

Optimal Monetary Policy with Endogenous Export Participation*

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Abstract: This paper studies optimal monetary policy in an open economy with firm heterogeneity and monopolistic competition. I consider a two-country dynamic general equilibrium model where firms make decisions to enter and exit the domestic and export markets. I show that endogenous export participation creates an incentive for policymakers to set high interest rates. This leads to high long-run inflation. Firm entry magnifies the welfare cost of inflation generating large gains to international monetary cooperation.

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1. Introduction

This paper studies optimal monetary policy in an open economy with firm heterogeneity and monopolistic competition. I consider a two-country dynamic general equilibrium model where firms make decisions to enter and exit the domestic and export markets.¹ Monetary policy affects firm entry, exit, and export decisions because firms fund working capital by borrowing from financial intermediaries.² There are two main results regarding optimal monetary policy. Endogenous export participation creates an incentive for policymakers to set higher interest rates than they would do if all firms were to export. This leads to high long-run inflation. Firm entry magnifies the welfare cost of inflation and there are large gains to international monetary cooperation.

Determining optimal monetary policy in the open economy requires understanding how countries interact through the terms of trade - the relative price of foreign to home output. In the closed economy, a monetary contraction creates a shortage of liquidity in the financial sector, reducing the supply of loanable funds. The resulting rise in the interest rate induces firms who need working capital to reduce output. The Friedman rule (a zero nominal interest rate) is the optimal monetary policy.³ In the open economy, with a given foreign monetary policy, a home monetary contraction improves the terms of trade, and with a given home monetary policy, an improvement in the terms of trade increases consumption. Thus, it can be optimal for the interest rate to depart from the Friedman rule in the open economy because the terms of trade provide a mechanism through which higher interest rates

¹As in Melitz (2003), new entrants can choose to produce a differentiated variety, or exit immediately. Firms that produce also choose whether or not to export.

²Working capital is used by firms to cover payments (e.g. the costs of inputs) prior to the realization of revenues. The working capital channel has recently been stressed by Christiano *et al.* (2011) and Jermann and Quadrini (2012).

³The optimality of the Friedman rule is not particular to the class of monetary model I consider. An extensive discussion is contained in Schmitt-Grohe and Uribe (2010).

indirectly raise household consumption.

Once heterogeneous firms make entry and exit decisions there is a second reason for the interest rate to depart from the Friedman rule. Due to selection, only high productivity firms - those which can generate enough revenue to pay market entry costs - export. With a given foreign monetary policy, a home monetary contraction increases the costs associated with production and market entry, and this lowers the rate of export participation. Higher interest rates therefore force weaker firms to exit the market, or shut-down altogether, whilst only the most productive firms continue to export. This process leads to a reallocation of labor towards productive firms and raises economy-wide productivity. Because labor can be reallocated within each country, when countries set monetary policy independently, there is an added incentive for policymakers to raise interest rates, and this leads to higher long-run inflation.

Two parameters play a key role in determining optimal monetary policy in my analysis. First, the lower the elasticity of substitution between home and foreign goods (Armington elasticity), the larger the improvement in the terms of trade, for a given increase in the interest rate. Since an improved terms of trade allows one country to import more, for a given amount of exports, a low elasticity raises the incentive of the policymaker to set a high interest rate. Second, with fixed costs of exporting, there is a productivity threshold which determines the rate of export participation. When the underlying dispersion of firm productivity (less firm heterogeneity) is low there are relatively many firms concentrated around the threshold. Marginal firms below the threshold are not too different from those firms that already export and the selection effect is strong. A strong selection effect allows for a relatively large increase in economy-wide productivity and this further raises the incentive to set a high interest rate.⁴

⁴Both the dispersion in firm productivity and the monopolistic markup alter firm heterogeneity. I

Endogenous entry also has implications for the welfare loss associated with setting monetary policy independently across countries. I compute the increase in steady-state consumption which an individual would require to be as well-off as under international monetary cooperation. I show analytically, at a given rate of long-run inflation, the welfare loss is an increasing function of the monopoly markup. This magnification effect occurs because inflation discourages firm entry into the domestic market. With Dixit-Stiglitz preferences, the monopoly markup is equated with consumer love-for-variety, and as the markup rises, so does the cost of inflation. Thus, when inflation results from strategic interaction between countries, the welfare loss from setting monetary policy independently departs from the standard model in two ways. Inflation is high due to selection and the welfare cost of inflation is magnified due to endogenous firm entry.

Finally, I consider the quantitative implications of the selection effect in my model by fixing heterogeneity in firm productivity and increasing the Armington elasticity for different monopolistic markups. In the baseline calibration, with an elasticity of 1.25 and a markup of 20%, annual inflation is 7.1%. The implied a welfare loss is 1.7% in units of consumption. I interpret my results in the context of a literature that seeks to evaluate the cost of long-run inflation when countries set monetary policy independently. Cooley and Quadrini (2003) develop a model in which inflation affects the relative price of intermediate inputs for a representative firm. There is less inflation and a smaller welfare loss from setting monetary policy independently; in their baseline calibration an annual 3.2% inflation generates a welfare loss of 0.7%. In the quantitative analysis, I also extend the model and allow for exogenous labor taxes, import tariffs, and international trade in financial assets.⁵ With ad-

stress the role of productivity dispersion in the discussion because the markup plays an independent role in determining optimal monetary policy.

⁵I do not consider the joint determination of fiscal instruments with monetary policy. In fixed-variety models, and in the presence of optimal tariffs, the role of monetary policy to change the terms of trade is redundant.

ditional domestic distortions, inflation falls, but the welfare cost of setting monetary policy independently remains high.

In the model I develop, prices are flexible, money is held for transactions purposes, and monetary policy generates real effects because households face restrictions in their choice of portfolio composition, consistent with a limited participation assumption.⁶ This source of monetary non-neutrality stands in contrast to the New Keynesian literature, which instead imposes restrictions on the setting of prices or wages, and focuses short-run stabilization policies.⁷ Recently, Bergin and Corsetti (2013) have developed an open economy New Keynesian model with a production relocation externality, partly inspired by Ossa's (2011) analysis of trade policy. They find the welfare gains to international monetary coordination (using stabilization rules) are magnified when there is firm entry, complementing the results I present here, which are based on firm heterogeneity and endogenous export participation.

The approach taken in this paper is also related to a growing literature that analyzes trade policy in new trade models with firm heterogeneity. For example, Demidova and Rodriguez-Clare (2009) show how trade policies can be used to correct for monopoly distortions in a small open economy and Felbermayr *et al.* (2013) study optimal tariffs in a two-country setting. Both of these studies show that the underlying distribution of productivity plays a key role in determining optimal policy.⁸ Indeed, the selection mechanism present in these

⁶Limited participation models of monetary policy are based on the idea that producers need to finance working capital. See Fuerst (1992).

⁷Corsetti *et al.* (2010) provide an overview of the sticky-price approach in the open economy, where the welfare gains to international monetary cooperation are thought to be small. In the limited participation framework of Cooley and Quadrini (2003), the welfare loss associated with losing the ability to react optimally to shocks is dominated by the benefits of reduced long-run inflation.

⁸For example, Demidova and Rodriguez-Clare (2009) show that, as firms become less heterogeneous (and firm productivity is less dispersed), the optimal import subsidy increases.

studies provides the rationale for high inflation in my analysis. In the sense that new trade models with firm heterogeneity add to an established trade policy literature my results contribute to a literature focused on the design of monetary policy in open economies.

My results also have a natural empirical interpretation because they show that falling inflation can be associated with increased export participation.⁹ Bergin and Lin (2012) have discussed the rise in the number of exported products across the European Union over the period 1990-2004. They suggest that forming a monetary union is equivalent to a news-shock to trade costs. They also provide empirical evidence that the extensive margin of exports across the EU (measured as the entry of new goods categories based on NBER-UN world trade data) responded aggressively prior to the adoption of the Euro, which is taken as confirmation of the news-shock hypothesis. The interpretation I offer to their results is that falling inflation and increased export participation is explained as the joint outcome of greater monetary cooperation between countries.

The remainder of the paper is organized as follows. In section 2, I develop a two-country monetary model with endogenous export participation. In section 3, I discuss the response on international relative prices to exogenous monetary shocks. I consider optimal non-cooperative monetary policy in section 4. I derive explicit expressions for inflation, both with and without endogenous export participation, and consider the welfare loss from policy competition. I undertake a quantitative analysis of the model in section 5.

2. Model Economy

This section outlines the model economy I use to study optimal monetary policy. There are two identical countries - home and foreign - each populated by a continuum of households

⁹Greater international trade in goods and falling inflation are the two most often cited benefits of monetary cooperation between countries. See the discussion in Frankel and Rose (2002).

with mass normalized to one. In each country, households supply labor to firms and deposit cash with financial intermediaries. Consumption is subject to a cash-in-advance constraint. There are many potential firms which may serve the domestic and export market. Firms are heterogenous in labor productivity, incur per-period fixed costs, and borrow from financial intermediaries to fund working capital. A government injects money into the economy, via financial intermediaries, using lump-sum transfers.

In what follows, I focus the exposition of the model on the home country, with the understanding that analogous expressions hold for the foreign country. Consumption, output, and the nominal price of the home/foreign output are denoted with h/f -subscripts. Asterisks denote foreign country variables.

2.1. Household Intratemporal Consumption

The overall consumption basket is an aggregate of home and foreign goods. Each good is defined over a continuum of differentiated varieties.

$$C_t = \left[\theta^{1-\xi} \left(\int_{\omega \in \Omega_t} c_{h,t}(\omega)^\sigma d\omega \right)^{\xi/\sigma} + (1-\theta)^{1-\xi} \left(\int_{\omega^* \in \Omega_t^*} c_{f,t}(\omega^*)^\sigma d\omega^* \right)^{\xi/\sigma} \right]^{1/\xi} \quad (1)$$

where $c_{h,t}(\omega)$ is the consumption of home variety $\omega \in \Omega_t$ and $c_{f,t}(\omega^*)$ is the consumption of foreign variety $\omega^* \in \Omega_t^*$. Differentiated varieties are substitutes, so $0 < \sigma < 1$, and the elasticity of substitution between varieties is $1/(1-\sigma) > 1$. The elasticity of substitution between home and foreign goods is defined as $\epsilon \equiv 1/(1-\xi) > 0$, where $\xi < 1$. The parameter $0 < \theta < 1$ determines the share of the home varieties in the consumption basket and I interpret this as a measure of openness.

Household consumption is characterized by the following demand curves,

$$c_{h,t}(\omega) = \theta \left(\frac{p_{h,t}(\omega)}{P_{h,t}} \right)^{-1/(1-\sigma)} \left(\frac{P_{h,t}}{P_t} \right)^{-\epsilon} C_t \quad (2)$$

and,

$$c_{f,t}(\omega^*) = (1 - \theta) \left(\frac{p_{f,t}(\omega^*)}{P_{f,t}} \right)^{-1/(1-\sigma)} \left(\frac{P_{f,t}}{P_t} \right)^{-\epsilon} C_t \quad (3)$$

where $p_{h,t}(\omega)$ is the home-currency price of home variety ω and $p_{f,t}(\omega^*)$ is the home-currency price of foreign variety ω^* . Price indexes for home and foreign goods are given by, $P_{h,t} = \left(\int_{\omega \in \Omega_t} p_{h,t}(\omega)^{\sigma/(\sigma-1)} d\omega \right)^{\sigma/(\sigma-1)}$ and $P_{f,t} = \left(\int_{\omega^* \in \Omega_t^*} p_{f,t}(\omega^*)^{\sigma/(\sigma-1)} d\omega^* \right)^{\sigma/(\sigma-1)}$. Finally, the consumer price index is, $P_t = [\theta P_{h,t}^{1-\epsilon} + (1 - \theta) P_{f,t}^{1-\epsilon}]^{1/(1-\epsilon)}$.

2.2. Firms

There are a continuum of firms in the home country, each producing a differentiated variety, $\omega \in \Omega_t$. Labor is the only factor of production. Firms are heterogeneous in productivity and use working capital loans to finance the payment of wages. Prior to entry, firms are identical and face a labor-intensive entry cost, $f_e > 0$. Upon entry, each firm draws its productivity level, $a \geq 1$, from a common distribution $g(a)$. With additional per-period fixed production and export costs - denoted $f_d > 0$ and $f_x > 0$ - once a firm knows its productivity level, it may choose to exit without production, produce only for the domestic market, or produce for both domestic and export markets. Firms enter, profits are realized, and all firms exit in a single period.¹⁰

I write the profits from potential domestic sales as, $\varphi_{h,t}(a) = (p_{h,t}(a) - \frac{W_t R_t}{a}) y_{h,t}(a) - f_d W_t R_t$, where $p_{h,t}(a) y_{h,t}(a)$ is domestic market firm-level revenue, $W_t R_t^b/a$ are the marginal costs of production, and $f_d W_t R_t$ are the fixed costs of production.¹¹ The nominal in-

¹⁰I relax this particular assumption in section 5.3.

¹¹I have also examined the case where costs are increasing in market penetration. Following Arkolakis (2010), we can write, $F(n_{d,t}(a)) \equiv f_d [1 - (1 - n_{d,t}(a))^{1-\chi}] / (1 - \chi)$, where $f_d > 0$ is a common component across firms, and $\chi \geq 0$ is a parameter that governs the proportion of home consumers the home firm reaches, denoted $n_{d,t}(a)$. The results are largely unchanged by this generalization because deviations from the Pareto distribution are greatest for the smallest firms.

terest rate affects both marginal and fixed costs because all inputs require working capital. In a similar fashion, the profits from potential export sales can be written as, $\varphi_{h,t}^*(a) = \left(\frac{e_t p_{h,t}^*(a)}{\iota \times \tau^*} - \frac{W_t R_t}{a} \right) y_{h,t}^*(a) - f_x e_t W_t^* R_t$, where e_t is the nominal exchange rate (the home-currency price of foreign currency), and $p_{h,t}^*(a)$ is the foreign-currency price of a home variety. In this formulation, export market revenue, $e_t p_{h,t}^*(a) y_{h,t}^*(a)$, is affected by a melting-iceberg trade (transportation) cost, $\iota \geq 1$, and a foreign import tariff, $\tau^* \geq 1$, and foreign wages appear because fixed costs pay for labor in the destination market, following Eaton *et al.* (2011).¹²

Each firm maximizes profit, $\varphi_{h,t}(a) + \varphi_{h,t}^*(a)$, subject to the (home and foreign) demand for its product, goods market constraints, $y_{h,t}(a) = c_{h,t}(a)$ and $y_{h,t}^*(a) = \iota c_{h,t}^*(a)$, and technology, $y_{h,t}(a) = a l_{h,t}(a)$ and $y_{h,t}^*(a) = a l_{h,t}^*(a)$. Optimal prices for the domestic and export market are,

$$p_{h,t}(a) = \left(\frac{1}{\sigma a} \right) W_t R_t \quad \text{and} \quad p_{h,t}^*(a) = (\iota \times \tau^*) \frac{p_{h,t}(a)}{e_t} \quad (4)$$

where $\sigma^{-1} > 1$ is the markup over marginal costs. Using the expressions in (4), potential profits can be decomposed into revenues and fixed costs. For example, profits from potential domestic sales can be re-written as, $\varphi_{h,t}(a) = (1 - \sigma) p_{h,t}(a) y_{h,t}(a) - f_d W_t R_t$, where $y_{h,t}(a) = c_{h,t}(a)$ is determined by the demand curve given in equation (2).

Due to fixed costs, firms with low productivity may decide not to produce or export. Since a firm will only produce (and export) if doing so earns nonnegative profit, there are two threshold levels of productivity, $a_{d,t} = \inf \{a : \varphi_{h,t}(a) > 0\}$ and $a_{x,t} = \inf \{a : \varphi_{h,t}^*(a) > 0\}$, which determine domestic and export market participation, respectively. The domestic and

¹²Barth and Ramey (2002) argue that a substantial fraction of firms' variable input costs are borrowed in advance and Manova (2013) provides evidence that firms face binding constraints in financing both fixed and variable export costs.

export market productivity thresholds are, in turn, implicitly determined by the following zero-profit productivity cut-off equations, $\varphi_{h,t}(a_{d,t}) = 0$ and $\varphi_{h,t}^*(a_{x,t}) = 0$, such that,

$$p_{h,t}(a_{d,t}) y_{h,t}(a_{d,t}) = \left(\frac{f_d}{1-\sigma} \right) W_t R_t \quad \text{and} \quad p_{h,t}^*(a_{x,t}) y_{h,t}^*(a_{x,t}) = (\iota \times \tau^*) \left(\frac{f_x}{1-\sigma} \right) W_t^* R_t \quad (5)$$

Thus, firms with productivity $a \in [1, a_{d,t}]$ exit without production, firms with $a \in [a_{d,t}, a_{x,t}]$ produce only for the domestic market, and firms with $a \in [a_{x,t}, \infty)$ produce for both domestic and export markets.¹³

Finally, each firm must pay a labor-intensive fixed cost to enter the domestic market. The total mass of entrants is determined by a free entry condition, which, in equilibrium, equates the expected profit from entering, given by $\int_{a_{d,t}}^{\infty} \varphi_{h,t}(a) g(a) da + \int_{a_{x,t}}^{\infty} \varphi_{h,t}^*(a) g(a) da$, to the cost of market entry, given by $f_e W_t R_t^b$. Denoting the mass of entrants in the home economy by $N_{e,t}$, the mass of active firms and the mass of exporting firms is,

$$N_{d,t} = [1 - G(a_{d,t})] N_{e,t} \quad \text{and} \quad N_{x,t} = \left[\frac{1 - G(a_{x,t})}{1 - G(a_{d,t})} \right] N_{d,t} \quad (6)$$

where $1 - G(a_{d,t})$ is the ex-ante probability of successful entry and $\frac{1 - G(a_{x,t})}{1 - G(a_{d,t})}$ is the ex-ante probability of exporting, conditional on successful entry.

2.3. Aggregation and International Relative Prices

From here onward, I assume a Pareto distribution for firm productivity, with shape parameter γ , and cumulative distribution, $G(a) = 1 - a^{-\gamma}$.¹⁴ This leads to the following relationship between the fraction of firms that export and the relative threshold productivity required to

¹³One can also determine market penetration (see footnote 11) as a function of firm productivity. For a firm with productivity $a > a_{d,t}$, market penetration is, $n_{d,t}(a) = 1 - (a_{d,t}/a)^{\sigma/(1-\sigma)\chi}$, such that, as $\chi \rightarrow 0$, and the cost of reaching an additional consumer is constant.

¹⁴A similar assumption is made, for example, in Helpman *et al.* (2004) and Chaney (2008). With monopolistic competition, the Pareto assumption leads to a power law in the distribution of firm sizes (di Giovanni and Levchenko, 2013).

participate in the domestic and export markets, $N_{x,t}/N_{d,t} = (a_{d,t}/a_{x,t})^\gamma$. Moreover, if average productivity in each market is defined as $a_j \equiv \left[\frac{1}{1-G(a_{j,t})} \int_{a_{j,t}}^{\infty} a^{\sigma/(1-\sigma)} g(a) da \right]^{(1-\sigma)/\sigma}$, for $j = \{d, x\}$, then the following relationship between the average productivity of firms in a market and the threshold productivity required to serve that market holds, $a_j = \left[\frac{\gamma - \sigma/(1-\sigma)}{\gamma} \right]^{(\sigma-1)/\sigma} a_j$. Using these results, I define new variables, which account for differences in the average price paid by the home (and foreign) consumer for the home and foreign goods.

Using the optimal firm pricing equations presented in (4), I aggregate the respective price indexes, which delivers,

$$\frac{e_t P_{h,t}^*}{P_{h,t}} = (\iota \times \tau^*) \left(\frac{N_{d,t}}{N_{x,t}} \right)^\lambda ; \quad \lambda \equiv \left(\frac{1-\sigma}{\sigma} \right) - \frac{1}{\gamma} \geq 0 \quad (7)$$

The left-hand side of equation (7) is the average price paid by the foreign household for exported varieties of the home good, relative to the average price paid by the home household, for all varieties of the home good.¹⁵ The right-hand side of (7) is comprised of exogenous trade costs and tariffs, and the (inverse) rate of export participation of home firms, adjusted for the distribution of firm sizes, which is captured by the parameter λ . This composite parameter has the following implications. With monopolistic competition and Pareto productivity the size distribution of firms follows a power law with exponent $\gamma / (\frac{\sigma}{1-\sigma})$. As $\gamma / (\frac{\sigma}{1-\sigma}) \rightarrow 1$, the distribution of firm sizes exhibits maximum dispersion and production occurs within an arbitrarily small mass of very productive firms. In this case, there is no selection effect, $\lambda \rightarrow 0$, and equation (7) reduces to $e_t P_{h,t}^* / P_{h,t} = \iota \times \tau^*$.

I now define three new variables that allow me to track the response of international relative prices to changes in monetary policy. First, the ratio of the consumer price index to the

¹⁵The analogous condition for the relative average price of the foreign good is given by, $P_{f,t}^* / (P_{f,t}/e_t) = (N_x^*/N_d^*)^\lambda / (\tau \times \iota^*)$.

domestic price index (the domestic price ratio) is,

$$\Phi_t = [\theta + (1 - \theta)(\chi_{h,t} \times T_t)^{1-\epsilon}]^{1/(1-\epsilon)} ; \quad \Phi_t \equiv \frac{P_t}{P_{h,t}} \quad (8)$$

The right-hand side of this expression captures the key distortions in the open economy.¹⁶ There are two (previously defined) parameters: $0 < \theta < 1$ determines the share of the home varieties in the consumption basket and $\epsilon > 0$ determines the elasticity of substitution between home and foreign goods. There are two (newly defined) endogenous variables. First, the rate of export participation, adjusted by the parameter λ . This is captured by $\chi_{h,t} \equiv e_t P_{h,t}^* / P_{h,t}$ and determined by equation (7). Second, the home country terms of trade, given by $T_t \equiv P_{f,t} / e_t P_{h,t}^*$. A fall in T_t represents an improvement in the terms of trade.

Notice that Φ_t is increasing in both $\chi_{h,t}$ and T_t and that a change in either variable has a weaker effect on Φ_t for higher values of ϵ . Anticipating the results on optimal monetary policy, the incentive to set high interest rates will depend on two separate effects: how consumer substitutes between home and foreign goods in utility and how changes in the mass of firms that decide to enter the export market affect average prices. Finally, I define the real exchange rate as, $q_t \equiv e_t P_t^* / P_t$. Using the definition of the domestic price ratio in equation (8), I write the real exchange rate in the following way, $q_t = \frac{\Phi_t^*}{\Phi_t} (\chi_{h,t} \times \chi_{f,t} \times T_t)$, where $\chi_{f,t}$ is the foreign analog of $\chi_{h,t}$. As such, I can also decompose any movement in the real exchange rate into the newly defined variables.

2.4. Household Intertemporal Decisions

Household intertemporal utility is,

$$\sum_{t=0}^{\infty} \beta^t [\ln(C_t) + \psi \ln(1 - L_t)] \quad (9)$$

¹⁶The analogous condition for the foreign economy is $\Phi_t^* \equiv P_t^* / P_{f,t}^*$.

where L_t is the total supply of labor and $\beta \in (0, 1)$ is the discount factor. The timing of events is based on Christiano *et al.*'s (1997) model of limited participation. At the beginning of a period households deposit cash with domestic financial intermediaries. Any remaining cash is used for (total) consumption, subject to the following cash-in-advance constraint,

$$P_t C_t \leq (1 - \tau_l) W_t L_t + M_t - D_t \quad (10)$$

where $D_t < M_t$ are household deposits of cash, M_t is the stock of money, $W_t L_t$ is nominal labor-income, and τ_l is a labor-income tax. The accumulation of cash (i.e., the cash the consumer has at the end-of-period t /beginning-of-period $t+1$) is,

$$\begin{aligned} e_t B_{f,t+1} - B_{h,t+1} - R_{b,t}^* e_t B_{f,t} + R_{b,t} B_{h,t} + M_{t+1} &\leq (1 - \tau_l) W_t L_t + (R_t - 1) D_t + M_t \\ &\quad + Tr_t + \zeta_t - P_t C_t \end{aligned} \quad (11)$$

where $B_{h,t+1}$ are one-period home currency bonds (paying out gross rate $R_{b,t}$) issued as debt by the home country and $B_{f,t+1}$ are one-period foreign currency bonds (paying out $R_{b,t}^*$) issued as debt by the foreign country.¹⁷ The term Tr_t is a lump-sum transfer from the revenue generated by the home import tariff (denoted $\tau \geq 1$) and labor-income tax, and ζ_t are the profits of financial intermediaries.

Households maximize lifetime utility subject to these two constraints. The first-order conditions imply,

$$w_t = \left(\frac{\psi}{1 - \tau_l} \right) \frac{C_t}{1 - L_t} \quad \text{and} \quad \mathbb{E}_{t-1} \left[\beta \left(\frac{P_t C_t}{P_{t+1} C_{t+1}} \right) R_t \right] = 1 \quad (12)$$

$$\mathbb{E}_t \left[\beta \left(\frac{P_{t+1} C_{t+1}}{P_{t+2} C_{t+2}} \right) R_{b,t+1} \right] = 1 \quad \text{and} \quad \mathbb{E}_t \left[\beta \left(\frac{P_{t+1} C_{t+1}}{P_{t+2} C_{t+2}} \right) R_{b,t+1}^* \frac{e_{t+1}}{e_t} \right] = 1 \quad (13)$$

where $w_t \equiv W_t / P_t$ is the wage measured in units of consumption. The first expression is a condition for the labor-leisure trade-off. The second expression is an Euler equation which

¹⁷Households hold assets denominated in foreign currency and issue debt in domestic currency. In writing this budget constraint I assume the free entry condition holds.

characterizes savings made through domestic financial intermediaries. The expectations operator, \mathbb{E}_{t-1} , appears in this Euler equation because household deposits with financial intermediaries are pre-determined. This source of monetary non-neutrality is consistent with the limited participation assumption. The final two conditions are Euler equations reflecting optimal decisions over holdings of home and foreign currency bonds.¹⁸

2.5. Equilibrium

The zero-profit productivity cut-off equations determine the mass of active firms in the domestic and export market. In the domestic market,

$$N_{d,t} = \left(\frac{s}{f_d W_t R_t} \right) P_{h,t} C_{h,t} \quad ; \quad s \equiv \left(\gamma - \frac{\sigma}{1-\sigma} \right) \left(\frac{1-\sigma}{\gamma} \right) \quad (14)$$

where $P_{h,t} C_{h,t}$ is the total expenditure on home varieties by the home household and $f_d W_t R_t$ is the fixed cost of production. An important element of (14) is the share parameter $s \leq 1$. As with the composite parameter λ , defined in equation (7), the share parameter depends on γ , which determines the dispersion of firm productivity. Again, it is instructive to consider the limiting case, where $\gamma / (\frac{\sigma}{1-\sigma}) \rightarrow 1$ and $s \rightarrow 0$. The correct interpretation of this case is that all entrants produce and $N_{d,t} = N_{e,t}$.¹⁹

I use the free entry condition to determine the mass of entrants in the domestic market. The key point to note is that we can write average (expected) profits in terms of productivity. For example, using the zero-profit productivity cut-off equation for domestic sales, we find,

¹⁸The equations in (13) also imply an uncovered interest rate parity condition holds. See Patureau (2007) for a quantitative analysis of exchange rate dynamics in the context of a limited participation model with international trade in financial assets. Also see Jorda and Salyer's (2003) analysis of term rates and monetary policy uncertainty.

¹⁹It is straightforward to show that the mass of exporting firms is, $N_{x,t} = \left(\frac{s/\tau^*}{f_x W_t R_t} \right) P_h^* C_h^*$. The case corresponding to Krugman (1980) can be recovered by assuming $f_x = f_d = 0$.

$\varphi_{h,t}(a) = \left[(\mathbf{a}_{d,t}/a_{d,t})^{\sigma/(1-\sigma)} - 1 \right] f_x W_t R_t$, with average profits from domestic sales defined as $\varphi_{h,t}(\mathbf{a}_{d,t})$. Using the analogous condition for exporters, I can express the total mass of entrants as a function of domestic and export sales,

$$\mathbf{N}_{e,t} = \left(P_{h,t} C_{h,t} + \frac{e_t}{\tau^\star} P_{h,t}^\star C_{h,t}^\star \right) \frac{\alpha_f}{W_t R_t} \quad (15)$$

where $\alpha_f \equiv f_e(\gamma/\sigma)$ and γ/σ represents the share of income that goes to labor for firm entry. Notice now that in the limiting case, $\gamma/(\frac{\sigma}{1-\sigma}) \rightarrow 1$, we find $\alpha_f \rightarrow f_e/(1-\sigma)$, and consistent with the expression for the mass of active firms, equation (15) implies greater consumption of home varieties or a falling wage bill generate an increase in firm entry, all else equal.

Production and fixed costs all require labor and total labor supplied (resources) can be written in terms of the mass of entrants and exporters,

$$L_t = \alpha_f \mathbf{N}_{e,t} - f_x \left[\left(\frac{e_t W_t^\star}{W_t} \right) \mathbf{N}_{x,t} - \mathbf{N}_{x,t}^\star \right] \quad (16)$$

In a similar fashion, firms fund working capital - which is complementary to labor - by borrowing from domestic financial intermediaries. The total amount of borrowing is $D_t + T_t$, where $T_t = M_{t+1} - M_t$ is a lump-sum transfer from the government. Combined with labor market clearing, borrowing can be re-expressed in terms of the mass of entrants as, $D_t + T_t = \alpha_f W_t \mathbf{N}_{e,t}$. This condition is the loans market equation for the economy.

Finally, using the household budget constraint, I obtain the nation-wide constraint.²⁰

$$b_{t,t+1} = \frac{e_t}{\tau^\star} P_{h,t}^\star C_{h,t}^\star - \frac{1}{\tau} P_{f,t} C_{f,t} - f_x (e_t W_t^\star \mathbf{N}_{x,t} - W_t \mathbf{N}_{x,t}^\star) \quad (17)$$

where,

$$b_{t,t+1} \equiv e_t (B_{f,t+1} - R_{b,t}^\star B_{f,t}) - (B_{h,t+1} - R_{b,t} B_{h,t})$$

²⁰Financial intermediaries pay interest on loans back to households at the end of the period. The total amount received by households is, $(T_t + D_t)R_t$, with profits equal to, $\zeta_t = T_t R_t$.

is the net international asset position of the economy. The right-hand side of equation (17) captures trade in goods and labor. Most important is the final term, $f_x(e_t W_t^* N_{x,t} - W_t N_{x,t}^*)$, which derives from the presence of the financial friction when there is an export participation decision. Bond market clearing conditions are $B_{f,t} = B_{f,t}^*$ and $B_{h,t} = B_{h,t}^*$.

2.6. Model Summary

Table 1 presents the key conditions for the model economy. All nominal variables are scaled by the beginning-of-period money stock in which these variables are denominated (either M_t or M_t^*). I use a lower-case for normalized variables, so, for example, $d_t \equiv D_t/M_t < 1$ is the predetermined stock of assets held by households in financial intermediaries.

===== Table 1 Here =====

Together with the definition of relative average prices $\chi_{h,t}$ (and $\chi_{f,t}$), the Euler equations - given by the second equation in (12), both equations in (13), their foreign counterparts, and the definition, $\Delta e_t \equiv \Delta q_t [\Delta p_t (1 + g_t) / \Delta p_t^* (1 + g_t^*)]$ (where, for variable, z_t , $\Delta z_t \equiv z_t/z_{t-1}$) - the equations in *table 1* form a 29 variable system which solve the model for given home and foreign growth rates of money, defined as $1 + g_t \equiv M_{t+1}/M_t$ and similarly for $1 + g_t^*$.

3. The Effects of Exogenous Monetary Shocks

In this section, I study the response of international relative prices to an exogenous home monetary shock. International relative prices play a key role in shaping the incentives faced by the policymaker when setting monetary policy because they determine the competitiveness of home goods in the export market

Using the conditions in table 1, I first derive a relation between the interest rate and the growth rate of money.

Lemma 1 Without international borrowing and lending ($b_{h,t} = b_{f,t} = 0$) or fiscal instruments ($\pi_t = 0$ and $\tau = 1$) the home nominal interest rate is determined by the home growth rate of money according to $R_t = (1 + g_t) / (d_t + g_t)$.

Proof See Appendix A.1. ■

This result is similar to Cooley and Quadrini (2003) but in a model with firm entry and firm heterogeneity. As in their analysis, given the stocks of deposits, it is possible to generate a unique relationship between the growth rate of money and the nominal interest rate in each country. Since the home nominal interest rate is not affected by the interest rate in the foreign country, the specification of the monetary policy instrument in terms of money growth or the nominal interest rate is equivalent.

To generate analytical results, in addition to the conditions described in lemma 1, I suppose an equal expenditure share on home and foreign goods in consumption ($\theta = 1/2$), no iceberg trade costs ($\iota = 1$), and an elasticity of substitution between home and foreign goods equal to one ($\epsilon = 1$). Using the definitions for international relative prices - $\chi_{h,t} \equiv e_t P_{h,t}^* / P_{h,t}$ and $T_t \equiv P_{f,t} / e_t P_{h,t}^*$ - I express the domestic price ratio in terms of export participation of home firms and the relative mass of entrants and exporters across countries,

$$\Phi_t = (\chi_{h,t} \times T_t)^{1/2} \quad (18)$$

where,

$$\chi_{h,t} = \left(\frac{N_{d,t}}{N_{x,t}} \right)^\lambda \quad \text{and} \quad T_t = \left(\frac{N_{e,t}}{N_{e,t}^*} \right)^{1+1/\gamma} \left(\frac{N_{x,t}}{N_{x,t}^*} \right)^\lambda$$

and $\lambda = \left(\frac{1-\sigma}{\sigma} \right) - \frac{1}{\gamma} \geq 0$. I begin by considering the impact of a home monetary contraction without selection. I suppose $\lambda \rightarrow 0$ and $f_d = f_x = 0$, such that all entrants produce (and export) and $\chi_{h,t} = 1$.²¹ The terms of trade are $T_t = (N_{e,t}/N_{e,t}^*)^{1/\sigma}$ and movements

²¹Section 4 of Christiano *et al.* (1997) contains a similar exercise for a closed economy model with a fixed mass of firms.

in the domestic price ratio are summarized by $\Phi_t = T_t^{1/2\sigma}$. We can determine the ratio of entrants across countries by noting that equations (15) - (17) imply firm entry is directly proportional to labor supply, and $N_{e,t} = L_t/\alpha_f$. Thus, the domestic price ratio is determined by the relative supply of labor across countries. Finally, the loans and deposit equations (see Table 1) imply labor supply is determined by the nominal interest rate according to, $L_t = 1/(1 + R_t)$. Thus, a home monetary contraction raises the interest rate, firm entry falls, and the terms of trade improve (both T_t and Φ_t fall).²²

I now reintroduce the selection effect ($\lambda > 0$). When firms make decisions to enter and exit the domestic and export markets monetary policy induces changes in average prices across countries. To make progress, I linearize the equilibrium conditions around a steady-state in which the nominal interest rate in both countries is equal to one. This leads to the following result.

Lemma 2 Assuming firm productivity has a Pareto distribution, international relative prices ($\chi_{h,t}$, T_t , and Φ_t) are more responsive to exogenous monetary shocks the lower the dispersion of firm productivity (given by γ^{-1}).

Proof See Appendix A.2. ■

Following a home monetary contraction the export participation rate of home firms falls. Firms that only sell domestically are less likely to stop production than exporting firms are to drop into the domestic market because they are less exposed to economic fluctuations in the foreign country. An response of the domestic price rate to the shock can be written as,

$$\hat{\Phi}_t = \left[\frac{1}{2} \left(1 + \frac{1}{\gamma} \right) \left(\frac{1}{1-s} \right) + \lambda \right] \hat{L}_t \quad (19)$$

²²That higher interest rates imply reduced firm entry is consistent with the evidence documented in Bergin and Corsetti (2008). Although slightly beyond the scope of the paper, this point is important because monetary models of the business cycle based on sticky-prices generate the opposite result for entry when entry costs are labor intensive (also see the discussion in Bilbiie *et al.* (2007)).

where $\lambda \geq 0$ and $s \leq 1$ are parameters defined in equations (7) and (14) and where a caret denotes the deviation of a variable from its steady-state value. Equation (19) shows that as firm sizes are more dispersed, either through a fall in γ^{-1} or σ , the domestic price ratio is more responsive to changes in monetary policy (the same holds for the terms of trade, \hat{T}_t). This result is important because, in the standard model, the incentive for the policymaker to use monetary policy, and alter the terms of trade, depends on the trade elasticity. With firm heterogeneity, the effective trade elasticity rises, but it only depends on productive heterogeneity (Chaney, 2008). In the monetary model I develop, two parameters (i.e., γ^{-1} and σ) appear equation (19) because changes in monetary policy affect marginal costs and market access costs. This feature of the model creates a disconnect between the trade elasticity and inflation.

4. Optimal Monetary Policy

In this section, I derive analytical results for optimal monetary policy under commitment. I begin by assuming each country has a fixed mass of firms. I show that, because the terms of trade provide a mechanism through which higher interest rates indirectly raise household consumption, a lower elasticity of substitution between home and foreign goods raises long-run inflation. I then allow for firm entry and exit in the domestic and export markets. In this case, due to selection, higher interest rates raise economy-wide productivity, and less dispersion in productivity (and firm sizes) leads to higher long-run inflation. Finally, I consider the welfare loss associated with setting monetary policy independently across countries. I show that firm entry magnifies the welfare cost of inflation by a factor equal to the net monopoly markup.

4.1. Optimal monetary policy when all firms export

For the purposes of this section I first note, when the mass of firms is fixed, labor is deter-

mined by the nominal interest rate according to, $L_t = 1 / [1 + (R_t/\sigma)]$.²³ I summarize the result for optimal monetary policy as follows.

Proposition 1 When all firms export, the optimal rate of inflation is decreasing in the elasticity of substitution between home and foreign goods (given by ϵ) and the monopolistic markup (given by σ^{-1}). Inflation is given by,

$$\pi = \beta\sigma \left(\frac{1}{1 - \frac{1}{2\epsilon}} \right). \quad (20)$$

Proof See Appendix A.3. ■

To understand how the elasticity of substitution affects optimal monetary policy it is necessary to determine the impact of higher interest rates on consumption.²⁴ Dropping time-subscripts, recall the home country terms of trade are $T \equiv P_f/eP_h^*$ (a fall in T represents an improvement in the terms of trade). Given the relative simplicity of the model specification, I first re-write the terms of trade as a function of labor supply,

$$T = \left(\frac{L}{L^*} \right)^{1/\epsilon} \quad (21)$$

A higher home interest rate reduces home labor supply and improves the terms of trade. The effectiveness of monetary policy is dictated by the Armington elasticity, $\epsilon > 0$. When the elasticity is relatively low, a small movement in the interest rate induces a relatively large improvement in the terms of trade. Since an improved terms of trade allows one country

²³ Assuming a fixed mass of firms requires an amendment to lemma 1, where the total mass of firms is endogenous (free entry). In this case, the home nominal interest rate is determined by the home growth rate of money according to $R_t = \sigma [(1 + g_t) / (d_t + g_t)]$. Labor supply is determined using the economy-wide resource constraint, balanced trade, and the firm optimal pricing equation. I continue to assume equal expenditure shares and no iceberg trade costs. These restrictions can be relaxed without changing the main point.

²⁴ Household welfare depends on both consumption and leisure, and higher interest rates generate an increase in leisure. Thus, the key to determining optimal monetary policy in the open economy is to pin-down how the terms of trade and interest rates impact consumption.

to import more, for a given amount of exports, a low elasticity will raise the incentive of the policymaker to set a high interest rate. This point becomes clearer once we write (total) consumption as a function of the terms of trade.

$$C = \frac{L}{\Phi(T)} \quad \text{where} \quad \Phi(T) = \left(\frac{1 + T^{1-\epsilon}}{2} \right)^{1/(\epsilon-1)} \quad (22)$$

The implications of equation (22) are the following: keeping the terms of trade constant, an increase in the domestic interest rate lowers consumption; keeping the foreign country's interest rate constant, an increase in the domestic interest rate improves the terms of trade; and keeping the domestic interest rate constant, an improvement in the terms of trade increases consumption. Thus, in the open economy, a monetary contraction has direct negative effect and an indirect positive effect on consumption. When the elasticity of substitution is relatively low, the indirect effect of a higher interest is strong, and the incentive to raise the interest rate is high. The outcome is higher long-run inflation.²⁵

A second parameter that determines long-run inflation is the monopolistic markup, given by $\sigma^{-1} > 1$. The markup is a distortion the policymaker wants to eliminate. To understand how the markup affects optimal monetary policy, it is sufficient to consider the closed economy, where the consumer price index is equal to $P = WR/\sigma$. By setting $R = \sigma$ the policymaker can subsidize monopolists and help undo the distortion. However, this is not possible because of the restriction on the interest rate, $R \geq 1$. As proposition 1 demonstrates, in the open economy, where $P_h = eP_h^* = WR/\sigma$, the policymaker has an incentive to raise interest rates because $P_h \neq P$. In this case, a higher markup (a lower value of σ)

²⁵Throughout this section I assume the mass of firms is fixed at one. Here I note that the optimal rate of inflation is unaffected by firm entry for the following reasons. Although the terms of trade become more sensitive to changes in the interest rate ($T = (L/L^*)^{1/\epsilon\sigma}$), the costs of financing firm entry also imply consumption is more sensitive to interest rates, given the terms of trade ($C \times \Phi(T) \propto L^{1/\sigma}$). Long-run inflation can be determined by, $\pi = \beta [(1/L) - 1]$.

simply acts to reduce the incentive generated through the terms of trade.²⁶

4.2. Optimal monetary policy with endogenous export participation

I now allow for firm entry and exit decisions in the domestic and export markets. To generate analytical results, I assume a unit elasticity of substitution between home and foreign goods ($\epsilon = 1$). I summarize the result for optimal monetary policy with firm heterogeneity as follows.

Proposition 2 When export participation is endogenous, the optimal rate of inflation is decreasing in the dispersion of firm productivity (given by γ^{-1}) and the monopolistic markup (given by σ^{-1}). Inflation is given by,

$$\pi = \beta\sigma^2 \left[1 + \lambda \left(1 - \frac{s}{2} \right) \right], \quad (23)$$

where $\lambda = \left(\frac{1-\sigma}{\sigma} \right) - \frac{1}{\gamma} \geq 0$ and $s = \left(\gamma - \frac{\sigma}{1-\sigma} \right) \left(\frac{1-\sigma}{\gamma} \right) \leq 1$.

Proof See Appendix A.4. ■

The influence of productivity dispersion on optimal monetary policy is summarized by the two composite parameters introduced previously. First, the parameter λ , defined in equation (7), which governs the response of relative average prices to changes in the rate of export participation, $\frac{N_{d,t}}{N_{x,t}}$. Second, the share parameter, s , defined in equation (14), which, in equilibrium, determines the total fraction of labor devoted to the fixed cost of exporting, $\frac{s}{2} = \frac{f_x N_x}{L}$.²⁷ For a given markup, less productivity dispersion leads to higher long-run inflation. However, a low (high) markup translates productivity differences into large (small) differences in firm size. As firm sizes become more dispersed, and there is more firm

²⁶Arseneau (2007) discusses this point in detail.

²⁷We can also generate a simple expression for the mass of exporters under optimal policy as a function of model parameters, $N_x = \frac{s/f_x}{1+\sigma^2[1+\lambda(1-\frac{s}{2})]}$. Under optimal policy, there is a greater mass of firms in the export market when firm productivity is less dispersed.

heterogeneity, inflation falls. In the limiting case of $\gamma / (\frac{\sigma}{1-\sigma}) \rightarrow 1$, where both s and λ approach zero, inflation is the same as the standard model.

The distribution of firm sizes is important for optimal monetary policy because higher interest rates force weaker firms to exit the market, or shut-down, and as labor is reallocated towards the most productive firms, (average) economy-wide productivity rises. The firm-size distribution matters because policymakers are concerned with marginal firms. When there are only small differences in firm sizes, firms below the productivity threshold are similar to those that export. In this case, there is a strong selection effect, and marginal firms are important for welfare. With greater dispersion in firm sizes, and a more skewed distribution, marginal firms are relatively dissimilar to exporters, and raising the interest rate generates a smaller gain in average productivity. It is also worth noting an important difference with the standard model. From proposition 1, inflation falls as the trade elasticity rises. Firm heterogeneity raises the trade elasticity, but inflation rises. This is because monetary policy affects both marginal costs and market access costs.

Finally, I determine how much additional inflation the selection mechanism generates by decompose inflation into that arising from the monopolistic markup, σ^{-1} , the intensive margin of trade, $1/2$, and the corresponding extensive margin, $[1 + \lambda(1 - s/2)]$. If we assume $\sigma = 1/2$, the optimal rate of inflation without selection is the Friedman rule (we can also see this by setting $\epsilon = 1$ in equation (20)). However, setting $\sigma = 1/2$ in equation (23) results in a rate of inflation above the Friedman rule because we then require $\gamma > 1$ to ensure the variance of firm sizes is finite. The alternative, i.e., the result under perfect competition, can be retrieved by assuming $\sigma \rightarrow 1$ (which, in turn, requires $\gamma \rightarrow \infty$). Since inflation must converge across model specifications, we can conclude that when markups are relatively high, selection generates relatively more inflation, and although inflation falls as the monopolistic markup rises, it does so at a slower rate than when all firms export.

4.3. The Welfare Loss of Independent Monetary Policy

I now consider the welfare loss associated with setting monetary policy independently across countries. I define the welfare loss, which is denoted by \mathcal{L} , as the additional consumption required to make an individual as well-off as they would be under international monetary cooperation.²⁸ Given the functional form of per-period household utility, I generate the following result.

Proposition 3 When firm entry is endogenous, the welfare cost of inflation is an increasing function of the monopolistic markup. The welfare loss is given by,

$$\mathcal{L} = \frac{\beta}{\pi} \left(\frac{1 + \pi/\beta}{2} \right)^{2+m} - 1, \quad (24)$$

where $m \equiv (1/\sigma) - 1 \geq 0$ is the net monopoly markup.

Proof See Appendix A.5. ■

Proposition 3 demonstrates that, at a given rate of inflation, the welfare loss of setting monetary policy independently only depends on the monopolistic markup. Without firm entry (as in section 4.1), we can generate the same expression as in (24), but with $m = 0$. A simple interpretation, therefore, is that the parameter m affects the welfare cost of inflation because, with Dixit-Stiglitz preferences, the net markup is identical to consumer love-for-variety. Higher (exogenous) rates of inflation reduce the number of products available to the consumer, and as love-for-variety rises, so inflation becomes more costly.²⁹ When inflation results from strategic interaction between countries, the welfare loss of independent monetary policy departs from the standard model in two ways. Inflation survives at high markups

²⁸Cooperation is equivalent to a supra-national agency running a closed economy. In an equilibrium with commitment, goods market distortions cannot be eliminated because there is a zero lower bound on the nominal interest rate, and the Friedman rule is optimal.

²⁹Wu and Zhang (2001) show that firm entry raises the welfare cost of inflation, albeit with an alternative entry mechanism.

(a high m) because of selection, and with a high markup, the welfare cost of inflation is magnified, due to firm entry.

My results suggest that firm entry and firm heterogeneity are important when evaluating the welfare gains to international monetary cooperation. This contrasts with the argument made in Arkolakis *et al.* (2012) that models with firm heterogeneity generate the same welfare formula as Armington models when computing the welfare gains from trade. Temporarily ignoring monetary frictions in my model, and comparing allocations with and without iceberg trade costs, I derive the following expression for welfare, $\mathcal{L}_\iota = \iota^{1/2} - 1$, for $\iota \geq 1$. This condition holds regardless of firm entry or firm heterogeneity.³⁰ Iceberg trade costs allow for an equivalence result across models because they create a wedge between production and consumption. However, in my model, this equivalence breaks-down because inflation affects the costs of production and market entry.

5. Quantitative Analysis

In this section, I undertake a quantitative analysis of the model. I focus on the results that suggest firm heterogeneity contributes to high long-run inflation and firm entry to a magnification effect of inflation onto welfare such that the gains to international monetary cooperation are large. I first compare my model with firm entry and firm heterogeneity to one with a fixed mass of firms. I then shut down selection and show firm entry generates a large welfare loss whilst selection drives high inflation. Finally, I consider the robustness of my results to the inclusion of long-lived firms, labor-income taxes, import tariffs, and international trade in financial assets.

5.1. Calibration

³⁰The details are contained in an online Appendix.

I calibrate model parameters under the assumption that countries cooperate. I set the discount factor at $\beta = 1/1.04$ and interpret a period as a year. I assume the consumption basket is mostly comprised of domestic goods and set $\theta = 0.67$. This figure is consistent with the Euro Area average reported in Andres *et al.* (2008). I adjust ψ so that households spend 40% of their time in work activities. Given the functional form of utility this implies a Frisch elasticity of labor supply equal to 1.5. In the benchmark calibration, I assume a 20% monopolistic markup ($\sigma^{-1} = 1.2$) and an elasticity of substitution across home and foreign goods ($\epsilon > 0$) of 1.25. I vary both of these values in the analysis below.

Firm heterogeneity is measured by the standard deviation of log plant sales, which, in the model, is given by $1/\left[\gamma - \left(\frac{\sigma}{1-\sigma}\right)\right]$. I match the value of 1.67 reported in Bernard *et al.* (2003) by using the shape parameter of the productivity distribution. For the benchmark case, this requires $\gamma = 5.6$, which implies a power law exponent on firm sales of $\gamma/\left(\frac{\sigma}{1-\sigma}\right) = 1.12$. The survival rate of entrants is given by $1 - G(a_d) = N_d/N_e$ and the rate of export participation is given by $1 - G(a_x) = N_x/N_d$. The exporter productivity premium is $(N_x/N_d)^{-1/\gamma}$. In the benchmark calibration, I set $f_e/f_d = 0.09$ to match the EU average survival rate of new firms of 85% reported by Eurostat and $f_d/f_x = 2.33$ to match the export participation rate of 21% reported in Bernard *et al.* (2003).³¹ This implies a productivity premium for exporting firms of 32.2%. I normalize f_e to unity.

The calibration also determines the labor share associated with different firm activities. I consider two statistics: the fraction of labor used to create a new firms (i.e., pay entry costs), given by N_e/L , and the employment share of exporters (including domestic sales), given by $N_x(l_h + l_h^*)/L$. The benchmark calibration implies values of 14.9% and 39.1% for these

³¹In the basic model specification all firms exit after a single period. The survival rate I use is based on the proportion of new firms who survive one year. Eurostat (business and demography statistics) report an annual death rate of 10%. I consider a version of the model with long-lived firms in section 5.3.

shares. The share of employment by entering firms is too high relative to the data because, in the basic model, all firms enter and exit in a single period. I address this issue in section 5.3 where I introduce long-lived firms. The share of labor used by exporting firms is very close to the figure of 40% reported Bernard *et al.* (2007). It is worth noting that as the markup falls, adjusting γ , f_e/f_d , and f_d/f_x appropriately, labor is reallocated away from entry and towards exporting.

5.2. The Role of the Firm Heterogeneity for Inflation and Welfare

I begin by assuming no international trade in financial assets and no fiscal instruments. To determine the relative strengths of the elasticity of substitution and firm selection I vary the markup between 10% and 30% (the productivity premium varies between 15.9% and 48.6%) and the elasticity of substitution between 1 and 1.5. Table 2 presents the long-run rate of inflation and associated welfare loss, when all firms export (denoted, “All Firms”), and when firm entry and export participation are endogenous (denoted, “Endogenous”).³²

===== Table 2 Here =====

There are three main results. First, in all cases, inflation is higher when export participation is endogenous. If we recall section 4.2, we can also explain why, for lower markups, inflation converges across model specifications: as the markup falls, and $\sigma \rightarrow 1$, we require, $\gamma \rightarrow \infty$, which eliminates selection. Moreover, a lower markup also reduces the employment share of new entrants. Second, a higher elasticity of substitution between goods reduces inflation (see equation (20) when all firms export). Thus, with a relatively high elasticity and markup, the Friedman rule may be optimal, even with endogenous export participation. Finally, the welfare loss associated with independent monetary policy is high. The typical estimate for the welfare cost of an exogenous 10% rate of inflation is between 0.5% and 1%, but in the

³²The policy problems of section 5 are discussed in Appendix B.

benchmark case the welfare loss of a 7.1% inflation is 1.7% in units of consumption.³³ With a higher markup, inflation is lower, but it is more costly.

I now show that whilst selection and firm heterogeneity lead to high inflation the welfare cost of inflation is determined by the monopolistic markup (as in Proposition 3). I do so by determining optimal monetary policy in a version of the model with firm entry where all firms export. I then compare the welfare loss of setting monetary policy independently across countries for different degrees of firm heterogeneity, with a given markup.³⁴ Table 3 presents three cases (“No Selection” is the model specification in which entrants all export).

===== Table 3 Here =====

The results in table 3 quantify the result of section 4.3. Analytically, I have shown that when there is firm entry, the welfare cost of inflation is magnified by a factor equal to the net monopoly markup. In table 3, without selection, long-run inflation is the same with or without firm entry. However, the welfare loss of inflation is higher (compare column 1/“All Firms” of table 3 with column 2 of table 2).³⁵ When firm heterogeneity falls (and as

³³Cooley and Hansen (1989) report a 0.5% welfare loss from a 10% inflation in an RBC-type economy. To understand this result consider the case where all firms export. The welfare loss of a 10.3% inflation is 2.4% (table 2). If we then reduce the markup (from 30%) to zero, but maintain inflation at the same rate, the welfare loss falls to 0.5%.

³⁴In table 2, the parameter γ is adjusted such that $\gamma / \left(\frac{\sigma}{1-\sigma} \right) = 1.12$. Using French data, Arkolakis (2011) estimates at $\gamma / \left(\frac{\sigma}{1-\sigma} \right) = 1.49$. Given his calibration of the markup, the implied standard deviation of log sales is 0.37. Using this value generates higher long-run inflation in my model because it also requires less dispersion in firm productivity (a lower γ^{-1}).

³⁵Arkolakis and Esposito (2014) have recently argued that the welfare gains from trade rise as labor supply becomes more elastic. In my model, the Frisch elasticity of labor supply can be written as, $\nu \equiv (u_L/L) / (u_{LL} - u_{CL}^2/u_{CC})$. I re-worked my calculation for additively separable utility, with $\nu = 0.5$. In the benchmark calibration the welfare loss fell by around 30%. However, this drop was smaller than in the model with fixed varieties.

selection rises), inflation rises, but the welfare cost of an extra percentage point of inflation remains almost constant. Thus, the welfare gain from an exogenous reduction in inflation is not related to heterogeneity, but neither is it the same as the standard model, which features a fixed mass of firms.

My quantitative results can be compared to studies that evaluate the welfare implications of impediments to firm entry and international trade when there is firm heterogeneity. For example, di Giovanni and Levchenko (2013) consider two experiments: a reduction in the fixed costs of entry and exporting and a reduction in (iceberg) trade costs. Neither experiment produces a large increase in welfare when the model is calibrated to match the distribution of firm sizes in the data.³⁶ However, fixing firm heterogeneity and raising the monopolistic markup does increase the welfare gain (Table 3, page 292). I find a very similar result. Higher markups lead to greater welfare losses from inflation. However, higher markups also lead to lower equilibrium inflation. Firm heterogeneity is relevant for this point because, by flattening the firm-size distribution, higher markups are consistent with greater selection, and increased selection leads to higher inflation.

5.3. The Role of Long-Lived Firms, Exogenous Fiscal Instruments, and International Trade in Financial Assets

I now consider a version of the model with long-lived firms.³⁷ I extend the baseline model such that an existing firm has a probability $\delta \in (0, 1)$ of exiting exogenously and a probability $1 - \delta$ of surviving to produce each period. Surviving firms can choose either to exit or to continue to produce and pay fixed costs. Entry requires the payment of sunk cost

³⁶It is worth noting that firm heterogeneity matters for whether fixed costs (which affect the behavior of marginal firms) or trade costs (which affect the behavior of marginal exporters) have a larger impact on welfare. This distinction is relevant because, in my model, inflation affects fixed and variable costs.

³⁷I follow Atkeson and Burstein's (2010) formulation (albeit without productivity dynamics). Details are presented in Appendix D. Also see Burstein and Melitz (2013).

which yields a new firm in the following period. In this environment, the steady-state mass of active firms is given by, $N_d = [1 - G(a_d)] N_e / \delta$, and the free entry condition reads, $\int_{a_d}^{\infty} \varphi_h(a) g(a) da + \int_{a_x}^{\infty} \varphi_h^*(a) g(a) da = \tilde{\beta} f_e$, where $\tilde{\beta} \equiv [1 - (1 - \delta) \beta] / \beta \delta$ adjusts for the exit rate and the discount factor. One implication of this change is that, as less firms die each period, less labor is used to pay for entry, and in a steady-state where countries cooperate, this implies, $N_e/L = 1 / \left[1 + \tilde{\beta} \left(\frac{\gamma}{\sigma} - 1 \right) \right]$.

I also introduce a simple asymmetry into the model. In all cases, I parameterize the international position of the home economy assuming net liabilities (assets) in domestic (foreign) currency at 50% (30%) of GDP.³⁸ I also assume exogenous labor-income taxes of 20% and import tariffs of 10%. The tax rate is consistent with typical Euro Area household taxes on labor, reported in Coenen *et al.* (2008), and the tariff rates are similar to those reported in Alvarez and Lucas (2007).³⁹ Table 4 presents the long-run rate of inflation and welfare loss for annual exit rates of 90%, 50%, and 10%, the latter of which is empirically plausible.

===== Table 4 Here =====

There are two main results. First, whilst labor-income taxes have a large impact on long-run inflation, the effect of import tariffs is considerably weaker. The reason the labor-income tax has a relatively large impact on inflation is that it works directly against the financial friction. Recall that monetary non-neutrality arises in the model because firms fund working capital

³⁸Based on recent Eurostat data a net international position of -20% of GDP for European countries is a little conservative. The results I report below are not very sensitive to these values.

³⁹The value of 20% labor-income tax rate is below the total tax rate reported by Lipinska and von Thadden (2012), but this includes firm contributions to social security. One caveat to the results of this section is the exogeneity of taxes and tariffs. In Basevi *et al.* (1990), for example, monetary policy under the threat of tariffs “converges to a cooperative-equivalent equilibrium, with zero tariffs and optimal monetary policy”.

- which is complementary to labor - by borrowing from financial intermediaries. Higher interest rates reduce the demand for labor. Higher labor taxes affect the incentive to raise interest rates because they impact individuals labor supply decisions. Tariffs, on the other hand, affect the revenue firms generate from exporting. The second result is that the introduction of long-lived firms has only a small impact on the welfare cost of inflation. This is important because less labor is now devoted to pay for entry costs. Thus, we can identify the cost of inflation as arising from distortions associated directly with monopolistic competition.⁴⁰

6. Conclusion

I study optimal monetary policy in an open economy with firm heterogeneity and monopolistic competition. I show analytically that two key parameters determine optimal monetary policy when firms make decisions to enter the domestic and export markets: the elasticity of substitution between home and foreign goods and the dispersion of firm productivity. Both a low elasticity of substitution and a small dispersion in productivity (which implies a small dispersion in firm sizes and a strong selection effect) create incentives to set high interest rates. This leads to high long-run inflation. I also show that the welfare cost of (exogenous) inflation depends on the monopolistic markup.

Because inflation remains high when there are relatively high markups, and because high markups are consistent with greater consumer love-for-variety, there are large welfare gains to international monetary cooperation. The results are robust to a number of extensions, including international trade in financial assets, exogenous fiscal instruments (labor-income taxation and revenue generating import-tariffs), and long-lived firms. A central theme of

⁴⁰Inflation is higher as the rate of exit falls because less resources are required to replace the existing stock of firms. The impact of this additional channel onto policy competition is mitigated as $\beta \rightarrow 1$ because, in this case, net income from the stock of assets held by the household is eliminated.

the paper is that, by ignoring the entry and export decisions of firms, any potential benefits of low inflation that arise through international monetary cooperation are underestimated.

Appendices

Appendix A.1 (*Proof of Lemma 1*)

Eliminating the home consumer price index, p_t , in equations ‘Loans’ and ‘Deposits’ from table 1 implies, $C_t = (1 - \tau_l) w_t L_t + \alpha_f w_t N_{e,t} \left(\frac{1+g_t}{d_t+g_t} - 1 \right)$. If we now suppose $b_{h,t} = b_{f,t} = 0$ and $\tau_l = 0$ and $\tau = 1$, ‘Free Entry’ can be written as, $\alpha_f N_{e,t} = (P_{h,t} C_{h,t} + e_t P_{h,t}^* C_{h,t}^*) / W_t R_t$, and ‘Trade Account’ as, $W_t (L_t - \alpha_f N_{e,t}) = P_{f,t} C_{f,t} - e_t P_{h,t}^* C_{h,t}^*$. Noting that total nominal consumption is $P_t C_t = P_{h,t} C_{h,t} + P_{f,t} C_{f,t}$, it is possible to eliminate exports from each condition, which results in, $C_t = w_t L_t + \alpha_f N_{e,t} w_t (R_t - 1)$. Eliminating consumption from the original equation implies, $R_t = (1 + g_t) / (d_t + g_t)$.

Appendix A.2 (*Proof of Lemma 2*)

Since money growth determines the interest rate in each country it is not necessary to use the equations ‘Loans’ and ‘Deposits’ to consider monetary shocks. Home and foreign monetary policy variables are R_t and R_t^* . Setting $\epsilon = 1$ in all remaining equations, and using ‘Domestic Firms’, ‘Export Firms’, and ‘Labor Supply’ to eliminate the variables $\{N_{d,t}, N_{x,t}, N_{d,t}^*, N_{x,t}^*, w_t, w_t^*\}$ leaves 9 equations. Now consider a steady-state with nominal interest rates equal to 1 in both countries. Since the economies are symmetric, the real exchange rate is $q = 1$ (steady-state variables are written without time subscripts). The equations ‘Resources’ and ‘Trade Account’ imply, $L = \alpha_f N_e = 1 / (1 + \psi)$ and $C = C^*$. I linearize all equations around the steady-state, assuming a one-off shock to the home nominal interest rate. I use a caret to denote the deviation of a variable from its steady-state value.

The key step involves solving for labor supply as a function of the shock. By elimination, I find $\widehat{L}_t = -\left(\frac{\psi}{1+\psi}\right) \widehat{R}_t$ and $\widehat{L}_t^* = 0$, where $\left(\frac{\psi}{1+\psi}\right) < 1$. When $\epsilon = 1$, this is enough to determine the response of the fraction of firms that exports. In the home economy, $\widehat{N}_{d,t} = \widehat{L}_t$ and $\widehat{N}_{x,t} = -\widehat{R}_t$. These conditions determine $\widehat{\chi}_{h,t}$. Determining the response of the terms of

trade requires pinning down entry and export patterns across countries. Entry can be written solely as a function of own-country labor supply, which automatically implies $\widehat{N}_{e,t}^* = 0$. For the home country, and assuming $\psi = 1$ for simplicity, $\widehat{N}_{e,t} = (\frac{1}{1-s}) \widehat{L}_t$, where $s \leq 1$. Finally, foreign export participation is determined by $N_{x,t}^* = -\widehat{L}_t$. Using all these results, $\widehat{T}_t = \left[\left(1 + \frac{1}{\gamma}\right) \left(\frac{1}{1-s}\right) + 3\lambda \right] \widehat{L}_t$. Finally, and for completeness, the response of consumption to the shock can be written as, $\widehat{C}_t = \frac{1}{2} \left[\left(\frac{3s-1}{s-1}\right) + \frac{1}{\gamma} \right] \widehat{L}_t$. This condition is also discussed in Appendix A.4.

Appendix A.3 (Proof of Proposition 1)

Fix the mass of firms in the home (foreign) economy at one. We can make the following observations. Labor supply is determined by the nominal interest rate according to, $L_t = [1 + (R_t/\sigma)]^{-1}$, so the specification of the monetary policy instrument in terms of the interest rate or labor supply is equivalent. Average prices are equal across countries, implying $P_{h,t}^*/P_{h,t} = P_{f,t}^*/P_{f,t} = 1/e_t$. The real exchange rate is unity, $q_t = 1$. Given these results, I write the home policy problem as the choice of $\{L_t, C_t, C_t^*, \Phi_t, \Phi_t^*\}_{t=0}^\infty$ to maximize equation (9) in the text, subject to four constraints: home and foreign resources, international relative prices, and balanced trade, with $\{L_t^*\}_{t=0}^\infty$ given. Since the problem is static, I drop time subscripts, and denote the lagrange multipliers associated with the four constraints as, λ_j , for $j = 1, \dots, 4$. The policy maker chooses $\{L, C, C^*, \Phi, \Phi^*\}$ and $\{\lambda_j\}_{j=1}^4$ to maximize,

$$\begin{aligned} & \ln(C) + \ln(1-L) + \lambda_1 [2L - (C + C^*) \Phi^\epsilon] + \lambda_2 [2L^* - (C^* + C) (\Phi^*)^\epsilon] \\ & + \lambda_3 (\Phi^{*\epsilon-1} + \Phi^{\epsilon-1} - 2) + \lambda_4 [C^* - C (2\Phi^{1-\epsilon} - 1)] \end{aligned}$$

with L^* given. Since countries are identical, in equilibrium, each country chooses the same policy, and $L = L^*$. Equating resource constraints, $\Phi = \Phi^*$, and since $2 = \Phi^{*\epsilon-1} + \Phi^{\epsilon-1}$, we find $\Phi = 1$. Balanced trade then implies consumption is equalized across countries, where $C = L$. Substituting these requirements into the first-order conditions, eliminating $\lambda_3 > 0$ and $\lambda_4 > 0$, we find $1/L = 2(\lambda_1 + \lambda_2)$ and $\lambda_1 = \lambda_2 + (\frac{\epsilon-1}{\epsilon})(\lambda_1 + \lambda_2)$. Using

$L = [1 + (R/\sigma)]^{-1}$ and $\pi = \beta R$ generates the result in the text.

For completeness, I determine optimal monetary policy with firm entry. Using the loans and deposits conditions, labor supply is determined by the nominal interest rate according to, $L_t = 1/(1 + R_t)$, and the specification of the monetary policy instrument in terms of the interest rate or labor supply is equivalent. Resources now read, $L_t = f_e N_{e,t} + N_t (c_{h,t} + c_{h,t}^*)$, where $N_{e,t} = N_t$, and free entry is, $N_t f_e = (1 - \sigma) (P_{h,t} C_{h,t} + e_t P_{h,t}^* C_{h,t}^*) / W_t R_t$, consistent with equation (15) in the text. Using the aggregated pricing equations we can link the mass of firms to total labor supply as, $L_t = [f_e / (1 - \sigma)] N_t$. I then re-write the resource constraint as, $2L_t^{1/\sigma} = \varsigma \Phi_t^\epsilon (C_t + C_t^*)$, where $\varsigma \equiv [(1 - \sigma) / f_e]^{(\sigma-1)/\sigma} / \sigma$, with an analogous version for the foreign economy. Since the two other constraints are unchanged, the policy problem is now identical to the case with a fixed mass of firms. Finally, although the markup, $1/\sigma$, appears in the exponent of labor supply, because free entry implies $L_t = 1/(1 + R_t)$, the optimal rate of inflation is same as without entry.

Appendix A.4 (Proof of Proposition 2)

With endogenous export participation, the specification of the monetary policy instrument in terms of the interest rate or labor supply is no longer equivalent. I can rework the home policy problem in the following way. Take the 9 equations outlined in Appendix B. Using ‘Entrants’, ‘Int’l Rel. Prices’ and ‘Real Ex. Rate’, eliminate the variables $\{N_{e,t}, N_{e,t}^*, q_t, \Phi_t^*\}$. The home policymaker now chooses $\{C_t, C_t^*, L_t, L_t^*, R_t\}_{t=0}^\infty$ to maximize (9), subject to five constraints, with $\{R_t^* \geq 1\}_{t=0}^\infty$ taken as given. Again, we can exploit symmetry and impose $R_t = R_t^*$, which implies allocations are equalized across countries and $q_t = 1$. However, given the specification of (9) the solution to this problem is equivalent to $\pi = \beta (\hat{L}_t / \hat{C}_t)$. Appendix A.2 provides an expression for \hat{L}_t / \hat{C}_t under the assumption nominal interest rates are equal to 1 in both countries (the Friedman rule). In general, the following condition

holds,

$$2R \left(\frac{\widehat{C}_t}{\widehat{L}_t} \right) = 3 + \left[\frac{1}{R - \frac{s}{2}(1+R)} \right] \left\{ \frac{s}{2} \left[1 - \left(1 + \frac{2}{\gamma} \right) R \right] - (3 + \lambda) R + \frac{R^2}{\sigma} \right\} \quad (25)$$

Imposing $\pi = \beta \left(\widehat{L}_t / \widehat{C}_t \right)$ and then $\pi = \beta R$ generates the expression used in the text. If we instead impose $R = 1$ on (25) we retrieve the condition in Appendix A.2.

Appendix A.5 (Proof of Proposition 3)

The welfare loss associated with policy competition - denoted by \mathcal{L} - is defined as the additional consumption required to make individuals as well-off as they would be under cooperation. Denote C^C (L^C) and C^N (L^N) as the allocations of consumption (labor) under the cooperative (where $R = 1$) or non-cooperative regime (where $R \geq 1$). The steady-state welfare loss is implicitly determined by,

$$\ln(C^C) + \ln(1 - L^C) = \ln[(1 + \mathcal{L}) C^N] + \ln(1 - L^N)$$

Solving for \mathcal{L} yields, $\mathcal{L} = (C^C/C^N) [(1 - L^C) / (1 - L^N)] - 1$. Because $R = R^*$, we have $C = C^*$ and $q = 1$. In this case, resources are, $L = \alpha_f \mathbf{N}_e$. Using the loans and deposit equations, $L = 1/(1+R)$, and thus, $L = L^*$, and $eW = W^*$. Using ‘Entrants’ in table 1, $f_d \mathbf{N}_e^{1/\gamma} = (s\theta\Gamma/\sigma) \mathbf{N}_d^{-\lambda-1} \Phi C$, where $\mathbf{N}_d = (f_x/f_d) \mathbf{N}_{x,t} = (s/2f_d) L$ and $\Phi = (f_x/f_d)^{\lambda/2}$. By substitution, and using, $\lambda \equiv (\frac{1-\sigma}{\sigma}) - \frac{1}{\gamma}$, we have, $C \propto L^{1/\sigma}$, where $1/\sigma$ is the markup. Because we are interested in the ratio of variables, all parameters drop out of the expression for \mathcal{L} , and with $\pi = \beta R$, we can express \mathcal{L} in terms of the optimal rate of inflation. For a given rate of inflation, the welfare loss is increasing in the markup. Once we fix the mass of firms (see conditions in Appendix A.3) and repeat these steps, we find the expression for \mathcal{L} is independent of the markup, at a given rate of inflation.

Appendix B (Optimal Monetary Policy)

In sections 5.2 and 5.3 I determine the optimal rate of inflation numerically. The results in table 2 (symmetric countries) are generated using the equations in table 1 in the following

way. I reduce the policy problem to one in 9 constraints, to maximize $\ln(C) + \psi \ln(1 - L)$, choosing $\{L, L^*, C, C^*, \Phi^*, \Phi^*, \chi_h, \chi_f, q, R\}$, with $R^* \geq 1$ given. This generates 19 first-order conditions (9 for the endogenous variables, 1 for the policy variable, and 9 for the lagrange multipliers associated with the constraints). Since countries are symmetric, I impose $R = R^*$, and use the MATLAB routine `fsolve.m` to find the zeros of this 19 equation system. The results in table 4 (asymmetric countries) are computed in a differently. Because $\epsilon = 1$, I can reduce the home policy problem to one in 5 constraints, to maximize $\ln(C) + \psi \ln(1 - L)$, choosing $\{L, L^*, C, C^*, \Phi^*, R\}$, with $R^* > 1$ given.⁴¹ This generates 11 first-order conditions. The foreign policy problem is subject to the same constraints, and the policy maker maximizes foreign utility, $\ln(C^*) + \psi^* \ln(1 - L^*)$, with $R \geq 1$ given. This also generates 11 first-order conditions. I use the 6 first-order conditions associated with endogenous variables and the policy variable. I use `fsolve.m` to find the zeros of this 17 equation system.

⁴¹In this case, I solve for the golden-rule rate of inflation. Faia and Monacelli (2004) discuss golden rule and modified golden rule outcomes in an open economy sticky-price setting and compare their results with Cooley and Quadrini's (2003) limited participation model.

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Table 1: Model Summary[‡]

	Home Economy	Foreign Economy
Resources	$L_t = \alpha_f \mathbf{N}_{e,t} - f_x \left[\left(\frac{q_t w_t^*}{w_t} \right) \mathbf{N}_{x,t} - \mathbf{N}_{x,t}^* \right]$	$L_t^* = \alpha_f \mathbf{N}_{e,t}^* - f_x \left[\left(\frac{w_t}{q_t w_t^*} \right) \mathbf{N}_{x,t}^* - \mathbf{N}_{x,t} \right]$
Free Entry	$\alpha_f \mathbf{N}_{e,t} = \frac{\Phi_t^{\epsilon-1}}{w_t R_t} \left[\theta C_t + \left(\frac{1-\theta}{\tau^*} \right) q_t C_t^* \left(\frac{\chi_{h,t}}{q_t} \right)^{1-\epsilon} \right]$	$\alpha_f \mathbf{N}_{e,t}^* = \frac{\Phi_t^{*\epsilon-1}}{w_t^* R^*} \left[\theta C_t^* + \left(\frac{1-\theta}{\tau} \right) \frac{C_t}{q_t} \left(\frac{q_t}{\chi_{f,t}} \right)^{1-\epsilon} \right]$
Labor Supply	$w_t = \alpha_\tau C_t / (1 - L_t)$	$w_t^* = \alpha_\tau^* C_t^* / (1 - L_t^*)$
Entrants	$\mathbf{N}_{e,t}^{1/\gamma} = \left(\frac{\Gamma}{\sigma} \frac{s\theta}{f_d} \right) \mathbf{N}_{d,t}^{-\lambda-1} C_t \Phi_t^\epsilon$	$\mathbf{N}_{e,t}^{*\gamma} = \left(\frac{\Gamma}{\sigma} \frac{s\theta}{f_d} \right) (\mathbf{N}_{d,t}^*)^{-\lambda-1} C_t^* (\Phi_t^*)^\epsilon$
Domestic Firms	$\mathbf{N}_{d,t} = \frac{s\theta}{f_d} \frac{C_t}{w_t R_t} \Phi_t^{\epsilon-1}$	$\mathbf{N}_{d,t}^* = \frac{s\theta}{f_d} \frac{C_t^*}{w_t^* R_t^*} \Phi_t^{*\epsilon-1}$
Export Firms	$\mathbf{N}_{x,t} = \frac{s(1-\theta)}{f_x \tau^*} \frac{C_t^*}{w_t^* R_t} \left(\frac{\chi_{h,t}}{q_t \Phi_t} \right)^{1-\epsilon}$	$\mathbf{N}_x^* = \frac{s(1-\theta)}{f_x \tau} \frac{C_t}{w_t R_t^*} \left(\frac{q_t}{\chi_{f,t} \Phi_t^*} \right)^{1-\epsilon}$
Loans	$p_t = (d_t + g_t) / \alpha_f w_t \mathbf{N}_{e,t}$	$p_t^* = (d_t^* + g_t^*) / \alpha_f w_t^* \mathbf{N}_{e,t}^*$
Deposits	$d_t = [(1 - \tau_l) w_t L_t - C_t] p_t + 1$	$d_t^* = [(1 - \tau_l^*) w_t^* L_t^* - C_t^*] p_t^* + 1$
Int'l Rel. Prices	$\Phi_t^{\star 1-\epsilon} = \theta + [(1 - \theta)^2 / (\Phi_t^{1-\epsilon} - \theta)] (\chi_h / \chi_f)^{1-\epsilon}$	
Real Ex. Rate	$q_t = \chi_{f,t} (\Phi_t^* / \Phi_t) [(\Phi_t^{1-\epsilon} - \theta) / (1 - \theta)]^{1/(1-\epsilon)}$	
Trade Account	$b_{t,t+1} = w_t (L_t - \alpha_f \mathbf{N}_{e,t}) + (1 - \theta) \left[\frac{q_t C_t^*}{\tau^*} \left(\frac{\chi_{h,t}}{q_t \Phi_t} \right)^{1-\epsilon} - \frac{C_t}{\tau} \left(\frac{q_t}{\chi_{f,t} \Phi_t^*} \right)^{1-\epsilon} \right]$	
Int'l Asset Position	$b_{t,t+1} \equiv (q_t / p_t^*) [(1 + g_t^*) b_{f,t+1} - R_{b,t}^* b_{f,t}] - (1/p_t) [(1 + g_t) b_{h,t+1} - R_{b,t} b_{h,t}]$	

[‡]The expressions in the table use the following variables and composite parameters. Variables: money growth, $1+g_t \equiv M_{t+1}/M_t$ and $1+g_t^* \equiv M_{t+1}^*/M_t^*$, scaled financial variables: $b_{f,t} \equiv B_{f,t}/M_t$, $b_{h,t} \equiv B_{h,t}/M_t^*$, $d_t \equiv D_t/M_t$, $d_t^* \equiv D_t^*/M_t^*$, and scaled consumer price indexes: $p_t \equiv P_t/M_t$, $p_t^* \equiv P_t^*/M_t^*$. Composite parameters: $s \equiv \left(\gamma - \frac{\sigma}{1-\sigma} \right) \left(\frac{1-\sigma}{\gamma} \right)$, $\lambda \equiv \left(\frac{1-\sigma}{\sigma} \right) - \frac{1}{\gamma}$, $\alpha \equiv f_e (\gamma/\sigma)$, $\Gamma \equiv \left[\left(\gamma - \frac{\sigma}{1-\sigma} \right) \frac{1}{\gamma} \right]^{(1-\sigma)/\sigma}$.

Table 2: Optimal Inflation and Welfare Varying the Monopolistic Markup[§]

Elas. of Sub'n: ϵ	Exporting	Markup: $\frac{1}{\sigma}$					
		10%		20%		30%	
		Inflation	Loss	Inflation	Loss	Inflation	Loss
1	All Firms	30.3	4.3	19.5	3.4	10.3	2.4
	Endogenous	30.9	4.8	20.9	4.5	13.1	3.9
1.25	All Firms	16.1	2.0	6.4	1.3	-1.8	0.3
	Endogenous	16.3	2.3	7.1	1.7	-0.5	0.6
1.5	All Firms	8.9	1.1	-0.1	0.4	-4.0	-
	Endogenous	9.0	1.3	0.1	0.5	-4.0	-

[§]Values calculated in the table for inflation and the welfare loss (in units of consumption) are in annual percentage terms. For each value of the markup (in ascending order) the implied calibrated productivity parameter, $\gamma = [\sigma / (1 - \sigma)] + 1 / 1.67$, is 10.599, 5.599, and 3.932.

Table 3: Optimal Inflation and Welfare Varying Firm Heterogeneity[¶]

Elas. of Sub'n: ϵ		Firm Heterogeneity: $\frac{1}{\gamma - \frac{\sigma}{1-\sigma}}$					
		No Selection		3		0.4	
		Inflation	Loss	Inflation	Loss	Inflation	Loss
1		19.5	4.3	20.3	4.4	24.0	5.2
1.25		6.4	1.5	6.9	1.6	8.6	1.9
1.5		-0.1	0.5	0.0	0.5	0.7	0.6

[¶]The markup is 20% in all cases. For the latter two cases $\gamma / \left(\frac{\sigma}{1-\sigma} \right)$ is equal to 1.07 and 1.5.

Table 4: Robustness of Results to Long-Lived Firms and Exogenous Fiscal Instruments^{||}

Tariffs and Taxes	Exogenous Exit Rate: δ					
	0.9		0.5		0.1	
	Inflation	Loss	Inflation	Loss	Inflation	Loss
10% Tariff: $\tau = 1.1$	19.1	4.0	19.6	4.2	23.5	5.3
20% Tax: $\tau_l = 0.2$	-0.2	0.3	-0.1	0.4	2.1	1.0
$\tau_l = 0.2$ and $\tau = 1.1$	-3.0	0.0	-2.6	0.1	-0.6	0.2

^{||}The markup is 20% and the elasticity of substitution across goods is one in all cases. The reported values are calibrated with $\gamma / \left(\frac{\sigma}{1-\sigma} \right) = 1.12$ (as in table 1).