rms}$ | $a_{peak}$ | $a_{95\%}$ | $a_{2.5\sigma}$ | $a_{rms}$ |
| **Sheffield footbridge** | | | | | | | | | | |
| Test 1 | 0.280 | 0.091 | 0.060 | 0.125 | 0.098 | 0.041 | 0.607 | 0.167 | 0.181 | 0.072 |
| Test 2 | 0.505 | 0.173 | 0.130 | 0.180 | 0.186 | 0.075 | 0.944 | 0.308 | 0.325 | 0.131 |
| Test 3 | 0.673 | 0.186 | 0.150 | 0.190 | 0.207 | 0.087 | 0.907 | 0.377 | 0.415 | 0.174 |
| **Podgorica footbridge** | | | | | | | | | | |
| Test 4 | 1.218 | 0.397 | 0.300 | 0.440 | 0.426 | 0.172 | 1.703 | 0.548 | 0.589 | 0.239 |
| Test 5 | 1.117 | 0.345 | 0.290 | 0.350 | 0.370 | 0.150 | 1.697 | 0.480 | 0.523 | 0.170 |
| Test 6 | 0.963 | 0.341 | 0.270 | 0.370 | 0.376 | 0.155 | 1.638 | 0.560 | 0.622 | 0.256 |